imph6h73h

February 9, 2024

[151]:
```python
# load data
from sklearn import datasets
boston = datasets.load_boston()
print(boston.keys())
```

```
dict_keys(['data', 'target', 'feature_names', 'DESCR', 'filename',
'data_module'])
```

```
/Users/prateek/anaconda3/envs/AnacondaTest/lib/python3.11/site-
packages/sklearn/utils/deprecation.py:87: FutureWarning: Function load_boston is
deprecated; `load_boston` is deprecated in 1.0 and will be removed in 1.2.

    The Boston housing prices dataset has an ethical problem. You can refer to
    the documentation of this function for further details.

    The scikit-learn maintainers therefore strongly discourage the use of this
    dataset unless the purpose of the code is to study and educate about
    ethical issues in data science and machine learning.

    In this special case, you can fetch the dataset from the original
    source::

        import pandas as pd
        import numpy as np

        data_url = "http://lib.stat.cmu.edu/datasets/boston"
        raw_df = pd.read_csv(data_url, sep="\s+", skiprows=22, header=None)
        data = np.hstack([raw_df.values[::2, :], raw_df.values[1::2, :2]])
        target = raw_df.values[1::2, 2]

    Alternative datasets include the California housing dataset (i.e.
    :func:`~sklearn.datasets.fetch_california_housing`) and the Ames housing
    dataset. You can load the datasets as follows::

        from sklearn.datasets import fetch_california_housing
        housing = fetch_california_housing()

    for the California housing dataset and::
```

```
        from sklearn.datasets import fetch_openml
        housing = fetch_openml(name="house_prices", as_frame=True)

    for the Ames housing dataset.
  warnings.warn(msg, category=FutureWarning)
```

[152]:
```python
# a form of summary of the data
feature = boston.data
price = boston.target
print('data size = ', feature.shape)
print('target size = ', price.shape)
print('feature attributes: ', boston.feature_names)
print(boston.DESCR)
```

```
data size =  (506, 13)
target size =  (506,)
feature attributes:  ['CRIM' 'ZN' 'INDUS' 'CHAS' 'NOX' 'RM' 'AGE' 'DIS' 'RAD'
'TAX' 'PTRATIO'
 'B' 'LSTAT']
.. _boston_dataset:

Boston house prices dataset
---------------------------

**Data Set Characteristics:**

    :Number of Instances: 506

    :Number of Attributes: 13 numeric/categorical predictive. Median Value
(attribute 14) is usually the target.

    :Attribute Information (in order):
        - CRIM     per capita crime rate by town
        - ZN       proportion of residential land zoned for lots over 25,000
sq.ft.
        - INDUS    proportion of non-retail business acres per town
        - CHAS     Charles River dummy variable (= 1 if tract bounds river; 0
otherwise)
        - NOX      nitric oxides concentration (parts per 10 million)
        - RM       average number of rooms per dwelling
        - AGE      proportion of owner-occupied units built prior to 1940
        - DIS      weighted distances to five Boston employment centres
        - RAD      index of accessibility to radial highways
        - TAX      full-value property-tax rate per $10,000
        - PTRATIO  pupil-teacher ratio by town
        - B        1000(Bk - 0.63)^2 where Bk is the proportion of black people
by town
```

```
        - LSTAT    % lower status of the population
        - MEDV     Median value of owner-occupied homes in $1000's

    :Missing Attribute Values: None

    :Creator: Harrison, D. and Rubinfeld, D.L.

This is a copy of UCI ML housing dataset.
https://archive.ics.uci.edu/ml/machine-learning-databases/housing/


This dataset was taken from the StatLib library which is maintained at Carnegie
Mellon University.

The Boston house-price data of Harrison, D. and Rubinfeld, D.L. 'Hedonic
prices and the demand for clean air', J. Environ. Economics & Management,
vol.5, 81-102, 1978.   Used in Belsley, Kuh & Welsch, 'Regression diagnostics
…', Wiley, 1980.   N.B. Various transformations are used in the table on
pages 244-261 of the latter.

The Boston house-price data has been used in many machine learning papers that
address regression
problems.

.. topic:: References

   - Belsley, Kuh & Welsch, 'Regression diagnostics: Identifying Influential
Data and Sources of Collinearity', Wiley, 1980. 244-261.
   - Quinlan,R. (1993). Combining Instance-Based and Model-Based Learning. In
Proceedings on the Tenth International Conference of Machine Learning, 236-243,
University of Massachusetts, Amherst. Morgan Kaufmann.
```

