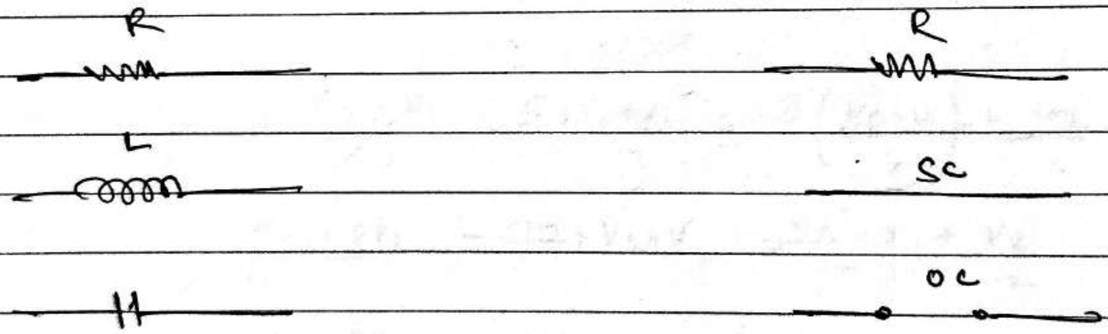
Transient

Note 1

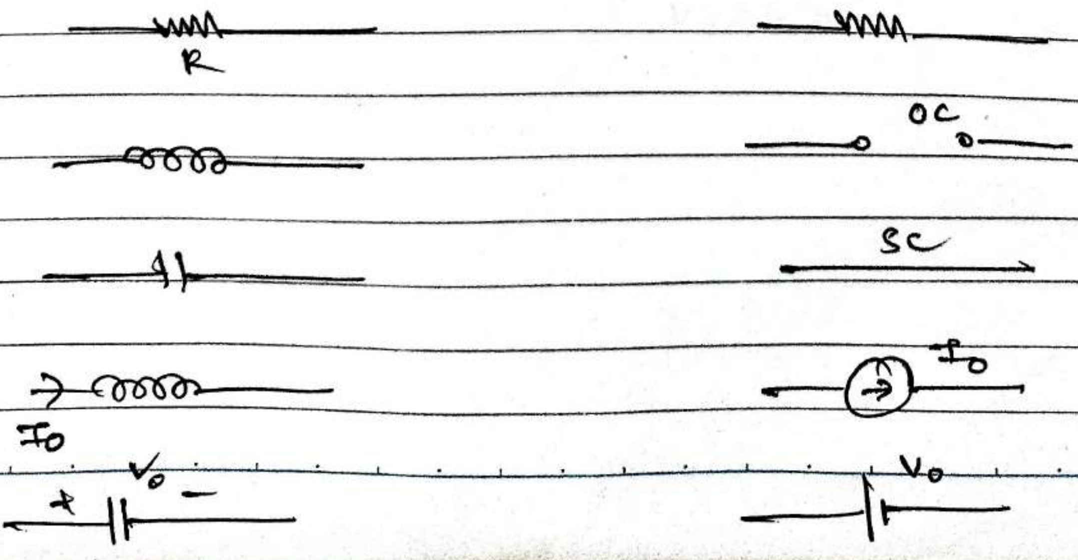
Component	Volt.	Current
①	IR	$\frac{V}{R}$
② L	$L\frac{di}{dt}$	$\frac{1}{L}\int_0^t v(t)\,dt$
③ C	$\frac{1}{C}\int_0^t i(t)\,dt$	$C\frac{dV}{dt}$

