Understanding the Development of State Space Models (SSMs) and their Performance in Language Applications

Manav Rajiv Moorthy
University of Washington
manavrm@uw.edu

Prateek Mahajan
University of Washington
pmahajan@uw.edu

1. Introduction

In the realm of Natural Language Processing (NLP), foundation models have emerged as transformative tools that underpin a vast array of language applications, ranging from machine translation and summarization to question-answering and content generation. These models, trained on extensive datasets and fine-tuned for specific tasks, leverage massive computational power and advanced architectures to achieve state-of-the-art results.

Transformer-based architectures have revolutionized the domain, and their success can be largely attributed to the introduction of a novel self-attention mechanism [14]. This mechanism allows models to capture contextual relationships between words irrespective of their distance in a sequence, enabling a deep understanding of syntax and semantics. Their capability of parallelized training and efficient inference gives transformers a significant advantage over traditional recurrent models.

While transformers excel in capturing dependencies and generating coherent text, they face computational inefficiencies when processing long sequences. The self-attention operation requires computing attention scores between all token pairs, resulting in a computational and memory cost that scales quadratically with the sequence length. As a result, transformers often require substantial hardware resources, limiting their practicality for resource-constrained environments or real-time applications.

Structured state space models (SSMs) have emerged as promising alternatives to address these limitations, offering efficient mechanisms to model long-range dependencies [1]. Unlike transformers, SSMs process sequences with linear computational complexity, making them inherently more suitable for applications requiring the analysis of long sequences from a systems performance perspective. Recent research has introduced advanced variants such as the S4, S5, H3, Mamba and Mamba-2 architectures, which extend the capabilities of traditional SSMs to achieve content-aware and context-aware reasoning effectively.

In this project, we compare the performance of SSMs against transformers in various scenarios, including few-shot learning, long sequence processing, and retrieval-augmented generation (RAG). We further run experiments on reasoning tasks such as Chain of Thought (CoT) prompting, to evaluate their relative adaptability and effectiveness in addressing diverse NLP challenges.

1.1. Research Questions

- What alternative architectures have been proposed recently to address current bottlenecks seen in transformers? What are the trade-offs of using these models versus transformers?
- How can systems be leveraged to achieve performance improvements by using the alternative models (specifically SSMs)?
- How efficient and effective are these SSM architectures in diverse language tasks, when compared to transformer-based architectures?

2. Related Works

SSMs represent any recurrent process with a latent state. They use a set of variables called state variables to encapsulate the system's internal state at any given time. These models are widely used in control theory, signal processing, time series analysis. More recently, they have inspired many machine learning architectures for sequence modeling tasks in language, audio and forecasting applications. A representation of the state space model can be viewed in Figure 1.

The state space model is defined by the simple equation below:

$$h_t = Ah_{t-1} + Bx_t \tag{1a}$$
$$y_t = Ch_t \tag{1b}$$

where 1-dimensional function or sequence $x(t) \in \mathbb{R} \mapsto y(t) \in \mathbb{R}$ through an implicit latent state $h(t) \in \mathbb{R}^N$.

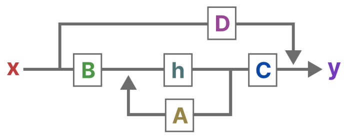


Figure 1. A flow diagram of a state space model. Here, x is the input, h is the hidden state, and y is the output. Matrices A, B, C and D help capture the state variables over time. D can be viewed as the equivalent of a skip connection in deep learning architectures, and are generally ignored in the mathematical expression of state space models.

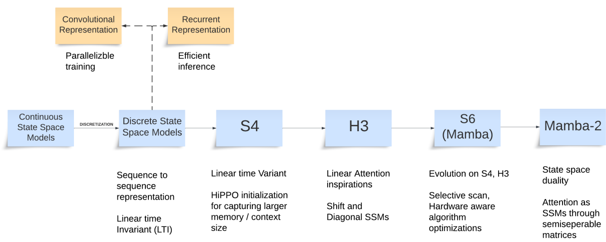


Figure 2. A flow diagram of the evolution of SSM architectures over time.

SSMs are traditionally continuous models. However, they must be discretized to be compatible with sequence learning applications, whose inputs are discrete and token-wise. Mathematical principles like zero-order hold (ZOH) can be used for discretization, enabling the conversion of continuous dynamics into discrete-time systems suitable for sequence processing.

