1) If X and Y have the joint pdf $f(x,y) \Rightarrow \frac{1}{3}(x+y)$ $0 \le x \le 1$, $0 \le y \le 2$.

Find i) $r(X,Y)$ ii) the two lines of regression iii) two regression curves for the means.

$$r(x,y) \Rightarrow \frac{\text{Covariance}(X,Y)}{\sigma_x \sigma_y}$$

$$\Rightarrow \frac{E(xy) - E(x)E(y)}{\sigma_x \sigma_y}$$

$$E(x) \Rightarrow \cancel{\int_{-\infty}^{\infty} x f(x)\, dx} \Rightarrow \int_{-\infty}^{\infty} x f_X(x)\, dx$$

since $f(x,y)$ is the joint pdf

$$f_x(X) \Rightarrow \int_0^2 \frac{1}{3}(x+y)\,dy$$

$$\Rightarrow \frac{1}{3}\int_0^2 (x+y)\,dy \Rightarrow \frac{1}{3}\left[xy + \frac{y^2}{2}\right]_0^2$$

$$\Rightarrow \frac{1}{3}\left[x(2) + \frac{4}{2}\right] \Rightarrow \frac{1}{3}\left[2x+2\right]$$

$$\boxed{f_x(X) \Rightarrow \frac{2}{3}(x+1)}$$

$$E(x) \Rightarrow \int_0^1 x \, \frac{2}{3}(x+1)\,dx$$

$$\Rightarrow \frac{2}{3}\int_0^1 (x^2+x)\,dx$$

$$\Rightarrow \frac{2}{3}\left[\frac{x^3}{3}+\frac{x^2}{2}\right]_0^1$$

$$\Rightarrow \frac{2}{3}\left[\frac{1}{3}+\frac{1}{2}\right] \Rightarrow \frac{2}{3}\left(\frac{5}{6}\right) \Rightarrow \frac{5}{9}$$

$$\boxed{E(X) \Rightarrow \frac{5}{9}}$$

$$E(y) \Rightarrow \int_{-\infty}^{\infty} y \, f_y(Y)\,dy$$

$$f_y(Y) \Rightarrow \int_0^1 \frac{1}{3}(x+y)\,dx$$

$$\Rightarrow \frac{1}{3}\int_0^1 (x+y)\,dx \Rightarrow \frac{1}{3}\left[\frac{x^2}{2}+xy\right]_0^1$$

$$\Rightarrow \frac{1}{3}\left[\frac{1}{2}+y\right]$$

$$f_y(Y) \Rightarrow \frac{1}{3}\left[\frac{1+2y}{2}\right] \Rightarrow \frac{2y+1}{6} \Rightarrow \frac{1}{6}(2y+1)$$

$$E(y) \Rightarrow \int_0^2 y\left(\frac{1}{6}(2y+1)\right)dy$$

$$\Rightarrow \frac{1}{6}\int_0^2 (2y^2+y)\,dy \Rightarrow \frac{1}{6}\left[\frac{2y^3}{3}+\frac{y^2}{2}\right]_0^2$$

$$\Rightarrow \frac{1}{6}\left[\frac{2(8)}{3}+\frac{\cancel{4}^{\,2}}{\cancel{2}}\right] \Rightarrow \frac{1}{6}\left[\frac{16}{3}+2\right]$$

$$\Rightarrow \frac{1}{6}\left[\frac{16+6}{3}\right] \Rightarrow \frac{1}{\cancel{6}_{3}}\left[\frac{\cancel{22}^{\,11}}{3}\right] \Rightarrow \frac{11}{9}$$

$$\boxed{E(y) = \frac{11}{9}}$$

$$E(x,y) \Rightarrow \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy \, f(x,y)\, dx\, dy$$

$$\Rightarrow \int_{0}^{2}\int_{0}^{1} xy \; \tfrac{1}{3}(x+y) \quad dx\, dy$$

$$\Rightarrow \tfrac{1}{3} \int_{0}^{2}\int_{0}^{1} (x^2y + xy^2)\, dx\, dy$$

$$\Rightarrow \tfrac{1}{3} \int_{0}^{2} \left[ \int_{0}^{1} (x^2y + xy^2)\, dx \right] dy$$

