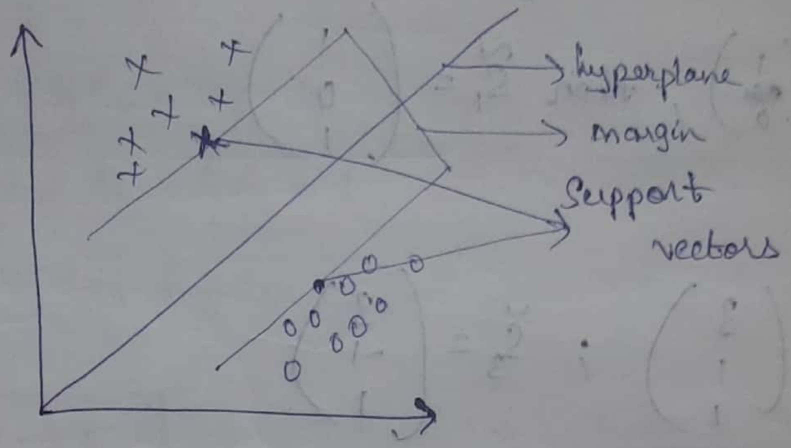
Support Vector Machine (SVM):-

* SVM is a discriminative classifier that is formally designed by a separative hyperplane.

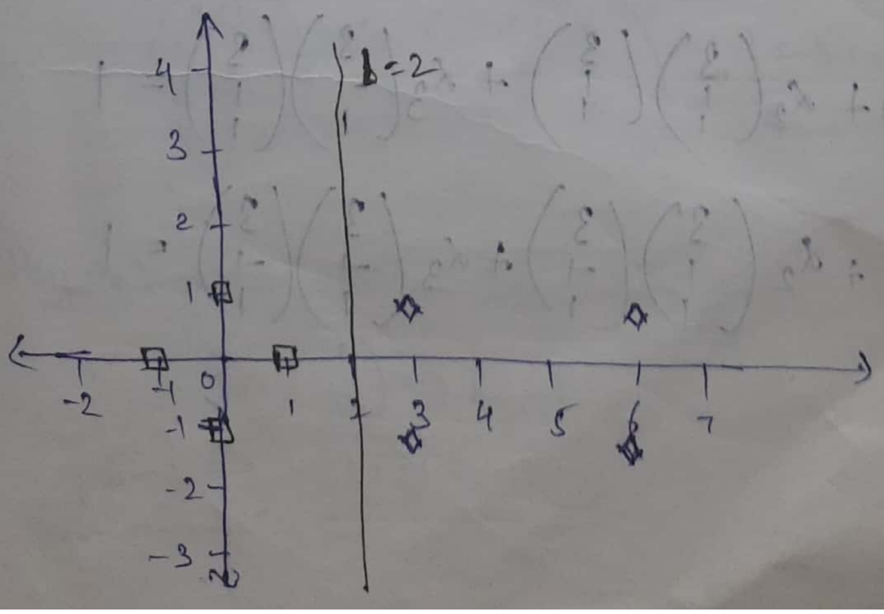


Suppose we are given the following positively labeled data points,

$$\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \end{pmatrix} \right\}$$

negatively labeled data points;

$$\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\}$$



(1) $\left\{ s_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, s_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, s_3 = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \right\}$

(2) Each vector is augmented with '1' as a bias input

$$s_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \text{ then } \tilde{s}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

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$$\tilde{s}_2 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} ; \tilde{s}_3 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$$

$$\alpha_1 \tilde{s}_1 \cdot \tilde{s}_1 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_1 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_1 = -1 \text{ (-ve sample)}$$

$$\alpha_1 \tilde{s}_1 \cdot \tilde{s}_2 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_2 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_2 = +1 \text{ (+ve sample)}$$

$$\alpha_1 \tilde{s}_1 \cdot \tilde{s}_3 + \alpha_2 \tilde{s}_2 \cdot \tilde{s}_3 + \alpha_3 \tilde{s}_3 \cdot \tilde{s}_3 = +1 \text{ (+ve sample}$$

$$\Downarrow$$

$$\alpha_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} = 1$$

$$\alpha_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = 1$$

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$$\alpha_1(1+0+1) + \alpha_2(3+0+1) + \alpha_3(3+0+1) = -1$$

$$\alpha_1(3+0+1) + \alpha_2(9+1+1) + \alpha_3(9-1+1) = 1$$

$$\alpha_1(3+0+1) + \alpha_2(9-1+1) + \alpha_3(9+1+1) = 1$$

$$2\alpha_1 + 4\alpha_2 + 4\alpha_3 = -1$$

$$4\alpha_1 + 11\alpha_2 + 9\alpha_3 = 1$$

$$4\alpha_1 + 9\alpha_2 + 11\alpha_3 = 1$$

$$\boxed{\begin{aligned} \alpha_1 &= -3.5 \\ \alpha_2 &= 0.75 \\ \alpha_3 &= 0.75 \end{aligned}}$$

$$\boxed{\bar{w} = \sum_i \alpha_i \tilde{S}_i}$$

$$\bar{w} = -3.5\begin{pmatrix}1\\0\\1\end{pmatrix} + 0.75\begin{pmatrix}3\\1\\1\end{pmatrix} + 0.75\begin{pmatrix}3\\-1\\1\end{pmatrix}$$

$$= \begin{pmatrix}1\\0\\-2\end{pmatrix} \quad \left.\begin{matrix}1\\0\end{matrix}\right\} \rightarrow w \quad \left.-2\right\} \rightarrow b$$

<u>Hyperplane equation</u> $\rightarrow$ $\boxed{y = wx + b}$

$$w = \begin{pmatrix}1\\0\end{pmatrix} \; ; \; b = -2$$

$$\boxed{b - 2 = 0 \Rightarrow \boxed{b = 2}}$$