EEE1024: Fundamentals of Electrical and Electronics Engineering

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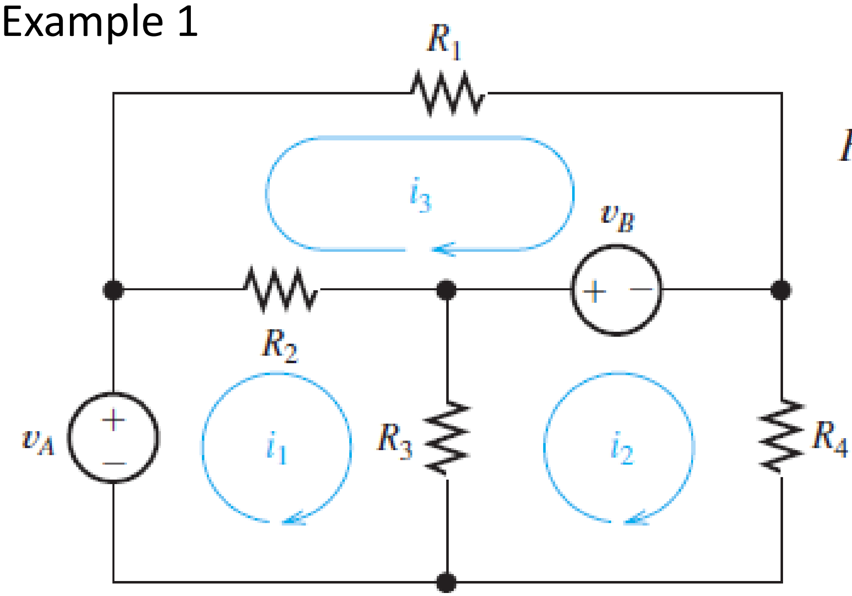
Mesh current method - 1

