SY. B.Tech Sem III
Branch - Computer
ITVS

Vector Differentiation
Practice Problems

DIV-A	DIV-B	Questions
1	64	Show that $[\bar{p}+\bar{q},\ \bar{q}+\bar{r},\ \bar{r}+\bar{p}]=(\bar{p}+\bar{q})\cdot[(\bar{q}+\bar{r})\times(\bar{r}+\bar{p})]=2[\bar{p}\ \bar{q}\ \bar{r}]$
2	65	If $\bar{l},\bar{m},\bar{n}$ are three non-coplanar vectors, prove that $[\bar{l}\ \bar{m}\ \bar{n}](\bar{a}\times\bar{b})=\begin{vmatrix}\bar{l}\cdot\bar{a} & \bar{l}\cdot\bar{b} & \bar{l}\\ \bar{m}\cdot\bar{a} & \bar{m}\cdot\bar{b} & \bar{m}\\ \bar{n}\cdot\bar{a} & \bar{n}\cdot\bar{b} & \bar{n}\end{vmatrix}$
3	66	Prove that the points (2, 1, 1), (0, 1, -3), (3, 2, -1) and (7, 2, 7) are coplanar.
4	67	Show that the vectors are coplanar: $2i-j+k,\ i+2j-3k,\ 3i+aj+5k\ \ if\ a=-4$
5	68	Prove that the points (1, 1, 1), (2, -1, 1), (3, 1, 2) and (5, 1, 3) are coplanar.
6	69	Prove that $i\times(\bar{a}\times i)+j\times(\bar{a}\times j)+k\times(\bar{a}\times k)=2\bar{a}$
7	70	Prove that $\bar{a}\times[\bar{b}\times(\bar{c}\times\bar{d})]=(\bar{b}\cdot\bar{d})(\bar{a}\times\bar{c})-(\bar{b}\cdot\bar{c})(\bar{a}\times\bar{d})$
8	71	Prove that $[\bar{b}\times\bar{c}\ \ \bar{c}\times\bar{a}\ \ \bar{a}\times\bar{b}]=[\bar{a}\ \bar{b}\ \bar{c}]^2$
9	72	Prove that $\bar{a}\times(\bar{b}\times\bar{c})+\bar{b}\times(\bar{c}\times\bar{a})+\bar{c}\times(\bar{a}\times\bar{b})=0$
10	73	Prove that $(\bar{a}\times\bar{b})\cdot(\bar{c}\times\bar{d})+(\bar{b}\times\bar{c})\cdot(\bar{a}\times\bar{d})+(\bar{c}\times\bar{a})\cdot(\bar{b}\times\bar{d})=0$
11	74	Prove that $(\bar{a}\times\bar{b})\cdot(\bar{c}\times\bar{d})=[\bar{a}\ \bar{c}\ \bar{d}]\bar{b}-[\bar{b}\ \bar{c}\ \bar{d}]\bar{a}$, where $\bar{a},\bar{b},\bar{c}$ are coplanar vectors.
12	75	Prove that $\bar{d}\cdot[\bar{a}\times[\bar{b}\times(\bar{c}\times\bar{d})]]=(\bar{b}\cdot\bar{d})[\bar{a}\ \bar{c}\ \bar{d}]$
13	76	If $\frac{d\bar{a}}{dt}=\bar{u}\times\bar{a}$ & $\frac{d\bar{b}}{dt}=\bar{u}\times\bar{b}$, prove that $\frac{d}{dt}[\bar{a}\times\bar{b}]=\bar{u}\times(\bar{a}\times\bar{b})$
14	77	If $\bar{r}=a\cos t\ i+a\sin t\ j+at\tan\alpha\ k$, show that $\left[\frac{d\bar{r}}{dt}\ \frac{d^2\bar{r}}{dt^2}\ \frac{d^3\bar{r}}{dt^3}\right]=a^3\tan\alpha$
15	78	Show that $\frac{d}{dt}\left[\bar{u}\ \frac{d\bar{u}}{dt}\ \frac{d^2\bar{u}}{dt^2}\right]=\left[\bar{u}\ \frac{d\bar{u}}{dt}\ \frac{d^3\bar{u}}{dt^3}\right]$
16	79	If $\bar{A}=(\sin t)i+(\cos t)j+tk,\ \bar{B}=(\cos t)i-(\sin t)j-3k,\ \bar{C}=2i+3j-k$ find $\frac{d}{dt}[\bar{A}\times(\bar{B}\times\bar{C})]$ at $t=0$
17	80	If $\bar{a}=i-2j-3k,\ \bar{b}=2i+j-k,\ \bar{c}=i+3j-k$ Find $\bar{a}\times(\bar{b}\times\bar{c})$ and $(\bar{a}\times\bar{b})\times\bar{c}$
GRADIENT and DIRECTIONAL DERIVATIVES		
18	81	Find $\nabla\phi$ if $\phi=3x^2y-y^3z^2$ at $(1,-2,1)$
19	82	Find the directional derivative of $\phi=x^2y+y^2z+z^2x^2$ at $P(1,2,1)$ in the direction of the normal to the surface $x^2+y^2-z^2x=1$ at $Q(1,1,1)$
20	83	Find the directional derivative of $\phi=2x^3y-3y^2z$ at $P(1,2,-1)$ in the direction towards $Q(3,-1,5)$. In what direction from P is the directional derivative maximum? Find the magnitude of maximum directional derivative.
21	84	Find the directional derivative of $\phi=x^4+y^4+z^4$ at $A(1,-2,1)$ in the direction of AB where B is $(2,6,-1)$. Also find the maximum directional derivative of ϕ at $(1,-2,1)$.
22	85	Find the directional derivative of $\phi=x^2y^2+y^2z^2+z^2x^2$ at $(1,1,-2)$ in the direction of the tangent to the curve $x=e^{-t},y=2\sin t+1,z=t-\cos t$ at $t=0$
23	86	Find the directional derivative of $\phi=e^{2x}\cos yz$ at (0, 0, 0) in the direction of the tangent to the curve $x=a\sin t,\ y=a\cos t,\ z=at$ at $t=\pi/4$.

