Linear D. Eqn.

$$\frac{dy}{dx} + P(x)y = Q(x)$$

soln is

$$y\, e^{\int P(x)\,dx} = \int e^{\int P(x)\,dx}\, Q(x)\, dx + C$$

$$\frac{dx}{dy} + P(y)x = Q(y)$$

$$x\, e^{\int P(y)\,dy} = \int e^{\int P(y)\,dy}\, Q(y)\, dy + C$$

$\frac{dy}{dx}+\left(\frac{1-2x}{x^2}\right)y=1$ D.E. is in the form $\frac{dy}{dx}+P(x)y=Q(x)$

$P=\frac{1-2x}{x^2}$ $\quad Q=1$ $\quad x^{-2}$

Sumas $\quad \left(\frac{x^{-1}}{-1}\right).$

$$y\,e^{\int P\,dx}=\int e^{\int P\,dx}\,Q\,dx+C \quad \text{—— (1)}$$

$$e^{\int P\,dx}=e^{\int \frac{1-2x}{x^2}dx}=e^{\int\left(\frac{1}{x^2}-\frac{2x}{x^2}\right)dx}$$

$$=e^{\frac{x^{-1}}{-1}-2\log x}$$

$$=e^{-1/x}\cdot e^{-2\log x}$$

$$=e^{-1/x}\,e^{\log x^{-2}}$$

$$=e^{-1/x}\,x^{-2}=\frac{e^{-1/x}}{x^2}$$

from (1)

$$y\,\frac{e^{-1/x}}{x^2}=\int\frac{e^{-1/x}}{x^2}(1)\,dx+C$$

$$-\frac{1}{x}=t \Rightarrow +\frac{1}{x^2}dx=dt$$

$$=\int e^t\,dt+C=e^t+C$$

$$y\,\frac{e^{-1/x}}{x^2}=e^{-1/x}+C$$

$$(1 + x + xy^2)dy + (y + y^3)dx = 0$$

$(1 + x + xy^2)dy + (y + y^3)dx = 0$

$\cdot \quad \frac{dx}{dy} + P(y)x = Q(y)$

$$1 + x(1+y^2) + y(1+y^2)\frac{dx}{dy} = 0$$

$$\frac{1 + x(1+y^2)}{y(1+y^2)} + \frac{dx}{dy} = 0.$$

$$\frac{1}{y(1+y^2)} + \frac{x\cancel{(1+y^2)}}{y\cancel{(1+y^2)}} + \frac{dx}{dy} = 0$$

$$\frac{dx}{dy} + \frac{x}{y} = -\frac{1}{y(1+y^2)}$$

$$P = \frac{1}{y} \qquad Q = -\frac{1}{y(1+y^2)}$$

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

soln is

$$x e^{\int P dy} = \int e^{\int P dy} Q \, dy + C \quad \text{--- } 1$$

$$e^{\int P dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y$$

from 1

$$xy = \int \cancel{y}\left(-\frac{1}{\cancel{y}(1+y^2)}\right)dy + C$$

$$xy = -\tan^{-1}(y) + C$$

$(1 + y^2)dx = (e^{\tan^{-1}y} - x)dy$

$$\frac{dx}{dy} = \frac{e^{\tan^{-1}y} - x}{1+y^2}$$

$$= \frac{e^{\tan^{-1}y}}{1+y^2} - \frac{x}{1+y^2}$$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1}y}}{1+y^2}$$

$$P = \frac{1}{1+y^2} \qquad Q = \frac{e^{\tan^{-1}y}}{1+y^2}$$

sol^n is

$$x\, e^{\int P\,dy} = \int e^{\int P\,dy}\, Q\, dy + c \quad \text{—— (1)}$$

$$e^{\int P\,dy} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

from (1)

$$x\, e^{\tan^{-1}y} = \int e^{\tan^{-1}y}\, \frac{e^{\tan^{-1}y}}{1+y^2}\, dy + c$$

$$x\, e^{\tan^{-1}y} = \int \frac{e^{2\tan^{-1}y}}{1+y^2}\, dy + c$$

put $2\tan^{-1}y = t$

$$\frac{2}{1+y^2}\, dy = dt \Rightarrow \frac{1}{1+y^2}\, dy = \frac{dt}{2}$$

$$= \int e^t \frac{dt}{2} + c$$

$$= \frac{e^t}{2} + c$$

$$x\, e^{\tan^{-1}y} = \frac{e^{2\tan^{-1}y}}{2} + c$$

Equation Reducible to Linear D. Eq^n.

$$f'(y)\frac{dy}{dx}+P(x)f(y)=Q(x)$$

$$f'(y)\frac{dy}{dx}+Pf(y)=Q$$

$$\frac{dy}{dx}+P(x)y=Q(x)$$

$$ye^{\int P\,dx}=\int e^{\int P\,dx}Q\,dx+c$$

$$f(y)=v$$

$$f'(y)\frac{dy}{dx}=\frac{dv}{dx}$$

$$\frac{dv}{dx}+Pv=Q$$

$$ve^{\int P\,dx}=\int e^{\int P\,dx}Q\,dx+c$$

$$f(y)e^{\int P\,dx}=\int e^{\int P\,dx}Q\,dx+c$$

②

$$f'(x)\frac{dx}{dy} + P(y)\, f(x) = Q(y)$$

$\underline{\underline{\quad}}$ $f(x) = v$

$$f'(x)\frac{dx}{dy} = \frac{dv}{dy}$$

$$\frac{dv}{dy} + P(y)v = Q(y)$$

$$v\, e^{\int P\,dy} = \int e^{\int P\,dy}\, Q\, dy + C$$

where $v = f(x)$

$\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$

$\frac{dy}{dx} + P(x)y = Q(x)$

$\frac{1}{\cos^2 y}\frac{dy}{dx} + \frac{x\,2\sin y\cancel{\cos y}}{\cos^{\cancel{2}} y} = x^3$

