EEE1024: Fundamentals of Electrical and Electronics Engineering

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RECAP

CONVERSIONS

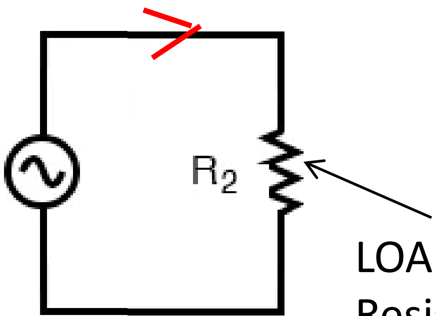
Sinusoidal form $\longleftrightarrow$ Phasors $\quad v(t) = V_m \cos(\omega t \pm \theta^o) \longleftrightarrow V = V\angle\theta^o$

Phasors $\longleftrightarrow$ Complex numbers (rectangular form) $\quad V = V\angle\theta^o \longleftrightarrow z = x + iy$

Phasors $\longleftrightarrow$ Sinusoidal form $\quad V = V\angle\theta^o \longleftrightarrow V_m \cos(\omega t \pm \theta^o)$

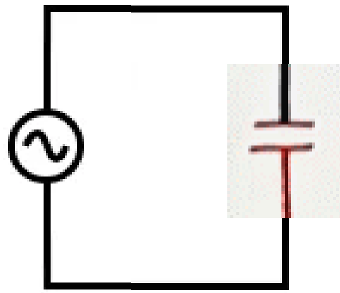
Resistance *Reactance* *Impedance*

Units for all 3 – same – Ohms (Ω)

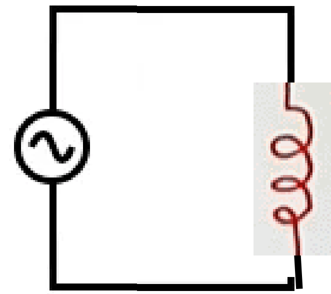


LOAD - Resistor

$$R = \frac{V}{I}$$



LOAD - pure Capacitor

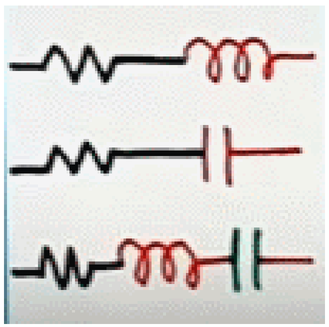


LOAD - pure Inductor

REACTANCE - X (Ω)

$$X = \frac{V}{I}$$

If LOAD -



R-L load

R-C load

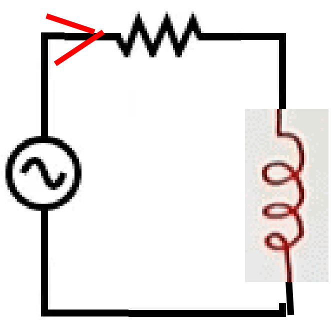
R-L-C load

IMPEDANCE – Z (Ω)

$$Z = \frac{V}{I}$$

COMPLEX IMPEDANCES – *INDUCTANCE(1)*

Inductive circuit



$$i_L(t) = I_m \sin(\omega t + \theta)$$ .... Sinusoidal current flowing through inductor

