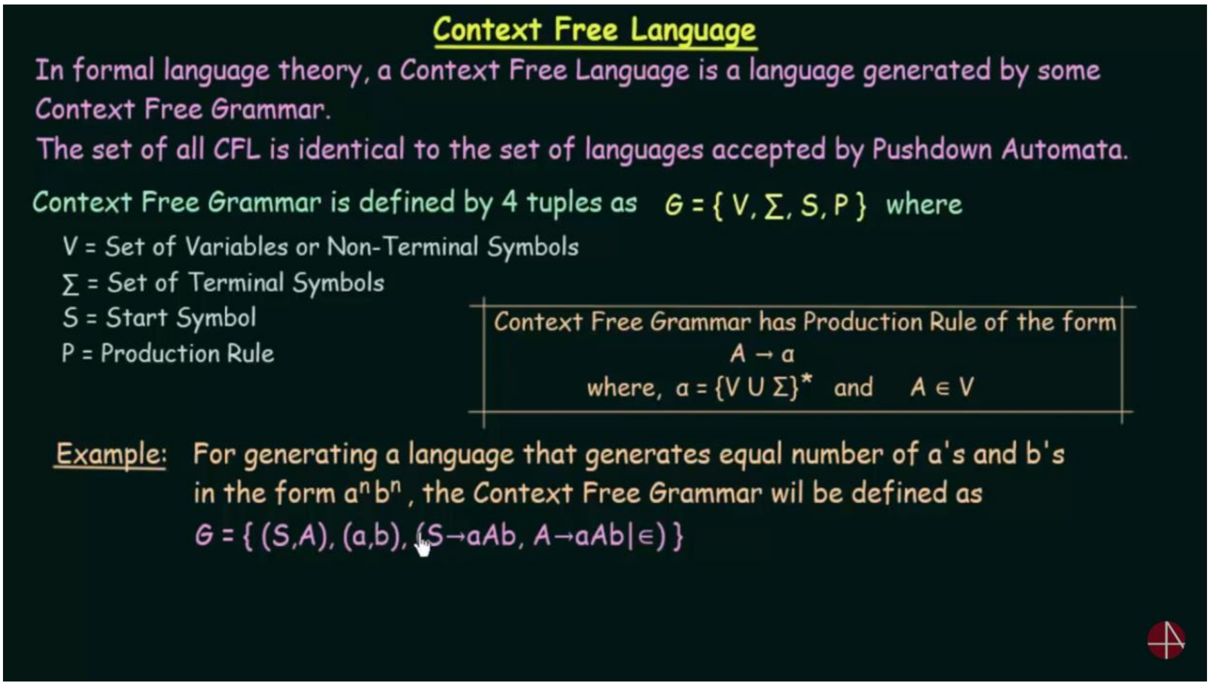


Left Most Derivation → Replace the **left** most non-terminal symbol in each step.
Right Most Derivation → Replace the **right** most non-terminal symbol in each step.

Parse Tree

Graphical representation of derivation where the
inner most nodes → non-terminal
leaves → terminal

Ambiguous Grammar

If a grammar has more than 1 parse tree either left-most/right-most derivation for a given input string, then that grammar → ambiguous grammar.

Simplification of Context free Grammar

1) **Remove useless symbol**
2) **Remove epsilon-production**
3) **Remove unit production**

After completing all the three process, then the final output of that grammar is the simplified form of context grammar.

Removing useless symbol :

1)

T → aaB | abA | aaT

A → aA

B → ab | b

C → ad

Ans: We cannot able to get a finite string. And we cannot able to reach C. So remove both.

T → aaB | aaT

B → ab | b

2)

```
S -> abS | abA | abB
A -> cd
B -> aB
C -> dc
```

Ans: S → abS | abA

A → cd

3)

$S \rightarrow AB \mid C$

$A \rightarrow 0B \mid C$

$B \rightarrow 1 \mid A0$

$C \rightarrow AC \mid C1$ ⇒ Goes infinitely

Solution

$S \rightarrow AB$

$A \rightarrow 0B$

$B \rightarrow 1 \mid A0$

Removing epsilon production:

(Write the grammar and then replace with epsilon)

1)

$S \rightarrow XYX$

$X \rightarrow 0X \mid \varepsilon$

$Y \rightarrow 1Y \mid \varepsilon$

Ans :

$S \rightarrow xyx \mid XY \mid YX \mid XX \mid X \mid Y$

$X \rightarrow 0X \mid 0$

$Y \rightarrow 1Y \mid 1$

2)

```
S -> ABCd
A -> BC
B -> bB | λ
C -> cC | λ
```

Ans:

```
S -> ABCd | ABd | ACd | BCd | Ad | Bd |Cd | d
A -> BC | B | C
B -> bB | b
C -> cC | c
```

Removing unit Production:

(Single non-terminal symbol in the right that is unit → remove that)

(Don't write the unit symbol , replace it)

1)

S → 0A | 1B | C
A → 0S | 00
B → 1 | A
C → 01

Ans:

S → 0A | 1B | 01
A → 0S | 00
B → 1 | 0S | 00
C → 01

2)

```
S -> Aa | B
A -> b | B
B -> A | a
```

(A is a single non-terminal symbol, so remove it)

Ans:

```
S -> Aa | b | a
A -> b | a
B -> a | b
```

3)

Question

$S \rightarrow a \mid AB$

$A \rightarrow ab \mid aB \mid B$

$B \rightarrow C \mid ae$

$C \rightarrow bb \mid K$

$K \rightarrow ab \mid b$

$S \rightarrow a \mid AB$

$A \rightarrow ab \mid aB \mid ae \mid bb \mid$ [illegible]

$B \rightarrow ae \mid bb \mid ab \mid b$

$C \rightarrow bb \mid ab \mid b$

$K \rightarrow ab \mid b$

4)

Question

$S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow C \mid b$

$C \rightarrow D$

$D \rightarrow E$

$E \rightarrow a$

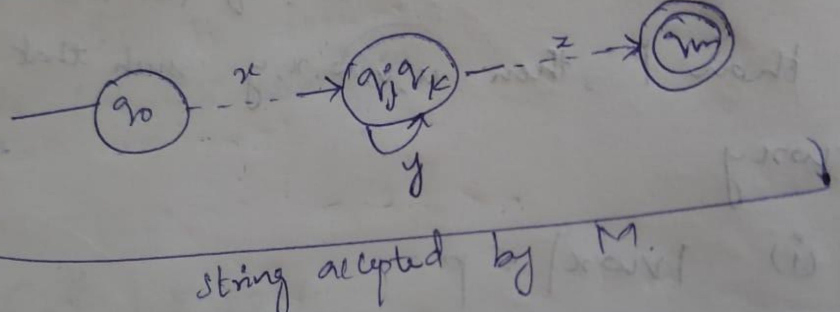
$S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow b \mid a$

$\left.\begin{array}{l} C \rightarrow a \\ D \rightarrow a \\ E \rightarrow a \end{array}\right\}$ It will automatically removed in the useless symbol.

Pumping lemma for regular set



Step 1: Assume 'L' be Regular & Let 'n' be the number states in FA

Step 2: Choose a string w such that $|w| \geq n$

use pumping lemma to write $w = xyz$ with

Application of context free grammar → Normal forms

$\boxed{\text{CNF}}$ Chomsky Normal form

$\boxed{\text{GNF}}$ Greibach Normal form

Chomsky Normal Form

<u>Chomsky Normal Form</u>

In Chomsky Normal Form (CNF) we have a restriction on the length of RHS; which is; elements in RHS should either be two variables or a Terminal.

A CFG is in Chomsky Normal Form if the productions are in the following forms:

$$A \rightarrow a$$

$$A \rightarrow BC$$

where A, B and C are non-terminals and a is a terminal

$$NT \rightarrow T$$

$$NT \rightarrow NT\ NT$$

Steps to convert a given CFG to Chomsky Normal Form:

Step 1: If the Start Symbol S occurs on some right side, create a new Start Symbol S' and a new Production $S' \to S$.

Step 2: Remove Null Productions. (Using the Null Production Removal discussed in previous Lecture)

Step 3: Remove Unit Productions. (Using the Unit Production Removal discussed in previous Lecture)

Step 4: Replace each Production $A \to B_1 \ldots\ldots\ldots B_n$ where $n > 2$, with $A \to B_1 C$ where $C \to B_2 \ldots\ldots B_n$
Repeat this step for all Productions having two or more Symbols on the right side.

Step 5: If the right side of any Production is in the form $A \to aB$ where 'a' is a terminal and A and B are non-terminals, then the Production is replaced by $A \to XB$ and $X \to a$.
Repeat this step for every Production which is of the form $A \to aB$

Question

5) $S \rightarrow ASB$

$A \rightarrow aAS \mid a \mid \varepsilon$

$B \rightarrow SbS \mid A \mid bb$

Solution :

*) S appears on the right side

$S' \rightarrow S$

$S \rightarrow ASB$

$A \rightarrow aAS \mid a \mid \varepsilon$

$B \rightarrow SbS \mid A \mid bb$

*) Remove useless symbol : No

*) Remove epsilon production : Indirectly $A \rightarrow \varepsilon$ and $B \rightarrow \varepsilon$

$S' \rightarrow S$

$S \rightarrow ASB \mid SB \mid AS \mid S$

$A \rightarrow aAS \mid a \mid aS$

$B \rightarrow SbS \mid A \mid bb$

*) Remove Null production:

$S' \rightarrow ASB \mid SB \mid AS$

$S \rightarrow ASB \mid SB \mid AS$

$A \rightarrow aAS \mid a \mid aS$

$B \rightarrow SbS \mid bb \mid aAS \mid a \mid aS$

*) Replace ~~$AS \rightarrow$~~ $X \rightarrow a$

$S' \rightarrow ASB \mid SB \mid AS$

$S \rightarrow ASB \mid SB \mid AS$

$A \rightarrow XAS \mid a \mid XS$

$B \rightarrow SbS \mid bb \mid XAS \mid a \mid XS$

$X \rightarrow a$

*) Replace $Y \rightarrow AS$

$S' \rightarrow YB \mid SB \mid AS$

$S \rightarrow YB \mid SB \mid AS$

$A \rightarrow XY \mid a \mid XS$

$B \rightarrow SbS \mid bb \mid XY \mid a \mid XS$

$X \rightarrow a$

$Y \rightarrow AS$

*) Replace $Z \rightarrow b$

$S' \rightarrow YB \mid SB \mid AS$

$S \rightarrow YB \mid SB \mid AS$

$A \rightarrow XY \mid a \mid XS$

$B \rightarrow SZS \mid XY \mid a \mid XS \mid ZZ$

$X \rightarrow a$

$Y \rightarrow AS$

$Z \rightarrow b$

*) Replace $P \rightarrow ZS$

$S' \rightarrow YB \mid SB \mid AS$

$S \rightarrow YB \mid SB \mid AS$

$A \rightarrow XY \mid a \mid XS$

~~$B \rightarrow SP \mid bb \mid XY \mid Z$~~ $B \rightarrow SP \mid ZZ \mid XY \mid a \mid XS$

$X \rightarrow a$

$Y \rightarrow AS$

$Z \rightarrow b$

$P \rightarrow ZS$

This is the required Chomsky Normal Form

Question

Question

$S \to abAB$

$A \to baB \mid \varepsilon$

$B \to Baa \mid A \mid \varepsilon$

Solution

*) Remove epsilon production:

Now $A \to \varepsilon$

$S \to abAB \mid abB$

$A \to baB$

$B \to Baa \mid \cancel{A} \mid \varepsilon$

Now $B \to \varepsilon$

$S \to abAB \mid abB \mid abA \mid ab$

$A \to baB \mid ba$

$B \to Baa \mid aa \mid A$

*) Remove unit production:

$S \to abAB \mid abB \mid abA \mid ab$

$A \to baB \mid ba$

$B \to Baa \mid aa \mid baB \mid ba$

*) Replace $X \to a$ and $Y \to b$

$S \to \boxed{XY}AB \mid \boxed{XY}B \mid \boxed{XY}A \mid XY$

$A \to \boxed{YX}B \mid YX$

$B \to BXX \mid XX \mid \boxed{YX}B \mid \boxed{YX}$

$X \to a$

$Y \to b$

* Replace $M \to XY$ and $N \to YX$

$S \to MAB \mid MB \mid MA \mid XY$

$A \to NB \mid YX$

$B \to BXX \mid XX \mid NB \mid YX$

$X \to a$

$Y \to b$

$M \to XY$

$N \to YX$

*) Replace $P \to BX$ and $Q \to MA$

$S \to QB \mid MB \mid MA \mid XY$

$A \to NB \mid YX$

$B \to PX \mid XX \mid NB \mid YX$

$X \to a$

$Y \to b$

$M \to XY$

$N \to YX$

$P \to BX$

$Q \to MA$

Question

Question $S \rightarrow aAD$

$A \rightarrow aB \mid bAB$

$B \rightarrow b$

$E \rightarrow d$

Chomsky

$\left(\begin{array}{l} NT \rightarrow T \\ NT \rightarrow NT\ NT \end{array} \right)$

Solution

*) Remove useless symbol

$S \rightarrow aAD$

$A \rightarrow aB \mid bAB$

$B \rightarrow b$

Applying techniques

*) Replace $(X \rightarrow a)$ and $(Y \rightarrow b)$

$S \rightarrow XAD$

$A \rightarrow XB \mid YAB$

$B \rightarrow Y$

$X \rightarrow a$

$Y \rightarrow b$

*) Replace $(Z \rightarrow XA)$ and $(T \rightarrow YA)$

$S \rightarrow ZD$

$A \rightarrow XB \mid TB$

$B \rightarrow Y$

$X \rightarrow a$

$Y \rightarrow b$

$Z \rightarrow XA$

$T \rightarrow YA$

Greibach normal form

Greibach Normal Form

A CFG is in Greibach Normal Form if the productions are in the following forms:

$$A \to b$$
$$A \to bC_1C_2 \ldots\ldots C_n$$

where $A, C_1, \ldots, C_n$ are Non-Terminals and b is a Terminal

Steps to convert a given CFG to GNF:

Step 1: Check if the given CFG has any Unit Productions or Null Productions and Remove if there are any (using the Unit & Null Productions removal techniques discussed in the previous lecture)

Step 2: Check whether the CFG is already in Chomsky Normal Form (CNF) and convert it to CNF if it is not. (using the CFG to CNF conversion technique discussed in the previous lecture)

Step 3: Change the names of the Non-Terminal Symbols into some A_i in ascending order of i

Steps to convert a given CFG to GNF:

Step 1: Check if the given CFG has any Unit Productions or Null Productions and Remove if there are any (using the Unit & Null Productions removal techniques discussed in the previous lecture)

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Step 3: Change the names of the Non-Terminal Symbols into some A_i in ascending order of i

Example:

$S \to CA \mid BB$
$B \to b \mid SB$
$C \to b$
$A \to a$

Replace:

S with A_1
C with A_2
A with A_3
B with A_4

We get:

$A_1 \to A_2A_3 \mid A_4A_4$
$A_4 \to b \mid A_1A_4$
$A_2 \to b$
$A_3 \to a$

$$\boxed{\begin{array}{l} NT \to T \\ NT \to T(NT)^* \end{array}}$$

Step 4: Alter the rules so that the Non-Terminals are in ascending order, such that,

If the Production is of the form $A_i \to A_j x$, then,

$i < j$ and should never be $i \geq j$

$$A_4 \to b \mid \underline{A_1} A_4$$

$$A_4 \to b \mid \underline{A_2} A_3 A_4 \mid A_4 A_4 A_4$$

$$A_4 \to b \mid b A_3 A_4 \mid A_4 A_4 A_4$$

Left Recursion

CNF	GNF
$NT \to T$	$NT \to T$
$NT \to NT\,NT$	$NT \to T\,(NT)^*$

Question

Question

$S \to CA \mid BB$

$B \to b \mid SB$

$C \to b$

$A \to a$

Replacing
S with A_1
C with A_2
A with A_3
B with A_4

Solution

$A_1 \to A_2A_3 \mid A_4A_4 \quad (1<2 \times 1<4)$ ✓

$A_4 \to b \mid A_1A_4 \quad (4>1)$ ×

$A_2 \to b$

$A_3 \to a$

$A_1 \to A_2A_3 \mid A_4A_4$

$A_4 \to b \mid A_2A_3A_4 \mid A_4A_4A_4$

$A_2 \to b$

$A_3 \to a$

$A_1 \to A_2A_3 \mid A_4A_4 \quad \cancel{bA_4} \; \cancel{A_2}$

$\underbrace{A_4}_{A} \to \underbrace{b}_{\beta_2} \mid \underbrace{A_4}_{A}\underbrace{A_4A_4}_{\alpha_1} \mid \underbrace{A_2A_3A_4}_{\beta_1}$

$\underbrace{A_4}_{A} \to \underbrace{A_4}_{A}\underbrace{A_4A_4}_{\alpha_1} \mid \underbrace{bA_3A_4}_{\beta_1} \mid \underbrace{b}_{\beta_2}$

$A \to \beta_1 \mid \beta_2 \mid \dots \beta_n$

$A \to \beta_1 z \mid \beta_2 z \mid \dots \beta_n z$

$z \to \alpha_1 \mid \alpha_2 \mid \dots \alpha_n$

$z \to \alpha_1 z \mid \alpha_2 z \mid \dots \alpha_n z$

Apply left recursion.

