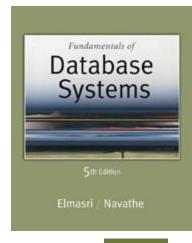


5th Edition

Elmasri / Navathe

## The Relational Algebra and Calculus





## **Chapter Outline**

- Relational Algebra
  - Unary Relational Operations
  - Relational Algebra Operations From Set Theory
  - Binary Relational Operations
  - Additional Relational Operations
  - Examples of Queries in Relational Algebra
- Relational Calculus
  - Tuple Relational Calculus
  - Domain Relational Calculus
- Example Database Application (COMPANY)
- Overview of the QBE language (appendix D)

## Relational Algebra Overview

- Relational algebra is the basic set of operations for the relational model
- These operations enable a user to specify basic retrieval requests (or queries)
- The result of an operation is a new relation, which may have been formed from one or more input relations
  - This property makes the algebra "closed" (all objects in relational algebra are relations)

## Relational Algebra Overview (continued)

- The algebra operations thus produce new relations
  - These can be further manipulated using operations of the same algebra
- A sequence of relational algebra operations forms a relational algebra expression
  - The result of a relational algebra expression is also a relation that represents the result of a database query (or retrieval request)

## Brief History of Origins of Algebra

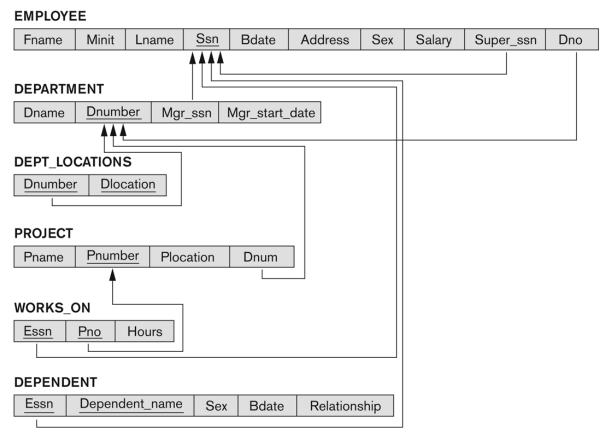
- Muhammad ibn Musa al-Khwarizmi (800-847 CE) wrote a book titled al-jabr about arithmetic of variables
  - Book was translated into Latin.
  - Its title (al-jabr) gave Algebra its name.
- Al-Khwarizmi called variables "shay"
  - "Shay" is Arabic for "thing".
  - Spanish transliterated "shay" as "xay" ("x" was "sh" in Spain).
  - In time this word was abbreviated as x.
- Where does the word Algorithm come from?
  - Algorithm originates from "al-Khwarizmi"
  - Reference: PBS (<a href="http://www.pbs.org/empires/islam/innoalgebra.html">http://www.pbs.org/empires/islam/innoalgebra.html</a>)

## Relational Algebra Overview

- Relational Algebra consists of several groups of operations
  - Unary Relational Operations
    - SELECT (symbol: σ (sigma))
    - PROJECT (symbol:  $\pi$  (pi))
    - RENAME (symbol: **p** (rho))
  - Relational Algebra Operations From Set Theory
    - UNION ( ∪ ), INTERSECTION ( ∩ ), DIFFERENCE (or MINUS, )
    - CARTESIAN PRODUCT (x)
  - Binary Relational Operations
    - JOIN (several variations of JOIN exist)
    - DIVISION
  - Additional Relational Operations
    - OUTER JOINS, OUTER UNION
    - AGGREGATE FUNCTIONS (These compute summary of information: for example, SUM, COUNT, AVG, MIN, MAX)

### Database State for COMPANY

All examples discussed below refer to the COMPANY database shown here.
 Figure 5.7
Referential integrity constraints displayed on the COMPANY relational database schema.



## Unary Relational Operations: SELECT

- The SELECT operation (denoted by σ (sigma)) is used to select a subset of the tuples from a relation based on a selection condition.
  - The selection condition acts as a filter
  - Keeps only those tuples that satisfy the qualifying condition
  - Tuples satisfying the condition are selected whereas the other tuples are discarded (filtered out)
- Examples:
  - Select the EMPLOYEE tuples whose department number is 4:

$$\sigma_{DNO=4}$$
 (EMPLOYEE)

Select the employee tuples whose salary is greater than \$30,000:

$$\sigma_{SALARY > 30,000}$$
 (EMPLOYEE)

## Unary Relational Operations: SELECT

- In general, the select operation is denoted by σ
   <selection condition>(R) where
  - the symbol σ (sigma) is used to denote the select operator
  - the selection condition is a Boolean (conditional) expression specified on the attributes of relation R
  - tuples that make the condition true are selected
    - appear in the result of the operation
  - tuples that make the condition false are filtered out
    - discarded from the result of the operation

# Unary Relational Operations: SELECT (contd.)

### SELECT Operation Properties

- The SELECT operation  $\sigma_{\text{selection condition}}(R)$  produces a relation S that has the same schema (same attributes) as R
- SELECT σ is commutative:
  - $\sigma_{\text{condition1}}(\sigma_{\text{condition2}}(R)) = \sigma_{\text{condition2}}(\sigma_{\text{condition1}}(R))$
- Because of commutativity property, a cascade (sequence) of SELECT operations may be applied in any order:
  - $\sigma_{\text{cond1}>}(\sigma_{\text{cond2}>} (\sigma_{\text{cond3}>} (R)) = \sigma_{\text{cond2}>} (\sigma_{\text{cond3}>} (\sigma_{\text{cond1}>} (R)))$
- A cascade of SELECT operations may be replaced by a single selection with a conjunction of all the conditions:
  - $\quad \sigma_{<cond1>}(\sigma_{<cond2>} (\sigma_{<cond3>}(R)) = \sigma_{<cond1>} (AND < cond2>} (AND < cond3>}(R)))$
- The number of tuples in the result of a SELECT is less than (or equal to) the number of tuples in the input relation R

