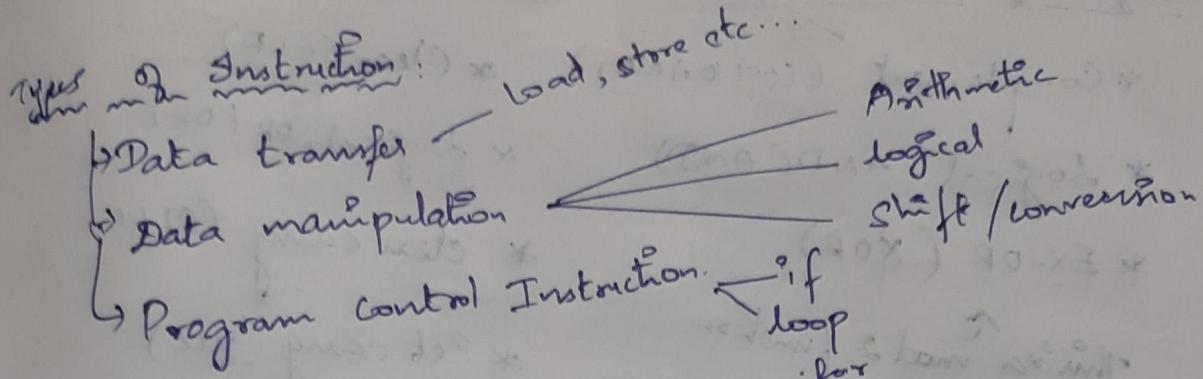


# Computer Organisation & Architecture



## Data transfer Instruction

→ Transfers the data

\* MOV      \* LOAD      \* STORE      \* Exchange      \* Input  
 ↓            ↓            ↓            ↓            ↓  
 copy      generally      from      swapping      for  
 MOV R<sub>1</sub>, R<sub>2</sub>      load from      register      device  
 MOV R<sub>1</sub>, <sub>so</sub>      to      to      XCHG R<sub>1</sub>, R<sub>2</sub>  
 (memory → register)      accumulator      memory      eg: STA

LDI <sub>soo</sub>

LD R<sub>1</sub>, R<sub>2</sub>

LD R<sub>1</sub>, X

\* Output      \* push      \* pop

in stack implementation

Arithmetic Instructions:      ALU performance of already available in hardware

\* ADD      \* Sub      \* Mul      \* DIV      \* INC      \* DEC

\* ADD with carry      \* Sub with borrow      \* Negate

Logical Instruction:  
 suppose : if it is  $\begin{array}{|c|c|} \hline 0 & 1 & 0 & 1 \\ \hline \end{array}$  its complements be  
 $\begin{array}{|c|c|} \hline 1 & 1 & 1 \\ \hline \end{array}$   $\leftarrow$   $\begin{array}{|c|c|} \hline 1 & 0 & 1 & 0 \\ \hline \end{array}$   $\leftarrow$   $\begin{array}{|c|c|} \hline 0 & 1 & 0 \\ \hline \end{array}$

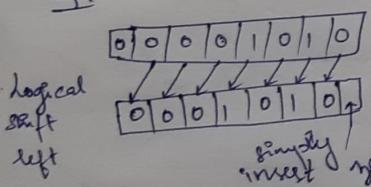
- \* Complement (COM1 OR NOT)
- \* Logical AND (AND)
- \* EX-OR (XOR)
- This is a mod 2 function
- $0+0=0$        $1+1=2 \text{ mod } 2$
- $1+0=1$        $=0$
- $0+1=1$
- \* Clear (CLR)
- \* logical-OR (OR)
- \* clear carry (CLRC)
- \* Set carry (SETC)

- \* Complement carry (CMC)
- \* Disable Interrupt (DI)
- represented by flag bit
- enable    → disable

### SHIFT INSTRUCTION:

- \* logical shift left

eg:-



The above example is the multiplication of 10 by 2.

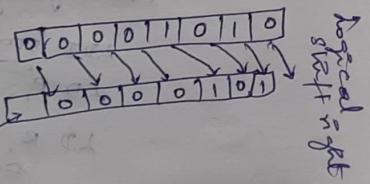
The number is 00001010 & decimal equivalent is 10.

00001010 is 10

After LSL,

The number is 00010100 & decimal equivalent of 00010100 is 20

- \* logical shift right



The above example is the division of 10 by 2.

The number is 000001010 & decimal equivalent is 5.

After LSR,

The number is 00000101 & decimal equivalent of number is 5

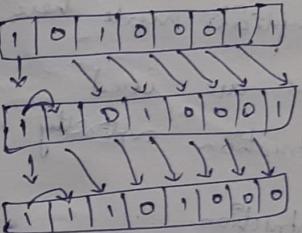
- \* Arithmetic shift left

generally used for signed bits

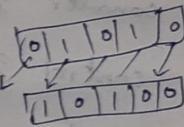
In 8 bit register, the MSB is the sign bit.

Note: ASL is same as LS2

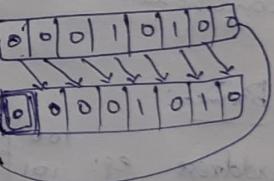
eg: for Arithmetic shift right :-



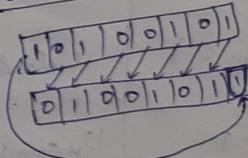
eg: for ASR :-



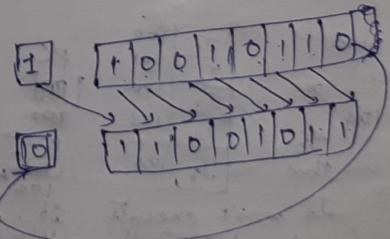
\* Rotate right :-



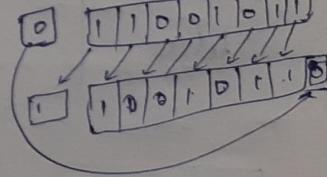
\* Rotate left :-



\* Rotate right through carry



\* Rotate left through carry



usual / General Instruction format

Addressing mode   Opcode   Operand

Instruction format for shift

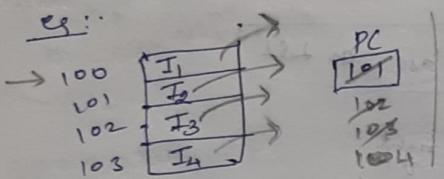
A.M   Type A   Type B   Count   Operand

## Program Control Instructions / Transfer of control

### Instruction

Instructions are executed in sequentially

e.g.:



means the instructions are executed sequentially

\* Program counter is in implicit mode here.

