MODE

Mode is the value of the variable, which occurs most frequently in the measurements. The word ‘Mode’ is derived from the French word “La Mode’ which signifies fashion. It is often used as a positional average in practice. Mode need not be unique. A data set may have two or more modes. If a data set contains two modes then the data is said to be bi-model data. Mode is usually denoted by X.

Case A :
Raw data : In this case, mode is defined as the value of the variable which occurs most frequently in the given data set.

Case B :
Discrete frequency distribution : In the case of discrete frequency distribution, mode is the value of the variable for which the frequency is maximum.

Case C :
Continuous frequency distribution : In the case of continuous frequency distribution, mode can be obtained by using the following formula.

$$\text{Mode} = \widehat{X} = L + \left(\frac{f - f_1}{2f - f_1 - f_2}\right)C$$

where L : Lower bound of the model class
f : Frequency of the model class
f_1 : Frequency of the class that precedes the model class
f_2 : Frequency of the class that succeeds model class
C : Length of the model class

Here, model class is the class that corresponds largest frequency.

Remarks :

In the following three cases, mode can not be obtained by using the above formula:

(a) When the highest frequency is observed at the beginning of the frequency table.
(b) When the highest frequency is observed at the ending of the frequency table.
(c) When two or more class intervals contain the same maximum frequencies.

However, in the above three cases, mode can be obtained by using either a method called ‘Grouping method’ or ‘empirical relationship between arithmetic mean, median and mode.

Empirical Relationship between A.M. Median and Mode

If mean, median and mode are equal for a frequency distribution, then the distribution is called symmetrical distribution. In a moderately asymmetrical distribution, mean, median and mode approximately satisfy the following empirical relationship:

(A.M. - Mode) = 3 (A.M. - Median)

Symbolically $(\bar{X} - \widehat{X}) = 3\,(\bar{X} - \widetilde{X})$

This relation is observed from experience and it is not mathematically derived. When we are given the values of any two of these three measures, then the third measure can be found from this relation. This relation is occasionally used to find mode, when A.M. and Median are known.

Mode = 3 Median – 2 A.M.

Uses of Mode :

Mode finds an important place in marketing studies, where a manager of a business concern is interested in knowing about the size which has the highest concentration of items. For example, in placing an order for shoes or ready made garments, the model size helps because this size and other sizes around it are in common demand. It is also used in dealing with non-quantitative data.

SOLVED PROBLEMS ON MODE

Problem: Find the mode for the following set of values of a variable :

2, 7, 3, 2, 1, 3, 2, 2, 5

Solution : In the given data the value 2 occurs most frequently than the other values. Hence,

mode of the given data is 2.

Problem: Determine the mode for the following data

1, 0, 2, 3, 6, 7, 5, 4, 8.

Solution : In the given data, no single value repeats more frequently, when compared to other

values. Therefore, we conclude that there is no mode for the above data.

Problem: Find the modal age of married women at first child birth :

Age at the birth of first child (in years)	13	14	15	16	17	18	19	20	21	22	23
No. of married women	37	150	300	360	270	435	160	200	85	65	25

Solution : From the given discrete frequency distribution, it is observed that the highest frequency is 435. The age of married women corresponding to this highest frequency is 18 years.

Hence, 18 years is the model age of married women at first child birth.

Problem Calculate the mode for the following data :

Class	130-134	135-139	140-144	145-149	150-154	155-159	160-164
Frequency	5	15	28	24	17	10	1

Solution :

Class	Frequency
130-134	5
135-139	$15 = f_1$
140-144 Model class	$28 = f$
145-149	$24 = f_2$
150-154	17
155-159	10
160-164	1
Total	100

The highest frequency corresponds the class 140 –144. Therefore, it is the model class.

From table, we have,

$L = 139.5$, $f = 28$, $f_1 = 15$, $f_2 = 24$ and $C = 5$

$$\text{Mode} = X = L + \left[\frac{f - f_1}{2f - f_1 - f_2}\right]C$$

$$= 139.5 + \left[\frac{28 - 15}{2x28 - 15 - 24}\right]5$$

$$= 139.5 + \frac{65}{17} = 139.5 + 3.8235$$
$$= 143.3235$$

Problem: In a moderately skewed distribution (Asymmetrical distribution) A.M. = 15 and Mode = 12. Find the value of the Median of the given distribution.

Solution : Consider the empirical relationship between mean, median and mode.

(Mean - Mode) = 3 (Mean - Median)

Hence, 15 - 12 = 3 (15 - Median)
Therefore, 3 Median = 45 – 3 = 42

Median = $\frac{42}{3} = 14$

Problem: Find the Mode for the following data :
2, 5, 3, 2, 1, 4, 6, 3, 7

Solution : Since 2 and 3 have maximum frequencies, (2 occurs 2 tm ice 3 occurs 2 tm ice) the given data is a bimodal data. Therefore, we use the empirical relationship between mean, median and mode.

$$A.M = \bar{x} = \frac{\sum x}{n} = \frac{33}{9} = 3.6667$$

To find median arrange the data in ascending order:
1, 2, 2, 3, 3, 4, 5, 6, 7
Median = 3
Consider the empirical relationship (AM. - Mode) = 3 (Mean - Median)
Substituting the values of A.M. and Median. we get.

