MODULE – VII : RELIABILITY

Reliability of a System (component) : Capacity of the System to function without breakdown.

Reliability → The probability that a system will perform properly for a specified time of period 't' under a given set of conditions.

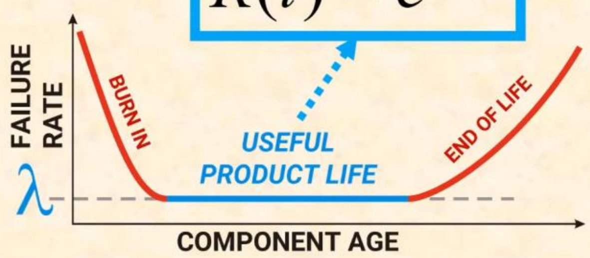
(or)

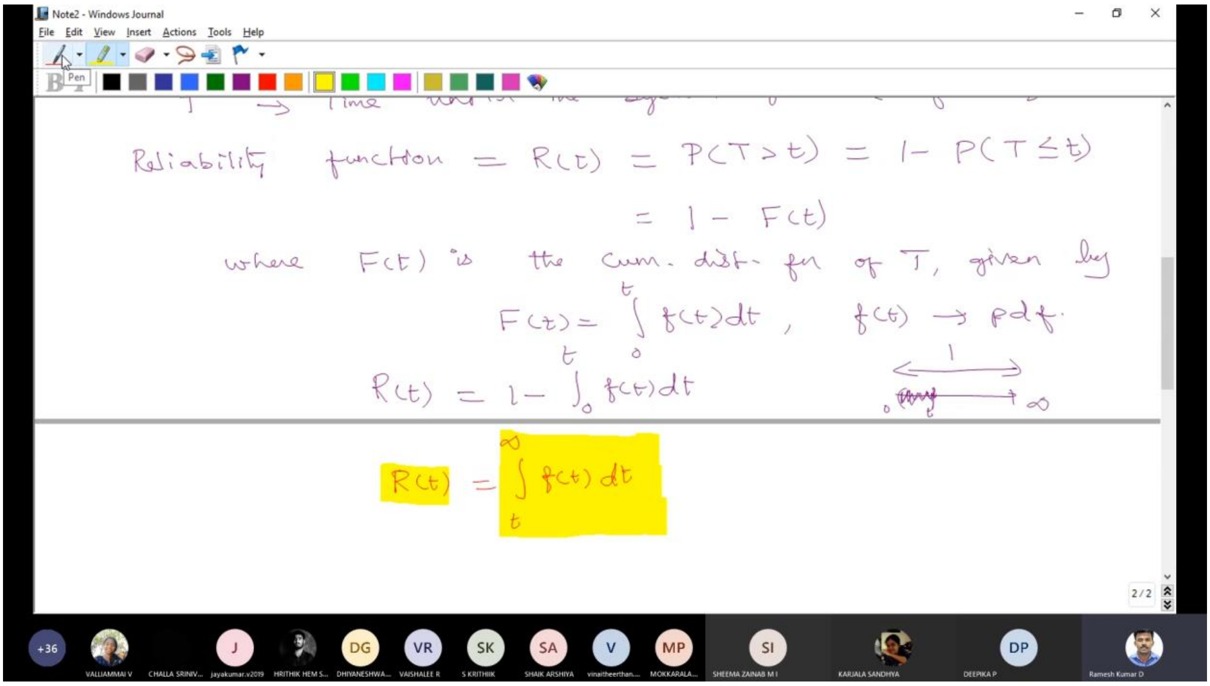
prob. that the System will not fail during the (0, t).

