WARWICK BUSINESS SCHOOL
THE UNIVERSITY OF WARWICK

# Masters Programmes: Group Assignment Cover Sheet

| | |
|---|---|
| **Student Numbers:**<br>Please list numbers of all group members | **5669869; 5617388; 5619385; 5651103; 2135253** |
| **Module Code:** | **IB9Y80** |
| **Module Title:** | **Asset Pricing** |
| **Submission Deadline:** | **16th December 2024, 12pm** |
| **Date Submitted:** | **15th December 2024** |
| **Word Count:** | **1,966** |
| **Number of Pages:** | **15** |
| **Question Attempted:**<br>*(question number/title, or description of assignment)* | **In reference to guidance questions, 2.1, 2.2 and 2.3** |
| **Have you used Artificial Intelligence (AI) in any part of this assignment?** | **Yes, we have used AI only to debug our MATLAB, enhance certain tests on Matlab and to explain certain terminologies and definitions more cohesively.** |

## Table of Content

## 1. Introduction

Asset pricing models are pivotal in quantifying systematic risks and their impact on asset returns. Traditional single-factor models, such as CAPM, focus on market beta (MKTB) to explain return variations. However, corporate bonds and industry portfolios often exhibit complexities that demand more nuanced models. Duration-adjusted CAPM (MKTDB) incorporates bond-specific risks, while the Momentum (MOM) factor captures behavioural anomalies linked to past performance trends.

### Research Focus

This study systematically explores these models to assess their explanatory power for portfolio returns. It evaluates single-factor CAPM variants, two-factor models and a multivariate framework integrating MOM. Using data from 50 portfolios, robust econometric techniques are employed, including GLS for heteroskedasticity adjustments and GRS tests to examine joint alpha significance.

The research aims to address the shortcomings of traditional models by integrating structural and behavioural dimensions, offering insights into their applicability to bond pricing. By employing advanced diagnostics and statistical tests, this study bridges theoretical concepts with empirical data, highlighting the strengths and limitations of each approach and emphasizing the potential of multifactor frameworks in capturing systematic and behavioural risks effectively.

## 2. Methodology

This study employs a multi-pronged methodological framework to evaluate the performance of asset pricing models, using MATLAB & STATA:

1) Portfolio returns
   - Returns for 50 portfolios serve as dependent variables in time-series regressions (Appendix 5-7).

2) Factor Construction
   - MKTB and MKTDB factors are directly utilized as proxies for systematic market and bond-specific risks.
   - The MOM factor is constructed using a 12-month rolling window to sort portfolios into deciles based on lagged returns. A one-month lag separates formation and holding periods to avoid biases.

3) Time-Series Regressions
   - Portfolio alphas and betas are estimated against MKTB, MKTDB, and MOM factors.
   - Conducted the Breusch-Pagan test to detect heteroskedasticity, checking the reliability of regression results.
   - GLS is used alongside OLS to address heteroskedasticity in residuals, ensuring robust parameter estimation.

4) Cross-Sectional Regressions
   - Average portfolio returns are regressed on betas from time-series regressions to estimate risk premia.
   - Fama-MacBeth regressions provide time-varying risk premia, validating the models' temporal consistency.

5) Diagnostics and Statistical Tests
   - GRS tests evaluate joint alpha significance, identifying model misspecifications.
   - White's test assesses residual heteroskedasticity, while Jarque-Bera tests evaluate normality.

6) Comparative Analysis
   - Single-factor models are compared with the multivariate framework to assess explanatory power, using $R^2$ values and residual diagnostics.

This structured approach integrates advanced econometric techniques to provide a comprehensive evaluation of the models

## 3. Empirical Results & Comparative Analysis

**Single-Factor Models (MKTB, MKTDB)**

The single-factor bond CAPM using the MKTB factor performed well in explaining portfolio returns. In time-series regressions, the MKTB model exhibited an average $R^2$ of 0.7731 (Appendix 10), indicating that it explained a substantial portion of the return variance. However, only 2 portfolios displayed statistically significant alphas, suggesting that abnormal returns were largely absent. Cross-sectional regressions for MKTB produced a significant risk premium ($\gamma_1$ = 0.00436, $p - value$ < 0.001) and an $R^2$ of 0.7512, reflecting the model's strong explanatory power in pricing risk premia. Despite these strengths, heteroskedasticity was detected in 46 portfolios through Breusch-Pagan (BP Test), indicating model misspecifications. (Appendix 1)

The duration-adjusted bond CAPM (MKTDB factor) exhibited a different performance profile. Time-series regressions yielded an average $R^2$ of 0.3962, significantly lower than MKTB, indicating limited explanatory power for individual portfolio returns (Appendix 11). This lower $R^2$ reflects the complexity of modelling individual portfolio returns over time, where idiosyncratic factors and stochastic variations dominate. Unlike MKTB, which better captures the rich market-driven variations due to its constant exposure to the market risk premium, MKTDB focuses on duration-based risks. One should not necessarily interpret MKTDB's lower $R^2$ as indicating poor performance. Rather than reflecting the wider market moves that MKTB represents, it reflects the factor's more limited scope, which captures systematic risks associated with bond duration adjustments. MKTDB identified 27 portfolios with significant alphas, highlighting persistent abnormal returns.

