Angle Modulation
1. Frequency
2. Phase Modulation

Angle Modulation

Table 7-1 Equations for Phase- and Frequency-Modulated Carriers

Type of Modulation	Modulating Signal	Angle-Modulated Wave, $m(t)$
(a) Phase	$v_m(t)$	$V_c \cos[\omega_c t + K v_m(t)]$
(b) Frequency	$v_m(t)$	$V_c \cos[\omega_c t + K_1 \int v_m(t)\, dt]$
(c) Phase	$V_m \cos(\omega_m t)$	$V_c \cos[\omega_c t + K V_m \cos(\omega_m t)]$
(d) Frequency	$V_m \cos(\omega_m t)$	$V_c \cos\left[\omega_c t + \frac{K_1 V_m}{\omega_m} \sin(\omega_m t)\right]$

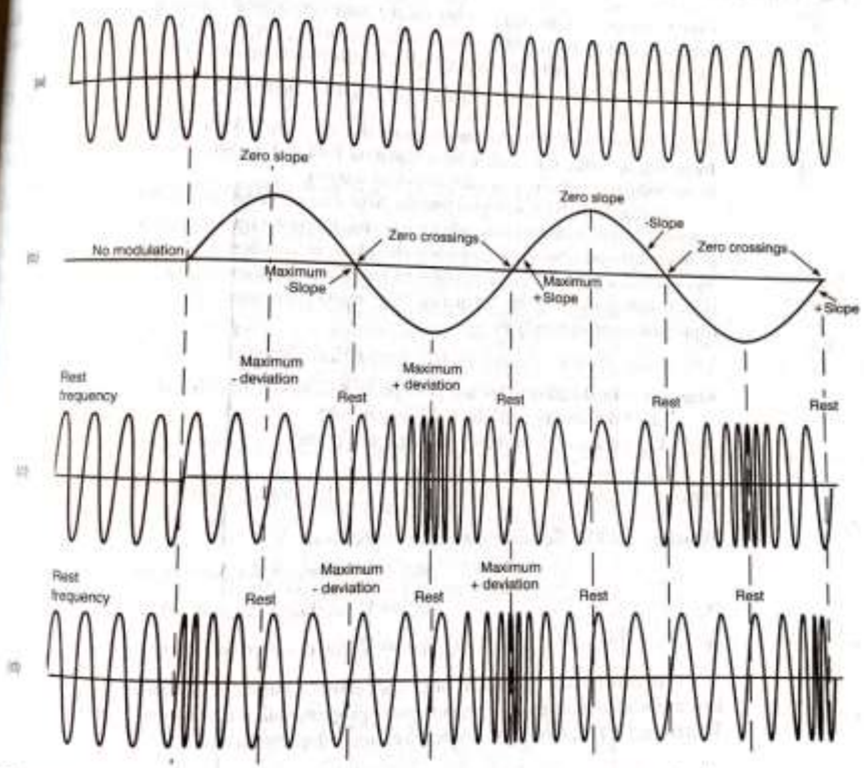


FIGURE 7-3 Phase and frequency modulation of a sine-wave carrier by a sine-wave signal: (a) unmodulated carrier; (b) modulating signal; (c) frequency-modulated wave; (d) phase-modulated wave

Equation of FM

Let f_c = Carrier frequency

$$\upsilon_m = V_m \cos \omega_m t = \text{Modulating signal}$$

Refer *figure 5.1.*

In FM, the carrier frequency is modulated. The new f is given as :

$$f = \text{New frequency} = f_c + \text{Deviation}$$

$$\text{where deviation} = k f_c \upsilon_m \quad \left(\begin{array}{l} \because \text{ frequency depends on} \\ \text{instantaneous value of } \upsilon_m \end{array} \right)$$