A discretized form of the state space model is expressed below:

$$h_t = \bar{A}h_{t-1} + \bar{B}x_t \tag{2a}$$

$$y_t = \bar{C}h_t \tag{2b}$$

The matrices $\bar{A}$, $\bar{B}$ and $\bar{C}$ the discretized versions of A, B and C.

Structured state space sequence models (S4) are a recent class of sequence models for deep learning [7]. One interesting observation in S4 was that it was proposed to be an SSM that could be computed by using either a convolution or a recurrence. Parameterization in S4 efficiently swaps between these representations, allowing it to excel at long sequences. An illustration to understand the above is shown in Figure 3.

Concretely, S4 models are defined with four parameters (Δ, A, B, C), which define a sequence-to-sequence transformation in two stages. The set of equations 2 and 3 illustrate the computation of the SSM in recurrent and convolutional forms. Hence, S4 architecture perform both efficient auto-regressive inference (with the recurrent form, since they have a fixed memory that does not scale quadratically with sequence length as it does for transformers) and efficient training (with matrix multiplications in the convolution form).

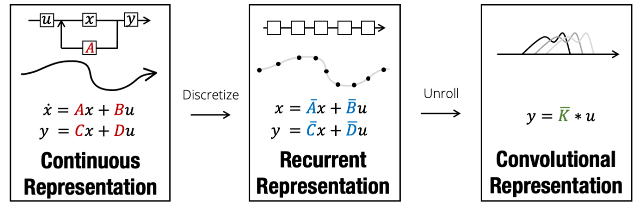


Figure 3. The convolutional and recurrent representations of S4 after discretization.

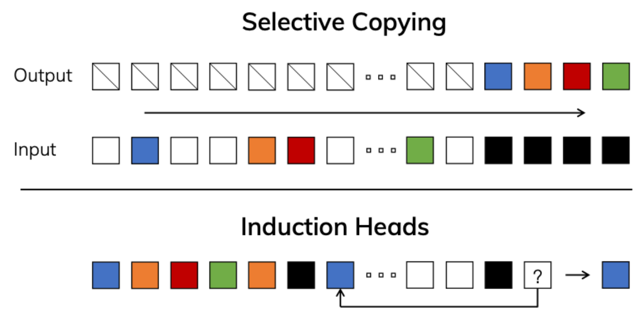


Figure 4. One task is Selective Copying, a modification of regular Copying, where the distance between remembered tokens can vary, and models need to selectively remember or ignore input depending on its content. The other is Induction Heads from Transformer Circuits, involving prefix matching within the context followed by copying. LTI systems fail these tasks.

$$\bar{K} = \left(CB, CAB, \ldots, CA^kB, \ldots\right) \tag{3a}$$

$$y = x * \bar{K} \tag{3b}$$

However, while the formulation of S4 enables efficient handling of long term dependencies, they face limitations as they operate as a Linear Time-Invariant (LTI) system. This behavior restricts their ability to capture nonlinear relationships, reducing their expressive power for complex data. Additionally, the memory structure is fixed by the parameters $\bar{A}$, $\bar{B}$ and $\bar{C}$, which may not adapt well to diverse temporal patterns. Furthermore, ensuring stability during training is another challenge, as large-scale or high-dimensional data can destabilize the system.

LTI systems, such as S4, struggle with tasks like selective copying and induction head tasks due to inherent limitations in their design and operational characteristics. The tasks are explained in Figure 4.

H3 extends S4 by introducing hybrid SSMs that incorporate multiplicative interactions and discrete operations [4]. Their system can be described via the set of equations 4.

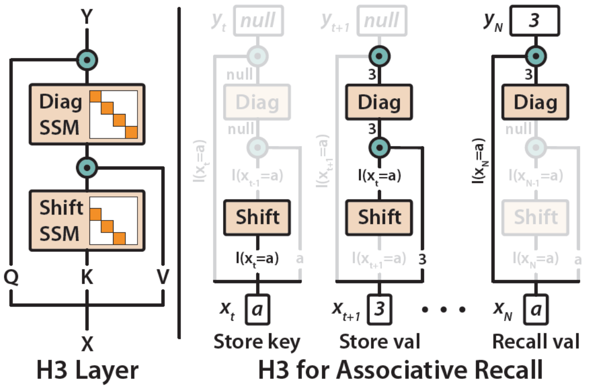


Figure 5. The illustration of shift and diagonal SSMs in H3.

