Hypothesis testing

Level of confidence (c) → tells us how sure we are , we have made the right thing.
Eg:99% confidence, we decided to reject the null hypothesis. Then it is concluded it is 99% sure with certainty that rejecting null hypothesis was correct.
Eg : If 50% of confidence, then we decided to reject the null hypothesis, then who is going to believe this???
If the level of confidence is higher than 90%, then only believing the null/alternate hypothesis can be concluded.

Level of significance (α) → 1-level of confidence.
if Level of confidence → 95%, then c=0.95
level of significance α → 1 – c → 1 – 0.95 → 0.05

Level of significance and level of confidence is concluding that How sure are we making the right decision or not all.

P-value method :

Decision Rules:

- If P-value $\leq \alpha$, reject the null hypothesis.
- If P-value $> \alpha$, do not reject the null hypothesis.

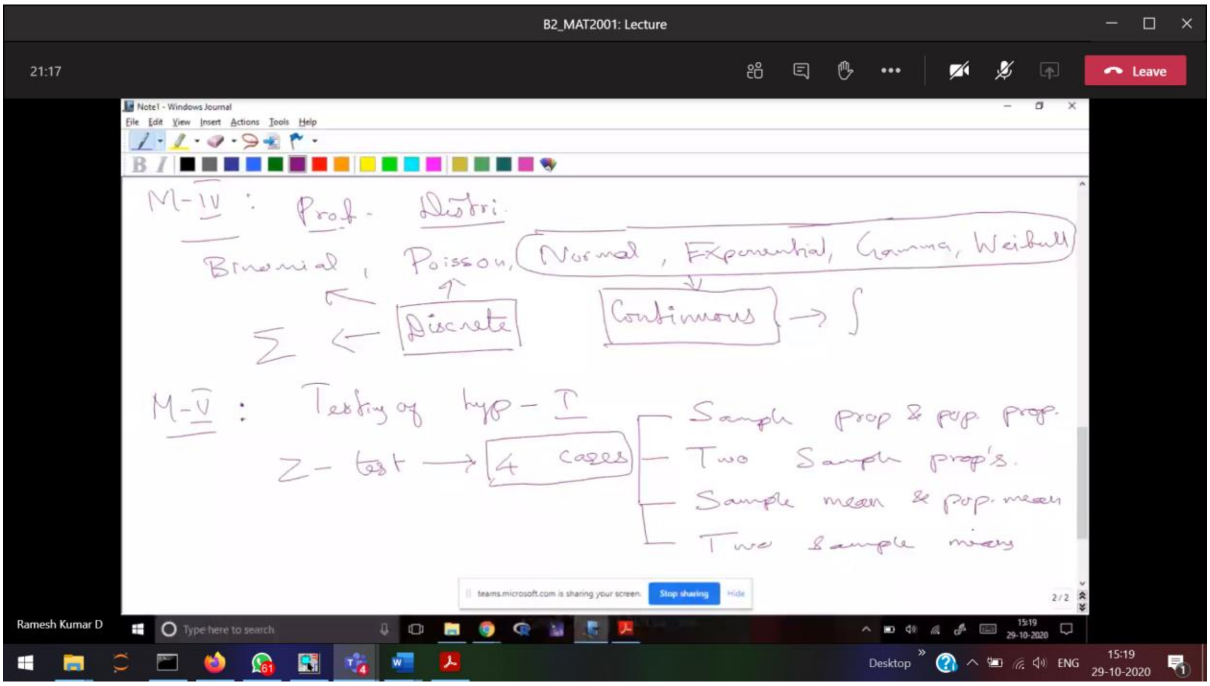
Standard Normal Distribution Table

z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.00000	0.00399	0.00798	0.01197	0.01595	0.01994	0.02392	0.02790	0.03188	0.03586
0.1	0.03983	0.04380	0.04776	0.05172	0.05567	0.05962	0.06356	0.06749	0.07142	0.07535
0.2	0.07926	0.08317	0.08706	0.09095	0.09483	0.09871	0.10257	0.10642	0.11026	0.11409
0.3	0.11791	0.12172	0.12552	0.12930	0.13307	0.13683	0.14058	0.14431	0.14803	0.15173
0.4	0.15542	0.15910	0.16276	0.16640	0.17003	0.17364	0.17724	0.18082	0.18439	0.18793
0.5	0.19146	0.19497	0.19847	0.20194	0.20540	0.20884	0.21226	0.21566	0.21904	0.22240
0.6	0.22575	0.22907	0.23237	0.23565	0.23891	0.24215	0.24537	0.24857	0.25175	0.25490
0.7	0.25804	0.26115	0.26424	0.26730	0.27035	0.27337	0.27637	0.27935	0.28230	0.28524
0.8	0.28814	0.29103	0.29389	0.29673	0.29955	0.30234	0.30511	0.30785	0.31057	0.31327
0.9	0.31594	0.31859	0.32121	0.32381	0.32639	0.32894	0.33147	0.33398	0.33646	0.33891
1.0	0.34134	0.34375	0.34614	0.34849	0.35083	0.35314	0.35543	0.35769	0.35993	0.36214
1.1	0.36433	0.36650	0.36864	0.37076	0.37286	0.37493	0.37698	0.37900	0.38100	0.38298
1.2	0.38493	0.38686	0.38877	0.39065	0.39251	0.39435	0.39617	0.39796	0.39973	0.40147
1.3	0.40320	0.40490	0.40658	0.40824	0.40988	0.41149	0.41308	0.41466	0.41621	0.41774
1.4	0.41924	0.42073	0.42220	0.42364	0.42507	0.42647	0.42785	0.42922	0.43056	0.43189
1.5	0.43319	0.43448	0.43574	0.43699	0.43822	0.43943	0.44062	0.44179	0.44295	0.44408
1.6	0.44520	0.44630	0.44738	0.44845	0.44950	0.45053	0.45154	0.45254	0.45352	0.45449
1.7	0.45543	0.45637	0.45728	0.45818	0.45907	0.45994	0.46080	0.46164	0.46246	0.46327
1.8	0.46407	0.46485	0.46562	0.46638	0.46712	0.46784	0.46856	0.46926	0.46995	0.47062
1.9	0.47128	0.47193	0.47257	0.47320	0.47381	0.47441	0.47500	0.47558	0.47615	0.47670
2.0	0.47725	0.47778	0.47831	0.47882	0.47932	0.47982	0.48030	0.48077	0.48124	0.48169
2.1	0.48214	0.48257	0.48300	0.48341	0.48382	0.48422	0.48461	0.48500	0.48537	0.48574
2.2	0.48610	0.48645	0.48679	0.48713	0.48745	0.48778	0.48809	0.48840	0.48870	0.48899
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983

3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49992	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
4.0	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998

