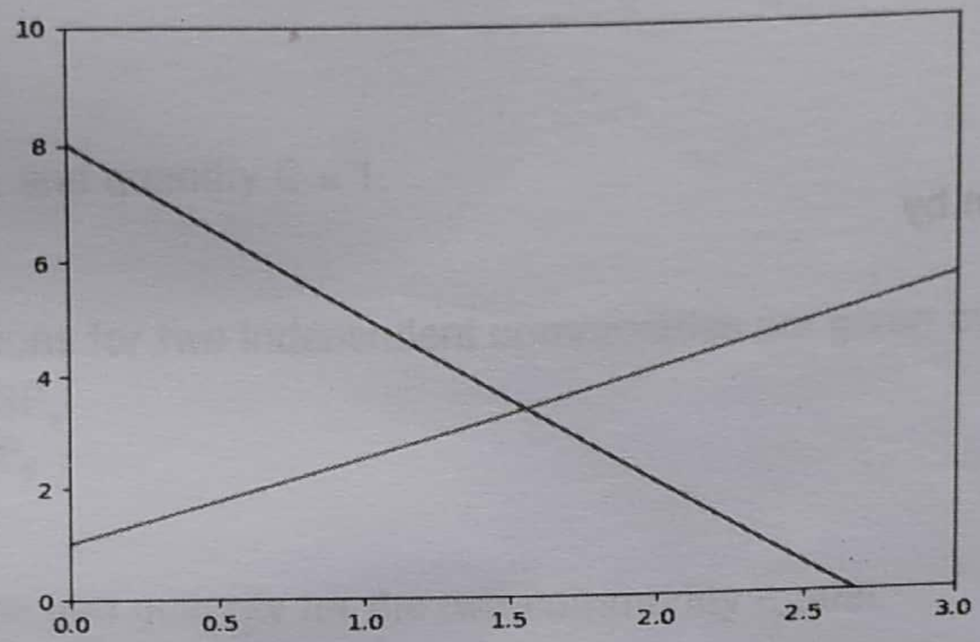
Market Equilibrium

We have seen that as the price increases the demand falls, and the supply rises. The price is determined by the interaction of the demand and supply. At the price, when the quantity demanded exactly matches with the quantity supplied, the market is said to be in **equilibrium**. The corresponding price and the quantity are called the equilibrium price and the equilibrium quantity. Graphically, at the point of equilibrium, the demand curve and the supply curve intersect each other.



Mathematically, at the point of equilibrium,

$$Q_D(P) = Q_S(P)$$

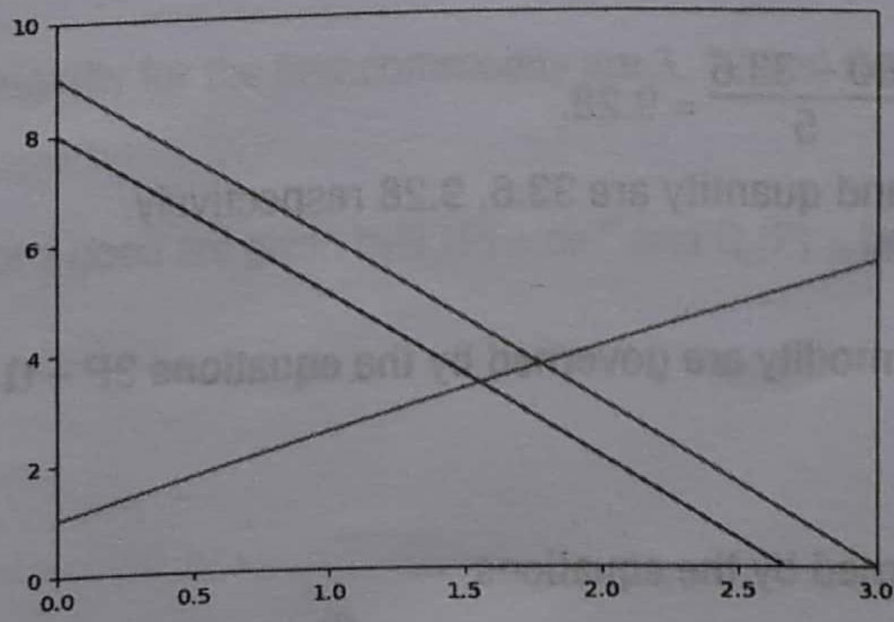
So, to determine the equilibrium price and the corresponding quantity, we solve the equation resulting from equating the demand and the supply function.

