

DIGITAL ASSIGNMENT-II

19M120017

A2 SLOT.

Question 1:- Add the following 2 phasors:

i) $V_1(t) = 10 \cos(\omega t) + 10 \sin(\omega t)$

Step 1: Convert all functions into cosine functions.

$$V_1(t) = 10 \cos(\omega t) + 10 \cos(\omega t - 90^\circ)$$

Step 2: Convert the sinusoidal form to phasor form

$$V_1(t) = 10 \angle 0^\circ + 10 \angle -90^\circ$$

Step 3: Convert phasors into rectangular form

$$\begin{aligned} V_1(t) &= 10(\cos 0^\circ + j \sin 0^\circ) + 10(\cos 90^\circ - j \sin 90^\circ) \\ &= 10(1 + j0) + 10(0 - j1) \end{aligned}$$

$$V_1(t) = 10 - j10$$

Step 4: Convert rectangular form to phasor form again

$$\begin{aligned} V_1(t) &= 10 - j10 \\ &= \sqrt{10^2 + 10^2} \tan^{-1}(-10/10) \\ &= \sqrt{200} \tan^{-1}(-1) \\ &= \sqrt{2} \cdot 10 \angle -45^\circ \end{aligned}$$

$$V_1(t) = 10\sqrt{2} \angle -45^\circ$$

$$V_1(t) = 14.14 \angle -45^\circ$$

Step 5: Convert phasor form to sinusoidal form

$$V_1(t) = 14.14 \cos(\omega t - 45^\circ)$$

$$V_1(t) = 14.14 \sin(\omega t + 45^\circ)$$

$$(ii) i_1(t) = 10 \cos(\omega t + 30^\circ) + 5 \sin(\omega t + 30^\circ)$$

Step 1: Convert all functions into cosine forms.

$$\begin{aligned} i_1(t) &= 10 \cos(\omega t + 30^\circ) + 5 \cos(\omega t + 30^\circ - 90^\circ) \\ &= 10 \cos(\omega t + 30^\circ) + 5 \cos(\omega t - 60^\circ) \end{aligned}$$

Step 2:

Convert the sinusoidal form to phasor

$$i_1(t) = 10 \angle 30^\circ + 5 \cos \angle -60^\circ$$

Step 3: Convert the phasor form to rectangular one

$$i_1(t) = 10(\cos 30^\circ + j \sin 30^\circ) + 5(\cos 60^\circ - j \sin 60^\circ)$$

$$= 10\left(\frac{\sqrt{3}}{2} + j \cdot \frac{1}{2}\right) + 5\left(\frac{1}{2} - j \frac{\sqrt{3}}{2}\right)$$

$$= 5\sqrt{3} + j \frac{10}{2} + \frac{5}{2} - \frac{5\sqrt{3}}{2} j$$

$$= 11.16 + 0.669j$$

$$i_1(t) = 11.16 + 0.669j$$

Step 4: Convert rectangular form to phasor form

$$i_1(t) = \sqrt{11.16^2 + (0.669)^2} \tan^{-1}\left(\frac{+0.669}{11.16}\right)$$

$$= 11.18 \angle 3.43^\circ$$

Step 5: Convert from phasor form to sinusoidal form

$$i_1(t) = 11.18 \cos(\omega t + 3.43^\circ)$$

$$(iii) i_2(t) = 20 \sin(\omega t + 90^\circ) + 15 \cos(\omega t - 60^\circ)$$

Step 1: Convert all functions into cosine

$$\begin{aligned} i_2(t) &= 20 \cos(\omega t + 90^\circ - 90^\circ) + 15 \cos(\omega t - 60^\circ) \\ &= 20 \cos(\omega t) + 15 \cos(\omega t - 60^\circ) \end{aligned}$$

Step 2: Convert sinusoidal function to phasor form

$$i_2(t) = 20 \angle 0^\circ + 15 \angle -60^\circ$$

Step 3: Convert phasor form to rectangular form

$$\begin{aligned} i_2(t) &= 20 \cos 0 + 20i \sin 0 + 15 \cos 60^\circ - 15i \sin 60^\circ \\ &= 20 + i0 + 15/2 - \frac{15\sqrt{3}}{2}i \\ &= 20 + 15/2 - \frac{15\sqrt{3}}{2}i \\ &= 27.5 - 12.99i \end{aligned}$$