STANDARD NORMAL DISTRIBUTION

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4965	.4985	.4986	.4986
3.0	.49865	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.49903	.4990	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993129	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995166	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4996631	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998	.4998
3.5	.4997674	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998409	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4998922	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999277	.4999	.4999	.4999	.4999	.4999	.4999	.5000	.5000	.5000
3.9	.4999519	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000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Note1 - Windows Journal
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M-IV : Prob. Distri.
Binomial, Poisson, Normal, Exponential, Gamma, Weibull
Σ ← Discrete
Continuous → ∫
M-V : Testing of hyp - I
Z - test → 4 cases
Sample prop & pop. prop.
Two Sample prop's.
Sample mean & pop. mean
Two Sample means
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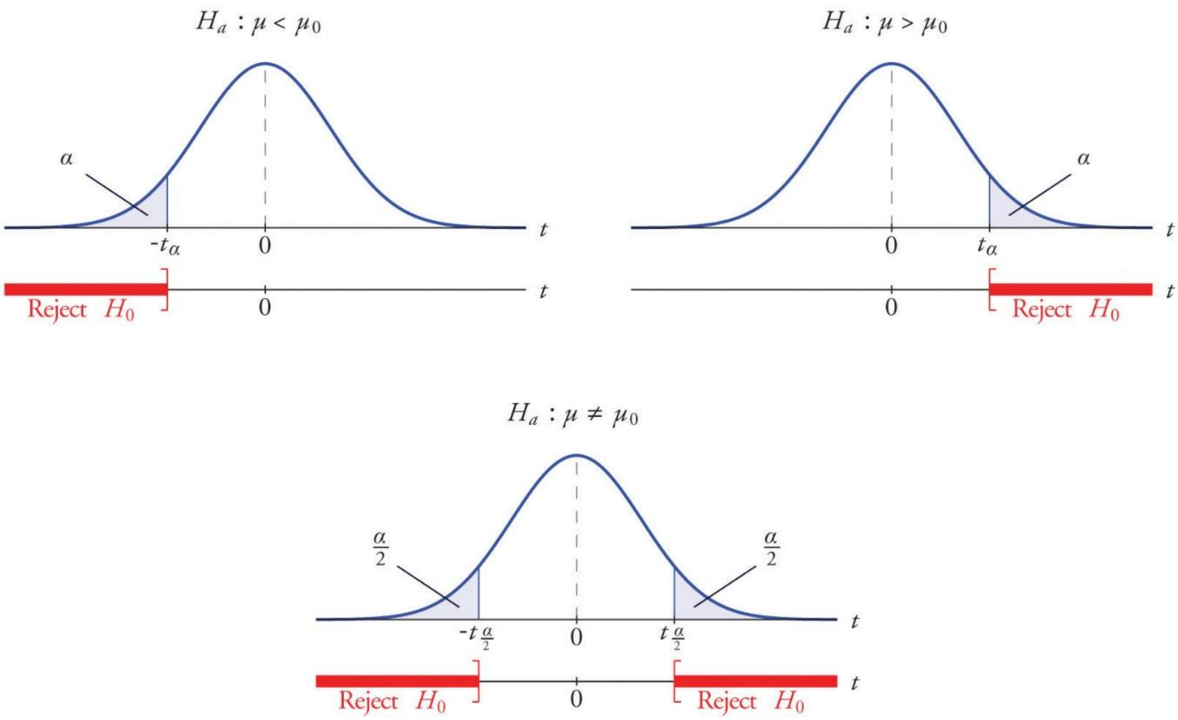

$H_a : \mu < \mu_0$
α
$-t_\alpha$
0
t
Reject H_0
$H_a : \mu > \mu_0$
t_α
$H_a : \mu \neq \mu_0$
$\frac{\alpha}{2}$
$-t_{\frac{\alpha}{2}}$
$t_{\frac{\alpha}{2}}$

Case 1: Test of significance of the difference between sample proportion and population proportion.

P - Population Proportion

p - Sample Proportion

n - Sample Size

(i) Test Statistic $z = \dfrac{p - P}{\sqrt{\dfrac{PQ}{n}}}$, where $Q = 1 - P$.

(ii) When P is not known, the 95 percent confidents limits for P are given by

$$p - 1.96\sqrt{\frac{pq}{n}} \leq P \leq p + 1.96\sqrt{\frac{pq}{n}}$$

Case 2: Test of significance of the difference between two sample proportions.

$$\text{Test Statistic } z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}},$$

where $P = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ and $Q = 1 - P$.

Case 3: Test of significance of the difference between sample mean and population mean.

$\overline{x}$ - Sample Mean, n - Sample Size, μ - Population Mean and σ - Population Standard Deviation (S.D.)

$$\text{Test Statistic } z = \frac{\overline{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$$

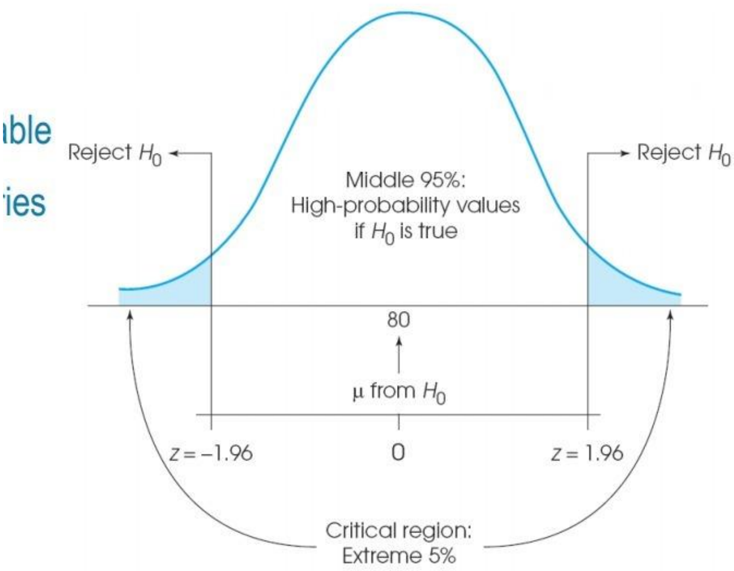
Note: If σ is not known, the sample S.D. 's' can be used as 's' tends to σ when n is sufficiently large.

Case 4: Test of significance of the difference between two sample means.

$$\text{Test Statistic } z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

Note:

1. If the samples are drawn from the same population, that is, $\sigma = \sigma_1 = \sigma_2$, then
$$z = \frac{\overline{x_1} - \overline{x_2}}{\sigma\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$
2. If σ_1 and σ_2 are not known, then σ_1 and σ_2 can be approximated by the sample S.D.'s s_1 and s_2. Now,
$$z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$



Z = Z(alpha) → H0 is rejected

1)

The CEO of a large electric utility claims that atleast 80 percent of his 10,00,000 customers are very satisfied with the service they receive. To test this claim, the local newspaper surveyed 100 customers, using simple random sampling. Among the sampled customers, 73 percent say they are very satisified. Based on these findings, can we reject the CEO's hypothesis that 80 percent of the customers are very satisfied? Use a 0.05 level of significance.

P = 0.8

p = 0.73

n = 100

H0 : P>=0.80 (at-least 80%)

H1 : P<0.8

In this problem, $P = 0.8, p = 0.73$ and $n = 100$.

Step 1: Null hypothesis (H_0) : $P \geq 0.80$, that is, atleat 80 percent customers are satisfied.

Alternative hypothesis (H_1) : $P < 0.80$

Step 2: Note that these hypotheses constitute a one-tailed (left-tailed) test.

Step 3: As it is given $\alpha = 0.05$ and one-tailed test, we have $z_\alpha = -1.645$.

Step 4: Test Statistic $z = \dfrac{p - P}{\sqrt{\dfrac{PQ}{n}}} = \dfrac{0.73 - 0.8}{\sqrt{\dfrac{0.8 \times 0.2}{100}}} = -1.75.$

Step 5: It can be viewed that $|z| = 1.75 > 1.645 = |z_\alpha|$. Therefore, we reject the null hypothesis H_0, that is, H_1 is accepted.

$\therefore$ The CEO's claim is wrong.

2)

Experience has shown that 20 percent of a manufactured product is of top quality. In one day's production of 400 articles, only 50 are of top quality. Show that either the production of the day chosen was not a representative sample or the hypothesis of 20 percent was wrong.

Experience
20% top quality

1 day's production
400 articles
50 article → top quality
(50/400)*100 → 12.5% (top quality)

P = 0.2
p = 0.125
n = 400
H0 : P=0.20 (correct)
H1 : P≠0.20 (wrong)

Step 1: Null Hypothesis (H_0) : $P = \frac{1}{5}$, that is, 20 percent of the products manufactured is of top quality.
Alternative Hypothesis (H_1) : $P \neq \frac{1}{5}$, that is, 20 percent of the products manufactured is not of top quality.
Step 2: Assume that $\alpha = 5\%$ and one can note that the type of test is two-tailed based on H_1.
Step 3: Since it is two-tailed test and $\alpha = 5\%$, $z_\alpha = 1.96$.

Step 4: Test Statistic $= z = \dfrac{p - P}{\sqrt{\dfrac{PQ}{n}}} = \dfrac{\frac{1}{8} - \frac{1}{5}}{\sqrt{\frac{1}{5} \times \frac{4}{5} \times \frac{1}{400}}} = -3.75.$

Step 5: Now, $|z| = 3.75 > 1.96 = |z_\alpha|$ which implies that H_0 is rejected (or H_1 is accepted).
$\therefore$ The production of the particular day chosen was not a representative sample.

3)

A recent article in a weekly magazine reported that a job awaits 33% of new college graduates. A survey of 200 recent graduates from your college revealed that 80 students had jobs. At a 99% level of confidence, can we conclude that a larger proportion of students at your college have jobs?

