UNIFORM DISTRIBUTION

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In statistics, uniform distribution is a term used to describe a form of probability distribution where every possible outcome has an equal likelihood of happening.

The probability is constant since each variable has equal chances of being the outcome.

TYPES OF UNIFORM DISTRIBUTION

Uniform distribution can be grouped into two categories based on the types of possible outcomes.

- **DISCRETE UNIFORM DISTRIBUTION**
- **CONTINIOUS UNIFORM DISTRIBUTION**

DISCRETE UNIFORM DISTRIBUTION

In probability theory and statistics, the discrete uniform distribution is a symmetric probability distribution wherein a finite number of values are equally likely to be observed.

Every one of n values has equal probability 1/n.

A r.v. is said to follow a uniform distribution if X takes all integral values 1, 2, 3, ………, n with $P(X = x_i) = \frac{1}{n}$, $\forall x_i = 1,2, \dots . n$

DISCRETE UNIFORM DISTRIBUTION

A r.v. is said to follow a uniform distribution if X takes all integral values 1, 2, 3, ………, n with $P(X = x_i) = \frac{1}{n}$, $\forall x_i = 1,2, \ldots .n$

e.g. 1) A coin is tossed then $P = \frac{1}{2}$ for both head and tail

2) A die is thrown then $P = \frac{1}{6}$ for all numbers obtained on die

MEAN AND VARIANCE

$$E(X) = \sum p_i x_i = \sum_{i=0}^{n} x_i \frac{1}{n}$$

$$= 1 \times \frac{1}{n} + 2 \times \frac{1}{n} + 3 \times \frac{1}{n} + \cdots \dots \dots .. + n \times \frac{1}{n}$$

$$= \frac{1}{n}(1 + 2 + 3 + \cdots \cdots + n) = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

$$E(X^2) = \frac{1}{n}(1^2 + 2^2 + \cdots \cdots + n^2)$$

$$= \frac{1}{n}\frac{n(n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{6}$$

$$Var(X) = E(X^2) - \left(E(X)\right)^2 = \frac{n^2-1}{12}$$

EXAMPLE-1

Roll a six faced fair die. Suppose X denote the number appear on the top of a die.

A) Find the probability that an even number appear on the top.

B) Find the probability that the number appear on the top is less than 3.

C) Compute Mean and Variance of X.

Solution:

Let X denote the number appear on the top of a die. Then the random variable X take the values $X = 1,2,3,4,5,6$ and X follows $U(1,6)$ distribution.

EXAMPLE-1

The probability mass function of random variable X is $P(X = x) = \frac{1}{6}, x = 1,2,3,4,5,6$

A) the probability that an even number appear on the top of the die is

$$= P(X = even\,number)$$

$$= P(X = 2) + P(X = 4) + P(X = 6) = 0.5$$

B) the probability that the number appear on the top of the die is less than 3 is

$$= P(X < 3) = P(X = 1) + P(X = 2) = \frac{1}{3}$$

C) Mean $= \frac{1+n}{2} = 3.5$

Variance $= \frac{n^2-1}{12} = \frac{35}{12} = 2.9167$

EXAMPLE-2

A telephone number is selected at random from a directory. Suppose X denote the last digit of selected telephone number. Find the probability that the last digit of the selected number is

A) 6 B) less than 3

C) greater than or equal to 8 t

D) Find Mean and Variance

Solution:

Let X denote the last digit of randomly selected telephone number. Then the random variable X take the values $X = 0,1,2,\cdots,9$.

All the numbers $0,1,2,\cdots,9$ are equally likely. Thus the random variable X follows a discrete uniform distribution $U(0,9)$

EXAMPLE-2

The probability mass function of random variable X is $P(X = x) = \frac{1}{10}, x = 0,1,2,3,4,5,6,7,8,9$

A) the probability that the last digit of the selected number is 6 $= P(X = 6) = \frac{1}{10} = 0.1$

B) the probability that the last digit of the selected telephone number is less than 3

$$= P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = \frac{3}{10}$$

$$= 0.3$$

C) the probability that the last digit of the selected telephone number is greater than or equal to 8 $= P(X \geq 8) = \mathrm{P}(X = 8) + P(X = 9) = 0.2$

D) Mean $= \frac{1+n}{2} = 5.5$

Variance $= \frac{n^2-1}{12} = \frac{99}{12} = 8.25$

CONTINUOUS UNIFORM DISTRIBUTION

The Uniform distribution is the simplest probability distribution, but it plays an important role in statistics since it is very useful in modeling random variables.