With an $R^2$ value of 0.867, surpassing the value obtained under MKTB, the cross-sectional regression analysis for MKTDB revealed a statistically significant intercept and slope ($\gamma_1$ = 0.00356, $p - value$ < 0.001). This higher $R^2$ suggests that MKTDB is more effective at capturing the relationship between average returns and factor betas across portfolios, as it is not influenced by the time-series idiosyncratic noise for each portfolio. The MKTDB factor may be capturing systematic risk well at the portfolio level, but it does not adequately model the dynamics of returns over time for individual portfolios. The high cross-sectional $R^2$ implies that MKTDB is relevant for pricing the cross-section of returns, explaining differences in average returns across portfolios. Nevertheless, the lower average time-series $R^2$ implies that MKTDB is not enough to fully represent the fluctuation in returns over time, suggesting that the factor does not account for other dynamics or idiosyncratic risks. (Appendix 2)

GLS adjustments reduced residual heteroskedasticity for both models, improving reliability. Post-GLS, significant heteroskedasticity persisted in 27 portfolios for MKTB and 15 for MKTDB. The GRS test rejected the null hypothesis for both models (MKTB $p-value$ = 0.0016, MKTDB $p-value$ = 0.00249), confirming unpriced risks. *While MKTB excelled in time-series analysis, MKTDB outperformed in cross-sectional regression.*

**Two-Factor Models (MOM with MKTB or MKTDB)**

The MKTDB & MOM model exhibits an $R^2$ of 0.8667 and an F-statistic of 152.77 ($p-value$ = 2.72e-21), indicating strong explanatory power in cross-sectional returns. Both MOM ($x_1$: 0.0017, $p-value$ = 1.77e-13) and MKTDB ($x_1$: 0.0035, p-value = 6.02e-18) are statistically significant, capturing systematic risks. The intercept (0.0012, $p-value$ = 3.20e-07) highlights unexplained average returns, while the BP test ($p-value$ = 0.00153) indicates residual heteroskedasticity. Residuals are approximately normally distributed (*Jarque-Bera* $p-value$ = 0.7179), confirming robust model specification.

The MKTB & MOM model achieves an $R^2$ of 0.8623 and an F-statistic of 147.18 ($p-value$ = 5.80e-21), with significant coefficients for both MKTB ($x_1$: 0.0020, $p-value$ = 4.02e-19) and MOM ($x_2$: 0.0031, $p-value$ = 2.01e-12). The intercept (0.0005, $p-value$ = 0.1080) is insignificant, suggesting the model captures average returns effectively. However, the BP test ($p-value$ = 9.41e-06) reveals heteroskedasticity in residuals. Normality is marginally violated (*Jarque-Bera* $p-value$ = 0.0273). (Appendix 3)

**Multi-Factor Model (MKTB, MKTDB, MOM):**

**Adding MOM to MKTB & MKTDB to check the explanatory power beyond the single-factor models.**

The multivariate model incorporating MKTB, MKTDB, and MOM achieves the highest explanatory power among all tested models, with an $R^2$ of 87.46% and an adjusted $R^2$ of 86.64%. The model's F-statistic of 107 ( $p-value$ = 9.43e-21) highlights the overall significance of the predictors. (Appendix 2)

All three factors exhibit statistically significant and theoretically consistent coefficients:

- MKTB: Positive risk premium ($\beta$ = 0.0029854, $p-value$ < 0.001), indicating robust market-driven explanatory power.

- MKTDB: Positive and significant ($\beta$ = 0.00341, $p - value$ < 0.001), capturing bond-specific duration risks.
- MOM: Positive and significant ($\beta$ = 0.0017775, $p - value$ < 0.001), validating the relevance of behavioural momentum effects.

Diagnostics further support the model's robustness. The BP test reveals heteroskedasticity ($Test\ Statistic$ = 18.56, $p - value$ = 3.37e-04), which requires attention, while the Jarque-Bera test confirms that residuals are approximately normally distributed ($Test\ Statistic$ = 2.026, $p - value$ = 0.363). Moreover, the reduction in significant alphas (6 portfolios) compared to MKTDB (27 portfolios) and MKTB (12 portfolios with MOM) indicates fewer unexplained returns under the multivariate framework.

Now, we evaluate the performance of various models. The comparison encompasses GRS tests, risk premia analysis, and diagnostic evaluations, including heteroskedasticity and robustness adjustments using GLS.

**GRS Test Comparisons Across Models**

The GRS test (Gibbons, 1989) highlights the multivariate model's superior pricing performance. With a GRS statistic of 1.6963 ($p - value$ = 0.00688), the multi-factor model outperforms MKTB and MKTDB This reflects fewer unexplained variations in returns, confirming the multivariate model's alignment with theoretical expectations. Both single-factor models struggle to capture the cross-sectional variance effectively, as indicated by higher GRS statistics.

**Performance Analysis of Risk Premia**

The multivariate model achieves the highest explanatory power, with an $R^2$ of 0.875 and an adjusted $R^2$ of 0.866. Each factor exhibits statistically significant risk premia with the correct theoretical signs where $\gamma$ MKTB, estimated at $\gamma$ 0.0030, confirming that higher market exposure is rewarded with higher returns, $\gamma$ MKTDB, estimated at $\gamma$ 0.0034, highlighting the importance of duration-adjusted market risk, $\gamma$ MOM estimated at $\gamma$ 0.0018, reinforcing the role of momentum as a systematic risk factor.

