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Q1. let $(X_1, X_2 \dots X_n)$ be a random sample of size 'n' taken from a normal population with parameters. mean $= \theta_1$ and variance $= \theta_2$. Find the maximum likelihood Estimates of these 2 parameters.

$$MLE(\theta_1) = \text{joint pdf}(x_1, x_2 \dots x_n \mid \theta_1)$$
$$MLE(\theta_2) = \text{joint pdf}(x_1, x_2 \dots x_n \mid \theta_2)$$

$$MLE(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

log on both sides.

$$\ln(L(\theta_1, \theta_2) = \cancel{\ln\left(\frac{1}{\sqrt{2\pi\theta_2}}\right)}$$

$$\sum_{i=1}^{n}\left[-\ln\left(\sqrt{2\pi\theta_2}\right) - \frac{(x_i - \theta_1)^2}{2\theta_2}\ln(e)\right]$$

$$= -n\ln\left(\sqrt{2\pi\theta_2}\right) - \Sigma\left(\frac{x_i^2 + (\theta_1)^2 - 2x_i\theta_1}{2\theta_2}\right)$$

$$= \boxed{-n\ln\left(\sqrt{2\pi\theta_2}\right) - \frac{\Sigma x_i^2}{2\theta_2} - \frac{n\theta_1^2}{2\theta_2} + \frac{2\theta_1 \Sigma x_i}{2\theta_2}}$$

a. differentiate w.r.t θ_1

$$\frac{-2n\theta_1}{2\theta_2} + \frac{1}{\theta_2}\sum x_i = 0$$

$$\cancel{\theta_1 = \sqrt{\frac{\sum x_i}{2n\theta_2}}}$$

$$\frac{-n\theta_1 + \sum x_i}{\theta_2} = 0$$

$$\boxed{\frac{\sum x_i}{n} = \theta_1}$$

$\theta_1 = \bar{x}$ {sample mean}

b. differentiate w.r.t θ_2

$$\cancel{\frac{-n}{\sqrt{2\pi\theta_2}} \times \frac{\sqrt{2\pi}}{\sqrt{\theta_2}} \times \frac{1}{2}}$$

$$\frac{d\left(-n\ln(\sqrt{2\pi\theta_2}) - \frac{\sum x_i^2}{2\theta_2} - \frac{n\theta_1^2}{2\theta_2} + \frac{2\theta_1\sum x_i}{2\theta_2}\right)}{d\theta_2}$$

$$= \frac{-n}{\sqrt{2\pi\theta_2}} \times \sqrt{2\pi} \times \frac{1}{2\sqrt{\theta_2}} + \frac{\sum x_i^2}{2\theta_2^2} + \frac{n\theta_1^2}{2\theta_2^2} - \frac{\theta_1\sum x_i}{\theta_2^2}$$

$= 0$

$$\Rightarrow \frac{-n\sqrt{2\pi}}{2\sqrt{2\pi}\times\theta_2} + \frac{\sum x_i^2 + n\theta_1^2 - 2\theta_1\sum x_i}{2\theta_2^2} = 0$$

$$\Rightarrow \frac{-n\theta_2 + \sum x_i^2 + n\bar{x}^2 - 2\bar{x}\sum x_i}{2\theta_2^2} = 0$$

$$\Rightarrow \theta_2 = \frac{\sum x_i^2}{n} + (\bar{x})^2 - 2\bar{x}\underbrace{\frac{\sum x_i}{n}}_{\to \bar{x}}$$

$$\theta_2 = E(x_i^2) + E^2(x) - 2E^2(x)$$

$$\theta_2 = E(x_i^2) + E^2(x) - 2E^2(x)$$

$$\theta_2 = E(x_i^2) - E^2(x)$$

$$\boxed{\theta_2 = \text{Variance}(x)}$$

$$\therefore \boxed{\begin{aligned}\theta_1 &= \text{mean}(x)\\ \theta_2 &= \text{variance}(x)\end{aligned}}$$

Q2. Let $X_1, X_2 \ldots X_n$ be a random sample from $B(m, \theta)$ dist. where $\theta \in (0,1)$ is unknown and m is a known positive integer. Compute θ using M.L.E.

pdf Binomial = $\cancel{{}^nC_m\, p^m (1-p)^{n-m}}$

$$\boxed{{}^mC_x\, \theta^x (1-\theta)^{m-x}}$$

$$L(\theta \mid x_1, \dots x_n) = \text{Joint pdf}(x_1, \dots\dots x_n \mid \theta)$$

$$= \prod_{i=1}^{n} {}^mC_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking ln on bothside

$$\sum_{i=1}^{n} \ln\left[{}^mC_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}\right]$$

$$= \sum_{i=1}^{n} \left[\ln({}^mC_{x_i}) + x_i \ln\theta + (m-x_i)\ln(1-\theta)\right]$$

Derivate w.r.t θ

$$= \frac{\sum x_i}{\theta} + \frac{nm - \sum x_i}{1-\theta} \times (-1) = 0$$

$$\sum x_i - \cancel{\theta \sum x_i} - nm\theta + \cancel{\theta \sum x_i} = 0$$

$$\frac{\sum x_i}{nm} = \theta$$

$$\boxed{\theta = \frac{\bar{x}}{m}}$$ mean of sample