Intersymbol Interference

- Intersymbol Interference (ISI)
  - Arises when the communication channel is *dispersive*

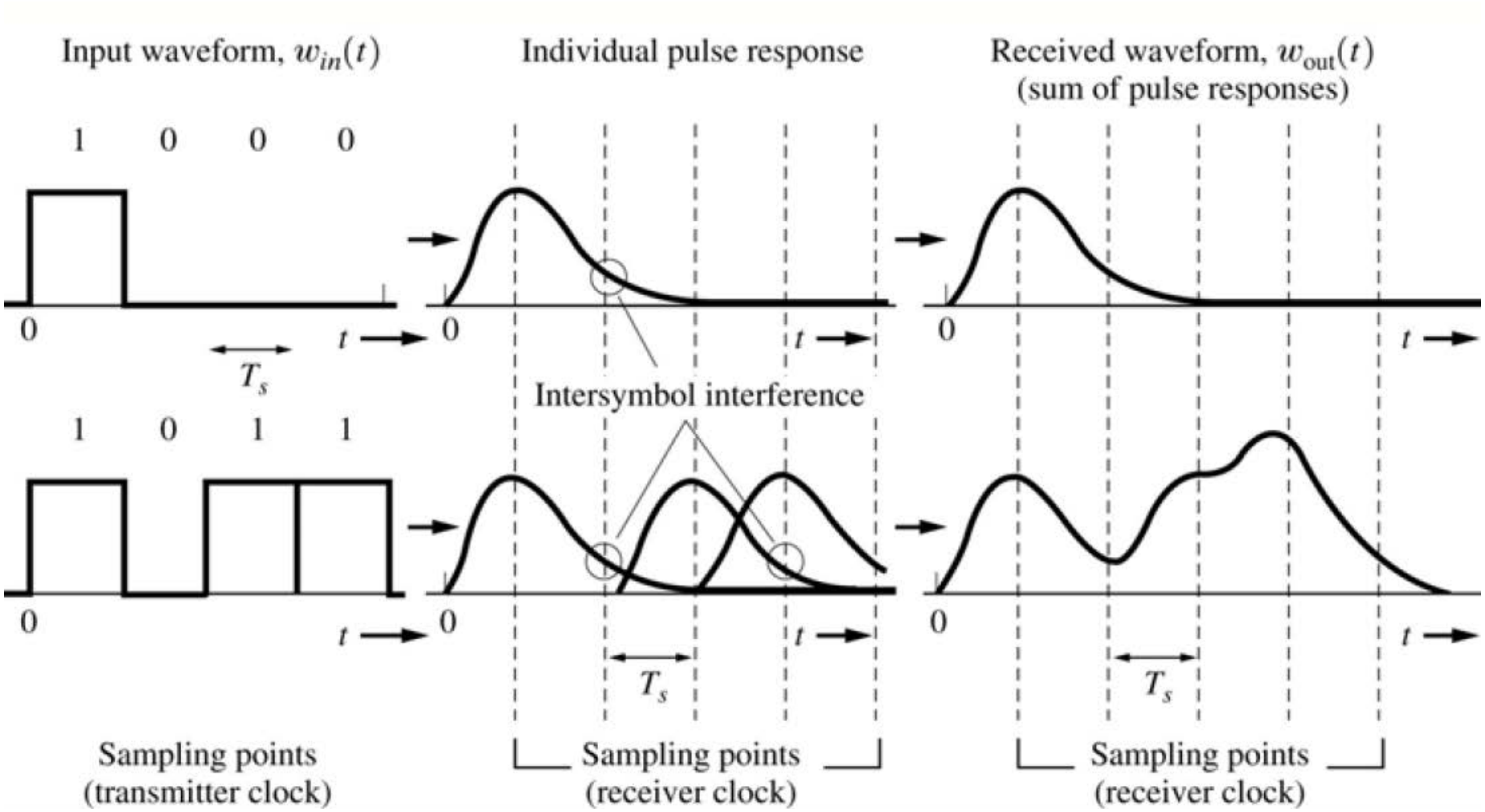
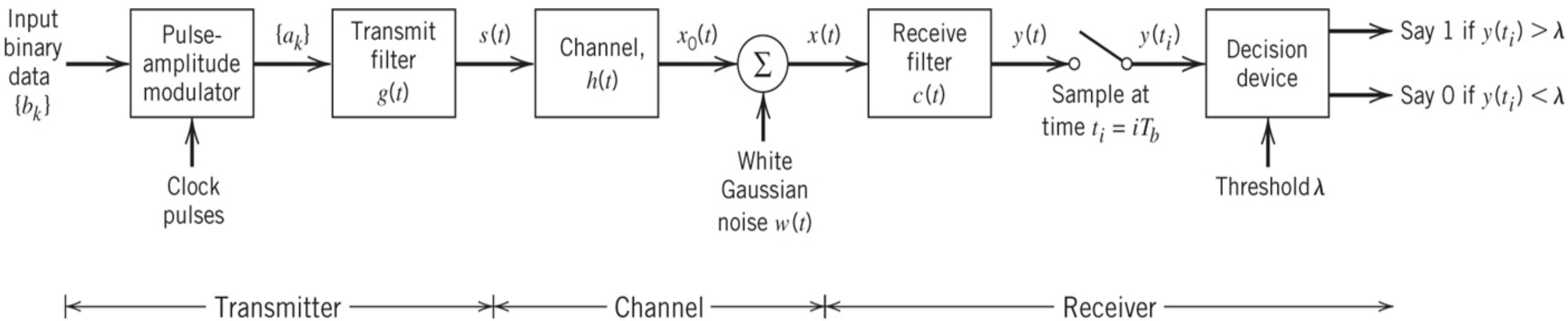