$$\Rightarrow \tfrac{1}{3} \int_{0}^{2} \left[ \frac{x^3y}{3} + \frac{x^2y^2}{2} \right]_0^1 dy$$

$$\Rightarrow \tfrac{1}{3} \int_{0}^{2} \left[ \tfrac{y}{3} + \tfrac{y^2}{2} \right] dy$$

$$\Rightarrow \tfrac{1}{3} \left[ \frac{y^2}{6} + \frac{y^3}{6} \right]_0^2 \Rightarrow \tfrac{1}{3}\left[ \frac{\cancel{4}^{\,2}}{\cancel{6}_{\,3}} + \frac{\cancel{8}^{\,4}}{\cancel{6}_{\,3}} \right]$$

$$\Rightarrow \tfrac{1}{3}\left[\tfrac{6}{3}\right]$$

$$\Rightarrow \tfrac{6}{15} \Rightarrow \tfrac{2}{3}$$

$$\boxed{E(x,y) \Rightarrow \tfrac{2}{3}}$$

$$Var(x) \Rightarrow E(x^2) - (E(x))^2$$

$$E(x^2) \Rightarrow \int_0^1 x^2 f_X(x)\, dx$$

$$\Rightarrow \int_0^1 x^2 \tfrac{2}{3}(x+1)\, dx$$

$$\Rightarrow \tfrac{2}{3}\int_0^1 (x^3+x^2)\, dx$$

$$\Rightarrow \tfrac{2}{3}\left[\frac{x^4}{4}+\frac{x^3}{3}\right]_0^1$$

$$\Rightarrow \tfrac{2}{3}\left[\tfrac{1}{4}+\tfrac{1}{3}\right] \Rightarrow \tfrac{2}{3}\left[\frac{3+4}{12}\right] \Rightarrow \tfrac{\cancel{2}}{3}\left(\tfrac{7}{\cancel{12}_{6}}\right)$$

$$\boxed{E(x^2) \Rightarrow \tfrac{7}{18}}$$

$$E(y^2) \Rightarrow \int_0^2 y^2 f_Y(y)\, dy$$

$$\Rightarrow \int_0^2 y^2\, \tfrac{1}{6}(2y+1)\, dy$$

$$\Rightarrow \tfrac{1}{6}\int_0^2 (2y^3+y^2)\, dy \Rightarrow \tfrac{1}{6}\left[\frac{\cancel{2}y^4}{\cancel{4}_2}+\frac{y^3}{3}\right]_0^2$$

$$\Rightarrow \tfrac{1}{6}\left[\cancel{\frac{16}{2}+\frac{8}{3}}\right] \cancel{\Rightarrow \tfrac{8}{6}\left(\tfrac{1}{2}+\tfrac{1}{3}\right)}$$

$$\Rightarrow \tfrac{4}{3}\left(\frac{3+2}{6}\right) \cancel{\Rightarrow \tfrac{\cancel{4}^{2}}{3}\left(\tfrac{5}{\cancel{6}_3}\right)} \Rightarrow \tfrac{10}{9}$$

$$\Rightarrow \tfrac{1}{6}\left[\tfrac{16}{2}+\tfrac{8}{3}\right] \Rightarrow \tfrac{1}{6}\left[8+\tfrac{8}{3}\right]$$

$$\Rightarrow \frac{8}{6}\left[1+\frac{1}{3}\right] \Rightarrow \cancel{\frac{1}{2}\left[\frac{3+1}{3}\right]}$$

$$\cancel{\Rightarrow \frac{1}{2}\left(\frac{4}{3}\right) \Rightarrow 4}$$

$$\Rightarrow \frac{4}{3}\left(1+\frac{1}{3}\right)$$

$$\Rightarrow \frac{4}{3}\left(\frac{3+1}{3}\right) \Rightarrow \frac{4}{3}\left(\frac{4}{3}\right) \Rightarrow \frac{16}{9}$$

$$\boxed{E(y^2) \Rightarrow \frac{16}{9}}$$

Variance $(x) \Rightarrow E(x^2) - (E(x))^2$

$$\Rightarrow \frac{7}{18} - \left(\frac{5}{9}\right)^2 \Rightarrow \frac{7}{18} - \frac{25}{81}$$