$$v_L(t) = L\frac{di_L(t)}{dt}$$ .... Voltage across inductor

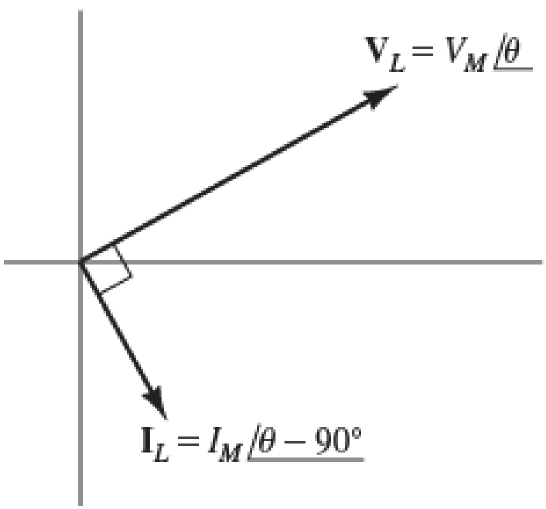
$$v(t) = L\frac{di}{dt}$$

$$v_L(t) = \omega L I_m \cos(\omega t + \theta)$$

Phasor for current - $\mathbf{I}_L = I_m \underline{/\theta - 90^\circ}$

Phasor for voltage - $\mathbf{V}_L = \omega L I_m \underline{/\theta}$

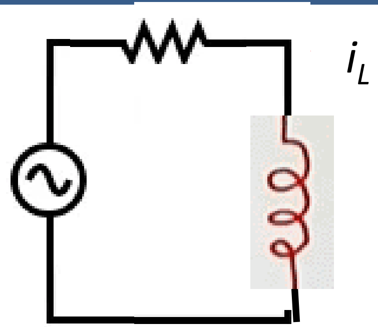
$$= V_m \underline{/\theta}$$

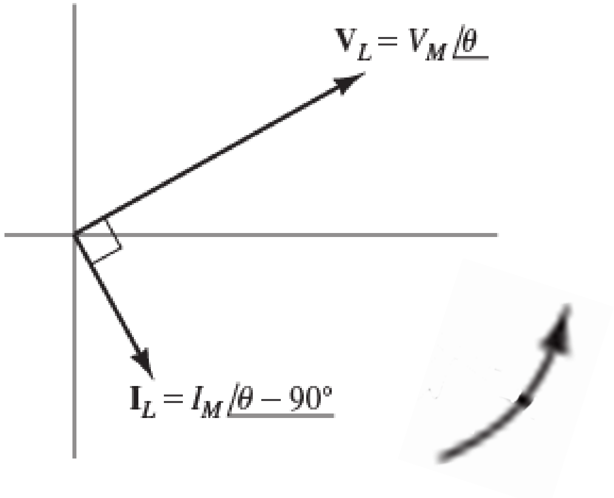


Current lags Voltage by 90^o

COMPLEX IMPEDANCES – *INDUCTANCE(2)*

Inductive circuit





$$\mathbf{V}_L = \omega L I_m \angle\theta$$

$$= \omega L I_m \angle(\theta^o + 90^o - 90^0)$$

$$\mathbf{V}_L = (\omega L \angle 90°) \times I_m \angle\theta - 90°$$

$$\mathbf{I}_L = I_m \angle\theta - 90°$$

$$\mathbf{V}_L = (\omega L \angle 90°) \times \mathbf{I}_L$$

$$\mathbf{V}_L = j\omega L \times \mathbf{I}_L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad j\omega L = \omega L \angle 90°$$

- Impedance of the inductance

$$Z_L$$

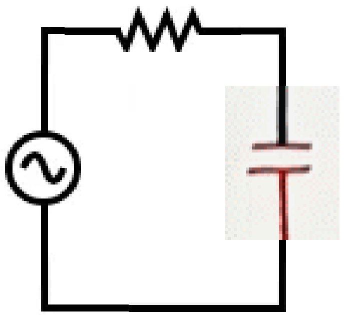
$$\boxed{Z_L = j\omega L = \omega L \angle 90°}$$

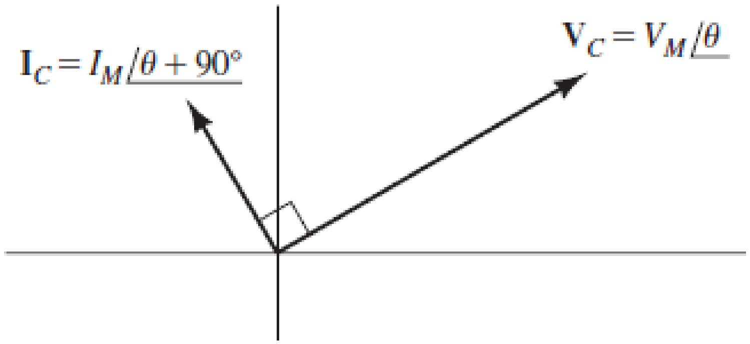
$$\boxed{\mathbf{V}_L = Z_L \mathbf{I}_L}$$

COMPLEXIMPEDANCES – *CAPACITANCE*

Capacitive circuit

$\mathbf{V}_C = \mathbf{Z}_C\mathbf{I}_C$ .... Like, $\mathbf{V}_L = \mathbf{Z}_L\mathbf{I}_L$





$$Z_C = -j\frac{1}{\omega C}$$
$$= \frac{1}{j\omega C}$$
$$= \frac{1}{\omega C}\angle -90°$$

