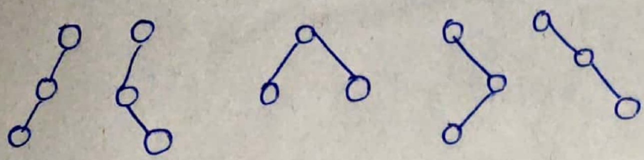
Number of Binary trees using N nodes

n => 3   o o o

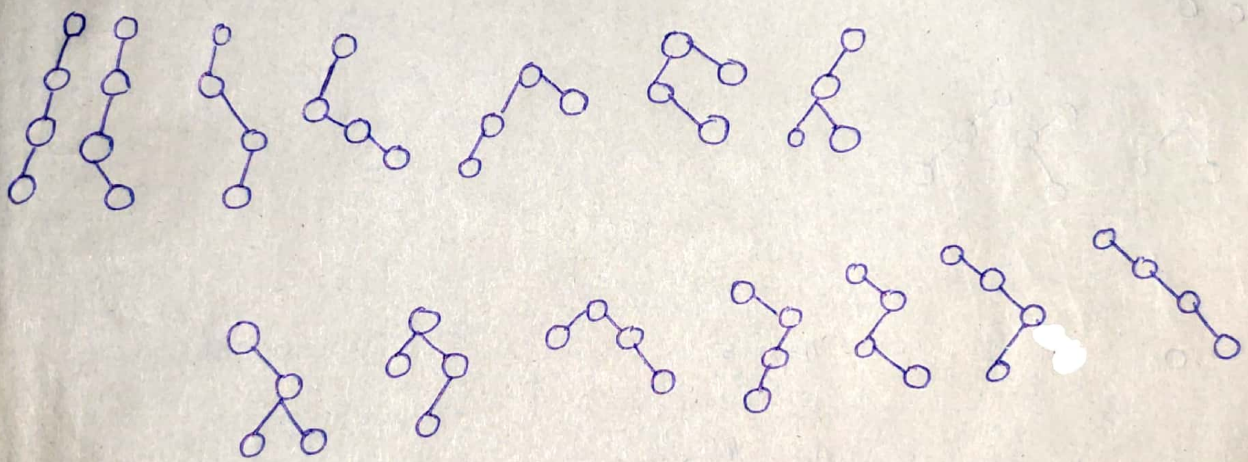


$\boxed{T(3) = 5}$

For 3 possible nodes, 5 different combinations can be obtained.

n = 4   o o o o

$\boxed{T(4) = 14}$



$$C(n, r) \Rightarrow \frac{n!}{r!\,(n-r)!}$$

Catalan numbers:

$$T(n) = \frac{2n C_n}{n+1} \Rightarrow T(5) = \frac{2*5 C_5}{6}$$

$$\Rightarrow \frac{10 C_5}{6} \Rightarrow \frac{\;\frac{10*9*8*7*6*5!}{5!\,5*4*3*2*1}\;}{6} \Rightarrow 42$$

$\boxed{T(5) = 42}$

Number of Maximum height trees } →

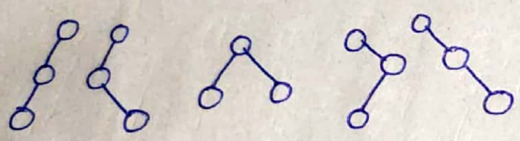
For $n=3 \Rightarrow 2^2 \Rightarrow 4$

$n=4 \Rightarrow 2^3 \Rightarrow 8$

Then for n trees $\Rightarrow 2^{n-1}$ number of maximum height trees can be obtained

Try using different formula

$n \Rightarrow 3$ o o o



n	0	1	2	3	4	5	6
$T(n) = \frac{2n_{C_n}}{n+1}$	1	1	2	5	14	42	

$$T(6) \Rightarrow \underbrace{(1*42) + (1*14) + (2*5)}_{\text{Forward}} + \underbrace{(5*2) + (14*1) + (42*1)}_{\text{Reverse}} \Rightarrow 132$$

Check

$$T(6) \Rightarrow \frac{2*6\ C_6}{7} \Rightarrow \frac{12C_6}{7} \Rightarrow \frac{\frac{12*11*10*9*8*7*\cancel{6!}}{1*2*3*4*5*6*\cancel{6!}}}{7} \Rightarrow 132$$

Generalizing

$$\left([T(0) * T(5)] + [T(1) * T(4)] + [T(2) * T(3)]\right)$$
$$+\left([T(3) * T(2)] + [T(4) * T(1)] + [T(5) * T(0)]\right)$$

