

Statistics for Engineers

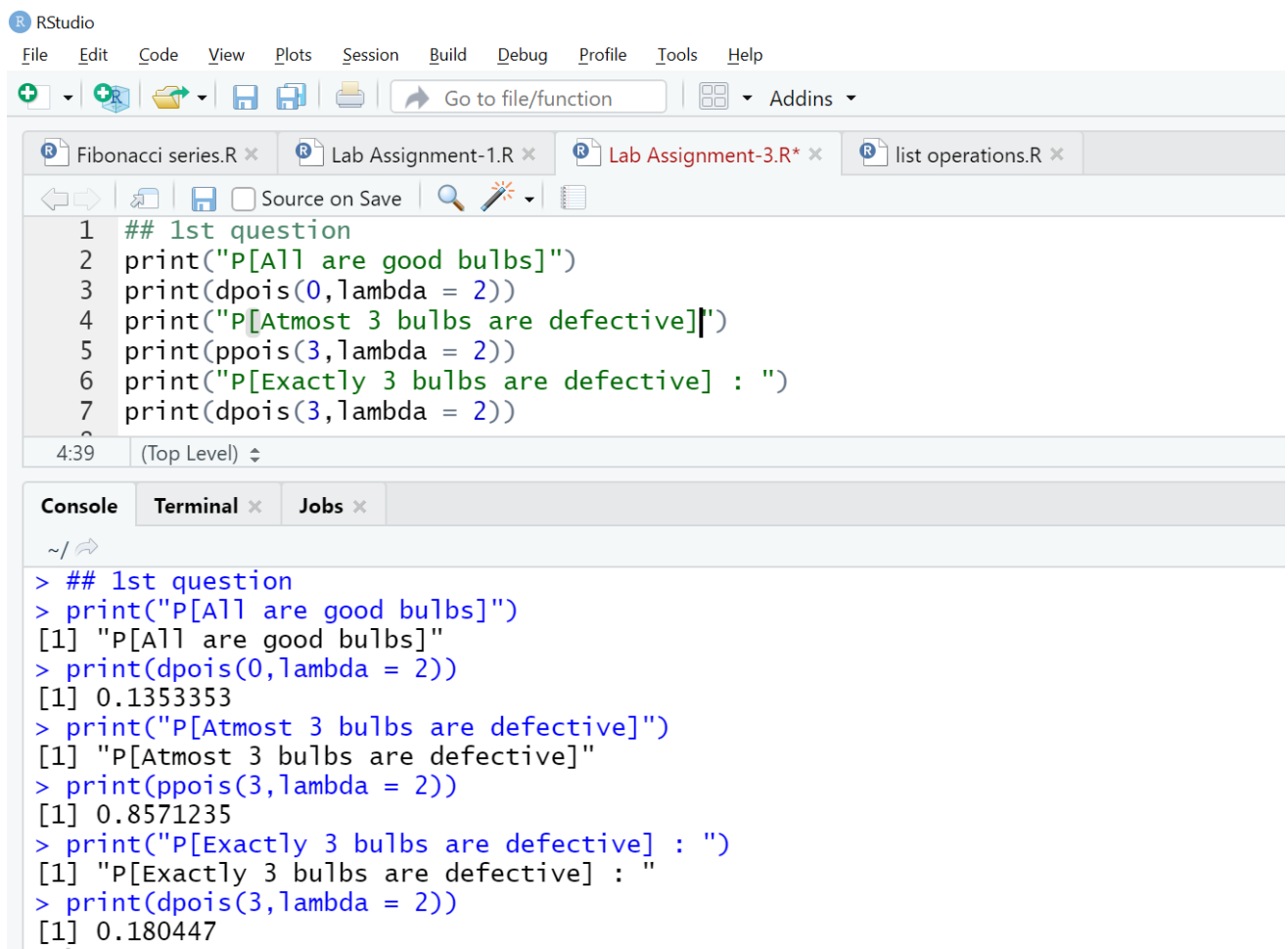
Lab assessment – 3

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1) Write a R code to solve the following questions

- (a) In a large consignment of electric bulbs 10 % are defective. A random sample of 20 is taken for inspection. Find the probability that
- (i) All are good bulbs,
 - (ii) At most there are 3 defective bulbs,
 - (iii) Exactly there are three defective bulbs.



