BIG M METHOD

Ex 3. Using the Penalty (Big M) method solve the following LPP

Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$

Subject to $x_1 + 2x_2 + 3x_3 = 15,$

$$2x_1 + x_2 + 5x_3 = 20,$$

$$x_1 + 2x_2 + x_3 + x_4 = 10,$$

$$x_1, x_2, x_3, x_4 \geq 10$$

Solution:

Introducing three artificial variables A_1, A_2, A_3 in the three equalities and assigning big penalty $-M$ in the object function for A_1, A_2, A_3.

We have,

Maximize

$$z = x_1 + 2x_2 + 3x_3 - x_4 - MA_1 - MA_2 - MA_3$$

Subject to $x_1 + 2x_2 + 3x_3 + A_1 = 15$

$2x_1 + x_2 + 5x_3 + A_2 = 20$

$x_1 + 2x_2 + x_3 + x_4 + A_3 = 10$

Adding M times the first, second and the third constraints to the object function, we get,

$$z = (1 + 4M)x_1 + (2 + 5M)x_2 + (3 + 9M)x_3$$
$$+ (-1 + M)x_4 - 45M$$

$$\therefore z - (1 + 4M)x_1 - (2 + 5M)x_2 - (3 + 9M)x_3$$
$$+ (1 - M)x_4 = -45M$$

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z									
	A_1									
	A_2									
	A_3									

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
	A_1	1	2	3	0	1	0	0	15	
	A_2	2	1	5	0	0	1	0	20	
	A_3	1	2	1	1	0	0	1	10	

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
	A_1	1	2	3	0	1	0	0	15	
	A_2	2	1	5	0	0	1	0	20	
	A_3	1	2	1	1	0	0	1	10	

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
	A_1	1	2	3	0	1	0	0	15	5
	A_2	2	1	5	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
	A_1	1	2	3	0	1	0	0	15	5
	A_2	2	1	5	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

1	z						—			
	A_1						—			
	x_3						—			
	A_3						—			

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

1	z						—			
	A_1						—			
	x_3	2/5	1/5	1	0	0	—	0	4	
	A_3						—			

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

1	z						—			
	A_1						—			
	x_3	2/5	1/5	1	0	0	—	0	4	
	A_3	3/5	9/5	0	1	0	—	1	6	

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

1	z						—			
	A_1	$-1/5$	$7/5$	0	0	1	—	0	3	
	x_3	$2/5$	$1/5$	1	0	0	—	0	4	
	A_3	$3/5$	$9/5$	0	1	0	—	1	6	

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

1	z	$\dfrac{1-2M}{5}$	$\dfrac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
	A_1	$-1/5$	$7/5$	0	0	1	—	0	3	
	x_3	$2/5$	$1/5$	1	0	0	—	0	4	
	A_3	$3/5$	$9/5$	0	1	0	—	1	6	

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
1	z	$\dfrac{1-2M}{5}$	$\dfrac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
	A_1	$-1/5$	$7/5$	0	0	1	—	0	3	
	x_3	$2/5$	$1/5$	1	0	0	—	0	4	
	A_3	$3/5$	$9/5$	0	1	0	—	1	6	

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

1	z	$\dfrac{1-2M}{5}$	$\dfrac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
	A_1	$-1/5$	$7/5$	0	0	1	—	0	3	15/7
	x_3	$2/5$	$1/5$	1	0	0	—	0	4	20
	A_3	$3/5$	$9/5$	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
	A_1	$-1/5$	$7/5$	0	0	1	—	0	3	15/7
	x_3	$2/5$	$1/5$	1	0	0	—	0	4	20
	A_3	$3/5$	$9/5$	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	$7/5$*	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	7/5*	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
2	z					—	—			
	x_2					—	—			
	x_3					—	—			
	A_3					—	—			

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	7/5*	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
2	z					—	—			
	x_2	$-1/7$	1	0	0	—	—	0	15/7	
	x_3					—	—			
	A_3					—	—			

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
1	z	$\dfrac{1-2M}{5}$	$\dfrac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	7/5*	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
2	z	$-\dfrac{6M}{7}$	0	0	$1-M$	—	—	0	$\dfrac{105-15M}{7}$	
	x_2	$-1/7$	1	0	0	—	—	0	15/7	
	x_3					—	—			
	A_3					—	—			

