qrnmj2fvp

February 9, 2024

[489]:
```
from sklearn import datasets
digits = datasets.load_digits()
print(digits.keys())
```

```
dict_keys(['data', 'target', 'frame', 'feature_names', 'target_names', 'images',
'DESCR'])
```

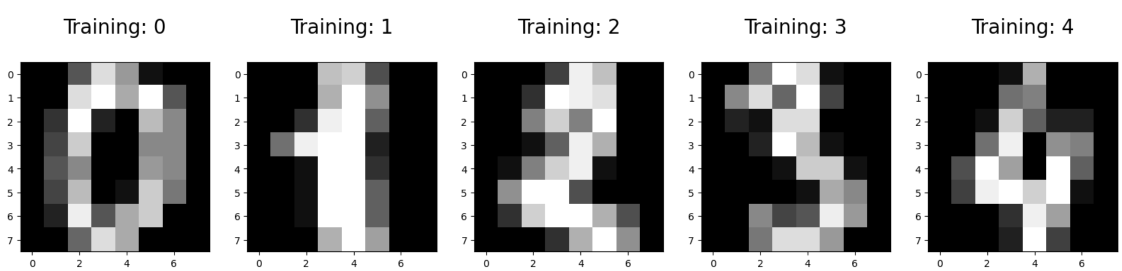
[490]:
```
# a form of summary of the data
print('data size = ', digits.data.shape)
print('target size = ', digits.target.shape)
print(digits.DESCR)
```

```
data size =  (1797, 64)
target size =  (1797,)
.. _digits_dataset:

Optical recognition of handwritten digits dataset
--------------------------------------------------

**Data Set Characteristics:**

    :Number of Instances: 1797
    :Number of Attributes: 64
    :Attribute Information: 8x8 image of integer pixels in the range 0..16.
    :Missing Attribute Values: None
    :Creator: E. Alpaydin (alpaydin '@' boun.edu.tr)
    :Date: July; 1998

This is a copy of the test set of the UCI ML hand-written digits datasets
https://archive.ics.uci.edu/ml/datasets/Optical+Recognition+of+Handwritten+Digit
s

The data set contains images of hand-written digits: 10 classes where
each class refers to a digit.

Preprocessing programs made available by NIST were used to extract
normalized bitmaps of handwritten digits from a preprinted form. From a
total of 43 people, 30 contributed to the training set and different 13
```

```
to the test set. 32x32 bitmaps are divided into nonoverlapping blocks of
4x4 and the number of on pixels are counted in each block. This generates
an input matrix of 8x8 where each element is an integer in the range
0..16. This reduces dimensionality and gives invariance to small
distortions.

For info on NIST preprocessing routines, see M. D. Garris, J. L. Blue, G.
T. Candela, D. L. Dimmick, J. Geist, P. J. Grother, S. A. Janet, and C.
L. Wilson, NIST Form-Based Handprint Recognition System, NISTIR 5469,
1994.

.. topic:: References

  - C. Kaynak (1995) Methods of Combining Multiple Classifiers and Their
    Applications to Handwritten Digit Recognition, MSc Thesis, Institute of
    Graduate Studies in Science and Engineering, Bogazici University.
  - E. Alpaydin, C. Kaynak (1998) Cascading Classifiers, Kybernetika.
  - Ken Tang and Ponnuthurai N. Suganthan and Xi Yao and A. Kai Qin.
    Linear dimensionalityreduction using relevance weighted LDA. School of
    Electrical and Electronic Engineering Nanyang Technological University.
    2005.
  - Claudio Gentile. A New Approximate Maximal Margin Classification
    Algorithm. NIPS. 2000.
```

[491]:

```python
%matplotlib inline
import matplotlib.pyplot as plt
import numpy as np

# show examples of dataset
plt.figure(figsize=(20,4))
for index, (image, label) in enumerate(zip(digits.data[0:5], digits.target[0:
 ↪5])):
    plt.subplot(1, 5, index + 1)
    plt.imshow(np.reshape(image, (8,8)), cmap=plt.cm.gray)
    plt.title('Training: %i\n' % label, fontsize = 20)
```