Figure from (Couch, 2007)

Intersymbol Interference

- Baseband Binary Data Transmission System



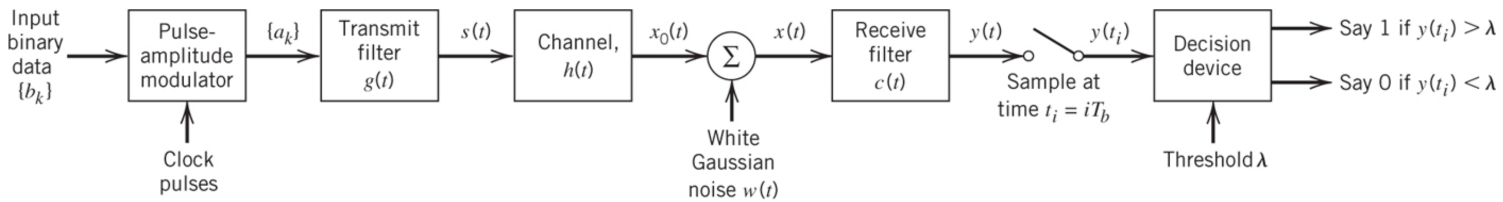
Pulse amplitude modulator $a_k = \begin{cases} +1 \text{ if } b_k = 1 \\ -1 \text{ if } b_k = 0 \end{cases}$

Transmission filter of impuse response $g(t)$

$$\rightarrow s(t) = \sum_{k=-\infty}^{\infty} a_k g(t - kT_b)$$

Figure from (Haykin & Moher, 2010)

Intersymbol Interference



Receiver filter output $\quad Y(t) = \mu \sum_{k=-\infty}^{\infty} a_k\, p(t - kT_b) + N(t)$

where $\quad \mu\, p(t) = g(t) * h(t) * c(t) \quad \text{with} \quad p(0) = 1$

$$\mu P(f) = G(f) H(f) C(f)$$

$$\rightarrow Y(iT_b) = \mu \sum_{k=-\infty}^{\infty} a_k\, p(iT_b - kT_b) + N(iT_b)$$

$$= \mu a_i + \underbrace{\mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k\, p((i-k)T_b)}_{\text{ISI}} + \underbrace{N(iT_b)}_{\text{channel noise}}$$

