*STATISTICS * FOR ENGINEERS *

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SLOT : $B_2 + TB_2$

DIGITAL ASSIGNMENT - I

1. If X and Y have the joint p.d.f $f(x,y) = \frac{1}{3}(x+y)$, $0 \le x \le 1, 0 \le y \le 2$, then find

i) $r(X,Y)$

soln:

$f(x,y) = \frac{1}{3}(x+y)$

$f_X(x) = \int_0^2 \frac{1}{3}(x+y)\,dy$

$= \frac{1}{3}\left[xy + \frac{y^2}{2}\right]_0^2$

$= \frac{2}{3}(x+1)$

$E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\,dx$

$= \int_0^1 x(x+1)\frac{2}{3}\,dx$

$= \frac{2}{3}\int_0^1 (x^2+x)\,dx$

$= \frac{2}{3}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1$

$= \frac{2}{3}\left[\frac{1}{3} + \frac{1}{2}\right]$

$E[X] = \frac{5}{9}$

$f_Y(y) = \int_0^1 \frac{1}{3}(x+y)\,dx$

$= \frac{1}{3}\left[\frac{x^2}{2} + xy\right]_0^1$

$= \frac{1}{3}\left[\frac{1}{2} + y\right]$

$= \frac{1}{6} + \frac{y}{3}$

$E[Y] = \int_{-\infty}^{\infty} y\, f_Y(y)\,dy$

$= \int_0^2 y\left(\frac{1}{6} + \frac{y}{3}\right)dy$

$= \int_0^2 \left(\frac{y}{6} + \frac{y^2}{3}\right)dy$

$= \left[\frac{y^2}{12} + \frac{y^3}{9}\right]_0^2$

$= \left(\frac{4}{12} + \frac{8}{9}\right) - 0$

$= \frac{3}{9} + \frac{8}{9}$

$E[Y] = \frac{11}{9}$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

$$= \int_0^1 x^2 \cdot \tfrac{2}{3}(x+1)\,dx$$

$$= \tfrac{2}{3}\int_0^1 (x^3+x^2)\,dx$$

$$= \tfrac{2}{3}\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_0^1$$

$$= \tfrac{2}{3}\left[\tfrac{1}{4} + \tfrac{1}{3}\right]$$

$$= \tfrac{7}{18}$$

$$Var[X] = E[X^2] - (E[X])^2$$

$$= \frac{7}{18} - \left(\tfrac{5}{9}\right)^2$$

$$= \tfrac{7}{18} - \frac{25}{81}$$

$$= \tfrac{13}{162}$$

$$= 0.0802$$

$$\sigma_x = \sqrt{Var[X]}$$

$$= \sqrt{0.0802}$$

$$= 0.2832$$

$$E[Y^2] = \int_{-\infty}^{\infty} y^2 f_Y(y)\,dy$$

$$= \int_0^2 y^2\left(\tfrac{1}{6} + \tfrac{y}{3}\right)dy$$

$$= \int_0^2 \left(\tfrac{y^2}{6} + \tfrac{y^3}{3}\right)dy$$

$$= \left[\frac{y^3}{18} + \frac{y^4}{12}\right]_0^2$$

$$= \left[\frac{\cancel{8}^{4}}{\cancel{18}_{9}} + \frac{\cancel{16}^{4}}{\cancel{12}_{3}}\right] - 0$$

$$= \frac{4+12}{9}$$

$$= \tfrac{16}{9}$$

$$Var[Y] = E[Y^2] - (E[Y])^2$$

$$= \tfrac{16}{9} - \left(\tfrac{11}{9}\right)^2$$

$$= \frac{16}{9} - \frac{121}{81}$$

$$= \frac{23}{81}$$

$$= 0.2839$$

$$\sigma_y = \sqrt{var[Y]}$$

$$= \sqrt{0.2839}$$

$$= 0.5328$$

$$E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f(x,y)\,dx\,dy$$

$$= \int_0^2\int_0^1 xy\left(\tfrac{1}{3}(x+y)\right)dx\,dy$$

$$= \int_0^2 \int_0^1 \left(\frac{x^2 y}{3} + \frac{xy^2}{3}\right) dx\,dy$$

$$= \int_0^2 \left[\frac{x^3 y}{9} + \frac{x^2 y^2}{6}\right]_0^1 dy$$

$$= \int_0^2 \left(y/9 + y^2/6\right) dy$$

$$= \left[y^2/18 + y^3/18\right]_0^2$$

$$= \frac{4}{18} + \frac{8}{18} = \frac{12}{18} = 2/3$$

$$E[XY] = 2/3$$

$$r(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sigma_x \cdot \sigma_y}$$