[492]:
```python
# train-test split
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(digits.data, digits.target,␣
 ↪test_size=0.25, random_state=8)
print(X_train[256], y_train[256])
```

```
[ 0.  0.  2. 16. 10.  0.  0.  0.  0.  0.  4. 16. 16.  5.  0.  0.  0.  0.
  8. 16. 16.  3.  0.  0.  0.  0.  9. 16. 16.  3.  0.  0.  0.  0.  8. 16.
 16.  3.  0.  0.  0.  0.  8. 16. 16.  1.  0.  0.  0.  0.  5. 16. 14.  0.
  0.  0.  0.  0.  1. 12. 16.  3.  0.  0.] 1
```

[493]:
```python
# 3.2 batch gradient descent (GD) for Logistic regression
def LogisticRegression_GD(X_train, y_train, learning_rate):
    loss = []
    dataset_size, data_attributes = X_train.shape
    print("Dataset Size : ", dataset_size)
    epsilon = 1.0*np.exp(-4)
    eta = 0.1
    c = 10
    W = np.random.normal(0, 1, (data_attributes*c))
    b = np.zeros(c)
    W.resize(data_attributes, c)
    while 1 :
        Z = np.matmul(X_train, W)
        for i in range(dataset_size):
            Z[i] = Z[i] + b
        k=0
        for i in range(dataset_size):
            for j in range(c):
                if(Z[i][j] > k) :
                    k = Z[i][j]
        Z = Z - k*np.ones(Z.shape)
        Z = np.exp(Z)
        Z_sum = np.sum(Z, axis = 1)
        for i in range(dataset_size):
            for j in range(c):
                Z[i][j] = Z[i][j]/Z_sum[i]
        Zlog = -np.log(Z)
        F = Zlog.sum()
        F = F/dataset_size
        W_2 = W**2
        W_2_reg = eta*W_2.sum()/2
        F = F + W_2_reg
        loss.append(F)
        if len(loss) >= 2 :
            if np.abs(loss[-1] - loss[-2]) < epsilon :
                break
```

```python
        #Gradient Descent Stuff
        gradient_desc_sum = 0;
        for i in range(dataset_size):
            error = np.zeros(c)
            for j in range(c):
                if(y_train[i] == j+1):
                    error[j] = (1 - Z[i][j])
                else :
                    error[j] = (-Z[i][j])
            temp = np.zeros((data_attributes, c))
            for j in range(data_attributes):
                for k in range(c):
                    temp[j][k] = X_train[i][j]*error[k]
            W = W + (learning_rate*temp/dataset_size)
        W = W - (learning_rate*eta*W)
    return W, b, np.array(loss)
```

