Effect of Feedback

1.30.4 Effect of Feedback on Overall Gain

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

Fig. 1.118

The overall transfer function of open loop system shown in fig 1.118 is

$$\frac{C(s)}{R(s)} = G(s)$$

The overall transfer function of closed loop system shown in fig 1.117 is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

For negative feedback the gain G(s) is reduced by a factor $\frac{1}{1 + G(s)\,H(s)}$. So due to nega feedback overall gain of the system reduces.

1.30.5. Effect of Feedback on Stability

Consider the open loop system with overall transfer function

$$G(s) = \frac{K}{s+T}$$

The pole is located at $s = -T$

Now, consider closed loop system with unity negative feedback. then overall transfer functio is given by

$$\frac{C(s)}{R(s)} = \frac{K}{s+(T+K)}$$

Now, closed loop pole is located at $s = -(T+K)$

Thus, feedback controls the time response by adjusting the location of the poles. The stabili depends upon the location of poles. Thus we can say the feedback affects the stability. Feedbac can improve the stability or may be harmful to stability if it is not properly design and apply.

1.31. THERMAL SYSTEMS

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Effect of Feedback

6.8 SENSITIVITY

The parameters of a control system may have a tendency to vary due to changing environment conditions and this variation in parameters affect the desired performance of a control system. The use of feedback in a control system reduces the effect of parameter variations. The term sensitivity in relation to control systems gives an assessment of the system performance as affected due to parameter variations.

6.8.1. Effect of transfer function parameter variations in an open-loop control system

In the case of open-loop system (Fig. 6.8.1) the overall transfer function is

$$M(s) = \frac{C(s)}{R(s)} = G(s)$$

A slight change say $\Delta G(s)$ in the transfer function $G(s)$ changes it to $[G(s) + \Delta G(s)]$ and let the corresponding change in the output be $\Delta C(s)$. The new output of the system is

$$C(s) + \Delta C(s) = [G(s) + \Delta G(s)]\ R(s)$$

or

$$C(s) + \Delta C(s) = G(s)R(s) + \Delta G(s)R(s)$$

$\because$

$$C(s) = G(s)R(s)$$

$\therefore$

$$C(s) + \Delta C(s) = C(s) + [\Delta G(s)]\ R(s)$$

or

$$\Delta C(s) = [\Delta G(s)]\ R(s) \qquad \text{...(6.111)}$$

Fig. 6.8.1. Open-loop system.

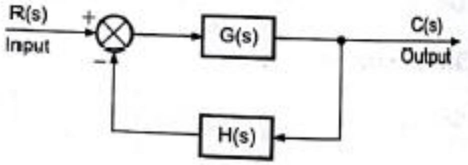

Fig. 6.8.2. Closed-loop control system.

The Eq. (6.111) gives the corresponding change in the output with reference to change $\Delta G(s)$ in the transfer function $G(s)$ of an open-loop system.

The change in a variable due to variations in the parameter of a control system is expressed in terms of sensitivity as below :

Let the variable in a control system which changes its value be A such as output and this change is considered due to parameter variation of element K such as gain or feedback, then the control system sensitivity is expressed as :

$$\text{Sensitivity} = \frac{\%\ \text{change in } A}{\%\ \text{change in } K}$$

In mathematical terms sensitivity is written as :

$$S_K^A = \frac{\partial A / A}{\partial K / K} \qquad \text{...(6.115)}$$

The notation S_K^A denotes sensitivity of variable A with respect to parameter K.

It is preferable that the sensitivity function S_K^A should be minimum.

... with respect to forward path

... of an open-loop system.