\* Program control instructions are used to execute the next instruction explicitly.

(randomly)

```
ep: f1()
main()
{
```

f1()

:

f2()

g:

f2()

### Branch Instruction

Unconditional Conditional

### Unconditional Branch

↳ target address

ep:

Jump 2000

B 3000

(Branch)

skip 104

In this

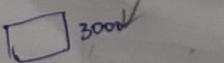
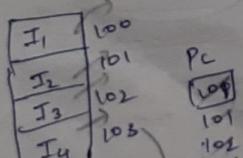
Call 3000

J4 won't execute

will automatically

skip to

104



### Conditional Branch

1) BE R<sub>1</sub>, R<sub>2</sub>, 2000 (Branch if equal R<sub>1</sub> = R<sub>2</sub>)

Suppose R<sub>1</sub> ≠ R<sub>2</sub>, in that case, next instruction will be executed

2) BNZ R<sub>1</sub>, 2000 [if not equal to zero, R<sub>1</sub> ≠ 0].

### INSTRUCTION FORMAT

Let's in C,

main ()

{

int a = 10;

int b = 20;

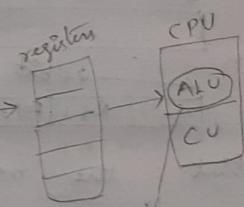
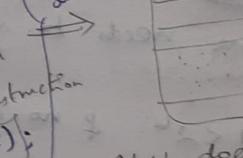
int c ; Data

c = a + b; Instruction

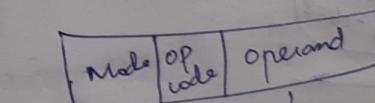
printf("%d", c);

}

converted into  
set of instructions



ALU doesn't know what to do with the data unless the opcode says/flags performing addition



Addressing mode

Address

\* Size of instruction set depends on the type of computer architecture

Common Bus size & Register size & IS

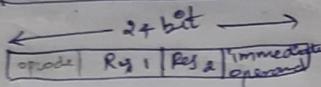
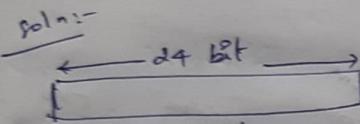
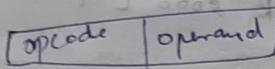
Qn:

A machine has 24 bit instruction format. It has 32 registers and each of which is 32-bit long. It needs to support 47 instructions.

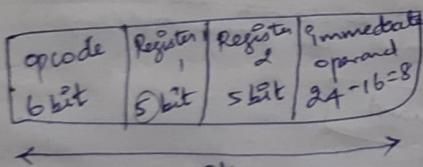
Each instruction has a register operand and one immediate operand. If the immediate operand is signed integer - the minimum value of immediate operand is

- 64
- 128
- 256
- 32

General format

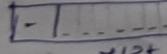


hint: the machine needs to support 49 instructions.  
 $\therefore 2^5 < 49 < 2^6$  & no. of registers = 32



so, it must be 24 bit  
 $2^8 = 256$  but here it is

signed bit, so the number format  
 be [-] . so it ranges from



-128 to 127

CPU Organisation

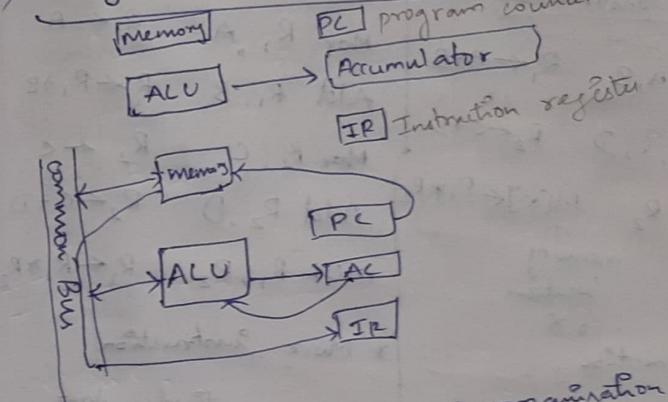
Single Accumulator

General Register

Stack organisation

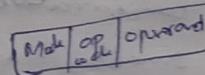
Single Accumulator

1) Single Accumulator CPU Organisation



single Accumulator

single address instruction



CPU organisation supports

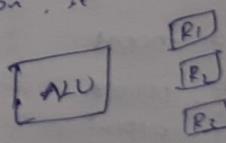
for eg:  $X = A + B$   
 the instructions are like  
 \* LDAA (Load A to accumulator)  
 \* ADD B LDA - Load accumulator  
 \* store X  $AC \leftarrow AC + B$

\* more here.

Note: No. of micro-operations  
 micro-operation ↑, range ↓

2) General Register:

\* General Register use 2 or 3 address instructions  
 \* Comparing to single accumulator CPU organisation, it is fast and costly.



Format:-

Mode	Op	Destin. Reg.	Source operand 1	Source operand 2
4 bit	4 bit	4 bit	4 bit	4 bit

8 address

Ex:-

$$x = (A+B) * (C+D)$$

### 2) Address Instruction

$$\text{Add } R_1, A, B \quad R_1 \leftarrow A+B$$

$$\text{Add } R_2, C, D \quad R_2 \leftarrow C+D$$

$$\text{MUL } R_1, R_2 \quad M[x] \leftarrow R_1 * R_2$$

memory location of  $x$

\* Less instruction

\* fast

\* costly

\* Instruction length, Bus usage  
are big

Dominates

### 3) Address Instruction:-

$$\text{Mov } R_1, A \quad R_1 \leftarrow A$$

$$\text{Add } R_1, B \quad R_1 \leftarrow R_1 + B$$

$$\text{Mov } R_2, C \quad R_2 \leftarrow C$$

$$\text{Add } R_2, D \quad R_2 \leftarrow C+D$$

$$\text{MUL } R_1, R_2 \quad R_1 \leftarrow R_1 * R_2$$

### More instruction

\* comparatively slow

\* comparatively cheap

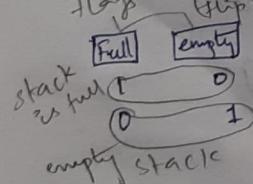
\* comparatively small

### 3) STACK ORGANIZATION:-

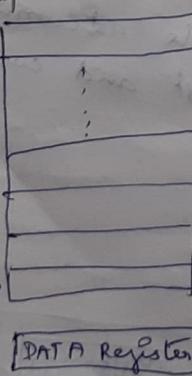
LIFO

#### Register STACK Organization:-

flags 64 word registers  
(top of stack)