Note 2 : 0^- condition



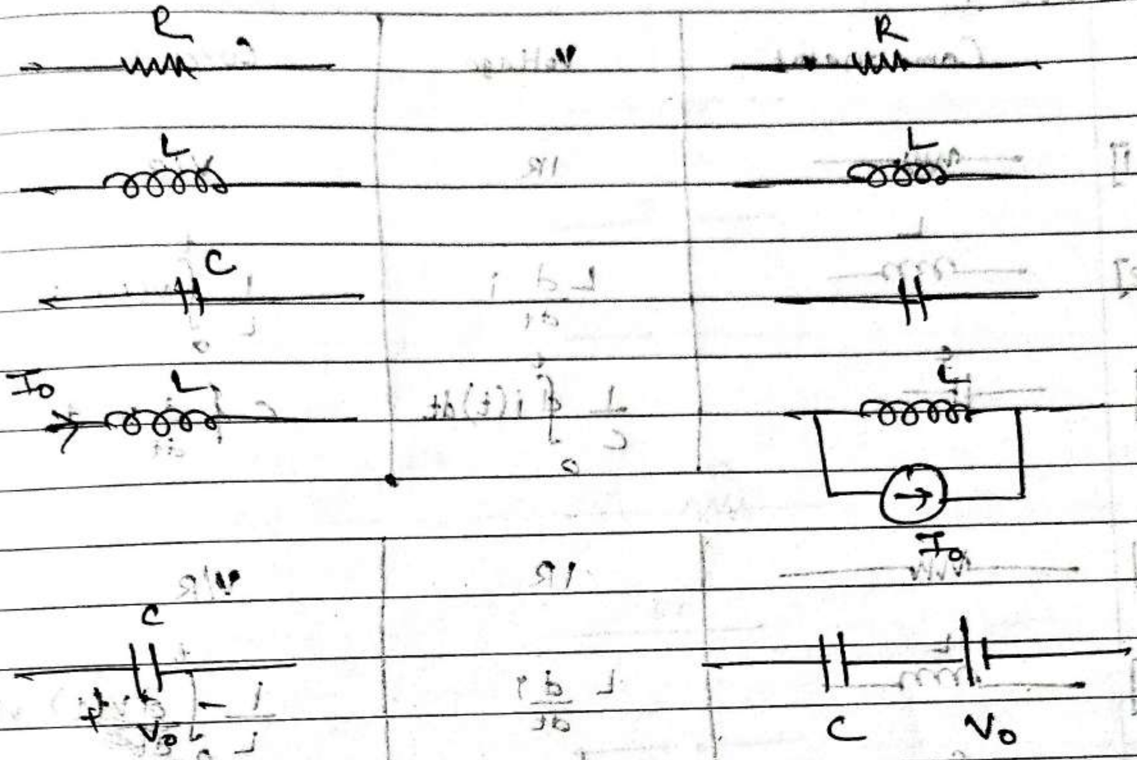
at 0^+ cond^n

Note 3



Date :

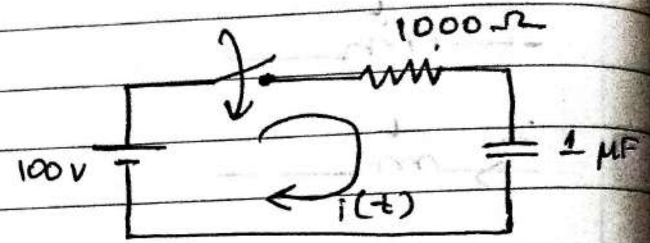
at $t = \infty$



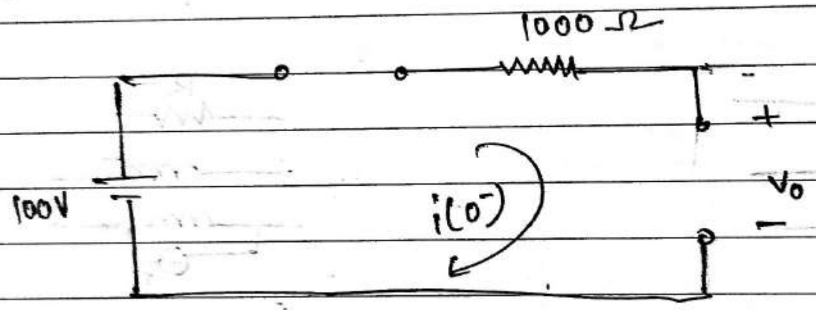
Type 1

Q At $t = 0^+$.