Bdate

1965-01-09

Ssn

123456789

Minit

В

Lname

Smith

Fname

John

Franklin	Т	Wong	33344	45555	55 1955-12-08		638	3 Voss, Houston, TX		М	400	00	8886	65555	5
Alicia	J	Zelaya	99988	37777	7 1968-01-19 3321 Castle, Spring,		pring, TX	F	250	00	9876	54321	4		
Jennifer	S	Walla	ce 98765	54321	321 1941-06-20 291		291	Berry, Bella	erry, Bellaire, TX		430	00	8886	65555	6 4
Ramesh	K	Naray	an 66688	34444	4 1962-09	-15	975	Fire Oak, F	łumble, T	х м	380	00	3334	45555	5
Joyce	Α	Englis	h 45345	53453	1972-07	-31	5631	Rice, Hou	uston, TX	F	250	00	3334	45555	5
Ahmad	V	Jabba	98798	37987	1969-03	-29	980	Dallas, Ho	uston, TX	М	250	00	9876	54321	4
James	E	Borg	8886	65555	1937-11	-10	450	Stone, Hou	uston, TX	M	550	00	NULI	L	1
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9876543	321	30	20.0	_	1234567	89	М	ichael		М	1988	3-01-	04	Son	
9876543	221	20	15.0		12345678	89	AI	ice		F	1988	3-12-	зо	Daugl	nter

Address

731 Fondren, Houston, TX

Salary

30000

Super\_ssn

333445555

Dno

5

Sex

М

# Examples of applying SELECT and PROJECT operations

### Figure 6.1

Results of SELECT and PROJECT operations. (a)  $\sigma_{\text{(Dno=4 AND Salary>25000) OR (Dno=5 AND Salary>30000)}}$  (EMPLOYEE). (b)  $\pi_{\text{Lname, Fname, Salary}}$  (EMPLOYEE). (c)  $\pi_{\text{Sex, Salary}}$  (EMPLOYEE).

### (a)

Fname	Minit	Lname	<u>Ssn</u>	Bdate	Address	Sex	Salary	Super_ssn	Dno
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	М	40000	888665555	5
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	М	38000	333445555	5

### (b)

Lname	Fname	Salary
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

### (c)

Salary
30000
40000
25000
43000
38000
25000
55000

## **Unary Relational Operations: PROJECT**

- PROJECT Operation is denoted by  $\pi$  (pi)
- This operation keeps certain columns (attributes) from a relation and discards the other columns.
  - PROJECT creates a vertical partitioning
    - The list of specified columns (attributes) is kept in each tuple
    - The other attributes in each tuple are discarded
- Example: To list each employee's first and last name and salary, the following is used:

 $\pi_{\text{LNAME, FNAME,SALARY}}(\text{EMPLOYEE})$ 

# Unary Relational Operations: PROJECT (cont.)

The general form of the *project* operation is:

$$\pi_{\text{}}(R)$$

- $\bullet$   $\pi$  (pi) is the symbol used to represent the *project* operation
- <attribute list> is the desired list of attributes from relation R.
- The project operation removes any duplicate tuples
  - This is because the result of the project operation must be a set of tuples
    - Mathematical sets do not allow duplicate elements.

# Unary Relational Operations: PROJECT (contd.)

### PROJECT Operation Properties

- The number of tuples in the result of projection  $\pi_{<\text{list}>}(R)$  is always less or equal to the number of tuples in R
  - If the list of attributes includes a key of R, then the number of tuples in the result of PROJECT is equal to the number of tuples in R
- PROJECT is not commutative
  - $\pi_{< \text{list1}>}$  ( $\pi_{< \text{list2}>}$  (R) ) =  $\pi_{< \text{list1}>}$  (R) as long as tist2> contains the attributes in tist1>

## Relational Algebra Expressions

- We may want to apply several relational algebra operations one after the other
  - Either we can write the operations as a single relational algebra expression by nesting the operations, or
  - We can apply one operation at a time and create intermediate result relations.
- In the latter case, we must give names to the relations that hold the intermediate results.

# Single expression versus sequence of relational operations (Example)

- To retrieve the first name, last name, and salary of all employees who work in department number 5, we must apply a select and a project operation
- We can write a single relational algebra expression as follows:
  - $\pi_{\text{FNAME, LNAME, SALARY}}(\sigma_{\text{DNO}=5}(\text{EMPLOYEE}))$
- OR We can explicitly show the sequence of operations, giving a name to each intermediate relation:
  - DEP5\_EMPS  $\leftarrow \sigma_{DNO=5}(EMPLOYEE)$
  - RESULT  $\leftarrow \pi$  FNAME, LNAME, SALARY (DEP5\_EMPS)

## Example of applying multiple operations and RENAME

### (a)

Fname	Lname	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

### (b) TEMP

Fname	Minit	Lname	<u>Ssn</u>	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	В	Smith	123456789	1965-01-09	731 Fondren, Houston,TX	М	30000	333445555	5
Franklin	Т	Wong	333445555	1955-12-08	638 Voss, Houston,TX	М	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble,TX	М	38000	333445555	5
Joyce	Α	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Figure 6.2

Results of a sequence of operations.

