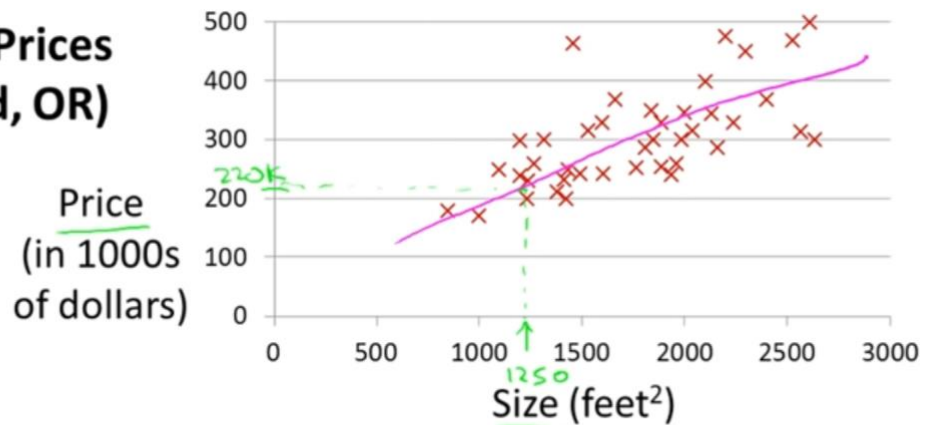


## Linear Regression model Part-1

### Housing Prices (Portland, OR)



#### Supervised Learning

Given the “right answer” for each example in the data.

#### Regression Problem

Predict real-valued output

*Classification: Discrete-valued output*

Andrew Ng

## Terminologies

### Training set of housing prices (Portland, OR)

Size in feet <sup>2</sup> ( $x$ )	Price (\$) in 1000's ( $y$ )
→ 2104	460
→ 1416	232
→ 1534	315
→ 852	178
...	...

*m = 47*

Notation:

- $m$  = Number of training examples
- $x$ 's = “input” variable / features
- $y$ 's = “output” variable / “target” variable
- $(x, y)$  - one training example
- $(x^{(i)}, y^{(i)})$  -  $i^{\text{th}}$  training example

$$\left\{ \begin{array}{l} x^{(1)} = 2104 \\ x^{(2)} = 1416 \\ y^{(1)} = 460 \end{array} \right.$$

Andrew Ng

## Terminology

Training set: Data used to train the model

$x$ size in feet <sup>2</sup>	$y$ price in \$1000's
(1) 2104	400
(2) 1416	232
(3) 1534	315
(4) 852	178
...	...
(47) 3210	870

$m = 47$

$x^{(1)} = 2104$     $y^{(1)} = 400$   
 $(x^{(1)}, y^{(1)}) = (2104, 400)$

$x^{(2)} = 1416$     $x^{(2)} \neq x^2$  not exponent

Notation:

$x$  = "input" variable  
feature

$y$  = "output" variable  
"target" variable

$m$  = number of training examples

$(x, y)$  = single training example

$(x^{(i)}, y^{(i)})$

$(x^{(i)}, y^{(i)})$  =  $i^{\text{th}}$  training example  
index (1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> ...)

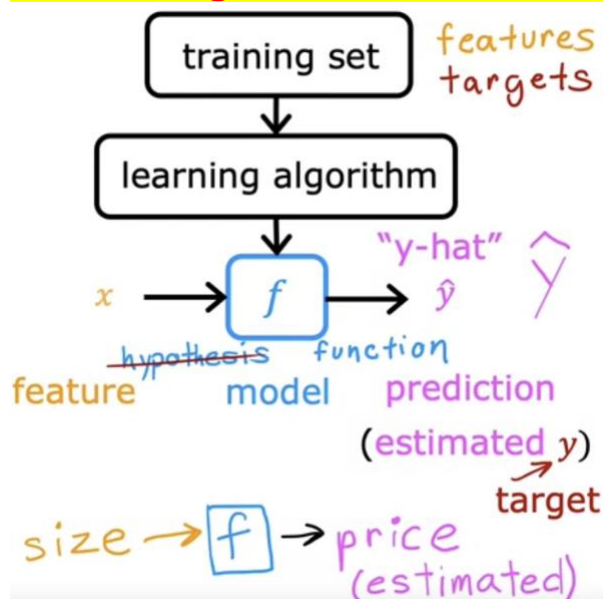
For linear regression, the model is represented by  $f_{w,b}(x) = wx + b$ . Which of the following is the output or "target" variable?

- ☐  $\hat{y}$ .
- ☐  $m$
- ☒  $y$
- ☐  $x$

✓ Correct

$y$  is the true value for that training example, referred to as the output variable, or "target".

