

# Anharmonic Potential Problem

Solve the s-wave radial Schrodinger equation for a particle of mass m:

$$\frac{d^2y}{dr^2} = A(r)u(r), A(r) = \frac{2m}{\hbar^2}[V(r) - E]$$

For the anharmonic oscillator potential

$$V(r) = \frac{1}{2}kr^2 + \frac{1}{3}br^3$$

for the ground state and first state energy (in MeV) of particle to an accuracy of three significant digits. Also, plot the corresponding wave functions. Choose  $m = 940 \text{ MeV}/c^2$ ,  $k = 100 \text{ MeV fm}^{-2}$ ,  $b = 0, 10, 30 \text{ MeV fm}^{-3}$  in these units,  $c\hbar = 197.3 \text{ MeV fm}$ . The ground state energy I expected to lie between 90 and 110 MeV for all three cases.

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In [51]: import numpy as np
import matplotlib.pyplot as plt
X=np.linspace(10e-100,10,1000)
r=X
h=X[2]-X[1]
m=940
k=100
hc=197.3
b_values=[0,10,30]
for b in b_values:
    print(f"For b={b}")
    H=np.zeros([1000,1000])
    for i in range(len(H)):
        try:
            H[i,i-1]=1
            H[i,i]=-2
            H[i,i+1]=1
        except:
            pass
    H=(1/h**2)*H
    H=((-hc**2)/(2*m))*H
    V=np.zeros([1000,1000])
    for i in range(1000):
        V[i,i]=(1/2)*k*r[i]**2+(1/3)*b*r[i]**3
    H=H+V
    eigenvalues,eigenvectors=np.linalg.eig(H)
    idx=eigenvalues.argsort()[ :]
    eigenvalues=eigenvalues[idx]
    eigenvectors=eigenvectors[:,idx]
    print("Energy values for Ground and first state are:",eigenvalues[:2])
    fig,axs=plt.subplots(2,2,figsize=(9.5,5.5))
    for i in range(2):
        axs[i,0].plot(r,eigenvectors[:,i], 'blue')
        axs[i,0].set_ylabel(r'\Psi(r)')
        axs[i,0].set_xlabel("$r$")
```

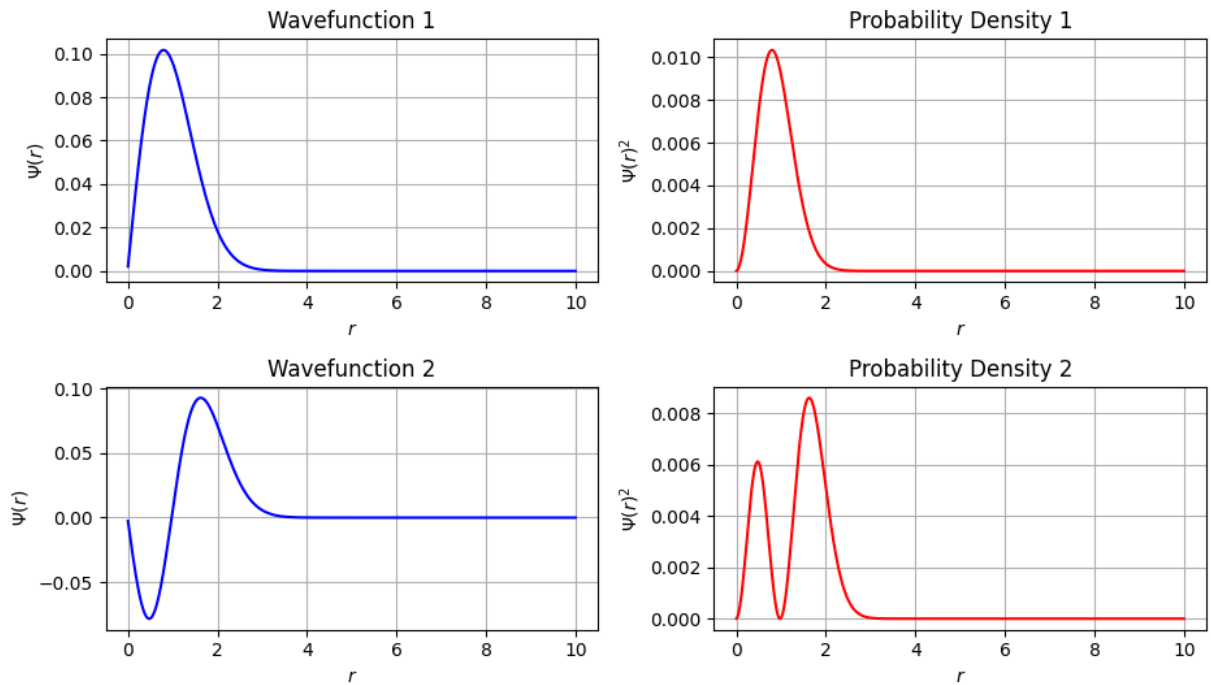
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    axs[i,0].set_title(f"Wavefunction {i+1}")
    axs[i,0].grid(True)
    axs[i,1].plot(r,eigenvectors[:,i]**2,'r')
    axs[i,1].set_ylabel(r'$\Psi(r)^2$')
    axs[i,1].set_xlabel("$r$")
    axs[i,1].set_title(f"Probability Density {i+1}")
    axs[i,1].grid(True)
    plt.tight_layout()
    plt.show()

