LPP

SIMPLEX METHOD

SIMPLEX METHOD

Simplex method, also called simplex technique or simplex algorithm was developed by G.B. Dantzig, an American mathematician.

It has the advantage of being universal.

i.e. any linear model for which the solution exists, can be solved by it.

In principle, it consists of starting with a

certain solution of which all that we know is that it is

feasible, i.e. it satisfies the non negativity conditions

$(x_j \geq 0, j = 1, 2, 3, \dots \dots, n)$, we then improve upon this

solution at consecutive stages, until, after a certain finite number of stages, we arrive at the optimal solution

The Simplex method provides an algorithm which consists of moving from one vertex of the region of feasible solutions to another in such a manner that the value of the objective function at the succeeding vertex is less (or more as the case may be) then at the preceeding vertex.

This procedure of jumping from one vertex to another is then repeated. since the number of vertices is finite, this methods leads to an optimal vertex in a finite number of steps.

The basis of the simplex method consist of two fundamental conditions

1. **The feasibility condition**: It ensures that if the starting solution is basic feasible, only basic feasible solutions will be obtained during computation
2. **The optimality condition**: It guarantees that only better solutions (as compared to the current solution) will be encountered

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Using Simplex method solve the following LPP

Maximize $z = 10x_1 + x_2 + x_3$

Subject to

$$x_1 + x_2 - 3x_3 \leq 10,$$
$$4x_1 + x_2 + x_3 \leq 20,$$
$$x_1, x_2, x_3 \geq 0$$

Solution: We first express the given problem in standard form

$$z - 10x_1 - x_2 - x_3 + 0s_1 + 0s_2 = 0$$
$$x_1 + x_2 - 3x_3 + s_1 + 0s_2 = 10$$
$$4x_1 + x_2 + x_3 + 0s_1 + s_2 = 20$$

The initial basic feasible solution

2 equations in 5 variable :

2 basic and 3 non basic variables

Let $x_1, x_2 \,\&\, x_3$ *be non basic and* s_1, s_2 *be basic*

$\therefore s_1 = 10\,, s_2 = 20$ is the initial basic solution

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	x_3	s_1	s_2		
0	z							
	s_1							
	s_2							

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	x_3	s_1	s_2		
0	z	-10	-1	-1	0	0	0	
	s_1	1	1	-3	1	0	10	
	s_2	4	1	1	0	1	20	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	x_3	s_1	s_2		
0	z	-10	-1	-1	0	0	0	
	s_1	1	1	-3	1	0	10	
	s_2	4	1	1	0	1	20	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	x_3	s_1	s_2		
0	z	-10	-1	-1	0	0	0	
	s_1	1	1	-3	1	0	10	10
	s_2	4	1	1	0	1	20	5

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	x_3	s_1	s_2		
0	z	-10	-1	-1	0	0	0	
	s_1	1	1	-3	1	0	10	10
	s_2	4*	1	1	0	1	20	5

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	x_3	s_1	s_2		
0	z	-10	-1	-1	0	0	0	
s_2 leaves	s_1	1	1	-3	1	0	10	10
x_1 enters	s_2	4*	1	1	0	1	20	5

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	x_3	s_1	s_2		
0	z	-10	-1	-1	0	0	0	
s_2 leaves	s_1	1	1	-3	1	0	10	10
x_1 enters	s_2	4*	1	1	0	1	20	5
1	z							
	s_1							
	x_1							

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	x_3	s_1	s_2		
0	z	-10	-1	-1	0	0	0	
s_2 leaves	s_1	1	1	-3	1	0	10	10
x_1 enters	s_2	4*	1	1	0	1	20	5
1	z							
	s_1							
	x_1	1	1/4	1/4	0	1/4	5	1

