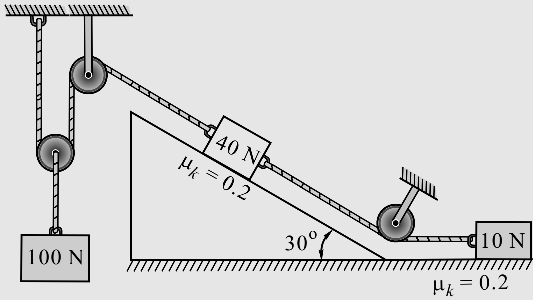
14

KINETICS OF PARTICLES - I NEWTON'S SECOND LAW/ D'ALEMBERT'S PRINCIPLE



14.1 Introduction to Kinetics

- **Kinetics :** *It is the study of geometry of motion with reference to the cause of motion*. Here we consider force and mass.

Basic Concepts

- **Particle :** *It is a matter with considerable mass but negligible dimension,* i.e., any object whose mass is considered but dimension is not considered.
- **Force :** *An external agency which changes or tends to change the state of rest or of uniform motion of a body upon which it acts is known as force*.
- **Mass :** *It is the quantity of matter contained in a body.*

  The quantity does not change on account of the position occupied by the body. The force of attraction exerted by the earth on two different bodies with the same mass will be same. *Mass is the property of body which measures its resistance to a change of motion*. Its SI unit is kg.
- **Weight :** *The gravitational force of attraction exerted by the earth on a body is known as the weight of the body*. This force exists whether the body is at rest or in motion. Since this attraction is a force, the weight of body should be expressed in Newton (N) in SI units.

14.2 Newton's Second Law of Motion

- **Newton's Second Law of Motion :** *If an external unbalanced force acts on a body, the momentum of the body changes. The rate of change of momentum is directly proportional to the force and takes place in the direction of motion.*

- **Momentum :** *The quantity of motion possessed by the body.* Linear momentum of a body is calculated as the product of mass and velocity of the body.

  Rate of change of momentum is directly proportional to the force, i.e.,

$$\frac{d}{dt}(m\bar{v}) \propto \bar{F}$$

$$\frac{d}{dt}(m\bar{v}) = k\bar{F}$$

$$m\frac{d\bar{v}}{dt} = k\bar{F}$$

$$m\bar{a} = k\bar{F}$$

when $m = 1$, $a = 1$, $F = 1$ then $k = 1$

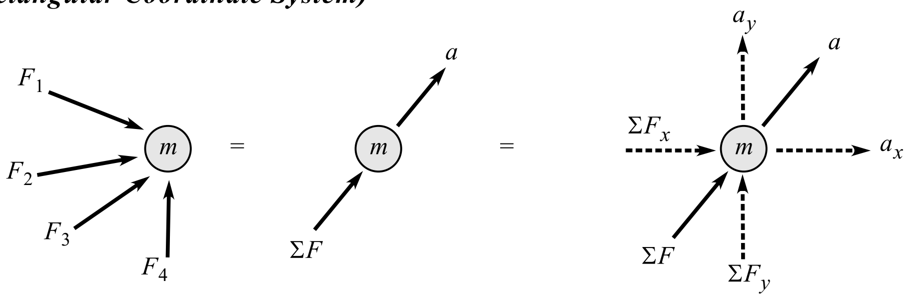
$$\therefore \bar{F} = m\bar{a}$$

In other words, we can also say that *if the resultant force acting on the particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of the resultant force.*

$$\bar{F} = m\bar{a}$$

For Rectilinear Motion

(Rectangular Coordinate System)



$$\sum F = ma$$

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

Fig. 14.2-i

For Curvilinear Motion

(Tangent and Normal Coordinate System)

a_t = Tangential component of acceleration

a_n = Normal component of acceleration

$$\sum F_t = ma_t = m\frac{dv}{dt}$$

$$\sum F_n = ma_n = m\frac{v^2}{\rho}$$

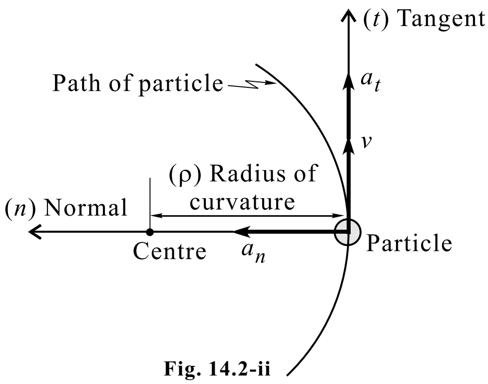


Fig. 14.2-ii

14.3 D'Alemberts' Principle (Dynamic Equilibrium)

- **Dynamic Equilibrium :** The force system consisting of external forces and inertia force can be considered to keep the particle in equilibrium. Since the resultant force externally acting on the particle is not zero, the particle is said to be in *dynamic equilibrium*.

- **D'Alemberts' Principle :** *The algebraic sum of external force (ΣF) and inertia force ($-ma$) is equal to zero.*

$$\sum F + (-ma) = 0$$

- **For Rectilinear Motion**

$$\sum F_x + (-ma_x) = 0 \quad \text{and} \quad \sum F_y + (-ma_y) = 0$$

- **For Curvilinear Motion**

$$\sum F_t + (-ma_t) = 0 \quad \text{and} \quad \sum F_n + (-ma_n) = 0$$

Note : Comparing D'Alemberts' Principle with Newton's Second Law

We understand Newton's Law as the original and D'Alembert had expressed the same concept in a different wording with adjustment of mathematical expression. So in this book we have solved problems considering Newton's Second Law.

How to Analyse a Problem ?

1. Draw a FBD of a particle showing all active and reactive force with all known and unknown values by considering geometrical angles if any. (*Similar to FBD in Statics*)
2. Show direction of acceleration and consider +ve sign convention along the direction of acceleration.
3. Assumption for direction of acceleration.
   a. If the friction is not given then one can assume any direction for acceleration. Positive answer means assumed direction is correct.
   b. If the friction is given then one has to carefully analyse the problem and assume the direction of acceleration. Here we must get +ve answer. In case if answer is negative then one should resolve the whole problem with change in the direction opposite to the assumed direction.
4. If more than one particles are involved in a system then find the kinematic relation between the particles (i.e., relation of displacement, velocity and acceleration).
5. For finding the kinematic relation of connected particles introduce the tension in each cord. Apply the Virtual Work Principle which says - *total virtual work done by internal force (tension) is zero*. Consider work done to be +ve if displacement and tension are in same direction and work done to be –ve if displacement and tension are in one direction.

14.4 Solved Problems Based on Rectilinear Motion

Problem 1

A crate of mass 20 kg is pulled up the inclined 20^o by force F which varies as per the graph shown in Fig. 14.1(a). Find the acceleration and velocity of the crate at $t = 5$ seconds, knowing that its velocity was 4 m/s at $t = 0$. Take $\mu_k = 0.2$.

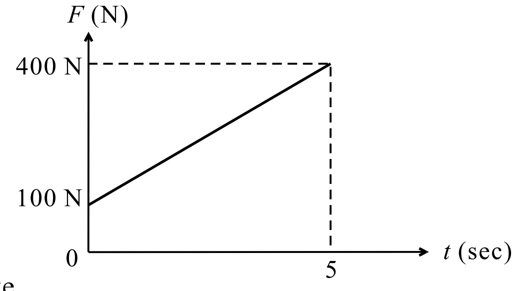


Fig. 14.1(a)

Solution

(i) From the graph F-t shown in Fig. 14.1(a), we have

$$y = mx + c$$

$$m = \frac{400 - 100}{5} = 60 \quad \text{and} \quad c = 100$$

$$\therefore \; F = 60t + 100$$

(ii) Consider the FBD of the crate

Refer to Fig. 14.1(b)

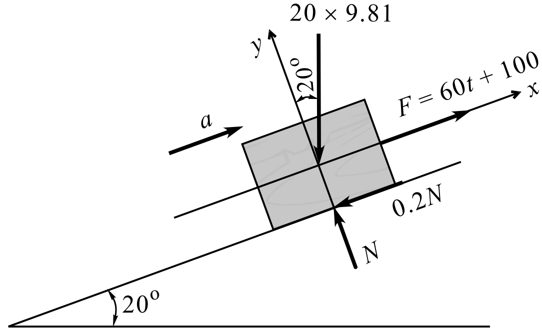


Fig. 14.1(b)

By Newton's second law, we have

$$\sum F_x = ma_x$$

$$F - 0.2N - 20 \times 9.81 \sin 20^o = 20 \times a$$

$$60t + 100 - 0.2 \times 20 \times 9.81 \cos 20^o$$
$$- 20 \times 9.81 \sin 20^o = 20 \times a$$

$$60t - 4 = 20a$$

$$a = 3t - 0.2 \quad \text{(Variable acceleration)}$$

$$a = 3 \times 5 - 0.2$$

$$a = 14.8 \text{ m/s}^2 \quad \textbf{\textit{Ans.}}$$

(iii) $a = \dfrac{dv}{dt}$

$$\therefore \; dv = a\,dt$$

$$\therefore \; dv = (3t - 0.2)\,dt$$

Integrating both sides

$$\int_{v_1 = 4 \text{ m/s}}^{v_2} dv = \int_{t_1 = 0}^{t_2 = 5} (3t - 0.2)\,dt$$

$$\left[v\right]_4^{v_2 = ?} = \left[\frac{3 \times t^2}{2} - 0.2t\right]_0^5$$

$$v_2 - 4 = \left[\frac{3 \times 5^2}{2} - 0.2 \times 5\right]$$

$$v_2 = 32.5 \text{ m/s} \quad \textbf{\textit{Ans.}}$$

Problem 2

A 50 kg block kept on the top of a 15^o sloping surface is pushed down the plane with an initial velocity of 20 m/s. If $\mu_k = 0.4$, determine the distance travelled by the block and the time it will take as it comes to rest.

Solution

(i) Consider the FBD of 50 kg block (Refer to Fig. 14.2)

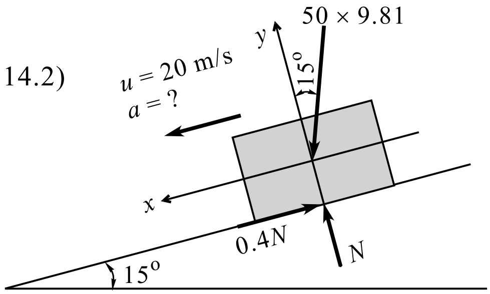


Fig. 14.2

(ii) By Newton's second law, we have

$$\sum F_y = ma_y = 0 \quad (\because\ a_y = 0)$$

$$N - 50 \times 9.81 \cos 15^o = 0$$

$$N = 50 \times 9.81 \cos 15^o$$

$$\sum F_x = ma_x$$

$$50 \times 9.81 \sin 15^o - 0.4 \times 50 \times 9.81 \cos 15^o = 50a$$

$$\therefore\ a = -1.25 \text{ m/s}^2 \text{ (Retardation)}$$

(iii) $u = 20$ m/s ; $v = 0$; $a = -1.25 \text{ m/s}^2$; $s = ?$; $t = ?$

$$v = u + at$$

$$0 = 20 + (-1.25)t$$

$$\therefore\ t = 16 \text{ seconds.}$$

$$s = ut + \frac{1}{2}at^2$$

$$s = 20 \times 16 + \frac{1}{2}(-1.25) \times (16)^2$$

$s = 160$ m ***Ans.***

Problem 3

An aeroplane has a mass of 25000 kg and its engines develop a total thrust of 40 kN along the run way. The force of air resistance to motion of aeroplane is given by $R = 2.25v^2$ where v is m/s and R is in Newton. Determine the length of runway required if the plane takes off and becomes airborne at a speed of 240 km/hr.

Solution

(i) Consider the FBD of the plane (Refer to Fig. 14.3)

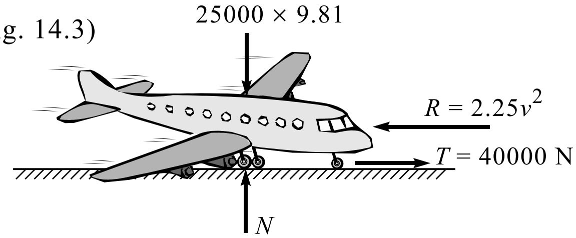


Fig. 14.3 : FBD of Plane

By Newton's second law, we have

$$\sum F_x = ma_x$$

$$40000 - 2.25v^2 = 25000a$$

$$40000 - 2.25v^2 = 25000 \times v\frac{dv}{ds}$$

$$\therefore\ ds = 25000\left(\frac{v\,dv}{40000 - 2.25v^2}\right)$$

Integrating both the sides, we get

$$\int_0^s ds = 25000\int_0^{66.67}\left(\frac{v\,dv}{40000 - 2.25v^2}\right) \qquad \left[v = 240 \times \frac{5}{18} = 66.67 \text{ m/s}\right]$$

$$s = \frac{25000}{-2.25 \times 2}\left[\log_e(40000 - 2.25\,v^2)\right]_0^{66.67}$$

$\therefore\ s = 1598.3$ m (Runway length) ***Ans.***

Problem 4

Two blocks A (mass 10 kg), B (mass 28 kg) are separated by 12 m, as shown in Fig. 14.4(a). If the blocks start moving, find the time 't' when the blocks collide. Assume μ = 0.25 for block A and plane and μ = 0.10 for block B and plane.

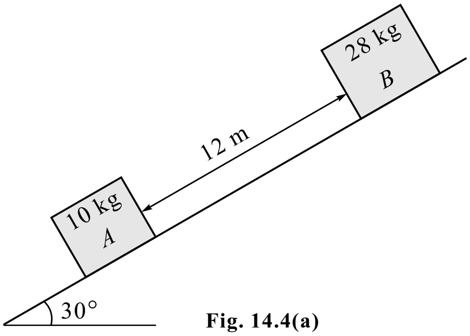


Fig. 14.4(a)

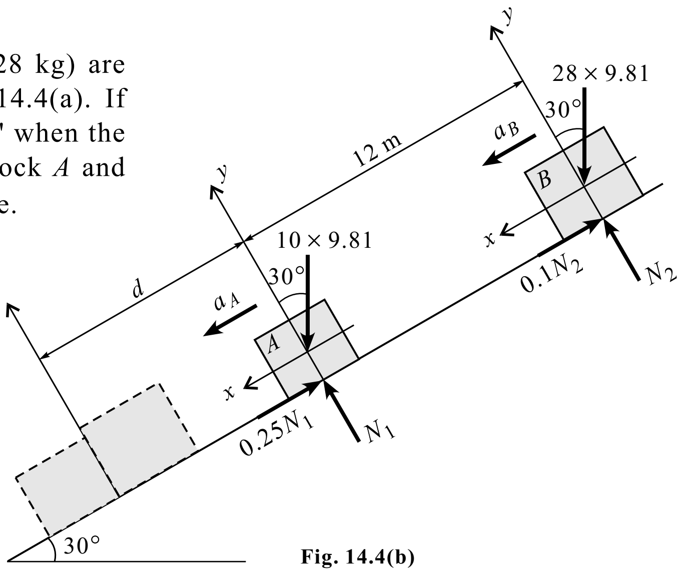


Fig. 14.4(b)

Solution

Refer to Fig. 14.4(b)

(i) Consider the FBD of Block *A*

By Newton's second law, we have

$\sum F_x = ma_x$

$10 \times 9.81 \sin 30^\circ - 0.25 \times 10 \times 9.81 \cos 30^\circ = 10a_A$

$a_A = 2.781 \text{ m/s}^2 \left(30^\circ \swarrow\right)$

(ii) Consider the FBD of Block *B*

By Newton's second law, we have

$\sum F_x = ma_x$

$29 \times 9.81 \sin 30^\circ - 0.1 \times 28 \times 9.81 \cos 30^\circ = 28a_B$

$a_B = 4.055 \text{ m/s}^2 \left(30^\circ \swarrow\right)$

(iii) Motion of Block *A*

$$d = 0 + \frac{1}{2} a_A t^2 \qquad \text{... (I)}$$

(iv) Motion of Block *B*

$$d + 12 = 0 + \frac{1}{2} a_B t^2 \qquad \text{... (II)}$$

(v) From Eqs. (I) and (II), we get

$$\frac{1}{2} \times 2.781 \times t^2 + 12 = \frac{1}{2} \times 4.055 \times t^2$$

$\therefore$ t = 4.34 seconds (Time when the blocks collide) ***Ans.***

Problem 5

An elevator being lowered into mine shaft starts from rest and attains a speed of 10 m/s within a distance of 15 metres. The elevator alone has a mass of 500 kg and it carries a box of mass 600 kg in it. Find the total tension in cables supporting the elevator, during this accelerated motion. Also find the total force between the box and the floor of the elevator.

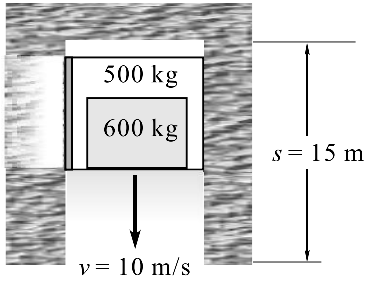


Fig. 14.5(a)

Solution

(i) Considering uniform acceleration of elevator

we have

$$v^2 = u^2 + 2as$$

$$10^2 = 0^2 + 2a \times 15$$

$$a = 3.33 \text{ m/s}^2$$ ***Ans.***

(ii) Considering the FBD of elevator with box

By Newton's second law, we have

$$\sum F_y = ma_y$$

$$(500 + 600)9.81 - T = (500 + 600)a$$

$$T = 1100 \times 9.81 - 1100 \times 3.33$$

$$T = 7128 \text{ N}$$ ***Ans.***

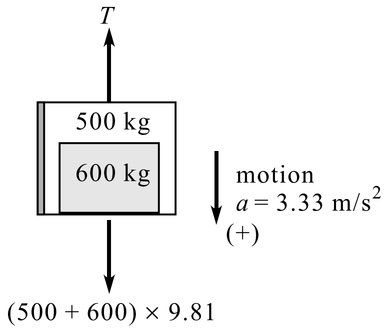


Fig. 14.5(b) : FBD of Elevator with Box

(iii) Consider the FBD of the box

Let N be the normal reaction exerted between the box and floor of the elevator.

$$\sum F_y = ma_y$$

$$600 \times 9.81 - N = 600 \times 3.33$$

$$N = 3888 \text{ N}$$ ***Ans.***

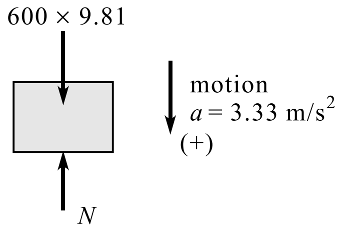


Fig. 14.5(c) : FBD of Box

Problem 6

Two weights $W_1 = 400$ N and $W_2 = 100$ N are connected by a string and move along a horizontal plane under the action of force $P = 200$ N applied horizontally to the weight W_1. The coefficient of friction between the weights and the plane is 0.25. Determine the acceleration of the weights and the tension in the string. Will the acceleration and tension in the string remain the same if the weights are interchanged ?

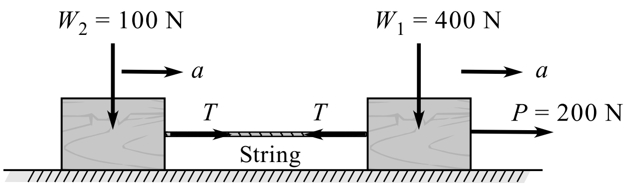


Fig. 14.6(a)

Solution

Note : Since both the block are connected by a single string, therefore, acceleration will remain same.

Case I

(i) Consider the FBD of Block W_1

By Newton's second law, we have

$$\sum F_y = ma_y = 0 \quad (\because a_y = 0)$$

$$N_1 - 400 = 0$$

$$N_1 = 400 \text{ N}$$

$$\sum F_x = ma_x$$

$$200 - T - \mu N_1 = \frac{400}{9.81} \times a$$

$$200 - T - 0.25 \times 400 = 40.78a$$

$$100 - T = 40.78a \qquad \text{..... (I)}$$

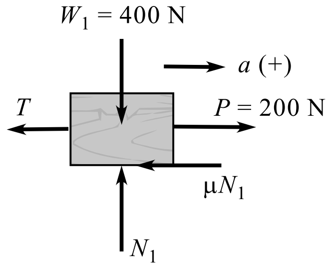


Fig. 14.6(b) : FBD of Block W_1

(ii) Consider the FBD of Block W_2

By Newton's second law, we have

$$\sum F_y = ma_y = 0 \quad (\because a_y = 0)$$

$$N_2 - 100 = 0$$

$$N_2 = 100 \text{ N}$$

$$\sum F_x = ma_x$$

$$T - \mu N_2 = \frac{100}{9.81} a$$

$$T - 0.25 \times 100 = 10.19a$$

$$T - 25 = 10.19a \qquad \text{..... (II)}$$

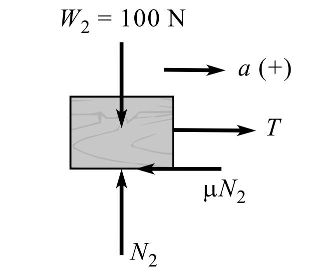


Fig. 14.6(c) : FBD of Block W_2

Solving Eqs. (I) and (II)

$$T = 39.98\text{N} \text{ and } a = 1.47 \text{ m/s}^2 \quad \textbf{\textit{Ans.}}$$

Case II : Weights Interchanged

Refer to Fig. 14.6(d).

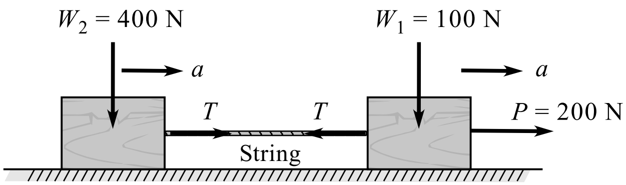


Fig. 14.6(d)

(i) Consider the FBD of Block W_1

By Newton's second law, we have

$$\sum F_y = a_y = 0 \quad (\because a_y = 0)$$

$$N_1 - 100 = 0$$

$$N_1 = 100 \text{ N}$$

$$\sum F_x = ma_x$$

$$200 - T - \mu N = \frac{100}{9.81} a$$

$$175 - T = 10.19a \qquad \text{..... (III)}$$

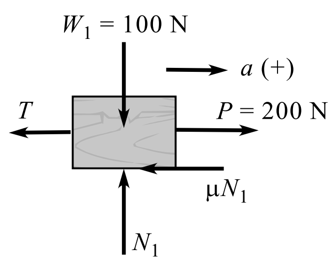


Fig. 14.6(e) : FBD of Block W_1

(ii) Consider the FBD of Block W_2

By Newton's second law, we have

$\sum F_y = ma_y = 0 \quad (\because a_y = 0)$

$N_2 - 400 = 0 \quad \therefore N_2 = 400$ N

$\sum F_x = ma_x$

$T - \mu N_2 = \dfrac{400}{9.81} a$

$T - 100 = 40.78a$ ..... (IV)

Solving Eqs. (III) and (IV)

$T = 160$ N and $a = 1.47$ m/s^2 ***Ans.***

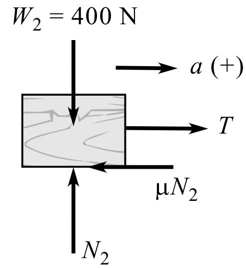


Fig. 14.6(f) : FBD of Block W_2

Referring to both the cases we can conclude that tension in string changes, if the position of weights is interchanged whereas acceleration remains same for both the cases.

Problem 7

The 100 kg crate shown in Fig. 14.7(a) is hoisted up by the incline using the cable and motor M. For a short time, the force in the cable is $F = 800\ t^2$ N where t is in seconds. If the crate has an initial velocity $v_1 = 2$ m/s when $t = 0$ s, determine the velocity when $t = 2$ s. The coefficient of kinetic friction between the crate and the incline is $\mu_k = 0.3$.

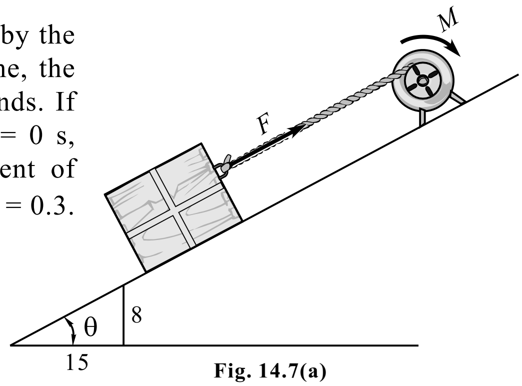


Fig. 14.7(a)

Solution

(i) Consider the FBD of the crate

By Newton's second law, we have

$\sum F_y = ma_y = 0 \quad (\because a_y = 0)$

$N - 100 \times 9.81 \cos\theta = 0$

$N = 865.53$ N $\quad \left(\tan\theta = \dfrac{8}{15} \quad \therefore \theta = 28.07^\circ\right)$

$\sum F_x = ma_x$

$F - 100 \times 9.81 \sin\theta - \mu_k N = 100 \times a$

$800\ t^2 - 100 \times 9.81 \times \sin\theta - 0.3 \times 865.53 = 100\ a$

$a = 8t^2 - 7.213$

$\dfrac{dv}{dt} = 8t^2 - 7.213 = 8t^2 - 7.213$

$$\int_{v_1 = 2\text{ m/s}}^{v_2} dv = \int_{t_1 = 0}^{t_2 = 2\text{ s}} (8t^2 - 7.213)\, dt$$

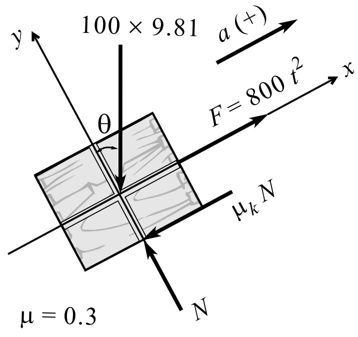


Fig. 14.7(b) : FBD of Crate

$$\Big[v\Big]_2^{v_2} = \left[\frac{8t^3}{3} - 7.213\right]_0^2 \quad \Rightarrow \quad v_2 - 2 = \left[\frac{8(2)^3}{3} - 7.213(2)\right] - \Big[0\Big]$$

$v_2 = 8.91$ m/s ***Ans.***

Problem 8

A body of mass 25 kg resting on a horizontal table is connected by string passing over a smooth pulley at the edge of the table to another body of mass 3.75 kg and hanging vertically, as shown in Fig. 14.8(a). Initially, the friction between 25 kg mass and the table is just sufficient to prevent the motion. If an additional 1.25 kg is added to the 3.75 kg mass, find the acceleration of the masses.