Excess of Demand and Excess of Supply

The situationof $Q_D > Q_S$, i.e., quantity demanded exceeds the quantity supply, there is an **excess of demand** by $Q_D - Q_S$, and is termed as **scarcity of goodsin** the market. The opposite situation when $Q_S > Q_D$ is when there is an **excess of supply** by $Q_S - Q_D$, termed as **surplus of goods** in the market..

Effect of Income in the Market Equilibrium

When the income increases, the demand generally increases. If the αY is the contribution of the income Y to the demand, the demand function $Q = Q_D(p)$ becomes $Q = Q_D(p) + \alpha Y$ and the demand curve moves to the right or to the left according as α is positive or negative. As a result, a new equilibrium point is attained.



EXAMPLE 1

The demand and supply function of a good are given by$P + 5Q = 80$, $P - 2Q = 10$.

a. Find the equilibrium price and quantity.

b. If the government deducts, as tax, 15% of the market price of each good, determine the new equilibrium price and quantity.

Solution:

a. Since the demand function is given by$P + 5Q = 80$,

$$Q = \frac{80 - P}{5}$$

$$\text{So, } Q_D(P) = \frac{80 - P}{5}$$

The supply function is given by

$$P - 2Q = 10$$

$$\text{i.e., } Q = \frac{P - 10}{2}$$

$$\text{So, } Q_S(P) = \frac{P - 10}{2}$$

For the equilibrium,

$$Q_D(P) = Q_S(P)$$

$$\text{or, } \frac{80 - P}{5} = \frac{P - 10}{2}$$

or, $160 - 2P = 5P - 50$

or, $7P = 210$

or, $P = 30$

Putting the value of P in any of the equations,

$$Q = 10$$

Hence, the equilibrium price and quantity are 30, 10 respectively.

Hence, the equilibrium price and quantity are 30, 10 respectively.

b. When 15% tax is imposed, the supplier will get 15% less revenue per unit. So, we replace P in the supply function by P – 15% of P = P – 0.15P = 0.85P. So, the supply function is

$$Q_S(P) = \frac{0.85P - 10}{2}$$

However, the demand function remains the same, i.e., $Q_D(P) = \frac{80 - P}{5}$

For the equilibrium,

$$Q_D(P) = Q_S(P)$$

or, $\frac{80 - P}{5} = \frac{0.85P - 10}{2}$

or, $160 - 2P = 4.25P - 50$

or, $6.25P = 210$

or, $P = 33.6 \text{ and } Q = \frac{80 - P}{5} = \frac{80 - 33.6}{5} = 9.28.$

Hence, the equilibrium price and quantity are 33.6, 9.28 respectively.

EXAMPLE 2

The supply and demand of a commodity are governed by the equations $3P - Q = 5$, $3P + Q^2 + 2Q = 9$. Determine the equilibrium price and quantity.

Solution:

The supply and demand are governed by the equations

$$3P - Q = 5 \qquad \text{... (i)}$$

$$3P + Q^2 + 2Q = 9 \qquad \text{... (ii)}$$

The equilibrium values of p and q can be obtained by solving (i) and (ii). Subtracting (i) from (ii),

$$Q^2 + 3Q = 4$$

or, $Q^2 + 3Q - 4 = 0$

or, $(Q + 4)(Q - 1) = 0$

This gives $Q = -4, 1$

As $Q \geq 0$, $Q = 1$

Substituting $q = 1$ in (i), we get

$$3P - 1 = 5$$

or, $3P = 6$

$\therefore$ $P = 2$

So the equilibrium price $P = 2$, and quantity $Q = 1$.

EXAMPLE 3

The demand and supply functions for two independent commodities are given by

$$Q_{D1}(P_1, P_2) = 30 - 2P_1 - 3P_2$$
$$Q_{D2}(P_1, P_2) = 40 - P_1 - 5P_2$$
$$Q_{S1}(P_1, P_2) = -3 + 5P_1$$
$$Q_{S2}(P_1, P_2) = -7 + 6P_2$$

Determine the equilibrium price and quantity for the two-commodity model.

