Energy Consumption Time-Series Analysis

Prateek Singh, Sumanth Reddy Thandra 2025-03-31

Data Pre-processing

Loading PJME hourly time-series data

```
setwd("F:/Studies/Master's Notes/Spring 2025/MA 5781 - Time Series Analysis/FP")
data <- read.csv("./data/PJME_hourly.csv", header = TRUE, sep = ",")
head(data)</pre>
```

```
## Datetime PJME_MW

## 1 2002-12-31 01:00:00 26498

## 2 2002-12-31 02:00:00 25147

## 3 2002-12-31 03:00:00 24574

## 4 2002-12-31 04:00:00 24393

## 5 2002-12-31 05:00:00 24860

## 6 2002-12-31 06:00:00 26222
```

```
any(is.na(data$PJME_MW))
```

```
## [1] FALSE
```

```
data$Datetime <- as.POSIXct(data$Datetime, tz = "UTC", format = "%Y-%m-%d %H:%M:%S")
data <- data[order(data$Datetime), ]

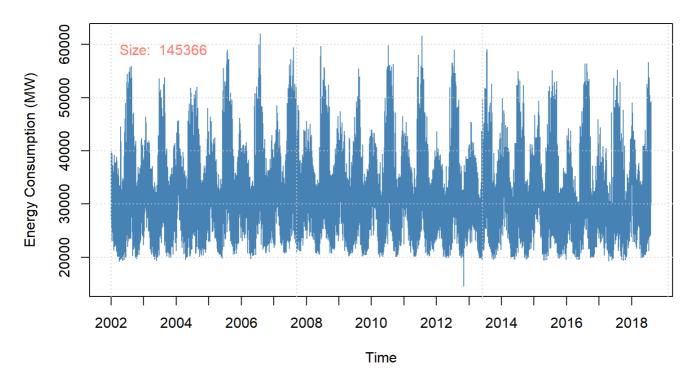
ts.energy.hourly <- ts(data$PJME_MW)
summary(ts.energy.hourly)</pre>
```

```
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 14544 27573 31421 32080 35650 62009
```

Hourly Data and Stationarity Check

```
plot(ts.energy.hourly, main = "Hourly Energy Consumption", xlab = "Time",
    ylab = "Energy Consumption (MW)",col = "steelblue", lwd = 1, xaxt = "n")
axis(1, at = (2002:2018 - 2002) * 365 * 24, labels = 2002:2018, las = 1)
text(x = 0.95, y = max(ts.energy.hourly) * 0.95,
    labels = paste("Size: ", length(ts.energy.hourly)), col = "salmon", pos = 4)
grid()
```

Hourly Energy Consumption



```
adf.test(ts.energy.hourly); kpss.test(ts.energy.hourly); pp.test(ts.energy.hourly);
```

```
##
## Augmented Dickey-Fuller Test
##
## data: ts.energy.hourly
## Dickey-Fuller = -24.065, Lag order = 52, p-value = 0.01
## alternative hypothesis: stationary
```

```
##
## KPSS Test for Level Stationarity
##
## data: ts.energy.hourly
## KPSS Level = 6.4861, Truncation lag parameter = 24, p-value = 0.01
```

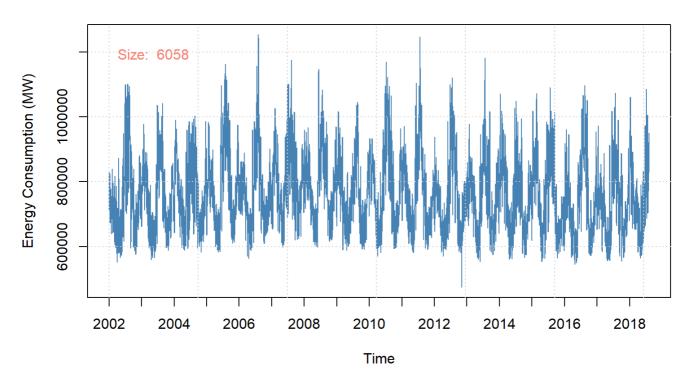
```
##
## Phillips-Perron Unit Root Test
##
## data: ts.energy.hourly
## Dickey-Fuller Z(alpha) = -1433.6, Truncation lag parameter = 24,
## p-value = 0.01
## alternative hypothesis: stationary
```

- 1. ADF (lag = 52) and PP (lag = 24) rejects the null \rightarrow suggest the series is stationary.
- 2. **KPSS (lag = 24)** rejects the null of stationarity → suggests the series is **non-stationary**.
- 3. Lag choices (24, 52) suggest daily/weekly patterns are influencing stationarity

Daily Data and Stationarity Check

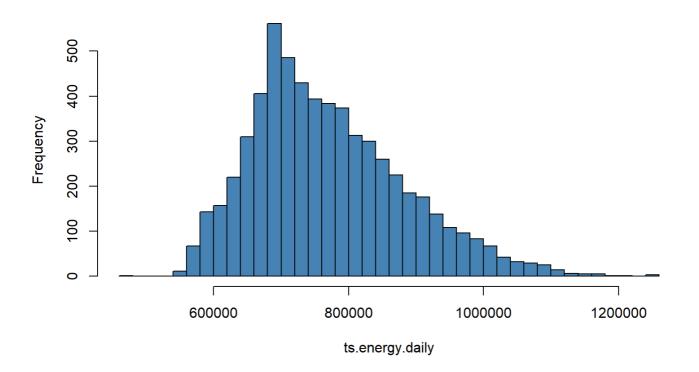
```
data$Date <- as.Date(data$Datetime)
data.daily <- aggregate(PJME_MW ~ Date, data = data, FUN = sum)
ts.energy.daily <- ts(data.daily[-nrow(data.daily), ]$PJME_MW)
plot(ts.energy.daily, main = "Daily Energy Consumption",xlab = "Time",
    ylab = "Energy Consumption (MW)", col = "steelblue", lwd = 1, xaxt = "n")
axis(1, at = (2002:2018 - 2002) * 365, labels = 2002:2018, las = 1)
text(x = 0.95, y = max(ts.energy.daily) * 0.95,
    labels = paste("Size: ", length(ts.energy.daily)), col = "salmon", pos = 4)
grid()</pre>
```

Daily Energy Consumption



hist(ts.energy.daily, main="Daily Energy Consumption Histogram",
 breaks="FD", col="steelblue")

Daily Energy Consumption Histogram



```
adf.test(ts.energy.daily); kpss.test(ts.energy.daily); pp.test(ts.energy.daily);
```

```
##
## Augmented Dickey-Fuller Test
##
## data: ts.energy.daily
## Dickey-Fuller = -7.656, Lag order = 18, p-value = 0.01
## alternative hypothesis: stationary
```

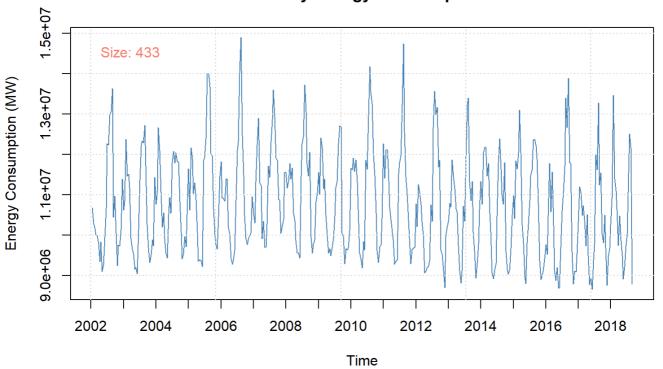
```
##
## KPSS Test for Level Stationarity
##
## data: ts.energy.daily
## KPSS Level = 0.83879, Truncation lag parameter = 11, p-value = 0.01
```

```
##
## Phillips-Perron Unit Root Test
##
## data: ts.energy.daily
## Dickey-Fuller Z(alpha) = -770.67, Truncation lag parameter = 11,
## p-value = 0.01
## alternative hypothesis: stationary
```

- 1. ADF (lag = 18) and PP (lag = 1) tests reject the null of non-stationarity → suggest the series is stationary.
- 2. **KPSS (lag = 11)** rejects the null of stationarity → suggests the series is **non-stationary**.
- 3. Lag 11–18 (days) suggests the tests are accounting for autocorrelation up to ~2.5 weeks.

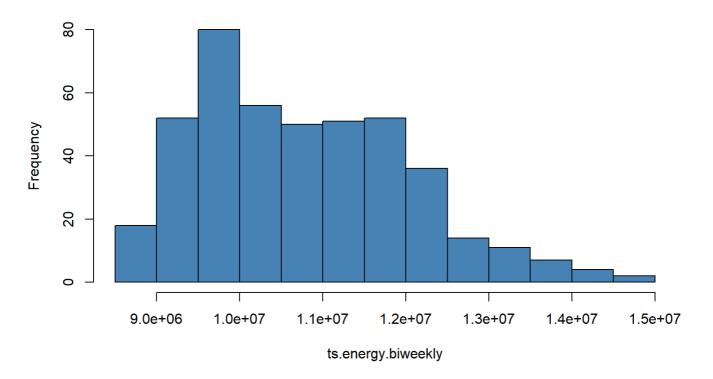
Biweekly Data and Stationarity Check

Biweekly Energy Consumption



hist(ts.energy.biweekly, main="Biweekly Energy Consumption Histogram",
 breaks="FD", col="steelblue")

Biweekly Energy Consumption Histogram



```
adf.test(ts.energy.biweekly); kpss.test(ts.energy.biweekly); pp.test(ts.energy.biweekly);
```

```
##
## Augmented Dickey-Fuller Test
##
## data: ts.energy.biweekly
## Dickey-Fuller = -9.51, Lag order = 7, p-value = 0.01
## alternative hypothesis: stationary
```

```
##
## KPSS Test for Level Stationarity
##
## data: ts.energy.biweekly
## KPSS Level = 0.29161, Truncation lag parameter = 5, p-value = 0.1
```

```
##
## Phillips-Perron Unit Root Test
##
## data: ts.energy.biweekly
## Dickey-Fuller Z(alpha) = -135.66, Truncation lag parameter = 5, p-value
## = 0.01
## alternative hypothesis: stationary
```

