Examples on Inverse Laplace Transform

10 July 2023
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Ex-1 :- Find $L^{-1}\left[\frac{1-\sqrt{s}}{s^2}\right]^2$

Solution :- $L^{-1}\left[\frac{1-\sqrt{s}}{s^2}\right]^2 = L^{-1}\left[\frac{1-2\sqrt{s}+s}{s^4}\right] = L^{-1}\left(\frac{1}{s^4}\right) - 2L^{-1}\left(\frac{1}{s^{7/2}}\right) + L^{-1}\left(\frac{1}{s^3}\right)$

But $L^{-1}\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{\Gamma n}$ or $\frac{t^{n-1}}{(n-1)!}$

$$\therefore L^{-1}\left[\frac{1-\sqrt{s}}{s^2}\right]^2 = \frac{t^3}{3!} - 2\frac{t^{5/2}}{\Gamma 7/2} + \frac{t^2}{2!}$$

$$= \frac{t^3}{6} - \frac{16}{15\sqrt{\pi}}t^{5/2} + \frac{t^2}{2}$$

Ex-2 : If $L[f(t)] = \frac{s+2}{s^2+2}$. Find $L[f'(t)]$

Solution :- By using the formula of Laplace of Derivatives

$$L[f'(t)] = -f(0) + sL[f(t)]$$

To find $f(0)$, we need to find $f(t)$

$$f(t) = L^{-1}\left[\frac{s+2}{s^2+2}\right] = L^{-1}\left[\frac{s}{s^2+2}\right] + 2L^{-1}\left[\frac{1}{s^2+2}\right]$$

$$= \cos\sqrt{2}t + 2\cdot\frac{1}{\sqrt{2}}\sin\sqrt{2}t$$

$$\therefore f(t) = \cos\sqrt{2}t + \sqrt{2}\sin\sqrt{2}t$$

$$\therefore f(0) = \cos(0) + \sqrt{2}\sin(0) = 1$$

$$\therefore L[f'(t)] = -f(0) + sL[f(t)]$$

$$= -1 + s\left[\frac{s+2}{s^2+2}\right] = \frac{\cancel{s^2 - 2 + s^2 + 2s}}{s^2+2}$$

$\therefore L[f'(t)] = \frac{\cancel{2(s-1)}}{s^2+2}$

Ex-3 If $L[f(t)] = \frac{s+3}{s^2+4}$, find $L[f'(t)]$

Home work Ans:- $\frac{3s-4}{s^2+4}$

Ex-4 Find the inverse Laplace Transform of

(i) $\frac{2s+3}{s^2+9}$ (ii) $\frac{1}{4s+5}$ (iii) $\frac{4s+15}{16s^2-25}$

Solution :- (i) $L^{-1}\left[\frac{2s+3}{s^2+9}\right] = 2L^{-1}\left[\frac{s}{s^2+9}\right] + 3L^{-1}\left[\frac{1}{s^2+9}\right]$

$= 2\cos 3t + 3 \cdot \frac{1}{3} \sin 3t$

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(ii) $L^{-1}\left[\frac{1}{4s+5}\right] = \frac{1}{4} L^{-1}\left[\frac{1}{s+5/4}\right] = \frac{1}{4} e^{-\frac{5}{4}t}$

(iii) $L^{-1}\left[\frac{4s+15}{16s^2-25}\right] = 4L^{-1}\left[\frac{s}{16s^2-25}\right] + 15L^{-1}\left[\frac{1}{16s^2-25}\right]$

$= \frac{4}{16} L^{-1}\left[\frac{s}{s^2-\frac{25}{16}}\right] + \frac{15}{16} L^{-1}\left[\frac{1}{s^2-\frac{25}{16}}\right]$

$= \frac{4}{16} L^{-1}\left[\frac{s}{s^2-\left(\frac{5}{4}\right)^2}\right] + \frac{15}{16} L^{-1}\left[\frac{1}{s^2-\left(\frac{5}{4}\right)^2}\right]$

$= \frac{4}{16}\cosh\left(\frac{5}{4}\right)t + \frac{15}{16} \cdot \frac{4}{5} \sinh\left(\frac{5}{4}\right)t$

$L^{-1}\left[\frac{4s+15}{16s^2-25}\right] = \frac{1}{4}\cosh\left(\frac{5}{4}\right)t + \frac{3}{4}\sinh\left(\frac{5}{4}\right)t$

Ex-5 Find $L^{-1}\left[\frac{1}{(s-3)^3}\right]$

$3t.2$

Ex-5 Find $L^{-1}\left[\frac{1}{(s-3)^3}\right]$

Solution $L^{-1}\left[\frac{1}{(s-3)^3}\right] = e^{3t} L^{-1}\left[\frac{1}{s^3}\right] = e^{3t} \cdot \frac{t^2}{2!} = \frac{e^{3t} t^2}{2}$

Ex-6 Find $L^{-1}\left[\frac{s}{s^2+2s+2}\right]$

Solution $L^{-1}\left[\frac{s}{s^2+2s+2}\right] = L^{-1}\left[\frac{s+1-1}{(s+1)^2+1}\right]$

$$= L^{-1}\left[\frac{s+1}{(s+1)^2+1}\right] - L^{-1}\left[\frac{1}{(s+1)^2+1}\right]$$

$$= e^{-t} L^{-1}\left[\frac{s}{s^2+1}\right] - e^{-t} L^{-1}\left[\frac{1}{s^2+1}\right]$$

$$= e^{-t}\cos t - e^{-t}\sin t = e^{-t}(\cos t - \sin t)$$

Ex-7 Find inverse Laplace Transform of $\frac{1}{(s+1)^2} + \frac{s-2}{s^2-4s+5} + \frac{s-2}{s^2-4s+3}$

Solution:- $L^{-1}\left[\frac{1}{(s+1)^2} + \frac{s-2}{s^2-4s+5} + \frac{s-2}{s^2-4s+3}\right]$

$$= L^{-1}\left[\frac{1}{(s+1)^2}\right] + L^{-1}\left[\frac{s-2}{s^2-4s+5}\right] + L^{-1}\left[\frac{s-2}{s^2-4s+3}\right]$$

$$= e^{-t} L^{-1}\left[\frac{1}{s^2}\right] + L^{-1}\left[\frac{s-2}{(s-2)^2+1}\right] + L^{-1}\left[\frac{s-2}{(s-2)^2-1}\right]$$

$$= e^{-t} L^{-1}\left[\frac{1}{s^2}\right] + e^{2t} L^{-1}\left[\frac{s}{s^2+1}\right] + e^{2t} L^{-1}\left[\frac{s}{s^2-1}\right]$$

$$= e^{-t} \cdot \frac{t}{1} + e^{2t}\cos t + e^{2t}\cosh t$$

$$= t e^{-t} + e^{2t}\cos t + e^{2t}\cosh t$$