Find i, $\frac{di}{dt}$ and $\frac{d^2 i}{dt^2}$ at $t = 0^+$



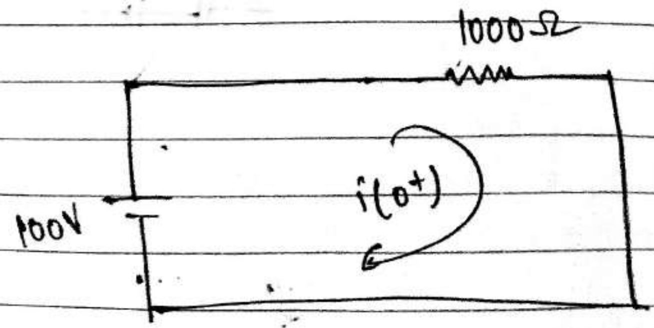
$\Rightarrow$ at $t = 0^-$



$i(0^-) = 0\ A$

$V_0(\cancel{\ }) = 0\ V$ No. cont. path

at $t = 0^+$



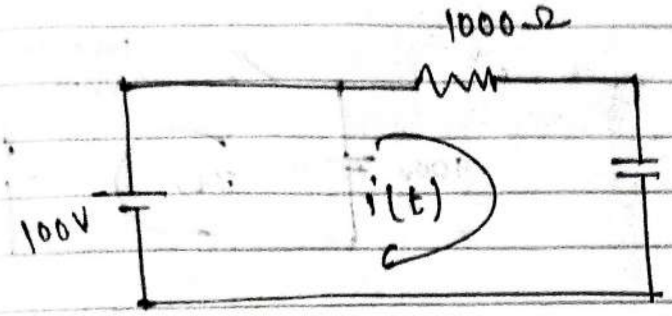
$\therefore i(0^+) = \frac{100}{1000}$

$= 0.1\ A$

$100 - 1000\, i(0^+) = 0$

Date :

$t = 0^{+}$



$$100 - 1000\, i(t) - \frac{1}{1\times10^{-6}} \int_0^t i(t)\, dt = 0$$

$$100 - 1000\, i(t) - 10^{6} \int_0^t i(t)\, dt = 0 \quad \text{—} (1)$$

Diff. eqⁿ (1) w.r.t 't'

$$0 - 1000 \frac{d\, i(t)}{dt} - 10^{6}\, i(t) = 0$$

$$1000 \frac{di(t)}{dt} = -10^{6}\, i(t)$$

$$\boxed{\frac{di(t)}{dt} = -1000\, i(t)} \quad \text{—} (2)$$

at $t = 0^{+}$ $\Rightarrow$ $\frac{di(0^{+})}{dt} = -1000\, i(0^{+}) = -1000(0.1)$

$$= -100 \frac{A}{s}$$

Diff eqⁿ (2)

$$\frac{d^2 i(t)}{dt^2} = -1000 \frac{di(t)}{dt}$$

at $t = 0^{+}$

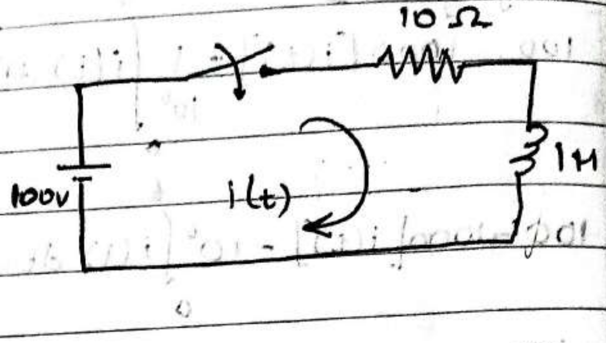
$$\frac{d^2 i(0^{+})}{dt^2} = -1000 \frac{di(0^{+})}{dt}$$

