NUMBER SYSTEM

- Introduction
- Basic Rules

INTRODUCTION

A number is a mathematical object used to count, measure, and label. The original examples are the natural numbers 1, 2, 3, and so forth. A notational symbol that represents a number is called a numeral.

Numbers can be classified into sets, called number systems, such as the natural numbers and the real numbers.

Types of Numbers:

1. Natural numbers

The most familiar numbers are the natural numbers or counting numbers: 1, 2, 3, and so on. Traditionally, the sequence of natural numbers started with 1

2. Integer

The negative of a positive integer is defined as a number that produces 0 when it is added to the corresponding positive integer.
The natural numbers form a subset of the integers. As there is no common standard for the inclusion or not of zero in the natural numbers, the natural numbers without zero are commonly referred to as positive integers, and the natural numbers with zero are referred to as non-negative integers.

3. Rational numbers

A rational number is a number that can be expressed as a fraction with an integer numerator and a positive integer denominator. Two different fractions may correspond to the same rational number; for example 1/2 and 2/4 are equal, that is: 1/2=2/4

4. Real numbers

The real numbers include all the measuring numbers. Real numbers are usually represented by using decimal numerals, in which a decimal point is placed to the right of the digit with place value 1.

5. Complex numbers
The real numbers can be extended to the complex numbers. The complex numbers consist of all numbers of the form a+ bi, where a and b are real numbers and i is the square root of −1.

BASIC CONCEPTS OF NUMBER SYSTEM

I. Divisibility Rule:

Divisibility rules of whole numbers are very useful because they help us to quickly determine if a number can be divided by 2, 3, 4, 5, 9, and 10 without doing long division.

When the numbers are large, use the following divisibility rules:

Rule #1: Divisibility by 2

A number is divisible by 2 if its last digit is 0, 2, 4, 6, or 8.

Rule # 2: Divisibility by 3:

A number is divisible by 3 if the sum of its digits is divisible by 3

Rule # 3: Divisibility by 4

A number is divisible by 4 if the number represented by its last two digits is divisible by 4.

Rule #4: Divisibility by 5

A number is divisible by 5 if its last digit is 0 ot 5.

Rule # 5: Divisibility by 6

A number is divisible by 6 if it is divisible by 2 and 3. Be careful! it is not one or the other. The number must be divisible by both 2 and 3 before you can conclude that it is divisible by 6.

Rule # 6: Divisibility by 7

To check divisibility rules for 7, study carefully the following two examples:

Is 348 divisible by 7?

Remove the last digit, which is 8. The number becomes 34. Then, Double 8 to get 16 and subtract 16 from 34.

34 − 16 = 18 and 18 is not divisible by 7. Therefore, 348 is not divisible by 7

Is 37961 divisible by 7?

1. Remove the last digit, which is 1. The number becomes 3796. Then, Double 1 to get 2 and subtract 2 from 3796.
   3796 − 2 = 3794, so still too big? Thus repeat the process.

2. Remove the last digit, which is 4. The number becomes 379. Then, Double 4 to get 8 and subtract 8 from 379.
   379 − 8 = 371, so still too big? Thus repeat the process.

3. Remove the last digit, which is 1. The number becomes 37. Then, Double 1 to get 2 and subtract 2 from 37.
   37 − 2 = 35 and 35 is divisible by 7. Therefore, 37961 is divisible by 7.

Rule #7: Divisibility by 8

A number is divisible by 8 if the number represented by its last three digits is divisible by 8.

Rule #8: Divisibility by 9

A number is divisible by 9 if the sum of its digits is divisible by 9.

Rule # 9: Divisibility by 10

A number is divisible by 10 if its last digits is 0

Example Problem-1:

Problem: Which of the following has most number of divisors?
A. 99 B. 101 C. 176 **D. 182**

Solution:
Divisors of 99=1, 3, 9,11,33,99
Divisors of 101=1,101
Divisors of 176=1, 2, 4, 8, 11, 22, 44, 88, 176
Divisors of 182=1, 2, 7, 13, 14, 26, 91, 182

II. Unit digit of numbers:

The concept of identifying the unit digit, we have to first familiarize with the concept of cyclicity.

