

1. Find the smallest natural number  $n$  which has the following properties:
  - (a) Its decimal representation has 6 as the last digit.
  - (b) If the last digit 6 is erased and placed in front of the remaining digits, the resulting number is four times as large as the original number  $n$ .
2. Determine all real numbers  $x$  which satisfy the inequality:

$$\sqrt{3-x} - \sqrt{x+1} > \frac{1}{2} \quad (1)$$

3. Consider the cube  $ABCD A' B' C' D'$  ( $ABCD$  and  $A' B' C' D'$  are the upper and lower bases, respectively, and edges  $AA'$ ,  $BB'$ ,  $CC'$ ,  $DD'$  are parallel). The point  $X$  moves at constant speed along the perimeter of the square  $ABCD$  in the direction  $ABCD A$ , and the point  $Y$  moves at the same rate along the perimeter of the square  $B' C' C B$  in the direction  $B' C' C B B'$ . Points  $X$  and  $Y$  begin their motion at the same instant from the starting positions  $A$  and  $B'$ , respectively. Determine and draw the locus of the midpoints of the segments  $XY$ .
4. Solve the equation  $\cos^2(x) + \cos^2(2x) + \cos^2(3x) = 1$
5. On the circle  $K$  there are given three distinct points  $A, B, C$ . Construct (using only straightedge and compasses) a fourth point  $D$  on  $K$  such that a circle can be inscribed in the quadrilateral thus obtained.
6. Consider an isosceles triangle. Let  $r$  be the radius of its circumscribed circle and  $\rho$  the radius of its inscribed circle. Prove that the distance  $d$  between the centers of these two circles is  $d = \sqrt{r(r - 2\rho)}$
7. The tetrahedron  $SABC$  has the following property: there exist spheres, each tangent to the edges  $SA, SB, SC, BCCA, AB$ , or to their extensions.

- (a) Prove that the tetrahedron  $SABC$  is regular.
- (b) Prove conversely that for every regular tetrahedron five such spheres exist.