DIV	V- DIV	- Questions
Α	В	Questions
1	64	Show that $\left[\overline{p} + \overline{q}, \ \overline{q} + \overline{r}, \ \overline{r} + \overline{p}\right] = \left(\overline{p} + \overline{q}\right) \cdot \left[\left(\overline{q} + \overline{r}\right) \times \left(\overline{r} + \overline{p}\right)\right] = 2\left[\overline{p} \ \overline{q} \ \overline{r}\right]$
2	65	If \overline{l} , \overline{m} , \overline{n} are three non-coplanar vectors, prove that
		$\left[\bar{l}\ \overline{m}\ \overline{n}\right]\left(\overline{a}\times\overline{b}\right) = \begin{vmatrix} \bar{l}\cdot\overline{a} & \bar{l}\cdot\overline{b} & \bar{l} \\ \overline{m}\cdot\overline{a} & \overline{m}\cdot\overline{b} & \overline{m} \end{vmatrix}$
		$ \overline{n} \cdot \overline{a} \overline{n} \cdot \overline{b} \overline{n} $
3	66	Prove that the points $(2, 1, 1)$, $(0, 1, -3)$, $(3, 2, -1)$ and $(7, 2, 7)$ are coplanar.
4	67	Show that the vectors are coplanar: $2i - j + k$, $i + 2j - 3k$, $3i + aj + 5k$ if $a = -4$
5	68	Prove that the points $(1, 1, 1)$, $(2, -1, 1)$, $(3, 1, 2)$ and $(5, 1, 3)$ are coplanar.
6	69	Prove that $i \times (\overline{a} \times i) + j \times (\overline{a} \times j) + k \times (\overline{a} \times k) = 2\overline{a}$
7	70	Prove that $\overline{a} \times \left[\overline{b} \times (\overline{c} \times \overline{d}) \right] = \left(\overline{b} \cdot \overline{d} \right) \left(\overline{a} \times \overline{c} \right) - \left(\overline{b} \cdot \overline{c} \right) \left(\overline{a} \times \overline{d} \right)$
8	71	Prove that $[\overline{b} \times \overline{c} \overline{c} \times \overline{a} \overline{a} \times \overline{b}] = [\overline{a} \ \overline{b} \ \overline{c}]^2$
9	72	Prove that $\overline{a} \times (\overline{b} \times \overline{c}) + \overline{b} \times (\overline{c} \times \overline{a}) + \overline{c} \times (\overline{a} \times \overline{b}) = 0$
10	73	Prove that $(a \times (b \times c) + b \times (c \times a) + c \times (a \times b) = 0$
11	74	Prove that $(\overline{a} \times \overline{b}) \cdot (\overline{c} \times \overline{d}) + (\overline{b} \times \overline{c}) \cdot (\overline{a} \times \overline{d}) + (\overline{c} \times \overline{a}) \cdot (\overline{b} \times \overline{d}) = 0$
12	75	Prove that $(\overline{a} \times \overline{b}) \cdot (\overline{c} \times \overline{d}) = [\overline{a} \ \overline{c} \ \overline{d}] \overline{b} - [\overline{b} \ \overline{c} \ \overline{d}] \overline{a}$, where $\overline{a}, \overline{b}, \overline{c}$ are coplanar vectors.
13		Prove that $\overline{d} \cdot \left[\overline{a} \times \left[\overline{b} \times (\overline{c} \times \overline{d}) \right] \right] = (\overline{b} \cdot \overline{d}) \left[\overline{a} \ \overline{c} \ \overline{d} \right]$
13	/6	$ \frac{d\overline{a}}{dt} = \overline{u} \times \overline{a} \& \frac{d\overline{b}}{dt} = \overline{u} \times \overline{b}, \text{ prove that } \frac{d}{dt} \left[\overline{a} \times \overline{b} \right] = \overline{u} \times \left(\overline{a} \times \overline{b} \right) $
14	77	$\frac{dt}{dt} \frac{dt}{dt} \left[\frac{u \times b}{dt} \right] = u \times \left(\frac{u \times b}{dt} \right)$
14	77	If $\overline{r} = a \cos t \ i + a \sin t \ j + at \tan \alpha \ k$, show that $\left[\frac{d\overline{r}}{dt} \ \frac{d^2 \overline{r}}{dt^2} \ \frac{d^3 \overline{r}}{dt^3} \right] = a^3 \tan \alpha$
		show that $\left \frac{\partial}{\partial t} \frac{\partial}{\partial t^2} \frac{\partial}{\partial t^3} \right = a^4 \tan \alpha$
15	78	$d \left[d\overline{u} d^{2}\overline{u} \right] \left[d\overline{u} d^{3}\overline{u} \right]$
		Show that $\frac{d}{dt} \left[\overline{u} \frac{d\overline{u}}{dt} \frac{d^2 \overline{u}}{dt^2} \right] = \left[\overline{u} \frac{d\overline{u}}{dt} \frac{d^3 \overline{u}}{dt^3} \right]$
16	79	If $\overline{A} = (\sin t)i + (\cos t)j + tk$, $\overline{B} = (\cos t)i - (\sin t)j - 3k$, $\overline{C} = 2i + 3j - k$ find
		d = (-1)
		$\frac{d}{dt} \left[\overline{A} \times \left(\overline{B} \times \overline{C} \right) \right] at \ t = 0$
17	80 =	$\overline{A} = i - 2j - 3k$, $\overline{b} = 2i + j - k$, $\overline{c} = i + 3j - k$ Find $\overline{a} \times (\overline{b} \times \overline{c})$ and $(\overline{a} \times \overline{b}) \times \overline{c}$
	DIENI A	nd DIRECTIONAL DERIVATIVES
18	81	Find $\nabla \phi_{\text{if}} \phi = 3x^2y - y^3z^2$ at $(1, -2, 1)$
19	82	Find the directional derivative of $\phi = x^2y + y^2z + z^2x^2$ at $P(1, 2, 1)$ in the direction of the
		normal to the surface $x^2 + y^2 - z^2x = 1$ at $Q(1, 1, 1)$
20	83	Find the directional derivative of $\phi = 2x^3y - 3y^2z$ at $P(1, 2, -1)$ in the direction towards
		That the directional derivative of $\psi = 2x \ y = 3y \ z \ di \ P(1, 2, -1)$ in the direction towards
		Q(3, -1, 5). In what direction from P is the directional derivative maximum? Find the magnitude of maximum directional derivative.