Solution:

For equilibrium,

$$Q_{D1} = Q_{S1} \quad \text{... (i)}$$
$$Q_{D2} = Q_{S2} \quad \text{... (ii)}$$

From (i),

$$30 - 2P_1 - 3P_2 = -3 + 5P_1$$

or, $7P_1 + 3P_2 = 33$ ... (iii)

From (ii),

$$40 - P_1 - 5P_2 = -7 + 6P_2$$

or, $P_1 + 11P_2 = 47$ ... (iv)

Multiplying (iv) by 7 and subtracting from (iii),

$$-74P_2 = -296$$

$\therefore$ $P_2 = 4$

Substituting the value of P_2 in (iv),

$$P_1 + 44 = 47$$

$\therefore$ $P_1 = 3$

Let Q_1 and Q_2 be the quantities corresponding to P_1 and P_2, then

$$Q_1 = -3 + 5(3) = 12$$
$$Q_2 = -7 + 6(4) = 17$$

Hence the equilibrium price and quantity for the first commodity are 3, 12 and those for the second commodity are 4, 17.

EXAMPLE 4

The demand and supply function of a good are given by $Q_D(P) = ae^{-cP}$ and $Q_S(P) = be^{dP}$. Determine the equilibrium price.

Solution:

For the equilibrium,

$Q_D(P) = Q_S(P)$

$ae^{-cP} = be^{dP}$

$$\text{or, } e^{cP + dP} = \frac{a}{b}$$

$$\text{or, } (c + d)P = \ln(a/b)$$

$$\therefore P = \frac{\ln(a/b)}{c + d}$$

EXAMPLE 5

The demand for a good is given by $P + 5Q = 80$, and its supply is given by $P - 2Q = 10$. Analyze the effect on the market when (a) $P = 20$, (b) $P = 40$

Solution:

Here, the quantity demanded

$$Q_D = \frac{80 - P}{5}$$

and quantity supplied

$$Q_S = \frac{P - 10}{2}$$

a. When $P = 20$, the quantity demanded $Q_D = \dfrac{80 - 20}{5} = 12$,

and the quantity supplied $Q_S = \dfrac{20 - 10}{2} = 5$

Since, $Q_D > Q_S$, there is excess of demand by 7 units.

b. When $P = 40$, the quantity demanded $Q_D = \dfrac{80 - 40}{5} = 8$,

and the quantity supplied $Q_S = \dfrac{40 - 10}{2} = 15$

Since, $Q_S > Q_D$, there is excess of supply by 7 units.

Exercise 23.2

1. The demand and supply functions of a good are: $Q_D(P) = 25 - \frac{1}{2}P$, $Q_S(P) = 2P - 50$. Determine the equilibrium price and quantity. Draw the functions on a graph and show the equilibrium point.

2. For the linear demand and supply functions given by $P = aq + b$, $P = cQ + d$, find the equilibrium price and quantity.

3. The demandand supply functions of a good are $Q_D(P) = 8 - P + \frac{1}{8}Y$, and $Q_S(P) = P - 2$, where Y is the income. Find the equilibrium price and quantity if $Y = 16$. If the income increases to 32, find the new equilibrium. Show the functions and equilibrium points in a graph.

4. The demand and supply functions of a good are given by $Q_D(P) = 16 - p/3$, $Q_S(P) = 2P - 46$ Find the equilibrium quantity if the government imposes a fixed tax of Rs. 4 on each good.

5. The demand of a certain good is given by $P_D = 100 - 0.5Q_D$, and the supply is given by $P_S = 10 + 0.5Q_S$. Find the equilibrium price and quantity if the government imposes a tax of 20% of the market price on each good.

6. The demand and supply functions for three independent commodities are given by

$Q_{D1} = 15 - P_1 + 2P_2 + P_3$ $\qquad$ $Q_{D2} = 9 + P_1 - P_2 - P_3$

$Q_{D3} = 8 + 2P_1 - P_2 - 4P_3$ $\qquad$ $Q_{S1} = -7 + P_1$

$Q_{S2} = -4 + 4P_2$ $\qquad$ $Q_{S3} = -5 + 2P_3$

Find the equilibrium price and quantity for the three-commodity model.

7. A supply function is given by $P = 2Q^2 + 10Q + 10$, and demand function by, $P = -Q^2 - 5Q + 52$. Determine the equilibrium Price and quantity.

8. The demand and supply functions of a good are: $Q_D(P) = 290 - \frac{1}{2}P$, $Q_S(P) = \frac{1}{3}P - 10$. Analyse the effect on the market when (a) $P = 354$, (b) $P = 372$.

