PCA

Number of samples ⇒ 6

Number of features ⇒ 2

X	1	2	3	4	5	6
Y	2	6	4	5	6	7

X	Y	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})$
1	2	-2.5	-3	6.25	9	~~6.25~~
2	6	-1.5	1	2.25	1	~~2.25~~
3	4	-0.5	-1	0.25	1	~~0.25~~
4	5	0.5	0	0.25	0	~~0~~
5	6	1.5	1	2.25	1	~~2.25~~
6	7	2.5	2	6.25	4	~~6.25~~
21	30			17.5	16	

$\bar{X} \Rightarrow \frac{21}{6} \Rightarrow 3.5$

$\bar{Y} \Rightarrow \frac{30}{6} \Rightarrow 5$

$$\text{Covariance}(X,X) \Rightarrow \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2$$

$$\Rightarrow \frac{1}{5} (17.5) \Rightarrow 3.5$$

Covariance (X,X) ⇒ 3.5

$$\text{Covariance}(Y,Y) \Rightarrow \frac{1}{N-1} \sum_{i=1}^{N} (y_i - \bar{y})^2$$

$$\Rightarrow \frac{1}{5} (16)$$

Covariance (Y,Y) ⇒ 3.2

$$\text{Covariance}(X,Y) \Rightarrow \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})$$

$$\Rightarrow \frac{1}{5} \times 13$$

$(y_i - \bar{y})$
1.5
-1.5
0.5
0
1.5
5
13

Covariance $(X,Y) \Rightarrow 2.6$

Covariance Matrix

$$\begin{matrix} 5 \\ \} \end{matrix} \begin{bmatrix} 3.5 & 2.6 \\ 2.6 & 3.2 \end{bmatrix}$$

$$\begin{bmatrix} xx & xy \\ xy & yy \end{bmatrix}$$

Eigen values & Eigen vectors

$$|5 - \lambda I| \Rightarrow 0$$

$$\begin{bmatrix} 3.5 - \lambda & 2.6 \\ 2.6 & 3.2 - \lambda \end{bmatrix}$$

$$\Rightarrow (3.5 - \lambda)(3.2 - \lambda) - (2.6)(2.6) = 0$$

$$\Rightarrow (3.5)(3.2) - 3.5\lambda - 3.2\lambda + \lambda^2 - 6.76 = 0$$

$$\lambda^2 - 6.7\lambda + 11.2 - 6.76 = 0$$

$$\lambda^2 - 6.7\lambda + 4.44 = 0$$

Use formula

1. $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\lambda_1 \Rightarrow 0.7456$ | $\lambda_2 = 5.9543$

$\lambda_1 \Rightarrow$ [redacted] 0.7456

$3.5 - 0.7456 \Rightarrow 2.7544$

$3.2 - 0.7456 \Rightarrow 2.4544$

$$(S - \lambda I)U = 0$$

$$\begin{bmatrix} 2.7544 & 2.6 \\ 2.6 & 2.4544 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2.7544\,u_1 + 2.6\,u_2 \Rightarrow 0$$

$$2.6\,u_1 + 2.4544\,u_2 \Rightarrow 0$$

$$5.3544\,u_1 + 5.0544\,u_2 = 0$$

$$5.0544\,u_2 = -5.3544\,u_1$$

$$u_2 = \frac{-5.3544}{5.0544}\,u_1$$

$$u_2 = 1.05935\,u_1$$

$u_2 = 1.05935$ $\quad$ $u_1 = 1$

Normalizing $\Rightarrow \sqrt{u_1^2 + u_2^2}$

$\Rightarrow \sqrt{1^2 + (1.05935)^2}$

$\Rightarrow 1.4567$

Eigen vectors $e_1^t = ($[redacted]

$u_1 \Rightarrow \frac{1}{1.4567}$ $\qquad$ $u_2 \Rightarrow \frac{1.05935}{1.4567}$

$e_1^t \Rightarrow (0.68644, 0.72722$

$u_1 \Rightarrow 0.68644$ $\qquad$ $u_2 \Rightarrow 0.72722$

$\lambda_2 = 5.9543$

$3.5 - 5.9543 \Rightarrow -2.4543$

$3.2 - 5.9543 \Rightarrow -2.7543$

$$(S - \lambda I)\, U = 0$$

$$\begin{pmatrix} -2.4543 & 2.6 \\ 2.6 & -2.7543 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2.4543\, u_1 + 2.6\, u_2 \Rightarrow 0$$

$$2.6\, u_1 - 2.7543\, u_2 \Rightarrow 0$$

$$0.1457\, u_1 - 0.1543\, u_2 \Rightarrow 0$$

$$0.1457\, u_1 = 0.1543\, u_2$$

$$u_2 = \frac{0.1457}{0.1543}\, u_1$$

$$u_2 = 0.94426\, u_1$$

$u_1 = 1$ $\qquad$ $u_2 = 0.94426$

Normalizing $\Rightarrow \sqrt{u_1^2 + u_2^2}$

$\Rightarrow \sqrt{1^2 + (0.94426)^2}$

$\Rightarrow \sqrt{1.8916}$

$\Rightarrow 1.37536$

$u_1 \Rightarrow \frac{1}{1.37536}$ $\qquad$ $u_2 \Rightarrow \frac{0.94426}{1.37536}$

$u_1 \Rightarrow 0.72708$ $\qquad$ $u_2 \Rightarrow 0.68655$

$e_2^t \Rightarrow (0.72708 \; ; \; 0.68655)$

For 1st sample

$e_1^t \Rightarrow \begin{pmatrix} x_i - \bar{x} \\ y_i - \bar{y} \end{pmatrix}$

$(0.72708, 0.68655) \begin{pmatrix} -2.5 \\ -3 \end{pmatrix} \Rightarrow -3.87735$

For 2nd sample

$(0.72708, 0.68655) \begin{pmatrix} -1.5 \\ +1 \end{pmatrix} \Rightarrow -0.40407$

For 3rd sample

$(0.72708, 0.68655) \begin{pmatrix} -0.5 \\ -1 \end{pmatrix} \Rightarrow -1.05009$

For 4th sample

$(0.72708, 0.68655) \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} \Rightarrow 0.36354$

For 5th sample

$(0.72708, 0.68655) \begin{pmatrix} 1.5 \\ 1 \end{pmatrix} \Rightarrow 1.77717$

For 6th sample

$(0.72708, 0.68655) \begin{pmatrix} 2.5 \\ 2 \end{pmatrix} \Rightarrow 3.1908$

x	1	2	3	4	5	6
y	2	6	4	5	6	7
PCA	-3.8775	-0.40407	-1.05009	0.36354	1.7717	3.1908