New college graduates
33% job offer

Recent survey
Out of 200 only 80 got job (40%)

P = 0.33
p = 0.4
n = 200
H0 : P=0.33 (correct)
H1 : P>0.33 (wrong)

Ans : College's claim is correct

Step 1: $H_0 : P = 0.33$ and $H_1 : P > 0.33$
Step 2: It is one-tailed (right-tailed) test.
Step 3: Here, $\alpha = 1\%$. So $z_\alpha = 2.33$.
Step 4:

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.4 - 0.33}{\sqrt{0.33 \times 0.67 \times \frac{1}{200}}} = 2.1021.$$

Step 5: $|z| = 2.1021 < 2.33 = |z_\alpha|$ implies that H_0 is not rejected. Therefore, we do not have enough evidence to state that a larger proportion of students at our college have jobs.

1. A salesman in a departmental store claims that at most 60 percent of the shoppers entering the store leaves without making a purchase. A random sample of 50 shoppers showed that 35 percent of them left without making a purchase. Are these sample results consistent with the claim of the salesman at a level of significance of 0.05?

Ho: p=0.6
Ha: p is not 0.6
x=35 when n=50 implies that p-hat=35/50=0.7
z(0.7)=(0.7-0.6)/sqrt[(0.6*0.4)/50)=0.1/sqrt0.0048=0.1/0.069=1.44

p value = 0.07
Since p value > alpha, Fail to Reject Ho.
Conclusion.
The sales clerk's opinon is about right.
Cheers,
Stan H.

2. A cubical die is thrown 900 times and a throw of three or four is observed 3240 times. Show that the die cannot be regarded as unbiased one and find the extreme limits between which the probability of a throw of three or four lies.

Example:

The fatality rate of typhoid patients is believed to be 17.26 per cent. In a certain year 640 patients suffering from typhoid were treated in a metropolitan hospital and only 63 patients died. Can you consider the hospital efficient?

General   Metropolitan Hospital

Death → 17.26%   640 patients

63 died   (63/640)*100 → 9.8%

P = 0.1726

p = 0.098

n = 640

H0 : P=0.1726   (believed)

H1 : P>p   (Efficient hospital → When the death rate of the hospital is low)

Ans : Hospital is efficient

$$z \Rightarrow \frac{p-P}{\sqrt{PQ/n}}$$

$$\Rightarrow \frac{0.098 - 0.1726}{\sqrt{\frac{(0.1726)(0.824)}{640}}} \Rightarrow \frac{-0.0742}{0.0149071} \Rightarrow -4.977$$

$$\boxed{z \Rightarrow -4.977}$$

$\alpha \Rightarrow 0.01$ $|Z_\alpha| \Rightarrow 2.33$

$|z| \Rightarrow 4.977$

$|Z_\alpha| < |z|$ (or) $|z| > |Z_\alpha|$

H_0 is ~~accepted~~ rejected and H_1 is accepted.

So the hospital is efficient.

Exercise:

A salesman in a departmental store claims that at most 60 percent of the shoppers entering the store leaves without making a purchase. A random sample of 50 shoppers showed that 35 of them left without making a purchase. Are these sample results consistent with the claim of the salesman? Use a level of significance of 0.05.

Salesman claims
Atmost 60%

Random Sample
(35/50)*100 → 70%

P = 0.6

p = 0.7

n = 50

H0 : P=0.6 (atmost) **[p=P]**

H1 : P>0.6 (without making a purchase) **[p>P]**

Ans : Sales clerk opinion is right

Question

b) A die is thrown 9000 times and the outcome of 3 or 4 is observed 3240 times. Show that the die cannot be regarded as an unbiased one and find the limits between which the probability of a throw of 3 or 4 lies.

Solution

H_0: the die is unbiased, i.e. $P = \frac{1}{3}$ (the probability of a throw of 3 or 4).

$H_a: P \neq \frac{1}{3}$.

Two-tailed test is to be used.

Let LOS be 5%. Therefore, $z_\alpha = 1.96$.

Although we may test the significance of the difference between the sample and population proportions, we shall test the significance of the difference between the number of successes X in the sample and that in the population.

When n is large, X follows $N(np, \sqrt{nPQ})$.

$$z = \frac{X - nP}{\sqrt{nPQ}} = \frac{3240 - 9000 \cdot \frac{1}{3}}{\sqrt{9000 \cdot \frac{1}{3} \cdot \frac{2}{3}}} = 5.37$$

$$|z| > z_\alpha$$

Therefore, the difference between X and nP is significant, i.e., H_0 is rejected.

That is, the dice cannot be regarded as unbiased.

If X follows $N(\mu, \sigma)$, then $P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) = 0.9974$.

The limits $\mu \pm 3\sigma$ are considered as the extreme (confidence) limits within which X lies.

Accordingly, the extreme limits for P are given by

$$\frac{|P - p|}{\sqrt{\frac{pq}{n}}} \leq 3$$

$$p - 3\sqrt{\frac{pq}{n}} \leq P \leq p + 3\sqrt{\frac{pq}{n}}$$

$$0.36 - 3\sqrt{\frac{0.36 \cdot 0.64}{9000}} \leq P \leq 0.36 + 3\sqrt{\frac{0.36 \cdot 0.64}{9000}}$$

$$0.345 \leq P \leq 0.375.$$

Case-2 (Difference of proportions)

Case 2: Test of significance of the difference between two sample proportions.

$$\text{Test Statistic } z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}},$$

where $P = \dfrac{n_1p_1 + n_2p_2}{n_1 + n_2}$ and $Q = 1 - P$.

Example

AAA investigated the question of whether a man or a woman would be more likely to stop and ask for directions. A random sample showed that 300 of 811 women would stop and ask for directions while 255 of 750 men indicated they would. At the $\alpha = .05$ significance level, test the claim that women are more likely to ask for directions.

women

$p_1 \Rightarrow 300/811$

$p_1 \Rightarrow 0.3699$

$n_1 \Rightarrow 811$

men

$p_2 \Rightarrow 255/750$

$p_2 \Rightarrow 0.34$

$n_2 \Rightarrow 750$

Significance level : $\alpha \Rightarrow 0.05$

$H_0 : p_1 \leq p_2$

$H_1 : p_1 > p_2$ (Women are more likely to ask for directions)

$\boxed{\text{Right tailed test}}$

$$P \Rightarrow \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$\Rightarrow \frac{(811 * (0.3699)) + (750 * (0.34))}{811 + 750}$$

$\boxed{P \Rightarrow 0.355}$

$Q \Rightarrow 1-P \Rightarrow 1-0.355$

$\boxed{Q \Rightarrow 0.6444}$

$$z \Rightarrow \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\Rightarrow \frac{(0.3699 - 0.34)}{\sqrt{(0.355) * (0.644)\left(\frac{1}{811} + \frac{1}{750}\right)}}$$

$$\Rightarrow \frac{0.0299}{\sqrt{0.22862 * 2.56637 * 10^{-3}}}$$

$$\Rightarrow \frac{0.0299}{0.0242224}$$

$$\boxed{z \Rightarrow 1.23}$$

$z_\alpha \Rightarrow 1.645$

$|z| < |z_\alpha|$

H_0 is accepted

H_1 is rejected. Therefore we cannot support women's claim are more likely to ask for the directions.

Suppose the RK Drug Company develops a new drug, designed to prevent Covid19. The company states that the drug is more effective for women than for men. To test this claim, they choose a a simple random sample of 100 women and 200 men from a population of 100,000 volunteers.
At the end of the study, 38% of the women caught a Covid19 and 51% of the men caught a Covid19. Based on these findings, can we conclude that the drug is more effective for women than for men? Use a 0.01 level of significance.

p1 → 38% and n1 → 100 (women)
p2 → 51% and n2 → 200 (men)
H0 → p1>=p2
H1 → p1<p2

Assume that p_1 represents the effectiveness of drug on women and p_2 represents the effectiveness of drug on men. In this problem, $n_1 = 100, p_1 = 0.38, n_2 = 200$ and $p_2 = 0.51$.
Step 1: Null hypothesis (H_0): $p_1 \geq p_2$
Alternative hypothesis (H_1): $p_1 < p_2$.
Step 2: Note that these hypotheses constitute a left-tailed test.
Step 3: As we have left-tailed test and the significance level is 0.01, $z_\alpha = -2.33$.
Step 4: Test Statistic $(z) = \dfrac{(p_1 - p_2)}{\sqrt{PQ\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$,

where $P = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$.
By simple calculation, one can note that $P = 0.467$ and $Q = 0.533$ which yields $z = -2.13$.

