## BIGM BIGM METHOD

## Ex 3. Using the Penalty (Big M) method solve the following LPP

Maximize 
$$z = x_1 + 2x_2 + 3x_3 - x_4$$
  
Subject to  $x_1 + 2x_2 + 3x_3 = 15$ ,  
 $2x_1 + x_2 + 5x_3 = 20$ ,  
 $x_1 + 2x_2 + x_3 + x_4 = 10$ ,  
 $x_1, x_2, x_3, x_4 \ge 10$ 

## **Solution:**

Introducing three artificial variables  $A_1, A_2, A_3$  in the three equalities and assigning big penalty -M in the object function for  $A_1, A_2, A_3$ .

We have,

Maximize

$$z = x_1 + 2x_2 + 3x_3 - x_4 - MA_1 - MA_2 - MA_3$$
Subject to 
$$x_1 + 2x_2 + 3x_3 + A_1 = 15$$

$$2x_1 + x_2 + 5x_3 + A_2 = 20$$

$$x_1 + 2x_2 + x_3 + x_4 + A_3 = 10$$

Adding *M* times the first, second and the third constraints to the object function, we get,

$$z = (1 + 4M)x_1 + (2 + 5M)x_2 + (3 + 9M)x_3 + (-1 + M)x_4 - 45M$$

$$z - (1 + 4M)x_1 - (2 + 5M)x_2 - (3 + 9M)x_3 + (1 - M)x_4 = -45M$$

Iteration	Basic			Coeff	ficients	of			RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z									
	$A_1$									
	$A_2$									
	$A_3$									

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Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1 – M	0	0	0	-45 <i>M</i>	_
	$A_1$	1	2	3	0	1	0	0	15	
	$A_2$	2	1	5	0	0	1	0	20	
	$A_3$	1	2	1	1	0	0	1	10	

Iteration	Basic		ı	Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$\chi_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1 – M	0	0	0	-45 <i>M</i>	
	$A_1$	1	2	3	0	1	0	0	15	
	$A_2$	2	1	5	0	0	1	0	20	
	$A_3$	1	2	1	1	0	0	1	10	

Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$\chi_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1 – M	0	0	0	-45 <i>M</i>	
	$A_1$	1	2	3	0	1	0	0	15	5
	$A_2$	2	1	5	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10

Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1 – M	0	0	0	-45 <i>M</i>	
	$A_1$	1	2	3	0	1	0	0	15	5
	$A_2$	2	1	5	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10

Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1-M	0	0	0	-45 <i>M</i>	
A <sub>2</sub> leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	γ.		Coefficient		$A_1$	$A_2$	$A_3$	RHS sol.	Rati o
0	Z	$\frac{x_1}{-(1+4M)}$	$\begin{array}{c c} x_2 \\ \hline -(2+5M) \end{array}$	$\begin{array}{c} \chi_3 \\ -(3+9M) \end{array}$	$X_4$ $1-M$	0	0	0	-45 <i>M</i>	$\dashv$
A <sub>2</sub> leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
1	Z									
	$A_1$									
	<i>x</i> <sub>3</sub>									
	$A_3$									

Iteration Number	Basic Var.	Υ.		Coefficient $x_3$		$A_1$	$A_2$	$A_3$	RHS sol.	Rati o
0	Z	$\frac{x_1}{-(1+4M)}$	$x_2$ $-(2+5M)$	-(3+9M)	$\frac{x_4}{1-M}$	0	0	0	-45 <i>M</i>	$\dashv$
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
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1	Z									
	$A_1$									
	$x_3$	2/5	1/5	1	0	0		0	4	
	$A_3$									

Iteration Number	Basic Var.	$x_1$	x <sub>2</sub>	Coefficient $x_3$	cs of $x_4$	$A_1$	$A_2$	$A_3$	RHS sol.	Rati o
0	Z	-(1 + 4M)	-(2+5M)	-(3 + 9M)	1 – M	0	0	0	-45 <i>M</i>	
A <sub>2</sub> leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
1	Z						_			
	$A_1$						_			
	$x_3$	2/5	1/5	1	0	0		0	4	
	$A_3$	3/5	9/5	0	1	0	_	1	6	

Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1-M	0	0	0	-45 <i>M</i>	
A <sub>2</sub> leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
1	Z						—			
	$A_1$	-1/5	7/5	0	0	1		0	3	
	$x_3$	2/5	1/5	1	0	0		0	4	
	$A_3$	3/5	9/5	0	1	0		1	6	

Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1 – M	0	0	0	-45 <i>M</i>	
A <sub>2</sub> leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
1	Z	$\frac{1-2M}{5}$	<u>-7-16<i>M</i></u> 5	0	1-M	0		0	12 – 9 <i>M</i>	
	$A_1$	- 1/5	7/5	0	0	1		0	3	
	$x_3$	2/5	1/5	1	0	0		0	4	
	$A_3$	3/5	9/5	0	1	0		1	6	

Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1 – M	0	0	0	-45 <i>M</i>	
A <sub>2</sub> leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10

1	Z	$\frac{1-2M}{5}$	<u>-7-16M</u> 5	0	1-M	0	0	12 – 9 <i>M</i>	
	$A_1$	-1/5	7/5	0	0	1	0	3	
	<i>x</i> <sub>3</sub>	2/5	1/5	1	0	0	0	4	
	$A_3$	3/5	9/5	0	1	0	1	6	

Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	<i>x</i> <sub>2</sub>	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1 – M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
1	Z	$\frac{1-2M}{5}$	<u>-7-16<i>M</i></u> 5	0	1-M	0		0	12 – 9 <i>M</i>	
	$A_1$	-1/5	7/5	0	0	1		0	3	15/7
	$x_3$	2/5	1/5	1	0	0		0	4	20
	$A_3$	3/5	9/5	0	1	0		1	6	10/3

Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1-M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
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1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1-M	0	 0	12 – 9 <i>M</i>	
	$A_1$	-1/5	7/5	0	0	1	 0	3	15/7
	$x_3$	2/5	1/5	1	0	0	0	4	20
	$A_3$	3/5	9/5	0	1	0	 1	6	10/3

Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1-M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10

1	Z	$\frac{1-2M}{5}$	<u>-7-16<i>M</i></u> 5	0	1-M	0	 0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	-1/5	7/5*	0	0	1	0	3	15/7
x <sub>2</sub> enters	$x_3$	2/5	1/5	1	0	0	0	4	20
	$A_3$	3/5	9/5	0	1	0	 1	6	10/3

Iteration	Basic			Coefficient	.s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1 + 4M)	-(2+5M)	-(3 + 9M)	1-M	0	0	0	-45M	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
	-									
1	Z	$\frac{1-2M}{5}$	<u>-7-16<i>M</i></u> 5	0	1-M	0	_	0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	-1/5	7/5*	0	0	1		0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0		0	4	20
	$A_3$	3/5	9/5	0	1	0		1	6	10/3
2	Z						_			
	<i>x</i> <sub>2</sub>					—		_		
	<i>x</i> <sub>3</sub>					_				
	$A_3$									

		1							1	
Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3 + 9M)	1-M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
							-	-		
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1-M	0		0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	-1/5	7/5*	0	0	1	_	0	3	15/7
$x_2$ enters	<i>x</i> <sub>3</sub>	2/5	1/5	1	0	0		0	4	20
	$A_3$	3/5	9/5	0	1	0		1	6	10/3
2	Z									
	$x_2$	-1/7	1	0	0			0	15/7	
	<i>x</i> <sub>3</sub>						_			
	$A_3$					_	_			

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Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3 + 9M)	1-M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1-M	0		0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	-1/5	7/5*	0	0	1		0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0	_	0	4	20
	$A_3$	3/5	9/5	0	1	0	_	1	6	10/3
2	Z	$-\frac{6M}{7}$	0	0	1-M			0	105–15 <i>M</i>	
	$x_2$	-1/7	1	0	0			0	15/7	
	<i>x</i> <sub>3</sub>									
	$A_3$									

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Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1 + 4M)	-(2+5M)	-(3 + 9M)	1-M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1-M	0		0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	-1/5	7/5*	0	0	1		0	3	15/7
$x_2$ enters	<i>x</i> <sub>3</sub>	2/5	1/5	1	0	0	_	0	4	20
	$A_3$	3/5	9/5	0	1	0	_	1	6	10/3
2	Z	$-\frac{6M}{7}$	0	0	1-M	_	_	0	105–15 <i>M</i>	
	$x_2$	-1/7	1	0	0	_		0	15/7	
	<i>x</i> <sub>3</sub>	3/7	0	1	0			0	25/7	
	$A_3$							-		

Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	o
0	Z	-(1 + 4M)	-(2+5M)	-(3 + 9M)	1-M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
					-					
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1-M	0	_	0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	-1/5	7/5*	0	0	1		0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0	_	0	4	20
	$A_3$	3/5	9/5	0	1	0		1	6	10/3
2	Z	$-\frac{6M}{7}$	0	0	1-M	_	_	0	105-15 <i>M</i>	
	$x_2$	-1/7	1	0	0		_	0	15/7	>
	<i>x</i> <sub>3</sub>	3/7	0	1	0			0	25/7	
	$A_3$	6/7	0	0	1			1	15/7	6

Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	<i>x</i> <sub>3</sub>	<i>x</i> <sub>4</sub>	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1+4M)	-(2+5M)	-(3 + 9M)	1-M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
					•		•		•	
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1-M	0		0	12 – 9 <i>M</i>	V
$A_1$ leaves	$A_1$	-1/5	7/5*	0	0	1		0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0		0	4	20
	$A_3$	3/5	9/5	0	1	0		1	6	10/3
		•								
2	Z	$-\frac{6M}{7}$	0	0	1 – M			0	105-15 <i>M</i>	× ×
	$x_2$	-1/7	1	0	0		_	0	15/7	Ž
	<i>x</i> <sub>3</sub>	3/7	0	1	0			0	25/7	
	$A_3$	6/7	0	0	1			1	15/7	>

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Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	<i>x</i> <sub>3</sub>	<i>x</i> <sub>4</sub>	$A_1$	$A_2$	$A_3$	sol.	o
0	Z	-(1+4M)	-(2+5M)	-(3 + 9M)	1-M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1-M	0		0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	-1/5	7/5*	0	0	1		0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0		0	4	20
	$A_3$	3/5	9/5	0	1	0		1	6	10/3
									-	
2	Z	$-\frac{6M}{7}$	0	0	1 – M		_	0	105–15 <i>M</i>	
	$x_2$	-1/7	1	0	0			0	15/7	_
	<i>x</i> <sub>3</sub>	3/7	0	1	0			0	25/7	_
	$A_3$	6/7	0	0	1			1	15/7	15/7

Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	0
0	Z	-(1 + 4M)	-(2+5M)	-(3 + 9M)	1-M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1-M	0		0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	-1/5	7/5*	0	0	1		0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0		0	4	20
	$A_3$	3/5	9/5	0	1	0		1	6	10/3
		•	•	•						
2	Z	$-\frac{6M}{7}$	0	0	1 – M	_	_	0	105–15 <i>M</i>	
	$x_2$	-1/7	1	0	0			0	15/7	_
	<i>x</i> <sub>3</sub>	3/7	0	1	0			0	25/7	_
	$A_3$	6/7	0	0	1			1	15/7	15/7

									,	
Iteration	Basic			Coefficient	s of				RHS	Rati
Number	Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	sol.	o
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1-M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1-M	0		0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	-1/5	7/5*	0	0	1		0	3	15/7
$x_2$ enters	<i>x</i> <sub>3</sub>	2/5	1/5	1	0	0	_	0	4	20
	$A_3$	3/5	9/5	0	1	0		1	6	10/3
		•								
2	Z	$-\frac{6M}{7}$	0	0	1 – M	_	_	0	105–15 <i>M</i>	
$A_3$ leaves	$x_2$	-1/7	1	0	0		_	0	15/7	_
$x_4$ enters	<i>x</i> <sub>3</sub>	3/7	0	1	0			0	25/7	_ [
	$A_3$	6/7 *	0	0	1			1	15/7	15/7

									1	
Iteration	Basic Var.			Coefficients		4	4		RHS sol.	Ratio
Number 0	Z	$\frac{x_1}{-(1+4M)}$	-(2+5M)	$x_3 - (3 + 9M)$	$\frac{x_4}{1-M}$	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	-45 <i>M</i>	
					<del>                                     </del>			0		
A <sub>2</sub> leaves	A <sub>1</sub>	1	2	3	0	1	0		15	5
$x_3$ enters	A <sub>2</sub>	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
		4 214	7. 4cM	Γ	i			i		
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1-M	0		0	12 - 9M	
$A_1$ leaves	$A_1$	<b>-1/5</b>	7/5*	0	0	1	—	0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0		0	4	20
	$A_3$	3/5	9/5	0	1	0		1	6	10/3
2	7	6 <i>M</i>	0	0	1 – <i>M</i>			0	  105-15 <i>N</i>	
	Z	7	U	U	1 — M				7	
$A_3$		1 /7	1					0	1	
leaves	$x_2$	-1/7	1	0	0				15/7	
$x_4$		0.75						0	05.75	
enters	$x_3$	3/7	0	1	0				25/7	-
	$A_3$	6/7	0	0	1*			1	15/7	15/7
	3	- /		_					- /	
3	Z									
	$x_2$									
	$x_3$									
	$x_4$						_	_		