These values surpass those of single-factor and two-factor models, confirming the significant role of momentum alongside market and duration-adjusted risks in explaining portfolio returns.

**Compared to the two-factor models, MOM enhances explanatory power significantly.** GLS regressions were employed to address residual heteroskedasticity, revealing improved fit and reduced significant heteroskedasticity. The White-test (WT) results for the multivariate model showed persistent heteroskedasticity in 38 portfolios. Despite this, GLS adjustments demonstrated that the multivariate model maintained robust risk premia, confirming its reliability.

Residual diagnostics underscore the multivariate model's robustness, with the Jarque-Bera test confirming normality ($p - value$ = 0.363). RMSE values further support the multi-factor model's competitive performance ($RMSE$ = 0.000596), closely matching the lowest among all models. The multivariate model demonstrates stronger explanatory power, evidenced by its broader inclusion of factors. However, it shows heteroskedasticity in 35 portfolios compared to MKTB (27 portfolios) and MKTDB (15 portfolios), reflecting higher complexity but greater coverage of risk factors.

By combining MOM (behavioural anomaly), MKTB (market risk), and MKTDB (duration risk), the multivariate model uniquely integrates both fundamental and behavioural risk drivers.

## 4. Discussion

### Implications of findings

The multivariate model incorporating MKTB, MKTDB, and MOM outperforms both single-factor models. Achieving the highest $R^2$ (0.875), it has greater explanatory power, showing portfolios with significant heteroskedasticity and improves pricing performance, as shown by the GRS test (1.6963, $p$ = 0.0068). The MOM factor captures systematic momentum effects, reinforcing its behavioral relevance in asset pricing. However, residual diagnostics reveal persistent heteroskedasticity in 38 portfolios post-GLS, indicating potential omitted variables such as liquidity, credit risks, or macroeconomic influences etc. (Kan, R., Robotti, C. and Shanken, J. 2013)

### Proposed Enhancements

While the multivariate model shows notable advancements, it still fails to completely explain corporate bond returns due to limitations. Addressing these gaps, requires targeted improvements to enhance pricing reliability and accuracy.

1. Incorporate Bond-Specific factors: Integrate measures such as credit spreads, liquid

proxies and default risk metrics etc to address any systematic risk that may exist unique to corporate bonds. (Gibbons, Ross, and Shanken, 1989)

2. Expand Macroeconomic indicators: GDP statistics, inflation expectations and monetary policy changes to name a few, can account for broader economic dynamics that can influence bonds. (Merton, 1973)
3. Nonlinear modelling: Employ nonlinear transformations to capture and model complex relations between risk factors, particularly for bonds that exhibit convex sensitivities. (Kan, R., Robotti, C. and Shanken, J. 2013)
4. Dynamic Factors: Utilizing regime-switching models or machine learning techniques to capture time-varying relations and improve predictive accuracy, otherwise missed by human tendencies. (Merton, 1973)
5. Behavioural Enhancements: Extending the MOM factors to incorporate anomalies like reversals, geographical indicators and overreactions of the market, to reflect invest driven trends. (Jegadeesh and Titman, 1993)
6. Robust Diagnostic Tools: Performing more advanced residual diagnostics, such as clustered standard errors or heteroskedasticity-robust methods, to ensure reliability in estimating risk premia. We can address significant residual patterns to minimize model misspecification. (Kan, R., Robotti, C. and Shanken, J. 2013)
7. Portfolio-Specific Adjustments: Customizing models to account for heterogeneity across portfolios. For instance, portfolios with extreme duration or credit exposure may require tailored risk factors or weight adjustments during MOM construction. (Carhart, 1997)

## 5. Conclusion

This study evaluated the performance of single-factor and multi-factor asset pricing models in explaining corporate bond and industry portfolio returns. While the MKTB and MKTDB factors provided useful insights, the inclusion of the MOM factor significantly enhanced explanatory power, emphasizing the importance of behavioural effects. However, persistent residual variance and heteroskedasticity highlight the need for additional bond-specific and macroeconomic factors. The findings underscore that no single model is comprehensive, and future asset pricing frameworks must integrate structural, behavioural, and economic dimensions to improve robustness, accuracy, and practical relevance for investment decision-making.

## 6. Appendix

### Appendix 1: Cross Sectional Analysis for MKTB.

| Parameter | Estimate | T-Statistic | P-Value |
|---|---|---|---|
| Avg_alpha | -0.00053567 | -1.4053 | 0.1664 |
| Avg_beta | 0.00436428 | 12.0394 | 0.0000 |
| Cross_Sectional_R_squared | 0.75122682 | - | - |
| GRS_Test_Statistic | 1.85955012 | - | - |
| GRS_p_value | 0.00163831 | - | - |