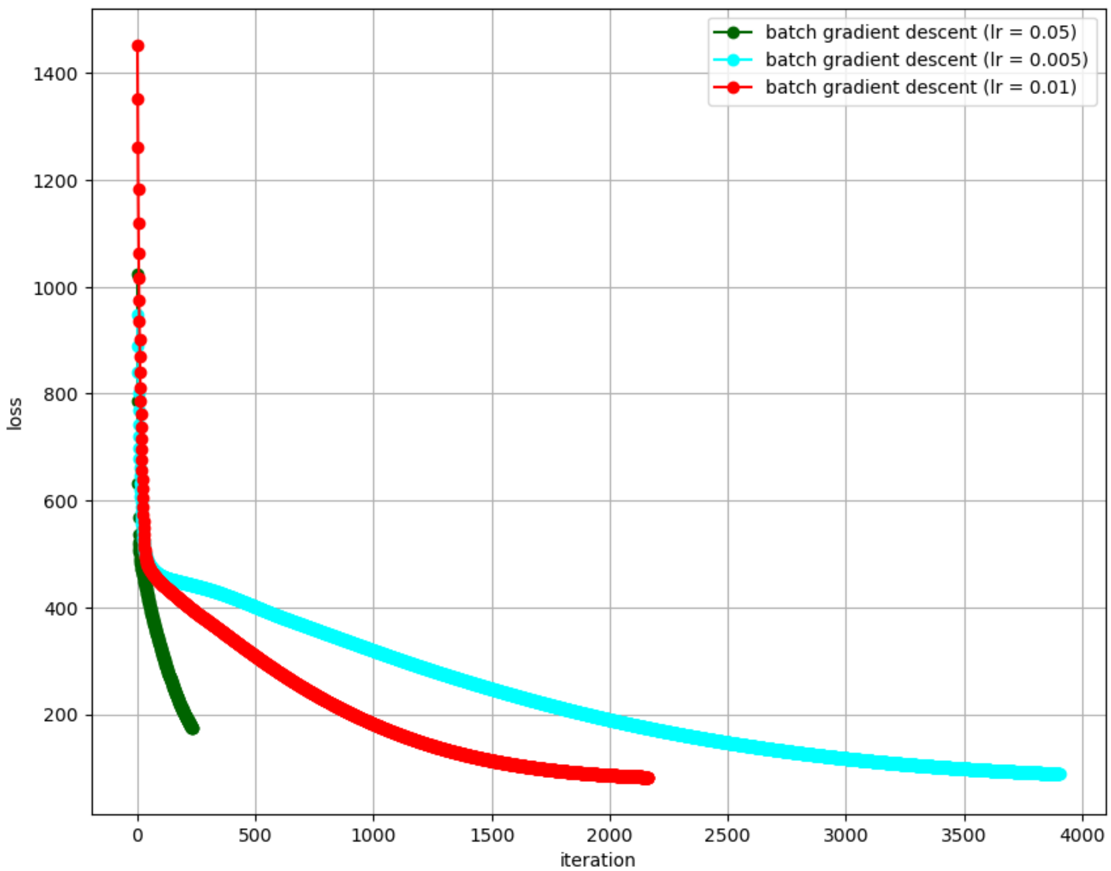
[495]:

```python
def calculate_precision(Y, W, X):
    Z = np.matmul(X, W)
    Z = np.exp(Z)
    c = 10
    dataset_size, data_attributes = X.shape
    Z_sum = np.sum(Z, axis = 1)
    for i in range(dataset_size):
            for j in range(c):
                Z[i][j] = Z[i][j]/Z_sum[i]
    argmax = np.argmax(Z, axis = 1)

    correct_predictions = 0
    for i in range(dataset_size):
        if Y[i] == (argmax[i] + 1):
            correct_predictions += 1
    precision = (correct_predictions/dataset_size)*100
    return precision

# evaluation of different learning rate
learning_rate = [5.0e-2, 5.0e-3, 1.0e-2]
cl = ['darkgreen', 'cyan', 'red']
fig, ax = plt.subplots(figsize=(10, 8))
for i in range(len(learning_rate)):

    print('----------------------------------------')
    print('learning rate =', learning_rate[i])

    W, b, loss_GD = LogisticRegression_GD(X_train, y_train, learning_rate[i])

    print("Final Loss Value : ", loss_GD[-1])
```

```python
    prec_train = calculate_precision(y_train, W, X_train)
    print('training precision =', prec_train)

    prec_test = calculate_precision(y_test, W, X_test)
    print('test precision =', prec_test)

    plt.plot(loss_GD, c = cl[i], ls = '-', marker = 'o', label = 'batch␣
↪gradient descent (lr = ' + str(learning_rate[i]) + ')')

plt.grid()
plt.legend()
plt.xlabel('iteration')
plt.ylabel('loss')
```

```
---------------------------------------
learning rate = 0.05
Dataset Size :  1347
Final Loss Value :  174.10307939002357
training precision = 80.69784706755753
test precision = 78.88888888888889
---------------------------------------
learning rate = 0.005
Dataset Size :  1347
Final Loss Value :  88.26890279017098
training precision = 88.2702301410542
test precision = 86.22222222222223
---------------------------------------
learning rate = 0.01
Dataset Size :  1347
Final Loss Value :  81.93692451199382
training precision = 88.71566443949519
test precision = 86.66666666666667
```

[495]: Text(0, 0.5, 'loss')