Example 1



$$R_2(i_1 - i_3) + R_3(i_1 - i_2) - v_A = 0$$ **KVL @ Mesh 1**

$$R_3(i_2 - i_1) + R_4 i_2 + v_B = 0$$ **KVL @ Mesh 2**

$$R_2(i_3 - i_1) + R_1 i_3 - v_B = 0$$ **KVL @ Mesh 3**

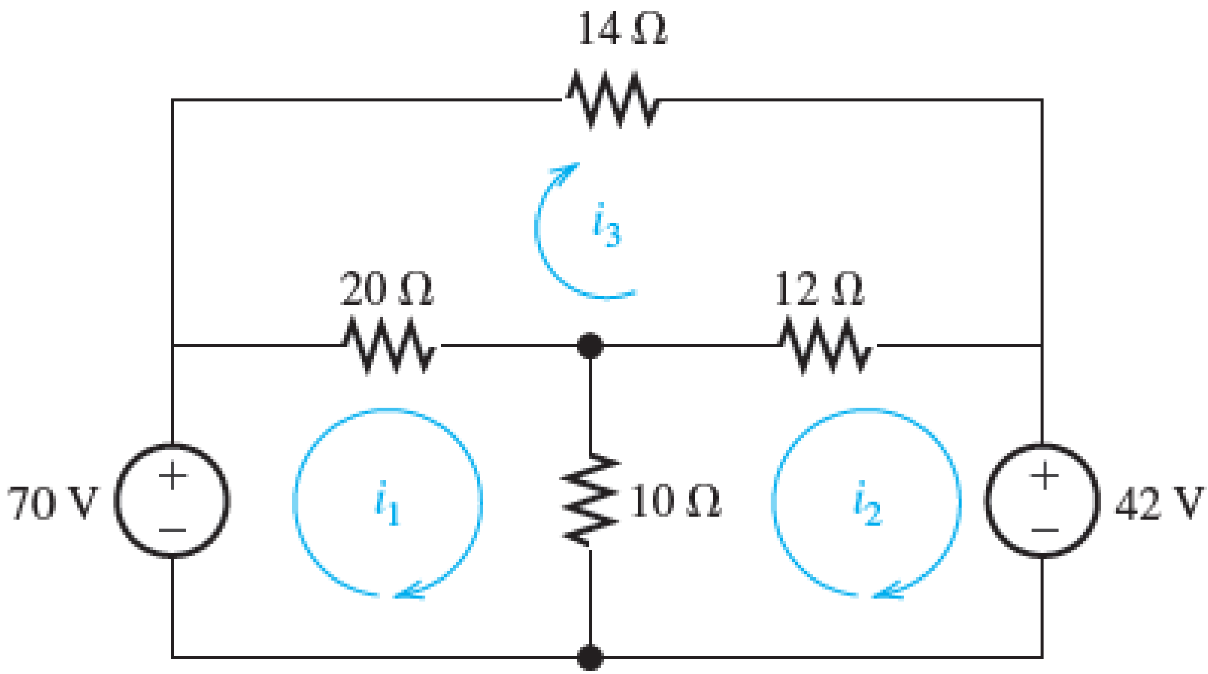
Matrix form

$$\begin{aligned} (R_2 + R_3)i_1 - R_3 i_2 - R_2 i_3 &= v_A \\ -R_3 i_1 + (R_3 + R_4) i_2 &= -v_B \\ -R_2 i_1 + (R_1 + R_2) i_3 &= v_B \end{aligned}$$

$$\begin{bmatrix} (R_2 + R_3) & -R_3 & -R_2 \\ -R_3 & (R_3 + R_4) & 0 \\ -R_2 & 0 & (R_1 + R_2) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_A \\ -v_B \\ v_B \end{bmatrix}$$

Mesh current method - 2

Example 2: Solve for currents in each element of this circuit



KVL @ Mesh 1: $20(i_1 - i_3) + 10(i_1 - i_2) - 70 = 0 \Rightarrow 30i_1 - 10i_2 - 20i_3 = 70$

KVL @ Mesh 2: $10(i_2 - i_1) + 12(i_2 - i_3) + 42 = 0 \Rightarrow -10i_1 + 22i_2 - 12i_3 = -42$

KVL @ Mesh 3: $20(i_3 - i_1) + 14i_3 + 12(i_3 - i_2) = 0 \Rightarrow -20i_1 - 12i_2 + 46i_3 = 0$

Mesh current method - 3

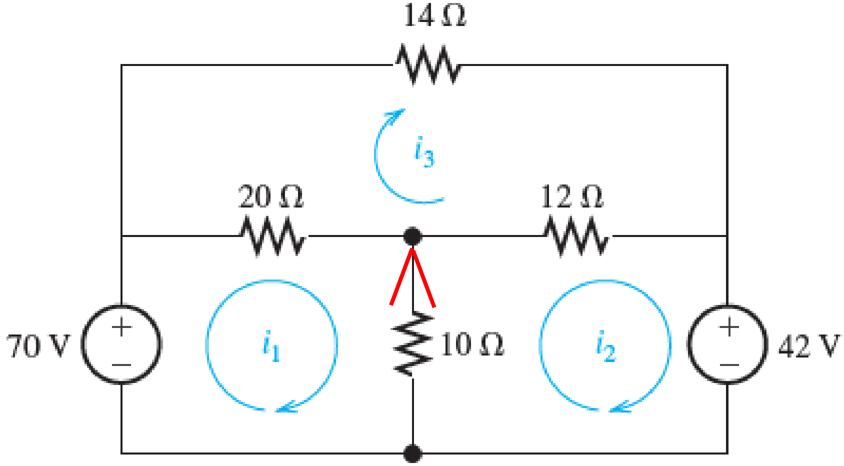
Example 2 continued

$$30i_1 - 10i_2 - 20i_3 = 70$$

$$-10i_1 + 22i_2 - 12i_3 = -42$$

$$-20i_1 - 12i_2 + 46i_3 = 0$$

$$\begin{bmatrix} 30 & -10 & -20 \\ -10 & 22 & -12 \\ -20 & -12 & 46 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 70 \\ -42 \\ 0 \end{bmatrix}$$