(a)  $\pi_{\text{Fname, Lname, Salary}}(\sigma_{\text{Dno=5}}(\text{EMPLOYEE})).$ 

(b) Using intermediate relations and renaming of attributes.

## Unary Relational Operations: RENAME

- The RENAME operator is denoted by ρ (rho)
- In some cases, we may want to rename the attributes of a relation or the relation name or both
  - Useful when a query requires multiple operations
  - Necessary in some cases (see JOIN operation later)

# Unary Relational Operations: RENAME (contd.)

- The general RENAME operation ρ can be expressed by any of the following forms:
  - ρ<sub>S (B1, B2, ..., Bn )</sub>(R) changes both:
    - the relation name to S, and
    - the column (attribute) names to B1, B1, .....Bn
  - ρ<sub>S</sub>(R) changes:
    - the relation name only to S
  - ρ<sub>(B1, B2, ..., Bn)</sub>(R) changes:
    - the column (attribute) names only to B1, B1, .....Bn

# Unary Relational Operations: RENAME (contd.)

- For convenience, we also use a shorthand for renaming attributes in an intermediate relation:
  - If we write:
    - RESULT  $\leftarrow \pi_{\text{FNAME, LNAME, SALARY}}$  (DEP5\_EMPS)
    - RESULT will have the same attribute names as DEP5\_EMPS (same attributes as EMPLOYEE)
  - · If we write:
    - RESULT (FN, LN, Sal,)  $\leftarrow \pi_{\text{FNAME, LNAME, SALARY}}$  (DEP5\_EMPS)
    - 3 attributes of RESULT are renamed to FN, LN, Sal respectively

# Relational Algebra Operations from Set Theory

- Type Compatibility of operands is required for the binary set operation UNION ∪, INTERSECTION ∩, and SET DIFFERENCE –,
- R1(A1, A2, ..., An) and R2(B1, B2, ..., Bn) are type compatible if:
  - they have the same number of attributes, and
  - the domains of corresponding attributes are type compatible (i.e. dom(Ai)=dom(Bi) for i=1, 2, ..., n).
- The resulting relation for R1∪R2 (also for R1∩R2, or R1–R2, see next slides) has the same attribute names as the first operand relation R1 (by convention)

## Example of the result of a UNION operation

### UNION Example

### Figure 6.3

Result of the UNION operation RESULT ← RESULT1 URESULT2.

### **RESULT1**

Ssn
123456789
333445555
666884444
453453453

### **RESULT2**

Ssn
333445555
888665555

### RESULT

Ssn
123456789
333445555
666884444
453453453
888665555

## Relational Algebra Operations from Set Theory: UNION

### Example:

- To retrieve the social security numbers of all employees who either work in department 5 (RESULT1 below) or directly supervise an employee who works in department 5 (RESULT2 below)
- We can use the UNION operation as follows:

$$\begin{aligned} \mathsf{DEP5\_EMPS} \leftarrow \sigma_{\mathsf{DNO=5}} \; (\mathsf{EMPLOYEE}) \\ \mathsf{RESULT1} \leftarrow \pi_{\; \mathsf{SSN}} (\mathsf{DEP5\_EMPS}) \\ \mathsf{RESULT2} (\mathsf{SSN}) \leftarrow \pi_{\mathsf{SUPERSSN}} (\mathsf{DEP5\_EMPS}) \end{aligned}$$

### RESULT ← RESULT1 ∪ RESULT2

 The union operation produces the tuples that are in either RESULT1 or RESULT2 or both

# Relational Algebra Operations from Set Theory:

### UNION Operation

- Binary operation, denoted by U
- The result of R ∪ S, is a relation that includes all tuples that are either in R or in S or in both R and S
- Duplicate tuples are eliminated
- The two operand relations R and S must be "type compatible" (or UNION compatible)
  - R and S must have same number of attributes
  - Each pair of corresponding attributes must be type compatible (have same or compatible domains)

# Example to illustrate the result of UNION, INTERSECT, and DIFFERENCE

### (a) STUDENT

Fn	Ln		
Susan	Yao		
Ramesh	Shah		
Johnny	Kohler		
Barbara	Jones		
Amy	Ford		
Jimmy	Wang		
Ernest	Gilbert		

### **INSTRUCTOR**

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(b)

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

(c)	Fn	Ln			
	Susan	Yao			
	Ramesh	Shah			

(d)	Fn	Ln
	Johnny	Kohler
	Barbara	Jones
	Amy	Ford
	Jimmy	Wang
	Ernest	Gilbert

(e)	Fname	Lname
	John	Smith
	Ricardo	Browne
	Francis	Johnson

### Figure 6.4

The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b) STUDENT  $\cup$  INSTRUCTOR. (c) STUDENT  $\cap$  INSTRUCTOR. (d) STUDENT - INSTRUCTOR.