9. The demand and supply function of a good are given by $Q_D(P) = ae^{-cP}$ and $Q_S(P) = be^{dP}$, show that the equilibrium quantity is $(a^d b^c)^{1/(c+d)}$.

☑ Answers:

1. $P = 30$, $Q = 10$
2. $\frac{bc - ad}{c - a}, \frac{b - d}{c - a}$
3. (6, 4), (7, 5)
4. 24.86, 7.71
5. 61.11, 77.78
6. 20, 5, 8; 13, 16, 11
7. $P = 38$, $Q = 2$

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Exercise 23.2

1. The demand and supply functions of a good are: $Q_D(p) = 25 - \frac{1}{2}p$, $Q_S(p) = 2p - 50$. Determine the equilibrium price and quantity. Draw the functions on a graph and show the equilibrium point.

Solution:

Here, $Q_D(p) = 25 - \frac{1}{2}p$, $Q_S(p) = 2p - 50$

for equilibrium condition, we have

$Q_D(p) = Q_S(p) \Rightarrow 25 - \frac{1}{2}p = 2p - 50$

or, $75 = 2p + \frac{1}{2}p$

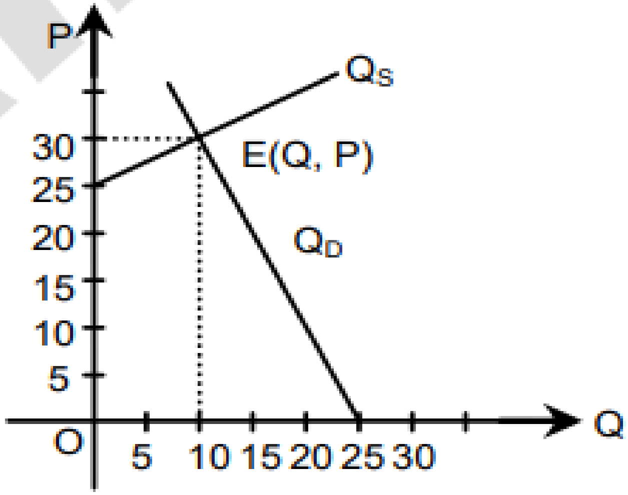
or, $75 = \frac{5p}{2}$

or, $p = 30$

put $p = 30$ in $25 - \frac{1}{2}p$ (or, $2p - 50$), we get,

$Q = 25 - \frac{1}{2} \times 30 = 10$

Hence the equilibrium price and quantity are 30 and 10 respectively.



for graph:

$Q_D(p) = 25 - \frac{1}{2}p$

Q	25	20
p	0	10

$Q_S(p) = 2p - 50$

Q	0	10
p	25	30

$\therefore P = 30, Q = 10$

2. For the linear demand and supply functions given by $p = aq + b$, $p = cq + d$, find the equilibrium price and quantity.

Solution:

Here, $p_d = aq + b$, $p_s = cq + d$

for market equilibrium,

$p_d = p_s \Rightarrow aq + b = cq + d$

or, $aq - cq = d - b$

or, $q(a - c) = d - b \therefore q = \dfrac{d - b}{a - c}$

Also, $p = aq + b = a\dfrac{(b - d)}{c - a} + b = \dfrac{ab - da + bc - ab}{c - a} = \dfrac{bc - ad}{c - a}$

3. The demandand supply functions of a good are $Q_D(p) = 8 - p + \frac{1}{8}Y$, and $Q_S(p) = p - 2$, where Y is the income. Find the equilibrium price and quantity if Y = 16. If the income increases to 32, find the new equilibrium. Show the functions and equilibrium points in a graph.

Solution:

Here, demand function $Q_D(p) = 8 - p + \frac{1}{8}y$,

supply function $Q_s(p) = p - 2$

If y = 16, for market equilibrium condition,

$Q_D = Q_s$

or, $8 - p + \frac{1}{8} \times 16 = p - 2$

or, $8 - p + 2 = p - 2$

or, $10 - p = p - 2$

or, $2p = 12 \therefore p = 6$

Now, $Q = p - 2 = 6 - 2 = 4 \therefore (p, Q) = (6, 4)$

Also if y = 32,

Then, $8 - p + \frac{1}{8} \times 32 = p - 2$

or, $8 - p + 4 = p - 2$

or, $2p = 14$

$\therefore p = 7$ and, $Q = 7 - 2 = 5$

$\therefore$ new equilibrium $(p, Q) = (7, 5)$

4. The demand and supply functions of a good are given by $Q_D(p) = 16 - p/3$, $Q_S(p) = 2p - 46$. Find the equilibrium quantity if the government imposes a fixed tax of Rs. 4 on each good.