6.8.2. Effect of forward path transfer function parameter variations in a closed-loop control system

In the case of closed-loop system the overall transfer function is

$$M(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \qquad \text{...(6.112)}$$

A slight change say $\Delta G(s)$ in the forward path transfer function $G(s)$ changes it to $G(s) + \Delta G(s)$ and let the corresponding change in the output be $\Delta C(s)$. The new output of the system is

$$C(s) + \Delta C(s) = \frac{G(s) + \Delta G(s)}{1 + [G(s) + \Delta G(s)]\ H(s)} \cdot R(s)$$

or

$$C(s) + \Delta C(s) = \frac{G(s) + \Delta G(s)}{1 + G(s)H(s) + [\Delta G(s)]\ H(s)} \cdot R(s) \qquad \text{...(6.113)}$$

As the change in $G(s)$ is considered slight, therefore $\Delta G(s) << G(s)$ and thus the term $[\Delta G(s)]H(s)$ can be neglected in comparison with $G(s)H(s)$.

$\therefore$

$$C(s) + \Delta C(s) \cong \frac{G(s) + \Delta G(s)}{1 + G(s)H(s)} R(s)$$

or

$$C(s) + \Delta C(s) \cong \frac{G(s)}{1 + G(s)H(s)} \cdot R(s) + \frac{\Delta G(s)}{1 + G(s)H(s)} R(s)$$

$\because$

$$C(s) = \frac{G(s)}{1 + G(s)H(s)} \cdot R(s)$$

$\therefore$

$$C(s) + \Delta C(s) \cong C(s) + \frac{\Delta G(s)}{1 + G(s)H(s)} \cdot R(s)$$

or

$$\Delta C(s) \cong \frac{\Delta G(s)}{1 + G(s)H(s)} \cdot H(s) \qquad \text{...(6.114)}$$

The Eq. (6.114) gives the corresponding change in the output with reference to change $\Delta G(s)$ in the forward path transfer function $G(s)$ of a closed-loop control system.

As $|\ 1 + G(s)H(s)\ | >> 1$ therefore, comparing (6.111) and (6.114) it is concluded that due to feedback the variation in the output caused by the change in the forward path transfer function is reduced by a factor $[1 + G(s)H(s)]$, whereas in the open-loop system this reduction is not indicated. Thus, the output variation is more sensitive to variations in $G(s)$ in case of open-loop system as compared to closed-loop system.

Effect of Feedback

It is preferable that the sensitivity function S_K^A should be minimum.

6.8.3. Sensitivity of overall transfer function $M(s)$ with respect to forward path transfer function $G(s)$

The sensitivity function for the overall transfer function $M(s)$ with respect to variation in $G(s)$ is written as

$$S_G^M = \frac{\partial M(s)/M(s)}{\partial G(s)/G(s)} \quad ...(6.116)$$

or

$$S_G^M = \frac{G(s)}{M(s)} \cdot \frac{\partial M(s)}{\partial G(s)}$$

(1) **Open-loop control system.** For the open-loop control system shown in Fig. 6.8.1. The overall transfer function is

$$M(s) = \frac{C(s)}{R(s)} = G(s)$$

or

$$\frac{M(s)}{G(s)} = 1 \quad ...(6.117)$$

differentiating $M(s)$ w.r.t. $G(s)$

$$\frac{\partial M(s)}{\partial G(s)} = 1 \quad ...(6.118)$$

substituting (6.117) and (6.118) in (6.116), the sensitivity of $M(s)$ w.r.t. $G(s)$ for an open-loop control system is obtained as

$$S_G^M = \frac{G(s)}{M(s)} \cdot \frac{\partial M(s)}{\partial G(s)} = 1 \quad ...(6.119)$$

(2) **Closed-loop control system.** For the closed-loop control system shown in Fig. 6.8.2 the overall transfer function is

$$M(s) = \frac{G(s)}{1 + G(s)H(s)} \quad ...(6.120)$$

differentiating w.r.t. $G(s)$

$$\frac{\partial M(s)}{\partial G(s)} = \frac{[1 + G(s)H(s)] - [G(s)H(s)]}{[1 + G(s)H(s)]^2}$$

or

$$\frac{\partial M(s)}{\partial G(s)} = \frac{1}{[1 + G(s)H(s)]^2} \quad ...(6.121)$$

substituting (6.120) and (6.121) in (6.116), the sensitivity of $M(s)$ w.r.t. $G(s)$ for a closed-loop control system is obtained as

