13. **Marginal probability** : "The probability of only one event that taken at a time when both are occurring jointly is called Marginal Probability".

14. **Joint probability :** The probability of occurrence of both the events A and B together, denoted by $P(A \cap B)$, is known as joint probability of A and B.

15. **Conditional probability** : The conditional probability of A for the given B is $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$ if $P(B) \neq 0$ and it is undefined if $P(B) = 0$.

A rearrangement of the above definition yields the following :

MR (Multiplication Rule).

$$P(A \cap B) = \begin{cases} P(B)\ P(A/B) & \text{if } P(B) \neq 0 \\ P(A)\ P(B/A) & \text{if } P(A) \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

16. **Explain the term "two events are independent".**

Two events are independent if any one of the following equivalent statements is true.

1. $P(A/B) = P(A)$ 2. $P(B/A) = P(B)$

3. $P(A \cap B) = P(A)\ P(B)$

17. **Type of Random variables :**
    1. Discrete random variables
    2. Continuous random variables

Discrete Random Variable	**Continuous Random Variable**
A r.v. X is discrete if it takes only discrete or countable values.	A r.v. X is said to be continuous if it takes all possible values between certain limits or in an internal which may be finite or infinite.

Discrete Random Variable	Continuous Random Variable
P.m.f. [Probability mass function] If X is a discrete r.v. then the function $\bar{P}(x) = P[X = x]$ is called the p.m.f. of X provided it satisfies the following conditions (i) $p(x_i) \geq 0, \quad \forall, i = 1,2,3,...$ (ii) $\sum_{i=1}^{\infty} p(x_i) = 1$	**p.d.f [Probability density function]** If X is a continuous r.v. such that $P\left\{x - \frac{1}{2}dx \leq X \leq x + \frac{1}{2}dx\right\} = f(x)\,dx$ then $f(x)$ is called the p.d.f of X provided $f(x)$ satisfies the following conditions. (i) $f(x) \geq 0$, (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ $p[a \leq X \leq b]$ (or) $p[a < X < b]$ (or) $p(a \leq X < b)$ (or) $p[a < X \leq b]$ $= \int_a^b f(x)\,dx.$
C.d.f [Cumulative distribution function] $F[x] = P[X \leq x] = \sum_{j}^{x} P_j$	**C.d.f [Cumulative distribution function]** $F[x] = P[-\infty < X \leq x]$ $= \int_{-\infty}^{x} f(x)\,dx$
Properties of cdf. 1. $F[x]$ is a non-decreasing function of x, (i.e.,,) if $x_1 < x_2$, then $F[x_1] \leq F[x_2]$. $P[X \leq x_1] \leq P[X \leq X_2]$ $= P[x \leq x_i] - P[X \leq x_{i-1}]$ 2. $F[-\infty] = 0$ and $F[\infty] = 1$. 3. If X is discrete R.V. taking values $x_1, x_2, ...$ where $x_1 < x_2 < ...$ $< x_{i-1} < x_i < ...$ then $P[X = x_i] = F[x_i] - F[x_{i-1}]$	**Properties of cdf.** 1. $F[x]$ is a non-decreasing function of x, (i.e.,) if $x_1 < x_2$, then $F[x_1] \leq F[x_2]$. 2. $F[-\infty] = 0$ and $F[\infty] = 1$. 3. If X is a continuous R.V, then $\frac{d}{dx} F[x] = f(x)$, at all points where $F[x]$ is differentiable.

