2.4 Mathematical Operations on Continuous Time Signals

2.4.1 Scaling of Continuous Time Signals

The two types of scaling continuous time signals are,

1. Amplitude Scaling
2. Time Scaling

1. Amplitude Scaling

The ***amplitude scaling*** is performed by multiplying the amplitude of the signal by a constant.

Let x(t) be a continuous time signal. Now Ax(t) is the amplitude scaled version of x(t), where A is a constant.

When $|A| > 1$, then Ax(t) is the amplitude magnified version of x(t) and when $|A| < 1$, then Ax(t) is the amplitude attenuated version of x(t).

Example : 1

Let, $x(t) = at + be^{-ct}$

Let $x_1(t)$ and $x_2(t)$ be the amplitude scaled versions of x(t), scaled by constants 4 and $\frac{1}{4}\left(\frac{1}{4} = 0.25\right)$ respectively.

Now, $x_1(t) = 4x(t) = 4(at + be^{-ct}) = 4at + 4be^{-ct}$

$x_2(t) = 0.25x(t) = 0.25(at + be^{-ct}) = 0.25at + 0.25be^{-ct}$

Example : 2

A continuous time signal and its amplitude scaled version are shown in fig 2.20.

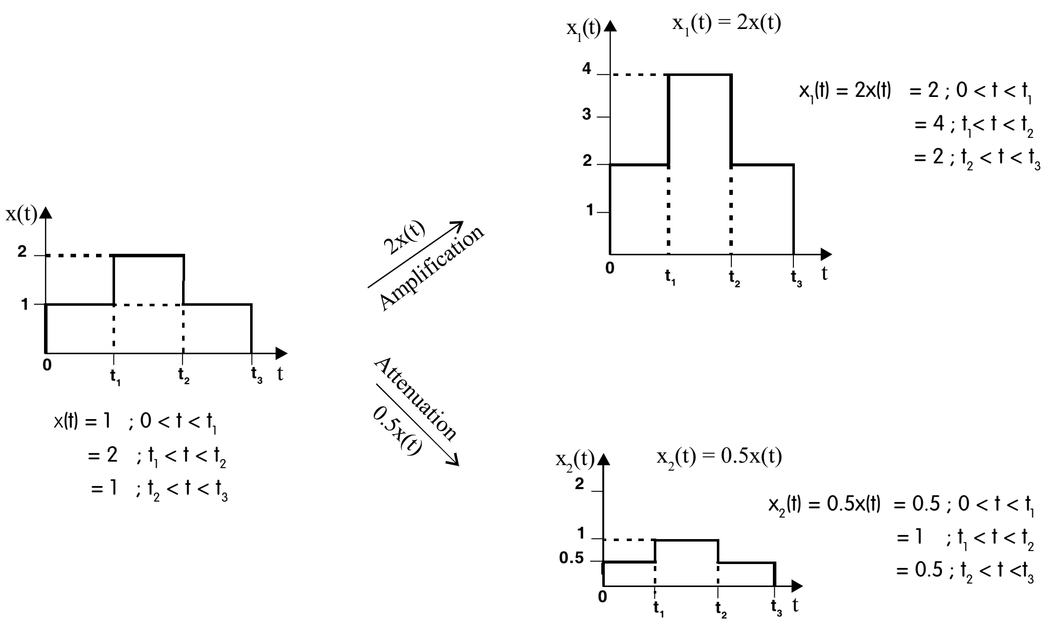


***Fig 2.20 :** A continuous time signal and its amplitude scaled version.*

2. *Time Scaling*

The ***time scaling*** is performed by multiplying the variable time by a constant.

If x(t) is a continuous time signal, then x(At) is the time scaled version of x(t), where A is a constant.

When $|A| > 1$, then x(At) is the time compressed version of x(t) and when $|A| < 1$, then x(At) is the time expanded version of x(t).

