(05)

01-04-2022

→ Paper discussion.

06-04-2022

→ Uncertainity.

- Based on evidence in KB agents predicts the future.

eg: in KB: A → B {if A is true then B is true}

But consider a situation where we are not sure about whether A is true or not then we cannot express this statement, this situation → uncertainity.

- So to represent uncertain knowledge, where we are not sure about the predicates, we need uncertain reasoning / probabilistic reasoning.

→ cause of uncertainity.

- information occured from unreliable sources
- Experimental errores
- Equipment fault
- Temperature variation
- Climate change

→ how to solve this → Probabilistic reasoning

- way of knowledge representation.

we combine probability theory with logic to handle the uncertainity

- we use probability in PR because it provides a way to handle uncertainity that is the result of someone's laziness & ignorance

→ Need of probabilistic reasoning in AI:

- when there are unpredictable outcomes
- when specifications / possibilities of predicates becomes too large to handle.

→ when an unknown error occurs during an experiment

2 ways to solve problems with uncertain knowledge → Bayes' rule
→ Bayesian statistics

→ probability, event (each possible outcome of variable), sample space (collection of all possible events), Random Variables → used to represent the event of objects in world.

prior probability, posterior probability

prior probability → probability computed before observing new info

posterior probability → probability that is calculated after all evidence / info has taken into account

conditional probability

probability of A under conditions of B.

$P(A \wedge B)$ → joint probability of A and B.

$P(B)$ → marginal probability of B.

$$\boxed{P(A/B) = \frac{P(A \wedge B)}{P(B)}}$$

08-04-2022

Bayes rule

way of finding probability when we know certain other probabilities

$$\boxed{P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}}$$

A & B are events
$P(B) \neq 0$.

P(A/B) is a conditional probability of event A occurring given that B is true

(also called as Posterior probability of A given B)

P(B/A) can be interpreted as likelihood of A given fixed B,

because $P(B/A) = L(A/B)$

Conditional probability of event B occurring given that A is true

$\boxed{P(A) \neq P(B)}$ are probabilities of observing A and B respectively without any given condition.

(known as marginal probability / prior probability)

- A and B must be different events

Q planning a picnic

① 50% of all rainy days start off cloudy
② But cloudy mornings are common. {abt 40% of days start cloudy}

③ and this is usually a dry month

(only 3 of 30 days tend to be rainy - or 10%)

what is the chance of rain during a cloudy day?

$$\boxed{P(rain/cloudy) = \frac{P(cloudy/rain) \times P(rain)}{P(cloudy)}}$$

$$\frac{50 \times 10}{40} = 12.5\%$$

Q from a standard deck of playing cards a single card is drawn Probability that card is king is 4/52. Then calculate posterior probability. P(king/Face). {which means face card is a king card?}

posterior probability → $\boxed{P(A/B) = \frac{P(B/A) \times P(A)}{P(B)}}$

$$P(king/face) = \frac{\underset{1}{P(face/king)} \times \underset{4/52}{P(king)}}{\underset{12/52}{P(face)}} \Rightarrow \frac{1 \times 1/13}{3/13} = 1/3$$

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Q What is probability that a patient has disease with stiffneck? it occurs 80% of time. the known probability that a patient has a disease is 1/30,000. and known probability that a patient has a stiff neck is 2%.

a = stiff neck   b = disease

$$P(b/a) = \frac{P(a/b) \times P(b)}{P(a)} \Rightarrow \frac{\frac{80}{100} \times \frac{1}{30,000}}{2/100} \Rightarrow 1/250 = 0.133$$

Q ABC companies produce 25% 35% 40% bolts out of total. 5% 4% 2% are defective. A ~~bolts~~ drawn at random from products. If bolt drawn is found defective, what is probability it is manufactured by B?

$$P(\text{company} = B / \text{bulb} = \text{defective}) = \frac{P(\text{defective}/B) \times P(B)}{P(\text{defective})}$$

$$P(B/\text{defective}) \Rightarrow \frac{4/100 \times 35/100}{\left(\frac{25}{100}\right)\left(\frac{5}{100}\right) + \left(\frac{35}{100}\right)\left(\frac{4}{100}\right) + \left(\frac{40}{100}\right)\left(\frac{2}{100}\right)} = \frac{28}{69}$$

Q Man is known to speak truth 3 out of 4 times. he says on a throw. "it is a six"? what is probability it is actually a six.

$P(E_1) = 1/6$ — occurance of 6 $P(E_2) = 5/6$ — occurance of number other than 6.

$P(A/E_1) = 3/4$ — truth $P(A/E_2) = 1/4$ — lie

$$\boxed{P\left(\frac{E_1}{A}\right) = \frac{P(A/E_1) \times P(E_1)}{P(A)}}$$

$$\frac{3/4 \times 1/6}{\left(\frac{1}{6}\right)\left(\frac{3}{4}\right) + \left(\frac{5}{6}\right)\left(\frac{1}{4}\right)} = 3/8$$

Q Bag A → 3R balls 4G Balls
Bag B → 4R balls 5 green balls
one ball is drawn at random from one of the bags & found to be red. what is probability that it was drawn from Bag A?

$$P(\text{Bag A}/\text{red}) = \frac{P(\text{red}/\text{Bag A}) \times P(\text{Bag A})}{P(\text{red})} \Rightarrow \boxed{\frac{3/7 \times 1/2}{\left(\frac{1}{2}\right)\left(\frac{3}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{9}\right)}} = 27/55$$

Q Student knew only 60% of questions in test each with 5 answers he simply guessed ~~the answers~~ while answering. what is probability that he knew answer to a question given that he answered correctly

$$\xrightarrow{0.60} A_1 \text{ (knew)} \xrightarrow{1} E_1 \rightarrow 0.60$$ Probability of correct answer given that he know the answer.

$$\xrightarrow{0.40} A_2 \text{ (not knew)} \xrightarrow{1/5} E_2 \rightarrow 0.08$$ Probability of correct answer given that he guessed the answer.