* Goal: to determine $p(t)$ for which ISI is completely eliminated

Intersymbol Interference

- Dispersive Nature of Telephone Channel

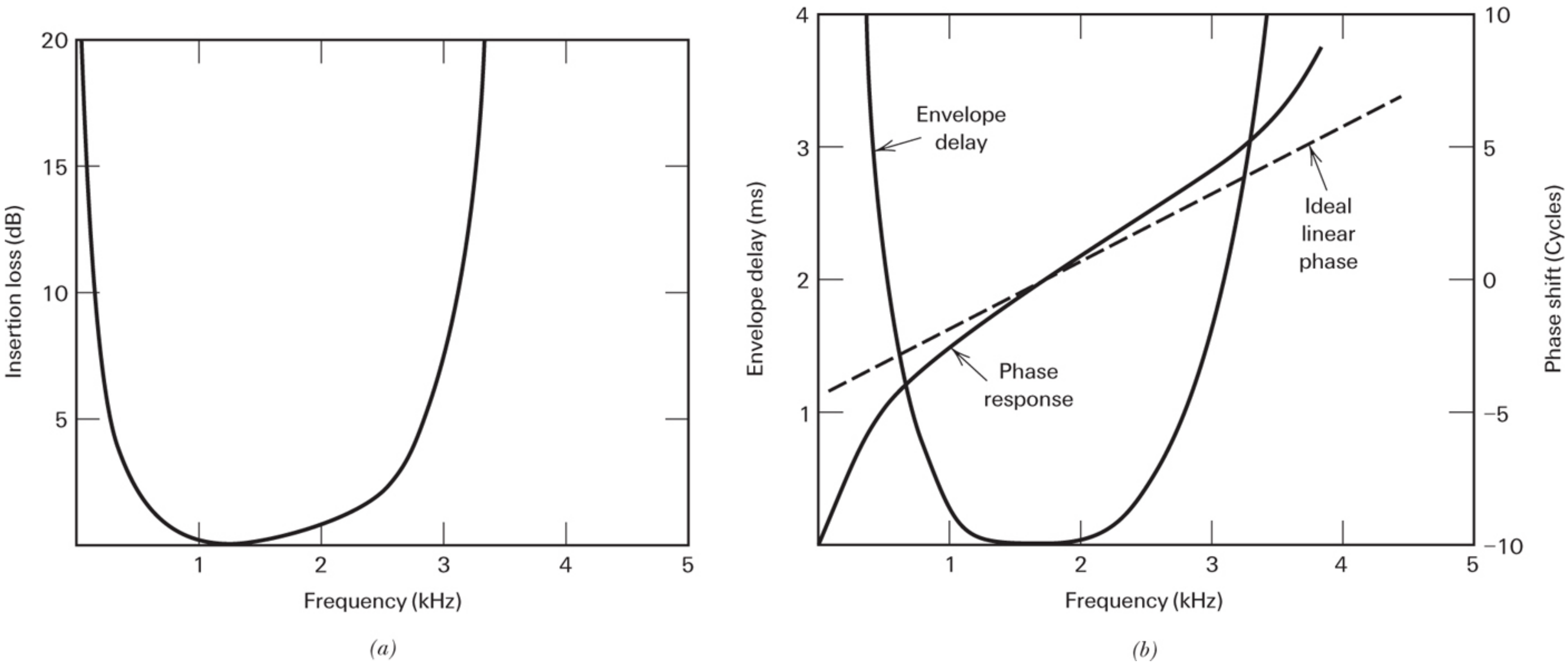


Figure from (Haykin & Moher, 2010)

Intersymbol Interference

- 1600 & 3200 bps transmission over telephone channel

Polar NRZ | Manchester

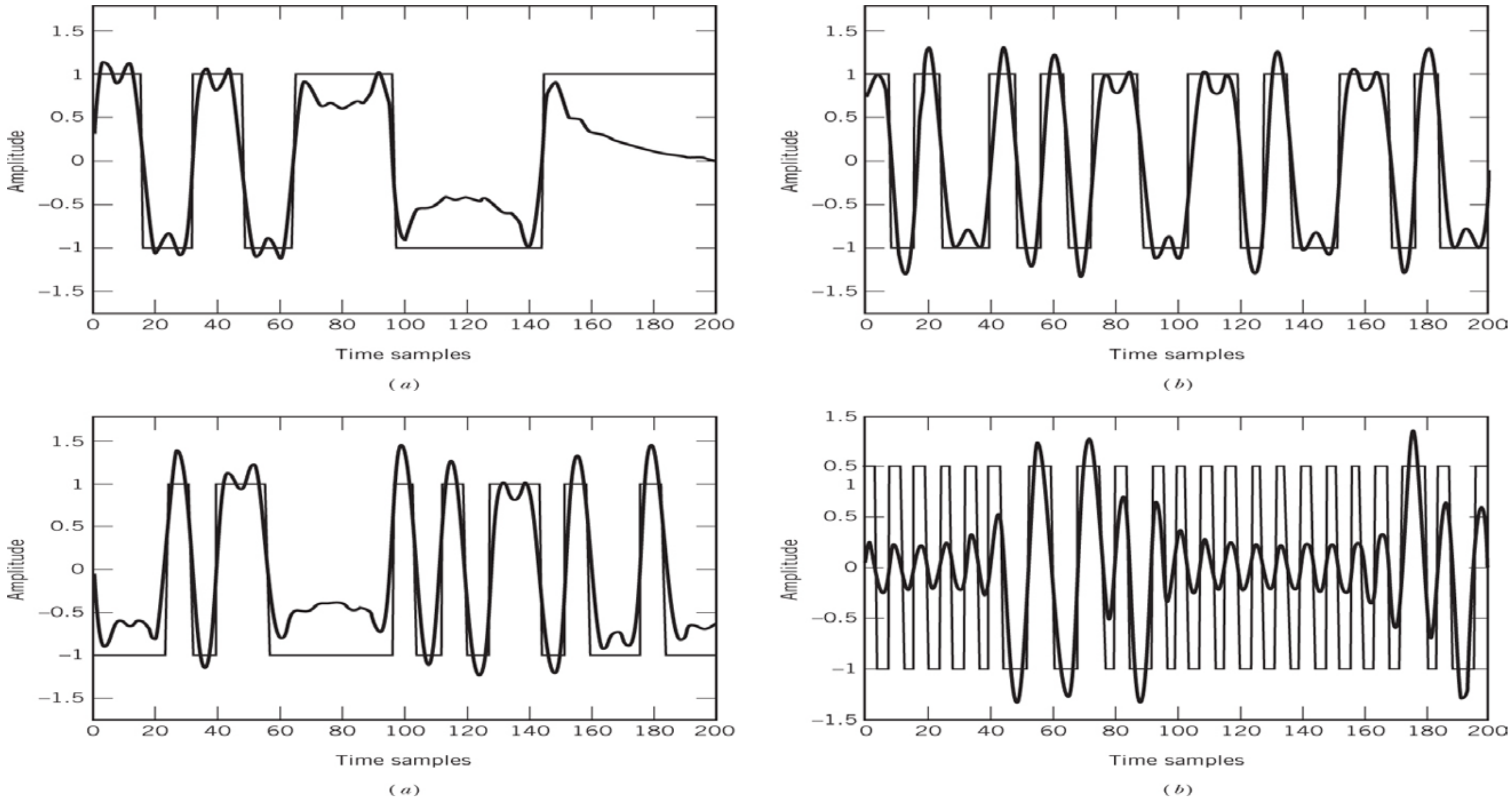


Figure from (Haykin & Moher, 2010)