$T(n) \Rightarrow \cancel{T(0) + T(1) + T(2) + T(3)}$

O ⇒ Increasing

— ⇒ Decreasing

$$\boxed{T(n) = \sum_{i=1}^{n} \left(T(i-1) * T(n-i)\right)}$$

∴ Catalan number for calculating the number of nodes

i) $T(n) \Rightarrow \dfrac{^{2n}C_n}{n+1}$ ← Combination formula

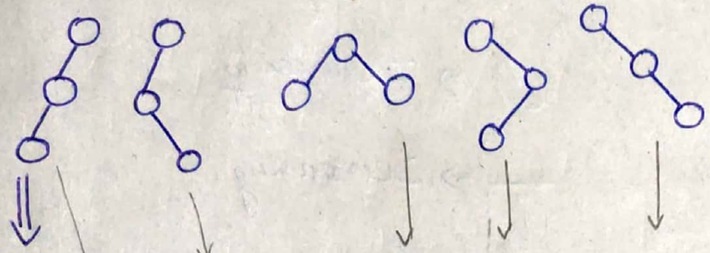
ii) $T(n) \Rightarrow \sum_{i=1}^{n} \left(T(i-1) * T(n-i)\right)$ ← Recursive formula

Calculating the number of labelled notes

Calculating the number of labelled nodes

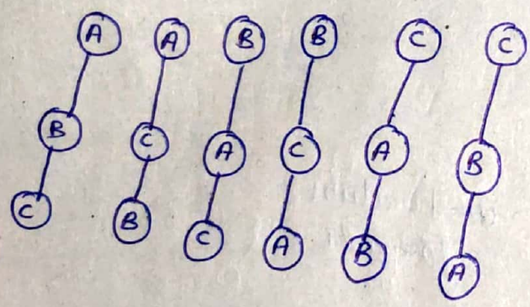
A B C

n = 3



$$\Rightarrow T(n) \; \frac{{}^{2n}C_n}{(n+1)}$$

Take this 3! 3! 3! 3! 3!



For 3 nodes 6 different combinations of labelled nodes can be obtained (ie) n = 3, then 3! can be obtained

then the number of labelled nodes $\Rightarrow T(n) = \frac{{}^{2n}C_n}{(n+1)} * n!$

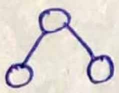
Height (vs) Nodes

Height $h=1$

Min

$n=2$

Max



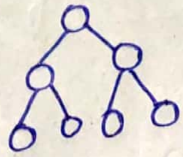
$n=3$

Height $h=2$

Min

$n=3$

Max



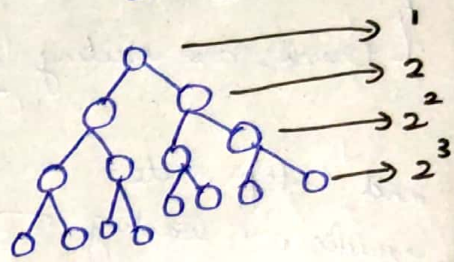
$n=7$

Height $h=3$

Min

$n=4$

Max



$n=15$

Formula for min & max ~~height~~ nodes for a particular height:

Min number of nodes $\Rightarrow$ $\boxed{n = h+1}$

Max number of nodes $\Rightarrow$ $\boxed{n = 2^{h+1} - 1}$

$$1 + 2 + 2^2 + 2^3 \Rightarrow 15$$

$\downarrow$

Similar to a GP series

GP series $\Rightarrow$ $a + ar + ar^2 + ar^3 + \dots + ar^h \Rightarrow \dfrac{a(r^{h+1}-1)}{(r-1)}$

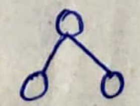
Here $a = 1$ $r = 2$

then $\dfrac{1(2^{h+1}-1)}{2-1} \Rightarrow 2^{h+1} - 1$

Formula for ~~the~~ min & max height with a particular given node:

Nodes $n=3$

Min



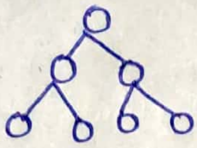
$\boxed{h=1}$

Max

$\boxed{h=2}$

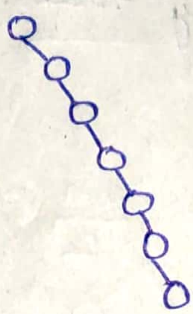
Nodes $n=7$

Min



$\boxed{h=2}$

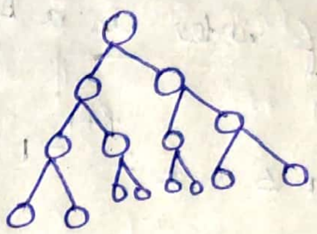
Max



$\boxed{h=6}$

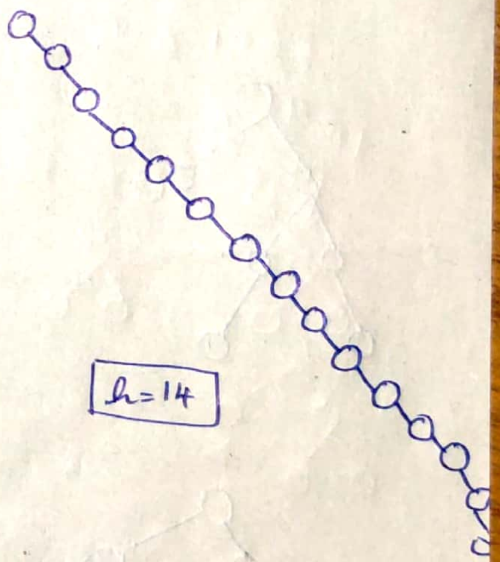
Nodes $n=15$

Min



$\boxed{h=3}$

Max



$\boxed{h=14}$

Max height $\Rightarrow$ $\boxed{h = n-1}$

Min height $\Rightarrow$ $\boxed{h = \log_2(n+1) - 1}$

$$n = 2^{h+1} - 1$$

$$2^{h+1} = n+1$$

$$\log(2^{h+1}) = \log(n+1)$$

$$\log_2(2^{h+1}) = \log_2(n+1)$$

$$h+1 = \log_2(n+1$$

$$h+1 \log_2(2) = \log_2(n+1)$$

$$h = \log_2(n+1) - 1$$

Height of a binary tree :