$A_4 \to bA_3A_4 \mid b$

$A_4 \to bA_3A_4z \mid bz$

and

$z \to A_4A_4$

$z \to A_4A_4z$

Combining together

$A_4 \to bA_3A_4 \mid b \mid bA_3A_4z \mid bz$

$z \to A_4A_4 \mid A_4A_4z$

$A_1 \rightarrow A_2A_3 \mid A_4A_4$

$A_4 \rightarrow bA_3A_4 \mid b \mid bA_3A_4Z \mid bZ$

$Z \rightarrow A_4A_4 \mid A_4A_4Z$

$A_2 \rightarrow b$

$A_3 \rightarrow a$

$A_1 \rightarrow bA_3 \mid bA_3A_4A_4 \mid bA_4 \mid bA_3A_4Z \mid bZA_4$

$A_4 \rightarrow bA_3A_4 \mid b \mid bA_3A_4Z \mid bZ$

$Z \rightarrow bA_3A_4A_4 \mid bA_4 \mid bA_3A_4ZA_4 \mid bZA_4 \mid$

$bA_3A_4A_4Z \mid bA_4Z \mid bA_3A_4ZA_4Z \mid bZA_4Z$

$A_2 \rightarrow b$

$A_3 \rightarrow a$

This is the required <u>GNF</u>

Question

Question

$S \to AB$

$A \to BS \mid b$

$B \to SA \mid a$

Replace $S \to S_1$

$A \to S_2$

$B \to S_3$

Solution

$S_1 \to S_2 S_3 \quad (1<2)$ ✓

$S_2 \to S_3 S_1 \mid b \quad (2<3)$ ✓

$S_3 \to \boxed{S_1} S_2 \mid a \quad (3<1)$ ✗

$S_1 \to S_2 S_3$

$S_2 \to S_3 S_1 \mid b$

$S_3 \to \boxed{S_2} S_3 S_2 \mid a \quad (3<2)$ ✗

$A \to \beta_1 \mid \beta_2 \mid \cdots \mid \beta_n$

$A \to \beta_1 Z \mid \beta_2 Z \mid \cdots \mid \beta_n Z$

$Z \to \alpha_1 \mid \alpha_2 \mid \cdots \alpha_n$

$Z \to \alpha_1 Z \mid \alpha_2 Z \mid \cdots \alpha_n Z$

$S_1 \to S_2 S_3$

$S_2 \to S_3 S_1 \mid b$

$\overset{A}{S_3} \to \overset{A}{S_3} \underbrace{S_1 S_3 S_2}_{\alpha_1} \mid \underbrace{b S_3 S_2}_{\beta_1} \mid \underbrace{a}_{\beta_2}$

Applying left recursion

$S_3 \to b S_3 S_2 \mid a$

$S_3 \to b S_3 S_2 Z \mid aZ$

$Z \to S_1 S_3 S_2$

$Z \to S_1 S_3 S_2 Z$

Combining

$S_1 \to S_2 S_3$

$S_2 \to \boxed{S_3} S_1 \mid b$

$S_3 \to b S_3 S_2 \mid a \mid b S_3 S_2 Z \mid aZ$

$Z \to S_1 S_3 S_2 \mid S_1 S_3 S_2 Z$

$S_1 \rightarrow \boxed{S_2} S_3$

$S_2 \rightarrow bS_3S_2S_1 \mid aS_1 \mid bS_3S_2ZS_1 \mid aZS_1 \mid b$

$S_3 \rightarrow bS_3S_2 \mid a \mid bS_3S_2Z \mid aZ$

$Z \rightarrow S_1S_3S_2 \mid S_1S_3S_2Z$

$S_1 \rightarrow bS_3S_2S_1S_3 \mid aS_1S_3 \mid bS_3S_2ZS_1S_3 \mid aZS_1S_3 \mid bS_3$

$S_2 \rightarrow bS_3S_2S_1 \mid aS_1 \mid bS_3S_2ZS_1 \mid aZS_1 \mid b$

$S_3 \rightarrow bS_3S_2 \mid a \mid bS_3S_2Z \mid aZ$

$Z \rightarrow \boxed{S_1}S_3S_2 \mid \boxed{S_1}S_3S_2Z$

$S_1 \rightarrow bS_3S_2S_1S_3 \mid aS_1S_3 \mid bS_3S_2ZS_1S_3 \mid aZS_1S_3 \mid bS_3$

$S_2 \rightarrow bS_3S_2S_1 \mid aS_1 \mid bS_3S_2ZS_1 \mid aZS_1 \mid b$

$S_3 \rightarrow bS_3S_2 \mid a \mid bS_3S_2Z \mid aZ$

$Z \rightarrow bS_3S_2S_1S_3S_3S_2 \mid aS_1S_3S_3S_2 \mid bS_3S_2ZS_1S_3S_3S_2 \mid aZS_1S_3S_3S_2 \mid bS_3S_3S_2 \mid$

$bS_3S_2S_1S_3S_3S_2Z \mid aS_1S_3S_3S_2Z \mid bS_3S_2ZS_1S_3S_3S_2Z \mid$

$aZS_1S_3S_3S_2Z \mid bS_3S_3S_2Z$

This is the required GNF

Question

Question

$S \to XB \mid AA$

$A \to a \mid SA$

$B \to b$

$X \to a$

Replace S with S_1

X with S_2

B with S_3

A with S_4

Solution

$S_1 \to S_2S_3 \mid S_4S_4$ (1<2) (1<4) ✓

$S_4 \to a \mid \boxed{S_1}S_4$ (4<1) ✗

$S_3 \to b$

$S_2 \to a$

$S_1 \to \boxed{S_2}S_3 \mid S_4S_4$

$S_4 \to a \mid S_2S_3S_4 \mid S_4S_4S_4$

$S_3 \to b$

$S_2 \to a$

$S_1 \to aS_3 \mid S_4S_4$

$S_4 \to a \mid \boxed{S_2}S_3S_4 \mid S_4S_4S_4$

$S_3 \to b$

$S_2 \to a$

$S_1 \to aS_3 \mid S_4S_4$

$S_4 \to a \mid aS_3S_4 \mid S_4S_4S_4$

$S_3 \to b$

$S_2 \to a$

$S_1 \rightarrow aS_3 \mid S_4S_4$

~~$S_4 \rightarrow S_4S_4S_4 \mid a \mid aS_3S_4$~~ $\quad \underbrace{S_4}_{A} \rightarrow \underbrace{S_4}_{A} \underbrace{S_4S_4}_{\alpha_1} \mid \underbrace{a}_{\beta_1} \mid \underbrace{aS_3S_4}_{\beta_2}$

$S_3 \rightarrow b$

$S_2 \rightarrow a$

Applying left recursion,

$A \rightarrow \beta_1 \mid \beta_2 \mid \dots \beta_n$

$A \rightarrow \beta_1 Z \mid \beta_2 Z \mid \dots \beta_n Z$

$Z \rightarrow \alpha_1 \mid \alpha_2 \mid \dots \alpha_n$

$Z \rightarrow \alpha_1 Z \mid \alpha_2 Z \mid \dots \alpha_n Z$

$S_4 \rightarrow a \mid aS_3S_4$

$S_4 \rightarrow aZ \mid aS_3S_4Z$

$Z \rightarrow S_4S_4$

$Z \rightarrow S_4S_4Z$

Combining

$S_4 \rightarrow a \mid aS_3S_4 \mid aZ \mid aS_3S_4Z$

$Z \rightarrow \boxed{S_4}S_4 \mid \boxed{S_4}S_4Z$

$S_1 \rightarrow aS_3 \mid \boxed{S_4}S_4$

$S_3 \rightarrow b$

$S_2 \rightarrow a$

$S_1 \rightarrow aS_3 \mid aS_4 \mid aS_3S_4S_4 \mid aZS_4 \mid aS_3S_4ZS_4$

$Z \rightarrow aS_4Z \mid aS_3S_4S_4Z \mid aZS_4Z \mid aS_3S_4ZS_4Z$

$aS_4 \mid aS_3S_4S_4 \mid aZS_4 \mid aS_3S_4ZS_4$

$S_2 \rightarrow a$

$S_3 \rightarrow b$

$S_4 \rightarrow a \mid aS_3S_4 \mid aZ \mid aS_3S_4Z$

This is the required GNF.

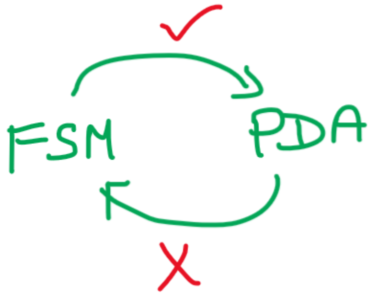
Pushdown automata

↳ context free grammar

↳ context free language

accepted by
i) NFA
ii) DFA

accepts
i) Regular grammar
ii) Regular language



- It is a finite automata with extra memory called stack - which helps pushdown automata to recognize context free language.

stack < pop, push

context free language.

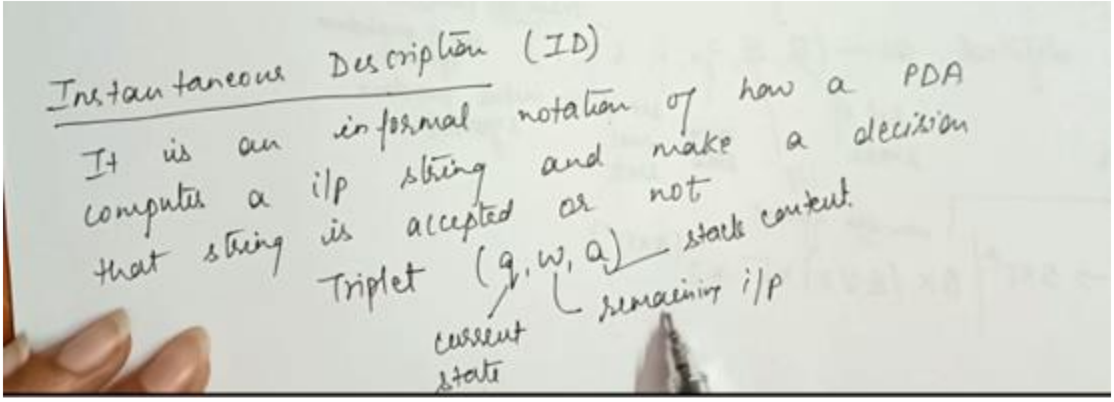
- Pushdown automata are used in theories about what can be computed by machines.
- They are more capable than finite state machine but less capable than Turing Machine.
- It can be defined as — $(Q, \Sigma, q_0, F, \delta, Z, \Gamma)$

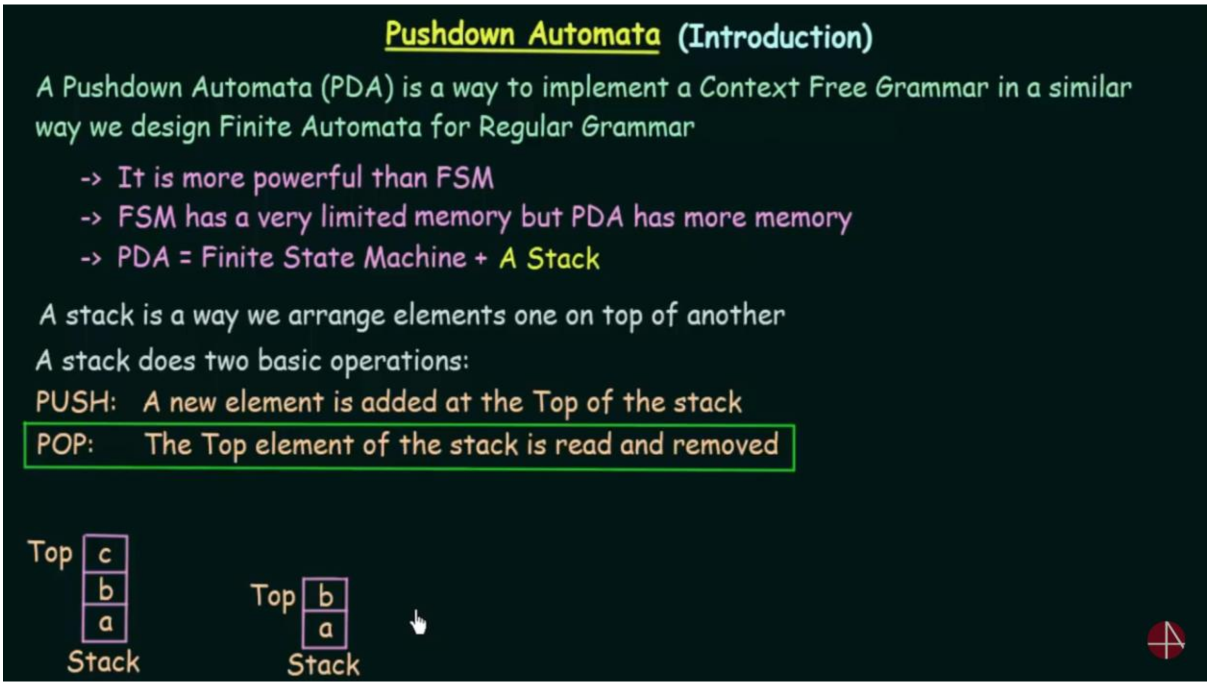
Q – set of states; Σ – i/p symbol; q_0 – initial state; F – set of final state; δ – Transition function; Z – initial pushdown symbol; Γ – set of pushdown symbol.