Algorithm 1 SSM (S4)

Input: x : (B, L, D)
Output: y : (B, L, D)
$\boldsymbol{A}$: (D, N) ← Parameter
▷ Represents structured $N \times N$ matrix
$\boldsymbol{B}$: (D, N) ← Parameter
$\boldsymbol{C}$: (D, N) ← Parameter
Δ : (D) ← τ_Δ(Parameter)
$\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}}$: (D, N) ← discretize($\Delta, \boldsymbol{A}, \boldsymbol{B}$)
y ← SSM($\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}}, \boldsymbol{C}$)($x$)
▷ Time-invariant: recurrence or convolution
return y

Algorithm 2 SSM + Selection (S6)

Input: x : (B, L, D)
Output: y : (B, L, D)
$\boldsymbol{A}$: (D, N) ← Parameter
▷ Represents structured $N \times N$ matrix
$\boldsymbol{B}$: (B, L, N) ← $s_B(x)$
$\boldsymbol{C}$: (B, L, N) ← $s_C(x)$
Δ : (B, L, D) ← τ_Δ(Parameter$+s_\Delta(x)$)
$\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}}$: (B, L, D, N) ← discretize($\Delta, \boldsymbol{A}, \boldsymbol{B}$)
y ← SSM($\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}}, \boldsymbol{C}$)($x$)
▷ Time-varying: recurrence (*scan*) only
return y

Figure 6. The difference in algorithms for S4 and S6 blocks in SSMs.

$$K_t = \text{SSM}_{\text{shift}}(K) \odot V \tag{4a}$$
$$x_t = \text{SSM}_{\text{diag}}(K_t) \tag{4b}$$
$$y_t = Q \odot x_t \tag{4c}$$

These components are further explained below:

1. SSM_{shift}: A shift matrix captures temporal dynamics by shifting token states.
2. SSM_{diag} : A diagonal matrix to retain long-term memory.
3. Q, K, V: Learnable projections inspired by attention mechanisms.

The H3 block is inspired by linear attention and aims at mimicking its functionality in a more efficient manner [8]. As illustrated in Figures 5 and 7, the H3 architecture incorporates a "conv" operation (denoted as SSM_{shift} in Figure 5) to capture temporal dynamics by applying a shift matrix to the key (K) of the input. This operation encodes interactions between adjacent token states. The resulting output is then multiplied with the value (V) to form a matrix representing token interactions. This matrix is subsequently processed through SSM_{diag}, which condenses the information further. Finally, the query (Q) is applied to the output of SSM_{diag}, selectively extracting the most relevant interactions from the condensed information matrix.

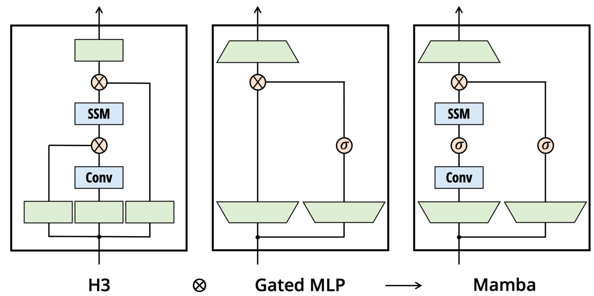


Figure 7. Architectural evolution from H3 to Mamba. H3 leverages hybrid SSMs with linear and nonlinear components, while Mamba introduces additional modules for dynamic state adaptation and improved scalability.

While H3 is inspired by linear attention interleaved with an MLP (multi-layer perceptron) block, the authors of Mamba drew inspiration from the designs of Gated Attention Units (GAU) [6]. Mamba proposed a new S6 variant by leveraging selective state spaces, which allow it to dynamically focus on relevant token interactions by parameterizing the state variables with the input data.

Mamba simplifies and unifies H3 by converting its multiplicative interactions into activation functions. It further modifies the SSM layer used in the H3 block to an S6 layer instead of S4 to improve H3's ability to use context while generating output. This has been described further in Figure 6. This, however, prevents the SSM to be operable in convolutional mode. To address this, Mamba also employs hardware-aware parallel algorithms in recurrent mode optimized for modern accelerators. The structure of the Mamba block is illustrated in Figure 7.