The screenshot shows the RStudio interface. The script editor contains the following R code:

```
1 ## 1st question
2 print("P[All are good bulbs]")
3 print(dpois(0,lambda = 2))
4 print("P[Atmost 3 bulbs are defective]")
5 print(ppois(3,lambda = 2))
6 print("P[Exactly 3 bulbs are defective] : ")
7 print(dpois(3,lambda = 2))
```

The console shows the output of the code:

```
> ## 1st question
> print("P[All are good bulbs]")
[1] "P[All are good bulbs]"
> print(dpois(0,lambda = 2))
[1] 0.1353353
> print("P[Atmost 3 bulbs are defective]")
[1] "P[Atmost 3 bulbs are defective]"
> print(ppois(3,lambda = 2))
[1] 0.8571235
> print("P[Exactly 3 bulbs are defective] : ")
[1] "P[Exactly 3 bulbs are defective] : "
> print(dpois(3,lambda = 2))
[1] 0.180447
```

a) Poisson distribution

$$p \Rightarrow 10\% \Rightarrow 0.1 \quad n \Rightarrow np$$
$$n \Rightarrow 20 \quad \Rightarrow 20 \left(\frac{10}{100} \right) \Rightarrow 2$$

i) $P[\text{all are good bulbs}]$

$$P(X=x) \Rightarrow \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(X=0) \Rightarrow \frac{e^{-2} (2)^0}{0!} \Rightarrow e^{-2} \Rightarrow 0.1353352$$

2) $P[\text{at most 3 bulbs are defective}]$

$$P[X \leq 3] \Rightarrow P[X=0] + P[X=1] + P[X=2] + P[X=3]$$

$$\Rightarrow \frac{e^{-2} (2)^0}{0!} + \frac{e^{-2} (2)^1}{1!} + \frac{e^{-2} (2)^2}{2!} + \frac{e^{-2} (2)^3}{3!}$$