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24	87	Find the directional derivative of $\phi = x^2 + y^2 + z^2$ in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ at $(1, 2, 3)$
25	88	Find the directional derivative of $\phi = x^2 y \cos z$ in the direction of the line $\bar{a} = 2i + 3j + 2k$ at $(1, 2, \pi/2)$
26	89	Find the acute angle between the surface $x^2 + y^2 + z^2 = 9$ *and* $z = x^2 + y^2 - 3$ *at* $(2, -1, 2)$
27	90	Find the angle between the two surfaces $x^2 + y^2 + a z^2 = 6$ *and* $z = 4 - y^2 + b x y$ *at* $(1, 1, 2)$
28	91	Find the rate of change of $\phi = xy + yz + zx$ *at* $(1, -1, 2)$ in the direction of the normal to the surface $x^2 + y^2 = z + 4$.
29	92	In what direction is the directional derivative of $\phi = 2xz - y^2$ *at* $(1, 3, 2)$ maximum? Find its magnitude.
30	93	Find the rate of change of $\phi = x y^2 + y z^3$ *at* $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ *at* $(-1, 2, 1)$
31	94	Find the angle between the normals to the surfaces $x^2 y + 2xz = 4$ *at* $(2, -2, 3)$ and to $x^3 + y^3 + 3xyz = 3$ *at* $(1, 2, -1)$
32	95	Find the constants a and b such that the surfaces $ax^2 - 2byz = (a+4)x$ will be orthogonal to the surface $4x^2 y + z^3 = 4$ *at* $(1, -1, 2)$
33	96	Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2 z + xy + y + 1 = z$ at (0, 1, 2) is $\cos^{-1}(1/\sqrt{3})$.
34	97	Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2 y + bz^3 = 4$ cut orthogonally at (1,-1, 2)
35	98	If the directional derivative of $\phi = ax^2 + by + 2z$ at (1, 1, 1) is maximum in the direction of $i + j + k$, find a &b.
36	99	Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the surface $x^2 + y^2 = 4 + xz$ at (1, 2, 1)
37	100	Find the directional derivative of $\emptyset = \frac{x}{x^2+y^2}$ at (0,1,1) in the direction of normal to the surface $x^2 + y^2 - z^2 x = 1$ at (1,1,1)
38	101	Find the constants a and b such that the directional derivative of $\emptyset = ax^2 + by^2 + cz^2$ at (1,1,2) has the maximum magnitude 4 in the direction parallel to x-axis.
DIFFERENTIAL OPERATORS		
39	102	If $\bar{a}$ is a constant vector such that $\lvert \bar{a} \rvert = a$ then prove that $\nabla \cdot \{ (\bar{a} \cdot \bar{r}) \bar{a} \} = a^2$
40	103	If $\bar{a}$ is a constant vector and $\bar{r} = xi + yj + zk$, prove that i) $div\,(\bar{a} \times \bar{r}) = 0$ ii) $div\,(\bar{a} \cdot \bar{r})\bar{a} = a^2$ iii) $div\,(\bar{a} \times \bar{r} \times \bar{a}) = 2a^2$ iv) $curl\,(\bar{a} \times \bar{r}) = 2\bar{a}$
41	104	If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\bar{r} \cdot \nabla\phi$, (ii) $\nabla \cdot \bar{F}, \nabla \times \bar{F}$ *where* $\bar{F} = \nabla\phi$
42	105	Prove that $\nabla\left(\frac{1}{r}\right) = -\frac{\bar{r}}{r^3}$.
43	106	Prove that $\nabla f(r) = \frac{f'(r)}{r}\bar{r}$ and hence, find f *if* $\nabla f = 2r^4 \bar{r}$.
44	107	Show that $\nabla\left[\frac{(\bar{a} \cdot \bar{r})}{r^n}\right] = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} \cdot \bar{r})\bar{r}}{r^{n+2}}$