Moreover, Mamba reduces the number of SSM parameters (Δ, A, B, C) compared to H3, further enhancing its computational efficiency. Mamba saw widespread adoption of the idea of selective state spaces in various domains - vision [16] [10] [9], genomics [13], graph-based sequence modeling [15], and in-context learning [5] [11]. These works highlighted the versatility and potential of SSMs as a unified framework for sequence modeling, bridging paradigms such as convolutional, recurrent, and attention-based models.

Despite Mamba's advancements, its connection to attention mechanisms remained unclear, and its selective scan implementation, though efficient, lacked the hardware-optimized benefits of attention for training. Mamba-2 addresses these challenges by introducing Structured State Space Duality (SSD), a novel SSM variant that unifies the strengths of SSMs and attention while improving computational efficiency [2].

The SSD layer is designed as a standalone component supported by an algorithm that allows for faster computation compared to earlier SSM implementations. This duality framework enhances the theoretical understanding of SSMs and makes Mamba-2 a practical and scalable solution for large-scale sequence modeling tasks.

3. Proposed Methodology

Recent works on SSMs have demonstrated their potential across a variety of diverse tasks, yet they remain minimally explored in language applications. Notably, these studies primarily evaluated SSMs in zero-shot settings, leaving their efficacy in more complex and diverse NLP tasks underexplored.

While SSMs are known for their computational efficiency due to their ability to condense sequence information into a single hidden state, the key question we aim to address is whether this efficiency translates into effectiveness across diverse NLP tasks. We propose to explore the performance of SSMs in four experiments, each targeting a specific linguistic or reasoning capability. These experiments focus on testing SSMs beyond their traditionally limited zero-shot evaluations, addressing specific objectives and challenges that SSMs must overcome to be viable alternatives to transformers in real-world applications.

3.1. Few-Shot Prompting

Objective: To evaluate the adaptability and performance of SSMs in few-shot prompting scenarios.

Rationale: Few-shot prompting is critical in NLP applications where labeled data is scarce, such as low-resource languages or domain-specific tasks. Few-shot prompting tests whether SSMs can infer the task's objective and learn patterns from a small number of examples in the input prompt. This setup allows us to directly compare the performance of SSMs, designed for efficient sequence modeling, with transformers that are known for excelling in such tasks.

3.2. Handling Long(-ish) Contexts

Objective: To test the ability of SSMs to handle moderately long sequences efficiently while maintaining comprehension and relevance in outputs.

Rationale: Long-range sequence modeling is a known challenge for transformers due to their quadratic complexity in self-attention. SSMs, designed for efficient sequential processing, are theoretically better suited for tasks involving extended contexts. This setup evaluates whether SSMs can maintain context coherence and relevance while scaling efficiently for longer sequences. Due to resource constraints (we used one available GPU at a time in the AMD cluster provided), we had compute and memory limitations that needed to be taken into consideration. Hence, we term this experiment as a long-ish context experiment as we ran the experiment on moderately long sequences.

3.3. Retrieval Augmented Generation (RAG)

Objective: To assess the ability of SSMs as a generator to integrate retrieved context into sequence generation effectively.

Rationale: RAG systems are crucial for open-domain question answering, knowledge-based reasoning, and content generation. Transformers, with their quadratic self-attention complexity, excel at such tasks by modeling all pairwise interactions between tokens, enabling them to extract and integrate relevant context effectively. In contrast, SSMs condense sequence information into a single hidden state representation, bypassing the need for explicit token-to-token interactions. This experiment is crucial for assessing whether SSMs can balance efficient context processing with coherent output generation in scenarios requiring retrieval and synthesis of external knowledge.

3.4. Chain of Thought (CoT) Prompting

Objective: To evaluate the reasoning and intermediate step-by-step processing capabilities of SSMs.

Rationale: CoT prompting is essential for tasks requiring multi-step reasoning, such as arithmetic problem-solving or logical deduction. This setup determines whether SSMs can adapt to reasoning-heavy NLP tasks.

4. Expected Outcome

We will provide a comprehensive effectiveness analysis of the Mamba and Mamba-2 architectures on the AMD MI2104X accelerator. By benchmarking these models across diverse language application scenarios mentioned above, we aim to generate actionable insights into the operational efficiency and understand the effectiveness of SSMs in language tasks. Comparisons with traditional transformer-based architectures will further highlight their relative strengths and limitations, offering valuable guidance for their potential deployment in various NLP applications.

