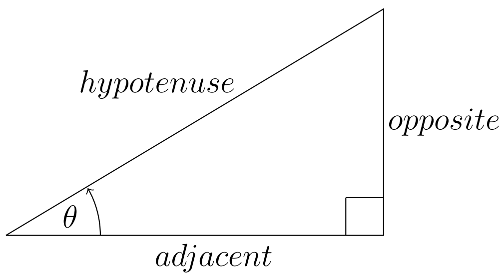
Trigonometric Formula Sheet
Definition of the Trig Functions

Right Triangle Definition

Assume that:

$0 < \theta < \frac{\pi}{2}$ or $0^\circ < \theta < 90^\circ$



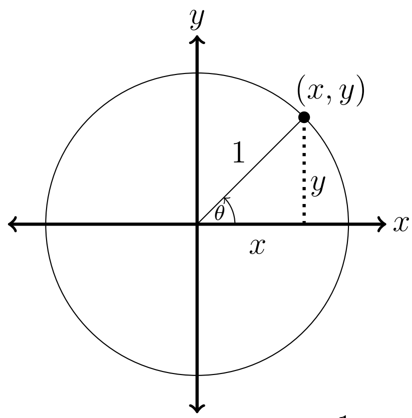
$$\sin\theta = \frac{opp}{hyp} \qquad \csc\theta = \frac{hyp}{opp}$$

$$\cos\theta = \frac{adj}{hyp} \qquad \sec\theta = \frac{hyp}{adj}$$

$$\tan\theta = \frac{opp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

Unit Circle Definition

Assume θ can be any angle.



$$\sin\theta = \frac{y}{1} \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Domains of the Trig Functions

$\sin\theta, \quad \forall\ \theta \in (-\infty, \infty)$

$\cos\theta, \quad \forall\ \theta \in (-\infty, \infty)$

$\tan\theta, \quad \forall\ \theta \neq \left(n + \frac{1}{2}\right)\pi,\ where\ n \in \mathbb{Z}$

$\csc\theta, \quad \forall\ \theta \neq n\pi,\ where\ n \in \mathbb{Z}$

$\sec\theta, \quad \forall\ \theta \neq \left(n + \frac{1}{2}\right)\pi,\ where\ n \in \mathbb{Z}$

$\cot\theta, \quad \forall\ \theta \neq n\pi,\ where\ n \in \mathbb{Z}$

Ranges of the Trig Functions

$$-1 \leq \sin\theta \leq 1$$
$$-1 \leq \cos\theta \leq 1$$
$$-\infty \leq \tan\theta \leq \infty$$

$$\csc\theta \geq 1\ and\ \csc\theta \leq -1$$
$$\sec\theta \geq 1\ and\ \sec\theta \leq -1$$
$$-\infty \leq \cot\theta \leq \infty$$

Periods of the Trig Functions

The period of a function is the number, T, such that f $(\theta + \text{T})$ = f (θ) .
So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\theta) \ \Rightarrow\ T = \frac{2\pi}{\omega}$$

$$\cos(\omega\theta) \ \Rightarrow\ T = \frac{2\pi}{\omega}$$

$$\tan(\omega\theta) \ \Rightarrow\ T = \frac{\pi}{\omega}$$

$$\csc(\omega\theta) \ \Rightarrow\ T = \frac{2\pi}{\omega}$$

$$\sec(\omega\theta) \ \Rightarrow\ T = \frac{2\pi}{\omega}$$

$$\cot(\omega\theta) \ \Rightarrow\ T = \frac{\pi}{\omega}$$

Dr. Rachana Desai

Identities and Formulas

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$
$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$
$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Even and Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$
$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$
$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Periodic Formulas

If n is an integer

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$
$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$
$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then:

$$\frac{\pi}{180^\circ} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180^\circ} \ \textbf{and} \ x = \frac{180^\circ t}{\pi}$$