3.6667 – Mode = 3 (3.667-3)
3.6667 – Mode = 2.0001
$\therefore$ Mode = 3.667 – 2.0001 = 1.6666

Problem: Compute Mode for the following data :

Size	0-4	4-8	8-12	12-16	16-20
Frequency	10	20	30	35	35

Solution: The highest frequency is 35 and it corresponds the two bottom most class intervals of the frequency table. Hence, the given distributions is a bimodal. In this case, we use the empirical relationship between A.M., Median and Mode.

Class	Frequency *f*	Mid values *x*	*fx*	Less than cumulative frequency
0-4	10	2	20	10
4-8	20	6	120	30
8-12	30	10	300	60 =m
12-16	35 = f	14	490	95
16-20	35	18	630	130
Total	130	-	1560	-

Form the table, $\sum fx$ = 1560; N = 130

$\therefore$ A.M. = $\bar{x} = \frac{\sum fx}{N} = \frac{1560}{130} = 12$

Since, N/2 = 130/2 = 65

12-16 is the median class

L=12, m = 60, f = 35, N = 130, C = 4

$$\text{Median} = L + \left(\frac{\frac{N}{2} - m}{f}\right)C$$

$$= 12 + \left[\frac{65-60}{35}\right]4 = 12 + \frac{20}{35} = 12 + 0.5714 = 12.5714$$

Median = 12.5714

Consider the empirical relationship

(A.M. - Mode) = 3 (Mean - Median)

12 – Mode = 3 (12 – 12.5714)

12 – Mode = 3 (- 0.5714)

12 – Mode = - 1.7142

$\therefore$ Mode = 12 + 1.7142

Mode = 13.7142

Problem: The median and mode of the following wage distribution are known to be Rs. 335 and Rs. 340 respectively. Find the three missing frequencies in the frequency distribution given below.

Wages (in Rs.)	Frequency

0-100	4
100-200	16
200-300	60
300-400	?
400-500	?
500-600	?
600-700	4
Total	230

Solution : Let f_1, f_2 and f_3 be the missing frequencies.

Wages (Rs.)	Frequency	Less than cumulative frequence
0-100	4	4
100-200	16	20
200-300	60	80
300-400 Median class	f_1	$80 + f_1$
400-500	f_2	$80 + f_1 + f_2$
500-600	f_3	$80 + f_1 + f_2 + f_3$
600-700	4	$84 + f_1 + f_2 + f_3$
Total	230	

We have, $230 = 84 + f_1 + f_2 + f_3$

$f_3 = 230 - 84 - f_1 - f_2$

$= 146 - (f_1 + f_2)$..........(1)

Since the median and mode are 355 and 340, both lie in the same class, viz., 300 – 400.

$$\text{Median} = 335 = 300 + \left[\frac{\frac{230}{2} - 80}{f_1}\right]100$$

$$335\text{-}300 = \frac{(115 - 80)100}{f_1} \Rightarrow 35 f_1 = 3500$$

$$\therefore f_1 = \frac{3500}{35} = 100$$

$$\text{Mode} = L + \left[\frac{100 - 60}{2 \times 100 - 60 - f_2}\right]100$$

$$40 = \frac{4,000}{140 - f_2} \Rightarrow 40(140 - f_2) = 4000$$

$$\Rightarrow 5600 - 40 f_2 = 4000$$

$$40 f_2 = 5600 - 4000 = 1600$$

$$\therefore f_2 = \frac{1600}{40} = 40$$

From (1), we get $f_3 = 146 - (100 + 46) = 6$

Hence, the missing frequencies are 100, 40 and 6 respectively

Requisites for an Ideal Measure of Central Tendency :

According to Prof. Yule, the following are the chief characteristics to be satisfied by an ideal measure of central tendency.

i) It should be rigidly defined
ii) It should be easy to understand and easy to calculate
iii) It should be based on all the observations.
iv) It should be suitable for further mathematical manipulations
v) It should be affected as little as possible by sampling fluctuations.
vi) It should not be affected much by extreme values.

Now, we shall consider the merits and demerits (or advantages and disadvantages) of Arithmetic mean, median and mode.

Measure of Central Tendency	Merits	Demerits :
a) Arithmetic Mean	1) It is rigidly defined. 2) It is easy to understand and easy to calculate. 3) It is based on all the observations. 4) It is suitable for further mathematical manipulations. 5) Of all the averages, it is affected least by sampling fluctuations.	1) It is affected very much by extreme values 2) It cannot be found for distributions with open end classes. 3) It cannot be used if measurements are qualitative.
b) Median	1) It is rigidly defined. 2) It is easy understand and easy to calculate. 3) It is not affected by extreme values. 4) It can be found for distributions with open end classes. 5) Median is the only average to be used while dealing with the qualitative data.	1) It is not based on all the observations. 2) It is not suitable for further mathematical manipulations. 3) It is affected by sampling fluctuations. 4) It is not so generally familiar as the arithmetic mean.
c) Mode	1) It is easy to understand and easy to calculate. 2) It is not affected by extreme classes. 3) It can be found for distribution with open end classes.	1) Mode is ill-defined 2) It is not based on all the observations. 3) It is not suitable for further mathematical manipulations.

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