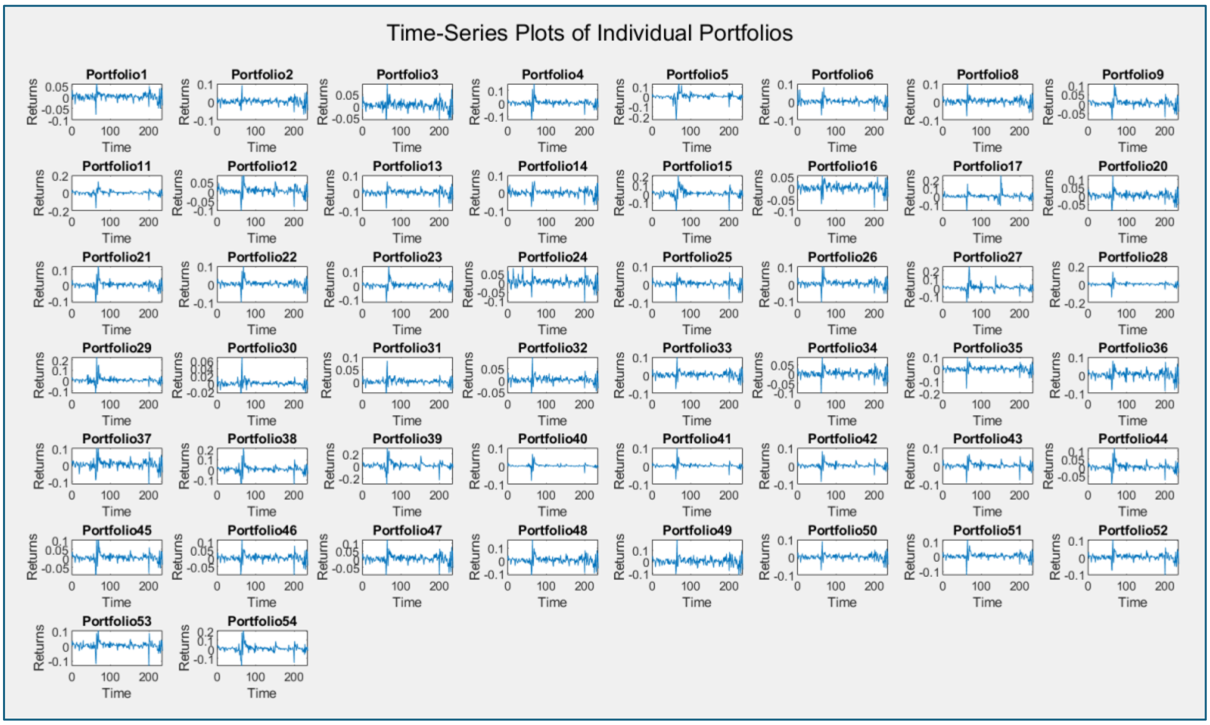
### Appendix 2: Cross Sectional Analysis for MKTDB

| Parameter | Estimate | T-Statistic | P-Value |
|---|---|---|---|
| Avg_alpha | 0.00116020 | 6.70197 | 0.0000 |
| Avg_beta | 0.00356076 | 17.72419 | 0.0000 |
| Cross_Sectional_R_squared | 0.86745705 | - | - |
| GRS_Test_Statistic | 1.80953481 | - | - |
| GRS_p_value | 0.00249913 | - | - |

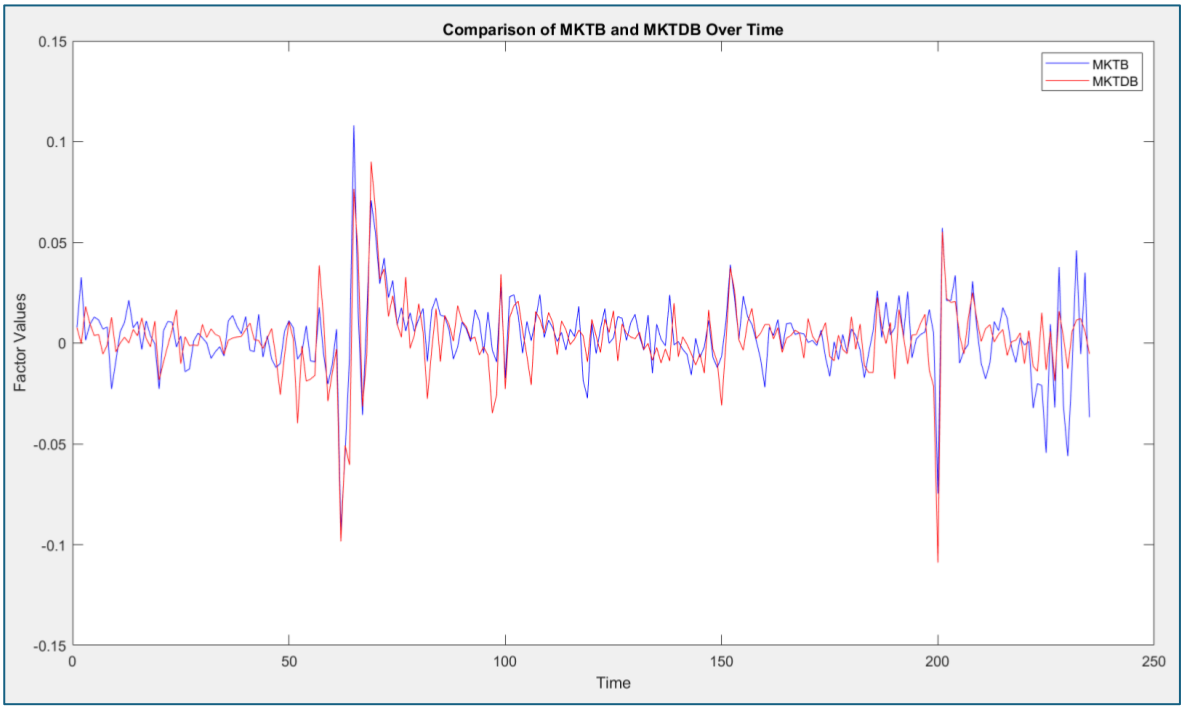
### Appendix 3: Cross Sectional Analysis for MKTDB-MOM, MKTB-MOM,MKTB-MKTDB compared with Multivariate Model