# Relational Algebra Operations from Set Theory: INTERSECTION

- INTERSECTION is denoted by ∩
- The result of the operation R ∩ S, is a relation that includes all tuples that are in both R and S
  - The attribute names in the result will be the same as the attribute names in R
- The two operand relations R and S must be "type compatible"

# Relational Algebra Operations from Set Theory: SET DIFFERENCE (cont.)

- SET DIFFERENCE (also called MINUS or EXCEPT) is denoted by –
- The result of R S, is a relation that includes all tuples that are in R but not in S
  - The attribute names in the result will be the same as the attribute names in R
- The two operand relations R and S must be "type compatible"

## Some properties of UNION, INTERSECT, and DIFFERENCE

- Notice that both union and intersection are commutative operations; that is
  - $R \cup S = S \cup R$ , and  $R \cap S = S \cap R$
- Both union and intersection can be treated as n-ary operations applicable to any number of relations as both are associative operations; that is
  - $R \cup (S \cup T) = (R \cup S) \cup T$
  - $(R \cap S) \cap T = R \cap (S \cap T)$
- The minus operation is not commutative; that is, in general
  - $R S \neq S R$

# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

- CARTESIAN (or CROSS) PRODUCT Operation
  - This operation is used to combine tuples from two relations in a combinatorial fashion.
  - Denoted by R(A1, A2, . . ., An) x S(B1, B2, . . ., Bm)
  - Result is a relation Q with degree n + m attributes:
    - Q(A1, A2, . . ., An, B1, B2, . . ., Bm), in that order.
  - The resulting relation state has one tuple for each combination of tuples—one from R and one from S.
  - Hence, if R has  $n_R$  tuples (denoted as  $|R| = n_R$ ), and S has  $n_S$  tuples, then R x S will have  $n_R$  \*  $n_S$  tuples.
  - The two operands do NOT have to be "type compatible"

# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT (cont.)

- Generally, CROSS PRODUCT is not a meaningful operation
  - Can become meaningful when followed by other operations
- Example (not meaningful):
  - FEMALE\_EMPS  $\leftarrow \sigma_{SEX='F'}(EMPLOYEE)$
  - EMPNAMES  $\leftarrow \pi_{\text{FNAME, LNAME, SSN}}$  (FEMALE\_EMPS)
  - EMP\_DEPENDENTS ← EMPNAMES x DEPENDENT
- EMP\_DEPENDENTS will contain every combination of EMPNAMES and DEPENDENT
  - whether or not they are actually related

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1965-01-09

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Lname

Smith

Fname

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K	Naraya	n 66688	34444	1962-09	-15	975 Fi	re Oak, H	lumble, T	X M	380	000	3334	145555	5 5
Α	English	45345	3453	1972-07	-31	5631 F	Rice, Hou	ıston, TX	F	250	000	3334	145555	5 5
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E	Borg	88866	35555	1937-11	-10	450 St	tone, Hou	ıston, TX	M	550	000	NUL	L	1
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			¬ ⊢										Daughter	
321	20	15.0		12345678	89	Alic	e		F	1988	3-12-	30	Daug	hter
	J S K A V E ENT me ch tration arters  ON 789 789 444 453 453 555 555 555 777 777 987	J   Zelaya   S   Wallac   K   Naraya   A   English   V   Jabbar   E   Borg     ENT	J   Zelaya   99988   S   Wallace   98765   K   Narayan   66688   A   English   45345   V   Jabbar   98798   E   Borg   88866   ENT	J Zelaya 999887777 S Wallace 987654321 K Narayan 666884444 A English 453453453 V Jabbar 987987987 E Borg 888665555  ENT  me Dnumber Mg sh 5 3334 tration 4 9876 arters 1 8886  DN  Pno Hours 8886  789 1 32.5 789 2 7.5 444 3 40.0 453 1 20.0 453 1 20.0 453 2 20.0 555 2 10.0 555 3 10.0 555 3 10.0 555 10 10.0 555 20 10.0 777 30 30.0 777 10 10.0 687 10 35.0 687 30 5.0	J Zelaya 999887777 1968-01 S Wallace 987654321 1941-06 K Narayan 666884444 1962-09 A English 453453453 1972-07 V Jabbar 987987987 1969-03 E Borg 888665555 1937-11  ENT The Dnumber Mgr_sn Sh 5 333445555 Stration 4 987654321 Barters 1 888665555  ON  Pno Hours 789 1 32.5 789 2 7.5 444 3 40.0 453 1 20.0 453 1 20.0 453 2 20.0 555 3 10.0 555 3 10.0 555 10 10.0 555 10 10.0 555 20 10.0 555 20 10.0 577 30 30.0 777 10 10.0 587 10 35.0 587 10 35.0 587 30 5.0	J       Zelaya       999887777       1968-01-19         S       Wallace       987654321       1941-06-20         K       Narayan       666884444       1962-09-15         A       English       453453453       1972-07-31         V       Jabbar       987987987       1969-03-29         E       Borg       888665555       1937-11-10         ENT         me       Dnumber       Mgr_ssn       Mgr_ssn         tration       4       987654321       1         arters       1       888665555       1         DN         DN         DN         Pno       Hours         789       1       32.5         789       2       7.5         4444       3       40.0         453       2       20.0         555       3       10.0         555       10       10.0         555       20       10.0         555       20       10.0         655       20       10.0         655       20       10.0         677       10       10.0         68	J   Zelaya   999887777   1968-01-19   3321   0     S	J Zelaya   999887777   1968-01-19   3321 Castle, Sign   S   Wallace   987654321   1941-06-20   291 Berry, Bellate   K   Narayan   666884444   1962-09-15   975 Fire Oak, Foundation   A   English   453453453   1972-07-31   5631 Rice, Houvey   Jabbar   987987987   1969-03-29   980 Dallas, Houvey   Borg   888665555   1937-11-10   450 Stone, Houvey   450 Stone, Houve	J Zelaya   999887777   1968-01-19   3321 Castle, Spring, TX   S   Wallace   987654321   1941-06-20   291 Berry, Bellaire, TX   K   Narayan   666884444   1962-09-15   975 Fire Oak, Humble, TA   English   453453453   1972-07-31   5631 Rice, Houston, TX   V   Jabbar   987987987   1969-03-29   980 Dallas, Houston, TX   E   Borg   888665555   1937-11-10   450 Stone, Houston, TX   ENT   Me   Dnumber   Mgr_ssn   Mgr_start_date	J Zelaya   999887777   1968-01-19   3321 Castle, Spring, TX   F   S   Wallace   987654321   1941-06-20   291 Berry, Bellaire, TX   F   K   Narayan   666884444   1962-09-15   975 Fire Oak, Humble, TX   M   A   English   453453453   1972-07-31   5631 Rice, Houston, TX   F   Y   Jabbar   987987987   1969-03-29   980 Dallas, Houston, TX   M   E   Borg   888665555   1937-11-10   450 Stone, Houston, TX   M   M   M   M   M   M   M   M   M	J Zelaya   999887777   1968-01-19   3321 Castle, Spring, TX   F   250	J   Zelaya   999887777   1968-01-19   3321 Castle, Spring, TX   F   25000     S   Wallace   987654321   1941-06-20   291 Berry, Bellaire, TX   F   43000     K   Narayan   666884444   1962-09-15   975 Fire Oak, Humble, TX   M   38000     A   English   453453453   1972-07-31   5631 Rice, Houston, TX   F   25000     V   Jabbar   987987987   1969-03-29   980 Dallas, Houston, TX   M   25000     E   Borg   888665555   1937-11-10   450 Stone, Houston, TX   M   25000     ENT	J Zelaya   999887777   1968-01-19   3321 Castle, Spring, TX   F   25000   98765	J Zelaya   999887777   1968-01-19   3321 Castle, Spring, TX   F   25000   987654321   S   Wallace   987654321   1941-06-20   291 Berry, Bellaire, TX   F   43000   888665555   K   Narayan   666884444   1962-09-15   975 Fire Oak, Humble, TX   M   38000   333445555   M   25000   267634321

