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# **LOGARITHMS**



#### INTRODUCTION



- In mathematics the logarithms is the inverse function to the exponentiation.
- The logarithm of a given number x is the exponent to which another fixed number, the base b, must be
  raised, to produce that number x
- since  $1000 = 10 \times 10 \times 10 = 10^3$ , the "logarithm to base 10" of 1000 is 3.
- For example: The logarithm of x to base b is denoted as log<sub>b</sub> x
- More explicitly the relation between logarithm and exponentiation is  $\log_b x = y$  exactly if  $b^y = x$
- The logarithm is denoted "log<sub>b</sub> x" (pronounced as "the logarithm of x to base b" or "the base-b logarithm of x" or (most commonly) "the log, base b, of x").



#### **IMPORTANT TERMS**



| Property | Definition                                 | Example  |
|----------|--|--|
| Product  | $\log_b mn = \log_b m + \log_b n$          | $\log_3 9x = \log_3 9 + \log_3 x$  |
| Quotient | $\log_b \frac{m}{n} = \log_b m - \log_b n$ | $\log_{\frac{1}{4}} \frac{4}{5} = \log_{\frac{1}{4}} 4 - \log_{\frac{1}{4}} 5$ |
| Power    | $\log_b m^p = p \cdot \log_b m$            | $\log_2 8^x = x \cdot \log_2 8$  |
| Equality | If $\log_b m = \log_b n$ , then $m = n$ .  | $log_8(3x-4) = log_8(5x+2)$<br>so, $3x - 4 = 5x+2$                             |



If log 27 = 1.431, then the value of log 9 is \_\_\_\_\_\_.

A. 0.934

B. 0.945

C. 0.954

D. 0.958

Answer: C





$$log(3^3) = 1.431$$

$$3 \log 3 = 1.431$$

$$log 3 = 0.477$$

$$\log 9 = \log(3^2) = 2 \log 3 = (2 \times 0.477) = 0.954.$$





 $\log_6(216) + [\log(42) - \log(6)] / \log(49)$ 

A. 7/2

B. 1/2

C. 4/3

D. 2/3

Answer: A





$$\log_6(216) + [\log(42) - \log(6)] / \log(49)$$

$$= \log_6(6^3) + \log(42/6) / \log(7^2)$$

$$= 3 + \log(7) / 2 \log(7) = 3 + \frac{1}{2}$$

$$= 7/2$$





Find the value of  $\log_2 2 + \log_2 2^2 + \log_2 2^3 + \dots + \log_2 2^n$ .

- A. n(n+1)/2
- B. n+1
- C. n
- D. 2n

Answer: A





$$log_2 2 + log_2 2^2 + log_2 2^3 + \dots + log_2 2^n$$
  
=  $log_2 2 + 2log_2 2 + 3log_2 2 + \dots + nlog_2 2$   
=  $1+2+3+\dots+n$   
=  $n(n+1)/2$ 





If  $\log 2 = 0.301$  and  $\log 3 = 0.4771$ , find the number of digits in  $48^{12}$ ?

A. 19

B. 20

C. 21

D. 24

Answer: C





We have  $\log 48^{12} = 12 \times \log 48 = 12 \times \log (2^4 \times 3)$ 

- $= 12 \times (4 \log 2 + \log 3)$
- $= 12 \times (4 \times 0.301 + 0.4771)$
- $= 12 \times (1.204 + 0.4771)$
- $= 12 \times 1.6811 = 20.1732$

Now the characteristic is 20, so the number of digits = 20 + 1 = 21.





Solve for 'x:' the equation is  $2\log_2 x - \log_2 (x - 2) = 3$ 

- A. 6
- B. 4
- C. 1
- D. 2

Answer: B





We have 
$$2\log_2 x - \log_2 (x - 2) = 3$$

$$\Rightarrow \log_2 x^2 - \log_2 (x - 2) = 3$$

$$\Rightarrow \log_2(x^2/x-2) = 3$$

$$\Rightarrow$$
 (x<sup>2</sup>/x-2) = 2<sup>3</sup> = 8

$$\Rightarrow$$
  $x^2 = 8(x-2)$ 

$$\Rightarrow x^2 - 8x + 16 = 0$$

$$\Rightarrow (x-4)^2 = 0$$

$$\Rightarrow$$
 x=4

Hence answer is option B





 $Log_{10} 3 + log_{10} (4x+1) = log_{10} (x+1) + 1$ 

A. 2/7

B. 7/2

C. 3/7

D. 7/3

Answer: B





$$\log_{10} 3 + \log_{10} (4x+1) = \log_{10} (x+1) + 1$$

$$Log_{10} 3 + log_{10} (4x+1) = log_{10} (x+1) + log_{10} 10$$

$$Log_{10} (3(4x+1)) = log_{10} (10(x+1))$$

$$\rightarrow$$
 3(4x+1)=10(x+1)

→ 
$$12x+3 = 10x+10$$

$$x = 7/2$$





If  $\log_{10} 2 = 0.3010$ , the value of  $\log_{10} 80$  is \_\_\_\_\_.