$$= -1000(-100) \frac{A}{s^2}$$

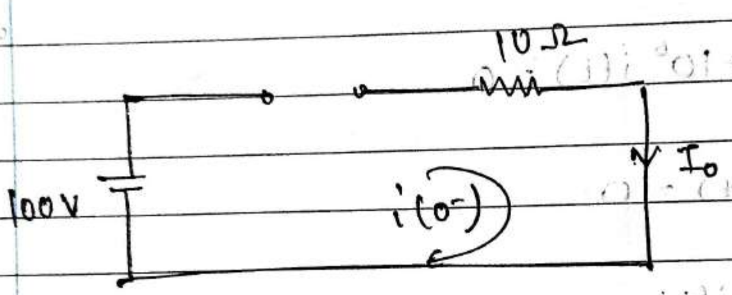
$$= 10^{5}\ A/s^2$$

2] Find

$i, \frac{di}{dt}$ and $\frac{d^2 i}{dt^2}$ at $t = 0^+$

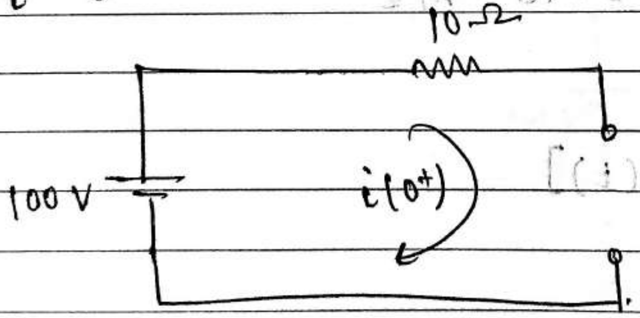


$\Rightarrow$ $t = 0^-$



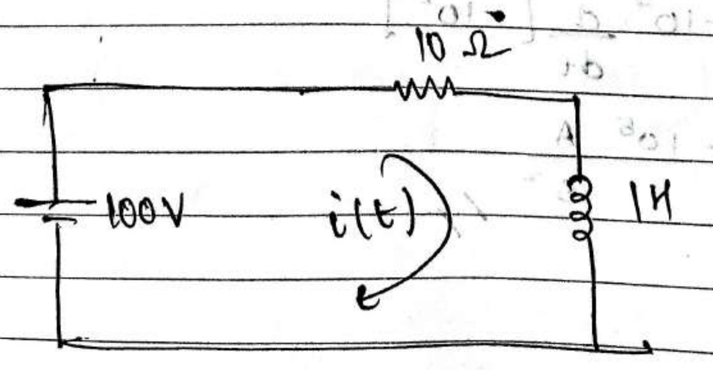
$i(0^-) = 0$ A $\quad \boxed{I_0 = 0 A}$

$t = 0^+$



$i(0^+) = 0$ A

$t = \infty$



Date :

KVL

$$100 - 10\,i(t) - 1\frac{d\,i(t)}{dt} = 0$$

$$\boxed{\frac{di(t)}{dt} = 100 - 10\,i(t)}$$

at $t = 0^+$

$$\frac{di(0^+)}{dt} = 100 - 10(0) = 100\ \frac{A}{s}$$

Diff. w.r.t t

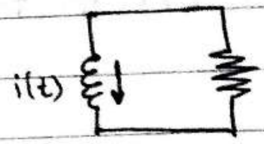
$$\frac{d^2 i(t)}{dt^2} = -10\frac{d\,i(t)}{dt} \quad \text{at } t = 0^+$$

$$= -10(100)$$

$$= -1000\ A/s^2$$

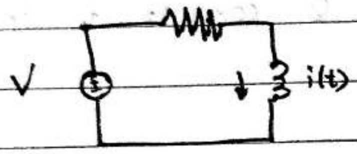
Types of Ckt   Std eqn

1] Simple RL



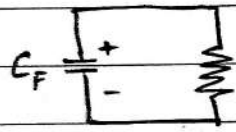
$$I(t) = I_0 e^{-\frac{Rt}{L}}$$

2] Driven RL



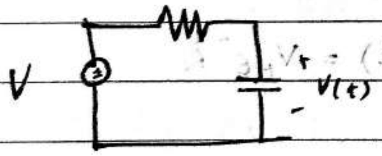
$$I(t) = \frac{V}{R} + \left[I_0 - \frac{V}{R}\right] e^{-\frac{Rt}{L}}$$

