BIG M METHOD

THE BIG M-METHOD

The method is due to Charnes and is based on the following considerations

If any one or some constraints are of greater than type then we have to subtract surplus variables i.e. we have to add $-s_1, -s_2, \ldots..$ etc to convert "the greater than or equal to" type inequality to equality.

But then we would not get a unit matrix.

To overcome this difficulty, we introduce in addition to surplus variables $-s_1, -s_2, \ldots..$ artificial variables $A_1, A_2 \ldots.$ with positive sign in these constraints.

- An artificial variable is introduced even when the constraints is of equality type.
- In the objective function we assign big penalty by subtracting $MA_1, MA_2, \ldots.$ if the objective function is of maximization type.

Consider the problem

Maximize $z = c_1x_1 + c_2x_2 + c_3x_3$

Subject to

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \geq b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \leq b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \leq b_3$$
$$x_1, x_2, x_3 \geq 0$$

- Since the first constraints is of "greater than or equal to type" $(\geq)$, we subtract s_1 and add an artificial variables A_1.
- In the objective function, we assign big penalty for this artificial variable A_1 i.e. we subtract MA_1 from the objective function. Thus, we have

Maximize $Z = c_1x_1 + c_2x_2 + c_3x_3 - MA_1$

Subject to

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 - s_1 + A_1 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + s_2 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + s_3 = b_3$$

$$x_1, x_2, x_3, s_1, s_2, s_3, A_1 \geq 0$$

- We now write the objective function free from artificial variable by adding M times, the first constraint to the objective function.

- Now, the objective function becomes

$$z = (c_1 + Ma_{11})x_1 + (c_2 + Ma_{12})x_2 + (c_3 + Ma_{13})x_3 - Ms_1 - Mb_1$$

$$\therefore$$

$$z - (c_1 + Ma_{11})x_1 - (c_2 + Ma_{12})x_2 - (c_3 + Ma_{13})x_3 + Ms_1 = -Mb_1$$

Now, we follow all the usual steps of simplex method as before. After the required number of iterations, we will find one of the following situations.

1. The artificial variables leaves the process and the optimality condition is satisfied by the basic variables. This is, then the **optimal basic feasible solutions**

2. Atleast one of the artificial variables remains in the basic with zero values and the optimality condition is satisfied. This is the **optimal basic feasible solution** (though **degenerate**) to the given problem

3. Atleast one of the artificial variables remains in the basis with non-zero value and the optimality condition is satisfied. This solution though satisfies optimality conditions is not an optimal solution since it contains large penalty M. This is not a solution but a **pseudo-Solution**

EX 1. Using penalty (Big-M or Charne's) method solve the following LPP

Maximize $z = 3x_1 - x_2$

Subject to $2x_1 + x_2 \geq 2,$

$$x_1 + 3x_2 \leq 3,$$

$$x_2 \leq 4,$$

$$x_1, x_2 \geq 0$$

Solution: We introduce the artificial variable A_1 in the first constraint and big penalty in the object function

Maximize $z = 3x_1 - x_2 - MA_1$ ..... (1)

Subject to

$2x_1 + x_2 - s_1 + A_1 = 2$ .... (2)

$x_1 + 3x_2 + s_2 = 3$ .... (3)

$x_2 + s_3 = 4$ .... (4)

We now eliminate the term $-MA_1$ from (1) by adding M times the first constraint to it

$$\therefore z = (3+2M)x_1 + (-1+M)x_2 - Ms_1 - 2M$$

$$\therefore z - (3+2M)x_1 - (-1+M)x_2 + Ms_1 = -2M$$

Setting decision variables $x_1 = 0, x_2 = 0$ & $s_1 = 0$

as the basic feasible solution is

$A_1 = 2, s_2 = 3, s_3 = 4.$

A_1 is greater than zero, in this case 2.

But it must not appear in the final solution. To achieve this we assign a large penalty $(-M)$ to A_1 in the object function (1).

