Predicate Calculus or Predicate logic :

In mathematics and Computer Programs, we Come across the statements involving Variables such as "$x > 10$", "$x = y + 5$" and "$x + y = z$". These statements are neither ture nor false, when the values of the variables are not specified.

The statement "x is greater than 10" has 2 parts.

The first part, the variable x, is the subject of the statement.

The second part "is greater than 10" which refers to a property that the subject can have, is called the predicate.

We can denote the statement "x is greater than 10" by the notation $P(x)$, where P denotes the predicate "is greater than 10" and x is the variable.

$P(x)$ is called the propositional function at x.

Once a value has been assigned to the variable x, the statement $P(x)$ becomes a proposition and has a truth value.

For example,

The truth value of $P(15)$ is T. $(\because P(15) \equiv \underbrace{15 > 10}_{\text{is T}})$

The truth value of $P(5)$ is F. $(\because 5 > 10$ is F$)$

The statements "$x = y + 5$" and "$x + y = z$" will be denoted by $P(x, y)$ and $P(x, y, z)$ respectively.

The logic based on the analysis of predicates in any statement is called predicate logic or predicate calculus.

Simple Statement function:

A simple statement function of one variable is defined to be an expression consisting of a predicate symbol and an individual variable.

Such a Statement function becomes a Statement when the Variable is replaced by the name of any object.

<u>Ex:</u>

If "x is a teacher" is denoted by $T(x)$, it is a Statement function. If x is replaced by John, then "John is a teacher" is a Statement.

<u>Compound Statement function</u>

A Compound Statement function

is obtained by Combining one or more Simple Statement functions by logical Connetives.

Ex:

$$M(x) \wedge H(x)$$

$$M(x) \rightarrow H(x)$$

$$M(x) \vee \neg H(x)$$

An extension of this idea to the Statement functions of two or more Variables is Straight forward.

Quantifiers:

certain declarative sentences involve words that indicate quantity such as "all, some, none or one". Since each of these words indicate quantity they are called quantifiers.

Consider the following statement

1. All isoceles triangles are equiangular
2. Some birds cannot fly
3. Not all vegetarians are healthy persons.
4. There is one and only one even prime integer.

5. Each rectangle is a parallelogram

After some thought, we realize that there are two main quantifiers, all and some, where some is interpreted to mean atleast one.

The quantifier "all" is called the universal quantifier and we shall denote it by $\forall x$ and read it as for all x or for every x or for each x.

The quantifier "Some" is called the existential quantifier. we denote it by $\exists x$ and it is read as There exists at least one x or for some x.

Universe of discourse:

Many mathematical statements assert that a property is true for all values of a variable in a particular domain, called the universe of discourse. Such a statement is expressed using a universal qunatification.

Ex:

Let $p(x)$ be the statement "$x+1 > x$" what is the truth value of the quantification $\forall x\, p(x)$ where the universe of discourse consists of all real numbers)

Solution:

Since $p(x)$ is true for all real numbers x, the quantification $\forall x\, p(x)$ is true.

Ex:

1. Something is Green.

It can be denoted by $(\exists x)(G(x))$

2. Something is not green.

It can be denoted by

$$(\exists x)\ (\neg G(x)).$$

Nagation of a Quantified Expression

If $p(x)$ is the statement "x has studied computer programming", then $\forall x\, p(x)$ means that "every student (in the class) has studied computer programming".

The negation of this statement is "It is not the case that every student in the class has studied computer programming" or equivalently, "There is a student in the class who has not studied computer programming" which is denoted by $\exists x \neg P(x)$.

Thus we see that

$$\neg \forall x\, P(x) \equiv \exists x \neg P(x)$$

Similarly, $\exists x\, p(x)$ means that "there is a student in the class who has studied computer programming".

The negation of this statement is "Every student in this class has not studied computer programming" which is denoted by $\forall x \neg p(x)$. Thus we get

$$\neg \exists x\, p(x) \equiv \forall x \neg P(x).$$

Further we note that

$\neg \forall x p(x)$ is true, when there is an x for which $p(x)$ is false.

" is false when $p(x)$ is true for every x.

Since $\neg \forall x p(x) \equiv \exists x \neg p(x)$

$\equiv \neg p(x_1) \vee \neg p(x_2) \vee \dots \vee \neg p(x_n)$

$\neg \exists x p(x)$ is true, when $p(x)$ is false for every x and false when there is an x for which $p(x)$ is true, since

$$\neg \exists x p(x) \equiv \forall x \neg p(x)$$

$$\equiv \neg p(x_1) \wedge \neg p(x_2) \wedge \dots \wedge \neg p(x_n)$$