$$\text{i.e. } f = f_c + k f_c V_m \cos \omega_m t \qquad \text{..... (1)}$$

$$\text{i.e. } 2\pi f = 2\pi f_c + 2\pi f_c k V_m \cos \omega_m t$$

$$\therefore \omega = \omega_c + \omega_c k V_m \cos \omega_m t$$

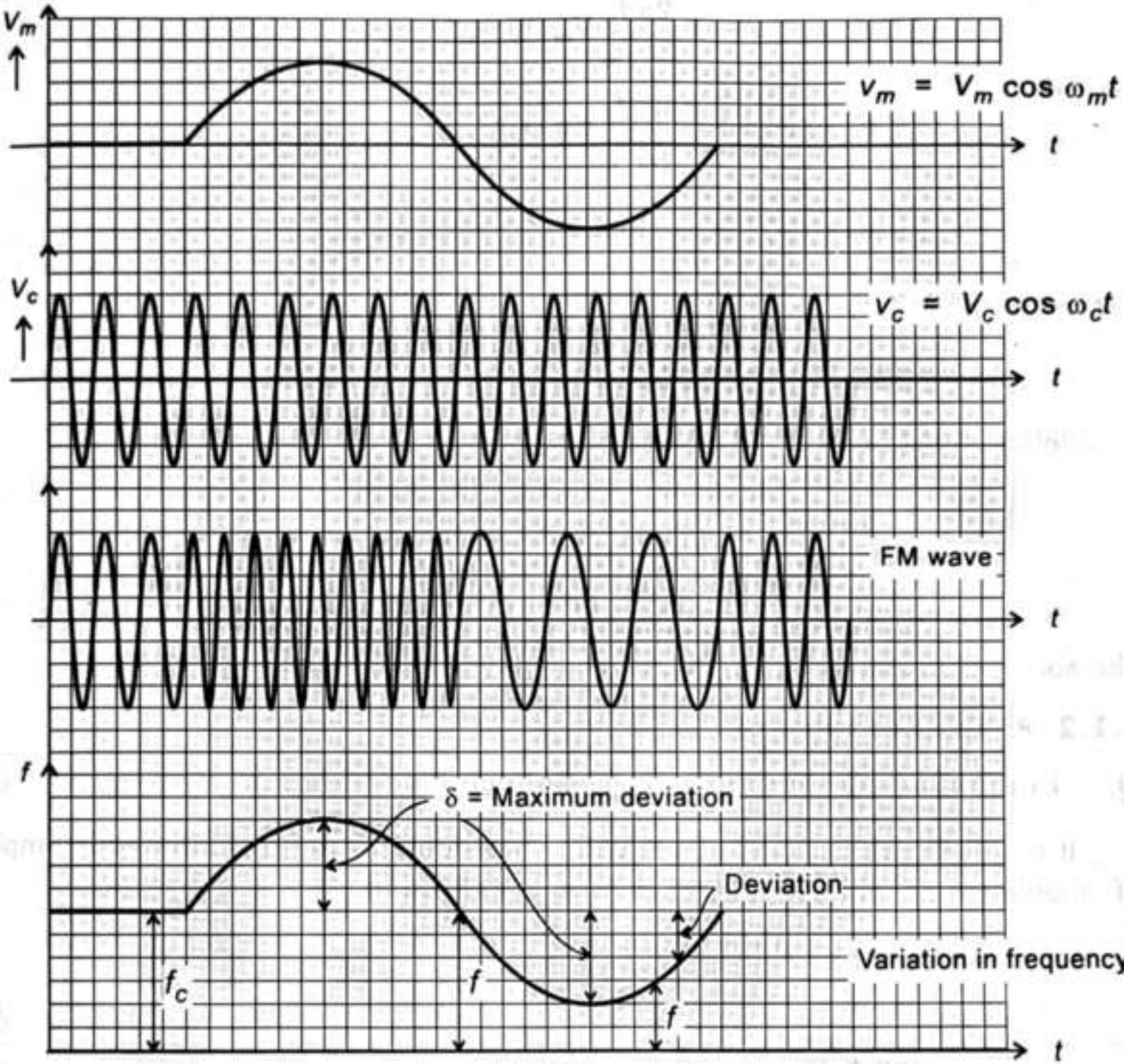
$$\text{where } \omega = 2\pi f$$

$$\therefore \omega = \omega_c[1 + k V_m \cos \omega_m t] \qquad \text{..... (2)}$$

In FM, the amplitude of the signal remains constant but the frequency varies according to modulating signal.