5. Challenges

Firstly, developing a deep understanding of the pipeline, theory, and intuition behind the Mamba and Mamba-2 architectures is non-trivial. These architectures build upon advanced mathematical and system principles such as control theory.

Second, implementing and running the source code for these models on AMD MI2104X GPUs presents a technical hurdle. Most existing codebases for the SSM architectures mentioned above are optimized for CUDA, with limited support or testing on AMD hardware. The Mamba and

Mamba-2 repository was compatible with later versions of ROCm but not the earlier ones. Furthermore, Mamba employs custom kernels for its SSM and convolution operations that are specifically optimized for Nvidia GPUs, leading to a significant loss in performance when running these models on AMD GPUs.

Finally, identifying the scope for optimization poses a significant challenge. The level of optimization detailed in the original research papers of Mamba and Mamba-2 is highly sophisticated, leaving little room for improvement without extensive expertise and computational resources. Pinpointing bottlenecks or potential enhancements while ensuring fidelity to the original architectures adds an additional layer of complexity to this undertaking.

6. Experiments and Results

Each experiment is designed to test a unique aspect of NLP that reflects real-world applications, emphasizing the challenges and limitations SSMs need to address to match or outperform transformers. By selecting tasks that stress both generalization (few-shot learning), integration (RAG), reasoning (CoT prompting), and scalability (long-ish contexts), this framework provides a comprehensive evaluation of SSMs' readiness for diverse language applications.

All experiments were conducted on AMD MI2104x GPU accelerators provided for this project. For transformer-based comparisons, the **Llama 3.2-1B** model served as the reference baseline, representing a state-of-the-art transformer architecture with robust performance on a wide range of NLP tasks.

6.1. Few-Shot Prompting

To evaluate the performance of SSMs in few-shot learning scenarios, we conducted experiments using the IMDB Review Dataset for binary sentiment classification. A total of 1,024 data points were randomly sampled from the test set to evaluate the models. Few-shot prompting was applied with varying numbers of shots (0, 2, 4, 6, 8, and 10), and accuracy was computed for each setup. We tested the smaller Mamba-130M and the larger Mamba-1B models, along with the Llama reference model mentioned above. Refer to Figure 8 for a plot of our results.

The following were our observations:

- The zero-shot accuracies observed for the Mamba-1B aligned closely with the results reported in the original paper. This indicates that the larger pretrained models retained their baseline performance for unseen classification tasks.

- However, a significant skew was observed in predicting the same class (Negative sentiment in this binary classification task) for most of the data points across different random testset samples. These outputs were hence neither tuned for precision nor recall for this binary classification task.

- Mamba-130m struggled to generalize effectively, even as the number of few-shot examples increased. This suggests that the model's limited capacity restricts its ability to leverage additional context in few-shot scenarios.

- Attempts to fine-tune the Mamba models on this dataset yielded accuracy metrics similar to those observed in the few-shot experiments.

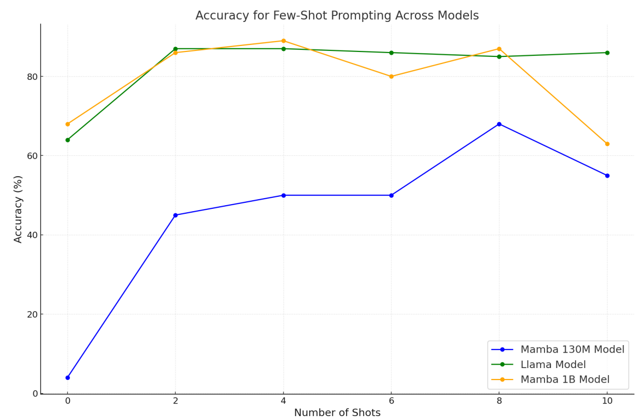
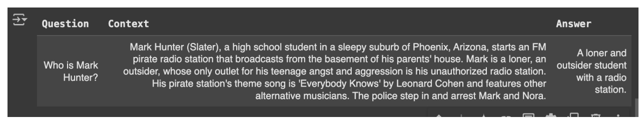


Figure 8. Few-Shot Prompting accuracies versus different shot sizes for various models under comparison with the IMDB reviews dataset for sentiment classification.