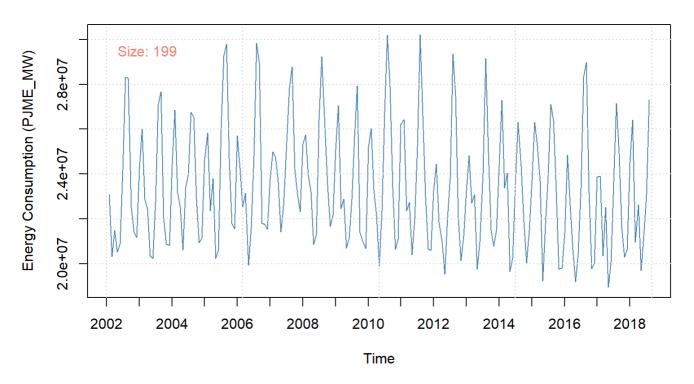
- ADF & PP Tests (p = 0.01) → Strong evidence against unit root (data is stationary).
- **KPSS Test (p = 0.1)** → Fails to reject level stationarity (supports stationarity)
- · The decomposed plots, show strong seasonality.

Monthly Data and Stationarity Check

```
data$YearMonth <- format(data$Datetime, "%Y-%m")
data.monthly <- aggregate(PJME_MW ~ YearMonth, data, sum)
data.monthly <- data.monthly[-nrow(data.monthly), ]
ts.energy.monthly <- ts(data.monthly$PJME_MW)

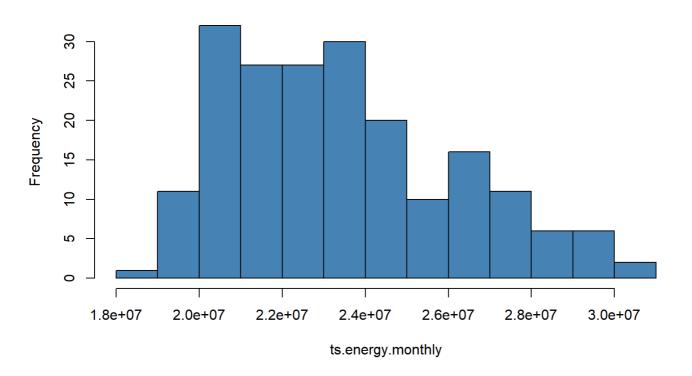
plot(ts.energy.monthly, main = "Monthly Energy Consumption Over Time",
    ylab = "Energy Consumption (PJME_MW)", xlab = "Time",
    col = "steelblue", lwd = 1, xaxt = "n")
axis(1, at = (2002:2018 - 2002) * 12, labels = 2002:2018, las = 1)
text(x = 0.95, y = max(ts.energy.monthly) * 0.975,
    labels = paste("Size:", length(ts.energy.monthly)), col = "salmon", pos = 4)
grid()</pre>
```

Monthly Energy Consumption Over Time



```
hist(ts.energy.monthly, main="Monthly Energy Consumption Histogram",
    breaks = "FD", col="steelblue")
```

Monthly Energy Consumption Histogram



```
adf.test(ts.energy.monthly); kpss.test(ts.energy.monthly); pp.test(ts.energy.monthly);
```

```
##
## Augmented Dickey-Fuller Test
##
## data: ts.energy.monthly
## Dickey-Fuller = -5.9851, Lag order = 5, p-value = 0.01
## alternative hypothesis: stationary
```

```
##
## KPSS Test for Level Stationarity
##
## data: ts.energy.monthly
## KPSS Level = 0.4485, Truncation lag parameter = 4, p-value = 0.05625
```

```
##
## Phillips-Perron Unit Root Test
##
## data: ts.energy.monthly
## Dickey-Fuller Z(alpha) = -64.248, Truncation lag parameter = 4, p-value
## = 0.01
## alternative hypothesis: stationary
```

- ADF & PP Tests (p = 0.01) → Strong evidence against unit root (data is stationary).
- KPSS Test (p = 0.5625) → Fails to reject level stationarity (supports stationarity), albeit borderline

Checking for Stochastic Trend

```
adf.test(ts.energy.biweekly); kpss.test(ts.energy.biweekly); pp.test(ts.energy.biweekly);
```

```
##
## Augmented Dickey-Fuller Test
##
## data: ts.energy.biweekly
## Dickey-Fuller = -9.51, Lag order = 7, p-value = 0.01
## alternative hypothesis: stationary
```

```
##
## KPSS Test for Level Stationarity
##
## data: ts.energy.biweekly
## KPSS Level = 0.29161, Truncation lag parameter = 5, p-value = 0.1
```

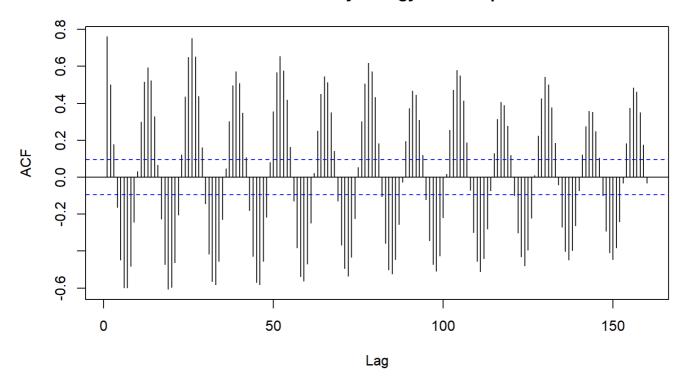
```
##
## Phillips-Perron Unit Root Test
##
## data: ts.energy.biweekly
## Dickey-Fuller Z(alpha) = -135.66, Truncation lag parameter = 5, p-value
## = 0.01
## alternative hypothesis: stationary
```

Auto-correlation Analysis and choosing d

```
eacf(ts.energy.biweekly)
```

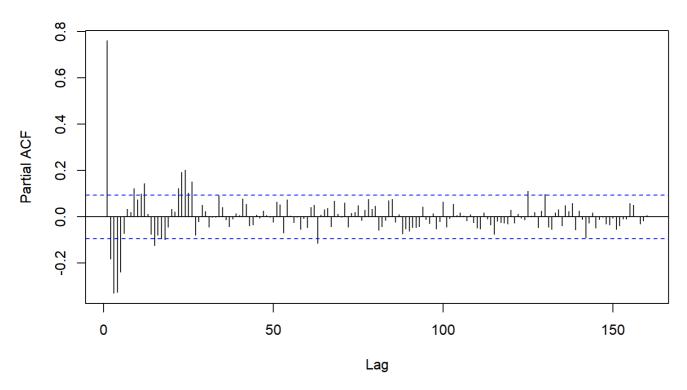
```
acf(ts.energy.biweekly, main = "ACF of Biweekly Energy Consumption", lag.max=160)
```

ACF of Biweekly Energy Consumption



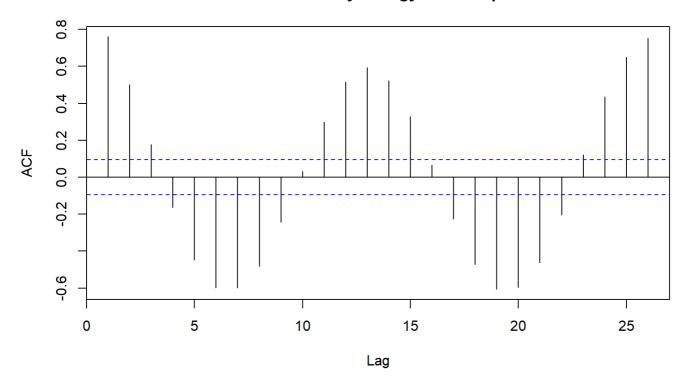
pacf(ts.energy.biweekly, main = "PACF of Biweekly Energy Consumption", lag.max=160)

PACF of Biweekly Energy Consumption



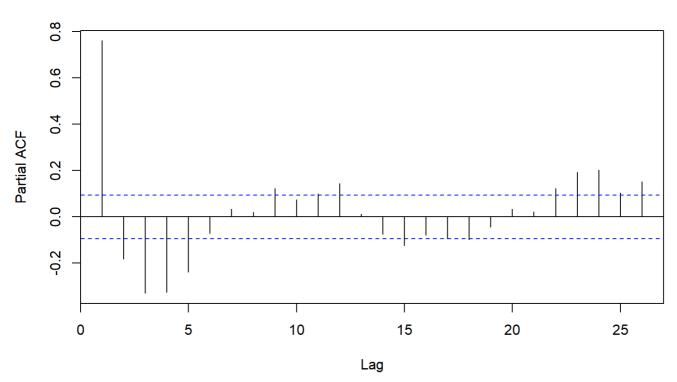
acf(ts.energy.biweekly, main = "ACF of Biweekly Energy Consumption")

ACF of Biweekly Energy Consumption



pacf(ts.energy.biweekly, main = "PACF of Biweekly Energy Consumption")

PACF of Biweekly Energy Consumption



1. For Non-Seasonal part

- 1. ACF (decay with no sharp cutoff) \rightarrow MA(1), MA(2)
- 2. PACF (sharp cutoff at Lag 1) \rightarrow AR(1)
- 3. From EACF \rightarrow
 - 1. AR(2): Most lags become insignificant (o) after Lag 1 \rightarrow AR(2) may be adequate.