$\therefore$ we can write equation of FM as

$$\upsilon_{FM} = A \sin \theta$$



V_m
$v_m = V_m \cos \omega_m t$
t
V_c
$v_c = V_c \cos \omega_c t$
t
FM wave
t
f
δ = Maximum deviation
Deviation
Variation in frequency
f_c
f
f
t

Fig. 5.1

Fig. 5.1

We can find value of θ as follows :

$$\omega = \frac{d\theta}{dt}$$

$$\therefore \ \theta = \int \omega \, dt$$

Substitute equation (2),

$$\therefore \ \theta = \int \omega_c [1 + k V_m \cos \omega_m t] \, dt$$

$$= \omega_c t + \omega_c k V_m \left[\frac{\sin \omega_m t}{\omega_m}\right] + \phi \quad \text{where, } \phi = \text{Phase}$$

$$\therefore \ \theta = \omega_c t + \omega_c k V_m \left[\frac{\sin \omega_m t}{\omega_m}\right] \quad ..... \{\phi = 0 \text{ at initial position}\}$$

$$\theta = \omega_c t + k \frac{2\pi f_c}{2\pi f_m} V_m \sin \omega_m t$$

$$\therefore \theta = \omega_c t + \frac{k V_m f_c}{f_m} \sin \omega_m t$$

$$\therefore \theta = \omega_c t + \frac{\delta}{f_m} \sin \omega_m t \qquad \text{..... (3)}$$

where δ = Maximum frequency deviation

$$\therefore v_{FM} = A \sin\left[\omega_c t + \frac{\delta}{f_m} \sin \omega_m t\right] \qquad \text{..... (4)}$$

Modulation index (m_f) for FM is defined as :

$$m_f = \frac{\text{Maximum frequency deviation}}{\text{Modulating frequency}} = \frac{\delta}{f_m}$$

$$\therefore \boxed{v_{FM} = A \sin [\omega_c t + m_f \sin \omega_m t]} \qquad \text{..... (5)}$$

The above expression gives frequency modulated carrier signal.

5.1.2 Phase Modulation

Q. Explain the relationship between FM and PM.

If the phase of carrier signal is varied according to the instantaneous amplitude of modulating signal, then it is called *phase modulation*.

$$\text{Let } v_{PM} = A \sin [\omega_c t + \phi]$$

$$\text{where } \phi = \text{Phase angle}$$

$$\text{and } \phi \propto \text{Modulating signal}$$

$$\therefore \phi = k\, v_m = k\, V_m \sin \omega_m t$$

$$\therefore v_{PM} = A \sin [\omega_c t + k\, V_m \sin \omega_m t]$$

$$\therefore v_{PM} = A \sin [\omega_c t + m_p \sin \omega_m t] \qquad \text{..... (6)}$$

$$\text{where } m_p = k\, V_m = \text{Phase modulation index}$$

and it is proportional to amplitude of modulating signal and independent of frequency.

Difference Between FM & PM

Q. Differentiate FM and PM.

Sr.	FM	PM
(1)	The frequency of the carrier is varied according to instantaneous value of modulating signal.	The phase of carrier is varied according to instantaneous value of modulating signal.
(2)	Equation : $v_{FM} = V_c \sin [\omega_c t + m_f \sin \omega_m t]$	Equation : $v_{PM} = V_c \sin [\omega_c t + m_p \sin \omega_m t]$
(3)	Modulation index depends upon the modulating frequency. $m_f = \frac{\delta}{f_m}$	Modulation index does not depend upon modulating frequency. $m_p = kV_m$
(4)	Noise immunity is better than both AM and PM	Noise immunity is better than AM but worse than FM.
(5)	**Applications** : Radio, TV transmission, wireless devices, point to point communication.	**Applications** : Used in Mobile systems.

