Problem Selected: B
Study of Quadcopter Stability in Wind

Team 298

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Abstract

This paper presents a detailed study highlighting various types of wind effects on the quadcopter dynamics. With that purpose in mind different mathematical models of winds have been properly explored. Besides their effectiveness for determining wind speed as well as quadcopter stability is presented in this paper.

To propose a solution for maximum wind speed with the constraint of maintaining quadcopter stability, several simulations have been performed varying the magnitude and direction of wind velocity. For simulation purposes,"Matlab" is used. These numerical simulations have strengthened our theoretical analysis.

In numerical simulations, we have endeavored to simulate the effects of several parameters on the wind speed. In this way, we can have an idea about the possible maximum wind speed. In fact we have tried to incorporate effects of varying wind speed, wind direction, and also deviations from the target location.

From these analyses, we can assert that the maximum wind speed for stable as well as the safe operation of quadcopter might be in the range from 3.41m/s to 5.62 m/s. The logical reasoning behind this inference has been extensively discussed in this paper.

In addition to that, we have tried to implement a quadcopter stability analysis with real world windy situations. In a nutshell, this paper presents a precise theoretical as well as numerical analysis of the quadcopter stability considering wind effects.

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1 Introduction

Since the last decade, we have been witnessing a tremendous increase in the usage of unmanned aerial vehicles (UAV) in various fields of applications. Quadcopter, a four-rotor wing device, is also a kind of UAV, and the four propulsors enable the device for vertical takeoff and landing. However, it is an under-actuated and dynamically unstable system for its six degrees of freedom and four controllable variables.

Nowadays, the quadcopter is becoming more popular for its dynamic capabilities, performance, and relatively small size. It is being used largely for conducting surveillance and military operations, even in unreachable areas and adverse conditions. To operate a quadcopter in these kinds of environments securely and reliably, it is essential to consider its safety under challenging conditions like varied and turbulent wind blows. Most of the quadcopters are relatively small, and high wind speed and strong wind gusts in the target location can seriously hamper their safety. Different wind conditions can seriously affect their positions and trajectories. These can even lead to dangerous situations when operating close to the target location or objects.

Now to analyze this problem, we have to model the system dynamically under different cases of wind disturbances and understand the effect on it to measure its safety level. In this paper, we have proposed

2 Notations and Symbols

Symbol	**Meaning**
ϕ, θ, ψ	Roll, pitch and yaw angles
u, v, w	x, y and z velocities in body axis
p, q, r	Roll, pitch and yaw rates
g	Gravitational acceleration parameter
T, F, G	Thrust, Net Force, Gravitational force
m	Quadcopter mass

3 Restatement of the Problem

As per the problem statement, we have an already designed quadcopter UAV of mass 1.5kg. It has four rotors arranged in square and each rotor is centered at a equal distance from its center of mass. Therefore, the whole system is surely a symmetric one. Each rotor can supply thrust of maximum seven Newtons when needed. Now we have to consider a practical case of wind disturbance on the copter. Wind flows of varied magnitude and direction are affecting the system and in this case, we have to find the threshold wind speed so that the system can operate safely and stay within 20 cm of a target location.

4 Assumptions

1. For the proposed analysis, we have overlooked the dramatic change of wind direction.
2. There should be a constant wind flow but direction might be changed.
3. We have assumed, not only a point to be the target rather than a desired route of the copter can be considered as a target. And in the starting, the copter's inclination is $\pi/4$.
4. For analyzing the force of wind acting on the copter, we should take the area on which air faces the resistance to flow smoothly.For different inclination angle of air and the copter different area should have been taken.But we have considered the same area for each analysis to avoid complexity.
5. For deviation of 20cm, we have assumed a spherical track whose radius is 20cm, and the target is at the center of the spherical track.
6. The copter has a controller which can automatically control the thrust of the propeller within it's limit to set itself at a stable position.

5 Related Physics

Firstly, this problem requires a detailed dynamic model of the quadcopter, which includes the effect of free-stream velocity on forces and moments experienced by the vehicle. It also requires a compact model of expected wind disturbances and an understanding of the effect of wind conditions on the vehicle. Instead of simply reacting to disturbances once they produce a measurable result in terms of position, velocity and acceleration error, the goal is to eliminate the effect of wind disturbances by predicting them as they occur.

5.1 How a Quadcopter works

Let's start with the basic mechanics. As we have discussed before, a quad rotor has four rotors that support the vehicle's weight. So each rotor spins and generates the thrust. The relationship between the thrust force and the angular velocity is almost quadratic. Every time a rotor spins, there's also a drag that the rotor has to overcome and that drag moment is also quadratic. The we have to design the motors so that they can overcome that drag moment.

