

Q How to actually solve to get complexity?  
Ans Few Few & ways :-

1) Plug & Chug

OR

2) Master's theorem.

OR

3) Akra Bazzi § 1996 § :- solve any

\*\* Akra Bazzi :-

$$\text{formula: } T(n) = O\left(n^p + n^p \int_1^n \frac{g(u)}{u^{p+1}} du\right)$$

Revise basic integration

Words  $\hookrightarrow$  Time complexity =  $T(n)$

$g(u)$  :- this "g funt" is basically a time complexity  
it-self. & we know that in time complexity  
like constants & more dominating terms  
are ignored.

we know that this going to be the simplest  
form of the function because all the  
less dominating terms and other things  
will be removed.

P:

$$a_1 b_1^P + a_2 b_2^P + \dots = 1$$

$$\text{i.e. } \sum_{j=1}^k a_j + b_j^P = 1 //$$

$$T(N) = T\left(\frac{N}{2}\right) + C$$

Q In constants why do we write  $O(1)$ ?

Ans Because we don't really care about constants  
for eg:- we can write  $O(1)$  or  $O(2k+c)$   
all of these are constant,  $\therefore$  we can ignore the constants

$\therefore$  Basically  $O((2k+c) \times 1)$

$\therefore$  Anything multiplied by anything, you can ignore the constant constants.

So, Hence anything in constants can be written as  $O(1)$ .

Eg: 1)  $T(N) = 2T\left(\frac{N}{2}\right) + (N-1)$

$$a_1 = 2$$

$$b_1 = \frac{1}{2}$$

$$g(x) = x^{N-1}$$

$$\therefore 2 \times \left(\frac{1}{2}\right)^P = 1$$

$$\therefore P = 1$$



Once you've found the P then substitute it into the Akra-Bazzi formula:-

$$T(n) = O\left(n' + n' \int_1^n \frac{u-1}{u^2} du\right)$$

$$= O\left(n + n \int_1^n \frac{1}{u} - \frac{1}{u^2} du\right)$$

$$= O\left(n + n \left[ \int_1^n \frac{du}{u} - \int_1^n \frac{du}{u^2} \right]\right) \quad \begin{matrix} = \log u \\ = -\frac{1}{u} \end{matrix}$$

$$= O\left(n + n \left[ (\log u) + \left[\frac{1}{u}\right] \right]\right)$$

↑  
put ↑.

$$= O\left(n + n \left[ \log n + \frac{1}{n} - 1 \right]\right)$$

$$= O\left(n + n \log n + \frac{1}{\cancel{n}} - \cancel{n}\right)$$

$$= O(n \log n + 1)$$

$$\therefore O(n \log n) \quad // \text{ Time complexity.}$$

So, for array of size  $N$  :-  
Merge sort complexity =  $O(N \log N)$ .

$$2) \quad T(N) = 2T\left(\frac{N}{2}\right) + \frac{8}{9}T\left(\frac{3N}{4}\right) + N^2$$

$$\therefore 2 \times \left(\frac{1}{2}\right)^P + \frac{8^2 \times \frac{3}{4}}{9^3} = 1$$

$$1 + \frac{2}{3} \quad \text{So, put } P = 2$$

$$2 \times \frac{1}{4^2} + \frac{8}{9} \times \frac{69}{16^2} = 2$$

$$\therefore \frac{1}{2} + \frac{1}{2} = 1$$

$$\therefore P = 2 //$$

Substitute

$$T(n) = O\left(n^2 + n^2 \int_1^n \frac{x^2}{3x^3} dx\right)$$

$$= O\left(n^2 + n^2 \log n\right) \quad \text{? ignore the less dominant term?}$$

$$= O\left(n^2 \log n\right)$$



If you're unable to find value of  $P$ :

$$①. T(n) = 3T\left(\frac{n}{3}\right) + 4T\left(\frac{n}{4}\right) + n^2$$

②  $P=1$  Case 1

$$= 3 \times \left(\frac{1}{3}\right) + 4 \times \left(\frac{1}{4}\right) = 1$$

$$\therefore 1 + 1 = 1$$

$$2 \neq 1$$

$\therefore 2 > 1$  This means

what? I need to increase the denominator

③  $P=2$  Case 2

$$3 \times \frac{1}{9} + 4 \times \frac{1}{16} = 1$$

$$\frac{1}{3} + \frac{1}{4} = \frac{7}{12} < 1$$

Hence,  $P$  is less than 2.

So,  $P$  actually lies in <sup>rd</sup> range 1 & 2

Note:

When  $P < \text{power of } (g(n))$  then your  
ans =  $g(n)$ .

Here,  $g(n) = n^2$

$p < 2$  {i.e. power of  $g(n)$ }

Hence, ans =  $O(g(n))$ .

$$T(n) = O \left( n^p + n^p \int_1^n \frac{u^2}{u^{p+1}} du \right)$$

$$= O \left( n^p + n^p \int_1^n u^{1-p} du \right)$$

$$= O(n^p + n^2)$$

$$\therefore p < 2$$

$\therefore n^p$  will become less dominating term  $n^2$  Hence, ignore.

$$\therefore O(n^2) //$$

Hence proved.