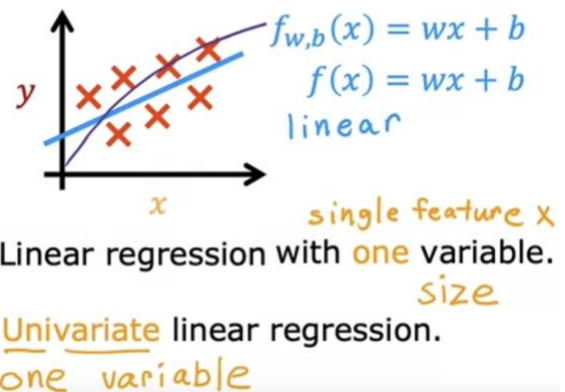
## Linear Regression model Part-2



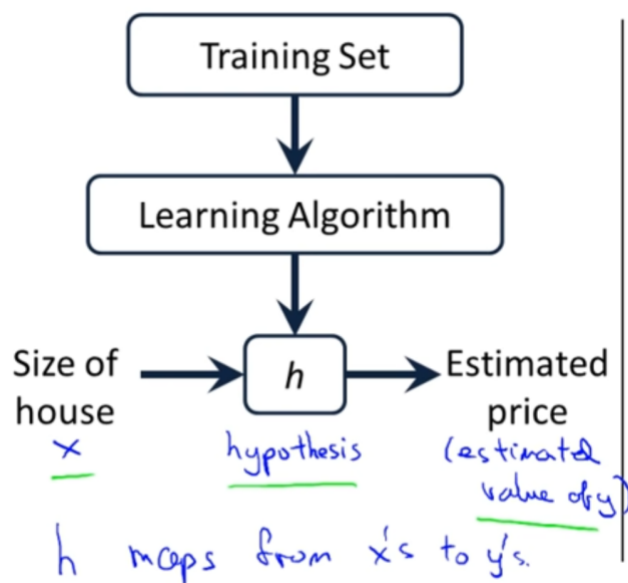
How to represent  $f$ ?

$$f_{w,b}(x) = wx + b$$

$$f(x)$$



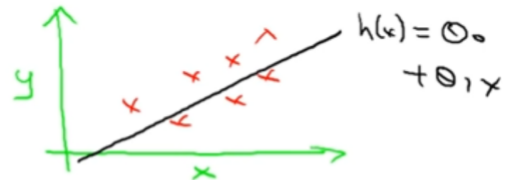
The function  $f$  is also known as function/hypothesis/model.



How do we represent  $h$  ?

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

Shorthand:  $h(x)$



Linear regression with one variable.  $(x)$   
Univariate linear regression.