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	x_3	s_1	s_2		
0	z	-10	-1	-1	0	0	0	
s_2 leaves	s_1	1	1	-3	1	0	10	10
x_1 enters	s_2	4*	1	1	0	1	20	5
1	z							
	s_1	0	3/4	-13/4	1	-1/4	5	
	x_1	1	1/4	1/4	0	1/4	5	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	x_3	s_1	s_2		
0	z	-10	-1	-1	0	0	0	
s_2 leaves	s_1	1	1	-3	1	0	10	10
x_1 enters	s_2	4*	1	1	0	1	20	5
1	z	0	6/4	6/4	0	10/4	50	
	s_1	0	3/4	-13/4	1	-1/4	5	
	x_1	1	1/4	1/4	0	1/4	5	

Since all the coefficients in the objective equation in the row of z are positive.
This is a optimal solution.
The values of the variables and of z are given by the RHS column

$$\therefore x_1 = 5, x_2 = 0, x_3 = 0, z_{max} = 50$$

Ex. 2 Solve the following LPP by simplex method

Maximize $z = x_1 + 4x_2$

Subject to

$$2x_1 + x_2 \leq 3,$$
$$3x_1 + 5x_2 \leq 9,$$
$$x_1 + 3x_2 \leq 5,$$
$$x_1, x_2 \geq 0$$

We first express the problem in the standard form

Maximize $z = x_1 + 4x_2 - 0s_1 - 0s_2 - 0s_3$

i.e. $z - x_1 - 4x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

Subject to

$$2x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 3$$

$$3x_1 + 5x_2 + 0s_1 + s_2 + 0s_3 = 9$$

$$x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 = 5$$

We now put this in the following table:

Simplex table

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-1	-4	0	0	0	0	
	s_1	2	1	1	0	0	3	
	s_2	3	5	0	1	0	9	
	s_3	1	3	0	0	1	5	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-1	-4	0	0	0	0	
	s_1	2	1	1	0	0	3	
	s_2	3	5	0	1	0	9	
	s_3	1	3	0	0	1	5	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-1	-4	0	0	0	0	
	s_1	2	1	1	0	0	3	3
	s_2	3	5	0	1	0	9	9/5= 1.8
	s_3	1	3	0	0	1	5	5/3= 1.67

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-1	-4	0	0	0	0	
	s_1	2	1	1	0	0	3	3
	s_2	3	5	0	1	0	9	9/5= 1.8
	s_3	1	3*	0	0	1	5	5/3= 1.67

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-1	-4	0	0	0	0	
	s_1	2	1	1	0	0	3	3
s_3 leaves	s_2	3	5	0	1	0	9	9/5= 1.8
x_2 enters	s_3	1	3*	0	0	1	5	5/3= 1.67

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-1	-4	0	0	0	0	
s_3 leaves	s_1	2	1	1	0	0	3	3
x_2 enters	s_2	3	5	0	1	0	9	9/5= 1.8
	s_3	1	3*	0	0	1	5	5/3= 1.67
1	z							
	s_1							
	s_2							
	x_2	1/3	1	0	0	1/3	5/3	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-1	-4	0	0	0	0	
s_3 leaves	s_1	2	1	1	0	0	3	3
x_2 enters	s_2	3	5	0	1	0	9	9/5= 1.8
	s_3	1	3*	0	0	1	5	5/3= 1.67
1	z							
	s_1							
	s_2	4/3	0	0	1	-5/3	2/3	
	x_2	1/3	1	0	0	1/3	5/3	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-1	-4	0	0	0	0	
s_3 leaves	s_1	2	1	1	0	0	3	3
x_2 enters	s_2	3	5	0	1	0	9	9/5= 1.8
	s_3	1	3*	0	0	1	5	5/3= 1.67
1	z							
	s_1	5/3	0	1	0	-1/3	4/3	
	s_2	4/3	0	0	1	-5/3	2/3	
	x_2	1/3	1	0	0	1/3	5/3	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-1	-4	0	0	0	0	
s_3 leaves	s_1	2	1	1	0	0	3	3
x_2 enters	s_2	3	5	0	1	0	9	9/5= 1.8
	s_3	1	3*	0	0	1	5	5/3= 1.67
1	z	1/3	0	0	0	4/3	20/3	
	s_1	5/3	0	1	0	-1/3	4/3	
	s_2	4/3	0	0	1	-5/3	2/3	
	x_2	1/3	1	0	0	1/3	5/3	