Solution:

Here, demand function $Q_D(p) = 16 - \frac{p}{3}$

supply function $Q_s(p) = 2p - 46$
government tax = Rs.4 on each goods
Now, new supply function after Rs.4
added is $Q_s(p) - 4 = 2p - 46$
or, $Q_s(p) = 2p - 42$
$\therefore$ equilibrim value is $Q_D(p) = Q_s(p)$

or, $16 - \frac{p}{3} = 2p - 42$

or, $48 - p = 6p - 126$
or, $7p = 174$
or, $p = \frac{174}{7}$

Now, equilibrium quantity is $Q = 16 - \frac{p}{3}$

or, $Q = 16 - \frac{174}{7 \times 3}$

or, $Q = 16 - \frac{58}{7} = \frac{112 - 58}{7} = \frac{54}{7}$

5. The demand of a certain good is given by $p_d = 100 - 0.5Q_d$, and the supply is given by $p_s = 10 + 0.5Q_s$. Find the equilibrium price and quantity if the government imposes a tax of 20% of the market price on each good.

Solution:

Here, demand function is $p_d = 100 - 0.5Q_d$
supply function is $p_s = 10 + 0.5Q_s$
now, when 20% tax is imposed, the supplier will get 20% less revenue per unit. So. we replace P in supply function by p – 20% of p = p – 0.2 p = 0.8p
So, the demand function $p_d = 100 - 0.5Q_d$ remains constant.
So, for market equilibrium,

$$p_d = p_{SN} \Rightarrow 100 - 0.5Q = \frac{10 + 0.58}{0.8}$$

or, $80 - 0.4Q = 10 + 0.5Q$

or, $0.9Q = 70$

or, $Q = \frac{70}{0.9} = 77.78$ and $p = 100 - 0.5Q = 100 - 0.5 \times 77.78 = 61.11$

Hence the equilibrium price and quantity are 61.11 and 77.78 respectively.

6. The demand and supply functions for three independent commodities are given by

$Q_{D1} = 15 - p_1 + 2p_2 + p_3$ $\quad$ $Q_{D2} = 9 + p_1 - p_2 - p_3$

$Q_{D3} = 8 + 2p_1 - p_2 - 4p_3$ $\quad$ $Q_{S1} = -7 + p_1$

$Q_{S2} = -4 + 4p_2$ $\quad$ $Q_{S3} = -5 + 2p_3$

Find the equilibrium price and quantity for the three-commodity model.

Solution:

Here, the demand and supply functions for three independent commodities are

$Q_{D1} = 15 - p_1 + 2p_2 + p_3$ $\quad$ $Q_{D2} = 9 + p_1 - p_2 - p_3$

$Q_{D3} = 8 + 2p_1 - p_2 - 4p_3$ $\quad$ $Q_{S1} = -7 + p_1$

$Q_{S2} = -4 + 4p_2$ $\quad$ $Q_{S3} = -5 + 2p_3$

For market equilibrium, we have

$Q_{D1} = Q_{S1}$

or, $15 - p_1 + 2p_2 + p_3 = -7 + p_1$

or, $-2p_1 + 2p_2 + p_3 = -22$ ... (i)

$Q_{D2} = Q_{S2}$

or, $9 + p_1 - p_2 - p_3 = -4 + 4p_2$

or, $p_1 - 5p_2 - p_3 = -13$ ... (ii)

and $Q_{D3} = Q_{S3}$

or, $8 + 2p_1 - p_2 - 4p_3 = -5 + 2p_3$

or, $2p_1 - p_2 - 6p_3 = -13$ ... (iii)

Solving the equation (i) and (ii) on addition, we get

$$\begin{array}{rrrr} -2p_1 & + 2p_2 & + p_3 & = -22 \\ p_1 & - 5p_2 & - p_3 & = -13 \\ \hline -p_1 & - 3p_2 & & = -35 \end{array}$$

or, $p_1 + 3p_2 = 35$ ... (iv)