21	84	Find the directional derivative of $\phi = x^4 + y^4 + z^4$ at $A(1, -2, 1)$ in the direction of AB
		where R is $(2,6,-1)$ Also find the maximum direction of AB
22	85	where B is $(2, 6, -1)$. Also find the maximum directional derivative of ϕ at $(1, -2, 1)$.
* [* **: **		Find the directional derivative of $\phi = x^2y^2 + y^2z^2 + z^2x^2$ at $(1, 1, -2)$ in the direction of the
23	86	tangent to the curve $x = e^{-t}$, $y = 2\sin t + 1$, $z = t - \cos t$ at $t = 0$
	50	Find the directional derivative of $\phi = e^{2x} \cos yz$ at $(0, 0, 0)$ in the direction of the tangent to the
		curve $x = a \sin t$, $y = a \cos t$, $z = at$ at $t = \pi/4$.

da t,

42 105 Prove that $\nabla \left(\frac{1}{r}\right) = -\frac{\overline{r}}{r^3}$. 43 106 Prove that $\nabla f(r) = \frac{f'(r)}{r}\overline{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.	24	87	Find the directional derivative of $\phi = x^2 + y^2 + z^2$ in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ at
Find the directional derivative of $\phi = x^2y\cos z$ in the direction of the line $\overline{a} = 2i + 3j + 2k$ at $(1, 2, \pi/2)$ Find the acute angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$ So Find the acute angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$ Find the rate of change of $\phi = xy + yz + zx$ at $(1, -1, 2)$ in the direction of the normal to the surface $x^2 + y^2 = z + 4$. In what direction is the directional derivative of $\phi = 2xz - y^2$ at $(1, 3, 2)$ maximum? Find its magnitude. Find the rate of change of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at $(-1, 2, 1)$ Find the rate of change of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at $(-1, 2, 1)$ Find the angle between the normals to the surfaces $x^2y + 2xz = 4$ at $(2, -2, 3)$ and to $x^3 + y^3 + 3xyz = 3$ at $(1, 2, -1)$ Find the constants and b such that the surfaces $ax^2 - 2byz = (a + 4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at $(1, -1, 2)$ Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}(1/\sqrt{3})$. 34	1		(1, 2, 3)
Find the acute angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$ Find the angle between the two surfaces $x^2 + y^2 + az^2 = 6$ and $z = 4 - y^2 + bxy$ at $(1, 1, 2)$ Find the rate of change of $\phi = xy + yz + zx$ at $(1, -1, 2)$ in the direction of the normal to the surface $x^2 + y^2 = z + 4$. Pind the rate of change of $\phi = xy + yz + zx$ at $(1, -1, 2)$ in the direction of the normal to the surface $x^2 + y^2 = z + 4$. Pind the rate of change of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the normal to the surface $x + y = x^2 + y^2 = x + 4$. Pind the rate of change of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the normal to the surface $x + y^2 + 3z + 3$	25	88	Find the directional derivative of $\phi = x^2 y \cos z$ in the direction of the line $\overline{a} = 2i + 3j + 2k$ at
Find the angle between the two surfaces $x^2 + y^2 + az^2 = 6$ and $z = 4 - y^2 + bxy$ at $(1, 1, 2)$ Find the rate of change of $\phi = xy + yz + zx$ at $(1, -1, 2)$ in the direction of the normal to the surface $x^2 + y^2 = z + 4$. In what direction is the directional derivative of $\phi = 2xz - y^2$ at $(1, 3, 2)$ maximum? Find its magnitude. In what direction is the directional derivative of $\phi = 2xz - y^2$ at $(1, 3, 2)$ maximum? Find its magnitude. Find the rate of change of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at $(-1, 2, 1)$ Find the angle between the normals to the surfaces $x^2y + 2xz = 4$ at $(2, -2, 3)$ and to $x^3 + y^3 + 3xyz = 3$ at $(1, 2, -1)$ Find the constants a and b such that the surfaces $ax^2 - 2byz = (a + 4)x$ will be orthogonal to the surface $ax^2y + 2xz + 4$ at $ax^2y + 2x^2y + 4 = 2x + 4$ at $ax^2y + 2x^2y + 4 = 2x + 4$ at $ax^2y + 2x^2y + 4 = 2x + 4$ at $ax^2y + 2x^2y + 4 = 2x + 4$ at $ax^2y + 2x^2y + 2x^2y + 4 = 2x + 4$ at $ax^2y + 2x^2y + $	26	90	$(1,2,\pi/2)$
27 $x^2 + y^2 + a z^2 = 6$ and $z = 4 - y^2 + bxy$ at $(1, 1, 2)$ 28 91 Find the rate of change of $\phi = xy + yz + zx$ at $(1, -1, 2)$ in the direction of the normal to the surface $x^2 + y^2 = z + 4$. 29 92 In what direction is the directional derivative of $\phi = 2xz - y^2$ at $(1, 3, 2)$ maximum? Find its magnitude. 30 93 Find the rate of change of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at $(-1, 2, 1)$ 31 94 Find the angle between the normals to the surfaces $x^2y + 2xz = 4$ at $(2, -2, 3)$ and to $x^3 + y^3 + 3xyz = 3$ at $(1, 2, -1)$ 32 95 Find the constants a and b such that the surfaces $x^2y + 2xz = 4$ at $(2, -2, 3)$ and to the surface $4x^2y + z^3 = 4$ at $(1, -1, 2)$ 33 96 Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}(1/\sqrt{3})$. 