Example : 1

Let, $x(t) = at + be^{-ct}$

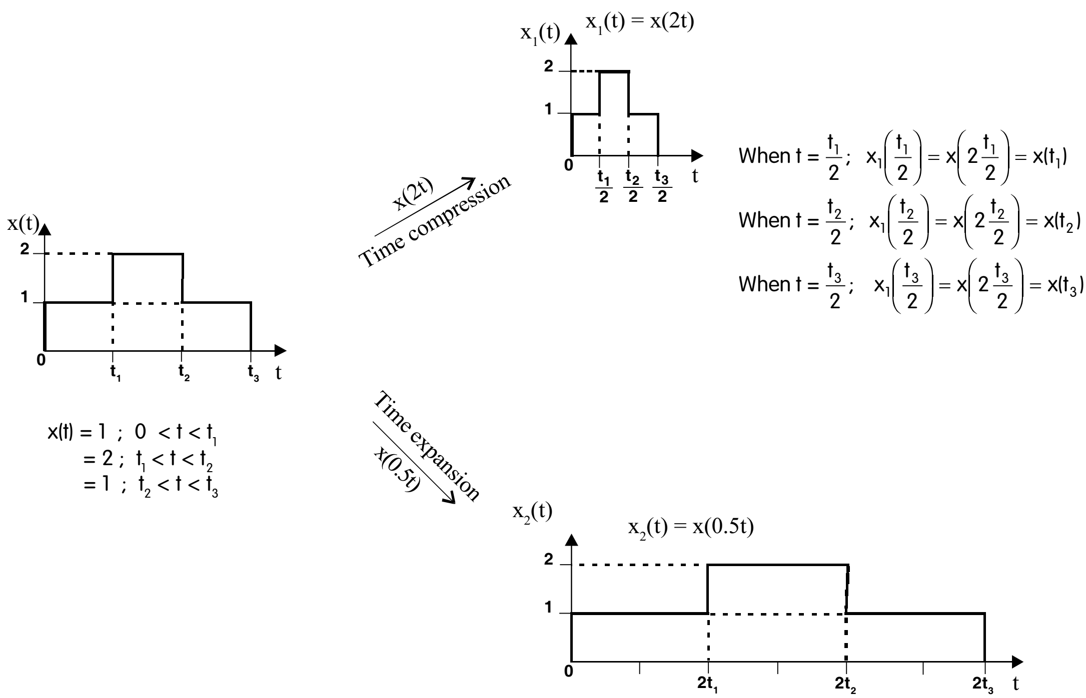
Let $x_1(t)$ and $x_2(t)$ be the time scaled versions of x(t), scaled by constants 4 and 1/4 (0.25)respectively.

Now, $x_1(t) = x(4t) = a \times 4t + be^{-c \times 4t} = 4at + be^{-4ct}$

$x_2(t) = x(0.25t) = a \times 0.25t + be^{-c \times 0.25t} = 0.25at + be^{-0.25ct}$

Example : 2

A continuous time signal and its time scaled version are shown in fig 2.21.



$x(t) = 1 \;;\; 0 < t < t_1$
$\quad = 2 \;;\; t_1 < t < t_2$
$\quad = 1 \;;\; t_2 < t < t_3$

When $t = \frac{t_1}{2}$; $x_1\left(\frac{t_1}{2}\right) = x\left(2\frac{t_1}{2}\right) = x(t_1)$

When $t = \frac{t_2}{2}$; $x_1\left(\frac{t_2}{2}\right) = x\left(2\frac{t_2}{2}\right) = x(t_2)$

When $t = \frac{t_3}{2}$; $x_1\left(\frac{t_3}{2}\right) = x\left(2\frac{t_3}{2}\right) = x(t_3)$

When $t = 2t_1$; $x_1(2t_1) = x(0.5 \times 2t_1) = x(t_1)$
When $t = 2t_2$; $x_1(2t_2) = x(0.5 \times 2t_2) = x(t_2)$
When $t = 2t_3$; $x_1(2t_3) = x(0.5 \times 2t_3) = x(t_3)$

Fig 2.21 : A continuous time signal and its time scaled version.

2.4.2 Folding (Reflection or Transpose) of Continuous Time Signals

The ***folding*** of a continuous time signal x(t) is performed by changing the sign of time base t in the signal x(t).

The folding operation produces a signal x(–t) which is a mirror image of the original signal x(t) with respect to the time origin t = 0.

Example : 1

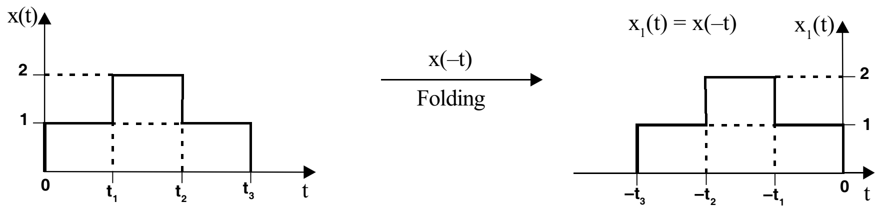
Let, $x(t) = at + be^{-ct}$

Let $x_1(t)$ be folded version of x(t).

Now, $x_1(t) = x(-t) = a(-t) + be^{-c(-t)} = -at + be^{ct}$

Example : 2

A continuous time signal and its folded version is shown in fig 2.22.



$x(t) = 1\ ;\quad 0 < t < t_1$

$\quad\;\; = 2\ ;\quad t_1 < t < t_2$

$\quad\;\; = 1\ ;\quad t_2 < t < t_3$

When $t = -t_1$; $x_1(-t_1) = x(-(-t_1)) = x(t_1)$

When $t = -t_2$; $x_1(-t_2) = x(-(-t_2)) = x(t_2)$

When $t = -t_3$; $x_1(-t_3) = x(-(-t_3)) = x(t_3)$

***Fig 2.22 :** A continuous time signal and its folded version.*

2.4.3 Time Shifting of Continuous Time Signals

The ***time shifting*** of a continuous time signal x(t) is performed by replacing the independent variable t by t – m, to get the time shifted signal x(t – m), where m represents the time shift in seconds.

