Analysis of Thin-Walled Cylinder

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Abstract

Objective:

1. To study the effect of application of axial as well as radial load on thin-walled circular cylindrical metal shells.
2. To obtain material properties: Critical Buckling Load and Young's Modulus of the specimen

Our original objective was to limit our study to open thin-walled cylinders, but since it was difficult to perform pressure vs strain analysis (as it was difficult to make the cylinder air-tight without generating significant longitudinal stress, in which case, we will have to consider this stress as well in our calculations).
We performed the buckling analysis for open thin-walled cylinder by chopping off the tapered ends of the can. We used the UTM machine to perform the experiment, and we obtained the Load vs Displacement curve. The experimental curve matches the theoretical curve, and we obtained the critical buckling load.
The second part of the experiment is to measure circumferential strain when the closed and sealed can is subjected to internal pressure. Originally, we wanted to measure both circumferential and longitudinal strains using strain gauges. One of the problems we faced was that the air compressor was present in FMFP lab, but the strain gauge machine was not present there. Secondly, since the can is too thin (1mm thickness), it came to our attention that strain gauges cannot be put on the surface, as the can might deform easily and the strain gauges will show wrong readings. Hence, we continued the experiment by measuring the change in diameter with pressure using Vernier Calipers. Through this, we obtained the Circumferential strain, and were able to plot Circumferential Stress vs Strain curve. Further we were able to calculate Young's Modulus.
We could not calculate Poisson's ratio because there was no method to measure longitudinal strain, while our setup was in FMFP lab. Hence, we have assumed a theoretical value for Poisson's ratio.

Category: (b) Material Property Measurement
Justification: We measured the critical buckling load and the Young's Modulus of Coca Cola Aluminum can (thin-walled cylinder approximation).

1. Introduction

We chose to study the effect of application of load on thin-walled circular cylindrical metal shells, because we felt that metal beverage can is one of the most commonly used item. We were curious about the material properties and its geometry, which allows it to stay intact when pressurized, but easily buckle when empty. In real life, this can be extended to applications in pressure cylinders, pipes and uses in industries, such as the main parts of aircrafts, rockets, and submarines.

Radial loading: pressurized fluid flowing through pipe
Axial loading: a structure may be present above a long pipe standing vertically, where buckling becomes important

2. Experimental Methodology

Material = Aluminum alloy, usually with 1-2% manganese + magnesium

<u>**Buckling Analysis:**</u>

- Construction of Specimen: A Coca Cola can was used as our specimen to emulate a thin cylinder/ shell. The ends were sawed off and rubbed with sand paper to make them smooth
- Dimensions of the shell were taken using a Vernier Calliper, namely the internal and external diameters and the length
- Buckling Analysis: The shell was placed vertically in the Universal Testing Machine – Lloyd 50kN and a compressive load was applied. We obtained the load vs displacement curve which helps us calculate the Young' Modulus and the Critical Buckling Load. The data was saved for analysis.

Pressure Analysis:

- The entire can is used for radial loading analysis. A hole was made at the bottom from which liquid was drained. A nozzle which would connect to an air compressor is inserted into the hole and secured using M-Seal and Bondtite. We marked two diametrically opposite points using a marker, and always aligned the jaws of Vernier Calipers to these markings to measure diameter.
- The specimen is then connected to an air compressor of capacity 10 bars using a connecting tube. A pressure gauge is mounted on the connecting tube.
- Once the valve is opened, air flows inside the can through the tube. On closing the valve, we wait for the pressure gauge to stabilise on a reading. Since the volume of tube is less compared to the volume of the can, we assumed that the pressure inside the can is the one detected by the pressure gauge. The reading is noted.
- At the instant, the diameter of the cylinder is measured using a Vernier Calliper and noted.
- All data is then further analysed to obtain material properties.

3. Results and Discussion

Outer Diameter in one direction = 57.16 mm
Outer Diameter in second direction = 56.6 mm
Initial Thickness = 0.98 mm
Thickness after sanding = 0.93 mm
Length of cylindrical section = 115 mm

Mean radius, r = $\frac{\frac{57.16+56.6}{2}-0.93}{2}$ = 27.98 mm

Ratio = $\frac{r}{t} = \frac{27.98}{0.93}$ = 30.08 (Hence, thin-walled cylinder approximation is valid)

Using Young's Modulus, E = 70 GPa and Poisson's ratio, v = 0.33 for predicting theoretical values

Least count of Vernier Calipers = 0.02 mm

Least count of Pressure gauge = (0.05 bar – we used this gauge upto 2 bars
0.2 bar – we used this gauge for measurements above 2 bars)

Buckling Analysis:

Critical Buckling Stress, $\sigma_{cr} = \frac{\eta\gamma Et}{r\sqrt{3(1-v^2)}}$

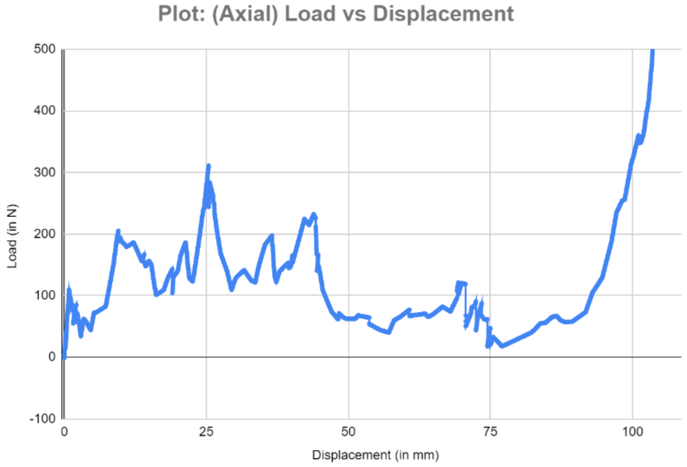
Where η = plasticity reduction factor (assumed to be 1)