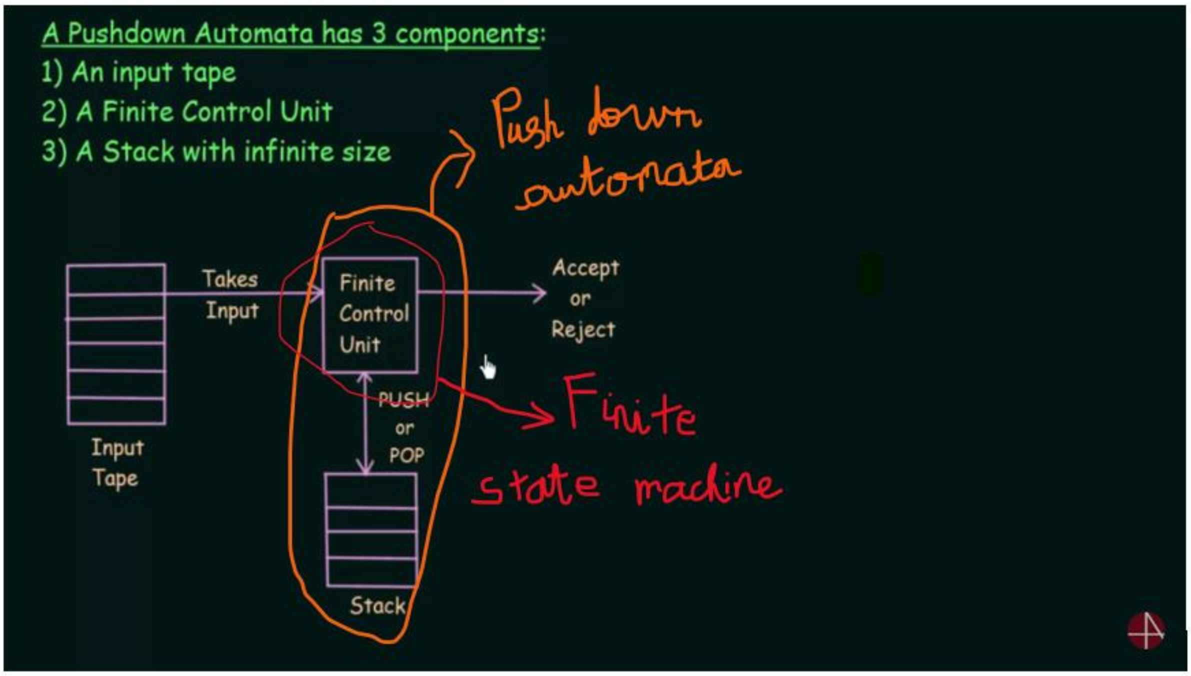
δ

Deterministic: $Q \times \{\Sigma \times \epsilon\} \times \Gamma \rightarrow Q \times \Gamma^*$

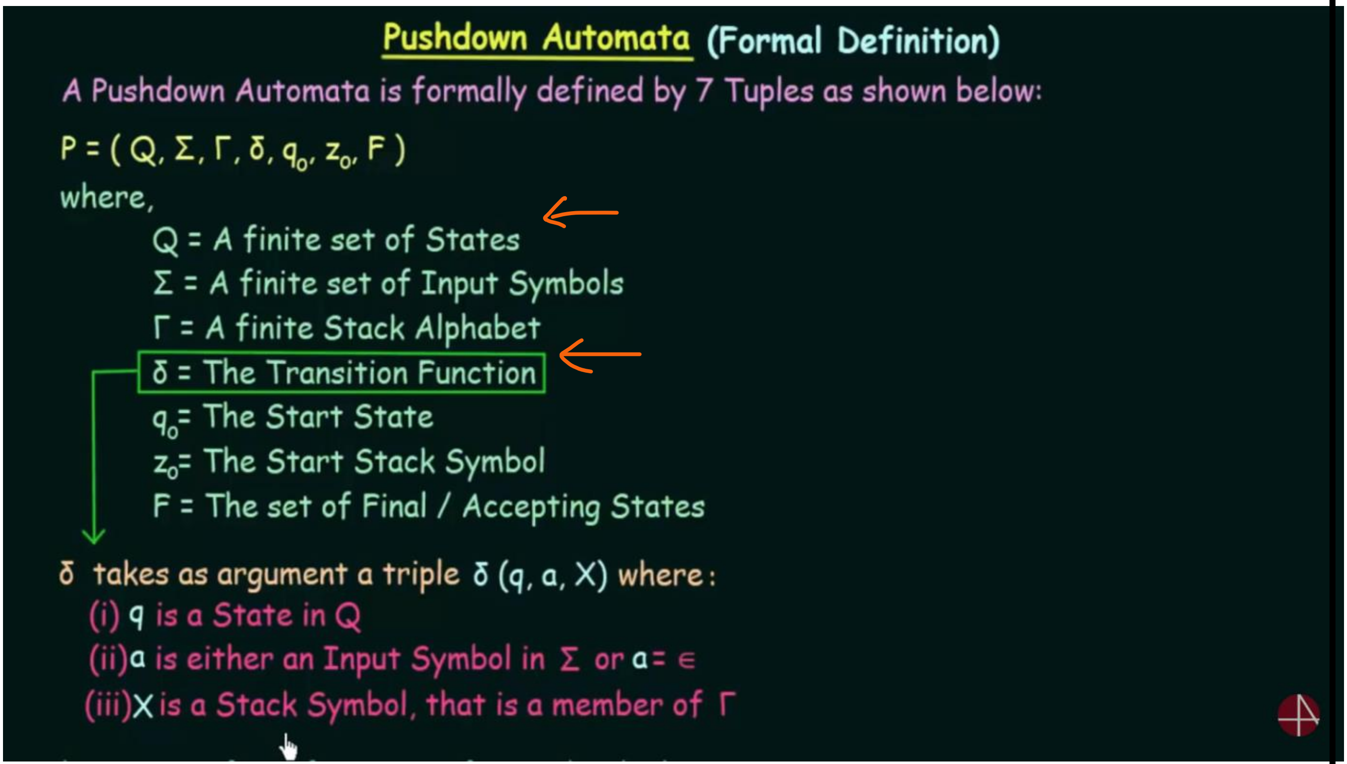
Non-deter: $Q \times \{\Sigma \cup \epsilon\} \times \Gamma \rightarrow 2^{(Q \times \Gamma^*)}$


Instantaneous Description (ID)
It is an informal notation of how a PDA
computes a i/p string and make a decision
that string is accepted or not
Triplet (q, w, α)
current state
remaining i/p
stack content


Pushdown Automata (Introduction)
A Pushdown Automata (PDA) is a way to implement a Context Free Grammar in a similar way we design Finite Automata for Regular Grammar
-> It is more powerful than FSM
-> FSM has a very limited memory but PDA has more memory
-> PDA = Finite State Machine + A Stack
A stack is a way we arrange elements one on top of another
A stack does two basic operations:
PUSH: A new element is added at the Top of the stack
POP: The Top element of the stack is read and removed
Top
c
b
a
Stack
Top
b
a
Stack


A Pushdown Automata has 3 components:
1) An input tape
2) A Finite Control Unit
3) A Stack with infinite size
Push down automata
Takes Input
Finite Control Unit
Accept or Reject
PUSH or POP
Input Tape
Stack
Finite state machine

Finite automata F = (Q,∑,q0,δ ,F)



The transition function for finite automata is entirely different from push-down automata.

FA = δ(Q, ∑)
PDA = δ(Q1, ∑, Z0) → (Q2, V)

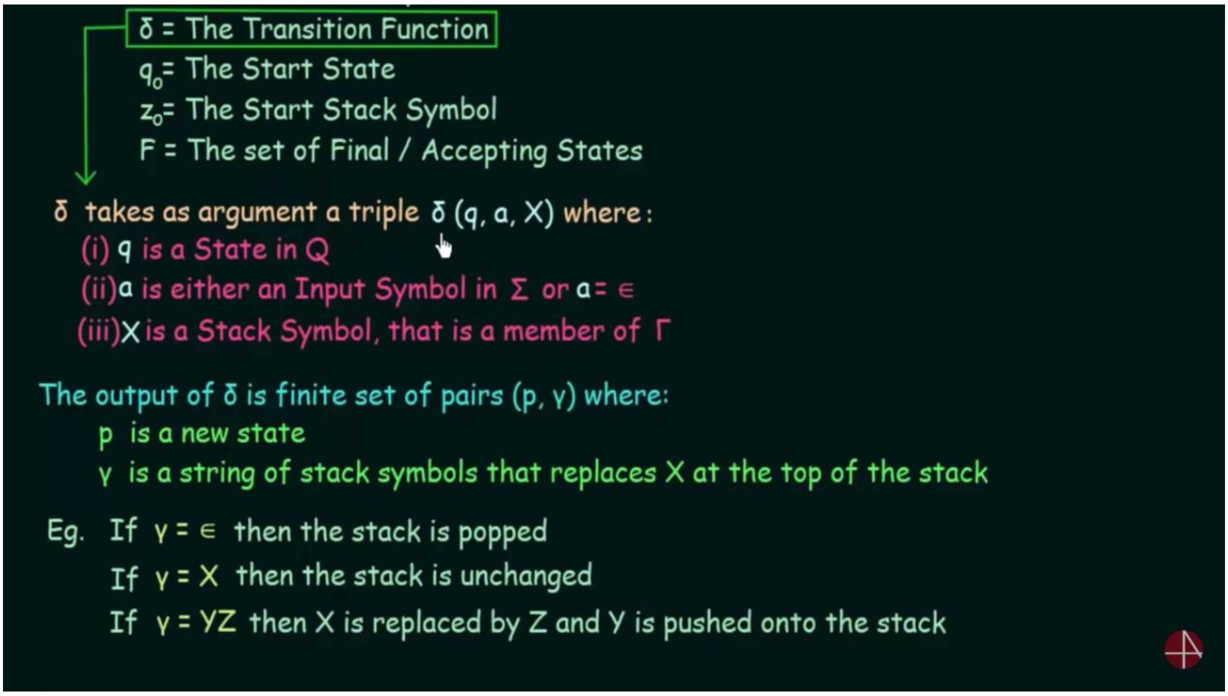
Q1 is one state and it is changed to Q2 which is another state.
∑ → one input alphabet
Z0 → top of the stack

1) δ(q1,a,b) → (q2,epsilon)
[a → input symbol(cursor) and b → top of the stack]
If V → epsilon (Whenever the stack's top and the cursor symbol are different they are cancelled eachother.

2) δ (q1,a,z) → (q2,az)
If V → az (Whenever the stack is empty (i.e denoted by z) and the cursor symbol is about to enter into the stack.

3) δ (q1,a,a) → (q2,aa)

If V → aa (Whenever the top of the stack and the cursor symbol are same then that cursor symbol will be added to the stack instead of cancelling.



Instantaneous Description

Instantaneous Description (ID) is an informal notation of how a **PDA** "computes" a input string and make a decision that string is accepted or rejected.

1)

$$L \Rightarrow \{ a^n b^n \mid n \geq 1 \}$$

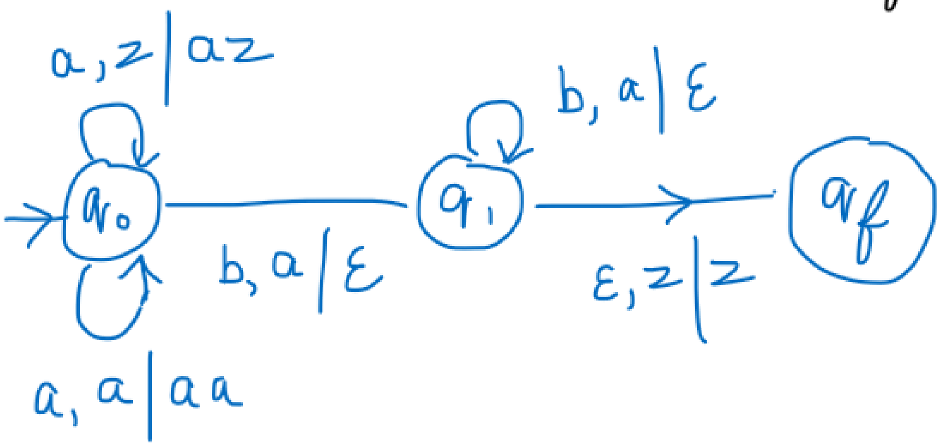
$$\delta(q_0, a, z) \Rightarrow (q_0, az)$$

$$\delta(q_0, a, a) \Rightarrow (q_0, aa)$$

$$\delta(q_0, b, a) \Rightarrow (q_1, \varepsilon)$$

$$\delta(q_1, b, a) \Rightarrow (q_1, \varepsilon)$$

$$\delta(q_1, \varepsilon, z) \Rightarrow (q_f, z)$$



2)

$L \Rightarrow \{a^n b^{2n} \mid n \geq 1\}$

$\delta(q_0, a, z) \Rightarrow (q_0, az)$

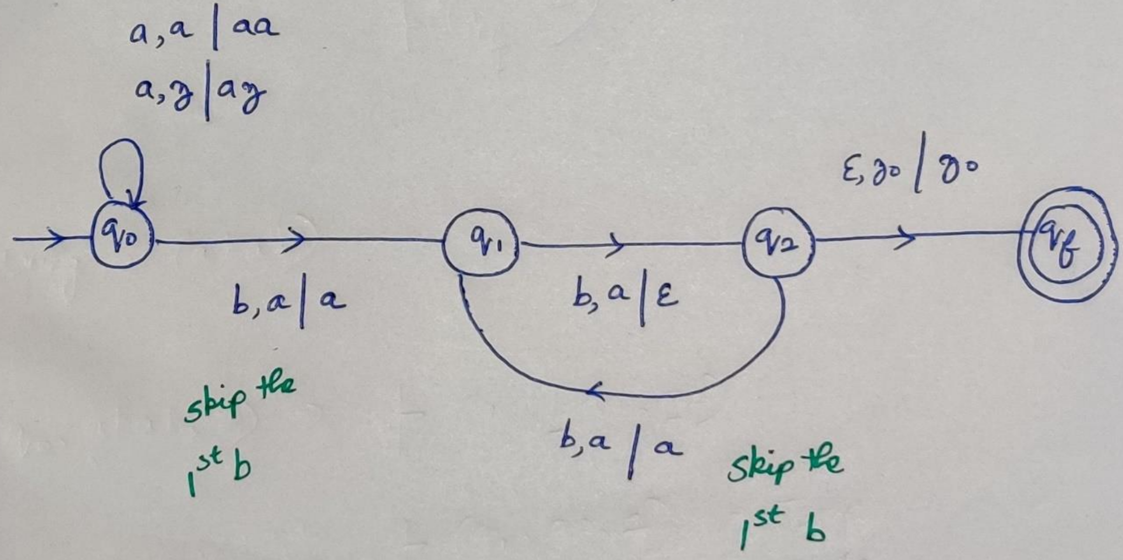
$\delta(q_0, a, a) \Rightarrow (q_0, aa)$

$\delta(q_0, b, a) \Rightarrow (q_1, a)$ Once you see 1st b, skip b

$\delta(q_1, b, a) \Rightarrow (q_2, \varepsilon)$ Once you see 2nd b, cancel it

$\delta(q_2, b, a) \Rightarrow (q_1, a)$

$\delta(q_2, \varepsilon, z_0) \Rightarrow (q_f, z_0)$



3)