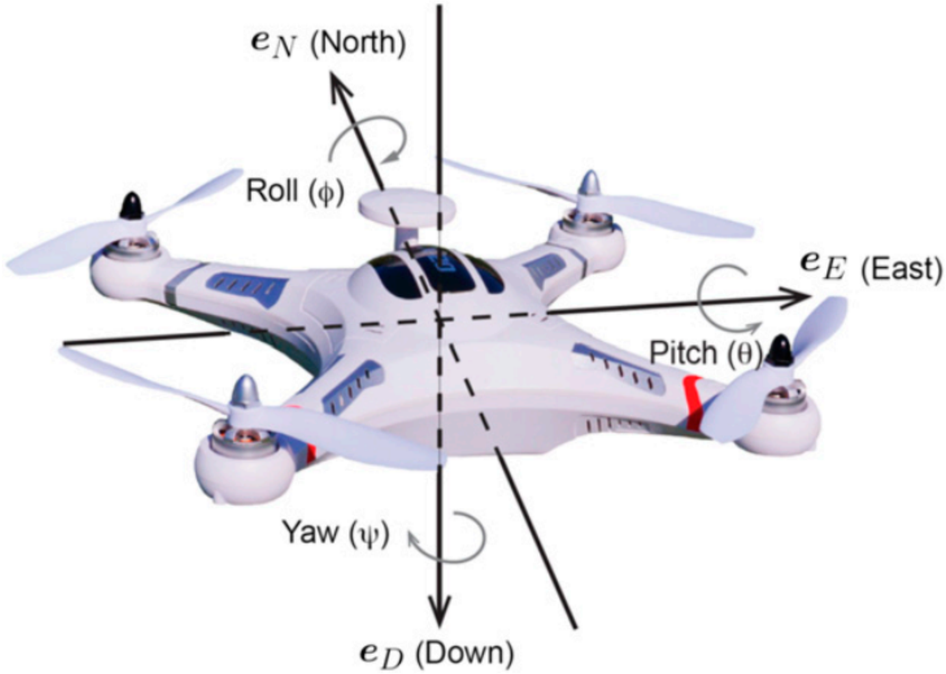

Figure 1: Axes system on the quadcopter. For small angles, roll (f), pitch (u), and yaw (c) are rotations about the $\boldsymbol{e_N}, \boldsymbol{e_E}$, and $\boldsymbol{e_D}$ axes, respectively. In the positioning above, the quadcopter's body axes (not shown) align with the inertial axes.

Three main forces act on the quadcopter system: thrust T, drag D, and gravitational force G. Let $\boldsymbol{e_N}, \boldsymbol{e_E}$, and $\boldsymbol{e_D}$ be unit vectors in the north–east–down inertial coordinates and $\boldsymbol{X_B}$, $\boldsymbol{Y_B}$, and $\boldsymbol{Z_B}$ be the unit vectors along the quadcopter's body axes. Roll (u), pitch (u), and yaw (c) are rotations that parameterize a rotation matrix that maps the quadrotor's body frame to the fixed inertial frame. When the ψ, $\boldsymbol{\theta}$, and $\boldsymbol{\phi}$ rotations are small, they can be approximated as rotations about the $\boldsymbol{e_N}, \boldsymbol{e_E}$, and $\boldsymbol{e_D}$ axes, respectively, as illustrated in Fig. 1. We assume that the roll, pitch, and yaw rotations are small, and that the plane of each rotor is parallel to the $\boldsymbol{X_B} - \boldsymbol{Y_B}$ plane. Thus, the total force acting on the quadcopter system is

$$\boldsymbol{F} = \boldsymbol{G} + \boldsymbol{D} + \boldsymbol{T} = \boldsymbol{mge_D} - \boldsymbol{De_v} - \sum_{i=1}^{4} \boldsymbol{T_i} \tag{1}$$

where $\boldsymbol{mge_D}$ is the gravitational force along the inertial $\boldsymbol{e_D}$ axis, $\boldsymbol{De_v}$ is the drag force opposite of the relative wind velocity $\boldsymbol{e_v}$ in inertial coordinates, and $\boldsymbol{T_i}$ is the thrust in the direction perpendicular to the plane of the ith rotor. So, when the robot is hovering, the rotor speeds compensate for the weight as Fig. 2. But there are also the drag forces and as the forces must be balanced, so the thrust force equals the combination of drag force and gravity as

$$\boldsymbol{T} = \boldsymbol{mge_D} - \boldsymbol{De_v} \tag{2}$$

In the absence of wind, the system does not experience any drag force and the thrust force

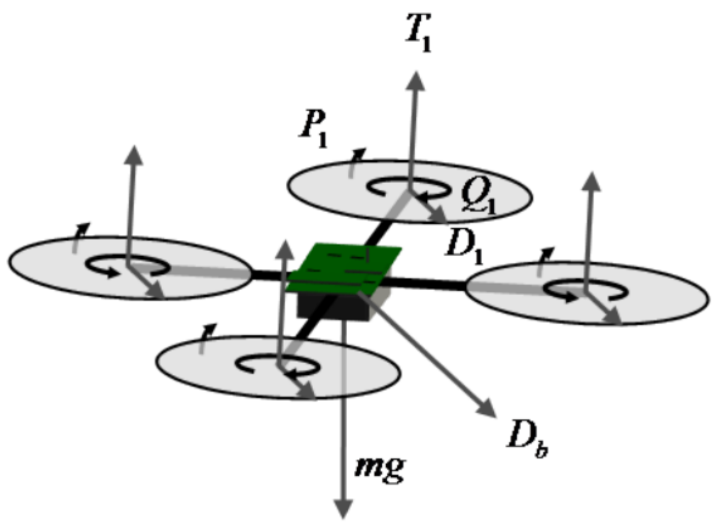

Figure 2: Hovering of a quadcopter with all the rotors spinning. They are generating the the thrusts to compensate for the weight.

opposes only the force of gravity, that is, T = mg. And as we know where the center of mass is, we can quickly calculate moments about the center of mass. The total moment is obtained by calculating the moments due to the forces exerted by the rotors and the reactions due to the rotors spinning in counterclockwise or clockwise directions. Those reactions are moments, and they add to the net moment In equilibrium, the resultant force is obviously zero. And the resultant moment is also zero. Forces in this scenario act only along $\boldsymbol{e_D}$.
But when these resultant forces in moments are non-zero, the system gets an acceleration. To keep things simple let's first consider the acceleration in the vertical direction. So in the vertical direction, again, every motor thrust is the same, and they'll add up to support the weight. But if we increase the motor speeds, then the robot accelerates up. If we decrease the motor speeds, obviously the robot will accelerate down. So a combination of motor thrusts and the weight determines which way the robot accelerates. Equations at the bottom tell us the net force and the net moment:

$$m\ddot{r} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ T_1 + T_2 + T_3 + T_4 \end{bmatrix} \tag{3}$$

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} L(T_2 - T_4) \\ L(T_3 - T_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} * I \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{4}$$

If we combine the net force and the net moment, with the Newton-Euler Equations we get these two sets of equations. On the right side, you see the total thrust and this thrust vector is known in the body fix frame. The matrix R is rotating this thrust vector to an inertia frame. At the bottom you see the net moment. Also known in the body fixed frame. But the equations as they've been written have components in the inertial frame on the top, and in a body fixed frame at the bottom. These are the Newton-Euler equations and these are the equations we've used to develop controllers and planners for our vehicles.