- A. 1.6020
- B. 1.9030
- C. 9030
- D. None of these

Answer: B





$$\log_{10} 80 = \log_{10} (8 *10)$$

$$= \log_{10} 8 + \log_{10} 10$$

$$= \log_{10} (2^{3}) + 1$$

$$= 3 \log_{10} 2 + 1$$

$$= (3 \times 0.3010) + 1$$

$$= 1.9030.$$





If  $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$ , then x is equal to:

- A. 1
- B. 3
- C. 5
- D. 10

Answer: B





$$\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$$

$$\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + \log_{10} 10$$

$$\log_{10} [5 (5x + 1)] = \log_{10} [10(x + 5)]$$

$$5(5x + 1) = 10(x + 5)$$

$$5x + 1 = 2x + 10$$

$$3x = 9$$

$$x = 3$$





If  $\log x + \log (x + 3) = 1$  then the value(s) of x will be, the solution of the equation

- A. x + x + 3 = 1
- B. x + x + 3 = 10
- C. x(x + 3) = 10
- D. x(x + 3) = 1

Answer: C





taking  $\log x = \log 10x$ using the law of logs Log A+log B=log AB we have Log x + log (x+3) = 1Log x(x+3)=1--(1)definition of logs Log ab=c⇒ac=b  $(1) \rightarrow 101 = x(x+3)$ x2+3x=10x2+3x-10=0factorising and solving (x+5)(x-2)=0 $\Rightarrow$ x=-5, or x=2 When solving with x=2 the equation becomes as option c





Find x if  $\log x = \log 1.5 + \log 12$ 

A. 18

B. 8

C. 15

D. 12

Answer: A







If  $2^{2x+4} - 17 \times 2^{x+1} = -4$ , then which of the following is true?

- A. x is a positive value
- B. x is a negative value
- C. x can be either a positive value or a negative value
- D. None of these







$$2^{x+4} - 17 * 2^{x+1} = -4$$
  
 $2^{x+1} = y$   
 $2^{2x+2} = y^2$   
 $2^2(2^{2x+2}) - 17 * 2^{x+1} = -4$   
 $4y^2 - 17y + 4 = 0$   
 $4y^2 - 16y - y + y = 0$   
 $4y (y - 4) - 1 (y - 4) = 0$   
 $Y = 4$   
 $2^{x+1}$   
 $x + 1 = 2 \text{ or } -2$   
 $x = 1 \text{ or } -3$ 

the answer is "x can be either a positive value or a negative value".





 $\log_3 x + \log_x 3 = 17/4$ . Find X?

- A. 3<sup>4</sup>
- B. 3<sup>1/8</sup>
- C. 3<sup>1/4</sup>
- D. 3<sup>1/3</sup>

Answer: C





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log_3x + log_x3 = 17/4 Let y = log_3x We know that log_x3 = 1/log 3(x) Thus the equation can be written as y + 1/y = 174 4y^2 + 4 = 17y 4y^2 + 4 - 17y = 0 y = 4 or \frac{1}{4} log_3x = 4 Then x = 3^4 If y = 1/4 log_3x = 1/4 x = 3^{1/4}
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 $log_a 4 + log_a 16 + log_a 64 + log_a 256 = 10$ . Then  $a = _____$ .

- A. 2
- B. 4
- C. 8
- D. 5

Answer: B





 $\log a^{(4+16+64+256)} = 10 \log a^{410} = 10 \text{ then } a=4$ 





 $Log_{15} 3375 \times log_4 1024 =$ \_\_\_\_\_\_.

- A. 1
- B. 18
- C. 4
- D. 15

Answer: D





 $\log_{15} 3375 \times \log_{4} 1024 = 3 \log_{15} 15 \times 5 \log_{4} 4 = 3 \times 5 = 15$ 





If  $\log 3 = 0.4771$ , find  $\log (0.81)^2 \times \log (27/10)^{2/3} \pi \log 9$ .

- A. 2.689
- B. -0.0552
- C. 2.2402
- D. 2.702

Answer: D