$\therefore$ $P(\text{knew}/\text{correct}) = \dfrac{P(\text{correct}/\text{knew}) \times P(\text{knew})}{P(\text{correct})}$

$\Rightarrow \dfrac{1 \times 0.60}{0.68} \Rightarrow 0.88235$

P(correct) = 0.68

Q Bag → 3W 5B, 4 balls are transferred into an empty bag. From this bag, a ball is drawn and is found to be white. what is probability that out of 4 balls transferred 3 are white and 1 is black

3W 5B → 0W 4B → $\dfrac{3c_0 \times 5c_4}{8c_4} \Rightarrow 1/14$

1W 3B — 6/14

2W 2B — 6/14

3W 0B — 1/14

probabilities of selecting a Bag.

$P(3W1B/\text{white}) \Rightarrow \dfrac{P(3W1B) \times P(\text{white}/3W1B)}{P(\text{white})}$

$P(3W1B) = 1/14$, $P(\text{white}/3W1B) = 3/4$

$P(\text{white})$

$\left(\frac{1}{14}\right)(0) + \left(\frac{6}{14}\right)\left(\frac{1}{4}\right) + \left(\frac{6}{14}\right)\left(\frac{2}{14}\right) + \left(\frac{1}{14}\right)\left(\frac{3}{4}\right)$

$= 1/7$

⇒ **Markov Model** {Present value can decide future value {purely based on Probabilities}}

• weather.
State 1: Rainy
State 2: Cloudy
State 3: Sunny

the weather of some city found following weather pattern.

(1)