Eye Pattern

- Eye pattern

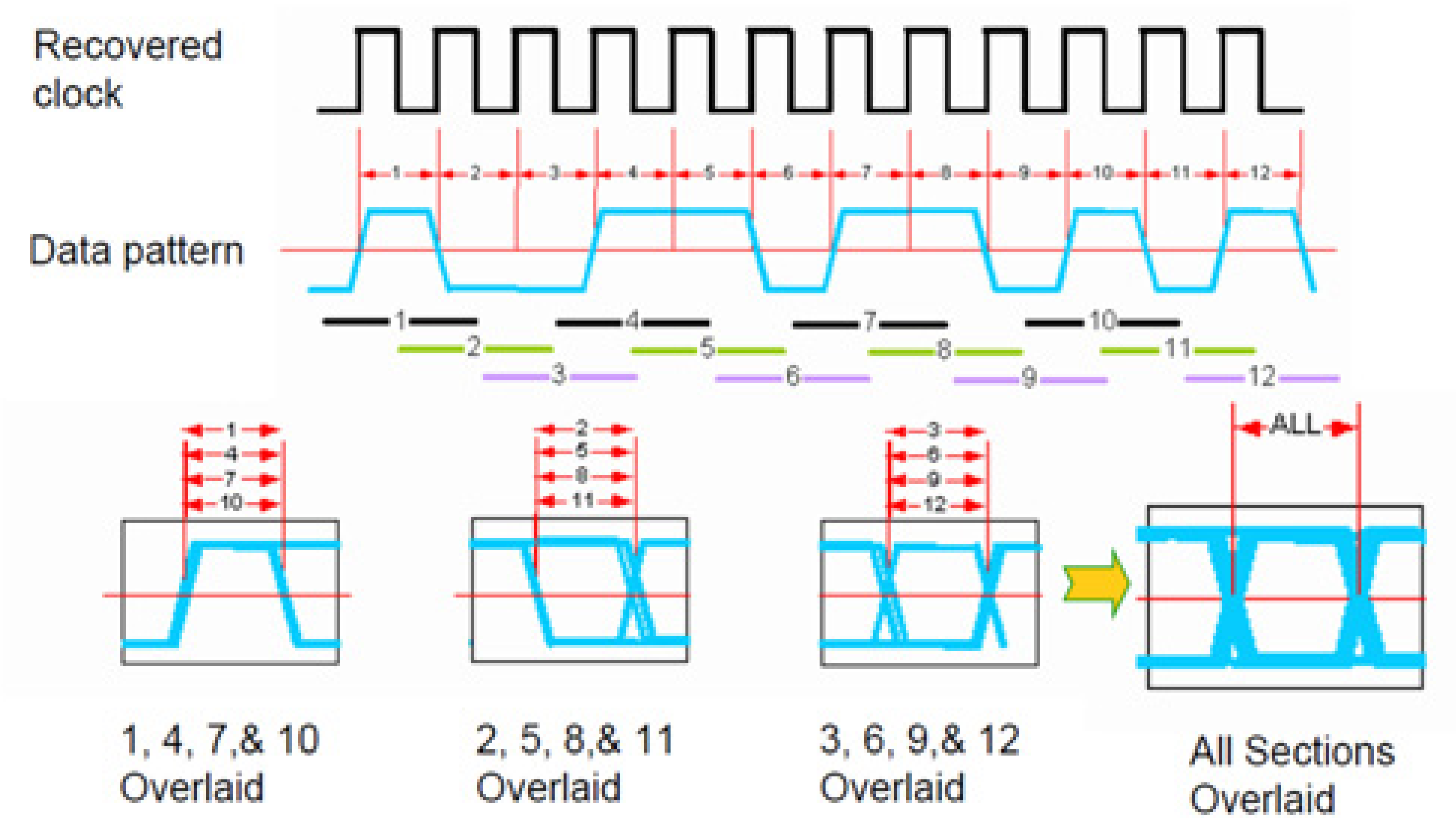


Figure from https://www.signalintegrityjournal.com/articles/432-s-parameters-signal-integrity-analysis-in-the-blink-of-an-eye

