H0 → there is no significant difference

H1 → there is a significant difference

1. Ten individuals are chosen at random from a population and their heights are found to be in inches 63,63,64,65,66,69,69,70,70,71 discuss the suggestion that the mean height in the universe is 65 inches given that for 9 degrees of freedom the value of *Student's t* and 5 percent level of significance is 2.262.

   Solution:

   For the calculation of sample mean and sample variance, we have taken the following into consideration

Serial no	x	$x-\bar{x}$	$(x-\bar{x})^2$
1	63	-4	16
2	63	-4	16
3	64	-3	9
4	65	-2	4
5	66	- 1	1
6	69	2	4
7	69	2	4
8	70	3	9
9	70	3	9
10	71	4	16
n = 10	$\Sigma x = 670$	-	$\Sigma (x-\bar{x})^2 = 88$

Sample mean, $\bar{x} = \frac{\sum x}{n} = \frac{670}{10} = 67$

Sample standard deviation

$$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n}} = \sqrt{\frac{88}{9}} = 3.13 inches$$

$$\textbf{Test static: } t = \frac{\bar{x}-\mu}{s/\sqrt{n}} \, or \, \frac{(x-\mu)\sqrt{n}}{s} \qquad = \frac{\bar{x}-M}{s}\sqrt{n} = \frac{(67-65)\sqrt{10}}{3.13} = 2.02$$

Ho: the mean of the universe is 65 inches.

The number of degrees of freedom = v = 10 - 1 = 9.
Tabulated value for 9 d.f. at *5%* level of significance is 2.262.

Since calculated value of t is less than tabulated value for 9 d.f. (2.02 < 2.262). This error could have arisen due to fluctuations and we may conclude that the data are consistent with the assumption of mean height in the universe of 65 inches.

2. Two independent samples of 8 and 7 items respectively had the following values of the variable (weight in ounces):

Sample 1:	9	11	13	11	15	9	12	14
Sample 2:	10	12	10	14	9	8	10	

Is the difference between the means of the sample significant? Given $t_{0.05} = 2.16$.

Solution:

Assumed mean of $x = 12$, Assumed mean of $y = 10$

x	$(x-12)$	$(x-12)^2$	y	$(y-10)$	$(y-10)^2$
9	–3	9	10	0	0
11	–1	1	12	2	4
13	1	1	10	0	0
11	–1	1	14	4	16
15	3	9	9	–1	1
9	–3	9	8	–2	4
12	0	0	10	0	0
14	2	4	–	–	–
94	–2	34	73	3	25

$$\bar{x} = \frac{\Sigma x}{n} = \frac{94}{8} = 11.75$$

$$\sigma_x^2 = \frac{\Sigma (x-12)^2}{n} - \left(\Sigma \frac{(x-12)}{n}\right)^2 = \frac{34}{8} - \left(\frac{-2}{8}\right)^2 = 4.1875$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{73}{7} = 10.43$$

$$\sigma_y^2 = \frac{\Sigma (y-10)^2}{n} - \left[\frac{\Sigma (y-10)}{n}\right]^2 = \frac{25}{7} - \left(\frac{3}{7}\right)^2 = 3.438$$

$$s = \sqrt{\frac{(x-\bar{x})^2 + \Sigma (y-\bar{y})^2}{n_1 + n_2 - 2}} = \sqrt{\frac{34+25}{8+7-2}} = \sqrt{\frac{59}{13}} = \sqrt{4.54} = 2.13$$

$$t = \frac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{11.75 - 10.43}{2.13\sqrt{\frac{1}{8} + \frac{1}{7}}} = \frac{1.32}{2.13\sqrt{0.268}} = \frac{1.32}{2.13 \times 0.518}$$

$$= \frac{1.32}{1.103} = 1.12$$

The 5% value of t for 13 degree of freedom is given to be 2.16. Since calculated value of t is 1.12 is less than 2.16, the difference between the means of samples is not significant.

A group of 10 rats fed on diet A and another group of 8 rats fed on diet B, recorded the following increase in weight (gms).

Diet A : 5, 6, 8, 1, 12, 4, 3, 9, 6, 10

Diet B : 2, 3, 6, 8, 10, 1, 2, 8

Does it show superiority of diet A over diet B. [A.U. N/D 2011]

Solution : Given : $n_1 = 10, n_2 = 8$

$$\Sigma x_1 = 5 + 6 + 8 + 1 + 12 + 4 + 3 + 9 + 6 + 10 = 64$$

$$\Sigma x_1^2 = 5^2 + 6^2 + 8^2 + 1^2 + 12^2 + 4^2 + 3^2 + 9^2 + 6^2 + 10^2 = 512$$

$$\Sigma x_2 = 2 + 3 + 6 + 8 + 10 + 1 + 2 + 8 = 40$$

$$\Sigma x_2^2 = 2^2 + 3^2 + 6^2 + 8^2 + 10^2 + 1^2 + 2^2 + 8^2 = 282$$

$$\bar{x}_1 = \frac{\Sigma x_1}{10} = \frac{64}{10} = 6.4$$

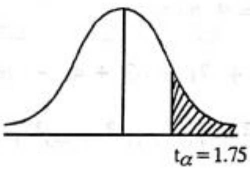
$$\bar{x}_2 = \frac{\Sigma x_2}{8} = \frac{40}{8} = 5$$

$$s_1^2 = \frac{\Sigma x_1^2}{n_1} - (\bar{x}_1)^2 = \frac{512}{10} - (6.4)^2 = 10.24$$

$$s_2^2 = \frac{\Sigma x_2^2}{n_2} - (\bar{x}_2)^2 = \frac{282}{8} - 25 = 10.25$$

$$S^2 = \left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right) = \frac{10\,(10.24) + 8\,(10.25)}{10 + 8 - 2} = 11.525$$