FM signal is given as

$$v_{FM} = A \sin [\omega_c t + m_f \sin \omega_m t]$$

Above signal can be expanded by using Bessel's function of the first kind $J_n(m_f)$ to get different component in FM.

$$v_{FM} = A\Big\{ J_0(m_f) \sin \omega_c t + J_1(m_f) [\sin (\omega_c + \omega_m) t - \sin (\omega_c - \omega_m) t] + J_2(m_f) [\sin (\omega_c + 2\omega_m) t + \sin (\omega_c - 2\omega_m) t] + \ldots\ldots \Big\}$$

From the above equation, it is seen that output consists of carrier frequency along with infinite number of sidebands.

If we substitute different values of m_f and n then the amplitude of each sideband can be calculated as :

$$J_n(m_f) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{m_f}{2}\right)^{2r+n}}{r! \, \overline{|n+r+1}}$$

$$= \left(\frac{m_f}{2}\right)^n \left\{ \frac{1}{n!} - \frac{\left(\frac{m_f}{2}\right)^2}{1!\,(n+1)!} + \frac{\left(\frac{m_f}{2}\right)^4}{2!\,(n+2)!} - \ldots\ldots \right\}$$

> **Note :** *To expand a sine of sine function i.e. a function like $f(x) = \sin (a + b \sin x)$ we need Bessel's function.*

Table 5.1 gives amplitude of number of sidebands for different values of m_f .

m_f	J_0	J_1	J_2	J_3	J_4	J_5	.....
0.0	1.0						
0.25	0.98	0.12	—	—	—		
0.5	0.94	0.03	—	—	—		
1.0	0.77	0.44	0.11	0.02	—		

Table 5.1

If $m_f = 1$ and $A = 1$ V, then

Amplitude of carrier = 0.77

Amplitude of 1st sideband $(\omega_c \pm \omega_m)$ = 0.44 V

Amplitude of 2nd sideband $(\omega_c \pm 2\omega_m)$ = 0.11 V

Amplitude of 3rd sideband $(\omega_c \pm 3\omega_m)$ = 0.02 V

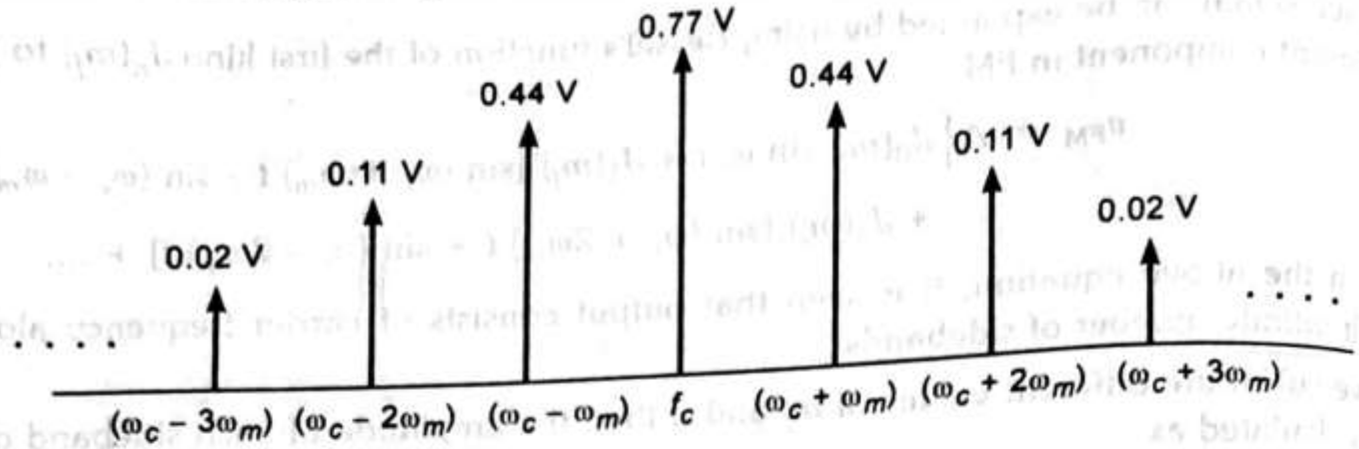


Fig. 5.2

5.3 Carson's Rule

This rule is used to estimate BW (bandwidth) of all angle modulated systems regardless of modulation index.