Eye Pattern

- Interpretation of eye pattern

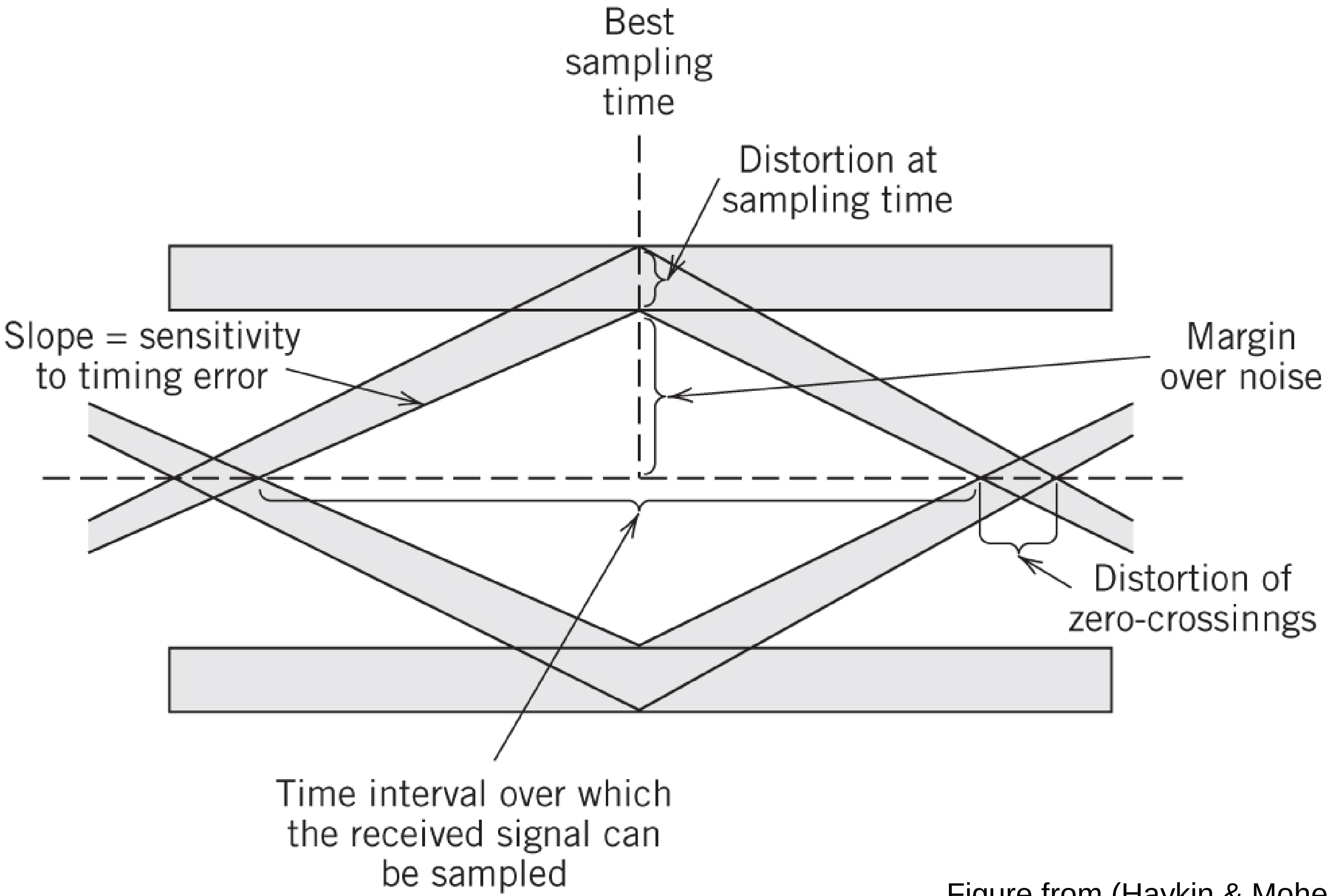
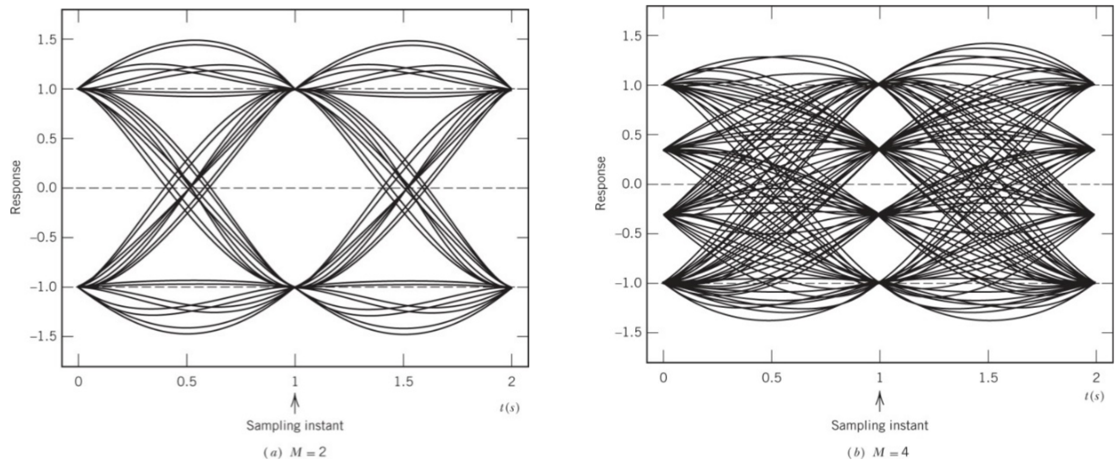


Figure from (Haykin & Moher, 2010)

