LPP

STANDARD FORM OF LPP

STANDARD FORM OF LPP

CHARACTERISTICS OF THE STANDARD FORM:

(i) The objective function is of maximization type

(ii) All constrains are expressed as equations

(iii) Right hand side of each constraint is non negative

(iv) All variables are non negative

NOTE :

(i) Minimization of a function Z is equivalent to Maximization of the negative expression of this function

i.e. $\text{Min } Z = -\text{Max } (-Z)$

(ii) If RHS of any constraint is negative, multiply that constraint by -1 to convert the RHS to positive.

(iii) If a variable is unrestricted in sign, then it can be expressed as difference of two non-negative variable

i.e. X_1 is unrestricted in sign, then

$X_1 = X_1' - X_1''$, where X_1', X_1'' are ≥ 0

(v) In standard form, all the constraints are expressed in equation, which is possible by introducing some additional variables called **slack variables** and **surplus variable** so that a system of simultaneously linear equations is obtained.

Convert the following LPP in the standard form

EX 1. Maximize $z = 3x_1 + 5x_2$

subject to $3x_1 + 2x_2 \leq 15,$

$$2x_1 + 5x_2 \geq 12,$$

$$x_1, x_2 \geq 0$$

Solution: Introducing the slack variables the problem can be converted to standard form as:

Maximize $z = 3x_1 + 5x_2 + 0s_1 + 0s_2$

Subject to $3x_1 + 2x_2 + s_1 + 0s_2 = 15,$

$2x_1 + 5x_2 + 0s_1 - s_2 = 12,$

$x_1, x_2, s_1, s_2 \geq 0$

Convert the following LPP in the standard form

EX 2. Minimize $z = -3x_1 + 2x_2 - x_3$

Subject to $x_1 - 3x_2 + 2x_3 \geq -6,$

$3x_1 + 4x_3 \leq 3,$

$-3x_1 + 5x_2 \leq 4,$

$x_{1,}x_2 \geq 0, x_3$ is unrestricted

Solution: Since the problem is of minimization type we write $z' = -z$, so that the objective function is of maximization type.

Since in the first constraints the right hand side is negative, we multiply it by (-1), so
that it becomes positive and of less than type.
Hence, we add slack variable $s_1 (\geq 0)$.
Since the second and the third constraints are
of less than type we add slack variables s_2 and s_3 both (≥ 0).

Since x_3 is unrestricted we write

$x_3 = x_3' - x_3''$ where $x_3' \geq 0, x_3" \geq 0$

Now the problem becomes

Maximize

$$z' = -z = 3x_1 - 2x_2 + x_3' - x_3 + 0s_1 + 0s_2 + 0s_3$$

Subject to

$$-x_1 + 3x_2 - 2x_3' + 2x_3" + s_1 + 0s_2 + 0s_3 = 6,$$

$$3x_1 + 0x_2 + 4x_3' - 4x_3" + 0s_1 + s_2 + 0s_3 = 3,$$

$$-3x_1 + 5x_2 + 0x'_3 - 0x_3" + 0s_1 + 0s_2 + s_3 = 4,$$

$$x_1, x_2, x_3', x_3", s_1, s_2, s_3 \geq 0.$$

Convert the following LPP in the standard form

EX 3. Minimize $z = 2x_1 + 3x_2$

Subject to $2x_1 - 3x_2 - x_3 = -4,$

$$3x_1 + 4x_2 - x_4 = -6,$$

$$2x_1 + 5x_2 + x_5 = 10,$$

$$4x_1 - 3x_2 + x_6 = 18$$

$$x_3, x_4, x_5, x_6 \geq 0$$

Solution:

Let $x_1 = y_1 - y_2$ and $x_2 = y_3 - y_4$,

$x_3 = y_5, x_4 = y_6,\ x_5 = y_7,\ x_6 = y_8$

The problem changes to

Maximize $z' = -z = -2y_1 + 2y_2 - 3y_3 + 3y_4$

Subject to $\quad -2y_1 + 2y_2 + 3y_3 - 3y_4 + y_5 = 4$

$$-3y_1 + 3y_2 - 4y_3 + 4y_4 + y_6 = 6$$

$$2y_1 - 2y_2 + 5y_3 - 5y_4 + y_7 = 10$$

$$4y_1 - 4y_2 - 3y_3 + 3y_4 + y_8 = 18$$

$$y_i \geq 0 \text{ for all } i = 1, 2, \dots \dots, 8$$

Covert the following LPP to standard form

EX 4. Maximize $z = 2x_1 + 3x_2 + 6x_3$

Subject to $3x_1 - 2x_2 + 4x_3 \leq 5,$

$$2x_1 + 5x_2 = 10,$$

$$x_1 + 2x_2 + x_3 \leq 2,$$

$$x_1, x_2, x_3 \geq 0$$

Also put the problem in matrix form.

Solution: Introducing slack variables s_1 and s_3 both (≥ 0), we have

Maximize $z = 2x_1 + 3x_2 + 6x_3 + 0s_1 + 0s_2$

Subject to $3x_1 - 2x_2 + 4x_3 + s_1 + 0s_2 = 5$

$2x_1 + 5x_2 + 0x_3 + 0s_1 + 0s_2 = 10$

$x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 = 2$

Thus, the problem in the matrix form becomes:

Maximize $z = CX$

Subject to $AX = B, \;\; X \geq 0$

Where, $A = \begin{bmatrix} 3 & -2 & 4 & 1 & 0 \\ 2 & 5 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & -1 \end{bmatrix},$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{bmatrix}, B = \begin{bmatrix} 5 \\ 10 \\ 2 \end{bmatrix}, C = \begin{bmatrix} 2 & 3 & 6 & 0 & 0 \end{bmatrix}$$