Introduction to Inverse Laplace Transform

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Definition: If $L\{f(t)\} = \emptyset(s) = \int_0^\infty e^{-st} f(t)dt$ then $f(t)$ is called the inverse Laplace transform of $\emptyset(s)$ and can be denoted as $L^{-1}\{\emptyset(s)\} = f(t)$

1. Table of Inverse Laplace Transforms:

$L(1) = \frac{1}{s}$	$L^{-1}\left(\frac{1}{s}\right) = 1$
$L(e^{-at}) = \frac{1}{s+a}$	$L^{-1}\left(\frac{1}{s+a}\right) = e^{-at}$
$L(e^{at}) = \frac{1}{s-a}$	$L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$
$L(t^{n-1}) = \frac{\lvert n}{s^n}$	$L^{-1}\left(\frac{1}{s^n}\right) = \frac{t^{n-1}}{\lvert n}$
$L(t^{n-1}) = \frac{(n-1)!}{s^n}$	$L^{-1}\left(\frac{1}{s^n}\right) = \frac{t^{n-1}}{(n-1)!}$
$L(\sin at) = \frac{a}{s^2+a^2}$	$L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{1}{a}\sin at$
$L(\cos at) = \frac{s}{s^2+a^2}$	$L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos at$
$L(\sin h\ at) = \frac{a}{s^2-a^2}$	$L^{-1}\left(\frac{1}{s^2-a^2}\right) = \frac{1}{a}\sin h\ at$
$L(\cos h\ at) = \frac{s}{s^2-a^2}$	$L^{-1}\left(\frac{s}{s^2-a^2}\right) = \cos h\ at$

2. Using shifting theorem:

We know that, If $L\{f(t)\} = \emptyset(s)$, then $L\{e^{-at}f(t)\} = \emptyset(s+a)$

This means if $f(t) = L^{-1}\{\emptyset(s)\}$ then $L^{-1}\{\emptyset(s+a)\} = e^{-at}f(t)$

i.e $L^{-1}\{\emptyset(s+a)\} = e^{-at}L^{-1}\{\emptyset(s)\}$

3. Method of partial fractions:

Whenever possible we express the given function $\emptyset(s)$ into the sum of linear or quadratic partial fractions as $\emptyset(s) = \frac{A}{(s+a)^r} + \frac{Bs+C}{(s^2+a^2)^r}$ and then use the standard results given above to find L^{-1}

(i) distinct linear factor $\frac{px+q}{(x-a)(x-b)}$	Express : $\frac{px+q}{(x-a)(x-b)} = \frac{A}{(x-a)} + \frac{B}{(x-b)}$
(ii) distinct linear factor $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	Express : $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{(x-a)} + \frac{B}{(x-b)} +$
(iii) repetitive linear factor $\frac{px+q}{(x-a)^2}$	Express : $\frac{px+q}{(x-a)^2} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2}$
(iv) repetitive linear factor $\frac{px^2+qx+r}{(x-a)^3}$	Express : $\frac{px^2+qx+r}{(x-a)^3} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-a)^3}$
(v) repetitive linear factor $\frac{px^2+qx+r}{(x-a)^2(x-b)}$	Express : $\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$
(vi) Linear & quadratic factor $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	Express: $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{(x-a)} + \frac{Bx+C}{(x^2+bx+c)}$