Solving (ii) and (iii) by multiplying (ii) by 6 on subtraction, we get,

$$\begin{array}{rrrr} 6p_1 & - 30p_2 & - 6p_3 & = -78 \\ 2p_1 & - p_2 & - 6p_3 & = -13 \\ - & + & + & + \\ \hline 4p_1 & - 29p_2 & & = -65 \end{array}$$

$\therefore$ $4p_1 - 29p_2 = -65$ ... (v)

Again solving (iv) and (v) by multiplying (iv) by 4 on subtraction, we get

$$4p_1 - 29p_2 = -65$$

$\therefore 4p_1 - 29p_2 = -65$ ... (v)

Again solving (iv) and (v) by multiplying (iv) by 4 on subtraction, we get

$$\begin{array}{r}4p_1 + 12p_2 = 140 \\ 4p_1 - 29p_2 = -65 \\ - \quad + \qquad + \\ \hline 41p_2 = 205\end{array}$$

$\therefore p_2 = \dfrac{205}{41} = 5$

put $p_2 = \dfrac{41}{8}$ in equation (iv)

then, $p_1 = 35 - 3 \times 5 = 20$

Again, substitute p_1 and p_2 in equation (ii), we get

$20 - 5 \times 5 - p_3 = -13$

or, $20 - 25 + 13 = p_3$

$\therefore p_3 = 8$

Also,

$Q_1 = -7 + p_1 = -7 + 20 = 13$

$Q_2 = -4 + 4p_2 = -4 + 4 \times 5 = 16$

$Q_3 = -5 + 2p_3 = -5 + 2 \times 8 = 11$

Hence, the equilibrium price and quantities are 20, 5, 8; 13, 16, 11.

7. A supply function is given by $p = 2q^2 + 10q + 10$, and demand function, by, $p = -q^2 - 5q + 52$. Determine the equilibrium price and quantity.

Solution:

Here, supply function is $p = 2q^2 + 10q + 10$

and demand function is $p = -q^2 - 5q + 52$

For market equilibrium, we have

$p_s = p_d$

$\Rightarrow 2q^2 + 10q + 10 = -q^2 - 5q + 52$

or, $3q^2 + 15q - 42 = 0$

or, $q^2 + 5q - 14 = 0$

or, $q^2 + (7 - 2)\,q - 14 = 0$

or, $q^2 + 7q - 2q - 14 = 0$

or, $q(q + 7) - 2(q + 7) = 0$

or, $(q + 7)(q - 2) = 0$

So, either $q = 2$ or, $q = -7$ (not possible).

and $p = 2 \times 2^2 + 10 \times 2 + 10 = 38$

Hence, the equilibrium price and quantity are 38 and 2.

8. The demand and supply function of a good are given by $Q_D(p) = ae^{-cp}$ and $Q_S(p) = be^{dp}$, show that the equilibrium quantity is $(a^d b^c)^{1/(c+d)}$.

Solution:

Here, demand and supply function are

$Q_D(p) = ae^{-cp}$ and $Q_s(p) = be^{dp}$

for equilibrium, $Q_d(p) = Q_s(p)$

or, $ae^{-cp} = be^{dp}$

or, $e^{cp + dp} = \frac{a}{b}$

or, $(c - d)p = \ln\left(\frac{a}{b}\right)$

or, $p = \frac{\ln\left(\frac{a}{b}\right)}{c + d}$

$$\text{Now, } Q = be^{dp} = be^{d\frac{\ln\left(\frac{a}{b}\right)}{c+d}} = be^{\frac{d}{c+d}\ln\left(\frac{a}{b}\right)} = be^{\ln\left(\frac{a}{b}\right)^{\frac{d}{c+d}}} \quad [\because p\ln x = \ln x^p]$$

$$= b\left(\frac{a}{b}\right)^{\frac{d}{c+d}} \quad [\because e^{\ln x} = x]$$

$$= \frac{b}{b^{\frac{d}{c+d}}} \,.\, a^{\frac{d}{c+d}} = b^{1 - \frac{d}{c+d}} \,.\, a^{\frac{d}{c+d}}$$

$$= b^{\frac{c+d-d}{c+d}} \,.\,.\, a^{\frac{d}{c+d}} = b^{\frac{c}{c+d}} \,.\, a^{\frac{d}{c+d}} = (a^d\, b^c)^{\frac{1}{c+a}} \text{ proved.}$$