[509]:
```
# 3.3 stochastic gradient descent (SGD) for Logistic regression

def LogisticRegression_SGD(X, y, batch_size, lr=1.0e-2, eta=2.0e-1, eps = 1.
↪0e-4, max_epoch=500):

    loss = []
    dataset_size, data_attributes = X_train.shape
    print("Dataset Size : ", dataset_size)
    epsilon = 1.0*np.exp(-4)
    eta = 0.1
    c = 10
    notstop = True
    W = np.random.normal(0, 1, (data_attributes*c))
    b = np.zeros(c)
    W.resize(data_attributes, c)
    batch_starting_point = 0
    epoch = 0
    # optimization loop
    while notstop and epoch < max_epoch:
```

```
        batch_ending_point = 0
        if batch_starting_point + batch_size <= dataset_size:
            batch_starting_point = 0

        Z = np.matmul(X[batch_starting_point:batch_starting_point+100], W)
        batch_starting_point = batch_starting_point+100
        for i in range(batch_size):
            Z[i] = Z[i] + b
        k=0
        for i in range(batch_size):
            for j in range(c):
                if(Z[i][j] > k) :
                    k = Z[i][j]
        Z = Z - k*np.ones(Z.shape)
        Z = np.exp(Z)
        Z_sum = np.sum(Z, axis = 1)
        for i in range(batch_size):
            for j in range(c):
                Z[i][j] = Z[i][j]/Z_sum[i]
        Zlog = -np.log(Z)
        F = Zlog.sum()
        F = F/batch_size
        W_2 = W**2
        W_2_reg = eta*W_2.sum()/2
        F = F + W_2_reg
        loss.append(F)
        if len(loss) >= 2 :
            if np.abs(loss[-1] - loss[-2]) < epsilon :
                break

        # half lr if not improving in 10 epochs
        if epoch > 10:
            if loss[epoch - 10] <= loss[epoch] - eps:
                lr *= 0.5
                print('reduce learning rate to', lr)

        # stop if not improving in 20 epochs
        if epoch > 20:
            if loss[epoch - 20] <= loss[epoch] - eps or abs(loss[epoch] - loss[epoch-1]) <= eps:
                notstop = False
                break

        #Gradient Descent Stuff
        gradient_desc_sum = 0;
        for i in range(batch_size):
            error = np.zeros(c)
```

```
            for j in range(c):
                if(y_train[i] == j+1):
                    error[j] = (1 - Z[i][j])
                else :
                    error[j] = (-Z[i][j])
            temp = np.zeros((data_attributes, c))
            for j in range(data_attributes):
                for k in range(c):
                    temp[j][k] = X_train[i][j]*error[k]
            W = W + (lr*temp/batch_size)
        W = W - (lr*eta*W)
        epoch += 1

    return (W, b, np.array(loss))
```

