MODULE - 2 : MICROWAVE COMPONENTS.

Desirable characteristics of microwave components

→ low SWR.
→ lower attenuation.
→ lower insertion loss.

eg: waveguide junctions, joints, corners, directional couplers, ferrite devices, phaseshifters etc.

Waveguide Microwave junction.

In a waveguide system it is necessary to split all or part of the microwave energy into particular directions. This is achieved by microwave junctions

Functions of a Mwave junction.

→ to split all or part of Mwave energy into particular directions.
→ To combine two or more signals.

Microwave junction is the interconnection of two or more microwave components.

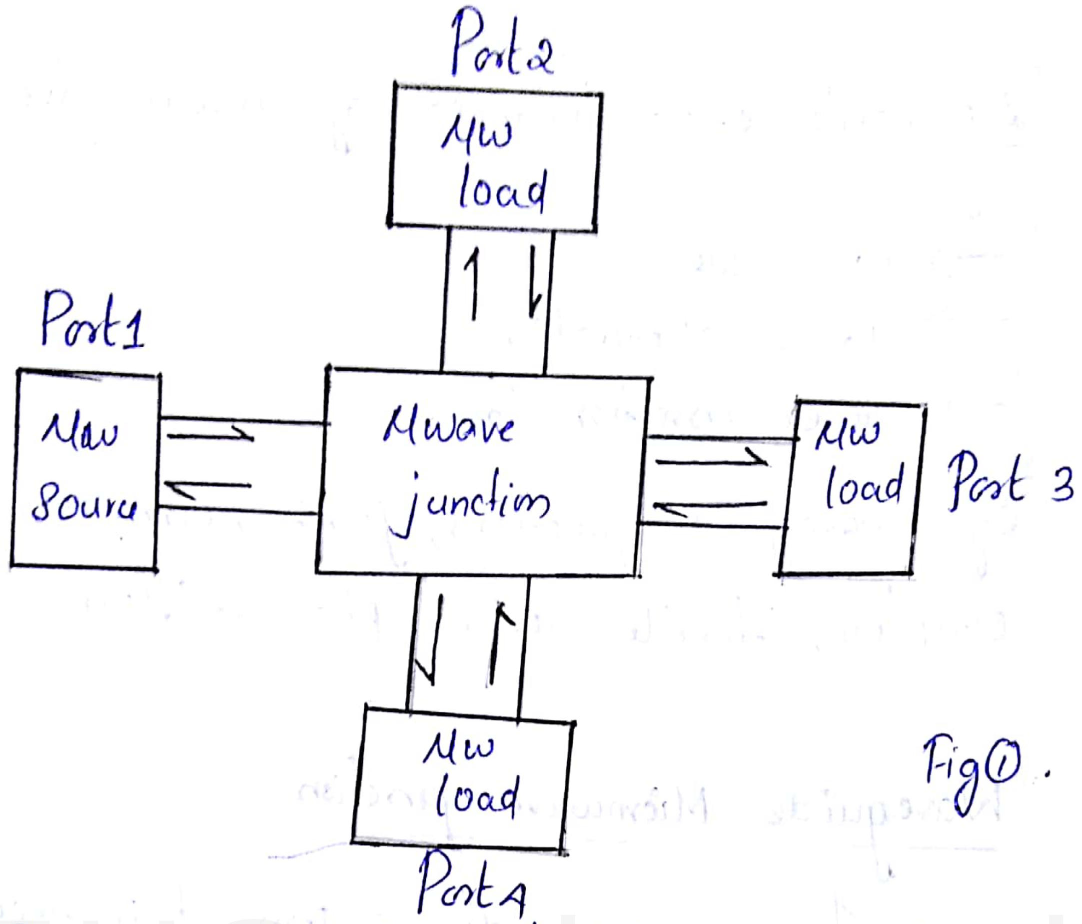


Fig ①.

The microwave junction shown in the figure has four ports. Microwave source is at port 1 & microwave loads at port 2, 3 & 4. (Thus Mwave junction is analogous to a traffic junction where a no: of roads meet on which vehicles enter or leave the traffic junction) When input from microwave source is applied to port 1 a part of it comes out of port 2, another part out of port 3 some part out of port 4 and the remaining part may come out of port 1 due to mismatch b/w port 1 and Mwave junction.

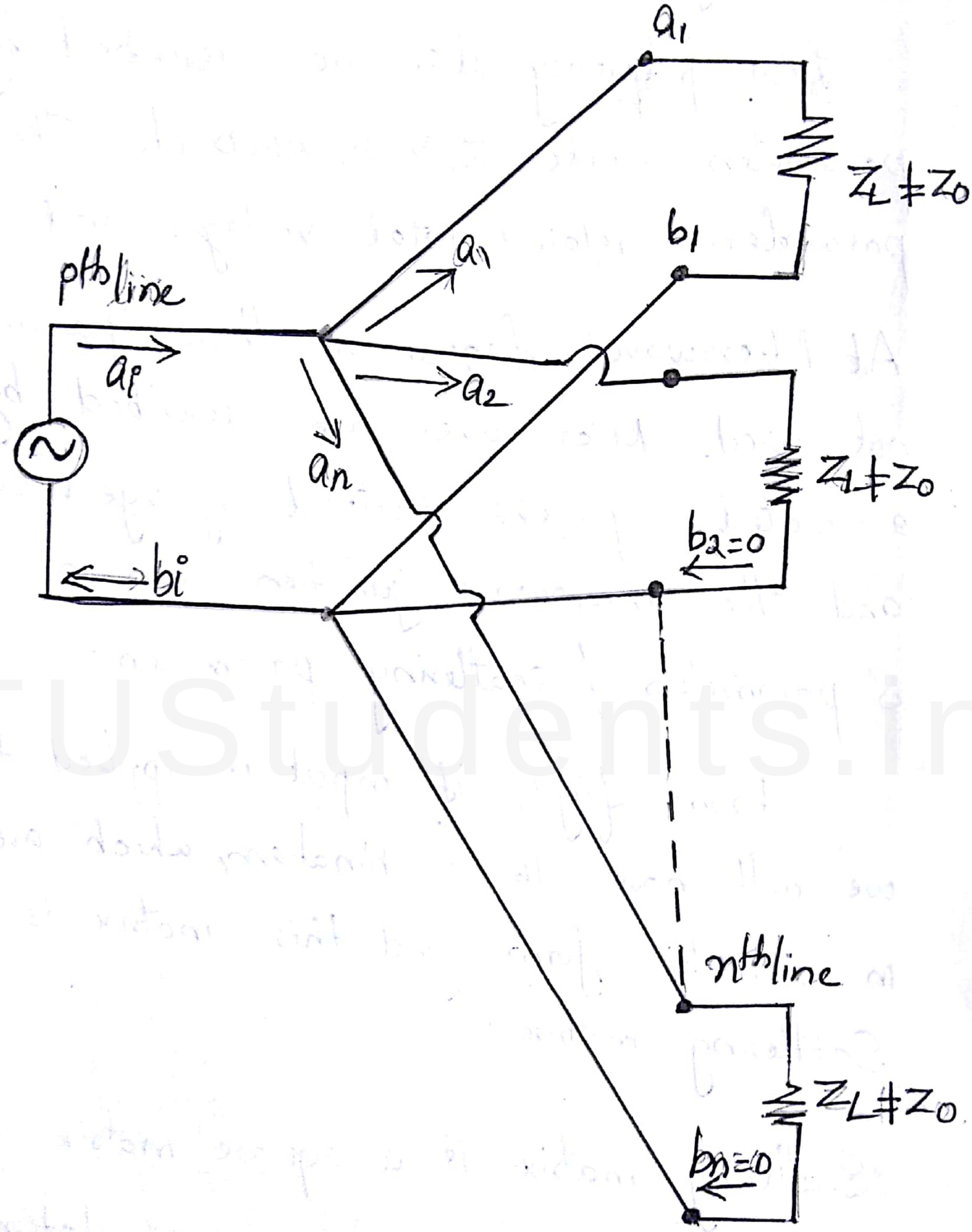
Scattering / S parameters:

Low frequency ckts are described by 2 port n/w parameters like Z, Y, H, ABCD etc. These n/w parameters relate total voltages and total currents. At Microwave frequencies these parameters are not used. Microwaves are described by their associated powers instead of vge & current and the microwave junction can be described by S parameters / scattering parameters.