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	7/5*	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
2	z	$-\frac{6M}{7}$	0	0	$1-M$	—	—	0	$\frac{105-15M}{7}$	
	x_2	$-1/7$	1	0	0	—	—	0	15/7	
	x_3	3/7	0	1	0	—	—	0	25/7	
	A_3					—	—			

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	7/5*	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3

2	z	$-\frac{6M}{7}$	0	0	$1-M$	—	—	0	$\frac{105-15M}{7}$	
	x_2	$-1/7$	1	0	0	—	—	0	15/7	
	x_3	3/7	0	1	0	—	—	0	25/7	
	A_3	6/7	0	0	1	—	—	1	15/7	

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	$7/5$*	0	0	1	—	0	3	15/7
x_2 enters	x_3	$2/5$	$1/5$	1	0	0	—	0	4	20
	A_3	$3/5$	$9/5$	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
2	z	$-\frac{6M}{7}$	0	0	$1-M$	—	—	0	$\frac{105-15M}{7}$	
	x_2	$-1/7$	1	0	0	—	—	0	15/7	
	x_3	$3/7$	0	1	0	—	—	0	25/7	
	A_3	$6/7$	0	0	1	—	—	1	15/7	

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	$7/5$*	0	0	1	—	0	3	15/7
x_2 enters	x_3	$2/5$	$1/5$	1	0	0	—	0	4	20
	A_3	$3/5$	$9/5$	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
2	z	$-\frac{6M}{7}$	0	0	$1-M$	—	—	0	$\frac{105-15M}{7}$	
	x_2	$-1/7$	1	0	0	—	—	0	15/7	—
	x_3	$3/7$	0	1	0	—	—	0	25/7	—
	A_3	$6/7$	0	0	1	—	—	1	15/7	15/7

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	$7/5$*	0	0	1	—	0	3	15/7
x_2 enters	x_3	$2/5$	$1/5$	1	0	0	—	0	4	20
	A_3	$3/5$	$9/5$	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
2	z	$-\frac{6M}{7}$	0	0	$1-M$	—	—	0	$\frac{105-15M}{7}$	
	x_2	$-1/7$	1	0	0	—	—	0	15/7	—
	x_3	$3/7$	0	1	0	—	—	0	25/7	—
	A_3	$6/7$	0	0	1	—	—	1	15/7	15/7

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	$7/5$*	0	0	1	—	0	3	15/7
x_2 enters	x_3	$2/5$	$1/5$	1	0	0	—	0	4	20
	A_3	$3/5$	$9/5$	0	1	0	—	1	6	10/3

		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
2	z	$-\frac{6M}{7}$	0	0	$1-M$	—	—	0	$\frac{105-15M}{7}$	
A_3 leaves	x_2	$-1/7$	1	0	0	—	—	0	15/7	—
x_4 enters	x_3	$3/7$	0	1	0	—	—	0	25/7	—
	A_3	$6/7$ *	0	0	1	—	—	1	15/7	15/7

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	$7/5*$	0	0	1	—	0	3	15/7
x_2 enters	x_3	$2/5$	$1/5$	1	0	0	—	0	4	20
	A_3	$3/5$	$9/5$	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
2	z	$-\dfrac{6M}{7}$	0	0	$1-M$	—	—	0	$\dfrac{105-15M}{7}$	
A_3 leaves	x_2	$-1/7$	1	0	0	—	—	0	15/7	—
x_4 enters	x_3	$3/7$	0	1	0	—	—	0	25/7	—
	A_3	$6/7$	0	0	$1*$	—	—	1	15/7	15/7

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
3	z					—	—	—		
	x_2					—	—	—		
	x_3					—	—	—		
	x_4					—	—	—		

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	7/5*	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3

2	z	$-\dfrac{6M}{7}$	0	0	$1-M$	—	—	0	$\dfrac{105-15M}{7}$	
A_3 leaves	x_2	$-1/7$	1	0	0	—	—	0	15/7	—
x_4 enters	x_3	3/7	0	1	0	—	—	0	25/7	—
	A_3	6/7	0	0	1*	—	—	1	15/7	15/7