Address

731 Fondren, Houston, TX

Sex

M

Salary

30000

Super\_ssn

333445555

Dno

5

# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT (cont.)

- To keep only combinations where the DEPENDENT is related to the EMPLOYEE, we add a SELECT operation as follows
- Example (meaningful):
  - FEMALE\_EMPS  $\leftarrow \sigma_{SFX='F'}(EMPLOYEE)$
  - EMPNAMES  $\leftarrow \pi_{\text{FNAME, LNAME, SSN}}$  (FEMALE\_EMPS)
  - EMP DEPENDENTS ← EMPNAMES x DEPENDENT
  - ACTUAL\_DEPS  $\leftarrow \sigma_{SSN=ESSN}(EMP\_DEPENDENTS)$
  - RESULT  $\leftarrow \pi_{\text{FNAME, LNAME, DEPENDENT_NAME}}$  (ACTUAL\_DEPS)
- RESULT will now contain the name of female employees and their dependents

#### FEMALE\_EMPS

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Alicia	J	Zelaya	999887777	1968-07-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291Berry, Bellaire, TX	F	43000	888665555	4
Joyce	Α	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

#### **EMPNAMES**

Fname	Lname	Ssn
Alicia	Zelaya	999887777
Jennifer	Wallace	987654321
Joyce	English	453453453

#### EMP DEPENDENTS

EMP_DE							
Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	
Alicia	Zelaya	999887777	333445555	Alice	F	1986-04-05	
Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	
Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	
Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	
Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	
Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	
Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	
Jennifer	Wallace	987654321	333445555	Alice	F	1986-04-05	
Jennifer	Wallace	987654321	333445555	Theodore	M	1983-10-25	
Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	
Jennifer	Wallace	987654321	987654321	Abner	М	1942-02-28	
Jennifer	Wallace	987654321	123456789	Michael	M	1988-01-04	
Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	
Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	
Joyce	English	453453453	333445555	Alice	F	1986-04-05	
Joyce	English	453453453	333445555	Theodore	M	1983-10-25	
Joyce	English	453453453	333445555	Joy	F	1958-05-03	
Joyce	English	453453453	987654321	Abner	M	1942-02-28	
Joyce	English	453453453	123456789	Michael	M	1988-01-04	
Joyce	English	453453453	123456789	Alice	F	1988-12-30	
Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	

#### ACTUAL\_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	

#### RESULT

	Fname	Lname	Dependent_name
ı	Jennifer	Wallace	Abner

## Binary Relational Operations: JOIN

- JOIN Operation (denoted by ⋈)
  - The sequence of CARTESIAN PRODECT followed by SELECT is used quite commonly to identify and select related tuples from two relations
  - A special operation, called JOIN combines this sequence into a single operation
  - This operation is very important for any relational database with more than a single relation, because it allows us combine related tuples from various relations
  - The general form of a join operation on two relations R(A1, A2, . . ., An) and S(B1, B2, . . ., Bm) is:

 $R_{\bowtie}$  <join condition>S

 where R and S can be any relations that result from general relational algebra expressions.