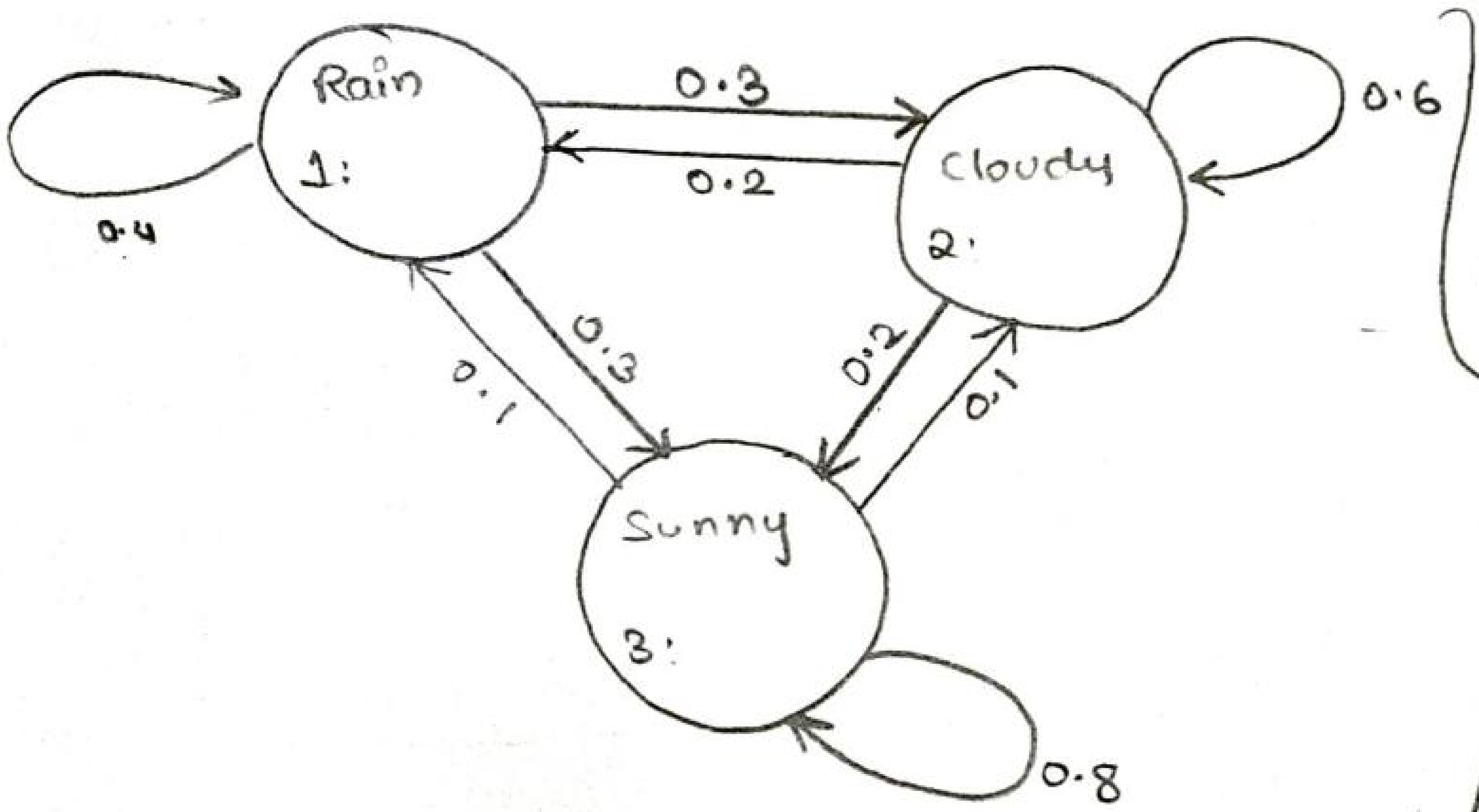
Today / Tommorrow

	Rainy	Cloudy	Sunny
Rainy	$\boxed{0.4}$	0.3	0.3
Cloudy	0.2	$\boxed{0.6}$	0.2
Sunny	0.1	0.1	$\boxed{0.8}$

if today is cloudy, 0.2 is the probability of tommorrow being Rainy.

(HMM) → hidden markov model

• we need to find hidden variables based on observed variables.

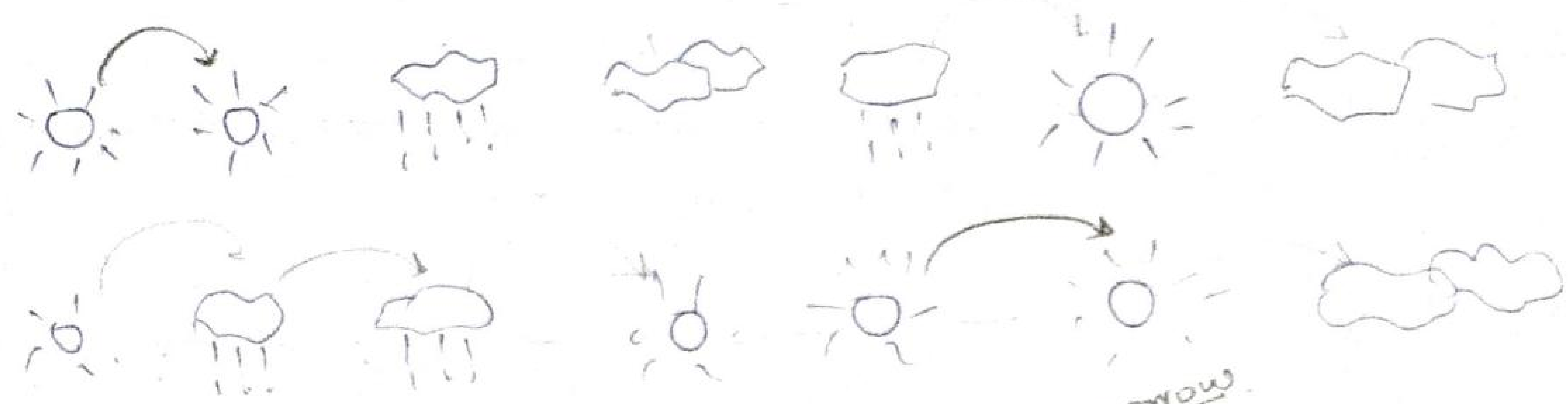


Graphical representation

• Each State corresponds to 1 observation and then sum of outgoing edge weight is 1.