Measuring Reliability

- **Reliability is based on the concept of a "mission"**
  - Reliability ***R(t)***: probability system still working since start of mission
  - A mission is ***t*** continuous operating hours between diagnostics
  - Constant Failure Rate λ (failures/hr)

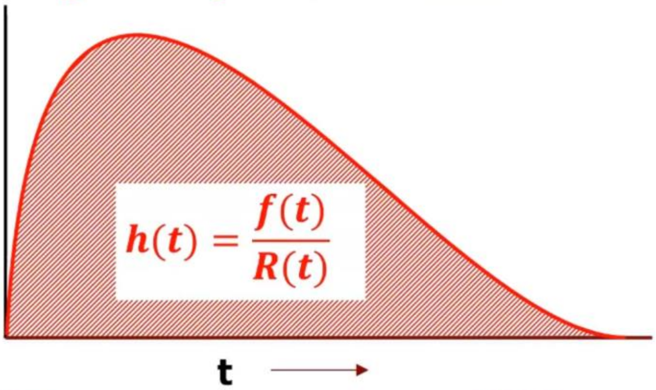
$$\boxed{R(t) = e^{-\lambda t}}$$





https://www.youtube.com/watch?v=0a-BF036Di4

Instantaneous hazard rate can be considered as how fast the components fail as a proportion of the surviving components.



Hazard Rate   Feb-2020

Hazard Rate Example

Consider a building which has 1200 electric switches. Suppose four switches fail in three months out of 1200. Thus the hazard rate per month per unit will be:

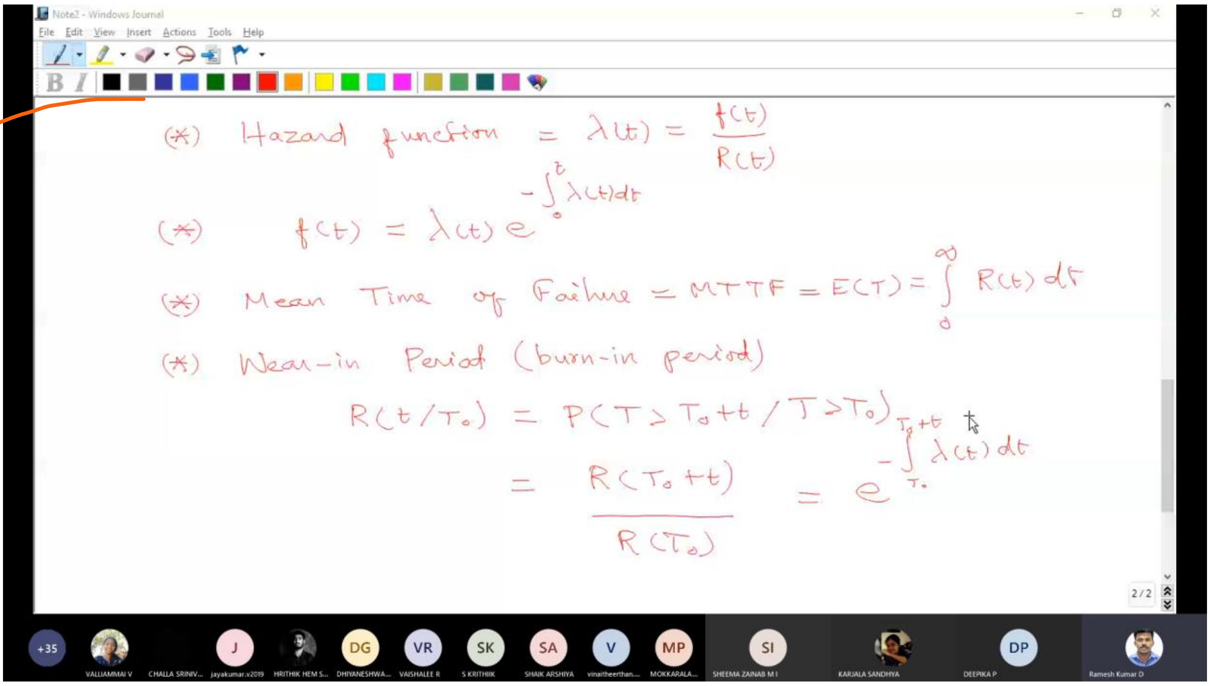
$$h(t) = \frac{4}{3 \times 1200} = 0.001111$$

Note: Hazard rate is often denoted as $\lambda(t)$.