$L \Rightarrow \{a^n b^{2n+1} \mid n \geq 1\}$

1st time — 2^{rd} time

$\delta(q_0, a, z_0) \Rightarrow (q_0, az_0)$

$\delta(q_0, a, a) \Rightarrow (q_0, aa)$

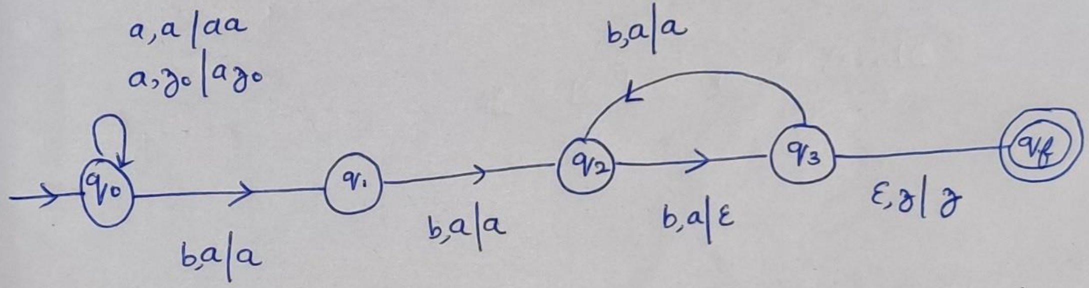
$\delta(q_0, b, a) \Rightarrow (q_1, a)$ Skip the 1^{st} b

$\delta(q_1, b, a) \Rightarrow (q_2, a)$ skip the 2^{rd} b

$\delta(q_2, b, a) \Rightarrow (q_3, \epsilon)$ Cut the 3^{rd} b — cut the 2^{rd} b

$\delta(q_3, b, a) \Rightarrow (q_2, a)$ — Skip the 1^{st} b and then

$\delta(q_3, \epsilon, z) \Rightarrow (q_f, z_0)$



Eg: String i) $\cancel{a}$ b b $\cancel{b}$

ii) $\cancel{a}\cancel{a}\cancel{a}$ b b $\cancel{b}$ b $\cancel{b}$ b $\cancel{b}$

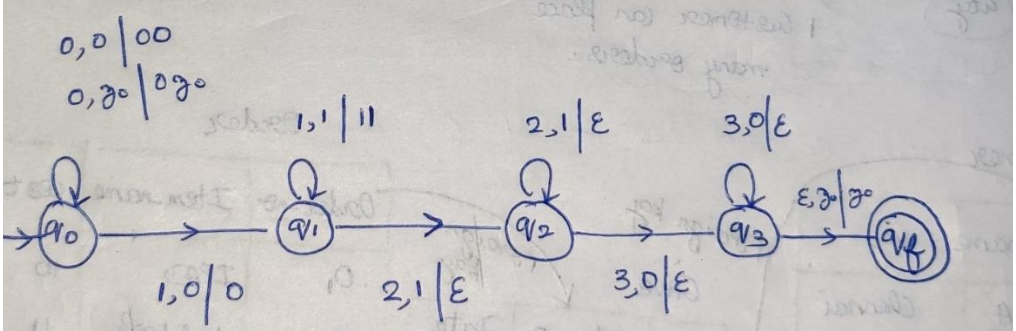
→ 1^{st} time skip 2 b's and cut the 3^{rd}

→ 2^{nd} time skip 1 b and cut the 2^{nd} b

4)

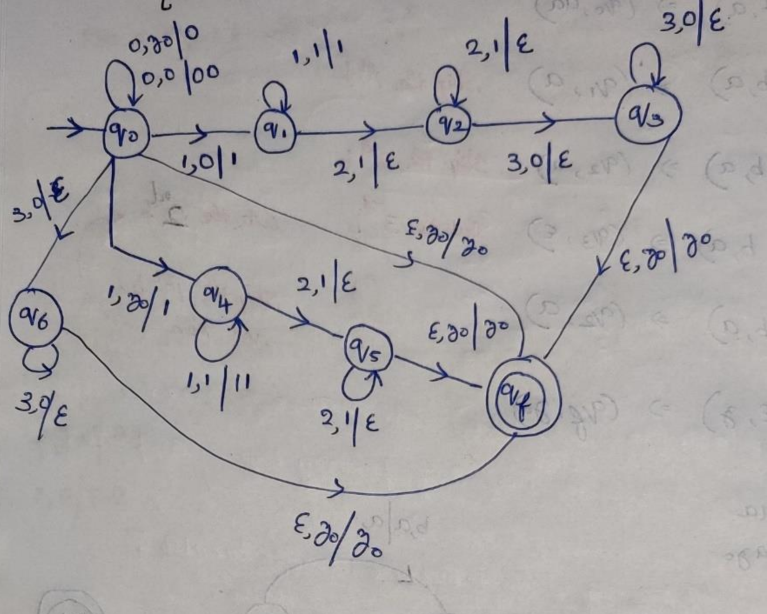
$L \Rightarrow \{0^n 1^m 2^m 3^n \mid n, m \geq 1\}$

There should be equal number of $\boxed{0\text{'s} \,\&\, 1\text{'s}}$ and $\boxed{1\text{'s} \,\&\, 2\text{'s}}$



5)

$L \Rightarrow \{0^n 1^m 2^m 3^n \mid n, m \geq 0\}$

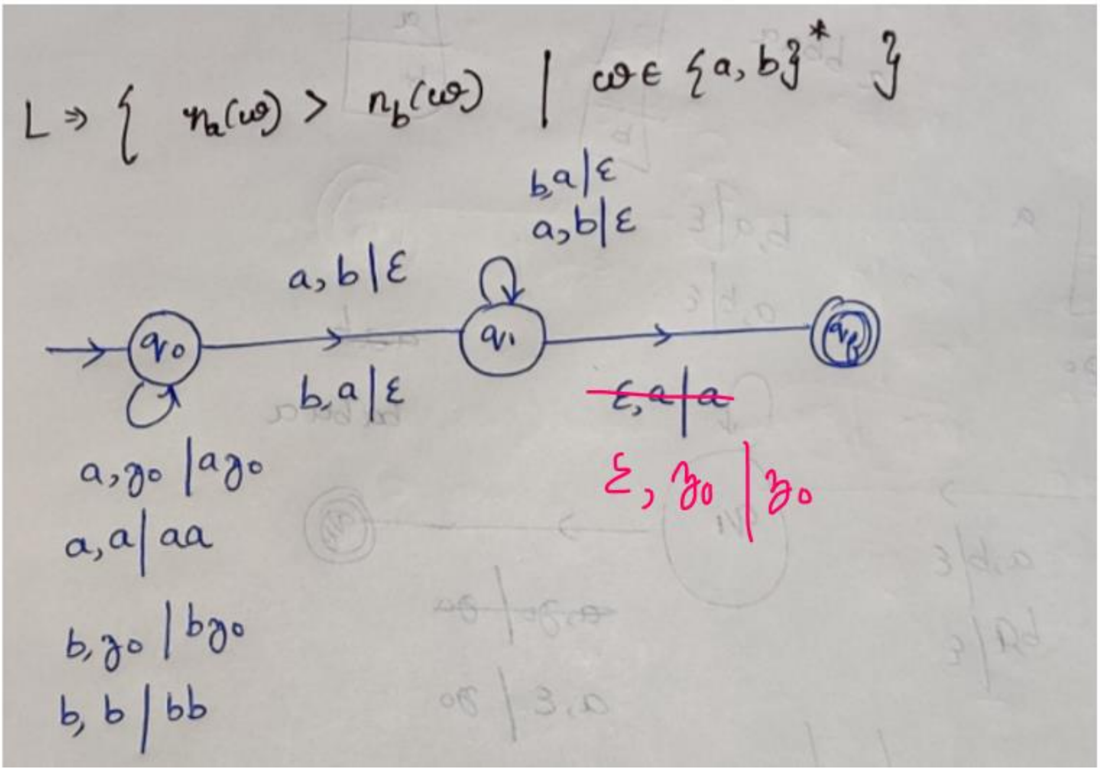


Case-1 : $n \geq 0$ then $1^m 2^m$

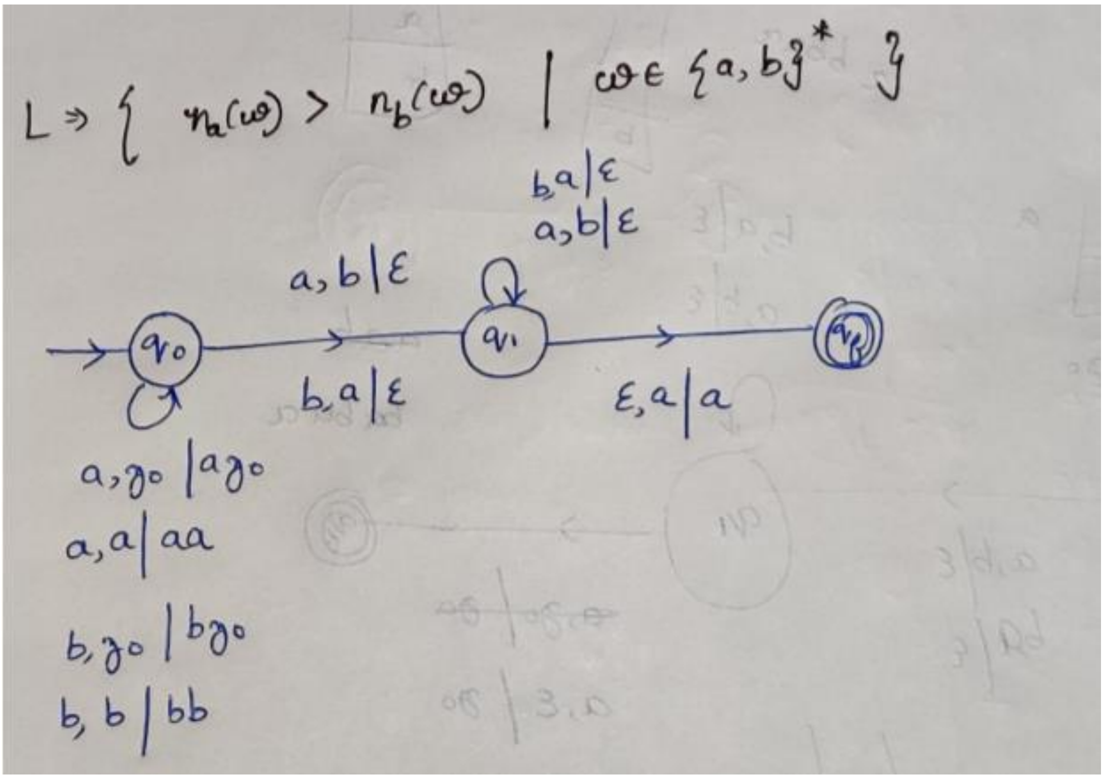
Case-2 : $m \geq 0$ then $0^n 3^n$

Case-3 : $m, n \neq 0$ then $0^n 1^m 2^m 3^n$

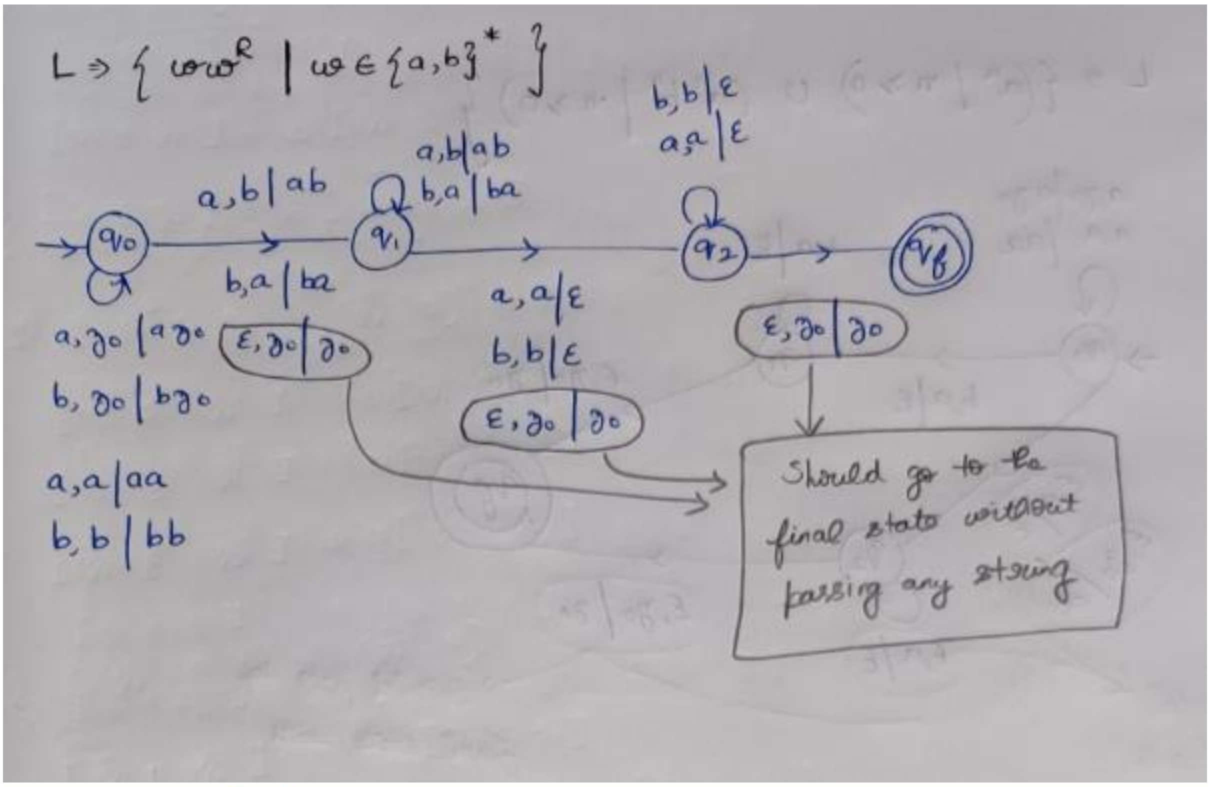
6) L = {na(w)=nb(w) | w belongs to {a,b}* }