34 97 Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1, -1, 2)$ 35 98 If the directional derivative of $\phi = ax^2 + by + 2z$ at $(1, 1, 1)$ is maximum in the direction of $i + j + k$, find a &b. 36 99 Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at $(1, 2, 1)$ 37 100 Find the directional derivative of $\theta = \frac{x}{x^2 + y^2}$ at $(0, 1, 1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1, 1, 1)$ 38 101 Find the constants a and b such that the directional derivative of $\theta = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ has the maximum magnitude 4 in the directional derivative of $\theta = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ has the maximum magnitude 4 in the directional part of $\theta = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ has the maximum magnitude 4 in the directional part of $\theta = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS 39 102 If \overline{a} is a constant vector and \overline{a} = $x^2 + y^2 + z^2$, prove that \overline	20		ture curtaces
surface $x^2 + y^2 = z + 4$. 29 92 In what direction is the directional derivative of $\phi = 2xz - y^2$ at $(1, 3, 2)$ maximum? Find its magnitude. 30 93 Find the rate of change of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at $(-1, 2, 1)$ 31 94 Find the angle between the normals to the surfaces $x^2y + 2xz = 4$ at $(2, -2, 3)$ and to $x^3 + y^3 + 3xyz = 3$ at $(1, 2, -1)$ 32 95 Find the constants a and b such that the surfaces $ax^2 - 2byz = (a + 4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at $(1, -1, 2)$. 33 96 Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}(1/\sqrt{3})$. 34 97 Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1, -1, 2)$ 35 98 If the directional derivative of $\phi = ax^2 + by + 2z$ at $(1, 1, 1)$ is maximum in the direction of $i + j + k$, find a &b. 36 99 Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at $(1, 2, 1)$ 37 100 Find the directional derivative of $\phi = \frac{x}{x^2 + y^2} = 4 + xz$ at $(0, 1, 1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1, 1, 1)$ 38 101 Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ 38 101 Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ 38 101 Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ 38 101 Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ 38 101 Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ 38 101 Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ 39 102 If \overline	27	90	$x^2 + y^2 + a z^2 = 6$ and $z = 4 - y^2 + b x y$ at $(1, 1, 2)$
29 92 In what direction is the directional derivative of $\phi = 2xz - y^2$ at $(1, 3, 2)$ maximum? Find its magnitude. 30 93 Find the rate of change of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at $(-1, 2, 1)$ 31 94 Find the angle between the normals to the surfaces $x^2y + 2xz = 4$ at $(2, -2, 3)$ and to $x^3 + y^3 + 3xyz = 3$ at $(1, 2, -1)$ 32 95 Find the constants a and b such that the surfaces $ax^2 - 2byz = (a + 4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at $(1, -1, 2)$. 33 96 Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}(1/\sqrt{3})$. 34 97 Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1, -1, 2)$. 35 98 If the directional derivative of $\phi = ax^2 + by + 2z$ at $(1, 1, 1)$ is maximum in the direction of $i + j + k$, find a &b. 36 99 Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $ax^2 + bx^2 = 4 + xz$ at $ax = (1, 2, 1)$ 37 100 Find the directional derivative of $ax = ax $	28	91	
Find the rate of change of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at $(-1, 2, 1)$ 31 94 Find the angle between the normals to the surfaces $x^2y + 2xz = 4$ at $(2, -2, 3)$ and to $x^3 + y^3 + 3xyz = 3$ at $(1, 2, -1)$ 32 95 Find the constants a and b such that the surfaces $ax^2 - 2byz = (a + 4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at $(1, -1, 2)$ Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}(1/\sqrt{3})$. Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1, -1, 2)$ 35 98 If the directional derivative of $\phi = ax^2 + by + 2z$ at $(1, 1, 1)$ is maximum in the direction of $i + j + k$, find a &b. 36 99 Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at $(1, 2, 1)$ Find the directional derivative of $\phi = \frac{x}{x^2 + y^2}$ at $(0, 1, 1)$ in the direction of normal to the surface $x^2 + y^2 = 4 + xz$ at $(1, 2, 1)$ Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS 39 102 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r}) \cdot \overline{a}\} = a^2$ 40 103 If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \times \overline{r}) \cdot \overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2a$ 41 104 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{r}$, $\nabla \times \overline{r}$ where $\overline{r} = \nabla \phi$ Prove that $\nabla f(r) = \frac{f'(r)}{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.	20	02	surface $x + y = z + 4$.