When wind (and therefore drag) is introduced into the system, the quadcopter tilts and T no longer aligns with the negative e_D axis. This tilt angle a between T and $-e_D$ (the up direction) is a combination of the roll and pitch angles. If the introduced wind is horizontal, that is, e_v lies on the $e_N - e_E$ plane and has both vertical and horizontal components:

$$T = T_v e_v + T_D e_D = |T| sin\alpha e_v + |T| cos\alpha e_D \tag{5}$$

Combining,

$$|T| sin\alpha = D \tag{6}$$

$$|T| cos\alpha = mg \tag{7}$$

A slight rearrangement to the ratio of and leads to a new definition of the drag force,

$$D = mgtan\alpha \tag{8}$$

The classic Raleigh drag equation of an object in air takes into account the object's drag coefficient C_D, air density ρ, the object's exposed area A, and the square of the relative wind velocity v,

$$D = \frac{1}{2} C_D \rho A v^2 \tag{9}$$

The number of controllable variables is equal to the number of propulsors which affect the position and attitude of the quadcopter in space. Quadcopter cannot move translationally without the finite rotation around one of the axes, i.e. without the inclination of the quadcopter. To achieve the inclination it is necessary to change the torque with respect to the one of the axes. In order to do so, it is necessary to increase or decrease the thrust on one or two propulsors. If the change of thrust happens on just one propulsors, that could cause instability in torque around the Z_B axis of rotation. To achieve stable flight, it is necessary to combine several high accuracy sensors with fast and robust control algorithms. Only moving parts on quadcopter are propellers on propulsors and which are fixed in a propulsors axis.

Quadcopter frame is a symmetric, light and thin construction that mechanically connects propulsors. Propulsor motor and propeller are directly connected, with all the propulsors axes being fixed and parallel. Propeller rotation causes airflow in the negative direction of the Z_B axis which results in thrust in the positive direction of the Z_B axis.The body of the quadcopter is assumed rigid and the structure is symmetric. The only thing that has direct influence on the quadcopter movement are each motor's RPM. Models for wind disturbances, quadrotor dynamics in wind and on-board measurements are presented and an estimation algorithm is developed for the current wind velocity experienced by the vehicle. This wind estimate is used to improve positioning accuracy by both eliminating the effect of wind on the feedback position control law and adding a wind compensator to mitigate the effect of the expected wind disturbance.

5.2 Constant wind

The constant wind refers to the average wind speed in a certain environment, which varies along with temporal and spatial variation. The value of the constant wind in different temporal condition or different spatial condition can be obtained by statistics. Normally, the constant wind is used for simulation tests of UAV, while it is unable to perfectly describe the flight environment. However, The constant wind does not really exist in nature because it is just the reference value of wind speed in a certain environment. In order to implement a more effective and practical analysis and simulation test, other kinds of wind must be added.

5.3 Turbulent Flow

Turbulent flow is a continuous random fluctuation, which is always accompanied by the constant wind. This is more likely to be a practical case. In the real world, the cause of turbulent flow is related to many factors, such as

- Wind shear,
- Heat exchange,
- Topographic factors, and
- The vortices of other aircrafts.

Stochastic process theory and method are often used to describe atmospheric turbulence in engineering applications. The turbulence models include Dryden model and Von Karman model, both of which depend on a large number of measurements and statistics. The difference is that Dryden model establishes the correlation function of turbulence before deriving the spectral function, while Von Karman, on the contrary, establishes the spectral function first and then deduces the correlation function of turbulence. After relevant research, there is no significant difference between the two models; only the slope of the high frequency band of the spectral function is different, so both of them can be used in solving engineering problems. The Dryden spectral functions can be described by these equations:

$$\phi_u(\Omega) = \sigma_u^2 \frac{L_u}{\pi} \cdot \frac{1}{1 + (L_u\omega)^2}$$
$$\phi_v(\Omega) = \sigma_v^2 \frac{L_v}{\pi} \cdot \frac{1 + 12(L_v\omega)^2}{(1 + 4(L_v\omega)^2)^2}$$
$$\phi_w(\Omega) = \sigma_w^2 \frac{L_w}{\pi} \cdot \frac{1 + 12(L_w\omega)^2}{(1 + 4(L_w\omega)^2)^2}$$

5.4 Interaction between wind and quadcopter

The influences of the wind on the quadcopter can be as follows:

1. Wind blows - u,v,w velocity components,
2. $\boldsymbol{\alpha}$ (Angle of attack), $\boldsymbol{\beta}$ (Sideslip angle) and airspeed $\boldsymbol{V_T}$ change,
3. Aerodynamic force changes.

The airspeed is a prerequisite for the dynamic model of a quadcopter, and if the system is kept static, all aerodynamic derivatives of the dynamic model will lose their meaning. The airspeed may be considered as the relative velocity of an airplane to the air, based on the inertial coordinate system, F^i, the airspeed V_a will be represented by the vector sum of the ground speed V_g and the airflow velocity V_f.

$$\boldsymbol{V_a = V_g + V_f} \tag{10}$$

The projection of V_a onto the body coordinate system F^b is given by

$$\boldsymbol{V_a^b = u_a * X_B + v_a * Y_B + w_a * Z_B} \tag{11}$$

where u_a, v_a, and w_a are the components of the airspeed projected onto the directions of $\boldsymbol{X_B}$ axis, $\boldsymbol{Y_B}$ axis, and $\boldsymbol{Z_B}$ axis of the body frame, respectively. We can express the angle of attack $\boldsymbol{\alpha}$ and sideslip angle $\boldsymbol{\beta}$ as

$$\boldsymbol{\alpha = tan^-1(\frac{w_a}{u_a})}$$
$$\boldsymbol{\beta = sin^-1(\frac{v_a}{\sqrt{u_a^2 + v_a^2 + w_a^2}})}$$

The values of some aerodynamic derivatives in the mathematical model of the quadcopter will be affected by $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. In short, all kinds of winds change the airspeed of the quadcopter, thus changing $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, which affects the values of aerodynamic derivatives, and finally causes disturbance to the system's resultant force and resultant torque. Generally, the most popular way to describe the effects of winds on this system is from the viewpoint of velocity. Mathematical models of winds introduced above just illustrate the velocity V_f distribution with space and time, and it will be directly considered by the equation (10).