Step 5: Comparing z and z_α values, we obtain

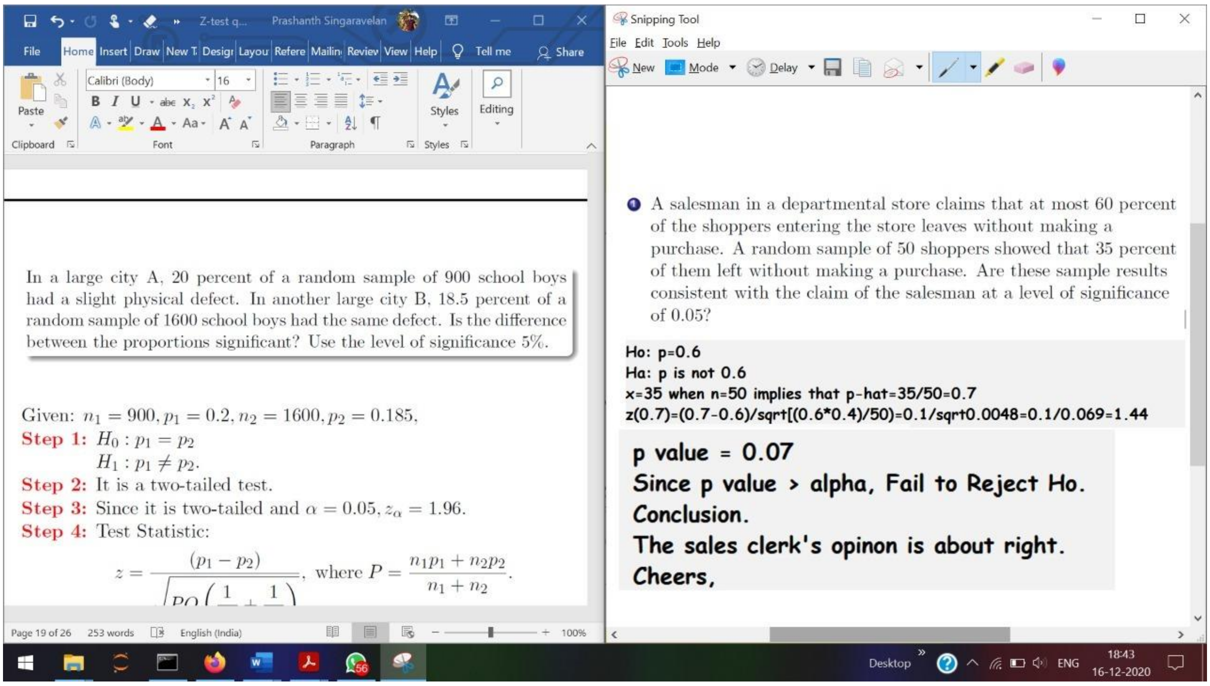
$$|z| = 2.13 < 2.33 = |z_\alpha|$$

which shows that H_0 is accepted. That is, the claim is true.

Exercise:

(1) Consider the previous problem (ie, problem 4) with same data and test the claim that the drug is equally effective for men and women.

Doubt



In a large city A, 20 percent of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5 percent of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant? Use the level of significance 5%.

Given: $n_1 = 900, p_1 = 0.2, n_2 = 1600, p_2 = 0.185,$

Step 1: $H_0 : p_1 = p_2$

$H_1 : p_1 \neq p_2.$

Step 2: It is a two-tailed test.

Step 3: Since it is two-tailed and $\alpha = 0.05, z_\alpha = 1.96.$

Step 4: Test Statistic:

$$z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \text{ where } P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}.$$

This implies that $z = \dfrac{(0.2 - 0.185)}{\sqrt{(0.1904)(0.8096)\left(\frac{1}{900} + \frac{1}{1600}\right)}} = 0.92.$

Step 5: Clearly, $|z| < |z_\alpha|$. Thus, H_0 is accepted, that is, H_1 is rejected.

956 children were born in a city A in one year out of which 52.5% were male, while 1406 children were born in cities A and B both out of which proportion of male was 0.496. Is the difference in the proportion of male children in two cities significant?

Sample-1 → City A
Sample-2 → City B
Population → Sample-1 + Sample-2

Here, $n_1 = 956, p_1 = 52.5\%, n_2 = 1406 - 956 = 450$ and $P = 0.496 \Rightarrow Q = 1 - P = 0.504$.
We have,

$$P = \frac{n_1p_1 + n_2p_2}{n_1 + n_2} \Rightarrow 0.496 = \frac{(956)(0.525) + (450)(p_2)}{956 + 450} \Rightarrow p_2 = 0.432$$

Step 1: $H_0 : p_1 = p_2$ and $H_1 : p_1 \neq p_2$.
Step 2: Two-tailed Test will be used in this case.
Step 3: Let $\alpha = 5\%$, then $z_\alpha = 1.96$. as the type of the test is two-tailed.
Step 4: Test Statistic:

$$z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 3.268.$$

Step 5: Note that $|z| > |z_\alpha|$ which shows that H_1 is accepted.

1. Before an increase in excise duty on tea, 800 people out of a sample of 1000 were consumers of tea. After the increase in duty, 800 people were consumers of tea in a sample of 1200 persons. Find whether there is significant decrease in the consumption of tea after the increase in duty.

n1=1000 n2=1200

p1 = 800/1000 → 0.8 p2=800/1200 → 0.667

H0 → p1=p2 (there is no significance decrease in the consumption of tea before and after increase in the duty)

H1 → p1>p2 (there is significant decrease in the consumption of tea after the increase in duty)

(right tailed test)

Example 2. Before an increase in excise duty on tea, 400 people out of a sample of 500 persons were found to be tea drinkers. After an increase in excise duty, 400 people were observed to be tea drinkers in a sample of 600 people. Test whether there is a significant change in the number of tea drinkers after increase in excise duty on tea.

Solution: In this example $X_1 = 400, n_1 = 500, X_2 = 400, n_2 = 600$

H_0: $P_1=P_2$ i.e., there is no change in the number of tea drinkers after increase in excise duty on tea

H_1: $P_1 \neq P_2$

Here we shall use standard normal deviate (Z) test for difference of proportions as under:

In our example $p_1 = 400/500=0.8$, $p_2 = 400/600=0.6667$

$q_1 = 1 - p_1 = 0.2$, $q_2 = 1 - p_2 = 0.333$

$$Z = \frac{(p_1 - p_2)}{\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}} = \frac{0.8 - 0.6667}{\sqrt{\frac{(0.8)(0.2)}{500} + \frac{(0.6667)(0.333)}{600}}} = \frac{0.1333}{0.0263} = 5.07$$

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Since calculated value of Z statistic is greater than 3, therefore H_0 is rejected at all levels of significance which implies that there is a significant change in the number of tea drinkers after increase in excise duty on tea. It is further observed that the number of tea drinkers have significantly declined after increase in excise duty on tea which is due to decrease in thevalue of p_2 (0.667) from the value of p_1 (0.8).

2. 15.5 percent of a random sample of 1600 undergraduates were smokers, whereas 20 percent of a random sample 900 postgraduates were smokers in a state. Can we conclude that less number of undergraduates were smokers than the postgraduates?

p1=0.155 p2=0.2

n1=1600 n2=900

H0 → p1=p2

H1 → p1<p2

$$z \Rightarrow \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$P \Rightarrow \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$\Rightarrow \frac{1600(0.155) + 900(0.2)}{1600 + 900}$$

$$\Rightarrow \frac{0.155 - 0.2}{\sqrt{(0.1712) * (0.8288)\left(\frac{1}{1600} + \frac{1}{900}\right)}}$$

$$\boxed{P \Rightarrow 0.1712}$$

$$Q \Rightarrow 1 - P$$

$$\boxed{Q \Rightarrow 0.8288}$$

$$\Rightarrow \frac{-0.045}{\sqrt{0.14189\left(\frac{1}{576}\right)}} \Rightarrow -2.8671$$

$$\boxed{z \Rightarrow -2.8671}$$

$\alpha \Rightarrow 0.05$

$$\boxed{z_\alpha \Rightarrow -1.645}$$

$|z| > |z_\alpha|$

H_0 is rejected

H_1 is accepted

We can conclude that less number of undergraduates were smokers than post graduates.