| Model | Intercept (p-value) | x1 Estimate (p-value) | x2 Estimate (p-value) | x3 Estimate (p-value) | R-squared | Adjusted R-squared | F-statistic (p-value) | Heteroskedasticity (p-value) | Normality (p-value) |
|---|---|---|---|---|---|---|---|---|---|
| **MKTB & MKTDB** | 0.0007 (0.0325) | 0.0030 (4.43e-12) | 0.0036 (9.98e-23) | - | 0.8734 | 0.8680 | 162.15 (8.04e-22) | 0.00048 | 0.7384 |
| **MKTDB & MOM** | 0.0012 (3.20e-07) | 0.0035 (6.02e-18) | 0.0017 (1.77e-13) | - | 0.8667 | 0.8610 | 152.77 (2.72e-21) | 0.00153 | 0.7179 |
| **MOM & MKTB** | 0.0005 (0.1080) | 0.0020 (4.02e-19) | 0.0031 (2.01e-12) | - | 0.8623 | 0.8565 | 147.18 (5.80e-21) | 0.00001 | 0.0273 |
| **Multivariate Model** | 0.0007 (0.0312) | 0.0030 (6.57e-12) | 0.0034 (1.28e-17) | 0.0018 (9.68e-14) | 0.8746 | 0.8664 | 83.53 (6.57e-12) | 0.00034 | 0.3631 |

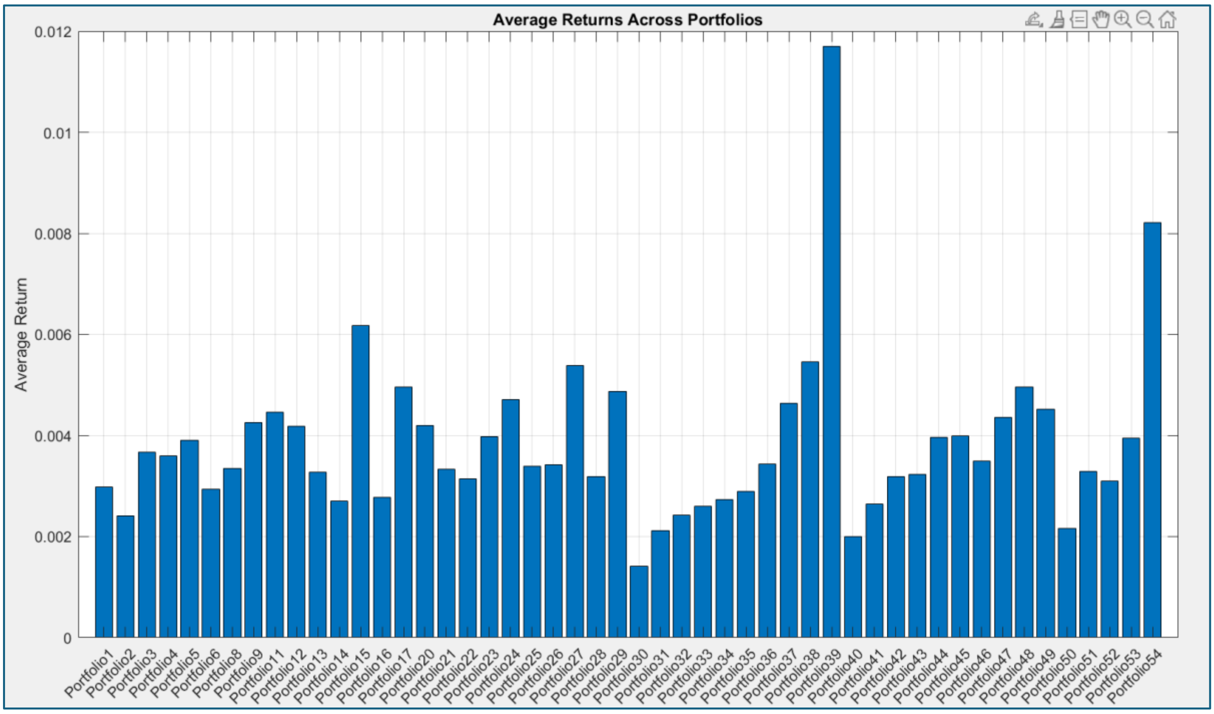
**Appendix 4: Plot of residuals of 50 portfolios.**