[153]:

```python
# more details of the data
import pandas as pd
df_feature = pd.DataFrame(feature, columns = boston.feature_names)
df_target = pd.DataFrame(price, columns =['MEDV'])
df_boston = pd.concat([df_feature, df_target,], axis = 1)
```

[154]:

```python
df_boston.head()
```

[154]:

```
      CRIM    ZN  INDUS  CHAS    NOX     RM   AGE     DIS  RAD    TAX  \
0  0.00632  18.0   2.31   0.0  0.538  6.575  65.2  4.0900  1.0  296.0
1  0.02731   0.0   7.07   0.0  0.469  6.421  78.9  4.9671  2.0  242.0
2  0.02729   0.0   7.07   0.0  0.469  7.185  61.1  4.9671  2.0  242.0
3  0.03237   0.0   2.18   0.0  0.458  6.998  45.8  6.0622  3.0  222.0
4  0.06905   0.0   2.18   0.0  0.458  7.147  54.2  6.0622  3.0  222.0
```

```
   PTRATIO       B  LSTAT  MEDV
0     15.3  396.90   4.98  24.0
1     17.8  396.90   9.14  21.6
2     17.8  392.83   4.03  34.7
3     18.7  394.63   2.94  33.4
4     18.7  396.90   5.33  36.2
```