3	z					—	—	—		
	x_2					—	—	—		
	x_3					—	—	—		
	x_4	6/7	0	0	1	—	—	—	15/7	

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	$7/5$*	0	0	1	—	0	3	15/7
x_2 enters	x_3	$2/5$	$1/5$	1	0	0	—	0	4	20
	A_3	$3/5$	$9/5$	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
2	z	$-\frac{6M}{7}$	0	0	$1-M$	—	—	0	$\frac{105-15M}{7}$	
A_3 leaves	x_2	$-1/7$	1	0	0	—	—	0	15/7	—
x_4 enters	x_3	$3/7$	0	1	0	—	—	0	25/7	—
	A_3	$6/7$	0	0	1*	—	—	1	15/7	15/7

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
3	z					—	—	—		
	x_2					—	—	—		
	x_3	$3/7$	0	1	0	—	—	—	25/7	
	x_4	$6/7$	0	0	1	—	—	—	15/7	

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	7/5*	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
2	z	$-\dfrac{6M}{7}$	0	0	$1-M$	—	—	0	$\dfrac{105-15M}{7}$	
A_3 leaves	x_2	$-1/7$	1	0	0	—	—	0	15/7	—
x_4 enters	x_3	3/7	0	1	0	—	—	0	25/7	—
	A_3	6/7	0	0	1*	—	—	1	15/7	15/7

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
3	z					—	—	—		
	x_2	-1/7	1	0	0	—	—	—	15/7	
	x_3	3/7	0	1	0	—	—	—	25/7	
	x_4	6/7	0	0	1	—	—	—	15/7	

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	$7/5$*	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
2	z	$-\frac{6M}{7}$	0	0	$1-M$	—	—	0	$\frac{105-15M}{7}$	
A_3 leaves	x_2	$-1/7$	1	0	0	—	—	0	15/7	—
x_4 enters	x_3	3/7	0	1	0	—	—	0	25/7	—
	A_3	6/7	0	0	1*	—	—	1	15/7	15/7

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
3	z	-6/7	0	0	0	—	—	—	90/7	
	x_2	-1/7	1	0	0	—	—	—	15/7	
	x_3	3/7	0	1	0	—	—	—	25/7	
	x_4	6/7	0	0	1	—	—	—	15/7	

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	$7/5$*	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3

2	z	$-\dfrac{6M}{7}$	0	0	$1-M$	—	—	0	$\dfrac{105-15M}{7}$	
A_3 leaves	x_2	$-1/7$	1	0	0	—	—	0	15/7	—
x_4 enters	x_3	3/7	0	1	0	—	—	0	25/7	—
	A_3	6/7	0	0	1*	—	—	1	15/7	15/7

3	z	-6/7	0	0	0	—	—	—	90/7	
	x_2	-1/7	1	0	0	—	—	—	15/7	
	x_3	3/7	0	1	0	—	—	—	25/7	
	x_4	6/7	0	0	1	—	—	—	15/7	

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	$7/5$*	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
2	z	$-\dfrac{6M}{7}$	0	0	$1-M$	—	—	0	$\dfrac{105-15M}{7}$	
A_3 leaves	x_2	$-1/7$	1	0	0	—	—	0	15/7	$-$
x_4 enters	x_3	3/7	0	1	0	—	—	0	25/7	$-$
	A_3	6/7	0	0	1*	—	—	1	15/7	15/7

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
3	z	-6/7	0	0	0	—	—	—	90/7	
	x_2	-1/7	1	0	0	—	—	—	15/7	-ve
	x_3	3/7	0	1	0	—	—	—	25/7	25/3
	x_4	6/7	0	0	1	—	—	—	15/7	15/6

Iteration Number	Basic Var.	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS sol.	Ratio
				Coefficients of						
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	7/5*	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3
2	z	$-\dfrac{6M}{7}$	0	0	$1-M$	—	—	0	$\dfrac{105-15M}{7}$	
A_3 leaves	x_2	$-1/7$	1	0	0	—	—	0	15/7	—
x_4 enters	x_3	3/7	0	1	0	—	—	0	25/7	—
	A_3	6/7	0	0	1*	—	—	1	15/7	15/7
3	z	-6/7	0	0	0	—	—	—	90/7	
	x_2	-1/7	1	0	0	—	—	—	15/7	-ve
	x_3	3/7	0	1	0	—	—	—	25/7	25/3
	x_4	6/7	0	0	1	—	—	—	15/7	15/6