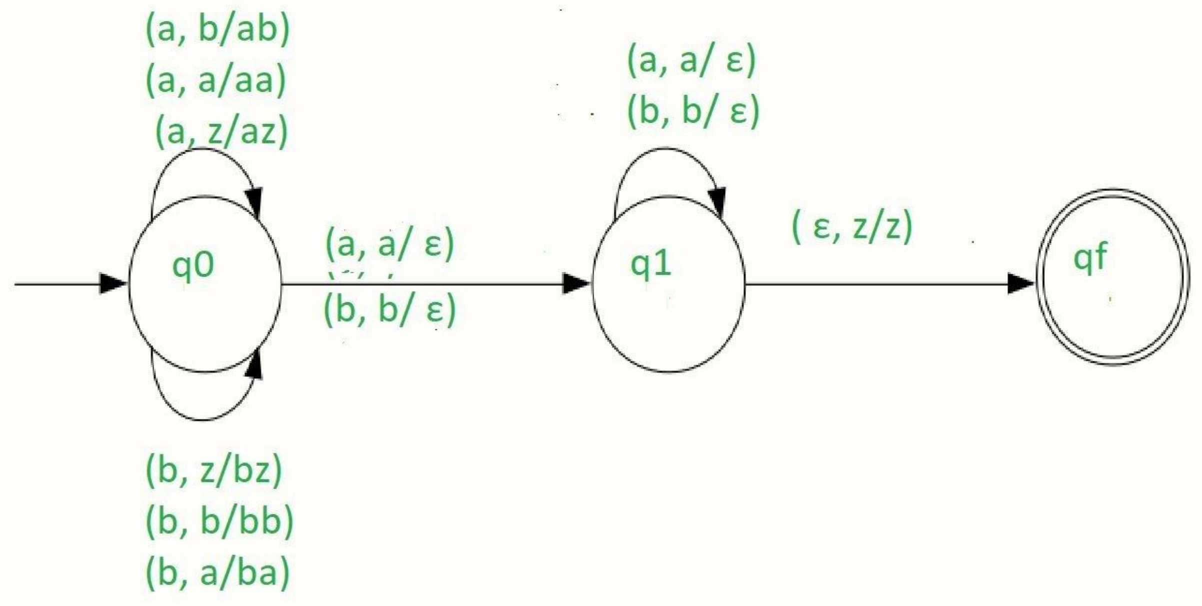


7) L = {na(w)>nb(w) | w belongs to {a,b}* }



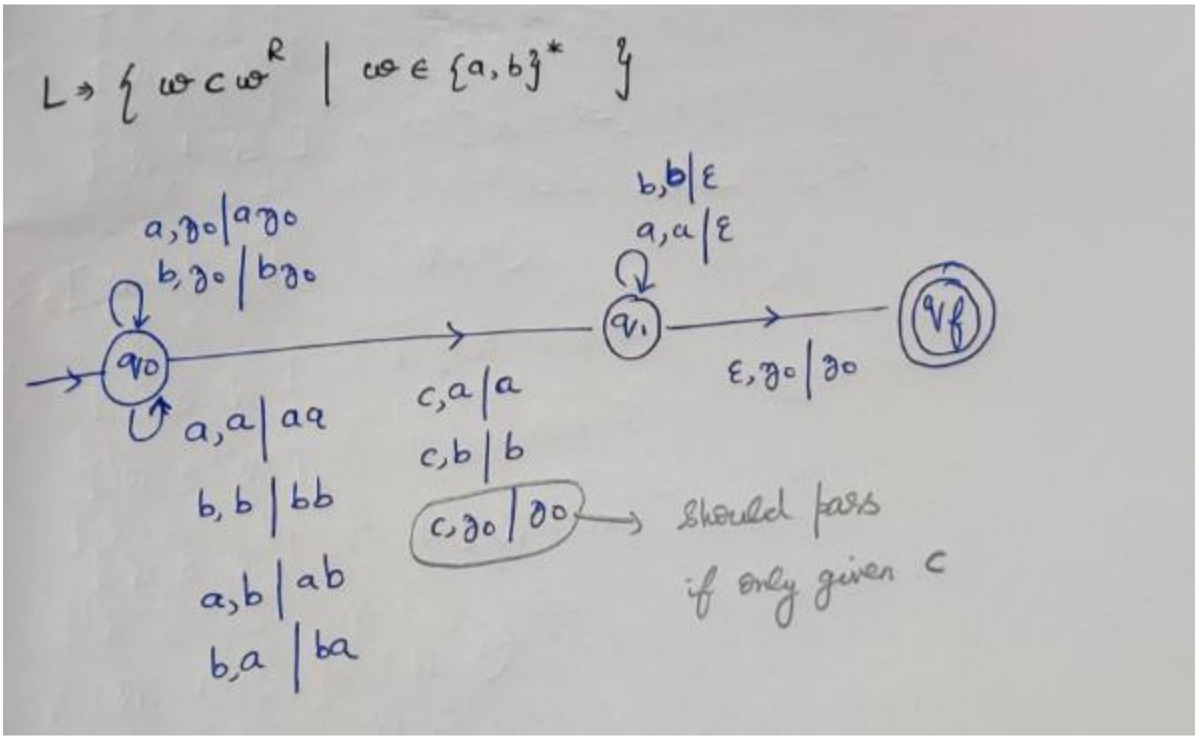
8) L = { ww^R | w belongs to {a,b}* }





Required NPDA

9) L = { wcw^R | w belongs to {a,b}* }



8) $L = \{ a^n b^n c^m \mid n,m \geq 1 \}$

9) $L = \{ a^n b^m c^n \mid n,m \geq 1 \}$

10) $L = \{ a^m b^n c^n \mid n,m \geq 1 \}$

11) $L = \{ a^n b^n c^m d^m \mid n,m \geq 1 \}$

12) $L = \{ a^n b^m c^m d^n \mid n,m \geq 1 \}$

13) $L = \{ a^{(m+n)} b^m c^n \mid m,n \geq 1 \}$

14) $L = \{ a^m b^{(m+n)} c^n \mid m,n \geq 1 \}$

L = { a^n b^n c^m d^m | n,m>=1 }

Deterministic Push Down Automata

L = { (a^n b^n | n>=1) **U (**a^n b^2n | n>=1) }
L = { ww^R | w belongs to {a,b}* }

There are context-free languages that are not accepted by any DPDA. For example, it can be shown that the languages

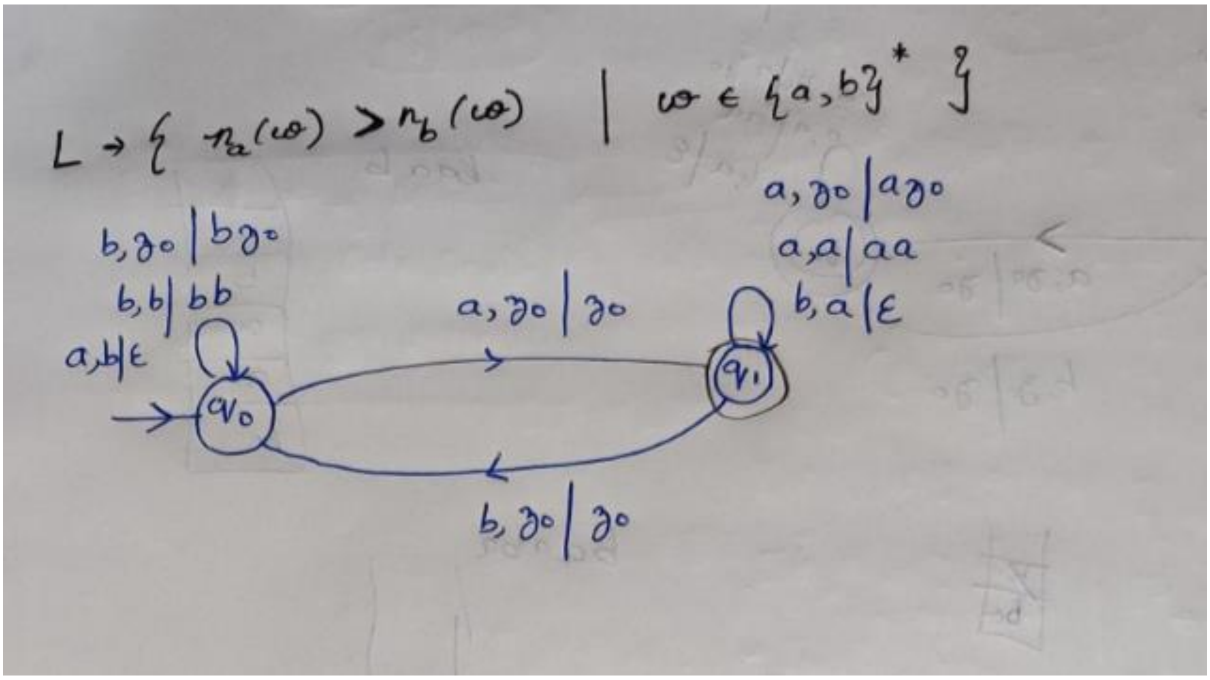
$$L_1 = \{a^n b^n \mid n \geq 1\} \cup \{a^n b^{2n} \mid n \geq 1\},$$

and

$$L_2 = \{ww^R \mid w \in \{a, b\}^*\},$$

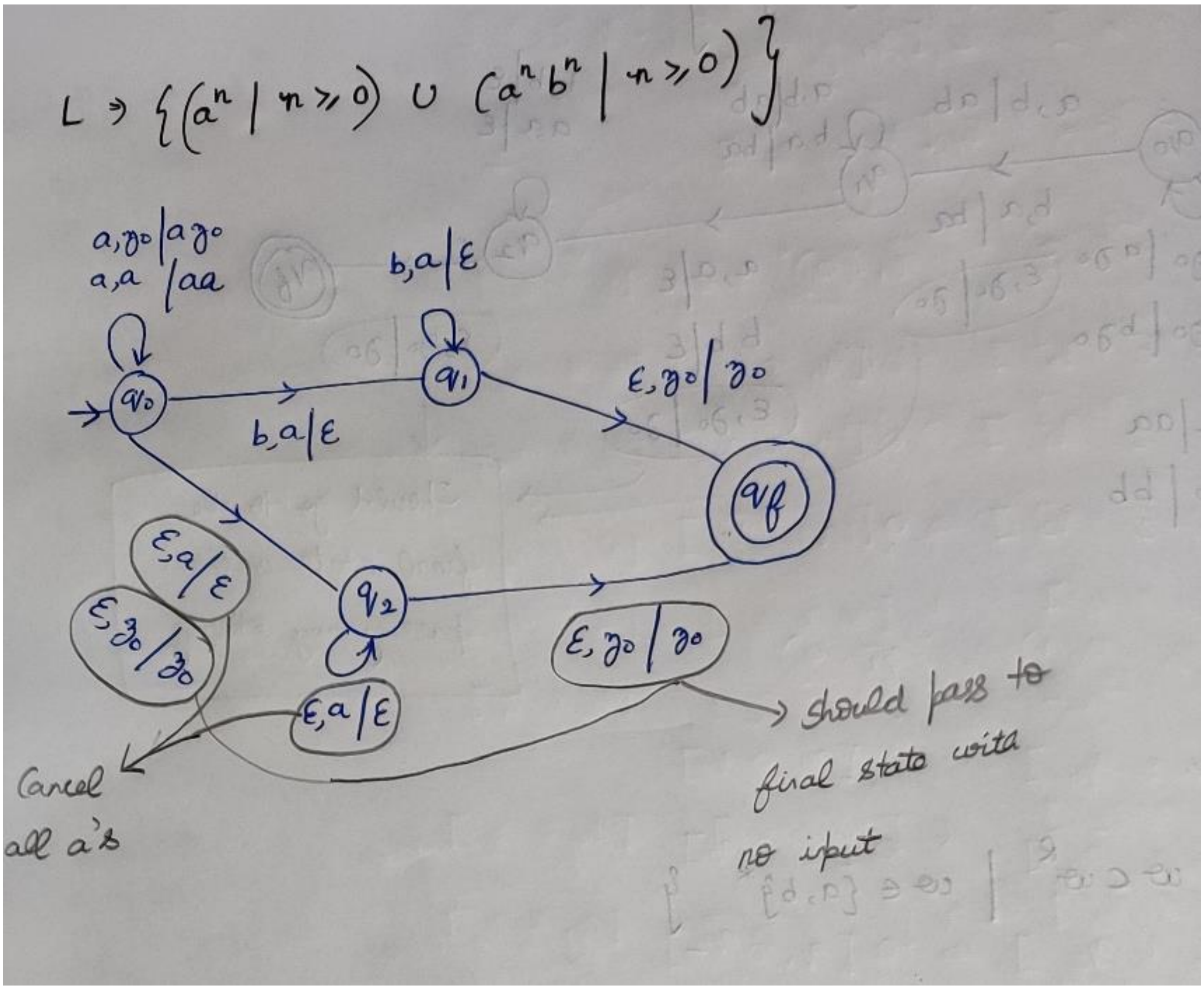
are not accepted by any DPDA.

1)L = {na(w)>nb(w) | w belongs to {a,b}* }



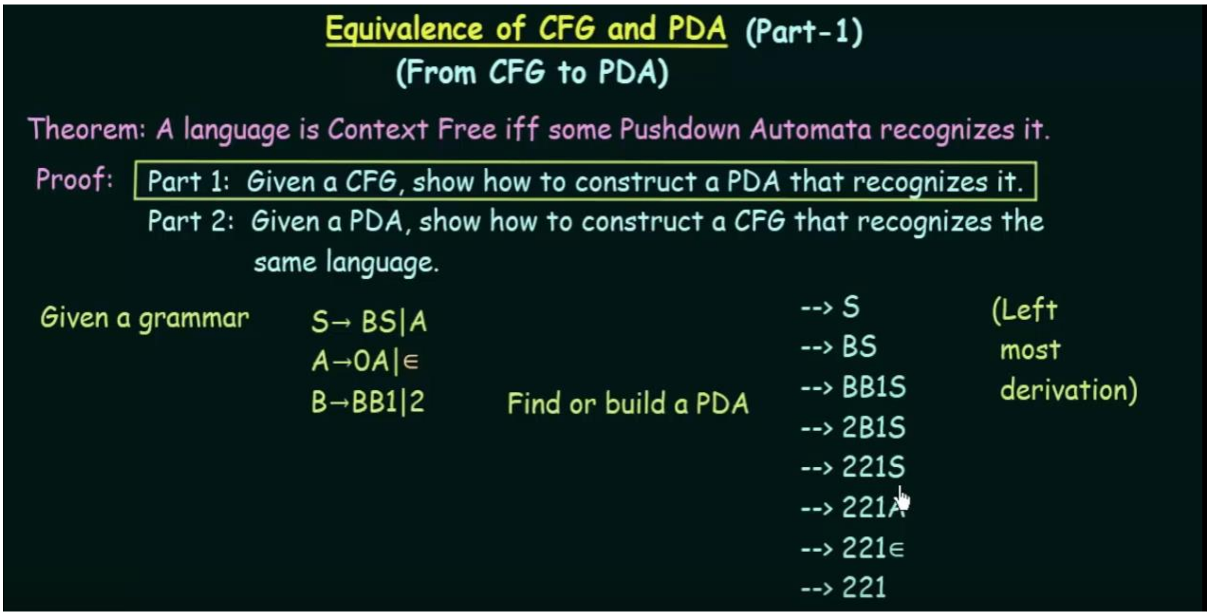
2)L = {na(w)=nb(w) | w belongs to {a,b}* }