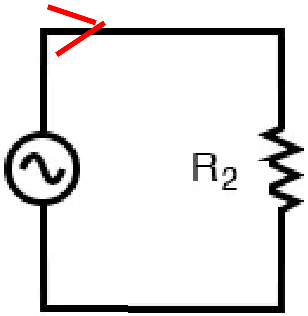
Current LEADS Voltage by 90°

If a phasor voltage Vc is $\mathbf{V}_C = V_m \angle \theta$

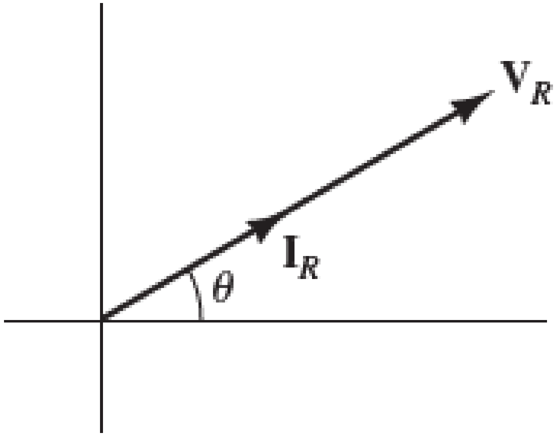
$$\mathbf{I}_C = \frac{\mathbf{V}_C}{Z_C} = I_m \angle \theta + 90°$$

COMPLEX IMPEDANCES – *RESISTANCE*

Resistive circuit



$$\mathbf{V}_R = R\mathbf{I}_R$$



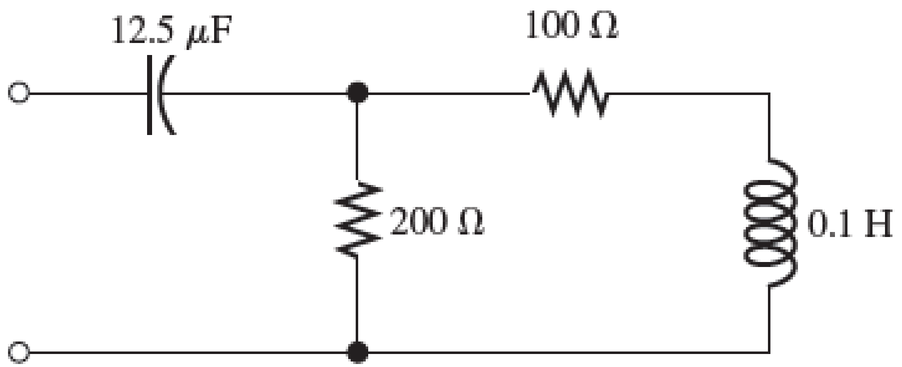
Voltage and Current phasors are ***- INPHASE***

COMBINING IMPEDANCES IN SERIES & PARALLEL(1)

IMPEDANCES of Capacitances/Inductances → Combined as → *Resistances*

Example –
Determine the complex impedance between terminals shpwn in the figure. ω = 1000 rad/s

Step 1: Calculate *impedances*



$$Z_c = \frac{-j}{\omega C}$$

$$Z_c = \frac{-j}{1000 \times 12.5 \times 10^{-6}}$$

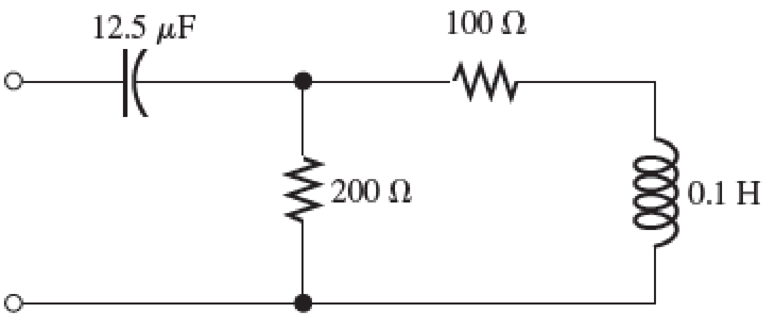
$$Z_c = \frac{-1}{1000 \times 12.5 \times 10^{-6}}$$

$$= -80j$$

$$Z_L = j\omega L$$

$$Z_L = j \times 1000 \times 0.1 = 100j$$

COMBINING IMPEDANCES IN SERIES & PARALLEL(2)



