Université Libre de Bruxelles

Reconfiguration problems

Complexity analysis

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Introductory problem : POWER SUPPLY Problem

Let C be a set of customers, P be a set of power stations and $G = (V, E)$ be a bipartite graph where $V = \{C \cup P\}$ with weights on the vertices.

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Introductory problem : POWER SUPPLY Problem

Let C be a set of customers, P be a set of power stations and $G = (V, E)$ be a bipartite graph where $V = \{C \cup P\}$ with weights on the vertices.
Can G be partitioned into subtrees, such that each subtree contains exactly one power supply P and that the demands of $P's$ customers is $\leq$ to its capacity?

Introduction

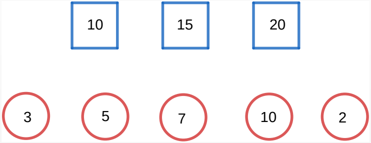

Figure: Input graph G where the blue vertices are the power stations and red vertices are the customers.

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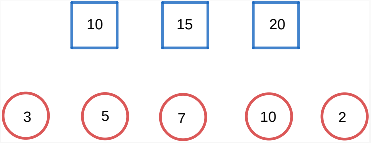

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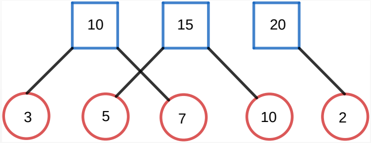

Figure: A feasible solution to the POWER SUPPLY problem.

Theorem

The POWER SUPPLY problem is $\mathcal{NP} - complete$.

POWER SUPPLY RECONFIGURATION
Problem

Suppose now that we are given two feasible solutions s_0 and s_t of the POWER SUPPLY problem and are asked: Can one solution be transformed into the other by moving only one customer at a time and always remaining feasible?

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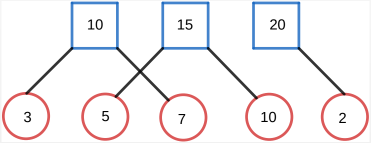

Figure: Feasible solution s_0.

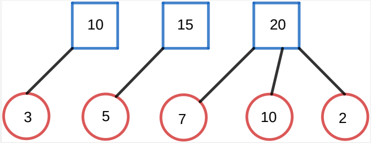

Figure: Feasible solution s_t.

POWER SUPPLY RECONFIGURATION
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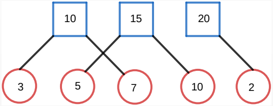

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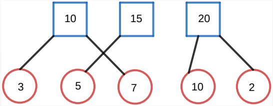

Figure: Intermediate feasible solution s_i where customer 10 is moved.

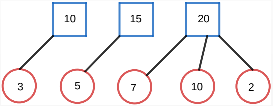

Figure: Target feasible solution s_t where customer 7 is moved from previous intermediate solution.

Theorem

POWER SUPPLY RECONFIGURATION problem is $PSPACE-complete$.

RECONFIGURATION problems

Defintion

Given two combinatorial configurations satisfying a problem while satisfying some constraints, is it possible to transform one configuration to another by modifying only one element at a time and that the intermediate solution remains satisfiable at all times.

Why do RECONFIGURATION problems spark interests?

Reconfiguration arises in countless problems that involve movement and change.

1. Problems in computational geometry such as morphing graph drawings.
2. Problems related to games and puzzles, such as the 15-puzzle, a topic of research since 1879.
3. Questions of evolvability: Can genotype y_0 evolve into genotype y_t via individual mutations each of which are of adequate fitness?
4. Most importantly reconfiguration problems yield insights into the structure of the solution space and connectivity of the set of feasible solutions.

SATISFIABILITY Problem

Definition

The SATISFIABILITY problem, also called SAT is to test whether a Boolean formula is satisfiable. An example of a Boolean formula is $\phi = (x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee \neg x_2 \vee x_3)$.

Theorem

Cook-Levin Theorem : SAT is $\mathcal{NP} - complete$.

The SAT RECONFIGURATION problem

Definition

Given a boolean formula ϕ and two feasible solutions s_0 and s_t, is there a way to transform the feasible solution s_0 to s_t while maintaining the two following constraints:

The SAT RECONFIGURATION problem

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Given a boolean formula ϕ and two feasible solutions s_0 and s_t, is there a way to transform the feasible solution s_0 to s_t while maintaining the two following constraints:

1. At each step, only one variable x_i of the boolean formula can be flipped.
2. Each intermediate solution x_k must be feasible.

Theorem

SAT RECONFIGURATION problem is PSPACE-complete.

Complexity

In general the reconfiguration problem of an $\mathcal{NP}-complete$ problem is $PSPACE - complete$. And the reconfiguration problem of a polynomial-time solvable problem is $PSPACE$. However there are exceptions to this general rule :

1. The $3-$coloring problem is $\mathcal{NP}-hard$ and its corresponding reconfiguration problem is solvable in polynomial time.
2. The Shortest path problem is solvable in polynomial time whereas it's corresponding reconfiguration problem is PSPACE-complete.

Goal

The goal of this thesis is to study the classifications established among the computational complexity of different types of reconfiguration problems.
And to find more about the properties of host problems that result in the pattern (seen in the previous slide) holding or not.

Open Questions

1. What is the connection between the complexity of reconfiguration problems and the complexity of the decision problem on the existence of configurations of a particular kind ?
2. Is the TRAVELLING SALESMAN RECONFIGURATION problem (where two tours are adjacent if they differ in two edges) PSPACE- complete?
3. What are the properties of host problems that result in the general pattern holding or not?

Thank you for your attention

The slides, resources and report can be found at:

`https://github.com/Prateeba/MEMO-F403`