In x(t – m), if m is positive, then the time shift results in a delay by m seconds. The ***delay*** results in shifting the original signal x(t) to right, to generate the time shifted signal x(t – m).

In x(t – m), if m is negative, then the time shift results in an advance of the signal by |m| seconds. The ***advance*** results in shifting the original signal x(t) to left, to generate the time shifted signal x(t – m).

Example : 1

Let, $x(t) = at + be^{-ct}$

Let $x_1(t)$ and $x_2(t)$ be time shifted version of x(t), shifted by m units of time.

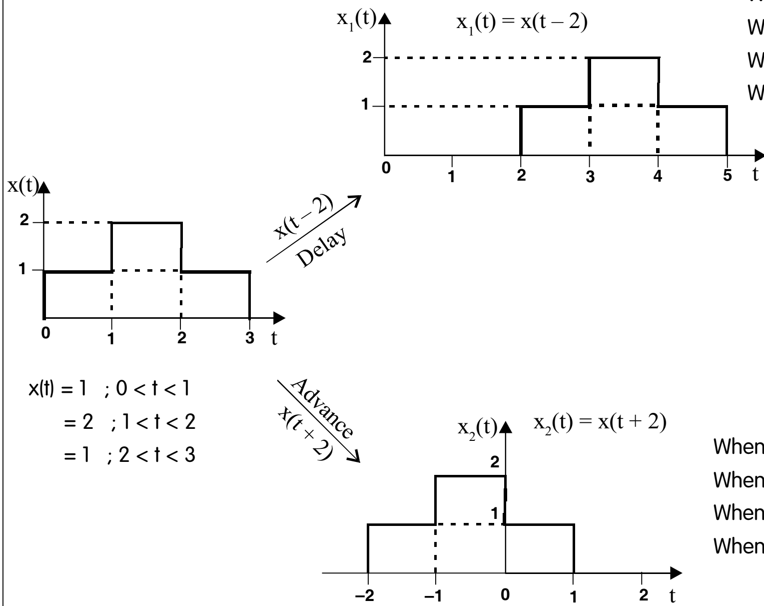
Let $x_1(t)$ be delayed version of x(t) and $x_2(t)$ be advanced version of x(t).

Now, $x_1(t) = a(t - m) + be^{-c(t-m)}$

$x_2(t) = a(t + m) + be^{-c(t+m)}$

Example : 2

A signal and its shifted version are shown in fig 2.23.



When t = 2; $x_1(2) = x(2-2) = x(0) = 1$
When t = 3; $x_1(3) = x(3-2) = x(1) = 1$
When t = 4; $x_1(4) = x(4-2) = x(2) = 2$
When t = 5; $x_1(5) = x(5-2) = x(3) = 1$

$x(t) = 1 \; ; 0 < t < 1$
$\quad = 2 \; ; 1 < t < 2$
$\quad = 1 \; ; 2 < t < 3$

When t = –2 ; $x_1(-2) = x(-2+2) = x(0) = 1$
When t = –1 ; $x_1(-1) = x(-1+2) = x(1) = 1$
When t = 0 ; $x_1(0) = x(0+2) = x(2) = 2$
When t = 1 ; $x_1(1) = x(1+2) = x(3) = 1$

***Fig 2.23 :** A continuous time signal and its shifted version.*

Delayed Unit Impulse Signal

The unit impulse signal is defined as,

$$\delta(t) = \infty \; ; t = 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$
$$= 0 \; ; \; t \neq 0$$

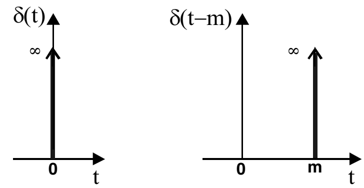


***Fig 2.24a** : Impulse.* ***Fig 2.24b** : Delayed impulse.*
***Fig 2.24 :** Impulse and delayed impulse signal.*

The unit impulse signal delayed by m units of time is denoted as $\delta(t-m)$, and it is defined as,

$$\delta(t-m) = \infty; \; t = m \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t-m)\, dt = 1$$
$$= 0 \; ; \; t \neq m$$