$$\Rightarrow e^{-2} \left[1 + 2 + \frac{2^2}{2} + \frac{8}{6} \right]$$

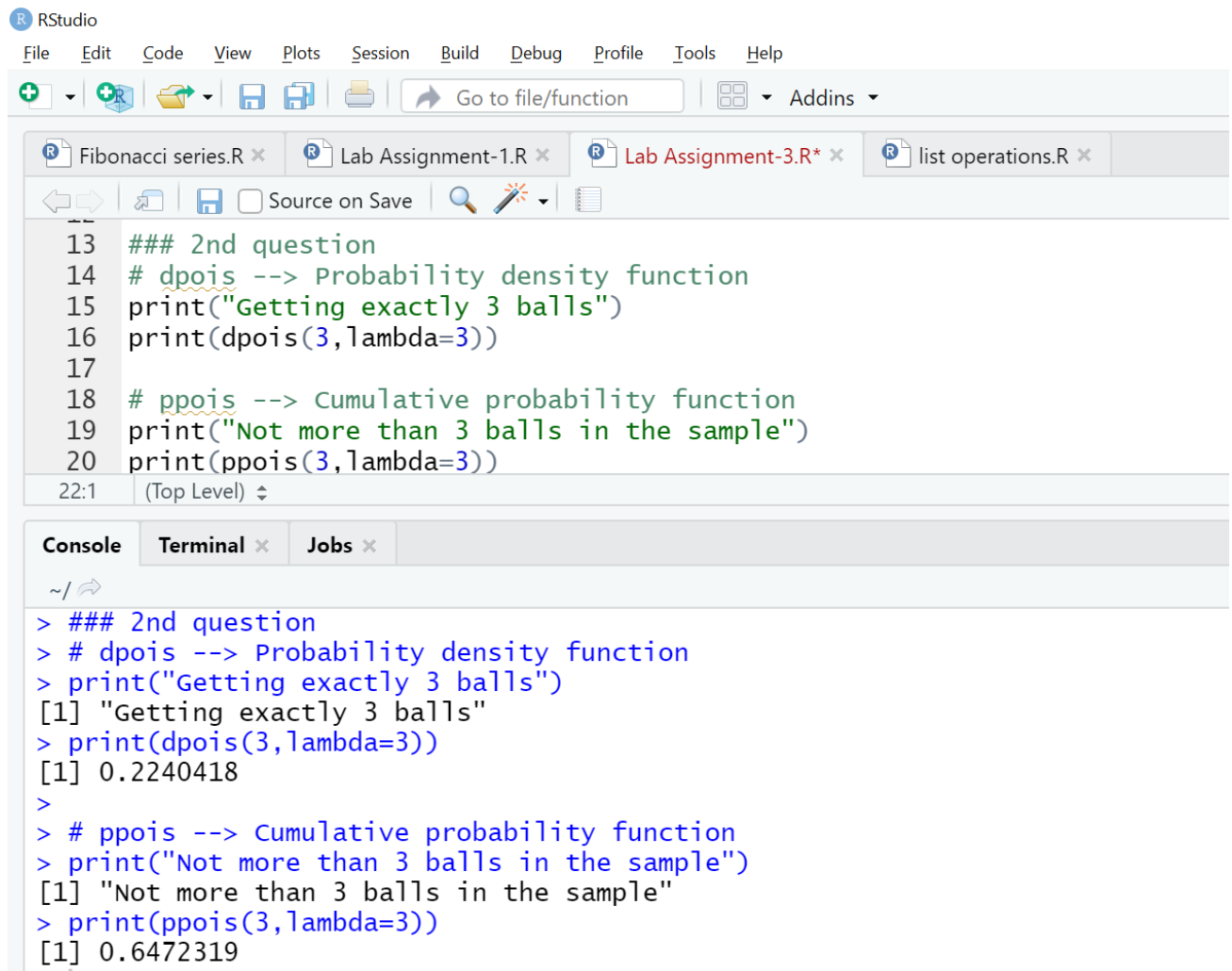
$$\Rightarrow e^{-2} [3 + 2 + 1.33]$$

$$\Rightarrow 0.85712$$

3) $P[X = \text{exactly 3 defective bulb}]$

$$P[X=3] \Rightarrow \frac{e^{-2} (2)^3}{3!} \Rightarrow \frac{e^{-2} (8)}{3 \times 2} \Rightarrow 0.18044$$

- (b) Out of 1000 balls 50 are red and the rest white. If 60 balls are picked at random, what is the probability of picking up (i) 3 red balls (ii) not more than 3 red balls in the sample. Assume poisson distribution for the number of red balls picked up in the sample.



The screenshot shows the RStudio environment. The source editor contains the following R code:

```
13 ### 2nd question
14 # dpois --> Probability density function
15 print("Getting exactly 3 balls")
16 print(dpois(3,lambda=3))
17
18 # ppois --> Cumulative probability function
19 print("Not more than 3 balls in the sample")
20 print(ppois(3,lambda=3))
```

The console shows the output of the code:

```
> ### 2nd question
> # dpois --> Probability density function
> print("Getting exactly 3 balls")
[1] "Getting exactly 3 balls"
> print(dpois(3,lambda=3))
[1] 0.2240418
>
> # ppois --> Cumulative probability function
> print("Not more than 3 balls in the sample")
[1] "Not more than 3 balls in the sample"
> print(ppois(3,lambda=3))
[1] 0.6472319
```