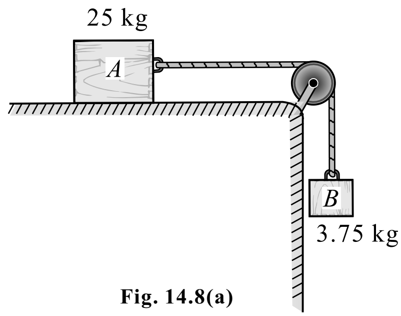


Fig. 14.8(a)

Solution

(i) Static equilibrium analysis

Consider the FBD of block *A*

$\sum F_y = 0$

$N - 25 \times 9.81 = 0$

$N = 245.25\text{N}$

$\sum F_x = 0$

$T - \mu N = 0$

$3.75 \times 9.81 - \mu \times 245.25 = 0$

$\mu = 0.15$

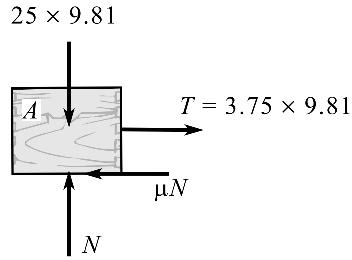


Fig. 14.8(b) : FBD of Block *A*

(ii) Dynamic equilibrium analysis

Assume $\mu_s = \mu_k = 0.15$

By Newton's second law, we have

Consider the *FBD* of block *A*

$\sum F_y = ma_y = 0 \quad (\because\ a_y = 0)$

$N - 25 \times 9.81 = 0$

$N = 245.25\text{N}$

$\sum F_x = ma_x$

$T - \mu N = 25a$

$T - 0.15 \times 245.25 = 25a$

$T = 36.79 + 25a$ ..... (I)

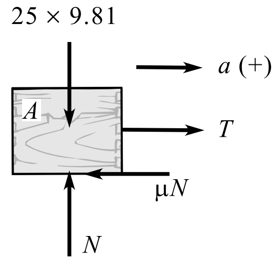


Fig. 14.8(c) : FBD of Block *A*

(iii) Consider the FBD of Block *B*

By Newton's second law, we have

$\sum F_y = ma_y$

$5 \times 9.81 - T = 5 \times a$

$T = 49.05 - 5a$ ..... (II)

Equating Eqs. (I) and (II), we get

$a = 0.409\ \text{m/s}^2$ and $T = 47.005$ N ***Ans.***

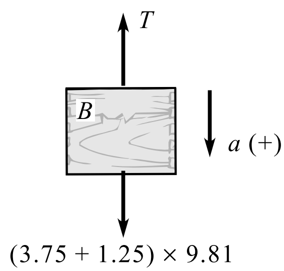


Fig. 14.8(d) : FBD of Block *B*

Problem 9

A horizontal force $P = 600$ N is exerted on block A of mass 120 kg, as shown in Fig. 14.9(a). The coefficient of friction between block A and the horizontal plane is 0.25. Block B has a mass of 30 kg and the coefficient of friction between it and the plane is 0.4. The wire between the two blocks makes 30° with the horizontal. Calculate the tension in the wire.

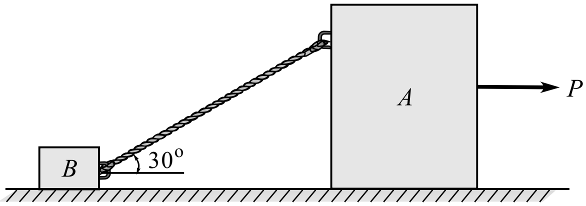


Fig. 14.9(a)

Solution

As both the blocks are connected by a single wire, the acceleration of both the blocks will be the same.

(i) Consider the FBD of Block *B*

By Newton's second law, we have

$$\sum F_y = ma_y = 0 \quad (\because a_y = 0)$$

$$N_B + T \sin 30^\circ - 30 \times 9.81 = 0$$

$$N_B = 294.3 - T \sin 30^\circ$$

$$\sum F_x = ma_x$$

$$T \cos 30^\circ - \mu_B N_B = 30 \times a$$

$$T \cos 30^\circ - 0.4\,(294.3 - T \sin 30^\circ) = 30a$$

$$T - 28.14a = 110.43 \qquad \text{..... (I)}$$

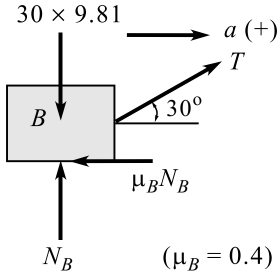


Fig. 14.9(b) : FBD of Block *B*

(ii) Consider the FBD of Block *A*

By Newton's second law, we have

$$\sum F_y = ma_y = 0 \quad (\because a_y = 0)$$

$$N_A - T \sin 30^\circ - 120 \times 9.81 = 0$$

$$N_A = 1177.2 + T \sin 30^\circ$$

$$\sum F_x = ma_x$$

$$600 - \mu_A N_A - T \cos 30^\circ = 120 \times a$$

$$600 - 0.25(1177.2 + T \sin 30^\circ) - T \cos 30^\circ = 120\,a$$

$$T + 121.09a = 308.476 \qquad \text{..... (II)}$$

Solving Eqs. (I) and (II), we get

$T = 147.78$ N and $a = 1.327$ m/s^2 ***Ans.***

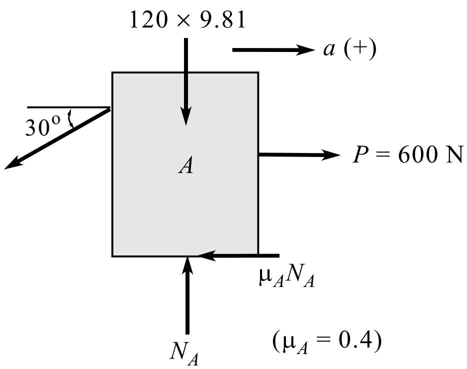


Fig. 14.9(c) : FBD of Block *A*

Problem 10

Masses A and B are 7.5 kg and 27.5 kg respectively as shown in Fig. 14.10(a). The coefficient of friction between A and the plane is 0.25 and between B and the plane is 0.1. What is the force between the two as they slide down the incline?

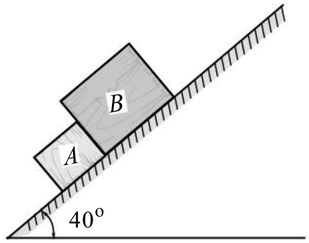


Fig. 14.10(a)

Solution

(i) Consider the FBD of Block *A*

By Newton's second law, we have

$$\sum F_y = ma_y = 0 \qquad (\because\ a_y = 0)$$

$$N_A - 7.5 \times 9.81 \cos 40^\circ = 0$$

$$N_A = 56.36 \text{ N}$$

$$\sum F_x = ma_x$$

$$P + 7.5 \times 9.81 \sin 40^\circ - 0.25 \times 56.36 = 7.5a$$

$$33.2 + P = 7.5a \qquad \text{..... (I)}$$

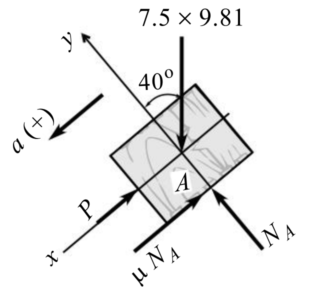


Fig. 14.10(b) : FBD of Block *A*

(ii) Consider the FBD of Block *B*

By Newton's second law, we have

$$\sum F_y = ma_y = 0 \qquad (\because\ a_y = 0)$$

$$N_B - 27.5 \times 9.81 \cos 40^\circ = 0$$

$$N_B = 206.66 \text{ N}$$

$$\sum F_x = ma_x$$

$$27.5 \times 9.81 \sin 40^\circ - P - 0.1 \times 206.66 = 27.5a$$

$$152.74 - P = 27.5a \qquad \text{..... (II)}$$

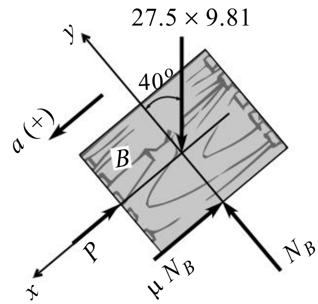


Fig. 14.10(c) : FBD of Block *B*

Solving Eqs. (I) and (II)

$P = 6.625$ and $a = 5.31 \text{ m/s}^2$ ***Ans.***

Problem 11

In the system of pulleys, the pulleys are massless and the strings are inextensible. Mass of A = 2 kg, mass of B = 4 kg and mass = 6 kg as shown in Fig. 14.11(a). If the system is released from rest, find **(i)** tension in each of the three string and **(ii)** acceleration of each of the three masses.

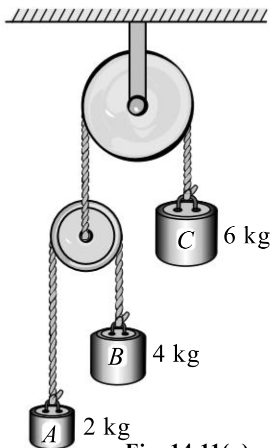


Fig. 14.11(a)

Solution

Assume the direction of motion of all block as above.

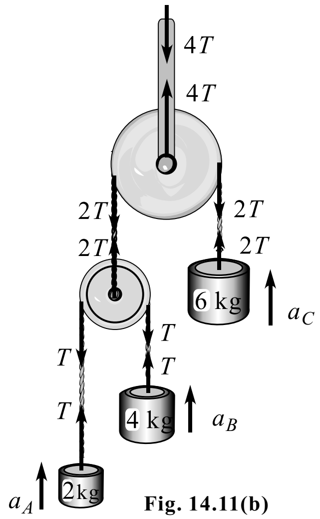


Fig. 14.11(b)

(i) Kinematic relation

$$Tx_A + Tx_B + 2Tx_C = 0$$

$$x_A + x_B + 2x_C = 0$$

Differentiating w.r.t. t

$$v_A + v_B + 2v_C = 0$$

Differentiating w.r.t. t again,

$$a_A + a_B + 2a_C = 0 \qquad \text{..... (I)}$$

(ii) Consider the FBD of Block *A*

$$\sum F_y = ma_y$$

$$T = 2 \times 9.81 = 2a_A$$

$$a_A = 0.5T - 9.81 \qquad \text{..... (II)}$$

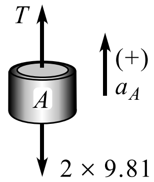


Fig. 14.11(c) : FBD of Block *A*

(iii) Consider the FBD of Block *B*

$$\sum F_y = ma_y$$

$$T - 4 \times 9.81 = 4a_B$$

$$a_B = 0.25T - 9.81 \qquad \text{..... (III)}$$

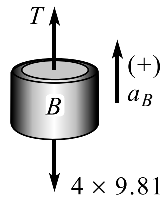


Fig. 14.11(d) : FBD of Block *B*

(iv) Consider the FBD of Block *C*

$$\sum F_y = ma_y$$

$$2T - 6 \times 9.81 = 6a_C$$

$$a_C = 0.33T - 9.81 \qquad \text{..... (IV)}$$

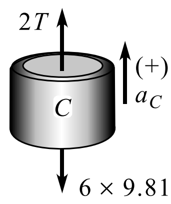


Fig. 14.11(e) : FBD of Block *C*

(v) Putting Eqs. (II), (III) and (IV) in Eq. (I)

$$a_A + a_B + a_C = 0$$

$$(0.5T - 9.81) + (0.25T - 9.81) + (0.33T - 9.81) = 0$$

$$0.5T + 0.25T + 0.33T - 9.81 - 9.81 - 9.81 = 0$$

$$1.08T - 29.43 = 0$$

$$T = 27.25 \text{ N} \quad \textbf{\textit{Ans.}}$$

(vi) From Eq. (I)

$$a_A = 0.5 \times 27.25 - 9.81$$

$$a_A = 3.82 \text{ m/s}^2 \ (\uparrow) \quad \textbf{\textit{Ans.}}$$

(vii) From Eq. (II)

$$a_B = 0.25 \times 27.25 - 9.81$$

$$a_B = -3 \text{ m/s}^2 \text{ (Wrong assumed direction)}$$

$$a_B = 3 \text{ m/s}^2 \ (\downarrow) \quad \textbf{\textit{Ans.}}$$

(viii) From Eq. (III)

$$a_C = 0.33 \times 27.25 - 9.81$$

$$a_C = -0.82 \text{ m/s}^2 \text{ (Wrong assumed direction)}$$

$$a_C = 0.82 \text{ m/s}^2 \ (\downarrow) \quad \textbf{\textit{Ans.}}$$

Problem 12

Determine the tension developed in chords attached to each block and the accelerations of the blocks when the system, shown in Fig. 14.12(a), is released from rest. Neglect the mass of the pulleys and chords.

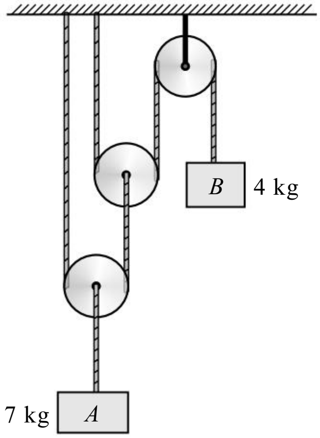


Fig. 14.12(a)

Solution

(i) Kinematic relation

Work done by internal forces $= 0$

$$4Tx_A - Tx_B = 0$$

$$4x_A - x_B = 0$$

Differentiating w.r.t. t

$$4v_A - v_B = 0$$

Differentiating w.r.t. t

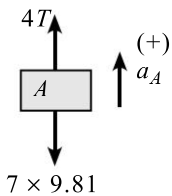
$$4a_A - a_B = 0 \qquad \text{..... (I)}$$

(ii) Consider the FBD of Block A

$$\sum F_y = ma_y$$

$$4T - 7 \times 9.81 = 7a_A$$

$$a_A = 0.5714T - 9.81 \qquad \text{..... (II)}$$



FBD of Block A

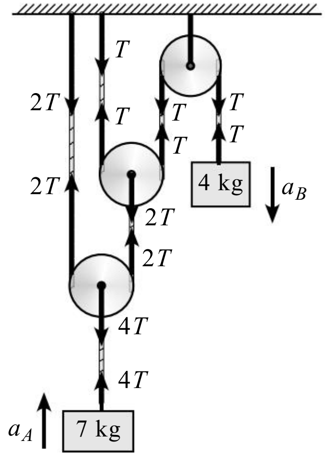


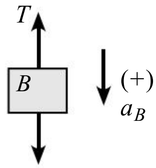
Fig. 14.12(b)

(iii) Consider the FBD of Block B

$$\sum F_y = ma_y$$

$$4 \times 9.81 - T = 4a_B$$

$$a_B = 9.81 - 0.25T \qquad \text{..... (III)}$$



FBD of Block B

(v) Putting Eqs. (II) and (III) in Eq. (I)

$$4(0.5714T - 9.81) - (9.81 - 0.25T) = 0$$

$$2.286T - 39.24 - 9.81 + 0.25T = 0$$

$T = 19.34$ N (Tension in cord attached to block A) ***Ans.***

$4T = 77.36$ N (Tension in cord attached to block B) ***Ans.***

(vi) From Eqs. (II) and (III), we get

$$a_A = 0.5714 \times 19.34 - 9.81$$

$a_A = 1.241 \text{ m/s}^2 \ (\uparrow)$ ***Ans.***

$$a_B = 9.81 - 0.25 \times 19.34$$

$a_B = 4.975 \text{ m/s}^2 \ (\downarrow)$ ***Ans.***

Problem 13

Block A = 100 kg, shown in Fig. 14.13(a), is observed to move upward with an acceleration of 1.8 m/s^2. Determine **(i)** mass of block B and **(ii)** the corresponding tension in the cable.

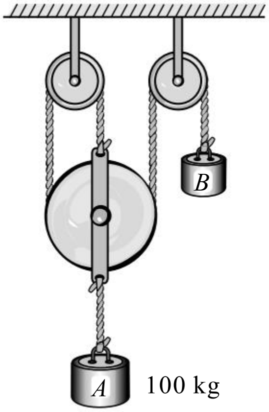


Fig. 14.13(a)

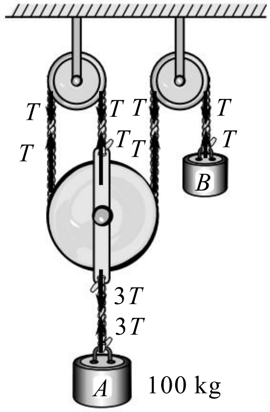


Fig. 14.13(b)

Solution

(i) Kinematic relation

Work done by internal forces = 0

$3Tx_A - Tx_B = 0$

$3x_A = x_B$

Differentiating w.r.t. t

$3v_A = v_B$

Differentiating w.r.t. t

$3a_A = a_B$

$\therefore \; a_B = 3 \times 1.8 \qquad (\because \; a_A = 1.8 \text{ m/s}^2)$

$a_B = 5.4 \text{ m/s}^2$

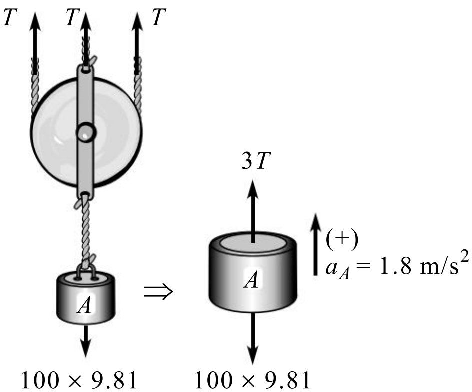


Fig. 14.13(c) : FBD of Block A

(ii) Consider the FBD of Block A

By Newton's second law, we have

$\sum F_y = ma_y$

$3T - 100 \times 9.81 = 100 \times 1.8$

$T = 387$ N ***Ans.***

(iii) Consider the FBD of Block B

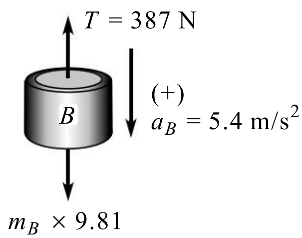


Fig. 14.13(d) : FBD of Block B

By Newton's second law, we have

$\sum F_y = ma_y$

$m_B \times 9.81 - T = m_B \times a_B$

$m_B\,(9.81 - 5.4) = 387$

$m_B = 87.76$ kg ***Ans.***

Problem 14

At a given instant the 50 N block A is moving downward with a speed of 1.8 m/s. Determine its speed 2 s later. Block B has a weight 20 N, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.2$. Neglect the mass of pulley's and chord. Use D'Alemberts principle.

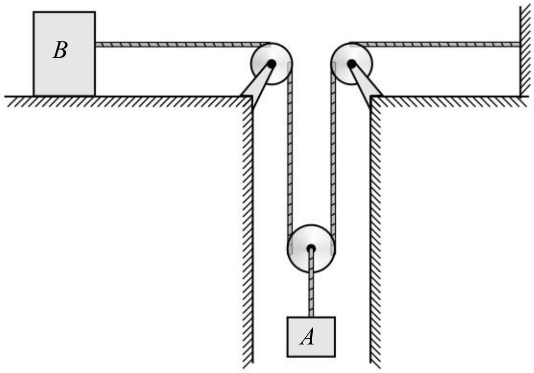


Fig. 14.14(a)

Solution

(i) Kinematic relation

$$Tx_B - 2Tx_A = 0$$

$$x_B - 2x_A = 0$$

Differentiating w.r.t. t

$$v_B - 2v_A = 0$$

Differentiating w.r.t. t again

$$a_B - 2a_A = 0$$

$$a_B = 2a_A \qquad \text{..... (I)}$$

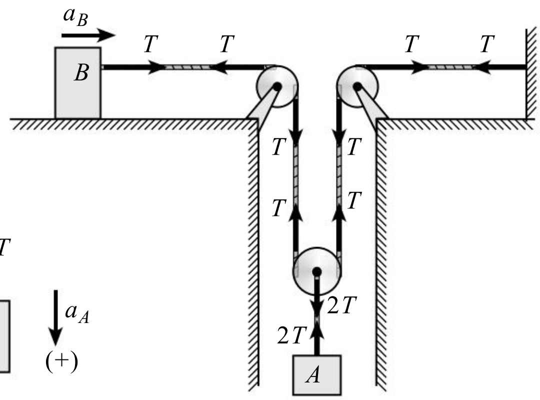


Fig. 14.14(b)

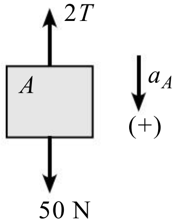
(ii) Consider the FBD of Block A

By D'Alembert's principle, we have

$$\sum F_y + (-ma_y) = 0$$

$$50 - 2T - \frac{50}{9.81}a_A = 0$$

$$T = 25 - 2.548\,a_A \qquad \text{..... (II)}$$



FBD of Block A

(iii) Consider the FBD of Block B

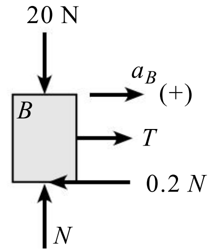
By D'Alembert's principle, we have

$$\sum F_x + (-ma_x) = 0$$

$$T - 0.2N - \frac{20}{9.81}a_B = 0$$

$$T - 0.2 \times 20 - \frac{20}{9.81}a_B = 0$$

$$T = 4 + 2.039\,a_B \qquad \text{..... (III)}$$



FBD of Block B

(iv) Equating Eqs. (II) and (III)

$$25 - 2.548\,a_A = 4 + 2.039\,a_B$$

$$2.548\,a_A + 2.039(2a_A) = 25 - 4$$

$$6.626\,a_A = 21$$

$$a_A = 3.169 \text{ m/s}^2\ (\downarrow)$$

(v) Speed = ? after 2 sec.

$$v = u + at$$

$$v_A = 1.8 + 3.169 \times 2 = 8.138 \text{ m/s}\ (\downarrow) \quad \textbf{\textit{Ans.}}$$

Problem 15

Two blocks, shown in Fig. 14.15(a), start from rest. If the cord is inextensible, friction and inertia of pulley are negligible, calculate acceleration of each block and tension in each cord. Consider coefficient of friction as 0.25.

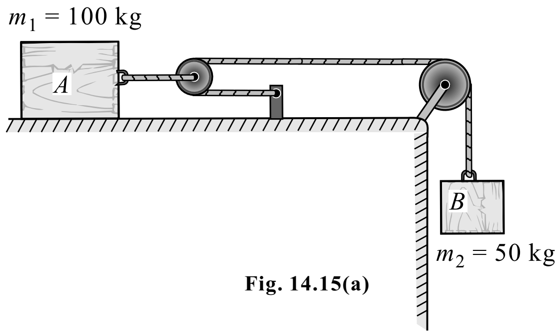


Fig. 14.15(a)

Solution

(i) Kinematic relation

Work done by internal forces $= 0$

$2T \times x_1 - Tx_2 = 0$

$2x_1 = x_2$

Differentiating w.r.t. t

$2v_1 = v_2$

Differentiating w.r.t. t again,

$2a_1 = a_2$

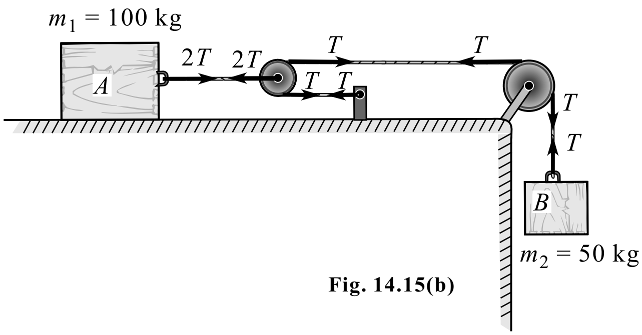


Fig. 14.15(b)

(ii) Consider the FBD of Block m_1

By Newton's second law, we have

$\sum F_y = ma_y = 0 \qquad (\because\ a_y = 0)$

$N_1 - 100 \times 9.81 = 0$

$N_1 = 981$ N

$\sum F_x = ma_x$

$2T - \mu N_1 = 100a_1$

$2T - 0.25 \times 981 = 100a_1$

$T = 122.625 + 50a_1 \qquad \text{..... (I)}$

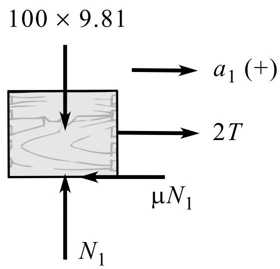


Fig. 14.15(c) : FBD of Block m_1

(iii) Consider the FBD of Block m_2

By Newton's second law, we have

$\sum F_y = ma_y$

$50 \times 9.81 - T = 50a_2$

$T = 490.5 - 100a_2 \qquad \text{..... (II)}$

Solving Eqs. (I) and (II), we get

$T = 245.25$ N; $\quad a_1 = 2.45$ m/s^2 ***Ans.***

$2T = 490.5$ N; $\quad a_2 = 4.9$ m/s^2 ***Ans.***

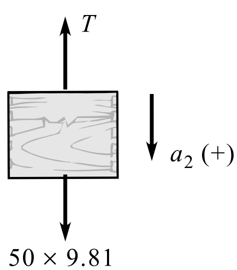


Fig. 14.15(d) : FBD of Block m_2

Problem 16

A system shown in Fig. 14.16(a) is at rest initially. Neglecting friction determine velocity of block A after it has moved 2.7 m when pulled by a force of 90 N.