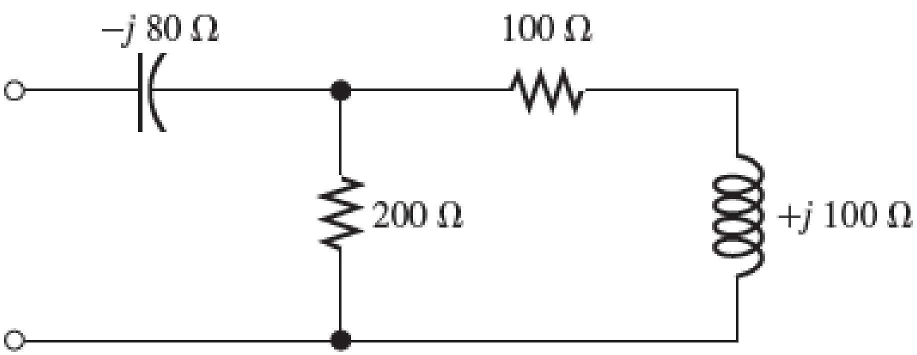
Step 1: Calculate *impedances- DONE*

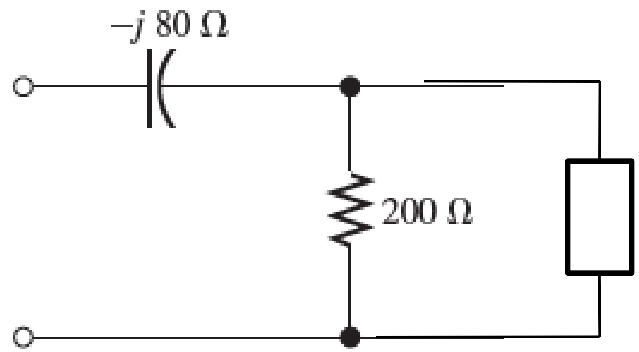
$$Z_c = -80\,j\ \Omega$$

$$Z_L = 100\,j\ \Omega$$

Step 2:
Find equivalent resistance in the "ALL RESISTANCE -like" Circuit

→ 100 Ω and 100j Ω - in series



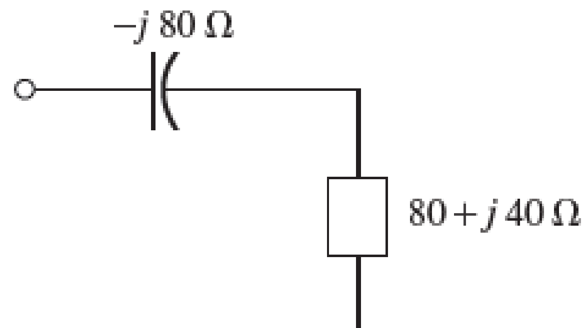


100+100j

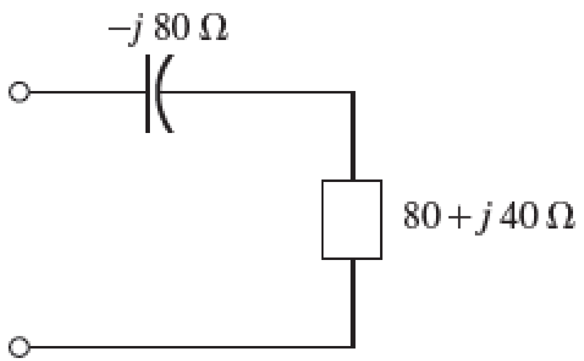
→ 200 Ω and 100+100j Ω - in parallel

$$= \frac{1}{\left(\frac{1}{200} + \frac{1}{100+100\,j}\right)}$$

=1/(1/200+1/(100+100i)) =80+40i



COMBINING IMPEDANCES IN SERIES & PARALLEL(3)



$$-j80 + 80 + j40 \quad = 80 - j40$$ ← ANS

Example 2: A voltage $v_L(t) = 100\cos(200t)$ is applied to a 0.25H inductance. Notice that ω=200 rad/s.

a) Find impedance of inductance, phasor current and phasor voltage (of inductor)
b) Draw phasor diagram

