# EEE1024: Fundamentals of Electrical and Electronics Engineering

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# RECAP

$$v(t) = V_m \sin(\omega t + \theta)$$

$$\omega = \frac{2\pi}{T} \qquad \omega = 2\pi f$$

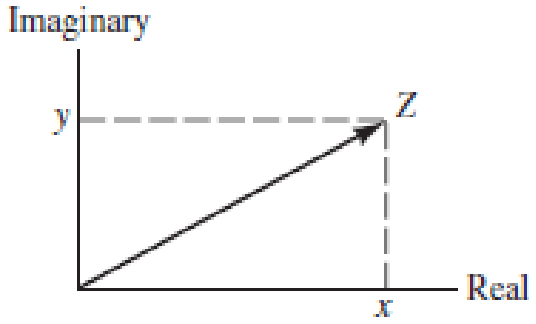
$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \qquad I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R} \qquad P_{\text{avg}} = I_{\text{rms}}^2 R$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}$$

# Revision of COMPLEX NUMBERS

## Rectangular form



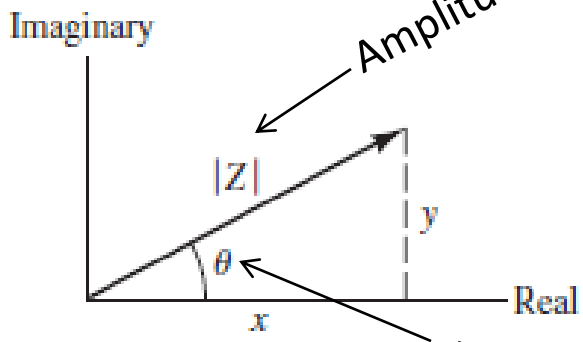

$$z = x + iy$$

Real Imaginary

$$z = x - iy$$

Complex conjugate

## Polar form




$$Z\angle\theta$$
$$= Z(\cos\theta + i\sin\theta)$$

$$|Z|^2 = x^2 + y^2$$
$$\tan(\theta) = \frac{y}{x}$$
$$x = |Z|\cos(\theta)$$
$$y = |Z|\sin(\theta)$$

## Euler's Identities

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$|e^{j\theta}| = \sqrt{\cos^2(\theta) + \sin^2(\theta)}$$
$$= 1$$

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$e^{-j\theta} = \cos(\theta) - j\sin(\theta)$$

# PHASORS

**PHASORS** → Voltages and Currents → As vectors in complex plane

VOLTAGES

$v_1(t) = V_1 \cos(\omega t + \theta_1)$

VOLTAGE PHASORS

$\mathbf{V}_1 = V_1 \angle \theta_1$

*Definition of phasor –*

*Real part of complex numbers, in polar form*

$v_2(t) = V_2 \sin(\omega t + \theta_2)$

$\mathbf{V}_2 = V_2 \angle \theta_2 - 90°$

$v_2(t) = V_2 \cos(\omega t + \theta_2 - 90°)$ ..... $\sin(\theta) = \cos(\theta - 90^o)$

CURRENTS

CURRENT PHASORS

$i_1(t) = I_1 \cos(\omega t + \theta_1)$

$\mathbf{I}_1 = I_1 \angle \theta_1$

$i_2(t) = I_2 \sin(\omega t + \theta_2)$

$\mathbf{I}_2 = I_2 \angle \theta_2 - 90°$

# ADDING AC VOLTAGES using phasors - I

$$v(t) = 10\cos(\omega t) + 5\sin(\omega t + 60°) + 5\cos(\omega t + 90°)$$

*STEP 1:*

*Convert all voltages into cosine function – use trigonometry*

$$\sin(\theta) = \cos(\theta - 90^o)$$

$$v(t) = 10\cos(\omega t) + 5\cos(\omega t + 60° - 90°) + 5\cos(\omega t + 90°)$$

$$v(t) = 10\cos(\omega t) + 5\cos(\omega t - 30°) + 5\cos(\omega t + 90°)$$

*STEP 2:*

*Write PHASORS for each voltage (or current)*

$$v(t) = 10\angle 0° + 5\angle -30° + 5\angle 90°$$

*STEP 3:*

*Convert PHASORS into complex numbers - rectangular form*

$$v(t) = 10(\cos 0 + i\sin 0) + 5(\cos(-30) + i\sin(-30)) + 5(\cos(90) + i\sin(90))$$