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z								
	A_1								
	s_2								
	s_3								

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3-2M$	$1-M$	M	0	0	0	$-2M$	
	A_1	2	1	-1	0	0	1	2	
	s_2	1	3	0	1	0	0	3	
	s_3	0	1	0	0	1	0	4	

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3 - 2M$	$1 - M$	M	0	0	0	$-2M$	
	A_1	2	1	-1	0	0	1	2	
	s_2	1	3	0	1	0	0	3	
	s_3	0	1	0	0	1	0	4	

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3-2M$	$1-M$	M	0	0	0	$-2M$	
	A_1	2	1	-1	0	0	1	2	$2/2=1$
	s_2	1	3	0	1	0	0	3	$3/1=3$
	s_3	0	1	0	0	1	0	4	$1/0=\cdots$

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3-2M$	$1-M$	M	0	0	0	$-2M$	
	A_1	2	1	-1	0	0	1	2	$2/2=1$
	s_2	1	3	0	1	0	0	3	$3/1=3$
	s_3	0	1	0	0	1	0	4	$1/0=\cdots$

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3 - 2M$	$1 - M$	M	0	0	0	$-2M$	
A_1 leaves	A_1	2*	1	-1	0	0	1	2	$2/2 = 1$
x_1 enters	s_2	1	3	0	1	0	0	3	$3/1 = 3$
	s_3	0	1	0	0	1	0	4	$1/0 = \cdots$

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3 - 2M$	$1 - M$	M	0	0	0	$-2M$	
A_1 leaves	A_1	2*	1	-1	0	0	1	2	$2/2 = 1$
x_1 enters	s_2	1	3	0	1	0	0	3	$3/1 = 3$
	s_3	0	1	0	0	1	0	4	$1/0 = \cdots$

1	z								
	x_1								
	s_2								
	s_3								

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3-2M$	$1-M$	M	0	0	0	$-2M$	
A_1 leaves	A_1	2*	1	-1	0	0	1	2	$2/2=1$
x_1 enters	s_2	1	3	0	1	0	0	3	$3/1=3$
	s_3	0	1	0	0	1	0	4	$1/0=\cdots$

1	z								
	x_1	1	$1/2$	$-1/2$	0	0	—	1	
	s_2								
	s_3								

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3-2M$	$1-M$	M	0	0	0	$-2M$	
A_1 leaves	A_1	2*	1	-1	0	0	1	2	$2/2=1$
x_1 enters	s_2	1	3	0	1	0	0	3	$3/1=3$
	s_3	0	1	0	0	1	0	4	$1/0=\cdots$

1	z	0	$5/2$	$-3/2$	0	0	—	3	
	x_1	1	$1/2$	$-1/2$	0	0	—	1	
	s_2								
	s_3								

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3-2M$	$1-M$	M	0	0	0	$-2M$	
A_1 leaves	A_1	2*	1	-1	0	0	1	2	$2/2=1$
x_1 enters	s_2	1	3	0	1	0	0	3	$3/1=3$
	s_3	0	1	0	0	1	0	4	$1/0=\cdots$
1	z	0	$5/2$	$-3/2$	0	0	—	3	
	x_1	1	$1/2$	$-1/2$	0	0	—	1	
	s_2	0	$5/2$	$1/2$	1	0	—	2	
	s_3								

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3-2M$	$1-M$	M	0	0	0	$-2M$	
A_1 leaves	A_1	2*	1	-1	0	0	1	2	$2/2=1$
x_1 enters	s_2	1	3	0	1	0	0	3	$3/1=3$
	s_3	0	1	0	0	1	0	4	$1/0=\cdots$

1	z	0	$5/2$	$-3/2$	0	0	—	3	
	x_1	1	$1/2$	$-1/2$	0	0	—	1	
	s_2	0	$5/2$	$1/2$	1	0	—	2	
	s_3	0	1	0	0	1	—	4	

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3-2M$	$1-M$	M	0	0	0	$-2M$	
A_1 leaves	A_1	2*	1	-1	0	0	1	2	$2/2=1$
x_1 enters	s_2	1	3	0	1	0	0	3	$3/1=3$
	s_3	0	1	0	0	1	0	4	$1/0=\cdots$

1	z	0	$5/2$	$-3/2$	0	0	—	3	
	x_1	1	$1/2$	$-1/2$	0	0	—	1	
	s_2	0	$5/2$	$1/2$	1	0	—	2	
	s_3	0	1	0	0	1	—	4	

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3-2M$	$1-M$	M	0	0	0	$-2M$	
A_1 leaves	A_1	2*	1	-1	0	0	1	2	$2/2=1$
x_1 enters	s_2	1	3	0	1	0	0	3	$3/1=3$
	s_3	0	1	0	0	1	0	4	$1/0=\cdots$