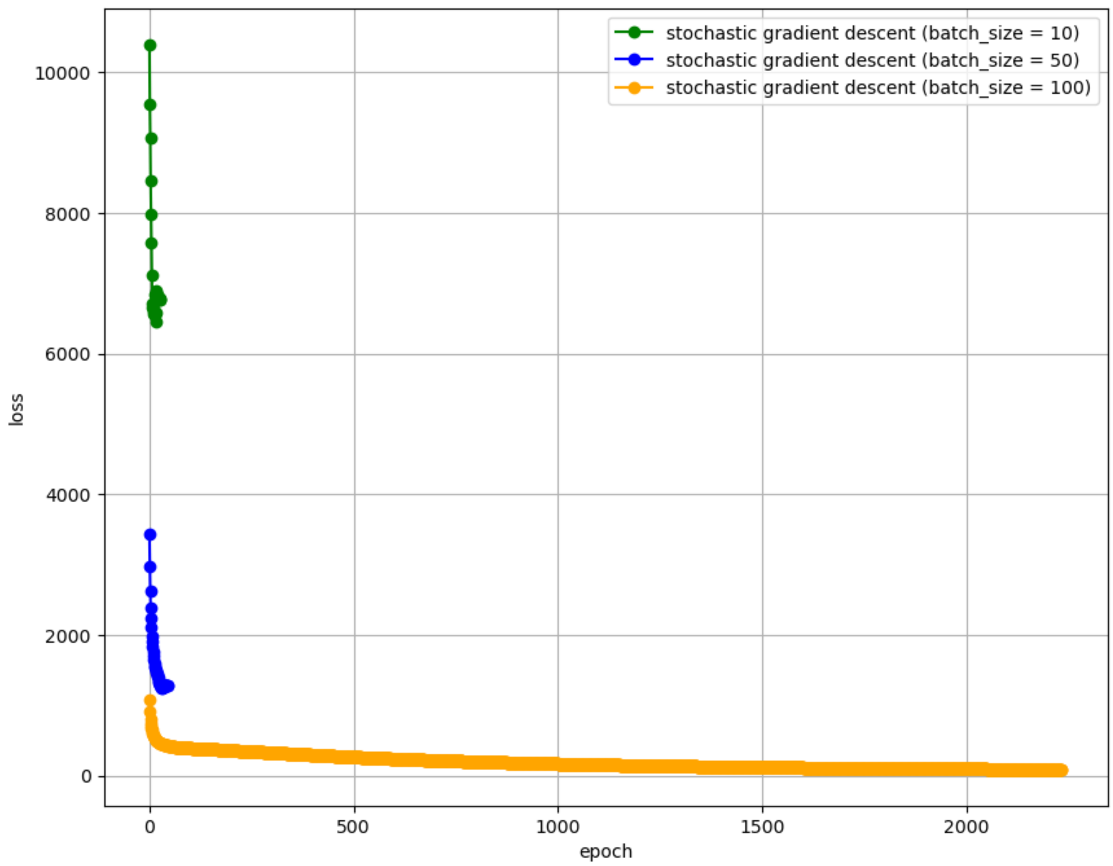
[527]:

```
# evaluation of different batch size
bs = [10, 50, 100]
cl = ['green', 'blue', 'orange']
# Standard Learning Rate
lr = [1.0e-2, 1.0e-2, 1.0e-2]
fig, ax = plt.subplots(figsize=(10, 8))

for i in range(len(bs)):

    print('----------------------------------------')
    print('batch_size =', bs[i])
    W, b, loss_SGD = LogisticRegression_SGD(X_train, y_train, bs[i], lr[i], eta␣
 ↪= 2.0e-1, eps = 1.0e-4, max_epoch = 2500)

    print("Final Loss Value : ", loss_SGD[-1])
    prec_train = calculate_precision(y_train, W, X_train)
    print('training precision =', prec_train)

    prec_test = calculate_precision(y_test, W, X_test)
    print('test precision =', prec_test)

    plt.plot(loss_SGD, c = cl[i], ls = '-', marker = 'o', label = 'stochastic␣
 ↪gradient descent (batch_size = ' + str(bs[i]) + ')')

plt.grid()
plt.legend()
plt.xlabel('epoch')
plt.ylabel('loss')
```

```
----------------------------------------
batch_size = 10
Dataset Size :  1347
```

```
reduce learning rate to 0.005
reduce learning rate to 0.0025
reduce learning rate to 0.00125
reduce learning rate to 0.000625
reduce learning rate to 0.0003125
reduce learning rate to 0.00015625
reduce learning rate to 7.8125e-05
reduce learning rate to 3.90625e-05
Final Loss Value :  6775.26035476595
training precision = 13.21455085374907
test precision = 15.777777777777777
---------------------------------------
batch_size = 50
Dataset Size :  1347
reduce learning rate to 0.005
reduce learning rate to 0.0025
reduce learning rate to 0.00125
reduce learning rate to 0.000625
reduce learning rate to 0.0003125
reduce learning rate to 0.00015625
reduce learning rate to 7.8125e-05
reduce learning rate to 3.90625e-05
Final Loss Value :  1290.2466192487811
training precision = 27.171492204899778
test precision = 23.333333333333332
---------------------------------------
batch_size = 100
Dataset Size :  1347
Final Loss Value :  91.59868972107643
training precision = 75.27839643652561
test precision = 74.22222222222223
```

```
[527]: Text(0, 0.5, 'loss')
```



[546]:
```
# evaluation of different batch size
bs = [10, 50, 100]
cl = ['green', 'blue', 'orange']
# Experimented Learning Rates
lr = [3.0e-1, 3.0e-1, 1.0e-2]
fig, ax = plt.subplots(figsize=(10, 8))

for i in range(len(bs)):

    print('----------------------------------------')
    print('batch_size =', bs[i])
    W, b, loss_SGD = LogisticRegression_SGD(X_train, y_train, bs[i], lr[i], eta␣
↪= 2.0e-1, eps = 1.0e-4, max_epoch = 2500)

    print("Final Loss Value : ", loss_SGD[-1])
    prec_train = calculate_precision(y_train, W, X_train)
    print('training precision =', prec_train)

    prec_test = calculate_precision(y_test, W,  X_test)
```