## Binary Relational Operations: JOIN (cont.)

- Example: Suppose that we want to retrieve the name of the manager of each department.
  - To get the manager's name, we need to combine each DEPARTMENT tuple with the EMPLOYEE tuple whose SSN value matches the MGRSSN value in the department tuple.
  - We do this by using the join operation.
  - DEPT\_MGR ← DEPARTMENT MGRSSN=SSN EMPLOYEE
- MGRSSN=SSN is the join condition
  - Combines each department record with the employee who manages the department
  - The join condition can also be specified as DEPARTMENT.MGRSSN= EMPLOYEE.SSN

## Example of applying the JOIN operation

### DEPT\_MGR

Dname	Dnumber	Mgr_ssn	 Fname	Minit	Lname	Ssn	
Research	5	333445555	 Franklin	Т	Wong	333445555	
Administration	4	987654321	 Jennifer	S	Wallace	987654321	
Headquarters	1	888665555	 James	E	Borg	888665555	

**Figure 6.6** Result of the JOIN operation

## Some properties of JOIN

- Consider the following JOIN operation:
  - R(A1, A2, ..., An)
     S(B1, B2, ..., Bm)
     R.Ai=S.Bj
  - Result is a relation Q with degree n + m attributes:
    - Q(A1, A2, . . ., An, B1, B2, . . ., Bm), in that order.
  - The resulting relation state has one tuple for each combination of tuples—r from R and s from S, but only if they satisfy the join condition r[Ai]=s[Bj]
  - Hence, if R has n<sub>R</sub> tuples, and S has n<sub>S</sub> tuples, then the join result will generally have less than n<sub>R</sub> \* n<sub>S</sub> tuples.
  - Only related tuples (based on the join condition) will appear in the result

## Some properties of JOIN

- The general case of JOIN operation is called a Theta-join: R
   S
   theta
- The join condition is called theta
- Theta can be any general boolean expression on the attributes of R and S; for example:
  - R.Ai<S.Bj AND (R.Ak=S.Bl OR R.Ap<S.Bq)</li>
- Most join conditions involve one or more equality conditions "AND"ed together; for example:
  - R.Ai=S.Bj AND R.Ak=S.Bl AND R.Ap=S.Bq

## Binary Relational Operations: EQUIJOIN

- EQUIJOIN Operation
- The most common use of join involves join conditions with equality comparisons only
- Such a join, where the only comparison operator used is =, is called an EQUIJOIN.
  - In the result of an EQUIJOIN we always have one or more pairs of attributes (whose names need not be identical) that have identical values in every tuple.
  - The JOIN seen in the previous example was an EQUIJOIN.

## Binary Relational Operations: NATURAL JOIN Operation

### NATURAL JOIN Operation

- Another variation of JOIN called NATURAL JOIN denoted by \* — was created to get rid of the second (superfluous) attribute in an EQUIJOIN condition.
  - because one of each pair of attributes with identical values is superfluous
- The standard definition of natural join requires that the two join attributes, or each pair of corresponding join attributes, have the same name in both relations
- If this is not the case, a renaming operation is applied first.

# Binary Relational Operations NATURAL JOIN (contd.)

- Example: To apply a natural join on the DNUMBER attributes of DEPARTMENT and DEPT\_LOCATIONS, it is sufficient to write:
  - DEPT\_LOCS ← DEPARTMENT \* DEPT\_LOCATIONS
- Only attribute with the same name is DNUMBER
- An implicit join condition is created based on this attribute:
   DEPARTMENT.DNUMBER=DEPT\_LOCATIONS.DNUMBER
- Another example: Q ← R(A,B,C,D) \* S(C,D,E)
  - The implicit join condition includes each pair of attributes with the same name, "AND"ed together:
    - R.C=S.C AND R.D.S.D
  - Result keeps only one attribute of each such pair:
    - Q(A,B,C,D,E)

## Example of NATURAL JOIN operation

(a)

Ы	RO.	J_	D	E	P	T
		_				

Pname	<u>Pnumber</u>	Plocation	Dnum	Dname	Mgr_ssn	Mgr_start_date
ProductX	1	Bellaire	5	Research	333445555	1988-05-22
ProductY	2	Sugarland	5	Research	333445555	1988-05-22
ProductZ	3	Houston	5	Research	333445555	1988-05-22
Computerization	10	Stafford	4	Administration	987654321	1995-01-01
Reorganization	20	Houston	1	Headquarters	888665555	1981-06-19
Newbenefits	30	Stafford	4	Administration	987654321	1995-01-01

#### (b)

#### **DEPT\_LOCS**

Dname	Dnumber	Mgr_ssn	Mgr_start_date	Location
Headquarters	1	888665555	1981-06-19	Houston
Administration	4	987654321	1995-01-01	Stafford
Research	5	333445555	1988-05-22	Bellaire
Research	5	333445555	1988-05-22	Sugarland
Research	5	333445555	1988-05-22	Houston

### Figure 6.7

Results of two NATURAL JOIN operations.

(a) PROJ\_DEPT ← PROJECT \* DEPT.

(b) DEPT\_LOCS ← DEPARTMENT \* DEPT\_LOCATIONS.

## Complete Set of Relational Operations

- The set of operations including SELECT  $\sigma$ , PROJECT  $\pi$ , UNION  $\cup$ , DIFFERENCE -, RENAME  $\rho$ , and CARTESIAN PRODUCT X is called a *complete set* because any other relational algebra expression can be expressed by a combination of these five operations.
- For example:
  - $R \cap S = (R \cup S) ((R S) \cup (S R))$
  - R  $\bowtie_{< join \ condition>} S = \sigma_{< join \ condition>} (R \ X \ S)$

## Binary Relational Operations: DIVISION

### DIVISION Operation

- The division operation is applied to two relations
- R(Z) ÷ S(X), where X subset Z. Let Y = Z X (and hence Z = X ∪ Y); that is, let Y be the set of attributes of R that are not attributes of S.
- The result of DIVISION is a relation T(Y) that includes a tuple t if tuples t<sub>R</sub> appear in R with t<sub>R</sub> [Y] = t, and with
  - $t_R[X] = t_s$  for every tuple  $t_s$  in S.