points the address  
(stack pointer)  
size: 6 bits



63 111111

3 000011

2 000010

1 000001

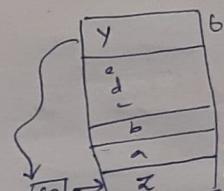
0 000000

Push

$$SP \leftarrow SP + 1$$

$$M[SP] \leftarrow DR$$

If ( $SP \neq 0$ )  
then ( $Full \leftarrow 1$ ) ( $Empty \leftarrow 0$ )



if  $SP$  reaches 64th location  
(i.e.  $(63)_{10} = (111111)_2$ ). Then by  
a instructions.  $SP + 1$  it  
becomes  $(64)_{10} = (1000000)_2$

overflow  $SP$   
 $10_2$  will be  
inserted to zero

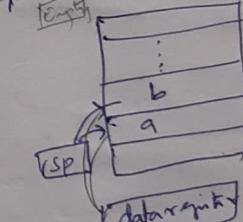
Push

$$SP \leftarrow SP + 1$$

$$M[SP] \leftarrow DR$$

If ( $SP = 0$ ) then ( $Full \leftarrow 1$ )

$Empty \leftarrow 0$



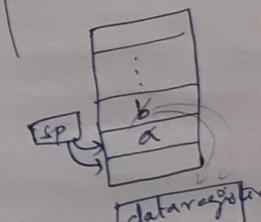
Pop

$$DR \leftarrow M[SP]$$

$$SP = SP - 1$$

If ( $SP = 0$ ) then ( $Empty \leftarrow 1$ )

$$Full \leftarrow 0$$



\* Stack organization uses address instruction

$$\text{eg: } (a+b) * (c+d)$$

Push a

Push b

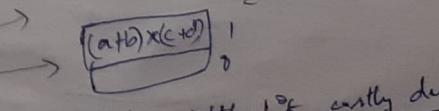
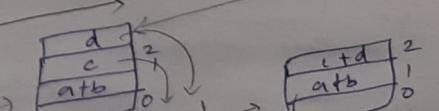
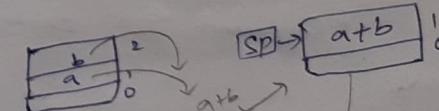
add  
yields address instruction

Push c

Push d

add

mult



\* This type of organization is little bit costly due to no 6-bit registers

## ADDRESSING MODES:-

The term addressing mode refers to the way in which the operand of an instruction is specified.

(Instruction format)

Data

→ 1st. (1) bit for mode 3  
retention  
 $1st. 2 \rightarrow 2$

then one nibble (4bit) used for

$16 > A \rightarrow DA$

(first four bits used) 000 ③

(remaining three bits) A10 ③

functions of buses are given below ④

→ [ ] bba [ ] after each bus

bus is your understand defined \*

what is retention

a delivery path of memory

the beginning is initiating with

(initiation state) A3

[ ] [ ] [ ]

$2nd. 1st. 1st.$   
 $2nd. 2nd. 2nd.$

busy stage 1st.

## Implied Addressing Mode:

\* Operand is specified implicitly in the definition of instruction

\* Used for yes and one address instruction

### eg. ① INCA [Increment Accumulator]

2 address → Add  $R_1, R_2$   
 Source → Destination  
 $R_1 \leftarrow R_1 + R_2$

But here INCA Destination cum source

$$AC \leftarrow AC + 1$$

### ② CRC [Clear the carry flag]

### ③ CLA [Complement Accumulator]

④ Add here no operand is mentioned but in stack organization it means add the last two data  $\boxed{R_1, R_2}$  Add  $\boxed{+}$

\* Benefit: Instruction size is small

## Immediate Addressing Mode:-

\* Operand is directly provided as constant

\* No computation is required to calculate EA (effective address)

eg: 

opcode	operand
--------	---------

 ← address →

Add 

$R_1$	#3
-------	----

$$R_1 \leftarrow R_1 + 3$$

## Application:-

### main()

```
{
  = 
  = 
  = 
  } =
```

② universal constant  
 ④

no need to load / store the value of those constants in some registers and then use in computation, we can directly load its value to the register, which is under computation.

Not only universal constant, the declaration of const.int a=10 in C can also use immediate addressing mode coz, its value is same throughout the program.

\* But int a=10 is not possible in immediate mode. coz it can be change. eg: int a=10; a=a+10;

## Note:-

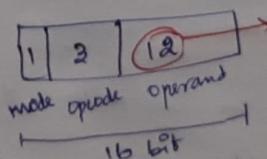
In immediate addressing, the operand (address) is neither to be fetched nor to be decoded and executed.

So less computational time. or ml.

## Limitation:-

The size of the instruction format depends on the range of the constant

eg:



It can hold only  $0 \text{ to } 2^{12}-1$ .

## Register Mode:

- \* Operand is present in the register
- \* Register no. is written in Instruction.

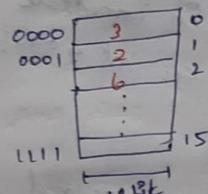
register no.

opcode / operand

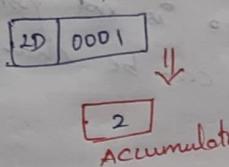
R<sub>1</sub>, R<sub>2</sub>, ...

Ex: LD R<sub>1</sub>; AC  $\leftarrow$  R<sub>1</sub>

Let say a system has 16 Registers



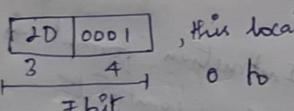
LD R<sub>1</sub>; AC  $\leftarrow$  R<sub>1</sub>



$$EA = [R]$$

### Benefits:-

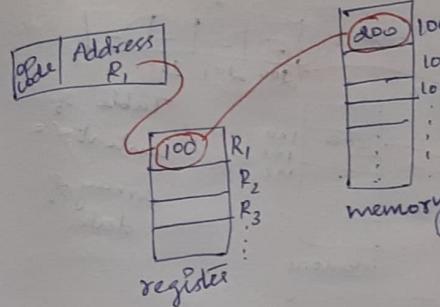
- \* Instruction set size is small
- number in range

e.g.:  , this location can hold  $0 \text{ to } 2^{12-1}$  numbers

- \* It is fast due to the usage of registers.