1. $H_0 : \mu_1 = \mu_2$
2. $H_1 : \mu_1 > \mu_2$ [One-tailed test (right)]
3. $\alpha = 5\%$ d.f. $= n_1 + n_2 - 2 = 10 + 8 - 2 = 16$
4. Critical region



5. The test statistic :

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{6.4 - 5}{\sqrt{11.525\left(\frac{1}{10} + \frac{1}{8}\right)}}$$

$$= \frac{1.4}{1.6103} = 0.869$$

6. Conclusion :

If $t < t_\alpha$, then we accept H_0; otherwise, we reject H_0.

Here, $0.869 < 1.75$

So, we accept H_0.

Hence, the difference is not significant, so we cannot conclude that diet A is superior to diet B.

The following are the number of sales which a sample of 9 salespeople of industrial chemicals in Gujarat and a sample of 6 sales people of industrial chemicals in Maharashtra made over a certain fixed period of time :

Gujarat :	59	68	44	71	63	46	69	54	48
Maharashtra :	50	36	62	52	70	41			

Assuming that the population sampled can be approximated closely with normal distributions having the same variance, test the null hypothesis $\mu_1 - \mu_2 = 0$ against the alternative hypothesis $\mu_1 - \mu_2 \neq 0$ at the 0.01 level of signifance. [A.U N/D 2009]

Solution : Given : $n_1 = 9,\ n_2 = 6$

$$\Sigma x_1 = 59 + 68 + 44 + 71 + 63 + 46 + 69 + 54 + 48 = 522$$

$$\Sigma x_1^2 = 59^2 + 68^2 + 44^2 + 71^2 + 63^2 + 46^2 + 69^2 + 54^2 + 48^2 = 31148$$

$$\Sigma x_2 = 50 + 36 + 62 + 52 + 70 + 41 = 311$$

$$\Sigma x_2^2 = 50^2 + 36^2 + 62^2 + 52^2 + 70^2 + 41^2 = 16925$$

$$\bar{x}_1 = \frac{\Sigma x_1}{9} = \frac{522}{9} = 58$$

$$\bar{x}_2 = \frac{\Sigma x_2}{6} = \frac{311}{6} = 51.83$$

$$s_1^2 = \frac{\Sigma x_1^2}{n_1} - (\bar{x}_1)^2 = \frac{31148}{9} - (58)^2 = 96.89$$

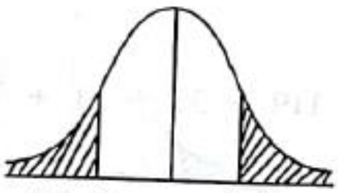
$$s_2^2 = \frac{\Sigma x_2^2}{n_2} - (\bar{x}_2)^2 = \frac{16925}{6} - (51.83)^2 = 134.48$$

$$S^2 = \left[\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right] = \frac{9\,(96.89) + 6\,(134.48)}{9 + 6 - 2}$$

$$= \frac{872.01 + 806.88}{13} = 129.15$$

$$S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right) = (129.15)\left(\frac{1}{9} + \frac{1}{6}\right) = (129.15)\left(\frac{5}{18}\right) = 35.88$$

1. $H_0 : \mu_1 = \mu_2$
2. $H_1 : \mu_1 \neq \mu_2$ [Two-tailed test]
3. $\alpha = 1\%$, d.f. $= n_1 + n_2 - 2 = 9 + 6 - 2 = 13$
4. Critical region :



5. The test statistic :

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{58 - 51.83}{\sqrt{35.88}} = 1.03$$

6. Conclusion :

If $-t_{\alpha/2} < t < t_{\alpha/2}$, then we accept H_0 ; otherwise, we reject H_0.

Here, $-3.102 < 1.03 < 3.012$

So, we accept H_0.

The following are the average weekly losses of working hours due to accidents in 10 industrial plants before and after an introduction of a safety program was put into operation.

Before :	45	73	46	124	33	57	83	34	26	17
After :	36	60	44	119	35	51	77	29	24	11

Use to 0.05 level of significance to test whether the safety program is effective.

