

Medial Object Extraction - A State of the Art

Yogesh Kulkarni^{1*}, Dr. Shailesh Deshpande²

¹ Software Engineer, Autodesk India Pvt Ltd, Pune, India

² Director, Intellection Software & Technologies Pvt. Ltd, India

*Corresponding author (e-mail: yogeshkulkarni@yahoo.com)

Owing to data-compatibility gap between Computer Aided Design (CAD) and Computer Aided Engineering (CAE), simplification is needed for CAD data to be processed by CAE applications. One of the important abstraction techniques is Dimension Reduction, achieved by extraction of Medial Object of parent geometry. Medial Object (curve in 2D, surface in 3D) is an entity equidistant from the boundaries and has one dimension less than the parent geometry. Medial Object has been used in diverse fields such as pattern recognition, robotic motion planning, shape matching, segmentation and finite element mesh generation. More recently there has been significant impetus for developing approaches for extraction of the Medial Object for use in automatic mesh generation, which are relatively robust and reliable to become part of commercial products. But still there is a wide scope for improvements in these approaches as they do not perform well for complex geometries-topologies and also for various application domains.

This paper attempts to provide the reader a complete picture of Medial Object extraction techniques through a systematic literature review. This presents a state of the art by organizing various activities required for the medial object extraction and also presents breadth and depth of various approaches followed by researchers all over the world.

1. Introduction

Design cycle cannot afford a very large computation time at early stages. The way to achieve quicker simulation without loss of much accuracy is by model simplification. It is done by retaining only important details-features and removing irrelevant ones. Reducing the dimensions of CAD models is also beneficial in some application areas. For example, if thin plate is represented as a sheet with additional data such as thickness at various points, in CAE domain, shell elements can be used in mesh generation. With this approach, there is negligible effect on the accuracy of the analysis but the computational time reduces dramatically [1].

Due to large number of publicly available research papers on this subject, this survey, by no means has taken all of them into account, but only a subset of them. The study of the available literature revealed that approaches adopted by various researchers can be classified as follows:

- Voronoi diagram based approaches
- Thinning based approaches
- Tracing based approaches
- Decomposition based approaches
- Pairing based approaches
- Mesh based approaches

The subsequent sections discuss about these approaches in terms of their formulation and about the contributions made by individual author(s).

2. Voronoi diagram based approaches

These approaches utilize Voronoi diagram (or its dual, Delaunay triangulation) in generating Medial Object which is also known as Medial Axis Transform (MAT).

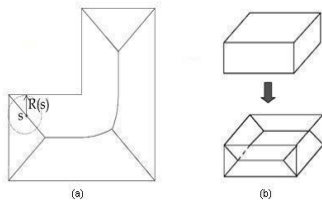


Figure 1.0 MAT 2D & 3D [2]

MAT can be regarded as a mathematical entity, represented by a skeletal shape, and having radius value at points on curve. Figure 1.a shows an 'L' shaped profile with MAT curves inside it. Radius $R(s)$ at point s gives distance of s from Boundary and also a Radius of a maximal circle touching the boundary. MAT also captures sharp corners of a boundary, where the radius value approaches to zero. Figure 1.b shows MAT in 3D, the medial surface

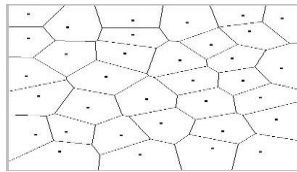


Figure 2.0 Voronoi Diagram

Voronoi diagram of points is a partition of space into cells, each of which consists of the points closer to one particular object than to any others. Figure 2.0 shows set of points and cells around them such that each point has equal influence area with respect to other neighboring cells. Brandt [4] used internal edges of Voronoi diagram generated after sampling boundaries to compute MAT.

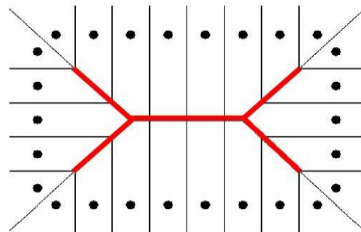


Figure 3.0 Voronoi Diagram [5]

Figure 3.0 shows a boundary represented by sample points (black dots). Around these points are Voronoi cells. The common cell boundaries, those internal to domain and those are not sharing adjacent cells, form set of lines called MAT.

Advantages of these approaches are that MAT can be computed for any shape. It's unique and invertible. The limitations of these approaches are that it creates unnecessary branches and their shapes are smaller than the original corresponding faces. A small notch appearing on the contour introduces a branch of medial axis extending to it.

Following are some approaches researched by researchers to overcome the limitations:

Ogniewicz [6] focused on pruning of the Voronoi diagram by associating the prominence values (based on distance and radius of medial circle) that measure the significance of existence of particular segments and then insignificant ones are removed.