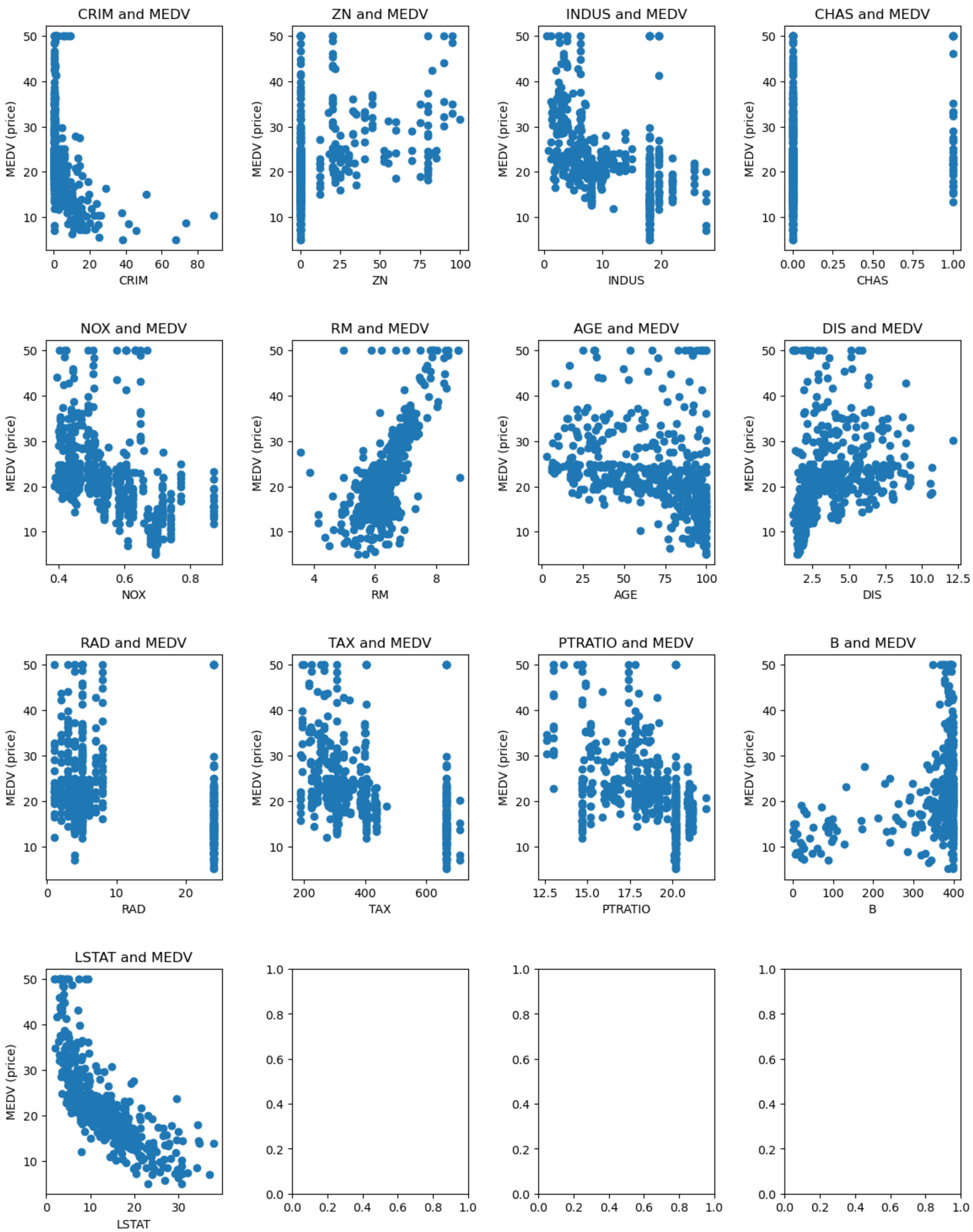
[155]:
```
df_boston.describe()
```

[155]:
```
             CRIM          ZN       INDUS        CHAS         NOX          RM  \
count  506.000000  506.000000  506.000000  506.000000  506.000000  506.000000
mean     3.613524   11.363636   11.136779    0.069170    0.554695    6.284634
std      8.601545   23.322453    6.860353    0.253994    0.115878    0.702617
min      0.006320    0.000000    0.460000    0.000000    0.385000    3.561000
25%      0.082045    0.000000    5.190000    0.000000    0.449000    5.885500
50%      0.256510    0.000000    9.690000    0.000000    0.538000    6.208500
75%      3.677083   12.500000   18.100000    0.000000    0.624000    6.623500
max     88.976200  100.000000   27.740000    1.000000    0.871000    8.780000

              AGE         DIS         RAD         TAX     PTRATIO           B  \
count  506.000000  506.000000  506.000000  506.000000  506.000000  506.000000
mean    68.574901    3.795043    9.549407  408.237154   18.455534  356.674032
std     28.148861    2.105710    8.707259  168.537116    2.164946   91.294864
min      2.900000    1.129600    1.000000  187.000000   12.600000    0.320000
25%     45.025000    2.100175    4.000000  279.000000   17.400000  375.377500
50%     77.500000    3.207450    5.000000  330.000000   19.050000  391.440000
75%     94.075000    5.188425   24.000000  666.000000   20.200000  396.225000
max    100.000000   12.126500   24.000000  711.000000   22.000000  396.900000

            LSTAT        MEDV
count  506.000000  506.000000
mean    12.653063   22.532806
std      7.141062    9.197104
min      1.730000    5.000000
25%      6.950000   17.025000
50%     11.360000   21.200000
75%     16.955000   25.000000
max     37.970000   50.000000
```

[156]:
```
# 2.1 how does each feature relate to the price
import matplotlib.pyplot as plt
plt.figure()
fig,axes = plt.subplots(4, 4, figsize=(14,18))
fig.subplots_adjust(wspace=.4, hspace=.4)
img_index = 0
for i in range(boston.feature_names.size):
```

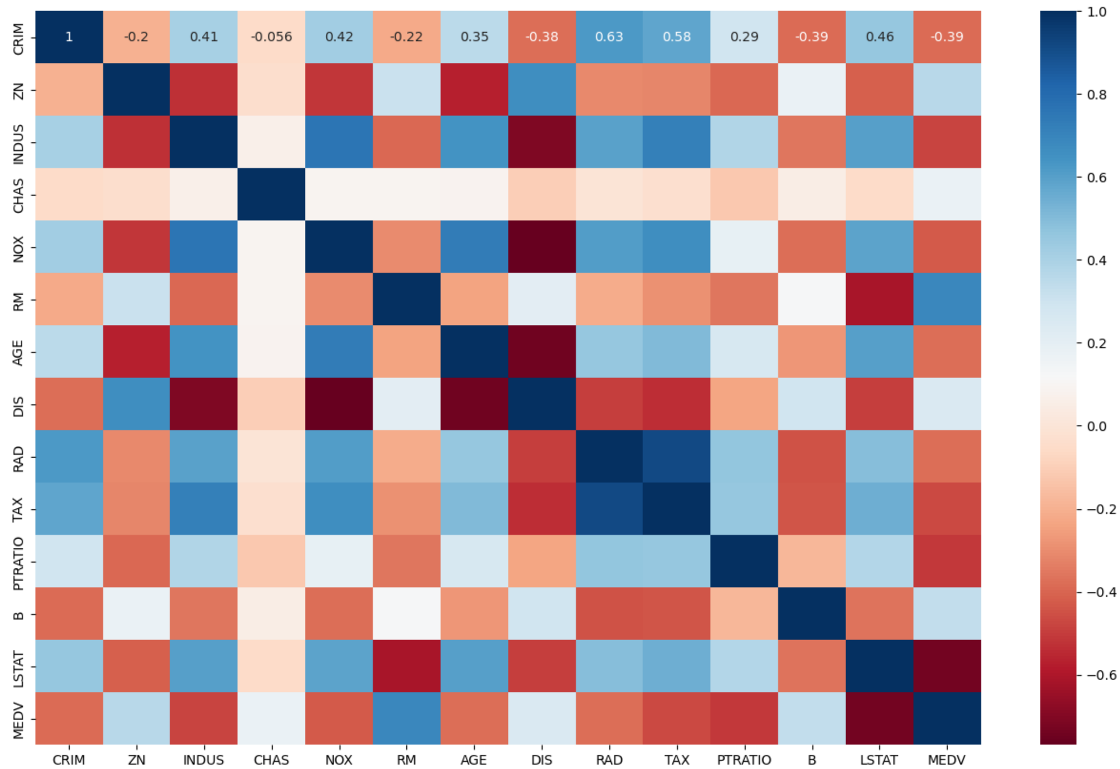
```
    row, col = i // 4, i % 4
    axes[row][col].scatter(feature[:,i], price)
    axes[row][col].set_title(boston.feature_names[i] + ' and MEDV')
    axes[row][col].set_xlabel(boston.feature_names[i])
    axes[row][col].set_ylabel('MEDV (price)')
plt.show()
```

```
<Figure size 640x480 with 0 Axes>
```