Eye Pattern

- Eye diagrams for binary and quaternary systems

No bandwidth limitation



Bandwidth limited channel

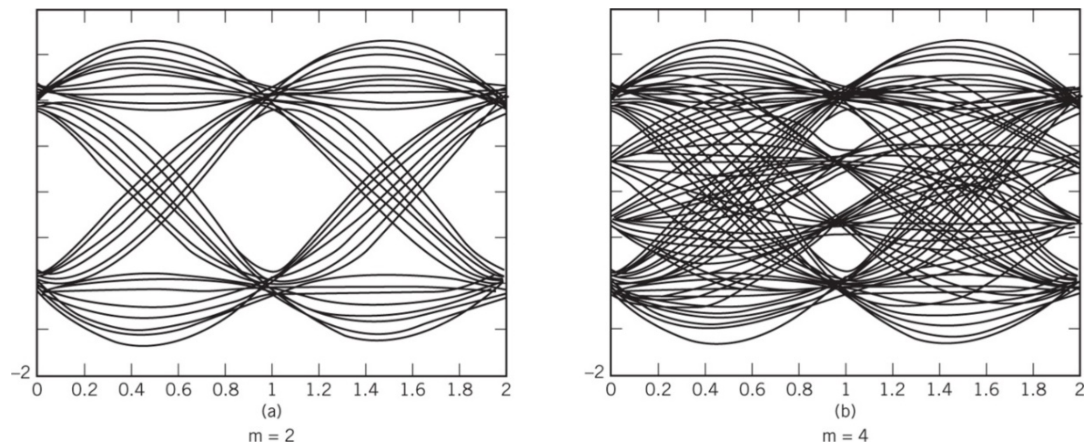


Figure from (Haykin & Moher, 2010)

Nyquist's Criterion for Distortionless Transmission

$$Y(iT_b) = \mu \sum_{k=-\infty}^{\infty} a_k \, p(iT_b - kT_b) + N(iT_b)$$

$$= \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k \, p(iT_b - kT_b) + N(iT_b)$$

Zero ISI when $p(iT_b - kT_b) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases}$

$\rightarrow\ Y(iT_b) = \mu a_i$ (Assuming $N(iT_b) = 0$)

Nyquist's Criterion for Distortionless Transmission

Zero ISI when $p(iT_b - kT_b) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases}$

Let $p_\delta(t) \equiv \sum_{m=-\infty}^{\infty} p(mT_b)\delta(t - mT_b)$

$$P_\delta(f) = R_b \sum_{n=-\infty}^{\infty} P(f - nR_b) \qquad (\ R_b = \frac{1}{T_b} : \text{bit rate}\ )$$

$$= \int_{-\infty}^{\infty} \left[\sum_{m=-\infty}^{\infty} p(mT_b)\delta(t - mT_b) \right] \exp(-j2\pi ft)dt$$

$$= \int_{\infty}^{-\infty} p(0)\delta(t) \exp(-j2\pi ft)dt = p(0) = 1$$

- Nyquist's criterion for distortionless baseband transmission

$$\boxed{\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b}$$

Ideal Nyquist Channel

$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b$$

$$P(f) = \frac{1}{2W} rect\left(\frac{f}{2W}\right) = \begin{cases} \frac{1}{2W}, & |f| \le W \\ 0\,, & otherwise \end{cases}$$

$$W = \frac{R_b}{2} = \frac{1}{2T_b} \quad \rightarrow R_b = 2W : \text{Nyquist rate}$$

$$p(t) = \frac{\sin(2\pi Wt)}{2\pi Wt} = \text{sinc}(2Wt)$$

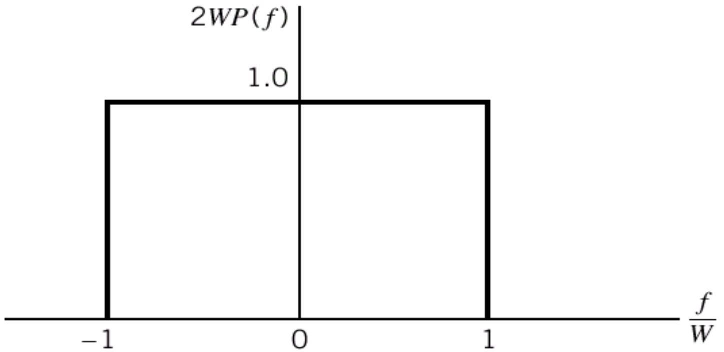
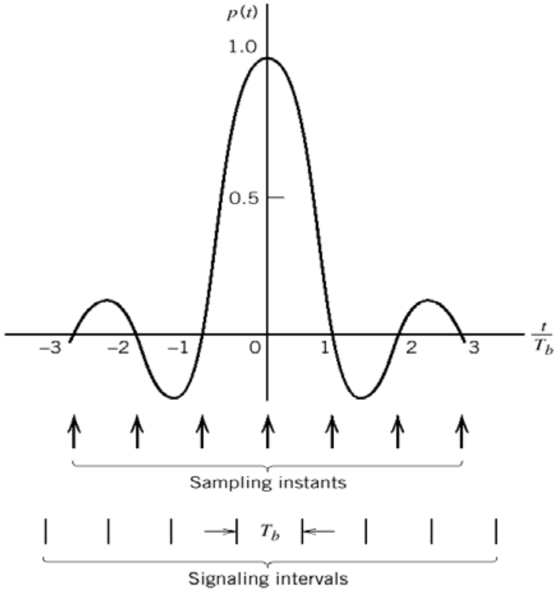


Figure from (Haykin & Moher, 2010)

Ideal Nyquist Channel

$$P(f) = \frac{1}{2W} rect\left(\frac{f}{2W}\right) \leftrightarrow p(t) = \text{sinc}(2Wt)$$