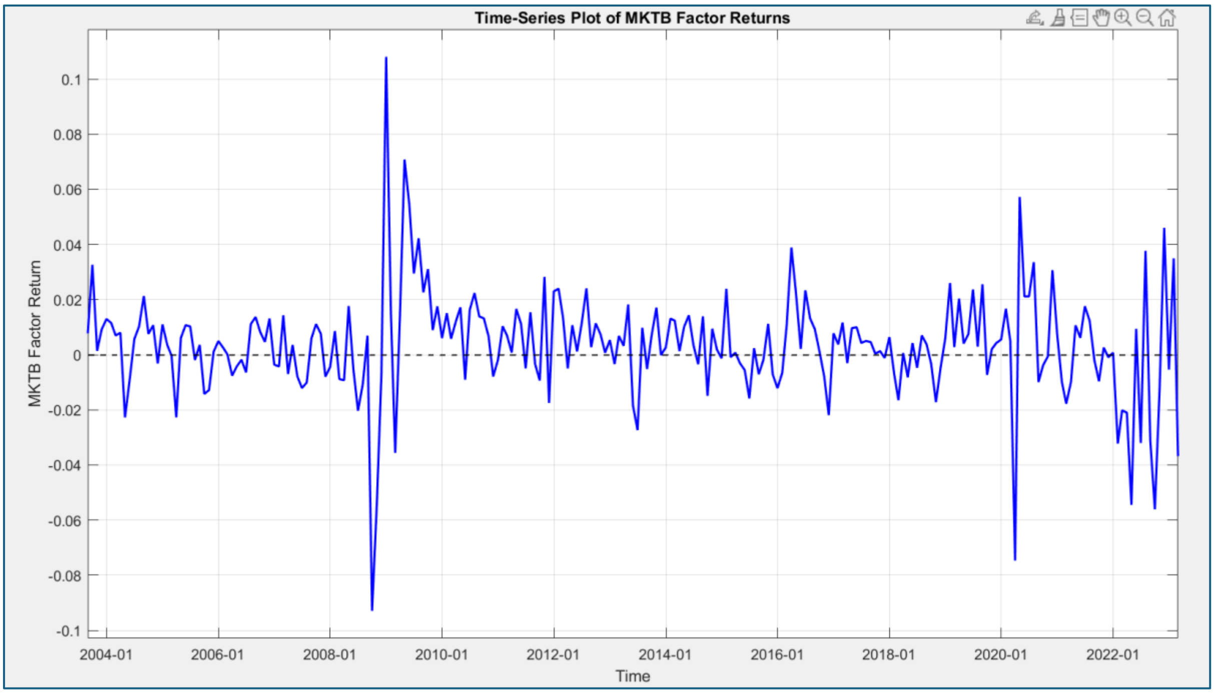

**Appendix 5: Comparison of MKTB & MKTDB over time.**



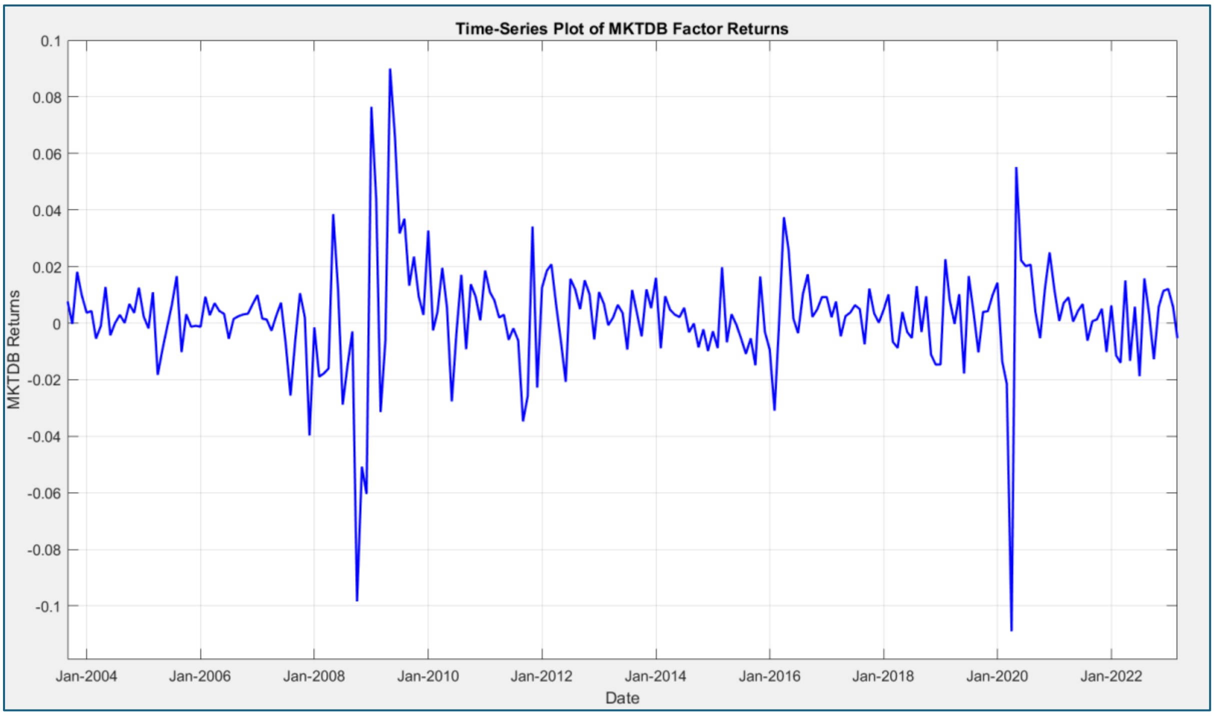
## Appendix 6: Average Returns across portfolios