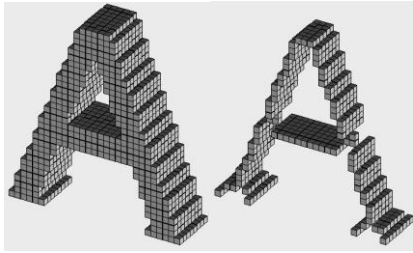
Amenta et al [7] claim convergence to the medial axis by using a relatively small subset of the Voronoi vertices called the poles (inner and outer). The inner poles converge to interior medial axis.

Dey & Zhao [8] have developed approximation algorithms those converge to medial axis by applying certain filter conditions to Voronoi diagram. These filters eliminate noisy component and are independent of scale and density.

Giesen et al [9] defined a scheme where smaller features get ignored. MAT is generated and then each medial ball is scaled up. When scaled-up balls are united smaller ones get absorbed. MAT is generated again. Then everything is scaled back to original values.

These approaches are used in applications where sample points are readily available (e.g., laser scans) or can be easily computed [10].

3. Thinning based approaches



The Thinning approaches simulate the grass-fire process. They perform thinning of an object by successively offsetting its boundary towards its interior till last layer remains. The amount of offset at each step depends on topology and connectivity of original object [10]. Figure 4.0 shows original object as well as result of *thinning* process.

Figure 4.0 Thinning [5]

Montanari [11] identified breakpoints where offset changes the topology of the object. By iteratively offsetting the boundary curves and efficiently identifying these breakpoints, one can locate all the critical points on the medial axis; together with geometry of the skeleton segments those connect them [10].

Advantage of these approaches is that it preserves topology of the shape, but there are numerous limitations such as, while removing layers in one particular direction in each pass, these approaches become sensitive to the order in which the different directions are processed and the resulting skeletons may not get centered within the object.

These approaches have wider applications in Image Processing as well as in voxel based solid models.

4. Tracing based approaches

Tracing approaches first identify a point on the medial axis and then complete the medial axis by traversing along associated boundary elements.

Chou [12] identified all the terminal (end) points on the medial axis. These are either at convex corners or centers of locally maximal positive curvatures. Bisectors are initiated from these terminal points along with associated contour elements. When two bisectors meet, signaled by the fact that mapping between a boundary point and a bisector point is unique, they merge and generate a new bisector. This process continues until a bisector can no longer be extended from a terminal point. The approach terminates when all bisectors are initiated and traversed from all terminal points.

Sherbrooke et al. [13] proposed an algorithm that constructs the medial axis transform of 3D polyhedral. All end points and branch points of skeletons are first computed and edges of skeletons are traversed from these points along the associated polygonal facets.

The skeleton faces are then identified by traversing along closed loops of skeleton edges.

These approaches have the added advantage that it may be extended to more complex curved geometries without changing the basic approach.

5. Decomposition based approaches

These approaches are based on Divide-and-Rule policy. Domain is subdivided into simpler ones for which Medial Axis can be computed and then all sub domains' medial axes are merged to get final result.

Choi [14] subdivides the domain (with holes) into smaller simply connected planar sub regions those overlap only at the joints where subdivision occurs. Approximated medial axes based on Bezier-Bernstein curves are computed for individual sub-regions. The domain boundary corresponding to the approximated MAT is then computed (in discrete steps) and compared with

the original domain geometry. If differences exceed a certain tolerance value, the sub-region is further subdivided at the medial axis point where the error is maximum [10].

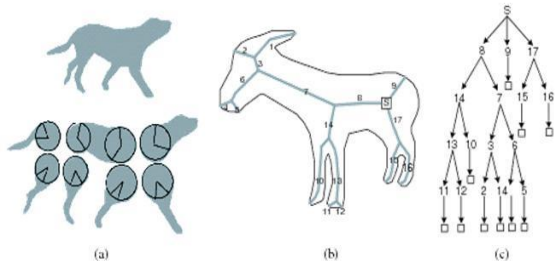


Figure 5.0 Decomposition Tree [22]

Figure 5.a shows complete domain and subdivision of it. Figure 5.b shows the final composition to form the medial axis. Figure 5.c shows the internal tree structure used for computation. Evans et al. [15] employed a top-down approach to construct the MAT trees for simply-connected curvilinear polygons.

Decomposition approach works for simple-shape domains but as complexity increases, division into sub domains may not be possible.

6. Pairing based approaches

Pairing approach involves constructing the 3D mid-surface for a part model by connecting/sewing the mid-surface patches obtained for 'pairs of surfaces'.

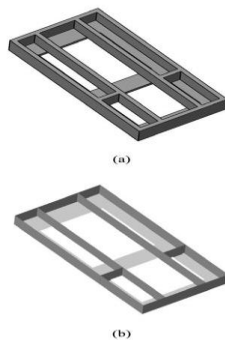


Figure 6.0 Part and its Midsurface

In Rezayat's [16] approach candidate surface pairs are identified on the solid model which represents thin walls on the part. A medial-surface is then constructed between each surface pair, and the resulting surfaces are trimmed and extended to form a consistent model. Figure 6.a shows parent geometry and Figure 6.b shows medial surface. Rezayat claims that this surface-pairing approach has benefits over other Medial Object extraction approaches because its resultant geometry requires less pruning and it preserves volume. However this approach has limitations in terms of identifying correct surface pairs.