Iteration				Coefficients	of					
Number	Basic Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	RHS sol.	Ratio
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1 – M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
			1	r	1	1			1	
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1 – M	0		0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	<b>- 1/5</b>	7/5*	0	0	1	_	0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0	—	0	4	20
	$A_3$	3/5	9/5	0	1	0	—	1	6	10/3
			1							
2	Z	6 <i>M</i>	0	0	1-M	_		0	105-15 <i>M</i>	
_		7							7	
$A_3$ leaves	$x_2$	-1/7	1	0	0	_		0	15/7	_
$x_4$ enters	$x_3$	3/7	0	1	0			0	25/7	_
	$A_3$	6/7	0	0	1*			1	15/7	15/7
3	Z									
	$x_2$									
	$x_3$									
	$x_4$	6/7	0	0	1				15/7	

Iteration				Coefficients	of					
Number	Basic Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	RHS sol.	Ratio
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1 – M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
	1		1	i	1	ī				
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1 – M	0	—	0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	<b>-1/5</b>	7/5*	0	0	1		0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0	—	0	4	20
	$A_3$	3/5	9/5	0	1	0		1	6	10/3
2	Z	6 <i>M</i>	0	0	1-M			0	105-15 <i>N</i>	
		7							7	1
$A_3$		1 /7	4					0	15/7	
leaves	$x_2$	-1/7	1	0	0				15/7	
$x_4$		0 /5		4				0	25 /5	
enters	$x_3$	3/7	0	1	0				25/7	-
	$A_3$	6/7	0	0	1*			1	15/7	15/7
									!	
3	Z									
	$x_2$									
	$x_3$	3/7	0	1	0				25/7	
	$x_4$	6/7	0	0	1				15/7	

Iteration				Coefficients						
Number	Basic Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	RHS sol.	Ratio
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1-M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
			•	r	1	1			1	
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1 – M	0	_	0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	-1/5	7/5*	0	0	1	_	0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0	_	0	4	20
	$A_3$	3/5	9/5	0	1	0		1	6	10/3
	<del></del>		_	<b>r</b>	<u> </u>					
2		6 <i>M</i>						0	105-15 <i>M</i>	
2	Z	$-\frac{377}{7}$	0	0	1-M				7	}
		,							/	
$A_3$		4 /5						0		
leaves	$ x_2 $	-1/7	1	0	0				15/7	_
leaves										
$x_4$		o /=		_				0		
	$ x_3 $	3/7	0	1	0				25/7	_
enters										
	$A_3$	6/7	0	0	1*			1	15/7	15/7
	<u> </u>	,			1				,	,
3	7									
<u> </u>	Z									
	$x_2$	-1/7	1	0	0	_			15/7	
	$x_3$	3/7	0	1	0				25/7	
	$x_4$	6/7	0	0	1				15/7	

Iteration				Coefficients	nf .					
Number	Basic Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	RHS sol.	Ratio
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1-M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
	1			<u> </u>	i	ı			1	
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1 – M	0		0	12 - 9M	
$A_1$ leaves	$A_1$	<b>-1/5</b>	7/5*	0	0	1	_	0	3	15/7
$x_2$ enters	<i>x</i> <sub>3</sub>	2/5	1/5	1	0	0	—	0	4	20
	$A_3$	3/5	9/5	0	1	0	—	1	6	10/3
	1				1				1	
		6 <i>M</i>	0		4 14			0	105-15 <i>N</i>	
2	Z	<del>- 7</del>	0	0	1-M	_			7	
A <sub>3</sub> leaves	<i>x</i> <sub>2</sub>	-1/7	1	0	0	_	_	0	15/7	_
x <sub>4</sub> enters	$x_3$	3/7	0	1	0			0	25/7	_
	$A_3$	6/7	0	0	1*			1	15/7	15/7
				-						
3	Z	-6/7	0	0	0				90/7	
	$x_2$	-1/7	1	0	0				15/7	
	$x_3$	3/7	0	1	0				25/7	
	$\chi_4$	6/7	0	0	1				15/7	