$$b) n \Rightarrow 1000$$

50 \Rightarrow red

950 \Rightarrow white

$$\lambda \Rightarrow np$$

$$\Rightarrow 60 \left(\frac{50}{1000} \right)$$

$$\Rightarrow \frac{30}{10} \Rightarrow 3$$

$$\boxed{\lambda = 3}$$

i) $P[\text{getting 3 red balls}]$

$$P(X=3) \Rightarrow \frac{e^{-3} (3)^3}{3!}$$

$$\Rightarrow \frac{e^{-3} (3 \times 3 \times 3)}{(3 \times 2)}$$

$$\Rightarrow 0.2240$$

ii) $P[\text{not more than 3 balls in sample}]$

$$P[X \leq 3] \Rightarrow 1 - P(X > 3)$$

$$\Rightarrow 1 - [P(X=0) + P(X=1) + P(X=2)]$$

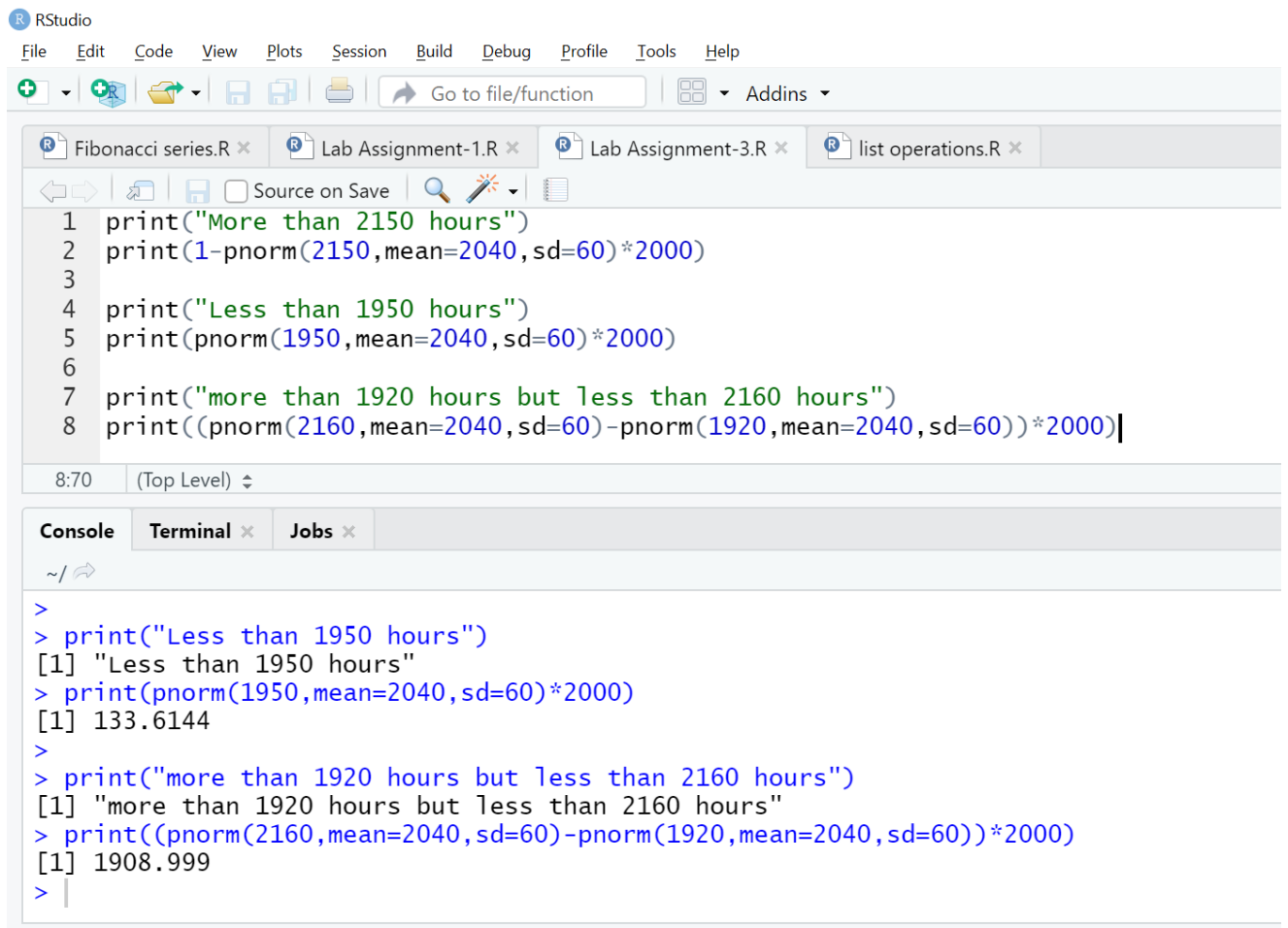
$$\Rightarrow 1 - \left[\frac{e^{-3} (3)^0}{0!} + \frac{e^{-3} (3)^1}{1!} + \frac{e^{-3} (3)^2}{2!} \right]$$

$$\Rightarrow 1 - e^{-3} \left[1 + 3 + \frac{9}{2} \right] \Rightarrow 1 - e^{-3} \left[\frac{11}{2} \right] \Rightarrow 0.5768$$

$$P(X \leq 3)$$

$$0.6473219$$

- (c) In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for
- (i) more than 2150 hours,
 - (ii) less than 1950 hours and
 - (iii) more than 1920 hours but less than 2160 hours.