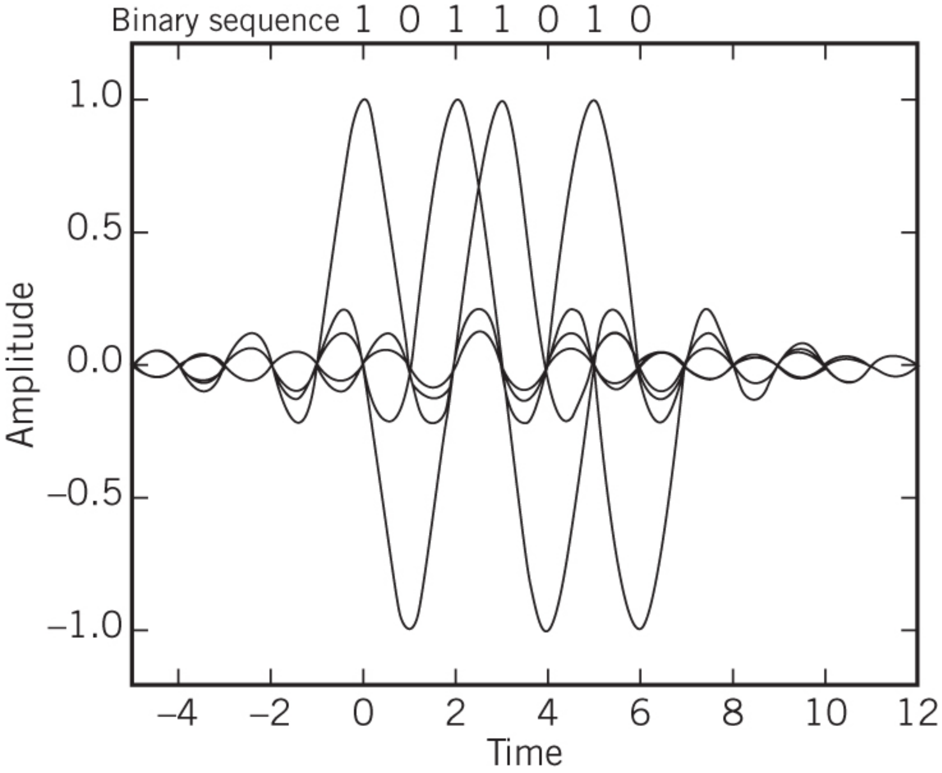


Figure from (Haykin & Moher, 2010)

Ideal Nyquist Channel

- Ideal Nyquist Channel
  - Advantage
    - Zero ISI with minimum bandwidth
  - Practical difficulty
    - Physically unrealizable
    - Timing error causes significant ISI
  - Solution
    - Pulse shaping to reduce ISI

Ideal Nyquist Channel

- Effect of Timing Error

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k p(t - kT_b) \;\Rightarrow\; y(\Delta t) = \mu \sum_{k=-\infty}^{\infty} a_k p(\Delta t - kT_b) \text{ for } iT_b = 0$$

$$y(\Delta t) = \mu \sum_{k=-\infty}^{\infty} a_k \text{sinc}[2W(\Delta t - kT_b)] = \mu \sum_{k=-\infty}^{\infty} a_k \text{sinc}[2W\Delta t - k] \qquad (2W = \frac{1}{T_b})$$

$$= \mu a_0 \text{sinc}(2W\Delta t) + \mu \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} a_k \frac{\sin(2\pi W\Delta t - \pi k)}{2\pi W\Delta t - \pi k}$$

Since $\sin(2\pi W\Delta t - \pi k) = \sin(2\pi W\Delta t)\cos(\pi k) - \cos(2\pi W\Delta t)\sin(\pi k)$

$$= \sin(2\pi W\Delta t)(-1)^k$$

$$y(\Delta t) = \mu a_0 \text{sinc}(2W\Delta t) + \underbrace{\frac{\mu \sin(2\pi W\Delta t)}{\pi} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{(-1)^k a_k}{(2W\Delta t - k)}}_{\text{ISI caused by timing error}}$$

Raised Cosine Filter

– Alleviates timing error problem at the expense of increased bandwidth (between W and $2W$)

$$P(f) = \begin{cases} \dfrac{1}{2W}, & 0 \le |f| \le f_1 \\ \dfrac{1}{4W}\left\{1 + \cos\left[\dfrac{\pi(|f| - f_1)}{2(W - f_1)}\right]\right\}, & f_1 \le |f| < 2W - f_1 \\ 0, & 2W - f_1 \le |f| \end{cases}$$

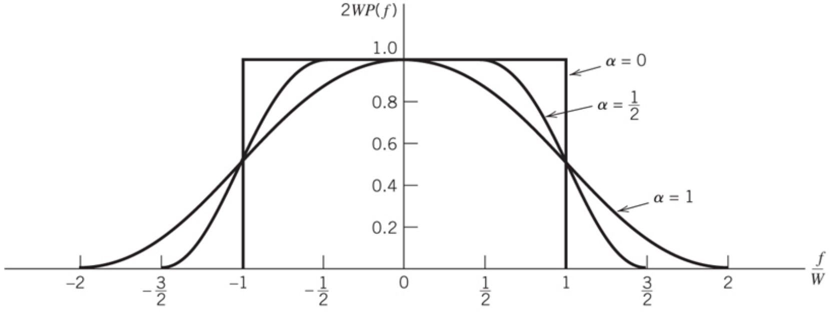
Rolloff factor $\alpha = 1 - \dfrac{f_1}{W}$ (Controls excess bandwidth)