```

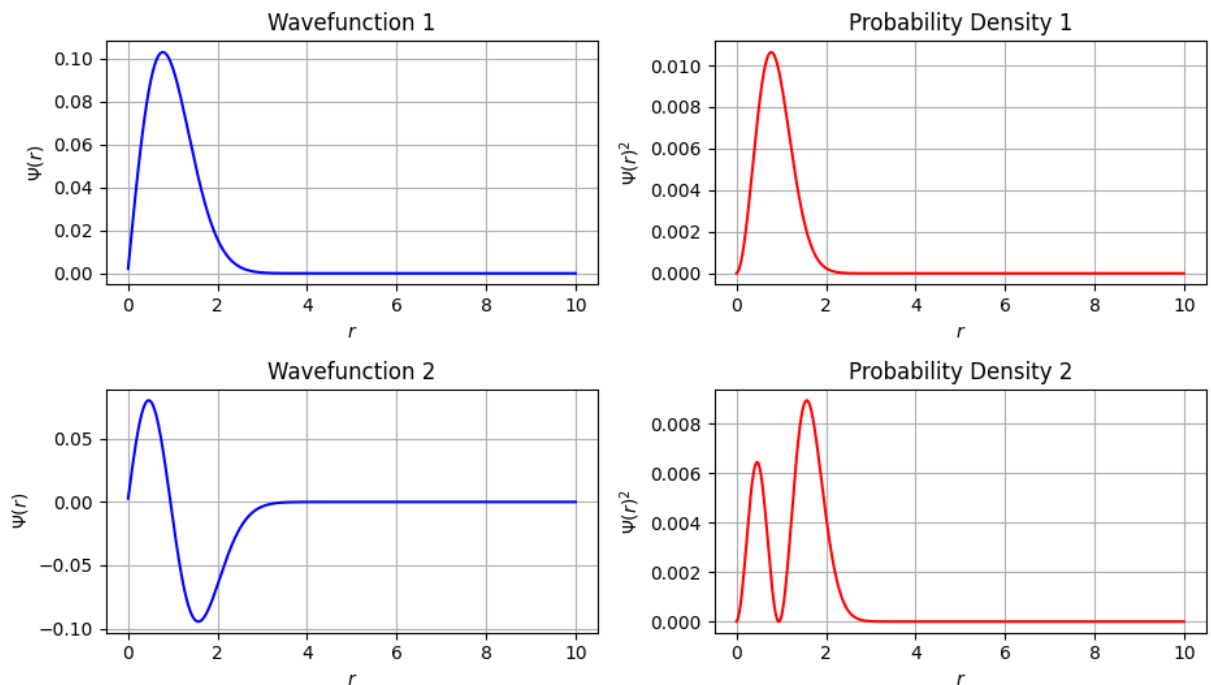
For  $b=0$

Energy values for Ground and first state are: [ 95.62451876 223.86968926]



For  $b=10$

Energy values for Ground and first state are: [ 99.27601065 236.50053529]



For  $b=30$

Energy values for Ground and first state are: [105.80437266 258.2243101 ]