3] Simple RC



$$V(t) = V_0 e^{-\frac{t}{RC}}$$

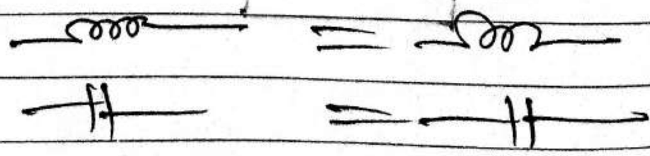
3] Driven RC



$$V(t) = V + [V_0 - V] e^{-\frac{t}{RC}}$$

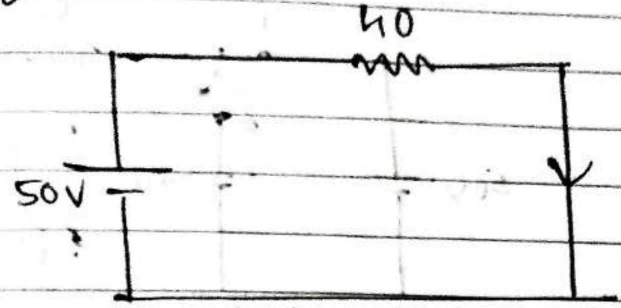
1   0

2   ∞ ⇒

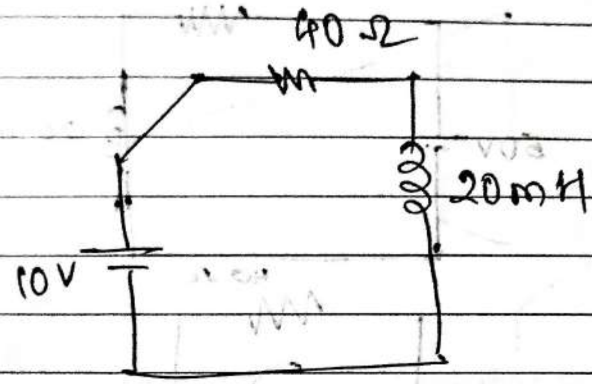


Date : ____________

$t = 0^-$



$I_0 = 1.25\ A$



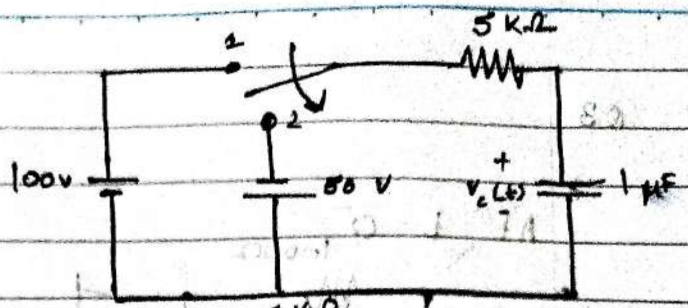
$$i(t) = \frac{V}{R} + \left[I_0 - \frac{V}{R} \right] e^{-\frac{Rt}{L}}$$

$$= \frac{10}{40} + \left(1.25 - \frac{10}{40} \right) e^{-\frac{40t}{20 \times 10^{-3}}}$$

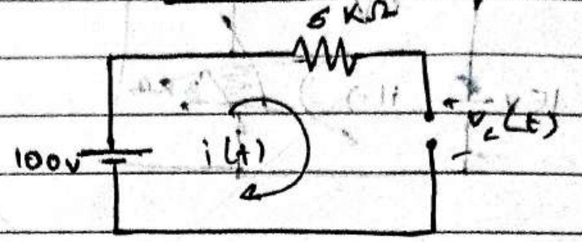
$$= 0.25 + 1\, e^{-2000t}$$

Date :

Q.



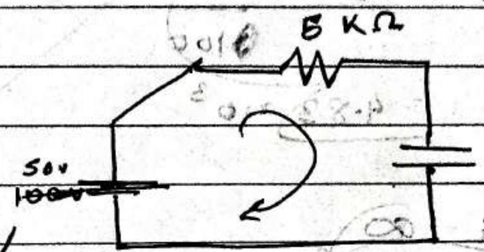
At $t = 0^-$



$i(t) = 0$

$v_c(t) = 100 - 5000 \cdot i(t)$

$= 100$ V



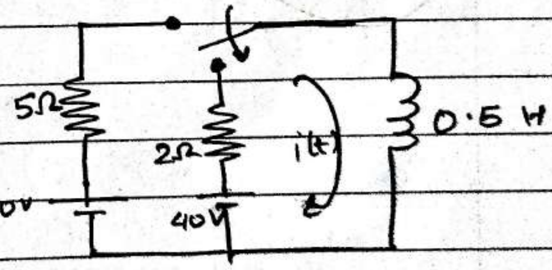
At $t = \infty$

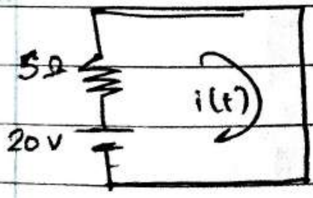
Driven RC

$V_c(t) = -50 + [100 + 50] e^{-t/5k \times 10^{-6}}$

$= -50 + 150 e^{-5 \times 10^{2} t}$

Q. At $t = 0^-$



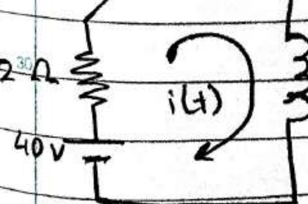


KVL

$20 - 5(i(t))$

$i(t) = 4$ A

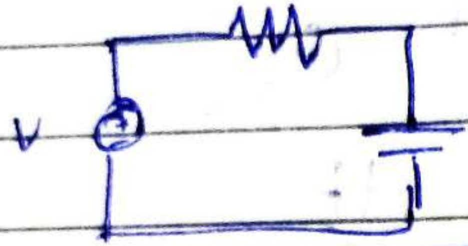
At $t = \infty$



Driven RL

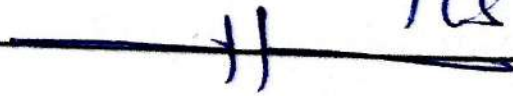
$i(t) = \frac{40}{2} + \left[4 - \frac{40}{2}\right] e^{-\frac{2t}{0.5}}$