- 2. MA(2): Lags beyond 2 become insignificant → MA(2) likely sufficient.
- 2. For Seasonal part
 - 1. ACF (decays slowly with not sharp cutoff till lag 14) → Differencing might be needed
 - 2. PACF (Cuts off after lag 1) \rightarrow AR(1) may be adequate

Candidate modelling (Based on ACF, PACF, EACF)

ARMA models

```
1. ARIMA(1,0,0)
```

- 2. ARIMA(1,0,1)
- 3. ARIMA(1,0,2)
- 4. ARIMA(2,0,0)
- 5. ARIMA(2,0,1)
- 6. ARIMA(2,0,2)
- 7. Auto ARIMA (Selected using auto.arima)

```
# Dropping Last aggregation (incompletete data)
ts.energy.biweekly <- ts.energy.biweekly[-length(ts.energy.biweekly)]

model.arima_1 <- arima(ts.energy.biweekly, order = c(1, 0, 0))
model.arima_2 <- arima(ts.energy.biweekly, order = c(1, 0, 1))
model.arima_3 <- arima(ts.energy.biweekly, order = c(1, 0, 2))
model.arima_4 <- arima(ts.energy.biweekly, order = c(2, 0, 0))
model.arima_5 <- arima(ts.energy.biweekly, order = c(2, 0, 1))
model.arima_6 <- arima(ts.energy.biweekly, order = c(2, 0, 2))
model.arima_ns_auto <- auto.arima(ts.energy.biweekly)

summary(model.arima_ns_auto)</pre>
```

```
## Series: ts.energy.biweekly
## ARIMA(2,0,2) with non-zero mean
##
## Coefficients:
##
           ar1
                     ar2
                              ma1
                                      ma2
##
        1.7124 -0.9274 -1.1413 0.4904 10778043.86
## s.e. 0.0225
                0.0218
                         0.0611 0.0514
                                              52544.37
##
## sigma^2 = 4.549e+11: log likelihood = -6409.8
## AIC=12831.61
                 AICc=12831.81
                                  BIC=12856.02
##
## Training set error measures:
##
                                                   MPE
                                                           MAPE
                                                                    MASE
                      ME
                              RMSE
                                        MΔF
## Training set -232.7162 670572.5 526771.1 -0.3685558 4.836914 0.807866
##
## Training set 0.05080482
```

```
## Model AIC BIC

## 6 ARMA(2,0,2) 12831.61 12856.02

## 7 Auto.ARIMA 12831.61 12856.02

## 5 ARMA(2,0,1) 12890.38 12910.72

## 3 ARMA(1,0,2) 12955.42 12975.76

## 4 ARMA(2,0,0) 12968.22 12984.49

## 2 ARMA(1,0,1) 12976.12 12992.40

## 1 ARMA(1,0,0) 12983.49 12995.70
```

```
results[order(results$BIC), ]
```

```
## Model AIC BIC
## 6 ARMA(2,0,2) 12831.61 12856.02
## 7 Auto.ARIMA 12831.61 12856.02
## 5 ARMA(2,0,1) 12890.38 12910.72
## 3 ARMA(1,0,2) 12955.42 12975.76
## 4 ARMA(2,0,0) 12968.22 12984.49
## 2 ARMA(1,0,1) 12976.12 12992.40
## 1 ARMA(1,0,0) 12983.49 12995.70
```

Best Performing Non-Seasonal Components

- 1. ARMA(2,0,2)
- 2. ARMA(1,0,2)
- 3. ARMA(1,0,2)

SARIMA models

- 1. SARIMA(2,0,2)(0,1,1)
- 2. SARIMA(2,0,2)(1,1,1)
- 3. SARIMA(2,0,2)(1,1,2)
- 4. SARIMA(2,0,1)(0,1,1)
- 5. SARIMA(2,0,1)(1,1,1)
- 6. SARIMA(2,0,1)(1,1,2)
- 7. Auto SARIMA (Selected using auto.arima)

```
ts.energy.biweekly \leftarrow ts(ts.energy.biweekly, start = c(2002, 1), frequency = 26)
model.sarima_1 \leftarrow arima(ts.energy.biweekly, order = c(2, 0, 2), seasonal = c(0, 1, 1))
model.sarima_2 \leftarrow arima(ts.energy.biweekly, order = c(2, 0, 2), seasonal = c(1, 1, 1))
model.sarima_3 \leftarrow arima(ts.energy.biweekly, order = c(2, 0, 2), seasonal = c(1, 1, 2))
model.sarima_4 \leftarrow arima(ts.energy.biweekly, order = c(2, 0, 1), seasonal = c(0, 1, 1))
model.sarima_5 \leftarrow arima(ts.energy.biweekly, order = c(2, 0, 1), seasonal = c(1, 1, 1))
model.sarima_6 \leftarrow arima(ts.energy.biweekly, order = c(2, 0, 1), seasonal = c(1, 1, 2))
model.sarima_auto <- auto.arima(ts.energy.biweekly)</pre>
summary(model.sarima auto)
## Series: ts.energy.biweekly
## ARIMA(1,0,1)(2,1,0)[26]
##
## Coefficients:
##
            ar1
                      ma1
                              sar1
                                       sar2
##
         0.5979 -0.2064 -0.5670 -0.3122
## s.e. 0.0955
                 0.1185
                            0.0512
                                     0.0515
## sigma^2 = 4.068e+11: log likelihood = -6006.07
## AIC=12022.14
                 AICc=12022.29
                                   BIC=12042.17
## Training set error measures:
                                                   MPE
##
                        ME
                               RMSE
                                       MAE
                                                           MAPE
                                                                      MASE
                                                                                 ACF1
## Training set -9641.886 615293.6 452939 -0.3225825 4.108352 0.7498197 0.00237244
results_sarima <- data.frame(
  Model = c("SARIMA(2,0,2)(0,1,1)", "SARIMA(2,0,2)(1,1,1)", "SARIMA(2,0,2)(1,1,2)",
            "SARIMA(2,0,1)(0,1,1)", "SARIMA(2,0,1)(1,1,1)", "SARIMA(2,0,1)(1,1,2)",
            "Auto.ARIMA"),
  AIC = c(AIC(model.sarima 1), AIC(model.sarima 2), AIC(model.sarima 3),
          AIC(model.sarima_4), AIC(model.sarima_5), AIC(model.sarima_6),
          AIC(model.sarima auto)),
  BIC = c(BIC(model.sarima 1), BIC(model.sarima 2), BIC(model.sarima 3),
          BIC(model.sarima_4), BIC(model.sarima_5), BIC(model.sarima_6),
          BIC(model.sarima_auto))
results_sarima[order(results_sarima$AIC), ]
##
                     Model
                                AIC
                                          BTC
```

```
## Model AIC BIC

## 2 SARIMA(2,0,2)(1,1,1) 11982.01 12010.05

## 3 SARIMA(2,0,2)(1,1,2) 11982.73 12014.79

## 4 SARIMA(2,0,1)(0,1,1) 11983.02 12003.05

## 5 SARIMA(2,0,1)(1,1,1) 11983.71 12007.75

## 6 SARIMA(2,0,1)(1,1,2) 11984.29 12012.33

## 1 SARIMA(2,0,2)(0,1,1) 11984.88 12008.92

## 7 Auto.ARIMA 12022.14 12042.17
```

```
results_sarima[order(results_sarima$BIC), ]
```

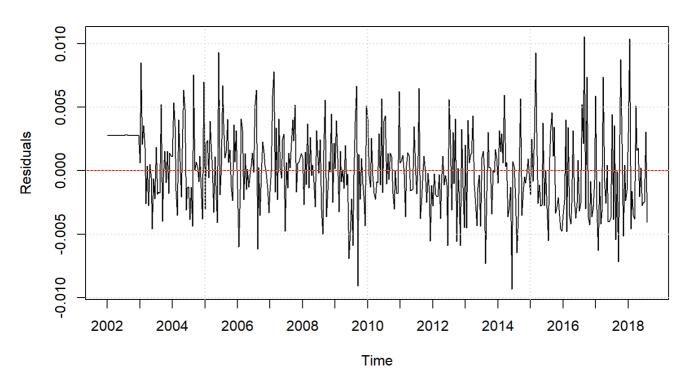
Best Performing Seasonal Models

- 1. SARIMA(2,0,1)[0,1,1]
- 2. SARIMA(2,0,2)[1,1,1]
- 3. SARIMA(2,0,1)(1,1,1)

Model Diagnostic: SARIMA(2,0,1)(0,1,1)[26]

Double log for transformation

Residuals After SARIMA Model

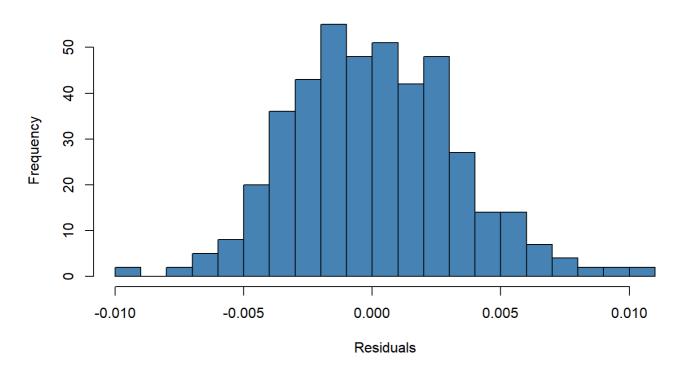


shapiro.test(residuals)

```
##
## Shapiro-Wilk normality test
##
## data: residuals
## W = 0.99304, p-value = 0.04306
```