Cyclicity of any number is about the last digit and how they appear in a certain defined manner.

Let’s take an example to clear this thing:

The cyclicity chart of 2 is:

$$2^1 = 2$$
$$2^2 = 4$$
$$2^3 = 8$$
$$2^4 = 16$$
$$2^5 = 32$$

Have a close look at the above. You would see that as 2 is multiplied every-time with its own self, the last digit changes. On the 4th multiplication, 25 has the same unit digit as 21. This shows us the cyclicity of 2 is 4, that is after every fourth multiplication, the unit digit will be two.

Cyclicity table:

The cyclicity table for numbers is given as below:

Number	Power Cycle	Cyclicity
0	0	1
1	1	1

2	2,4,8,6	4
3	3,9,7,1	4
4	4,6	2
5	5	1
6	6	1
7	7,9,3,1	4
8	8,4,2,6	4
9	2,1	2

Example Problem-2:

Problem: The digit in the unit place of the number 7^{95} X 3^{58} is
A. 7 B. 2 C. 6 D.4

Solution:

The Cyclicity table for 7 is as follows:
7^1 =7
7^2 =49
7^3 = 343
7^4 = 2401
7^5 = 16807

Let's divide 95 by 4: the remainder is 3. Thus, the last digit of 7^{95} is equals to the last digit of 7^3 i.e. 3.

The Cyclicity table for 3 is as follows:
3^1 =3
3^2 =9
3^3 = 27
3^4 = 81
3^5 = 243

Let's divide 58 by 4, the remainder is 2. Hence the last digit will be 9.

Therefore, unit's digit of (7^{95} X 3^{58}) is unit's digit of product of digit at unit's place of 7^{95} and 3^{58} = 3 * 9 = 27. Hence option 1 is the answer.

III. Remainders:

When a number is divided by any another number (divisor) we get quotient and remainder if the number is not divisible.
Let's take a simple example
On dividing 9 with 4 we get 2 as the quotient and 1 as the remainder.

Example Problem-3:

Problem: What will be the remainder when 63 + 67 +81 is divided by 11?

Solution:

Instead to add up all, let's do it separately.

63/11 = 5 will become as the quotient and 8 will be the remainder.

When 67/11 = 6 will become as the quotient and 1 will be the remainder.

81/11 = 7 will become as the quotient and 4 will be the remainder.

Now the remainders are 8, 1, and 4 respectively By adding all the remainders we come to know that the um of remainders is greater than the divisor so therefore again the sum of remainder is divided by the divisor i.e. by 11

On dividing 13 by 11, 1 will be the quotient and 2 will be the remainder

Hence the remainder on dividing 63 +67 +81 by 11 will be **2**

Example Problem-4:

Problem: What is the minimum two digit number which when divided by 3,4,5 leaves 1,2,3 as the remainder respectively ?

Solution:

From the question we can observe that a common number is decreased each time.

That means 2 is subtracted from 3, 4 and 5 to get the remainders 1, 2, 3

That means if a number which is completely divisible by 3and if we subtract 2 from the number then 1 will be the remainder.

This case is also applicable for rest of the two cases.

So think of a common number of three of them and remove 2 from it so that we can get the remainders 1, 2, 3 respectively.

That number would be the minimum common difference of three of them i.e. 60

If we subtract 2 from 60, we are left with 58 and when this 58 is divided by 3, 4, 5 then 1, 2, 3 will be the remainder respectively.

POWER RANGERS-1

1. The average temperature for the first 9 days was 20ºC and that of the first 5 was 25º C and that for the last 5 days was 15ºC. What was the temperature on the fifth day?

Solution:

The usual method will take more than 45 seconds to solve the problem. So we can use speed math.

Speed Math: (5 seconds)
Since the average temperatures for the first 5 days and the last 5 days is ‘equidistant’ from the average
temperature for the 9 days, we can simply take the arithmetic mean of 25 and 15 as

$\frac{25+15}{2} = \frac{40}{2}$ = 20ºC

2. Find the remainder when 3^{75} is divided by 5.

Solution: Speed Math

Step 1:
Express power in the form, 4k + x where x = 1,2,3,4. In this case 75 = 4k + 3

Step 2:
Take the power cycle of 3 which is 3,9,7,1. Since the form is 4k + 3, take the third digit of the cycle, which is 7.