Method 1: Calculator

Method 2: Manual calculations

Method 3: MATLAB!

```
>> R = [30 -10 -20; -10 22 -12; -20 -12 46];
>> V = [70; -42; 0];
>> I = R\V
I =
    4.0000
    1.0000
    2.0000
```

$$i_1 = 4A$$
$$i_2 = 1A$$
$$i_3 = 2A$$

Current through 10 Ω = i_{10}

$$i_2 - i_1 = 1 - 4 = -3A$$

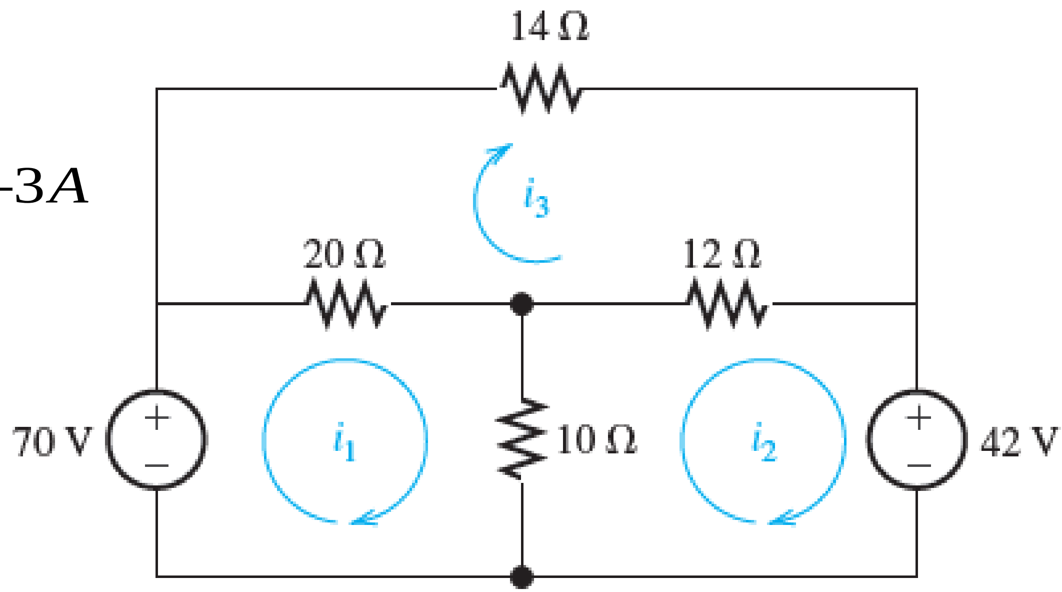
Mesh current method - 2

Example 2: a) Find the current through the 10 Ω resistor of the given circuit.

b) Find the power supplied by 42V voltage source

c) Find the power delivered to the 10 Ω resistor using mesh currents analysis in the figure shown

$$i_1 = 4A$$
$$i_2 = 1A \quad i_2 - i_1 = 1 - 4 = -3A$$
$$i_3 = 2A$$



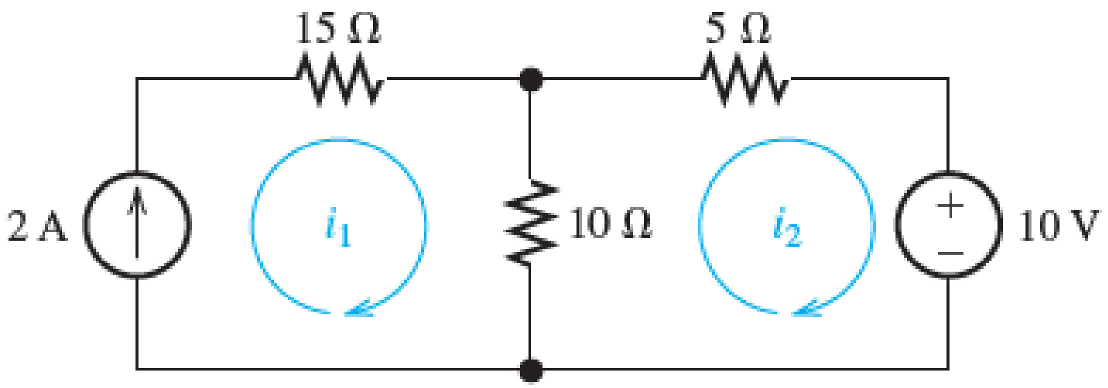
b) P = $V * i_2$

= 42W

c) Either mesh 1 or mesh 2 can be used.