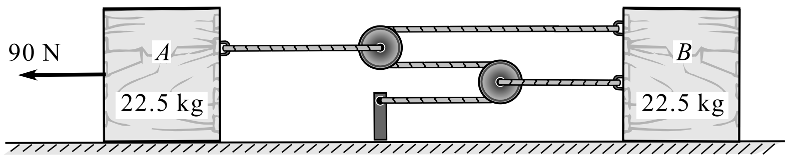


Fig. 14.16(a)

Solution

(i) Kinematic relation

Work done by internal forces $= 0$

$\therefore\ 3Tx_B - 2Tx_A = 0$

$3x_B = 2x_A$

Differentiating w.r.t. t

$3v_B = 2v_A$

Differentiating w.r.t. t

$3a_B = 2a_A$

$$a_B = \frac{2}{3}a_A$$

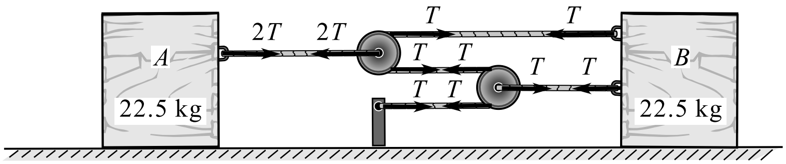


Fig. 14.16(b)

(ii) Consider the FBD of Block A

By Newton's second law, we have

$\sum F_y = ma_y = 0 \quad (\because\ a_y = 0)$

$N_A - 22.5 \times 9.81 = 0$

$N_A = 22.5 \times 9.81 = 220.725$ N

$\sum F_x = ma_x$

$90 - 2T = 22.5 \times a_A$ ..... (I)

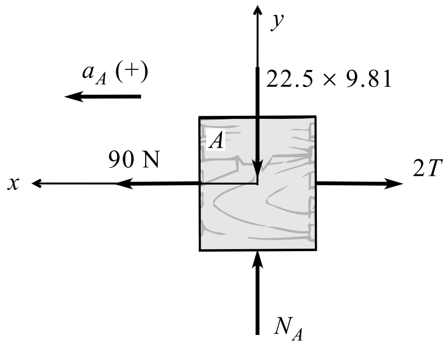


Fig. 14.16(c) : FBD of Block A

(iii) Consider the FBD of Block B

By Newton's second law, we have

$\sum F_y = ma_y \quad (\because\ a_y = 0)$

$N_B - 22.5 \times 9.81 = 0 \quad \therefore\ N_B = 220.725$ N

$\sum F_x = ma_x$

$T + 2T = 22.5 \times a_B$

$3T = 22.5a_B$

$$T = \frac{22.5a_B}{3} \qquad \text{..... (II)}$$

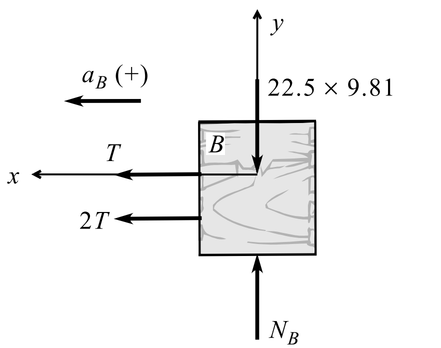


Fig. 14.16(d) : FBD of Block B

Putting Eq. (II) in Eq. (I)

$$90 - 2\left(\frac{22.5a_B}{3}\right) = 22.5a_A$$

From kinematic relation $a_B = \frac{2}{3}a_A$

$$90 - 2 \times \frac{22.5}{3} \times \frac{2}{3} a_A = 22.5 a_A$$

$$90 = 32.5 a_A$$

$$a_A = 2.77 \text{ m/s}^2$$

$$a_B = \frac{2}{3} a_A = \frac{2}{3} \times 2.77 \quad \therefore \; a_B = 1.85 \text{ m/s}^2$$

(iv) $x_A = 2.7$ m; $a_A = 2.77$ m/s^2, $u = 0$

$$v^2 = u^2 + 2as = 0 + 2 \times 2.77 \times 2.7$$

$v = 3.87$ m/s ***Ans.***

Problem 17

Masses $A = 5$ kg, $B = 10$ kg and $C = 20$ kg are connected as shown in Fig. 14.17(a) by inextensible cord passing over massless and frictionless pulleys. The coefficients of friction for mass A and B and ground is 0.2. If the systems is released from rest, find the acceleration. a_A, a_B and a_C and tension T in the cord. Present your answer in tabular form.

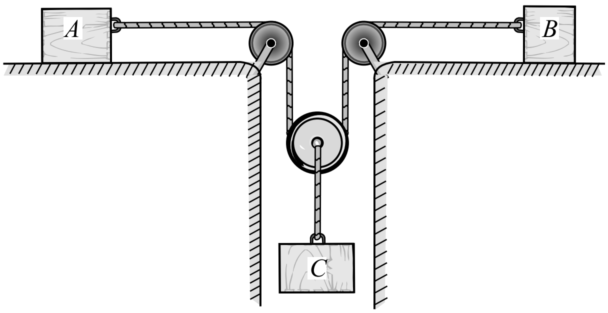


Fig. 14.17(a)

Solution

(i) Kinematic relation

$$Tx_A + Tx_B - 2Tx_C = 0$$

$$x_A + x_B - 2x_C = 0$$

Differentiating w.r.t. t

$$v_A + v_B - 2v_C = 0$$

Differentiating w.r.t. t again

$$a_A + a_B - 2a_C = 0$$

$$a_C = \frac{a_A + a_B}{2}$$

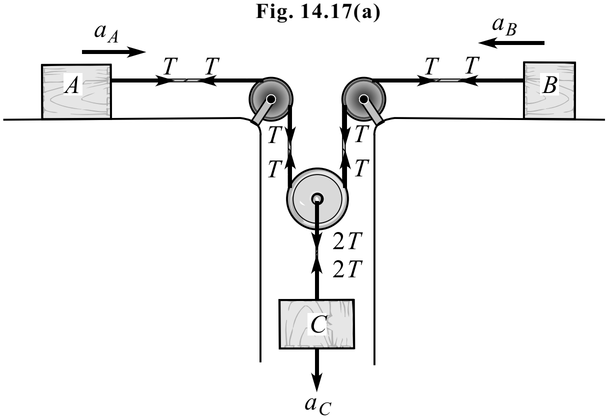


Fig. 14.17(b)

(ii) Consider the FBD of Block *A*

By Newton's second law, we have

$$\sum F_y = ma_y = 0 \qquad (\because a_y = 0)$$

$$N_A - 5 \times 9.81 = 0 \qquad \therefore \; N_A = 49.05 \text{ N}$$

$$\sum F_x = ma_x$$

$$T - \mu N_A = 5 \times a_A$$

$$T - 0.2 \times 49.05 = 5a_A$$

$$T - 9.81 = 5a_A$$

$$a_A = \frac{T - 9.81}{5} \qquad \ldots\ldots \text{(I)}$$

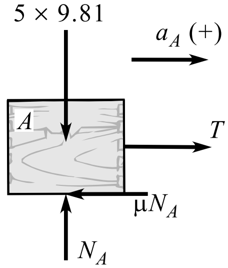


Fig. 14.17(c) : FBD of Block *A*

(iii) Consider the FBD of Block *B*

By Newton's second law, we have

$$\sum F_y = ma_y = 0 \qquad (\because a_y = 0)$$

$$N_B - 10 \times 9.81 = 0$$

$$N_B = 98.1 \text{ N}$$

$$\sum F_x = ma_x$$

$$T - \mu N_B = 10a_B$$

$$T - 0.2 \times 98.1 = 10a_B$$

$$T - 19.62 = 10a_B$$

$$a_B = \frac{T - 19.62}{10} \qquad \text{..... (II)}$$

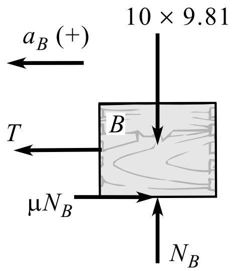


Fig. 14.17(d) : FBD of Block *B*

(iv) Consider the FBD of Block *C*

By Newton's second law, we have

$$\sum F_y = ma_y$$

$$20 \times 9.81 - 2T = 20a_C$$

$$20 \times 9.81 - 2T = 20\left[\frac{a_A + a_B}{2}\right]$$

$$196.2 - 2T = 10(a_A + a_B) \qquad \text{..... (III)}$$

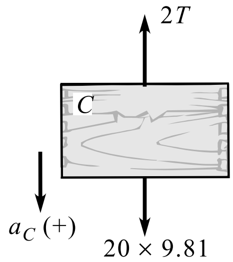


Fig. 14.17(e) : FBD of Block *C*

Putting Eqs. (I) and (II) in (III)

$$196.2 - 2T = 10\left[\left(\frac{T - 9.81}{5}\right) + \left(\frac{T - 19.62}{10}\right)\right]$$

$$196.2 - 2T = 2(T - 9.81) + (T - 19.62)$$

$$5T = 235.44$$

$$T = 47.09 \text{ N} \quad \textbf{\textit{Ans.}}$$

From Eq. (I)

$$a_A = \frac{47.09 - 9.81}{5} = 7.456 \text{ m/s}^2$$

From Eq. (II)

$$a_B = \frac{47.09 - 19.62}{10} = 2.75 \text{ m/s}^2$$

From kinematic relation

$$a_C = \frac{a_A + a_B}{2} = \frac{7.456 + 2.75}{2} = 5.103 \text{ m/s}^2$$

a_A	a_B	a_C	T
7.456 m/s^2	2.75 m/s^2	5.103 m/s^2	47.09 N

Ans.

Problem 18

Masses A (5 kg), B (10 kg), C (20 kg) are connected, as shown in the Fig. 14.18(a) by inextensible cord passing over massless and frictionless pulleys. The coefficient of friction for masses A and B with ground is 0.2. If the system is released from rest, find the acceleration of the blocks and tension in the cords.

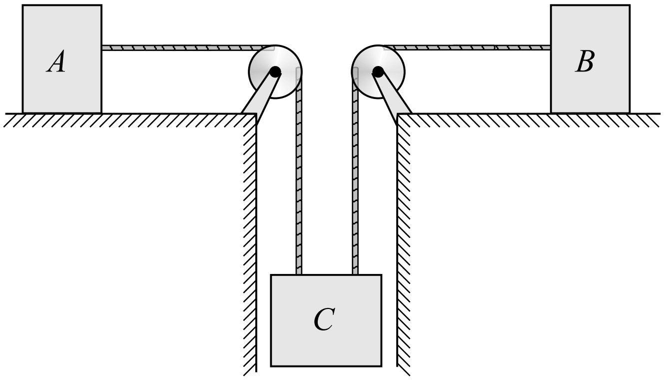


Fig. 14.18(a)

Solution

(i) Kinematic relation

All the three blocks are connected directly to each other.

$\therefore$ Acceleration of all the three blocks will be same.

$$a_A = a_B = a_C = a \quad \text{..... (I)}$$

(ii) Consider the FBD of Block A

By Newton's second law, we have

$$\sum F_x = ma_x$$

$$T_1 - 0.2N_A = 5a_A$$

$$T_1 - 0.2 \times 5 \times 9.81 = 5a_A$$

$$T_1 = 5a_A + 9.81 \quad \text{..... (II)}$$

(iii) Consider the FBD of Block B

By Newton's second law, we have

$$\sum F_x = ma_x$$

$$T_2 - 0.2N_B = 10a_B$$

$$T_2 = 0.2 \times 10 \times 9.81 + 10a_B$$

$$T_2 = 19.62 + 10a_B \quad \text{..... (III)}$$

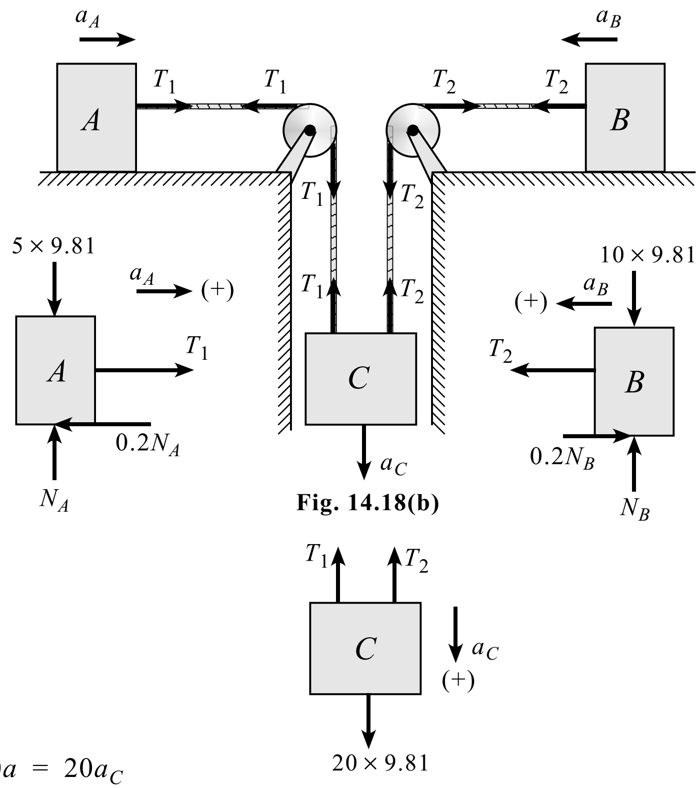


Fig. 14.18(b)

(iv) Consider the FBD of Block C

By Newton's second law, we have

$$20 \times 9.81 - T_1 - T_2 = 20a_C$$

From Eq. (II) and (III)

$$20 \times 9.81 - 5a - 9.81 - 19.62 - 10a = 20a_C$$

$$35a = 166.77$$

$$a = 4.765 \text{ m/s}^2$$

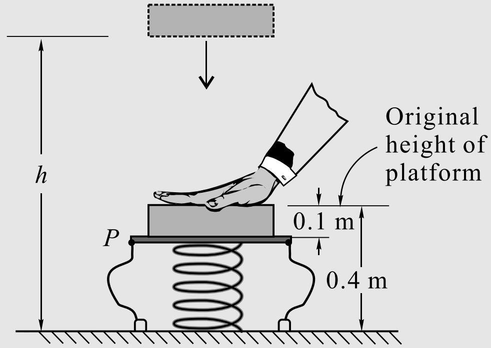
(v) Substituting the value of a in Eqs. (II) and (III), we get

$T_1 = 5 \times 4.765 + 9.81$ $\qquad$ $T_2 = 19.62 + 10 \times 4.765$

$T_1 = 33.63$ N ***Ans.*** $\qquad$ $T_2 = 67.27$ N ***Ans.***

15

Kinetics of Particles - II Work n Energy Principle



15.1 Introduction

In the previous chapter, problems were solved using *Newton's Second Law*. In this chapter, we are going to approach by *Work Energy Principle*. This method is advantageous over Newton's second law when the problem involves *force*, *velocity* and *displacement*, rather than *acceleration*. Also when spring force is involved one should prefer work energy principle to solve the problem.

15.2 Work Done by a Force

If a particle is subjected to a force F and particle is displaced by s from position ① to position ② then work done U is the product of force and displacement.

Work done = Force × Displacement

$U = F \times s$

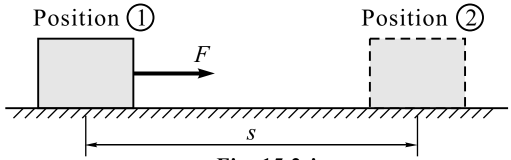


Fig. 15.2-i

If a particle is subjected to a force F at an angle θ with horizontal and the particle is displaced by s from position ① to position ② then work done U is the product of force component in the direction of displacement $F \cos \theta$ and displacement s.

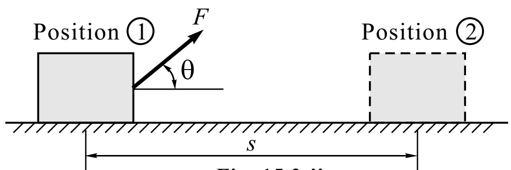


Fig. 15.2-ii

Work done = Component of force in direction of displacement × Displacement

$U = F \cos \theta \times s$

Important Points About Work Done

1. *Work done by a force is positive if the direction of force and the direction of displacement both are in same direction.* **Example :** Work done by force of gravity is positive when a body moves from an upper position to lower position.
2. *Work done by a force is negative if the direction of force and the direction of displacement both are in opposite direction.* **Example :** Work done by force of gravity is negative when a body moves from a lower position to a higher position.
3. *Work done by a force is zero if either the displacement is zero or the force acts normal to the displacement.* **Example :** Gravity force does not work when body moves horizontally.
4. *Work is a scalar quantity.*
5. *Unit of work is N.m or Joule (J).*

15.3 Work Done by Weight Force

In Fig. 15.3-i, a particle of mass m is displaced from position ① to position ② then work done is given by

Work done = Component of weight in the direction of displacement × Displacement

$$U = mg \sin\theta \times s$$

Work done = Weight force × Displacement in the direction of weight force

$$U = mg \times s \sin\theta$$

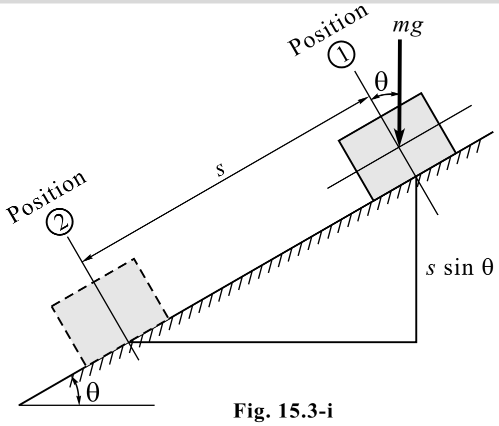


Fig. 15.3-i

15.4 Work Done by Frictional Force

In Fig. 15.4-i, a particle of mass m is moves from position ① to position ②, then work done on frictional force is given by

Work done = –Frictional force × Displacement

$$U = -\mu N \times s$$

Note : (i) Work done by friction force is –ve because direction of frictional force and displacement are opposite.

(ii) Work done by normal reaction (N) and component of weight force perpendicular to inclined plane ($mg \cos\theta$) is zero.

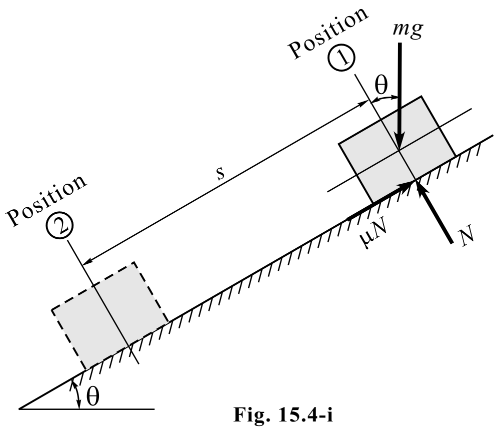


Fig. 15.4-i

15.5 Work Done by Spring Force

Consider a spring of stiffness k as shown in Fig. 15.5-i, a undeformed (free/original) length.

Let x_1 be the deformation of spring at position ①.

Let x_2 be the deformation of spring at position ②.

$\therefore$ Spring force $F = -k \times x$

where k is the spring stiffness (N/m)

x is the deformation of spring (m)

–ve sign indicates direction of spring force acts towards original position.

Work done = Spring force × Deformation

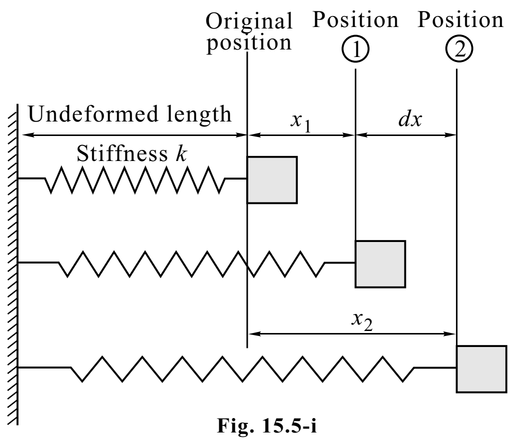


Fig. 15.5-i

$$U = \int_{x_1}^{x_2} -kx \, dx$$

$$\therefore \ U = -\frac{1}{2} k\left(x_2^2 - x_1^2\right)$$

$$\therefore \ U = \frac{1}{2} k\left(x_1^2 - x_2^2\right)$$

15.6 Work - Energy Principle

Work done by the forces acting on a particle during some displacement is equal to the change in kinetic energy during that displacement.

Proof

Consider the particle having mass m is acted upon by a force F and moving along a path which can be rectilinear or curvilinear as shown in Fig. 15.6-i.

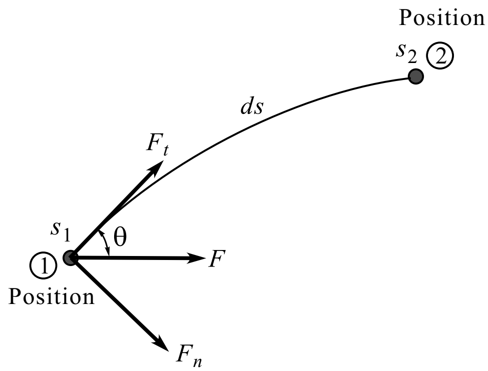


Fig. 15.6-i

Let v_1 and v_2 be the velocities of the particle at position ① and position ② and the corresponding displacement s_1 and s_2 respectively.

By Newton's second law, we have

$$\sum F_t = ma_t$$

$$F \cos\theta = ma_t = m\frac{dv}{dt}$$

$$F \cos\theta = m\frac{dv}{ds} \times \frac{ds}{dt}$$

$$F \cos\theta = mv \times \frac{dv}{ds}$$

$$F \cos\theta \, ds = mv \, dv$$

Integrating both sides, we have

$$\int_{s_1}^{s_2} F\cos\theta \, ds = \int_{v_1}^{v_2} mv \, dv$$

$$\therefore \ U_{1\text{-}2} = \frac{1}{2} mv_1^2 - \frac{1}{2} mv_2^2$$

Work done = Change in Kinetic Energy

Kinetic Energy of a Particle

It is the energy possessed by a particle by virtue of its motion.

If a particle of mass m is moving with the velocity v, its kinetic energy is given by

$$KE = \frac{1}{2} mv^2$$ Unit of KE is N.m or Joule (J)

Potential Energy of a Particle

It is the energy possessed by a particle by virtue of its position.

If a particle of mass m is moving from position ① to position ②, then

Work done = Weight force × Displacement

$U = mgh = \text{PE}$

Unit of PE is N.m or Joule (J)

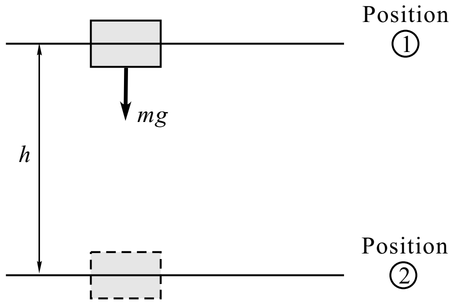


Fig. 15.6-ii

Note : Work done by weight force will be negative if moved from lower position to upper position.

Conservative Forces

*If the work of a force is moving a particle between two positions is independent of the path followed by the particle and can be expressed as a change in its potential energy, then such forces is called as **conservative forces**.*

Example : Weight force of particle (gravity force), spring force and elastic force.

Non-Conservative Forces

*The forces in which the work is dependent upon the path followed by the particles is known as **non-conservative forces**.*

Example : Frictional force, viscous force.

Principle of Conservation of Energy

*When a particle is moving from position ① to position ② under the action of only conservative forces (i.e., when frictional force does not exist) then by **energy conservation principle** we say that the total energy remains constant.*

Total energy = Kinetic energy + Potential energy + Strain energy of spring

$$\text{Total energy} = \frac{1}{2} mv^2 \pm mgh + \frac{1}{2} kx^2$$

Power

It is defined as the rate of doing work.