Any number divided by 5, the remainder will be that of the unit digit divided by 5. Hence the remainder is 2.
Sometime, you may get a question in the term of variables, where you need to substitute values to get the answer in the fastest possible way.

3. What is the first non-zero integer from the right in $8330^{1957} + 8370^{1982}$?

A) 3 B) 1 C) 9 D) none of these

Solution: Speed Math

8370^{1982}will end with more number of zeroes.
So we need to consider only the first part.
Rightmost non-zero integer of the expression will be = unit digit of 833^{1957} = unit digit of 3^{1957}.
Since 1957 = 4k +1, take the first digit in the power cycle of 3, which is 3.

PRACTICE PROBLEMS (EXPLANATORY ANSWERS AT THE END)

DIVISIBILITY TEST

1. Which one of the following numbers is divisible by 8 and 11?
A. 12496 B. 414206
C. 999000 D. 38400

2. Which one of the following numbers is divisible by 7 and 9?
A. 803619 B. 203861
C. 552951 D. 339927

3. How many of the following numbers divide 5!?
[A] 3 [B] 5 [C] 10 [D] 15 [E] 16
A. 2 B. 3 C. 4 D. 5

4. What is the number of digits in the smallest number consisting of only 1's and 0's and divisibly by 45?
A. 9 B. 10 C. 12 D. 45

5. How many three digit numbers less than 600 are divisible by the first 3 prime numbers?
A. 30 B. 20 C. 17 D. 16

6. How many numbers are there from 100 to 200, which are divisible by 2 or 3?
A. 67 B. 84 C. 68 D. 73

7. A number 2P4Q is divisible by 8 find the minimum value of P+Q?
A. 0 B. 8 C. 16 D. 18

8. What must be subtracted from 3687 so that it becomes divisible by both 5 & 7?
A. 2 B. 0 C. 10 D. 12

9. 3710 is divisible by which of the following?
A. 5 B. 7
C. 6 D. Both (A) & (B)

10. The largest natural number by which the product of three consecutive even natural numbers is always divisible by:
A. 16 B. 24 C. 48 D. 96

UNIT/LAST DIGIT OF THE EXPRESSION X^Y

1. The digit in the unit's place in the expansion of $(123 \times 456 \times 789)$ is?
A. 2 B. 3 C. 6 D. 9

2. What is the last digit of the expression 777^{777}?
A. 3 B. 1 C. 7 D. 9

3. The digit in the unit's place, if 12^{12} is multiplied by 13^{13} is?
A. 6 B. 8 C. 2 D. 4

4. The unit's digit of the expression $11^1 + 12^2 + 13^3 + \cdots + 16^6$ is?
A. 9 B. 7 C. 1 D. 0

5. The unit's digit of the product $3^{1001} \times 7^{22002} \times 13^{333003}$ is?
A. 1 B. 3 C. 5 D. 9

6. The unit's digit of the product $34^{234!} \times 3456^{123456!}$ is?
A. 4 B. 8 C. 1 D. 6

7. What is the number of trailing zeroes after the last significant digit of N, if N = 5 × 10 × 15 ×………..× 50?
A. 5 B. 8 C. 7 D. 9

8. If N = 1! + 2! + 3! + ... + 2010! then what is the digit in the unit's place of N?
A. 3 B. 2 C. 1 D. 0

9. The last digit of 8^{233} is?
A. 8 B. 4 C. 2 D. 6

10. The last digit of 4^{456} is?
A. 2 B. 4 C. 6 D. 8

REMAINDER

1. What is the remainder when $112 \times 115 \times 117$ is divided by 11?
A. 1 B. 2 C. 4 D. 10

2. What is the remainder when 2^{25} is divided by 3?
A. 1 B. 2 C. 0 D. 3

3. What is the remainder when $17^{25} + 21^{25}$ is divided by 19?
A. 0 B. 2 C. 17 D. 18

4. A number when divided by 255 leaves a remainder 70. What will be the remainder, when the same number is divided by 17?
A. 2 B. 3 C. 4 D. None