$$S_G^M = \frac{G(s)}{M(s)} \cdot \frac{\partial M(s)}{\partial G(s)} = \frac{G(s)}{\frac{G(s)}{1 + G(s)H(s)}} \cdot \frac{1}{[1 + G(s)H(s)]^2}$$

or

$$S_G^M = \frac{1}{1 + G(s)H(s)} \quad ...(6.122)$$

Comparing sensitivity functions (6.122) and (6.119) it is observed that the sensitivity of the overall transfer function w.r.t. forward path transfer function in the case of closed-loop control system is reduced by a factor $[1 + G(s)H(s)]$ as compared to open-loop system.

6.8.4. Sensitivity of overall transfer fu...

... $[1 + G(s)H(s)]$ as compared to open-loop system.

6.8.4. Sensitivity of overall transfer function $M(s)$ with respect to feedback path transfer function

The sensitivity function for the overall transfer function $M(s)$ with respect to variations in the feedback path transfer function $H(s)$ is written as

$$S_H^M = \frac{\partial M(s)/M(s)}{\partial H(s)/H(s)}$$

or

$$S_H^M = \frac{H(s)}{M(s)} \cdot \frac{\partial M(s)}{\partial H(s)} \quad ...(6.123)$$

The overall transfer function $M(s)$ for the closed-loop control system (Fig. 6.8.2) is

$$M(s) = \frac{G(s)}{1 + G(s)H(s)} \quad ...(6.124)$$

differentiating w.r.t. $H(s)$

$$\frac{\partial M(s)}{\partial H(s)} = \frac{[G(s)]^2}{[1 + G(s)H(s)]^2} \quad ...(6.125)$$

Substituting (6.124) and (6.125) in (6.123) the sensitivity of $M(s)$ w.r.t. $H(s)$ for a closed-loop control system is obtained as

$$S_H^M = -\frac{G(s)H(s)}{1 + G(s)H(s)} \quad ...(6.126)$$

Comparing sensitivity functions (6.122) and (6.126) it is concluded that a closed-loop control system is more sensitive to variations in feedback path parameters than the variations in forward path parameters therefore, the specifications of feedback elements in a closed-loop control system should be more rigid as compared to that of forward path elements.

Effect of Feedback

6.8.5. Effect of feedback on time constant of a control system

In a control system the time response is improved by adjusting the feedback parameters. Consider an open-loop control system whose transfer function is

$$\frac{C(s)}{R(s)} = \frac{K}{1+sT} \qquad \ldots(6.127)$$

where K is the gain and T the time constant.

The time response of the transfer function (6.127) subjected to unit step input is given by

$$c(t) = \frac{K}{T}(1-e^{-t/T}) \qquad \ldots(6.128)$$

If the system is made closed-loop with a feedback path transfer function $H(s) = b$. The closed-loop (overall) transfer function is

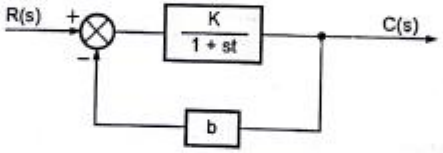

Fig. 6.8.4. Closed-loop system.

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{1+sT}}{1+\frac{K}{1+sT}\cdot b} \quad \text{or} \quad \frac{C(s)}{R(s)} = \frac{K}{1+sT+Kb}$$

or

$$\frac{C(s)}{R(s)} = \frac{K}{1+Kb}\left[\frac{1}{1+s\left(\frac{T}{1+Kb}\right)}\right] \qquad \ldots(6.129)$$

The time response of the closed-loop transfer function subjected to unit step input is given by

$$c(t) = \frac{K}{1+Kb}\left[1-e^{\frac{-\frac{t}{T}}{1+K_b}}\right] \qquad \ldots(6.130)$$

The time constant in the case of open-loop system is T and that in closed-loop case is $T/(1+Kb)$. Since $K > 1$ and b being positive $T/(1+Kb) < T$, indicating improvement in time response. However, the overall gain is reduced.