CHAPTER 2

Continuous Time Signals and Systems

2.1 Introduction

Time is an important independent variable required to measure/monitor any activity. Hence, whatever phenomena we observe in nature are always measured as a function of time.

Time is a continuous independent variable represented by the letter ‘t’. Any physical observation or a measure is a continuous function of time represented by x(t) and called ***signal.*** The signal x(t) is called the continuous time (CT) signal which is a function of the independent variable, time. In x(t),the unit of time and the unit of the value of x(t) at any time is not considered but only the numerical values are considered.

The signal x(t) can be used to represent any physical quantity, and the start of any observation or a measure of the physical quantity is taken as time $t = 0$. The time after the start of observation is taken as positive time and the time before the start of observation is taken as negative time.

The physical devices which are all sources of continuous time signals are called ***continuous time systems***. The standard continuous time signals, mathematical operation on continuous time signals and classification of continuous time signals are discussed in this chapter. The mathematical representation of continuous time systems and their analyses are also presented in this chapter. Wherever required, the discussion on LTI systems are presented separately.

2.2 Standard Continuous Time Signals

1. Impulse signal

The impulse signal is a signal with infinite magnitude and zero duration, but with an area of A. Mathematically, impulse signal is defined as,

$$\text{Impulse Signal, } \delta(t) = \infty \; ; \; t = 0 \qquad \text{and} \qquad \int_{-\infty}^{+\infty} \delta(t)\, dt = A$$
$$= 0 \; ; \; t \neq 0$$

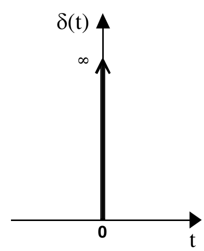

Fig 2.1 : *Impulse signal (or Unit Impulse signal).*

The unit impulse signal is a signal with infinite magnitude and zero duration, but with unit area. Mathematically, unit impulse signal is defined as,

$$\text{Unit Impulse Signal, } \delta(t) = \infty \; ; \; t = 0 \qquad \text{and} \qquad \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$
$$= 0 \; ; \; t \neq 0$$

2. Step signal

The step signal is defined as,

$$x(t) = A \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

The unit step signal is defined as,

$$x(t) = u(t) = 1 \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

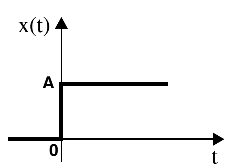

Fig 2.2 : *Step signal.*

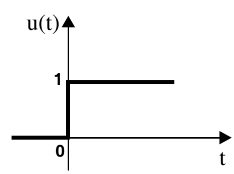

Fig 2.3 : *Unit step signal.*

3. Ramp signal

The ramp signal is defined as,

$$x(t) = At \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

The unit ramp signal is defined as,

$$x(t) = t \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

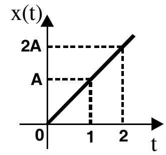

Fig 2.4 : *Ramp signal.*

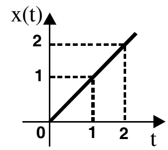

Fig 2.5 : *Unit ramp signal.*

4. Parabolic signal

The parabolic signal is defined as,

$$x(t) = \frac{At^2}{2} \; ; \; \text{for } t \geq 0$$
$$= 0 \; ; \; t < 0$$

The unit parabolic signal is defined as,

$$x(t) = \frac{t^2}{2} \; ; \; \text{for } t \geq 0$$
$$= 0 \; ; \; t < 0$$

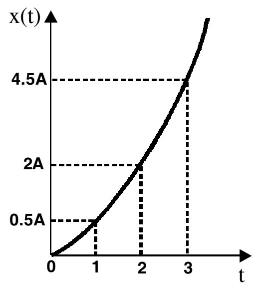

Fig 2.6 : *Parabolic signal.*

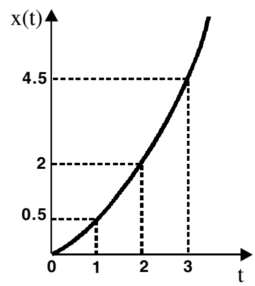

Fig 2.7 : *Unit parabolic signal.*

5. Unit pulse signal

The unit pulse signal is defined as,

$$x(t) = \Pi(t) = u\left(t + \frac{1}{2}\right) - u\left(t - \frac{1}{2}\right)$$