The uniform distribution is a continuous probability distribution and is concerned with events that are equally likely to occur.

The continuous random variable X is said to be uniformly distributed, or having rectangular distribution on the interval [a,b].

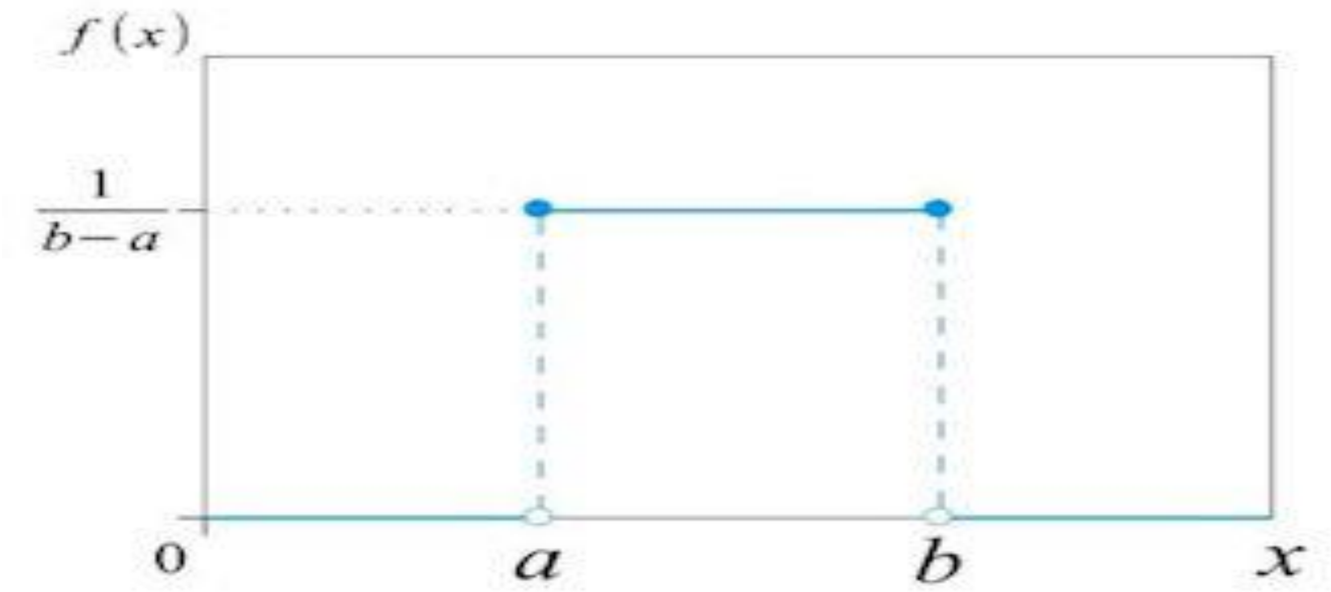
CONTINUOUS UNIFORM DISTRIBUTION

The probability density function for a uniform distribution taking values in the range a to b is

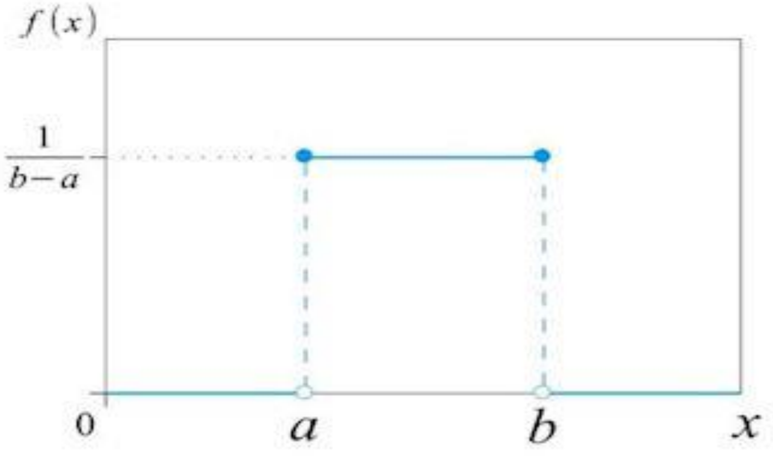
$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a \le x \le b \\ 0 \quad , & otherwise \end{cases}$$

CONTINUOUS UNIFORM DISTRIBUTION

The graph of the probability distribution function for continuous random variable for a uniform distribution is as given below.