Hazard Rate Example

Suppose In the next two months, six more switches fail! Remember, these are out of 1196 as four had failed within first three months! Thus the hazard rate during this period will be:

$$h(t) = \frac{6}{2 \times 1196} = 0.002508$$



(*) Hazard function $= \lambda(t) = \dfrac{f(t)}{R(t)}$

(*) $f(t) = \lambda(t)\, e^{-\int_0^t \lambda(t)\,dt}$

(*) Mean Time of Failure $= MTTF = E(T) = \int_0^\infty R(t)\,dt$

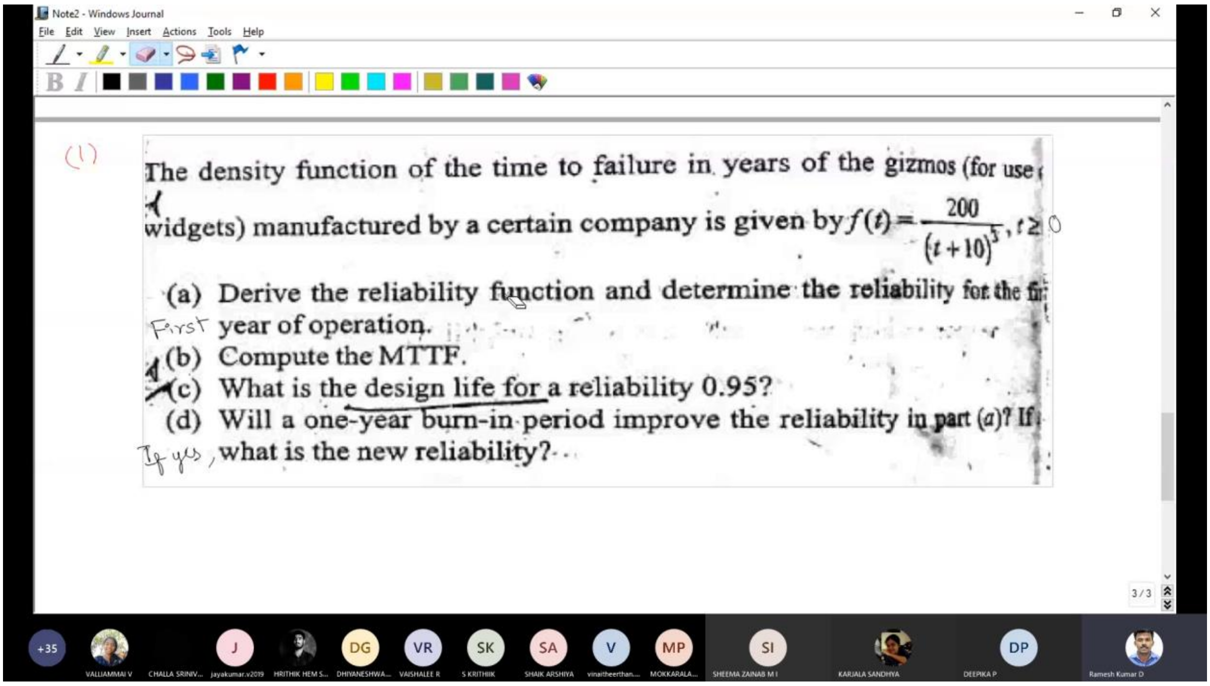
(*) Wear-in Period (burn-in period)

$$R(t/T_0) = P(T > T_0+t \,/\, T > T_0) = \frac{R(T_0+t)}{R(T_0)} = e^{-\int_{T_0}^{T_0+t} \lambda(t)\,dt}$$

$$R(t) = e^{-\int_0^t \lambda(t)\,dt}$$

And

Problem-1



(1) The density function of the time to failure in years of the gizmos (for use on widgets) manufactured by a certain company is given by $f(t) = \dfrac{200}{(t+10)^3}$, $t \geq 0$