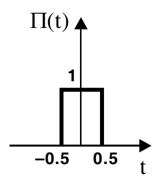

Fig 2.8 : *Unit pulse signal.*

6. Sinusoidal signal

Case i : ***Cosinusoidal signal***

The cosinusoidal signal is defined as,

$$x(t) = A\cos(\Omega_0 t + \phi)$$

$$\text{where, } \; \Omega_0 = 2\pi F_0 = \frac{2\pi}{T} = \text{Angular frequency in rad/sec}$$

$$F_0 = \text{Frequency in cycles/sec or Hz}$$

$$T = \text{Time period in sec}$$

When $\phi = 0$, $\quad x(t) = A\cos\Omega_0 t$

When $\phi = \text{Positive}$, $\quad x(t) = A\cos(\Omega_0 t + \phi)$

When $\phi = \text{Negative}$, $\quad x(t) = A\cos(\Omega_0 t - \phi)$

Case ii : Sinusoidal signal

The sinusoidal signal is defined as,

$$x(t) = A\ \sin(\Omega_0 t + \phi)$$

where, $\Omega_0 = 2\pi F_0 = \dfrac{2\pi}{T}$ = Angular frequency in rad/sec

F_0 = Frequency in cycles/sec or Hz

T = Time period in sec

When $\phi = 0$, $x(t) = A \sin \Omega_0 t$

When ϕ = Positive, $x(t) = A\ \sin(\Omega_0 t + \phi)$

When ϕ = Negative, $x(t) = A\ \sin(\Omega_0 t - \phi)$

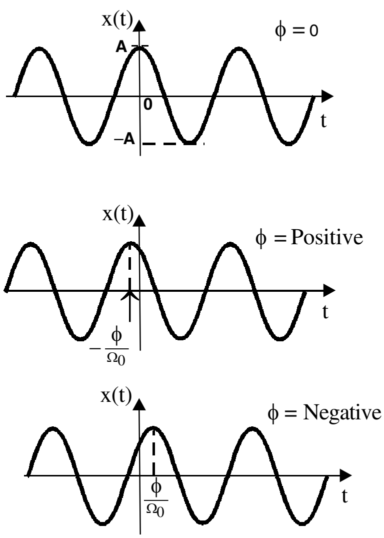

Fig 2.9 : Cosinusoidal signal.

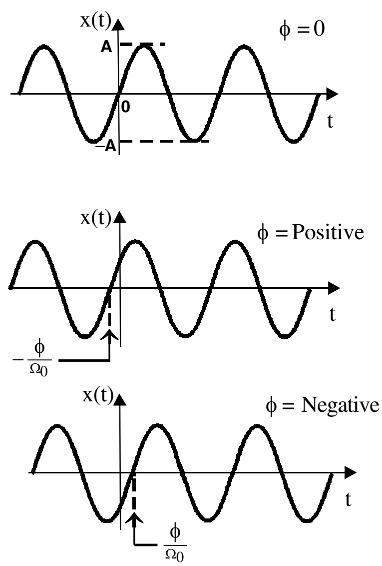

Fig 2.10 : Sinusoidal signal.

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7. Exponential signal

Case i : Real exponential signal

The real exponential signal is defined as,

$$x(t) = A\ e^{bt}$$

where, A and b are real

Here, when b is positive, the signal x(t) will be an exponentially rising signal; and when b is negative the signal x(t) will be an exponentially decaying signal.

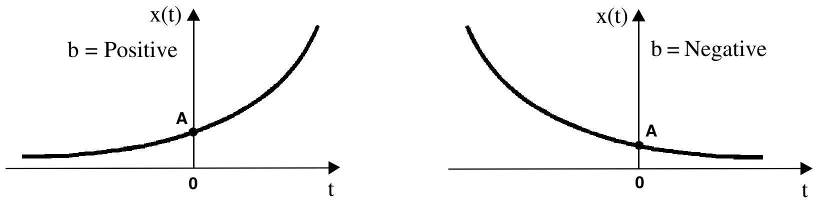

Fig 2.11 : *Real exponential signal.*

***Case ii :* Complex exponential signal**

The complex exponential signal is defined as,

$$x(t) = A\,e^{j\Omega_0 t}$$

where, $\Omega_0 = 2\pi F_0 = \dfrac{2\pi}{T}$ = Angular frequency in rad/sec

F_0 = Frequency in cycles/sec or Hz

T = Time period in sec

The complex exponential signal can be represented in a complex plane by a rotating vector, which rotates with a constant angular velocity of Ω_0 rad/sec.