1 i/p = 4 o/p
4 i/p = 4×4 = 16

From fig 1, if input is applied to all ports we will have 16 combinations, which are represented in a matrix form and this matrix is called Scattering matrix.

Scattering matrix is a square matrix which gives all the combinations of power relationships b/w various input and output ports of a Mwave junction. The elements of this matrix called Scattering coeffecients / S parameters.



Consider a junction of 'n' no: of transmission lines where, the ith line is terminated in a source, a_i be the incident wave at the junction due to source at ith line. It divides among $(n-1)$

lines as $a_1, a_2, \ldots a_n$. If the first line is not terminated at the characteristic impedence, Z_0, there will be reflected wave b_1 going back into the junction. If all (n-1) lines are terminated in an impedence other than Z_0 then there will be reflections into the junction from every line & hence the total contribution to the outward travelling wave in ith line is

$$b_i = S_{i1} a_1 + S_{i2} a_2 + \ldots \ldots + S_{in} a_n$$

i from 1 to n

$$\therefore b_1 = S_{11} a_1 + S_{12} a_2 + \ldots + S_{1n} a_n$$

$$b_2 = S_{21} a_1 + S_{22} a_2 + \ldots \ldots + S_{2n} a_n$$

$\vdots$

$$b_n = S_{n1} a_1 + S_{n2} a_2 + \ldots \ldots + S_{nn} a_n$$

In Matrix form

$$\underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}}_{\text{o/p from diff lines}} = \underbrace{\begin{bmatrix} S_{11} & S_{12} & \ldots & S_{1n} \\ S_{21} & S_{22} & \ldots & S_{2n} \\ \vdots & & & \\ S_{n1} & S_{n2} & \ldots & S_{nn} \end{bmatrix}}_{\text{S Matrix}} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}}_{\text{i/p i/p to diff lines.}}$$

ie

$$[b] = [S][a]$$

where

[a] => rep the i/ps to particular port.

[b] => rep the o/p out of various ports.

S_{ij} => rep the scattering coeff resulting due to i/p at i^{th} port and o/p taken from j^{th} port.

S_{ii} => denotes how much of power is reflected back from the junction into the i^{th} port when i/p is applied at i^{th} port itself.

* Properties of S Matrix

1. [S] is always a square matrix of order (n×n)
2. [S] is a symmetric matrix.

   ie $S_{ij} = S_{ji}$.
3. [S] is a unitary matrix.

   ie $[S][S^*] = [I]$

   where $[S^*]$ => complex conjugate of [S].

   [I] => unit matrix / identity matrix of the same order as [S].

4. The sum of the product of each term of any row (or column) multiplied by the complex conjugate of the corresponding terms of any other row (or column) is zero.

ie $\sum_{i=1}^{n} S_{ik} S_{ij}^{*} = 0 \quad k \neq j. \quad \left(\begin{array}{l} k = 1,2,3 \dots n \\ j = 1,2,3 \dots n \end{array} \right)$

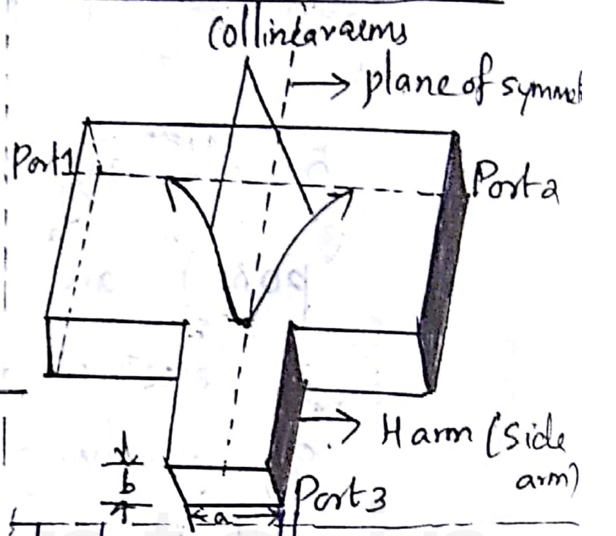
5. If any one terminal / reference planes (say k^{th} port) are moved away from the junction by an electric distance $\beta_k l_k$, each of the coeffecients S_{ij} involving k will be multiplied by a factor $e^{-j\beta_k l_k}$.

MICROWAVE TEE JUNCTIONS.

The 'Tee' junction is an interconnection of three waveguides in the form of capital letter 'T'. Various microwave tee junctions are

1. H Plane Tee
2. E Plane Tee
3. E-H Plane Tee
4. Magic Tee
5. Rat Race Ring.

H Plane Tee (fig)



1. H Plane Tee.

H Plane Tee is formed by cutting a rectangular slot along the width of the main waveguide and attaching another waveguide - the side arm called H arm. Ports 1 and 2 are called Collinear arms [Collinear ports] and port 3 is called H arm or side arm. All the three arms of H Plane Tee lie in the plane of MF, the magnetic field divides itself into arms ∴ This junction is also called as 'Current Junction'.

There are 3 possible inputs and 3 possible outputs. Hence a H plane Tee is completely described by a S matrix of order 3x3.

To determine S parameters.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Condition 1.

Because of the plane of symmetry of junction

$S_{13} = S_{23}$.

Condition 2.

Since port 3 is perfectly matched to the junction

$S_{33} = 0$.

Condition 3.

From symmetric property

$S_{12} = S_{21}$

$S_{13} = S_{31}$

$S_{23} = S_{32}$.

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix}$$

Condition 4

From unitary property

$$[S][S^*] = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R_1C_1

$$S_{11} \cdot S_{11}^* + S_{12} \cdot S_{12}^* + S_{13} \cdot S_{13}^* = 1.$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{———} \quad (1).$$

R_2C_2

$$S_{12} \cdot S_{12}^* + S_{22} \cdot S_{22}^* + S_{13} \cdot S_{13}^* = 1.$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \text{———} \quad (2).$$

R_3C_3.

$$S_{13} \cdot S_{13}^* + S_{13} \cdot S_{13}^* = 1.$$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \quad \text{———} \quad (3).$$

R_3C_1

$$S_{13} \cdot S_{11}^* + S_{13} \cdot S_{12}^* + 0 = 0 \quad \text{———} \quad (4).$$

From (3)

$$2|S_{13}|^2 = 1.$$

$$|S_{13}|^2 = \frac{1}{2}$$

$$|S_{13}| = 1/\sqrt{2} \text{ ——— } (5)$$

equating eqn (1) and (2)

$$|S_{11}|^2 + \cancel{|S_{12}|^2} + \cancel{1/2} = \cancel{|S_{12}|^2} + |S_{22}|^2 + \cancel{1/2}$$

$$\therefore \underline{\underline{S_{11} = S_{22}}} \text{ ——— } (6)$$

From (4).

$$S_{13} \cdot S_{11}^* + S_{13} \cdot S_{12}^* = 0.$$

$$S_{13} [S_{11}^* + S_{12}^*] = 0.$$

$$S_{11}^* + S_{12}^* = 0.$$

$$S_{11} + S_{12} = 0.$$

$$S_{11} = -S_{12} \text{ ——— } (7)$$

From (1).

$$|S_{11}|^2 + |S_{12}|^2 = \frac{1}{2}.$$

$$2|S_{11}|^2 = \frac{1}{2}.$$

$$|S_{11}|^2 = \frac{1}{4}.$$

$$S_{11} = \underline{\underline{1/2}} \text{ ——— } (8)$$

$$S_{11} = S_{22}$$

$$\therefore S_{22} = 1/2 \text{ ——— } (9)$$

$$S_{11} = -S_{12}.$$

$$\therefore S_{12} = -1/2 \text{ ——— } (10)$$

$$\therefore [S] = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ \frac{1}{\sqrt{2}} & 1/\sqrt{2} & 0 \end{bmatrix}$$

$$[b] = [S][a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1/2\, a_1 & -1/2\, a_2 & 1/\sqrt{2}\, a_3 \\ -1/2\, a_1 & 1/2\, a_2 & 1/\sqrt{2}\, a_3 \\ \frac{1}{\sqrt{2}} a_1 & \frac{1}{\sqrt{2}} a_2 & 0 \end{bmatrix}$$

$$\therefore b_1 = 1/2\, a_1 - 1/2\, a_2 + 1/\sqrt{2}\, a_3$$

$$b_2 = -1/2\, a_1 + 1/2\, a_2 + 1/\sqrt{2}\, a_3$$

$$b_3 = 1/\sqrt{2}\, a_1 + 1/\sqrt{2}\, a_2 + 0.$$

Case I

If i/p is applied to port ③ only

$$\therefore a_1 = a_2 = 0.$$

then $b_1 = 1/\sqrt{2}\, a_3$.