$$P = V(i_1 - i_2) \qquad P = (i_1 - i_2)^2 R$$
$$P_{10} = (4-1)^2 \times 10$$
$$= 90W$$

Mesh current method – *Current Sources*



@ Mesh 1,

$$i_1 = 2\text{ A}$$

@ Mesh 2,

$$10(i_2 - i_1) + 5i_2 + 10 = 0$$

$$10i_2 - 20 + 5i_2 = -10$$

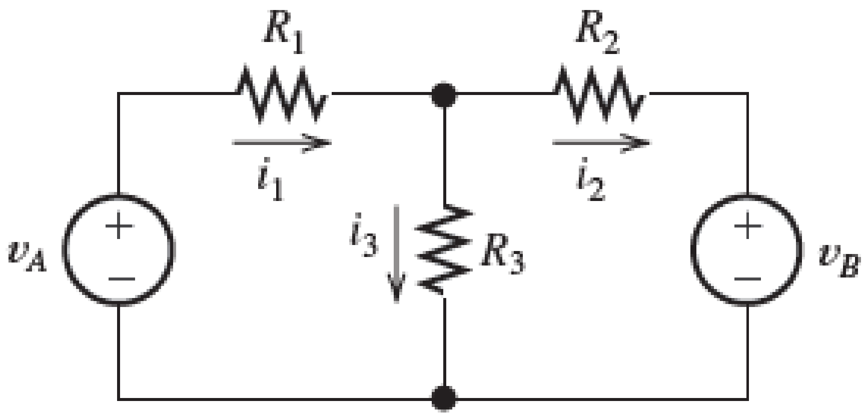
$$15i_2 = 10$$ or $i_2 = 0.67\text{A}$

DO NOT APPLY KVL to LOOP/MESH having a CURRENT SOURCE!

THEVENIN'S Theorem

MAXIMUM POWER TRANSFER Theorem -

Need for Thevenin's Theorem



- Find V across R_3
- Find V across terminal A and B and current through R_3 for 50 values of R_3

??

Thevenin's Theorem

Léon *Charles Thévenin*

Complicated circuit – segregated into fixed part and variable part

Entire fixed part of a network is replaced by -
an equivalent voltage source and a series resistor,
called as *Thevenin Equivalent Circuit*

" A linear bi-directional two terminal network can be replaced by an equivalent network consisting of a voltage source V_{th} (or V_t) connected in series with a resistor R_{th} (or R_t)"