Delayed Unit Step Signal

The unit step signal is defined as,

$$u(t) = 1 \; ; \text{ for } \; t \geq 0$$
$$= 0 \; ; \text{ for } \; t < 0$$

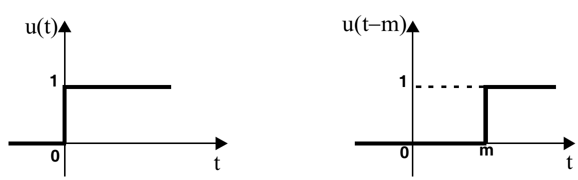


***Fig 2.25a** : Unit step signal.* ***Fig 2.25b** : Delayed unit step signal.*
***Fig 2.25 :** Unit step and delayed unit step signal.*

The unit step signal delayed by m units of time is denoted as $u(t-m)$, and it is defined as,

$$u(t-m) = 1 \; ; t \geq m$$
$$= 0 \; ; t < m$$

2.4.4 Addition of Continuous Time Signals

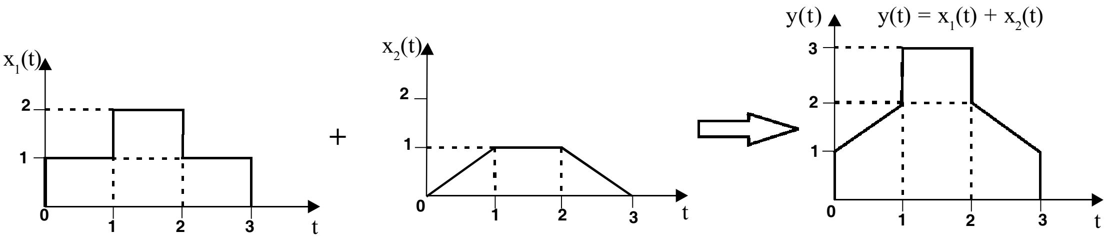
The ***addition*** of two continuous time signals is performed by adding the value of the two signals corresponding to the same instant of time.

The sum of two signals $x_1(t)$ and $x_2(t)$ is a signal y(t), whose value at any instant is equal to the sum of the value of these two signals at that instant.

i.e., $y(t) = x_1(t) + x_2(t)$

Example :

Graphical addition of two continuous time signals is shown in fig 2.26.



$x_1(t) = 1 \;\; ; 0 < t < 1$
$\quad = 2 \;\; ; 1 < t < 2$
$\quad = 1 \;\; ; 2 < t < 3$

$x_2(t) = t \;\; ; 0 < t < 1$
$\quad = 1 \;\; ; 1 < t < 2$
$\quad = 3 - t \; ; 2 < t < 3$

For t = 0 to 1 ; $y(t) = x_1(t) + x_2(t) = 1 + t$
For t = 1 to 2 ; $y(t) = x_1(t) + x_2(t) = 2 + 1 = 3$
For t = 2 to 3 ; $y(t) = x_1(t) + x_2(t) = 1 + 3 - t = 4 - t$

Fig 2.26 : *Addition of two continuous time signals.*

2.4.5 Multiplication of Continuous Time Signals

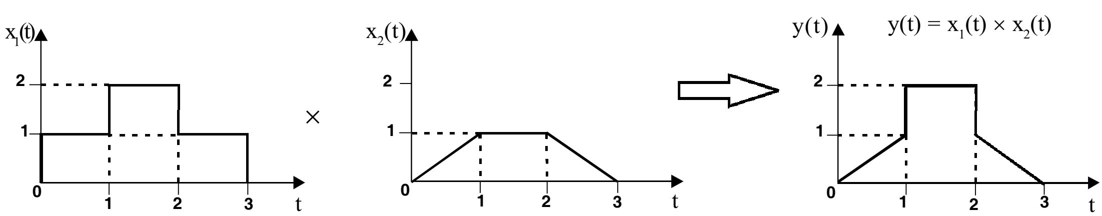
The ***multiplication*** of two continuous time signals is performed by multiplying the value of the two signals corresponding to the same instant of time.

The product of two signals $x_1(t)$ and $x_2(t)$ is a signal y(t), whose value at any instant is equal to the product of the values of these two signals at that instant.

i.e., $y(t) = x_1(t) \times x_2(t)$

Example :

Graphical multiplication of two continuous time signals is shown in fig 2.27.



$x_1(t) = 1 \; ; 0 < t < 1$
$\quad = 2 \; ; 1 < t < 2$
$\quad = 1 \; ; 2 < t < 3$

$x_2(t) = t \; ; \; 0 < t < 1$
$\quad = 1 \; ; 1 < t < 2$
$\quad = 3 - t \; ; 2 < t < 3$

For t = 0 to 1; $y(t) = x_1(t) \times x_2(t) = 1$ ´ $t = t$
For t = 1 to 2; $y(t) = x_1(t) \times x_2(t) = 2$ ´ $1 = 2$
For t = 2 to 3; $y(t) = x_1(t) \times x_2(t) = 1$ ´ $(3 - t) = 3 - t$

Fig 2.27 : *Multiplication of two continuous time signals.*

2.4.6 Differentiation and Integration of Continuous Time Signals

Differentiation is a mathematical operation used to estimate the rate of change of a continuous time signal at any instant of time.

