15/9/20

CAT-2 MAT2001

Statistics for Engineers

Slot :

Timing : 2:30pm to 3:15pm

Name : Prasharth S

Reg NO : 19MIDOO20

1)

Y \ X	-1	+1	P(y)
0	1/8	3/8	4/8
1	2/8	2/8	4/8
P(X)	3/8	5/8	1

~~Cov → E(X)E(Y) - E(XY)~~

$E[x] \Rightarrow \sum x_i (p_i) \Rightarrow -1\left(\frac{3}{8}\right) + 1\left(\frac{5}{8}\right) \Rightarrow \frac{2}{8} \Rightarrow \frac{1}{4}$

$E[Y] \Rightarrow \sum y_i (p_i) \Rightarrow 0\left(\frac{4}{8}\right) + 1\left(\frac{4}{8}\right) \Rightarrow \frac{4}{8} \Rightarrow \frac{1}{2}$

$E[x^2] \Rightarrow \sum x_i^2 (p_i) \Rightarrow (1)\left(\frac{3}{8}\right) + 1\left(\frac{5}{8}\right) \Rightarrow \frac{8}{8} \Rightarrow \frac{8}{8} \Rightarrow 1$

$E[Y^2] \Rightarrow \sum y_i^2 (p_i) \Rightarrow 0\left(\frac{4}{8}\right) + 1\left(\frac{4}{8}\right) \Rightarrow \frac{4}{8} \Rightarrow \frac{4}{8} \Rightarrow \frac{1}{2}$

$E[xy] \Rightarrow \sum \sum x_i y_i (p(x_i)\, p(y_i))$

$\Rightarrow 0 + (1)\frac{3}{8} + (0)1\left(\frac{2}{8}\right) + 1\left(\frac{2}{8}\right)$

$E[xy] \Rightarrow -\frac{2}{8} + \frac{2}{8} \Rightarrow 0$

$\sigma_x^2 \Rightarrow E[X^2] - (E[X])^2$

$\Rightarrow 1 - (1/4)^2 \Rightarrow 1 - 1/16 \Rightarrow 1 - 0.0625 \Rightarrow 0.9375$

$\sigma_x^2 \Rightarrow 0.9375$; $\boxed{\sigma x \Rightarrow 0.9682}$

$\sigma_y^2 \Rightarrow E[Y^2] - (E[Y])^2$

$\Rightarrow 1/2 - (1/2)^2$

$\sigma_y^2 \Rightarrow 1/2 [1 - 1/2] \Rightarrow 1/2 (1/2) \Rightarrow 1/4$

$\sigma_y \Rightarrow \sqrt{1/4} \Rightarrow \sqrt{0.25} \Rightarrow 0.5$

$\boxed{\sigma_y \Rightarrow 0.5}$

$\text{Covariance}(X,Y) \Rightarrow E[XY] - (E[X]E[Y])$

$\Rightarrow 0 - (1/4)(1/2) \Rightarrow -1/8$

$\rho_{xy} \Rightarrow \dfrac{\text{Covariance}(X,Y)}{\sigma_x \sigma_y} \Rightarrow \dfrac{-1/8}{(0.9682) * (0.5)} \Rightarrow \dfrac{-1/8}{0.4841}$

$\Rightarrow \dfrac{-0.125}{0.4841} \Rightarrow 0.258211$

$\boxed{\rho_{xy} \Rightarrow 0.258211}$

5)

University	Affliated college
$n_1 \Rightarrow 400$	$n_2 \Rightarrow 500$
Passed $\Rightarrow 300$	Passed $\Rightarrow 300$
$P_1 \Rightarrow 0.75$	$P_2 \Rightarrow 0.6$

Step-1:

Null hypothesis : $H_0 \Rightarrow P_1 = P_2$

Alternate hypothesis : $H_1 \Rightarrow P_1 > P_2$

Step-2:

It is right tailed hypothesis

Step-3: $Z_\alpha \Rightarrow 2.33$

Step 4:

$$Z \Rightarrow \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$P \Rightarrow \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$$

$$\Rightarrow \frac{(400 * 0.75) + (500 * 0.6)}{(500 + 400)}$$

$$\Rightarrow \frac{300 + 300}{900} \Rightarrow \frac{600}{900}$$

$P \Rightarrow 0.666$

$Q \Rightarrow 0.333$

$$Z \Rightarrow \frac{0.75 - 0.6}{\sqrt{(0.666)(0.33)\left(\frac{1}{400} + \frac{1}{500}\right)}}$$

Steps:

$$\Rightarrow \frac{0.15}{(0.278)(0.0045)} \Rightarrow \frac{2700}{569.2099} \Rightarrow 4.73$$

Step 5:

$|z| \Rightarrow 4.73$

$|z_\alpha| \Rightarrow 2.33$

$|z| > |z_\alpha|$ $H_1 \Rightarrow$ accepted

$H_0 \Rightarrow$ rejected

∴ Proportion of passed student in university students is higher than affliated college students.