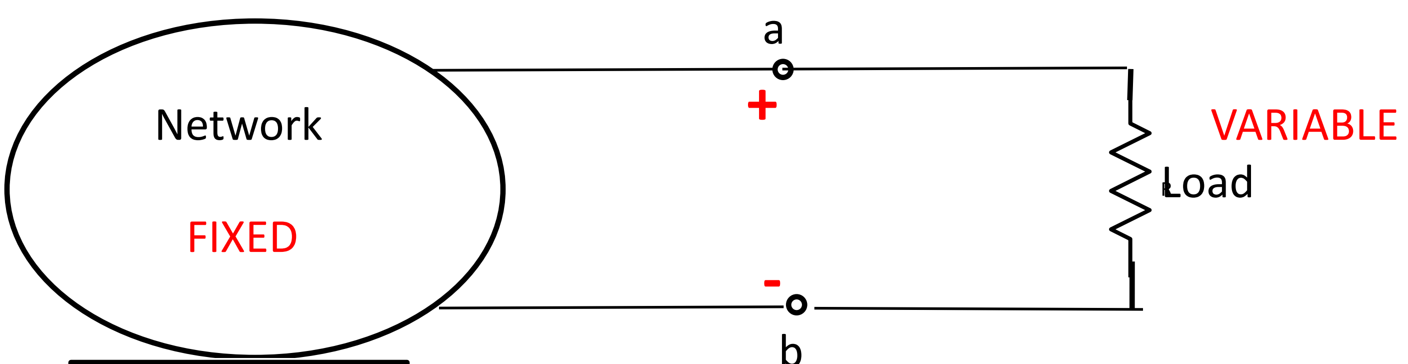
Thevenin Equivalent Circuit

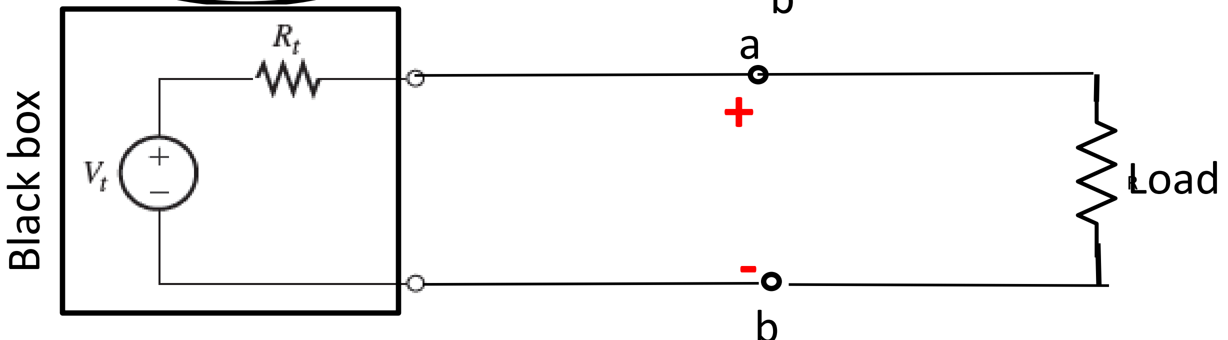
Network can be replaced by an equivalent network consisting of a voltage source V_t connected in series with a resistor R_t

What is V_t and R_t?

V_t – *Open Circuit voltage at the terminals*

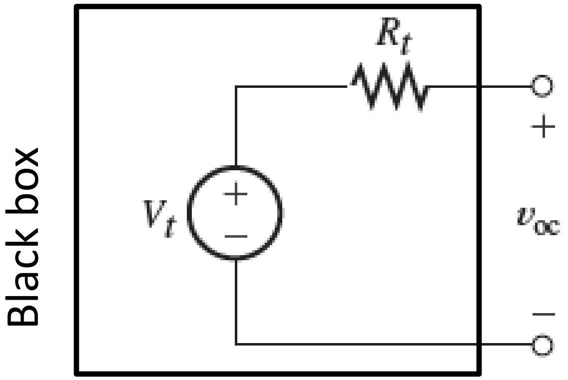
R_t – *Input/ equivalent resistance at the terminals when sources (independent) are turned off*





Thevenin Equivalent Circuit

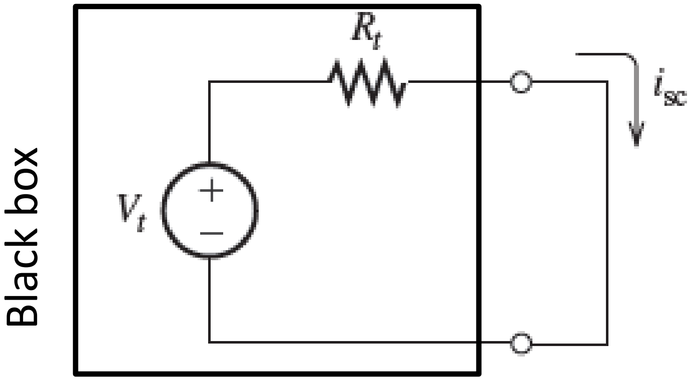
Open-Circuit terminals



No current flows through this circuit

$$V_t = V_{oc}$$

Short-Circuit terminals



Current flows through this circuit & voltage across terminals is zero

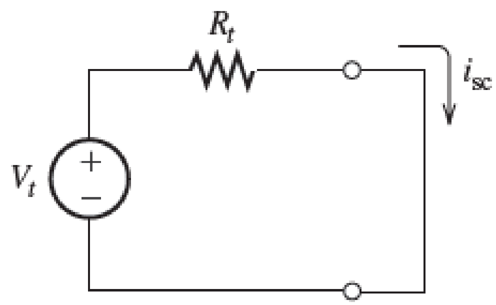
$$R_t = \frac{V_t}{i_{sc}}$$

$$R_t = \frac{V_{oc}}{I_{sc}}$$

Finding Thevenin Resistance Directly

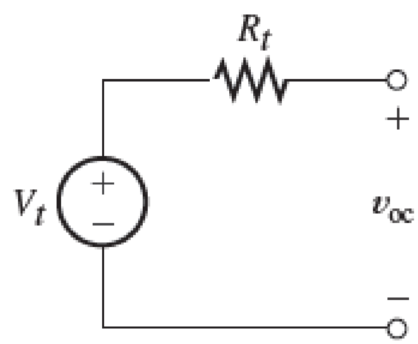
Zeroing method!