5. A number N when divided by 5 leaves remainder 1and when divided by 6 leaves remainder 3. What is the remainder when N is divided by 30?
A. 16 B. 15 C. 51 D. 21

6. What is the remainder when 30^{80} is divided by 17?
A. 13 B. 16 C. 1 D. None

7. What is the remainder when $(1^1 + 2^2 + 3^3 + \cdots + 100^{100})$ is divided by 4?
A. 3 B. 2 C. 1 D. 0

8. Find the largest 4 digit number which when divided by 10, 15 and 25 will leave a remainder 2?
A. 2504 B. 5008
C. 9902 D. 9802

9. What is the remainder, if $111 \times 112 \times 113 \times - - - \times 115$ is divided by 480?
A. 2 B. 5 C. 1 D. 0

10. A number when divided by 247 leaves 41 a remainder. What would be the remainder when the same number is divided by 19?
A. 3 B. 0 C. 4 D. 5

SOLUTION WITH EXPLNATORY ANSWER

DIVISIBILITY TEST

1. For a number to be divisible by 8, the new number formed by its last 3 digits has to be divisible by 8.
So 1st we check divisibility by 8 :
12496 → 496 is divisible by 8, hence 12496 is also divisible by 8.
414206 → 206 is not divisible by 8, hence 414206 will also be not divisible by 8.
999000 → 000 is divisible by 8, hence 999000 will also be divisible by 8.
38400 → 400 is divisible by 8, hence 38400 will also be divisible by 8.
So option2 can be eliminated.

For divisibility by 11, the difference between the sum of even digits and the sum of odd digits of the number, should be divisible by 11.
12496→ (6+4+1) – (9+2) = 0 (which is divisible by 11, so the number will be divisible by 11.)
Hence the correct answer is **option 1.**

2. For any number to be divisible by 9, sum of all of its digits should be 9 or a multiple of 9.
803619 → (8+0+3+6+1+9) = 27 (divisible by 9)
So 803619 will be divisible by 9.
Similarly we can see that 552951 is also divisible by 9.
Now for checking divisibility by 7, we can check it for 803619, and find that it is not divisible by 7, so only 552951 can be the correct answer.
Hence the correct answer is **option 3.**

3. 5! Is simply 120 which is equal to 3x5x2x2x2.
So out of the given numbers, 3, 5, 10 and 15 will divide 5! i.e. 120.
Hence the correct answer is **option 3.**

4. For any number to be divisible by 45, it should be individually divisible by factors of 45 i.e. 9 and 5.
Now as we are given that the number can consist of only '1' and '0' digits, so for it to be divisible by 5, its unit digit can only be '0'.

Now, for the number to be divisible by 9, sum of its all digits should be a multiple of 9 (9, 18, 27..etc).
So for the smallest such number, sum of all of its digits should be 9.
Now since the number consists of only '0' and '1', we can see that smallest such number can only be 1111111110 (i.e. 9 times 1 followed by a 0)
So it will consist of 10 digits.
Hence the correct answer is **option 2.**

5. 1st 3 prime numbers are 2, 3 and 5. So for any number to be divisible by all 3 of them, it should also be divisible by the LCM of all 3 of them, i.e. 30.
So the numbers will be { 30, 60, 90, 120….. }
Since it is given that the numbers need to be 3 digits numbers and also less than 600, so such numbers will be { 120, 150, 180, ….., 570 }
This is an AP with a=120 and d=30. Now we have to find the number of terms in it.
So we can apply the formula:
$l = a + (n-1)d$
$\rightarrow 570 = 120 + (n-1)30$
$\rightarrow 450 = (n-1)30$
$\rightarrow n = 16.$
Hence the correct answer is **option 4.**

6. Numbers divisible by either 2 or 3 = (numbers divisible by 2 + numbers divisible by 3) - numbers divisible by both 2 and 3 (i.e. by 6)
Divisible by 2 { 100, 102, 104…. 200 } :-
$l = a + (n-1)d$
$\rightarrow 200 = 100 + (n-1)2$
$\rightarrow n = 51.$
Divisible by 3 { 102, 105…. 198 } :-
$198 = 102 + (m-1)3$
$\rightarrow m = 33.$
Divisible by 6 { 102, 108,…. 198 } :-
$198 = 102 + (p-1)6$
$\rightarrow p = 17.$
So answer = n + m – p = 51 + 33 – 17 = 67.
Hence the correct answer is **option 1.**

7. Here the number is 2P4Q is divisible by 8.
So consider the last three digit and it should be divisible by 8.
Considering the minimum value of P,Q
Such that (P+Q) is minimum . ie 0.

