Geometric Mechanics and Differential Geometry

From a Robotics/Engineering Perspective

Howie Choset and Ross Hatton

Course Web Site

- http://geometricmechanics.org
- http://www.cs.cmu.edu/~biorobotics/geometricmechanics/

Overview

- Problem-driven exploration of concepts in differential geometry and geometric mechanics.
- Simple physical examples for intuition
- Rigorous notions of length and curvature that form the foundations of differential geometry.
- Relationships between familiar constructs in vector calculus and their generalizations in differential geometry.
- Some topics
  - tangent and cotangent spaces;
  - differential forms;
  - Lie groups, algebras, and brackets;
  - distance metrics;
  - Hodge-Helmholtz decompositions.
- Example systems
  - nonholonomic mechanics,
  - low Reynolds number swimming,
  - nertial control systems such as satellites.

Syllabus

29-Aug	Joints/Links, configuration space, intro to velocity	Direct product, isomorphism (homeomorphisms),diffeomorphism, manifold, groups, Lie group, tangent spaces, vector fields	1.1-1.2, 1.4		Howie
31-Aug	Rigid bodies in SE(2)	Semi-direct products, linear algebra (vector algebra, matrix anxiety, null spaces, index notation?)	1.2-1.4		Howie
					Howie
5-Sep	Labor Day				No class
7-Sep	SE(2) velocities, spatial and body velocities	lifted actions, lifted action examples, geodesics	1.5		Howie
12-Sep	SE(2) velocities	adjoint and exponential map	1.6		Ross
14-Sep	Finish chapter 1 day	frames, notation	1	HW1 Assigned	Ross
19-Sep	Articulated sytems, fixed base systems (new ross notation), proximal vs. medial vs. distal, start mobile articulated systems	holonomic constraints	2.1-2.2		Howie
21-Sep	mobile articulated systems, generalized body frames		2.3-2.4		Ross
26-Sep	IROS				No class
28-Sep	IROS				No class
3-Oct	Jacobians	Review adjoints	2.5		Howie
5-Oct	Jacobian examples		2.5	HW 1 Due HW 2 Assigned	Howie
10-Oct	Kinematic Locomotion up to differential car - Pfaffian constraints and Connection, connection vectorfields	directional linearity, linearity, nonholonomic constraints	3.1-3.3	HW1-A,B	Howie
12-Oct	Connection vectorfields, no-slide and inertial constraints locomotion	one-forms, co-vectors, Noether's Theorem	3.4	HW1-C	Howie
17-Oct	Connection vectorfields, no-slide and inertial constraints locomotion	Noether's Theorem revisited	3.4	HW1-D	Howie
19-Oct	Locomotion in fluids		3.4	HW1-E	Ross?
24-Oct	Gaits	Basic definition of non-communitive and non conservative effects, Lie bracket, curl and Stokes Theorem	4.foo	HW 2 Due HW 3 Assigned	Howie
26-Oct	Gaits - unify concepts	review one-forms, two forms, reexplain curl, Lie brackets, and Stokes theorem again	4.bar	HW 2-A,B	Howie
31-Oct	Ross Code			HW 2-C,D	Howie
2-Nov	Chap 3 and 4 Review			HW 3 DUE Assign HW 4 (code)	Howie
7-Nov	Murray and Walsh Papers				Students
19-Nov	Coordinate Optimization	Hodge-Helmoltz Decomposition	5	Re-assign HW 4	Howie
14-Nov	Hatton (swim) and Meli Papers				Students
16-Nov	Gait efficiency	Distance metrics, geodesics, curvature	5		Howie
21-Nov	Pre-thanksgiving				No class
23-Nov	Pre-Thanksgiving				No class
28-Nov	Kelly Paper			HW 4 due	Students
30-Nov	Finish Chap 5, Begin Mode Shapes				Ross
5-Dec	TBD				
7-Dec	TBD				

Textbook

- We are working on it

Homework

- Handed out at the end of each chapter
- Due at the beginning of the next
- Feel free to work in teams
- Graded for good-faith effort
- Present answers to class
- Last assignment is code
- Papers also are homework

Papers

- Nonholonomic Motion Planning: Steering Using Sinusoids
  R. Murray and S. Sastry
  IEEE Transactions on Automatic Control 38 700-716 (1993)

- On Reorienting Linked Rigid Bodies Using Internal Motions
  G. Walsh and S. Sastry
  Robotics and Automation, IEEE Transactions on 11 139-146 (1995)

- Motion Planning for an Articulated Body in a Perfect Planar Fluid
  J. B. Melli and C. W. Rowley and D. S. Rufat
  SIAM Journal of Applied Dynamical Systems 5 650-669 (2006)

- And two more…