Carson's rule states that we can neglect any frequency component (i.e. sideband) beyond n such that $n = m_f + 1$ and m_f is modulation index.

$$\therefore \text{ BW of FM } = 2n\, f_m = 2(m_f + 1)\, f_m \quad \text{where } m_f = \frac{\delta}{f_m}$$

$$= 2\left(\frac{\delta}{f_m} + 1\right) f_m$$

$$\text{BW of FM } = 2(\delta + f_m)$$

Difference Between Narrow and wide Band FM

Q. Compare Wideband and Narrow band FM.

Sr.	Wideband FM	Narrowband FM
(1)	It is type of FM where m_f normally exceeds unity $m_f > 1$.	It is type of FM where m_f slightly greater than 0 but less than 1.
(2)	Maximum deviation $\delta = 75$ kHz.	Maximum deviation $\delta = 5$ kHz
(3)	Range of modulating frequency f_m is 30 Hz to 15 kHz.	Range of modulating frequency f_m is upto 3 kHz.
(4)	Bandwidth is almost 15 times greater than that of narrowband FM.	Bandwidth is comparatively less.
(5)	Due to large bandwidth effect of noise is more.	Effect of noise less.
(6)	There are more than two sidebands having significant amplitude.	There are only two sidebands.

Table 5.2

Noise Triangle

If the signal noise frequency falls within the passband of the receiver then it affects the output of the receiver.

The noise frequency gets added to the FM signal and produces unwanted changes as shown in *figure 5.3*.