ASSIGNMENT

Example 3: A voltage $v_C(t) = 100\cos(200t)$ is applied to a 100µF capacitance.

a) Find impedance of capacitance, phasor current and phasor voltage (of capacitor)
b) Draw phasor diagram

CIRCUIT ANALYSIS with PHASORS and COMPLEX IMPEDANCES

KIRCHOFF'S LAWS

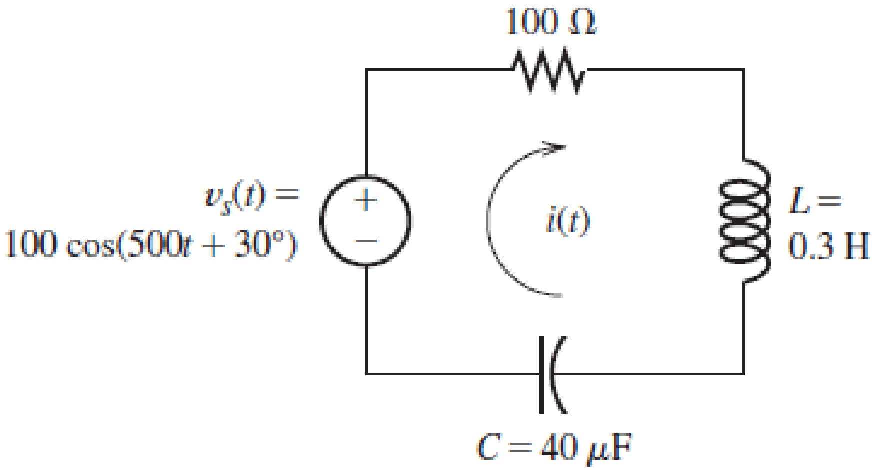
NODE VOLTAGE ANALYSIS

MESH CURRENT ANALYSIS

Steady state AC analysis of an AC circuit

Example:

a) Find the steady state current in the given circuit
b) Find the voltage across each element in the circuit and construct a phasor diagram



Step 1: Calculate *impedances*

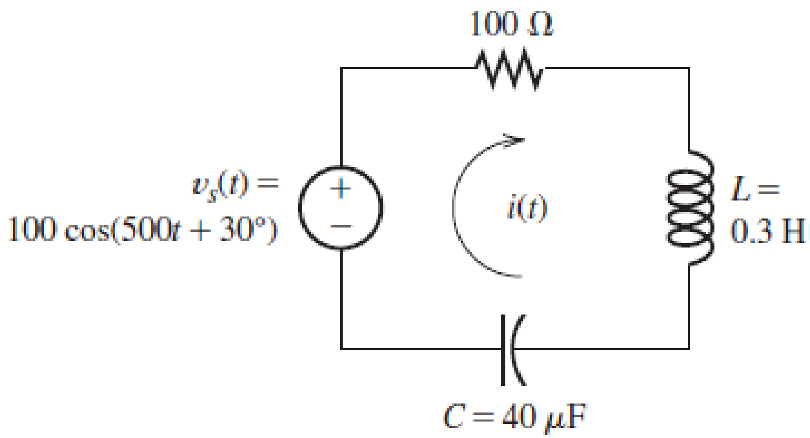
$$Z_L = j\omega L = j500 \times 0.3 = j150\ \Omega$$

$$Z_C = -j\frac{1}{\omega C} = -j\frac{1}{500 \times 40 \times 10^{-6}} = -j50\ \Omega$$