One thousand articles from a factory are examined and 30 were found defective. 1500 similar articles from the second factory were examined where 300 were found defective. Can it be reasonably concluded that the products of the first factory are inferior to the second?

$n_1 \Rightarrow 1000$

Defective $\Rightarrow 30$

$p_1 \Rightarrow 30/1000$

$\boxed{p_1 \Rightarrow 0.03}$

$n_2 \Rightarrow 1500$

Defective $\Rightarrow 300$

$p_2 \Rightarrow 300/1500 \Rightarrow 0.2$

$\boxed{p_2 \Rightarrow 0.2}$

Null hypothesis : $\cancel{p_1 \neq p_2}$ $p_1 = p_2$

Alternate hypothesis : $p_1 < p_2$ (left tail)

$$P \Rightarrow \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$\Rightarrow \frac{1000(0.03) + 1500(0.2)}{1000 + 1500}$$

$$\boxed{P \Rightarrow 0.132}$$

$$Q \Rightarrow 0.868$$

$$Z \Rightarrow \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\Rightarrow \frac{0.03 - 0.2}{\sqrt{(0.132) * (0.868) * \left(\frac{1}{1000} + \frac{1}{1500}\right)}}$$

$$z \Rightarrow \cancel{-0.11}\ -12.302$$

$|z| \Rightarrow 12.302$

$\alpha \Rightarrow 0.05$ $|z_\alpha| \Rightarrow 1.645$

$|z| > |z_\alpha|$ H_0 is rejected & H_1 is accepted.

It is clear that products from the 1st factory are inferior to the second.

Case-3(Test of significance of the difference between the sample and population mean)

Case 3: Test of significance of the difference between sample mean and population mean.

$\overline{x}$ - Sample Mean, n - Sample Size, μ - Population Mean and σ - Population Standard Deviation (S.D.)

$$\text{Test Statistic } z = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Note: If σ is not known, the sample S.D. 's' can be used as 's' tends to σ when n is sufficiently large.

1) An insurance agent has claimed that the average age of policy holders who insure through him is less than the average for all agents which is 30.5 years. A random sample of 100 policy holders who had insured through him reveal that the mean and S.D. are 28.8 years and 6.35 years respectively. Test his claim at 5% level of significance.

Here, $n = 100, \mu = 30.5, s = 6.35, \overline{x} = 28.8$ and $\alpha = 5\%$.

Step 1: Null Hypothesis $(H_0) : \mu = 30.5$

Alternative Hypothesis $(H_0) : \mu < 30.5$

Step 2: Type of test is one-tailed (left-tailed).

Step 3: Note that $z_\alpha = -1.645$ as it is one-tailed and $\alpha = 0.05$.

Step 4: The test statistic:

$$z = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{28.8 - 30.5}{6.23/10} = -2.68.$$

Step 5: The computed value $z = -2.68$ falls in the critical/rejection area, that is, $|z| > |z_\alpha|$. So we reject H_0.

The same question without sample mean standard deviation

Example 9 : An insurance agent has claimed that the average age of policy holders who issue through him is less than the average for all agents which is 30.5 years. A random sample of 100 policy holders who had issued through him gave the following age distribution.

Age	16-20	21-25	26-30	31-35	36-40
No. of persons	12	22	20	30	16

(100)

Class interval	frequency (f)	x (Mid)	$d \Rightarrow \frac{x-28}{5}$	fd	fd^2
15.5 - 20.5	12	18	-2	-24	48
20.5 - 25.5	22	23	-1	-22	22
25.5 - 30.5	20	28	0	0	0
30.5 - 35.5	30	33	1	30	30
35.5 - 40.5	16	38	2	32	64
				$\Sigma fd \Rightarrow 16$	$\Sigma fd^2 \Rightarrow 164$

$$\text{Mean} \Rightarrow A + \left(\frac{\Sigma fd}{N}\right) c$$

$$\Rightarrow 28 + \left(\frac{16}{100}\right) * 5$$

$$\boxed{\text{Mean} \Rightarrow 28.8}$$

$$\text{Standard deviation} \Bigg\} \ \sigma \Rightarrow \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} * C$$

$$\Rightarrow \sqrt{\frac{164}{100} - \left(\frac{16}{100}\right)^2} * 5$$

$$\Rightarrow \sqrt{1.64 - 0.0256} * 5$$

$$\Rightarrow 1.2705 * 5$$

$$\boxed{\sigma \Rightarrow 6.35}$$

Calculate the Arithmetic mean and Standard deviation of this distribution and use these values to test his claim at 5% level of significance.

Solution : Take $A = 28,\ d_i = x_i - A$

$$\therefore \quad \text{A.M.} = \bar{x} = A + \frac{h\sum f_i d_i}{N} = 28 + \frac{5\times 16}{100} = 28.8$$

$$S.D : S = h\sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} = 5 \,.\, \sqrt{\frac{164}{100} - \left(\frac{16}{100}\right)^2} \quad [\because h = 5]$$

$$= 6.35$$

1. **Null Hypothesis H_0 :** The sample is drawn from a population with mean μ i.e. $\bar{x}$ and μ do not differ significantly where $\mu = 30.5$ years.

2. **Alternative Hypothesis H_1 :** $\mu < 30.5$ years (left tail test)

Now, $\bar{x} = 28.8$, $S = 6.35$, $\mu = 30.5$ years and $n = 100$

3. **The test statistic** is, $Z = \dfrac{\bar{x}-\mu}{S/\sqrt{n}} = \dfrac{28.8 - 30.5}{6.35/\sqrt{100}} = -\ 2.677$

$\therefore \quad |Z| = 2.68.$

Tabulated value of Z at 5% level of significance is 1.645 (left tail test).

Here calculated $Z >$ tabulated Z.

$\therefore$ The Null hypothesis H_0 is rejected.

i.e., $\bar{x}$ and μ differ significantly.

i.e., The sample is not drawn from a population with mean $\mu = 30.5$ years

2) A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that, in the population, the mean height is 165 cm and S.D. is 10 cm? LOS is 1%.

From the sample's view it is asked that in population also, is the mean height=165. [Comparing the sample mean and population mean]

1. A sample of 900 members has a mean 3.4 cm and S.D. 2.61 cm. Is the sample from a large population of mean 3.25 cm and S.D. of 2.61 cm? (Test at 5% level of significance.)

Here, $n = 160, \overline{x} = 160, \mu = 165$ and $\sigma = 10$.

Step 1: $H_0 : \overline{x} = \mu$ and $H_1 : \overline{x} \neq \mu$.

Step 2: In this case, we will use two-tailed test based on H_1.

Step 3: Type of test and LOS value imply that $|z_\alpha| = 2.58$.

Step 4: The test statistic:

$$z = \frac{\overline{x} - \mu}{\sigma\sqrt{n}} = \frac{160 - 165}{10/\sqrt{100}} = -5.$$

Step 5: Comapring the tabulated and calculated values of z, we have $|z| > |z_\alpha|$. That is, we reject H_0 at $\alpha = 0.01$.

3)

The guaranteed average life of a certain type of electric light bulbs is 1000 hours with a S.D. of 125 hours. It is decided to sample the output so as to ensure that 90 percent of the bulbs do not fall short of the guaranteed average by more than 2.5 percent. What must be the minimum size of the sample?

Given: $\mu = 1000$ hours and $\sigma = 125$ hours.
Since we do not want the sample mean to be less than the guaranteed average, ie, $\mu = 1000$, by more than 2.5 percent, we have,

$$\overline{x} > 1000 - 2.5\% \text{ of } 1000 = 1000 - 25 = 975.$$

Let n be the size of the sample then

$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} \text{ follows } N(0,1), \text{ as sample is sufficiently large.}$$

We want

$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} > \frac{975 - 1000}{125/\sqrt{n}} > -\frac{\sqrt{n}}{5}.$$

According to the given condition, we have

$$P(z > -\sqrt{n}/5) = 0.90 \Rightarrow P(0 < z < -\sqrt{n}/5) = 0.40$$

$\therefore \sqrt{n}/5 = 1.28$ (From Normal Probability Table) which implies $n = 41$ approximately.

4) A sample of 900 members has a mean 3.4 cm and S.D. 2.61 cm. Is the sample from a large population of mean 3.25 cm and S.D. of 2.61 cm? (Test at 5% level of significance.)

