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HCF AND LCM

LCM

LCM:

Least Common Multiple (LCM) of two or more numbers is the smallest number that is a multiple of all the numbers.

Example: LCM of 3 and 4 = 12 because 12 is the smallest multiple which is common to 3 and 4 (In other words, 12 is the smallest number which is divisible by both 3 and 4)

How to find out LCM using prime factorization method

- We can find LCM using prime factorization method in the following steps

  Step1 : Express each number as a product of prime factors.
  Step2 : LCM = The product of highest powers of all prime factors

LCM

Find the LCM of 14, 36, 42

Step-1: Express each number as a product of prime factors.

14=2*7

36= 2^2*3^2

42=2*3*7

Common indices are=2,3,7

Hence LCM =2^2*3^27**

LCM

How to find out LCM using Division Method (shortcut)

Step 1: Write the given numbers in a horizontal line separated by commas.

Step 2: Divide the given numbers by the smallest prime number which can exactly divide at least two of the given numbers.

Step 3: Write the quotients and undivided numbers in a line below the first.

Step 4: Repeat the process until we reach a stage where no prime factor is common to any two numbers in the row.

Step 5: LCM = The product of all the divisors and the numbers in the last line.

HCF

Highest Common Factor (H.C.F) or Greatest Common Measure(G.C.M) or Greatest Common Divisor (G.C.D) of two or more numbers is the greatest number which divides each of them exactly.

Example: HCF or GCM or GCD of 60 and 75 = 15 because 15 is the highest number which divides both 60 and 75 exactly.

How to find out HCF using prime factorization method

Step-1: Express each number as a product of prime factors. (Reference: Prime Factorization and how to find out Prime Factorization)

Step-2: HCF is the product of all common prime factors using the least power of each common prime factor.

HCF

Example 1: Find out HCF of 60 and 75 (Prime Factorization and how to find out Prime Factorization)

Step-1: Express each number as a product of prime factors.

60=2^3 *3* 5
75=3 * 5

Step-2: HCF is the product of all common prime factors using the least power of each common prime factor.
Here, common prime factors are 3 and 5

Hcf common factors-3,5

The least power of 3 here = 3

The least power of 5 here = 5 **Hence, HCF = 3 × 5 = 15**

HCF

How to find out HCF using prime factorization method - By dividing the numbers (shortcut)

Step 1: Write the given numbers in a horizontal line separated by commas.

Step 2: Divide the given numbers by the smallest prime number which can exactly divide all of the given numbers.

Step 3: Write the quotients in a line below the first.

Step 4: Repeat the process until we reach a stage where no common prime factor exists for all of the numbers.

Step 5: We can see that the factors mentioned in the left side clearly divides all the numbers exactly and they are common prime factors. Their product is the HCF

How to calculate LCM and HCF for fractions:

The product of LCM and HCF of any two given natural numbers is equivalent to the product of the given numbers.

LCM × HCF = Product of the Numbers

Suppose A and B are two numbers, then.

LCM (A & B) × HCF (A & B) = A × B

HCF of co-prime numbers is 1. Therefore LCM of given co-prime numbers is equal to the product of the numbers.

LCM of Co-prime Numbers = Product Of The Numbers

Least Common Multiple (L.C.M.)for fractions=LCM of Numerators / HCF of Denominators

Highest Common Multiple (L.C.M.)for fractions=HCF of Numerators / LCM of Denominators

HCF and LCM

How to calculate LCM and HCF for fractions:

EXAMPLE 1: Find out LCM of 1/2 3/8 , 3/4

LCM=LCM(1,3,3)/HCF(2,8,4)

=3/2

Example 2: Find out HCF of 3/5, 6/11,9/20

HCF=HCF(3,6,9)/LCM(5,11,20)

=3/220

SHORT TRICKS

The H.C.F of two or more numbers is smaller than or equal to the smallest number of given numbers