2)L = { (a^n | n>=0) U (a^nb^n | n>=0) }



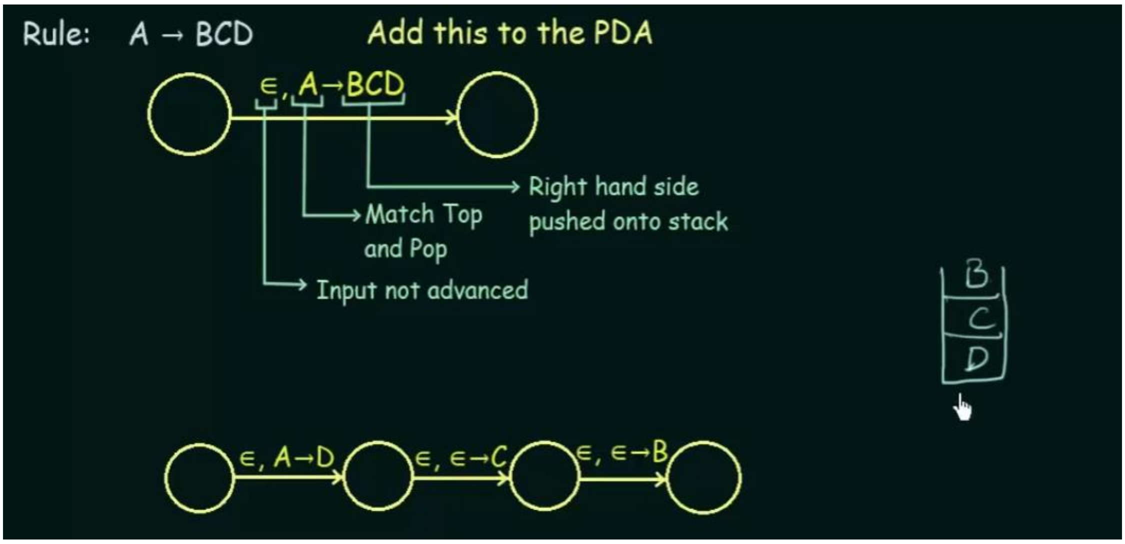
Equivalance of CFG and PDA

Context free grammar to Push Down Automata



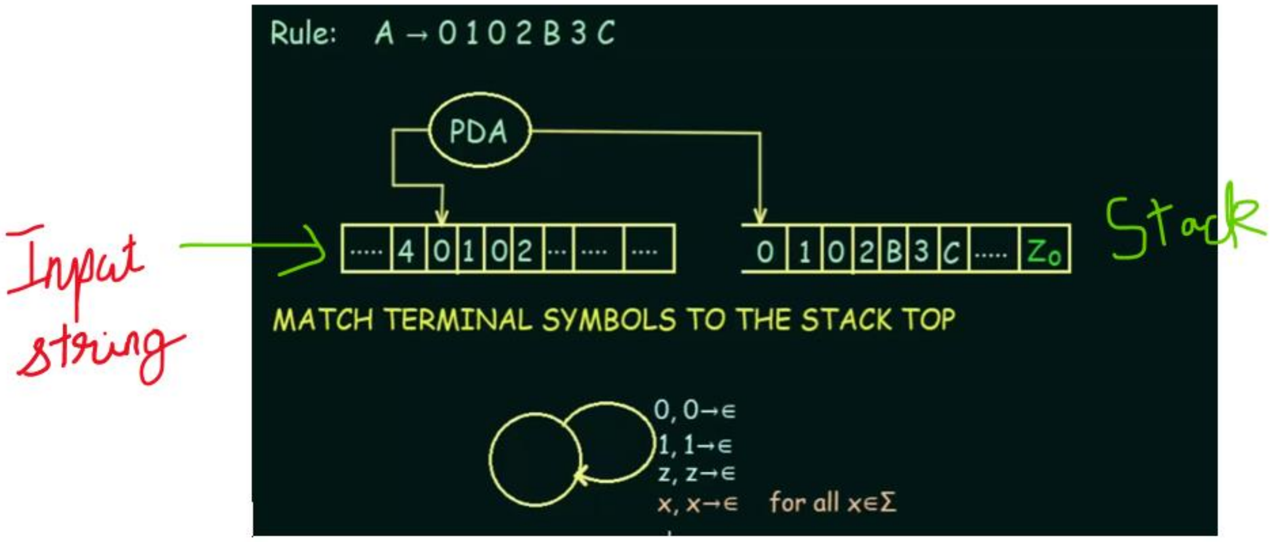
When the top of the stack meets the production rule, then push all the elements into the stack.

Non-Terminal on the top of the stack



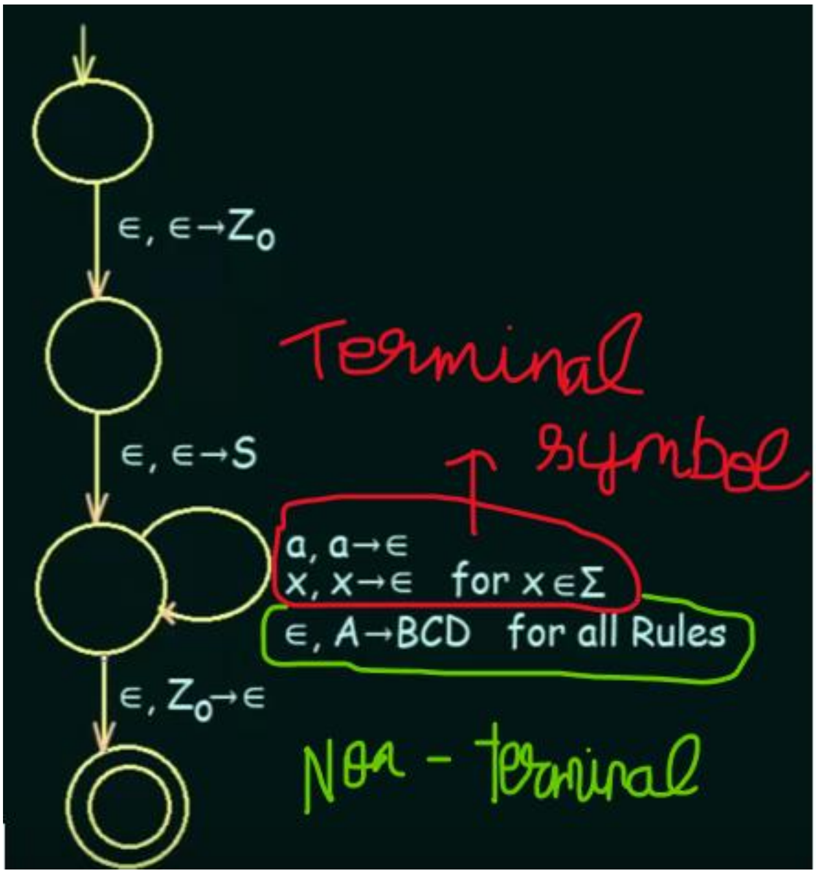
Push the elements into the stack in the reverse order.
(i.e) Rule: A→BCD, then push D,C,B. By looking the stack from the top to bottom it will look as BCD. Don't advance the input.

Terminal on the top of the stack



Assume that A was on the top of the stack, and matched with a production rule A→0102B3C. After matching we should pop that A and push the right hand-side elements into the stack.

Then we have to look whether top of the stack matches with the input symbol, if matching then cancel it (i.e pop it out) and proceed for the next input element (i.e Advancing the input symbol) and look on the top of the stack.



Question

$S \to aSA \mid a$ Input string $\Rightarrow aaabbb$

$A \to bB$

$B \to b$

$\delta(q, \varepsilon, z_0) \Rightarrow (q, S)$

$(q, \varepsilon, S) \Rightarrow (q, aSa)$

$(q, \varepsilon, S) \Rightarrow (q, a)$

$\delta(q, a, a) \Rightarrow (q, \varepsilon)$

$\delta(q, b, b) \Rightarrow (q, \varepsilon)$

$(q, \varepsilon, A) \to (q, bB)$

$(q, \varepsilon, B) \to (q, b)$

$S \to aSA$

$a\,aSA\,a$

$S \to aSA$

$\to aS\,bB$

$\to a\,aSA\,bB$

$\to aaa\,A\,bB$

$\to aaa\,bB\,bB$

$\to aaa\,bbb\,B$

Instantaneous description

$\delta(q, \varepsilon aaabbb, z_0) \Rightarrow (q, \varepsilon aaabbb, Sz_0)$

$\Rightarrow (q, aaabbb, aSAz_0)$

$\Rightarrow (q, \cancel{a}\,\varepsilon aabbb, SAz_0)$

$\Rightarrow (q, aabbb, aSAAz_0)$

$\Rightarrow (q, \varepsilon abbb, SAAz_0)$

$\Rightarrow (q, abbb, aAAz_0)$

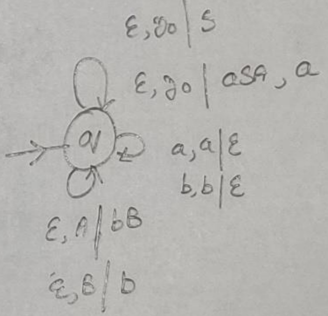
$\Rightarrow (q, \varepsilon bbb, bBAz_0)$

$\Rightarrow (q, bbb, bBAz_0)$

$\Rightarrow (q, \varepsilon bb, bAz_0)$

$\Rightarrow (q, \varepsilon b, bBz_0)$

$\Rightarrow (q, \varepsilon, Bz_0)$ This string is Not accepted.



Question

$Q \rightarrow aAA$

$A \rightarrow aQ \mid bQ \mid a$

String accepted $\}$ $aba^4 \Rightarrow abaaaa$

$\delta(q, \varepsilon, z_0) \Rightarrow (q, Q)$ start symbol

$\delta(q, \varepsilon, Q) \Rightarrow (q, aAA)$

$\delta(q, \varepsilon, A) \Rightarrow (q, aQ)$

$\delta(q, \varepsilon, A) \Rightarrow (q, bQ)$

$\delta(q, \varepsilon, A) \Rightarrow (q, a)$

For terminal symbols

$\delta(q, a, a) \Rightarrow (q, \varepsilon)$

$\delta(q, b, b) \Rightarrow (q, \varepsilon)$

$Q \Rightarrow a\underline{A}A$

$ab\underline{Q}A$

$ab\,a\underline{A}A\,A$

$abaa\underline{A}A$

$abaaa\underline{A}$

$abaaaa$

Instantaneous description

$\delta(q, \varepsilon abaaaa, z_0) \Rightarrow (q, \varepsilon abaaaa, \underline{Q}z_0)$

$\Rightarrow (q, abaaaa, aAAz_0)$

$\Rightarrow (q, \varepsilon baaaa, \underline{A}Az_0)$

$\Rightarrow (q, baaaa, bQAz_0)$

$\Rightarrow \cancel{(q, baaaa, aAAAz_0)} \Rightarrow (q, \varepsilon aaaa, QAz_0)$

$\Rightarrow (q, aaaa, aAAAz_0)$

$\Rightarrow (q, \varepsilon aaa, AAAz_0)$

$\Rightarrow (q, aaa, aAAz_0)$

$\Rightarrow (q, \varepsilon aa, AAz_0)$

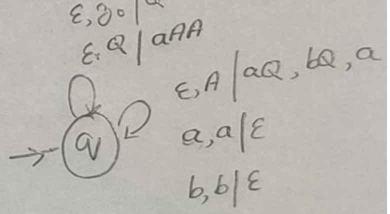
$\Rightarrow (q, aa, aAz_0)$

$\Rightarrow (q, \varepsilon a, Az_0)$

$\Rightarrow (q, a, az_0)$

$\Rightarrow (q, \varepsilon, z_0)$

$\Rightarrow (q_f, z_0)$



The string is accepted.

Question

$S \rightarrow 0S1 \mid A$

$A \rightarrow 1A0 \mid S \mid \varepsilon$

Input string } 010011

$\delta(q, \varepsilon, z_0) \Rightarrow (q, S)$

$\delta(q, \varepsilon, S) \Rightarrow (q, 0S1)$

$\delta(q, \varepsilon, S) \Rightarrow (q, A)$

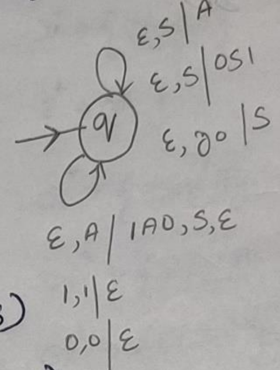
$\delta(q, \varepsilon, A) \Rightarrow (q, 1A0)$

$\delta(q, \varepsilon, A) \Rightarrow (q, S)$

$\delta(q, \varepsilon, A) \Rightarrow (q, \varepsilon)$

$\delta(q, 1, 1) \Rightarrow (q, \varepsilon)$

$\delta(q, 0, 0) \Rightarrow (q, \varepsilon)$



Instantaneous description

$\delta(q, \varepsilon 010011, z_0) \Rightarrow (q, \varepsilon 010011, S)$

$\Rightarrow (q, \cancel{0}10011, \cancel{0}S1)$

$\Rightarrow (q, \varepsilon 10011, S1)$

$\Rightarrow (q, \cancel{1}0011, \cancel{1}A01)$

$\Rightarrow (q, \varepsilon 0011, A01)$

$\Rightarrow (q, 0011, S01)$

$\Rightarrow (q, \cancel{0}011, \cancel{0}S1\ 01)$

$\Rightarrow (q, \varepsilon 011, S101)$

$\Rightarrow (q, \cancel{0}11, \cancel{0}S1\ 101)$

$\Rightarrow (q, \varepsilon 11, S101)$

$\Rightarrow (q, \cancel{1}1, \cancel{1}A0\ 1101)$

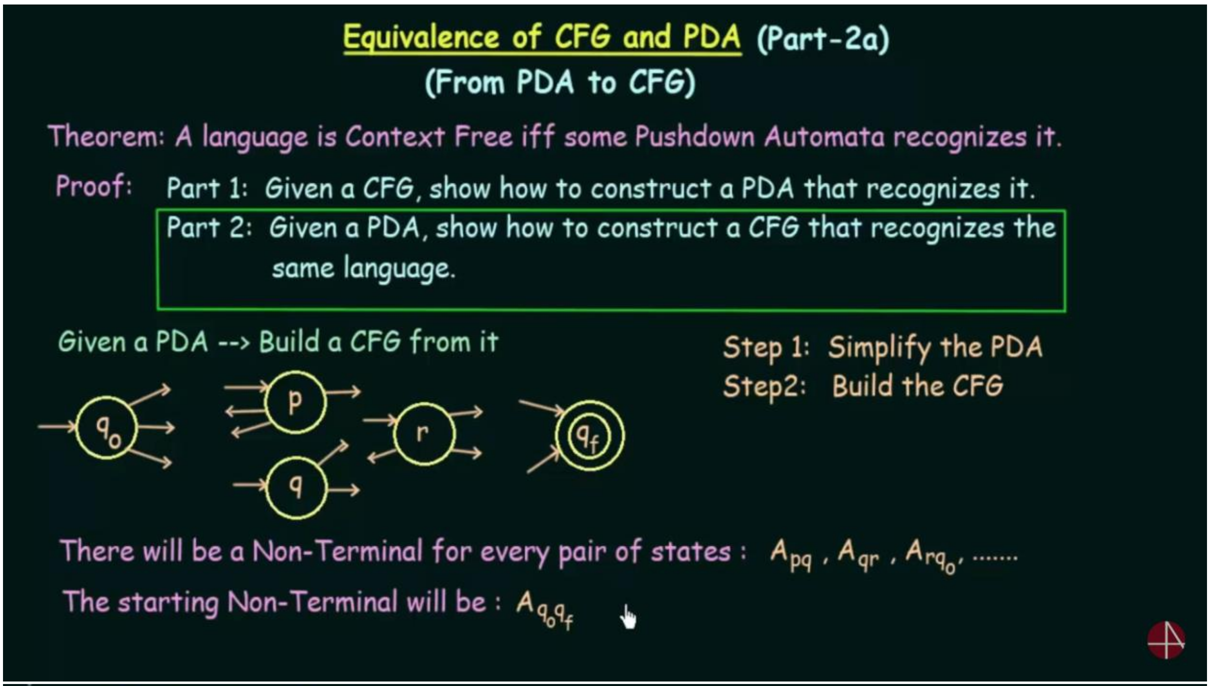
$\Rightarrow (q, \varepsilon 1, A01101)$