$= 20 - 16 e^{-4t}$



$v(t) = V + \left[V_0 - V \right.$

current source

component	Laplace
	
L 	LS 
C 	1/CS 
volt source  V	 V/S

$$L^{-1}\left[1\right] = \delta(t)$$

$$L^{-1}\left[\frac{1}{s}\right] = u(t) \qquad L\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{(n-1)!}$$

$$L^{-1}\left[\frac{1}{s+a}\right] = e^{-at}\,u(t)$$

$$L^{-1}\left[\frac{1}{s-a}\right] = e^{at}\,u(t)$$

$$L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at\;u(t)$$

$$L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at\;u(t)$$

Partial frac

$$\frac{Nr}{(s+a)(s+b)} = \frac{A}{s+a} + \frac{B}{s+b}$$

$$\frac{Nr}{(s+a)(s+b)^2} = \frac{A}{s+a} + \frac{B}{(s+b)} + \frac{C}{(s+b)^2}$$

$$\frac{Nr}{(s+a)(s^2+bs+c)} = \frac{A}{s+a} + \frac{Bs+C}{s^2+bs+c}$$

eg: $\dfrac{s+29}{(s+4)(s^2+9)} = \dfrac{A}{s+4} + \dfrac{Bs+C}{s^2+9}$

Xing by $(s+4)(s^2+9)$

$s+29 = A(s^2+9) + (Bs+C)$

Put $s=-4$

$25 = 25A$

$A = 1$

put $s+29 =$ Xing by $(s+4)(s^2+9)$

$s+29 = A(s^2+9) + (Bs+C)(s+4)$

$s+29 = As^2 + A9 + Bs^2 + 4Bs + sC + 4C$

$s+29 = s^2(A+B) + s(4B+C) + A9 + 4C$

comparing coeff.

$0 = A+B$

$1 = 4B+C$

$29 = 9A+4C$

we get $A = 1$

$B = -1$

$C = 5$

Date

$s^2 - 4s + 4$ $s^2 - 2s - 2s + 4$

$s^2 - 2s + s - 2$

$s^2 - s - 2$

Q Find $L^{-1}\left[\frac{5s^2 - 15s - 11}{(s+1)(s-2)^2}\right]$

Let

$$\frac{5s^2 - 15s - 11}{(s+1)(s-2)^2} = \frac{A}{(s+1)} + \frac{B}{(s-2)} + \frac{C}{(s-2)^2}$$

$$5s^2 - 15s - 11 = A(s-2)^2 + B(s+1)(s-2) + C(s+1)$$

$$= As^2 - 4As + 4A + Bs^2 - Bs - 2B + sC + C$$

$$\cancel{= s^2(A+B) - s(4A - B) + 4A}$$

$$5s^2 - 15s - 11 = s^2(A+B) - s(4A + B - C) + 4A - 2B + C$$

comparing we get

$5 = A + B$

$15 = 4A + B - C$

$-11 = 4A - 2B + C$

$\therefore A = 1 \quad B = 4 \quad C = -7$

$$\Rightarrow \frac{1}{(s+1)} + \frac{4}{(s-2)} - \frac{7}{(s-2)^2}$$

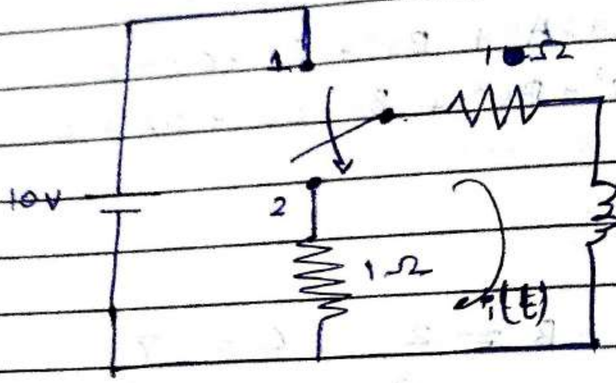
$$\therefore e^{-t}\ L^{-1}\left[\frac{1}{s+1}\right] + 4L^{-1}\left[\frac{1}{s-2}\right] - 7L^{-1}\left[\frac{1}{(s-2)^2}\right]$$

$$\Rightarrow e^{-t} + 4e^{2t} - 7e^{2t} L^{-1}\left[\frac{1}{s^2}\right]$$

Q $\cancel{L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)}\right]}$

$$F(s) = \frac{s}{(s^2+1)(s^2+4)} = \frac{As+C}{(s^2+1)} + \frac{Bs+d}{(s^2+4)}$$