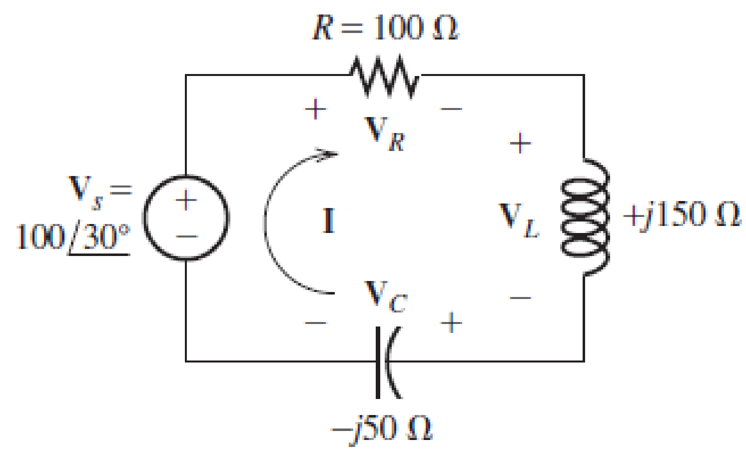
Step 2: Replace v(t) by phasors

$$\mathbf{V}_s = 100\ \angle 30°$$

STEADY STATE ANALYSIS of an AC Circuit



Step 2: Replace v(t) and i(t) by phasors



Step 3a: Use KVL

$$-V_s + 100I + (150i)I + (-50i)I = 0$$

$$V_s = (100 + 150i - 50i)I$$

$$V_s = (100 + 100i)I$$

$$I = \frac{V_s}{100 + 100i} \quad = \frac{100\angle 30}{100 + 100i}$$

$$100\angle 30 = 100(\cos(30) + i\sin(30))$$
$$= 100(0.866 + i(0.5))$$
$$= 86 + 50i$$

$$I = \frac{86 + 50i}{100 + 100i}$$

$$I = \frac{(86 + 50i)}{(100 + 100i)} \times \frac{(100 - 100i)}{(100 - 100i)} \quad = 0.68 - 0.18i$$

STEADY STATE ANALYSIS of an AC Circuit

Step 3b: Use the properties of a series circuit

Series circuit – Current remains same!

$$Z_{eq} = R + Z_L + Z_C$$

$$Z = \frac{V}{I}$$

$$Z_{eq} = 100 + j150 - j50 = 100 + j100$$

$$Z_{eq} = 141.4 \angle 45^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{Z} = \frac{100 \angle 30^\circ}{141.4 \angle 45^\circ} = 0.707 \angle -15^\circ$$

$$i(t) = 0.707 \cos(500t - 15^\circ)$$

Step 4: Calculate voltages across each element

$$\mathbf{V}_R = R \times \mathbf{I} \qquad = 100 \times 0.707 \angle -15^\circ = 70.7 \angle -15^\circ$$

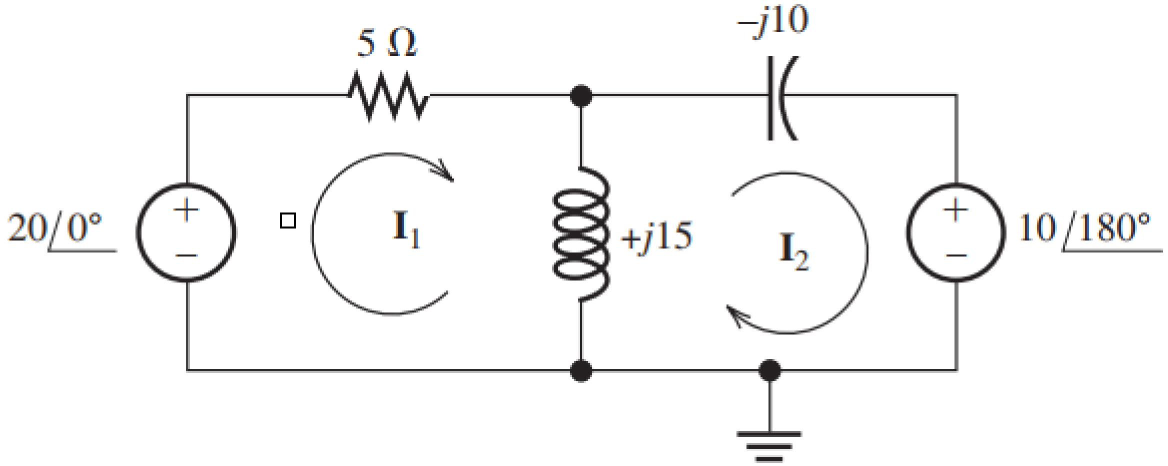
$$\mathbf{V}_L = j\omega L \times \mathbf{I} \qquad = \omega L \angle 90^\circ \times \mathbf{I} = 150 \angle 90^\circ \times 0.707 \angle -15^\circ = 106.1 \angle 75^\circ$$

$$\mathbf{V}_C = -j\frac{1}{\omega C} \times \mathbf{I} \qquad = \frac{1}{\omega C} \angle -90^\circ \times \mathbf{I} = 50 \angle -90^\circ \times 0.707 \angle -15^\circ$$

$$= 35.4 \angle -105^\circ$$

CIRCUIT ANALYSIS with PHASORS and COMPLEX IMPEDANCES

MESH CURRENT ANALYSIS



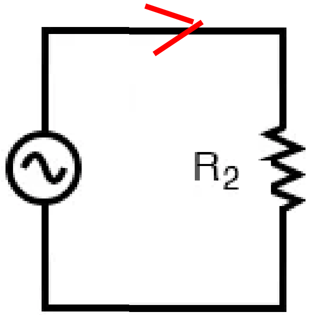
@Loop1 $$-20\angle 0 + I_1(5) + (I_1 - I_2)15j = 0$$