Unfortunately, in most conditions, the induced velocity of a certain wind field is nonuniform, like the gust wind field and propeller vortex field, which means the velocity V_f is not a constant value, and it changes along with the different parts of the copter. To solve this problem, Dogan and co. propose an averaging technique to compute the effective wind velocity and wind gradient, to derive the relationships between the variation of air speed and effective wind, and between the variation of body angular velocities and wind gradient.

6 Case Studies and Simulation Results

Now, we have designed a model to simulate and analyze this problem using MATLAB ver. R2019a software and by varying the necessary parameters we have got some interesting results. Here we are going to discuss some simulation results for wind velocity obtained from our designed model in MATLAB considering all the necessary conditions and situations.

6.1 Wind Flow of Different Directions

We have considered a couple of cases in which wind can create an effect on a quadcopter's dynamics and make a distance deviation from it's target point or target path. For example wind can flow laterally, vertically, and also the direction and magnitude of wind can be changed. We have made some assumptions while simulating various parameters through MATLAB.

6.1.1 Case - 1: Wind flow inclined at 30^0 with vertical axis

Wind flows Laterally from the side of the copter. It's natural that when the wind flows the copter will be inclined in an angle thus the thrust will be divided into two parts. One will neutralize the gravitational force and another one will try to neutralize the wind force. The more wind flows the more thrust of the copter will be created until it does not cross it's limit. We have simulated the wind speed for different values of inclined angle with different values of thrust of the rotors. Then we have found the maximum wind speed at the limiting value of thrust of 4 propellers of the copter. The limiting value of the copter's

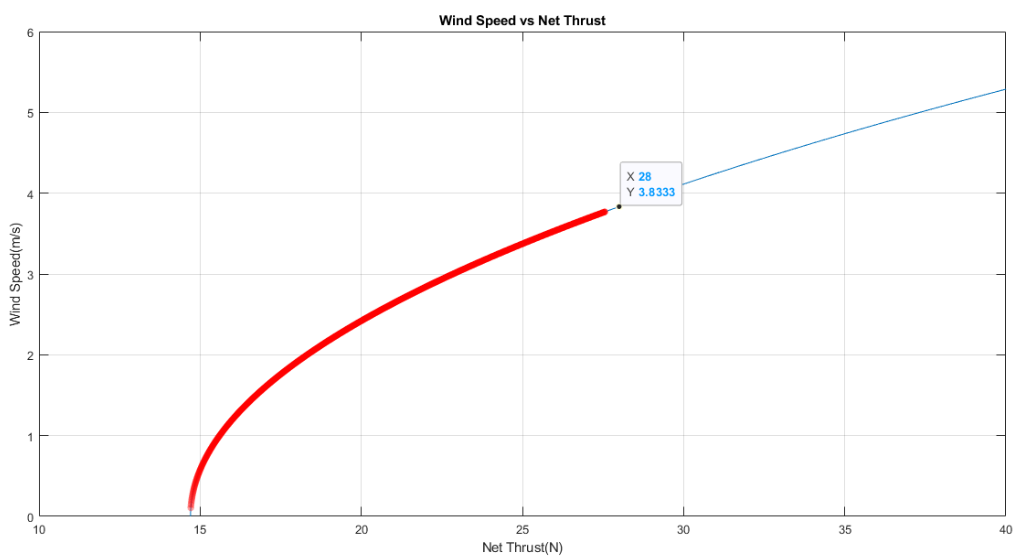

Figure 3: A simulation based graph between Wind Speed (inclined at $\pi/6$ radian with vertical axis) and overall net thrust of the copter.

thrust was 28N (7N each of 4 rotors). Figure 3 shows that the copter can be stable at **3.833m/s** speedy wind flow inclined at $\pi/6$ radian.

6.1.2 Case - 2: Wind flow inclined at 60^0 with vertical axis

In this case we have assumed that the inclination angle of the wind flow is $\pi/3$.

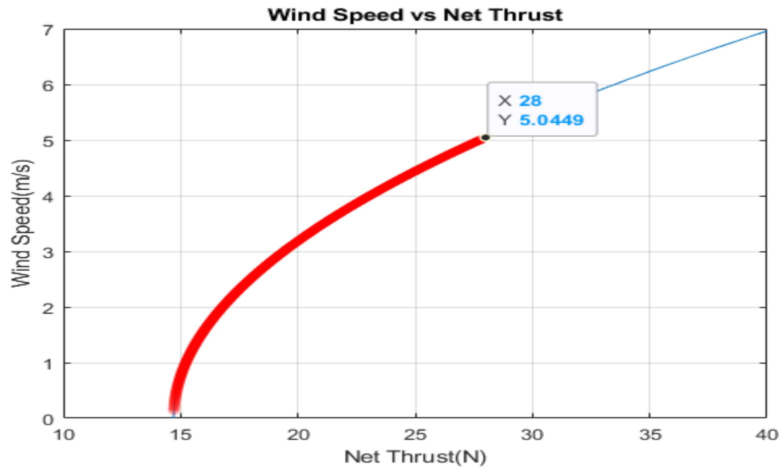

Figure 4: A simulation based graph between Wind Speed (inclined at $\pi/3$ radian with vertical axis) and overall net thrust of the copter.

This figure shows that the copter can be stable at **5.0449m/s** speedy wind flow which is inclined at $\pi/3$ radian.

6.1.3 Case - 3: Wind Flow Vertically Downward

In this case we have assumed that wind is flowing directly downward and the copter will directly oppose the wind force as well as gravitational force. Since the thrust is limited to 28N, there must be a maximum speed in which the copter will be stable or it will fall in ground. From this simulation based graph Figure 5 we can find the maximum speed and that is **3.5673 m/s**.