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

$$\text{Power} = \frac{\text{Force} \times \text{Displacement}}{\text{Time}}$$

$$\text{Power} = \text{Force} \times \text{Velocity}$$

Unit of power is Joule/second (J/s) or watt (W)

$1 \text{ W} = 1 \text{ J/s} = 1 \text{ Nm/s}$

One metric horse power = 735.5 watt

15.7 Solved Problems

Problem 1

A force of 500 N is acting on a block of mass 50 kg resting on a horizontal surface as shown in Fig. 15.1(a). Determine its velocity after the block has travelled a distance of 10 m. Coefficient of kinetic friction $\mu_k = 0.5$.

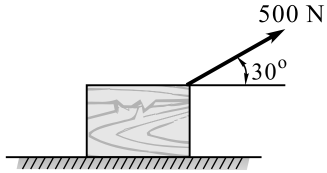


Fig. 15.1(a)

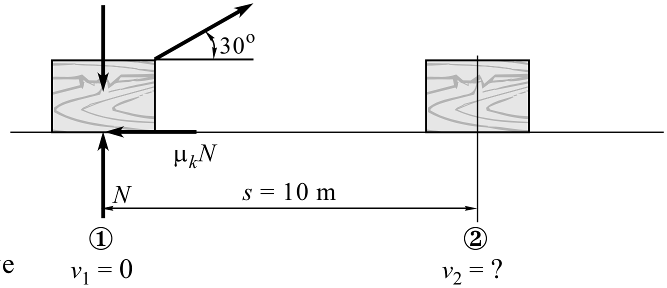


Fig. 15.1(b)

Solution

(i) $\Sigma F_y = ma_y = 0 \ (\because a_y = 0)$

$N_1 - 50 \times 9.81 + 500 \sin 30^\circ = 0$

$N = 240.5$ N

(ii) By principle of work energy, we have

Work done = Change in KE

$$500 \cos 30^\circ \times 10 - \mu_k N \times 10 = \frac{1}{2} \times 50 \times v_2^2 - 0$$

$$500 \cos 30^\circ \times 10 - 0.5 \times 240.5 \times 10 = 25 \times v_2^2$$

$v_2 = 11.185$ m/s ***Ans.***

Problem 2

Block A has a mass of 2 kg and has a velocity of 5 m/s up the plane shown in Fig. 15.E2. Use the principle of work energy; locate the rest position of the block.

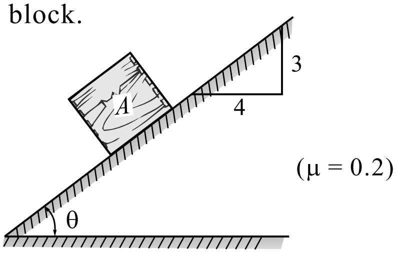


Fig. 15.2(a)

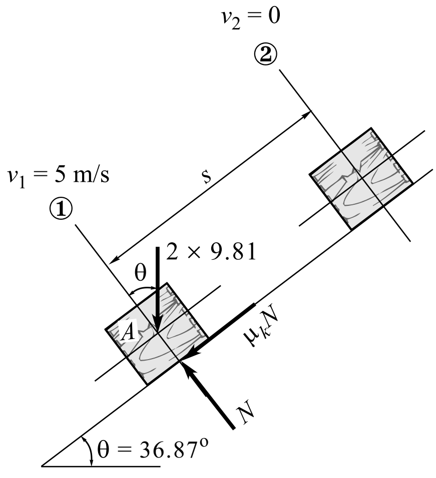


Fig. 15.2(b)

Solution

(i) By principle of work energy, we have

Work done = Change in KE

$$-2 \times 9.81 \sin \theta \times s - \mu N \times s = 0 - \frac{1}{2} \times 2 \times 5^2$$

$$-2 \times 9.81 \sin 36.87^\circ \times s - 0.2 \times 2 \times 9.81 \cos 36.87^\circ \times s = -\frac{1}{2} \times 2 \times 5^2$$

$s = 1.68$ m ***Ans.***

Problem 3

A mass of 20 kg is projected up an inclined of 26° with velocity of 4 m/s, as shown in Fig. 15.E3. If $\mu = 0.2$, **(i)** find maximum distance that the package will move along the plane and **(ii)** What will be the velocity of the package when it comes back to initial position ?

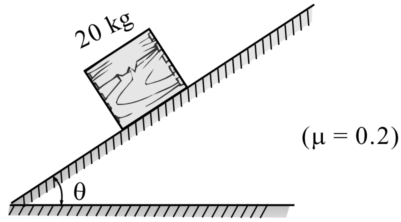


Fig. 15.3(a)

Solution

Case (i) Upward motion, from ① to ②

Refer to Fig. 15.3(b).

At position ①

$v_1 = 4$ m/s

At position ②

$v_2 = 0$

By work energy principle, we have

Work done = Change in KE

$$-20 \times 9.81 \sin 26^\circ \times d - 0.2 \times 20 \times 9.81 \cos 26^\circ \times d = 0 - \frac{1}{2} \times 20 \times 4^2$$

$$d(86 + 35.37) = 160$$

$d = 1.32$ m ***Ans.***

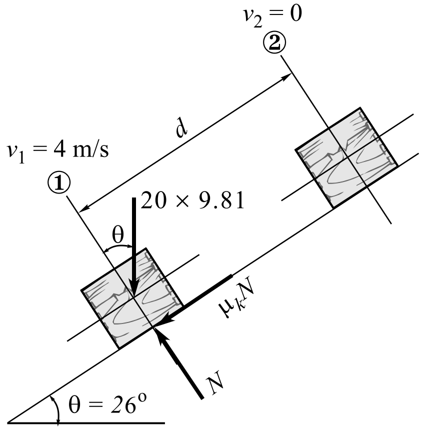


Fig. 15.3(b)

Case (ii) Downward motion, from ② to ①

Refer to Fig. 15.3(c).

At position ①

$v_1 = 0$

At position ②

$v_2 = ?$

Displacement $d = 1.32$ m

By work energy principle, we have

Work done = Change in KE

$$20 \times 9.81 \sin 26^\circ \times 1.32 - 0.2 \times 20 \times 9.81 \cos 26^\circ \times 1.32 = \frac{1}{2} \times 20 v_2^2 - 0$$

$v_2 = 2.59$ m/s ***Ans.***

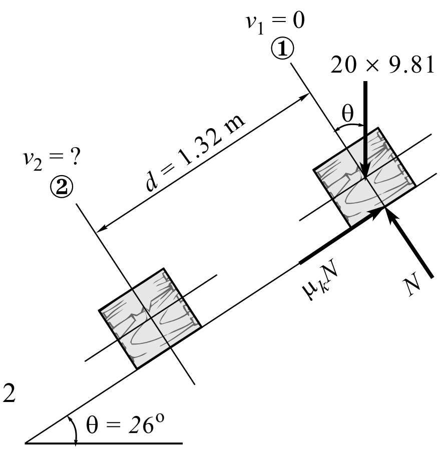


Fig. 15.3(c)

Problem 4

The 0.8 kg collar slides with negligible friction on the fixed rod in the vertical plane as shown in Fig. 15.E4. If the collar starts from the rest at A under the action of constant 8 N horizontal force. Calculate the velocity as it hits the stop at B.

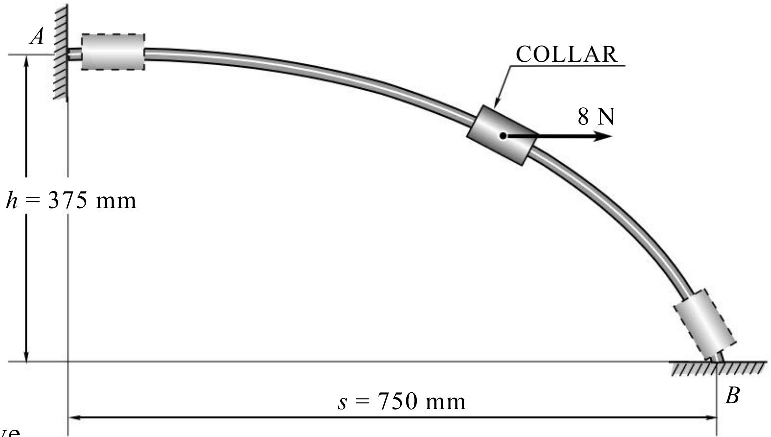


Fig. 15.4

Solution

At position A : $v_A = 0$

At position B : $v_B = ?$

By work energy principle, we have

Work done = Change in KE

$$mgh + 8 \times s = \frac{1}{2} \times 0.8 \times v_B^2 - 0$$

$$0.8 \times 9.81 \times 0.375 + 8 \times 0.75 = 0.4\, v_B^2$$

$v_B = 4.728$ m/s ***Ans.***

Problem 5

(i) Determine the distance in which a car moving at 90 kmph can come to rest after the power is switched off if μ between tyres and road is 0.8.

(ii) Determine also the maximum allowable speed of a car, if it is to stop in the same distance as above on ice road where the coefficient of friction between tyres and road is 0.08.

Solution

(i) μ = 0.8

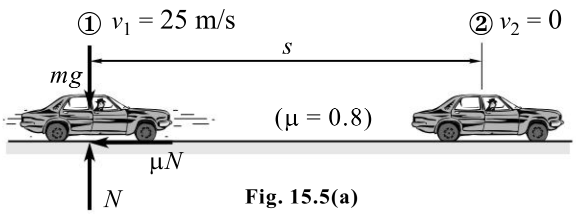


Fig. 15.5(a)

At position 1

$$v_1 = 90 \times \frac{1000}{3600} = 25 \text{ m/s}$$

At position 2 $v_2 = 0$

By work energy principle, we have

Work done = Change in KE

$$-\mu N \times s = 0 - \frac{1}{2} \times m \times (25)^2$$

$$-0.8 \times mg \times s = -\frac{1}{2}\, m \times 625$$

$s = 39.82$ m ***Ans.***

(ii) For Ice Road

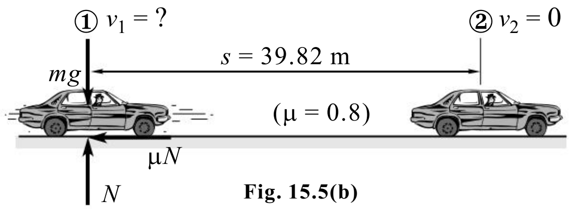


Fig. 15.5(b)

$\mu = 0.08$, $v_1 = ?$ $v_2 = 0$, $s = 39.82$ m

By work energy principle, we have

Work done = Change in KE

$$-\mu N \times s = 0 - \frac{1}{2} \times m \times v_1^2$$

$$-0.08 \times mg \times 39.82 = -\frac{1}{2}\, m \times v_1^2$$

$v_1 = 7.91$ m/s ***Ans.***

Problem 6

A block of mass 8 kg slides freely on a smooth vertical rod as shown in Fig. 15.6(a). The mass is released from rest at a distance of 500 mm from the top of the spring. The spring constant is 60 N/mm. Determine the velocity of block when the spring has compressed through 20 mm. The free length of the spring is 400 mm.

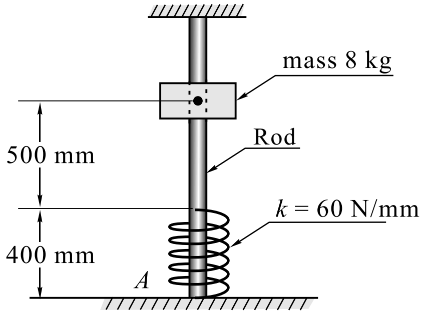


Fig. 15.6(a)

Solution

Given : $k = 60$ N/mm $= 60000$ N/m

At position ① : $v_1 = 0$, $x_1 = 0$

At position ② : $v_2 = ?$, $x_2 = 0.02$ m

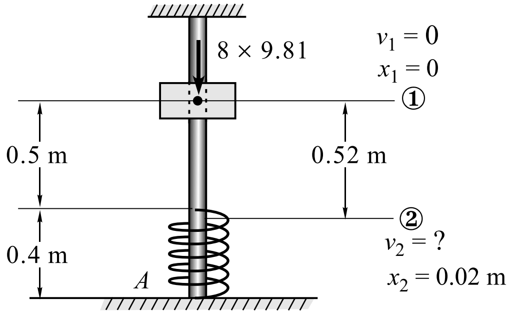


Fig. 15.6(b)

Total displacement = 500 + 20 = 520 mm

$s = 0.52$ m

By work energy principle, we have

Work done = Change in kinetic energy

$$8 \times 9.81 \times 0.52 + \frac{1}{2} \times 60000 \times (0 - 0.02^2) = \frac{1}{2} \times 8 \times v_2^2 - 0$$

$$28.8096 = 4\, v_2^2$$

$v_2 = 2.684$ m/s ***Ans.***

Problem 7

Collar of mass 15 kg is at rest at '*A*'. It can freely slide on a vertical smooth rod *AB*. The collar is pulled up with a constant force *F* = 600 *N* applied as shown in Fig. 15.E7. Unstretched length of spring is 1 m. Calculate velocity of the collar when it reaches position *B*. *Given* : Spring constant *k* = 3 N/mm. *AC* is horizontal.

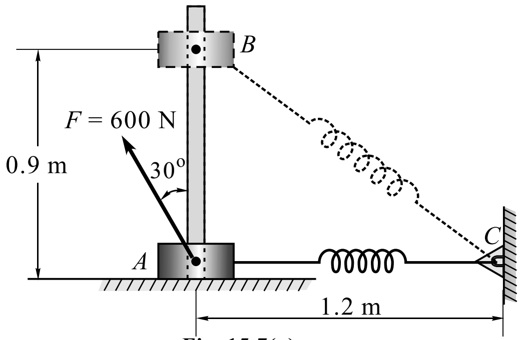


Fig. 15.7(a)

Solution

Given : $k = 3$ N/mm $= 3000$ N/m

At position ①

$v_1 = 0$

$x_1 = 1.2 - 1 = 0.2$ m

At position ②

$v_2 = 0$

$x_2 = (BC) - 1 = 1.5 - 1$

$\quad = 0.5$ m

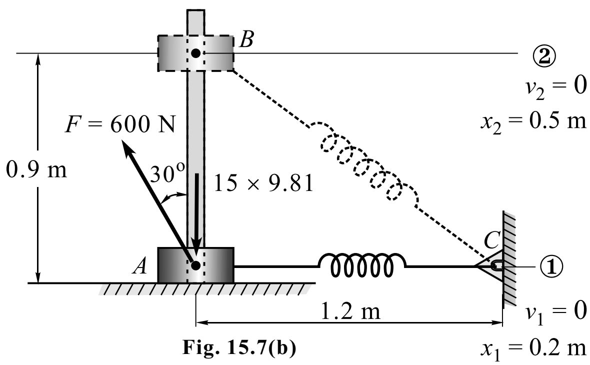


Fig. 15.7(b)

By work energy principle

Work done = Change in kinetic energy

$$600 \cos 30^\circ \times 0.9 - 15 \times 9.81 \times 0.9 + \frac{1}{2} \times 3000(0.2^2 - 0.5^2)$$
$$= \frac{1}{2} \times 15 \times v_2^2 - 0$$

$$20.22 = 7.5\, v_2^2$$

$v_2 = 1.64$ m/s (↑) ***Ans.***

Problem 8

A collar A of mass 10 kg moves in vertical guide as shown in Fig. 15.8(a). Neglecting the friction between the guide and the collar, find its velocity when it passes through position ② after starting from rest in position ①. The spring constant is 200 N/m and the free length of the spring is 200 mm.

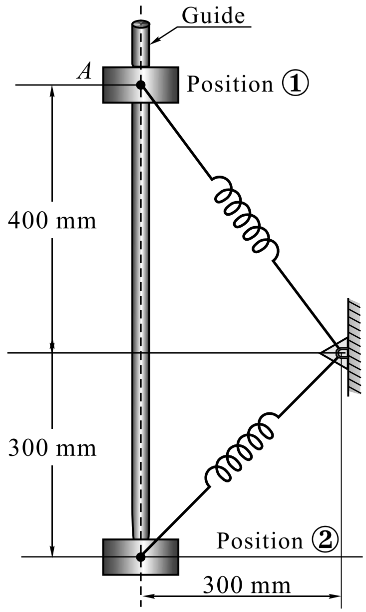


Fig. 15.8(a)

Solution

Given : $k = 200$ N/m

Free length of spring = 200 mm = 0.2 m

At position ①

$x_1 = 500 - 200 = 300$ mm

$x_1 = 0.3$ m

$v_1 = 0$

At position ②

$x_2 = 424.26 - 200 = 224.26$ mm

$x_2 = 0.224$ m

$v_2 = ?$

By work energy principle, we have

Work done = Change in kinetic energy

$$10 \times 9.81 \times 0.7 + \frac{1}{2} \times 200 \times (0.3^2 - 0.224^2) = \frac{1}{2} \times 10 \times v_B^2 - 0$$

$$72.65 = 5\, v_B^2$$

$v_2 = 3.81$ m/s ***Ans.***

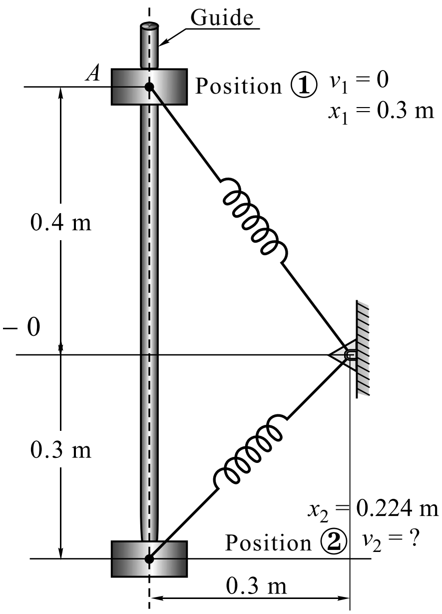


Fig. 15.8(b)

Problem 9

The mass m = 1.8 kg slides from rest at A along the frictionless rod bent into a quarter circle. The spring with modulus k = 16 N/m has an unstretched length of 400 mm.

(i) Determine the speed of m at B.

(ii) If the path is elliptical, what is the speed at B.

Solution

(i) At position ①

$v_1 = 0$

$x_1 = (600 - 400) = 200$ mm

$x_1 = 0.2$ m

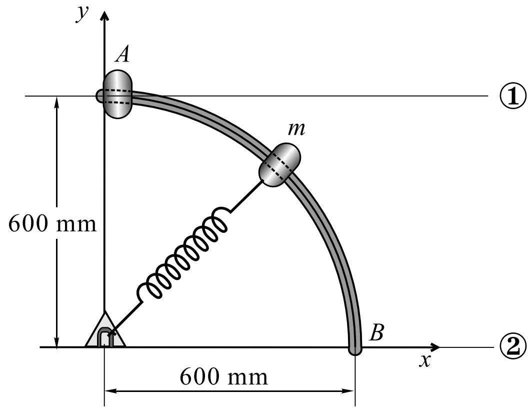


Fig. 15.9(a)

At position ②

$v_2 = ?$

$x_2 = (600 - 400) = 200$ mm

$x_2 = 0.2$ m

By work energy principal, we have

Work done = Change in kinetic energy

$$mgh + \frac{1}{2}k(x_1^2 - x_2^2) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$1.8 \times 9.81 \times 0.6 + \frac{1}{2} \times 16 \times (0.2^2 - 0.2^2) = \frac{1}{2} \times 1.8 \times v_2^2 - 0$$

$v_2 = 3.43$ m/s

(ii) At position ①

$v_1 = 0$

$x_1 = (600 - 400) = 200$ mm

$x_1 = 0.2$ m

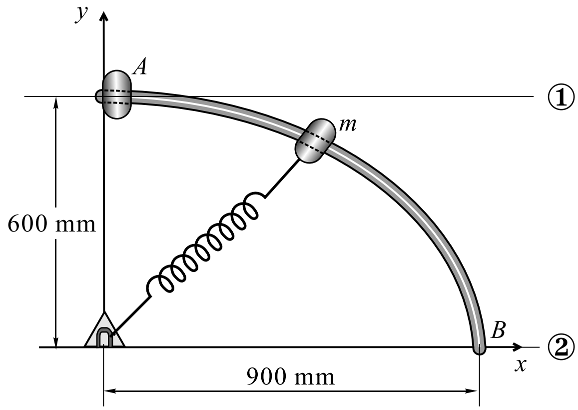


Fig. 15.9(b)

At position ②

$v_2 = ?$

$x_2 = (900 - 400) = 500$ mm

$x_2 = 0.5$ m

By work energy principal, we have

Work done = Change in kinetic energy

$$mgh + \frac{1}{2}k(x_1^2 - x_2^2) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$1.8 \times 9.81 \times 0.6 + \frac{1}{2} \times 16 \times (0.2^2 - 0.5^2) = \frac{1}{2} \times 1.8 \times v_2^2 - 0$$

$v_2 = 3.15$ m/s ***Ans.***

Problem 10

The slider of mass 1 kg attached to a spring of stiffness 400 N/m and unstretched length 0.5 m is released from *A* as shown in Fig. 15.10(a). Determine the velocity of the slider as it passes through *B* and *C*. Also compute the distance beyond *C* where the slider will come to the rest.

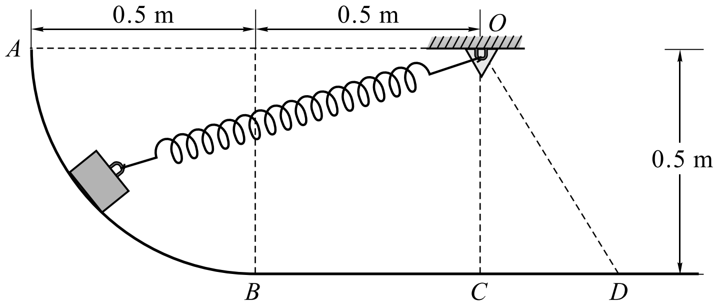


Fig. 15.10(a)

Solution

Redrawing the given Fig. 15.10(b).

Method I

(i) From *A* to *B*

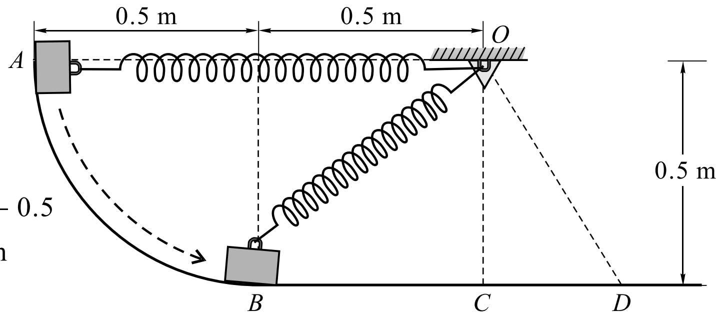


Fig. 15.10(b)

At position *A*	**At position *B***
$v_A = 0$	$v_B = ?$
$x_A = 0.5$ m	$x_B = 0.707 - 0.5$
	$x_B = 0.207$m

Displacement $s = 0.5$ m

By work energy principal, we have

Work done = Change in kinetic energy

$$1 \times 9.81 \times 0.5 + \frac{1}{2} \times 400\,(0.5^2 - 0.207^2) = \frac{1}{2} \times 1 \times v_B^2 - 0$$

$v_B = 9.63$ m/s ***Ans.***

(ii) From *A* to *C*

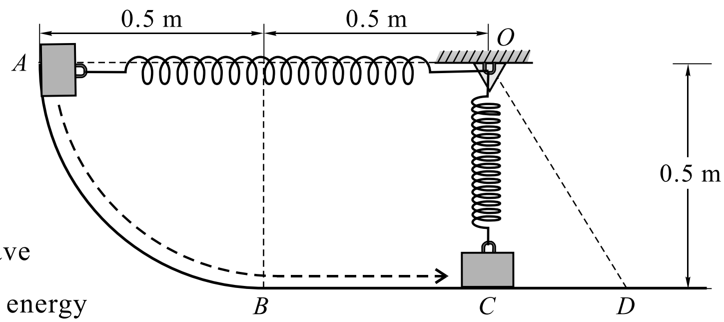


Fig. 15.10(c)

At position *A*	**At position *C***
$v_A = 0$	$v_C = ?$
$x_A = 0.5$ m	$x_C = 0$

Displacement $s = 0.5$ m

By work energy principal, we have

Work done = Change in kinetic energy

$$1 \times 9.81 \times 0.5 + \frac{1}{2} \times 400\,(0.5^2 - 0^2) = \frac{1}{2} \times 1 \times v_C^2 - 0$$

$v_C = 10.48$ m/s ***Ans.***

(iii) From *A* to *D*

At position *A*	**At position *D***
$v_A = 0$	$v_D = 0$
$x_A = 0.5$ m	$x_D = ?$

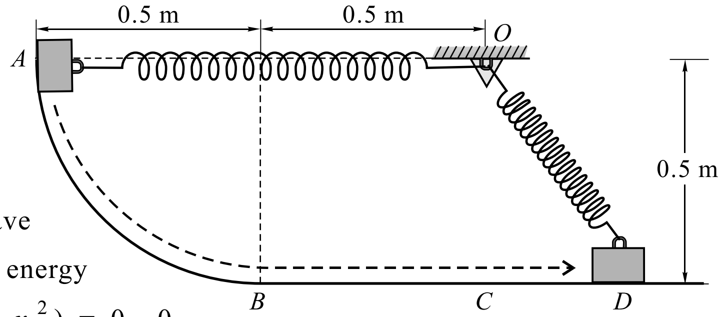


Fig. 15.10(d)

Displacement $s = 0.5$ m

By work energy principal, we have

Work done = Change in kinetic energy

$$1 \times 9.81 \times 0.5 + \frac{1}{2} \times 400\,(0.5^2 - x_D^2) = 0 - 0$$

$x_D = 0.524$ ***Ans.***

OD = Unstretched length of spring + x_D = $0.5 + 0.524 = 1.024$ m ***Ans.***

Distance beyond C to D

$$CD = \sqrt{(OD)^2 - (OC)^2} = \sqrt{(1.024)^2 - (0.5)^2} = 0.89 \text{ m}$$ ***Ans.***

Method II

Total Energy = KE ± PE + SE

$$TE = \frac{1}{2}mv^2 \pm mgh + \frac{1}{2}kx^2$$

(i) At position *A* ($v_A = 0,\ x_A = 0.5,\ h = 0$)

$$TE = 0 + 0 + \frac{1}{2} \times 400 \times 0.5^2 = 50 \text{ J}$$

(ii) At position *B* ($v_B = ?,\ x_B = 0.207,\ h = -0.5$m)

TE = KE ± PE + SE

$$50 = \frac{1}{2} \times 1 \times v_B^2 - 1 \times 9.81 \times 0.5 + \frac{1}{2} \times 400 \times 0.207^2$$

$v_B = 9.63$ m/s ***Ans.***

(iii) At position *C* ($v_C = ?,\ x_C = 0,\ h = -0.5$m)

TE = KE ± PE + SE

$$50 = \frac{1}{2} \times 1 \times v_C^2 - 1 \times 9.81 \times 0.5 + \frac{1}{2} \times 400 \times 0$$

$v_C = 10.48$ m/s ***Ans.***

(iv) At position *D* ($v_D = ?,\ x_D = ?,\ h = -0.5$m)

TE = KE ± PE + SE

$$50 = 0 - 1 \times 9.81 \times 0.5 + \frac{1}{2} \times 400\, x_D^2$$

$x_D = 0.524$ m ***Ans.***

OD = Unstrected length of spring + x_D^2 = $0.5 + 0.524 = 1.024$ m ***Ans.***

Distance beyond C to D

$$CD = \sqrt{(OD)^2 - (OC)^2} = \sqrt{(1.024)^2 - (0.5)^2} = 0.89 \text{ m}$$ ***Ans.***

Problem 11

A 1 kg collar is attached to a spring and slides without friction along a circular rod which lies in horizontal plane as shown in Fig. 15.E11. The spring has a constant $k = 250$ N/m and is undeformed when collar is at B. Knowing that collar passes through point D with a speed of 1.6 m/s, determine the speed of the collar when it passes through point C and point B.