$$\Rightarrow 0.388 - \cancel{0.555}\ \cancel{0.0617}\ 0.308$$

$$\Rightarrow \cancel{-0.167}\ 0.08$$

Variance $(Y) \Rightarrow E(Y^2) - (E(Y))^2$

$$\Rightarrow \frac{16}{9} - \left(\frac{11}{9}\right)^2 \Rightarrow \frac{16}{9} - \frac{121}{81}$$

$$\Rightarrow 1.777 - 1.4938$$

$$\Rightarrow 0.2839$$

$\sigma_x \Rightarrow \sqrt{\text{Var}(x)}$

$\Rightarrow \sqrt{0.08}$

$\boxed{\sigma_x \Rightarrow 0.2828}$

$\sigma_y \Rightarrow \sqrt{\text{Var}(Y)}$

$\Rightarrow \sqrt{0.2839}$

$\boxed{\sigma_y \Rightarrow 0.5328}$

$$r(x,y) \Rightarrow \frac{\text{Covariance}(x,y)}{\sigma_x \sigma_y}$$

$$\Rightarrow \frac{E(xy) - E(x)E(y)}{\sigma_x \sigma_y}$$

$$\Rightarrow \frac{2/3 - \left(\left(5/9\right)\left(11/9\right)\right)}{(0.2828)\ (0.5328)}$$

$$\Rightarrow \frac{0.66 - 0.67901}{0.15067}$$

$$\Rightarrow \frac{-0.01234}{0.15067}$$

$$\boxed{r(x,y) \Rightarrow -0.0819}$$

ii) the two lines of regression

Lines of regression $\underline{y \text{ on } x}$

$$(y-\bar{y}) \Rightarrow r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$$

$$\boxed{\begin{array}{l} \bar{x} \Rightarrow E(x) \Rightarrow 5/9 \\ \bar{y} \Rightarrow E(y) \Rightarrow 11/9 \\ r \Rightarrow -0.0819 \\ \sigma_x \Rightarrow 0.2828 \\ \sigma_y \Rightarrow 0.5328 \end{array}}$$

$$y - 11/9 \Rightarrow (-0.0819)\left(\frac{\cancel{0.2828}\ 0.5382}{0.2828}\right)(x-5/9)$$

$$\left(y-11/9\right) \Rightarrow -0.15586\left(x-5/9\right)$$

$$y-1.22 \Rightarrow -0.15586x + 0.086591$$

$$\boxed{y + 0.15586x = 1.306591}$$

Lines of regression $\underline{x \text{ on } y}$

$$(x-\bar{x}) \Rightarrow r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$$

$$\left(x-5/9\right) \Rightarrow (-0.0819)\frac{0.2828}{0.5382}\left(y-11/9\right)$$

$$\left(x-5/9\right) \Rightarrow -0.04303\left(y-11/9\right)$$

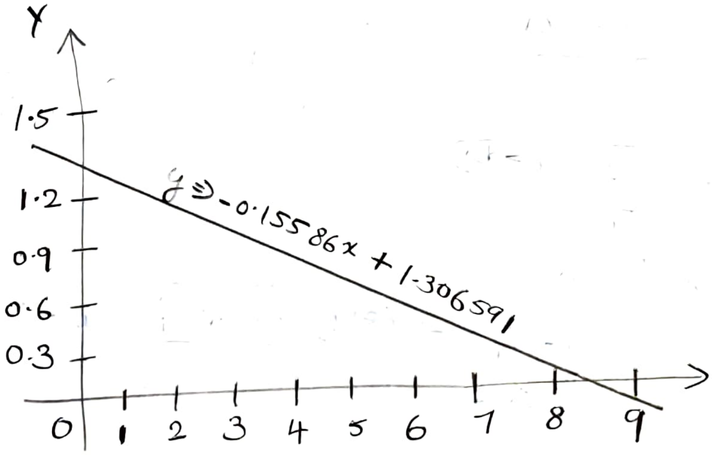
$$(x-0.55) \Rightarrow -0.04303y + 0.052592$$