[157]:
```python
# 2.2 correlation matrix
import seaborn as sns
fig, ax = plt.subplots(figsize=(16, 10))
correlation = df_boston.corr()
sns.heatmap(correlation, annot = True, cmap = 'RdBu')
```

```
plt.show()
correlation
```



[157]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE
CRIM	1.000000	-0.200469	0.406583	-0.055892	0.420972	-0.219247	0.352734
ZN	-0.200469	1.000000	-0.533828	-0.042697	-0.516604	0.311991	-0.569537
INDUS	0.406583	-0.533828	1.000000	0.062938	0.763651	-0.391676	0.644779
CHAS	-0.055892	-0.042697	0.062938	1.000000	0.091203	0.091251	0.086518
NOX	0.420972	-0.516604	0.763651	0.091203	1.000000	-0.302188	0.731470
RM	-0.219247	0.311991	-0.391676	0.091251	-0.302188	1.000000	-0.240265
AGE	0.352734	-0.569537	0.644779	0.086518	0.731470	-0.240265	1.000000
DIS	-0.379670	0.664408	-0.708027	-0.099176	-0.769230	0.205246	-0.747881
RAD	0.625505	-0.311948	0.595129	-0.007368	0.611441	-0.209847	0.456022
TAX	0.582764	-0.314563	0.720760	-0.035587	0.668023	-0.292048	0.506456
PTRATIO	0.289946	-0.391679	0.383248	-0.121515	0.188933	-0.355501	0.261515
B	-0.385064	0.175520	-0.356977	0.048788	-0.380051	0.128069	-0.273534
LSTAT	0.455621	-0.412995	0.603800	-0.053929	0.590879	-0.613808	0.602339
MEDV	-0.388305	0.360445	-0.483725	0.175260	-0.427321	0.695360	-0.376955

	DIS	RAD	TAX	PTRATIO	B	LSTAT	MEDV
CRIM	-0.379670	0.625505	0.582764	0.289946	-0.385064	0.455621	-0.388305
ZN	0.664408	-0.311948	-0.314563	-0.391679	0.175520	-0.412995	0.360445

```
INDUS   -0.708027  0.595129  0.720760  0.383248 -0.356977  0.603800 -0.483725
CHAS    -0.099176 -0.007368 -0.035587 -0.121515  0.048788 -0.053929  0.175260
NOX     -0.769230  0.611441  0.668023  0.188933 -0.380051  0.590879 -0.427321
RM       0.205246 -0.209847 -0.292048 -0.355501  0.128069 -0.613808  0.695360
AGE     -0.747881  0.456022  0.506456  0.261515 -0.273534  0.602339 -0.376955
DIS      1.000000 -0.494588 -0.534432 -0.232471  0.291512 -0.496996  0.249929
RAD     -0.494588  1.000000  0.910228  0.464741 -0.444413  0.488676 -0.381626
TAX     -0.534432  0.910228  1.000000  0.460853 -0.441808  0.543993 -0.468536
PTRATIO -0.232471  0.464741  0.460853  1.000000 -0.177383  0.374044 -0.507787
B        0.291512 -0.444413 -0.441808 -0.177383  1.000000 -0.366087  0.333461
LSTAT   -0.496996  0.488676  0.543993  0.374044 -0.366087  1.000000 -0.737663
MEDV     0.249929 -0.381626 -0.468536 -0.507787  0.333461 -0.737663  1.000000
```

