AEC 2.1

10 January 2022 22:47

Todays topic:
Useful Frequency response concepts

✳ Useful frequency response concepts:

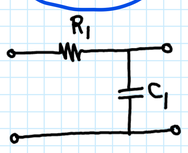
- Poles and zeros
- Bode's Rules
- Finding Poles by inspection

① $H(s) \xrightarrow{s=j\omega} H(j\omega) \longrightarrow |H(j\omega)|$ → Frequency response

Magnitude of transfer function

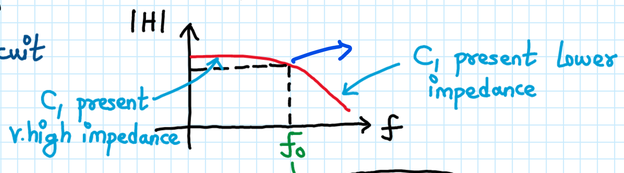
Prof Razavi

eg



Simple RC circuit

$$|H| = \frac{1}{\sqrt{R_1^2 C_1^2 \omega^2 + 1}}$$



Quick conclusion:

a) Capacitors generally limit the bandwidth

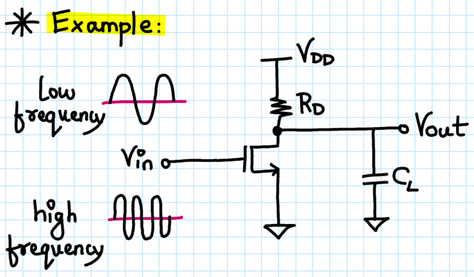
$X_C = \frac{1}{2\pi f C}$

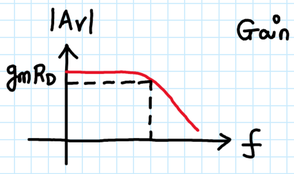
$\frac{1}{2\pi R_1 C_1}$ → To maximize the bandwidth, there is a need to minimize R_1C_1

b) As far as speed of the circuit is concerned → capacitors are generally our enemy (most cases) → We minimize C wherever they appear in the path of the signal as it goes from input to the output.

1

✳ Example:





Gain = Output amplitude / Input amplitude

C_L → Limit the bandwidth

- How does this capacitor C_L affect the speed of the circuit ?
- Amplitude of output signal goes down as f increases

✳ Poles and Zeros:

$H(s) = \frac{N(s)}{D(s)}$ → Roots of numerator (zero's)

→ Roots of denominator (Poles)

✳ Bode's Rules: (helps to plot approximate behavior of frequency response)

What does this mean?

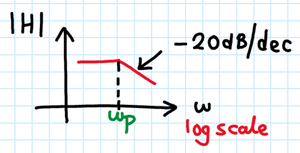
If frequency goes up by a factor of 10 → magnitude changes by a factor of 10 or 20dB

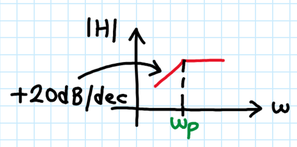
Gain = 10 → $20\log_{10}10 = 20dB$

① As ω passes a pole frequency → the slope of |H| decreases by 20dB/dec

Magnitude in logarithm scale

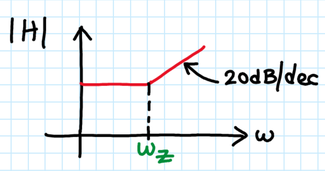
increase in frequency by a factor of 10

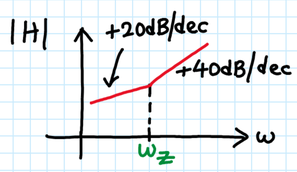




2

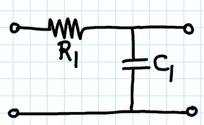
② As ω passes a zero frequency, the slope of |H| increases by 20dB/dec





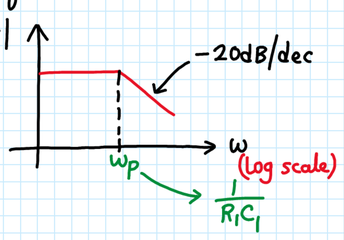
③ With Bode rules, it's much easier to plot frequency response of circuits

④ Example:



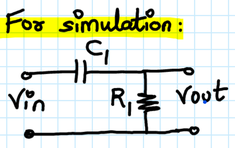
$H(s) = \frac{1}{R_1C_1 s + 1}$

Pole frequency $\omega_p = -\frac{1}{R_1C_1}$



3

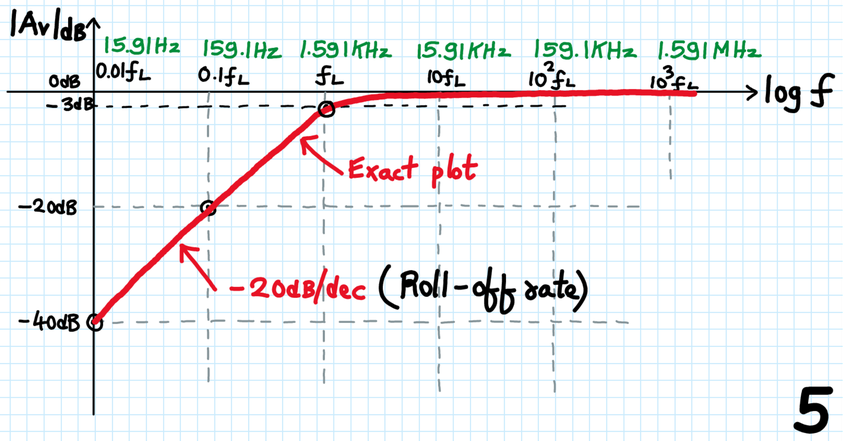
For simulation:



$R_1 = 1k\Omega$

$C_1 = 0.1\mu F$

$f_L = \frac{1}{2\pi R_1 C_1} = 1.591\ kHz$



5

✳ Concept building: Low frequency analysis of simple RC circuit

HPF

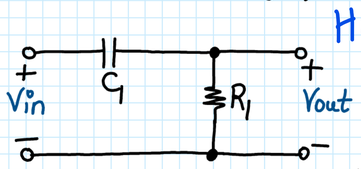
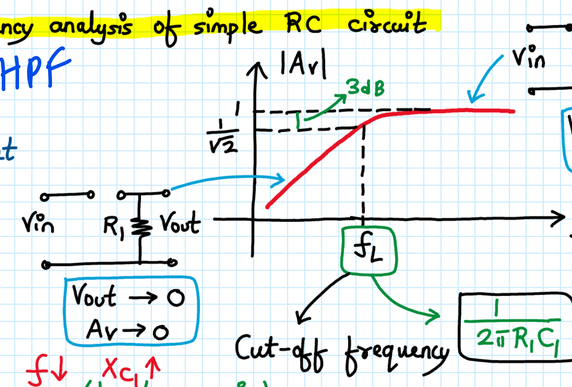


fig 2.1.1

$X_{C_1} = \frac{1}{2\pi f C_1}$

Capacitive impedance



f↓ X_{C_1}↑ (Low frequencies)
i.e f → 0, X_{C_1} → ∞
i.e C_1 behaves as open-circuit

f↑ X_{C_1}↓
i.e f → ∞, X_{C_1} → 0 (high frequencies)
i.e C_1 behaves as short-circuit

① $V_{out} = \frac{R_1}{R_1 + X_{C_1}} V_{in}$ .... by voltage divider rule (From circuit fig 2.1.1)

② $\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_1}{\sqrt{R_1^2 + X_{C_1}^2}}$

For special case: $X_{C_1} = R_1$

70.7% Vin

③ $|A_V| = \frac{R_1}{\sqrt{2R_1^2}} = \frac{1}{\sqrt{2}} \rightarrow V_{out} = \frac{1}{\sqrt{2}} V_{in} = 0.707 V_{in}$

i.e $|A_V| = \frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{2}} = 0.707 \big|_{X_{C_1} = R_1}$