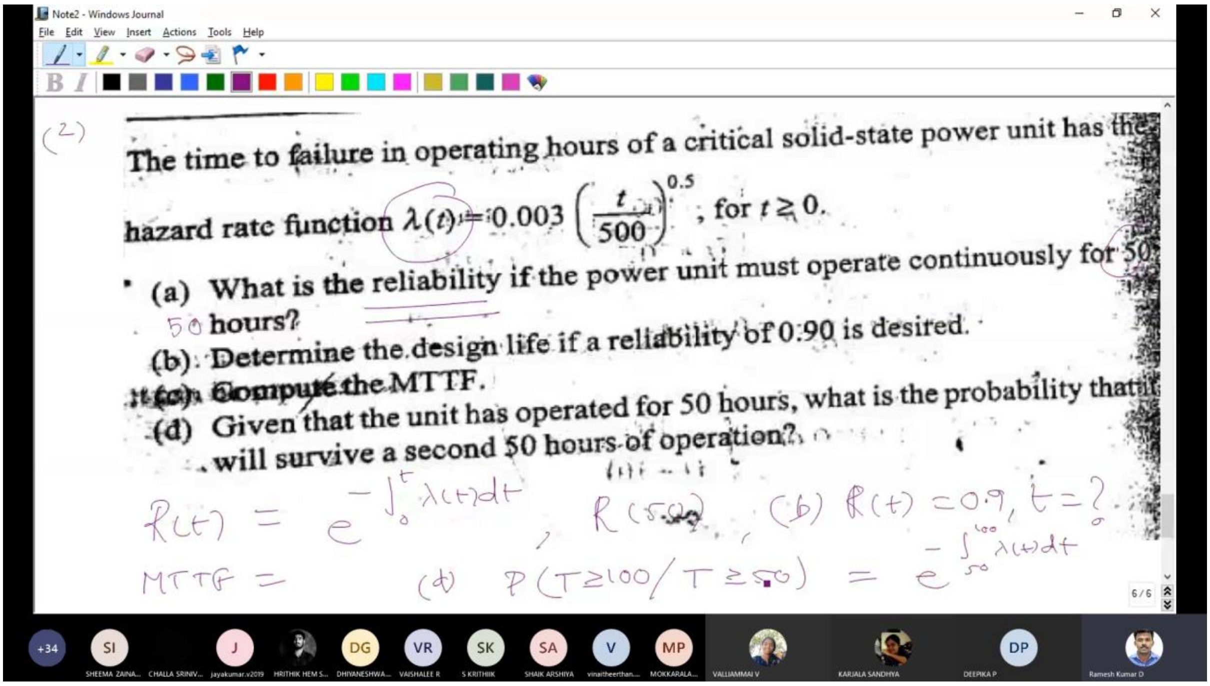
(a) Derive the reliability function and determine the reliability for the first year of operation.

(b) Compute the MTTF.

(c) What is the design life for a reliability 0.95?

(d) Will a one-year burn-in period improve the reliability in part (a)? If yes, what is the new reliability?

Problem-2



(2) The time to failure in operating hours of a critical solid-state power unit has the hazard rate function $\lambda(t) = 0.003\left(\frac{t}{500}\right)^{0.5}$, for $t \geq 0$.

(a) What is the reliability if the power unit must operate continuously for 50 hours?

(b) Determine the design life if a reliability of 0.90 is desired.

(c) Compute the MTTF.