$b_2 = 1/\sqrt{2}\, a_3$.

$b_3 = 0$.

When TE_{10} mode is allowed to propogate into port 3, the waves that come out of port 1 and 2 are equal in magnitude and phase. The EF lines do not change their direction when they come out of ports 1 and 2, and hence this Tee jn is called H plane Tee.

Let P_3 be the i/p power at port 3. P_3 divides equally b/w ports 1 and 2 in phase

$\therefore P_3 = P_1 + P_2$.

Also $P_1 = P_2$

$\therefore P_3 = 2P_1$ or $2P_2$

Power gain at port 1 in decibels.

$$= 10 \log_{10}\left(\frac{P_1}{P_3}\right).$$

$$= 10 \log_{10}\left(\frac{P_1}{2P_1}\right)$$

$$= 10 \log_{10}\left(1/2\right) = 10 \log_{10} 0.5 = \underline{\underline{-3\,dB}}$$

ie $\boxed{\text{Supplied power } -3\,dB = \text{o/p power}}$

∴ H plane Tee is also called 3dB splitter as power from ports 1 and 2 is 3dB less of i/p power at port 3.

Case 2

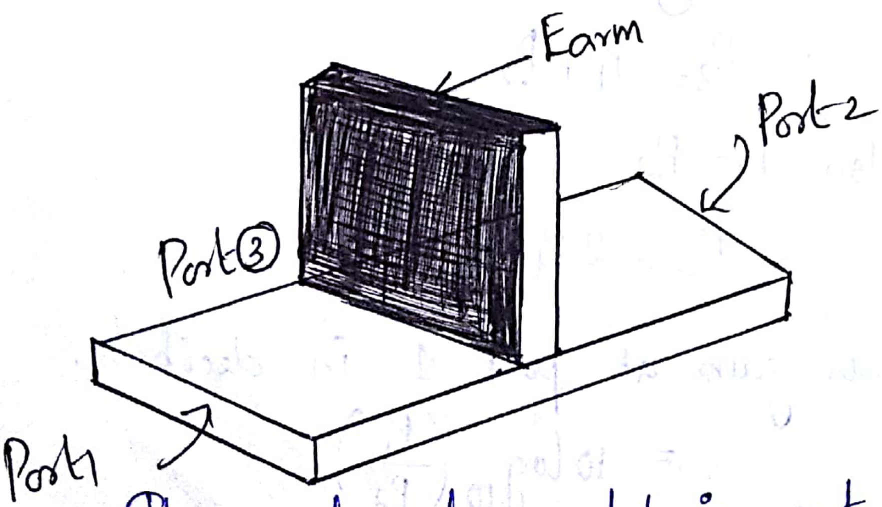
$a_1 = a_2 = a$.

$a_3 = 0$.

then $b_1 = 0$, $b_2 = 0$, $b_3 = 2\left(\frac{1}{\sqrt{2}}a\right) = \sqrt{2}a$

o/p at port 3 is the addition of i/ps at port 1 and port 2 and these are added in phase

2. E Plane Tee



The rectangular slot is cut along the broader dimension of a waveguide and a side arm is attached to form E Plane Tee. Port 1 and 2 are called collinear arms. Port 3 is called E arm.

When TE_{10} mode is made to propogate into port 3, the two o/ps at port 1 and port 2 will have a phase shift of $180°$. Since EF changes their direction when they come out of port 1 and 2 it is called E plane Tee. E plane Tee is also called voltage junction / series junction.

the S matrix $[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$

Condition 1

Because of the plane of symmetry of junction

$S_{23} = -S_{13}$.

Condition 2

Since port 3 is perfectly matched $S_{33} = 0$.

Condition 3

From symmetry property

$S_{12} = S_{21}$

$S_{13} = S_{31}$

$S_{23} = S_{32}$

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix}$$

④ $[S][S^*] = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1C_1 \rightarrow$

$$S_{11}.S_{11}^* + S_{12}.S_{12}^* + S_{13}.S_{13}^* = 1$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{——— ①}$$

$R_2C_2 \rightarrow$

$$S_{12}.S_{12}^* + S_{22}.S_{22}^* + -S_{13}.-S_{13}^* = 1$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \text{——— ②}$$

$R_3C_3 \rightarrow$

$$S_{13}.S_{13}^* + -S_{13}.-S_{13}^* + 0 = 1$$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \quad \text{——— ③}$$

From ③

$$2|S_{13}|^2 = 1$$

$$|S_{13}|^2 = 1/2$$

$$\therefore \quad |S_{13}| = 1/\sqrt{2} \quad \text{———} \quad ④.$$

Sub in ① and ②

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} = 1$$

$$\Rightarrow |S_{11}|^2 + |S_{12}|^2 = 1/2 \quad \text{———} \quad ⑤.$$

lly $2 \longrightarrow$

$$|S_{12}|^2 + |S_{22}|^2 = 1/2 \quad \text{———} \quad ⑥.$$

$\underline{R_3 C_1} \longrightarrow$

$$S_{13} \cdot S_{11}^* - S_{13} \cdot S_{12}^* = 0$$

$$S_{13} [S_{11}^* - S_{12}^*] = 0.$$

$$\Rightarrow S_{13} = 0 \quad S_{11}^* - S_{12}^* = 0 \quad \text{ie} \quad \underline{\underline{S_{11} = S_{12}}} \quad - ⑦$$

Sub ⑦ in ⑤

$$|S_{12}|^2 + |S_{11}|^2 = \frac{1}{2}$$

$$\Rightarrow \quad S_{11} = 1/2$$

$$S_{12} = 1/2$$

$$S_{13} = 1/\sqrt{2}.$$

Equating ⑤ and ⑥.

$$|S_{22}| = 1/2.$$

ie $[S] = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$

$[b] = [S][a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$\therefore \; b_1 = 1/2\, a_1 + 1/2\, a_2 + 1/\sqrt{2}\, a_3$

$b_2 = 1/2\, a_1 + 1/2\, a_2 - 1/\sqrt{2}\, a_3$

$b_3 = 1/\sqrt{2}\, a_1 - 1/\sqrt{2}\, a_2$ -

Case 1

$a_1 = a_2 = 0, \quad a_3 \neq 0.$

input is applied to port 3 only.

then $b_1 = 1/\sqrt{2}\, a_3$

$b_2 = -1/\sqrt{2}\, a_2$

$b_3 = 0.$

An i/p at port 3 equally divides b/w port 1 and 2, but introduces a phase shift of 180°.

Case 2

$a_1 = a_2 = a$, $a_3 = 0$.

then $b_1 = a$, $b_2 = a$ $b_3 = 0$.

i.e. Zero o/p comes from port 3.

