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Sensing Assignment - Question 1.

1) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, what is $C(A)$?

$C(A)$ = column space of matrix A = span of col vectors of matrix A

- span of n independent column vectors is $\mathbb{R}^n$.

column vectors of matrix A -

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, v_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

since v_1 & v_2 are basis vectors, basis vectors are always independent.

Hence span of 2 indep. vectors is $\mathbb{R}^2$.

$$\boxed{C(A) = \mathbb{R}^2}$$

2) $B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

column vectors - $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

- two vectors v_1 & v_2 are linearly dependent if $c_1\vec{v}_1 + c_2\vec{v}_2 = 0$ for some $c_1, c_2 \neq 0$.

hence.

$$c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$c_1 + 2c_2 = 0 \Rightarrow c_1 = -2c_2$$
$$2c_1 + 4c_2 = 0 \Rightarrow c_1 = -2c_2$$

since $c_1 = -2c_2$ so $\bar{v}_2$ can be represented by $\bar{v}_1$, $(\vec{v}_2 = 2\vec{v}_1)$ hence $\bar{v}_1$ & $\bar{v}_2$ are linearly dependent.

so $\boxed{C(B) = \mathbb{R}^1}$

3) $D = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$

Column vectors $\bar{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\bar{v}_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $\bar{v}_3 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

dependency in $\bar{v}_1, \bar{v}_2$ & $\bar{v}_3$:-

$$c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$c_1 + 2c_2 + 3c_3 = 0 \quad -(1)$$

$$4c_3 = 0 \Rightarrow c_3 = 0 \quad -(2)$$

By eq^n (1) & (2) -

$$c_1 + 2c_2 + 3(0) = 0$$

$$c_1 = -2c_2$$

hence $\bar{v}_2$ can be represented by $\bar{v}_1$

so $\bar{v}_1$ & $\bar{v}_2$ are dependent and $\bar{v}_3$ is independent from both,

so span of column vectors is $\mathbb{R}^2$

$$C(D) = \mathbb{R}^2$$