2

2 weeks weather



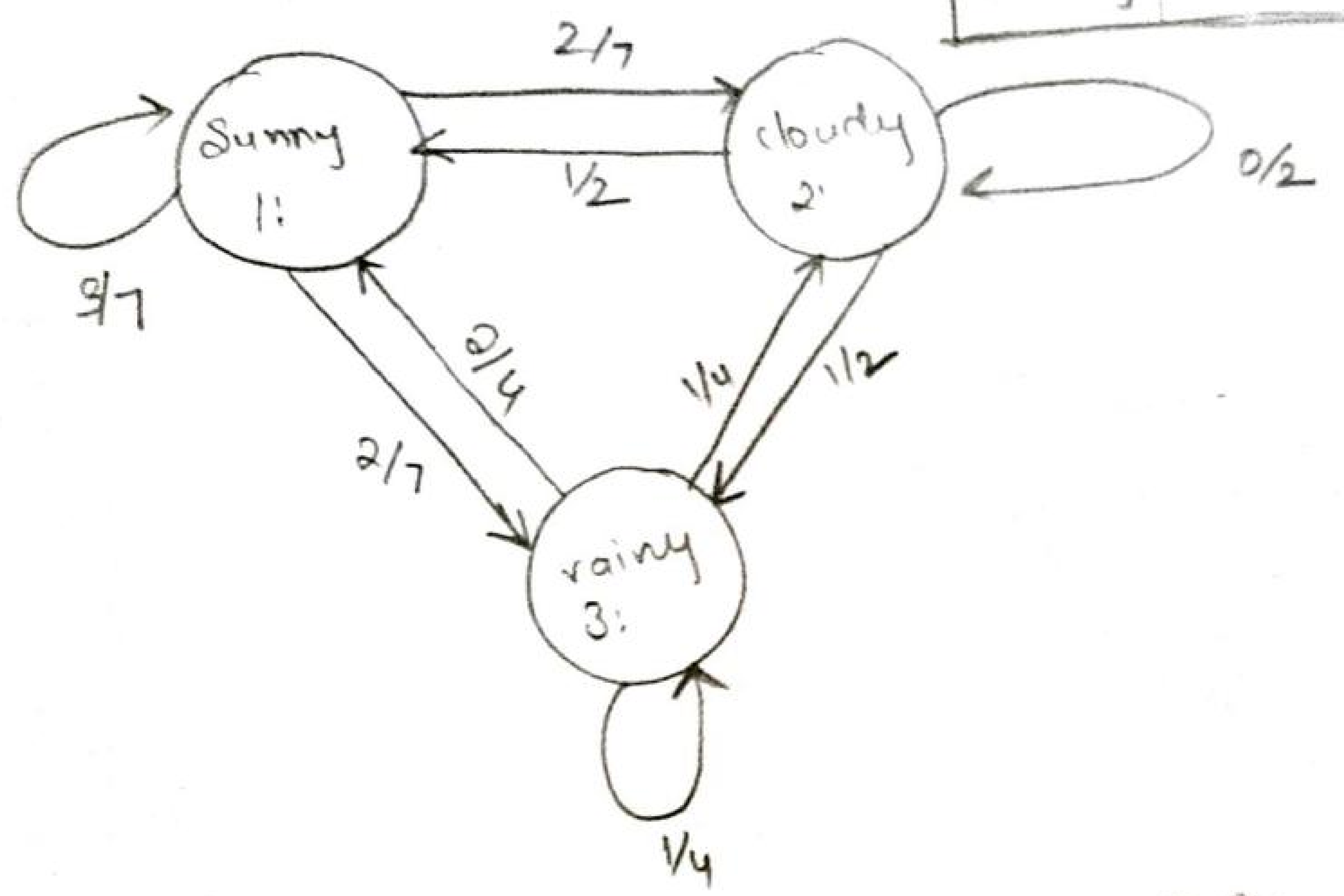
Tomorrow

	rainy	Sunny	cloudy	
rainy	$1/4 = 0.25$	$2/4 = 0.5$	$1/4 = 0.25$	— 4
Sunny	$2/7 =$	$3/7 =$	$2/7 =$	— 7
cloudy	$1/2 = 0.5$	$1/2 = 0.5$	$0/2 = 0$	— 2

Today

(total transitions: 13)

Sunny → Sunny (2/13)



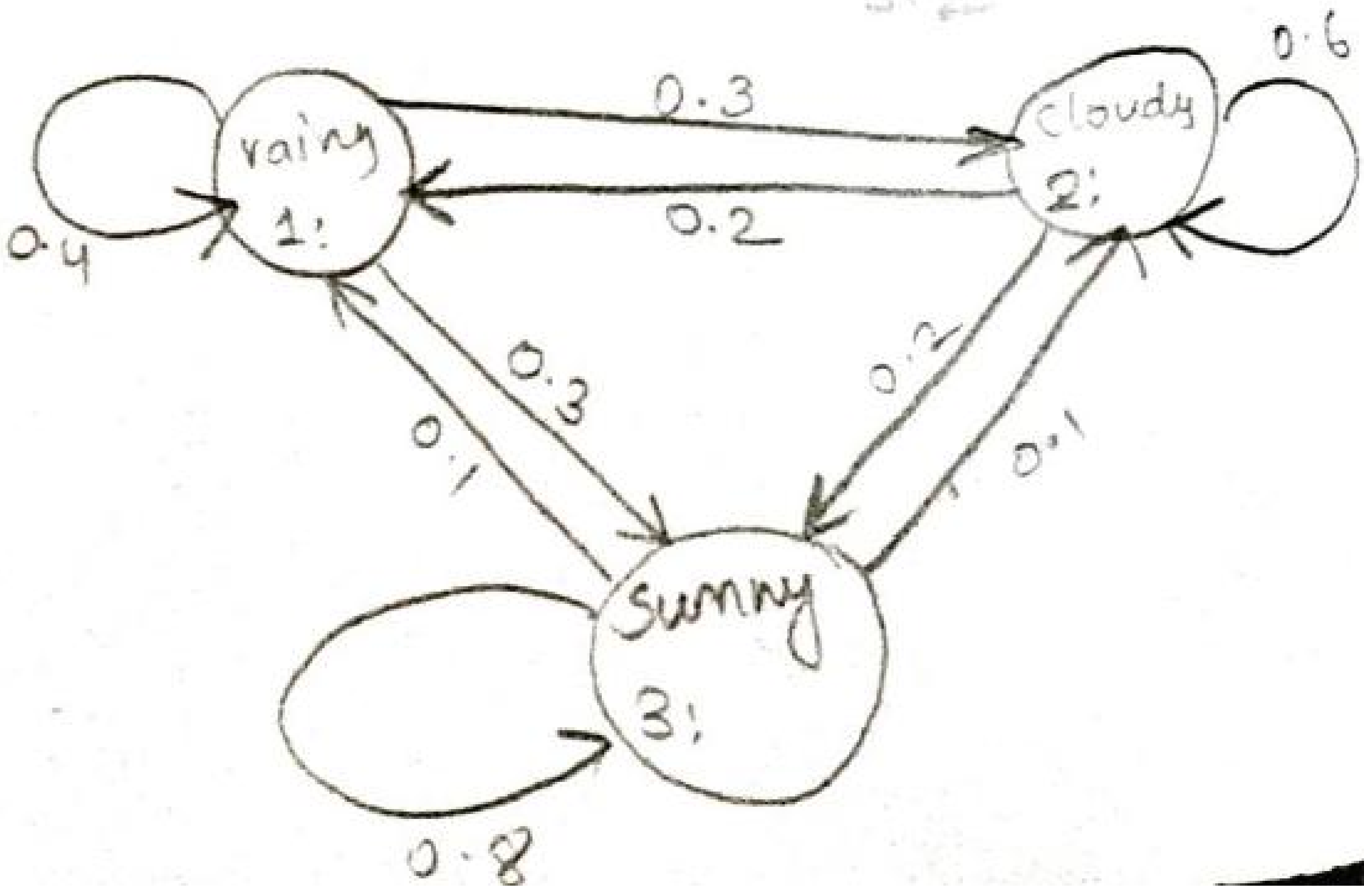
$0.8 \times 0.8 \times 0.1 \times 0.4 \times 0.3 \times 0.1 \times 0.2 = 1.536 \times 10^{-4} = 0.0001536$