$\sec^2 y\frac{dy}{dx} + x\;2(\tan y) = x^3$

$\tan y = v \Rightarrow \sec^2 y\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dv}{dx} + x\,2v = x^3$

$P = 2x \qquad Q = x^3$

Sol^n is

$v e^{\int P dx} = \int e^{\int P dx} Q\,dx + c$ — (1)

$e^{\int P dx} = e^{\int 2x dx} \quad e^{\cancel{2}\frac{x^2}{\cancel{2}}} = e^{x^2}$

from (1)

$v e^{x^2} = \int e^{x^2} x^3 dx + c$

$x^2 = t$
$\Rightarrow 2x\,dx = dt$

$= \int e^{x^2}(x^2)\cdot x\,dx + c$

$= \int e^t\; t\;\frac{dt}{2} + c$

$= \frac{1}{2}\int t e^t dt + c$

$v e^{x^2} = \frac{1}{2}\left[(t)e^t - (1)e^t\right] + c$

$v e^{x^2} = \frac{1}{2}\left[x^2 e^{x^2} - e^{x^2}\right] + c$

$\tan y\; e^{x^2} = \frac{1}{2}(x^2 e^{x^2} - e^{x^2}) + c.$

$$\frac{dy}{dx} = e^{x-y}(e^x - e^y)$$

$$f'(y)\frac{dy}{dx} + P(x) f(y) = Q(x)$$

$$\frac{dy}{dx} = \frac{e^x}{e^y}(e^x - e^y)$$

$$\frac{dy}{dx} + P(x) y = Q(x)$$

$$e^y \frac{dy}{dx} = e^x(e^x - e^y) = e^{2x} - e^x \cdot e^y$$

$$e^y \frac{dy}{dx} + e^x \cdot e^y = e^{2x}.$$

$$e^y = v \Rightarrow e^y \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dv}{dx} + e^x v = e^{2x}. \qquad P = e^x \qquad Q = e^{2x}.$$

Soln is $v\, e^{\int P\,dx} = \int e^{\int P\,dx} Q\, dx + c$ —— (1)

$$e^{\int P\,dx} = e^{\int e^x dx} = e^{e^x}$$

from (1)

$$v\, e^{e^x} = \int e^{e^x} e^{2x}\, dx + c$$

put $e^x = t \Rightarrow e^x dx = dt$

$$v\, e^{e^x} = \int e^{e^x} e^x e^x\, dx + c$$

$$= \int e^t\, t\, dt + c$$

$$= \int t e^t\, dt + c$$

$$v e^{e^x} = [t\, e^t - (1) e^t] + c$$

$$e^y e^{e^x} = [e^x e^{e^x} - e^{e^x}] + c.$$

$\frac{dy}{dx} = \frac{y^3}{e^{2x}+y^2}$

$$\frac{dx}{dy} = \frac{e^{2x}+y^2}{y^3}$$

$$\frac{dx}{dy} \mp P(y)x = Q(y)$$

$$f'(x)\frac{dx}{dy} + P(y)f(x) = Q(y)$$

$$\frac{dx}{dy} = \frac{e^{2x}}{y^3} + \frac{1}{y}$$

$$e^{-2x}\frac{dx}{dy} = \frac{1}{y^3} + \frac{e^{-2x}}{y}$$

$$e^{-2x}\frac{dx}{dy} - \frac{e^{-2x}}{y} = \frac{1}{y^3}$$

$$e^{-2x} = v \Rightarrow -2e^{-2x}\frac{dx}{dy} = \frac{dv}{dy}$$

$$\Rightarrow e^{-2x}\frac{dx}{dy} = -\frac{1}{2}\frac{dv}{dy}$$

$$-\frac{1}{2}\frac{dv}{dy} - \frac{1}{y}v = \frac{1}{y^3}$$

$\times(-2)$
$$\frac{dv}{dy} + \frac{2}{y}v = \frac{-2}{y^3}$$

$$P = \frac{2}{y} \qquad Q = \frac{-2}{y^3}$$

$$v\,e^{\int P\,dy} = \int e^{\int P\,dy}\,Q\,dy + C \quad \text{—— (1)}$$

$$e^{\int P\,dy} = e^{\int \frac{2}{y}dy} = e^{2\log y} = e^{\log y^2} = y^2$$

from (1)

$$\therefore\ v y^2 = \int y^{\cancel{2}}\left(-\frac{2}{y^{\cancel{3}}}\right)dy + C$$

$$v y^2 = -\int \frac{2}{y}\,dy + C$$

$$v y^2 = -2\log y + C$$

$$e^{-2x}y^2 = -2\log y + C$$