$$\cancel{x+0.04303y} = \cancel{0.02408} \qquad x + \cancel{0.04303y}$$

$$\boxed{x-0.04303y-0.02408=0} \qquad \boxed{x+0.04303y+0.60259 \Rightarrow 0}$$

ii) two regression ~~line~~ curves of the mean

y on x




$y \Rightarrow -0.15586x + 1.306591$

slope $\Rightarrow -0.15586$

intercept $\Rightarrow 1.306591$

$x + 0.04303y + 0.60259 \Rightarrow 0$

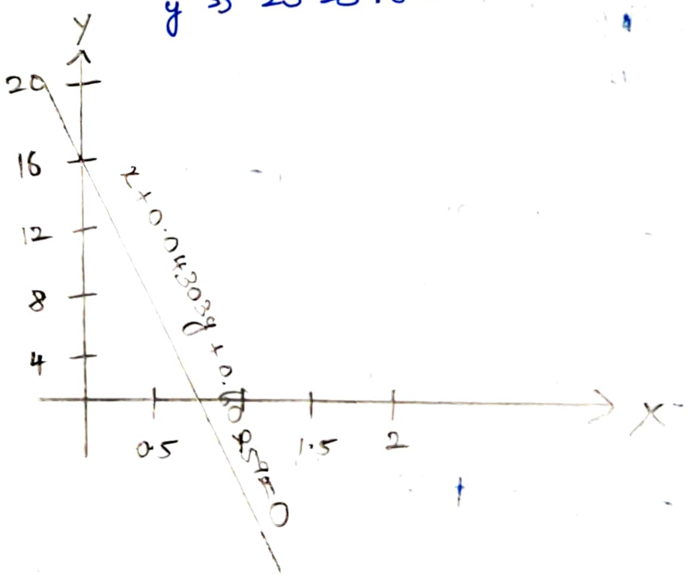
x on y

$y \Rightarrow \dfrac{-x - 0.60259}{0.04303}$

$\cancel{0.04303y = x \pm 0.02408}$

$y \Rightarrow \cancel{23.2396x} - 0.5596$

$y \Rightarrow -23.239x$



2) X, Y, Z ⇒ uncorrelated

Mean ⇒ 0 $\quad U = X + Y$

SD ⇒ 5, 13, 9 $\quad V = Y + Z$

Find the correlation coefficient bt U and V

$$r(U, V) \Rightarrow \frac{\text{Covariance}(U, V)}{\sigma_u \sigma_v}$$

$$r_{(u,v)} \Rightarrow \frac{E(UV) - E(U)\,E(V)}{\sigma_u \sigma_v}$$

$$E(UV) \Rightarrow E\left[(x+y)(y+z)\right]$$

$$\Rightarrow E\left[xy + xz + y^2 + z\right]$$

$$\Rightarrow E[xy] + E[xz] + E[y^2] + E[z]$$

$$E(U,V) \Rightarrow 0 + 0 + E[y^2] + 0$$

?

$$\text{Variance}(X) \Rightarrow E(x^2) - (E(x))^2$$

$$(5)^2 \Rightarrow E(x^2) - 0$$

$$\boxed{E(x^2) \Rightarrow 25}$$

$$\text{Variance}(Y) \Rightarrow E(y^2) - (E(y))^2$$

$$(13)^2 \Rightarrow E(y^2)$$

$$\boxed{E(y^2) \Rightarrow 169}$$

$$\text{Variance}(z) \Rightarrow E(z^2) - (E(z))$$

$$(9)^2 \Rightarrow E(z^2) - 0$$

$$\boxed{E(z^2) \Rightarrow 81}$$

$$E(u,v) \Rightarrow E(y^2)$$

$$\boxed{E(u,v) \Rightarrow 169}$$

$$E(u) \Rightarrow E(x+y) \Rightarrow E(x) + E(y)$$

$$\Rightarrow 0+0$$

$$\boxed{E(u) \Rightarrow 0}$$

$$E(v) \Rightarrow E(y+z) \Rightarrow E(y) + E(z)$$

$$\boxed{E(v) \Rightarrow 0}$$

$$r_{u,v} \Rightarrow \frac{81 - 0 - 0}{\sigma_u \sigma_v \; ?}$$

$$\sigma_u \Rightarrow \sqrt{Var(u)}$$

$$Var(u) \Rightarrow E(u^2) - (E(u))^2 \Rightarrow E(u^2) - 0$$

$$\Rightarrow E(x+y)^2 \Rightarrow E(x^2 + 2xy + y^2) \Rightarrow \left(E(x^2) + 2E(x)E(y) + E(y^2)\right)$$