From calculations, γ = 0.737

$\sigma_{cr} = \frac{0.737\times 70\times 10^9 \times 0.93}{27.98\times\sqrt{3(1-0.33^2)}}$ = 1049 MPa

Experimentally, F_{cr} = 310 N

$$\sigma_{cr} = \frac{F_{cr}}{2\pi rt} = \frac{310}{2\pi\times 0.02798\times 0.00093} = 1.896\ MPa$$

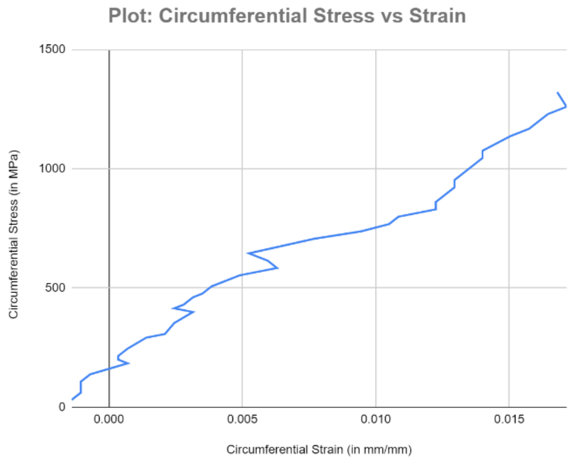


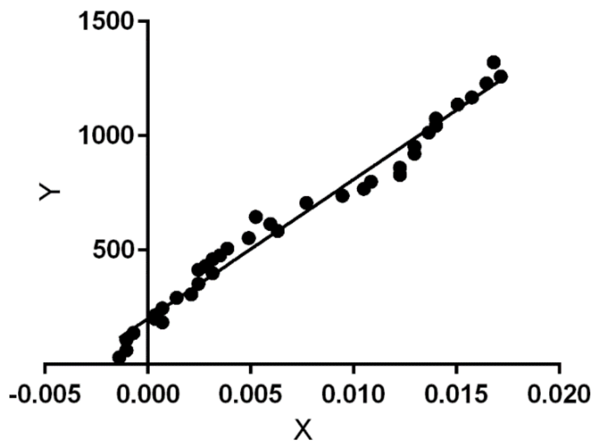
There is a huge difference in the experimental and theoretical critical stress, because we have assumed the plasticity reduction factor to be unity. The state of the can would be severely plastic close to the critical buckling load. But the formula to calculate η requires complicated calculations and data, that is not available with us.

Pressure Analysis:

Circumferential Stress, $\sigma_c = \frac{pr}{t}$ Longitudinal Stress, $\sigma_l = \frac{pr}{2t}$

Circumferential Strain, $\varepsilon_c = \frac{\sigma_c - v\sigma_l}{E} = \frac{(\frac{pr}{t}-\frac{vpr}{2t})}{E} = (\frac{pr}{tE})(1-\frac{v}{2})$





Using an online Linear regression tool, we got the slope = 60.89 GPa

$Slope = \frac{E}{1-\frac{\nu}{2}} = 60.89 => \frac{E}{1-\frac{0.33}{2}} = 60.89 =>$ E = 50.843 GPa

Percentage error in the calculation of Young's Modulus = $\frac{70-50.843}{70} \times 100$ = 27.37 %

4. Summary and Conclusions

It is observed that the experimentally obtained Load vs displacement curve for buckling of can, is similar to the theoretical curve – there is a maximum initially peaking at critical buckling load, then there is a dip and then again, an increase in the load. The major reason behind the curve being not smooth is the presence of uneven edges at the end, which were inevitable as it was difficult to hold the can and cut its tapered ends using a mini hand saw.

We obtained the Young's modulus from the experiment for pressure analysis. Since the can is made of an Aluminum alloy, its expected modulus should have been greater than that for pure Aluminum (70 GPa). We obtained the modulus to be 50 GPa (error of 30%). The major sources of error include:

(a) The pressure gauge was connected to the tube that was putting air into the can. We did not make another hole to insert the pressure gauge, as it would reduce the strength of the can to sustain enough internal pressure for the experiment.

(b) The cross-section of the cylinder is not cylindrical throughout, which adds further perturbations to the formula we used.

(c) The diameter was measured using Vernier Calipers, which is not accurate enough to give reliable readings.

Thin-walled cylinders are able to sustain relatively large amounts of internal pressure as compared to axial loads. Manganese and Magnesium are added in small amount in the Aluminum alloy to improve strength and durability.

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Author Contribution:

Pratham – Conceptualization, Experiment, Analysis, Report, Proposal Presentation, PPTs Making
Disha – Conceptualization, Experiment, Analysis, Report, Final Presentation, PPTs Making
Mansi – Conceptualization, Experiment, Report, Editing, Review Presentation
Paryas –Experiment, Editing, PPTs making

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