surface $x \log z - y^2 + 4 = 0$ at $(-1, 2, 1)$ 31 94 Find the angle between the normals to the surfaces $x^2y + 2xz = 4$ at $(2, -2, 3)$ and to $x^3 + y^3 + 3xyz = 3$ at $(1, 2, -1)$ 32 95 Find the constants a and b such that the surfaces $ax^2 - 2byz = (a + 4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at $(1, -1, 2)$. 33 96 Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}(1/\sqrt{3})$. 34 97 Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1, -1, 2)$ 35 98 If the directional derivative of $\phi = ax^2 + by + 2z$ at $(1, 1, 1)$ is maximum in the direction of $i + j + k$, find a &b. 36 99 Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at $(1, 2, 1)$ 37 100 Find the directional derivative of $\phi = \frac{x}{x^2 + y^2}$ at $(0, 1, 1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1, 1, 1)$ 38 101 Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ 39 102 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r})\overline{a}\} = a^2$ 40 103 If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 2\overline{a}$ 41 104 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find $(i)\overline{r} \cdot \nabla \phi$, $(ii)\overline{v} \cdot \overline{r}$, $\overline{v} \times \overline{r}$ where $\overline{F} = \nabla \phi$ 42 105 Prove that $\nabla f(r) = \frac{f'(r)}{r}\overline{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.	29	92	
surface $x \log z - y^2 + 4 = 0$ at $(-1, 2, 1)$ 31 94 Find the angle between the normals to the surfaces $x^2y + 2xz = 4$ at $(2, -2, 3)$ and to $x^3 + y^3 + 3xyz = 3$ at $(1, 2, -1)$ 32 95 Find the constants a and b such that the surfaces $ax^2 - 2byz = (a + 4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at $(1, -1, 2)$. 33 96 Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}(1/\sqrt{3})$. 34 97 Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1, -1, 2)$ 35 98 If the directional derivative of $\phi = ax^2 + by + 2z$ at $(1, 1, 1)$ is maximum in the direction of $i + j + k$, find a &b. 36 99 Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at $(1, 2, 1)$ 37 100 Find the directional derivative of $\phi = \frac{x}{x^2 + y^2}$ at $(0, 1, 1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1, 1, 1)$ 38 101 Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ 39 102 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r})\overline{a}\} = a^2$ 40 103 If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 2\overline{a}$ 41 104 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find $(i)\overline{r} \cdot \nabla \phi$, $(ii)\overline{v} \cdot \overline{r}$, $\overline{v} \times \overline{r}$ where $\overline{F} = \nabla \phi$ 42 105 Prove that $\nabla f(r) = \frac{f'(r)}{r}\overline{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.	30	93	Find the rate of change of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the normal to the
Find the angle between the normals to the surfaces $x^2y + 2xz = 4$ at $(2, -2, 3)$ and to $x^3 + y^3 + 3xyz = 3$ at $(1, 2, -1)$ Find the constants a and b such that the surfaces $ax^2 - 2byz = (a + 4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at $(1, -1, 2)$. Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}(1/\sqrt{3})$. Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1, -1, 2)$ If the directional derivative of $\phi = ax^2 + by + 2z$ at $(1, 1, 1)$ is maximum in the direction of $i + j + k$, find a &b. Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $ax^2 + by^2 = 4 + xz$ at $(1, 2, 1)$ Find the directional derivative of $\phi = \frac{x}{x^2 + y^2}$ at $(0, 1, 1)$ in the direction of normal to the surface $ax^2 + bx^2 - bx^2 + cx^2 = 1$ at $(1, 1, 1)$ Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cx^2 = 1$ at $(1, 1, 1)$ Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cx^2 = 1$ at $(1, 1, 1)$ Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cx^2 = 1$ at $(1, 1, 1)$ Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cx^2 = 1$ at $(1, 1, 1)$ If $\phi = x^2 + y^2 - z^2x = 1$ at $(1, 1, 1)$ If $\phi = x^2 + by^2 + cx^2 = 1$ at $(1, 1, 1)$ in the direction of normal to the surface $ax + bx + $			
32 95 Find the constants a and b such that the surfaces $ax^2 - 2byz = (a + 4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at $(1, -1, 2)$. 33 96 Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}(1/\sqrt{3})$. 34 97 Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1, -1, 2)$ 35 98 If the directional derivative of $\phi = ax^2 + by + 2z$ at $(1, 1, 1)$ is maximum in the direction of $i + j + k$, find a &b. 36 99 Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at $(1, 2, 1)$ 37 100 Find the directional derivative of $\phi = \frac{x}{x^2 + y^2}$ at $(0, 1, 1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1, 1, 1)$ 38 101 Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS 103 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r}) \overline{a}\} = a^2$ 103 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $ \overline{a} = a^2$ 101 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{r}$, $\nabla \times \overline{r}$ where $\overline{r} = \nabla \phi$ 105 Prove that $ \nabla \left(\frac{1}{r}\right) = -\frac{\overline{r}}{r^3}$.	31	94	Sin late $x \log 2^{-y}$ $y + 1^{-y}$ out $(2, 2, 2)$
Find the constants a and b such that the surfaces $ax^2 - 2byz = (a + 4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at $(1,-1,2)$. Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at $(0,1,2)$ is $\cos^{-1}(1/\sqrt{3})$. Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1,-1,2)$ If the directional derivative of $\phi = ax^2 + by + 2z$ at $(1,1,1)$ is maximum in the direction of $i+j+k$, find a &b. Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at $(1,2,1)$ Find the directional derivative of $\phi = \frac{x}{x^2 + y^2}$ at $(0,1,1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1,1,1)$ Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at $(1,1,2)$ has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $ \nabla \cdot \{(\overline{a} \cdot \overline{r}) \overline{a}\} = a^2$ If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r}) \overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{r}$, $\nabla \times \overline{r}$ where $\overline{r} = \nabla \phi$ Prove that $\nabla f(r) = \frac{f'(r)}{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.			