3

Q What is the probability that the weather for 20-04-2022 next 7 days will be the Sun → sunny → Sunny → Rainy → Rainy → Sunny → cloudy → Sunny when today is sunny?

0.8 0.1 0.4 0.3 0.1 0.2

	Sunny	rainy	cloudy
Sunny	0.8	0.1	0.1
rainy	0.3	0.4	0.3
Cloudy	0.2	0.2	0.6



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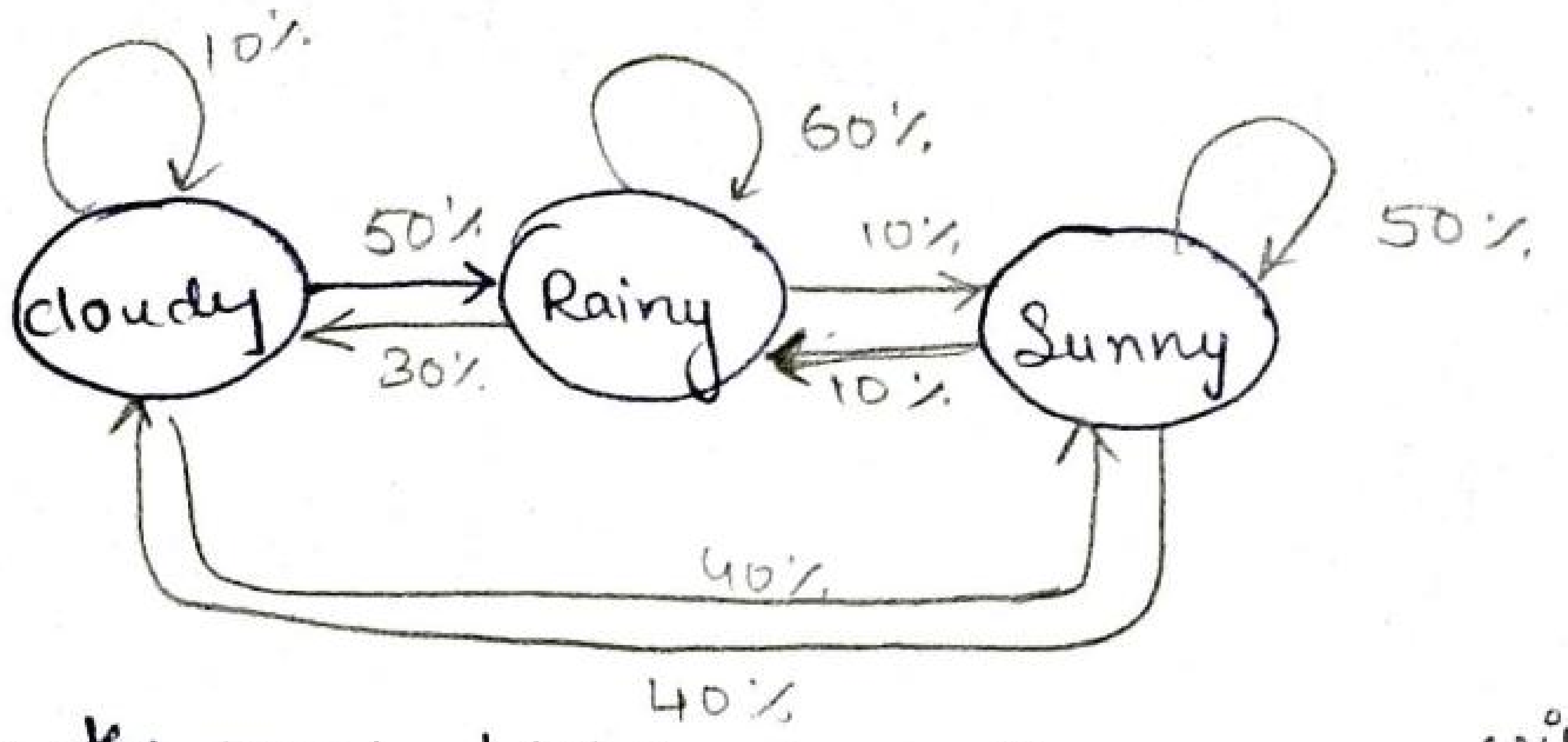

Graphical representation

What is the probability that "wednesday will be cloudy", if today (mon) is Sunny.