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
0	z	$-(1+4M)$	$-(2+5M)$	$-(3+9M)$	$1-M$	0	0	0	$-45M$	
A_2 leaves	A_1	1	2	3	0	1	0	0	15	5
x_3 enters	A_2	2	1	5*	0	0	1	0	20	4
	A_3	1	2	1	1	0	0	1	10	10

1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	7/5*	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3

2	z	$-\dfrac{6M}{7}$	0	0	$1-M$	—	—	0	$\dfrac{105-15M}{7}$	
A_3 leaves	x_2	$-1/7$	1	0	0	—	—	0	15/7	—
x_4 enters	x_3	3/7	0	1	0	—	—	0	25/7	—
	A_3	6/7	0	0	1*	—	—	1	15/7	15/7

3	z	-6/7	0	0	0	—	—	—	90/7	
	x_2	-1/7	1	0	0	—	—	—	15/7	-ve
	x_3	3/7	0	1	0	—	—	—	25/7	25/3
	x_4	6/7	0	0	1	—	—	—	15/7	15/6

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Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	$7/5*$	0	0	1	—	0	3	15/7
x_2 enters	x_3	$2/5$	$1/5$	1	0	0	—	0	4	20
	A_3	$3/5$	$9/5$	0	1	0	—	1	6	10/3
2	z	$-\frac{6M}{7}$	0	0	$1-M$	—	—	0	$\frac{105-15M}{7}$	
A_3 leaves	x_2	$-1/7$	1	0	0	—	—	0	15/7	—
x_4 enters	x_3	$3/7$	0	1	0	—	—	0	25/7	—
	A_3	$6/7$	0	0	$1*$	—	—	1	15/7	15/7
3	z	-6/7	0	0	0	—	—	—	90/7	
x_4 leaves	x_2	-1/7	1	0	0	—	—	—	15/7	-ve
x_1 enters	x_3	3/7	0	1	0	—	—	—	25/7	25/3
	x_4	6/7*	0	0	1	—	—	—	15/7	15/6

Iteration Number	Basic Var.	Coefficients of							RHS sol.	Ratio
		x_1	x_2	x_3	x_4	A_1	A_2	A_3		
1	z	$\frac{1-2M}{5}$	$\frac{-7-16M}{5}$	0	$1-M$	0	—	0	$12-9M$	
A_1 leaves	A_1	$-1/5$	$7/5^*$	0	0	1	—	0	3	15/7
x_2 enters	x_3	2/5	1/5	1	0	0	—	0	4	20
	A_3	3/5	9/5	0	1	0	—	1	6	10/3
2	z	$-\frac{6M}{7}$	0	0	$1-M$	—	—	0	$\frac{105-15M}{7}$	
A_3 leaves	x_2	$-1/7$	1	0	0	—	—	0	15/7	—
x_4 enters	x_3	3/7	0	1	0	—	—	0	25/7	—
	A_3	6/7	0	0	1^*	—	—	1	15/7	15/7
3	z	-6/7	0	0	0	—	—	—	90/7	
x_4 leaves	x_2	-1/7	1	0	0	—	—	—	15/7	-ve
x_1 enters	x_3	3/7	0	1	0	—	—	—	25/7	25/3
	x_4	$6/7^*$	0	0	1	—	—	—	15/7	15/6
4	z	0	0	0	1	—	—	—	15	
	x_2	0	1	0	1/6	—	—	—	5/2	
	x_3	0	0	1	-1/2	—	—	—	5/2	
	x_1	1	0	0	7/6	—	—	—39	5/2	

Since all the coefficients in the objective equation in the row of z are positive.
This is a optimal solution.
The values of the variables and of z are given by the RHS column

$$\therefore x_1 = \frac{5}{2}, x_2 = \frac{5}{2}, x_3 = \frac{5}{2}, z_{max} = 15$$