the surface $4x^2y + z^3 = 4$ at $(1,-1,2)$. Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at $(0,1,2)$ is $\cos^{-1}(1/\sqrt{3})$. Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1,-1,2)$. If the directional derivative of $\phi = ax^2 + by + 2z$ at $(1,1,1)$ is maximum in the direction of $i+j+k$, find a &b. Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at $(1,2,1)$ Find the directional derivative of $\emptyset = \frac{x}{x^2 + y^2}$ at $(0,1,1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1,1,1)$ Find the constants a and b such that the directional derivative of $\emptyset = ax^2 + by^2 + cz^2$ at $(1,1,2)$ has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r}) \overline{a}\} = a^2$ If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r}) \overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ In the finding of a is a in		1	
the surface $4x^2y + z^3 = 4$ at $(1,-1,2)$. Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at $(0,1,2)$ is $\cos^{-1}(1/\sqrt{3})$. Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1,-1,2)$. If the directional derivative of $\phi = ax^2 + by + 2z$ at $(1,1,1)$ is maximum in the direction of $i+j+k$, find a &b. Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at $(1,2,1)$ Find the directional derivative of $\emptyset = \frac{x}{x^2 + y^2}$ at $(0,1,1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1,1,1)$ Find the constants a and b such that the directional derivative of $\emptyset = ax^2 + by^2 + cz^2$ at $(1,1,2)$ has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r}) \overline{a}\} = a^2$ If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r}) \overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ In the finding of a is a in	32	95	Find the constants a and b such that the surfaces $ax^2 - 2byz = (a+4)x$ will be orthogonal to
Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}\left(1/\sqrt{3}\right)$. Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1, -1, 2)$ Find the directional derivative of $\phi = ax^2 + by + 2z$ at $(1, 1, 1)$ is maximum in the direction of $i + j + k$, find a &b. Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at $(1, 2, 1)$ Find the directional derivative of $\emptyset = \frac{x}{x^2 + y^2}$ at $(0, 1, 1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1, 1, 1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1, 1, 1)$ has the maximum magnitude 4 in the directional derivative of $\emptyset = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS 102 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r}) \overline{a}\} = a^2$ 103 If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ 104 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{r}$, $\nabla \times \overline{r}$ where $\overline{r} = \nabla \phi$ 105 Prove that $\nabla f(r) = \frac{f'(r)}{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.			
$x^{2}z + xy + y + 1 = z \text{ at } (0, 1, 2) \text{ is } \cos^{-1}(1/\sqrt{3}).$ $97 \qquad \text{Find the constants a, b such that the surfaces } 5x^{2} - 2yz - 9x = 0 \text{ & } ax^{2}y + bz^{3} = 4 \text{ cut orthogonally at } (1,-1,2)$ $98 \qquad \text{If the directional derivative of } \phi = ax^{2} + by + 2z \text{ at } (1, 1, 1) \text{ is maximum in the direction of } i + j + k, \text{ find a &b.}$ $36 \qquad 99 \qquad \text{Find the constants a and b such that the surface } ax^{2} - bxy + xz = 10 \text{ is orthogonal to the surface } x^{2} + y^{2} = 4 + xz \text{ at } (1,2,1)$ $37 \qquad 100 \qquad \text{Find the directional derivative of } \phi = \frac{x}{x^{2} + y^{2}} \text{ at } (0,1,1) \text{ in the direction of normal to the surface } x^{2} + y^{2} - z^{2}x = 1 \text{ at } (1,1,1)$ $38 \qquad 101 \qquad \text{Find the constants a and b such that the directional derivative of } \phi = ax^{2} + by^{2} + cz^{2} \text{ at } (1,1,2) \text{ has the maximum magnitude 4 in the direction parallel to x-axis.}}$ $01 \qquad \text{DIFFERENTIAL OPERATORS}$ $39 \qquad 102 \qquad \text{If } \overline{a} \text{ is a constant vector such that } \overline{a} = a \text{ then prove that } \nabla \cdot \{(\overline{a} \cdot \overline{r}) \overline{a}\} = a^{2}$ $103 \qquad 16 \qquad \text{if } \overline{a} \text{ is a constant vector and } \overline{r} = xi + yj + zk \text{, prove that } \text{ i) } div (\overline{a} \times \overline{r}) = 0 \text{ ii) } div (\overline{a} \cdot \overline{r}) \overline{a} = a^{2} \text{ iii) } div (\overline{a} \times \overline{r} \times \overline{a}) = 2a^{2} \text{ iv) } \text{ curl } (\overline{a} \times \overline{r}) = 2\overline{a}$ $41 \qquad 104 \qquad 16 \qquad 16 \qquad x^{3} + y^{3} + z^{3} - 3xyz, \text{ find } (i) \overline{r} \cdot \nabla \phi \text{, (ii) } \nabla \cdot \overline{F}, \nabla \times \overline{F} \text{ where } \overline{F} = \nabla \phi$ $42 \qquad 105 \qquad \text{Prove that } \nabla f(r) = \frac{f'(r)}{r} \overline{r} \text{ and hence, find } f \text{ if } f f$	33	96	
Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1,-1,2)$ 15 98 If the directional derivative of $\phi = ax^2 + by + 2z$ at $(1, 1, 1)$ is maximum in the direction of $i + j + k$, find a &b. 16 99 Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at $(1, 2, 1)$ 17 100 Find the directional derivative of $\phi = \frac{x}{x^2 + y^2}$ at $(0, 1, 1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1, 1, 1)$ 18 101 Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ has the maximum magnitude 4 in the direction parallel to x-axis. 19 102 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r}) \overline{a}\} = a^2$ 103 If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ 104 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{r}$, $\nabla \times \overline{r}$ where $\overline{r} = \nabla \phi$ 105 Prove that $\nabla f(r) = \frac{\overline{r}}{r^3}$.			
orthogonally at (1,-1, 2) If the directional derivative of $\phi = ax^2 + by + 2z$ at (1, 1, 1) is maximum in the direction of $i + j + k$, find a &b. 36 99 Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at (1, 2, 1) 37 100 Find the directional derivative of $\phi = \frac{x}{x^2 + y^2}$ at (0,1,1) in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at (1,1,1) 38 101 Find the constants a and b such that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at (1,1,2) has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS 39 102 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r}) \overline{a}\} = a^2$ 40 103 If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r}) \overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ 41 104 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{F}$, $\nabla \times \overline{F}$ where $\overline{F} = \nabla \phi$ 42 105 Prove that $\nabla f(r) = \frac{f'(r)}{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.			$x^{2}z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}(1/\sqrt{3})$.
If the directional derivative of $\phi = ax^2 + by + 2z$ at (1, 1, 1) is maximum in the direction of $i + j + k$, find a &b. 99 Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at (1, 2, 1) 37 100 Find the directional derivative of $\emptyset = \frac{x}{x^2 + y^2}$ at (0,1,1) in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at (1,1,1) 38 101 Find the constants a and b such that the directional derivative of $\emptyset = ax^2 + by^2 + cz^2$ at (1,1,2) has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS 39 102 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r}) \overline{a}\} = a^2$ 40 103 If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r}) \overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ 41 104 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{F}$, $\nabla \times \overline{F}$ where $\overline{F} = \nabla \phi$ Prove that $\nabla f(r) = \frac{f'(r)}{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.	34	97	Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut
$i+j+k$, find a &b. 36 99 Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at $(1, 2, 1)$ 37 100 Find the directional derivative of $\emptyset = \frac{x}{x^2 + y^2}$ at $(0, 1, 1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1, 1, 1)$ 38 101 Find the constants a and b such that the directional derivative of $\emptyset = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS 39 102 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r})\overline{a}\} = a^2$ 40 103 If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r})\overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ 41 104 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{F}$, $\nabla \times \overline{F}$ where $\overline{F} = \nabla \phi$ 42 105 Prove that $\nabla f(r) = \frac{r}{r^3}$.	25	- 00	
Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at $(1, 2, 1)$ Find the directional derivative of $\emptyset = \frac{x}{x^2 + y^2}$ at $(0, 1, 1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1, 1, 1)$ Find the constants a and b such that the directional derivative of $\emptyset = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r}) \cdot \overline{a}\} = a^2$ If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r}) \cdot \overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{F}$, $\nabla \times \overline{F}$ where $\overline{F} = \nabla \phi$ Prove that $\nabla f(r) = \frac{r}{r^3}$. Prove that $\nabla f(r) = \frac{f'(r)}{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.	33	98	If the directional derivative of $\phi = ax^2 + by + 2z$ at (1, 1, 1) is maximum in the direction of
surface $x^2 + y^2 = 4 + xz$ at $(1, 2, 1)$ Find the directional derivative of $\emptyset = \frac{x}{x^2 + y^2}$ at $(0, 1, 1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1, 1, 1)$ Find the constants a and b such that the directional derivative of $\emptyset = ax^2 + by^2 + cz^2$ at $(1, 1, 2)$ has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r}) \overline{a}\} = a^2$ If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r}) \overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{F}$, $\nabla \times \overline{F}$ where $\overline{F} = \nabla \phi$ Prove that $\nabla f(r) = \frac{f'(r)}{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.		. The same	