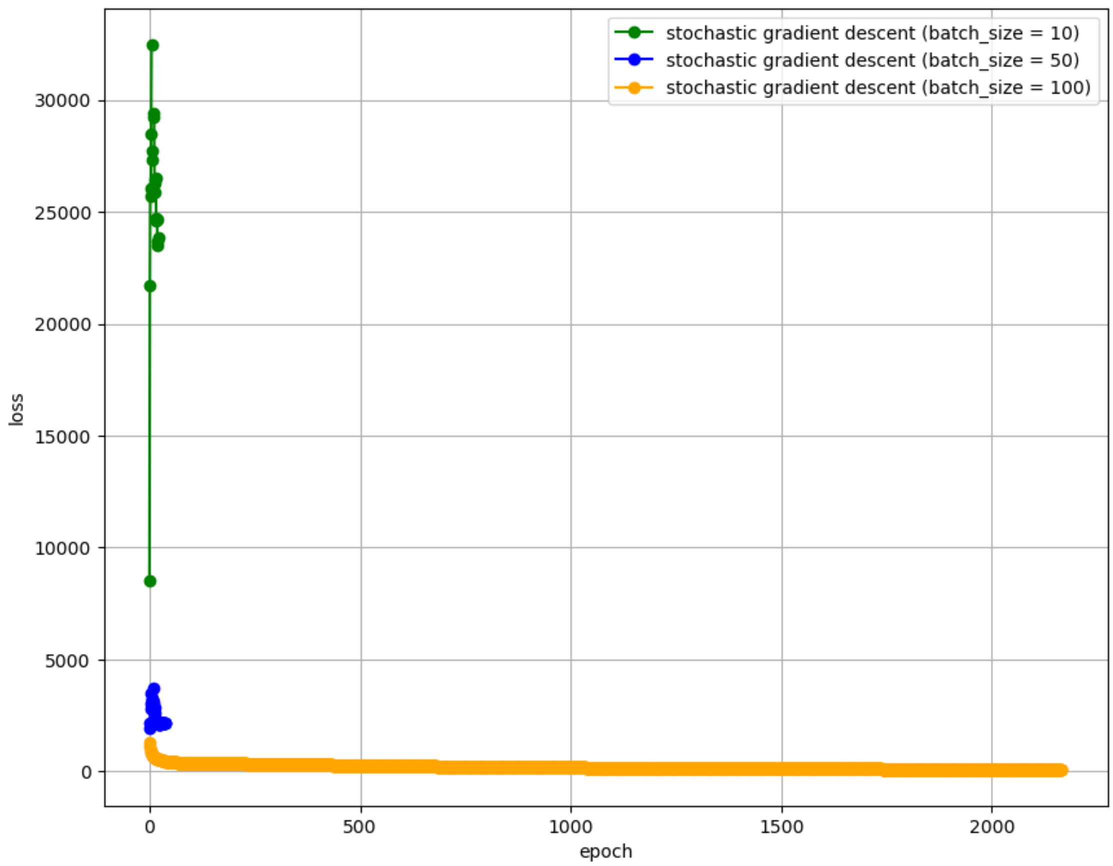
```
    print('test precision =', prec_test)

    plt.plot(loss_SGD, c = cl[i], ls = '-', marker = 'o', label = 'stochastic␣
 ↪gradient descent (batch_size = ' + str(bs[i]) + ')')

plt.grid()
plt.legend()
plt.xlabel('epoch')
plt.ylabel('loss')
```

```
----------------------------------------
batch_size = 10
Dataset Size :  1347
reduce learning rate to 0.15
reduce learning rate to 0.075
Final Loss Value :  23823.882027049494
training precision = 32.44246473645137
test precision = 35.55555555555556
----------------------------------------
batch_size = 50
Dataset Size :  1347
```

```
/var/folders/c9/3q_3q36n4k9_t4gnwrhyd7s80000gn/T/ipykernel_72269/3995326649.py:3
8: RuntimeWarning: divide by zero encountered in log
  Zlog = -np.log(Z)
```

```
reduce learning rate to 0.15
reduce learning rate to 0.075
reduce learning rate to 0.0375
reduce learning rate to 0.01875
reduce learning rate to 0.009375
reduce learning rate to 0.0046875
reduce learning rate to 0.00234375
reduce learning rate to 0.001171875
reduce learning rate to 0.0005859375
reduce learning rate to 0.00029296875
reduce learning rate to 0.000146484375
Final Loss Value :  2185.15948244421
training precision = 70.2301410541945
test precision = 70.0
----------------------------------------
batch_size = 100
Dataset Size :  1347
Final Loss Value :  91.29047788240926
training precision = 73.57089829250185
test precision = 72.0
```

[546]:
```
Text(0, 0.5, 'loss')
```



stochastic gradient descent (batch_size = 10)
stochastic gradient descent (batch_size = 50)
stochastic gradient descent (batch_size = 100)
30000
25000
20000
15000
10000
5000
0
loss
0
500
1000
1500
2000
epoch


```
[ ]:
```