 For a tuple t to appear in the result T of the DIVISION, the values in t must appear in R in combination with every tuple in S.

## Example of DIVISION

(a) SSN\_PNOS

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

### SMITH\_PNOS

Pno
1
2

#### **SSNS**

Ssn
123456789
453453453

### (b)

Α	В
a1	b1
a2	b1
аЗ	b1
a4	b1
a1	b2
аЗ	b2
a2	b3
аЗ	b3
a4	b3
a1	b4
a2	b4
аЗ	b4

#### S

Α
a1
a2
аЗ

#### Т

В
b1
b4

Figure 6.8

The DIVISION operation. (a) Dividing SSN\_PNOS by SMITH\_PNOS. (b)  $T \leftarrow R \div S$ .

## Recap of Relational Algebra Operations

**Table 6.1**Operations of Relational Algebra

Operation	Purpose	Notation
SELECT	Selects all tuples that satisfy the selection condition from a relation <i>R</i> .	$\sigma_{\langle \text{selection condition} \rangle}(R)$
PROJECT	Produces a new relation with only some of the attributes of <i>R</i> , and removes duplicate tuples.	$\pi_{< attribute \ list>}(R)$
THETA JOIN	Produces all combinations of tuples from $R_1$ and $R_2$ that satisfy the join condition.	$R_1 \bowtie_{< \text{join condition}>} R_2$
EQUIJOIN	Produces all the combinations of tuples from $R_1$ and $R_2$ that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{<\text{join condition}>} R_2,$ OR $R_1 \bowtie_{(<\text{join attributes 1>}),}$ $(<\text{join attributes 2>})$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of $R_2$ are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1*_{<\text{join condition}>} R_2,$ OR $R_1*_{(<\text{join attributes 1>}),}$ ( $<\text{join attributes 2>})$ $R_2$ OR $R_1*_{R_2}$
UNION	Produces a relation that includes all the tuples in $R_1$ or $R_2$ or both $R_1$ and $R_2$ ; $R_1$ and $R_2$ must be union compatible.	$R_1 \cup R_2$
INTERSECTION	Produces a relation that includes all the tuples in both $R_1$ and $R_2$ ; $R_1$ and $R_2$ must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in $R_1$ that are not in $R_2$ ; $R_1$ and $R_2$ must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of $R_1$ and $R_2$ and includes as tuples all possible combinations of tuples from $R_1$ and $R_2$ .	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in $R_1$ in combination with every tuple from $R_2(Y)$ , where $Z = X \cup Y$ .	$R_1(Z) \div R_2(Y)$

# Additional Relational Operations: Aggregate Functions and Grouping

- A type of request that cannot be expressed in the basic relational algebra is to specify mathematical aggregate functions on collections of values from the database.
- Examples of such functions include retrieving the average or total salary of all employees or the total number of employee tuples.
  - These functions are used in simple statistical queries that summarize information from the database tuples.
- Common functions applied to collections of numeric values include
  - SUM, AVERAGE, MAXIMUM, and MINIMUM.
- The COUNT function is used for counting tuples or values.

## Aggregate Function Operation

- Use of the Aggregate Functional operation  $\mathcal{F}$ 
  - $\mathcal{F}_{\text{MAX Salary}}$  (EMPLOYEE) retrieves the maximum salary value from the EMPLOYEE relation
  - $\mathcal{F}_{\text{MIN Salary}}$  (EMPLOYEE) retrieves the minimum Salary value from the EMPLOYEE relation
  - $\mathcal{F}_{\text{SUM Salary}}$  (EMPLOYEE) retrieves the sum of the Salary from the EMPLOYEE relation
  - $\mathcal{F}_{\text{COUNT SSN, AVERAGE Salary}}$  (EMPLOYEE) computes the count (number) of employees and their average salary
    - Note: count just counts the number of rows, without removing duplicates

## Using Grouping with Aggregation

- The previous examples all summarized one or more attributes for a set of tuples
  - Maximum Salary or Count (number of) Ssn
- Grouping can be combined with Aggregate Functions
- Example: For each department, retrieve the DNO, COUNT SSN, and AVERAGE SALARY
- A variation of aggregate operation  $\mathcal{F}$  allows this:
  - Grouping attribute placed to left of symbol
  - Aggregate functions to right of symbol
  - DNO FCOUNT SSN, AVERAGE Salary (EMPLOYEE)
- Above operation groups employees by DNO (department number) and computes the count of employees and average salary per department

## Examples of applying aggregate functions and grouping

### Figure 6.10

The aggregate function operation.

- (a)  $\rho_{R(\text{Dno, No_of_employees, Average\_sal})}$  ( $\rho_{\text{Dno}}$  Count Ssn, Average Salary (EMPLOYEE)). (b)  $\rho_{\text{Dno}}$  Count Ssn, Average Salary (EMPLOYEE).
- (c)  $\mathfrak{J}_{\mathsf{COUNT}}$  Ssn, AVERAGE Salary (EMPLOYEE).