Differentiation is denoted by the operator $\frac{d}{dt}$.

Therefore, the differentiation of a continuous time signal x(t) is denoted by $\frac{d}{dt}x(t)\left(\text{or }\frac{dx(t)}{dt}\right)$.

The differentiation of a continuous time signal x(t) is defined as,

$$\frac{dx(t)}{dt} = \underset{\Delta t \to 0}{Lt} \frac{x(t) - x(t-\Delta t)}{\Delta t}$$

Integration, is the inverse process of differentiation. More appropriately. Integration is the process of identifying the signal from its differentiation.

The integration is denoted by the operaor $\int .... dt$

Therefore, the integration of a continuous time signal x(t) is denoted by $\int x(t)\, dt$.

Differentiation and integration of standard continuous time signals are listed in table 2.1

Table 2.1 : Differentiation and Integration of Standard Continuous Time Signals

Signal, x(t)	**Differentiation of x(t), $\frac{dx(t)}{dt}$**	**Integration of x(t), $\int x(t)\, dt$**
$\delta(t)$	—	1
$u(t)$	$\delta(t)$	t
t	$u(t)$	$\frac{t^2}{2}$
t^2	2t	$\frac{t^3}{3}$
$\sin t$	$\cos t$	$-\cos t$
$\cos t$	$-\sin t$	$\sin t$
e^{-at}	$-ae^{-at}$	$\frac{e^{-at}}{-a}$
e^{at}	ae^{at}	$\frac{e^{at}}{a}$
$\sin\Omega_0 t$	$\Omega_0 \cos\Omega_0 t$	$\frac{-\cos\Omega_0 t}{\Omega_0}$
$\cos\Omega_0 t$	$-\Omega_0 \sin\Omega_0 t$	$\frac{\sin\Omega_0 t}{\Omega_0}$

The standard signals such as impulse, step, ramp and parabolic signals are related through integration and differentiation as shown below.

$$\underset{\text{(Impulse)}}{\delta(t)} \xrightarrow{\text{integration}} \underset{\text{(Unit step)}}{u(t)} \xrightarrow{\text{integration}} \underset{\text{(Unit ramp)}}{t} \xrightarrow{\text{integration}} \underset{\text{(Unit parabolic)}}{\frac{t^2}{2}}$$

$$\underset{\text{(Impulse)}}{\delta(t)} \xleftarrow{\text{differentiation}} \underset{\text{(Unit step)}}{u(t)} \xleftarrow{\text{differentiation}} \underset{\text{(Unit ramp)}}{t} \xleftarrow{\text{differentiation}} \underset{\text{(Unit parabolic)}}{\frac{t^2}{2}}$$

Note : $$\left.\frac{d}{dt}u(t)\right|_{t=0} = \left.\frac{\Delta u}{\Delta t}\right|_{t=0} = \left.\frac{u(0^+) - u(0^-)}{\Delta t}\right|_{t=0} = \left.\frac{1-0}{0}\right|_{t=0} = \infty\Big|_{t=0} = Impulse$$

Example 2.5

A continuous time signal is defined as,

$$x(t) = t \quad ; \; 0 \le t \le 3$$
$$\quad = 0 \quad ; \; t > 3$$

Sketch the waveform of $x(-t)$ and $x(2-t)$.

Solution

The given signal is shown in fig 1.

The signal $x(-t)$ is the folded version of $x(t)$. The signal $x(-t)$ is shown in fig 2.

The signal $x(2-t) = x(-t+2)$ is the advanced version of the folded signal. The signal $x(-t+2)$ is shown in fig 3.

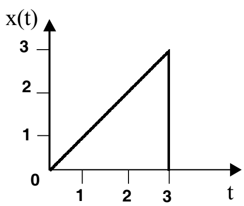


Fig 1 : x(t).

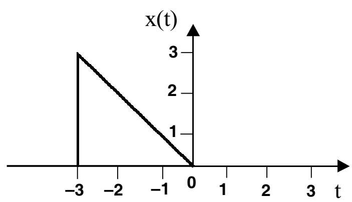


Fig 2 : x(−t).

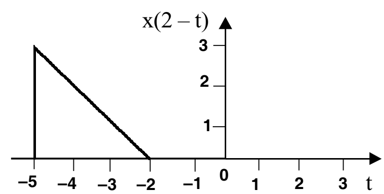
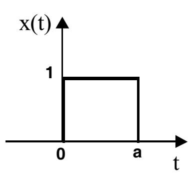


Fig 3 : x(2 − t).