Half Angle Formulas

$$\sin\theta = \pm\sqrt{\frac{1 - \cos(2\theta)}{2}}$$

$$\cos\theta = \pm\sqrt{\frac{1 + \cos(2\theta)}{2}}$$

$$\tan\theta = \pm\sqrt{\frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

Cofunction Formulas

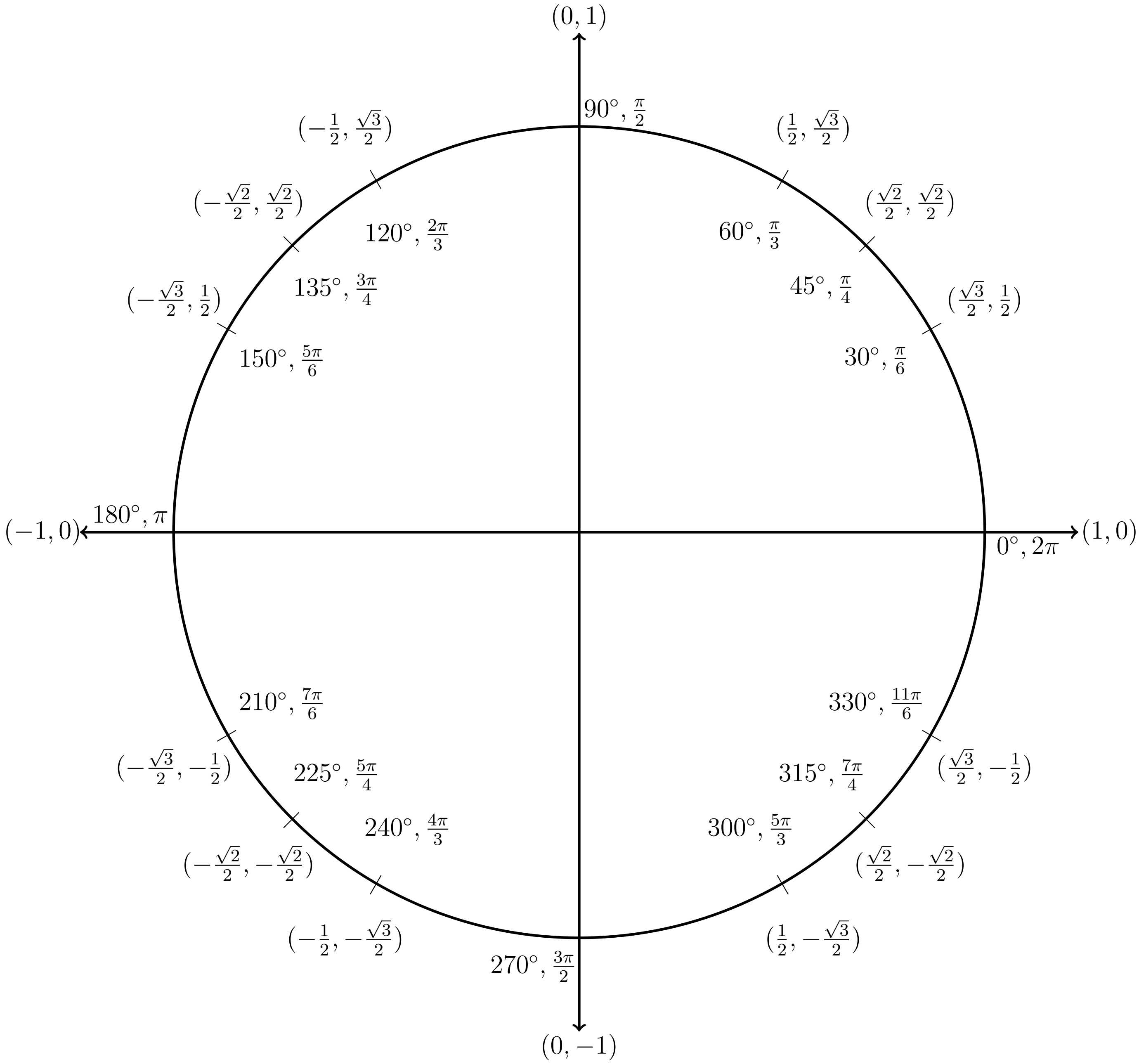
$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Dr. Rachana Desai

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos\theta = x$ *and* $\sin\theta = y$

Dr. Rachana Desai