$$= \frac{E[XY] - E[X].E[Y]}{\sigma_x \cdot \sigma_y}$$

$$= \frac{2/3 - (5/9)(11/9)}{(0.2832)(0.5328)}$$

$$= \frac{2/3 - \frac{55}{81}}{0.1509}$$

$$= \frac{54 - 55}{81(0.1509)}$$

$$= \frac{-1}{81(0.1509)}$$

$$r(X,Y) = -0.0818$$

(ii) lines of Regression :-

<u>Y on X</u> :-

$$(y-\bar{y}) = r_{xy}\frac{\sigma_y}{\sigma_x}(x-\bar{x})$$

$$y-11/9 = (-0.0818)\left(\frac{0.5328}{0.2832}\right)(x-5/9)$$

$$y-11/9 = -0.1539(x-5/9)$$

$$y = -0.1539x + \frac{(0.1539)(5)}{9} + 11/9$$

$$y = -0.1539x + 1.3077$$

<u>X on Y</u> :-

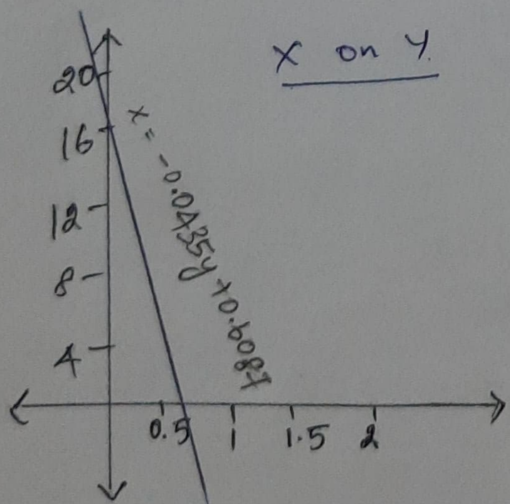
$$(x-\bar{x}) = r_{xy}\frac{\sigma_x}{\sigma_y}(y-\bar{y})$$

$$x-5/9 = (-0.0818)\left(\frac{0.2832}{0.5328}\right)(y-11/9)$$

$$x-5/9 = -0.0435(y-11/9)$$

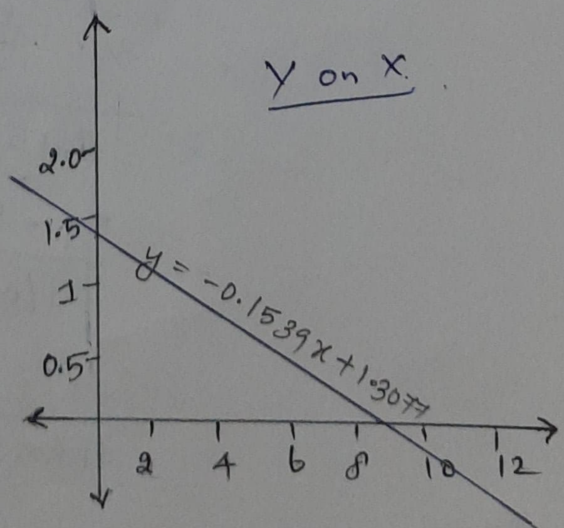
$$x = -0.0435y + 0.6087$$

(iii) the two regression curves for means



$Y=0 \Rightarrow x = 0.6087$

$x=0 \Rightarrow Y = 16.39$



$y=0 \Rightarrow x = 8.49$

$x=0 \rightarrow y = 1.3077$

2) If X, Y and Z are uncorrelated r.v's with zero mean and S.D's 5, 13 and 9 respectively and if $U = X + Y$ and $V = Y + Z$, find the correlation co-efficient between U and V.

Soln:

Given that Z, X, Y are uncorrelated and their means are zero.