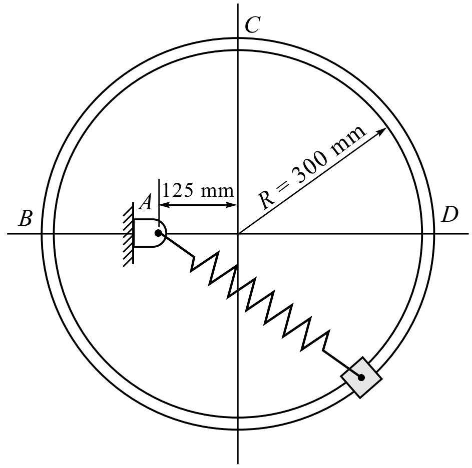


Fig. 15.11(a)

Solution

Undeformed length of spring is at B,

$= 300 - 125 = 175$ mm

$= 0.175$ m

Deformation of spring at position D,

$= 125 + 300 - 175 = 250$ mm

$= 0.25$ m

Deformation of spring at position C,

$= \sqrt{125^2 + 300^2} - 175$

$= 325 - 175 = 150$ mm

$= 0.15$ m

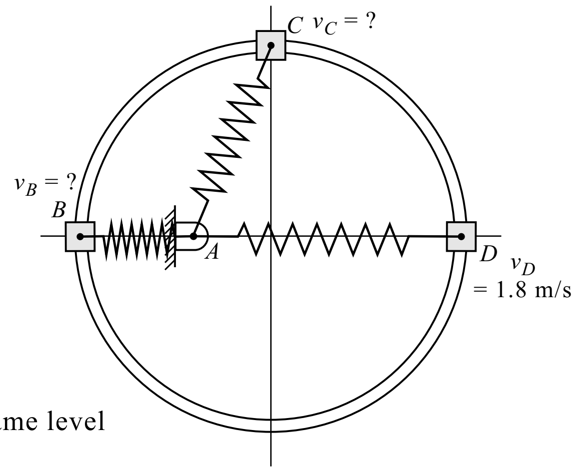


Fig. 15.11(b)

By principle of conservation of energy, we have

Total energy at any position remains constant.

P.E. throughout the ring is zero because it is at same level (horizontal).

Total energy at position D = Total energy at position B

(KE + PE + SE) at D = (KE + PE + Spring energy) at B

$$\frac{1}{2} \times 1 \times 1.8^2 + 0 + \frac{1}{2} \times 250 \times 0.25^2 = \frac{1}{2} \times 1 \times v_B^2 + 0 + \frac{1}{2} \times 250 \times 0^2$$

$$9.43 = 0.5 v_B^2$$

$v_B = 4.343$ m/s ***Ans.***

Total energy at position D = Total energy at position C

$$9.43 = \frac{1}{2} \times 1 \times v_C^2 + 0 + \frac{1}{2} \times 250 \times 0.15^2$$

$$9.43 = 0.5 v_C^2 + 2.8125$$

$\therefore\ v_C = 3.638$ m/s ***Ans.***

Problem 12

A 20 N block is released from rest. It slides down the inclined having $\mu = 0.2$ as shown in Fig. 15.12(a). Determine the maximum compression of the spring and the distance moved by the block when the energy is released from compressed spring. Springs constant $k = 1000$ N/m.

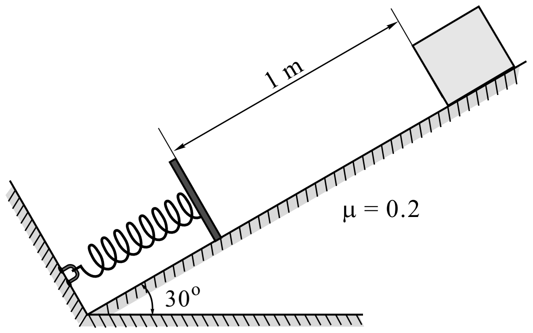


Fig. 15.12(a)

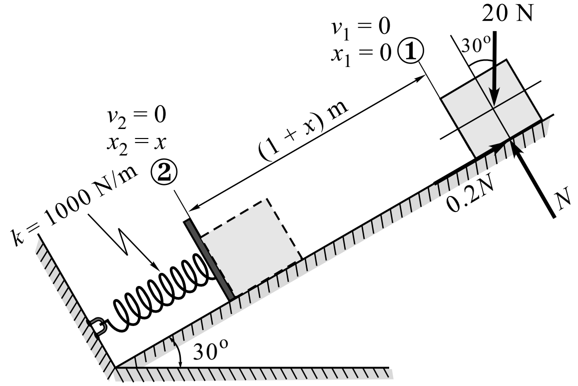


Fig. 15.12(b)

Solution

Part (i) Maximum compression of the spring

Let x be the maximum deformation of spring at position ② where the block comes to rest ($v_2 = 0$).

By work energy principal, we have

Work done = Change in kinetic energy

$$\frac{1}{2} \times 1000(0^2 - x^2) + 20 \sin 30^\circ (1 + x) - 0.2 \times 20 \cos 30^\circ (1 + x) = 0 - 0$$

$\therefore\ x = 0.121$ m ***Ans.***

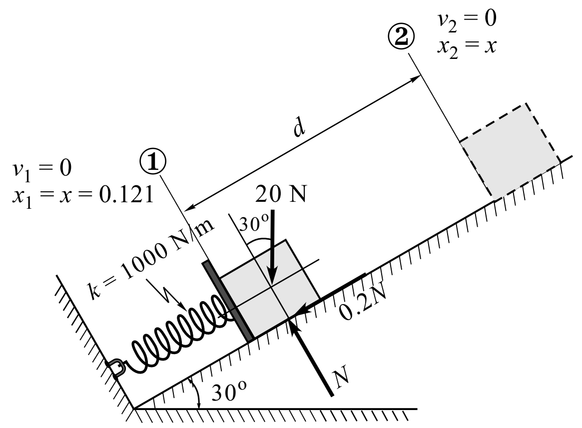


Fig. 15.12(c)

Part (ii) Distance moved by the block

By work energy principal, we have

Work done = Change in kinetic energy

$$\frac{1}{2} \times 1000(0.121^2 - 0^2) - 20 \sin 30^\circ \times d - 0.2 \times 20 \cos 30^\circ \times d = 0 - 0$$

$\therefore\ d = 0.5437$ m ***Ans.***

Problem 13

A spring is used to stop a 100 kg package which is moving down a 30° incline. The spring has a constant $k = 30$ kN/m and is held by cables so that it is initially compressed 90 mm. If the velocity of package is 5 m/s when it is 9 m from the spring, determine the maximum additional deformation of spring in bringing the package to rest. Assume coefficient of friction as 0.2.

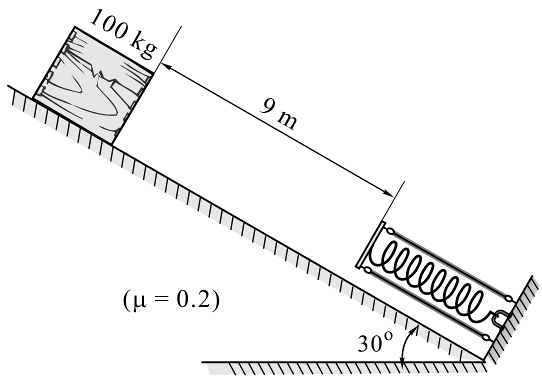


Fig. 15.13(a)

Solution

Let x be the maximum additional deformation of spring in bringing the package to rest.

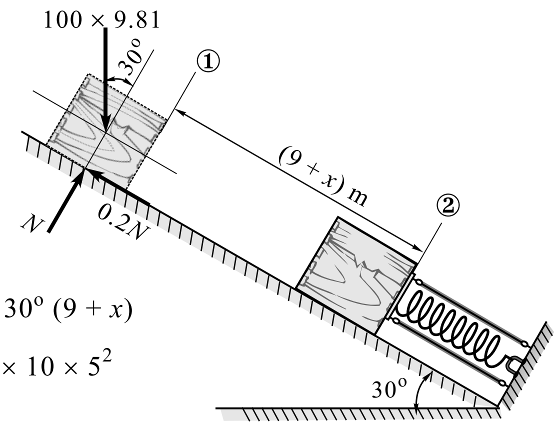


Fig. 15.13(b)

At position ①	**At position ②**
$v_1 = 5$ m/s	$v_2 = 0$
$x_1 = 0.09$ m	$x_2 = (x + 0.09)$ m

Displacement $s = (9 + x)$ m, $k = 30{,}000$ N/m

By work energy principal, we have

Work done = Change in kinetic energy

$$100 \times 9.81 \sin 30^\circ \times (9 + x) - 0.2 \times 100 \times 9.81 \cos 30^\circ (9 + x)$$

$$+ \frac{1}{2} \times 30000[(0.09)^2 - (x + 0.09)^2] = 0 - \frac{1}{2} \times 10 \times 5^2$$

$$15000x^2 + 2379.41x - 2885.31 = -1250$$

$$15000x^2 + 2379.41x - 1635.31 = 0$$

Solving quadratic equation, we get

$x = 0.4517$ m ***Ans.***

Problem 14

Two springs each having stiffness of 0.5 N/cm are connected to ball B having a mass of 5 kg in a horizontal position producing initial tension of 1.5 m in each spring as shown in Fig. 15.E14(a). If the ball is allowed to fall from rest what will be its velocity after it has fallen through a height of 15 cm.

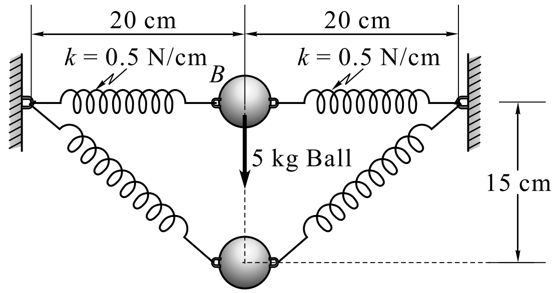


Fig. 15.14(a)

Solution

Method I

Initial position Tension = 1.5 N

$T = kx$

$1.5 = (0.5)(x)$

$x = 3$ cm (Deformation in initial position)

$\therefore$ Free length of spring $= 20 - 3 = 17$ cm

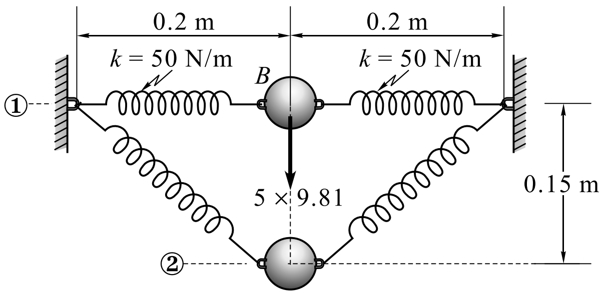


Fig. 15.14(b)

At position ①	**At position ②**
$v_1 = 0$	$v_2 = ?$
$x_1 = 3$ cm	$x_2 = (25 - 17) = 8$ cm
$\therefore x_1 = 0.03$ m	$x_2 = 0.08$ m
Displacement $h = 15$ cm	Spring constant $k = 0.5$ N/cm
$\therefore h = 0.15$ m	$\therefore k = 50$ N/m

By principle of work energy, we have

Work done = Change in kinetic energy

$$5 \times 9.81 \times 0.15 + \left[\frac{1}{2} \times 50(0.03^2 - 0.08^2)\right] \times 2 = \frac{1}{2} \times 5 \times v_2^2 - 0$$

$v_2 = 1.68$ m/s ***Ans.***

Method II

By principle of conservation of energy

Total energy = KE + P.E. + S.E.

Total energy remains constant at any position.

Total energy at position ① = Total energy at position ②

(KE + P.E. + S.E.) at position ① = (KE + P.E. + S.E.) at position ②

$$\frac{1}{2} \times 5 \times 0^2 + 5 \times 9.81 \times 0 + \frac{1}{2} \times 50 \times 0.03^2 = \frac{1}{2} \times 5 \times v_2^2 - 5 \times 9.81 \times 0.15 + \frac{1}{2} \times 50 \times 0.08^2$$

$$0.0225 = 2.5v_2^2 - 7.3575 + 0.16$$

$v_2 = 1.69$ m/s ***Ans.***

Problem 15

An 8 kg plunger is released from rest in the position shown in Fig. 15.15(a) and is stopped by two nested spring. The constant of the outer spring is k_o = 3 k N/m and constant of the inner spring k_i = 10 kN/m. Determine the maximum deflection of the outer spring.

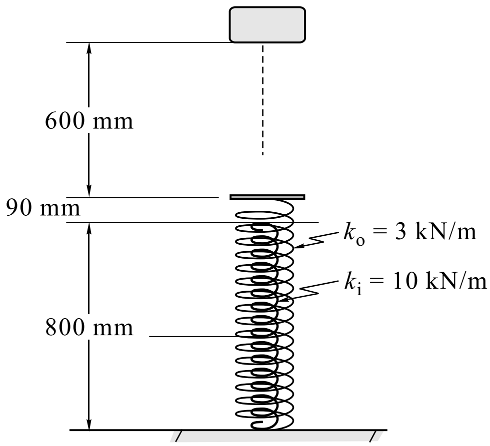


Fig. 15.15(a)

Solution

Let x be the deformation of inner spring.

At position ①	At position ②
$v_1 = 0$	$v_2 = 0$
$x_{i1} = 0$	$x_{i2} = x$
$x_{o1} = 0$	$x_{o2} = (0.09 + x)$

Displacement $s = (0.69 + x)$

By work energy principle, we have

Work done = Change in kinetic energy

$$8 \times 9.81 \times (0.69 + x) + \frac{1}{2} \times 3000[0^2 - (0.09 + x)^2] + \frac{1}{2} \times 10000(0^2 - x^2) = 0 - 0$$

$x = 0.067$ m = 67 mm

∴ The maximum additional deformation of outer spring will be

$d = x + 90$

$d = 157$ mm ***Ans.***

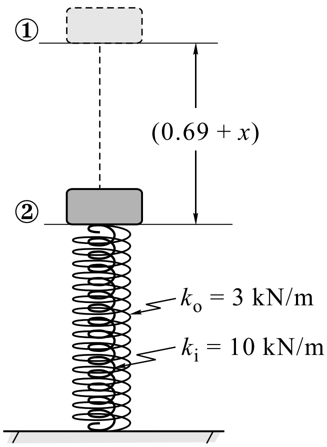


Fig. 15.15(b)

Exercises

[I] Problems

1. The 17.5 kN automobile shown in Fig. 15.E1, is travelling down the 10º inclined road at a speed of 6 m/s. If the driver wishes to stop his car, determine how far 's' his tyres skid on the road if he jams on the brakes, causing his wheels to lock. the coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.5$.

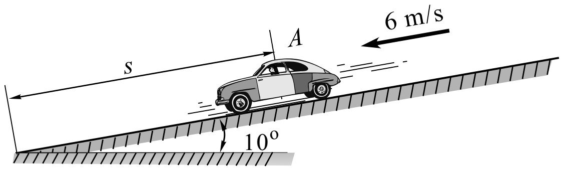


Fig. 15.E1

[***Ans.*** $s = 5.75$ m]

2. Packages are thrown down on incline at A with a velocity of 1.2 m/s as shown in Fig. 15.E2. The package slide along the surface ABC to a conveyer belt which moves with a velocity of 2.4 m/s. Knowing that $\mu_k = 0.25$ between the packages and the surface ABC, determine the distance 'd' if the packages are to arrive at C with a velocity of 2.4 m/s.

[***Ans.*** $d = 6.08$ m]

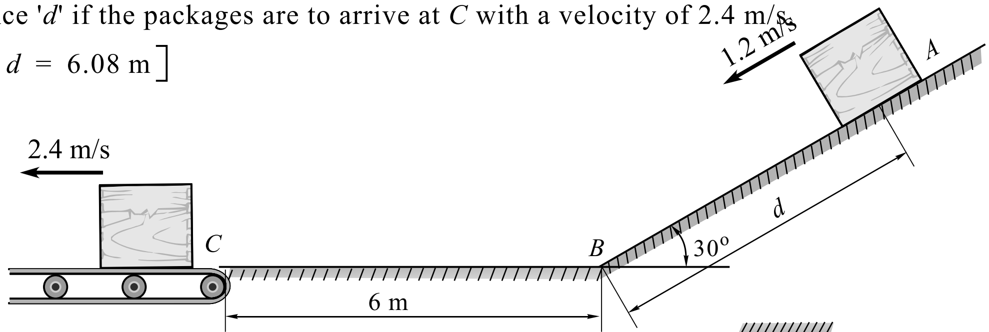


Fig. 15.E2

3. The 6 kg cylindrical collar is released from rest in the position shown in Fig. 15.E3 and drops onto the spring. Calculate the velocity v of the cylinder when the spring has been compressed 50 mm.

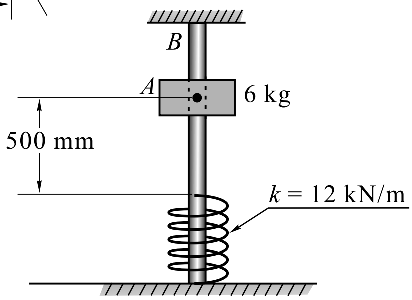


Fig. 15.E3

[***Ans.*** $v = 2.41$ m/s]

4. A 24 N package is placed with no initial velocity at the top of a incline shown in Fig. 15.E4. Knowing that μ_k between the package and the surface is 0.25. Determine **(a)** how far the package will slide on the horizontal portion, **(b)** the maximum velocity reached by the package and **(c)** the amount of the energy dissipated due to friction between A and B.

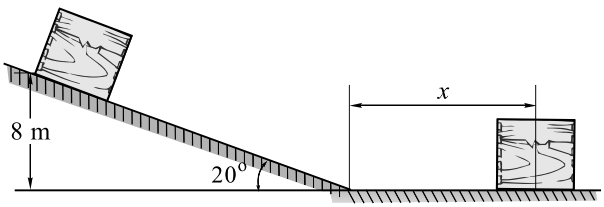


Fig. 15.E4

[***Ans.*** **(a)** 10.02 m, **(b)** 7.02 m/s and **(c)** 131.97 J.]

5. A 20 N block slides with initial velocity of 2 m/s down an inclined plane on to a spring of modulus 1000 N/m for a distance of 1 m as shown in Fig. 15.E5. Find maximum compression of the spring neglecting friction.

[***Ans.*** 177.63 mm]

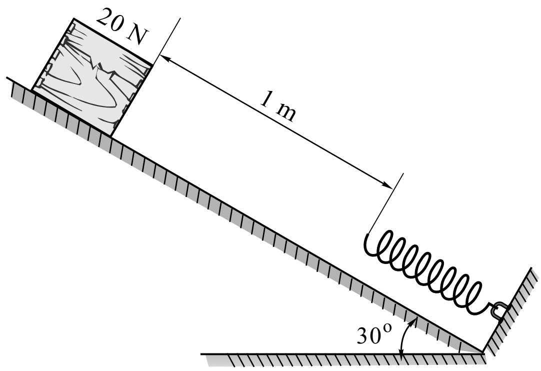


Fig. 15.E5

6. A collar of mass 5 kg can slide along a vertical bar as shown in Fig. 15.E6. The spring attached to the collar is in undeformed state of length 20 cm and stiffness 500 N/m. If the collar is suddenly released, find the velocity of the collar if it moves 15 cm down as shown in the figure.

[***Ans.*** 1.66 m/s]

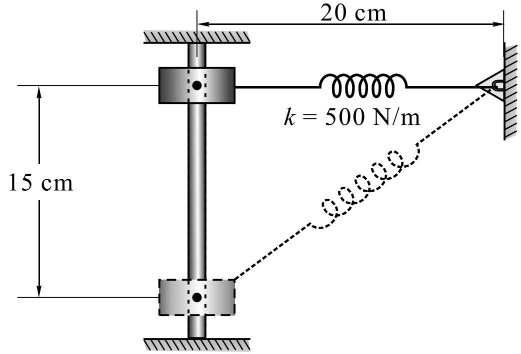


Fig. 15.E6

7. A 10 kg collar slides without friction along a vertical road as shown in Fig. 15.E7. The spring attached to the collar has an undeformed length of 100 mm and a constant of 500 N/m. If the collar is released from rest in position ①, determine its velocity after it has moved 150 mm to position ②.

[***Ans.*** $v = 1.54$ m/s]

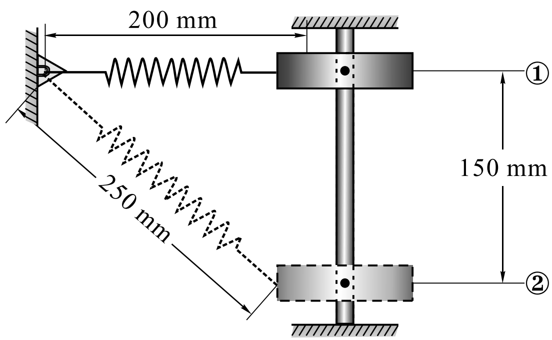


Fig. 15.E7

8. The collar has a mass of 20 kg and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown in Fig. 15.E8. If each spring has an uncompressed length of 1 m and the collar has a speed of 2 m/s when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.

[***Ans.*** $s = 0.73$ m]

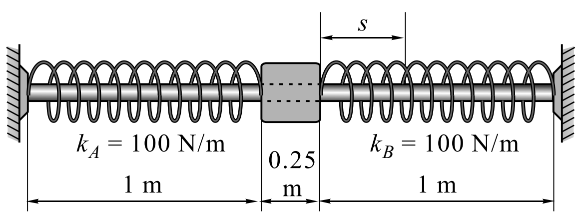


Fig. 15.E8

9. A 10 kg block rests on the horizontal surface shown in Fig. 15.E9. The spring, which is not attached to the block, has a stiffness $k = 500$ N/m and is initially compressed 0.2 m from C to A. After the block is released from rest at A, determine its velocity when it passes point D. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$

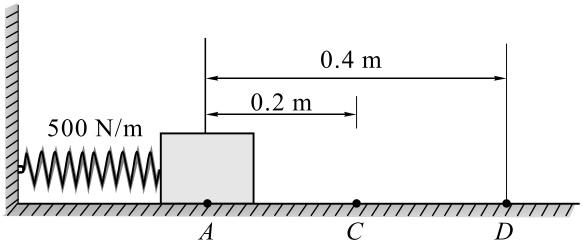


Fig. 15.E9

[*Ans.* 0.656 m/s]

10. Figure 15.E10 shows a wagon weighing 500 kN starts from rest, runs 30 m down one per cent grade and strikes the bumper post. If the rolling resistance of the track is 5 N/km. Find the velocity of the wagon when it strikes the post.

If the bumper spring which compresses 1 mm for every 15 kN, determine by how much this spring will be compressed.