$$\Rightarrow 25 + 2(0) + 169$$

$$Var(u) \Rightarrow 194$$

$$\boxed{\sigma_u \Rightarrow \sqrt{194} \Rightarrow 13.928}$$

$$\sigma_y \Rightarrow \sqrt{Var(v)}$$

$$Var(v) \Rightarrow E(v^2) - \left(E(v)\right)^2 \Rightarrow E(v^2) - 0 \Rightarrow E(v^2)$$

$$\Rightarrow E(y+z)^2 - 0$$

$$\Rightarrow E\left[y^2 + 2yz + z^2\right]$$

$$\Rightarrow E(y^2) + 2E(yz) + E(z^2)$$

$$\Rightarrow 169 + 2(0) + 81$$

$$Var(v) \Rightarrow 250$$

$$\sigma_v \Rightarrow \sqrt{250} \Rightarrow 15.811$$

$$\boxed{\sigma_v \Rightarrow 15.811}$$

$$r_{(u,v)} \Rightarrow \frac{169}{13.928 * 15.811} \Rightarrow 0.7674$$

$$\boxed{r_{u,v} \Rightarrow 0.7674}$$

3) Expected life length of the component ⇒ 100 h | 150 h

| | Process 1 | Process 2 |
|---|---|---|
| Expected life | 100 h | 150 h |
| Single Cost | ₹ 10 | ₹ 20 |

If life < 200h, a loss of ₹ 50 is to be borne by the manufacturer.

$P_i$ ⇒ probability of producing a component which lasts less than the guarranteed life span of 200 h

$E[x_1] \Rightarrow 100$ $\qquad$ $E[x_2] \Rightarrow 150$

$\lambda_1 \Rightarrow \frac{1}{E(x_1)} \Rightarrow \frac{1}{100}$ $\qquad$ $\lambda_2 \Rightarrow \frac{1}{E(x_2)} \Rightarrow \frac{1}{150}$

$\boxed{\lambda_1 \Rightarrow 1/100}$ $\qquad$ $\boxed{\lambda_2 \Rightarrow 1/150}$

Probability function of the two process

$f_1 \Rightarrow \lambda_1 e^{-\lambda_1 x}$

$\boxed{f_1 \Rightarrow 1/100 \, e^{-1/100 \, x}}$

$f_2 \Rightarrow \lambda_2 e^{-\lambda_2 x}$

$\boxed{f_2 \Rightarrow \frac{1}{150} e^{-\frac{1}{150} x}}$

i) Produced by process 1

$t \Rightarrow \frac{x}{100}$

$P(X < 200) \Rightarrow \int_0^{200} f_1 \, dx$

$dt \Rightarrow \frac{dx}{100}$

$\Rightarrow \int_0^{200} \frac{1}{100} e^{-x/100} dx$

$x$ from 0 to 200

$t$ from 0 to 2

$\Rightarrow \int_0^{2} e^{-t} dt$

$\Rightarrow [e^{-t}]_0^2 \Rightarrow -[e^{-2} - e^{-0}]$

$\Rightarrow -(0.13533 - 1)$

$\boxed{P(X<200) \Rightarrow +0.86466}$

86% product fails before 200h

So, Mean cost of producing component by process 1

$\Rightarrow$ cost of producing successful product + loss %

$\Rightarrow 10 + (0.8647) * 50$

Mean cost $\Rightarrow$ 53.235 $\longrightarrow$ ①

ii) Produced by process 2

$P(X < 200) \Rightarrow \int_0^{200} f_2 \, dx$ $\qquad t \Rightarrow \frac{x}{150}$

$\Rightarrow \int_0^{200} \frac{1}{150} e^{-1/150 \, x} dx$ $\qquad dt \Rightarrow \frac{dx}{150}$

$\Rightarrow \frac{1}{150} \int_0^{200} e^{-1/150 \, x} dx$

$x$ from 0 to 200

$t$ from 0 to 1.33

$\Rightarrow \int_0^{1.33} e^{-t} dt \Rightarrow -\left[e^{-t}\right]_0^{1.33}$

$\Rightarrow -\left[e^{-(1.33)} - e^{-(0)}\right]$

$\Rightarrow -\left[0.2644 - 1\right]$

$\boxed{P(X<200) \Rightarrow 0.7355}$ $\qquad$ 73% product fails before 200h

So, Mean cost of producing component by Process 2

$\Rightarrow$ Cost of producing successful product + loss %

$\Rightarrow 20 + (0.7355) * 50$

$\boxed{\text{Mean cost} \Rightarrow 56.82}$ $\longrightarrow$ ②

From ① & ② Process 1's amount ₹ 53.235 (loss) is less.