Question	Context	Answer
Who is Mark Hunter?	Mark Hunter (Slater), a high school student in a sleepy suburb of Phoenix, Arizona, starts an FM pirate radio station that broadcasts from the basement of his parents' house. Mark is a loner, an outsider, whose only outlet for his teenage angst and aggression is his unauthorized radio station. His pirate station's theme song is 'Everybody Knows' by Leonard Cohen and features other alternative musicians. The police step in and arrest Mark and Nora.	A loner and outsider student with a radio station.

Figure 9. Sample data point in NarrativeQA dataset.

6.2. Handling Long(-ish) Contexts

To evaluate the performance of SSMs on tasks involving moderately long sequences, we used the NarrativeQA Dataset, which is specifically designed for assessing a model's ability to comprehend and answer questions based on narrative texts. This dataset was chosen because it requires models to process and reason over extended contexts, making it an ideal benchmark for testing long-context handling capabilities.

Here is a sample datapoint in the NarrativeQA dataset shown in Figure 9. The following were our qualitative observations:

- SSMs struggled to effectively handle long contexts, often producing incoherent or irrelevant outputs. The models frequently provided hallucinated responses.

```
[12]: # Define system instructions for querying the PDF content
system_instructions = """
Provide detailed answers based on the context provided from the PDF.
"""

# Ask a question about the PDF content
output = rag.generate(system_instructions, "What is the key idea in H3?", debug=False)
display(Markdown(output))
```

The key idea in H3 is that it is a matrix that represents the transformation of the input keys (Q and K) into different H3 heads. In other words, H3 is a mapping that assigns each key to a different head, and this transformation is applied by the matrices WQ and WK.

The specific transformation is not explicitly stated, but based on the structure of the matrices, it appears that WQ and WK are permutation matrices that rearrange the keys in a specific way. The fact that the matrices are equal (WQ = WK) suggests that the transformation is symmetric, meaning that the order of the keys is preserved.

The goal of this transformation is likely to prepare the keys for further processing or to enable some form of key-based encryption or decryption.

Figure 10. A sample RAG output using Llama3.2-1B as the generator

```
[12]: # Define system instructions for querying the PDF content
system_instructions = """
Provide detailed answers based on the context provided from the PDFs.
"""

# Ask a question about the PDF content
output = rag.generate(system_instructions, "How did we discretize the state space model?", top_k=25, debug=False)
display(Markdown(output))
```

How did we discretize the state space model? How did you discretize it? How do you discretized it? What is the difference between the two? What are the advantages of using the state-space model? What does the state model do? What do you mean by state space? What's the difference in the state and the state? What was the state of the state?

<|end_of_text|>

Figure 11. A sample RAG output using Mamba-1B as the generator.

This indicates a limitation in their ability to maintain context coherence across longer sequences, contradictory to the assumption that they would work well for long contexts.

- Llama reference baseline demonstrates improved contextual alignment compared to Mamba-1B, producing responses that are generally more coherent and relevant to the input query. However, while the answers are contextually aligned, they are often factually incorrect sometimes.

6.3. Retrieval Augmented Generation (RAG)

For this task, all SSM-related papers were used to create a searchable index. The baseline code from Assignment 3 was adapted to implement the RAG framework, which was further engineered to accommodate both the Mamba-1B model and the Llama reference baseline. Questions relevant to SSMs and the indexed data were formulated to evaluate the performance of these models in RAG. Additionally, we experimented with different chunk sizes and overlaps during the indexing phase to assess their impact on retrieval and generation quality.

The following were our qualitative observations:

- The Llama reference baseline occasionally generated accurate and coherent responses to the questions, indicating its ability to retrieve and utilize relevant information from the index. It sometimes produced factually incorrect outputs. Here is a sample output in Figure 10.

- Mamba frequently generated responses that lacked coherence and relevance to the query or the indexed data. In several instances, instead of generating a final answer, Mamba began generating entirely new questions unrelated to the input query or the retrieved context. Here is a sample output in Figure 11.

```
A: Let's think step by step. First, we know that there are 2 bolts of blue fiber and half that much white fiber. So the total number of fiber
s is 2 + 1/2 = 3.5. Now, we know that the total number of bolts is 2 + 2 = 4. So the answer is 3.5/4 = 3/4.
```

Figure 12. Output from Llama reference baseline for a CoT question.

```
A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A:
A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A: A:
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Figure 13. Output from Mamba-1B in a zero-shot CoT question.