CONTINUOUS UNIFORM DISTRIBUTION



The length of the rectangular base is $(b - a)$
As we know that the total area under the curve of pdf should be 1, we have
Height of the rectangle = $\frac{1}{b-a}$

The value of pfd is usually obtained by integration but in case of uniform distribution the area is of a rectangular form and hence, the value equals area of the rectangle

MEAN AND VARIANCE

$$E(X) = \int_a^b x\, f(x)dx$$

$$= \int_a^b x \frac{1}{b-a}\ \mathrm{dx} = \frac{b+a}{2}$$

Mean $= \frac{b+a}{2}$

$$E(X^2) = \int_a^b x^2\, f(x)dx = \frac{b^2+ab+a^2}{3}$$

$$Var\ (X) = E(X^2) - [E(X)]^2$$

$$= \frac{(b-a)^2}{12}$$

EXAMPLE-3

Suppose in a quiz there are 30 participants. A question is given to all 30 participants and the time allowed to answer it is 25 seconds. Find the number of probable participants respond within 6 seconds.

Solution: Given interval of probability distribution $= [0\ seconds, 25\ seconds]$

Density of probability $f(x) = \frac{1}{25-0} = \frac{1}{25}$

The probability $P(x < 6) = \int_0^6 f(x)dx = \frac{6}{25}$

There are 30 participants in the quiz

Hence the participants likely to answer it in 6 seconds $= \frac{6}{25} \times 30 \approx 7$

EXAMPLE-4

Suppose a flight is about to land and the announcement says that the expected time to land is 30 minutes. Find the probability of getting flight land between 25 to 30 minutes.

Solution: Given interval of probability distribution $= [0\,minutes, 30\,minutes]$

Density of probability $f(x) = \frac{1}{30-0} = \frac{1}{30}$

The probability $P(25 < x < 30) = \int_{25}^{30} f(x)dx = \frac{5}{30} = \frac{1}{6}$

Hence the probability of getting flight land between 25 minutes to 30 minutes = 0.16

EXAMPLE-5

Suppose a random number N is taken from 690 to 850 in uniform distribution. Find the probability number N is greater than 790.

Solution: Given interval of probability distribution $= [690, 850]$

Density of probability $f(x) = \frac{1}{850-690} = \frac{1}{160}$

The probability

$P(790 < x < 850) = \int_{790}^{850} f(x)dx = \frac{60}{160} = \frac{3}{8}$

Hence the probability of getting number N greater than 790 = 0.375

EXAMPLE-6

Suppose a train is delayed by approximately 60 minutes. What is the probability that train will reach by 57 minutes to 60 minutes?

Solution: Given interval of probability distribution $= [0\, minutes, 60\, minutes]$

Density of probability $f(x) = \frac{1}{60-0} = \frac{1}{60}$

The probability $P(57 < x < 60) = \int_{57}^{60} f(x)dx = \frac{3}{60} = \frac{1}{20}$

Hence the probability of train to reach between 57 to 60 minutes = 0.05

EXAMPLE-7

If X is uniformly distributed in $-2 \leq x \leq 2$.

Find (a) $P(X < 1)$ (b) $P\left(|X-1| \geq \frac{1}{2}\right)$ (c) Mean and Variance

Solution: Given interval of probability distribution $= [-2, 2]$

Density of probability $f(x) = \frac{1}{2-(-2)} = \frac{1}{4}$

(a)The probability $P(x < 1) = \int_{-2}^{1} f(x)dx = \frac{3}{4}$

(b) $P\left(|X-1| \geq \frac{1}{2}\right)$

$= P\left(-2 \leq X \leq \frac{1}{2}\right) + P\left(\frac{3}{2} \leq X \leq 2\right) = \frac{3}{4}$

(c) Mean $= \frac{2+(-2)}{2} = 0$

Variance $= \frac{[2-(-2)]^2}{12} = \frac{4}{3}$