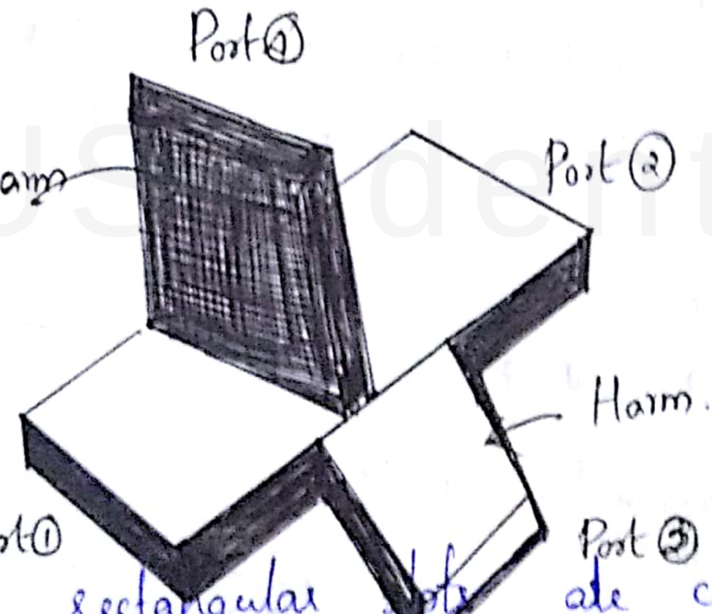
Equal i/ps at port 1 and 2 results no o/p in ports.

Case 3

$a_1 \neq 0$, $a_2 = a_3 = 0$.

$b_1 = \frac{1}{2} a_1$, $b_2 = \frac{1}{2} a_2$, $b_3 = \frac{1}{\sqrt{2}} a_1$.

3) E-H Plane Tee.



Here rectangular slots are cut both along width & breadth of a long waveguide & side arms are attached. Port 1 and 2 are called Collinear arms. Here port 3 is called H arm. The 4 port Tee junction combines the power dividing properties of both H plane Tee & E plane Tee.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

Conditions

① $S_{23} = S_{13}$ (H plane Tee property)

② $S_{24} = -S_{14}$ (E plane Tee property)

③ $S_{34} = S_{43} = 0$

port 3 and 4 are called isolated ports. ie i/p at port 3 cannot come out of port 4.

④ $S_{33} = S_{44} = 0$

port 3 and 4 are perfectly matched to junction.

⑤ From symmetric property

$S_{12} = S_{21}$, $S_{13} = S_{31}$, $S_{14} = S_{41}$, $S_{32} = S_{23}$, $S_{34} = S_{43}$, S_{24}

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix}$$

(6) $[S][S^*] = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 C_1$

$$S_{11} \cdot S_{11}^* + S_{12} \cdot S_{12}^* + S_{13} \cdot S_{13}^* + S_{14} \cdot S_{14}^* = 1$$

$R_2 C_2$

$$S_{12} \cdot S_{12}^* + S_{22} \cdot S_{22}^* + S_{13} \cdot S_{13}^* + S_{14} \cdot S_{14}^* = 1$$

$R_3 C_3$

$$S_{13} \cdot S_{13}^* + S_{13} \cdot S_{13}^* = 1$$

$R_4 C_4$

$$S_{14} \cdot S_{14}^* + -S_{14} \cdot -S_{14}^* = 1$$

$R_4 C_1$

$$S_{14} S_{11}^* + -S_{14} \cdot S_{12}^* = 0.$$

ie $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$ —— ①.

$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$ —— ②.

$|S_{13}|^2 + |S_{13}|^2 = 1$

$2|S_{13}|^2 = 1 \Rightarrow S_{13} = 1/\sqrt{2}$ —— ③.

$|S_{14}|^2 + |S_{14}|^2 = 1$

$2|S_{14}|^2 = 1 \Rightarrow S_{14} = 1/\sqrt{2}$ —— ④.

Sub ③ and ④ in ① and ②.

$|S_{11}|^2 + |S_{12}|^2 + 1/2 + 1/2 = 1$

$|S_{11}|^2 + |S_{12}|^2 = 0$ —— ⑤

||ly $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + 1/2 = 1$ —— ⑥

From $R_4 C_1$

$S_{14} S_{11}^* - S_{14} \cdot S_{12}^* = 0$

$S_{14}(S_{11}^* - S_{12}^*) = 0$

$S_{14} \neq 0$ ($1/\sqrt{2}$ value)

$\therefore S_{11}^* = S_{12}^* \Rightarrow \underline{\underline{S_{11} = S_{12}}}$

$\therefore$ Eqn ⑤ becomes

$|S_{11}|^2 + |S_{12}|^2 = 0.$

$|S_{11}|^2 + |S_{11}|^2 = 0 \Rightarrow \underline{\underline{S_{11} = 0}}$ $\therefore \underline{\underline{S_{12} = 0}}$

ie port 1 is perfectly matched to junction

Eqn ⑥ becomes

$$0 + |S_{22}|^2 + |1/\sqrt{2}|^2 + |1/\sqrt{2}|^2 = 1$$

$$|S_{22}|^2 = 0.$$

ie $S_{22} = 0$ ie port 2 is perfectly matched to jn

$$\therefore [S] = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

$$[b] = [S][a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

ie $b_1 = 1/\sqrt{2}\,(a_3 + a_4)$

$b_2 = 1/\sqrt{2}\,(a_3 - a_4)$

$b_3 = 1/\sqrt{2}\,(a_1 + a_2)$

$b_4 = 1/\sqrt{2}\,(a_1 - a_2)$

*4. Magic Tee

In a Hybrid Tee, $S_{11} = 0$ and $S_{22} = 0$. This implies that both ① and ② are perfectly matched to the junction. In any 4 port junction if 2 ports are perfectly matched to the junction, then the remaining 2 ports are automatically matched to the junction. Such a junction where all the four ports are perfectly matched to the junction is called 'Magic Tee'.

Case 1

$a_3 \neq 0, \quad a_1 = a_2 = a_4 = 0.$

$$\left.\begin{aligned} b_1 &= \tfrac{1}{\sqrt{2}} a_3 \\ b_2 &= \tfrac{1}{\sqrt{2}} a_3 \end{aligned}\right\} \Rightarrow \text{property of H plane tee.}$$

$$b_3 = b_4 = 0.$$

Case 2

$a_4 \neq 0, \quad a_1 = a_2 = a_3 = 0.$

$$\left.\begin{aligned} b_1 &= \tfrac{1}{\sqrt{2}} a_4 \\ b_2 &= -\tfrac{1}{\sqrt{2}} a_4 \end{aligned}\right\} \Rightarrow \text{E plane property.}$$

$$b_3 = b_4 = 0$$

Case 3

$a_1 \neq 0$, $a_2 = a_3 = a_4 = 0$

$b_1 = 0$, $b_2 = 0$, $b_3 = \frac{1}{\sqrt{2}} a_1$, $b_4 = \frac{1}{\sqrt{2}} a_1$

When power is fed to port 1, nothing comes out of port 2, even though they are collinear arms. Hence ports 1 and 2 are called isolated ports. An i/p at port 2 cannot come out of port 1. Similarly E and H ports are also isolated ports.

Case IV

$a_3 = a_4$; $a_1 = a_2 = 0$.

then $b_1 = \frac{1}{\sqrt{2}}(2a_3) = \sqrt{2}\, a_3$.

$b_2 = 0$

$b_3 = 0$

$b_4 = 0$.

Equal i/ps at port 3 & 4 results in an o/p which is the sum of 2 i/ps (additive property)

Case V

$a_1 = a_2$; $a_3 = a_4 = 0$.