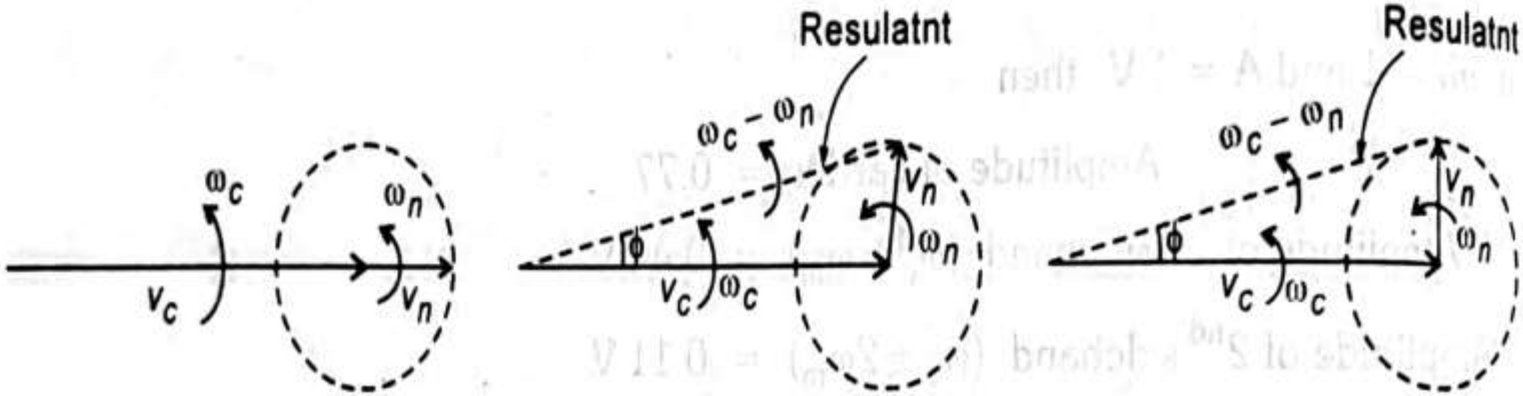


Fig. 5.3

As we can see from figure 5.3,

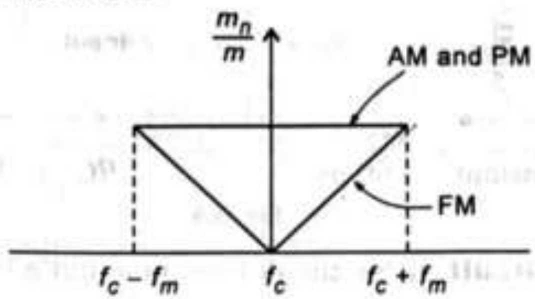
In case (i) : Amplitude changes but phase change is zero.

In case (ii) : Amplitude as well as phase changes.

In case (iii) : Amplitude changes and phase changes to its maximum.

The amplitude change can be eliminated by Amplitude limiter. The phase change can not be eliminated. But it can be minimized using pre-emphasis and de-emphasis (explained later). Noise triangle thus shows how noise is related to frequency of the incoming signal.

Noise triangle is shown in *figure 5.4*.



Here,

$m = m_a$ for AM

$m = m_p$ for PM

$m = m_f$ for FM

Fig. 5.4

- Noise triangle is a graph of frequency versus $\frac{\text{Modulation index of noise}}{\text{Modulation index of signal}}$.
- The noise modulation index remains almost constant for all frequencies.
- In AM, modulation index is $\frac{V_m}{V_c}$ i.e. it is independent of frequency. Hence noise in AM is almost constant.
- Similarly we can say that, noise in PM is almost constant. Since, $m_p = kV_m$.
- For FM, modulation index m_f is $\frac{\delta}{f_m}$.

  Hence as frequency f_m increases, the modulation index m_f decreases.
- Hence $\frac{m_n}{m_f}$ increases. This gives shape of Triangle. Hence known as *Noise Triangle*.

Q.1. ***Why is pre-emphasis required in FM generation. Explain the working of pre-emphasis and de-emphasis.***

Q.2. ***Explain pre-emphasis and de-emphasis in F.M.***

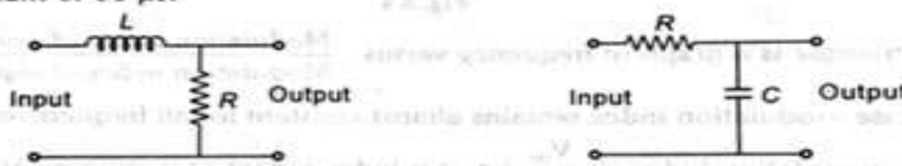
- From noise triangle of FM, we can see that the effect of noise increases if higher audio frequencies are used for frequency modulation.

- To overcome above problem, the higher audio frequencies are boosted at the transmitter and accordingly attenuated at the receiver.
- The artificial boosting of higher order frequencies at the transmitter is done with the help of pre-emphasis circuit.
- Similarly attenuation of the boosted frequencies at receiver is done with the help of de-emphasis circuit.
- **Pre-emphasis Circuit** : This circuit consists of high pass circuit with time constant of 60 μs.

$\frac{L}{R}$ = Time constant = 60 μs　　　　RC = Time constant = 60 μs

Fig. 5.5

- **De-emphasis Circuit** : This circuit is nothing but a low pass circuit with a time constant of 60 μs.



$\frac{L}{R}$ = 60 μs　　　　RC = 60 μs

Fig. 5.6

- The pre-emphasis and de-emphasis in done according to the pre-defined curve shown in *figure 5.7*.