We are given that

$$n = 900, \bar{X} = 3.4 \text{ cm}, \mu_0 = 3.25 \text{ cm and } \sigma = 2.61 \text{ cm}$$

Here, we wish to test that the sample comes from a large population of bolts with mean (μ) 3.25cm. So our claim is $\mu = 3.25$cm and its complement is $\mu \neq 3.25$cm. Since the claim contains the equality sign so we can take the claim as the null hypothesis and the complement as the alternative hypothesis. Thus,

$$H_0 : \mu = \mu_0 = 3.25 \text{ and } H_1 : \mu \neq 3.25$$

Since the alternative hypothesis is two-tailed, so the test is two-tailed test.

Here, we want to test the hypothesis regarding population mean when population SD is unknown, so we should use t-test if the population of bolts known to be normal. But it is not the case. Since the sample size is large ($n > 30$) so we can go for Z-test instead of t-test as an approximate. So test statistic is given by

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

$$= \frac{3.40 - 3.25}{2.61/\sqrt{900}} = \frac{0.15}{0.087} = 1.72$$

The critical (tabulated) values for two-tailed test at 5% level of significance are $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$.

Since calculated value of test statistic Z (= 1.72) is less than the critical value (= 1.96) and greater than critical value (= −1.96), that means it lies in non-rejection region, so we do not reject the null hypothesis i.e. we support the claim at 5% level of significance.

Thus, we conclude that sample does not provide us sufficient evidence against the claim so we may assume that the sample comes from the population of bolts with mean 3.25cm.

Question

5)

❷ A normal population has a mean of 6.48 and S.D. of 1.5. In a sample of 400 members, mean is 6.75. Is the difference significant?

Example 1. *A normal population has a mean of 6.8 and standard deviation of 1.5. A sample of 400 members gave a mean of 6.75. Is the difference significant?*

Sol. H_0: There is no significant difference between $\bar{x}$ and μ.

H_1: There is significant difference between $\bar{x}$ and μ.

Given $\mu = 6.8$, $\sigma = 1.5$, $\bar{x} = 6.75$ and $n = 400$

$$| z | = \left| \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \right| = \left| \frac{6.75 - 6.8}{1.5/\sqrt{900}} \right| = | -0.67 | = 0.67$$

Conclusion. As the calculated value of $| z | < z_\alpha = 1.96$ at 5% level of significance, H_0 is accepted, *i.e.*, there is no significant difference between $\bar{x}$ and μ.

Question

6)

❸ A principal at a college claims that the students in his college are above average intelligence. A random sample of 30 students' IQ level scores have a mean score of 112.5. Is there sufficient evidence to support the principal's claim? The mean population IQ is 100 with a standard deviation of 15. LOS is 5%.

- Sample size n = 30
- Population mean $\mu = 100$
- Population standard deviation $\sigma = 15$
- Level of significance $\alpha = 0.05$
- Sample mean $\bar{x} = 112$

Next, we state our null and alternative hypotheses as follows:

$H_0 : \mu = 100$
$H_1 : \mu > 100$

To test the above hypotheses, we calculate the z-statistic, as follows:

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{112 - 100}{15/\sqrt{30}} = 4.38$$

Using a z-score table, the area to the right of z = 4.38 is 0.0001. This is the P-value. So, the P-value = 0.0001. Because the P-value is less than the level of significance, our statistic lies in the critical region. Therefore, we have sufficient evidence to reject the null hypothesis.

Question

7) A machine part was designed to withstand an average pressure of 120 units. A random sample of size 100 from a large batch was tested and it was found that the average pressure which these parts can withstand is 105 units with a standard deviation of 20 units. Test whether the batch meets the specification.

Solution

Step 1 → $H_0 : \mu = 120,\ H_1 : \mu < 120$

Step 2 → $S.E. = \dfrac{S}{\sqrt{n}} = \dfrac{20}{\sqrt{100}} = 2$ *[since σ is not known]*

Step 3 → $Z = \dfrac{\bar{x} - \mu}{S.E._{\bar{x}}} = \dfrac{105 - 120}{2} = -7.5$

Step 4 → At 1% level, the critical value of Z for one tailed test is -2.33.

Step 5 → Decision: since the computed value is less than the table value, we reject H_0 and conclude that the batch does not meet the specification.

Question

8) The average number of defective articles per day in a certain factory is claimed to be less than the average of all the factories. The average of all factories is 30.5.

A random sample of 100 days showed the following distribution:

Class limits:	16 – 20	21 – 25	26 – 30	31 – 35	36 – 40
No. of days:	12	22	20	30	16

Is the average less than the figure for all the factories? LOS is 1%.

(100)

Class interval	frequency (f)	x (Mid)	$d \Rightarrow \frac{x-28}{5}$	fd	fd^2
15.5 - 20.5	12	18	-2	-24	48
20.5 - 25.5	22	23	-1	-22	22
25.5 - 30.5	20	28	0	0	0
30.5 - 35.5	30	33	1	30	30
35.5 - 40.5	16	38	2	32	64
				$\Sigma fd \Rightarrow 16$	$\Sigma fd^2 \Rightarrow 164$

$$\text{Mean} \Rightarrow A + \left(\frac{\Sigma fd}{N}\right) c$$

$$\Rightarrow 28 + \left(\frac{16}{100}\right) * 5$$

$$\boxed{\text{Mean} \Rightarrow 28.8}$$

Standard deviation $\Big\{ \sigma \Rightarrow \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} * C$

$\Rightarrow \sqrt{\frac{164}{100} - \left(\frac{16}{100}\right)^2} * 5$

$\Rightarrow \sqrt{1.64 - 0.0256} * 5$

$\Rightarrow 1.2705 * 5$

$\boxed{\sigma \Rightarrow 6.35}$

Sample	Population
$\bar{x} = 28.8$	$\mu = 30.5$
$\sigma = 6.35$	
$n \Rightarrow 100$	

$\alpha \Rightarrow 1\%$

$H_0 : \mu = 30.5$

$H_1 : \mu < 30.5$ (left - tailed)

$$z \Rightarrow \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$\Rightarrow \frac{28.8 - 30.5}{6.35 / \sqrt{100}}$$

$$\Rightarrow \frac{-1.7}{0.635} \Rightarrow -2.671$$

$\boxed{z \Rightarrow -2.671}$

$z_\alpha \Rightarrow -2.33$, $|z_\alpha| \Rightarrow 2.33$

$|z| > |z_\alpha|$ H_0 is rejected

The average is less than the figures of all factories.

Case-4

Test of significance of the difference between two sample means

Case 4: Test of significance of the difference between two sample means.

$$\text{Test Statistic } z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Note:

1. If the samples are drawn from the same population, that is, $\sigma = \sigma_1 = \sigma_2$, then
$$z = \frac{\overline{x_1} - \overline{x_2}}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$
2. If σ_1 and σ_2 are not known, then σ_1 and σ_2 can be approximated by the sample S.D.'s s_1 and s_2. Now,
$$z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Given: $n_1 = 1000, \overline{x_1} = 67.5, n_2 = 2000, \overline{x_2} = 68$ and $\sigma_1 = \sigma_2 = 2.5$.

Step 1: $H_0 : \overline{x_1} = \overline{x_2}$ and $H_1 : \overline{x_1} \neq \overline{x_2}$.

Step 2: Here, type of test is two-tailed.

Step 3: At $\alpha = 5\%$, tabulated value $z_\alpha = 1.96$ for two-tailed.

Step 4: The test statistic is given by

$$z = \frac{\overline{x_1} - \overline{x_2}}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{67.5 - 68}{2.5\sqrt{\frac{1}{1000} + \frac{1}{2000}}} = -5.16.$$

Step 5: Note that $|z| > |z_\alpha|$ which shows that H_0 is rejected.

Question

The average marks scored by 32 boys is 72 with a S.D. of 8, while that for 36 girls is 70 with a S.D. of 6. Test at 1% level of significance whether the boys perform better than girls.

Here, $n_1 = 32, \overline{x_1} = 72, s_1 = 8, n_2 = 36, \overline{x_2} = 70$ and $s_2 = 6$.

Step 1: $H_0 : \overline{x_1} = \overline{x_2}$
$H_1 : \overline{x_1} > \overline{x_2}$.

Step 2: Use one-tailed (right-tailed) test.

Step 3: The tabulated $z = 2.33$ at $\alpha = 0.01$ when the nature of test is right-tailed.