$\longrightarrow$ $B_T = 2W - f_1 = W(1+\alpha)$

$\longrightarrow$ $p(t) = \mathrm{sinc}(2Wt) \cdot \dfrac{\cos(2\pi\alpha Wt)}{1 - 16\alpha^2 W^2 t^2}$

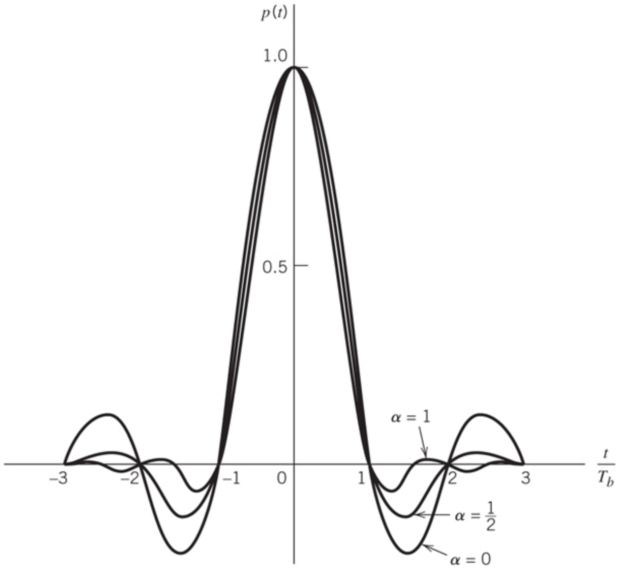
Raised Cosine Filter

– Responses for different rolloff factors



(a)

$$P(f) = \begin{cases} \dfrac{1}{2W}, & 0 \le |f| \le f_1 \\ \dfrac{1}{4W}\left\{1 + \cos\left[\dfrac{\pi(|f| - f_1)}{2(W - f_1)}\right]\right\}, & f_1 \le |f| < 2W - f_1 \\ 0, & 2W - f_1 \le |f| \end{cases}$$



(b)

$$p(t) = \mathrm{sinc}(2Wt) \cdot \frac{\cos(2\pi\alpha W t)}{1 - 16\alpha^2 W^2 t^2}$$

Figure from (Haykin & Moher, 2010)

Raised Cosine Filter

When $\alpha = 1$ (perfect rolloff characteristic)

$$P(f) = \begin{cases} \frac{1}{4W}[1+\cos(\frac{\pi f}{2W})], & 0 < |f| < 2W \\ 0, & |f| \geq 2W \end{cases} \quad \Leftrightarrow \quad p(t) = \frac{\mathrm{sinc}(4Wt)}{1-16W^2t^2}$$

Useful properties for synchronization (when $\alpha = 1$)

At $t = \pm\frac{T_b}{2} = \pm\frac{1}{4W}, \quad p(t) = 0.5$

$\longrightarrow$ Pulse width at half amplitude = T_b

$$p(t) = 0 \qquad t = \pm\frac{3}{2}T_{b,} \pm\frac{5}{2}T_{b,} \cdots$$

as well as $p(t)=0$ at $t = \pm T_{b,} \pm 2T_{b,} \cdots$

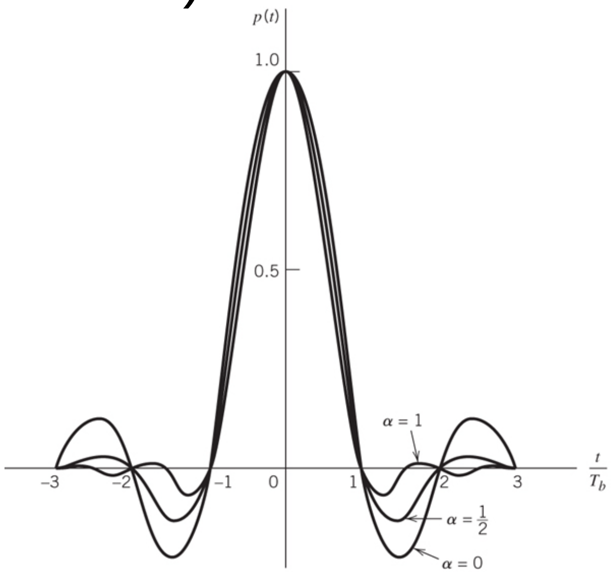


Figure from (Haykin & Moher, 2010)

BW Requirement of the T1 System

- 24 voice channels (8kHz, 8bit μ-law)

$$R_b = 1.544\,(\text{Mbps})$$

$$T_b = \frac{1}{1{,}544{,}000}\,(\text{sec}) = 0.647\,(\mu s)$$

$$B_T = W(1+\alpha) = \frac{1}{2T_b}(1+\alpha) = 722(1+\alpha)\,(\text{kHz})$$

– Ideal Nyquist channel $(\alpha = 0): 722\text{ kHz}$

– Full-cosine rolloff $(\alpha = 1): 1.544\text{ MHz}$

cf. Analog FDM based on SSB : $B_T = 24 \times 4 = 96\text{ kHz}$

Topics Covered

- Introduction to Communication
- Amplitude Modulation
- Phase & Frequency Modulation
- Random Process and Noise
- Noise in Analog Modulation
- Digital Representation of Analog Signals
- Baseband Transmission of Digital Signals

Remaining Issues

- Band-pass Transmission of Digital Signals
- Information Theory and Error Control Coding

References for Figures

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