i.e. $E[X] = E[Y] = E[Z] = 0$

$Var[X] = E[X^2] - [E(X)]^2 = E[X^2]$

III^ly

$Var[Y] = E[Y^2]$ and $Var[Z] = E[Z^2]$

$\therefore E[X^2] = (\sigma_x)^2 = (5)^2 = 25$

$E[Y^2] = (\sigma_y)^2 = (13)^2 = 169$

$E[Z^2] = (\sigma_z)^2 = (9)^2 = 81$

since the given $\sigma_x = 5$, $\sigma_y = 13$ and $\sigma_z = 9$

Since X and Y are uncorrelated,

$Cov(X, Y) = 0$

$E[XY] - E[X]E[Y] = 0$

$\therefore E[XY] = E[X]E[Y] = 0$

III^ly

$E[YZ] = 0$ &

$E[ZX] = 0$

To find $r(U, V)$:-

$$r(U, V) = \frac{cov(U, V)}{\sigma_U \sigma_V}$$

$$= \frac{E[UV] - E[U]E[V]}{\sigma_U \sigma_V}$$

$E(U) = E[X+Y] = E[X] + E[Y] = 0 + 0 = 0$

$E(V) = E[Y+Z] = E[Y] + E[Z] = 0 + 0 = 0$

$\therefore E[U^2] = E[(X+Y)^2] = E[X^2] + E[Y^2] + 2E[XY]$

$= 25 + 169 + 2(0)$

$= 194$

$E[V^2] = E[(Y+Z)^2] = E[Y^2] + E[Z^2] + 2E[YZ]$

$= 169 + 81 + 2(0)$

$= 250$

$Var(U) = E(U^2) - (E(U))^2$

$= 194 - 0$

$Var(U) = 194$

$\sigma_U = \sqrt{194}$

$\sigma_U = 13.93$

$Var(V) = E[V^2] - (E[V])^2$

$= 250 - 0$

$Var[V] = 250$

$\sigma_V = \sqrt{250}$

$\sigma_V = 15.81$

$E(UV) = E[(x+y)(y+z)]$

$= E[XY] + E[XZ] + E[Y^2] + E[YZ]$

$= 0 + 0 + 169 + 0$

$E[UV] = 169$

$\therefore \gamma(U,V) = \dfrac{Cov[U,V]}{\sigma_U \sigma_V} = \dfrac{E(UV) - E(U)E(V)}{\sigma_U \sigma_V}$

$= \dfrac{169 - (0)(0)}{(13.93)(15.81)}$

$\gamma(U,V) = 0.7674$

3) The life length X of an electronic component follows an exponential distribution. There are two processes by which the component may be manufactured. The expected life length of the component is 100 h if process I is used to manufacture while it is 150 h, if process II is used. The cost of the manufacturing a single component process one is Rs. 10 while it is Rs. 20 for process II. Moreover if the component lasts less than the guaranteed life of 200 h, a loss of Rs. 50 is to be borne by the manufacturer. Which process is advantageous to the manufacturer?

Soln:-

P_k = probability of producing a component which lasts less than the guarranteed life span of 200 h.

Given: $E[X_1] = 100$, $E[X_2] = 150$

$$\therefore \lambda_1 = \frac{1}{E[X_1]} = \frac{1}{100} \quad \& \quad \lambda_2 = \frac{1}{E[X_2]} = \frac{1}{150}$$

The probability function of the two processes are

$$f_1 = \lambda_1 e^{-\lambda_1 x} = \frac{1}{100} e^{-1/100 x}$$

$$f_2 = \lambda_2 e^{-\lambda_2 x} = \frac{1}{150} e^{-1/150 x}$$

The the probability of the component which meets the life time of less than 200 h

(i) produced by process $\hat{I}$

$t = x/100$

$\frac{dt}{dx} = \frac{1}{100}$

$100\,dt = dx$

$100\,(e^{t})_{0}^{200}$

$$p(x < 200) = \int_0^{200} f_1\,dx$$

$$= \int_0^{200} \frac{1}{100} e^{-x/100}\,dx$$

$$= \frac{1}{100}\left(-100\left(e^{-200/100} - e^{0}\right)\right)$$

$$= -(e^{-2} - 1)$$

$$p(x < 200) = +0.8647$$

So Mean cost of produing component by process $\hat{I}$, = cost of producing successful product + loss %.

$$= 10 + (+0.8647)\,50$$

$$= 53.235$$

(ii) produced by process $\hat{II}$

$$p(x < 200) = \int_0^{200} f_2\,dx$$

$$= \int_0^{200} \frac{1}{150} e^{-x/150}\,dx$$

$$= \frac{1}{150}\left(-150\left(e^{-200/150} - e^{0}\right)\right)$$

$$= -(-0.7364)$$

$$= 0.7364$$

So the expected mean cost of producing component by process $\hat{II}$,

$=$ cost of producing successful product + loss %

$= 20 + (0.7364) 50$

$= 56.82$

Comparing the result of (i) and (ii), it is obvious that the process $\hat{I}$ is advantageous to the manufacturer.

4). If the density function of a continuous r.v X is $f(x) = ce^{-b(x-a)}, a \leq x$ where a, b, c are constants. show that $b = c = 1/\sigma$ and $a = \mu - \sigma$ where $\mu = E[X]$ and $\sigma = Var[X]$