So, we can take P = 0 and Q = 0.
So, number formed is 2040 and is divisible by 8.
So, P + Q = 0 + 0 = 0.
Hence, the correct answer is **option 1.**

8. Here we need to take LCM of 5 & 7 both and it is 35.
So, dividing 3687 by 35 leaves remainder 12.
So, now subtracting 12 from 3687 gives 3675 which is divisible by both 5 and 7.
Hence, the correct answer is **option 4.**

9. Applying divisibility test of 5 and 7 we can say that 3710 is divisible by both 5 and 7.
Hence the correct answer is **option 4.**

10. Required number = (2×4×6)
= 48.
Hence, the correct answer is **option 3.**

UNIT/LAST DIGIT OF THE EXPRESSION X^Y

1. Last digit of (123 x 456 x 789) =
last digit of (3 x 6 x 9)
= last digit (162)
= 2
Hence the correct answer is **option 1.**

2. Last digit of 777^{777} = last digit of 7^{777}
= last digit of 7.7^{776} = 7
Hence the correct answer is **option 3.**

3. Unit digit of $12^{12}.13^{13}$ = unit digit of $2^{12}.3^{13}$
= unit digit of $2^4.3$ = unit digit of 48 = 8
Hence the correct answer is **option 2.**

4. Unit digit of $11^1 + 12^2 + 13^3 + \cdots + 16^6$
= unit digit of $1^1 + 2^2 + 3^3 + \cdots + 6^6$
= unit digit of 1 + 4 + 7 + 6 + 5 + 6
= unit digit of 29
= 9.
Hence the correct answer is **option 1.**

5. Unit digit of $3^{1001}\text{x}\,7^{22002}\text{x}\,13^{333003}$ =
Unit digit of $3^1\text{x}\,7^2\text{x}\,13^3$ =
Unit digit of $3^1\text{x}\,7^2\text{x}\,3^3$ =
Unit digit of 3 x 9 x 7 = 9
Hence the correct answer is **option 4.**

6. Unit digit of $34^{234!} \text{ x } 3456^{123456!}$ =
Unit digit of $4^4 \text{ x } 6^4$ =
Unit digit of $6 \text{ x } 6 = 6$.
Hence the correct answer is **option 4.**

7. Given $N = 5 \times 10 \times 15 \times \dots\dots\dots\dots \times 50 = 5^{12} \times 2^8 \times 3^3 \times 7$

We know that, the number of zeros of $5^m \times 2^n$ is $m, if\ m < n$ and $n, if\ m > n$

Now, number of zeros of N is '8'.
Hence the correct answer is **option (2)**

8. For $N \geq 5$ the last digit of $N!$ is $'0'$.

Now, last digit of $N = 1! + 2! + 3! + 4! = 1 + 2 + 6 + 24 = 33$ is $'3'$

Therefore last digit of $N = 1! + 2! + 3! + 4! + 5! \dots\dots\dots\dots + 2010! = 3 + 0 = 3$
Hence the correct answer is **option (1)**

9. $U(8^{233}) = U(8^{232} \times 8^1) = U(8^1) = 8$
Hence the correct answer is **option (1)**

10. $U(4^{456}) = U(4^4) = 6$
Hence the correct answer is **option (3)**

REMAINDER

1. We know that, Number ≡ *Remainder* (mod *Divisor*)
112 ≡ 2(mod 11)
115 ≡ 5(mod 11)
117 ≡ 7(mod 11)
Now, 112*115*117 ≡ 2*5*7(mod 11)
≡70(mod11)
≡ 4(mod11)
Hence the correct answer is **option (3)**