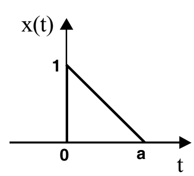
Example 2.6

Sketch the even and odd parts of the following signals.

a)



b)

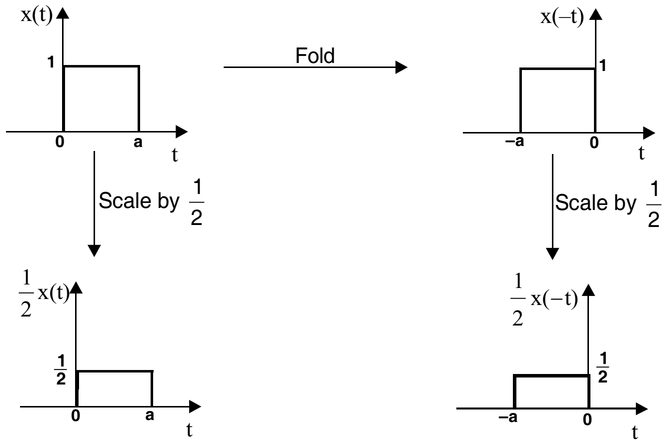


Solution

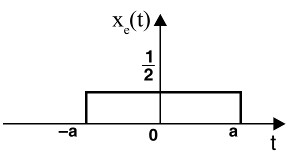
a) The even part of the signal is given by, $$x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}\,x(t) + \frac{1}{2}x(-t) \quad .....(1)$$

The odd part of the signal is given by, $$x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}\,x(t) - \frac{1}{2}x(-t) \quad .....(2)$$

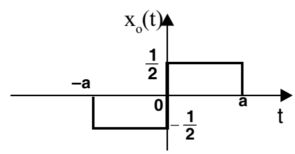
From equations (1) and (2), it is observed that the even and odd parts of the signal can be obtained from the folded and scaled versions of the signal. Hence the given signal is folded, scaled and then graphically added and subtracted to get the even and odd parts as shown below.



$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}\,x(t) + \frac{1}{2}x(-t)$$



$$x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}\,x(t) - \frac{1}{2}x(-t)$$



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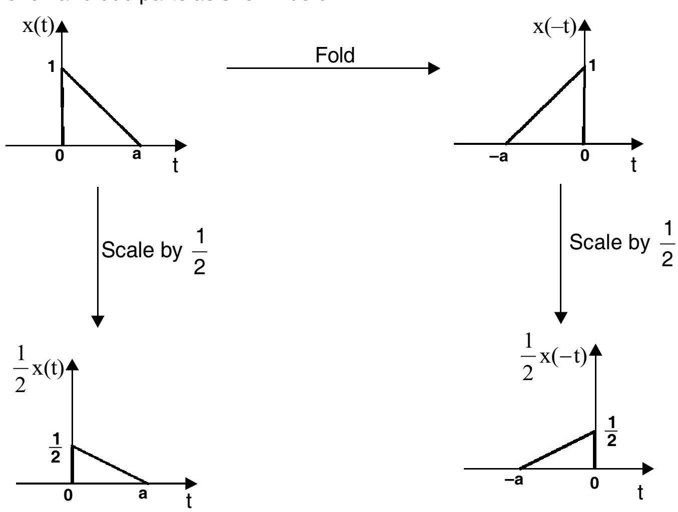
b) The even part of the signal is given by, $x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}\,x(t) + \frac{1}{2}x(-t)$ .....(1)

The odd part of the signal is given by, $x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}\,x(t) - \frac{1}{2}x(-t)$ .....(2)

From equations (1) and (2), it is observed that the even and odd parts of the signal can be obtained from the folded and scaled versions of the signal. Hence the given signal is folded, scaled and then graphically added and subtracted to get the even and odd parts as shown below.



Program 2.4

Write a MATLAB program to perform Amplitude scaling, Time scaling, and Time shift on the signal x(t) = 1+t; for t = 0 to 2.

Program to declare the given signal as function y(t)

```
% declare the given signal as function y(t)
function x = y(t)
x=(1.0 + t).*(t>=0 & t<=2);
```

Note: The above program should be stored as a separate file in the current working directory