3) $\lambda \Rightarrow 1/2$

$$P(x>2) \Rightarrow \int_2^\infty \lambda e^{-\lambda x} dx$$

$$\Rightarrow \int_2^\infty 1/2\, e^{-1/2 x} dx \Rightarrow 1/2 \left[\frac{e^{-1/2 x}}{-1/2} \right]_2^\infty$$

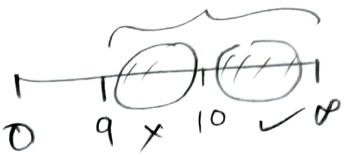
$$\Rightarrow -1 \left[e^{-1/2 x} \right]_2^\infty$$

$$\Rightarrow -1 \left[e^{-\infty} - e^{-1} \right]$$

$$\Rightarrow -1 \left[0 - e^{-1} \right] \Rightarrow -1 \left[-(0.3678) \right]$$

$P(x>2) \Rightarrow 0.3678$

$$\Rightarrow \frac{P(x>9) \cap P(x>10)}{P(x>9)}$$



$$\Rightarrow \frac{P(x>10)}{P(x>9)}$$

$$P(x>10) \Rightarrow \frac{1}{2}\int_{10}^{\infty} e^{-\frac{1}{2}x}\,dx$$

$$\Rightarrow \frac{1}{2}\left[\frac{e^{-\frac{1}{2}x}}{\left[-\frac{1}{2}\right]}\right]_{10}^{\infty}$$

$$\Rightarrow -1\left[e^{-\infty} - e^{-5}\right]$$

$$\Rightarrow -1\left[0 - 6.737 \times 10^{-3}\right]$$

$$P(x>10) \Rightarrow 0.006737$$

$$P(x>9) \Rightarrow \frac{1}{2}\int_{9}^{\infty} e^{-\frac{1}{2}x}\,dx$$

$$\Rightarrow -1\left[e^{-\infty} - e^{-9/2}\right]$$

$$\Rightarrow -1\left[0 - 0.0111\right]$$

$$\Rightarrow -1\left[-0.0111\right]$$

$$P(x>9) \Rightarrow 0.0111$$

$$\Rightarrow \frac{0.006737}{0.0111} \Rightarrow 0.606936$$

Conditional probability $\Rightarrow 0.606936$

4) $\mu \Rightarrow 33$

$\sigma \Rightarrow 3$

i) Rs: 100 if $30 < X < 35$

$P(30 < X < 35) \Rightarrow P(-1 < Z < 0.6667)$

$\Rightarrow 0.34134 + 0.24537$

$\Rightarrow 0.58671$

$X \Rightarrow 30$

$Z \Rightarrow \frac{X-\mu}{\sigma}$

$\Rightarrow \frac{30-33}{3}$

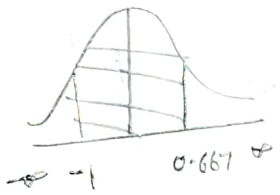
$Z \Rightarrow -3/3 \Rightarrow -1$

$X \Rightarrow 35$

$Z \Rightarrow \frac{35-33}{3}$

$\Rightarrow 2/3$

$\Rightarrow 0.6667$



ii) Rs: 50 if $(25 < X < 30)$ (or) $(35 \leq X < 40)$

$P(25 < X \leq 30) \Rightarrow P(-2.666 < Z \leq -1)$

$P(35 \leq X < 40) \Rightarrow P(0.6667 < Z < 2.33)$

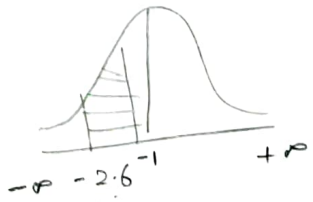
$P(-2.666 < Z < -1) \Rightarrow 0.49609 - 0.34134$

$\Rightarrow 0.15475$

$Z \Rightarrow \frac{25-33}{3}$

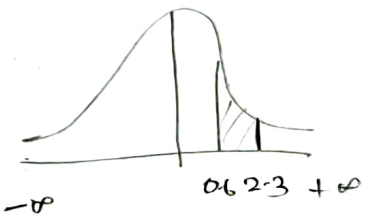
$\Rightarrow -2.666$

$Z \Rightarrow \frac{40-33}{3} \Rightarrow 2.33$



$P(0.6667 < Z < 2.33) \Rightarrow 0.49010 - 0.22575$

$\Rightarrow 0.26435$



Profit $\Rightarrow 0.58671 * 100 \Rightarrow$ ₹58.671

2)

y	2	4	6	8
x_1	10	14	8	2
x_2	3	6	9	1

y	x_1	x_2	x_1y	x_1^2	x_1x_2	x_2y	x_2^2
2	10	3	20	100	30	6	9
4	14	6	56	196	84	24	36
6	8	9	48	64	72	54	81
8	2	1	16	4	2	8	1
20	34	19	140	364	188	92	127

$$\Sigma y \Rightarrow na_0 + a_1 \Sigma x_1 + a_2 \Sigma x_2$$

$$20 \Rightarrow 4a_0 + a_1(34) + a_2(19)$$

$$\boxed{20 \Rightarrow 4a_0 + 34a_1 + 19a_2}$$

$$\Sigma x_1 y = a_0 \Sigma x_1 + a_1 \Sigma x_1^2 + a_2 \Sigma x_1 x_2$$

$$140 = a_0(34) + a_1(364) + a_2(188)$$

$$\boxed{140 = 30a_0 + 361a_1 + 188a_2}$$

$$\Sigma x_2 y = a_0 \Sigma x_2 + a_1 \Sigma x_1 x_2 + a_2 \Sigma x_2^2$$

$$\boxed{92 = a_0(19) + a_1(188) + a_2(127)}$$

$a_0 = 5.8236$

$a_1 = -0.085882$

$a_2 = -0.019704$

$$\boxed{y = 5.8236 - 0.085882 x_1 - 0.019704 x_2}$$