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Vector Differentiation
Practice Problems

45	108	Prove that $\nabla r^n = n r^{n-2}\bar{r}$
46	109	Prove that $\nabla\cdot(\nabla\times\bar{F})=0$ *where* $\bar{F}$ is a vector point function.
47	110	Prove that $\nabla\left\{\nabla\cdot\dfrac{\bar{r}}{r}\right\}=-\dfrac{2}{r^3}\bar{r}$
48	111	Prove that $\nabla\cdot\left(r\,\nabla\dfrac{1}{r^3}\right)=\dfrac{3}{r^4}$
49	112	Prove that $\nabla\cdot\left(r\,\nabla\dfrac{1}{r^n}\right)=\dfrac{n(n-2)}{r^{n+1}}$
50	113	Prove that $div\ grad\ r^n = n(n+1)r^{n-2}$
51	114	Prove that $\nabla\times\left(\dfrac{\bar{a}\times\bar{r}}{r^n}\right)=\dfrac{(2-n)\bar{a}}{r^n}+\dfrac{n(\bar{a}\cdot\bar{r})\bar{r}}{r^{n+2}}$
52	115	Prove that $\nabla\log r=\dfrac{\bar{r}}{r^2}$ and hence, show that $\nabla\times(\bar{a}\times\nabla\log r)=2\dfrac{(\bar{a}\cdot\bar{r})\bar{r}}{r^4}$, where $\bar{a}$ is a constant vector.
DIVERGENCE AND CURL		
53	116	Find $div\,\bar{F}$ *and* $curl\,\bar{F}$ *where* $\bar{F}=\dfrac{xi-yj}{x^2+y^2}$
54	117	If $\bar{A}=\nabla(xy+yz+zx)$, *find* $\nabla\cdot\bar{A}$ *and* $\nabla\times\bar{A}$
55	118	If $\bar{F}=(\bar{a}\cdot\bar{r})\bar{r}$ *where* $\bar{a}$ is constant vector, find $curl\ \bar{F}$ and P.T. it is perpendicular to $\bar{a}$.
56	119	Prove that $\bar{F}=\dfrac{\bar{r}}{r^3}$ is both irrotational and solenoidal.
57	120	A vector field $\bar{F}$ is given by $\bar{F}=(y\sin z-\sin x)i+(x\sin z+2yz)j+(xy\cos z+y^2)k$ Prove that it is irrotational and hence, find its scalar potential.
58	121	A vector field is given by $\bar{F}=(x^2+xy^2)i+(y^2+x^2y)j$. Show that $\bar{F}$ irrotational and find its scalar potential.
59	122	If $\nabla\phi=(y^2-2xyz^3)i+(3+2xy-x^2z^3)j+(6z^3-3x^2yz^2)k$, *find* ϕ *where* $\phi(1,0,1)=2$
60	123	Find the value of n for which the vector $r^n\bar{r}$ is solenoidal, where $\bar{r}=xi+yj+zk$
61	124	Prove that $\nabla\cdot\left\{\dfrac{f(r)}{r}\bar{r}\right\}=\dfrac{1}{r^2}\dfrac{d}{dr}\left[r^2 f(r)\right]$ hence or otherwise prove that $div(r^n\bar{r})=(n+3)r^n$
62	125	Show that $\bar{F}=(y^2-z^2+3yz-2x)i+(3xz+2xy)j+(3xy-2xz+2z)k$ is both solenoidal & irrotational.
63	126	If $\bar{r}$ is the position vector of point (x,y,z) and r is the modulus of $\bar{r}$, then prove that $r^n\bar{r}$ is an irrotational vector for any value of n but solenoidal only if $n=-3$.

Vector Differentiation

Practice Problems

D1	127,D7	If $\bar{f}=(x+y+1)i+j-(x+y)k$, prove that $\bar{f}\cdot curl\,\bar{f}=0$
D2	128,D8	Define irrotational field and hence check whether the vector field $\overline{F}=(x+2y+4z)i+(2x-3y-z)j+(4x-y+2z)k$ is irrotational.
DIFFERENTIAL OPERATORS		
D3	D9	With usual notation, prove that $\nabla^2\left[\nabla\cdot\frac{\bar{r}}{r^2}\right]=\frac{2}{r^4}$
D4	D10	Show that $\nabla^4 r^2\log r=\frac{6}{r^2}$
D5	D11	Prove that $\nabla^2 f(r)=\frac{d^2f}{dr^2}+\frac{2}{r}\frac{df}{dr}$
D6	D12	Prove that $\nabla^2(r^2\log r)=5+6\log r$