AI1103 : Assignment 3

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Download all python codes from

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https://github.com/prashanthsriram-s/AI1103/tree/main/Assignment3/codes/
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and latex codes from

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https://github.com/prashanthsriram-s/AI1103/tree/main/Assignment3/Assignment3.tex
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Problem statement

(AS3)=UGC / MATH (mathA June 2017), Q.118 Suppose the random varible X has the following probability density function

$$f(x) = \begin{cases} \alpha(x-\mu)^{\alpha-1}e^{-(x-\mu)^\alpha} & x > \mu \\ 0 & x \le \mu, \end{cases} \tag{0.0.1}$$

Where $\alpha > 0, -\infty < \mu < \infty$. Which of the following statements are correct? The Hazard function of X is
(A)an increasing function for all $\alpha > 0$
(B) a decreasing function for all $\alpha > 0$
(C)an increasing function for some $\alpha > 0$
(D) a decreasing function for some $\alpha > 0$

Solution

The Hazard function of X,

$$\lambda(X) = \frac{f(x)}{S(x)} \tag{0.0.2}$$

where $S(x)$ is the survival function given by,

$$S(x) = P(X \ge x) = 1 - F(x) = \int_x^\infty f(t)dt \tag{0.0.3}$$

Lemma 0.1.

$$S(x) = \begin{cases} e^{-(x-\mu)^\alpha} & x > \mu \\ 1 & x \le \mu \end{cases} \tag{0.0.4}$$

Proof.

$$\int f(t)dt = \int \alpha(t-\mu)^{\alpha-1}e^{-(t-\mu)^\alpha}dt \tag{0.0.5}$$

$$= -e^{-(t-\mu)^\alpha} + C \tag{0.0.6}$$

If $x > \mu$,

$$S(x) = \int_x^\infty \alpha(t-\mu)^{\alpha-1}e^{-(t-\mu)^\alpha}dt \tag{0.0.7}$$

$$= -e^{-(t-\mu)^\alpha}]_x^\infty \tag{0.0.8}$$

$$= e^{-(x-\mu)^\alpha} \tag{0.0.9}$$

If $x \le \mu$,

$$S(x) = \int_x^\mu f(t)dt + \int_\mu^\infty f(t)dt \tag{0.0.10}$$

$$= 0 + e^{-(\mu-\mu)^\alpha} \tag{0.0.11}$$

$$= 1 \tag{0.0.12}$$

From (0.0.9) and (0.0.12), we get $S(x)$ as,

$$S(x) = \begin{cases} e^{-(x-\mu)^\alpha} & x > \mu \\ 1 & x \le \mu \end{cases} \tag{0.0.13}$$

□

From (0.0.2) and (0.0.13), we get

$$\lambda(x) = \begin{cases} \alpha(x-\mu)^{\alpha-1} & x > \mu \\ 0 & x \le \mu \end{cases} \tag{0.0.14}$$

So,
if $\alpha > 1$, $\lambda(x)$ is an increasing function and

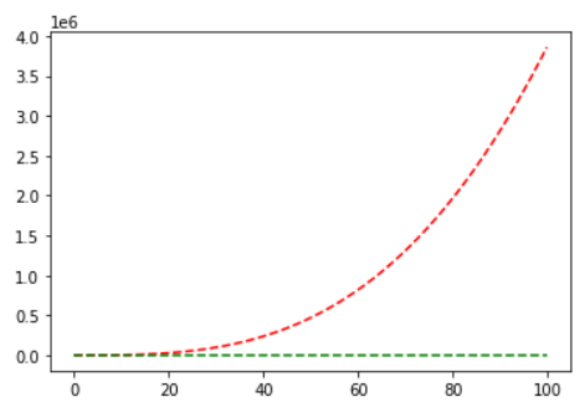

Fig. 0: $\alpha = 2$ for red. $\alpha = 1$ for green, $\mu = 1$ for both

if $0 < \alpha < 1$, $\lambda(x)$ is a decreasing function and for $\alpha = 1$, $\lambda(x) = 1$, a constant function.

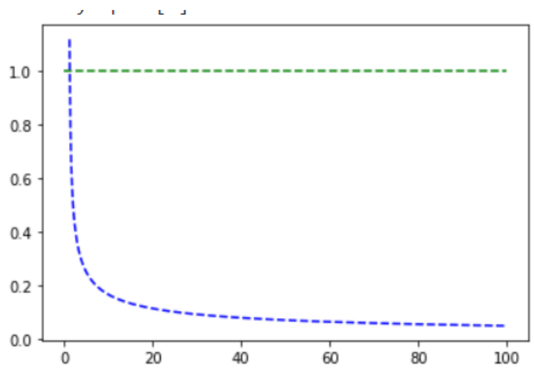

Fig. 0: $\alpha = 0.5$ for blue. $\alpha = 1$ for green, $\mu = 1$ for both

So, for some values of α, it is increasing, for some it is decreasing function

Therefore, answer is (C) and (D)