Program to perform amplitude & time scaling and time shift on y(t)

```
%To perform Amplitude scaling, Time scaling and Time shift
%on the signal x(t)=1.0+t; for t= 0 to 2
%include y.m file in work directory which declare the given signal
%as function y(t)

tmin=-3; tmax=5; dt=0.2;
t=tmin:dt:tmax;     %set a time vector

y0 =y(t);           %assign the given signal as y0
y1 =1.5*y(t);       %compute the amplified version of x(t)
y2 =0.5*y(t);       %compute the attenuated version of x(t)
y3=y(2*t);          %compute the time compressed version of x(t)
y4=y(0.5*t);        %compute the time expanded version of x(t)
y5=y(t-2);          %compute the delayed version of x(t)
y6=y(t+2);          %compute the advanced version of x(t)

%compute the min and max value for y-axis
ymin=min([min(y0), min(y1), min(y2), min(y3), min(y4),
min(y5),min(y6)]);
ymax=max([max(y0), max(y1), max(y2), max(y3), max(y4),
max(y5),max(y6)]);

%plot the given signal and amplitude scaled signal
subplot(3,3,1);plot(t,y0);axis([tmin tmax ymin ymax]);
xlabel('t');ylabel('x(t)');title('Signal x(t)');
subplot(3,3,2);plot(t,y1);axis([tmin tmax ymin ymax]);
xlabel('t');ylabel('x1(t)');title('Amplified signal 1.5x(t)');
subplot(3,3,3);plot(t,y2);axis([tmin tmax ymin ymax]);
xlabel('t');ylabel('x2(t)');title('Attenuated signal 0.5x(t)');

%plot the given signal and time scaled signal
subplot(3,3,4);plot(t,y0);axis([tmin tmax ymin ymax]);
xlabel('t');ylabel('x(t)');title('Signal x(t)');
subplot(3,3,5);plot(t,y3);axis([tmin tmax ymin ymax]);
xlabel('t');ylabel('x3(t)');title('Time comp. signal x(2t)');
subplot(3,3,6);plot(t,y4);axis([tmin tmax ymin ymax]);
xlabel('t');ylabel('x4(t)');title('Time expan. signal x(0.5t)');

%plot the given signal and time shifted signal
subplot(3,3,7);plot(t,y0);axis([tmin tmax ymin ymax]);
xlabel('t');ylabel('x(t)');title('Signal x(t)');
subplot(3,3,8);plot(t,y5);axis([tmin tmax ymin ymax]);
xlabel('t');ylabel('x5(t)');title('Delayed signal x(t-2)');
subplot(3,3,9);plot(t,y6);axis([tmin tmax ymin ymax]);
xlabel('t');ylabel('x6(t)');title('Advanced signal x(t+2)');
```

OUTPUT

The input and output waveforms of program 2.4 are shown in fig P2.4.

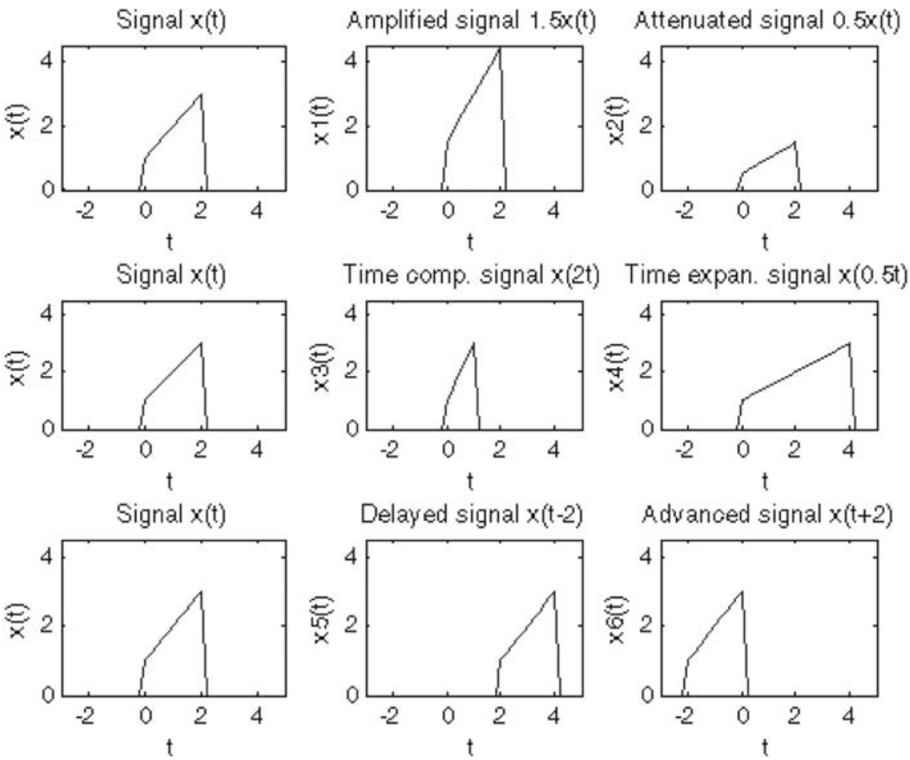


Fig P2.4 : Input and Output waveforms of program 2.4.