- The smallest number which is exactly divisible by a, b and c is L.C.M of a, b, c.
- The L.C.M of two or more numbers is greater than or equal to the greatest number of given numbers.
- The smallest number which when divided by a, b and c leaves a remainder R in each case. Required number = (L.C.M of a, b, c) + R
- The greatest number which divides a, b and c to leave the remainder R is H.C.F of (a – R), (b – R) and (c – R)

HCF AND LCM

- The greatest number which divide x, y, z to leave remainders a, b, c is H.C.F of (x – a), (y – b) and (z – c)
- The smallest number which when divided by x, y and z leaves remainder of a, b, c (x – a), (y – b), (z – c) are multiples of M
  Required number = (L.C.M of x, y and z) – M

Question: 01

If the sum of two numbers is 55 and the H.C.F. and L.C.M. of these numbers are 5 and 120 respectively, then the sum of the reciprocals of the numbers is equal to:

A. 11/120
B. 55/501
C. 11/60
D. 12/55

Answer: A

Explanation:

Correct Op: A

As we know that LCM * HCF = Product of two Numbers Thus a + b = 55 and ab = 5 x 120 = 600 Thus 1/a + 1/b = (b + a)/(ab) = 55/600 = 11/120

Question: 02

Find the greatest number which on dividing 1661 and 2045, leaves a remainder of 10 and 13 respectively?

A. 91

B. 127

C. 137

D. 140

Answer: B

Explanation:

In this type of question, its obvious we need to calculate the HCF, trick is HCF of (1661 – 10) and (2045 -13) = HCF (1651, 2032) = 127

Question: 03

Six bells commence tolling together and toll at the intervals of 2,4,6,8,10,12 seconds respectively. In 60 minutes how many times they will toll together?

A. 16

B. 15

C. 31

D. 30

Answer:C

Explanation:

Correct Op: C

LCM of 2-4-6-8-10-12 is 120 seconds, that is 2 minutes. Now 60/2 = 30 Adding one bell at the starting it will 30+1 = 31

Question: 04

There are three numbers, these are co-prime to each other are such that the product of the first two is 551 and that of the last two is 1073. What will be the sum of three numbers :

A. 67

B. 85

C.129

D. 87

Answer: B

Explanation:

Correct Op: C

As given the questions these numbers are co primes, so there is only 1 as their common factor. It is also given that two products have the middle number in common. So, middle number = H.C.F. of 551 and 1073 = 29; So first number is : 551/29 = 19 Third number = 1073/29 = 37 So sum of these numbers is = (19 + 29 + 37) = 85

Question: 05

The traffic lights at three different road crossings change after every 40 sec, 72 sec and 108 sec respectively. If they all change simultaneously at 5 : 20 : 00 hours, then find the time at which they will change simultaneously.

A. 5 : 28 : 00 hrs
B. 5 : 30 : 00 hrs
C. 5 : 38 : 00 hrs
D. 5 : 40 : 00 hrs

Answer: B

Explanation:

Correct Option: (b)

Traffic lights at three different road crossings change after every 40 sec, 72 sec and 108 sec respectively.

Therefore, find the L.C.M. of 40, 72 and 108.

L.C. M. of 40, 72 and 108 = 1080

The traffic lights will change again after 1080 seconds = 18 min

The next simultaneous change takes place at 5 : 38 : 00 hrs.

Question: 06

A rectangular courtyard 4.55 meters long and 5.25 meters wide is paved exactly with square tiles of same size. Find the largest size of the tile used for this purpose?

A. 25 cm

B. 45 cm

C. 21 cm

D. 35 cm

Answer:D

Explanation:

Correct Option: (d)

Here, we are asked to find the largest size of tile. Therefore, calculate H.C.F.

Step 1: Covert numbers without decimal places i.e 455 cm and 525 cm

Step 2: Find the H.C.F. of 455 and 525

H.C.F. of 455 and 525 = 35 cm

Hence, the largest size of the tile is 35 cm.