1-	٠.	
(a	1)	

Dno	No_of_employees	Average_sal
5 4		33250
4	3	31000
1	1	55000

(b)

)	Dno Count_ssn		Average_salary			
	5	4	33250			
	4	3	31000			
	1	1	55000			

(c)

Count_ssn	Average_salary		
8	35125		

# Illustrating aggregate functions and grouping

Figure 8.6 Results of GROUP BY and HAVING. (a) Q24. (b) Q26.

(a)	Fname	Minit	Lname	<u>Ssn</u>		Salary	Super_ssn	Dno				Dno	Count (*)	Avg (Salary)
	John	В	Smith	123456789		30000	333445555	5		Г	-	5	4	33250
	Franklin	Т	Wong	333445555		40000	888665555	5			<b>-</b>	4	3	31000
	Ramesh	K	Narayan	666884444		38000	333445555	5	]		<b>-</b>	1	1	55000
	Joyce	Α	English	453453453	]	25000	333445555	5				Result	of Q24	
	Alicia	J	Zelaya	999887777		25000	987654321	4	$\Box$					
	Jennifer	S	Wallace	987654321		43000	888665555	4	1	_	]			
	Ahmad	٧	Jabbar	987987987		25000	987654321	4						
	James	E	Bong	888665555		55000	NULL	1		_				

Grouping EMPLOYEE tuples by the value of Dno

- Recursive Closure Operations
  - Another type of operation that, in general, cannot be specified in the basic original relational algebra is recursive closure.
    - This operation is applied to a recursive relationship.
  - An example of a recursive operation is to retrieve all SUPERVISEES of an EMPLOYEE e at all levels — that is, all EMPLOYEE e' directly supervised by e; all employees e' directly supervised by each employee e'; all employees e'' directly supervised by each employee e''; and so on.

- Although it is possible to retrieve employees at each level and then take their union, we cannot, in general, specify a query such as "retrieve the supervisees of 'James Borg' at all levels" without utilizing a looping mechanism.
  - The SQL3 standard includes syntax for recursive closure.

(Borg's SSN is 888665555)

SUPERVISION	SSN1	SSN2	
	123456789	333445555	
	333445555	888665555	
	999887777	987654321	
	987654321	888665555	
	666884444	333445555	
	453453453	333445555	
	987987987	987654321	

RESULT 1	SSN		
	333445555		
	987654321		

(Supervised by Borg)

SSN
123456789
999887777
666884444
453453453
987987987

(Supervised by Borg's subordinates)

RESULT	SSN
	123456789
	999887777
	666884444
	453453453
	987987987
	333445555
	987654321

(RESULT1 ∪ RESULT2)

- The OUTER JOIN Operation
  - In NATURAL JOIN and EQUIJOIN, tuples without a matching (or related) tuple are eliminated from the join result
    - Tuples with null in the join attributes are also eliminated
    - This amounts to loss of information.
  - A set of operations, called OUTER joins, can be used when we want to keep all the tuples in R, or all those in S, or all those in both relations in the result of the join, regardless of whether or not they have matching tuples in the other relation.

- The left outer join operation keeps every tuple in the first or left relation R in R →S; if no matching tuple is found in S, then the attributes of S in the join result are filled or "padded" with null values.
- A similar operation, right outer join, keeps every tuple in the second or right relation S in the result of R ⋉ S.
- A third operation, full outer join, denoted by keeps all tuples in both the left and the right relations when no matching tuples are found, padding them with null values as needed.

### RESULT

Fname	Minit	Lname	Dname
John	В	Smith	NULL
Franklin	Т	Wong	Research
Alicia	J	Zelaya	NULL
Jennifer	S	Wallace	Administration
Ramesh	K	Narayan	NULL
Joyce	Α	English	NULL
Ahmad	V	Jabbar	NULL
James	E	Borg	Headquarters

### Figure 6.12

The result of a LEFT OUTER JOIN operation.

### OUTER UNION Operations

- The outer union operation was developed to take the union of tuples from two relations if the relations are not type compatible.
- This operation will take the union of tuples in two relations R(X, Y) and S(X, Z) that are partially compatible, meaning that only some of their attributes, say X, are type compatible.
- The attributes that are type compatible are represented only once in the result, and those attributes that are not type compatible from either relation are also kept in the result relation T(X, Y, Z).

- Example: An outer union can be applied to two relations whose schemas are STUDENT(Name, SSN, Department, Advisor) and INSTRUCTOR(Name, SSN, Department, Rank).
  - Tuples from the two relations are matched based on having the same combination of values of the shared attributes— Name, SSN, Department.
  - If a student is also an instructor, both Advisor and Rank will have a value; otherwise, one of these two attributes will be null.
  - The result relation STUDENT\_OR\_INSTRUCTOR will have the following attributes:

STUDENT\_OR\_INSTRUCTOR (Name, SSN, Department, Advisor, Rank)

# Examples of Queries in Relational Algebra

• Q1: Retrieve the name and address of all employees who work for the 'Research' department.

Q6: Retrieve the names of employees who have no dependents.

```
ALL_EMPS \leftarrow \pi ssn(EMPLOYEE)

EMPS_WITH_DEPS(SSN) \leftarrow \pi essn(DEPENDENT)

EMPS_WITHOUT_DEPS \leftarrow (ALL_EMPS - EMPS_WITH_DEPS)

RESULT \leftarrow \pi lname, fname (EMPS_WITHOUT_DEPS * EMPLOYEE)
```

## **Chapter Summary**

- Relational Algebra
  - Unary Relational Operations
  - Relational Algebra Operations From Set Theory
  - Binary Relational Operations
  - Additional Relational Operations
  - Examples of Queries in Relational Algebra