1	z	0	$5/2$	$-3/2$	0	0	—	3	
	x_1	1	$1/2$	$-1/2$	0	0	—	1	-2
	s_2	0	$5/2$	$1/2$	1	0	—	2	4
	s_3	0	1	0	0	1	—	4	—

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3-2M$	$1-M$	M	0	0	0	$-2M$	
A_1 leaves	A_1	2*	1	-1	0	0	1	2	$2/2=1$
x_1 enters	s_2	1	3	0	1	0	0	3	$3/1=3$
	s_3	0	1	0	0	1	0	4	$1/0=\cdots$

1	z	0	$5/2$	$-3/2$	0	0	—	3	
	x_1	1	$1/2$	$-1/2$	0	0	—	1	-2
	s_2	0	$5/2$	$1/2$	1	0	—	2	4
	s_3	0	1	0	0	1	—	4	—

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
0	z	$-3-2M$	$1-M$	M	0	0	0	$-2M$	
A_1 leaves	A_1	2^*	1	-1	0	0	1	2	$2/2=1$
x_1 enters	s_2	1	3	0	1	0	0	3	$3/1=3$
	s_3	0	1	0	0	1	0	4	$1/0=\cdots$

1	z	0	$5/2$	$-3/2$	0	0	—	3	
s_2 leaves	x_1	1	$1/2$	$-1/2$	0	0	—	1	-2
s_1 enters	s_2	0	$5/2$	$1/2$ *	1	0	—	2	4
	s_3	0	1	0	0	1	—	4	—

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
1	z	0	$5/2$	$-3/2$	0	0	—	3	
s_2 leaves	x_1	1	$1/2$	$-1/2$	0	0	—	1	-2
s_1 enters	s_2	0	$5/2$	$1/2$ *	1	0	—	2	4
	s_3	0	1	0	0	1	—	4	—
							—		
2	z								
	x_1								
	s_1								
	s_3								

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
1	z	0	$5/2$	$-3/2$	0	0	—	3	
s_2 leaves	x_1	1	$1/2$	$-1/2$	0	0	—	1	-2
s_1 enters	s_2	0	$5/2$	$1/2$ *	1	0	—	2	4
	s_3	0	1	0	0	1	—	4	—
							—		
2	z								
	x_1								
	s_1	0	5	1	2	0	—	4	
	s_3								

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
1	z	0	5/2	−3/2	0	0	—	3	
s_2 leaves	x_1	1	1/2	−1/2	0	0	—	1	−2
s_1 enters	s_2	0	5/2	1/2 *	1	0	—	2	4
	s_3	0	1	0	0	1	—	4	—
							—		
2	z								
	x_1								
	s_1	0	5	1	2	0	—	4	
	s_3	0	1	0	0	1	—	4	

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
1	z	0	5/2	−3/2	0	0	—	3	
s_2 leaves	x_1	1	1/2	−1/2	0	0	—	1	−2
s_1 enters	s_2	0	5/2	1/2 *	1	0	—	2	4
	s_3	0	1	0	0	1	—	4	—
							—		
2	z								
	x_1	1	3	0	1	0	—	3	
	s_1	0	5	1	2	0	—	4	
	s_3	0	1	0	0	1	—	4	

Iteration Number	Basic Variable	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	s_3	A_1		
1	z	0	5/2	−3/2	0	0	—	3	
s_2 leaves	x_1	1	1/2	−1/2	0	0	—	1	−2
s_1 enters	s_2	0	5/2	1/2 *	1	0	—	2	4
	s_3	0	1	0	0	1	—	4	—
							—		
2	z	0	10	0	3	0	—	9	
	x_1	1	3	0	1	0	—	3	
	s_1	0	5	1	2	0	—	4	
	s_3	0	1	0	0	1	—	4	

Since all the coefficients in the objective equation in the row of z are positive.

This is a optimal solution.

