SMALL SAMPLE TEST - 3

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NOT INDEPENDENT SAMPLES

In some cases the samples may not be different.

We may test the effectiveness of a drug on the same group of persons.

We may test the effectiveness of coaching on the same batch of students.

In each cases the sample is the same for two tests. The samples are not independent and above formula for testing of hypothesis cannot be used.

In such cases we calculate the t- statistic as explained below

We first find the difference of the corresponding values of the two sets of data then find the mean Difference $\bar{x}$ and standard deviation of differences s. We then define

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n-1}} = \frac{\bar{x}}{s/\sqrt{n-1}}$$

$$\boldsymbol{or}\ t = \frac{\bar{x}-\mu}{S/\sqrt{n}} = \frac{\bar{x}}{S/\sqrt{n}}$$

Where, $\mu = 0$ is the null hypothesis

$s =$ S.D. of the sample, $S =$ unbiased estimator of σ

Note: Taking the null hypothesis $\mu = 0$ for differences amounts to the null hypothesis of equality of means $\mu_1 = \mu_2$ of the two populations.

EXAMPLE-1

A certain injection administered to 12 patients resulted in the following changes of blood pressure: $5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4$ can it be concluded that the injection will be in general accompanied by an increase in blood pressure?

We first calculate $\bar{x}$ and s^2

$\bar{x} = 2.58$, $\sum(x_i - \bar{x})^2 = 104.92$,

$s^2 = \frac{\sum(x_i - \bar{x})^2}{n} = 8.74$

The null hypothesis $H_0: \mu = 0$

Alternative Hypothesis $H_a: \mu > 0$

Calculation of test statistic: Since the sample size is small, we use students t –distribution

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

$\therefore t = 2.89$

(The positive sign of t denotes the increase in the level i.e. in the blood pressure)

Level of significance: $\alpha = 0.05$

Critical value: The value of t_α for 5% level of significance for $v(= 12 - 1) = 11$ degrees of freedom for one tailed test

$= t_\alpha$ at 10% LOS for two tailed test $= 1.796$

Decision: Since the calculated value of $t = 2.89$ is greater than the critical value $t_\alpha = 1.769$,

the null hypothesis is rejected

There is rise in B.P.

EXAMPLE-2

Ten school boys were given a test in Statistics and their scores were recorded. They were given a month special coaching and a second test was given to them in the same subject at the end of the coaching period. Test if the marks given below give evidence to the fact that the students are benefited by coaching.

Marks in Test I	70	68	56	75	80	90	68	75	56	58
Marks in Test II	68	70	52	73	75	78	80	92	54	55

We first calculate $x =$ the differences between marks in test II and marks in test I and from these we calculate $\bar{x}$ and s^2.

x	-2	2	-4	-2	-5	-12	12	17	-2	-3

$\bar{x} = 0.1$, $\sum(x_i - \bar{x})^2 = 678.999$,

$$s^2 = \frac{\sum(x_i - \bar{x})^2}{n} = 67.90$$

- **Null hypothesis** $H_0: \mu = 0$
- **Alternative Hypothesis** $H_a: \mu > 0$

Calculation of test statistic: Since the sample size is small, we use t –distribution

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

$|t| = 0.04$

Level of significance: $\alpha = 0.05$

Critical value: The value of t_α for 5% level of significance for $v = 12 - 1 = 9$ degrees of freedom is 1.83 for one tailed test

Decision: Since the calculated value of $|t| = 0.004$ is less than the critical value $t_\alpha = 1.83$, the hypothesis is accepted

The students are not benefitted by coaching

EXAMPLE-3

In a certain experiment to compare two types of pig – foods A and B, the following results of increasing weights were obtained.

Pig Number	1	2	3	4	5	6	7	8
Increase in weight X kg by A	49	53	51	52	47	50	52	53
Increases in weight Y kg by B	52	55	52	53	50	54	54	53

Assuming that the two sample of pigs are independent, can we conclude that food B is better than food A.

Examine the case if the same set of pigs were used in both the cases.

For first part

We first calculate $\bar{x}_1, \bar{x}_2, \sum(x_1-\bar{x}_1)^2, \sum(x_2-\bar{x}_2)^2$

$\bar{x}_1 = 50.875$ $\bar{x}_2 = 52.875$

$\sum(x_1-\bar{x}_1)^2 = 30.875$ $\sum(x_2-\bar{x}_2)^2 = 16.875$

Null hypothesis $H_0: \mu_1 = \mu_2$

Alternative Hypothesis $H_a: \mu_1 < \mu_2$

Calculation of test statistic: $S_p = \sqrt{\frac{\sum(x_1-\bar{x}_1)^2+\sum(x_2-\bar{x}_2)^2}{n_1+n_2-2}}$

$= \sqrt{3.41}$

$S.E. = s_p \times \sqrt{\frac{1}{n_1}+\frac{1}{n_2}} =$

- 0.92
- $t = \frac{\bar{x}_1-\bar{x}_2}{S.E.}$
- $= -2.17$

- **Level of significance**: $\alpha = 0.05$
- **Critical value**: The table value of t at $\alpha = 0.05$ for $v = 8 + 8 - 2 = 14$ degree of freedom is 1.76
- **Decision**: Since the computed value of $t = -2.17$ is less than the table value $t_\alpha = -1.76$,
- the hypothesis is rejected at 5% level of significance (One tailed test)

If the same set of pigs were used in the two tests: we first calculate the differences between the weights in the two tests and from these we calculate $\bar{x}$ and s^2

x	-3	-2	-1	-1	-3	-4	-2	0	

- $\bar{x} = -2$
- $\sum(x_i - \bar{x})^2 = 12$
- $s^2 = \frac{\sum(x_i - \bar{x})^2}{n} = 1.5$
- **The null hypothesis** $H_0: \mu = 0$
- **Alternative Hypothesis** $H_a: \mu < 0$
- **Calculation of test statistic:**
- $t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$
- $= -4.32$

- **Level of significance**: $\alpha = 0.05$
- **Critical value**: The value of t_α for 5% level of significance for $v = 8 - 1 = 7$ degrees of freedom is -1.89 (One tailed test)
- **Decision**: Since the calculated value of $t = -4.32$ is less than the critical value $t_\alpha = -1.89$, the hypothesis is rejected
- **Food B is superior to food A**