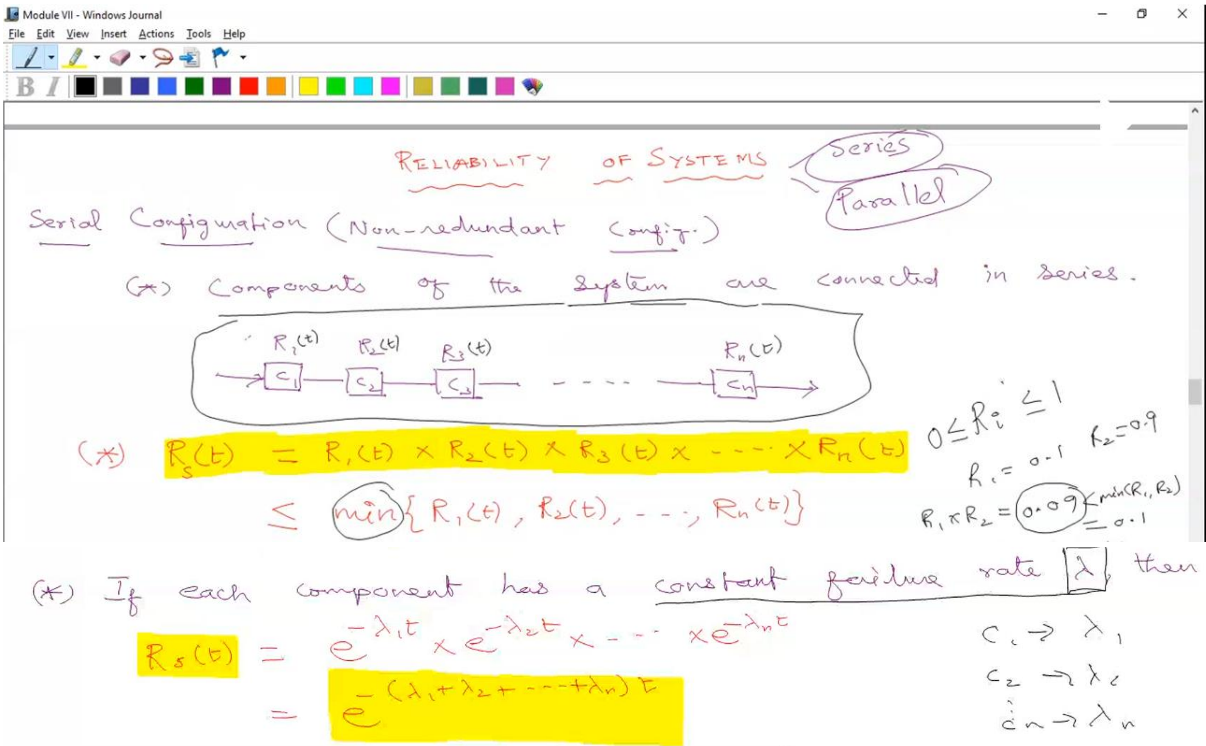
(d) Given that the unit has operated for 50 hours, what is the probability that it will survive a second 50 hours of operation?

$R(t) = e^{-\int_0^t \lambda(t)dt}$, $R(50)$, (b) $R(t) = 0.9$, $t = ?$

MTTF = (d) $P(T \geq 100 / T \geq 50) = e^{-\int_{50}^{100} \lambda(t)dt}$

Reliability of Systems

Series Configuration (Non-redundant Configuration) :



RELIABILITY OF SYSTEMS — Series / Parallel

Serial Configuration (Non-redundant config.)

(*) Components of the system are connected in series.

$R_1(t)$ $R_2(t)$ $R_3(t)$ ... $R_n(t)$

$\rightarrow [C_1] - [C_2] - [C_3] - \cdots - [C_n] \rightarrow$

(*) $R_s(t) = R_1(t) \times R_2(t) \times R_3(t) \times \cdots \times R_n(t)$

$\leq \min\{R_1(t), R_2(t), \ldots, R_n(t)\}$

$0 \leq R_i \leq 1$

$R_1 = 0.1$ $R_2 = 0.9$

$R_1 \times R_2 = 0.09 < \min(R_1, R_2) = 0.1$

(*) If each component has a constant failure rate $\boxed{\lambda}$ then

$R_s(t) = e^{-\lambda_1 t} \times e^{-\lambda_2 t} \times \cdots \times e^{-\lambda_n t}$

$= e^{-(\lambda_1 + \lambda_2 + \cdots + \lambda_n)t}$

$C_1 \rightarrow \lambda_1$

$C_2 \rightarrow \lambda_2$

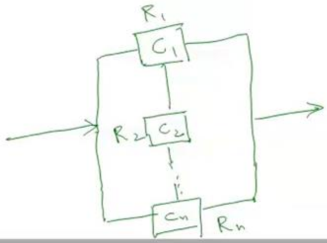
$\vdots$

$C_n \rightarrow \lambda_n$

Parallel Configuration (Redundant configuration)

Parallel Configuration (Redundant config.)