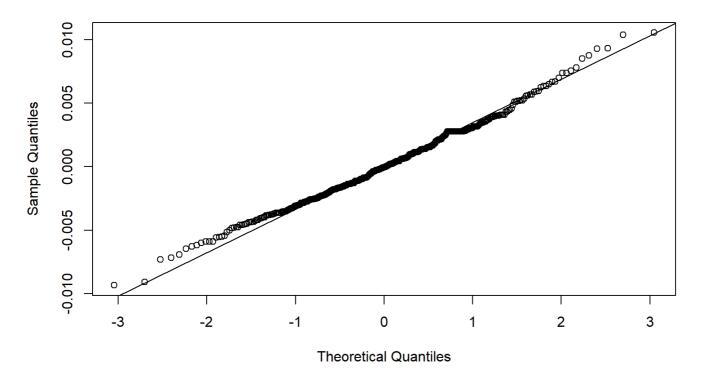
```
hist(residuals, main = "Histogram of Residuals",
    col="steelblue", xlab = "Residuals", breaks = "FD")
```

Histogram of Residuals



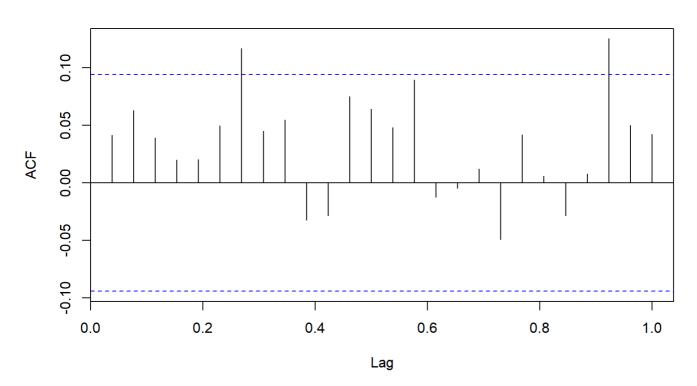
qqnorm(residuals, main = "Normal Q-Q Plot of Residuals"); qqline(residuals)

Normal Q-Q Plot of Residuals



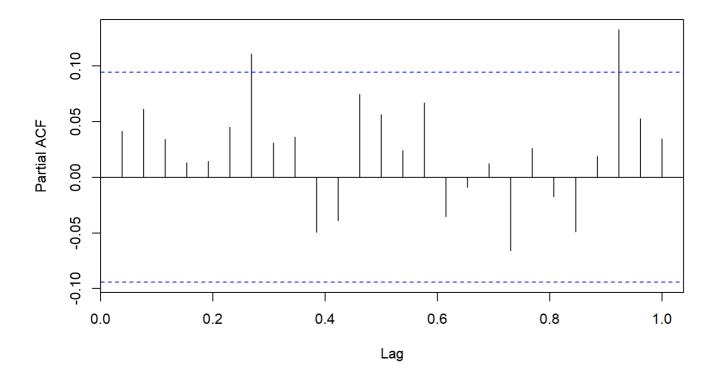
acf(residuals, main = "ACF of Residuals")

ACF of Residuals



pacf(residuals, main = "PACF of Residuals");

PACF of Residuals

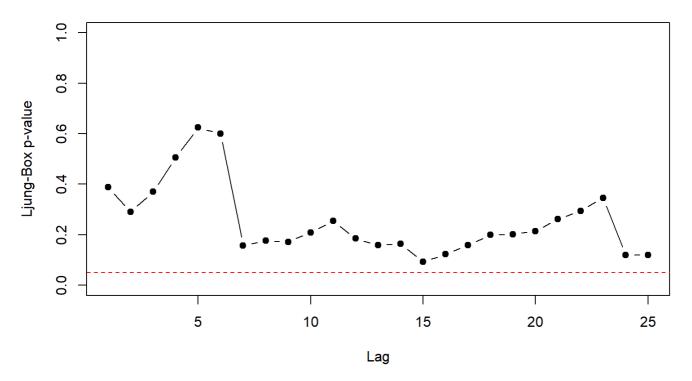


```
Box.test(residuals, lag = 20, type = "Box-Pierce")
```

```
##
## Box-Pierce test
##
## data: residuals
## X-squared = 23.981, df = 20, p-value = 0.2432
```

```
plot(1:25,
      sapply(1:25, function(lag) Box.test(residuals, lag = lag, type = "Ljung-Box")$p.value),
      main = "Ljung-Box Test p-values Across Lags", xlab = "Lag", ylab = "Ljung-Box p-value",
      type = "b", pch = 19, ylim = c(0, 1))
abline(h = 0.05, col = "red", lty = 2)
```

Ljung-Box Test p-values Across Lags

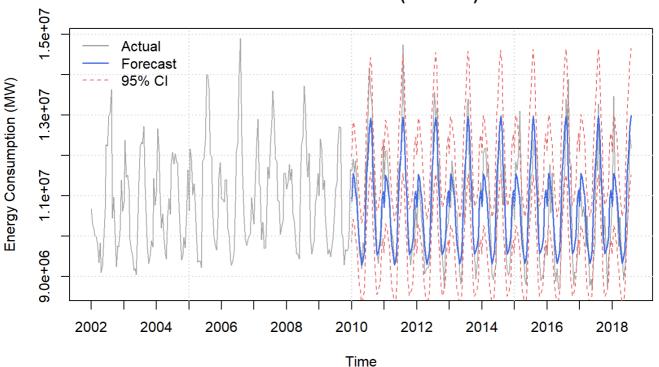


- Residual Mean: The residuals have a mean close to zero, indicating no significant bias in the model.
- **Homoscedasticity**: Residuals seem to have constant variance, but there are signs of increased fluctuations after 2010.
- **Normality**: The Shapiro-Wilk test shows that residuals are border-line normally distributed (p-value = 0.04306).
- **Independence**: ACF and PACF plots show no significant autocorrelation, and the Box-Ljung test confirms the independence of residuals (p-value = 0.2432).
- **Post-2010 Changes**: The model might not fully capture changes after 2010, suggesting the need for further decomposition and trend visualization.

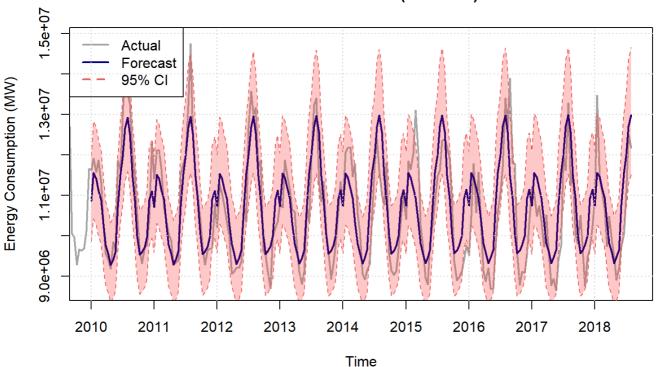
Prototype modelling: SARIMA(2,0,1)(0,1,1)[26]

```
train <- window(ts.energy.biweekly, end = time(ts.energy.biweekly)[208])</pre>
test <- window(ts.energy.biweekly, start = time(ts.energy.biweekly)[208 + 1])</pre>
model.final.sarima \leftarrow arima(log(log((train))), order = c(2, 0, 1), seasonal = c(0, 1, 1))
forecast.raw <- predict(model.final.sarima, n.ahead = length(test))</pre>
forecast.pred <- exp(exp(forecast.raw$pred))</pre>
forecast.upper <- exp(exp(forecast.raw$pred + 1.96 * forecast.raw$se))</pre>
forecast.lower <- exp(exp(forecast.raw$pred - 1.96 * forecast.raw$se))</pre>
forecast.time <- as.Date(as.numeric(time(test)))</pre>
plot(ts.energy.biweekly, main = "Actual vs Forecast (SARIMA)",
                xlab = "Time", ylab = "Energy Consumption (MW)",
                col = "darkgray", lwd = 1, xaxt = "n")
lines(forecast.time, forecast.pred, col = "royalblue", lwd = 1.5)
lines(forecast.time, forecast.upper, col = "indianred2", lty = 2, lwd = 1)
lines(forecast.time, forecast.lower, col = "indianred2", lty = 2, lwd = 1)
axis(1, at = t \leftarrow time(ts.energy.biweekly)[d \leftarrow !duplicated(y \leftarrow floor(time(ts.energy.biweekly)[d \leftarrow floor
y)))],
                labels = y[d], las = 1)
legend("topleft", legend = c("Actual", "Forecast", "95% CI"),
                      col = c("darkgray", "royalblue", "indianred2"),
                       lty = c(1, 1, 2), lwd = c(1.5, 1.5, 1), bty = "n")
grid()
```

Actual vs Forecast (SARIMA)



Actual vs Forecast (SARIMA)



Forecasting and Performance Metrics

```
actual_values <- as.numeric(test)
forecasted_values <- forecast.pred
residuals <- actual_values - forecasted_values

mae <- mean(abs(residuals))
rmse <- sqrt(mean(residuals^2))
mape <- mean(abs(residuals / actual_values)) * 100
r_squared <- 1 - (sum(residuals^2) / sum((actual_values - mean(actual_values))^2))

metrics <- data.frame(
   Metric = c("MAE", "RMSE", "MAPE (%)", "R-squared"),
   Value = round(c(mae, rmse, mape, r_squared), 3)
)
print(metrics, row.names = FALSE)</pre>
```

```
## Metric Value

## MAE 585420.028

## RMSE 739120.764

## MAPE (%) 5.492

## R-squared 0.681
```