[158]:
```
# train-test split
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(feature, price, test_size=0.
 ↪3, random_state=8)
```

[159]:
```
# 2.3 linear regression and ridge regression
import numpy as np

def least_square(X, y):
    #TODO
    theta = np.matmul(X.transpose(),X)
    theta = np.linalg.inv(theta)
    theta = np.matmul(theta,X.transpose())
    theta = np.matmul(theta, y)
    return theta

def ridge_reg(X, y, eta):
    #TODO
    theta = np.matmul(X.transpose(),X)
    num_rows, num_columns = theta.shape
    theta = theta + (eta/2)*np.identity(num_rows)
    theta = np.linalg.inv(theta)
    theta = np.matmul(theta,X.transpose())
    theta = np.matmul(theta, y)
    return theta

# apply linear regression
theta = least_square(X_train, y_train)
df_theta = pd.DataFrame(zip(boston.feature_names,␣
 ↪theta),columns=['Feature','Coeff'])
print("Linear Regression Output")
display(df_theta)

df_theta_r_temp = df_theta
```

```python
# apply ridge regression
for i in range(25):
    theta_r = ridge_reg(X_train, y_train, i)
    df_theta_r = pd.DataFrame(zip(boston.feature_names,␣
 ↪theta_r),columns=['Feature','Coeff'])
    print("Ridge Regression Output with eta = ", i)
    if(i == 20):
        df_theta_r_temp = df_theta_r
    display(df_theta_r)

# Take eta as 20, for the right balance of training vs testing dataset accuracy
df_theta_r = df_theta_r_temp
```

```
Linear Regression Output

    Feature     Coeff
0      CRIM -0.099324
1        ZN  0.052251
2     INDUS  0.004516
3      CHAS  2.957261
4       NOX  1.127938
5        RM  5.854198
6       AGE -0.014957
7       DIS -0.920844
8       RAD  0.159519
9       TAX -0.008934
10  PTRATIO -0.435674
11        B  0.014905
12    LSTAT -0.474751

Ridge Regression Output with eta =  0

    Feature     Coeff
0      CRIM -0.099324
1        ZN  0.052251
2     INDUS  0.004516
3      CHAS  2.957261
4       NOX  1.127938
5        RM  5.854198
6       AGE -0.014957
7       DIS -0.920844
8       RAD  0.159519
9       TAX -0.008934
10  PTRATIO -0.435674
11        B  0.014905
12    LSTAT -0.474751

Ridge Regression Output with eta =  1

    Feature     Coeff
```

0	CRIM	-0.099514
1	ZN	0.052396
2	INDUS	0.005971
3	CHAS	2.901961
4	NOX	0.928628
5	RM	5.852968
6	AGE	-0.014481
7	DIS	-0.920441
8	RAD	0.159970
9	TAX	-0.008921
10	PTRATIO	-0.433651
11	B	0.014964
12	LSTAT	-0.474664

Ridge Regression Output with eta = 2

	Feature	Coeff
0	CRIM	-0.099660
1	ZN	0.052550
2	INDUS	0.006999
3	CHAS	2.846734
4	NOX	0.810144
5	RM	5.848060
6	AGE	-0.014077
7	DIS	-0.919409
8	RAD	0.160346
9	TAX	-0.008919
10	PTRATIO	-0.431701
11	B	0.015015
12	LSTAT	-0.474969

Ridge Regression Output with eta = 3

	Feature	Coeff
0	CRIM	-0.099782
1	ZN	0.052709
2	INDUS	0.007809
3	CHAS	2.792744
4	NOX	0.731256
5	RM	5.841327
6	AGE	-0.013709
7	DIS	-0.918069
8	RAD	0.160677
9	TAX	-0.008922
10	PTRATIO	-0.429775
11	B	0.015062
12	LSTAT	-0.475464