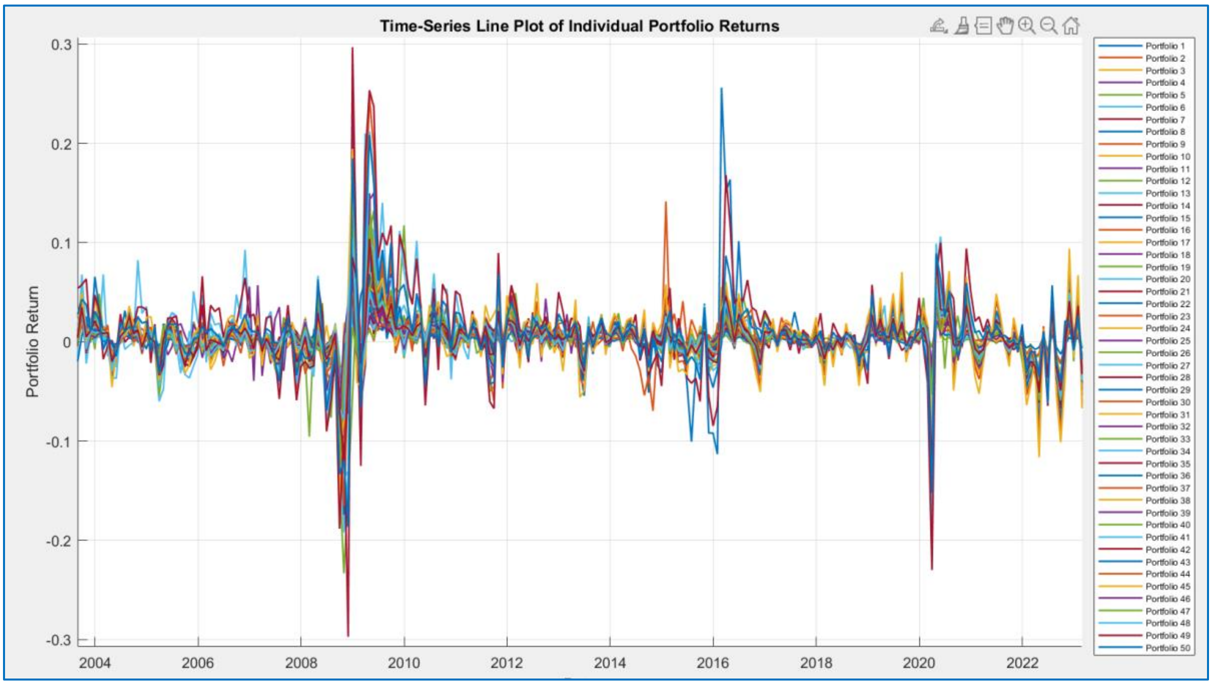

## Appendix 7: Time series plots of MKTB Factor returns



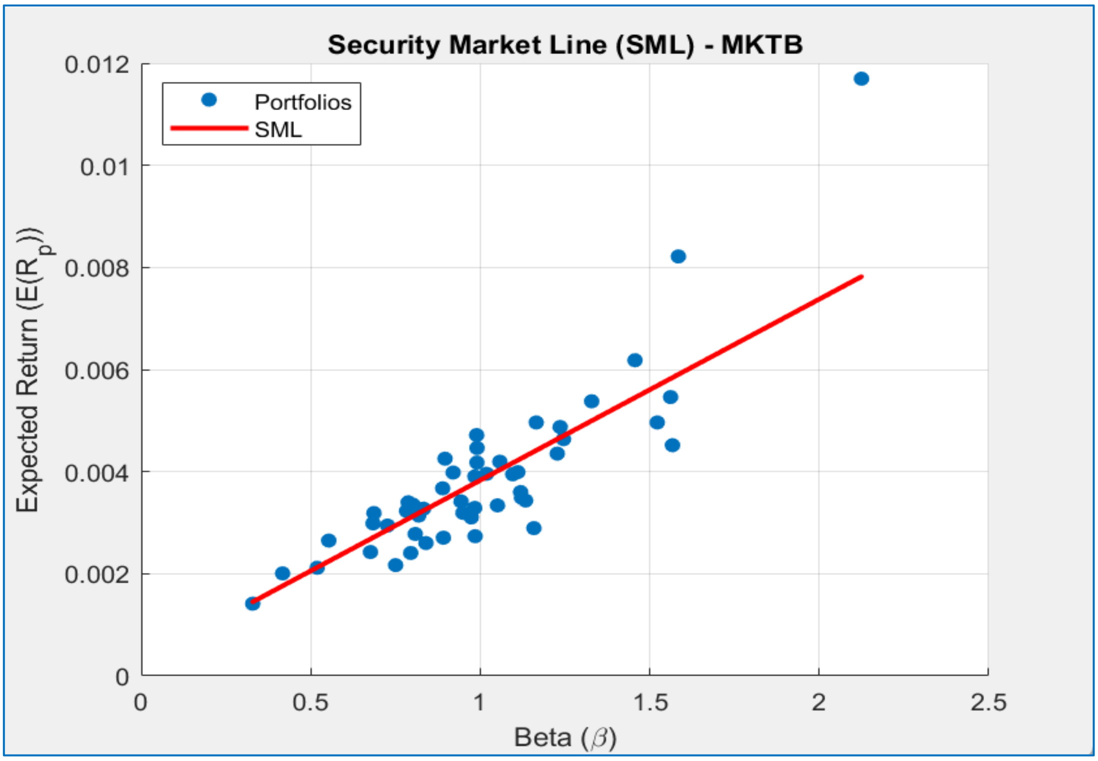
## Appendix 8: Time series plots of MKTDB Factor returns