The complex exponential signal can be resolved into real and imaginary parts as shown below,

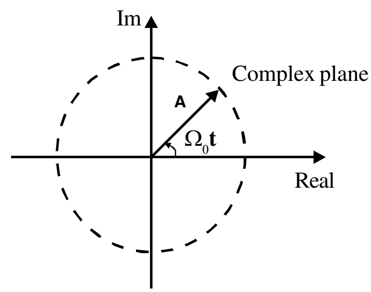

Fig 2.12 : *Complex exponential signal.*

$$x(t) = A\,e^{j\Omega_0 t} = A\,(\cos\Omega_0 t + j\sin\Omega_0 t)$$

$$= A\cos\Omega_0 t + jA\sin\Omega_0 t$$

$$\therefore \quad A\cos\Omega_0 t = \text{Real part of } x(t)$$

$$A\sin\Omega_0 t = \text{Imaginary part of } x(t)$$

From the above equation, we can say that a complex exponential signal is the vector sum of two sinusoidal signals of the form $\cos\Omega_0 t$ and $\sin\Omega_0 t$.

8. Exponentially rising/decaying sinusoidal signal

The exponential rising/decaying sinusoidal signal is defined as,

$$x(t) = A\,e^{bt}\sin\Omega_0 t$$

where, $\Omega_0 = 2\pi F_0 = \dfrac{2\pi}{T}$ = Angular frequency in rad/sec

F_0 = Frequency in cycles/sec or Hz

T = Time period in sec

Here, A and b are real constants. When b is positive, the signal x(t) will be an exponentially rising sinusoidal signal; and when b is negative, the signal x(t) will be an exponentially decaying sinusoidal signal.

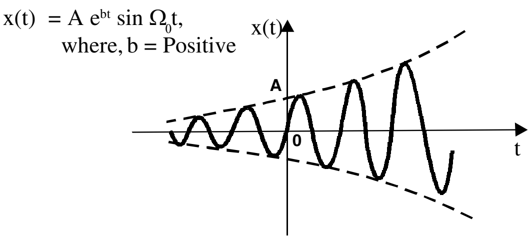

Fig 2.13 : Exponentially rising sinusoid.

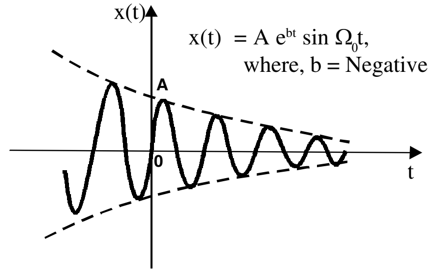

Fig 2.14 : Exponentially decaying sinusoid.

9. Triangular pulse signal

The Triangular pulse signal is defined as

$$x(t) = \Delta_a(t) = 1 - \frac{|t|}{a} \quad ; \quad |t| \le a$$
$$= 0 \quad ; \quad |t| > a$$

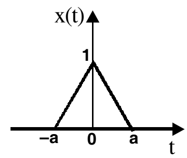

Fig 2.15 : Triangular pulse signal.

10. Signum signal

The Signum signal is defined as the sign of the independent variable t. Therefore, the Signum signal is expressed as,

$$x(t) = \text{sgn}(t) = 1 \quad ; \quad t > 0$$
$$= 0 \quad ; \quad t = 0$$
$$= -1 \quad ; \quad t < 0$$

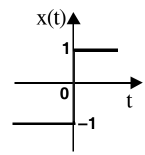

Fig 2.16 : Signum signal.

11. Sinc signal

The Sinc signal is defined as,

$$x(t) = \text{sinc}(t) = \frac{\sin t}{t} \quad ; \quad -\infty < t < \infty$$

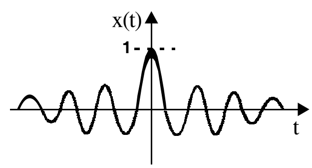

Fig 2.17 : Sinc signal.

12. Gaussian signal

The Gaussian signal is defined as,

$$x(t) = g_a(t) = e^{-a^2t^2} \quad ; \quad -\infty < t < \infty$$

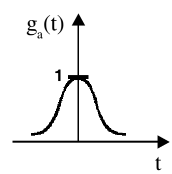

Fig 2.18 : Gaussian signal.

Let, $h_o(t)$ = Overall impulse response of cascaded system.