M T W

① Sunny - Sunny - cloudy $\Rightarrow 0.5 \times 0.4 = 0.2$ *
   0.5   0.4

② Sunny - Rainy - cloudy $\Rightarrow 0.3 \times 0.1 = 0.03$
   0.1   0.3

③ Sunny - cloudy - cloudy $\Rightarrow 0.4 \times 0.1 = 0.04$
   0.4   0.1

0.2
0.03
0.04
0.27

∴ 27% possibility of getting cloudy on wednesday

Applications of HMM.

user identification

- for identifying human's mindset — if a person visit restaurant during afternoon for biriyani, then hidden detail is he likes Biriyani.
- prediction of diagnosis of disease inside body.
  (eg: identifying cancer based on its symptoms)

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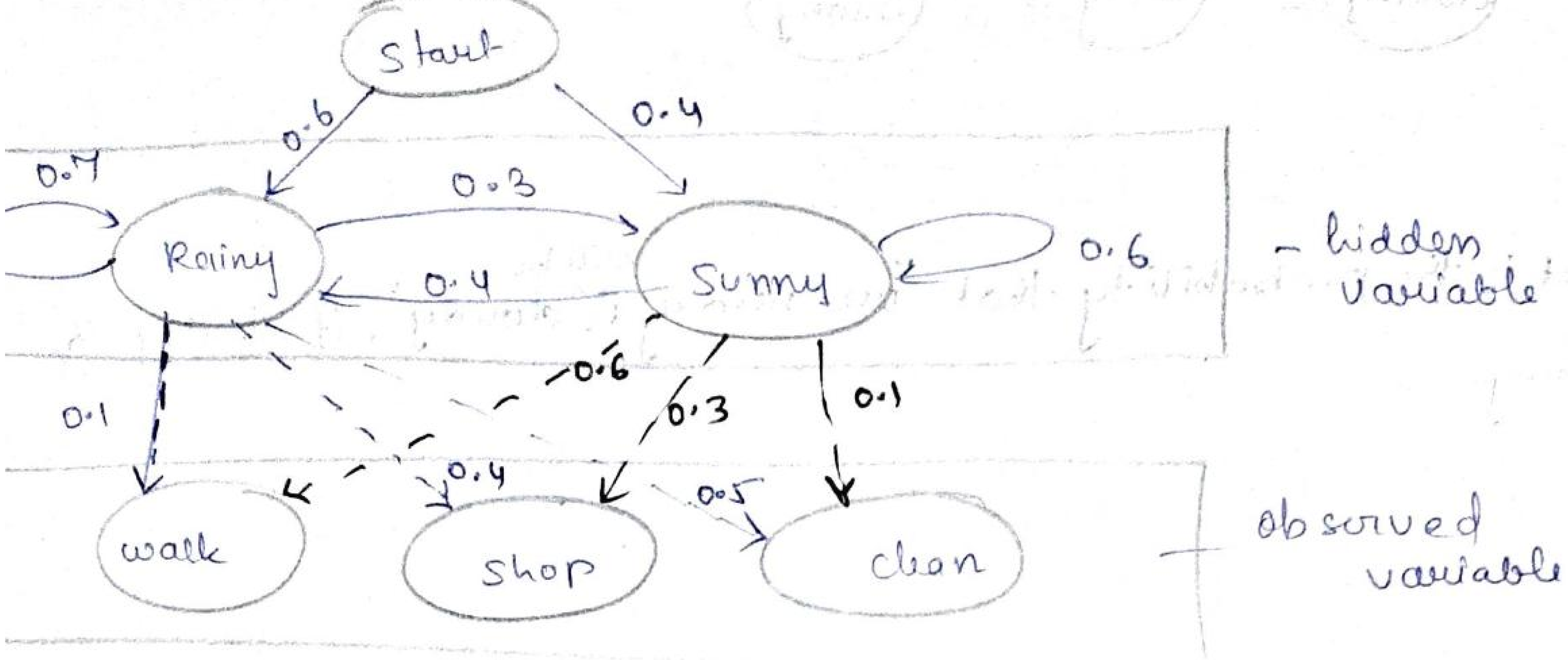


States: Rainy, sunny.

observation: walk, shop, clean

start probability: Rainy (0.6) sunny (0.4)