Ridge Regression Output with eta = 4

	Feature	Coeff

```
0      CRIM -0.099889
1        ZN  0.052870
2     INDUS  0.008487
3      CHAS  2.740394
4       NOX  0.674718
5        RM  5.833546
6       AGE -0.013363
7       DIS -0.916557
8       RAD  0.160976
9       TAX -0.008926
10  PTRATIO -0.427853
11        B  0.015106
12    LSTAT -0.476063
```

```
Ridge Regression Output with eta =  5
```

```
    Feature     Coeff
0      CRIM -0.099986
1        ZN  0.053032
2     INDUS  0.009079
3      CHAS  2.689813
4       NOX  0.632042
5        RM  5.825104
6       AGE -0.013033
7       DIS -0.914939
8       RAD  0.161248
9       TAX -0.008932
10  PTRATIO -0.425927
11        B  0.015148
12    LSTAT -0.476724
```

```
Ridge Regression Output with eta =  6
```

```
    Feature     Coeff
0      CRIM -0.100074
1        ZN  0.053194
2     INDUS  0.009608
3      CHAS  2.641017
4       NOX  0.598560
5        RM  5.816216
6       AGE -0.012713
7       DIS -0.913252
8       RAD  0.161499
9       TAX -0.008938
10  PTRATIO -0.423992
11        B  0.015189
12    LSTAT -0.477424
```

```
Ridge Regression Output with eta =  7
```

```
    Feature     Coeff
```

0	CRIM	-0.100155
1	ZN	0.053356
2	INDUS	0.010089
3	CHAS	2.593974
4	NOX	0.571497
5	RM	5.807010
6	AGE	-0.012402
7	DIS	-0.911518
8	RAD	0.161731
9	TAX	-0.008945
10	PTRATIO	-0.422048
11	B	0.015228
12	LSTAT	-0.478151

Ridge Regression Output with eta = 8

	Feature	Coeff
0	CRIM	-0.100231
1	ZN	0.053517
2	INDUS	0.010532
3	CHAS	2.548625
4	NOX	0.549095
5	RM	5.797569
6	AGE	-0.012097
7	DIS	-0.909751
8	RAD	0.161946
9	TAX	-0.008951
10	PTRATIO	-0.420093
11	B	0.015267
12	LSTAT	-0.478894

Ridge Regression Output with eta = 9

	Feature	Coeff
0	CRIM	-0.100302
1	ZN	0.053679
2	INDUS	0.010943
3	CHAS	2.504903
4	NOX	0.530189
5	RM	5.787950
6	AGE	-0.011799
7	DIS	-0.907961
8	RAD	0.162146
9	TAX	-0.008956
10	PTRATIO	-0.418128
11	B	0.015304
12	LSTAT	-0.479650

Ridge Regression Output with eta = 10

	Feature	Coeff

```
0      CRIM -0.100368
1        ZN  0.053839
2     INDUS  0.011329
3      CHAS  2.462736
4       NOX  0.513974
5        RM  5.778194
6       AGE -0.011505
7       DIS -0.906153
8       RAD  0.162332
9       TAX -0.008962
10  PTRATIO -0.416153
11        B  0.015341
12    LSTAT -0.480413
```

```
Ridge Regression Output with eta =  11
```

```
    Feature     Coeff
0      CRIM -0.100431
1        ZN  0.053999
2     INDUS  0.011691
3      CHAS  2.422052
4       NOX  0.499876
5        RM  5.768328
6       AGE -0.011216
7       DIS -0.904333
8       RAD  0.162505
9       TAX -0.008967
10  PTRATIO -0.414168
11        B  0.015378
12    LSTAT -0.481181
```

```
Ridge Regression Output with eta =  12
```

```
    Feature     Coeff
0      CRIM -0.100490
1        ZN  0.054158
2     INDUS  0.012033
3      CHAS  2.382781
4       NOX  0.487477
5        RM  5.758376
6       AGE -0.010930
7       DIS -0.902504
8       RAD  0.162666
9       TAX -0.008971
10  PTRATIO -0.412174
11        B  0.015414
12    LSTAT -0.481953
```

```
Ridge Regression Output with eta =  13
```

```
    Feature     Coeff
```

```
0      CRIM -0.100545
1        ZN  0.054317
2     INDUS  0.012357
3      CHAS  2.344856
4       NOX  0.476462
5        RM  5.748354
6       AGE -0.010648
7       DIS -0.900668
8       RAD  0.162816
9       TAX -0.008975
10  PTRATIO -0.410173
11        B  0.015449
12    LSTAT -0.482726
```

Ridge Regression Output with eta = 14

```
    Feature     Coeff
0      CRIM -0.100598
1        ZN  0.054475
2     INDUS  0.012665
3      CHAS  2.308210
4       NOX  0.466590
5        RM  5.738275
6       AGE -0.010370
7       DIS -0.898828
8       RAD  0.162955
9       TAX -0.008979
10  PTRATIO -0.408164
11        B  0.015484
12    LSTAT -0.483500
```