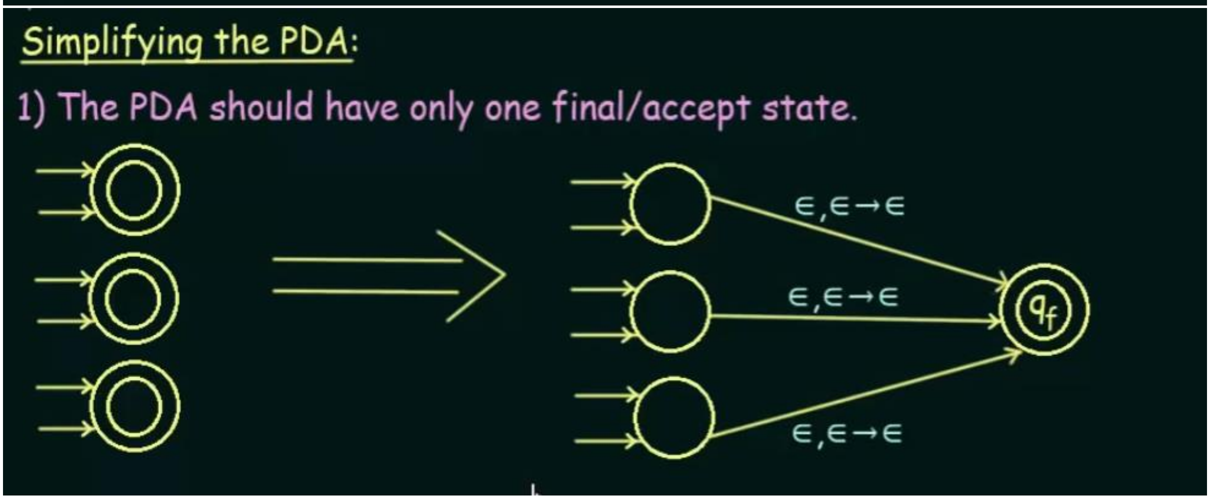
$\Rightarrow (q, \cancel{1}, \cancel{1}A0\ 01101)$

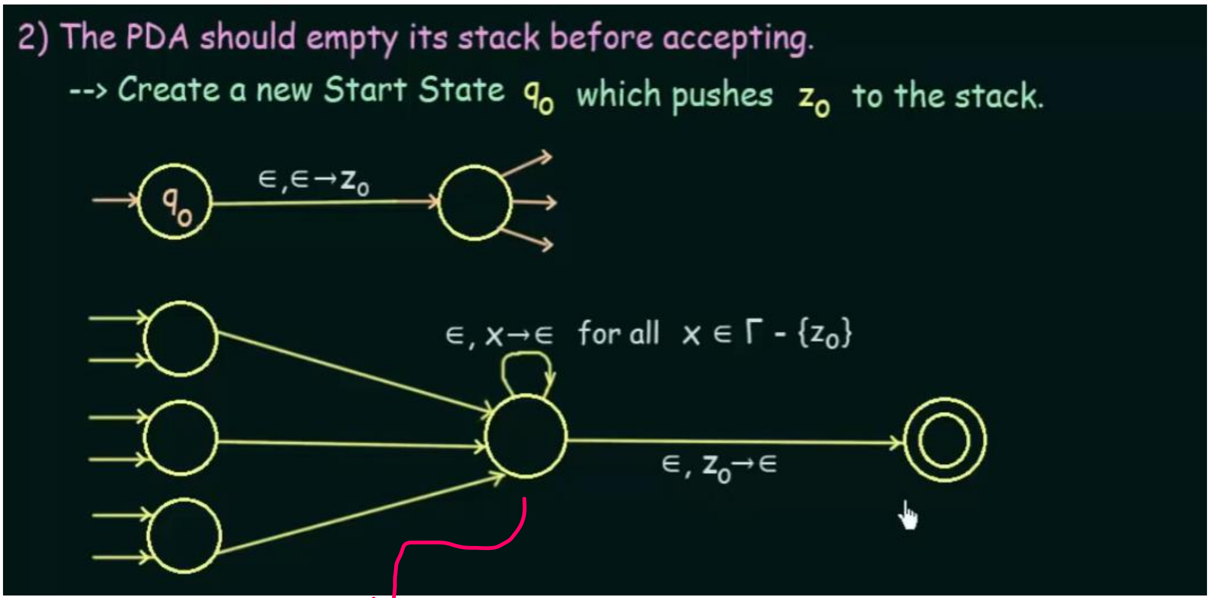
$\Rightarrow (q, \varepsilon, A001101)$

$\boxed{\text{String Not accepted}}$

Push Down Automata to Context free grammar

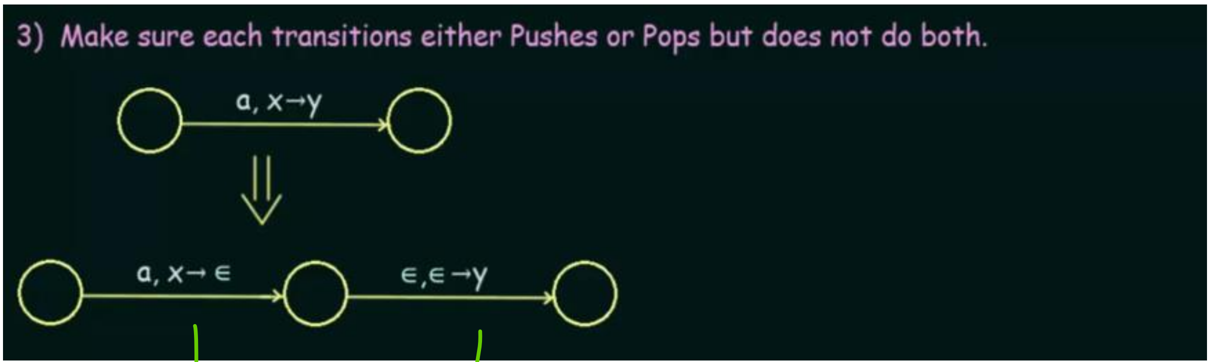




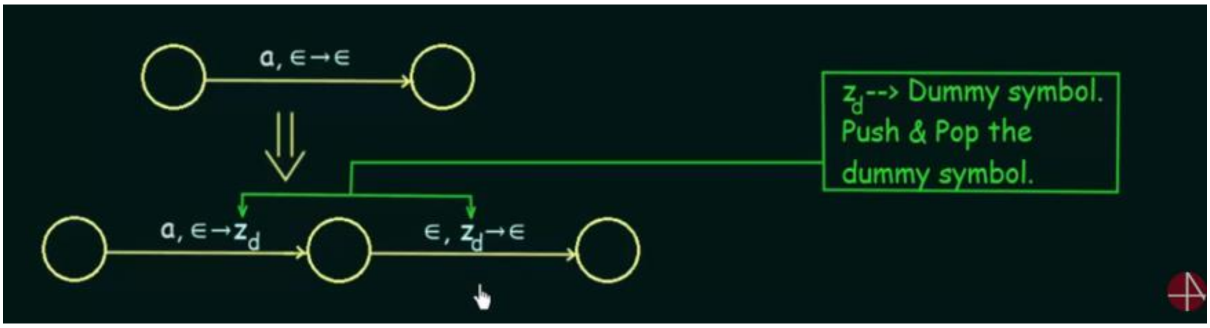


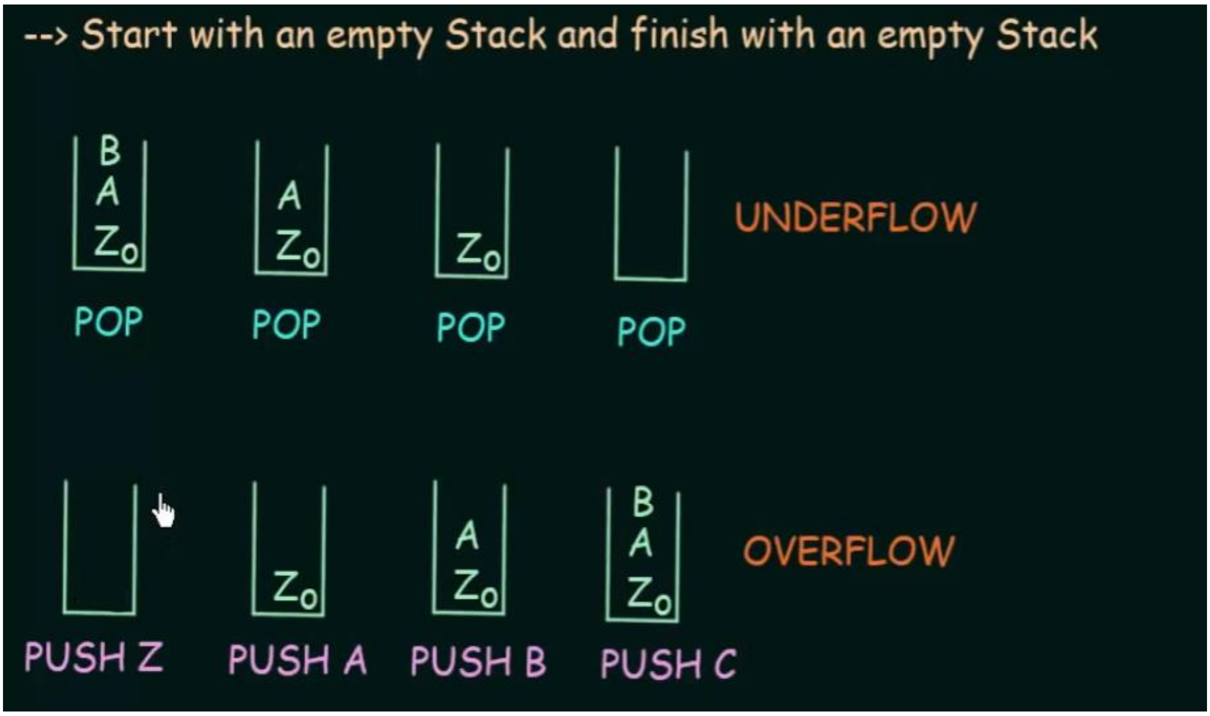
Pop all the elements from the stack except z0.

a,x|y → then a is read, x is popped out of the stack and y is pushed into the stack. This should be not there. In each transition there must be either push/pop not the both.



a is read and x is popped out y is pushed





Underflow occurs when we try to pop out the elements from an empty stack.

Overflow occurs when we try to push the elements into the stack beyond its limit. This is not related to pda, since in pda we will not fix any limit to the stack.

The productions in P are induced by move of PDA as follows:-

1) S productions are given by —

$$S \rightarrow [q_0 \; Z_0 \; q] \quad \text{for every } q \in Q.$$

2) For every popping move

$\delta(q, a, Z) = (q', \epsilon)$ induces production

$$[q \; Z \; q'] \rightarrow a$$

For each push move

$\delta(q, a, Z) = (q_1, Z_1 Z_2 \cdots Z_m)$ induces many productions

$$[q \; Z \; q'] \rightarrow a \; [q_1 \; Z_1 \; q_2][q_2 \; Z_2 \; q_3] \cdots [q_m \; Z_m \; q']$$

where each state $q', q_1, q_2 \cdots q_m$ can be any state in

1)

Generate CFG for the given PDA where M is defined as

$M \Rightarrow \{\{q_0, q_1\}; \{0,1\}; \{x, z_0\}; \delta, q_0, z_0, q_1\}$

where δ is given as follows:

1) $\delta(q_0, 1, z_0) \Rightarrow (q_0, x z_0)$
2) $\delta(q_0, 1, x) \Rightarrow (q_0, xx)$
3) $\delta(q_0, 0, x) \Rightarrow (q_0, x)$
4) $\delta(q_0, \varepsilon, x) \Rightarrow (q_1, \varepsilon)$
5) $\delta(q_1, \varepsilon, x) \Rightarrow (q_1, \varepsilon)$
6) $\delta(q_1, 0, x) \Rightarrow (q_1, xx)$
7) $\delta(q_1, 0, z_0) \Rightarrow (q_1, \varepsilon)$

$[q_0 x q_0] \rightarrow 1[q_0 x q_1][q_1 x q_0]$

$[q_0 x q_1] \rightarrow 1[q_0 x q_0][q_0 x q_1]$

$[q_0 x q_1] \rightarrow 1[q_0 x q_1][q_1 x q_1]$

iii) $\delta(q_0, 0, x) = (q_0, x)$

$[q_0 x q_0] \rightarrow 0[q_0 x q_0]$

$[q_0 x q_1] \rightarrow 0[q_0 x q_1]$

v) $\delta(q_0, \varepsilon, x) = (q_1, \varepsilon)$ (pop)

$[q_0 \; x \; q_1] \rightarrow \varepsilon$

) $\delta(q_1, \varepsilon, x) = (q_1, \varepsilon)$ (pop)

$[q_1 \; x \; q_1] \rightarrow \varepsilon$

vi) $\delta(q_1, 0, x) \Rightarrow (q_1, xx)$ (push 0 as x)

$[q_1 \; x \; q_0] \longrightarrow 0[q_1 \; x \; q_0] \; [q_0 \; x \; q_0]$

$[q_1 \; x \; q_0] \longrightarrow 0[q_1 \; x \; q_1] \; [q_1 \; x \; q_0]$

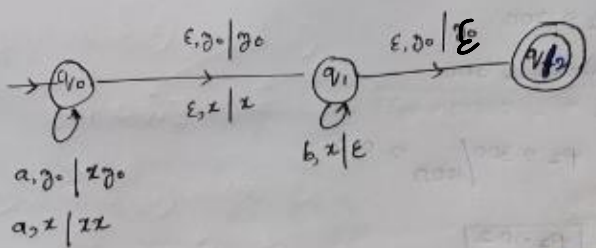
$[q_1 \; x \; q_1] \longrightarrow 0[q_1 \; x \; q_0] \; [q_0 \; x \; q_1]$

$[q_1 \; x \; q_1] \longrightarrow 0[q_1 \; x \; q_1] \; [q_1 \; x \; q_1]$

vii) $\delta(q_1, 0, z_0) \Rightarrow (q_1, \varepsilon)$ (pop)

$[q_1 \; z_0 \; q_1] \longrightarrow 0$

2)



There are totally 3 states

Solution:

Transition diagram:

$\delta(q_0, a, z_0) \Rightarrow (q_0, \underline{xz_0}) \Rightarrow 3^2 \Rightarrow 9$ (2 inputs)

$\delta(q_0, a, x) \Rightarrow (q_0, \underline{xx}) \Rightarrow 3^2 \Rightarrow 9$

$\delta(q_0, \varepsilon, z_0) \Rightarrow (q_1, \underline{z_0}) \Rightarrow 3^1 \Rightarrow 3$

$\delta(q_0, \varepsilon, x) \Rightarrow (q_1, \underline{x}) \Rightarrow 3^1 \Rightarrow 3$

$\delta(q_1, b, x) \Rightarrow (q_1, \underline{\varepsilon}) \Rightarrow 3^0 \Rightarrow 1$

$\delta(q_1, \varepsilon, z_0) \Rightarrow (q_2, \underline{\varepsilon}) \Rightarrow 3^0 \Rightarrow 1$