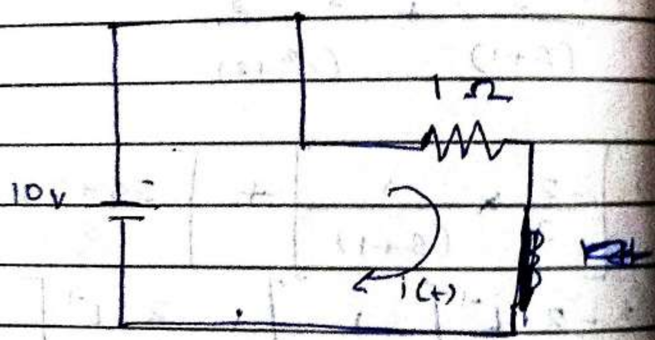
Q



At $t = -0$

$i(t) = \frac{10}{1}$

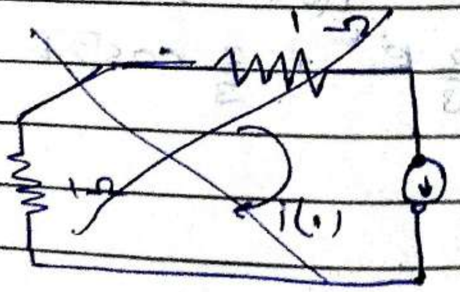
$= 10\ A$



~~At t = +0~~

$i(t) = 0$

~~V(t)~~ =

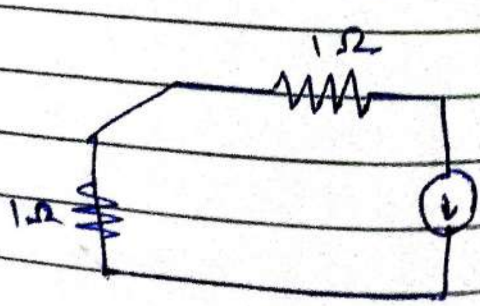


~~At t = ∞~~

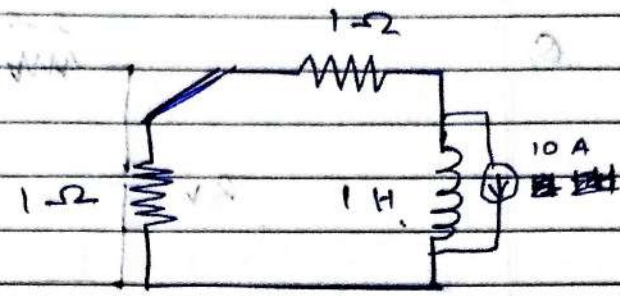
At $t = +0$

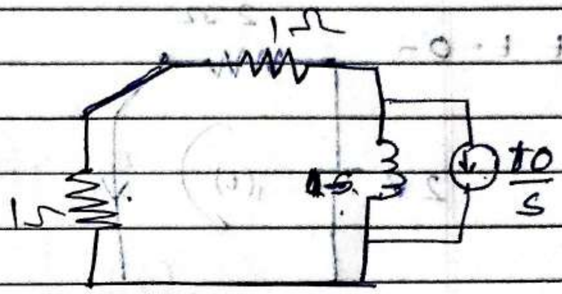
$i(t) = 10\ A$

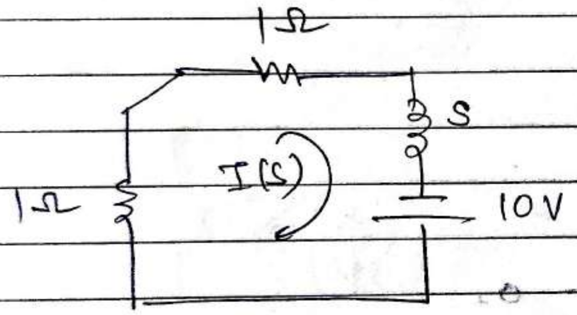
$v(t) = 20\ V$



At $t = \infty$







$$-2\,I(S) - S\,I(S) + 10 = 0$$

$$10 = S\,I(S) + 2\,I(S)$$

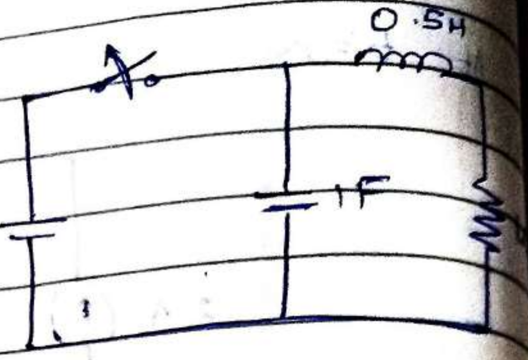
$$10 = I(S) \cdot (S+2)$$

$$I(S) = \frac{10}{S+2}$$

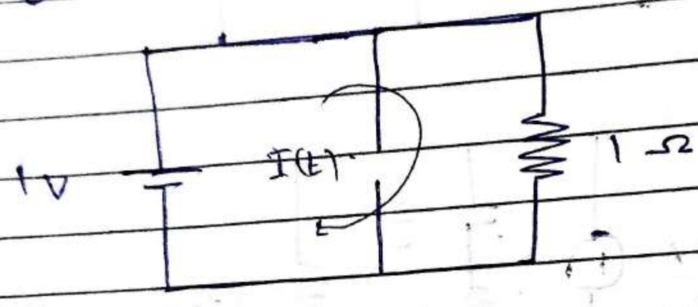
$$i(t) = \mathcal{L}^{-1}\left[\frac{10}{S+2}\right]$$

$$\boxed{i(t) = 10\,e^{-2t}\,u(t)}$$

Q.

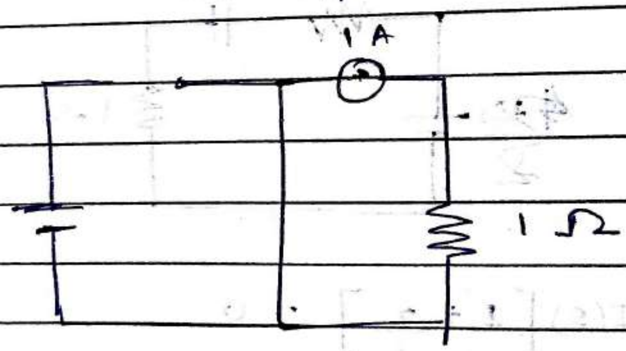


At $t = 0^-$



$I(t) = \frac{1}{1} = 1\ A.$