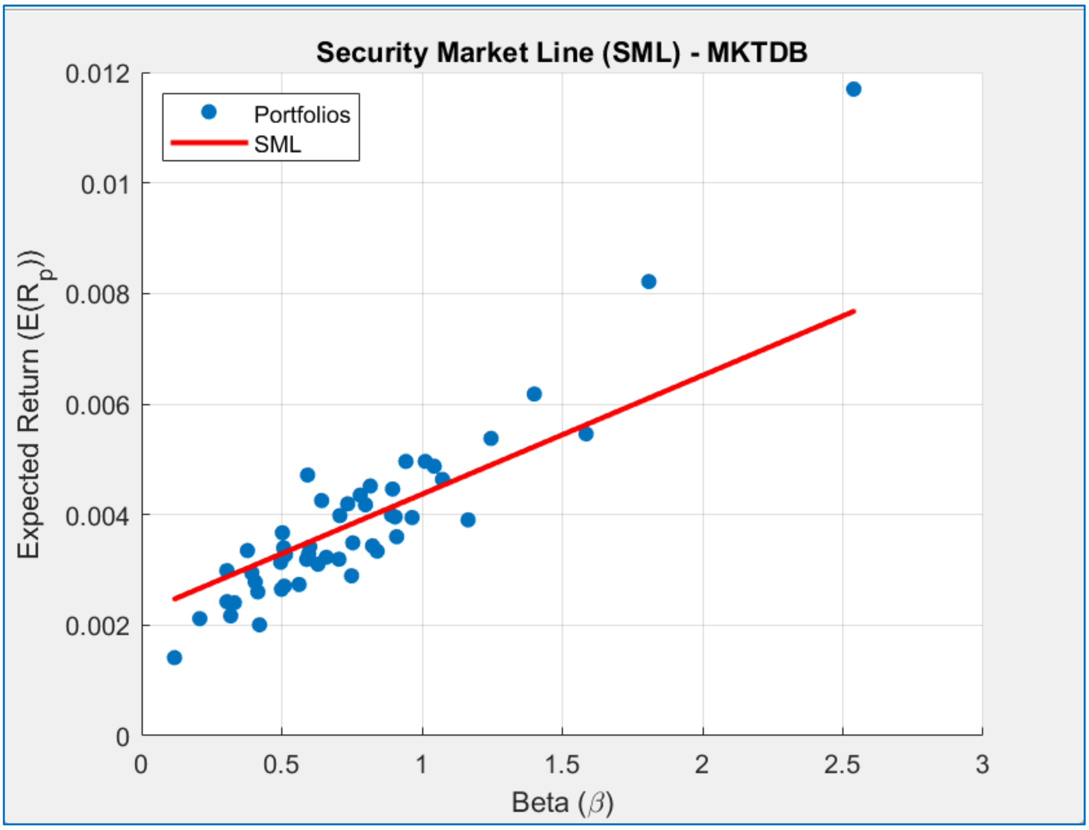

## Appendix 9: Time-series line plot of Individual Portfolio returns



**Appendix 10: Security Market Line MKTB**




**Appendix 11: Security Market Line MKTDB**



## 7. Reference List


Breusch, T. S., & Pagan, A. R. (1979). "A Simple Test for Heteroscedasticity and Random Coefficient Variation." *Econometrica*, 47(5), 1287-1294.

Gibbons, M. R., Ross, S. A., & Shanken, J. (1989). "A Test of the Efficiency of a Given Portfolio." *Econometrica, 57*(5), 1121-1152.

Aitken, A. C. (1935). *On Least Squares and Linear Combinations of Observations*. Proceedings of the Royal Society of Edinburgh, 55, 42-48.

Jegadeesh, N., & Titman, S. (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance*, 48(1), 65-91.

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1), 3-56.

Fama, E. F., & French, K. R. (1996). Multifactor Explanations of Asset Pricing Anomalies. *Journal of Finance*, 51(1), 55-84.

Merton, R. C. (1973). An Intertemporal Capital Asset Pricing Model. *Econometrica*, 41(5), 867-887.

Carhart, M. M. (1997): "On Persistence in Mutual Fund Performance". *Journal of Finance* 52(1), 57–82.

Wooldridge, J.M (2008): “Introductory Econometrics: A Modern Approach”, 4th edition, South Western College.

Dickerson, A., Mueller, P. and Robotti, C. (2023): “Priced Risk in Corporate Bonds”. *Journal of Financial Economics*, 150(2), 103707.

van Binsbergen, J.H., Nozawa, Y. and Schwert, M. (2024): “Duration-based Valuation of Corporate Bonds”. *The Review of Financial Studies.*

Kan, R., Robotti, C. and Shanken, J. (2013): “Pricing model performance and the two-pass cross-sectional regression methodology.*” The Journal of Finance*, 68(6), pp.2617-2649.

Jostova, G., Nikolova, S., Philipov, A. and Stahel, C.W. (2013): “Momentum in Corporate Bond Returns.” *The Review of Financial Studies*, 26(7), pp.1649-1693.

*Fama MacBeth: Useful resources for Fama MacBeth (FMB) / two-stage regression methodology; https://www.cesarerobotti.com/research/*
*Contains a plethora of useful papers and includes various Matlab code files for the two-pass FMB regression.*