6.1.4 Case - 4: Horizontal Wind Flow

Here the wind direction is considered from the side. That means the wind is flowing now from any of the sides of the copter and it's flowing horizontally. Here the copter will systematically make itself inclined to create opposite thrust to neutralize the effect of wind. We have found from figure 6 the copter can be stable at most **4.78m/s** speedy wind when it's flowing horizontally.

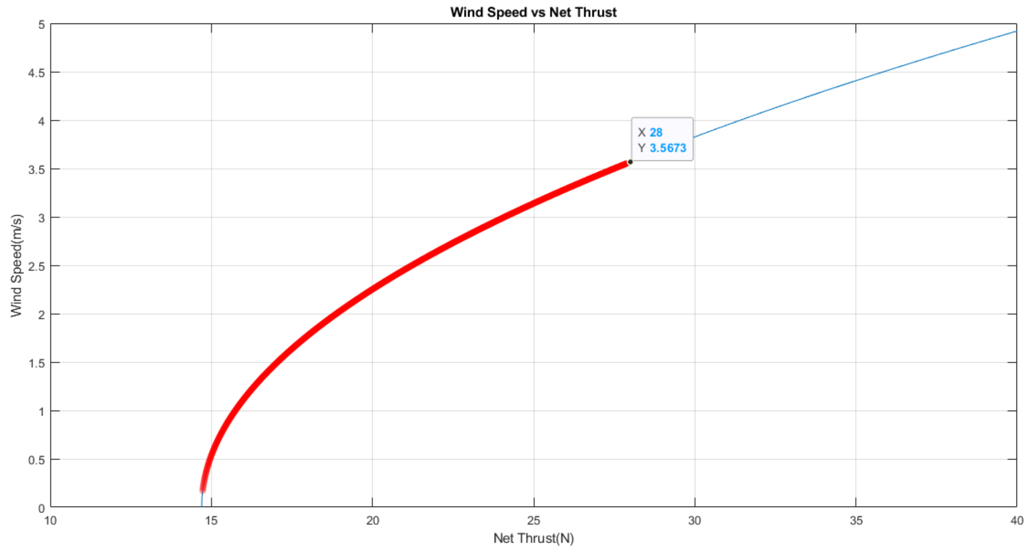

Figure 5: A simulation based graph between Wind Speed (flowing vertically downward) and overall thrust of the copter.

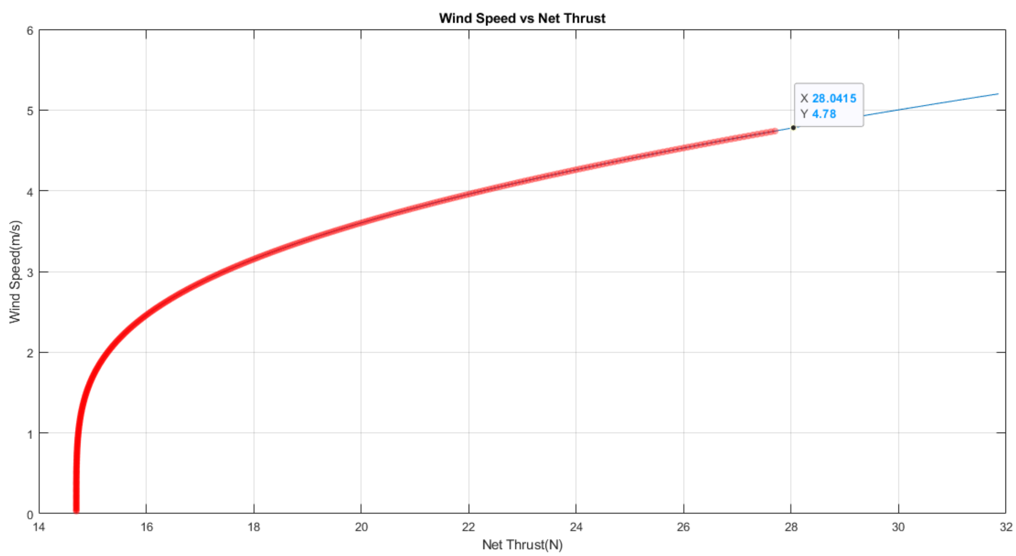

Figure 6: A simulation based graph between Wind Speed (flowing horizontally) and overall net thrust of the copter.

6.2 Wind Speed Versus Drag Coefficient:

For our problem, we have included the effects of various types of wind environments such as steady flow, stormy, turbulent flow etc. Since these different sorts of winds have different velocity and as such their drag coefficients(C_D) also vary. Hence varying drag coefficient in a fixed range mimics different types of wind conditions. For analyzing the effect of drag coefficient , we have applied this mathematical formula:

$$v^2 = \hat{c} tan\alpha \tag{12}$$

$$where, \hat{c} = \frac{2mg}{C_D \rho A} \tag{13}$$

Now, varying the drag coefficient as we have said before for simulating different kinds of winds we have obtained a plot like this: It can be clearly observed from this graph

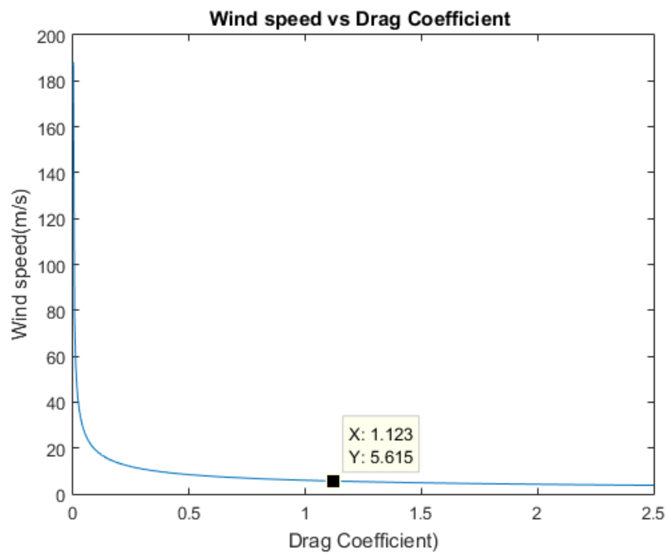

Figure 7: Wind speed vs drag coefficient

that with the increase of drag coefficient the wind speed decreases. The stability of the quadcopter has to be ensured. For serving that purpose we have to take the drag coefficient to be **1.123**. And from the graph it's clear that the wind speed should be less than or equal to **5.5615 m/s**. And the most interesting point is that this limiting value matches our analysis of wind speed with respect to other parameters. This ensures that our analysis coincides with each other.Hence to maintain quadcopter stability wind speed has to be less than or equal to the aforementioned value.