So More advantageous is Process 1.

4) Density function of the random variable $X$ $\Big\}$ $f(x) \Rightarrow ce^{-b(x-a)}$ $a \le x$

Show that $b = c = \frac{1}{\sigma}$ ; $a = \mu - \sigma$ ; $\mu = E(X)$

$\sigma = Var(X)$

We know that

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{a}^{\infty} ce^{-b(x-a)}\, dx = 1$$

$$c\int_{a}^{\infty} (e^{-bx} \cdot e^{ab})\, dx = 1$$

$$ce^{ab}\int_{a}^{\infty} (e^{-bx})\, dx = 1$$

$$ce^{ab}\left[\frac{e^{-bx}}{-b}\right]_{a}^{\infty} = 1$$

$$\frac{-ce^{ab}}{b}\left[e^{-b(\infty)} - e^{-ab}\right] \Rightarrow 1$$

$$\frac{-ce^{ab}}{b}(-e^{-ab}) \Rightarrow 1$$

$$-c = -b$$

$$\boxed{c = b}$$

$$E(x) \Rightarrow \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$\int dv = \int e^{-bx} dx$$

$$u = x \qquad v \Rightarrow \frac{e^{-bx}}{-b}$$

$$u' = 1 \qquad v_1 \Rightarrow \frac{e^{-bx}}{b^2}$$

$$\Rightarrow \int_a^{\infty} x\, c e^{-b(x-a)} dx$$

$$\Rightarrow c \int_a^{\infty} x e^{-b(x-a)} dx$$

$$\Rightarrow c e^{ab} \int_a^{\infty} x e^{-bx} dx$$

$$\Rightarrow c e^{ab} \left[ x\left(\frac{-e^{-bx}}{b}\right) - \frac{e^{-bx}}{b^2} \right]_a^{\infty}$$

$$\Rightarrow c e^{ab} \left[ \frac{-\infty\, e^{-b(\infty)}}{b} - \frac{e^{-b(\infty)}}{b^2} - \left( \frac{a(-e^{-ab})}{b} - \frac{e^{-ab}}{b^2} \right) \right]$$

$$\Rightarrow c e^{ab} \left[ 0 + \frac{a e^{-ab}}{b} + \frac{e^{-ab}}{b^2} \right]$$

$$\Rightarrow c e^{ab} \left( \frac{a e^{-ab}}{b} + \frac{e^{-ab}}{b^2} \right)$$

$$\Rightarrow c e^{ab} \left( \frac{b^2 a e^{-ab} + b e^{-ab}}{b^3} \right) \Rightarrow c e^{ab} \frac{b}{b^3} \left( ab e^{-ab} + e^{-ab} \right)$$

$$\Rightarrow \frac{c e^{ab} e^{-ab} (ab + 1)}{b^2}$$

$$\Rightarrow \frac{c}{b^2} (ab + 1)$$

$$\boxed{b = c}$$

$$\Rightarrow \frac{b}{b^2} (ab + 1)$$

$$\frac{\cancel{b}}{b\cancel{x}}(ab+1) \Rightarrow \frac{a\cancel{b}}{\cancel{b}} + \frac{1}{b} \Rightarrow a + \frac{1}{b}$$

$$\boxed{E(x) \Rightarrow a + \frac{1}{b}}$$

$$\int dv = \int e^{-bx} dx$$

$u = x^2$ $\quad$ $v \Rightarrow \frac{e^{-bx}}{-b}$

$u' = 2x$ $\quad$ $v_1 \Rightarrow \frac{e^{-bx}}{b^2}$

$u'' = 2$ $\quad$ $v_2 \Rightarrow \frac{-e^{-bx}}{b^3}$

$$E[x^2] \Rightarrow \int_a^{\infty} x^2 f(x) dx$$

$$\Rightarrow \int_a^{\infty} x^2 c e^{-b(x-a)} dx$$

$$\Rightarrow c e^{ab} \int_a^{\infty} x^2 e^{-bx} dx$$

$$\Rightarrow ce^{ab}\left[\frac{+x^2 e^{-bx}}{(-b)} - \frac{2x e^{-bx}}{b^2} + 2\left(\frac{-e^{-bx}}{b^3}\right)\right]_a^{\infty}$$