④ At frequency $f_L \rightarrow X_{C_1} = R_1 \rightarrow$ the output will be 70.7% of the input

i.e $X_{C_1} = \frac{1}{2\pi f_L C_1} = R_1$

i.e $f_L = \frac{1}{2\pi R_1 C_1}$ ..... Cut-off frequency

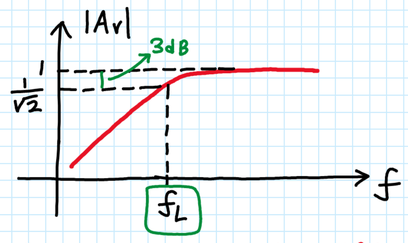
⑤ $20\log_{10}\left(\frac{1}{\sqrt{2}}\right) = -3dB$ → There is a 3dB drop in gain from the midband level when $f = f_L$

$20\log_{10}(1) = 0dB$

⑥ $A_V = \frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 - jX_{C_1}} = \frac{1}{1 - j\left(\frac{X_{C_1}}{R_1}\right)} = \frac{1}{1 - j\left(\frac{1}{2\pi f C_1 R_1}\right)} = \frac{1}{1 - j\left(\frac{f_L}{f}\right)}$

i.e $A_V = \frac{1}{1 - j\left(\frac{f_L}{f}\right)}$ ..... (2.1.10) ; where $f_L = \frac{1}{2\pi R_1 C_1}$ → Lower cut-off frequency

⑦ $|A_V| = \frac{1}{\sqrt{1 + \left(\frac{f_L}{f}\right)^2}}$



i) At f=0 → |Av| = 0
→ As f↑ → |Av| ↑

ii) At $f = f_L \rightarrow |A_V| = \frac{1}{\sqrt{2}} = 0.707$

iii) For $f >> f_L \rightarrow |A_V| \approx 1$

f_L is the frequency at which gain fall to 0.707 times its maximum value

⑧ Magnitude expressed in dB

$|A_V|_{dB} = 20\log_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{f_L}{f}\right)^2}}\right)$ ..... (2.1.11)

$f = 10^2 f_L$

i) In low frequency range → As f↑ → $|A_V|_{dB}$ also ↑

ii) At $f = 0.01 f_L$ (10000Hz) → $|A_V|_{dB} = -40dB$ ✓

iii) At $f = 0.1 f_L \rightarrow |A_V|_{dB} = -20dB$ ✓
(f is ↑sed by 10 fold ie by one decade)

→ It means that when frequency is increased by one decade gain increases by 20dB

Av = 0dB

iv) At $f = f_L \rightarrow |A_V|_{dB} = -3dB$

v) For $f >> f_L \rightarrow |A_V|_{dB} \approx 0dB$ (i.e Gain = 1)
eg $10f_L$, $10^2 f_L$, $10^3 f_L$

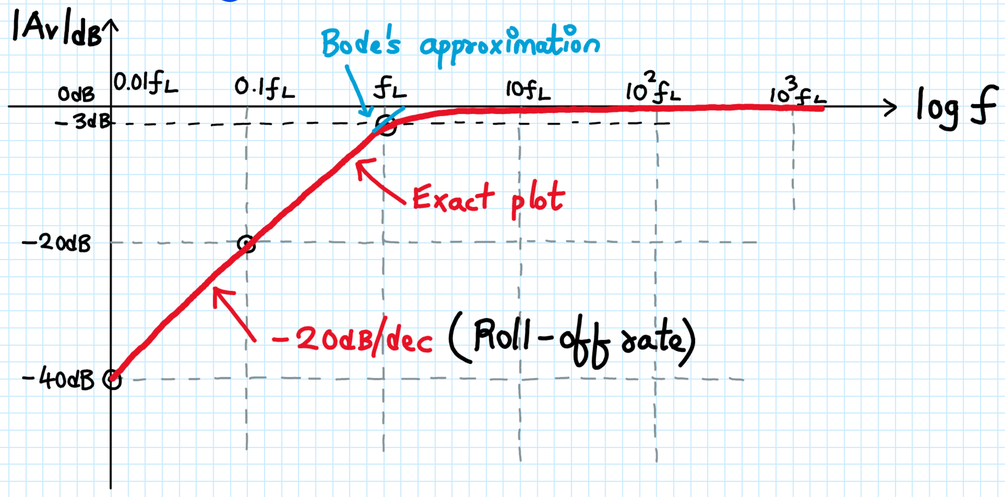


fig 2.1.2: Detailed frequency response plot

⑨ Different names for f_L:

- lower cut-off frequency / -3dB frequency / Corner frequency / break frequency

4