Splitting Time Series (Post 2011)

```
data.daily$Date <- as.POSIXct(data.daily$Date)
data.daily.post11 <- data.daily[data.daily$Date >= as.POSIXct("2012-01-01 00:00:00"), ]
data.daily.post11 <- data.daily.post11[-1, ]
data.daily.post11$Week <- data.daily.post11$Week - 522
data.daily.post11$Biweekly <- data.daily.post11$Biweekly - 261

# Aggregating and Pre-processing (Removing first and Last observation)
data.post11.weekly <- aggregate(PJME_MW ~ Week, data = data.daily.post11, FUN = sum)
data.post11.weekly <- data.post11.weekly[-c(1, nrow(data.post11.weekly)), ]
data.post11.weekly$Week <- data.post11.weekly$Week - 1

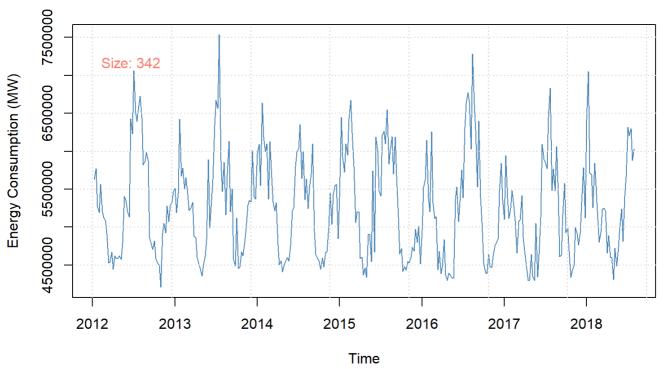
data.post11.biweekly <- aggregate(PJME_MW ~ Biweekly, data = data.daily.post11, FUN = sum)
data.post11.biweekly <- data.post11.biweekly[-c(1, nrow(data.post11.biweekly)), ]
data.post11.biweekly <- data.post11.biweekly$Biweekly - 1

ts.post11.weekly <- ts(data.post11.weekly$PJME_MW)
ts.post11.biweekly <- ts(data.post11.biweekly$PJME_MW)</pre>
```

Data Pre-processing (Post 2011)

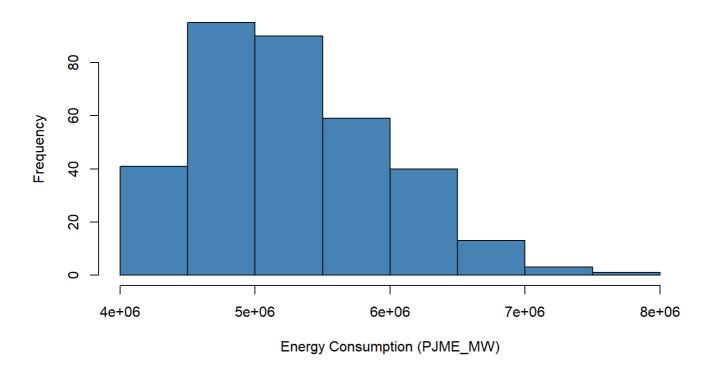
Plotting Weekly Data and Stationarity Check (Post 2011)

Weekly Energy Consumption



```
hist(ts.post11.weekly, main = "Histogram of Weekly Energy Consumption", xlab = "Energy Consumption (PJME_MW)", breaks = "FD", col = "steelblue")
```

Histogram of Weekly Energy Consumption



```
adf.test(ts.post11.weekly); kpss.test(ts.post11.weekly); pp.test(ts.post11.weekly);
```

```
##
## Augmented Dickey-Fuller Test
##
## data: ts.post11.weekly
## Dickey-Fuller = -7.8954, Lag order = 6, p-value = 0.01
## alternative hypothesis: stationary
```

```
##
## KPSS Test for Level Stationarity
##
## data: ts.post11.weekly
## KPSS Level = 0.049962, Truncation lag parameter = 5, p-value = 0.1
```

```
##
## Phillips-Perron Unit Root Test
##
## data: ts.post11.weekly
## Dickey-Fuller Z(alpha) = -85.679, Truncation lag parameter = 5, p-value
## = 0.01
## alternative hypothesis: stationary
```

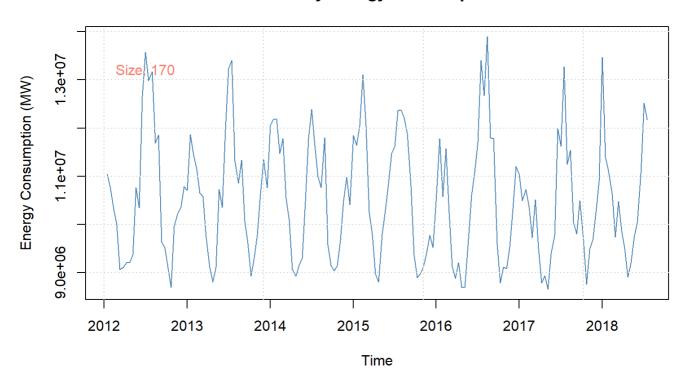
Plotting Bi-Weekly Data and Stationarity Check (Post 2011)

```
plot(ts.post11.biweekly,
    main = "Biweekly Energy Consumption",
    xlab = "Time", ylab = "Energy Consumption (MW)",
    col = "steelblue", lwd = 1, xaxt = "n")

text(x = 0.95, y = max(ts.post11.biweekly) * 0.95,
    labels = paste("Size:", length(ts.post11.biweekly)),
    col = "salmon", pos = 4)

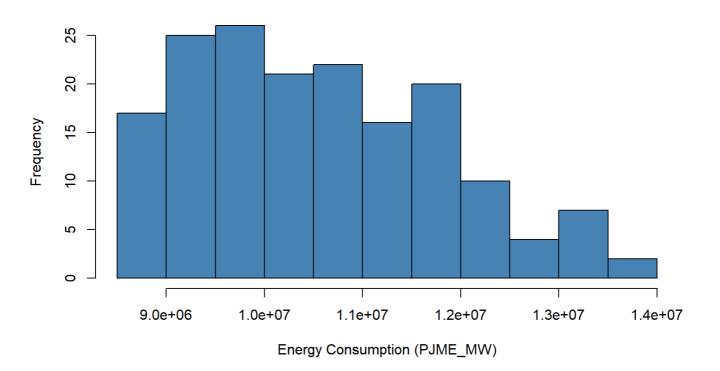
axis(1, at = (2012:2018 - 2012) * 26, labels = 2012:2018, las = 1)
grid()
```

Biweekly Energy Consumption



```
hist(ts.post11.biweekly, main = "Histogram of Post 2011 Bi-Weekly Energy Consumption", xlab = "Energy Consumption (PJME_MW)", breaks = "FD", col = "steelblue")
```

Histogram of Post 2011 Bi-Weekly Energy Consumption



```
adf.test(ts.post11.biweekly); kpss.test(ts.post11.biweekly); pp.test(ts.post11.biweekly);
```

```
##
## Augmented Dickey-Fuller Test
##
## data: ts.post11.biweekly
## Dickey-Fuller = -8.571, Lag order = 5, p-value = 0.01
## alternative hypothesis: stationary
```

```
##
## KPSS Test for Level Stationarity
##
## data: ts.post11.biweekly
## KPSS Level = 0.04428, Truncation lag parameter = 4, p-value = 0.1
```

```
##
## Phillips-Perron Unit Root Test
##
## data: ts.post11.biweekly
## Dickey-Fuller Z(alpha) = -58.024, Truncation lag parameter = 4, p-value
## = 0.01
## alternative hypothesis: stationary
```

- Biweekly data (Post-2011) exhibited stronger stationarity characteristics than the weekly series, leading to the decision to continue with biweekly stochastic trend analysis
- However, full-period biweekly data (2002–2018) still displayed stronger stationarity overall

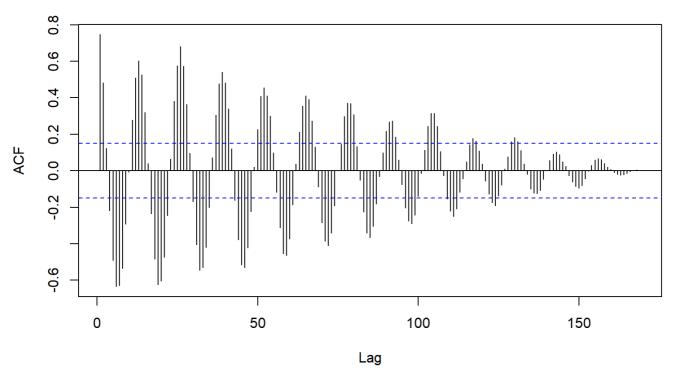
Auto-correlation Analysis and choosing d (Post 2011)

```
eacf(ts.post11.biweekly)
```

```
## AR/MA
## 0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x o x x x x x x x o x x x x
## 1 x x o x x x x x x x o x x x x
## 2 x x o o o o o o o o o o o
## 3 x o o o o o o o o o o o o o
## 4 x x o o o o o o o o o o o
## 5 x x o o o o x o o o o o o o
## 6 x o x o o o x o o o o o o o
## 7 x x x x x x x x x x o o o o o o
## 7 x x x o x x x x x x x o o o o o o o
```

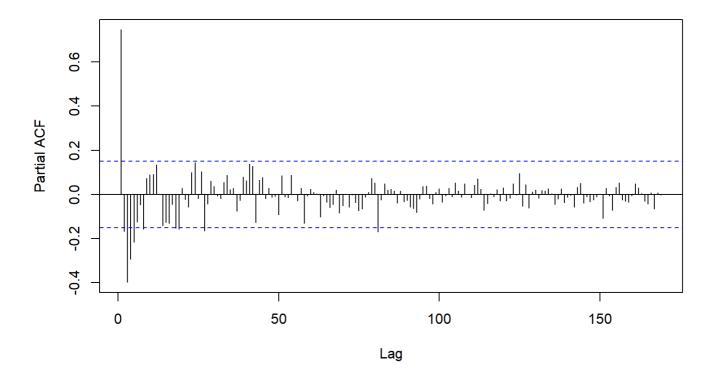
acf(ts.post11.biweekly, main = "ACF of Biweekly Energy Consumption", lag.max=260)