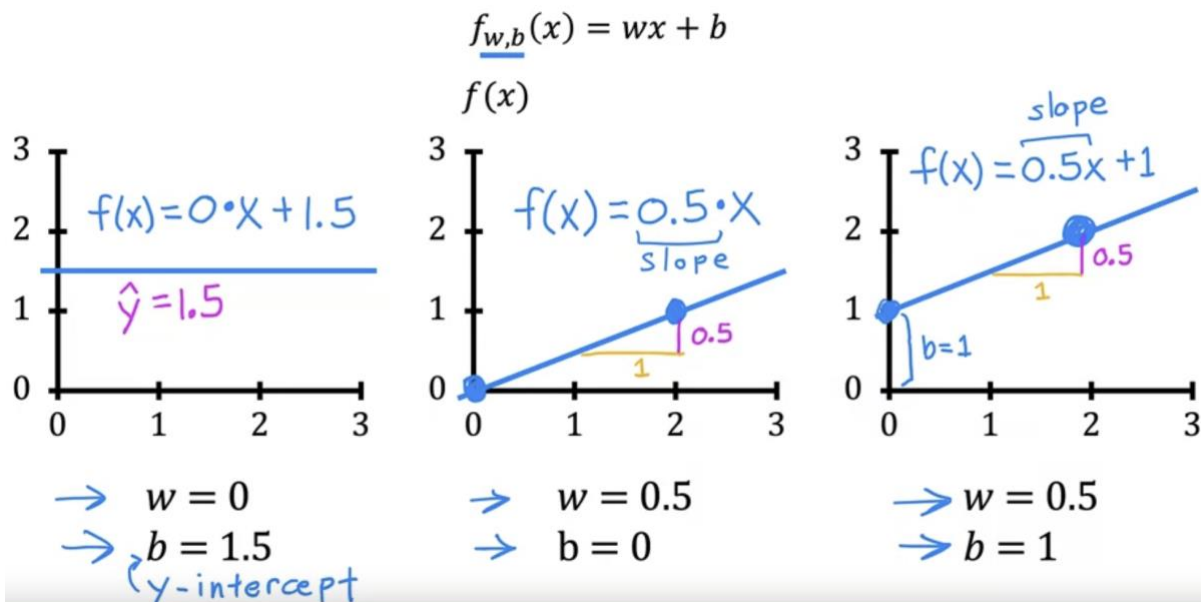
Andrew Ng

## Model Representation

Refer 3\_Model\_Representation\_Solution.ipynb file

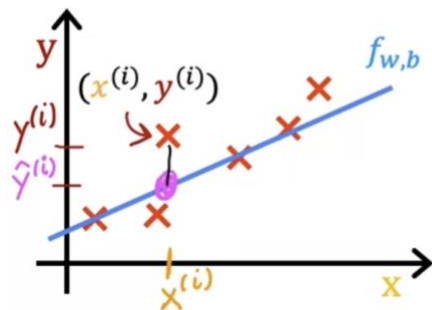
## Cost Function

Lets see the effect of slope and constant



## Cost function/Squared error function/Residual Sum of Squares

Average square of the distance between the observed and the predicted value



$$\hat{y}^{(i)} = f_{w,b}(x^{(i)}) \leftarrow$$

$$f_{w,b}(x^{(i)}) = wx^{(i)} + b$$

Cost function: Squared error cost function

$$J(w,b) = \frac{1}{2m} \sum_{i=1}^m \left( \underset{\text{error}}{\hat{y}^{(i)}} - y^{(i)} \right)^2$$

$m$  = number of training examples

$$J(w,b) = \frac{1}{2m} \sum_{i=1}^m (f_{w,b}(x^{(i)}) - y^{(i)})^2$$

Find  $w, b$ :

$\hat{y}^{(i)}$  is close to  $y^{(i)}$  for all  $(x^{(i)}, y^{(i)})$ .

The cost function used for linear regression is

$$J(w,b) = \frac{1}{2m} \sum_{i=1}^m (f_{w,b}(x^{(i)}) - y^{(i)})^2$$

Which of these are the parameters of the model that can be adjusted?

- ☒  $w$  and  $b$
- ☐  $f_{w,b}(x^{(i)})$
- ☐  $w$  only, because we should choose  $b=0$
- ☐  $\hat{y}$

✓ Correct

## Cost function Intuition

model:

$$f_{w,b}(x) = wx + b$$

parameters:

$$w, b$$

cost function:

$$J(w, b) = \frac{1}{2m} \sum_{i=1}^m (f_{w,b}(x^{(i)}) - y^{(i)})^2$$

goal:

$$\underset{w, b}{\text{minimize}} J(w, b)$$

simplified

$$f_w(x) = wx$$

$$b = \emptyset$$

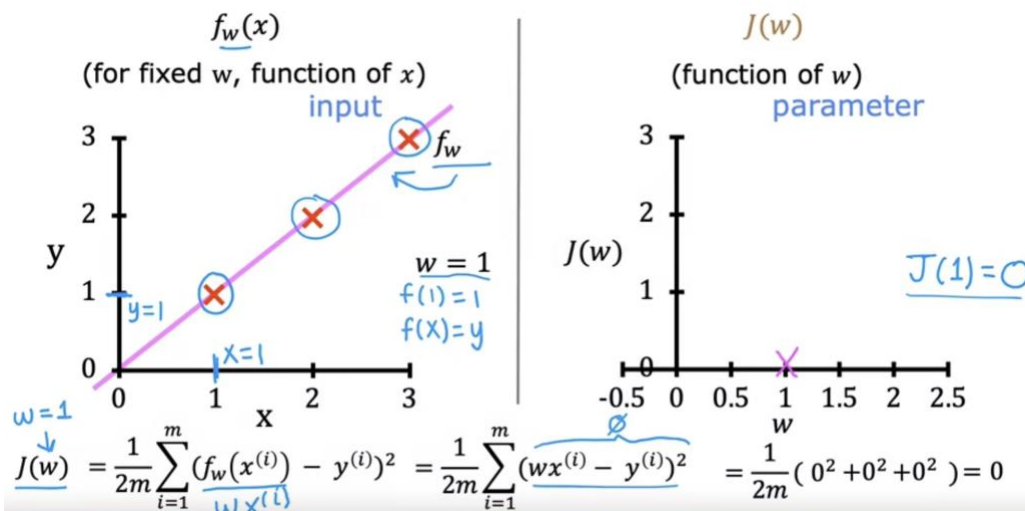
$w$

$$J(w) = \frac{1}{2m} \sum_{i=1}^m (f_w(x^{(i)}) - y^{(i)})^2$$

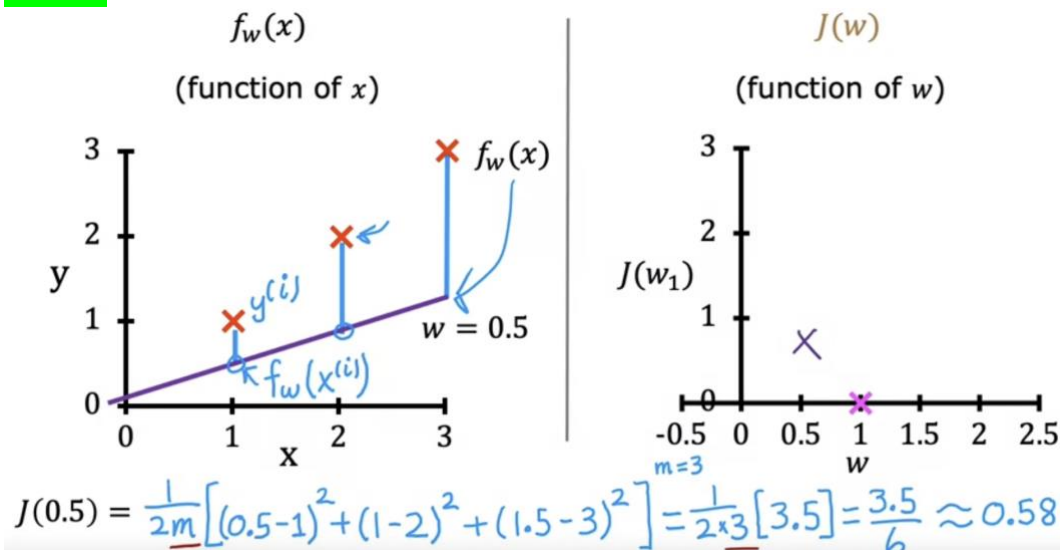
$$\underset{w}{\text{minimize}} J(w)$$

Currently put  $b=0$

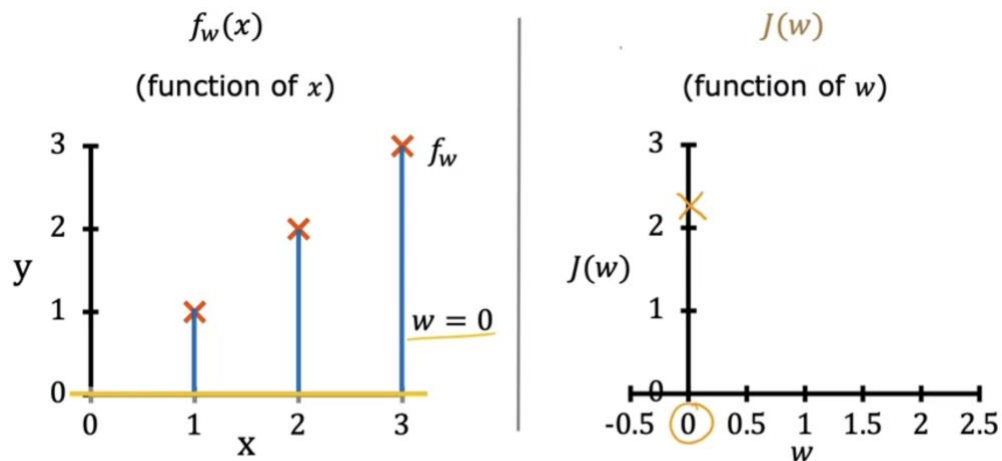
**$w=1$**



**$w=0.5$**

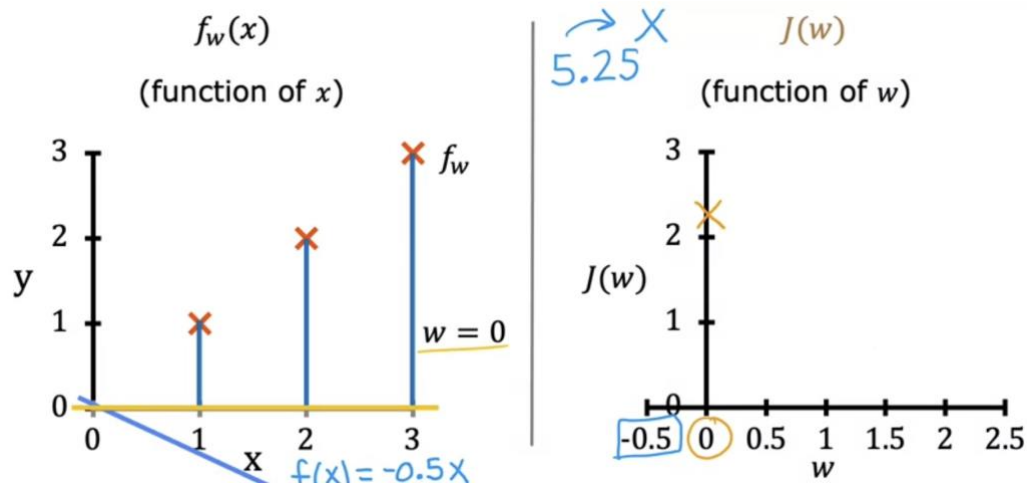