The screenshot shows the RStudio environment. The script editor contains the following R code:

```
1 print("More than 2150 hours")
2 print(1-pnorm(2150,mean=2040,sd=60)*2000)
3
4 print("Less than 1950 hours")
5 print(pnorm(1950,mean=2040,sd=60)*2000)
6
7 print("more than 1920 hours but less than 2160 hours")
8 print((pnorm(2160,mean=2040,sd=60)-pnorm(1920,mean=2040,sd=60))*2000)|
```

The console shows the execution of the code:

```
>
> print("Less than 1950 hours")
[1] "Less than 1950 hours"
> print(pnorm(1950,mean=2040,sd=60)*2000)
[1] 133.6144
>
> print("more than 1920 hours but less than 2160 hours")
[1] "more than 1920 hours but less than 2160 hours"
> print((pnorm(2160,mean=2040,sd=60)-pnorm(1920,mean=2040,sd=60))*2000)
[1] 1908.999
> |
```

Normal Distribution

c) $n \Rightarrow 2000$

$\mu \Rightarrow 2040$

$\sigma \Rightarrow 60$

i) More than 2150 hours

$$P(X \geq 2150) \Rightarrow P(Z \geq 1.833)$$

$$\Rightarrow 0.5 - P(Z \leq 1.833)$$

$$Z \Rightarrow \frac{X - \mu}{\sigma}$$

$$\Rightarrow 0.5 - [0.4664]$$

$$Z \Rightarrow \frac{2150 - 2040}{60}$$

$$\Rightarrow 0.5336 \Rightarrow 0.0336$$

$$Z \Rightarrow 1.833$$

For 2000 bulbs

$$\Rightarrow \frac{0.5336 * 2000}{0.0336} \Rightarrow 67.2$$

ii) Less than 1950 hours

$$Z \Rightarrow \frac{1950 - 2040}{60}$$

$$Z \Rightarrow -1.5$$

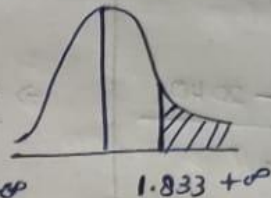
$$P(X \leq 1950) \Rightarrow P(Z \leq -1.5) \Rightarrow P(Z \leq -1.5)$$

$$\Rightarrow 0.5 - (P(Z \leq -1.5))$$

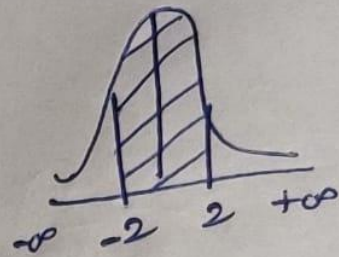
$$\Rightarrow 0.5 - (0.4332)$$

$$\Rightarrow 0.0668 * 2000$$

$$\Rightarrow 133.6$$



$$c) P[1920 \leq X \leq 2160]$$



$$z_1 \Rightarrow \frac{x - \mu}{\sigma}$$

$$\Rightarrow \frac{1920 - 2040}{60}$$

$$z_1 \Rightarrow -2$$

$$z_2 \Rightarrow \frac{x - \mu}{\sigma}$$

$$\Rightarrow \frac{2160 - 2040}{60}$$

$$z_2 \Rightarrow 2$$

$$P(1920 \leq X \leq 2160) \Rightarrow P(-2 \leq Z \leq 2)$$

$$\Rightarrow 2 * P(Z \leq 2)$$

$$\Rightarrow 2 * 0.4772$$

$$\Rightarrow 0.9544 * 2000$$

$$\Rightarrow 1908.88$$