Now, $$h_o(t) = h_1(t) * h_2(t) = \int_{\lambda=0}^{t} h_1(\lambda)h_2(t-\lambda)\,d\lambda = \int_{\lambda=0}^{t} e^{-a\lambda}e^{-b(t-\lambda)}d\lambda$$

$$= \int_{\lambda=0}^{t} e^{-a\lambda}e^{-bt}e^{b\lambda}\,d\lambda = e^{-bt}\int_{\lambda=0}^{t} e^{-(a-b)\lambda}d\lambda = e^{-bt}\left[\frac{e^{-(a-b)\lambda}}{-(a-b)}\right]_0^t = e^{-bt}\left[\frac{e^{-(a-b)t}}{-(a-b)} - \frac{e^0}{-(a-b)}\right]$$

$$= \frac{1}{a-b}\left[-e^{-at}+e^{-bt}\right]\ ;t\geq 0 \quad = \frac{1}{a-b}\left[-e^{-at}+e^{-bt}\right]u(t)$$

Q2.20 ***Find the overall impulse response of the system shown in fig Q2.20. Take, $h_1(t) = t\,u(t)$; $h_2(t) = 3\,u(t)$; $h_3(t) = 2\,u(t)$.***

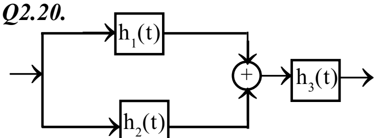

Fig Q : 2.20.

Solution

Overall impulse response, $$h_o(t) = \left[h_1(t)+h_2(t)\right] * h_3(t) = \left[h_1(t) * h_3(t)\right] + \left[h_2(t) * h_3(t)\right]$$

$$= \int_{\lambda=0}^{t} h_1(\lambda)h_3(t-\lambda)\,d\lambda + \int_{\lambda=0}^{t} h_2(\lambda)h_3(t-\lambda)\,d\lambda$$

$$= \int_{\lambda=0}^{t}\lambda\times 2\,d\lambda + \int_{\lambda=0}^{t}3\times 2\,d\lambda = 2\int_{\lambda=0}^{t}\lambda\,d\lambda + 6\int_{\lambda=0}^{t}d\lambda = 2\left[\frac{\lambda^2}{2}\right]_0^t + 6\left[\lambda\right]_0^t$$

$$= 2\left[\frac{t^2}{2}-0\right] + 6\left[t-0\right] = t^2+6t\ ;\ t\geq 0 = \left[t^2+6t\right]u(t)$$

Q2.21 ***What is the importance of convolution?***

The convolution operation can be used to determine the response of an LTI system for any input from the knowledge of its impulse response. [The response of an LTI system is given by convolution of input and impulse response of the LTI system].

2.13 MATLAB Programs

Program 2.1

Write a MATLAB program to generate standard signals like unit impulse, unit step, unit ramp, parabolic, sinusoidal, triangular pulse, signum, sinc and Gaussian signals.

```
%****************** program to plot some standard signals

tmin=-5; dt=0.1; tmax=5;
t=tmin:dt:tmax;              %set a time vector

%****************** unit impulse signal
x1=1;
x2=0;
x=x1.*(t==0)+x2.*(t~=0);     %generate unit impulse signal
subplot(3,3,1);plot(t,x);    %plot the generated unit impulse signal
xlabel('t');ylabel('x(t)');title('unit impulse signal');

%****************** unit step signal
x1=1;
x2=0;
x=x1.*(t>=0)+x2.*(t<0);      %generate unit step signal
subplot(3,3,2);plot(t,x);    %plot the generated unit step signal
xlabel('t');ylabel('x(t)');title('unit step signal');

%****************** unit ramp signal
```

```
x1=t;
x2=0;
x=x1.*(t>=0)+x2.*(t<0);        %generate unit ramp signal
subplot(3,3,3);plot(t,x);      %plot the generated unit ramp signal
xlabel('t');ylabel('x(t)');title('unit ramp signal');

%****************** parabolic signal
A=0.4;
x1=(A*(t.^2))/2;
x2=0;
x=x1.*(t>=0)+x2.*(t<0);        %generate parabolic signal
subplot(3,3,4);plot(t,x);      %plot the generated parabolic signal
xlabel('t');ylabel('x(t)');title('parabolic signal');

%****************** sinusoidal signal
T=2;                           %declare time period
F=1/T;                         %compute frequency
x=sin(2*pi*F*t);               %generate sinusoidal signal
subplot(3,3,5);plot(t,x);      %plot the generated sinusoidal signal
xlabel('t');ylabel('x(t)');title('sinusoidal signal');

%****************** triangular pulse signal
a=2;
x1=1-abs(t)/a;
x2=0;
x=x1.*(abs(t)<=a)+x2.*(abs(t)>a);  %generate triangular pulse signal
subplot(3,3,6);plot(t,x);          %plot the triangular pulse signal
xlabel('t');ylabel('x(t)');title('triangular pulse signal');

%****************** signum signal
```