Zeroing a voltage source → Reduce voltage to zero



SHORT-CIRCUIT

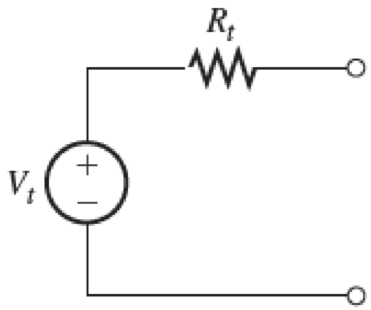
Zeroing a current source → Reduce current to zero



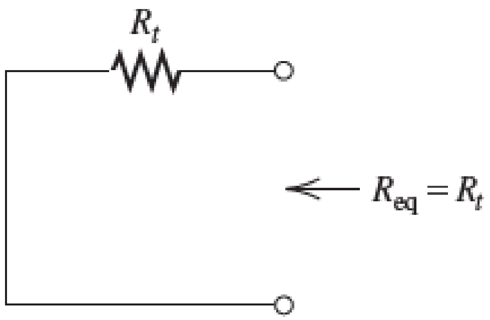
OPEN-CIRCUIT

Finding Thevenin Resistance Directly

Thevenin equivalent:
Original network



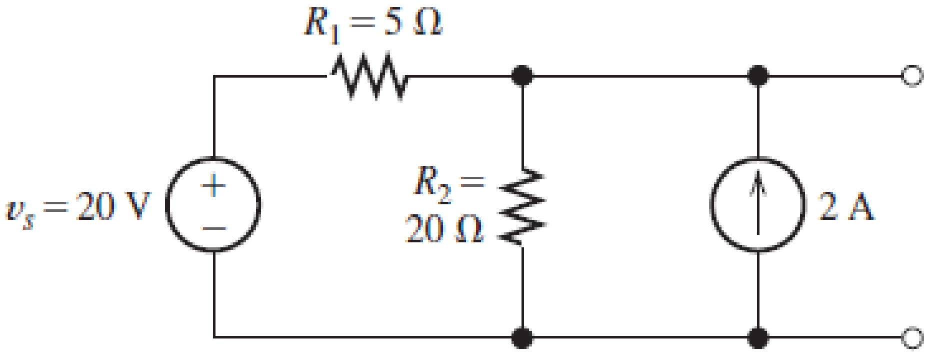
After Zeroing



To find Rt;

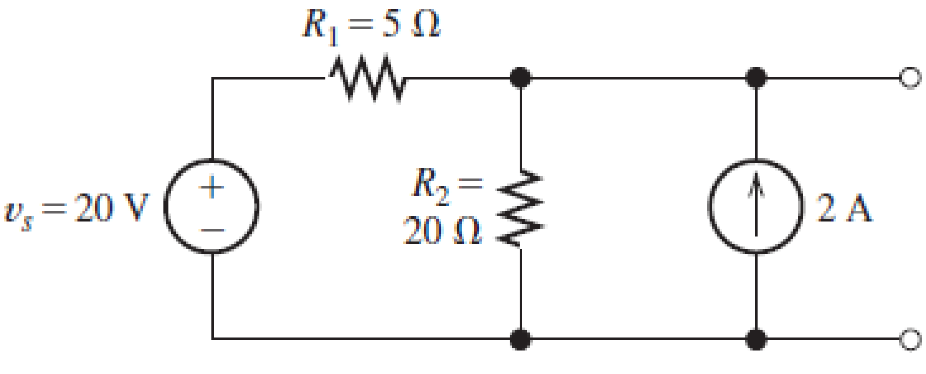
1. Zero sources in orginal
2. Compute R between terminals