- Since, all the coefficients in the z row are positive, the optimum solution is obtained.
- The required results are given by the RHS solution column.
- Thus, $x_1 = 0, x_2 = 5/3\,, z_{max} = 20/3$

EX 3. Solve the following LPP by Simplex method

Maximize $z = 4x_1 + 10x_2$

Subject to $2x_1 + x_2 \leq 10,$

$$2x_1 + 5x_2 \leq 20,$$

$$2x_1 + 3x_2 \leq 18,$$

$$x_1, x_2 \geq 0$$

Solution: We first express the given problem in standard form

Maximize $z = 4x_1 + 10x_2 + 0s_1 + 0s_2 + 0s_3$

i.e. $z - 4x_1 - 10x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

Subject to $2x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 10$

$2x_1 + 5x_2 + 0s_1 + s_2 + 0s_3 = 20$

$2x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 = 18,$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

We put this information in tabular form as follows

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z							
	s_1							
	s_2							
	s_3							

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
	s_1	2	1	1	0	0	10	
	s_2	2	5	0	1	0	20	
	s_3	2	3	0	0	1	18	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
	s_1	2	1	1	0	0	10	
	s_2	2	5	0	1	0	20	
	s_3	2	3	0	0	1	18	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
	s_1	2	1	1	0	0	10	10
	s_2	2	5*	0	1	0	20	4
	s_3	2	3	0	0	1	18	6

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
	s_1	2	1	1	0	0	10	10
	s_2	2	5*	0	1	0	20	4
	s_3	2	3	0	0	1	18	6

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5*	0	1	0	20	4
	s_3	2	3	0	0	1	18	6

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5*	0	1	0	20	4
	s_3	2	3	0	0	1	18	6
1	z							
	s_1							
	x_2							
	s_3							

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5*	0	1	0	20	4
	s_3	2	3	0	0	1	18	6
1	z							
	s_1							
	x_2	$2/5$	1	0	$1/5$	0	4	
	s_3							

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5*	0	1	0	20	4
	s_3	2	3	0	0	1	18	6
1	z							
	s_1							
	x_2	$2/5$	1	0	$1/5$	0	4	
	s_3	$4/5$	0	0	$-3/5$	1	6	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5*	0	1	0	20	4
	s_3	2	3	0	0	1	18	6
1	z							
	s_1	$8/5$	0	1	$-1/5$	0	6	
	x_2	$2/5$	1	0	$1/5$	0	4	
	s_3	$4/5$	0	0	$-3/5$	1	6	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5*	0	1	0	20	4
	s_3	2	3	0	0	1	18	6
1	z	0	0	0	2	0	40	
	s_1	$8/5$	0	1	$-1/5$	0	6	
	x_2	$2/5$	1	0	$1/5$	0	4	
	s_3	$4/5$	0	0	$-3/5$	1	6	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5*	0	1	0	20	4
	s_3	2	3	0	0	1	18	6
1	z	0	0	0	2	0	40	
	s_1	$8/5$	0	1	$-1/5$	0	6	
	x_2	$2/5$	1	0	$1/5$	0	4	
	s_3	$4/5$	0	0	$-3/5$	1	6	

Since all the coefficients in the objective equation in the row of z are positive. This is a optimal solution. The values of the variables and of z are given by the least column $\therefore x_1 = 0, x_2 = 4, z_{max} = 40$

But further considerations show that s_1 may leave and x_1 may enter

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5*	0	1	0	20	4
	s_3	2	3	0	0	1	18	6
1	z	0	0	0	2	0	40	
	s_1	$8/5$	0	1	$-1/5$	0	6	
	x_2	$2/5$	1	0	$1/5$	0	4	
	s_3	$4/5$	0	0	$-3/5$	1	6	