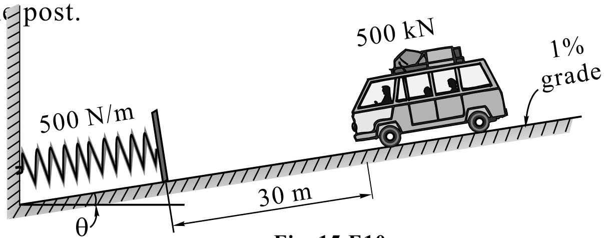


Fig. 15.E10

[*Ans.* 1.716 m/s; 100.2 mm]

11. In Fig. 15.E11, the block P of weight 50 N is pulled so that the extension in the spring is 10 cm. The stiffness of the spring is 4 N/cm and the coefficient of friction between the block and the plane O-X is $\mu = 0.3$.

Find the **(a)** velocity of the block as the spring returns to its undeformed state and **(b)** maximum compression in the spring.

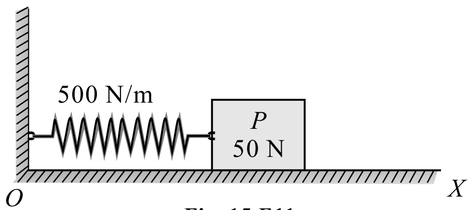


Fig. 15.E11

[*Ans.* $v = 0.443$ m/s and $x = 2.5$ cm.]

12. Figure 15.E12 shows a collar A having mass of 5 kg that can slide without friction on a pipe. If it is released from rest at the position shown, where the spring is unstretched, what speed will the collar have after moving 50 mm? Take spring constant as 2000 N/m.

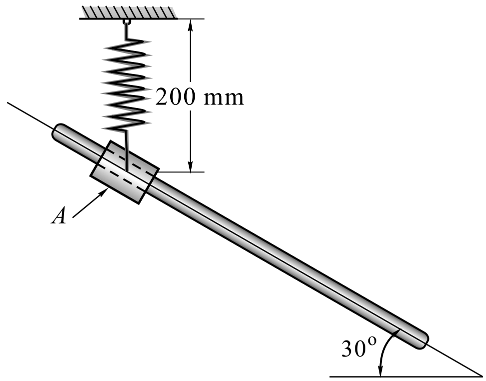


Fig. 15.E12

[*Ans.* 0.5 m/s]

13. The 25 N cylinder is falling A with a speed v_A = 3 m/s on to the platform as shown in Fig. 15.E13. Determine the maximum displacement of the platform, caused by the collision. The spring has an unstretched length of 0.53 m and is originally kept in compression by the 0.3 m long cables attached to the platform. Neglect the mass of the platform and spring and any energy lost during the collision.

[***Ans.*** 0.022 m]

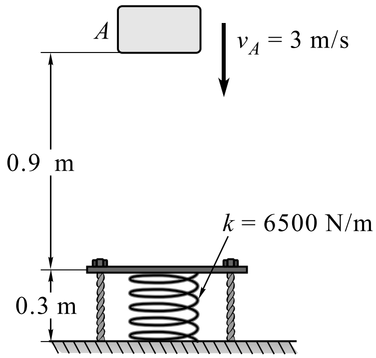


Fig. 15.E13

14. The platform P, shown in Fig. 15.E14 has negligible mass and it tied down so that the 0.4 m long cords keep the spring compressed 0.6 m when nothing is on the platform. If a 2 kg block is placed on the platform and released from rest after the platform is pushed down 0.1 m, determine the maximum height 'h' the block rises in the air, measured from the ground.

[***Ans.*** 0.963 m]

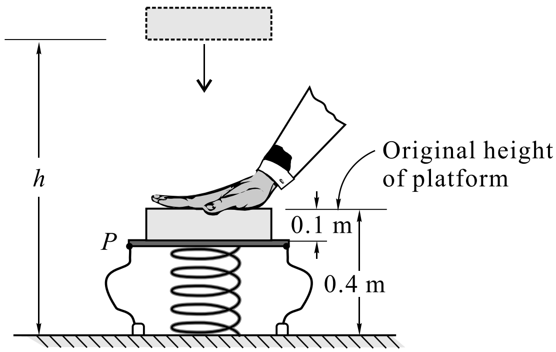


Fig. 15.E14

15. The 30 N ball is fired from a tube by a spring having a stiffness K = 4000 N/m as shown in Fig. 15.E15. Determine how, far the spring must be compressed to fire the ball from the compressed position to a height of 2.4 m, at which point it has a velocity of 1.8 m/s.

[***Ans.*** 0.196 m]

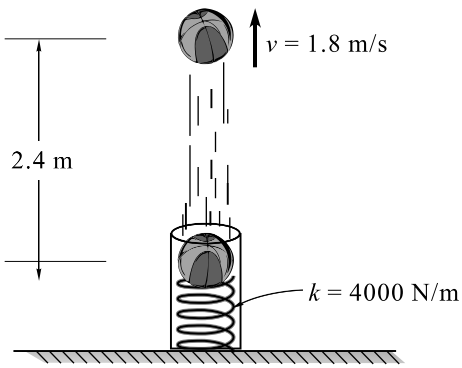


Fig. 15.E15

16. If a mass 'm' hangs freely stretches the spring a distance 'C' m, as shown in Fig. 15.E16. Show that if the mass is suddenly released the spring stretches a distance '$2C$' before mass starts to return upward (To show $S = 2C$).

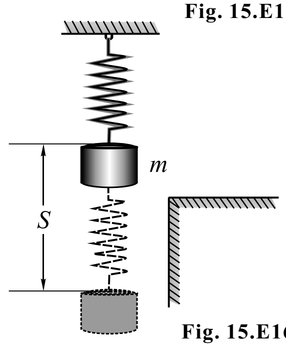


Fig. 15.E16

17. A 3 kg collar is attached to a spring and slides without friction in a vertical plane along the curved rod *ABC*, as shown in Fig. 15.E17. The spring is undeformed when collar is at *C* and its constant is 600 N/m. If the collar is released at *A* with no initial velocity, determine the velocity **(a)** as it passes through *B* and **(b)** as it releases *C*.

[**Ans.** $v_B = 2.33$ m/s and $v_C = 1.23$ m/s.]

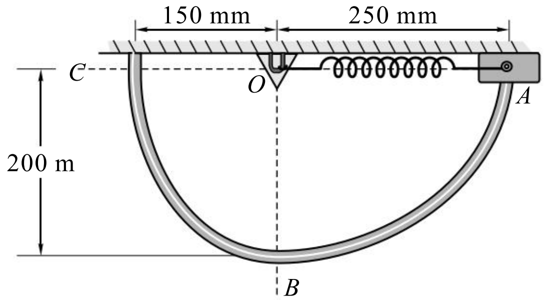


Fig. 15.E17

18. A 1.5 kg collar is attached to a spring and slides without friction along a circular rod in a horizontal plane as shown in Fig. 15.E18. The spring has an undeformed length of 150 mm and a constant k = 400 N/m. Knowing that the collar is in equilibrium at *A* and is given a slight push to get it moving, determine the velocity of the collar **(a)** as it passes through *B* and **(b)** as it passes through *C*.

[**Ans.** $v_B = 3.46$ m/s and $v_C = 4.47$ m/s.]

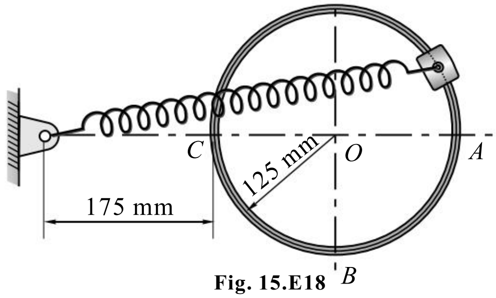


Fig. 15.E18

19. A 7 kg collar *A* slides with negligible friction on the fixed vertical shaft as shown in Fig. 15.E19. When the collar is released from rest at the bottom position shown, it moves up the shaft under the action of constant force F = 200 N applied to the cable. Calculate the stiffness k, which the spring must have if its maximum compression is to be limited to 75 mm. The position of the small pulley at *B* is fixed.

[**Ans.** $k = 8.79$ kN/m]

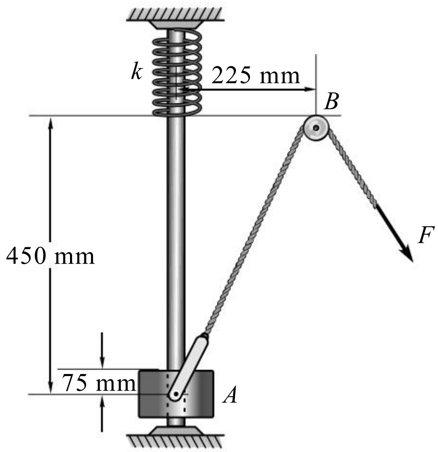


Fig. 15.E19

20. The 15 kg collar *A* is released from rest in the position shown in Fig. 15.E20 and slides with negligible friction up the fixed rod inclined 30° from the horizontal under the action of a constant force P = 200 N applied to the cable. Calculate the required stiffness k of the spring sot that its maximum deflection equals 180 mm. The position of the small pulley at *B* is fixed.

[**Ans.** $k = 1957$ kN/m]

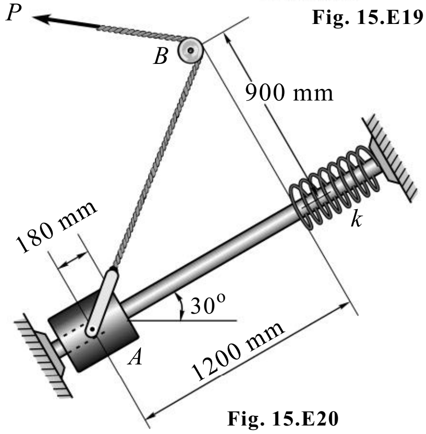


Fig. 15.E20

[II] Review Questions

1. Explain the following :
   (a) Work done by a force
   (b) Work done by a weight force
   (c) Work done by a frictional force
   (d) Work done by a spring force
2. State and prove work energy principle.
3. State the principle of conservation of energy.
4. Explain the term power.

[III] Fill in the Blanks

1. Work done by a force is ________ if the direction of force and the direction of displacement both are in opposite direction.
2. Work is a ________ quantity.
3. Work done by frictional force is always ________.
4. The energy possessed by a particle by virtue of its motion is called ________.
5. Work done by weight force will be ________ if moved from lower position to upper position.

[IV] Multiple-choice Questions

Select the appropriate answer from the given options.

1. Work done by normal reaction and component of weight force perpendicular to inclined plane is ________.

   (a) positive **(b)** negative **(c)** zero **(d)** none of these
2. The energy possessed by a particle by virtue of its position is called as _________.

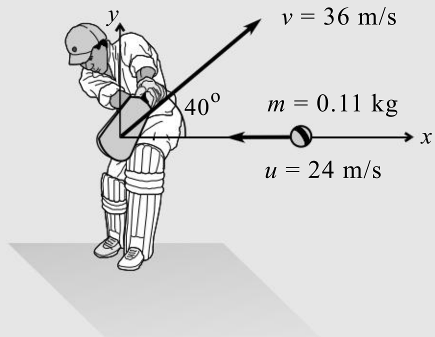
   (a) potential energy **(b)** kinetic energy **(c)** strain energy **(d)** heat energy
3. Frictional force in a system is considered as ________ force.

   (a) conservative **(b)** non-conservative **(c)** neutral **(d)** virtual
4. Work done by the forces in a system may be _________.

   (a) positive **(b)** negative **(c)** zero **(d)** any one of these

16

KINETICS OF PARTICLES - III IMPULSE MOMENT PRINCIPLE AND IMPACT



16.1 Introduction

In the first part of kinetics, we had used the term *force*, *mass* and *acceleration* to solve the problem by *Newton's Second Law* whereas in the second part of kinetics, we have used the term *force*, *velocity* and *displacement* to solve the problem by *Work Energy Principle*. In this chapter, we are going to use the term *force*, *time* and *velocity* to solve the problem by *Impulse Momentum Principle*.

16.2 Principle of Impulse and Momentum

Let F be the force acting on a particle having mass m and producing an acceleration a.

By Newton's second law of motion, we have

$$F = ma$$

$$F = m\frac{dv}{dt} \qquad \left(\because a = \frac{dv}{dt}\right)$$

$$F\,dt = m\,dv$$

Integrating both sides

$$\therefore \int_{t_1}^{t_2} F\,dt = \int_{v_1}^{v_2} m\,dv$$

$$\therefore \int_{t_1}^{t_2} F\,dt = mv_2 - mv_1$$

The term $\int_{t_1}^{t_2} F\,dt$ is called *impulse* and its unit is N-s (Newton-second).

The term mass × velocity is called *momentum*.

So, we have Impulse = Final momentum – Initial momentum

Since the velocity is a vector quantity, impulse is also a vector quantity.

Impulse of Force

When a large force acts over a small finite period the force is called as an *impulse force*.

Impulse of force F acting over a time interval from t_1 to t_2 is defined as

$$I = \int_{t_1}^{t_2} F\,dt$$

Points to be considered :

1. When impulse force acts on the system, non-impulsive force such as weight of the bodies is neglected.
2. When the impulsive forces acts for very small time, impulse due to external forces is zero.
3. The internal forces between the particles need not be considered as the sum of impulses or internal forces are zero.

In Component Form

$$\int_{t_1}^{t_2} F_x \, dt = mv_{x_2} - mv_{x_1}$$

and $$\int_{t_1}^{t_2} F_y \, dt = mv_{y_2} - mv_{y_1}$$

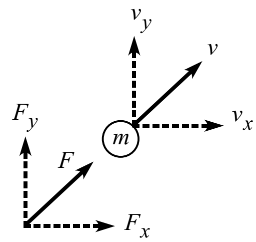


Fig. 16.2-i

The component of the resultant linear impulse along any direction is equal to change in the component of momentum in that direction.

16.3 Principle of Conservation Momentum

In a system, if the resultant force is zero, the impulse momentum equation reduces to final momentum equal to initial momentum. Such situation arises in many cases because the force system consists of only action and reaction on the elements of the system. The resultant force is zero, only when entire system is considered, but not when the free body of each element of the system is considered.

If a man jumps off a boat, the action of the man is equal and opposite to the reaction of the boat. Hence, the resultant is zero in the system.

$$\text{Initial momentum} = \text{Final momentum}$$

Similar equation holds good when we consider the system of a gun and shell.

16.4 Impact

Phenomenon of collision of two bodies, which occurs for a very small interval of time and during which two bodies exert very large force on each other, is called an *impact*.

Line of Impact

The common normal to the surfaces of two bodies in contact during the impact is called *line of impact*. Line of impact is perpendicular to common tangent.

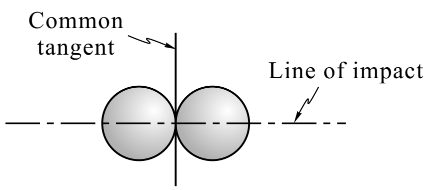


Fig. 16.4-i

Central Impact

When the mass centres C_1 and C_2 of the colliding bodies lie on the line of impact, it is called *central impact*.

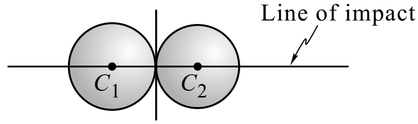


Fig. 16.4-ii

Non-Central Impact

When the mass centres C_1 and C_2 of the colliding bodies do not lie on the line of impact, it is called *non-central* or *eccentric impact*.

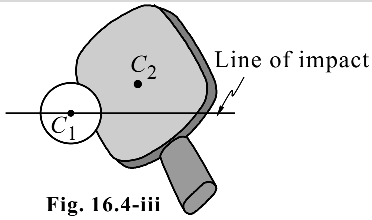


Fig. 16.4-iii

Direct Central Impact

When the direction of motion of the mass centres of two colliding bodies is along the line of impact then we say it is *direct central impact*. Here, the velocities of two bodies collision are collinear with the line of impact.

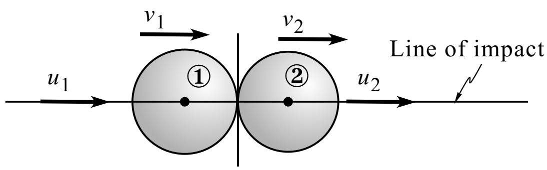


Fig. 16.4-iv

u_1 = Velocity of body ① before collision

u_2 = Velocity of body ② before collision

v_1 = Velocity of body ① after collision

v_2 = Velocity of body ② after collision

m_1 = Mass of body ①

m_2 = Mass of body ②

1. Total momentum of system is conserved along the line of impact.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

2. Coefficient of restitution relation

$$e = -\left[\frac{v_2 - v_1}{u_2 - u_1}\right]$$

Oblique Central Impact

When the direction of motion of the mass centres of one or two colliding bodies is not along the line of impact (i.e., at the same angle with the line of impact) then we say it is *oblique central impact*. Here the velocities of two bodies collision are not collinear with the line of impact.

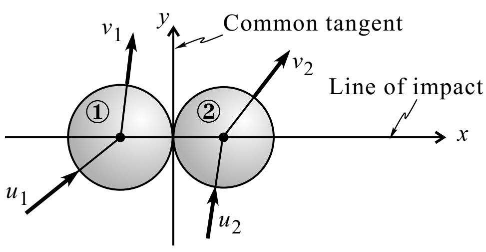


Fig. 16.4-v

1. The component of the total momentum of the two bodies along the line of impact is conserved.

$$m_1u_{1x} + m_2u_{2x} = m_1v_{1x} + m_2v_{2x}$$

2. Coefficient of restitution relation along the line of impact is

$$e = -\left[\frac{v_{2x} - v_{1x}}{u_{2x} - u_{1x}}\right]$$

3. Component of the momentum along the common tangent is conserved, which means the component of the velocities along the common tangent remains unchanged.

$$m_1 u_{1y} = m_1 v_{1y} \qquad \therefore\ u_{1y} = v_{1y}$$

$$m_2 u_{2y} = m_2 v_{2y} \qquad \therefore\ u_{2y} = v_{2y}$$

Note : Here the x-axis is line of impact which can be treated as normal (n) and y-axis is the common tangent which can be treated as tangent (t).

16.5 Coefficient of Restitution (e)

When two bodies collide for a very small interval of time, there will be phenomena of *Deformation* and *Restitution* (*Regain*) of shape.

By Impulse Momentum Principle for the process of deformation of the colliding body of mass m_1, we have

$$m_1 u_1 - \int F_D \, dt = m_1 u \qquad \text{... (16.1)}$$

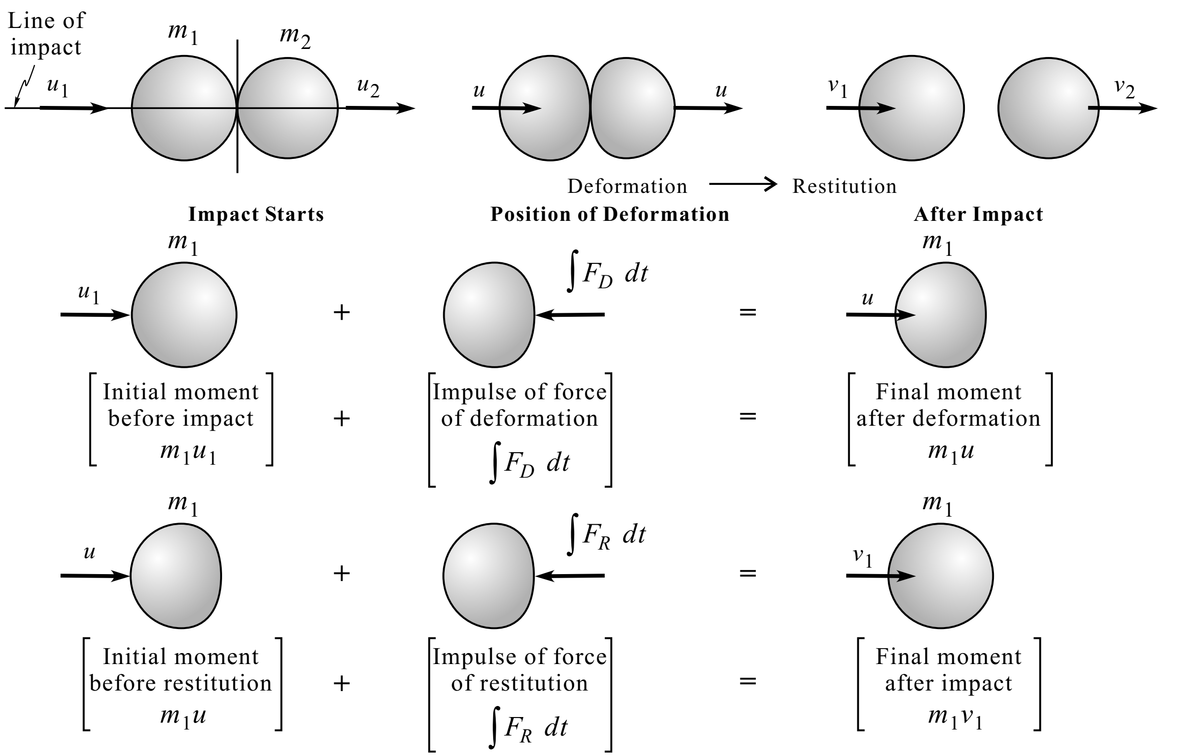


Fig. 16.5-i

Similarly, by Impulse Momentum Principle for the process of restitution of body of mass m_1, we have

$$m_1 u - \int F_R \, dt = m_1 v_1 \qquad \text{... (16.2)}$$

From Eqs. (16.1) and (16.2), we have

$$\int F_D \, dt = m_1 u_1 - m_1 u \quad \text{and} \quad \int F_R \, dt = m_1 u - m_1 v_1$$

$$\therefore \quad \frac{\int F_R \, dt}{\int F_D \, dt} = \frac{m_1(u - v_1)}{m_1(u_1 - u)} = \frac{u - v_1}{u_1 - u} = e \qquad \text{... (16.3)}$$

Similarly, by Impulse Momentum Principle for the process of deformation and restitution, we have

$$\frac{\int F_R \, dt}{\int F_D \, dt} = \frac{v_2 - u}{u - u_2} \qquad \text{... (16.4)}$$

From Eqs. (16.3) and (16.4), we get

$$\left[\frac{u - v_1 + v_2 - u}{u_1 - u + u - u_2}\right] = e$$

$$v_2 - v_1 = e(u_1 - u_2)$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$

$$\therefore \quad e = -\left[\frac{v_2 - v_1}{u_2 - u_1}\right]$$

Classification of Impact Based on Coefficient of Restitution

1. **Perfectly Elastic Impact**
   - **(a)** Coefficient of restitution $e = 1$.
   - **(b)** Momentum is conserved along the line of impact

     $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

   - **(c)** KE is conserved. No loss of KE

     $$\therefore \text{ Total KE before impact} = \text{Total KE after impact}$$

     $$\frac{1}{2} m_1u_1^2 + \frac{1}{2} m_2u_2^2 = \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2$$

2. **Perfectly Plastic Impact**
   - **(a)** Coefficient of restitution $e = 0$.
   - **(b)** After impact both the bodies collide and move together.
   - **(c)** Momentum is conserved

     $$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

     where v is the common velocity after impact.
   - **(d)** There is loss of KE during impact. Thus, KE is not conserved.

     $$\text{Loss of KE} = \text{Total KE before impact} - \text{Total KE after impact}$$

     $$\text{Loss of KE} = \left(\frac{1}{2} m_1u_1^2 + \frac{1}{2} m_2u_2^2\right) - \left(\frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2\right)$$

3. **Semi-elastic Impact**

   Coefficient of restitution $(0 < e < 1)$

16.6 Solved Problems Based on Impact

Problem 1

Two particles of masses 10 kg and 20 kg are moving along a straight line towards each other at velocities of 4 m/s and 1 m/s, respectively, as shown in Fig.16.1. If e = 0.6, determine the velocities of the particles immediately after their collision. Also find the loss of kinetic energy.

Solution

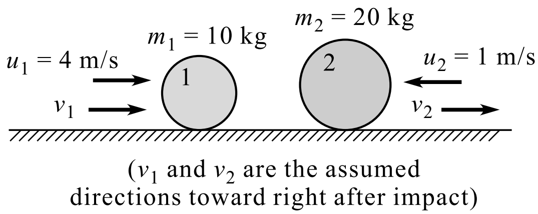


Fig. 16.1

(i) By law of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$10 \times 4 + 20 \times (-1) = 10v_1 + 20v_2$$

$$20 = 10v_1 + 20v_2$$

$$v_1 + 2v_2 = 2 \quad \ldots \text{(I)}$$

(ii) By coefficient of restitution, we have

$$e = -\left[\frac{v_2 - v_1}{u_2 - u_1}\right]$$

$$0.6 = -\left[\frac{v_2 - v_1}{-1 - 4}\right]$$

$$v_2 - v_1 = 3 \quad \ldots \text{(II)}$$

Solving Eqs. (I) and (II), we get

$v_2 = 1.667$ m/s ($\rightarrow$) ***Ans.***

$v_1 = -1.333$ m/s

$\therefore\ v_1 = 1.333$ m/s ($\leftarrow$) ***Ans.***

(iii) Loss of KE = Initial KE – Final KE

$$\text{Loss of KE} = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

$$\text{Loss of KE} = \left[\frac{1}{2}\times 10 \times 4^2 + \frac{1}{2}\times 20 \times (-1)^2\right] - \left[\frac{1}{2}\times 10 \times (-1.333)^2 + \frac{1}{2}\times 20 \times 1.667^2\right]$$

$$\text{Loss of KE} = 90 - 36.67$$

$$= 53.33 \text{ J}$$

$$\%\text{ loss in KE} = \frac{\text{Loss in KE}}{\text{Initial KE}} \times 100$$

$$= \frac{53.33}{90} \times 100$$

$\therefore$ % loss in KE = 59.27 % ***Ans.***

Problem 2

A 50 gm ball is dropped from a height of 600 mm on a small plate as shown in Fig.16.2(a). It rebounds to a height of 400 mm when the plate directly rests on the ground and to a height of 250 mm when a foam rubber mat is placed between the plate and the ground. Determine **(i)** the coefficient of restitution between the plate and the ground and **(ii)** the mass of the plate.