(*) Components of the System are connected in parallel.



$\boxed{C_3}$ is not working

It'll work

(*) $R_p(t) = 1 - (1-R_1)(1-R_2)\cdots(1-R_n)$ ✓

$\geq \max(R_1, R_2, \ldots, R_n)$

(*) λ – constant failure rate

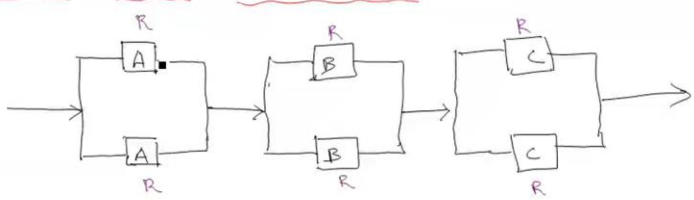
$$R_p(t) = 1 - (1-e^{-\lambda_1 t})(1-e^{-\lambda_2 t})\cdots(1-e^{-\lambda_n t})$$

(*) $\lambda_1, \lambda_2 \rightarrow$ Constant failure rates

$$MTTF = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1+\lambda_2}$$

Parallel Series Configuration

Parallel Series Configuration $\longrightarrow$



Subsystem 1:

$1-(1-R)(1-R)$

$= 1-(1-R)^2$ — (1)

Subsys 2: $1-(1-R)^2$ — (2)

Subsys 3: $1-(1-R)^2$ — (3)

$$R(t) = \left[1-(1-R)^2\right]^3$$

In general

$$R(t) = \left(1-(1-R)^n\right)^m$$

where $n \rightarrow$ no. of components are connected parallely with R in each Subsystem.

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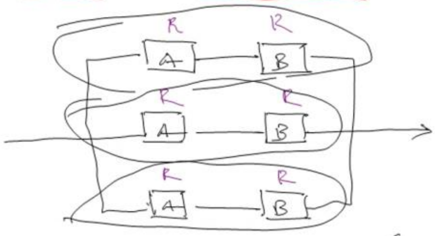
$m \rightarrow$ no. of Subsystems

Series – Parallel configuration

Series – Parallel Configuration

$R_s(t) = R_1 \times R_2 \times \cdots \times R_n$



$\rightarrow (1-R^2)$ ✓

$\rightarrow (1-R^2)$ ✓

$\rightarrow (1-R^2)$ ✓

$R(t) = 1-(1-R^2)^3$ ✓

$1-(1-R^2)(1-R^2)(1-R^2)$

$1-(1-R^2)^3$

In general

$R(t) = 1-(1-R^n)^m$

where m → no. of subsystems connected parallely.

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n → Number of components connected serially

Problem -1

An electronic circuit consists of 5 silicon transistors, 3 silicon diodes, 10 comp... sition resistors and 2 ceramic capacitors connected in series configuration. T... hourly failure rate of each component is given below:

Silicon transistor	: $\lambda_t = 4 \times 10^{-5}$
Silicon diode	: $\lambda_d = 3 \times 10^{-5}$
Composition resistor	: $\lambda_r = 2 \times 10^{-4}$
Ceramic capacitor	: $\lambda_c = 2 \times 10^{-4}$

Calculate the reliability of the circuit for 10 hours, when the components fo... llow exponential distribution.

Problem – 2

There are 16 components in a non-redundant system. The average reliabili... each component is 0.99. In order to achieve at least this system reliabili... using a redundant system with 4 identical new components, what should be... least reliability of each now component?