Step 4: The test statistic is given by

$$z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{72 - 70}{\sqrt{\frac{8^2}{32} + \frac{6^2}{36}}} = 1.15.$$

Step 5: Since $|z| = 1.15 < 2.33 = |z_\alpha|$, H_0 is accepted.
$\therefore$ The boys do not perform better than girls.

Question

The mean height of 50 male students who showed above average participation in college atheletics was 68.2 inches with a S.D. of 2.5 inches while 50 male students showed no interest in such participation had a mean height of 67.5 inches with a S.D. of 2.8 inches.

(a) Test the hypothesis that male students who participate in college athletics are taller than other male students

(b) By how much should the sample size of each of the two groups be increased in order that the observed difference of 0.7 inches in the mean heights be significant at the 5% level of significance.

In this problem, $n_1 = 50, \overline{x_1} = 68.2, s_1 = 2.5, n_2 = 50, \overline{x_2} = 67.5, s_2 = 2.8$.

(a) **Step 1:** $H_0 : \overline{x_1} = \overline{x_2}$
$H_1 : \overline{x_1} > \overline{x_2}$.

Step 2: The nature of the test is one-tailed (right-tailed) according to H_1.

Step 3: Observe that $z_\alpha = 1.645$ at $\alpha = 5\%$.

Step 4: The test statistic is

$$z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \frac{68.2 - 67.5}{\sqrt{\dfrac{(2.5)^2}{50} + \dfrac{(2.8)^2}{50}}} = 1.32.$$

Step 5: As $|z| = 1.32 < 1.645 = |z_\alpha|$, we accept H_0, that is, we conclude that the college athletes are not taller than other male students.

(b) The difference between the mean heights of two groups, each of size n will be significant at 5% level of significance if $z \geq 1.645$. That is,

$$\begin{aligned} \frac{68.2 - 67.5}{\sqrt{\dfrac{(2.5)^2}{n} + \dfrac{(2.8)^2}{n}}} &\geq 1.645 \\ \frac{0.7}{\sqrt{\dfrac{14.09}{n}}} &\geq 1.645 \\ \frac{0.7\sqrt{n}}{3.754} &\geq 1.645 \\ n &\geq 77.83 \approx 78. \end{aligned}$$

Hence, the sample size of the two groups should be increased by atleast $78 - 50 = 28$, in order that the difference between the mean heights of the two groups is significant.

Question

Two samples drawn from two different populations gave the following results:

	$Size$	$Mean$	$S.D.$
$Sample\ I$	100	582	24
$Sample\ II$	100	540	28

Test the hypothesis, at 5% level of significance, that the difference of the means of the population is 35.

A simple sample of heights of 6400 English men has a mean of 170 cm and a S.D. of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm and a S.D. of 6.3 cm. Do the data indicate that Americans are, on the average, taller than the Englishmen?

$$n_1 = 6400,\ \bar{x}_1 = 170 \quad \text{and} \quad s_1 = 6.4$$

$$n_2 = 1600,\ \bar{x}_2 = 172 \quad \text{and} \quad s_2 = 6.3$$

$$H_0 : \mu_1 = \mu_2 \quad \text{or} \quad \bar{x}_1 = \bar{x}_2,$$

i.e. the samples have been drawn from two different populations with the same mean.

$$H_1 : \bar{x}_1 < \bar{x}_2 \quad \text{or} \quad \mu_1 < \mu_2.$$

Left-tailed test is to be used.

Let LOS be 1%. $\therefore z_\alpha = -2.33$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

[$\because \sigma_1 \simeq s_1$ and $\sigma_2 \simeq s_2$. Refer to Note 2 under Test 4]

$$= \frac{170 - 172}{\sqrt{\dfrac{(6.4)^2}{6400} + \dfrac{(6.3)^2}{1600}}} = -11.32$$

Now $\quad |z| > |z_\alpha|$

$\therefore$ The difference between $\bar{x}_1$ and $\bar{x}_2$ (or μ_1 and μ_2) is significant at 1% level.

i.e. H_0 is rejected and H_1 is accepted.

i.e. The Americans are, on the average, taller than the Englishmen.

Test the significance of the difference between the means of the samples, drawn from two normal populations with the same S.D. from the following data:

Table 9.2

	Size	*Mean*	*S.D.*
Sample 1	100	61	4
Sample 2	200	63	6

$$H_0 : \bar{x}_1 = \bar{x}_2 \quad \text{or} \quad \mu_1 = \mu_2$$

$$H_1 : \bar{x}_1 \neq \bar{x}_2 \quad \text{or} \quad \mu_1 \neq \mu_2$$

Two-tailed test is to be used.

Let LOS be 5% $\therefore z_\alpha = 1.96$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_2} + \frac{s_2^2}{n_1}}}$$

[Refer to Note 3 under Test 4; The populations have the same S.D.]

$$= \frac{61 - 63}{\sqrt{\frac{4^2}{200} + \frac{6^2}{100}}} = -3.02$$

Now $\quad |z| > z_\alpha$

$\therefore$ The difference between $\bar{x}_1$ and $\bar{x}_2$ (or μ_1 and μ_2) is significant at 5% level.

i.e. H_0 is rejected and H_1 is accepted.

i.e. The two normal populations, from which the samples are drawn, may not have the same mean, though they may have the same S.D.

Different concept

The S.D. of a random sample of 1000 is found to be 2.6 and the S.D. of another random sample of 500 is 2.7. Assuming the samples to be independent, find whether the two samples could have come from populations with the same S.D.

$$n_1 = 1000, \quad s_1 = 2.6 \; ; \quad n_2 = 500, \quad s_2 = 2.7$$

$$H_0 : s_1 = s_2 \quad (\text{or } \sigma_1 = \sigma_2)$$

$$H_1 : s_1 \neq s_2$$

Two tailed test is to be used.

Let LOS be 5%. $\therefore$ $z_\alpha = 1.96$

$$z = \frac{s_1 - s_2}{\sqrt{\dfrac{s_1^2}{2n_2} + \dfrac{s_2^2}{2n_1}}}, \text{ since } \sigma \text{ is not known.}$$

$$= \frac{2.6 - 2.7}{\sqrt{\dfrac{(2.6)^2}{1000} + \dfrac{(2.7)^2}{2000}}} = -0.98$$

Now $\quad |z| < z_\alpha$.

$\therefore$ The difference between s_1 and s_2 (and hence between σ_1 and σ_2) is not significant at 5% level.

i.e. H_0 is accepted.

i.e. the two samples could have come from populations with the same S.D.

The average marks scored by 32 boys is 72 with a S.D. of 8, while that for 36 girls is 70 with a S.D. of 6. Test at 1% level of significance whether the boys perform better than girls.

$$H_0 : \bar{x}_1 = \bar{x}_2 \quad (\text{or } \mu_1 = \mu_2)$$
$$H_1 : \bar{x}_1 > \bar{x}_2$$

Right-tailed test is to be used.

$$\text{LOS} = 1\% \quad \therefore z_\alpha = 2.33$$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

(The two populations are assumed to have S.D.'s $\sigma_1 \simeq s_1$ and $\sigma_2 \simeq s_2$)

$$= \frac{72 - 70}{\sqrt{\frac{8^2}{32} + \frac{6^2}{36}}} = 1.15$$

$$|z| < z_\alpha.$$

$\therefore$ The difference between $\bar{x}_1$ and $\bar{x}_2$ (μ_1 and μ_2) is not significant at 1% level.
i.e. H_0 is accepted and H_1 is rejected.
i.e. Statistically, we cannot conclude that boys perform better than girls.

P-value Method

- Besides listing an a value, many computer statistical packages give a P-value for hypothesis tests.
- The P-value is the actual probability of getting the sample mean value or a more extreme sample mean value in the direction of the alternative hypothesis ($>$ or $<$) if the null hypothesis is true.
- The P-value is the actual area under the standard normal distribution curve (or other curve, depending on what statistical test is being used) representing the probability of a particular sample mean or a more extreme sample mean occurring if the null hypothesis is true.

Decision Rules:

- If P-value $\leq \alpha$, reject the null hypothesis.
- If P-value $> \alpha$, do not reject the null hypothesis.