[A.U. N/D 2008]

Solution :

$$\Sigma x_1 = 45 + 73 + 46 + 124 + 33 + 57 + 83 + 34 + 26 + 17$$
$$= 538$$

$$\Sigma x_1^2 = 45^2 + 73^2 + 46^2 + 124^2 + 33^2 + 57^2 + 83^2 + 34^2 + 26^2 + 17^2$$
$$= 38194$$

$$\Sigma x_2 = 36 + 60 + 44 + 119 + 35 + 51 + 77 + 29 + 24 + 11$$
$$= 486$$

$$\Sigma x_2^2 = 36^2 + 60^2 + 44^2 + 119^2 + 35^2 + 51^2 + 77^2 + 29^2 + 24^2 + 11^2$$
$$= 32286$$

$$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{538}{10} = 53.8$$

$$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{486}{10} = 48.6$$

$$s_1^2 = \frac{\Sigma x_1^2}{n_1} - (\bar{x}_1)^2 = \frac{38194}{10} - (53.8)^2$$
$$= 3819.4 - 2894.44$$
$$= 924.96$$

$$s_2^2 = \frac{\Sigma x_2^2}{n_2} - (\bar{x}_2)^2 = \frac{32286}{10} - (48.6)^2$$

$$= 3228.6 - 2361.96$$

$$= 866.64$$

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{(10)\,(924.96) + (10)\,(866.64)}{10 + 10 - 2}$$

$$= \frac{9249.6 + 8666.4}{18} = 995.33$$

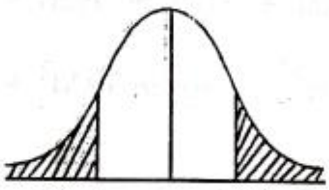
$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ [Two-tailed test]

$\alpha = 5\%$, d.f. $= n_1 + n_2 - 2 = 10 + 10 - 2 = 18$

Critical region :

$\alpha \Rightarrow \frac{5\%}{2} \Rightarrow 0.025$



$-t_{\alpha/2} = -2.101$ $\quad$ $t_{\alpha/2} = 2.101$

The test statistic :

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{53.8 - 48.6}{\sqrt{995.33\left(\frac{1}{10} + \frac{1}{10}\right)}} = 0.369$$

Conclusion :

If $-t_{\alpha/2} < t < t_{\alpha/2}$, then we accept H_0 ; otherwise, we reject H_0

Here, $-2.101 < 0.369 < 2.101$

So, we accept H_0.

The following random samples are measurements of the heat producing capacity (in millions of calories per ton) of specimen's of coals from two mines.

Mine 1:	8,260	8,130	8,350	8,070	8,340	
Mine 2:	7,950	7,890	7,900	8,140	7,920	7,840

Use the 0.01 level of significance to test whether the difference between the means of these two samples is significant.

[A.U. N/D 2008]

Solution :

$$\Sigma x_1 = 8260 + 8130 + 8350 + 8070 + 8340 = 41150$$

$$\Sigma x_1^2 = 8260^2 + 8130^2 + 8350^2 + 8070^2 + 8340^2 = 338,727,500$$

$$\Sigma x_2 = 7950 + 7890 + 7900 + 8140 + 7920 + 7840 = 47650$$

$$\Sigma x_2^2 = 7950^2 + 7890^2 + 7900^2 + 8140^2 + 7920^2 + 7840^2 = 378,316,200$$

$$\bar{x}_1 = \frac{\Sigma x_1}{5} = \frac{41150}{5} = 8230$$

$$\bar{x}_2 = \frac{47640}{6} = 7940$$

$$s_1^2 = \frac{\Sigma x_1^2}{n_1} - (\bar{x}_1)^2 = \frac{338,727,500}{5} - (8230)^2$$

$$= 67745500 - 67732900$$

$$= 12,600$$

$$s_2^2 = \frac{\Sigma x_2^2}{n_2} - (\bar{x}_2)^2 = \frac{378316200}{6} - (7940)^2$$

$$= 63052700 - 63043600$$

$$= 9,100$$

$$S^2 = \left[\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right]$$

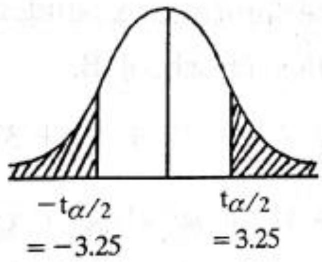
$$= \frac{(5)(12600) + (6)(9100)}{5 + 6 - 2}$$

$$= \frac{117600}{9}$$

$$S^2 = 13066.67$$

1. $H_0 : \mu_1 = \mu_2$
2. $H_1 : \mu_1 \neq \mu_2$ [Two-tailed test]
3. $\alpha = 1\%$, d.f. $= n_1 + n_2 - 2 = 5 + 6 - 2 = 9$
4. Critical region :

$\alpha \Rightarrow \frac{1\%}{2} \Rightarrow 0.005$



5. The test statistic :

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{8230 - 7940}{\sqrt{13066.67\left(\frac{1}{5} + \frac{1}{6}\right)}} = 4.19$$

6. Conclusion :

If $-t_{\alpha/2} < t < t_{\alpha/2}$, then we accept H_0 ; otherwise, we reject H_0.

Here, $-3.25 < 4.19 \nless 3.25$

So, we reject H_0.