@Loop2 $$10\angle 180 + (I_2 - I_1)15j + I_2(-10j) = 0$$

A C Power Calculations

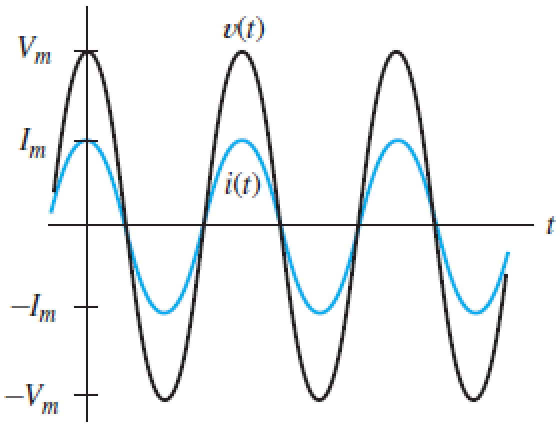
Current, Voltage and POWER

RESISTIVE Load

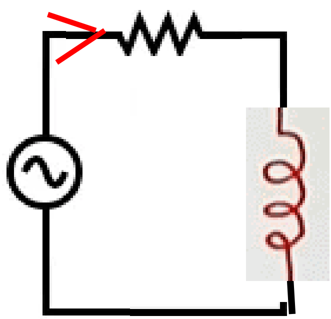


$$v(t) = V_m \cos(\omega t) \quad i(t) = I_m \cos(\omega t)$$

$$p(t) = v(t)i(t) = V_m I_m \cos^2(\omega t)$$



INDUCTIVE Load



$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t - 90°) = I_m \sin(\omega t)$$

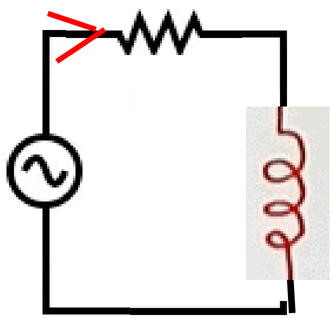
$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t) \sin(\omega t)$$

Using the trigonometric identity $\cos(x)\sin(x) = (1/2)\sin(2x)$

$$p(t) = \frac{V_m I_m}{2} \sin(2\omega t)$$

Current, Voltage and POWER

CAPACITIVE
Load



$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t + 90°) \;=\; -I_m \sin(\omega t)$$

$$p(t) = v(t)i(t) = -V_m I_m \cos(\omega t)\sin(\omega t)$$

$$= -\frac{V_m I_m}{2}\sin(2\omega t)$$

Using the trigonometric identity $\cos(x)\sin(x) = (1/2)\sin(2x)$

POWER calculations for a general RLC Load (1)

RLC load where phase θ can be any value from $-90°$ to $+90°$

$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t - \theta)$$

$$p(t) = V_m I_m \cos(\omega t) \cos(\omega t - \theta)$$

Average Power $P = \frac{V_m I_m}{2} \cos(\theta)$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta)$$

Average Power – ACTIVE power

Units: W

$\cos(\theta)$ – POWER FACTOR

POWER calculations for a general RLC Load (2)

RLC load where phase θ can be any value from -90° to $+90^\circ$

ACTIVE power $P = V_{\text{rms}} I_{\text{rms}} \cos(\theta)$ Units: W

POWER FACTOR $\cos(\theta)$ For phase θ = 0º

For phase θ value other than 0º

$$\theta = \theta_v - \theta_i$$

θ_v = Phase of voltage

θ_i = Phase of current

REACTIVE Power $Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta)$ Units: VAR

APPARENT Power $P^2 + Q^2 = (V_{\text{rms}} I_{\text{rms}})^2$ Units: VA