Step 4: Convert rectangular form to phasor form

$$\begin{aligned} i_2(t) &= \sqrt{(27.5)^2 + (12.99)^2} \tan^{-1} \left(\frac{-12.99}{27.5} \right) \\ &= 30.41 \angle -25.28^\circ \end{aligned}$$

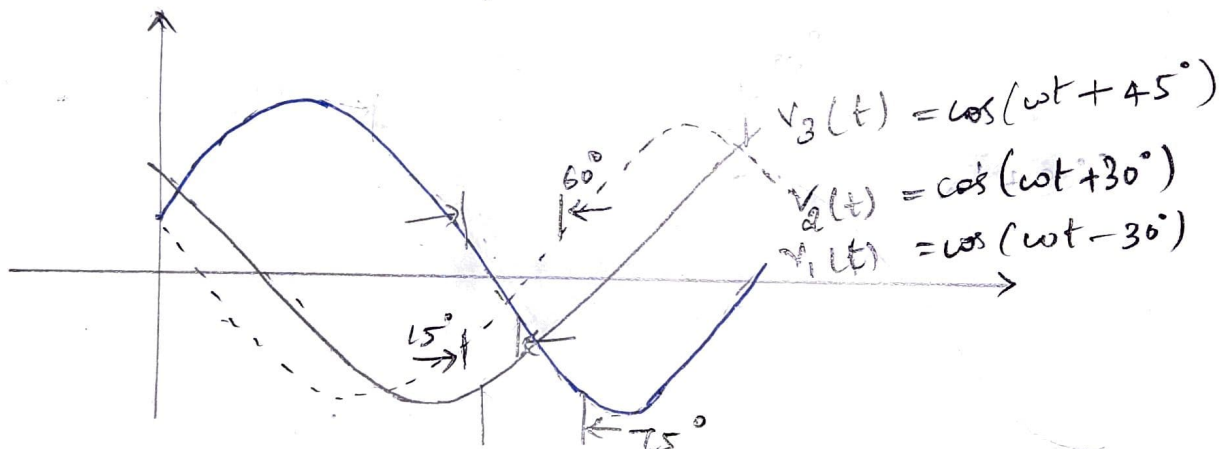
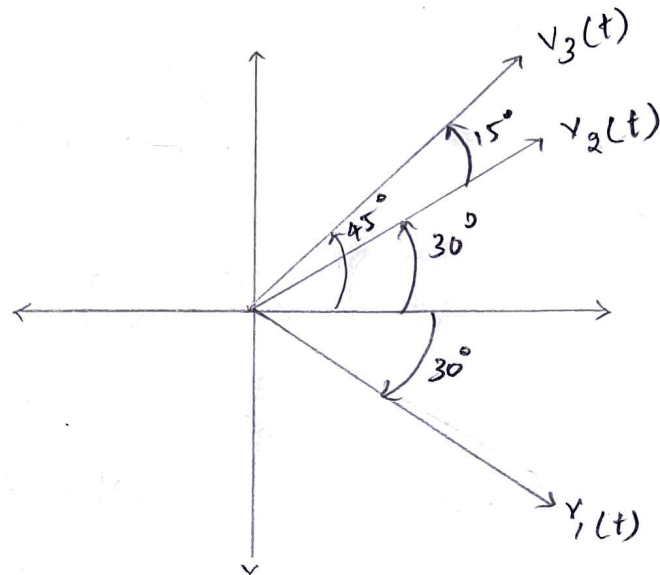
Step 5: Convert phasor form to sinusoidal form

$$\begin{aligned} i_2(t) &= 30.41 \cos(\omega t - 25.28^\circ) \\ &= 30.41 \sin(\omega t - 25.28^\circ + 90^\circ) \\ &= 30.41 \sin(\omega t + 64.72^\circ) \end{aligned}$$

2) State the ^{phase} relationship b/w each pair of voltages.

$$V_1(t) = \cos(\omega t - 30^\circ) \quad V_2(t) = \cos(\omega t + 30^\circ)$$

$$V_3(t) = \cos(\omega t + 45^\circ)$$



Relationship:

(i) V_2 leads V_1 by 60°

(ii) V_3 leads V_2 by 15°

(iii) V_3 leads V_1 by 75°
(or)