## Registers \* Indirect Mode:-

Registers contain address of operand rather than operand itself



$$\boxed{\text{Effective Address} = [CR_1]}$$

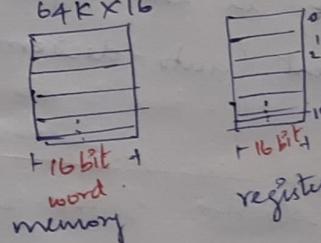
1) LD (R<sub>1</sub>)  
AC  $\leftarrow M[CR_1]$

2) Add R<sub>1</sub>, (R<sub>2</sub>)  
R<sub>1</sub>  $\leftarrow R_1 + M[R_2]$

### Benefit:-

- \* Large storage due to memory

e.g.: 64Kx16



$$64K = 2^6 \times R^{10}$$

$$= 2^{16}$$

To access the registers, we just need 4 bits. So ultimately the instruction size is small.

To directly access the memory:-

$$\begin{array}{c} \text{opcode} \quad 16 \text{ bit} \\ \hline x \text{ bit} \end{array} = x + 16 \text{ bit} \quad \text{large}$$

To indirectly access through the memory.

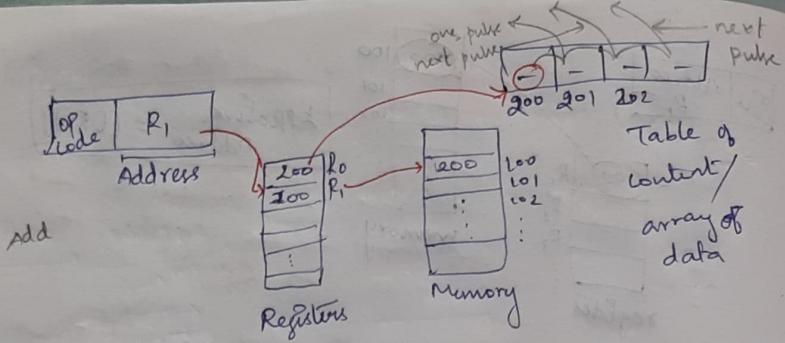
$$\begin{array}{c} \text{opcode} \quad \text{addr bit} \\ \hline x \text{ bit} \end{array} = x + 4 \text{ bit} \quad \text{small}$$

Note:- But it need some computation

## Auto Increment & Decrement Mode:-

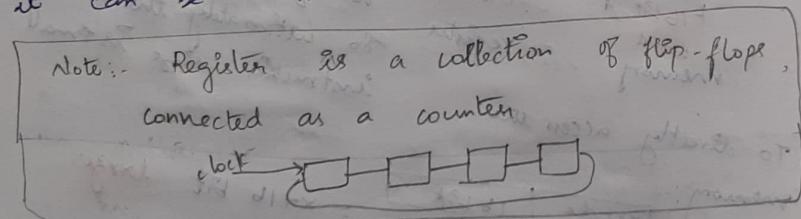
\* Special case of register indirect Addressing

Mode



→ Used to access continuous/sequential data [Table of content / array of data].

here the start address is 200, to access 201, 202, ... there is no need for the location (201, 202, 203, ...) present in the registers. Log of the auto increment (auto-decrement) it can be accessed.

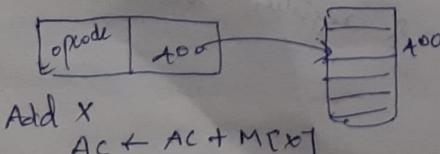


Instruction : LD (R<sub>i</sub>) +      | Autodecrement  
 $AC \leftarrow M[R_i] +$       |  $LD - (R_i)$

### DIRECT ADDRESSING MODE (Absolute addressing mode)

\* Actual address is given in instruction

\* Use to access variables



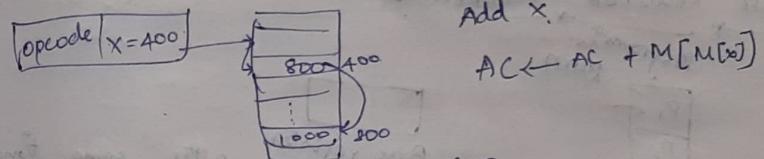
Add X  
 $AC \leftarrow AC + M[400]$

LD 400

$AC \leftarrow M[400]$

### INDIRECT ADDRESSING MODE :-

- \* Used to implement pointers and passing parameters
- \* 2 memory access required.



Add X.

$$AC \leftarrow AC + M[M[X]]$$

Here Effective Address = 800

$$EA = M[X]$$

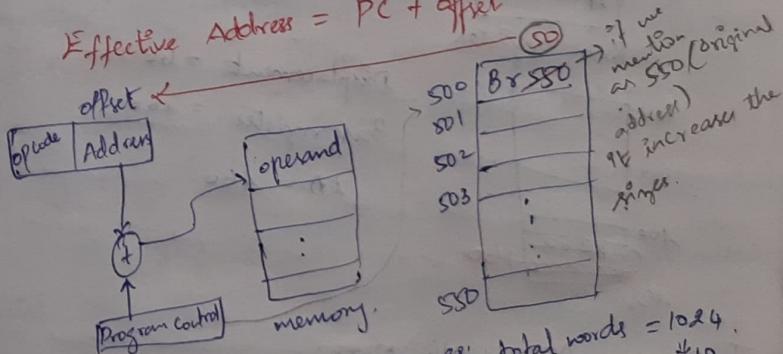
$$\text{int } *a = &b \quad &b = 800$$

fun(a) + fun(&f) also we RAM

### RELATIVE ADDRESSING MODE:

Program Control instruction like Branch, Jump use this type of addressing.

$$\text{Effective Address} = PC + \text{offset}$$



for eg: total words = 1024.