ACF of Biweekly Energy Consumption



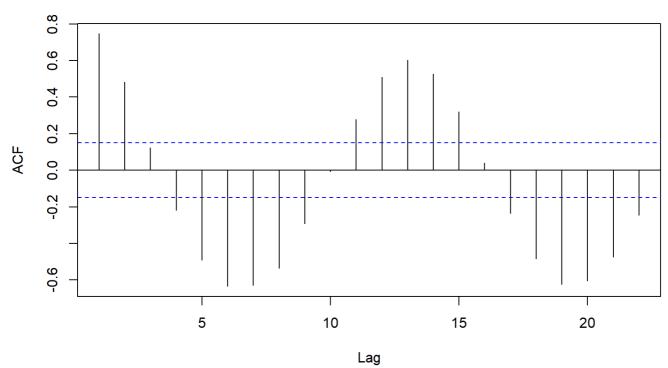
pacf(ts.post11.biweekly, main = "PACF of Transformed Energy Consumption", lag.max=260)

PACF of Transformed Energy Consumption



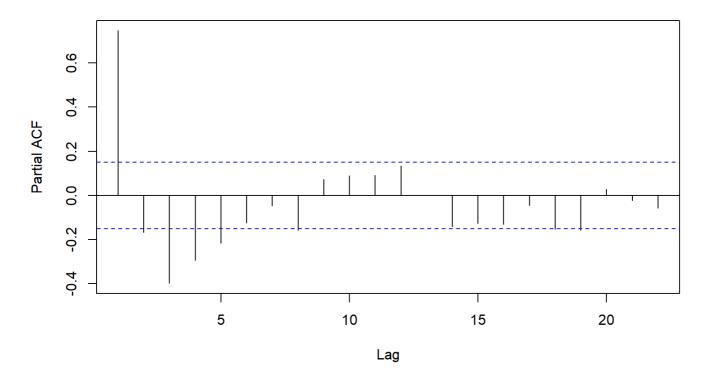
acf(ts.post11.biweekly, main = "ACF of Biweekly Energy Consumption")

ACF of Biweekly Energy Consumption



pacf(ts.post11.biweekly, main = "PACF of Biweekly Energy Consumption")

PACF of Biweekly Energy Consumption



1. For Non-Seasonal part

- ACF (decay with no sharp cutoff) \rightarrow MA(1), MA(2)
- PACF (sharp cutoff at Lag 1) \rightarrow AR(1)

2. From EACF \rightarrow

- AR(2): Most lags become insignificant (o) after Lag 1 → AR(2) should be sufficient.
- MA(2): Lags beyond 2 become insignificant \rightarrow MA(2) likely sufficient.

3. For Seasonal part

- ACF (decays slowly with not sharp cutoff till lag 14) → Differencing might be needed
- PACF (Cuts off after lag 1) \rightarrow AR(1) may be adequate

Candidate Modelling (Post 2011)

ARIMA models

- 1. ARIMA(0,0,1)
- 2. ARIMA(0,0,2)
- 3. ARIMA(1,0,0)
- 4. ARIMA(1,0,1)
- 5. ARIMA(1,0,2)
- 6. ARIMA(2,0,0)
- 7. ARIMA(2,0,1)
- 8. ARIMA(2,0,2)

```
model.post.arima_1 < -arima(ts.post11.biweekly, order = c(0, 0, 1))
model.post.arima_2 \leftarrow arima(ts.post11.biweekly, order = c(0, 0, 2))
model.post.arima_3 <- arima(ts.post11.biweekly, order = c(1, 0, 0))
model.post.arima_4 <- arima(ts.post11.biweekly, order = c(1, 0, 1))</pre>
model.post.arima_5 <- arima(ts.post11.biweekly, order = c(1, 0, 2))</pre>
model.post.arima_6 <- arima(ts.post11.biweekly, order = c(2, 0, 0))</pre>
model.post.arima_7 <- arima(ts.post11.biweekly, order = c(2, 0, 1))</pre>
model.post.arima_8 < - arima(ts.post11.biweekly, order = c(2, 0, 2))
model.post.arima_ns_auto <- auto.arima(ts.post11.biweekly)</pre>
summary(model.arima_ns_auto)
## Series: ts.energy.biweekly
## ARIMA(2,0,2) with non-zero mean
##
## Coefficients:
##
            ar1
                     ar2
                               ma1
                                       ma2
                                                    mean
##
         1.7124 -0.9274 -1.1413 0.4904 10778043.86
## s.e. 0.0225
                  0.0218
                           0.0611 0.0514
                                                52544.37
##
## sigma^2 = 4.549e+11: log likelihood = -6409.8
## AIC=12831.61
                  AICc=12831.81
                                   BIC=12856.02
##
## Training set error measures:
##
                       ME
                               RMSE
                                         MAE
                                                     MPE
                                                             MAPF
                                                                       MASE
## Training set -232.7162 670572.5 526771.1 -0.3685558 4.836914 0.807866
##
                       ACF1
## Training set 0.05080482
results <- data.frame(
```

```
## Model AIC BIC

## 8 ARMA(2,0,2) 5064.824 5083.638

## 9 Auto.ARIMA 5064.824 5083.638

## 7 ARMA(2,0,1) 5085.821 5101.500

## 5 ARMA(1,0,2) 5114.198 5129.877

## 6 ARMA(2,0,0) 5124.838 5137.381

## 4 ARMA(1,0,1) 5127.187 5139.731

## 3 ARMA(1,0,0) 5127.421 5136.828

## 2 ARMA(0,0,2) 5135.915 5148.458

## 1 ARMA(0,0,1) 5179.251 5188.659
```

```
results[order(results$BIC), ]
```

```
## Model AIC BIC
## 8 ARMA(2,0,2) 5064.824 5083.638
## 9 Auto.ARIMA 5064.824 5083.638
## 7 ARMA(2,0,1) 5085.821 5101.500
## 5 ARMA(1,0,2) 5114.198 5129.877
## 3 ARMA(1,0,0) 5127.421 5136.828
## 6 ARMA(2,0,0) 5124.838 5137.381
## 4 ARMA(1,0,1) 5127.187 5139.731
## 2 ARMA(0,0,2) 5135.915 5148.458
## 1 ARMA(0,0,1) 5179.251 5188.659
```

Best Performing Seasonal Models (Post 2011)

- 1. ARMA(2,0,2)
- 2. ARMA(2,0,1)

SARIMA Models

- 1. SARIMA(2,0,2)(0,1,1)
- 2. SARIMA(2,0,2)(1,1,1)
- 3. SARIMA(2,0,2)(1,1,2)
- 4. SARIMA(2,0,1)(0,1,1)
- 5. SARIMA(2,0,1)(1,1,1)
- 6. SARIMA(2,0,1)(1,1,2)

```
ts.post11.biweekly <- ts(ts.post11.biweekly, start = c(2012, 1), frequency = 26) model.post.sarima_1 <- arima(ts.post11.biweekly, order = c(2, 0, 2), seasonal = c(0, 1, 1)) model.post.sarima_2 <- arima(ts.post11.biweekly, order = c(2, 0, 2), seasonal = c(1, 1, 1)) model.post.sarima_3 <- arima(ts.post11.biweekly, order = c(2, 0, 2), seasonal = c(1, 1, 2)) model.post.sarima_4 <- arima(ts.post11.biweekly, order = c(2, 0, 1), seasonal = c(0, 1, 1)) model.post.sarima_5 <- arima(ts.post11.biweekly, order = c(2, 0, 1), seasonal = c(1, 1, 1)) model.post.sarima_6 <- arima(ts.post11.biweekly, order = c(2, 0, 1), seasonal = c(1, 1, 2)) model.post.sarima_auto <- auto.arima(ts.post11.biweekly) summary(model.post.sarima_auto)
```

```
## Series: ts.post11.biweekly
## ARIMA(1,0,1)(0,1,1)[26]
##
## Coefficients:
##
            ar1
                     ma1
                              sma1
##
         0.6190 -0.2516 -0.5986
## s.e. 0.1427
                  0.1727
                           0.1424
##
## sigma^2 = 4.679e+11: log likelihood = -2143.42
## AIC=4294.83
                AICc=4295.12
                                BIC=4306.71
##
##
  Training set error measures:
##
                                                    MPE
                                                           MAPE
                                                                      MASE
                       ME
                               RMSE
                                         MAE
## Training set -15535.82 622978.6 448054.3 -0.3752769 4.13975 0.7368919
##
                        ACF1
## Training set -0.004517471
```

```
## Model AIC BIC

## 6 SARIMA(0,1,2)(0,0,1) 4288.874 4309.662

## 5 SARIMA(2,0,1)(1,1,2) 4290.744 4308.562

## 3 SARIMA(2,0,2)(1,1,1) 4290.867 4314.625

## 2 SARIMA(2,0,2)(0,1,1) 4291.909 4312.698

## 7 Auto.ARIMA 4294.834 4306.713

## 4 SARIMA(2,0,1)(1,1,1) 4296.010 4310.859

## 1 SARIMA(2,0,2)(1,0,2) 4296.876 4314.695
```

```
results[order(results$BIC), ]
```

```
## Model AIC BIC

## 7 Auto.ARIMA 4294.834 4306.713

## 5 SARIMA(2,0,1)(1,1,2) 4290.744 4308.562

## 6 SARIMA(0,1,2)(0,0,1) 4288.874 4309.662

## 4 SARIMA(2,0,1)(1,1,1) 4296.010 4310.859

## 2 SARIMA(2,0,2)(0,1,1) 4291.909 4312.698

## 3 SARIMA(2,0,2)(1,1,1) 4290.867 4314.625

## 1 SARIMA(2,0,2)(1,0,2) 4296.876 4314.695
```

Best Performing Seasonal Model (Post 2011)