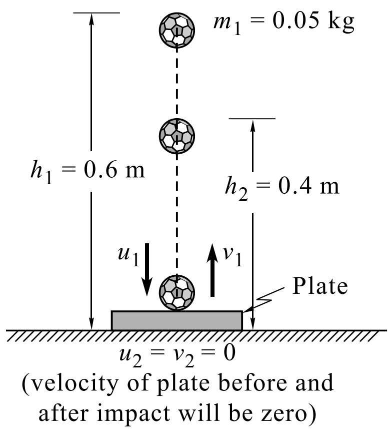


Fig. 16.2(a)

Solution

(i) The plate is kept directly on the ground

$$u_1 = \sqrt{2gh_1}\ (\downarrow)\ \text{(velocity before impact)}$$

$$u_1 = \sqrt{2 \times 9.81 \times 0.6}$$

$$u_1 = 3.43 \text{ m/s } (\downarrow)$$

$$v_1 = \sqrt{2gh_2}\ (\uparrow)\ \text{(velocity after impact)}$$

$$v_1 = \sqrt{2 \times 9.81 \times 0.4}$$

$$v_1 = 2.8 \text{ m/s } (\uparrow)$$

Coefficient of restitution

$$e = -\left[\frac{v_2 - v_1}{u_2 - u_1}\right] = -\left[\frac{0 - 2.8}{0 - (-3.43)}\right]$$

$e = 0.816$ ***Ans.***

(ii) The plate is kept on the foam rubber mat

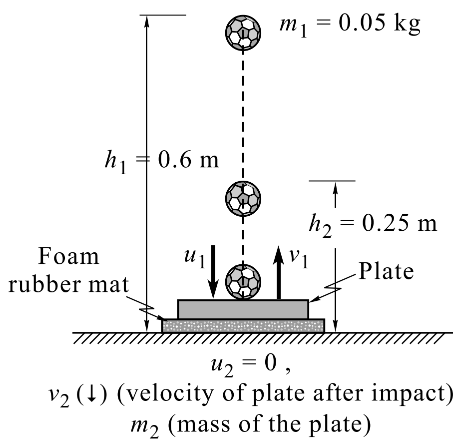


Fig. 16.2(b)

$$u_1 = \sqrt{2 \times 9.81 \times 0.6}$$

$$u_1 = 3.43 \text{ m/s } (\downarrow)$$

$$v_1 = \sqrt{2 \times 9.81 \times 0.25}$$

$$v_1 = 2.215 \text{ m/s } (\uparrow)$$

Coefficient of restitution gives

$$e = -\left[\frac{v_2 - v_1}{u_2 - u_1}\right]$$

$$0.816 = -\left[\frac{-v_2 - 2.215}{0 - (-3.43)}\right]$$

$$\therefore\ v_2 = 0.584 \text{ m/s } (\downarrow)$$

(iii) By law of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$0.05 \times (-3.43) + m_2 \times 0 = 0.05 \times 2.215 + m_2 \times (-0.584)$$

$\therefore\ m_2 = 0.483$ kg (mass of the plate) ***Ans.***

Problem 3

Two smooth spheres ① and ② having a mass of 2 kg and 4 kg, respectively collide with initial velocities, as shown in Fig. 16.3(a). If the coefficient of restitution for the spheres is e = 0.8, determine the velocities of each sphere after collision.

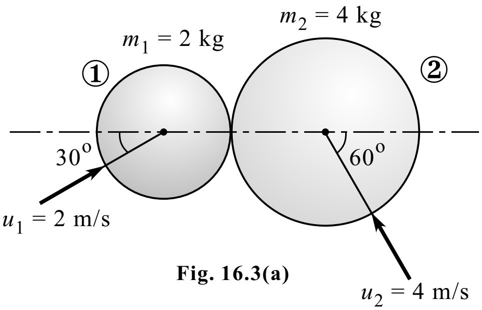


Fig. 16.3(a)

Solution

(i) By law of conservation of momentum along line of impact, we have

$$m_1u_{1x} + m_2u_{2x} = m_1v_{1x} + m_2v_{2x}$$

$$2 \times 2 \cos 30^o + 4 \times (-4 \cos 60^o) = 2(-v_{1x}) + 2v_{2x}$$

$$-v_{1x} + 2v_{2x} = -2.268 \qquad \text{... (I)}$$

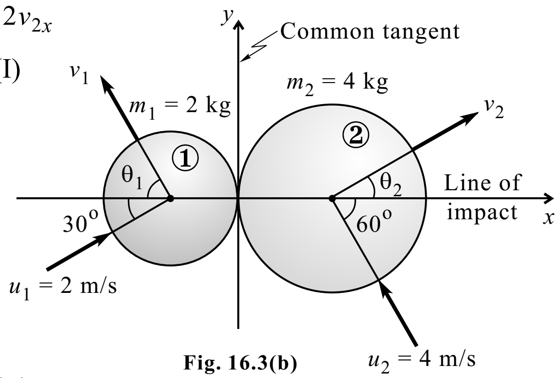


Fig. 16.3(b)

Coefficient of restitution along the line of impact gives

$$e = -\left[\frac{v_{2x} - v_{1x}}{u_{2x} - u_{1x}}\right]$$

$$0.8 = -\left[\frac{v_{2x} - (-v_{1x})}{-4 \cos 60^o - 2 \cos 30^o}\right]$$

$$v_{2x} + v_{1x} = 2.986 \text{ m/s} \qquad \text{... (II)}$$

Solving Eqs. (I) and (II), we get

$$v_{1x} = 2.747 \text{ m/s } (\leftarrow) \quad \text{and} \quad v_{2x} = 0.239 \text{ m/s } (\rightarrow) \quad \textit{Ans.}$$

(ii) Component of velocity before and after impact along common tangent is conserved

$$v_{1y} = 2 \sin 30^o \qquad v_{2y} = 4 \sin 60^o$$

$$v_{1y} = 1 \text{ m/s } (\uparrow) \qquad v_{2y} = 3.464 \text{ m/s } (\uparrow)$$

For v_1, we have

$$\tan \theta_1 = \frac{v_{1y}}{v_{1x}} = \frac{1}{2.747}$$

$$\theta_1 = 20^o$$

$$v_1 = \sqrt{{v_{1x}}^2 + {v_{1y}}^2}$$

$$= \sqrt{2.747^2 + 1^2}$$

$$v_1 = 2.923 \text{ m/s } \left(\theta_1 \nwarrow\right) \quad \textit{Ans.}$$

Velocity of sphere ①

For v_2, we have

$$\tan \theta_2 = \frac{v_{2y}}{v_{2x}} = \frac{3.464}{0.239}$$

$$\theta_2 = 86.05^o$$

$$v_2 = \sqrt{{v_{2x}}^2 + {v_{2y}}^2}$$

$$= \sqrt{0.239^2 + 3.464^2}$$

$$v_2 = 3.472 \text{ m/s } \left(\nearrow \theta_2\right) \quad \textit{Ans.}$$

Velocity of sphere ②

Problem 4

Two balls of same mass 0.5 kg moving with velocities as shown in Fig. 16.4(a), collide. If after collision ball ② travels along a line 30° counter clockwise from y-axis, determine the coefficient of restitution.

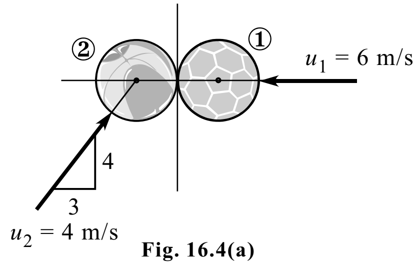


Fig. 16.4(a)

Solution

(i) By law of conservation of momentum along line of impact, we have

$$m_1 u_{1x} + m_2 u_{2x} = m_1 v_{1x} + m_2 v_{2x}$$

$$u_{1x} + u_{2x} = v_{1x} + v_{2x}$$

$$-6 + 4\cos 53.13^\circ = v_{1x} + (-v_2 \cos 60^\circ)$$

$$v_{1x} - 0.5 v_2 = -3.6$$

$$2v_{1x} - v_2 = -7.2 \qquad \text{... (I)}$$

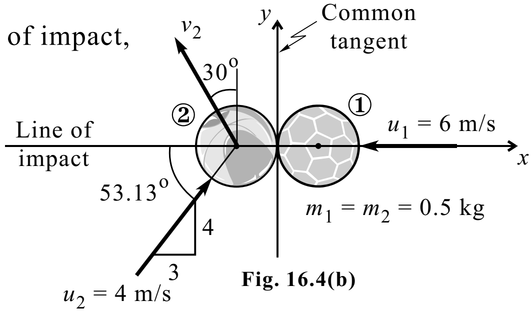


Fig. 16.4(b)

(ii) Component of velocity before and after impact along common tangent is conserved

$$v_2 \sin 60^\circ = 4 \sin 53.13^\circ$$

$$v_2 = 3.7 \text{ m/s}$$

From Eq. (I)

$$2v_{1x} - 3.7 = -7.2$$

$$v_{1x} = -1.75 \text{ m/s} \qquad \therefore v_{1x} = 1.75 \text{ m/s } (\leftarrow)$$

$$\because v_{1y} = 0 \quad \therefore v_1 = v_{1x} = 1.75 \text{ m/s } (\leftarrow)$$

Coefficient of restitution along the line of impact gives

$$e = -\left[\frac{v_{2x} - v_{1x}}{u_{2x} - u_{1x}}\right]$$

$$e = -\left[\frac{-3.7\cos 60 - (-1.75)}{4\cos 53.13 - (-6)}\right]$$

$\therefore e = 0.012$ ***Ans.***

Problem 5

A billiard ball, shown in Fig. 16.5 moving with a velocity of 5 m/s strikes a smooth horizontal plane at an angle 45° with the horizontal. If $e = 0.6$, between ball and plane what is the velocity with which the ball rebounds ?

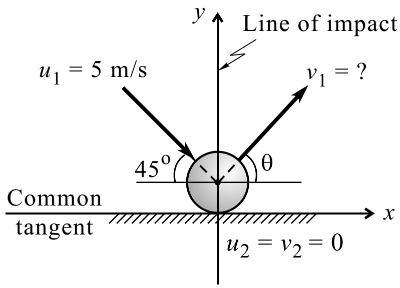


Fig. 16.5

Solution

(i) Coefficient of restitution along the line of impact gives

$$e = -\left[\frac{v_{2y} - v_{1y}}{u_{2y} - u_{1y}}\right]$$

$$0.6 = -\left[\frac{0 - v_{1y}}{0 - (-5 \sin 45^{\circ})}\right]$$

$$v_{1y} = 2.12 \text{ m/s } (\uparrow)$$

(ii) Component of velocity before and after impact along common tangent is conserved

$$v_{1x} = 5 \cos 45^{\circ} \qquad \therefore v_{1x} = 3.54 \text{ m/s } (\rightarrow)$$

$$\tan \theta = \frac{v_{1y}}{v_{1x}} = \frac{2.12}{3.54} \qquad \therefore \theta = 30.92^{\circ}$$

$$v_1 = \sqrt{{v_{1x}}^2 + {v_{1y}}^2} = \sqrt{3.54^2 + 2.12^2}$$

$$\therefore v = 4.126 \text{ km/hr } (\angle 30.92^{\circ})$$

The ball rebounds with a velocity 4.126 m/s at an angle of 30.92° w.r.t. x-axis. ***Ans.***

Problem 6

A ball is thrown against a wall with a velocity u forming an angle 30° with the horizontal as shown in Fig. 16.6(a). Assuming frictionless conditions and $e = 0.5$, determine the magnitude and direction of velocity of ball after it rebounds from the wall.

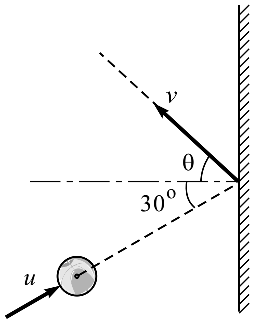


Fig. 16.6(a)

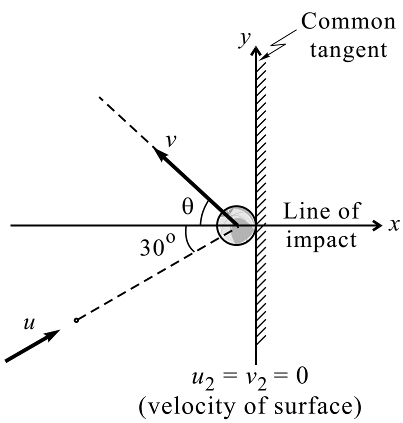


Fig. 16.6(b)

Solution

(i) Coefficient of restitution along the line of impact gives

$$e = -\left[\frac{v_{2x} - v_{1x}}{u_{2x} - u_{1x}}\right]$$

$$0.5 = -\left[\frac{0 - (-v_1 \cos \theta)}{0 - u_1 \cos 30^{\circ}}\right]$$

$$0.433u_1 = v_1 \cos \theta \qquad \text{... (I)}$$

(ii) Component of velocity before and after along common tangent is conserved

$$u_1 \sin 30 = v_1 \sin \theta \qquad \text{... (II)}$$

Dividing Eq. (I) by Eq. (II)

$$\frac{0.433u_1}{u_1 \sin 30} = \frac{v_1 \cos \theta}{v_1 \sin \theta}$$

$$\tan \theta = \frac{\sin 30^{\circ}}{0.433} \qquad \therefore \theta = 49.12^{\circ}$$

Putting the value in Eq. (I), we get

$$0.433u_1 = v_1 \cos 49.12^{\circ}$$

$$v_1 = 0.6616u$$

The ball rebounds with velocity $0.661u$ at an angle of 49.12°. ***Ans.***

Problem 7

A ball is dropped on an inclined plane from a height of 3 m vertically down, and the ball is observed to move horizontally with a velocity v, as shown in Fig. 16.7(a). If the coefficient of restitution is $e = 0.6$, determine the inclination of the plane and the velocity of the ball after impact.

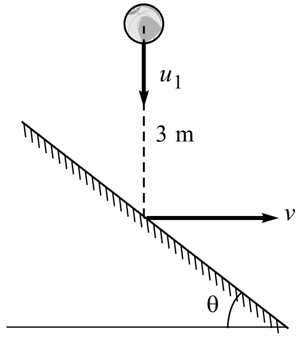


Fig. 16.7(a)

Solution

(i) Velocity before impact

$$u_1 = \sqrt{u^2 + 2gh} = \sqrt{0 + 2 \times 9.81 \times 3}$$

$$u_1 = 7.672 \text{ m/s} \quad (\downarrow)$$

(ii) Coefficient of restitution along the line of impact gives

$$e = -\left[\frac{v_{2y} - v_{1y}}{u_{2y} - u_{1y}}\right]$$

$$0.6 = -\left[\frac{0 - v_1 \sin\theta}{0 - (-7.672 \cos\theta)}\right]$$

$$v_1 \sin\theta = 4.603 \cos\theta$$

$$v_1 = 4.603 \cot\theta \qquad \text{... (I)}$$

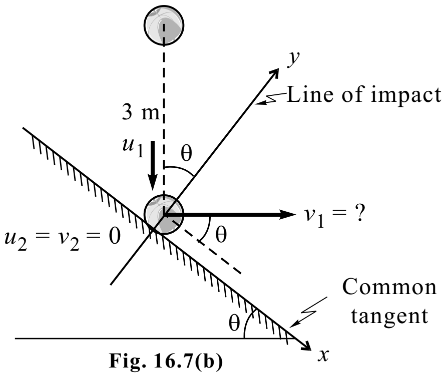


Fig. 16.7(b)

(iii) Component of velocity along common tangent before and after impact is conserved

$$v_1 \cos\theta = 7.672 \sin\theta$$

$$v_1 = 7.672 \tan\theta \qquad \text{... (II)}$$

From Eqs. (I) and (II), we get

$$4.603 \cot\theta = 7.672 \tan\theta$$

$$\tan^2\theta = \frac{1}{1.667} \quad \therefore \ \theta = 37.76^\circ$$ (Inclination of the plane) ***Ans.***

From Eq. (II), we get

$$v_1 = 7.672 \tan 37.76^\circ \quad \therefore \ v_1 = 5.942 \text{ m/s} (\rightarrow)$$ (Velocity of the ball after impact) ***Ans.***

Problem 8

Ball A of mass m is released from rest and slides down a smooth bowl and strikes another ball B of mass $m/4$, which is resting at bottom of the bowl, as shown in Fig. 16.8. Determine height h from which ball A should be released, so that after direct central impact, ball B just leaves the bowls surface. Take coefficient of restitution $e = 0.8$.

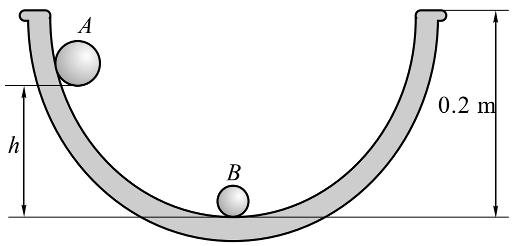


Fig. 16.8

Solution

$$m_A = m_1 = m \ ; \ u_A = u_1 = \sqrt{2gh} \ ; \ v_A = v_1 = \ ?$$

$$m_B = m_2 = 0.25m \ ; \ u_B = u_2 = 0 \ ; \ v_B = v_2 = \sqrt{2g \times 0.2} = 1.981 \text{ m/s}$$

(i) By law of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m\sqrt{2gh} + 0 = mv_1 + 0.25mv_2$$

$$\sqrt{2gh} = v_1 + 0.25 \times 1.981$$

$$\sqrt{2gh} = v_1 + 0.495 \quad \text{... (I)}$$

(ii) Coefficient of restitution gives

$$e = -\left[\frac{v_2 - v_1}{u_2 - u_1}\right] \Rightarrow 0.8 = -\left[\frac{1.981 - v_1}{0 - \sqrt{2gh}}\right]$$

$$0.8\sqrt{2gh} = 1.981 - v_1$$

$$\sqrt{2gh} = 2.476 - 1.25v_1 \quad \text{... (II)}$$

Comparing Eqs. (I) and (II), we get

$$v_1 + 0.495 = 2.476 - 1.25v_1$$

$v_1 = 0.88$ m/s ***Ans.***

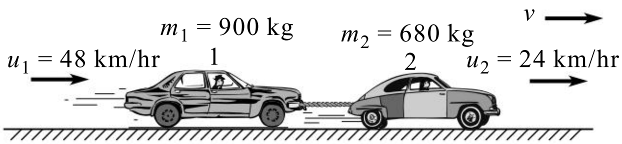
Substituting in Eq. (I), we get

$$\sqrt{2gh} = 0.88 + 0.495$$

$\therefore\ h = 0.096$ m ***Ans.***

Problem 9

A 900 kg car is travelling 48 km/hr couples to 680 kg car travelling 24 km/hr in the same directions, as shown in Fig. 16.9. What is their common speed after coupling.



(v is the common speed after coupling)

Fig. 16.9

Solution

By law of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$900 \times 48 + 680 \times 24 = (900 + 680)v$$

$v = 37.67$ km/hr (→) ***Ans.***

Problem 10

A 20 gm bullet is fired with a velocity of magnitude 600 m/s into a 4.5 kg block of wood, which is stationary as shown in Fig. 16.10. Knowing that the coefficient of kinetic friction between the block and the floor is 0.4. Determine **(i)** how far the block will move and **(ii)** the percentage of the initial energy lost in friction between the block and the floor.

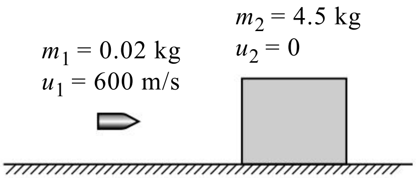


Fig. 16.10(a)

Solution

(i) By law of conservation of energy, we have

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v_1$$

$0.02 \times 600 + 0 = (0.02 + 4.5)v_1$

$v_1 = 2.655$ m/s (→)

(velocity of bullet and block together after impact)

(ii) By work energy principal, we have

Work done = Change in kinetic energy

$$-0.4 \times 4.52 \times 9.81 \times d = 0 - \frac{1}{2} \times 4.52 \times 2.655^2$$

$\therefore\ d = 0.9$ m ***Ans.***

(iii) Initial KE $= \frac{1}{2} \times 0.02 \times 600^2 = 3600$ J

Energy lost in friction $= 0.4 \times 4.52 \times 9.81 \times 0.9 = 15.93$ J

$\therefore$ percentage loss $= \frac{15.93}{3600} = 0.44\ \%$ ***Ans.***

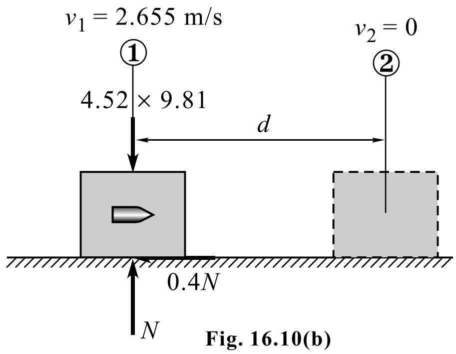


Fig. 16.10(b)

Problem 11

A 750 kg hammer of a drop hammer pile driver falls from a height of 1.2 m onto the top of a pile, as shown in Fig. 16.11. The pile is driven 100 mm into the ground. Assume perfectly plastic impact, determine the average resistance of the ground to penetration. Assume mass of pile as 2250 kg.

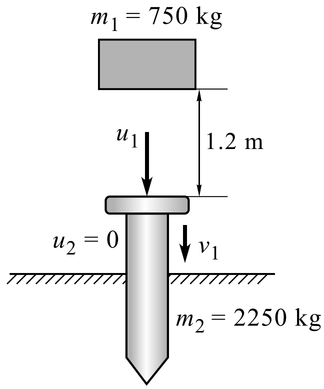


Fig. 16.11(a)

Solution

(i) Velocity before impact

$m_1 = 750$ kg

$u_1 = \sqrt{u^2 + 2gh} = \sqrt{0 + 2 \times 9.81 \times 1.2}$

$u_1 = 4.852$ m/s (↓)

$m_2 = 2250$ kg ; $u_2 = 0$

(ii) For perfectly plastic impact

$m_1u_1 + m_2u_2 = (m_1 + m_2)v_1$

$750 \times 4.852 + 2250 \times 0 = (750 + 2250)v_1$

$v_1 = 1.213$ m/s (↓)

After impact, both hammer and pile will move together with velocity $v_1 = 1.213$ m/s (↓)

(iii) By work energy principal, we have

Work done = Change in kinetic energy

$$(750 + 2250) \times 9.81 \times 0.1 - R \times 0.1 = 0 - \frac{1}{2} \times (750 + 2250) \times 1.213^2$$

$R = 51482.3$ N (↑)

Average resistance force of the ground $R = 51482.3$ N (↑) ***Ans.***

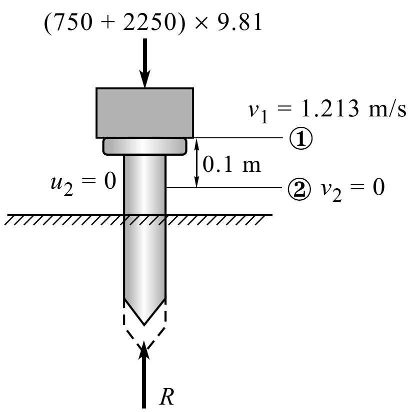


Fig. 16.11(b)

Problem 12

A bullet of mass 10 gm is moving with a velocity of 100 m/s and hits a 2 kg bob of a simple pendulum, horizontally, as shown in Fig. 16.12(a). Determine the maximum angle θ through which the pendulum string 0.5 m long may swing if **(i)** the bullet gets embedded in the bob and **(ii)** the bullet escapes from the other end of the bob with a velocity 10 m/s.

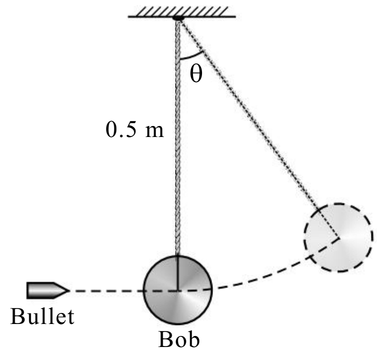


Fig. 16.12(a)

Solution

Case I : Find θ when the bullet gets embedded in the bob (Perfectly plastic impact)

(i) By law of conservation of momentum, we have

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v_1$$

$$0.01 \times 100 + 2 \times 0 = (0.01 + 2) v_1$$

$$v_1 = 0.4975 \text{ m/s } (\rightarrow)$$

Velocity of bullet and bob together after impact $v_1 = 0.4975$ m/s (→).