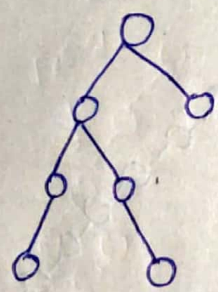
$$\log_2(n+1) - 1 \leq h \leq n-1$$

$O(\log n)$ $\quad$ $O(n)$

Number of nodes in a binary tree :

$$h+1 \leq n \leq 2^{h+1} - 1$$

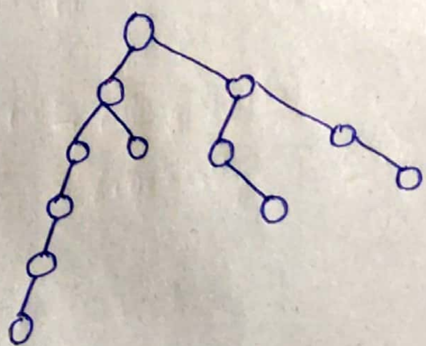
Relationship b/t internal & external Nodes



$deg(2) \Rightarrow 2$

$deg(1) \Rightarrow 2$

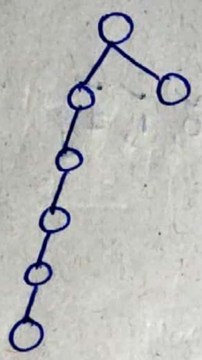
$deg(0) \Rightarrow 3$



$deg(2) \Rightarrow 3$

$deg(1) \Rightarrow 5$

$deg(0) \Rightarrow 4$



deg (2) ⇒ 1

deg (1) ⇒ 4

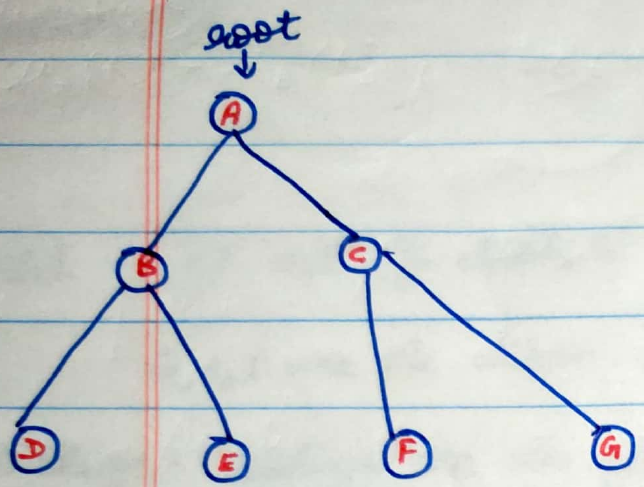
deg (0) ⇒ 2

Therefore $1 + deg(2) = deg(0)$

Hence $\boxed{deg(0) = 1 + deg(2)}$

Binary Tree

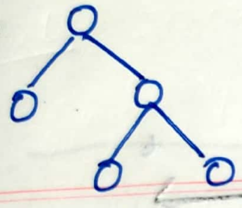
Tree of degree 2, Every node should have atmost 2 ~~—~~ nodes



degree (T) = 2

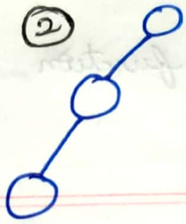
children can be either 0/1/2 and not more than 2.

(1)



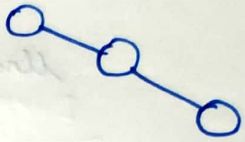
$\boxed{\checkmark}$

(2)



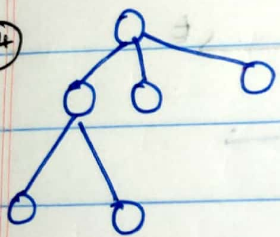
Left skewed tree $\boxed{\checkmark}$

(3)



Right skewed tree $\boxed{\checkmark}$

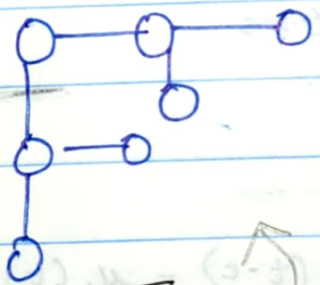
(4)



$\boxed{\times}$

Root Node is having more than 3 children

(5)



$\boxed{\checkmark}$

This is also possible

The same eg binary tree

(1) is drawn here.