Example 1: Find Thevenin resistance, short circuit current and Thevenin equivalent circuit for the circuit shown below

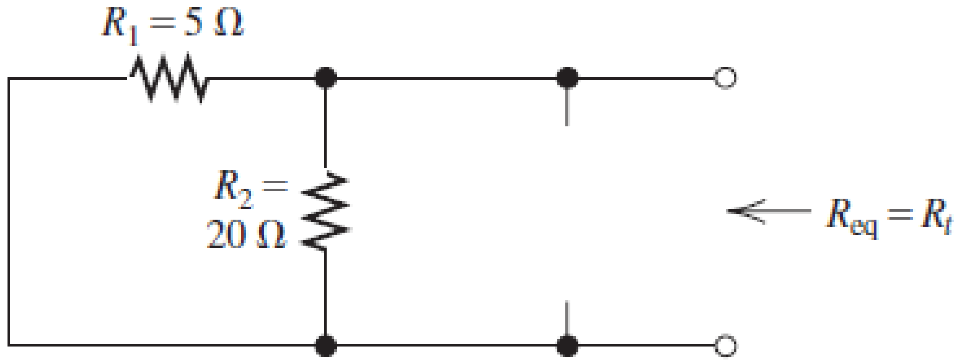


Thevenin Resistance - Example

Original circuit

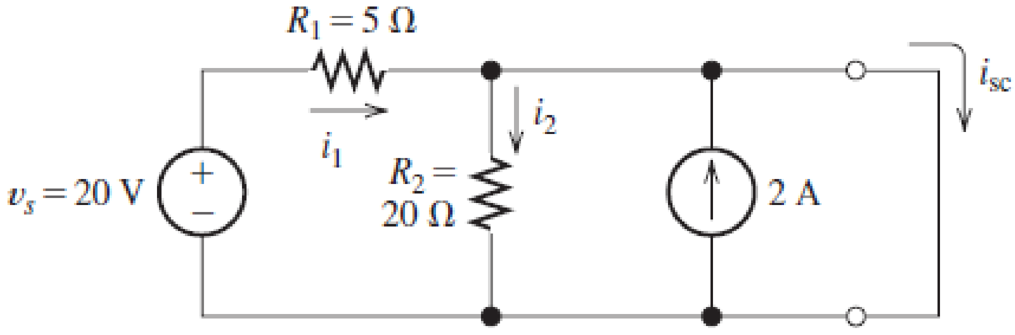


Circuit with sources zeroed



$$R_t = R_{eq} = \frac{1}{1/R_1 + 1/R_2} = \frac{1}{1/5 + 1/20} = 4\ \Omega$$

Thevenin Equivalent Circuit - Example



In short circuit,
V across R_2 = 0 =>
I through R_2 = 0

i_2 = 0

V_{R1} = 20V

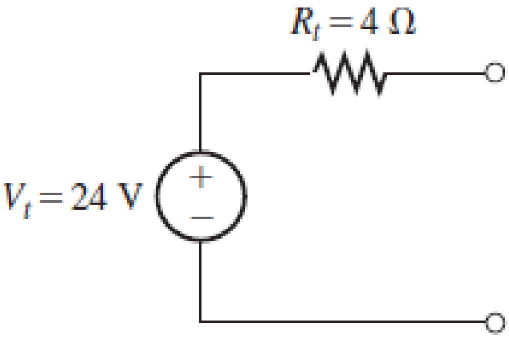
$$i_1 = \frac{v_s}{R_1} = \frac{20}{5} = 4 \text{ A}$$

KCL @ Node joining top ends of R_2 and 2A source

$$i_1 + 2 = i_2 + i_{sc}$$

$$i_{sc} = 6 \text{ A}.$$

$$R_t = \frac{V_{oc}}{I_{sc}} \qquad V_t = R_t i_{sc} = 4 \times 6 = 24 \text{ V}$$



Acknowledgements

1. H. Hayt, J.E. Kemmerly and S. M. Durbin, ‘Engineering Circuit Analysis’, 6/e, Tata McGraw Hill, New Delhi, 2011
2. Allan R. Hambley, ‘Electrical Engineering - Principles & Applications, Pearson Education, First Impression, 6/e, 2013