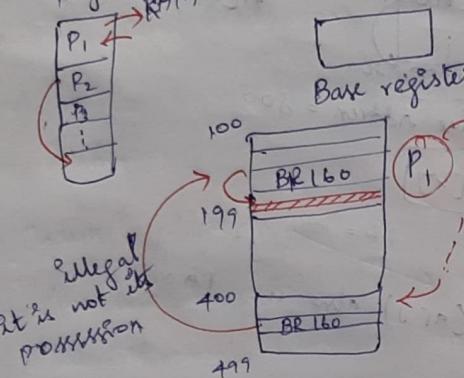
Suppose if the PC points 501 & if we add the offset to it then it becomes 551 so it will collapse entirely, hence we should add D-1 offset to the PC's pointing address.  
 $i.e. PC \Rightarrow 501 + 49 = 550$

## Benefits:

- \* Used for program relocation while the PC goes sequentially

BASE REGISTER ADDRESSING:-

Programs in RAM

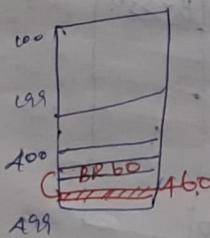


$$160 = 100 + 60$$

$$\therefore \text{BR to } 60$$

Qn:

Base registers



$$\text{Displacement} = 60$$

$$\text{Base register} = 400$$

$$\therefore \text{EA} = 460$$

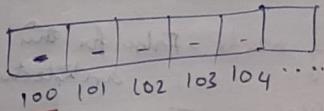
## INDEXED ADDRESSING MODE:-

- \* Use to access or implement array efficiently

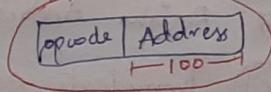
\* Multiple registers required to implement (one of them is index register)

\* Any element can be accessed without changing instruction

e1:-



fixed



Effective Address =

Base Address + IR

$$100 + 4$$

$$= 104$$

$$\begin{array}{|c|c|} \hline \text{Index register} & \text{Op code} \\ \hline 46 & 100 + 6 \\ \hline \end{array}$$

$\therefore 106$

Qn:

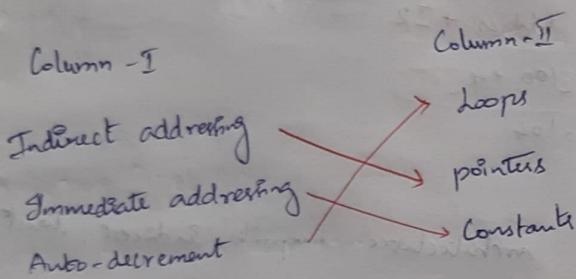
A certain architecture supports indirect, direct and register addressing mode for use in identifying operands for arithmetic instructions. Which one of the following can not be achieved with a single instruction.

- Specifying a register no in instruction such that register contain value of an operand that will be used by operation
- Specifying a register no in instruction such that register will serve as destination of operations output
- Specifying an operand value in instruction such that value will be used by operation
- Specifying a memory location with that it contains value of operand that will be used by operation.

Ans:-

- Register Addressing Mode - True
- Register Addressing Mode - True
- Immediate Addressing Mode - False for this architecture
- Direct Addressing Mode - True

27 Column - I



3) In the absolute addressing mode

- The operand is inside the instruction
- The address of the operand is inside the instruction
- The register containing the address of the operand is specified inside the instruction
- The location of the operand is implicit

Ans: ②

Q. Which of the following addressing modes are suitable for program relocation at run time?

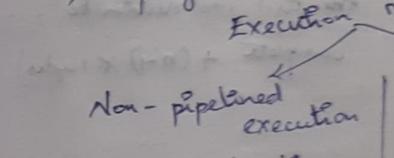
- Absolute
- Base
- Relative
- Indirect

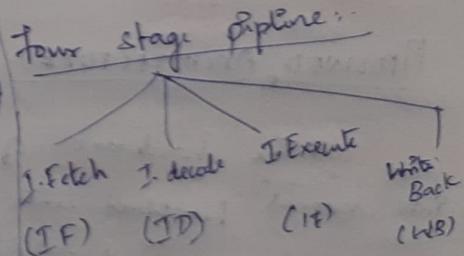
Ans: ② & ③

⑥ Array Implementation  
Writing relocatable code  
Passing array as parameters

indirect addressing  
indexed  
Base register

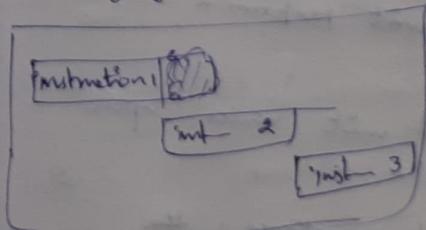
### PIPELINING IN COMPUTER ARCHITECTURE:

- ⇒ A program consists of several number of instructions
- 
- \* All the instruction of a program are executed sequentially one after other
  - \* new instruction executes only after the previous one has executed completely
  - \* highly inefficient



Phase time diagram:

clock cycle						
	1	2	3	4	5	
Stages	I <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>			
S <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>				
S <sub>2</sub>		I <sub>2</sub>	I <sub>3</sub>			
S <sub>3</sub>			I <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>	
S <sub>4</sub>				I <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>



$$\text{Time taken} = n \times t$$

$$= 3 \times t$$

$$= 3t$$

→ Time for one execution

$$\text{Time taken} = 6 - \text{clock cycle}$$

## Calculation for 3 Instructions

non-pipelined

for

$$\text{Time taken} = 3 \times t$$

$$\begin{aligned} \text{one instruction} &= 3 \times 4 \\ &= 12 \text{ clock cycles} \\ &\quad \left[ \begin{array}{l} \text{IF} \\ \text{ID} \\ \text{IE} \\ \text{WB} \end{array} \right] \quad \left[ \begin{array}{l} \text{clock} \\ \text{cycles} \end{array} \right] \end{aligned}$$

Pipelined

$$\text{Time taken} = 6 \text{ clock cycles}$$

$$= (\text{time taken for the first instruction to complete} + (n-1) \times 1 \text{ cycles})$$

## INSTRUCTION PIPELINING:-

\* It is a technique that implements a form of parallelism called as instruction level parallelism within a single processor.

## Pipelined Architecture:-

\* The hardware of the CPU is split up into several functional units  
\* Each functional unit performs a dedicated task  
\* No. of functional unit varies upon processors.

\* functional unit  $\rightarrow$  stages of the pipeline

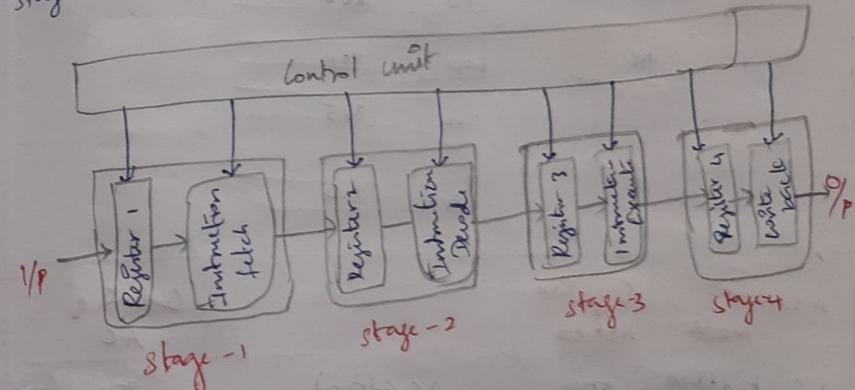
\* Control unit manages all the state using control signal

\* Registers in each stage hold the data

\* Global clock that synchronizes the working of all the stages.