**w=0**

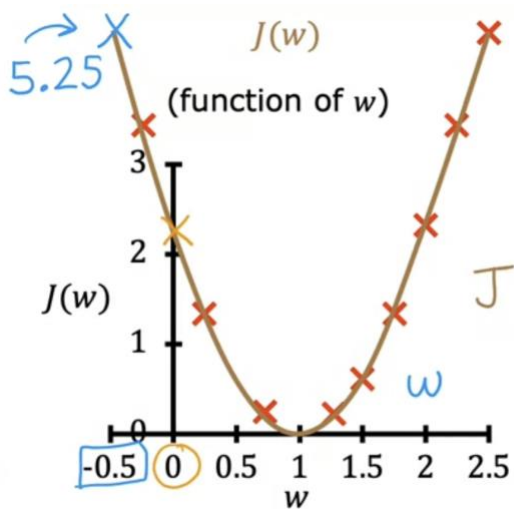


$$J(0) = \frac{1}{2m} (1^2 + 2^2 + 3^2) = \frac{1}{6} [14] \approx 2.3$$

**w=-0.5**



As we move on with different values of  $w$ , and the corresponding  $J$  value we can obtain this curve



So we want to choose  $w$ , which minimises  $J(w)$

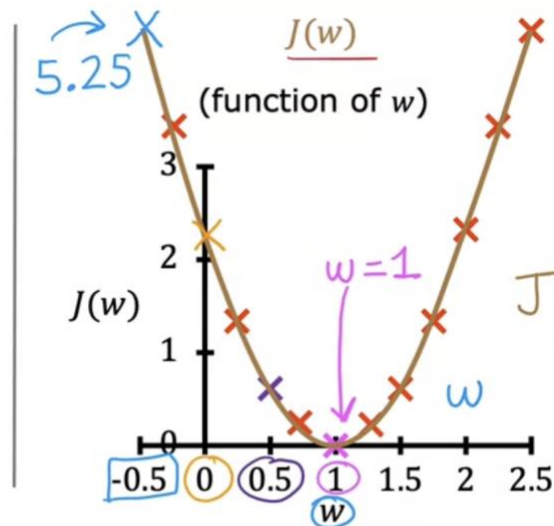


goal of linear regression:

$$\underset{w}{\text{minimize}} J(w)$$

general case:

$$\underset{w,b}{\text{minimize}} J(w, b)$$



choose  $w$  to minimize  $J(w)$

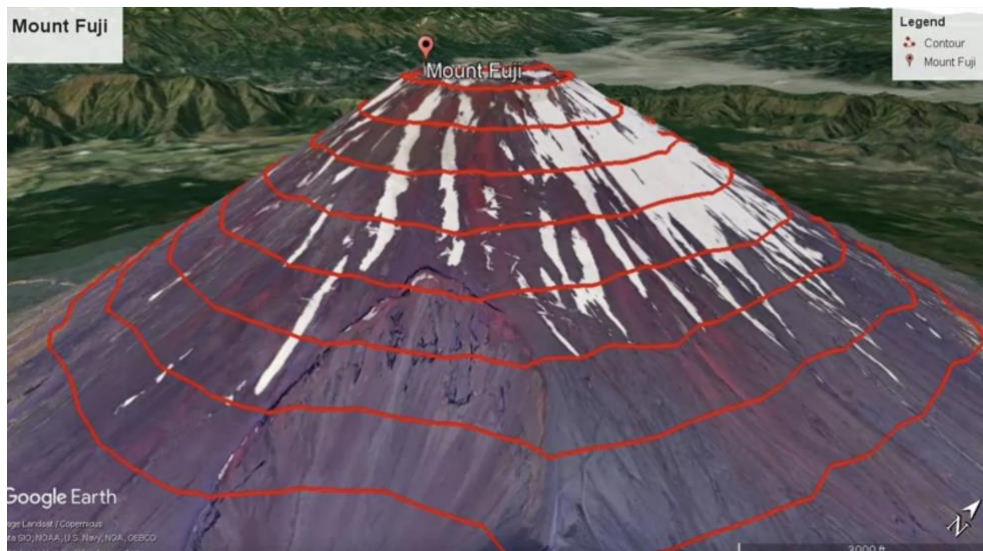
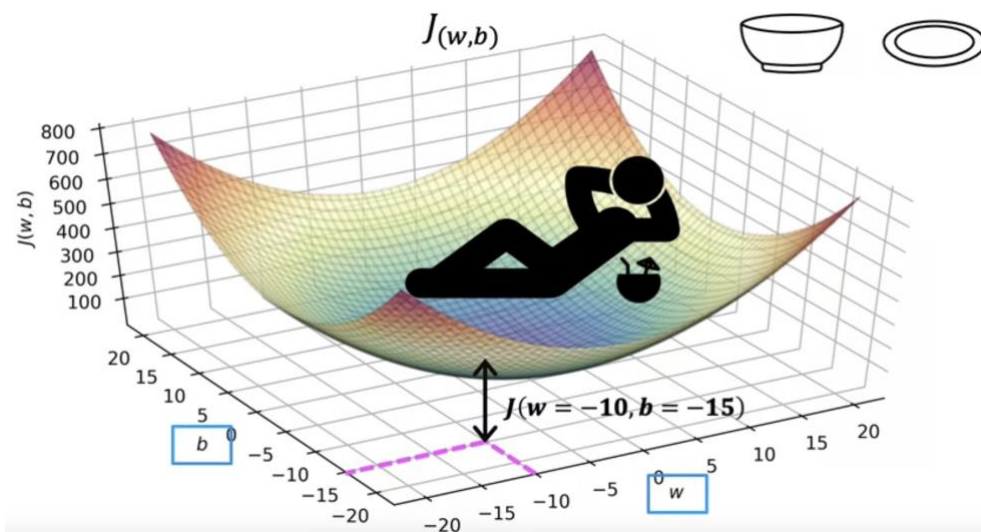
When does the model fit the data relatively well, compared to other choices for parameter  $w$ ?

- ☐ When  $f_w(x)$  is at or near a minimum for all the values of  $x$  in the training set.
- ☐ When  $w$  is close to zero.
- ☒ When the cost  $J$  is at or near a minimum.
- ☐ When  $x$  is at or near a minimum.

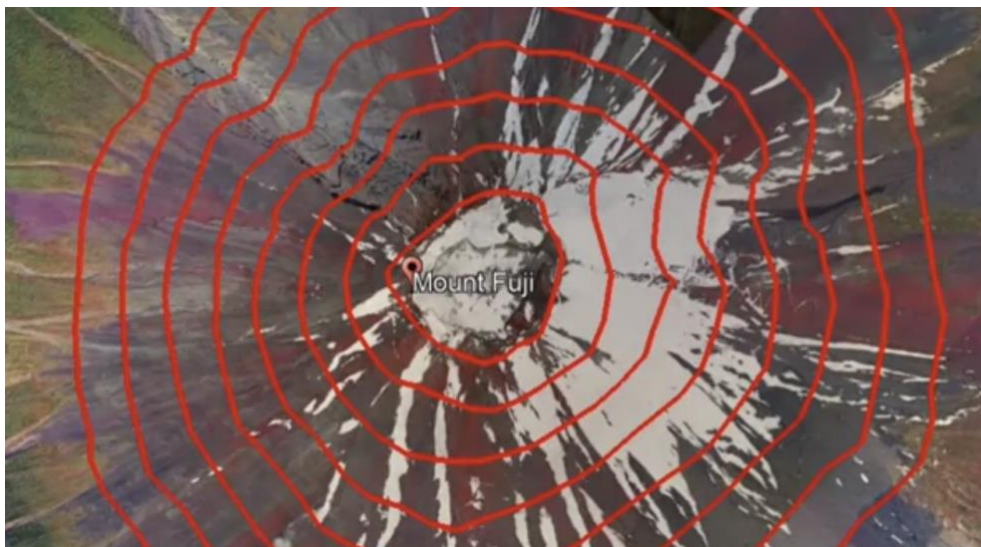
✓ **Correct**

When the cost is relatively small, closer to zero, it means the model fits the data better compared to other choices for  $w$  and  $b$ .