6.3 Wind Speed Versus Drone Inclination Angle:

For observing the effect of inclination angle on wind speed, this angle is varied in a fixed interval. The simulated result exactly matches with our theoretical analysis.

$$v^2 = \frac{2mg}{C_D \rho A} tan\alpha \tag{14}$$

Here, α = inclination angle.

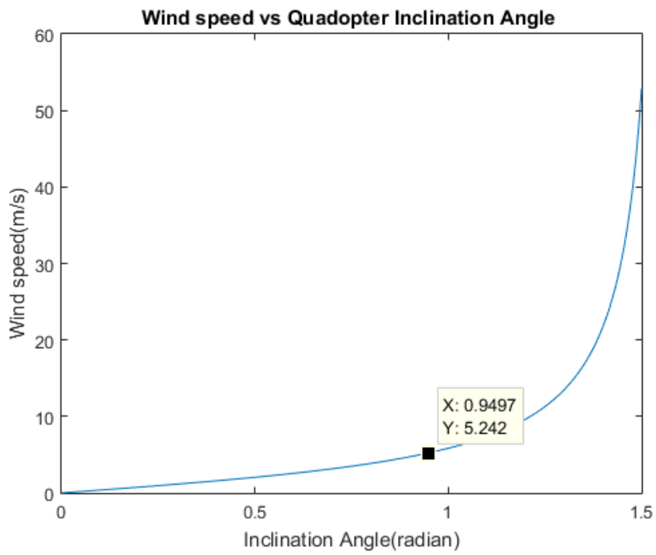

Figure 8: Wind speed vs inclination angle

Observing the plot,it can be said that wind speed increases drastically with the drone inclination angle. During this part of the analysis, we have varied inclination angle within 0 to $\pi/2$. Since at the point of $\pi/2$, wind speed jumps to infinity,we have taken a value slightly less than $\pi/2$. For safe operation of quadcopter,inclination angle should be 0.9497 radian. Hence it can be inferred that wind speed should be less than **5.242 m/s**.

6.4 Deviation Versus Wind Speed:

In this problem the quadcopter should remain within 20cm of the target location, which indicates that if it somehow goes beyond this radius, it will be considered as an unstable scenario for the quadcopter.
Now, for a safe navigation we need a navigation model of the quadcopter.

$$\dot{x} = v_a cos\psi$$
$$\dot{x} = v_a sin\psi$$
$$\dot{\psi} = \frac{a}{v_a}$$

Where v_a represents the speed of the copter, ψ is the flight path angle and **a** represents the lateral acceleration which acts as a guidance parameter.

$$\dot{d} = v_a sin(\psi_r - \psi) \tag{15}$$

Here equation 15 highlights the deviation for no wind condition or wind with negligible speed. Actually from this equation deviation is calculated with necessary mathematical

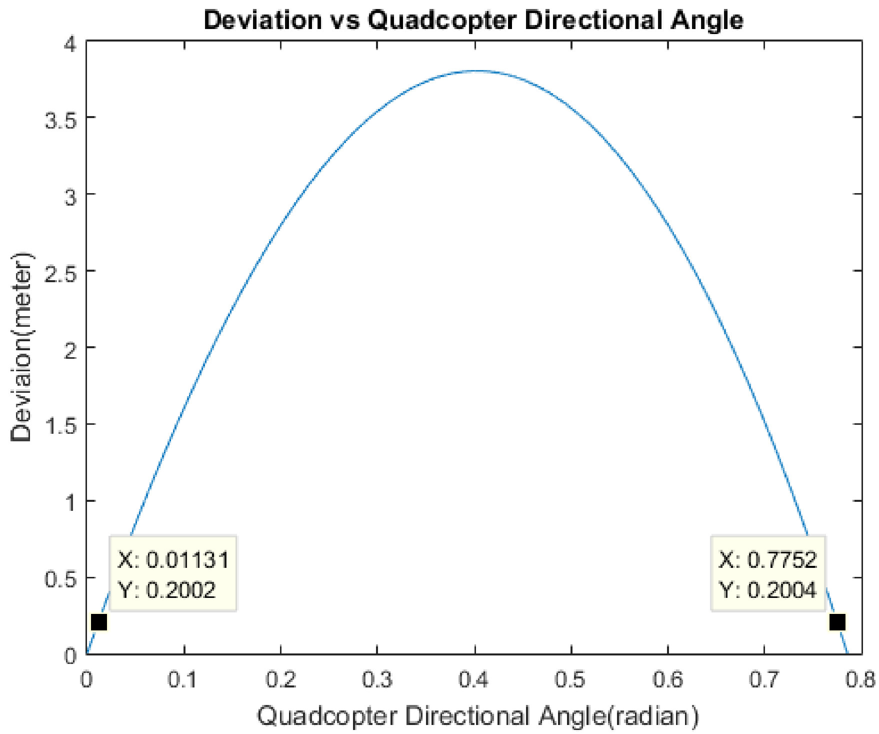

Figure 9: Change of deviation with respect to the directional angle ψ

manipulation. In this equation represents the drone directional angle or the angle that the trajectory of the drone makes with reference to the ground.
The quadcopter can't go beyond the 20cm range for safe operation.Analyzing this plot, it can be asserted that our drone is safe for directional angle less than 0.01131 radian and greater than .7752 radian. If the directional angle takes some value between 0.01131 and 0.7752, the deviation will be greater than 20cm. This will make the quadcopter unstable.