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5*	0	1	0	20	4
	s_3	2	3	0	0	1	18	6
1	z	0	0	0	2	0	40	
	s_1	$8/5$	0	1	$-1/5$	0	6	$15/4$
	x_2	$2/5$	1	0	$1/5$	0	4	10
	s_3	$4/5$	0	0	$-3/5$	1	6	$15/2$

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5*	0	1	0	20	4
	s_3	2	3	0	0	1	18	6
1	z	0	0	0	2	0	40	
	s_1	$8/5$	0	1	$-1/5$	0	6	$15/4$
	x_2	$2/5$	1	0	$1/5$	0	4	10
	s_3	$4/5$	0	0	$-3/5$	1	6	$15/2$

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	z	-4	-10	0	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5*	0	1	0	20	4
	s_3	2	3	0	0	1	18	6
1	z	0	0	0	2	0	40	
s_1 leaves	s_1	$8/5$*	0	1	$-1/5$	0	6	$15/4$
x_1 enters	x_2	$2/5$	1	0	$1/5$	0	4	10
	s_3	$4/5$	0	0	$-3/5$	1	6	$15/2$

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
1	z	0	0	0	2	0	40	
s_1 leaves	s_1	$8/5$*	0	1	$-1/5$	0	6	$15/4$
x_1 enters	x_2	$2/5$	1	0	$1/5$	0	4	10
	s_3	$4/5$	0	0	$-3/5$	1	6	$15/2$
2	z							
	x_1							
	x_2							
	s_3							

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
1	z	0	0	0	2	0	40	
s_1 leaves	s_1	$8/5^*$	0	1	$-1/5$	0	6	$15/4$
x_1 enters	x_2	$2/5$	1	0	$1/5$	0	4	10
	s_3	$4/5$	0	0	$-3/5$	1	6	$15/2$
2	z							
	x_1	1	0	$5/8$	$-1/8$	0	$15/4$	
	x_2							
	s_3							

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
1	z	0	0	0	2	0	40	
s_1 leaves	s_1	$8/5$	0	1	$-1/5$	0	6	$15/4$
x_1 enters	x_2	$2/5$	1	0	$1/5$	0	4	10
	s_3	$4/5$	0	0	$-3/5$	1	6	$15/2$
2	z	0	0	0	$-1/5$	0	40	
	x_1	1	0	$5/8$	$-1/8$	0	$15/4$	
	x_2							
	s_3							

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
1	z	0	0	0	2	0	40	
s_1 leaves	s_1	$8/5$	0	1	$-1/5$	0	6	$15/4$
x_1 enters	x_2	$2/5$	1	0	$1/5$	0	4	10
	s_3	$4/5$	0	0	$-3/5$	1	6	$15/2$
2	z	0	0	0	2	0	40	
	x_1	1	0	$5/8$	$-1/8$	0	$15/4$	
	x_2	0	1	$-1/4$	$1/4$	0	$5/2$	
	s_3							

Iteration Number	Basic Variables	Coefficients of					RHS Solution	Ratio
		x_1	x_2	s_1	s_2	s_3		
1	z	0	0	0	2	0	40	
s_1 leaves	s_1	$8/5$	0	1	$-1/5$	0	6	$15/4$
x_1 enters	x_2	$2/5$	1	0	$1/5$	0	4	10
	s_3	$4/5$	0	0	$-3/5$	1	6	$15/2$
2	z	0	0	0	2	0	40	
	x_1	1	0	$5/8$	$-1/8$	0	$15/4$	
	x_2	0	1	$-1/4$	$1/4$	0	$5/2$	
	s_3	0	0	$-1/2$	$-1/2$	1	3	

$\therefore x_1 = 15/4\,, x_2 = 5/2\,, z_{max} = 40$

This is an alternative solution. But this does not improve the above optimal solution

