

Type of family	Age group	Income status	Will they buy a car
Nuclear	Young	Low	Yes
Extended	Old	Medium	No
Childless	Middle-aged	Low	No
Childless	Young	Medium	Yes
Single Parent	Young	Medium	Yes
Childless	Young	Low	No
Nuclear	Old	High	Yes
Nuclear	Middle-aged	Medium	Yes
Extended	Middle-aged	High	Yes
Single Parent	Old	Low	No

Gini Impurity for 'Type of family' feature:

$$G_1(\text{class} = \text{'Nuclear'}) \Rightarrow 1 - [(P(\text{yes}))^2 + (P(\text{No}))^2]$$

$$\Rightarrow 1 - \left[\left(\frac{3}{3}\right)^2 + 0\right]$$

$$\Rightarrow 0$$

$$G_1(\text{class} = \text{'Childless'})$$

$$\Rightarrow 1 - [(P(\text{yes}))^2 + (P(\text{No}))^2]$$

$$\Rightarrow 1 - \left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]$$

$$\Rightarrow 1 - \left[\frac{1}{4} + \frac{1}{4}\right] \Rightarrow 1 - \frac{1}{2}$$

$$\Rightarrow \frac{1}{2}$$

$$G_1(\text{class} = \text{'childless'}) \Rightarrow 1 - [P(\text{yes})^2 + P(\text{no})^2]$$

$$\Rightarrow 1 - \left[ \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 \right]$$

$$\Rightarrow 1 - \left[ \frac{1}{9} + \frac{4}{9} \right] \Rightarrow 1 - \left[ \frac{5}{9} \right]$$

$$\Rightarrow \frac{4}{9}$$

$$G_1(\text{class} = \text{'single parent'}) \Rightarrow 1 - [P(\text{yes})^2 + P(\text{no})^2]$$

$$\Rightarrow 1 - \left[ \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \right]$$

$$\Rightarrow 1 - \left[ \frac{1}{4} + \frac{1}{4} \right]$$

$$\Rightarrow 1 - \left(\frac{1}{2}\right) \Rightarrow 0.5$$

Gini Impurity for the feature 'Age Group'

$$G_1(\text{class} = \text{'Young'}) \Rightarrow 1 - [P(\text{yes})^2 + P(\text{no})^2]$$

$$\Rightarrow 1 - \left[ \left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 \right]$$

$$\Rightarrow 1 - \left[ \frac{9}{16} + \frac{1}{16} \right] \Rightarrow 1 - \frac{10}{16} \Rightarrow \frac{4}{16} \Rightarrow \frac{1}{4}$$

$$G_1(\text{class} = \text{'old'}) \Rightarrow 1 - \left[ \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 \right]$$

$$\Rightarrow \frac{4}{9}$$

$$G_1(\text{class} = \text{'Middle-aged'}) \Rightarrow 1 - \left[ \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 \right] \Rightarrow 1 - \left[ \frac{4}{9} + \frac{1}{9} \right]$$

$$\Rightarrow \frac{4}{9}$$

Weighted sum of Gini Impurity for the feature 'Type of family'

$$\Rightarrow \left( \frac{3}{10} \times 0 \right) + \left( \frac{2}{10} \times \frac{1}{2} \right) + \left( \frac{3}{10} \times \frac{4}{9} \right) + \left( \frac{2}{10} \times \frac{1}{2} \right)$$

$$\Rightarrow 0 + \frac{1}{10} + \frac{2}{15} + \frac{1}{10}$$

$$\Rightarrow \frac{2}{10} + \frac{2}{15} \Rightarrow \frac{1}{3} \Rightarrow 0.33$$

Weighted sum of Gini Impurity for the feature 'Age Group'

$$\Rightarrow \left( \frac{4}{10} \times \frac{3}{8} \right) + \left( \frac{2}{10} \times \frac{4}{9} \right) + \left( \frac{3}{10} \times \frac{4}{9} \right)$$

$$\Rightarrow \frac{3}{20} + \frac{2}{15} + \frac{2}{15} \Rightarrow \frac{3}{20} + \frac{4}{15} \Rightarrow \frac{5}{12} \Rightarrow 0.41667$$