6.5 Effect of Wind Speed on deviation:

This analysis highlights the effect of wind on quadcopter. For this purpose, wind speed and direction of wind has to be included in the navigation model mentioned above. Considering the wind effect navigation model takes the following shape.

$$\dot{x} = v_a cos\psi + v_w cos\psi_w$$
$$\dot{x} = v_a sin\psi + v_w sin\psi_w$$
$$\dot{\psi} = \frac{a}{v_a}$$

where v_w is the wind speed and ψ_w is the direction vector of the wind. And the equation of deviation takes the following form:

$$\dot{d} = v_a sin(\psi_r - \psi) + v_w sin(\psi_r - \psi_w) \tag{16}$$

For analysis, wind speed and direction of wind has been varied separately. Looking

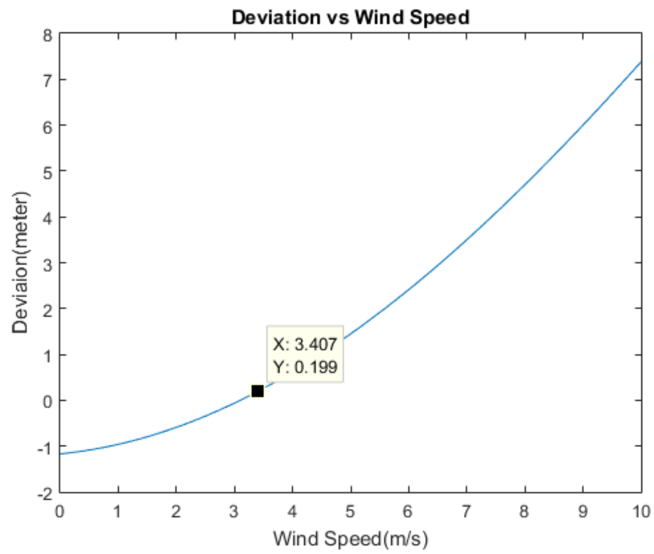

Figure 10: Variation of deviation with the change in speed of wind.

at the plot figure 10, it's clear that to be within the radius of 20cm, wind speed can be maximum 3.407 m/s.Here we have made direction of wind constant. Again, for analyzing the effect of different wind direction, ψ_w is varied from 0 to 2π. From the plot figure 11, it

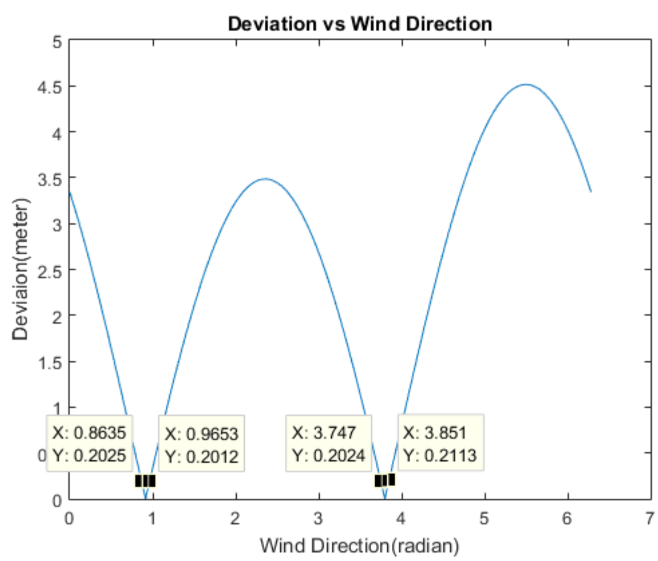

Figure 11: Effect of wind direction on deviation.

is quite obvious that angle should be in the range of 0.8635-0.9653 radian and 3.747-3.851 rad.Thus wind direction has a significant control in deviation.

6.6 Change of lateral acceleration with respect to deviation:

Deviation has an impact over lateral acceleration.For observing this, the following equation has been used:

$$a = k_1(\psi_r - \psi_i nit) + k_2 d \tag{17}$$

For stability of the drone, k_1 and k_2 must be greater than zero. For our solution, we have assumed k_1=15 and k_2 =1. Observing this plot, we can conclude that the quadcopter can hover safely with a lateral acceleration of 4.127m/s^2.Because if the lateral acceleration crosses this limiting value ,then the quadcopter will go beyond 20cm from the target which will lead to instability of the system.

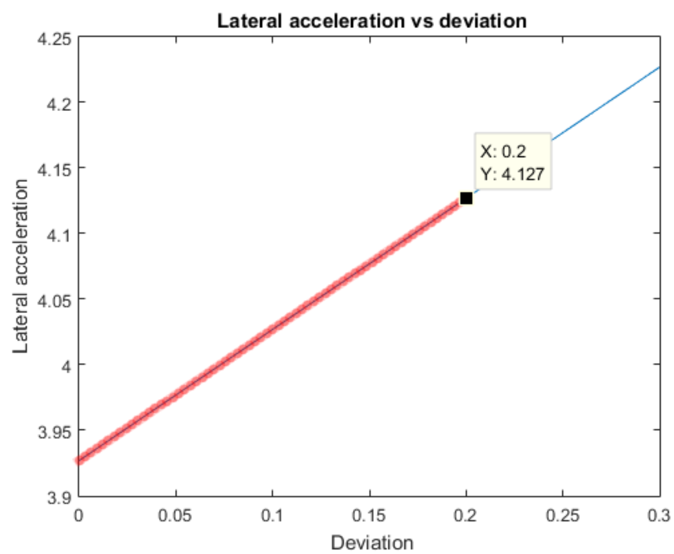

Figure 12: Variation of lateral acceleration with respect to deviation.