$b_1 = 0$,

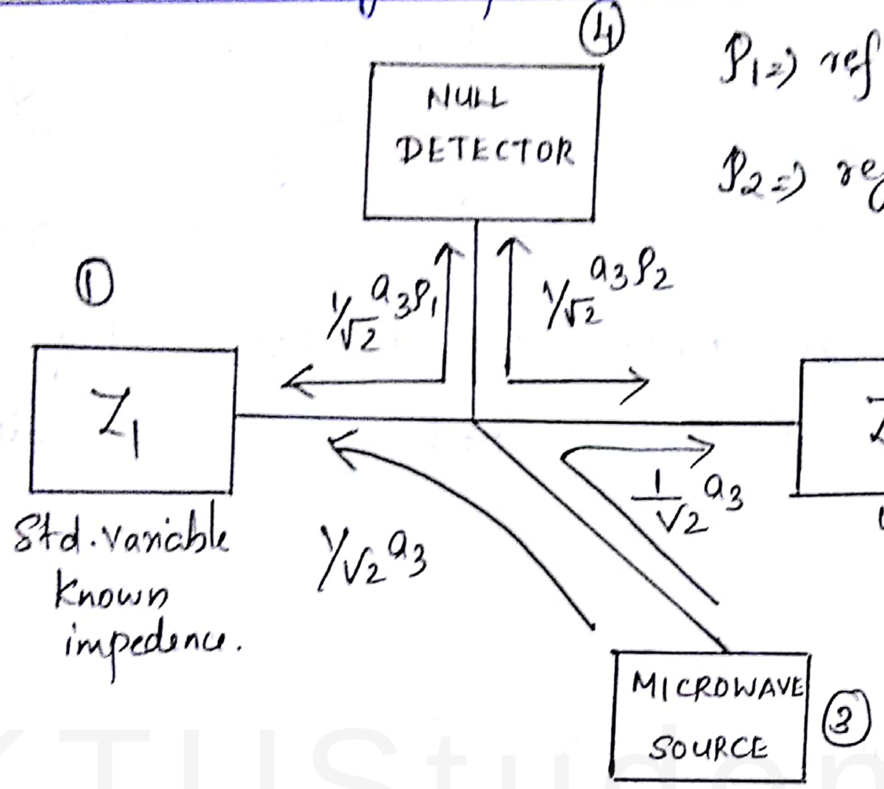
$b_2 = 0$

$b_3 = \sqrt{2} a_1$

$b_4 = 0$.

Equal i/ps at (1) and (2) results in a sum o/p at port 3.

1. Measurement of impedence



$\rho_1 \Rightarrow$ reflection coeffecient of Z_1

$\rho_2 \Rightarrow$ reflection coeffecient of Z_2.

Microwave source is connected in arm 3, null detector in arm 4, unknown impedence in arm 2 and a std variable (known impedence) in arm 1. Magic Tee is used in the form of a bridge. Power from microwave source (a_3) gets divided equally b/w arms (1) and (2). Impedence in these arms are not equal to the characteristic impedence. Hence there will be reflections from arms (1) and (2). The resultant wave into the null detector is

$$\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} a_3 \rho_1\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} a_3 \rho_2\right) = \frac{1}{2} a_3 (\rho_1 - \rho_2)$$

For perfect balancing of the bridge

$$\frac{1}{2} a_3 (\rho_1 - \rho_2) = 0.$$

$$\Rightarrow \rho_1 = \rho_2$$

In terms of Z

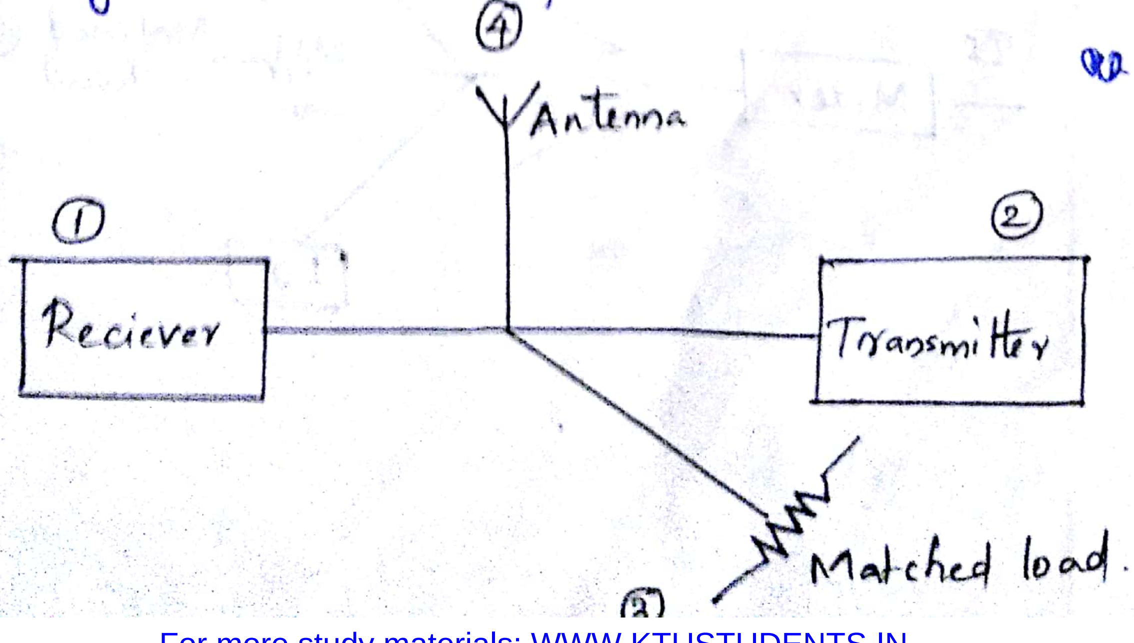
$$\frac{Z_1 - Z_3}{Z_1 + Z_3} = \frac{Z_2 - Z_3}{Z_2 + Z_3}$$

$$\Rightarrow Z_1 = Z_2$$

ie $R_1 + jX_1 = R_2 + jX_2$.

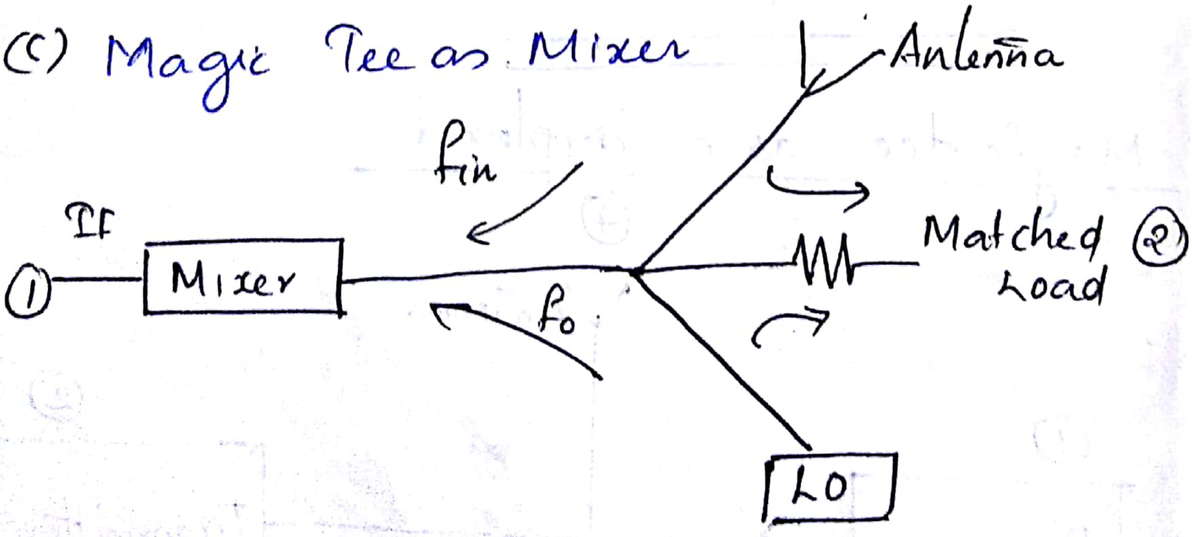
$\Rightarrow R_1 = R_2$ & $X_1 = X_2$

2. Magic tee as a duplexer.

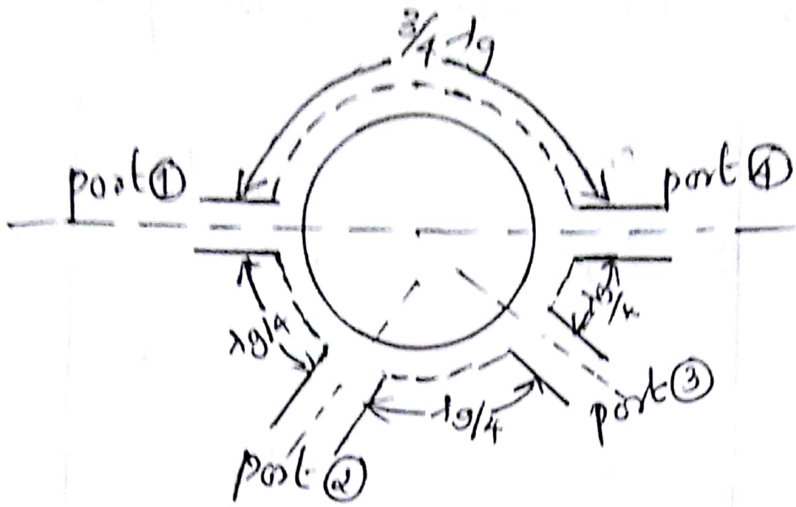


Here transmitter & Rxr are connected to ports (2) and (1), antenna in E arm and a matched load in H arm. During transmission half the power reaches the antenna from where it is radiated into space. Other half reaches the load where it is absorbed without reflections. No transmitted power reaches the Rxr since ports 1 and 2 are isolated. During reception half the recieved power goes to the reciever & other half to the Txr. Txr & Rxr are isolated during reception & transmission.