The values of the variables and of z are given by the RHS column

$\therefore x_1 = 3, x_2 = 0$ and $z_{max} = 9$

EX . Use Penalty method to solve the following LPP

Minimize $z = 2x_1 + 3x_2$

Subject to $x_1 + x_2 \geq 5,$

$$x_1 + 2x_2 \geq 6,$$

$$x_1, x_2 \geq 0$$

Since the given problem is of minimization type we convert it into maximization type

Maximize $z' = (-z) = -2x_1 - 3x_2$

Subject to $x_1 + x_2 \geq 5,$

$x_1 + 2x_2 \geq 6$

We now introduce slack and artificial variables and the penalties in the object function

Maximize $z' = -2x_1 - 3x_2 - MA_1 - MA_2$

Subject to $x_1 + x_2 - s_1 + A_1 = 5$

$x_1 + 2x_2 - s_2 + A_2 = 6$

We now eliminate the terms $-MA_1$ and $-MA_2$ from the object function by adding M times the first and second constraints to the object function

$$z' = -2x_1 - 3x_2 - MA_1 - MA_2 + Mx_1 + Mx_2 - Ms_1 + MA_1 - 5M + Mx_1 + 2Mx_2 - Ms_2 + MA_2 - 6M$$

$$\therefore z' = (-2 + 2M)x_1 + (-3 + 3M)x_2 - Ms_1 - Ms_2 - 0A_1 - 0A_2 - 11M$$

$$\therefore z' + (2 - 2M)x_1 + (3 - 3M)x_2 + Ms_1 + Ms_2 + 0A_1 + 0A_2 = -11M$$

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'								
	A_1								
	A_2								

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
	A_1	1	1	-1	0	1	0	5	
	A_2	1	2	0	-1	0	1	6	

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
	A_1	1	1	-1	0	1	0	5	
	A_2	1	2	0	-1	0	1	6	

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
	A_1	1	1	-1	0	1	0	5	$5/1=5$
	A_2	1	2	0	-1	0	1	6	$6/2=3$

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
	A_1	1	1	-1	0	1	0	5	$5/1=5$
	A_2	1	2	0	-1	0	1	6	$6/2=3$

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
A_2 leaves	A_1	1	1	-1	0	1	0	5	$5/1=5$
x_2 enters	A_2	1	2*	0	-1	0	1	6	$6/2=3$

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
A_2 leaves	A_1	1	1	-1	0	1	0	5	$5/1=5$
x_2 enters	A_2	1	2*	0	-1	0	1	6	$6/2=3$

1	z'								
	A_1								
	x_2								

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
A_2 leaves	A_1	1	1	-1	0	1	0	5	$5/1=5$
x_2 enters	A_2	1	2*	0	-1	0	1	6	$6/2=3$

1	z'								
	A_1								
	x_2	$1/2$	1	0	$-1/2$	0	—	3	

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
A_2 leaves	A_1	1	1	-1	0	1	0	5	$5/1=5$
x_2 enters	A_2	1	2*	0	-1	0	1	6	$6/2=3$

1	z'								
	A_1	$1/2$	0	-1	$1/2$	1	—	2	
	x_2	$1/2$	1	0	$-1/2$	0	—	3	

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
A_2 leaves	A_1	1	1	-1	0	1	0	5	$5/1=5$
x_2 enters	A_2	1	2*	0	-1	0	1	6	$6/2=3$

1	z'	$\frac{1-M}{2}$	0	M	$\frac{3-M}{2}$	0	—	$-9-2M$	
	A_1	$1/2$	0	-1	$1/2$	1	—	2	
	x_2	$1/2$	1	0	$-1/2$	0	—	3	

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
A_2 leaves	A_1	1	1	-1	0	1	0	5	$5/1=5$
x_2 enters	A_2	1	2*	0	-1	0	1	6	$6/2=3$

1	z'	$\frac{1-M}{2}$	0	M	$\frac{3-M}{2}$	0	—	$-9-2M$	
	A_1	$1/2$	0	-1	$1/2$	1	—	2	
	x_2	$1/2$	1	0	$-1/2$	0	—	3	

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
A_2 leaves	A_1	1	1	-1	0	1	0	5	$5/1=5$
x_2 enters	A_2	1	2*	0	-1	0	1	6	$6/2=3$

1	z'	$\frac{1-M}{2}$	0	M	$\frac{3-M}{2}$	0	—	$-9-2M$	
	A_1	$1/2$	0	-1	$1/2$	1	—	2	4
	x_2	$1/2$	1	0	$-1/2$	0	—	3	6

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
A_2 leaves	A_1	1	1	-1	0	1	0	5	$5/1=5$
x_2 enters	A_2	1	2*	0	-1	0	1	6	$6/2=3$