Iteration	Basic Var.			Coefficients					RHS sol.	Ratio
Number		$x_1$	x <sub>2</sub>	<i>x</i> <sub>3</sub>	<i>x</i> <sub>4</sub>	$A_1$	$A_2$	A <sub>3</sub>	4574	
0	Z	-(1 + 4M)	-(2+5M)	-(3+9M)	1 – M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	A <sub>2</sub>	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
	1	4.214	T 46M							
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1-M	0		0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	<b>- 1/5</b>	7/5*	0	0	1		0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0		0	4	20
	$A_3$	3/5	9/5	0	1	0	_	1	6	10/3
2	Z	<u>6M</u>	0	0	1-M			0	105-15 <i>N</i>	
	2	7		0	1 1/1				7	
$A_3$ leaves	$x_2$	-1/7	1	0	0			0	15/7	_
x <sub>4</sub> enters	$x_3$	3/7	0	1	0			0	25/7	_
	$A_3$	6/7	0	0	1*	_		1	15/7	15/7
3	Z	-6/7	0	0	0				90/7	
	$x_2$	-1/7	1	0	0				15/7	
	$x_3$	3/7	0	1	0				25/7	
	$x_4$	6/7	0	0	1				15/7	

	1								1	
Iteration	Basic Var.			Coefficients	1				RHS sol.	Ratio
Number	20010 1011	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$		
0	Z	-(1 + 4M)	-(2 + 5M)	-(3 + 9M)	1 – M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1 – M	0		0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	-1/5	7/5*	0	0	1		0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0	—	0	4	20
	$A_3$	3/5	9/5	0	1	0		1	6	10/3
2	Z	$-\frac{6M}{7}$	0	0	1-M		_	0	105–15 <i>N</i>	
$A_3$ leaves	<i>x</i> <sub>2</sub>	-1/7	1	0	0		_	0	15/7	_
x <sub>4</sub> enters	$x_3$	3/7	0	1	0		_	0	25/7	_
	$A_3$	6/7	0	0	1*			1	15/7	15/7
3	Z	-6/7	0	0	0				90/7	
	$x_2$	-1/7	1	0	0				15/7	-ve
	$x_3$	3/7	0	1	0				25/7	25/3
	$x_4$	6/7	0	0	1				15/7	15/6

Iteration				Coefficients	nf.					
Number	Basic Var.	$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	RHS sol.	Ratio
0	Z	-(1+4M)	-(2+5M)	-(3+9M)	1-M	0	0	0	-45 <i>M</i>	
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
				1	1	1				
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1 – M	0	_	0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	− 1/5	7/5*	0	0	1		0	3	15/7
$x_2$ enters	<i>x</i> <sub>3</sub>	2/5	1/5	1	0	0	_	0	4	20
	$A_3$	3/5	9/5	0	1	0		1	6	10/3
	1			<b>T</b>	1					
2		6 <i>M</i>	0					0	 105–15 <i>M</i>	
2	Z	$-\frac{311}{7}$	0	0	1-M				7	
$A_3$	26	1 /7	1	0	0			0	15/7	
leaves	$x_2$	-1/7	1	U	0				15/7	_
$x_4$	24	2 /7	0	1				0	)   25 /7	
enters	$x_3$	3/7	0		0				25/7	_
	$A_3$	6/7	0	0	1*			1	15/7	15/7
3	Z	-6/7	0	0	0				90/7	
	$x_2$	-1/7	1	0	0				15/7	-ve
		2 /7	0	4					25 /7	25/3
	$\chi_3$	3/7	0	1	0				25/7	
	$x_4$	6/7	0	0	1				15/7	15/6