Ridge Regression Output with eta = 15

```
    Feature     Coeff
0      CRIM -0.100648
1        ZN  0.054632
2     INDUS  0.012958
3      CHAS  2.272783
4       NOX  0.457674
5        RM  5.728152
6       AGE -0.010094
7       DIS -0.896985
8       RAD  0.163084
9       TAX -0.008982
10  PTRATIO -0.406149
11        B  0.015518
12    LSTAT -0.484274
```

Ridge Regression Output with eta = 16

```
    Feature     Coeff
```

```
0      CRIM -0.100695
1        ZN  0.054789
2     INDUS  0.013238
3      CHAS  2.238517
4       NOX  0.449568
5        RM  5.717993
6       AGE -0.009822
7       DIS -0.895141
8       RAD  0.163205
9       TAX -0.008985
10  PTRATIO -0.404128
11        B  0.015552
12    LSTAT -0.485046
```

```
Ridge Regression Output with eta =  17
```

```
    Feature     Coeff
0      CRIM -0.100740
1        ZN  0.054945
2     INDUS  0.013505
3      CHAS  2.205356
4       NOX  0.442153
5        RM  5.707805
6       AGE -0.009552
7       DIS -0.893296
8       RAD  0.163316
9       TAX -0.008988
10  PTRATIO -0.402102
11        B  0.015585
12    LSTAT -0.485818
```

```
Ridge Regression Output with eta =  18
```

```
    Feature     Coeff
0      CRIM -0.100782
1        ZN  0.055100
2     INDUS  0.013761
3      CHAS  2.173249
4       NOX  0.435333
5        RM  5.697594
6       AGE -0.009285
7       DIS -0.891451
8       RAD  0.163420
9       TAX -0.008990
10  PTRATIO -0.400072
11        B  0.015618
12    LSTAT -0.486588
```

```
Ridge Regression Output with eta =  19
```

```
    Feature     Coeff
```

```
0      CRIM -0.100823
1        ZN  0.055254
2     INDUS  0.014006
3      CHAS  2.142145
4       NOX  0.429030
5        RM  5.687367
6       AGE -0.009020
7       DIS -0.889607
8       RAD  0.163516
9       TAX -0.008991
10  PTRATIO -0.398037
11        B  0.015651
12    LSTAT -0.487356
```

```
Ridge Regression Output with eta =  20
```

```
    Feature     Coeff
0      CRIM -0.100861
1        ZN  0.055408
2     INDUS  0.014242
3      CHAS  2.112001
4       NOX  0.423179
5        RM  5.677128
6       AGE -0.008757
7       DIS -0.887765
8       RAD  0.163604
9       TAX -0.008993
10  PTRATIO -0.396000
11        B  0.015683
12    LSTAT -0.488121
```

```
Ridge Regression Output with eta =  21
```

```
    Feature     Coeff
0      CRIM -0.100898
1        ZN  0.055561
2     INDUS  0.014468
3      CHAS  2.082771
4       NOX  0.417726
5        RM  5.666881
6       AGE -0.008497
7       DIS -0.885925
8       RAD  0.163685
9       TAX -0.008994
10  PTRATIO -0.393960
11        B  0.015715
12    LSTAT -0.488884
```

```
Ridge Regression Output with eta =  22
```

```
    Feature     Coeff
```

0	CRIM	-0.100933
1	ZN	0.055713
2	INDUS	0.014685
3	CHAS	2.054414
4	NOX	0.412625
5	RM	5.656629
6	AGE	-0.008239
7	DIS	-0.884088
8	RAD	0.163760
9	TAX	-0.008994
10	PTRATIO	-0.391918
11	B	0.015747
12	LSTAT	-0.489645

Ridge Regression Output with eta = 23

	Feature	Coeff
0	CRIM	-0.100966
1	ZN	0.055865
2	INDUS	0.014894
3	CHAS	2.026893
4	NOX	0.407837
5	RM	5.646376
6	AGE	-0.007983
7	DIS	-0.882254
8	RAD	0.163829
9	TAX	-0.008994
10	PTRATIO	-0.389874
11	B	0.015778
12	LSTAT	-0.490402