Gini Impurity for the feature 'Income Status'

$$G_1(\text{class} = \text{'Low'}) \Rightarrow 1 - [ (P(\text{yes}))^2 + (P(\text{No}))^2 ]$$

$$\Rightarrow 1 - [ (1/4)^2 + (3/4)^2 ]$$

$$\Rightarrow 3/8$$

$$G_1(\text{class} = \text{'Medium'}) \Rightarrow 1 - [ (P(\text{yes}))^2 + (P(\text{No}))^2 ]$$

$$\Rightarrow 1 - [ (3/4)^2 + (1/4)^2 ]$$

$$\Rightarrow 3/8$$

$$G_1(\text{class} = \text{'High'}) \Rightarrow 1 - [ (P(\text{yes}))^2 + (P(\text{No}))^2 ]$$

$$\Rightarrow 1 - [ (2/2)^2 + (0)^2 ]$$

$$\Rightarrow 0$$

Weighted sum of Gini Impurity for the feature 'Income Status'

$$\Rightarrow 4/10 * 3/8 + 4/10 * 3/8 + 2/10 * 0$$

$$\Rightarrow 3/20 + 3/20 \Rightarrow 6/20 \Rightarrow 0.30$$

Weighted sum of  
Gini Impurity }

$$\text{Type of Family} \Rightarrow 0.33$$

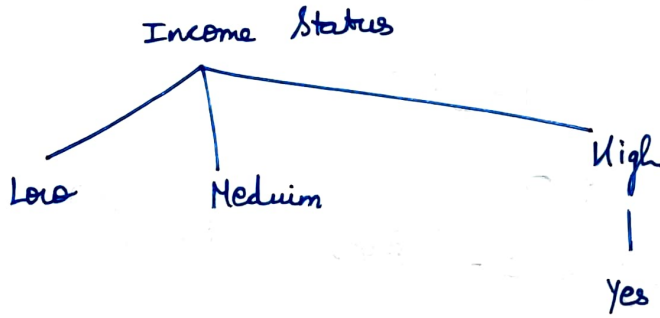
$$\text{Age Group} \Rightarrow 0.41667$$

$$\text{Income Status} \Rightarrow 0.30$$

splitting on the  
root Node

'Income status'

Lowest  
Highest Weighted  
gini impurity



[requires No further  
splitting]

Low Income status

Type of family	Age Group	Will they buy a Car
Nuclear	Young	Yes
Child-less	Middle-aged	No
Child-less	Young	No
Single Parent	Old	No

Medium income status

Type of family	Age Group	Will they buy a Car
Extended	Old	No
Child-less	Young	Yes
Single-parent	Young	Yes
Nuclear	Middle-aged	Yes



Low income status

Gini Impurity for  
'Type of family' }

$$G(\text{class} = \text{'Nuclear'}) \Rightarrow 1 - [(P(\text{yes}))^2 + (P(\text{No}))^2]$$

$$\Rightarrow 1 - [1+0]$$

$$\Rightarrow 0$$

$$G(\text{class} = \text{'child-less'}) \Rightarrow 1 - [(P(\text{yes}))^2 + (P(\text{No}))^2]$$

$$\Rightarrow 1 - [0 + \frac{2}{2}]$$

$$\Rightarrow 0$$

$$G(\text{class} = \text{'single parent'}) \Rightarrow 1 - [(P(\text{yes}))^2 + (P(\text{No}))^2]$$

$$\Rightarrow 1 - [0+1]$$

$$\Rightarrow 0$$

Weighted sum of gini impurity for "Type of family"

$$\Rightarrow \left(\frac{1}{4} * 0\right) + \left(\frac{2}{4} * 0\right) + \left(\frac{1}{4} * 0\right)$$

$$\Rightarrow 0$$

Gini Impurity for 'Age Group' feature

$$G(\text{class} = \text{'Young'}) \Rightarrow 1 - \left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]$$