1)

1. The average weight of all residents in town XYZ is 168 lbs. A nutritionist believes the true mean to be different. She measured the weight of 36 individuals and found the mean to be 169.5 lbs with a standard deviation of 3.9. (a) State the null and alternative hypotheses. (b) At a 95% confidence level, is there enough evidence to discard the null hypothesis? (Use the p-value method)

Given

$\mu \Rightarrow 168$

Sample

$n \Rightarrow 36$

$\bar{x} \Rightarrow 169.5$

$\sigma \Rightarrow 3.9$

$C \Rightarrow 95\% \Rightarrow 0.95$

$\alpha \Rightarrow 5\% \Rightarrow 0.05$

Solution:

Null hypothesis $\Rightarrow$ $H_0 : \mu = 168$

Alternate hypothesis $\Rightarrow$ $H_1 : \mu \neq 168$ (Two tail test)

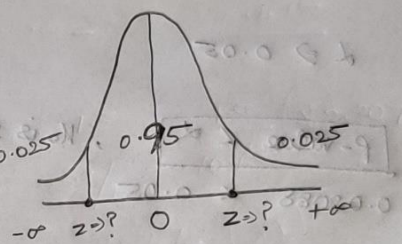
Now $Z_\alpha \Rightarrow 1.96$

$$Z \Rightarrow \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$\Rightarrow \frac{169.5 - 168}{3.9/\sqrt{36}} \Rightarrow \frac{169.5 - 168}{3.9/6}$$

$$\Rightarrow \frac{1.5}{0.65} \Rightarrow 2.30769$$

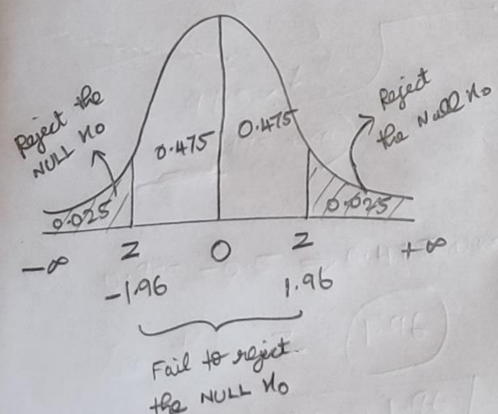
$$\boxed{Z \Rightarrow 2.30769}$$



0.95
0.025
0.975 $\Rightarrow$ See the point for this area.

0.975 $\rightarrow$ 0.475, 0.5

For Area $\Rightarrow$ 0.475 $\boxed{Z \Rightarrow 1.96}$



Z lies in the rejection 2.30769.

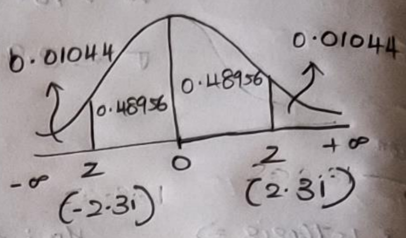
$|Z_\alpha| < |Z|$

H_0 is rejected.

Using p-value method

*) Plot the z value in the graph calculated from the traditional method.

*) Find the area (from 0 to 2.31)



$$\begin{array}{r} p\text{-value} \Rightarrow 0.01044 \\ 0.01044 \\ \hline 0.02088 \end{array}$$

0 to z (2.31) $\Rightarrow 0.48956$

2.31 to $\infty \Rightarrow 0.5 - 0.48956$
$\Rightarrow 0.01044$

$\alpha \Rightarrow 0.05$

$\boxed{p\text{-value} < \alpha}$, H_0 is rejected.

$0.02088 < 0.05$

2) An insurance agent has claimed that the average age of policy holders who insure through him is less than the average for all agents which is 30.5 years. A random sample of 100 policy holders who had insured through him reveal that the mean and S.D. are 28.8 years and 6.35 years respectively. Test his claim at 5% level of significance.

Given

$\mu \Rightarrow 30.5$

Sample

$n \Rightarrow 100$

$\bar{x} \Rightarrow 28.8$

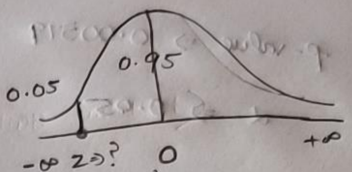
$\sigma \Rightarrow 6.35$

$\alpha \Rightarrow 5\%$

Solution

$H_0 \Rightarrow \mu = 30.5$

$H_1 \Rightarrow \mu < 30.5$ (left tailed test)



Find this area?

$0.5 - 0.05 \Rightarrow 0.45$

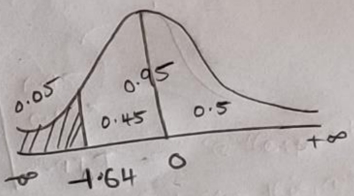
For this area, find z-value

z-value $\Rightarrow -1.64$

$z_\alpha \Rightarrow -1.64$

$$z \Rightarrow \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$\Rightarrow \frac{28.8 - 30.5}{6.35/\sqrt{100}} \Rightarrow \frac{-1.7}{6.35/10}$$



$$z \Rightarrow \frac{-1.7}{0.635} \Rightarrow -2.677165$$

$|z| \Rightarrow 2.677165$

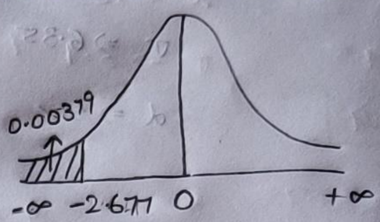
$|z_\alpha| \Rightarrow 1.64$

$|z| > |z_\alpha|$ H_0 is rejected.

Using p-value method

*) Use the z-value, calculated from traditional method.

*) $0.5 - 0.49621 \Rightarrow 0.00379$



p-value $\Rightarrow 0.00379$

$\alpha \Rightarrow 0.05$

p-value $< \alpha$, then H_0 is rejected

A genetic experiment involving peas yielded one sample of offspring consisting of 434 green peas and 173 yellow peas. Use a 0.05 significance level to test the claim that under the same circumstances, 24% of offspring peas will be yellow. Identify the null hypothesis, alternative hypothesis, test statistic, P-value, conclusion about the null hypothesis, and final conclusion that addresses the original claim. Use the P-value method.

Solution:

Sample

$\alpha \Rightarrow 5\% \Rightarrow 0.05$

green peas $\Rightarrow$ 434
yellow peas $\Rightarrow$ 173 $\Big\}\ n \Rightarrow 607$

Population proportion

$P \Rightarrow 24\% \Rightarrow 0.24$

Sample proportion (yellow peas)

$p \Rightarrow 173/(434+173) \Rightarrow 0.28$

$\boxed{p = 0.28}$

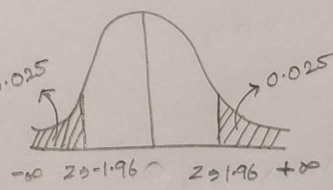
$H_0)\ P = 0.24$
$H_1)\ P \neq 0.24$
(two tailed test)

$P \Rightarrow 0.24 \quad Q \Rightarrow 0.76$

$$z \Rightarrow \frac{p-P}{\sqrt{PQ/n}} \Rightarrow \frac{0.28-0.24}{\sqrt{\frac{(0.24)*(0.76)}{607}}}$$

$$\Rightarrow \frac{0.04}{0.0173} \Rightarrow 2.596$$

$\boxed{z \Rightarrow 2.596}$



$\alpha \Rightarrow 5\%$

$|z_\alpha| \Rightarrow -1.96$

$|z| > |z_\alpha|$

H_0 is rejected

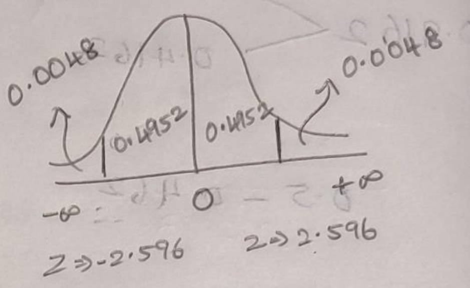
H_1 is accepted

Using p-value method.

$P \Rightarrow 0.0096$

So ~~$P \Rightarrow 1.9904$~~ $\alpha \Rightarrow 0.05$

$$\begin{array}{r} 0.0048 \\ 0.0048 \\ \hline 0.0096 \end{array}$$



From $-\infty$ to 2.596

↳ area is 0.9952

0.5   0.4952

$P < \alpha$

H_0 is rejected and

H_1 is accepted

(5) A national magazine claims that the average college student watches less television than the general public. The national average is 29.4 hours per week, with a standard deviation of 2 hours. A sample of 30 college students has a mean of 27 hours. Is there enough evidence to support the claim at a = 0.01?