Although these approaches may not yield good results for complex parts these appear to be popular ones for Midsurface generation in variety of commercial products like I-DEAS®, Unigraphics NX® etc. These approaches typically have manual intervention possible so that limitations of automatic pairing can be overcome.

7. Mesh based approaches

Mesh is used to generate medial surfaces. Chordal Axis Transform (CAT) is one such approach. In this approach, a discrete model (i.e., a tetrahedral mesh without any interior nodes) is used instead of working directly on a CAD model. The tetrahedral mesh of the CAD model is generated using a bubble packing approach. The bubble packing approach automatically places the nodes only on the top and bottom surface, without any interior nodes or nodes on the lateral surface. This is accomplished by setting the sphere radius more than half the maximum wall thickness. This bypasses the relatively difficult task of identifying the top and the bottom surface of the CAD model. The smart cutting of tetrahedrals using CAT yields correct topology at the non-manifold region in the raw mid-surface. As the raw mid-surface is not directly suitable for engineering purposes, it is trimmed using a smart clean-up procedure and then re-meshed [17].

8. Conclusions

Approaches for extraction of the Medial Object have been studied and classified in this paper. More details have been provided for approaches based on Voronoi diagram as these have been widely researched and more active category. As complexity of shape increases, intricacies related to topology preservation, robustness against perturbations on boundary and methodic complexities deter from having simple generalized solutions.

References

- [1] A. Thakur, A. G. Banerjee, S. K. Gupta, A Survey of cad model simplification techniques for physics-based simulation applications, *Computer. Aided Des.* 41 (2) (2009) 65-80.
- [2] Sheen DP; Son TG; Ryu CH; Lee SH; Lee KW, "Dimension reduction of solid models by mid-surface generation" *Int Journal of CAD/CAM*, vol. 7, no. 1, pp. 71-80, 2007.
- [3] Blum Harry, A Transformation for Extracting New Descriptors of Shape. In *Models for the Perception of Speech and Visual Form*, MIT Press, 1967.
- [4] J. W. Brandt, V. R. Algazi, Continuous skeleton computation by Voronoi diagram, *CVGIP: Image Understanding*, 1992, vol 55, no 3, pages 329-338.
- [5] K. Palyi, , " <http://www.inf.u-szeged.hu/~palyagi/skel/skel.html>" Oct 2009.
- [6] Ogniewicz R, Automatic medial axis pruning by mapping characteristics of boundaries evolving under the euclidean geometric heat flow onto Voronoi skeletons, *Rep* 95 (4).
- [7] Nina Amenta; Sunghee Choi; Ravi Krishna Kolluri, The power crust, *Proceedings of the sixth ACM symposium on Solid modeling and applications* (2001) 249266.
- [8] Tamal Dey ;Wulue Zhao. Approximate medial axis as a Voronoi sub-complex *Computer-Aided Design*, 2004, 36, 195–202
- [9] Joachim Giesen; Balint Miklos;Mark Pauly;Camille Wormser. The Scale Axis Transform, *SCG'09*, 2009
- [10] Ju-Hsien Kao, Process Planning For Additive/Subtractive Solid Freeform Fabrication Using Medial Axis Transform, Ph.D. thesis, Stanford university, June 1999.
- [11] Montanari, U. Continuous skeletons from digitized images, *Journal of the Association for Computing Machinery*, 1969, 16, 534-549
- [12] J. J. Chou, "Voronoi diagrams for planar shapes," *IEEE Computer Graphics and Applications*, vol. 15, no. 2, pp. 52-59, 1995.
- [13] E. C. Sherbrooke; N.M. Patrikalakis; E. Brisson. An algorithm for the medial axis transform of 3d polyhedral solids *IEEE Trans on Vis and Com Graphics*, 1996, 2, 44-61
- [14] H. I. Choi; S. W. Choi; H. P. Moon. New algorithm for medial axis transform of plane domain *Graphical Models and Image Processing*, 1997, 59, 463-483
- [15] G. Evans; A. Middleditch; N. Miles, Stable computation of the 2d medial axis transform, *International Journal of Computational Geometry & Applications* 8 (5-6) (1998) 577-598.
- [16] Mohsen Rezayat, "Midsurface abstraction from 3d solid models: general theory and applications" *Computer Aided Design*, vol. 28, no. 11, pp. 905-15, 1996.
- [17] WR Quadros, An approach for extracting non-manifold mid-surfaces of thin-wall solids using chordal axis transform" *Eng. with Com-put.*, vol. 24, no. 3, pp. 305-319, 2008.