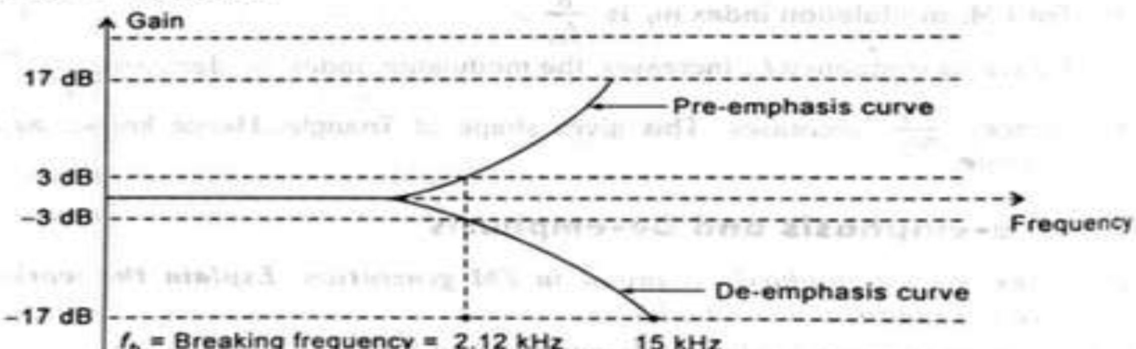


Fig. 5.7

- When the signal at the transmitter is boosted, care should be taken not to over modulate the higher order frequencies. Otherwise distortion will take place.

Difference between AM & FM

Sr.	AM	FM
(1)	In amplitude modulation, the amplitude of carrier is varied w.r.t instantaneous amplitude of modulating signal.	In frequency modulation frequency of carrier is varied w.r.t instantaneous amplitude of modulating signal.
(2)	Equation : $v_{AM} = [V_c + mV_c \sin \omega_m t] \sin \omega_c t$	Equation : $v_{FM} = V_c \sin [\omega_c t + m_f \sin \omega_m t]$
(3)	Modulation index $m = \dfrac{V_m}{V_c}$	Modulation index $m_f = \dfrac{\delta}{f_m}$
(4)	Bandwidth $2 f_m$.	Bandwidth $= 2[\delta + f_m]$.
(5)	Bandwidth is much less than FM.	Bandwidth is large.

Difference between AM & FM

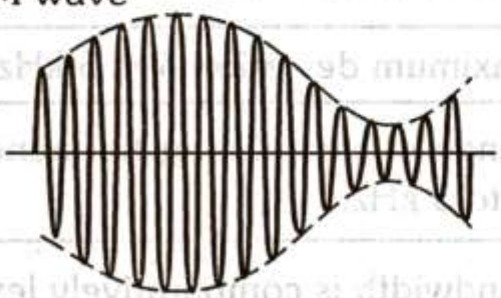
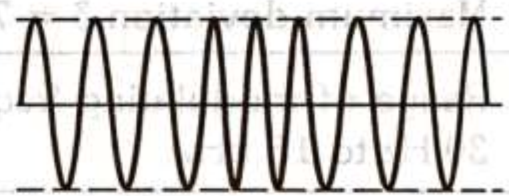
(6)	Frequency range of modulated signal f_s = 540 kHz to 1650 kHz.	Frequency range of modulated signal f_s = 88 MHz to 108 MHz.
(7)	Less complex.	More complex.
(8)	Information is contained in amplitude variation of carrier.	Information is contained in frequency variation of carrier.
(9)	AM wave 	FM wave 
(10)	**Applications** : Radio and TV transmission.	**Applications** : Radio and TV transmission, point to point communication.

Table 5.2

Am	FM
(1) Defn	(1) Defn
(2) eqn	(2) eqn
(3) $m = \frac{V_m}{V_c}$	(3) $m_f = \frac{\delta}{f_m}$
(4) $BW = 2f_m$	(4) $BW = 2(\delta + f_m)$
(5) BW is muchless	(5) BW is large
(6) freq range of modulated signal = 540KHz - 1650KHz	(6) freq. range = 88 MHz to 108 MHz
(7) Less complex	(7) more complex
(8) Information is in Amplitude variation of carrier	(8) Information is in freq. variation of carrier
(9) Radio & TV transmission	(9) Radio & TV transmissing, point to point comma wirless devizes
(10) more noise	(10) less noise