(c) Magic Tee as Mixer



5. Rat Race Junction.



$$\frac{\lambda g}{2} + \frac{\lambda g}{4} + \frac{3\lambda g}{4}$$

$$\frac{2\lambda g + \lambda g + 3\lambda g}{4} = \frac{6\lambda g}{4}$$

This is a 4 port junction. The 4 ports are connected in the form of a annular ring at proper intervals by series / parallel junction. The mean circumference of total race is 1.5 λg.

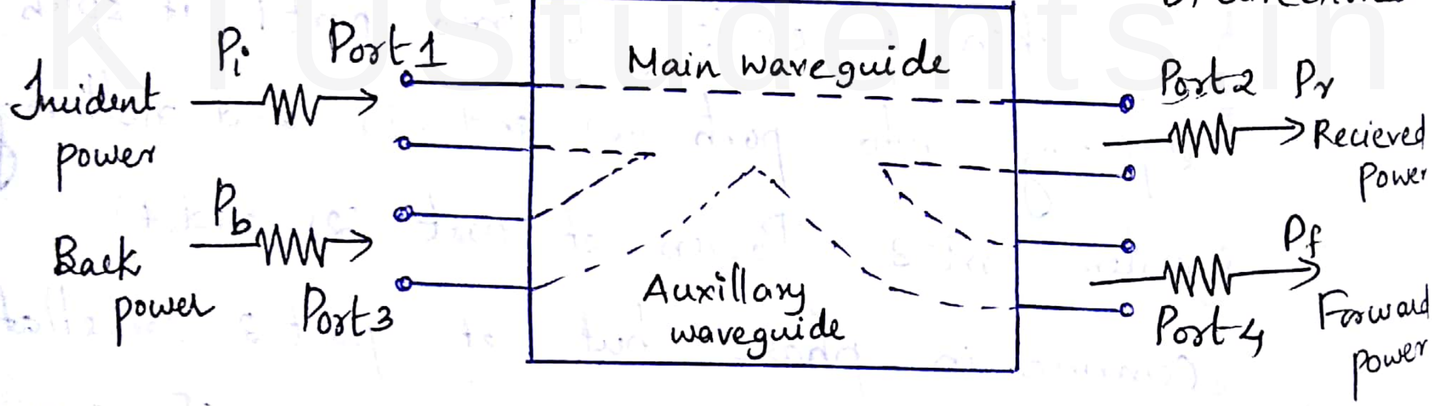
When power is fed into port 1 it splits equally into ports (2) and (4) and nothing enters port 3. Powers at port (2) and (4) combine in phase, but at port 3 cancellation occurs due to λg/2 path difference. For similar reasons any i/p applied at port 3 is equally divided b/w ports (2) and (4), but o/p out of port 1 will be zero.

The Rat race ring can be used for combining 2 s/ls or dividing a single s/l into two equal halves. If two unequal s/ls are applied at port 1 an o/p proportional to their sum will emerge from port 2 and 4.

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

DIRECTIONAL COUPLER

Directional Coupler is used to couple the microwave power which may be unidirectional or bi directional.



Directional coupler is a 4 port wave guide junction consisting of a primary main waveguide and a secondary auxillary waveguide. If all the four ports are provided with match terminations then the properties of directional coupler can be summarised as follows.

Property 1.

A portion of power travelling from port 1 to port 2 is coupled to port 4 but not to port 3.

Property 2.

A portion of power travelling from port 2 to port 1 is coupled to port 3 but not to port 4.

Property 3

A portion of power incident on port 3 is coupled to port 2, but not to port 1.

Property 4.

A portion of power incident on port 4 is coupled to port 1, but not to port 2.

Property 5:

Ports (1) and (3), as well as ports (2) and (4) are decoupled.

The performance of directional coupler is defined in terms of 3 parameters

① Coupling factor [C]

It is defined as the ratio of incident power to forward power measured in dB.

$$\boxed{C = 10\log_{10}\frac{P_i}{P_f}.}$$

Coupling factor is the measure of how much of incident power is being sampled.

② Directivity (D)

It is defined as the ratio of forward power to backward power expressed in dB.

$$\boxed{D = 10\log_{10}\left(\frac{P_f}{P_b}\right)}$$

Directivity is the measure of how well the directional coupler distinguishes b/w forward & reverse travelling power.

③ Isolation [I]

Isolation describes the directive properties of the directional coupler. It is defined as the ratio of incident power to back power expressed in dB.

$$\boxed{I = 10\log_{10}\left(\frac{P_i}{P_b}\right)}$$

$$\boxed{I_{dB} = C_{dB} + D_{dB}}$$

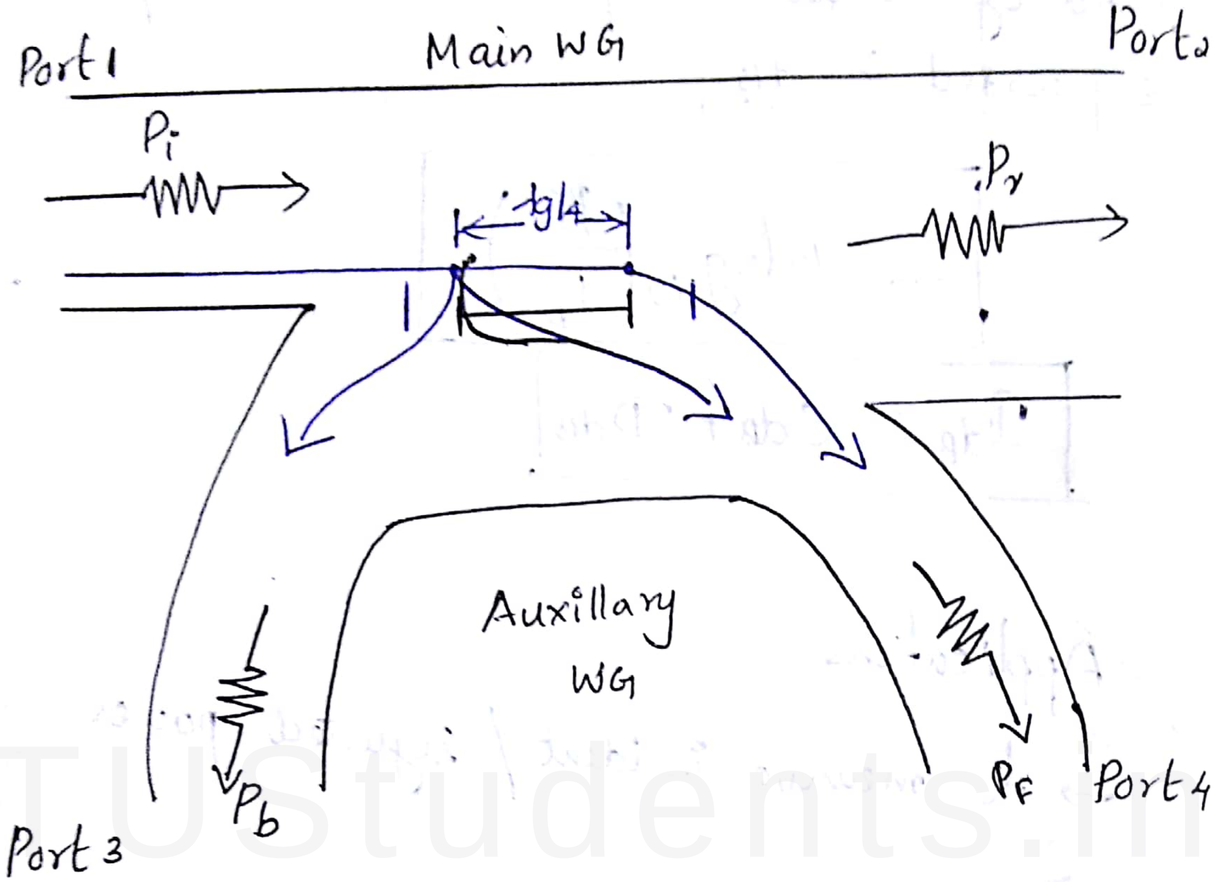
Applications.

