Fourier Transforms Questions

1. Find Fourier Sine Transform for $f(x) = \frac{e^{-ax}}{x}$

2. Find Fourier Sine Transform for $f(x) = \begin{cases} -e^{kx} & \text{for} \quad x < 0 \\ e^{-kx} & \text{for} \quad x > 0 \end{cases}$ and hence prove that, $\int_0^\infty \frac{\omega \sin \omega x}{\omega^2 + k^2} d\omega = \frac{\pi}{2} \cdot e^{-kx}$ If $x > 0, k > 0$

3. Find Fourier Cosine Transform of $f(x) = x, \quad 0 < x < a$
$= 0, \qquad x > a$

4. Find Fourier Sine Transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$

5. Find Fourier Sine Transform for $f(x) = \begin{cases} 1 & \text{for } 0 \le x \le \pi \\ 0 & \text{for} \quad x > \pi \end{cases}$ and Hence evaluate $\int_0^\infty \frac{1 - \cos \pi\lambda}{\lambda} \cdot \sin x\lambda \, d\lambda$