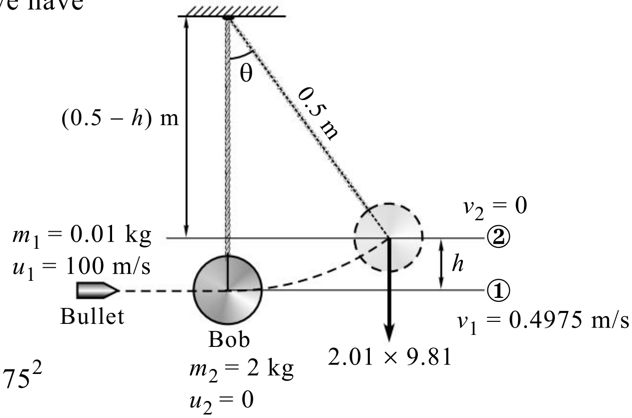


Fig. 16.12(b)

(ii) By work energy principal, we have

Work done = Change in kinetic energy

$$-2.01 \times 9.81 \times h = 0 - \frac{1}{2} \times 2.01 \times 0.4975^2$$

$$\therefore \; h = 0.0127 \text{ m}$$

(iii) $$\cos\theta = \frac{0.5 - h}{0.5} = \frac{0.5 - 0.0127}{0.5}$$

$$\therefore \; \theta = 12.94^{\circ}$$ ***Ans.***

Case II : Find θ when the bullet escapes from the other end of the bob with velocity 10 m/s

(i) By law of conservation of momentum, we have

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$0.01 \times 100 + 2 \times 0 = 0.01 \times 10 + 2 \times v_2$$

$$v_2 = 0.45 \text{ m/s } (\rightarrow)$$

(ii) By work energy principal, we have

Work done = Change in kinetic energy

$$-2 \times 9.81 \times h = 0 - \frac{1}{2} \times 2 \times 0.45^2$$

$$\therefore \; h = 0.01032 \text{ m}$$

(iii) $$\cos\theta = \frac{0.5 - h}{0.5} = \frac{0.5 - 0.01032}{0.5}$$

$$\therefore \; \theta = 11.66^{\circ}$$ ***Ans.***

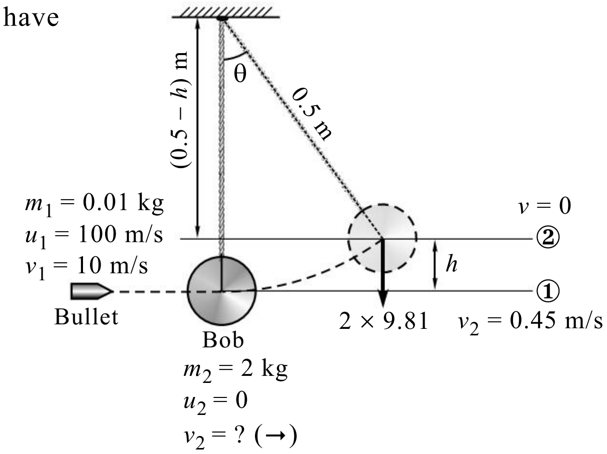


Fig. 16.12(c)

Problem 13

Determine the horizontal velocity u_A shown in Fig. 16.13(a) at which we must throw the ball so that it bounces once on the surface and then lands into the cup at C. Take the coefficient of restitution between ball and surface as $e = 0.6$ and neglect the size of the cup.

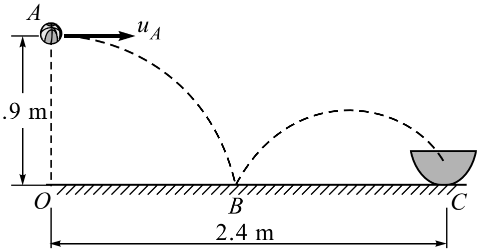


Fig. 16.13(a)

Solution

(i) Consider motion from A to B

Vertical motion

$$h = u\,t + \frac{1}{2}gt^2$$

$$0.9 = 0 + \frac{1}{2} \times 9.81 \times t_1^2$$

$$t_1 = 0.428 \text{ seconds}$$

$$v^2 = u^2 + 2gh$$

$$v = \sqrt{2 \times 9.81 \times 0.9} = 4.202 \text{ m/s}$$

$\therefore$ $u_1 = 4.202$ m/s $(\downarrow)$ (velocity before impact at B)

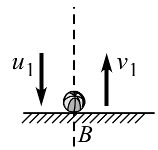
(ii) Consider the impact at B

Coefficient of restitution gives

$$e = -\left[\frac{v_2 - v_1}{u_2 - u_1}\right]$$

$$0.6 = -\left[\frac{0 - v_1}{0 - (-4.202)}\right]$$

$\therefore$ $v_1 = 2.52$ m/s $(\uparrow)$ (velocity of ball after impact)



$u_2 = v_2 = 0$
(velocity of surface before and after impact)

Fig. 16.13(b)

(iii) Consider motion from B to C

Vertical motion

$$h = u\,t + \frac{1}{2}gt^2$$

$$0 = 2.52 \times t_2 - \frac{1}{2} \times 9.81 \times t_2^2$$

$$t_2 = 0.514 \text{ seconds}$$

(iv) Consider projectile motion from A to B and B to C

Total time $t = t_1 + t_2 = 0.428 + 0.514$

$t = 0.942$ seconds

In projectile motion, horizontal motion happens with constant velocity

$\therefore$ Displacement = Velocity × Time

$2.4 = u_A \times 0.942$

$\therefore$ $u_A = 2.548$ m/s $(\rightarrow)$ ***Ans.***

Problem 14

A small steel ball is to be projected horizontally such that it bounces twice on the surface and lands into a cup placed at a distance of 8 m, as shown in Fig. 16.14. If the coefficient of restitution for each impact is 0.8, determine the velocity of projection 'u' of the ball.

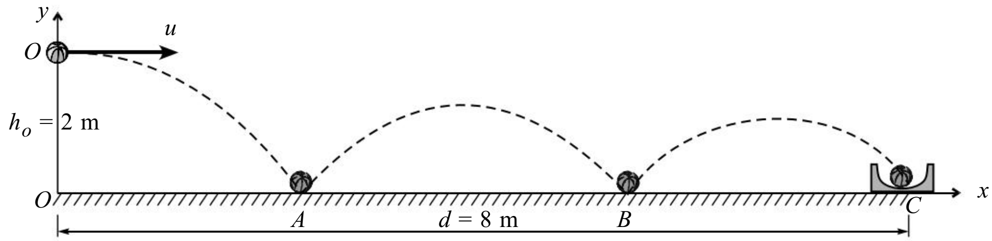


Fig. 16.14

Solution

(i) Consider motion from *O* to *A*

$$u_{1y} = \sqrt{2 \times 9.81 \times 2} \qquad \therefore\ u_{1y} = 6.264 \text{ m/s} \quad (\downarrow)$$

$$v_{1yA} = ?\ (\uparrow)\ ; \quad u_{2y} = v_{2y} = 0 \ \text{(velocity of flow before and after impact)}$$

(ii) Impact at *A*

$$e = -\left[\frac{v_{2y} - v_{1y}}{u_{2y} - u_{1y}}\right] = -\left[\frac{0 - v_{1yA}}{0 - (-6.264)}\right] = 0.8$$

$$v_{1yA} = 5.01 \text{ m/s} \ (\uparrow)$$

(iii) Impact at *B*

$$e = -\left[\frac{v_{2y} - v_{1y}}{u_{2y} - u_{1y}}\right] = -\left[\frac{0 - v_{1yB}}{0 - (-5.01)}\right] = 0.8$$

$$v_{1yB} = 4 \text{ m/s} \ (\uparrow)$$

(iv) Time from *O* to *A*

$$h = u\,t + \frac{1}{2}gt^2$$

$$2 = 0 + \frac{1}{2} \times 9.81 \times t_1^2$$

$$t_1 = 0.6386 \text{ seconds}$$

(v) Time from *A* to *B*

$$h = u\,t + \frac{1}{2}gt^2$$

$$0 = 5.01 \times t_2 - \frac{1}{2} \times 9.81 \times t_2^2$$

$$t_2 = 1.021 \text{ seconds}$$

(vi) Time from *B* to *C*

$$h = u\,t + \frac{1}{2}gt^2$$

$$0 = 4 \times t_3 - \frac{1}{2} \times 9.81 \times t_3^2$$

$$t_3 = 0.8155 \text{ seconds}$$

(vii) Total time

$$t = t_1 + t_2 + t_3 = 0.6386 + 1.021 + 0.8155$$

$$t = 2.475 \text{ seconds}$$

(viii) For projectile motion and oblique impact component of velocity in horizontal direction through the motion remains constant.

$\therefore$ Displacement = Velocity × Time

$8 = u \times 2.475$

$\therefore\ u = 3.232 \text{ m/s} \ (\rightarrow)$ ***Ans.***

16.7 Solved Problems Based on Impulse and Momentum

Problem 15

A cannon gun is nested by three springs each of stiffness 250 kN/cm, as shown in Fig. 16.15(a). The gun fires a 500 kg shell with a muzzle velocity of 1000 m/s. Calculate the total recoil and the maximum force developed in each spring if the gun has a mass of 80,000 kg.

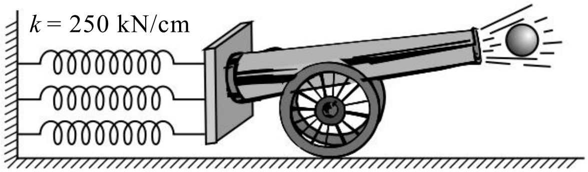


Fig. 16.15(a)

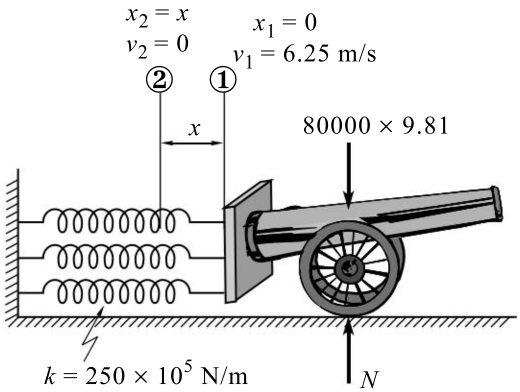


Fig. 16.15(b)

Solution

(i) By law of conservation of momentum, we have

Initial momentum = Final momentum

$$0 = 500 \times 1000 + 80000 \times v_{gun}$$

$$v_{gun} = -6.25 \text{ m/s}$$

$$\therefore\ v_{gun} = 6.25 \text{ m/s } (\leftarrow)$$ ***Ans.***

(iii) By work energy principal, we have

Work done = Change in kinetic energy

$$3\left[\frac{1}{2} \times 250 \times 10^5 (0^2 - x^2)\right] = 0 - \frac{1}{2} \times 80000 \times 6.25^2$$

$x = 0.204$ m (maximum compression of spring)

Spring force $F = kx$

$$\therefore\ F = 250 \times 10^5 \times 0.204$$

$$\therefore\ F = 51.025 \times 10^5 \text{ N}$$ ***Ans.***

Problem 16

Figure 16.16(a) shows a block of weight $W = 10$ N sliding down from rest on a rough inclined plane. The coefficient of friction $\mu = 0.2$ and $\theta = 30^\circ$. Calculate **(i)** the impulse of the forces acting in the interval $t = 0$ to $t = 5$ seconds, **(ii)** the velocity at the end of 5 sec and **(iii)** Distance covered by the block.

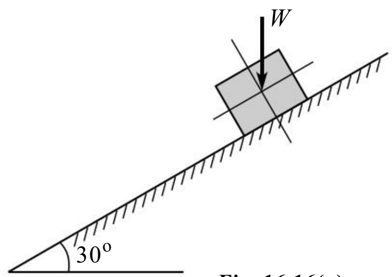


Fig. 16.16(a)

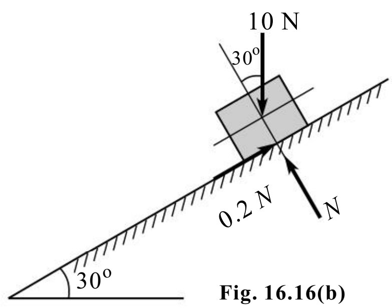


Fig. 16.16(b)

Solution

(i) Impulse = Net force × Time interval

$$\text{Impulse} = (10 \sin 30^\circ - 0.2 \times 10 \cos 30^\circ) \times 5$$

Impulse = 16.34 N-s ***Ans.***

(ii) By impulse momentum principal, we have

Impulse = Change in momentum

$$16.34 = \frac{10}{9.81}(v - 0)$$

$$v = 16.03 \text{ m/s} \quad (30^\circ) \quad \textbf{\textit{Ans.}}$$

(iii) $$d = \left(\frac{u+v}{2}\right) t$$

$$d = \frac{0 + 16.03}{2} \times 5$$

$$d = 40.075 \text{ m} \quad \textbf{\textit{Ans.}}$$

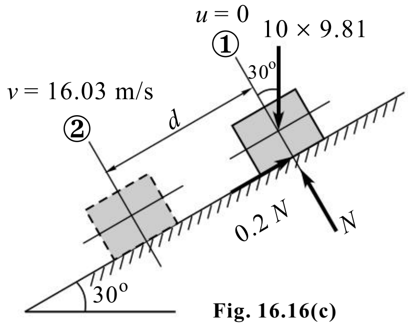


Fig. 16.16(c)

Problem 17

A ball of mass 110 gm is moving towards a batsman with a velocity of 24 m/s as shown in Fig. 16.17(a). The batsman hits the ball by the bat, the ball attains a velocity of 36 m/s. If ball and bat are in contact for a period of 0.015 seconds, determine the average impulsive force exerted on the ball during the impact.

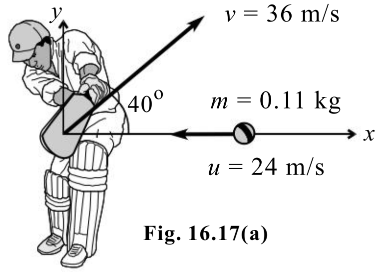


Fig. 16.17(a)

Solution

(i) Velocities before impact and after impact

$$u_x = 24 \text{ m/s } (\leftarrow)$$

$$u_y = 0$$

$$v_x = 36 \cos 40^\circ \text{ m/s } (\rightarrow)$$

$$v_y = 36 \sin 40^\circ \text{ m/s } (\uparrow)$$

(ii) By impulse momentum principle, we have

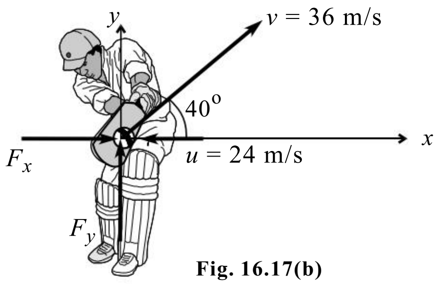


Fig. 16.17(b)

$$F_x \times t = m\,(v_x - u_x)$$

$$F_x \times 0.015 = 0.11 \times [36 \cos 40^\circ - (-24)]$$

$$\therefore \; F_x = 378.24 \text{ N } (\rightarrow)$$

$$F_y \times t = m\,(v_y - u_y)$$

$$F_y \times 0.015 = 0.11 \times [36 \sin 40^\circ - 0]$$

$$\therefore \; F_y = 169.7 \text{ N } (\uparrow)$$

(iii) Resultant force exerted

$$\tan\theta = \frac{F_y}{F_x} = \frac{169.7}{378.24}$$

$$\theta = 24.16^\circ$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{378.24^2 + 169.7^2}$$

$$F = 414.56 \text{ N} \quad (24.16^\circ) \quad \textbf{\textit{Ans.}}$$

Problem 18

A boy of mass 60 kg and a girl of mass 50 kg dive off the end of a boat of mass 160 kg with a horizontal velocity of 2 m/s relative to the boat, as shown in Fig. 16.18. Considering the boat to be initially at rest, find its velocity just after **(i)** both the boy and girls dive off simultaneously and **(ii)** the boy dives first followed by the girl.

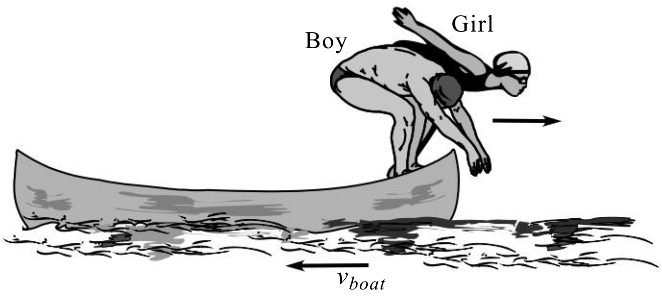


Fig. 16.18

Solution

(i) Both boy and girl dive off simultaneously

When boy and girl will jump together towards the right, the boat will move in opposite direction, i.e., towards left.

Here, velocity of boy and girl is 2 m/s relative to the boat

$$\therefore\ v_{boy/boat} = v_{boy} - v_{boat}$$

$$2 = v_{boy} - (-v_{boat})$$

$$v_{boy} = 2 - v_{boat}$$

and $v_{girl/boat} = v_{girl} - v_{boat}$

$$2 = v_{girl} - (-v_{boat})$$

$$v_{girl} = 2 - v_{boat}$$

By conservation of momentum principle to the system of boy, girl and boat

Initial momentum = Final momentum

$$0 = (\text{mass} \times \text{velocity})_{boy} + (\text{mass} \times \text{velocity})_{girl} + (\text{mass} \times \text{velocity})_{boat}$$

$$0 = 60(2 - v_{boat}) + 50(2 - v_{boat}) + 160(-v_{boat})$$

$$0 = 120 - 60v_{boat} + 100 - 50v_{boat} - 160v_{boat}$$

$$-220 = -270v_{boat}$$

$$v_{boat} = 1.227 \text{ m/s } (\leftarrow)$$ ***Ans.***

(ii) The boy dives first followed by the girl

Here, if boy is jumping first and girl is still on boat.

By conservation of momentum principle

Initial momentum = Final momentum

$$0 = (\text{Mass} \times \text{Velocity})_{boy} + (\text{Mass} \times \text{Velocity})_{boat}$$

$$0 = 60(2 - v_{boat}) + 160(-v_{boat})$$

$$0 = 120 - 60v_{boat} - 160v_{boat}$$

$$-220v_{boat} = -120$$

$$v_{boat} = 0.5455 \text{ m/s } (\leftarrow)$$ ***Ans.***

Later, the girl jumps from the boat when the boat is moving back with velocity 0.5455 m/s.

By conservation of momentum principle

Initial momentum = Final momentum

$$(\text{mass} \times \text{velocity})_{boat} = (\text{mass} \times \text{velocity})_{girl} + (\text{mass} \times \text{velocity})_{boat}$$

$$160(-0.5455) = 50(2 - v_{boat}) + 160(-v_{boat})$$

$$-87.28 = 100 - 50v_{boat} - 160v_{boat}$$

$$-210v_{boat} = -187.28$$

$$v_{boat} = 0.8918 \text{ m/s } (\leftarrow)$$ ***Ans.***

Problem 19

A boy having a mass of 60 kg and a girl having a mass of 50 kg stand motionless at the end of a boat, which has a mass of 30 kg, as shown in Fig. 16.19. If they exchange their positions, determine the final positions of boat. Neglect friction.

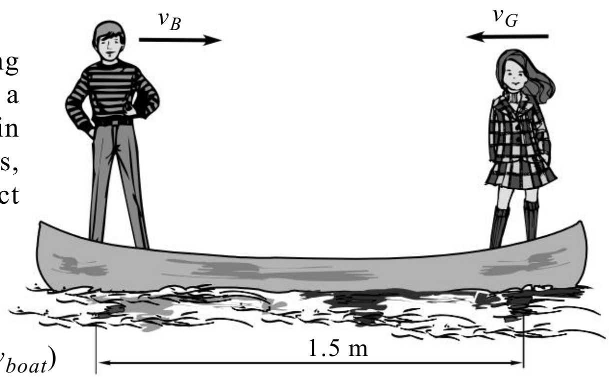


Fig. 16.19

Solution

Initial momentum = Final momentum

$$0 = m_B v_B + m_G(-v_G) + (m_B + m_G + m_{boat})(v_{boat})$$

$$0 = 60 \times \left(\frac{1.5}{t}\right) - 50 \times \left(\frac{1.5}{t}\right) + (60 + 50 + 30) \times \left(\frac{x}{t}\right)$$

$$x = -0.107 \text{ m}$$

$\therefore\ x = 0.107 \text{ m } (\leftarrow)$ (Backward displacement of boat) ***Ans.***

Problem 20

A particle of mass 1 kg is acted upon by a force F which varies, as shown in Fig. 16.20(a). If initial velocity of the particle is 10 m/s determine **(i)** what is the maximum velocity attained by the particle and **(ii)** the time when particle will be at point of reversal.

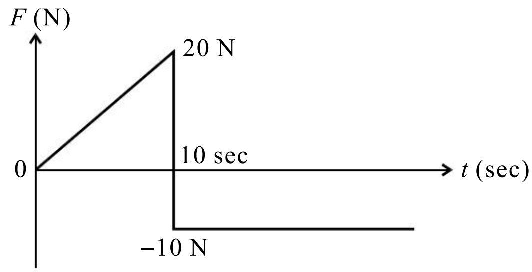


Fig. 16.20(a)

Solution

(i) For v_{max}

At force F = 20 N the velocity of particle will be maximum

$$\text{Change in velocity} = \frac{\text{Area under } F\text{-}t \text{ diagram}}{\text{Mass}}$$

$$v_{max} - 10 = \frac{\frac{1}{2} \times 10 \times 20}{1}$$

$\therefore\ v_{max} = 110$ m/s ***Ans.***

(ii) Let t be the time when particle will be at the point of reversal, where velocity $v = 0$

$$\text{Change in velocity} = \frac{\text{Area under } F\text{-}t \text{ diagram}}{\text{Mass}}$$

$$0 - 10 = \frac{\frac{1}{2} \times 10 \times 20 - 10(t - 10)}{1}$$

$$-10 = 100 - 10t + 100 \Rightarrow 10t = 210$$

$t = 21$ seconds ***Ans.***

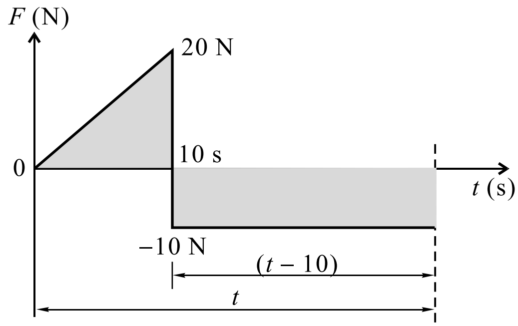


Fig. 16.20(b)

Problem 21

A body which is initially at rest, at the origin is subjected to a force varying with time, as shown in Fig. 16.21(a). Find the time **(i)** when the body again comes to rest and **(ii)** when it comes again to its original position.

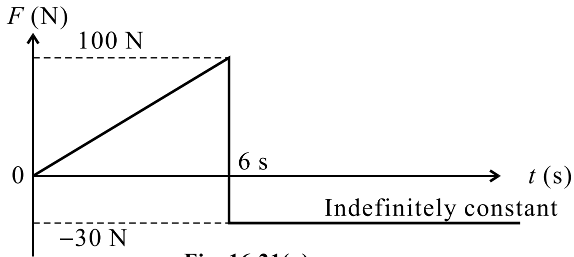


Fig. 16.21(a)

Solution

Let t be the time taken by the body to come to rest

(i) $\int F\, dt = 0$

Area under F-t curve

$$\frac{1}{2} \times 6 \times 100 + (-30)(t - 6) = 0$$

$t = 16$ seconds ***Ans.***

At $t = 16$ seconds body will again come to rest.

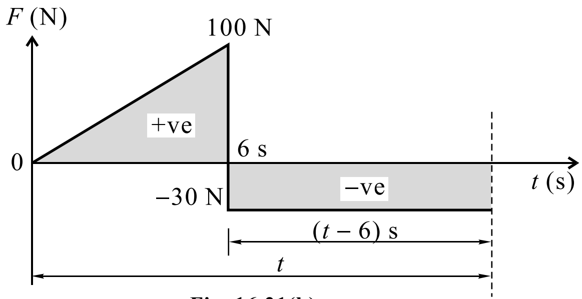


Fig. 16.21(b)

(ii) Let T be the time when body again comes to its original position (i.e., $s = 0$)

Displacement = Moment of area of F-t curve about P

$$0 = \left(\frac{1}{2} \times 6 \times 100\right)(T - 4) - 30(T - 6)\left(\frac{T - 6}{2}\right)$$

$T = 27.83$ seconds ***Ans.***

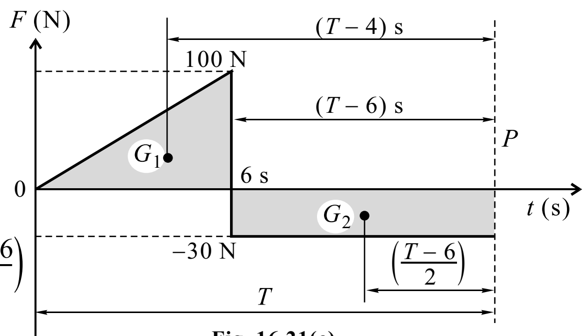


Fig. 16.21(c)