2. We know that, Number ≡ *Remainder* (mod *Divisor*)
2 ≡ -1 (mod 3)
$2^{25} \equiv (-1)^{25} (mod\ 3)$
$\equiv -1\ (mod\ 3)$
$\equiv 2\ (mod\ 3)$
Hence the correct answer is **option (2)**

3. We know that, $a^n + b^n$ is always divisible by $(a + b)$, if n is positive odd integer
Now, $17^{25} + 21^{25}$ is always divisible by $(17 + 21) = 38$ and it is divisible by 19.
Therefore reminder is 0.
Hence the correct answer is **option (1)**

4. We know that, $Dividend = divisor \times quotient + remainder$
Given that, $N = 255q + 70$
And $N = 17(15q + 4) + 2$
Here when we divided by 17 then the remainder is 2
Hence the correct answer is **option (1)**

5. We know that, $Dividend = divisor \times quotient + remainder$
$N \ = \ 5q + 1$
$N = 6q' + 3$
Now, $5q + 1 = 6q' + 3$
$5q - 6q' = 2$
The solution sets are
(4,3),(10,8),(14,13),(22,18) and so on.
By substituting q = 10 then N = 51.
And when 51 is divided by 30, remainder is 21.
Hence the correct answer is **option (4)**

6. Given that, $30^{80} = (2 \times 15)^{80} = 2^{80} \times 15^{80}$
We know that, Number ≡ *Remainder* (mod *Divisor*)
$2^4 \ \equiv -1 \ (mod\ 17)$
$2^{80} \ \equiv (-1)^{20} (mod\ 17)$
$\equiv 1 \ (mod\ 17)$

$15 \ \equiv -2 \ (mod\ 17)$
$15^4 \ \equiv (-2)^4 (mod\ 17)$
$\equiv -1 \ (mod\ 17)$
$15^{80} \ \equiv (-1)^{20} (mod\ 17)$
$\equiv 1 \ (mod\ 17)$
Now,
$30^{80} = 2^{80} \times 15^{80} \equiv 1 (mod\ 17)$
Hence the correct answer is **option (3)**

7. Given that, Remainder = $\frac{1^1+2^2+3^3+4^4 \ldots\ldots\ldots\ldots\ldots +100^{100}}{4} = \frac{1+4+27+256+3125 \ldots\ldots\ldots\ldots\ldots\ldots\ldots +100^{100}}{4}$

We know that,
$Remainder\ of\ \frac{1}{4} = 1$
$Remainder\ of\ \frac{4}{4} = 0$
$Remainder\ of\ \frac{27}{4} = 3$
$Remainder\ of\ \frac{256}{4} = 0$
$Remainder\ of\ \frac{3125}{4} = 1$ and the pattern continues after 4th term.

Now, Remainder =
$\frac{(1+0+3+0)+(1+0+3+0)\ldots\ldots\ldots\ldots\ 25\ times}{4} = 0$

Hence the correct answer is option (4)

8. Here, we need to use following formula.

LCM (x, y, z) + r = N (required number)

LCM(10, 15, 20) = 150.

Now looking at the options we can see that we need to take multiple of 150 to get the largest 4 digit number.

so, $150 \times 65 = 9750$

And $150 \times 66 = 9900$

So, here we need to take the largest number,

So, **9900 + 2 = 9902**

And when divided by the given numbers it leaves 2 as a remainder in each case.

Hence, the correct answer is **option 3**.

9. $480 = 2^5 \times 3^1 \times 5^1$

$111 = 3 \times 37$

$112 = 2^4 \times 7$

$114 = 2 \times 3 \times 19$

$115 = 5 \times 23$

so, $\frac{111 * 112 * 113 * 114 * 115}{480} = \frac{3*37*2^4*7*113*2*3*19*5*23}{2^5*3*5}$

So, numerator is exactly divisible by denominator.
Therefore the remainder is 0

Hence, the correct answer is **option 4.**

10. When number N is divided by 247 it leaves with the remainder 41 and let's say the quotient is Q.

So, number can be formed as,

N = 247Q + 41

And we need to find the remainder when the same number is divided by 19.

So, we need to divide 41 by 19 and look at the remainder.

So, we leave with the remainder **3.**

Hence, the correct answer is **option 1.**