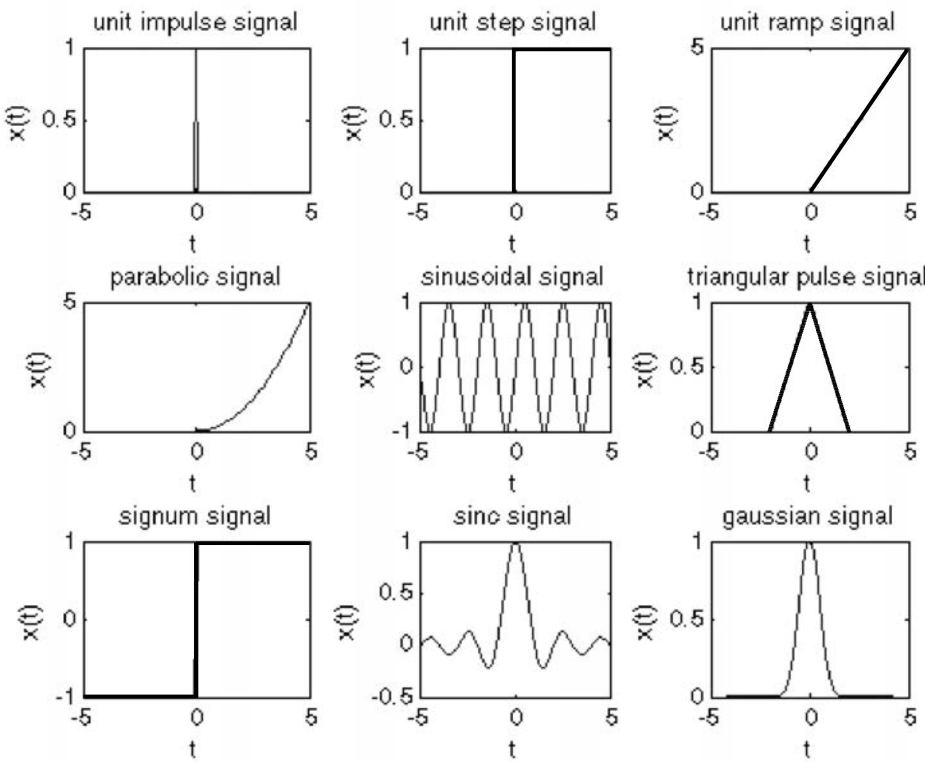

Fig P2.1 : *Output waveforms of program 2.1.*

```
x1=1;
x2=0;
x3=-1;
x=x1.*(t>0)+x2.*(t==0)+x3.*(t<0); %generate signum signal
subplot(3,3,7);plot(t,x);         %plot the generated signum signal
xlabel('t');ylabel('x(t)');title('signum signal');

%****************** sinc Pulse
x=sinc(t);                        %generate sinc Pulse
subplot(3,3,8);plot(t,x);         %plot the generated sinc Pulse
xlabel('t');ylabel('x(t)');title('sinc signal');

%****************** gaussian signal
a=2;
x=exp(-a.*(t.^2));                %generate gaussian signal
subplot(3,3,9);plot(t,x);         %plot the generated gaussian signal
xlabel('t');ylabel('x(t)');title('gaussian signal');
```

OUTPUT

The output waveforms of program 2.1 are shown in fig P2.1.

Program 2.2

Write a MATLAB program to find the even and odd parts of the signal $x(t)=e^{2t}$.

```
%To find the even and odd parts of the signal, x(t)=exp(2*t)

tmin=-3; tmax=3; dt=.1;
t=tmin:dt:tmax;     %set a time vector

x1=exp(2*t);        %generate the given signal
x2=exp(-2*t);       %generate the time folded signal

if(x2==x1)
     disp('"The given signal is even signal"');
else if (x2==(-x1))
     disp('"The given signal is odd signal"');
else
     disp('"The given signal is neither even nor odd signal"');
end
end

xe=(x1+x2)/2;        %compute even part
xo=(x1-x2)/2;        %compute odd part

ymin=min([min(x1), min(x2), min(xe), min(xo)]);
ymax=max([max(x1), max(x2), max(xe), max(xo)]);

subplot(2,2,1);plot(t,x1);axis([tmin tmax ymin ymax]);
xlabel('t');ylabel('x1(t)');title('signal x(t)');

subplot(2,2,2);plot(t,x2);axis([tmin tmax ymin ymax]);
xlabel('t');ylabel('x2(t)');title('signal x(-t)');

subplot(2,2,3);plot(t,xe);axis([tmin tmax ymin ymax]);
xlabel('t');ylabel('xe(t)');title('even part of x(t)');

subplot(2,2,4);plot(t,xo);axis([tmin tmax ymin ymax]);
xlabel('t');ylabel('xo(t)');title('odd part of x(t)');
```