## Visualize the Cost function



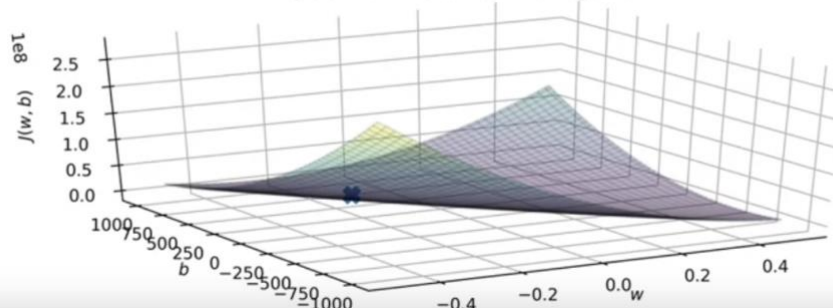
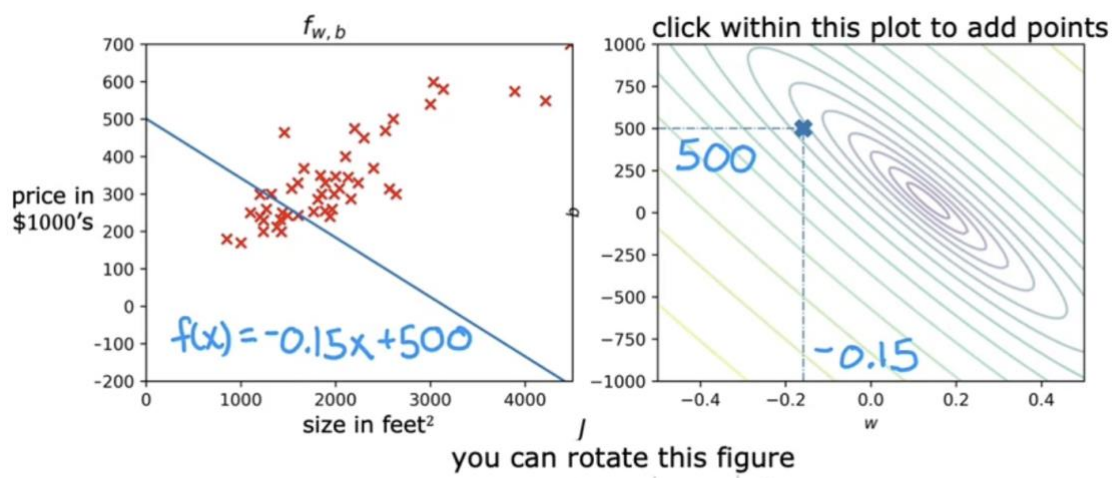
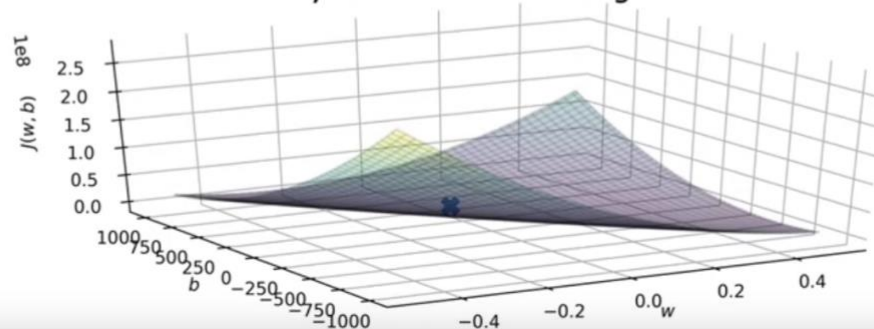
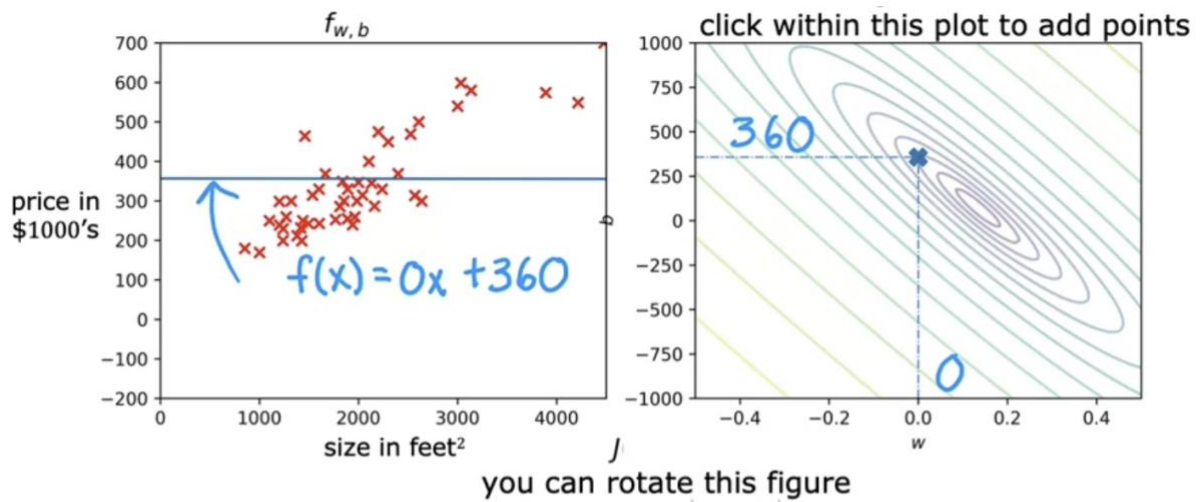
3D bowl shaped surface plot → 2D contour plots