Starting state:

$S \rightarrow [q_0\ z_0\ q_0] \mid [q_0\ z_0\ q_1] \mid [q_0\ z_0\ q_2]$

i) $\delta(q_0, a, z_0) \Rightarrow (q_0, xz_0)$

$[q_0\ z_0\ q_0] \Rightarrow a\ [q_0\ x\ q_0]\ [q_0\ z_0\ q_0]$

$[q_0\ z_0\ q_0] \Rightarrow a\ [q_0\ x\ q_1]\ [q_1\ z_0\ q_0]$

$[q_0\ z_0\ q_0] \Rightarrow a\ [q_0\ x\ q_2]\ [q_2\ z_0\ q_0]$

$[q_0\ z_0\ q_1] \Rightarrow a\ [q_0\ x\ q_0]\ [q_0\ z_0\ q_1]$

$[q_0\ z_0\ q_1] \Rightarrow a\ [q_0\ x\ q_1]\ [q_1\ z_0\ q_1]$

$[q_0\ z_0\ q_1] \Rightarrow a\ [q_0\ x\ q_2]\ [q_2\ z_0\ q_1]$

$[q_0\ z_0\ q_2] \Rightarrow a\ [q_0\ x\ q_0]\ [q_0\ z_0\ q_2]$

$[q_0\ z_0\ q_2] \Rightarrow a\ [q_0\ x\ q_1]\ [q_1\ z_0\ q_2]$

$[q_0\ z_0\ q_2] \Rightarrow a\ [q_0\ x\ q_2]\ [q_2\ z_0\ q_2]$

ii) $\delta(q_0, a, x) \Rightarrow (q_0, xx)$

$[q_0 x q_0] \Rightarrow a\,[q_0 x q_0]\;[q_0 x q_0]$

$[q_0 x q_0] \Rightarrow a\,[q_0 x q_1]\;[q_1 x q_0]$

$[q_0 x q_0] \Rightarrow a\,[q_0 x q_2]\;[q_2 x q_0]$

$[q_0 x q_1] \Rightarrow a\,[q_0 x q_0]\;[q_0 x q_1]$

$[q_0 x q_1] \Rightarrow a\,[q_0 x q_1]\;[q_1 x q_1]$

$[q_0 x q_1] \Rightarrow a\,[q_0 x q_2]\;[q_2 x q_1]$

$[q_0 x q_2] \Rightarrow a\,[q_0 x q_0]\;[q_0 x q_2]$

$[q_0 x q_2] \Rightarrow a\,[q_0 x q_1]\;[q_1 x q_2]$

$[q_0 x q_2] \Rightarrow a\,[q_0 x q_2]\;[q_2 x q_2]$

iii) $\delta(q_0, \varepsilon, z_0) \Rightarrow (q_1, z_0)$

$[q_0 z_0 q_0] \Rightarrow \varepsilon\,[q_1 z_0 q_0]$

$[q_0 z_0 q_1] \Rightarrow \varepsilon\,[q_1 z_0 q_1]$

$[q_0 z_0 q_2] \Rightarrow \varepsilon\,[q_1 z_0 q_2]$

iv) $\delta(q_0, \varepsilon, x) \Rightarrow (q_1, x)$

$[q_0 x q_0] \Rightarrow \varepsilon\,[q_1 x q_0]$

$[q_0 x q_1] \Rightarrow \varepsilon\,[q_1 x q_1]$

$[q_0 x q_2] \Rightarrow \varepsilon\,[q_1 x q_2]$

v) $\delta(q_1, b, x) \Rightarrow (q_1, \varepsilon)$

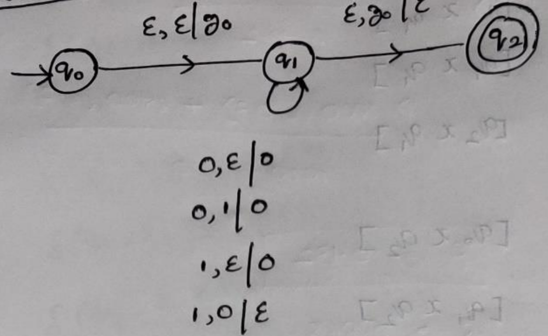
$[q_1\ x\ q_1] \Rightarrow b$

vi) $\delta(q_1, \varepsilon, z_0) \Rightarrow (q_2, \varepsilon)$

$[q_1\ z_0\ q_2] \Rightarrow \varepsilon$

3)

Question



Solution

Transition function,

$\delta(q_0, \varepsilon, \varepsilon) \rightarrow (q_1, \underline{z_0})$ 3'

$\delta(q_1, 0, \varepsilon) \Rightarrow (q_1, \underline{0})$ 3'

$\delta(q_1, 0, 1) \Rightarrow (q_1, \underline{0})$ 3'

$\delta(q_1, 1, \varepsilon) \Rightarrow (q_1, \underline{0})$ 3'

$\delta(q_1, 1, 0) \Rightarrow (q_1, \varepsilon)$ 3°

$\delta(q_1, \varepsilon, z_0) \Rightarrow (q_2, \varepsilon)$ 3°

Start state

$S \rightarrow [q_0\ z_0\ q_0] \mid [q_0\ z_0\ q_1] \mid [q_0\ z_0\ q_2]$

i) $\delta(q_0, \varepsilon, \varepsilon) \Rightarrow (q_1, z_0)$

$[q_0 \varepsilon q_0] \Rightarrow \varepsilon [q_1 z_0 q_0]$

$[q_0 \varepsilon q_1] \Rightarrow \varepsilon [q_1 z_0 q_1]$

$[q_0 \varepsilon q_2] \Rightarrow \varepsilon [q_1 z_0 q_2]$

ii) $\delta(q_1, 0, \varepsilon) \Rightarrow (q_1, 0)$

$[q_1 \varepsilon q_0] \Rightarrow 0 [q_1 0 q_0]$

$[q_1 \varepsilon q_1] \Rightarrow 0 [q_1 0 q_1]$

$[q_1 \varepsilon q_2] \Rightarrow 0 [q_1 0 q_2]$

iii) $\delta(q_1, 0, 1) \Rightarrow (q_1, 0)$

$[q_1 1 q_0] \Rightarrow 0 [q_1 0 q_0]$

$[q_1 1 q_1] \Rightarrow 0 [q_1 0 q_1]$

$[q_1 1 q_2] \Rightarrow 0 [q_1 0 q_2]$

iv) $\delta[q_1, 1, \varepsilon] \Rightarrow (q_1, 0)$

$[q_1 \varepsilon q_0] \Rightarrow 1 [q_1 0 q_0]$

$[q_1 \varepsilon q_1] \Rightarrow 1 [q_1 0 q_1]$

$[q_1 \varepsilon q_2] \Rightarrow 1 [q_1 0 q_2]$

v) $\delta(q_1, 1, 0) \Rightarrow (q_1, \varepsilon)$

$[q_1 0 q_1] = 1$

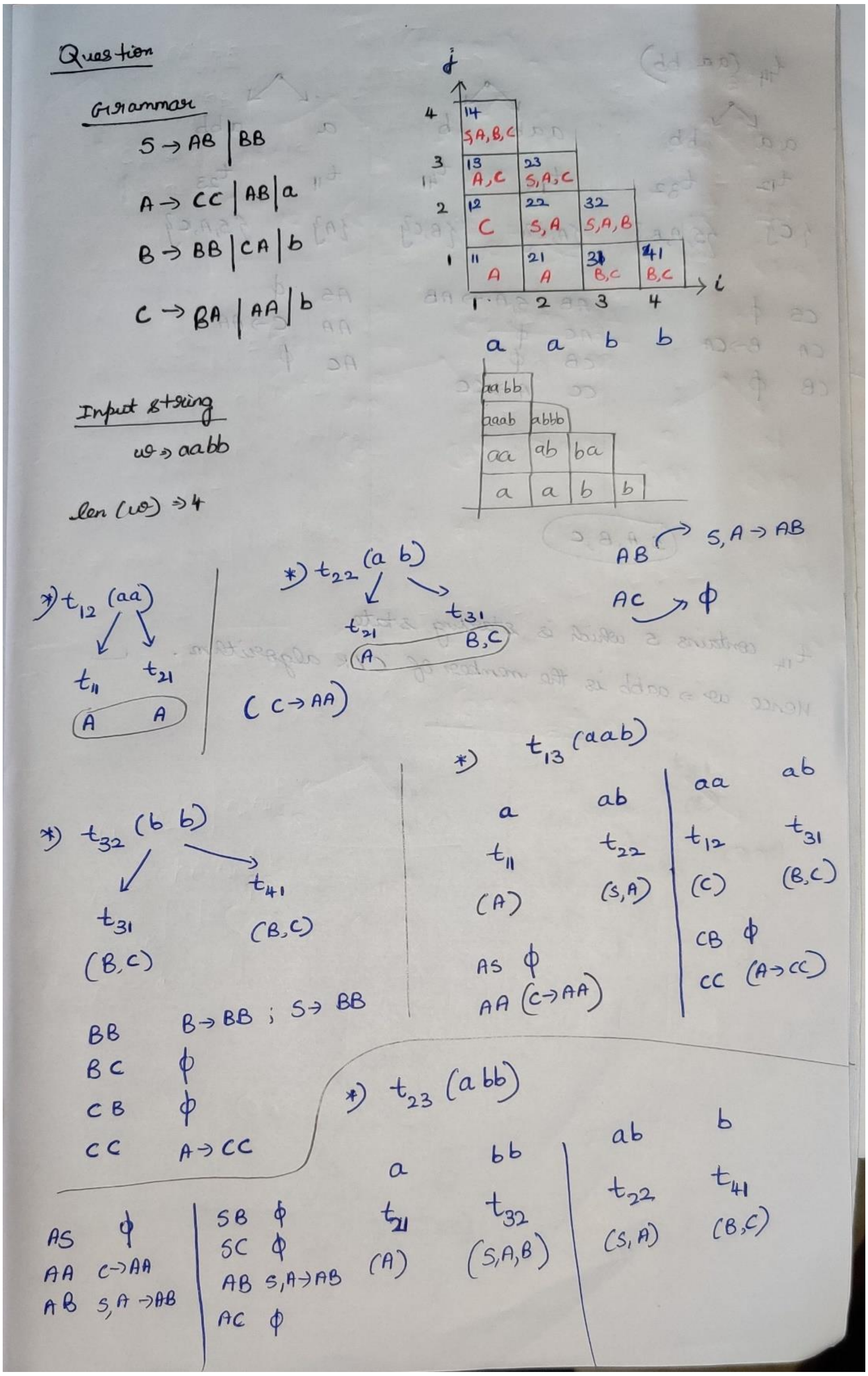
vi) $\delta(q_1, \varepsilon, z_0) \Rightarrow (q_2, \varepsilon)$

$[q_1 z_0 q_2] \Rightarrow \varepsilon$

Cocke–Younger–Kasami algorithm

- It is specific for Context free grammar.
- The grammar should be in Chomsky Normal Form, if not convert it to Chomsky Normal Form.

Question



t_{14} (aa bb)

aa bb

t_{12} t_{32}

$\{C\}$ $\{S, A, B\}$

CS ϕ

CA $B \to CA$

CB ϕ

aab b

t_{13} t_{41}

$\{A, C\}$ $\{B, C\}$

AB $S, A \to AB$

AC ϕ

CB ϕ

CC $A \to CC$

a abb

t_{11} t_{23}

$\{A\}$ $\{S, A, C\}$

AS ϕ

AA $C \to AA$

AC ϕ

$\{S, A, B, C\}$

t_{14} contains S which is starting state

Hence $w \Rightarrow$ aabb is the member of CYK algoritham.

Question

Question

$S \to AB \mid BC$

$A \to BA \mid a$

$B \to CC \mid b$

$C \to AB \mid a$

Input string

$w \Rightarrow baaba$

$|w| \Rightarrow 5$

baaba				
baab	aaba			
baa	aab	aba		
ba	aa	ab	ba	
b	a	a	b	a

j \ i	1	2	3	4	5
5	15 SCA				
4	14 ϕ	24 SAC			
3	13 ϕ	23 B	33 B		
2	12 A,S	22 B	32 S,C	42 A,S	
1	11 B	21 A,C	31 A,C	41 B	51 A,C
	b	a	a	b	a

t_{12} (ba)

t_{11} (B) $\quad t_{21}$ (A,C)

BA ($A \to BA$)

BC ($S \to BC$)

↳ (A,S)

t_{22} (aa)

t_{21} (A,C) $\quad t_{31}$ (A,C)

AA ϕ

AC ϕ

CA ϕ

CC ($B \to CC$)

t_{32} (ab)

t_{31} (A,C) $\quad t_{41}$ (B)

AB $\quad S \to AB$, $C \to AB$

CB $\quad \phi$

t_{42} (ba)

t_{41} (B) $\quad t_{51}$ (A,C)

BA ($A \to BA$)

BC ($S \to BC$)

t_{13} (baa)

t_{12} (A,S) $\quad t_{22}$ (B)

AB ($S, C \to AB$)

SB (ϕ)

t_{13} (baa)

ba a   b aa

t_{42} t_{51} t_{41}

———— x ———— x ————

t_{13} (baa)

ba a   b aa

(A,S) (A,C)   (B) (B)

AA
AC
SA
SC

$\} \phi$

BB ϕ

———— x ———— x ————

t_{23} (aab)

aa b   a ab

t_{22} t_{41} t_{21} t_{32}

(B) (B)   (A,C) (S,C)

BB $\Rightarrow \phi$

AS $\Rightarrow \phi$
AC $\Rightarrow \phi$
CS $\Rightarrow \phi$
CC $\Rightarrow$ ($B \rightarrow CC$)

———— x ———— x ————

t_{33} (aba)

ab a
t_{32} (S, C) t_{31} (A, C)

SA ⇒ ϕ
SC ⇒ ϕ
CA ⇒ ϕ
CC ⇒ B → CC

a ba
t_{31} (A, C) t_{42} (A, S)

AA ⇒ ϕ
AS ⇒ ϕ
CA ⇒ ϕ
CS ⇒ ϕ

—— × —— × ——

t_{14} (baab)

ba ab
t_{12} (A, S) t_{32} (S, C)

AS ⇒ ϕ
AC ⇒ ϕ
SS ⇒ ϕ
SC ⇒ ϕ

b aab
t_{11} (B) t_{23} (B)

BB ⇒ ϕ

baa b
t_{14} ϕ t_{11} (B)

ϕ

—— × —— × ——

t_{24} (aaba)

aa ba
t_{22} (B) t_{42} (A, S)

BA ⇒ (A → BA)
BS ⇒ ϕ

aab a
t_{23} (B) t_{21} (A, C)

BA ⇒ (A → BA)
BC ⇒ (S → BC)

a aba
t_{21} (A, C) t_{33} (B)

AB (S → AB) (C → AB)
CB ϕ

{S, A, C}

t_{15} (baaba)

baa | ba

$t_{13}(\phi)$ $t_{12}(A,S)$

ϕ

ba | aba

$t_{12}(A,S)$ $t_{33}(B)$

AB (S,C → AB)

SB (ϕ)

{S, A, C}

b | aaba

$t_{11}(B)$ $t_{24}(SAC)$

BS (ϕ)

BA (A → BA)

BC (S → BC)

baab | a

ϕ $t_{2?}$ (A, S

ϕ