Different Scenarios	**Maximum Acceptable Wind Speed (m/s)**
Wind flowing direct downward	3.57
Wind flowing direct Horizontally	4.75
Wind flowing with $\pi/3$ inclination	5.05
Wind flowing with $\pi/3$ inclination	3.83
For different drag coefficients	5.62
Deviation of 20 cm at varied wind direction	3.41

We can observe that for different cases of phenomena ,different maximum wind speed is acceptable. If we want to evaluate the maximum speed of wind for which the quadcopter will be stable for any case ,then the least maximum has to be selected. Consequently, the maximum wind speed is **3.41 m/s**.

7 Strength and Weakness

7.1 Strength

- This proposed solution mainly highlights the variation of wind speed with respect to a number of parameters like inclination angle,thrust,deviation,drag coefficient.By doing this we can get an idea of wind speed range which will ensure the safety of the quadcopter.
- We have analyzed the change of wind speed from the viewpoint of thrust.Besides, to make our solution more robust, we have calculated multiple wind speed direction, which mimics the real life scenario.
- Another important criteria of our problem is the 20cm restriction from the target location.Our proposed solution has highlighted this fact in a very unique way with the help of a number of intuitive plots.
- For including different types of wind effects, we have changed the value of drag coefficients.This approach highlights the real world wind conditions and makes our solution more realistic.

7.2 Weakness

- If we keenly observe the equations,we can realize that the better representation of those dynamics equations might be some 3-dimensional plots like mesh plot ,contour plot etc.Hence, 3D plot would make our numerical solution more attractive.
- During this analysis, we have made a number of assumptions which can slightly differ from the actual scenario.
- For numerical analysis, we have assumed the directional angle,expected directional angle of the quadcopter. In order to make the solution more logical,these might be taken as variable.

8 Conclusions

Quadcopter, a wonder of modern robotics technology, has become the buzzword for the study of dynamics.In the proposed solution, thorough analysis of quadcopter dynamics has been done. Moreover various sorts of wind effect phenomena have been incorporated in the analysis. Besides to make our solution more precise, we have done some numerical simulations through “Matlab”. From these plots,we can have an idea of wind speed for safe stable operation of quadcopter.Moreover we have presented the effect of various parameters on windspeed with the help of a number of logical assumptions.Besides various wind condition have been considered in the analysis which helps to match our solution with the real case studies.

References

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9 Appendix

9.1 Matlab Codes

Lateral Acceleration vs Deviation Code:

```
clc;
clear all;
close all;
%k1 and k2 should be greater than for stable condition.

k1 = 15;
k2 =1;
a_r= pi/4; % drone expected directional angle

a_init = pi/6;
d=linspace(0,0.3,100) // deviation of the drone
a=k1*(a_r-a_init) + k2.*d; // lateral acceleration as Function of
    deviation
plot(d,a)

title(' Lateral acceleration vs deviation')
xlabel(' Deviation')
ylabel('Lateral acceleration')
```

Deviation vs Wind Speed

```
clc;
clear all;
close all;

%with wind in 3D
a_r = pi/4
a = pi/6
a_w= pi/3
va = linspace(0, 10,200)
vw = linspace(0,10,200)

d = va*(sin(a_r-a))+vw.*sin(a_r-a_w)
plot(vw,abs(d))
title('Deviation vs Wind Direction')
xlabel('Wind Direction(radian)')
ylabel('Deviaion(meter)')
```

Deviation vs Wind Direction

```
clc;
clear all;
close all;

%with wind in 3D
a_r = pi/4
a = pi/6
a_w=linspace(0,2*pi,5000)
va=1.988 ;
vw = 1.24;

d = va*(sin(a_r-a))+vw.*sin(a_r-a_w)
plot(a_w,abs(d))
title('Deviation vs Wind Direction')
xlabel('Wind Direction(radian)')
ylabel('Deviaion(meter)')
```

Wind Speed vs Thrust (Case: 1):

```
clc;
clear all;
close all;
T = linspace(14.7,40,200);
w= 14.7;
inclination = pi/6;
dens = 1.1839;
Cd = 1.124;
A = 1.5708;
v = abs(sqrt((T-w)./(.5*Cd*dens*A.*cos(inclination))));
plot(T,v)

title("Wind Speed vs Net Thrust")
xlabel("Net Thrust(N)")
ylabel("Wind Speed(m/s)")
grid on
```

Wind Speed vs Thrust (Case: 2):

```
clc;
clear all;
close all;
T = linspace(14.7,40,200);
w= 14.7;
inclination = pi/3;
dens = 1.1839;
Cd = 1.124;
A = 1.5708;
v = abs(sqrt((T-w)./(.5*Cd*dens*A.*cos(inclination))));
plot(T,v)

title("Wind Speed vs Net Thrust")
xlabel("Net Thrust(N)")
ylabel("Wind Speed(m/s)")
grid on
```

Wind Speed vs Thrust (Case: 3/Vertical):

```
clc;
clear all;
close all;
T = linspace(14.7,40,200);
w= 14.7;
inclination = 0;
dens = 1.1839;
Cd = 1.124;
A = 1.5708;
v = abs(sqrt((T-w)./(.5*Cd*dens*A.*cos(inclination))));
plot(T,v)

title("Wind Speed vs Net Thrust")
xlabel("Net Thrust(N)")
ylabel("Wind Speed(m/s)")
grid on
```

Wind Speed vs Thrust (Case: 4/Lateral):

```
clc;
clear all;
close all;
v = [0:0.01:5.2];
w= 14.7;
dens = 1.1839;
Cd = 1.124;
A = 1.5708;

k = 0.5*Cd*dens*A*(v.^2);
T = k./sin(atan(k./w));

plot(T,v)

title("Wind Speed vs Net Thrust")
xlabel("Net Thrust(N)")
ylabel("Wind Speed(m/s)")
grid on
```

Wind Speed vs Drag Coefficient:

```
clc;
clear all;
close all;

w=14.7;
cd=linspace(0.001,2.5,500)
dens=1.1839;
A=1.5708;
inclination=0.9817

v=sqrt((2*w)./(cd*dens*A))*tan(inclination)
plot(cd,v)

title('Wind speed vs Drag Coefficient ')
xlabel('Drag Coefficient)')
ylabel('Wind speed(m/s)')
```