$$\Rightarrow \frac{1}{2}$$

$$G(\text{class} = \text{'Old'}) \Rightarrow 1 - [0+1]$$

$$\Rightarrow 0$$

$$G(\text{class} = \text{'medium aged group'}) \Rightarrow 1 - [1+0]$$

$$\Rightarrow 0$$

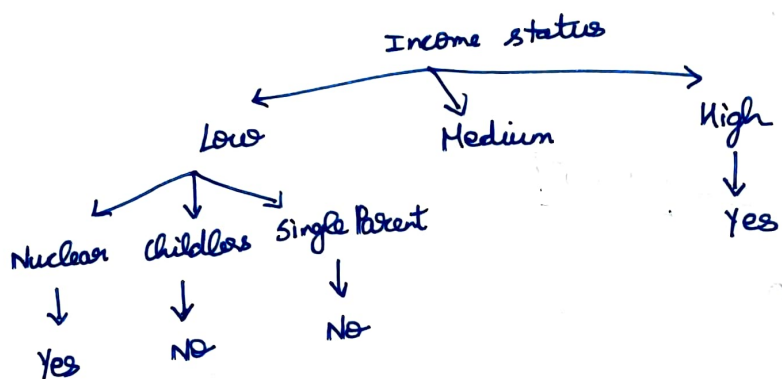
Weighted sum of Gini impurity for 'Age Group' feature

$$\Rightarrow \left(\frac{2}{4} * \frac{1}{2}\right) + \left(\frac{1}{4} * 0\right) + \left(\frac{1}{4} * 0\right)$$

$$\Rightarrow \frac{1}{2}$$

Weighted sum of Gini Impurity  $\begin{cases} \text{Type of family} \Rightarrow 0 \\ \text{Age group} \Rightarrow \frac{1}{2} \end{cases}$

so splitting on Type of family



Medium income status

Gini Impurity for 'Type of family'

$$G(\text{class} = \text{'Extended'}) \Rightarrow 1 - \left[ \left(P(\text{yes})\right)^2 + \left(P(\text{No})\right)^2 \right]$$

$$\Rightarrow 1 - [0 + 1] \Rightarrow 0$$

$$G(\text{class} = \text{'Child-less'}) \Rightarrow 1 - \left[ \left(\frac{2}{4}\right)^2 + 0^2 \right] \Rightarrow 0$$

$$G(\text{class} = \text{'Single-parent'}) \Rightarrow 1 - [0 + 1] \Rightarrow 0$$

$$G(\text{class} = \text{'Nuclear'}) \Rightarrow 1 - [1 + 0] \Rightarrow 0$$

Weighted sum of Gini Impurity for 'Type of family'

$$\Rightarrow \left(\frac{1}{4} * 0\right) + \left(\frac{1}{4} * 0\right) + \left(\frac{1}{4} * 0\right) + \left(\frac{1}{4} * 0\right)$$

$$\Rightarrow 0$$

Gini impurity for 'Age Group' feature:

$$\begin{aligned} G(\text{class} = \text{'Extended' Old}) &\Rightarrow 1 - [(P(\text{yes}))^2 + (P(\text{No}))^2] \\ &\Rightarrow 1 - [0 + 1] \\ &\Rightarrow 0 \end{aligned}$$

$$\begin{aligned} G(\text{class} = \text{'Young'}) &\Rightarrow 1 - [(P(\text{yes}))^2 + (P(\text{No}))^2] \\ &\Rightarrow 1 - \left[ \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \right] \\ &\Rightarrow \frac{1}{2} \end{aligned}$$

$$\begin{aligned} G(\text{class} = \text{'Middle Aged'}) &\Rightarrow 1 - [1^2 + 0^2] \\ &\Rightarrow 0 \end{aligned}$$

Weighted sum of Gini impurity for 'Age - Group' feature:

$$\Rightarrow \left(\frac{1}{4} * 0\right) + \left(\frac{2}{4} * \frac{1}{2}\right) + \left(\frac{1}{4} * 0\right) \Rightarrow \frac{1}{4}$$

Weighted sum of Gini Impurity for 'Age-group' feature }  $\begin{cases} \rightarrow \text{'Type of family'} \Rightarrow 0 \\ \rightarrow \text{'Age Group'} \Rightarrow \frac{1}{4} \end{cases}$

so splitting of 'Type of family' feature.