→ to measure incident / reflected power.

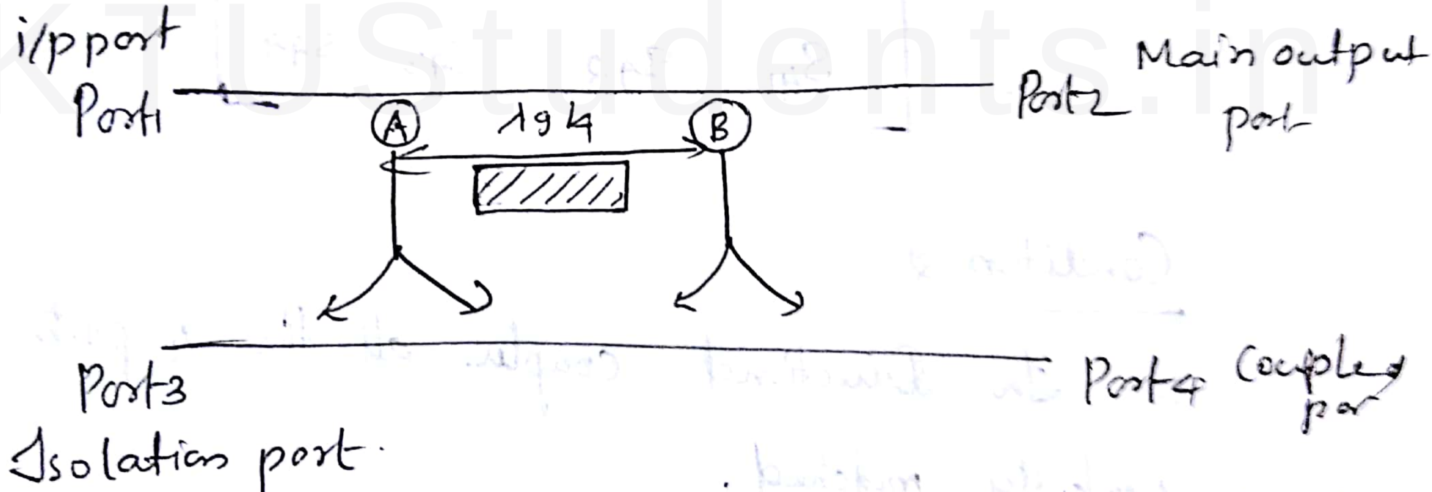
→

→ To provide signal path to a reciever etc.

TWO HOLE DIRECTIONAL COUPLER



It consists of two waveguides, the main waveguide and an auxillary waveguide with two tiny holes common b/w them. The two holes are at a distance of $\lambda g/4$, where λg is the guide wavelength. At the position of 2nd hole, the two leakages out of holes 1 and 2 are in phase & hence they add up contributing the forward power, P_F. At the position of 1st hole, the two leakages are out of phase by 180° and they cancel each other making the backward power, $P_B = 0$ ideally. The magnitude of the power coming out of two hole depends upon the dimension of hole. The degree of coupling is determined by the size & location of the holes in the waveguide walls.



It is used for measuring the unidirectional power & how much power is coupled.

S Matrix of Directional Coupler:

Condition 1

Directional coupler is a 4 port device.

$\therefore$ S Matrix is a 4x4 square matrix.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

Condition 2

In Directional coupler all the 4 ports are perfectly matched.

ie $S_{11} = S_{22} = S_{33} = S_{44} = 0$.

Condition 3

By symmetric property

$S_{12} = S_{21}$, $S_{13} = S_{31}$, $S_{14} = S_{41}$

$S_{32} = S_{23}$, $S_{42} = S_{24}$, $S_{43} = S_{34}$.

Condition 4

Ideally back power = 0. ($P_b = 0$)

ie $S_{13} = S_{31} = 0$

Also no coupling b/w ports (2) and (4)

ie $S_{24} = S_{42} = 0$.

$$\therefore [S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

Condition 5

By unitary property.

$[S][S^*] = [I]$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\underline{R_1C_1} \rightarrow$

$$|S_{12}|^2 + |S_{14}|^2 = 1 \quad \text{------------} \quad (1)$$

$\underline{R_2C_2} \rightarrow$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \quad \text{------------} \quad (2)$$

$\underline{R_3C_3} \rightarrow$

$$|S_{23}|^2 + |S_{34}|^2 = 1 \quad \text{------------} \quad (3)$$

$\underline{R_1C_3} \rightarrow$

$$S_{12} \cdot S_{23}^* + S_{14} \cdot S_{34}^* = 0 \quad \text{------------} \quad (4)$$

Equating (1) and (2).

$$|S_{12}|^2 + |S_{14}|^2 = |S_{12}|^2 + |S_{23}|^2$$

$$\therefore S_{14} = S_{23} \quad \text{------------} \quad (5)$$

Equating (2) and (3)

$$|S_{12}|^2 + |S_{23}|^2 = |S_{23}|^2 + |S_{34}|^2$$

$$\therefore S_{12} = S_{34} \quad \text{------------} \quad (6)$$

Let S_{12} be real & +ve

$\therefore S_{12} = S_{34} = S_{43} = p$ —— (7)

Sub in eqn: (4).

$p \cdot S_{23}^* + S_{14} \cdot p^* = 0$. $p^* = p$

$p S_{23}^* + S_{23} \cdot p = 0$ $[\because S_{14} = S_{23}]$ Since p is +ve

$p(S_{23} + S_{23}^*) = 0$.

$p \neq 0$

ie $S_{23} + S_{23}^* = 0$

$S_{23} = -S_{23}^*$

If $S_{23} = jq$

then $S_{23}^* = -jq$ —— (8)

Sub in eq: (2)

$|S_{12}|^2 + |S_{23}|^2 = 1$

$p^2 + |jq|^2 = 1$

$p^2 + q^2 = 1$ —— (9).

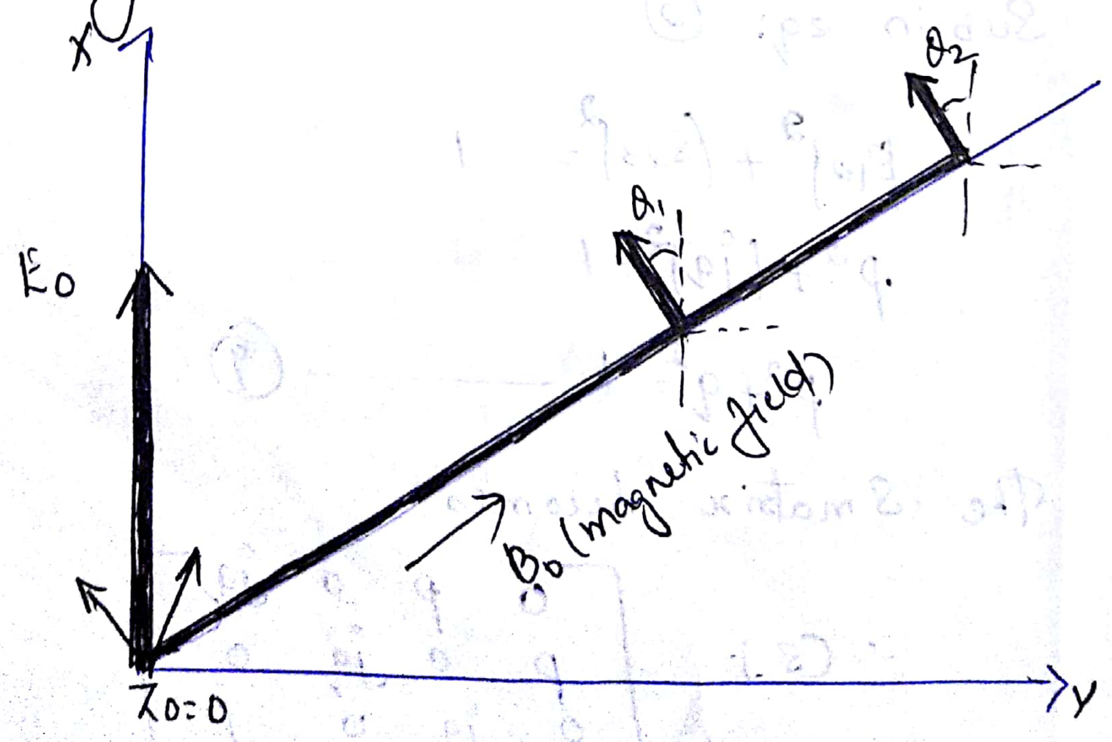
The S matrix becomes

$$\therefore [S] = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$$ //

FERRITE DEVICES.