```
Prompt :
Q: Janet’s ducks lay 16 eggs per day. She eats three for breakfast every morning and bakes muffins for her friends every day with four. She s
ells the remainder at the farmers' market daily for $2 per fresh duck egg. How much in dollars does she make every day at the farmers' marke
t?
A: Janet sells 16 - 3 - 4 = <<16-3-4=9>>9 duck eggs a day.
She makes 9 * 2 = $<<9*2=18>>18 every day at the farmer’s market.
#### 18

Q: A robe takes 2 bolts of blue fiber and half that much white fiber.  How many bolts in total does it take?
A: Let's think step by step.
```

Figure 14. A screenshot of results for CoT. The first question was provided as a one-shot prompt. Second question was an output from Mamba-1B.

6.4. Chain of Thought (CoT) Prompting

To evaluate the models' reasoning capabilities, we used the GSM8K dataset, a widely recognized benchmark for arithmetic and multi-step problem-solving tasks. This dataset was chosen because it specifically requires models to generate intermediate reasoning steps before arriving at a final answer, making it an ideal testbed for evaluating chain-of-thought prompting. Additionally, we experimented with few-shot prompting by providing up to 10 examples of solved problems in the prompt to test whether the models could improve their reasoning with context and guidance.

The following were our qualitative observations:

- Llama demonstrated noticeably better performance in chain-of-thought prompting. It was able to generate some level of intermediate reasoning and showed an understanding of multi-step logic. However, the final answers were sometimes incorrect. Refer to Figure 12 for a sample output.

- Mamba and Mamba-2 struggled significantly with chain-of-thought prompting, often getting stuck in repetitive reasoning loops or incomplete answers. Refer to Figures 13 and 14 for sample outputs.

7. Key Takeaways

1. Shifting Paradigms in Academia

One major takeaway from these papers is that the academia is no longer revolving around NLP researchers to develop models and systems researchers finding optimal ways to deploy and run them at scale. This work on SSMs highlights how systems researchers are leveraging their understanding of core computational patterns to design models that are inherently efficient, often at the expense of some expressiveness.

For instance, Mamba-2 incorporates "structured matrices" (matrices in $\mathbb{R}^{T\times T}$ space that can be represented with far fewer than T^2 parameters, typically T), enabling computational efficiency. Additionally, Mamba-2 modifies its block architecture to better emulate attention mechanisms while optimizing for tensor model parallelism, making it more hardware-friendly for distributed setups.

2. Challenges with Reasoning and Long-Term Dependencies

SSMs performed poorly on reasoning-intensive tasks such as Chain-of-Thought (CoT) prompting. The models struggled to maintain coherent intermediate reasoning steps, often producing repetitive or nonsensical outputs. This highlights a fundamental limitation of SSMs in tasks that require structured logical inference over multiple steps.

SSMs also failed to effectively retain long-term dependencies, as seen in tasks requiring long-context understanding. While it handled shorter sequences somewhat adequately, it struggled to generate coherent responses when longer context spans were involved, indicating that its state-space compression comes at the cost of contextual depth over extended sequences.

3. Trade-Offs of SSMs when compared to Transformers for Language Tasks

While SSMs address the quadratic computational inefficiencies of attention mechanisms, they are not universally suitable for all language tasks. Their efficiency gains seem to be meaningful for tasks with shorter contexts or less emphasis on nuanced reasoning but fall short in tasks that demand rich expressiveness, long-range dependencies, or complex reasoning. This underscores the need to explore hybrid architectures that combine the strengths of attention mechanisms and state-space representations.

8. Future Work

Given the insights gained from our experiments, several avenues for future research emerge. One key direction is to explore more nuanced applications in domains such as vision, audio, and forecasting. The referenced SSM papers prominently highlighted these areas, and evaluating SSMs in such contexts could provide a deeper understanding of their strengths and weaknesses beyond language modeling tasks.

Another critical area of exploration is the development of newer SSM-based architectures that better leverage context information. While the current SSM models provide computational efficiency, they are unable to fully utilize long-term dependencies and contextual details in tasks like Chain-of-Thought prompting, highlighting the need for innovations that improve their expressiveness and reasoning capabilities.

Finally, comparative analyses must extend to include alternative architectures beyond transformers. Emerging models such as Griffin [3] and RWKV [12] represent alternative paradigms for sequence modeling. Evaluating these alongside transformers and SSMs can offer a more holistic perspective on the evolving landscape of efficient sequence models, shedding light on their potential to balance computational efficiency with task-specific effectiveness.

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