Iteration	Basic Var.		<u> </u>	Coefficients	1				RHS sol.	Ratio
Number		<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	<i>x</i> <sub>3</sub>	<i>x</i> <sub>4</sub>	$A_1$	$A_2$	A <sub>3</sub>		$\blacksquare$
0	Z	-(1 + 4M)	-(2+5M)	-(3+9M)	1 – M	0	0	0	-45 <i>M</i>	igwdown
$A_2$ leaves	$A_1$	1	2	3	0	1	0	0	15	5
$x_3$ enters	$A_2$	2	1	5*	0	0	1	0	20	4
	$A_3$	1	2	1	1	0	0	1	10	10
·			i	i	1			1	ī	
1	Z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	1-M	0	_	0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	-1/5	7/5*	0	0	1	_	0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0	—	0	4	20
	$A_3$	3/5	9/5	0	1	0	—	1	6	10/3
2	Z	$-\frac{6M}{7}$	0	0	1-M		_	0	105–15 <i>N</i>	
A <sub>3</sub> leaves	<i>x</i> <sub>2</sub>	-1/7	1	0	0			0	15/7	_
$x_4$ enters	$x_3$	3/7	0	1	0			0	25/7	-
	$A_3$	6/7	0	0	1*		—	1	15/7	15/7
			<u> </u>	1	1				<u> </u>	
3	Z	-6/7	0	0	0		_		90/7	
	$x_2$	-1/7	1	0	0				15/7	-ve
	$x_3$	3/7	0	1	0	_	_		25/7	25/3
	$x_4$	6/7	0	0	1			_	15/7	15/6

	1								1	
Iteration Number	Basic Var.		Coefficients of							Ratio
		$x_1$	$x_2$	$x_3$	$x_4$	$A_1$	$A_2$	$A_3$	RHS sol.	Natio
1	Z	$\frac{1-2M}{5}$	<u>-7-16<i>M</i></u> 5	0	1 – M	0	_	0	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	<b>-1/5</b>	7/5*	0	0	1	—	0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0	_	0	4	20
	$A_3$	3/5	9/5	0	1	0	—	1	6	10/3
2	Z	$-\frac{6M}{7}$	0	0	1-M	_	_	0	$\frac{105-15M}{7}$	
$A_3$ leaves	<i>x</i> <sub>2</sub>	-1/7	1	0	0	_	_	0	15/7	-
$x_4$ enters	<i>x</i> <sub>3</sub>	3/7	0	1	0	_	_	0	25/7	-
	$A_3$	6/7	0	0	1*	_		1	15/7	15/7
3	Z	-6/7	0	0	0				90/7	
x <sub>4</sub> leaves	<i>x</i> <sub>2</sub>	-1/7	1	0	0				15/7	-ve
x <sub>1</sub> enters	<i>x</i> <sub>3</sub>	3/7	0	1	0	_	_		25/7	25/3
	$\chi_4$	6/7*	0	0	1			—	15/7	15/6

Iteration	Basic Var.	Coefficients of							RHS sol.	Ratio
Number 1	Z	<i>x</i> <sub>1</sub> <u>1-2<i>M</i></u>	<i>x</i> <sub>2</sub> -7-16 <i>M</i>	0	$x_4$ $1-M$	<i>A</i> <sub>1</sub> 0	A <sub>2</sub>	<i>A</i> <sub>3</sub>	12 – 9 <i>M</i>	
$A_1$ leaves	$A_1$	- 1/5	5 7/5*	0	0	1	_	0	3	15/7
$x_2$ enters	$x_3$	2/5	1/5	1	0	0		0	4	20
$\chi_2$ enters	$A_3$	3/5	9/5	0	1	0		1	6	10/3
113   3/3   3/3   0   1								-		10/3
2	Z	$-\frac{6M}{7}$	0	0	1 - M	_	_	0	105-15 <i>M</i>	
$A_3$ leaves	<i>x</i> <sub>2</sub>	-1/7	1	0	0	_		0	15/7	_
$x_4$ enters	<i>x</i> <sub>3</sub>	3/7	0	1	0	_	—	0	25/7	-
	$A_3$	6/7	0	0	1*		_	1	15/7	15/7
3	Z	-6/7	0	0	0				90/7	
$x_4$ leaves	$x_2$	-1/7	1	0	0	_			15/7	-ve
x <sub>1</sub> enters	<i>x</i> <sub>3</sub>	3/7	0	1	0	_			25/7	25/3
	$x_4$	6/7*	0	0	1	_		_	15/7	15/6
4	Z	0	0	0	1				15	
	$x_2$	0	1	0	1/6				5/2	
	$x_3$	0	0	1	-1/2			_	5/2	
	$x_1$	1	0 Nand	ini Rai 0	7/6			<del>-39</del> -	5/2	

Since all the coefficients in the objective equation in the row of z are positive. This is a optimal solution.

The values of the variables and of z are given by the RHS column

$$\therefore x_1 = \frac{5}{2}, x_2 = \frac{5}{2}, x_3 = \frac{5}{2}, z_{max} = 15$$