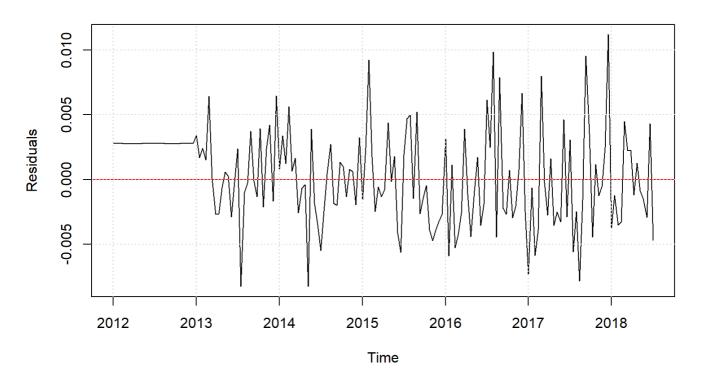
- 1. SARIMA(0,1,2)(0,0,1)
- 2. SARIMA(1,0,1)(0,1,1)
- 3. SARIMA(2,0,1)(1,1,2)
- SARIMA(2,0,1)[0,1,1] Initial best candidate is 4th and 5th best choice now by BIC and AIC.
- Finalized model (SARIMA(1,0,1)(0,1,1)): Previous suggested no differencing for non-seasonal part

Model Diagnostic: (Post 2011)

SARIMA(1,0,1)(0,1,1)[26]

Double Log transformation

Residuals After SARIMA Model

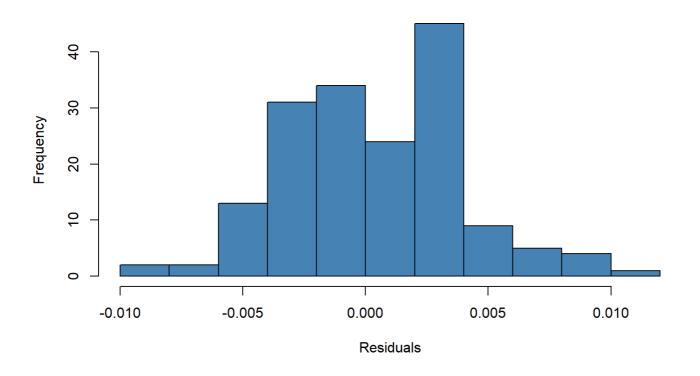


```
shapiro.test(residuals)
```

```
##
## Shapiro-Wilk normality test
##
## data: residuals
## W = 0.98262, p-value = 0.03193
```

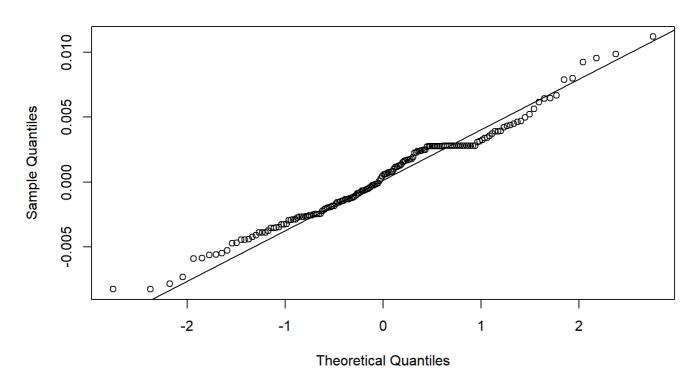
hist(residuals, main = "Histogram of Residuals", col="steelblue",
 xlab = "Residuals", breaks = "FD")

Histogram of Residuals



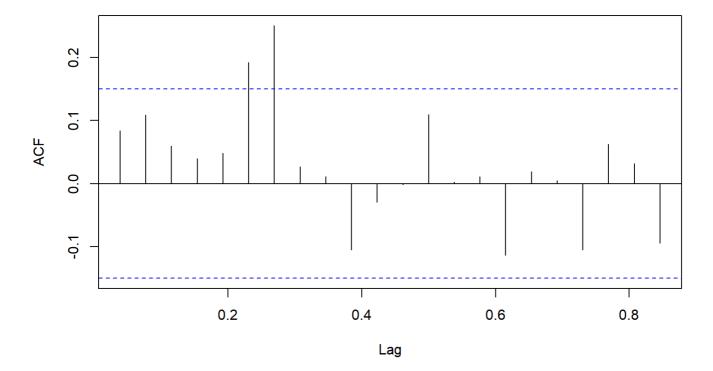
qqnorm(residuals, main = "Normal Q-Q Plot of Residuals"); qqline(residuals)

Normal Q-Q Plot of Residuals



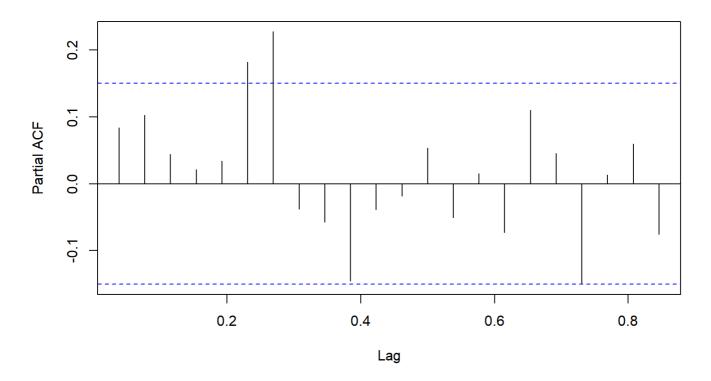
acf(residuals, main = "ACF of Residuals")

ACF of Residuals



pacf(residuals, main = "PACF of Residuals");

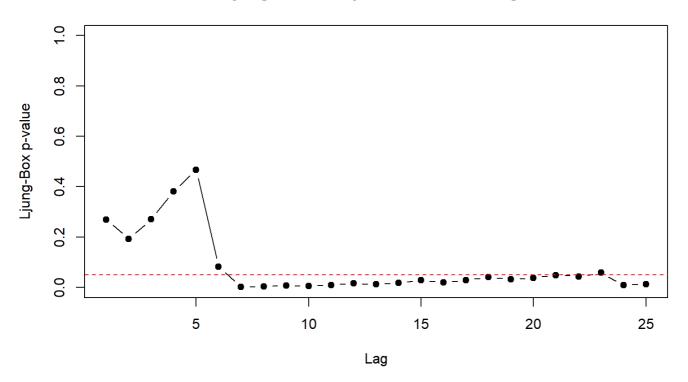
PACF of Residuals



Box.test(residuals, lag = 20, type = "Box-Pierce")

```
##
## Box-Pierce test
##
## data: residuals
## X-squared = 30.54, df = 20, p-value = 0.06157
```

Ljung-Box Test p-values Across Lags



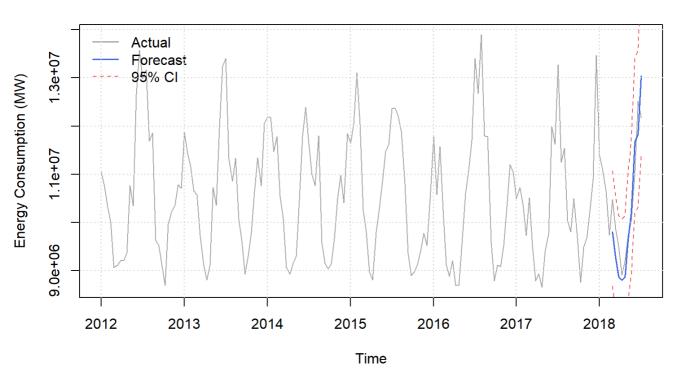
- Residual Mean: The residuals have a mean close to zero, indicating no significant bias in the model.
- **Homoscedasticity**: Residuals seem to have constant variance, but there are signs of increased fluctuations after 2010.
- **Normality**: The Shapiro-Wilk test shows that residuals are bordeline normally distributed (p-value = 0.2328).
- **Independence**: ACF and PACF plots show some correlation autocorrelation after lag 5, and the Box-Ljung test confirms the dependence of residuals (p-value = 6.083e-07).
- **Post-2010 Changes**: The model might not fully capture changes after 2010, suggesting the need for further decomposition and trend visualization.

Prototype Modelling: (Post 2011)

SARIMA(1,0,1)(0,1,1)

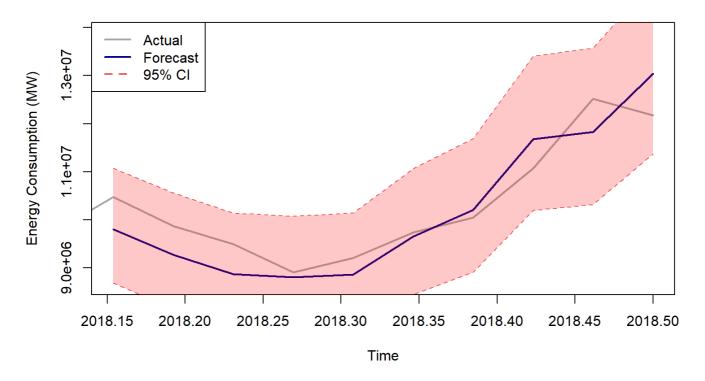
```
train <- window(ts.post11.biweekly, end = time(ts.post11.biweekly)[160])</pre>
test <- window(ts.post11.biweekly, start = time(ts.post11.biweekly)[160 + 1])</pre>
model.post.final.sarima \leftarrow arima(log(log(train)), order = c(1, 0, 1), seasonal = c(0, 1, 1))
forecast.raw <- predict(model.post.final.sarima, n.ahead = length(test))</pre>
forecast.pred <- exp(exp(forecast.raw$pred))</pre>
forecast.upper <- exp(exp(forecast.raw$pred + 1.96 * forecast.raw$se))</pre>
forecast.lower <- exp(exp(forecast.raw$pred - 1.96 * forecast.raw$se))</pre>
plot(ts.post11.biweekly, main = "Actual vs Forecast (SARIMA)", xlab = "Time",
     ylab = "Energy Consumption (MW)",col = "darkgray", lwd = 1, xaxt = "n")
lines(forecast.pred, col = "royalblue", lwd = 1.5)
lines(forecast.upper, col = "indianred2", lty = 2, lwd = 1)
lines(forecast.lower, col = "indianred2", lty = 2, lwd = 1)
axis(1, at = t <- time(ts.post11.biweekly)[d <- !duplicated(y <- floor(time(ts.post11.biweekly)</pre>
y)))],
     labels = y[d], las = 1)
legend("topleft", legend = c("Actual", "Forecast", "95% CI"),
col = c("darkgray", "royalblue", "indianred2"),
       lty = c(1, 1, 2), lwd = c(1.5, 1.5, 1), bty = "n")
grid()
```