Soln:

$$f(x) = ce^{-b(x-a)}, \quad a \leq x$$

WKT,

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{a}^{\infty} ce^{-b(x-a)} = 1$$

$$\int_{a}^{\infty} ce^{-bx+ab} = 1$$

$$c e^{ab} \int_a^{\infty} e^{-bx}\, dx = 1$$

$$\frac{c e^{ab}}{-b} \left[e^{-\infty} - e^{-ab} \right] = 1$$

$$\frac{c e^{ab}}{-b} (-e^{-ab}) = 1$$

$$\frac{c}{b} e^{0} = 1$$

$$c = b$$

$$E[x] = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_a^{\infty} x\, c\, e^{-b(x-a)}\, dx$$

$$= c e^{ab} \int_a^{\infty} x e^{-bx} \qquad u = x,\ dx = \int e^{-bx} dx$$

$$v = e^{-bx}/-b$$

$$= c e^{ab} \left[\frac{x e^{-bx}}{-b} - \frac{e^{-bx}}{b^2} \right]_a^{\infty} \qquad u' = 1,\ v'' = \frac{e^{-bx}}{b^2}$$

$$= c e^{ab} \left[\frac{a e^{-ab}}{+b} + \frac{e^{-ab}}{b^2} \right]$$

$$= \frac{c e^{ab} e^{-ab}}{b^2} (ab + 1) \qquad \frac{c}{b^2}(ab+1)$$

$$= \frac{cab}{b^2} + \frac{c}{b^2} \quad \text{where } c = b$$

$$= a + 1/b$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_a^\infty c e^{-b(x-a)} x^2 \, dx$$

$u = x^2$, $u' = 2x$, $u'' = 2$ | $dv = e^{-bx}$, $v = \frac{e^{-bx}}{-b}$, $v_1 = \frac{e^{-bx}}{b^2}$, $v_2 = \frac{e^{-bx}}{-b^3}$

$$= c e^{ab} \int_a^\infty x^2 e^{-bx} \, dx$$

$$= c e^{ab} \left[\frac{x^2 e^{-bx}}{-b} - \frac{2x e^{-bx}}{b^2} + \frac{2 e^{-bx}}{-b^3} \right]_a^\infty$$

$$= c e^{ab} \left[(0) - \left(\frac{a^2 e^{-ab}}{-b} - \frac{2a e^{-ab}}{b^2} + \frac{2 e^{-ab}}{-b^3} \right) \right]$$

$$= c e^{ab} \left[\frac{a^2 e^{-ab}}{b} + \frac{2a e^{-ab}}{b^2} + \frac{2 e^{-ab}}{b^3} \right]$$

$$= \frac{c e^{ab} e^{-ab}}{b^3} \left[a^2 b^2 + 2ab + 2 \right]$$

$$= \frac{c a^2 b^2}{b^3} + \frac{2abc}{b^3} + \frac{2c}{b^3} \qquad [c = b]$$

$$= a^2 + \frac{2ab^2}{b^3} + 2/b^2$$

$$Var[X] = a^2 + \frac{2ab^2}{b^3} + 2/b^2 - (a + 1/b)^2 = E[X^2] - (E[X])^2$$

$$= a^2 + \frac{2a}{b} + 2/b^2 - (a^2 + 2a/b + 1/b^2)$$

$$= 2/b^2 - 1/b^2$$

$$= 1/b^2$$

$$Var[X] = 1/b^2 \Rightarrow \sigma = \sqrt{1/b^2} = 1/b$$

Since $c = b$, $b = c = 1/\sigma$ & $\mu = a + 1/b$

i.e. $\Rightarrow a = \mu - 1/b = \mu - \sigma$

Hence proved

5). A study of prices of rice at Chennai and Vellore gave the following data

	Chennai	Vellore
μ	19.5	17.75
σ	1.75	2.5

Also, the co-efficient of correlation between the two is 0.8. Estimate the most likely price of rice (i) at Chennai corresponding to the price of 18 at Vellore (ii) at Vellore corresponding to the price of 17 at Chennai.

Soln:-

$\bar{x} = 19.5$, $\bar{y} = 17.75$, $\sigma_x = 1.75$, $\sigma_y = 2.5$

$r_{xy} = 0.8$

(i) x on y :-

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - \bar{x} = r_{xy}\frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

$$x - 19.5 = (0.8)\left(\frac{1.75}{2.5}\right)(y - 17.75)$$

$$x - 19.5 = 0.56(y - 17.75)$$

when $y = 18$,

$$x - 19.5 = 0.56(18 - 17.75)$$

$$x = 0.14 + 19.5$$

$$\boxed{x = ₹\ 19.64}$$

(ii) y on x :-

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - \bar{y} = r_{yx}\frac{\sigma_y}{\sigma_x}(x - \bar{x})$$

$$y - 17.75 = 0.8 \times \frac{2.5}{1.75}(x - 19.5)$$

$$y - 17.75 = 8/7\,(x - 19.5)$$

when $x = 17$,

$$y - 17.75 = 8/7\,(17 - 19.5)$$

$$y = \frac{-20}{7} + 17.75$$

$$\boxed{y = ₹\ 14.8928}$$