$$= 10(1+0) + 5(0.866 + i(-0.5)) + 5(0 + i(1))$$

$$= 10 + 5*0.866 - i(2.5) + 5i$$

$$v(t) = 14.33 + i*2.5$$

# ADDING AC VOLTAGES using phasors - II

$$v(t) = 10\cos(\omega t) + 5\sin(\omega t + 60°) + 5\cos(\omega t + 90°)$$

*STEP 3:*

*Convert PHASORS into rectangular form of complex numbers.*

$$v(t) = 14.33 + i * 2.5$$

*STEP 4:*

*Convert from rectangular form back to PHASORS (form)* $Z\angle\theta$

We need, $Z, \theta$

$$v(t) = \sqrt{14.33^2 + 2.5^2}\ \tan^{-1}(2.5/14.33)$$

$$v(t) = 14.54\angle 9.89$$

$$|Z|^2 = x^2 + y^2$$

$$\tan(\theta) = \frac{y}{x}$$

$$x = |Z|\cos(\theta)$$

$$y = |Z|\sin(\theta)$$

*STEP 5:*

*Convert PHASORS back into sinusoidal form*

$$v(t) = 14.54\cos(\omega t + 9.89)$$

# ADDING using phasors - Practice

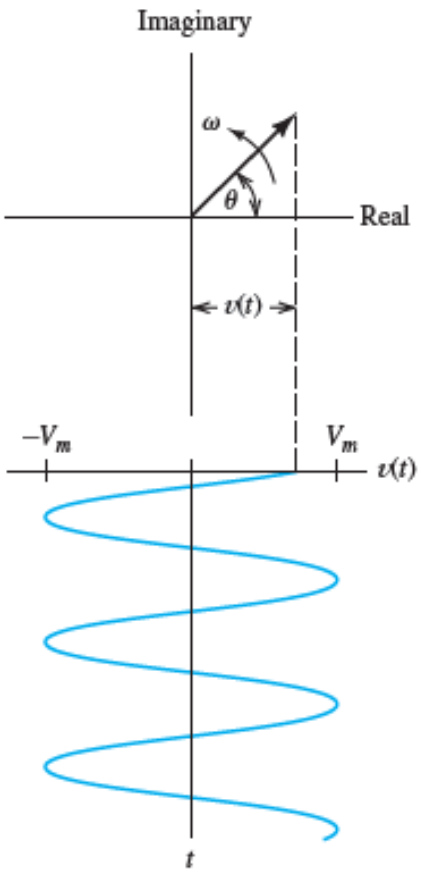
Q1) $v_1(t) = 10\cos(\omega t) + 10\sin(\omega t)$

Q2) $i_1(t) = 10\cos(\omega t + 30°) + 5\sin(\omega t + 30°)$

Q3) $i_2(t) = 20\sin(\omega t + 90°) + 15\cos(\omega t - 60°)$

ASSIGNMENT

# PHASE RELATIONSHIPS

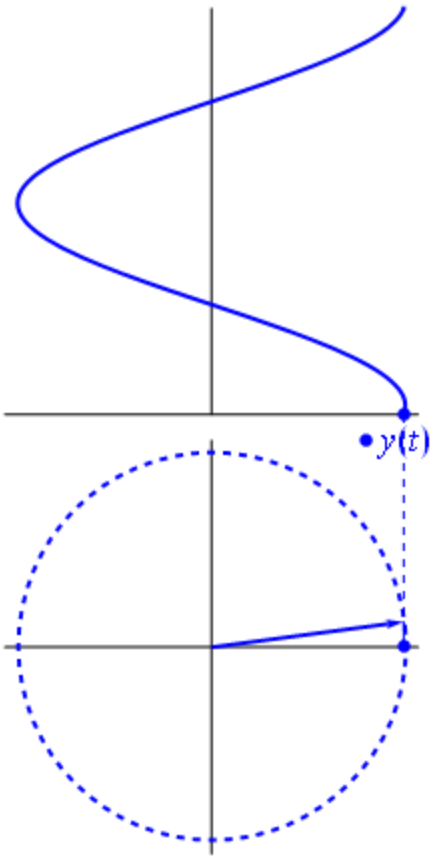



$$v_1(t) = 3\cos(\omega t + 40°)$$

$$\mathbf{V}_1 = 3\angle 40°$$

$$v_2(t) = 4\cos(\omega t - 20°)$$

$$\mathbf{V}_2 = 4\angle -20°$$




PHASORS rotate Counter-clockwise!

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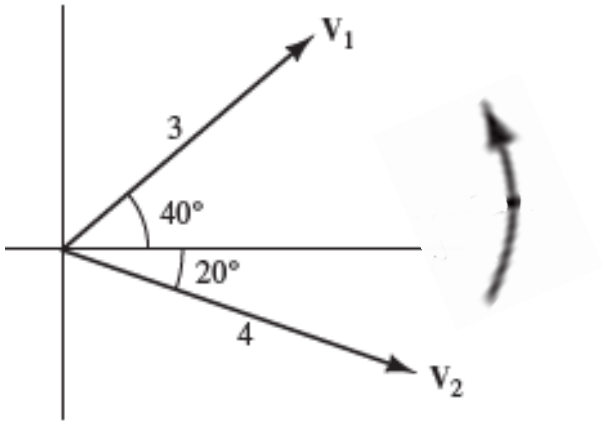


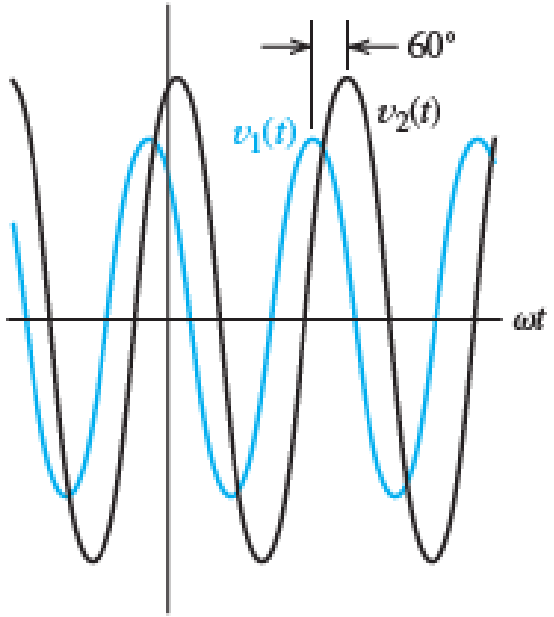

# PHASE RELATIONSHIPS - Practice

3 voltages are given as -

$$v_1(t) = \cos(\omega t - 30^o)$$

$$v_1(t) = \cos(\omega t + 30^o)$$

$$v_1(t) = \cos(\omega t + 45^o)$$

State the phase relationship between each pair of voltages

ASSIGNMENT

# Acknowledgements


1. Allan R. Hambley, 'Electrical Engineering - Principles & Applications, Pearson Education, First Impression, 6/e, 2013
2. https://en.wikipedia.org/wiki/Phasor