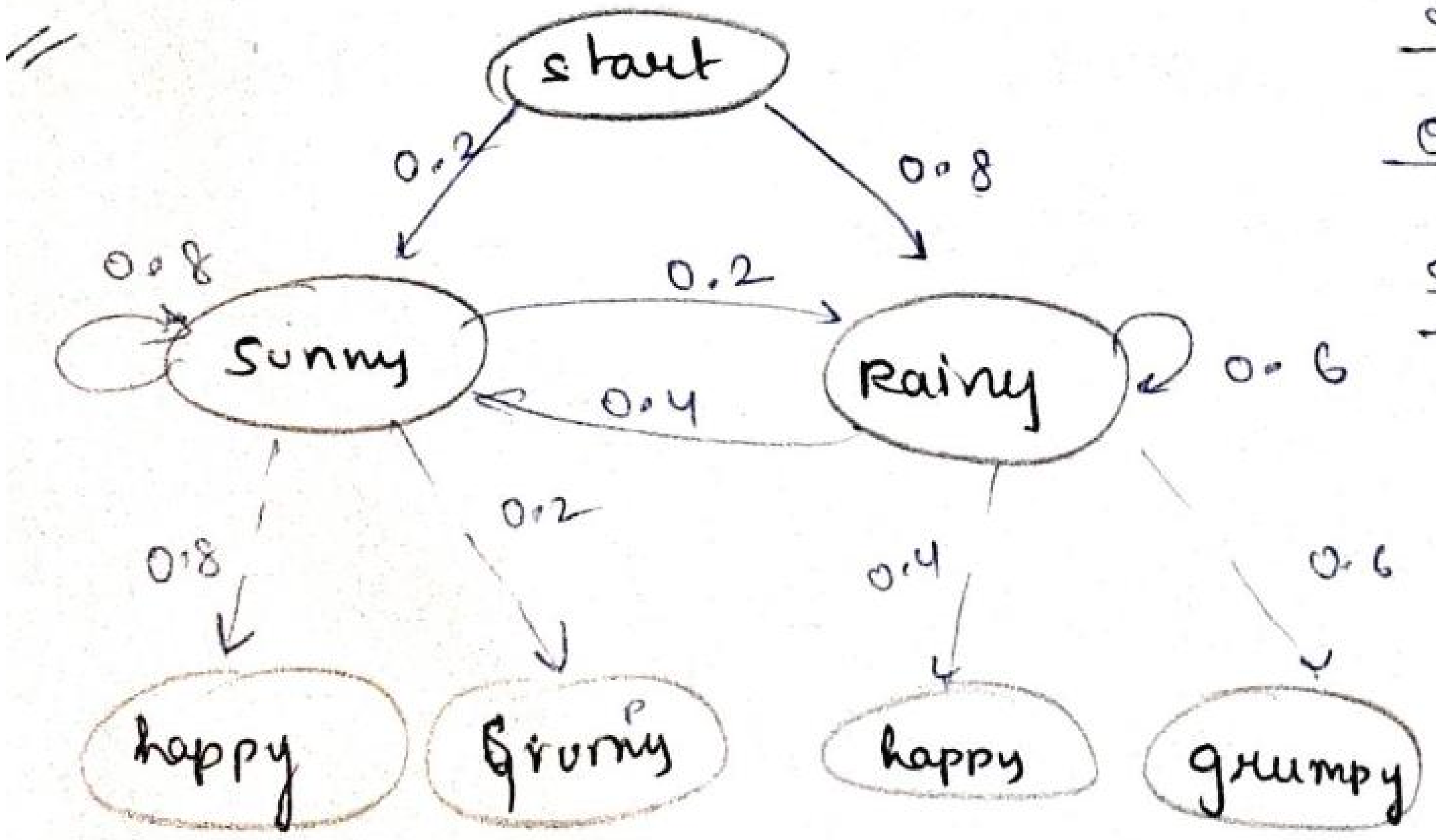
Transition probability:

	Rainy	Sunny
Rainy	0.7	0.3
Sunny	0.4	0.6

Emission probability

	walk	shop	clean
Rain	0.1	0.4	0.5
Sunny	0.6	0.3	0.1

//



States: Sunny rainy

observation: happy, grumpy

start probabilities: Sunny (0.2) rainy (0.8)

Transition probability:

	Sunny	Rainy
Sunny	0.8	0.2
Rainy	0.4	0.6

Emission prob.