Ridge Regression Output with eta = 24

	Feature	Coeff
0	CRIM	-0.100997
1	ZN	0.056015
2	INDUS	0.015095
3	CHAS	2.000170
4	NOX	0.403330
5	RM	5.636124
6	AGE	-0.007729
7	DIS	-0.880422
8	RAD	0.163892
9	TAX	-0.008994
10	PTRATIO	-0.387829
11	B	0.015809
12	LSTAT	-0.491157

[160]:
```python
# 2.4 evaluation
def pred_fn(X, theta):
    #TODO
    pred = np.matmul(X, theta)
    return pred

def root_mean_square_error(pred, y):
    #TODO
    diff_matrix = y - pred
    rmse = diff_matrix**2
    rmse = rmse.sum()
    rmse = rmse/np.size(y)
    rmse = np.sqrt(rmse)
    return rmse

pred_linear_train = pred_fn(X_train,df_theta.loc[:,"Coeff"])
pred_linear_test = pred_fn(X_test,df_theta.loc[:,"Coeff"])
rmse_linear_train = root_mean_square_error(pred_linear_train, y_train)
rmse_linear_test = root_mean_square_error(pred_linear_test, y_test)
print("Training Set RMSE of Linear Regression : ", rmse_linear_train)
print("Testing Set RMSE of Linear Regression : ", rmse_linear_test)

pred_ridge_train = pred_fn(X_train,df_theta_r.loc[:,"Coeff"])
pred_ridge_test = pred_fn(X_test,df_theta_r.loc[:,"Coeff"])
rmse_ridge_train = root_mean_square_error(pred_ridge_train, y_train)
rmse_ridge_test = root_mean_square_error(pred_ridge_test, y_test)
print("Training Set RMSE of Ridge Regression : ", rmse_ridge_train)
print("Testing Set RMSE of Ridge Regression : ", rmse_ridge_test)
```

```
Training Set RMSE of Linear Regression :  4.820626531838223
Testing Set RMSE of Linear Regression :  5.209217510530916
Training Set RMSE of Ridge Regression :  4.829777333975097
Testing Set RMSE of Ridge Regression :  5.189347305423606
```

[161]:
```python
# 2.5 linear models of top-3 features
X_train_top3 = np.stack((X_train[:,10],X_train[:,5],X_train[:,12])).transpose()
X_test_top3 = np.stack((X_test[:,10],X_test[:,5],X_test[:,12])).transpose()
# linear regression using top-3 features
theta_top3 = least_square(X_train_top3, y_train)
df_theta_top3 = pd.DataFrame(zip(boston.feature_names,␣
 ↪theta_top3),columns=['Feature','Coeff'])
pred_linear_train_top3 = pred_fn(X_train_top3,df_theta_top3.loc[:,"Coeff"])
pred_linear_test_top3 = pred_fn(X_test_top3,df_theta_top3.loc[:,"Coeff"])
rmse_linear_train_top3 = root_mean_square_error(pred_linear_train_top3, y_train)
rmse_linear_test_top3 = root_mean_square_error(pred_linear_test_top3, y_test)
print("Training Set RMSE of Linear Regression for top 3 features : ",␣
 ↪rmse_linear_train_top3)
```

```
print("Testing Set RMSE of Linear Regression for top 3 features : ",␣
↪rmse_linear_test_top3)
# ridge regression using top-3 features
theta_r_top3 = ridge_reg(X_train_top3, y_train, 20.0)
df_theta_r_top3 = pd.DataFrame(zip(boston.feature_names,␣
↪theta_r_top3),columns=['Feature','Coeff'])
pred_ridge_train_top3 = pred_fn(X_train_top3,df_theta_r_top3.loc[:,"Coeff"])
pred_ridge_test_top3 = pred_fn(X_test_top3,df_theta_r_top3.loc[:,"Coeff"])
rmse_ridge_train_top3 = root_mean_square_error(pred_ridge_train_top3, y_train)
rmse_ridge_test_top3 = root_mean_square_error(pred_ridge_test_top3, y_test)
print("Training Set RMSE of Ridge Regression for top 3 features and eta = 20.0 :
↪ ", rmse_ridge_train_top3)
print("Testing Set RMSE of Ridge Regression for top 3 features and eta = 20.0 :␣
↪", rmse_ridge_test_top3)
```

```
Training Set RMSE of Linear Regression for top 3 features :  5.273361751695365
Testing Set RMSE of Linear Regression for top 3 features :  5.494723646664577
Training Set RMSE of Ridge Regression for top 3 features and eta = 20.0 :
5.276310228536866
Testing Set RMSE of Ridge Regression for top 3 features and eta = 20.0 :
5.477573443118745
```

[ ]:

```

```