\* At the beginning of each clock cycle, each stage takes the input from its register.

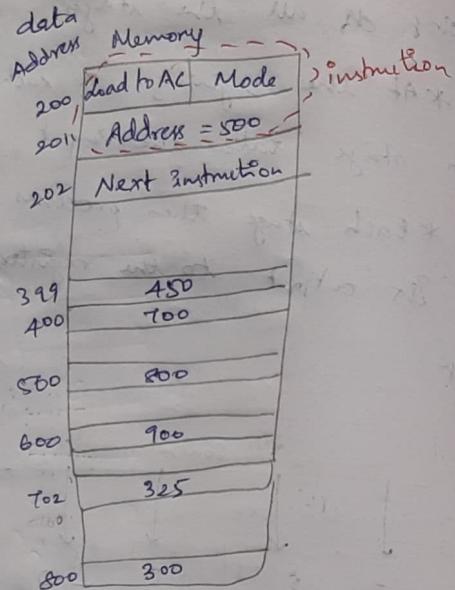
\* Each stage then processes the data and feed its output to the register to the next stage.



Mode	Algorithm	Advantage	Disadvantage
Immediate	Operand = 1	No memory reference	Limited operand magnitude
Direct	EA = A	Simple	Limited address space
Indirect	EA = (A)	Large Address space	multiple memory reference
Register	EA = R	No memory Reference	Limited address space
Register Indirect	EA = (R)	large address space	Extra memory space
Displacement	EA = A + (R)	Flexibility	Complexity
Stack	EA = Top of stack	No memory Reference	Limited applicability

Find the effective address and the content of

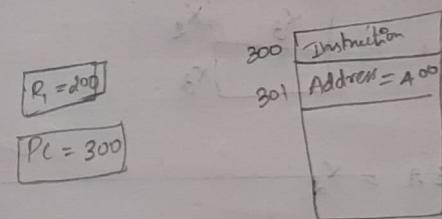
AC for the given



27. An instruction is stored at location 300 with 9 bits address field at location 301.

The address field has value 400. A processor register  $R_1$  contains the number 200.

Evaluate the EA if the addressing mode of the instruction is (a) direct (b) immediate (c) relative (d) register indirect (e) index with  $R_1$  as the index register.



Addressing Mode	Effective Address	Content of AC	No.	Mode	EA	Content of AC	
Direct address	500	AC ← (500)	800	a)	direct	500	AC ← [500]
Immediate (general)	201	AC ← 500	500	b)	immediate	301	AC ← 400
Indirect address	800	AC ← ((500))	300	c)	relative	700	AC ← (PC + 400)
Relative	(702)	AC ← (PC + 500) <small>(contents of PC add 500 to get address of next instruction)</small>	325	d)	register indirect	200	AC ← (R1)
Indexed	600	AC ← (XR + 500)	900	e)	indexed	600	AC ← (R1 + 400)
Registers	-	AC ← R1	400				
Register Indirect Auto-increment	400	AC ← (R1)	100				
Auto-decrement	400	AC ← (R1) +	700				
	399	AC ← -(R1)	450				

28) Let the address stored in the program counter designated by the symbol  $X_1$ . The instruction stored in  $X_1$  has the address part (operand reference)  $X_2$ . The operand needed to execute the instruction is stored in the memory word with address  $X_3$ . An index register contains

the value  $x_4$ . What is the relationship b/w these various quantity if the addressing mode of the instruction is

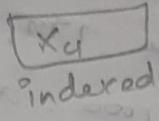
- (a) direct
- (b) indirect
- (c) indexed

- (b) indirect
- (c) PC relative

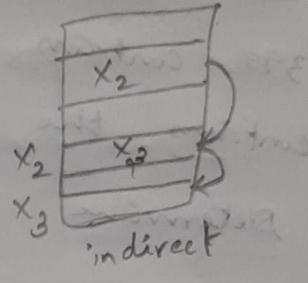
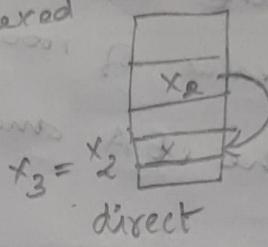
a)  $x_3 = x_2$



b)  $x_3 = (x_2)$



c)  $x_3 = (x_1 + 1) + x_2$   
next instruction



d)  $x_3 = x_2 + x_4$

4). An addressing field in an instruction contains decimal value 14. where is the corresponding operand located for

- a) Immediate addressing  $\rightarrow 14$  (The address field)
- b) Direct addressing  $\rightarrow [14]$  memory location 14.
- c) Indirect  $\rightarrow$  The memory location whose address is in memory location 14
- d) Register  $\rightarrow$  Register 14.
- e) Register indirect  $\rightarrow$  The memory location whose address is in register 14.

opcode	Address = 14
--------	--------------

Consider a 16-bit processor in which the following appears in main memory, starting at location 200

200	Load to AC mode
201	500
202	Next instruc

The first part of the first word. The mode field specifies an addressing mode and

if appropriate, indicates a source register, assuming that when used, the source register is R<sub>1</sub>; which has a value of 400. There is base register whose value is 100. The value of 500 in location 101 may be part of the address calculation. Assume that location 399 contains the value 222, location 400 contains the value of 1000, and so on.

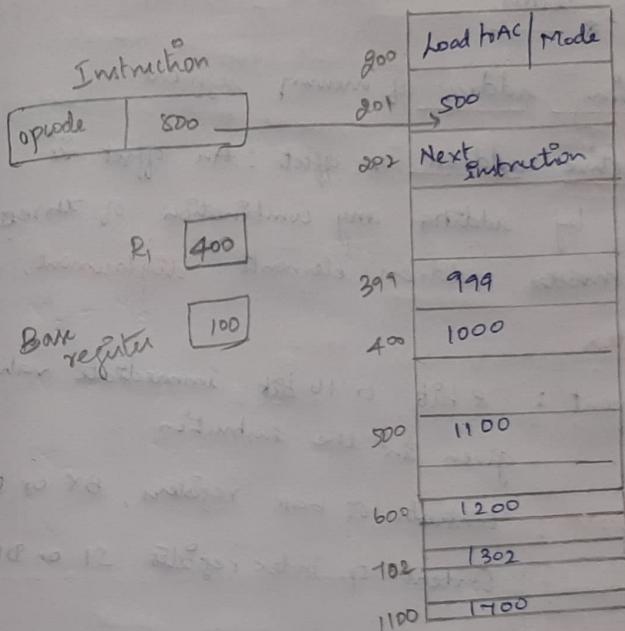
Determine the EA and operand.