surface $x^2 + y^2 - z^2x = 1$ at $(1,1,1)$ 38 101 Find the constants a and b such that the directional derivative of $\emptyset = ax^2 + by^2 + cz^2$ at $(1,1,2)$ has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS 39 102 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r})\overline{a}\} = a^2$ 40 103 If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r})\overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ 41 104 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{F}$, $\nabla \times \overline{F}$ where $\overline{F} = \nabla \phi$ 42 105 Prove that $\nabla \left(\frac{1}{r}\right) = -\frac{\overline{r}}{r^3}$. 43 106 Prove that $\nabla f(r) = \frac{f'(r)}{r}\overline{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.	36	99	$\int \int $
has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS 39 102 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r})\overline{a}\} = a^2$ 40 103 If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r})\overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ 41 104 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{F}$, $\nabla \times \overline{F}$ where $\overline{F} = \nabla \phi$ 42 105 Prove that $\nabla \left(\frac{1}{r}\right) = -\frac{\overline{r}}{r^3}$. 43 106 Prove that $\nabla f(r) = \frac{f'(r)}{r}\overline{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.	37	100	
has the maximum magnitude 4 in the direction parallel to x-axis. DIFFERENTIAL OPERATORS 39 102 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r})\overline{a}\} = a^2$ 40 103 If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r})\overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ 41 104 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{F}$, $\nabla \times \overline{F}$ where $\overline{F} = \nabla \phi$ 42 105 Prove that $\nabla \left(\frac{1}{r}\right) = -\frac{\overline{r}}{r^3}$. 43 106 Prove that $\nabla f(r) = \frac{f'(r)}{r}\overline{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.	v.		
DIFFERENTIAL OPERATORS 39 102 If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r})\overline{a}\} = a^2$ 40 103 If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r})\overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ 41 104 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{F}$, $\nabla \times \overline{F}$ where $\overline{F} = \nabla \phi$ 42 105 Prove that $\nabla \left(\frac{1}{r}\right) = -\frac{\overline{r}}{r^3}$. 43 106 Prove that $\nabla f(r) = \frac{f'(r)}{r}\overline{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.	38	101	Find the constants a and b such that the directional derivative of $\emptyset = ax^2 + by^2 + cz^2$ at (1,1,2)
If \overline{a} is a constant vector such that $ \overline{a} = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r})\overline{a}\} = a^2$ If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r})\overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{F}$, $\nabla \times \overline{F}$ where $\overline{F} = \nabla \phi$ Prove that $\nabla \left(\frac{1}{r}\right) = -\frac{\overline{r}}{r^3}$. Prove that $\nabla f(r) = \frac{f'(r)}{r}\overline{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.	DIE	FERENT	
If \overline{a} is a constant vector and $\overline{r} = xi + yj + zk$, prove that i) $div(\overline{a} \times \overline{r}) = 0$ ii) $div(\overline{a} \cdot \overline{r}) \overline{a} = a^2$ iii) $div(\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl(\overline{a} \times \overline{r}) = 2\overline{a}$ If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{F}$, $\nabla \times \overline{F}$ where $\overline{F} = \nabla \phi$ Prove that $\nabla \left(\frac{1}{r}\right) = -\frac{\overline{r}}{r^3}$. Prove that $\nabla f(r) = \frac{f'(r)}{r} \overline{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.			
ii) $div (\overline{a} \cdot \overline{r}) \overline{a} = a^2$ iii) $div (\overline{a} \times \overline{r} \times \overline{a}) = 2a^2$ iv) $curl (\overline{a} \times \overline{r}) = 2\overline{a}$ 41	40	103	
41 104 If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{r} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{F}$, $\nabla \times \overline{F}$ where $\overline{F} = \nabla \phi$ 42 105 Prove that $\nabla \left(\frac{1}{r}\right) = -\frac{\overline{r}}{r^3}$. 43 106 Prove that $\nabla f(r) = \frac{f'(r)}{r}\overline{r}$ and hence, find f if $\nabla f = 2r^4\overline{r}$.	70	103	
Prove that $\nabla \left(\frac{1}{r}\right) = -\frac{\overline{r}}{r^3}$. Prove that $\nabla f(r) = \frac{f'(r)}{r} \overline{r}$ and hence, find f if $\nabla f = 2r^4 \overline{r}$.			ii) $div (\bar{a} \cdot \bar{r}) \bar{a} = a^2 \text{ iii}) \ div (\bar{a} \times \bar{r} \times \bar{a}) = 2a^2 \text{ iv}) \ curl (\bar{a} \times \bar{r}) = 2\bar{a}$
Prove that $\nabla \left(\frac{1}{r}\right) = -\frac{\overline{r}}{r^3}$. Prove that $\nabla f(r) = \frac{f'(r)}{r} \overline{r}$ and hence, find f if $\nabla f = 2r^4 \overline{r}$.	41	104	If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\overline{F} \cdot \nabla \phi$, (ii) $\nabla \cdot \overline{F}$, $\nabla \times \overline{F}$ where $\overline{F} = \nabla \phi$
Prove that $\nabla f(r) = \frac{f'(r)}{r} \overline{r}$ and hence, find f if $\nabla f = 2r^4 \overline{r}$.	42	105	
P P			
Show that $\nabla \left[\frac{\left(\overline{a} \cdot \overline{r} \right)}{r^n} \right] = \frac{\overline{a}}{r^n} - \frac{n \left(\overline{a} \cdot \overline{r} \right) \overline{r}}{r^{n+2}}$	43	106	P .