Actual vs Forecast (SARIMA)



```
plot(ts.post11.biweekly, main = "Actual vs Forecast (SARIMA)",
    ylab = "Energy Consumption (MW)", xlab = "Time",
    col = "darkgray", lwd = 2, xlim = range(time(test)))
lines(forecast.pred, col = "darkblue", lwd = 2)
polygon(c(time(test), rev(time(test))), c(forecast.upper, rev(forecast.lower)),
    col = rgb(1, 0, 0, 0.2), border = NA)
lines(forecast.upper, col = "indianred2", lty = 2)
lines(forecast.lower, col = "indianred2", lty = 2)
legend("topleft", legend = c("Actual", "Forecast", "95% CI"),
    col = c("darkgray", "darkblue", "indianred2"), lty = c(1, 1, 2), lwd = 2)
```

Actual vs Forecast (SARIMA)



Forecasting and Performance Metrics (Post 2011)

```
res <- as.numeric(test) - forecast.pred
mape <- mean(abs(res / as.numeric(test))) * 100
r_squared <- 1 - (sum(res^2) / sum((as.numeric(test) - mean(as.numeric(test)))^2))
metrics <- data.frame(
  Metric = c("MAE", "RMSE", "MAPE (%)", "R-squared"),
  Value = round(c(mean(abs(res)), sqrt(mean(res^2)), mape, r_squared), 3))
print(metrics, row.names = FALSE)</pre>
```

```
## Metric Value

## MAE 474926.190

## RMSE 544114.244

## MAPE (%) 4.451

## R-squared 0.778
```

Visualizing Both Forecasting results

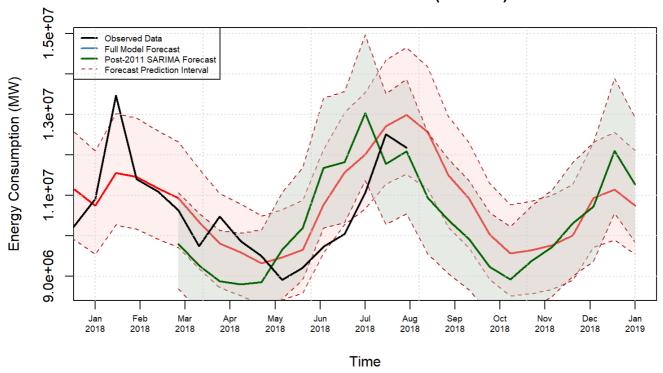
- **Data Full** SARIMA(2,0,1)(0,1,1)
- Post 2011 SARIMA(1,0,1)(1,1,1)

```
for.full.raw <- predict(model.final.sarima, n.ahead = 235)
for.full.pred <- exp(exp(for.full.raw$pred))
for.full.upper <- exp(exp(for.full.raw$pred + 1.96 * for.full.raw$se))
for.full.lower <- exp(exp(for.full.raw$pred - 1.96 * for.full.raw$se))

for.post.raw <- predict(model.post.final.sarima, n.ahead = 23)
for.post.pred <- exp(exp(for.post.raw$pred))
for.post.upper <- exp(exp(for.post.raw$pred + 1.96 * for.post.raw$se))
for.post.lower <- exp(exp(for.post.raw$pred - 1.96 * for.post.raw$se))</pre>
```

```
plot(ts.energy.biweekly, main = "Model SARIMA Forecasts (2018-19)",
    xlab = "Time", ylab = "Energy Consumption (MW)",
    col = "black", lwd = 2, xaxt = "n", xlim = c(2018, 2019))
polygon(c(time(for.full.upper), rev(time(for.full.lower))),
        c(for.full.upper, rev(for.full.lower)),
        col = adjustcolor("mistyrose", alpha.f = 0.4), border = NA)
lines(for.full.pred, col = "red", lwd = 2)
lines(for.full.upper, col = "firebrick", lty = 2)
lines(for.full.lower, col = "firebrick", lty = 2)
polygon(c(time(for.post.upper), rev(time(for.post.lower))),
        c(for.post.upper, rev(for.post.lower)),
        col = adjustcolor("honeydew3", alpha.f = 0.4), border = NA)
lines(for.post.pred, col = "darkgreen", lwd = 2)
lines(for.post.upper, col = "firebrick", lty = 2)
lines(for.post.lower, col = "firebrick", lty = 2)
lines(ts.energy.biweekly, col = "black", lwd = 2)
months_seq <- seq(from = as.Date("2018-01-01"), to = as.Date("2019-12-01"), by = "1 month")
month_ticks <- as.numeric(format(months_seq, "%Y")) + (as.numeric(format(months_seq, "%m")) -</pre>
1) / 12
axis(1, at = month_ticks, labels = format(months_seq, "%b\n%Y"), cex.axis = 0.6, las = 1)
legend("topleft", legend = c("Observed Data", "Full Model Forecast",
                             "Post-2011 SARIMA Forecast", "Forecast Prediction Interval"),
       col = c("black", "steelblue", "darkgreen", "firebrick"),
       lty = c(1, 1, 1, 2), lwd = c(2, 2, 2, 1), bg = "white", cex = 0.6)
grid()
```

Model SARIMA Forecasts (2018-19)



Forecasting till May 2023 (Covid-19 Era)

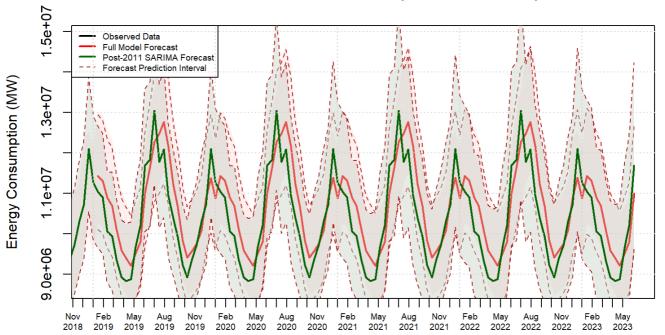
```
n.periods <- 115
for.full.raw <- predict(model.final.sarima, n.ahead = 235 + n.periods)
for.full.pred <- exp(exp(for.full.raw$pred))
for.full.upper <- exp(exp(for.full.raw$pred + 1.96 * for.full.raw$se))
for.full.lower <- exp(exp(for.full.raw$pred - 1.96 * for.full.raw$se))

for.post.raw <- predict(model.post.final.sarima, n.ahead = 23 + n.periods)
for.post.pred <- exp(exp(for.post.raw$pred))
for.post.upper <- exp(exp(for.post.raw$pred + 1.96 * for.post.raw$se))
for.post.lower <- exp(exp(for.post.raw$pred - 1.96 * for.post.raw$se))

com.pred <- 0.75 * tail(for.full.pred, n.periods) + 0.25 * tail(for.post.pred, n.periods)
com.upper <- 0.75 * tail(for.full.upper, n.periods) + 0.25 * tail(for.post.lower, n.periods)
com.lower <- 0.75 * tail(for.full.lower, n.periods) + 0.25 * tail(for.post.lower, n.periods)</pre>
```

```
plot(ts.energy.biweekly, main = "Model SARIMA Forecasts (Covid-19 Period)",
    xlab = "Time", ylab = "Energy Consumption (MW)",
    col = "black", lwd = 2, xaxt = "n", xlim = c(2019, 2023 + 5/12))
polygon(c(time(for.full.upper), rev(time(for.full.lower))),
        c(for.full.upper, rev(for.full.lower)),
        col = adjustcolor("mistyrose", alpha.f = 0.4), border = NA)
lines(com.pred, col = "red", lwd = 2)
lines(com.upper, col = "firebrick", lty = 2)
lines(com.lower, col = "firebrick", lty = 2)
polygon(c(time(for.post.upper), rev(time(for.post.lower))),
        c(for.post.upper, rev(for.post.lower)),
        col = adjustcolor("honeydew3", alpha.f = 0.4), border = NA)
lines(for.post.pred, col = "darkgreen", lwd = 2)
lines(for.post.upper, col = "firebrick", lty = 2)
lines(for.post.lower, col = "firebrick", lty = 2)
lines(ts.energy.biweekly, col = "black", lwd = 2)
months_seq <- seq(from = as.Date("2018-01-01"), to = as.Date("2023-05-01"), by = "1 month")
month_ticks <- as.numeric(format(months_seq, "%Y")) + (as.numeric(format(months_seq, "%m"))-</pre>
1)/12
axis(1, at = month_ticks, labels = format(months_seq, "%b\n%Y"), cex.axis = 0.6, las = 1)
legend("topleft", legend = c("Observed Data", "Full Model Forecast",
                             "Post-2011 SARIMA Forecast", "Forecast Prediction Interval"),
       col = c("black", "red", "darkgreen", "firebrick"),
       lty = c(1, 1, 1, 2), lwd = c(2, 2, 2, 1), bg = "white", cex = 0.6)
grid()
```

Model SARIMA Forecasts (Covid-19 Period)



Time