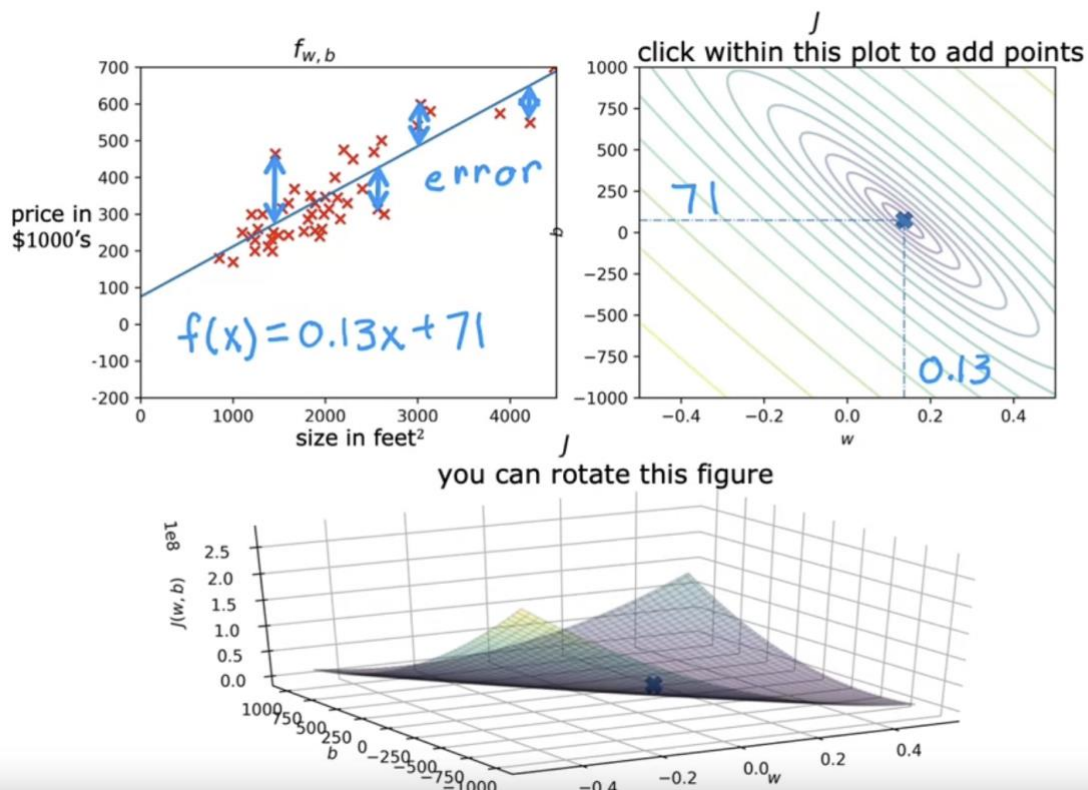


View directly above the mountain.









This is pretty close to the centre of the small eclipse.

How better the fit lines corresponds to the points on the graph of  $j$  that are closer to the minimum possible cost for the cost function  $J(w, b)$

## Quiz

1. Which are the two common types of supervised learning? (Choose two)

1 / 1 point

☒ Classification

☒ Correct

Classification predicts from among a limited set of categories (also called classes). These could be a limited set of numbers or labels such as "cat" or "dog".

☐ Clustering

☒ Regression

☒ Correct

Regression predicts a number among potentially infinitely possible numbers.

2.

1 / 1 point

Which of these is a type of unsupervised learning?

☐ Regression

☐ Classification

☒ Clustering

☒ Correct

Clustering groups data into groups or clusters based on how similar each item (such as a hospital patient or shopping customer) are to each other.

## References

[Linear regression model part 1 | Coursera](#)