	happy	grumpy
Sunny	0.8	0.2
Rainy	0.4	0.6

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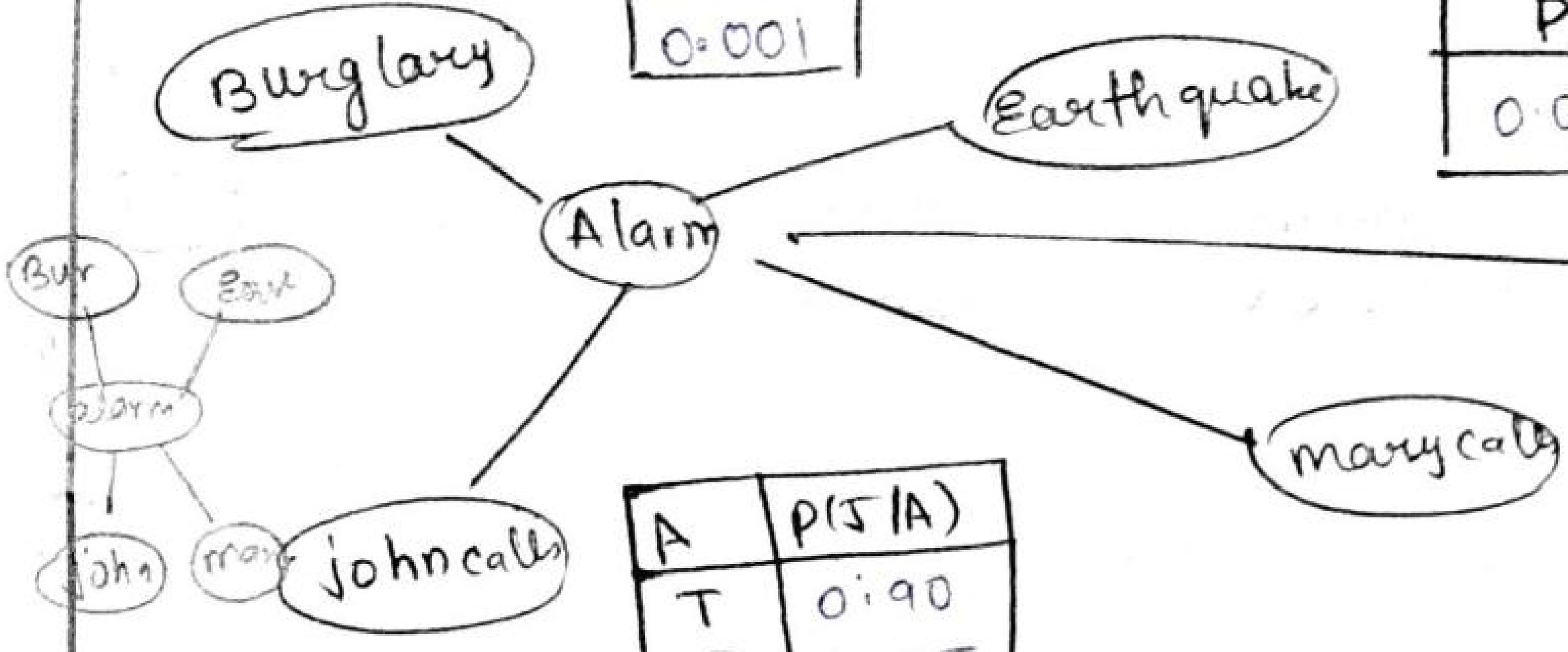
(17)

Bayesian Belief networks



P(B)
0.001

P(E)
0.002

B	E	P(A|B,E)
T	T	0.95
T	F	0.94
F	T	0.29
F	F	0.001

A	P(J|A)
T	0.90
F	0.05

A	P(M|A)
T	0.70
F	0.01

Q what is probability that alarm has sounded but niether a burglary nor an earthquake has occured & both john & merry call?

alarm sound ✓ ~~P(J∧M∧A~~
burglary ✗ → 0.999 × 0.998 × 0.001 × 0.90 × 0.7.
earthquake ✗
johne mary ✓

$= \boxed{0.00062}$

Q what is probability that john call?

john call (alarm ✓) . john call (alarm ✗)

$0.90 [\quad]$ $0.05 [\quad]$

(0.95)(0.001)(0.002) + (0.94)(0.001)(0.998) + (0.29)(0.999)(0.002) + (0.001)(0.998)(0.992)

= 0.0000019 + 0.00093812 + 0.00057942 + 0.00099016

$= \boxed{0.002509}$

0.05 (0.001)(0.002) + (0.06)(0.001)(0.998) + (0.71)(0.999)(0.002) + (0.999)(0.999)(0.998)

$= \boxed{0.9974}$

= 0.0521

Q what is probability that there is burglary given that john & mary calls.

john call (~~alarm~~ burglary ✓

$\boxed{P(b|j,m)} = \alpha\, P(b)\ P(j|a)\ P(m|a)\ P(a|b,e)\ P(e)$

$\Rightarrow \alpha\, P(b)\, P(j|a) \cdot P(m|a) \times \{ \begin{matrix} \text{alarm} \\ b✓ e✓ \end{matrix} + \begin{matrix} \text{alarm} \\ b✓ e× \end{matrix} \}$

$= \alpha P(b) \{ \text{alarm}✓ \{ \begin{matrix} \text{alarm}✓ \\ b✓ e✓ \end{matrix} + \begin{matrix} \text{alarm}✓ \\ b✓ e× \end{matrix} \} + \text{alarm}× \{ \begin{matrix} \text{alarm}× \\ b✓ e✓ \end{matrix} + \begin{matrix} \text{alarm}× \\ b✓ e× \end{matrix} \} \}$

$= \alpha \begin{bmatrix} (0.90)(0.70)\,[(0.95)(0.001)(0.002) + (0.94)(0.001)(0.998)] + \\ (0.05)(0.01)\,[(0.05)(0.001)(0.002) + (0.71)(0.998)(0.001)] \end{bmatrix}$

$= \alpha * (0.00059)$

find $P(\neg b|j,m) \longrightarrow \alpha * 0.0015$

$\boxed{\alpha = \dfrac{1}{P(b|j,m) + P(\neg b,j,m)}}$

$\alpha = \dfrac{1}{0.00059 + 0.0015}$ //

$= \underline{\underline{478.5}}$

$P(b|j,m) = 478.5 * 0.0059$ ($\alpha * P(b|j,m)$)

burgler given john & mary

burgler
john
ma

$P(b|j,m) + P(\neg b|j,m) = 1$

$\boxed{P(b,j,m)}$ joint probability

$\Rightarrow P(b) \times \underbrace{P(j|b) \times P(m|b)}_{\text{conditional probabilies}}$