Discrete Random Variable	Continuous Random Variable
Mean $(\mu) = E[X] = \sum_i x_i\, p(x_i)$ **Variance** $(\sigma^2) = \text{Var}(X)$ $= E[X^2] - [E[X]]^2$ $E[X^2] = \sum_i x_i^2\, p(x_i)$ **S.d** of X $= \sqrt{\text{Var}(X)}$ (+ve) **Moments** $\mu_r' = E[X^r] = \sum_{i=1}^{n} x_i^r\, p_i$ (about the origin) $= \sum_{i=1}^{n} (x_i - A)^r\, p_i$ (about any point $x = A$) $= \sum_{i=1}^{n} (x_i - mean)^r\, p_i$ (about the $(\bar{x})$) First four moments about the origin are $\mu_1' = E[X] =$ mean, $\mu_2' = E[X^2]$, $\mu_3' = E[X^3]$, $\mu_4' = E[X^4]$ $\mu_1 = 0$ first moment about mean $\mu_2 = E[X-\mu]^2 =$ variance of X $\mu_3 = E[X-\mu]^3$ $\mu_4 = E[X-\mu]^4$	**Mean** $(\mu) = E[X] = \int_{-\infty}^{\infty} x f(x)\, dx$ **Variance** $(\sigma^2) = \text{Var}[X]$ $= E[X^2] - [E(X)]^2$ $E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx$ **S.d.** of X $= \sqrt{\text{Var}(X)}$ (+ve) **Moments** $\mu_r' = E[X^r] = \int_{-\infty}^{\infty} x^r f(x)\, dx$ [about the origin] $= \int_{-\infty}^{\infty} (x - A)^r f(x)\, dx$ [about any point A] $= \int_{-\infty}^{\infty} (x - \bar{x})^r f(x)\, dx$ [about the mean] First four moments about the origin are $\mu_1' = E[X] =$ mean, $\mu_2' = E[X^2]$, $\mu_3' = E[X^3]$, $\mu_4' = E[X^4]$ $\mu_1 = 0$ first moment about mean $\mu_2 = E[X-\mu]^2 =$ variance of X $\mu_3 = E[X-\mu]^3$ $\mu_4 = E[X-\mu]^4$
M.G.F $M_X[t] = E[e^{tx}]$ $= \sum_x e^{tx}\, p(x)$	**M.G.F.** $M_x(t) = E[e^{tx}]$ $= \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$

TWO DIMENSIONAL RANDOM VARIABLES

Discrete Random Variable	Continuous Random Variable
If the possible values of (X, Y) are finite or countably infinite, then (X, Y) is called a two dimensional discrete random variable	If the values of (X, Y) is an uncountably infinite set, (i.e.,) the range space is a region in the XY plane, then (X, Y) is called the two-dimensional continuous random variable.
Joint probability mass function	**Joint Probability density function**
$P(x_i, y_i) = P[X=x_i ; Y=y_j]$	$P\left[x-\frac{dx}{2} \leq X \leq x+\frac{dx}{2}, y-\frac{dy}{2} \leq Y \leq y+\frac{dy}{2}\right] = f(x, y)\,dx\,dy$
(i) $p(x_i, y_i) \geq 0$, $\forall\ i = 1, 2, \ldots ;\ j = 1, 2, \ldots$ (ii) $\sum_i \sum_j p(x_i y_j) = 1$	(i) $f(x, y) \geq 0, \forall\ (x, y) \in R$ (ii) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\,dx\,dy = 1$ (or) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\,dy\,dx = 1$
Marginal probability mass function $f(x) = P_X(x_i) = P(X=x_i) = \sum_{i=1}^{m} p(x_i, y_i) = p_i$ $f(y) = P_Y(y_i) = P[Y = y_i] = p_j = \sum_j p_{ij}$	**Marginal density function** $f(x) = f_X(x) = \int_{-\infty}^{\infty} f(x, y)\,dy$ $f(y) = f_Y(y) = \int_{-\infty}^{\infty} f(x, y)\,dx$

Discrete Random Variable	Continuous Random Variable
c.d.f. $F_{XY}(x, y) = P[X \leq x, Y \leq y]$	**c.d.f.** $F[x, y] = \int_{-\infty}^{x} \int_{-\infty}^{y} f(x, y)\, dx\, dy$
Properties	**Properties**
1. $0 \leq P(x_i, y_i) \leq 1$	1. $0 \leq F(x, y) \leq 1$
2. $\sum_i \sum_j P(x_i, y_j) = 1$	2. $F(\infty, \infty) = 1$
3. $P(x_i) = \sum_i P(x_i, y_i)$	3. $F[-\infty, y] = 0$
4. $P(y_i) = \sum_i P(x_i, y_i)$	4. $F[x, -\infty] = 0$
	5. $f(x, y) = \frac{\partial^2}{\partial x \partial y} F(xy)$
Conditional probability mass function (i) $P_{Y/X}(y) = \frac{P_{XY}(x, y)}{P_X(x)}$ (ii) $\sum_{R_x} P_{Y/X}(y) = 1$	**Conditional probability density function** (i) $f_{Y/X}(y) = \frac{f(x, y)}{f_X(x)}$ (ii) $\int_{R_X} f_{Y/X}(y)\, dy = 1$
Note : $P_{XY}(x, y) = P(x, y)$	$f_X(x) = f(x)$
$P_Y(y) = P(y)$	$f_Y(y) = f(y)$
$P_X(x) = P(x)$	$f_X(x) = f(x)$
$P_{X/Y}(x/y) = P(x/y)$	$f_{X/Y}(x/y) = f(x/y)$
$P_{Y/X}(y/x) = P(y/x)$	$f_{Y/X}(y/x) = f(y/x)$
$P(x, y) = P(y, x)$	$f(x, y) = f(y, x)$