Determine the EA and

Mode	EA	Operand
Direct	500	1100
Immediate	201	800
Indirect	1100	1700
PC relative	$201 + 1 + 500 = 702$	1302
Displacement	$500 + 100 = 600$	1200
Register	R1	400
Register indirect	400	1000
Autoindexing with increment using R1	400	1000

\*  $R_1$  is incremented after the execution

of the instruction



- b). Memory values and a one address machine  
 with  
 an accumulator is given as

Word 20	contains 40	20	40
Word 30	contains 50	30	50
Word 40	contains 60	40	60
Word 50	contains 70	50	70

What values do the following instructions load into

- a. Load IMMEDIATE 20
  - b. Load DIRECT 20
  - c. Load INDIRECT 20
  - d. Load IMMEDIATE 30
  - e. Load DIRECT 30
  - f. Load INDIRECT 30

- AC ← 20
- AC ← 40
- AC ← 60
- AC ← 30
- AC ← 80
- AC ← 70

## IMPORTANT terms:-

- \* Starting address of memory segment
- \* Effective address or offset : An offset is determined by adding any combination of three addressing modes address elements: displacement, base, index.

Displacement : 8 bit or 16 bit immediate value given in the instruction

Base : Contents of base registers, BX or BP

Index : Content of index register SI or DI.

## Symbols:-

for immediate addressing : #

Direct / absolute addressing : [ ]

Indirect addressing : @ on ()

## Examples:-

1) Implied : CLC (used to reset carry flag to 0)

2) Immediate : MOV AL, 35H

3) Register : MOV AX, CX

4) Register indirect : MOV AX, [BX]

5) Auto indexed :

↑ increment mode : Add R<sub>1</sub>, (R<sub>2</sub>) +  
↓ decrement mode : Add R<sub>1</sub>, -(R<sub>2</sub>)

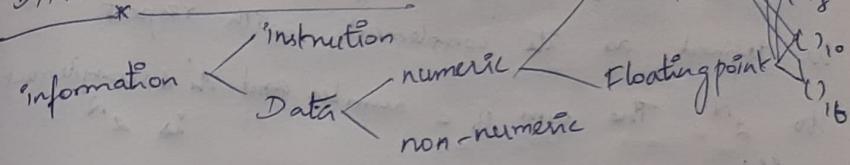
6) Direct : Add AL, [0301]

7) Indirect : Add AL, @301 (0301)

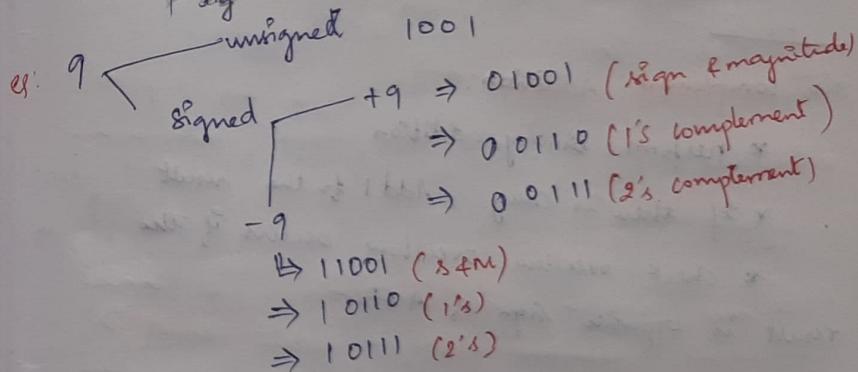
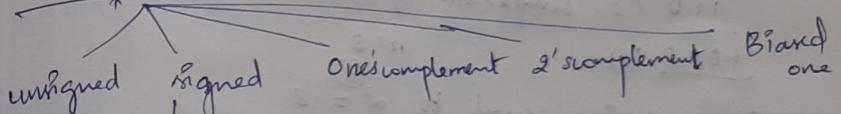
8) Based indexed addressing : MOV AX, [BX+SI]

9) Indexed addressing : Add AX, [SI+05]

## DATA REPRESENTATION:



## FIXED POINT REPRESENTATION:



## Reason for calling it as 1's complement:-

\* Complementing a single bit = To subtracting it from 1

$$0' \Rightarrow 1 \text{ and } 1 - 0 = 1$$

$$1' \Rightarrow 0 \text{ and } 1 - 1 = 0$$

\* Complementing each bit of an n-bit number is equivalent to subtracting it from  $2^n - 1$

e.g: 01101, here n=5  $\Rightarrow 2^{n-1} = 2^4 = 16_{10} = 1111_2$

$$\begin{array}{r}
 1111 \\
 01101 \\
 \hline
 10010
 \end{array}$$

\* The two's complement of an  $N$ -bit number is defined as its complement with respect to  $2^N$ . The sum of a number and its 2's complement is  $2^N$ .

e.g:

$$\begin{array}{r} 010 \quad (+) \\ 110 \quad (2's \text{ complement of } 010) \\ \hline 1000 \quad (8) \end{array}$$

$n=3$

$$8 \text{ which is equal to } 2^3 = 8$$

### 1's complement Addition:-

- \* perform binary addition
- \* if there is carry, Add 1 to the result
- \* check overflow: overflow occurs if the result is opposite sign of  $A + B$

Q:-

Note:- \* first do unsigned addition with the numbers, including the sign bits

\* Add the carry to the result

$\begin{array}{r} 0111 \quad (+7) \\ (+) \quad 1011 \\ \hline 10000 \end{array}$	$\begin{array}{r} 0010 \quad (-4) \\ (-) \quad 1 \\ \hline 0011 \end{array}$	$\begin{array}{r} 0011 \quad (+3) \\ (+) \quad 0101 \\ \hline 0100 \end{array}$
--	--	---

### 1's complement subtraction:-

- \* Take 1's complement of B by inverting all bits
- \* Add the 1's complement of B to A.

$$A - B = A + (-B)$$

### 2's complement Addition:-

\* Perform Binary addition

\* Ignore the carry out of the MSB

\* Check for overflow: overflow occurs if the 'carry in' and 'carry out' of the MSB are different, or if result is opposite sign of A and B