$$\Rightarrow ce^{ab}\left[-\frac{x^2 e^{-bx}}{b} - \frac{2x e^{-bx}}{b^2} - \frac{2e^{-bx}}{b^3}\right]_a^{\infty}$$

$$\Rightarrow ce^{ab}\left[0 - \left(\frac{-a^2 e^{-ab}}{b} - \frac{2ae^{-ab}}{b^2} - \frac{2e^{-ab}}{b^3}\right)\right]_a$$

$$\Rightarrow \frac{ce^{ab}e^{-ab}}{b}\left[+a^2 + \frac{2a}{b} + \frac{2}{b^2}\right]$$

$$\Rightarrow c/b\left[\frac{+b^2a^2 + 2ab + 2}{b^2}\right]$$

$$\Rightarrow \frac{c}{b}\left[\frac{a^2b^2+2ab+2}{b^2}\right] \quad (c=b)$$

$$\Rightarrow a^2 + \frac{2ab^2}{b^3} + \frac{2}{b^2}$$

$$\boxed{E(x^2) \Rightarrow a^2 + \frac{2ab^2}{b^3} + \frac{2}{b^2}}$$

$$\text{Variance } (X) \Rightarrow E(X^2) - (E(X))^2$$

$$\Rightarrow a^2 + \frac{2a}{b} + \frac{2}{b^2} - \left(a + \frac{1}{b}\right)^2$$

$$\Rightarrow a^2 + 2\frac{a}{b} + \frac{2}{b^2} - \left(a^2 + 2a\frac{1}{b} + \frac{1}{b^2}\right)$$

$$\Rightarrow \cancel{a^2} + \cancel{\frac{2a}{b}} + \frac{2}{b^2} - \cancel{a^2} - \cancel{\frac{2a}{b}} - \frac{1}{b^2}$$

$$V(x) \Rightarrow \frac{1}{b^2}$$

$$\sigma \Rightarrow \sqrt{V(X)}$$

$$\Rightarrow \sqrt{1/b^2} \Rightarrow 1/b$$

$$\sigma = 1/b$$

$b = 1/\sigma$ ; so $\boxed{b = c = 1/\sigma}$ & $\mu \Rightarrow a + 1/b$

$$a \Rightarrow \mu - 1/b$$

Hence proved $\boxed{a \Rightarrow \mu - \sigma}$

5)

| | Chennai | Vellore |
|---|---|---|
| Mean | 19.5 | 17.75 |
| SD | 1.75 | 2.5 |

$r_{xy} \Rightarrow 0.8$

$\bar{x} \Rightarrow 19.5$ | $\sigma_x \Rightarrow 1.75$

$\bar{y} \Rightarrow 17.75$ | $\sigma_y \Rightarrow 2.5$

x on y

$$(x-\bar{x}) = r_{xy} \frac{\sigma_x}{\sigma_y} (y-\bar{y})$$

$$(x-19.5) = 0.8 \frac{1.75}{2.5} (y-17.75)$$

$$x-19.5 = 0.56(y-17.75)$$

$$x-19.5 = 0.56y - 9.94$$

$$x - 0.56y = \cancel{29.44}\ 9.56$$

$$\boxed{x-0.56y = 9.56}$$

When $y = 18$

$$x - 0.56(18) = 9.56$$

$$x = 19.64$$

The price of rice at Chennai is ₹ 19.64

$y$ on $x$

$$(y-\bar{y}) \Rightarrow r_{xy} \frac{\sigma_y}{\sigma_x} (x-\bar{x})$$

$$(y-17.75) = 0.8 \frac{2.5}{1.75} (x-19.5)$$

$$(y-17.75) = 1.14285 (x-19.5)$$

$$(y-17.75) \Rightarrow 1.14285x - 22.2857$$

$$\boxed{1.14285x - y = 4.535}$$

When $x = 17$

$$1.14285(17) - y = 4.535$$

$$y = 14.89345$$

The price of Rice at Vellore is ₹ 14.8934