**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans. - You have a normal distribution with  mu= 45 and SD = 8.0. Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have X ≤ 50 as he started 10 min late. So

P(X > 50).

P(X > 50) = 1 - P(X ≤ 50).

You need to change this to a question about the standard normal variable Z using the transformation

Z = (X - mu)/SD = (X - 45)/8.0

Thus the first question can be answered by using the normal table to find

P(X ≤ 50) = P(Z ≤ (50 - 45)/8.0) = P(Z ≤ 0.625)

Z≤0.625 Now from Table it gives 73.4 approx

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

OR

from statistics import NormalDist

NormalDist(mu=45, sigma=8).cdf(50)

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans. - We have a normal distribution with mu= 38 and SD= 6.

Let X be the number of employees. So according to question

a)Probability of employees greater than age of 44= P(X>44)

P(X > 44) = 1 - P(X ≤ 44).

Z = (X -µ)/ SD= (X - 38)/6

Thus the question can be answered by using the normal table to find

P(X ≤ 44) = P (Z ≤(44 - 38)/6) = P(Z ≤ 1)=84.1345%

Probability that the employee will be greater than age of 44 = 100-84.1345=15.86%

So the probability of number of employees between 38-44 years of age = P(X<44)-0.5=84.1345-0.5= 34.1345%

Therefore the statement that more employees at the processing center are older than 44 than between 38 and 44 is TRUE.

b) Probability of employees less than age of 30 = P(X<30).

Z = (X - µ )/ SD= (30 - 38)/6

Thus the question can be answered by using the normal table to find

P (X < 30) = P(Z < (30 - 38)/6) = P(Z < -1.333)=9.12%

So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48(or 36 employees).

Therefore the statement B of the question is also TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans. As we know that if X <" N(µ1, Ã 1^2 ), and Y <" N(µ2, Ã 2^2 ) are two independent random variables then X + Y <" N(µ1 + µ2, Ã 1^2 + Ã 2^2 ) , and X " Y <" N(µ1 " µ2, Ã 1^2 + Ã 2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z <" N(aµ1 + bµ2, a^2Ã 1^2 + b^2Ã 2^2 ).

Therefore in the question

2X1~ N(2 u,4 Ã ^2) and

X1+X2 ~ N(µ + µ, Ã ^2 + Ã ^2 ) ~ N(2 u, 2Ã ^2 )

2X1-(X1+X2) = N( 4µ,6 Ã ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans. Ans. Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (I e. 1-0.99).

The Probability towards left from a = -0.005 (I e. 0.01/2).

The Probability towards right from b = +0.005 (I e. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- ¼ ) / Ã

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* Ã + ¼ = X

Z (-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z (+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans. import numpy as np

from scipy import stats

from scipy.stats import norm

A.

total mean 5+7=12

total profit =12\*45=540millions

SD = np.sqrt((9)+(16)) # from both divisions

stats.norm.interval(0.95,540,225)) # range

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

B.

from z table, 5 percentile = -1.645

X= 540+(-1.645)\*(225)=170 approx

C

# Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3)

0.04779

# Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4)

0.04005

Probability of Division 1 making a loss in a given year is more than Division 2