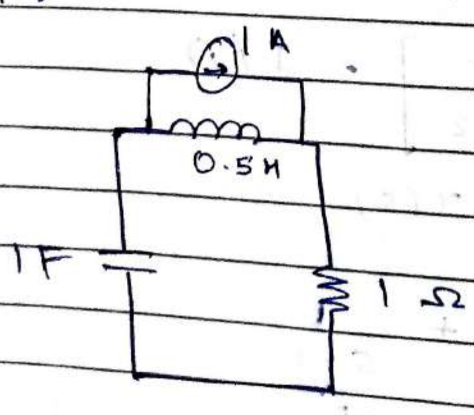
At $t = 0^+$



$i(t) = 1\ A$

$v(t) = 1\ V$

At $t = \infty$



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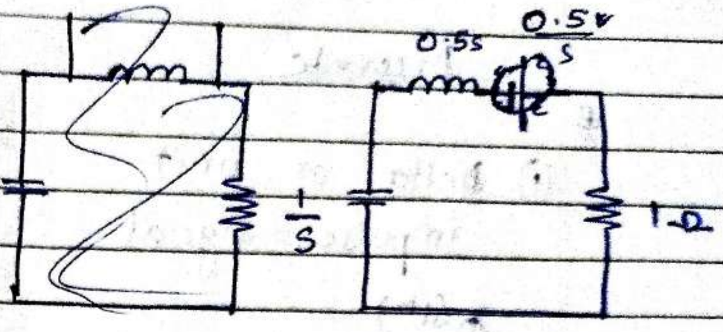
At $t = \infty$



$$\cancel{-\frac{1}{s} - 0.5s} \quad \frac{0.5}{s} - I(s)\left[\frac{1}{s} + 0.5s + 1\right] = 0$$

$$\frac{0.5}{s} = I(s)\left[\frac{1 + 0.5s^2 + s}{s}\right]$$

$$\frac{0.5}{1 + 0.5s^2 + s} = I(s)$$

$$\therefore I(s) = \cancel{0.5 \quad 0.5 \div 1 + 0.5s^2 + s}$$

$$= \frac{1}{2} \times \frac{2}{2 + s^2 + s}$$

$$= \frac{1}{2 + s^2 + 2s}$$

$$= \frac{1}{s^2 + 2s + 2} = \frac{1}{s^2 + 2s + 1 + 1}$$

$$=$$

$$\cancel{(s+1)s} = \frac{1}{(s+1)^2 + 1}$$

$$= \cancel{\frac{A}{(s+1)^2 + 1} + \frac{B}{(s+1)^2}} \quad \times 2 \qquad e^{-t}\, L^{-1}\left[\frac{1}{s^2+1}\right]$$

$$1 = \cancel{A + B(s+1)^2}$$