1	z'	$\frac{1-M}{2}$	0	M	$\frac{3-M}{2}$	0	—	$-9-2M$	
	A_1	$1/2$	0	-1	$1/2$	1	—	2	4
	x_2	$1/2$	1	0	$-1/2$	0	—	3	6

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
A_2 leaves	A_1	1	1	-1	0	1	0	5	$5/1=5$
x_2 enters	A_2	1	2*	0	-1	0	1	6	$6/2=3$

1	z'	$\frac{1-M}{2}$	0	M	$\frac{3-M}{2}$	0	—	$-9-2M$	
A_1 leaves	A_1	$1/2^*$	0	-1	$1/2$	1	—	2	4
x_1 enters	x_2	$1/2$	1	0	$-1/2$	0	—	3	6

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
1	z'	$\frac{1-M}{2}$	0	M	$\frac{3-M}{2}$	0	—	$-9-2M$	
A_1 leaves	A_1	$1/2^*$	0	-1	$1/2$	1	—	2	4
x_1 enters	x_2	$1/2$	1	0	$-1/2$	0	—	3	6
							—		
2	z'					—	—		
	x_1					—	—		
	x_2					—	—		

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
A_2 leaves	A_1	1	1	-1	0	1	0	5	$5/1=5$
x_2 enters	A_2	1	2*	0	-1	0	1	6	$6/2=3$
1	z'	$\frac{1-M}{2}$	0	M	$\frac{3-M}{2}$	0	—	$-9-2M$	
A_1 leaves	A_1	$1/2^*$	0	-1	$1/2$	1	—	2	4
x_1 enters	x_2	$1/2$	1	0	$-1/2$	0	—	3	6
							—		
2	z'					—	—		
	x_1	1	0	-2	1	—	—	4	
	x_2					—	—		

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
A_2 leaves	A_1	1	1	-1	0	1	0	5	$5/1=5$
x_2 enters	A_2	1	2*	0	-1	0	1	6	$6/2=3$

1	z'	$\frac{1-M}{2}$	0	M	$\frac{3-M}{2}$	0	—	$-9-2M$	
A_1 leaves	A_1	$1/2^*$	0	-1	$1/2$	1	—	2	4
x_1 enters	x_2	$1/2$	1	0	$-1/2$	0	—	3	6

2	z'					—	—		
	x_1	1	0	-2	1	—	—	4	
	x_2	0	1	1	-1	—	—	1	

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
A_2 leaves	A_1	1	1	-1	0	1	0	5	$5/1=5$
x_2 enters	A_2	1	2^*	0	-1	0	1	6	$6/2=3$

1	z'	$\frac{1-M}{2}$	0	M	$\frac{3-M}{2}$	0	—	$-9-2M$	
A_1 leaves	A_1	$1/2^*$	0	-1	$1/2$	1	—	2	4
x_1 enters	x_2	$1/2$	1	0	$-1/2$	0	—	3	6
							—		

2	z'	0	0	1	1	—	—	-11	
	x_1	1	0	-2	1	—	—	4	
	x_2	0	1	1	-1	—	—	1	

Iteration Number	Basic Var.	Coefficients of						RHS solution	Ratio
		x_1	x_2	s_1	s_2	A_1	A_2		
0	z'	$2-2M$	$3-3M$	M	M	0	0	$-11M$	
A_2 leaves	A_1	1	1	-1	0	1	0	5	$5/1=5$
x_2 enters	A_2	1	2*	0	-1	0	1	6	$6/2=3$

1	z'	$\frac{1-M}{2}$	0	M	$\frac{3-M}{2}$	0	—	$-9-2M$	
A_1 leaves	A_1	$1/2^*$	0	-1	$1/2$	1	—	2	4
x_1 enters	x_2	$1/2$	1	0	$-1/2$	0	—	3	6
							—		

2	z'	0	0	1	1	—	—	-11	
	x_1	1	0	-2	1	—	—	4	
	x_2	0	1	1	-1	—	—	1	

Since all the coefficients in the objective equation in the row of z are positive.

This is a optimal solution.

The values of the variables and of z are given by the RHS column

$\therefore x_1 = 4, x_2 = 1$ and $z'_{max} = -11$

$\therefore z_{min} = 11$