### 2's complement subtraction:-

$$A - B = A + (-B)$$

\* Take 2's complement of B by inverting all the bits and adding 1

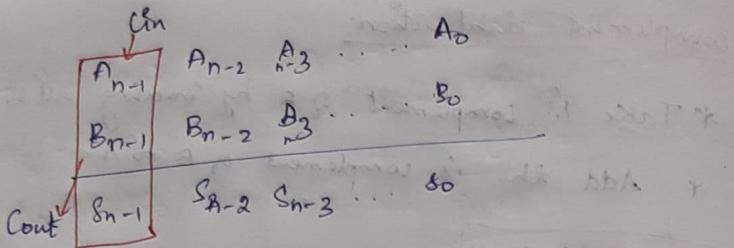
\* Add the 2's complement of B to A

### Overflow detection:-

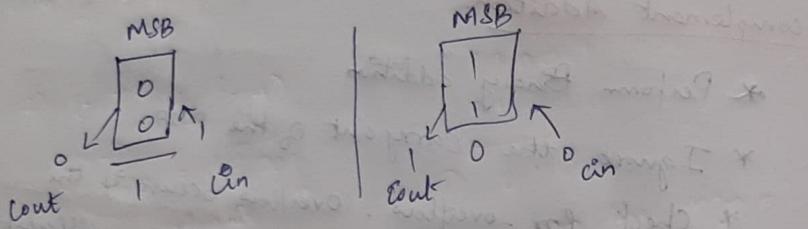
Occurs when i) 2 negative numbers are added and an answer comes as +ve.

ii) 2 +ve numbers are added and an answer comes as -ve.

⇒ So overflow can be detected by checking MSB of two operand and answer.



\* Overflow occurs when  $Cin + Cout = 1$



\* Overflow bit in 2's complement subtraction is ignored

Overflow  $\rightarrow$  when the addition of 2 binary numbers yields a result that is greater than the max. possible value

Underflow  $\rightarrow$  when the addition / subtraction of 2 binary numbers yields a result that is less than the min. possible value

Overflow

Assume 4 bit restriction and 2's complement.

$$\text{Max. possible value: } 2^{4-1} - 1 = 7$$

$$6_{10} + 3_{10} = 9_{10}$$

$$\begin{array}{r} 0110_2 \\ + 0011_2 \\ \hline 1001_2 \end{array} \quad \boxed{-7} \quad \times$$

Underflow

$$\text{Min. possible value: } -(2^{4-1}) = -8$$

$$-5_{10} + -5_{10} = -10_{10}$$

$$\begin{array}{r} 1011_2 \\ + 1011_2 \\ \hline 0110_2 \end{array} \quad \boxed{-5} \quad \times$$

Causes:

i) Carry generated but no overflow

$$\begin{array}{r} 1101 \\ + 1011 \\ \hline 1000 \end{array} \quad (-3) \quad (-5) \quad (-8)$$

(ii) carry and overflow

$$\begin{array}{r} 1101 \\ + 1010 \\ \hline 0111 \end{array} \quad (-3) \quad (-6) \quad (+7)$$

ii) No carry and no overflow

$$\begin{array}{r} 1101 \\ + 0010 \\ \hline 0111 \end{array} \quad (-3) \quad (+2) \quad (-1)$$

(iv) no carry but overflow

$$\begin{array}{r} 0111 \\ + 0011 \\ \hline 1010 \end{array} \quad (+7) \quad (+3) \quad (-6)$$

$Cin \oplus Cout \Rightarrow \text{overflow}$

shortcut to check the overflow:

\*  $x + y$  are the sign bits of two numbers

\*  $z$  is the sign bit of result

$$\begin{aligned} \bar{x} \cdot \bar{y} \cdot z + x \cdot y \cdot \bar{z} &= 0 \text{ if no overflow} \\ \bar{x} \cdot \bar{y} \cdot z + x \cdot y \cdot \bar{z} &= 1 \text{ (overflow)} \end{aligned}$$

Binary Arithmetic using 2's complement:

\* If the resultant sum contains carry out from the sign bit, then discard it in order to get the correct value.

\* If Resultant sum is +ve, keep it as it

\* If it is -ve, then take 2's complement of the result in order to get the magnitude

Ex:

$$1) (+7) + (+4)$$

$$+7_{10} = 00111_2$$

$$+4_{10} = 00100_2$$

$$(+7) + (+4) = 01011_2$$

$$+ve$$

$$01011_2 = +11_{10}$$

$$2) (-7) + (-4)$$

$$\begin{array}{c} \textcircled{-7} \\ +4 \\ \hline \end{array} = 1\textcircled{1}001_2$$

extra bit  
must also be 1

$$(-7)_2 = \textcircled{0}11100_2$$

$$\begin{array}{r} 11001 \\ 11100 \\ \hline \textcircled{1}0101 \end{array}$$

Carry out from the sign bit  
discard it

$$(-7) + (-4) = \textcircled{1}0101_2 = -5$$

Since the result is in  $-ve$  then 2's complement

$$\text{complement for } 0101 \text{ is } 1010 + 1 = 1011_2 = (11)_{10}$$

$$\therefore (-7) + (-4) = 10101_2 = -11$$

$$3) (+7) - (+4)$$

$$7 - 4 = 7 + (-4)$$

$$(+7) + (-4)$$

$$00111$$

$$11100$$

$$\underline{\textcircled{0}0011_2} = 3_{10}$$

2's complement of

$$+7_{10} = 00111_2$$

$$+4_{10} = 00100_2$$

$$-4_{10} = 11100_2$$

$$4) (+4) - (+7)$$

$$(+4) + (-7) \Rightarrow 00100$$

$$11001$$

$$\underline{\textcircled{0}1101}$$

$$-7_{10} = 11001_2$$

$$01101_2 = -3_{(10)}$$

$$1101$$

$$\underline{\textcircled{0}011}$$

Decimal	S.M.	1's complement	2's complement
7	0111	0111	0111
6	0110	0110	0110
5	0101	0101	0101
4	0100	0100	0100
3	0011	0011	0011
2	0010	0010	0010
1	0001	0001	0001
0	0000	0000	0000
-1	1000	1011	-
-2	1001	1110	1111
-3	1011	1101	1110
-4	1100	1011	1100
-5	1101	1010	1011
-6	1110	1001	1010
-7	1111	1000	1001
-8	-	-	1000

- \* Two number is same in all types of representation
- \* 2's complement  $\rightarrow$  asymmetric range  $+7$  to  $-8$ .
- \* 2's complement addition algorithm is simple