(i) V_1 lags V_2 by 60°

(ii) V_2 lags V_3 by 15°

(iii) V_1 lags V_3 by 75°

Q3) A voltage $V_L(t) = 100 \cos(200t)$ is applied to a 0.25 H inductance. Notice that $\omega = 200 \text{ rad/s}$

a) Find the impedance, phasor current and phasor voltage (of inductor)

$$V_L(t) = 100 \cos(200t)$$

$$\Rightarrow V_L(t) = \omega L I_m \cos(\omega t + \theta)$$

$$\therefore \theta = 0^\circ, \omega = 200 \text{ rad/s}$$

$$V_m = \omega L I_m = 100$$

$$I_m = \frac{100}{\omega L} = \frac{100}{200 \times 1/4} = 2 \text{ A}$$

$$V_L = V_m \angle 0^\circ$$

$$V_L = 100 \angle 0^\circ \text{ V}$$

$$I_L = I_m \angle 0^\circ - 90^\circ$$

$$= 2 \angle -90^\circ$$

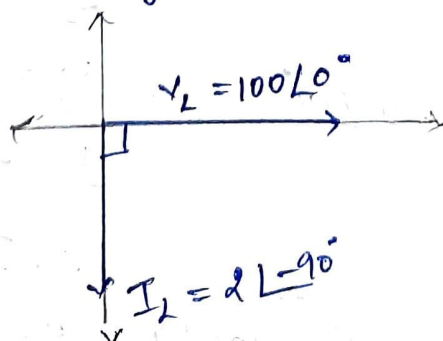
$$I_L = 2 \angle -90^\circ \text{ A}$$

$$V_L = Z_L I_L$$

$$Z_L = \frac{V_L}{I_L} = \frac{100 \angle 0^\circ}{2 \angle -90^\circ} = 50 \angle 0^\circ - (-90^\circ)$$

$$Z_L = 50 \angle 90^\circ \Omega$$

b) Draw phasor diagram



$\therefore V_L$ leads I_L by 90°

$$V_L = 100 \cos(200t) \text{ V}$$

$$I_L = 2 \sin(200t) \text{ A}$$

4) A voltage $V_c(t) = 100 \cos(200t)$ is applied to a $100 \mu\text{F}$ capacitance.

a) Find impedance of a capacitance, phasor current and phasor voltage of capacitor

$$V_c(t) = 100 \cos(200t)$$

$$\Rightarrow V_c(t) = \frac{1}{\omega C} I_m \cos(\omega t + \theta)$$

$$\therefore \theta = 0^\circ, \omega = 200,$$

$$V_m = \frac{1}{\omega C} I_m = 100$$

$$I_m = 100 \times \omega C$$

$$= 100 \times 200 \times 100 \times 10^{-6}$$

$$I_m = 2 \text{ A}$$

$$I_c = I_m \angle \theta + 90^\circ$$

$$I_c = 2 \angle 0 + 90 = 2 \angle 90^\circ \text{ A}$$

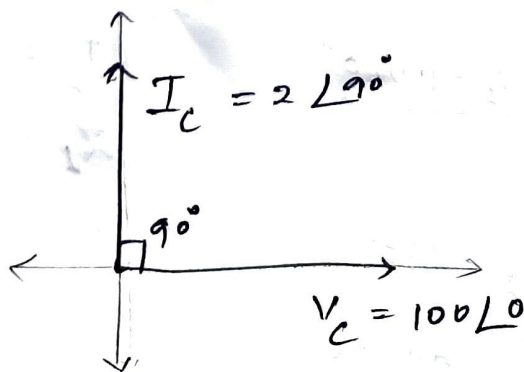
$$V_c = V_m \angle \theta$$

$$V_c = 100 \angle 0^\circ \text{ V}$$

$$Z_c = \frac{V_c}{I_c} = \frac{100 \angle 0}{2 \angle 90} = 50 \angle 0 - 90$$

$$Z_c = 50 \angle -90^\circ \Omega$$

b) Draw Phasor diagram



$$Z_c = 50 \sin(200t) \Omega$$

$$V_c = 100 \cos(200t)$$

$$I_c = 2 \sin(200t)$$

$\therefore I_c$ leads V_c by 90°