Thus we have two solutions

$\therefore x_1 = 0, x_2 = 2, s_1 = 6, s_2 = 0, s_3 = 6,$

$z_{max} = 40$

and $x_1 = 15/4\,, x_2 = 5/2\,, s_1 = 0, s_2 = 0, s_3 = 3,$

$z_{max} = 40$

If there are two solutions to a problem then there are infinite number solutions

Let $X = \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}, X_1 = \begin{bmatrix} 0 \\ 4 \\ 6 \\ 0 \\ 6 \end{bmatrix}, X_2 = \begin{bmatrix} 15/4 \\ 5/2 \\ 0 \\ 0 \\ 3 \end{bmatrix}$

then $X = \lambda X_1 + (1-\lambda)X_2$ for $0 \leq \lambda \leq 1$

i.e. $X = \begin{bmatrix} \frac{15}{4}(1-\lambda) \\ 4\lambda + \frac{5}{2}(1-\lambda) \\ 6\lambda \\ 0 \\ 6\lambda + 3(1-\lambda) \end{bmatrix}$

Ex 4. Solve the following LPP by Simplex method

Maximize $z = 4x_1 + x_2 + 3x_3 + 5x_4$

Subject to $-4x_1 + 6x_2 + 5x_3 + 4x_4 \leq 20,$

$-3x_1 - 2x_2 + 4x_3 + x_4 \leq 10,$

$-8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20,$

$x_1, x_2, x_3, x_4 \geq 0$

Solution: We first express the problem in standard form

$$z - 4x_1 - x_2 - 3x_3 - 5x_4 = 0$$
$$-4x_1 + 6x_2 + 5x_3 + 4x_4 + s_1 = 20$$
$$-3x_1 - 2x_2 + 4x_3 + x_4 + s_2 = 10$$
$$-8x_1 - 3x_2 + 3x_3 + 2x_4 + s_3 = 20$$

Iteration Number	Basic Variables	Coefficients of							RHS Solution	Ratio
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
0	z									
	s_1									
	s_2									
	s_3									

Iteration Number	Basic Variables	Coefficients of							RHS Solution	Ratio
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
0	z	-4	-1	-3	-5	0	0	0	0	
	s_1	-4	6	5	4	1	0	0	20	
	s_2	-3	-2	4	1	0	1	0	10	
	s_3	-8	-3	3	2	0	0	1	20	

Iteration Number	Basic Variables	Coefficients of							RHS Solution	Ratio
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
0	z	-4	-1	-3	-5	0	0	0	0	
	s_1	-4	6	5	4	1	0	0	20	
	s_2	-3	-2	4	1	0	1	0	10	
	s_3	-8	-3	3	2	0	0	1	20	

Iteration Number	Basic Variables	Coefficients of							RHS Solution	Ratio
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
0	z	-4	-1	-3	-5	0	0	0	0	
	s_1	-4	6	5	4	1	0	0	20	5
	s_2	-3	-2	4	1	0	1	0	10	10
	s_3	-8	-3	3	2	0	0	1	20	10

Iteration Number	Basic Variables	Coefficients of							RHS Solution	Ratio
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
0	z	-4	-1	-3	-5	0	0	0	0	
	s_1	-4	6	5	4	1	0	0	20	5
	s_2	-3	-2	4	1	0	1	0	10	10
	s_3	-8	-3	3	2	0	0	1	20	10

Iteration Number	Basic Variables	Coefficients of							RHS Solution	Ratio
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
0	z	-4	-1	-3	-5	0	0	0	0	
s_1 leaves	s_1	-4	6	5	4*	1	0	0	20	5
x_4 enters	s_2	-3	-2	4	1	0	1	0	10	10
	s_3	-8	-3	3	2	0	0	1	20	10

Iteration Number	Basic Variables	Coefficients of							RHS Solution	Ratio
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
0	z	-4	-1	-3	-5	0	0	0	0	
s_1 leaves	s_1	-4	6	5	4*	1	0	0	20	5
x_4 enters	s_2	-3	-2	4	1	0	1	0	10	10
	s_3	-8	-3	3	2	0	0	1	20	10
1	z									
	x_4									
	s_2									
	s_3									