Program 2.5

Write a MATLAB program to perform addition and multiplication on the following two signals.

```
xa(t)=1; 0<t<1          xb(t)=t;   0<t<1
     =2; 1<t<2               =1;   1<t<2
     =1; 2<t<3               =3–t; 2<t<3
```

```
%To perform addition and multiplication of the following two signals
%1) xa(t)=1;0<t<1           2) xb(t)=t;0<t<1
%         =2;1<t<2                    =1;1<t<2
%         =1;2<t<3                    =3-t;2<t<3

tmin=-1; tmax=5; dt=0.1;
t=tmin:dt:tmax;    %Set a time vector

x1=1;
x2=2;
x3=3-t;
xa=x1.*(t>0&t<1)+x2.*(t>=1&t<=2)+x1.*(t>2&t<3);
xb=t.*(t>0&t<1)+x1.*(t>=1&t<=2)+x3.*(t>2&t<3);
xadd=xa+xb;          %Add the two signals
xmul=xa.*xb;         %Multiply two signals

xmin=min([min(xa), min(xb), min(xadd), min(xmul)]);
xmax=max([max(xa), max(xb), max(xadd), max(xmul)]);
subplot(2,3,1);plot(t,xa);axis([tmin tmax xmin xmax]);
xlabel(‘t’);ylabel(‘xa(t)’);title(‘Signal xa(t)’);
subplot(2,3,2);plot(t,xb);axis([tmin tmax xmin xmax]);
xlabel(‘t’);ylabel(‘xb(t)’);title(‘Signal xb(t)’);
```

```
subplot(2,3,3);plot(t,xadd);axis([tmin tmax xmin xmax]);
xlabel('t');ylabel('xadd(t)');title('Sum of xa(t) and xb(t)');
subplot(2,3,4);plot(t,xa);axis([tmin tmax xmin xmax]);
xlabel('t');ylabel('xa(t)');title('Signal xa(t)');

subplot(2,3,5);plot(t,xb);axis([tmin tmax xmin xmax]);
xlabel('t');ylabel('xb(t)');title('Signal xb(t)');
subplot(2,3,6);plot(t,xmul);axis([tmin tmax xmin xmax]);
xlabel('t');ylabel('xmul(t)');title('Product of xa(t) and xb(t)');
```

OUTPUT

The input and output waveforms of program 2.5 are shown in fig P2.5.

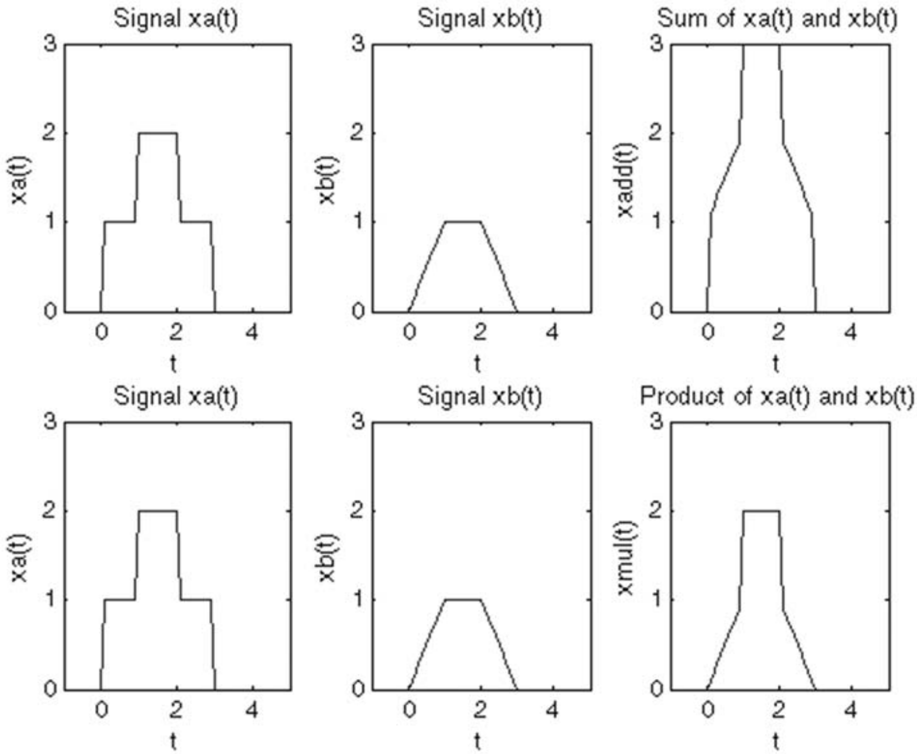


Fig P2.5 : *Input and Output waveforms of program 2.5.*

Program 2.6

Write a MATLAB program to perform convolution of the following two signals.

x1(t)=1; 1<t<10 x2(t)=1; 2<t<10

```
%*****************Program to perform convolution of two signals
%*****************x1(t)=1; t= 1 to 10 and x2(t)=1; t= 2 to 10

tmin=0; tmax=10; dt=0.01;
t=tmin:dt:tmax;                 %set time vector for given signal

x1=1.*(t>=1 & t<=10);           %generate signal x1(t)
x2=1.*(t>=2 & t<=10);           %generate signal x2(t)
```