$\begin{bmatrix} r^n \end{bmatrix} r^n r^{n+2}$	44	107	Show that $\nabla \left[\frac{(\bar{a} \cdot \bar{r})}{\bar{a} \cdot \bar{r}} \right] = \frac{\bar{a}}{\bar{a}} - \frac{n(\bar{a} \cdot \bar{r})\bar{r}}{\bar{a} \cdot \bar{r}}$
		No. of	$\begin{bmatrix} r^n \end{bmatrix} r^n r^{n+2}$

Vector Differentiation Practice Problems

45	108	Prove that $\nabla r^n = n r^{n-2} \overline{r}$
46	109	Prove that $\nabla \cdot (\nabla \times \overline{F}) = 0$ where \overline{F} is a vector point function.
47	110	Prove that $\nabla \left\{ \nabla \cdot \frac{\overline{r}}{r} \right\} = -\frac{2}{r^3} \overline{r}$
48	111	Prove that $\nabla \cdot \left(r \nabla \frac{1}{r^3} \right) = \frac{3}{r^4}$
49	112	Prove that $\nabla \cdot \left(r \nabla \frac{1}{r^n} \right) = \frac{n(n-2)}{r^{n+1}}$
50	113	Prove that $\operatorname{div} \operatorname{grad} r^n = n(n+1)r^{n-2}$
51	114	Prove that $\nabla \times \left(\frac{\overline{a} \times \overline{r}}{r^n}\right) = \frac{(2-n)\overline{a}}{r^n} + \frac{n(\overline{a} \cdot \overline{r})\overline{r}}{r^{n+2}}$
52	115	Prove that $\nabla \log r = \frac{\overline{r}}{r^2}$ and hence, show that $\nabla \times (\overline{a} \times \nabla \log r) = 2 \frac{(\overline{a} \cdot \overline{r}) \overline{r}}{r^4}$, where \overline{a} is
DIV	ERGEN	a constant vector. CE AND CURL
53	116	$div \overline{F} and curl \overline{F} where \overline{F} = \frac{xi - yj}{x^2 + y^2}$ Find
54	117	If $\overline{A} = \nabla(xy + yz + zx)$, find $\nabla \cdot \overline{A}$ and $\nabla \times \overline{A}$
55	118	If $\overline{F} = (\overline{a} \cdot \overline{r}) \overline{r}$ where \overline{a} is constant vector, find $\overline{curl} \overline{F}$ and P.T. it is perpendicular to \overline{a} .
56	119	Prove that $\overline{F} = \frac{\overline{r}}{r^3}$ is both irrotational and solenoidal.
57	120	A vector field \overline{F} is given by $\overline{F} = (y \sin z - \sin x)i + (x \sin z + 2yz)j + (xy \cos z + y^2)k \text{ Prove that it is irrotational and hence, find its scalar potential.}$
58	121	A vector field is given by $\overline{F} = (x^2 + xy^2)i + (y^2 + x^2y)j$. Show that \overline{F} irrotational and find its scalar potential.
59	122	If $\nabla \phi = (y^2 - 2xyz^3)i + (3 + 2xy - x^2z^3)j + (6z^3 - 3x^2yz^2)k$, find ϕ where $\phi(1, 0, 1) = 2$
60	123	Find the value of n for which the vector $r^n \overline{r}$ is solenoidal, where $\overline{r} = xi + yj + zk$
61	124	Prove that $\nabla \cdot \left\{ \frac{f(r)}{r} \overline{r} \right\} = \frac{1}{r^2} \frac{d}{dr} [r^2 f(r)]$ hence or otherwise prove that $\operatorname{div}(r^n \overline{r}) = (n+3)r^n$
62	125	Show that $\overline{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both solenoidal & irrotational.
63	126	If \overline{r} is the position vector of point (x, y, z) and r is the modulus of \overline{r} , then prove that $r^n \overline{r}$ is an irrotational vector for any value of n but solenoidal only if $n = -3$.

Vector Differentiation Practice Problems

D1	127,D 7	If $\bar{f} = (x+y+1)i + j - (x+y)k$, prove that $\bar{f} \cdot curl \bar{f} = 0$		
D2	128,D 8	Define irrotational field and hence check whether the vector field $\overline{F} = (x + 2y + 4z)i + (2x - 3y - z)j + (4x - y + 2z)k$ is irrotational.		
DIFF	DIFFERENTIAL OPERATORS			
D3	D9	With usual notation, prove that $\nabla^2 \left[\nabla \cdot \frac{\overline{r}}{r^2} \right] = \frac{2}{r^4}$		
D4	D10	Show that $\nabla^4 r^2 \log r = \frac{6}{r^2}$		
D5	DII	Prove that $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r} \frac{df}{dr}$		
D6	D12	Prove that $\nabla^2(r^2 \log r) = 5 + 6 \log r$		