Iteration Number	Basic Variables	Coefficients of							RHS Solution	Ratio
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
0	z	-4	-1	-3	-5	0	0	0	0	
s_1 leaves	s_1	-4	6	5	4*	1	0	0	20	5
x_4 enters	s_2	-3	-2	4	1	0	1	0	10	10
	s_3	-8	-3	3	2	0	0	1	20	10
1	z									
	x_4	-1	$3/2$	$5/4$	1	$1/4$	0	0	5	
	s_2									
	s_3									

Iteration Number	Basic Variables	Coefficients of							RHS Solution	Ratio
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
0	z	-4	-1	-3	-5	0	0	0	0	
s_1 leaves	s_1	-4	6	5	4*	1	0	0	20	5
x_4 enters	s_2	-3	-2	4	1	0	1	0	10	10
	s_3	-8	-3	3	2	0	0	1	20	10
1	z	-9	$13/2$	$13/4$	0	$5/4$	0	0	25	
	x_4	-1	$3/2$	$5/4$	1	$1/4$	0	0	5	
	s_2									
	s_3									

Iteration Number	Basic Variables	Coefficients of							RHS Solution	Ratio
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
0	z	-4	-1	-3	-5	0	0	0	0	
s_1 leaves	s_1	-4	6	5	4*	1	0	0	20	5
x_4 enters	s_2	-3	-2	4	1	0	1	0	10	10
	s_3	-8	-3	3	2	0	0	1	20	10
1	z	-9	$13/2$	$13/4$	0	$5/4$	0	0	25	
	x_4	-1	$3/2$	$5/4$	1	$1/4$	0	0	5	
	s_2	-2	$-7/2$	$-11/4$	0	$-1/4$	0	1	5	
	s_3									

Iteration Number	Basic Variables	Coefficients of							RHS Solution	Ratio
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
0	z	-4	-1	-3	-5	0	0	0	0	
s_1 leaves	s_1	-4	6	5	4	1	0	0	20	5
x_4 enters	s_2	-3	-2	4	1	0	1	0	10	10
	s_3	-8	-3	3	2	0	0	1	20	10
1	z	-9	$13/2$	$13/4$	0	$5/4$	0	0	25	
	x_4	-1	$3/2$	$5/4$	1	$1/4$	0	0	5	
	s_2	-2	$-7/2$	$-11/4$	0	$-1/4$	0	1	5	
	s_3	-6	-6	$1/2$	0	$-1/2$	1	0	10	

Iteration Number	Basic Variables	Coefficients of							RHS Solution	Ratio
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
0	z	-4	-1	-3	-5	0	0	0	0	
s_1 leaves	s_1	-4	6	5	4	1	0	0	20	5
x_4 enters	s_2	-3	-2	4	1	0	1	0	10	10
	s_3	-8	-3	3	2	0	0	1	20	10
1	z	-9	$13/2$	$13/4$	0	$5/4$	0	0	25	
	x_4	-1	$3/2$	$5/4$	1	$1/4$	0	0	5	
	s_2	-2	$-7/2$	$-11/4$	0	$-1/4$	0	1	5	
	s_3	-6	-6	$1/2$	0	$-1/2$	1	0	10	

Iteration Number	Basic Variables	Coefficients of							RHS Solution	Ratio
		x_1	x_2	x_3	x_4	s_1	s_2	s_3		
0	z	-4	-1	-3	-5	0	0	0	0	
s_1 leaves	s_1	-4	6	5	4	1	0	0	20	5
x_4 enters	s_2	-3	-2	4	1	0	1	0	10	10
	s_3	-8	-3	3	2	0	0	1	20	10
1	z	-9	$13/2$	$13/4$	0	$5/4$	0	0	25	
	x_4	-1	$3/2$	$5/4$	1	$1/4$	0	0	5	-5
	s_2	-2	$-7/2$	$-11/4$	0	$-1/4$	0	1	5	$-5/2$
	s_3	-6	-6	$1/2$	0	$-1/2$	1	0	10	$-5/3$

Since all entries in the ratio column are negative the problem has unbounded solution