Question: 07

John, Smith and Kate start at same time, same point and in same direction to run around a circular ground. John completes a round in 250 seconds, Smith in 300 seconds and Kate in 150 seconds. Find after what time will they meet again at the starting point?

A. 30 min
B. 25 min
C. 20 min
D. 15 min

Answer: B

Explanation:

Correct Option: (b)

L.C.M. of 250, 300 and 150 = 1500 sec

Dividing 1500 by 60 we get 25, which mean 25 minutes.

John, Smith and Kate meet after 25 minutes.

Question: 08

Find the least number, which when divided by 12, 15, 20 and 54 leaves a remainder of 8 in each case.

A. 548

B. 540

C. 532

D. 524

Answer:A

Explanation:

Correct Option: (a)

We are given that, the least number, when divided by 12, 15, 20 and 54 leaves a remainder of 8 in each case.

Therefore, add remainder 8 to the L.C.M. of divisors.

The required least number = (L.C.M. of 12, 15, 20 and 54) + remainder (8)

L.C.M. of 12, 15, 20 and 54

L.C.M. = 2 × 3 × 2 × 5 × 9 × 1 = 540

The required least number = (540) + (8) = 548

Question: 09

Find the largest number of 4-digits divisible by 12, 15 and 18

A. 9900

B. 9750

C. 9450

D. 9000.

Answer: A

Explanation:

Correct Option: (a)

Largest 4-digit number is 9999.

Required largest number must be divisible by the L.C.M. of 12, 15 and 18

L.C.M. of 12, 15 and 18

$12 = 2 \times 2 \times 3$

$15 = 5 \times 3$

$18 = 2 \times 3 \times 3$

L.C.M. = 180

Now divide 9999 by 180, we get remainder as 99

The required largest number = (9999 – 99) =9900

Number 9900 is exactly divisible by 180.

Question: 10

If the product and H.C.F. of two numbers are 4107 and 37 respectively, then find the greater number.

A. 111
B. 222
C. 332
D. 452

Answer: A

Explanation:

Product of two numbers = Product of their H.C.F. and L.C.M

Product of two numbers = 4107

4107 = 37 × L.C.M

LCM=4107/ 37=111

The greatest number= 111

Question: 11

Find the greatest possible length which can be used to measure exactly the lengths 4 m 95 cm, 9 m and 16 m 65 cm.

A. 45 cm

B. 35 cm

C. 42 cm

D. 20 cm

Answer: A

Explanation:

Required length = H.C.F. of 495 cm, 900 cm and 1665 cm.

495 = 32 x 5 x 11, 900 = 22x 32 x 52, 1665 = 32 x 5 x 37.

H.C.F. = 32 x 5 = 45.

Hence, required length = 45 cm.

Question: 12

Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case.

A. 35
B. 40
C. 20
D. 30

Answer: A

Explanation:

Required number = H.C.F. of (132 – 62), (237 – 132) and (237 – 62)

= H.C.F. of 70, 105 and 175 = 35.

Question: 13

A heap of coconuts is divided into groups of 2, 3 and 5 and each time one coconut is left over.

The least number of Coconuts in the heap is?

A. 31

B. 41

C. 51

D. 61

Answer: A

Explanation:

Explanation:

LCM = 30 => 30 + 1 = 31

Question: 14

A man was employed on the promise that he will be paid the highest wages per day. The contract money to be paid was Rs. 1189. Finally he was paid only Rs. 1073. For how many days did he actually work?

A. 39

B. 40

C. 37

D. 35

Answer: C

Explanation:

Explanation:

HCF of 1189, 1073 = 29

1073/29 = 37

Question: 15

A merchant has three different types of milk: 435 liters, 493 liters and 551 liters. Find the least number of casks of equal size required to store all the milk without mixing.

A. 51

B. 61

C. 47

D. 45

Answer:A

Explanation:

Explanation:

HCF of 435, 493, 551 = 29

(453/29) + (493/29) + (551/29) = 51