Ferrites are non metallic materials with resistivities 10^{14} times greater than metals. They are oxide based compounds having a general composition of the form $MeO\,Fe_2O_3$. ie a mixture of metal oxide with ferric oxide. They have some characteristics of ~~ceramic~~ ceramic insulator. Ferrites have atoms with large no: of spinning electrons resulting in strong magnetic properties.

Faraday Rotation.



Consider an infinite lossless medium. A plane TEM wave, ie linearly polarised is made to propogate through ferrite in z direction. The plane of polarisation of this wave will rotate with distance. This phenomenon is known as Faraday Rotation.

$$\boxed{\text{Angle of rotation}, \theta = \frac{l}{2}(\beta_+ - \beta_-)}$$

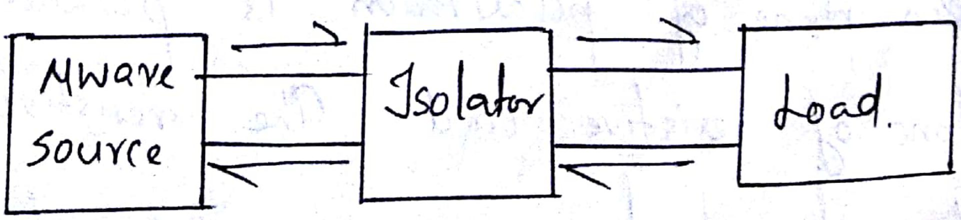
where 'l' is the length of ferrite rod.

$\beta_+^{\circ} \Rightarrow$ phase shift of right circularly polarised wave wrto some reference.

$\beta_-^{\circ} \Rightarrow$ phase shift of left circularly polarised wave wrto some reference.

Devices using Faraday Rotation.

① Isolator.

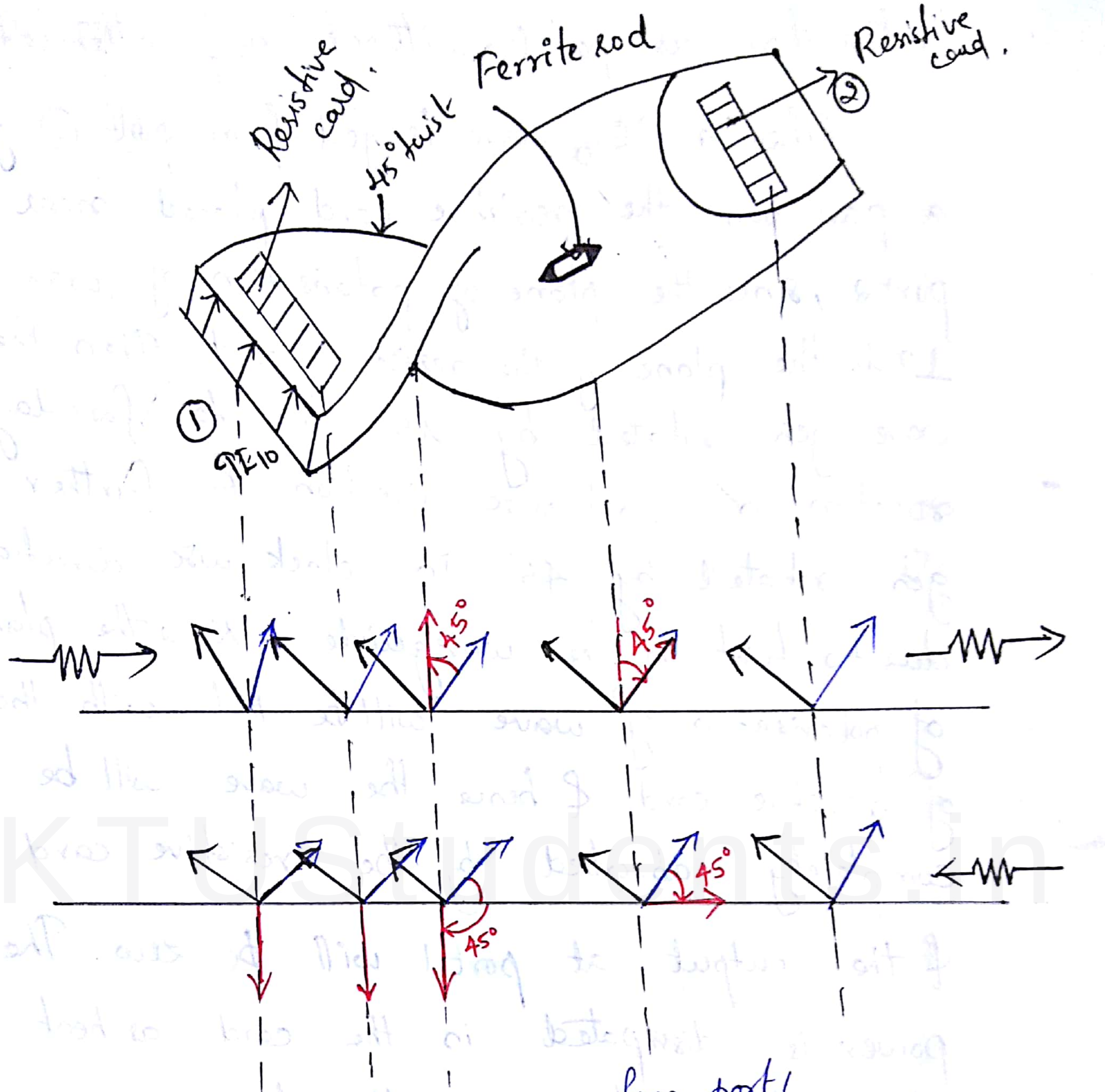


An Isolator is a two port device which provides very small amount of attenuation for transmission from port 1 to port 2, but provides maximum attenuation for transmission from port 2 to port 1.

When isolator is inserted between the generator & load, the generator is coupled to load with zero attenuation & the reflection from load side is completely absorbed by the isolator without affecting the generator o/p.

Operation :

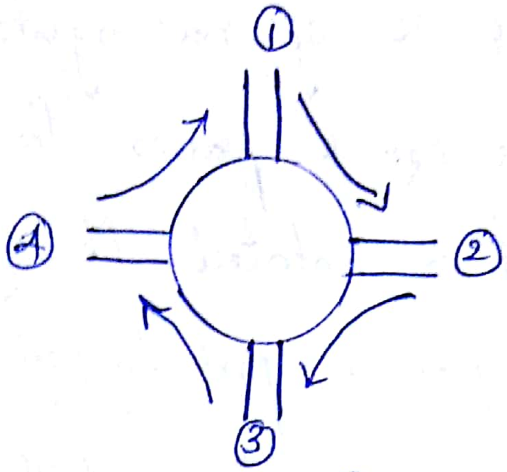
An isolator make use of 45° twist, 45° faraday rotation by ferrite rod and a resistive card is employed which absorbs any wave when plane of polarisation is parallel to the plane of resistive card. The resistive card does not absorb any wave whose plane of polarisation is $\perp^r$ to its own plane.

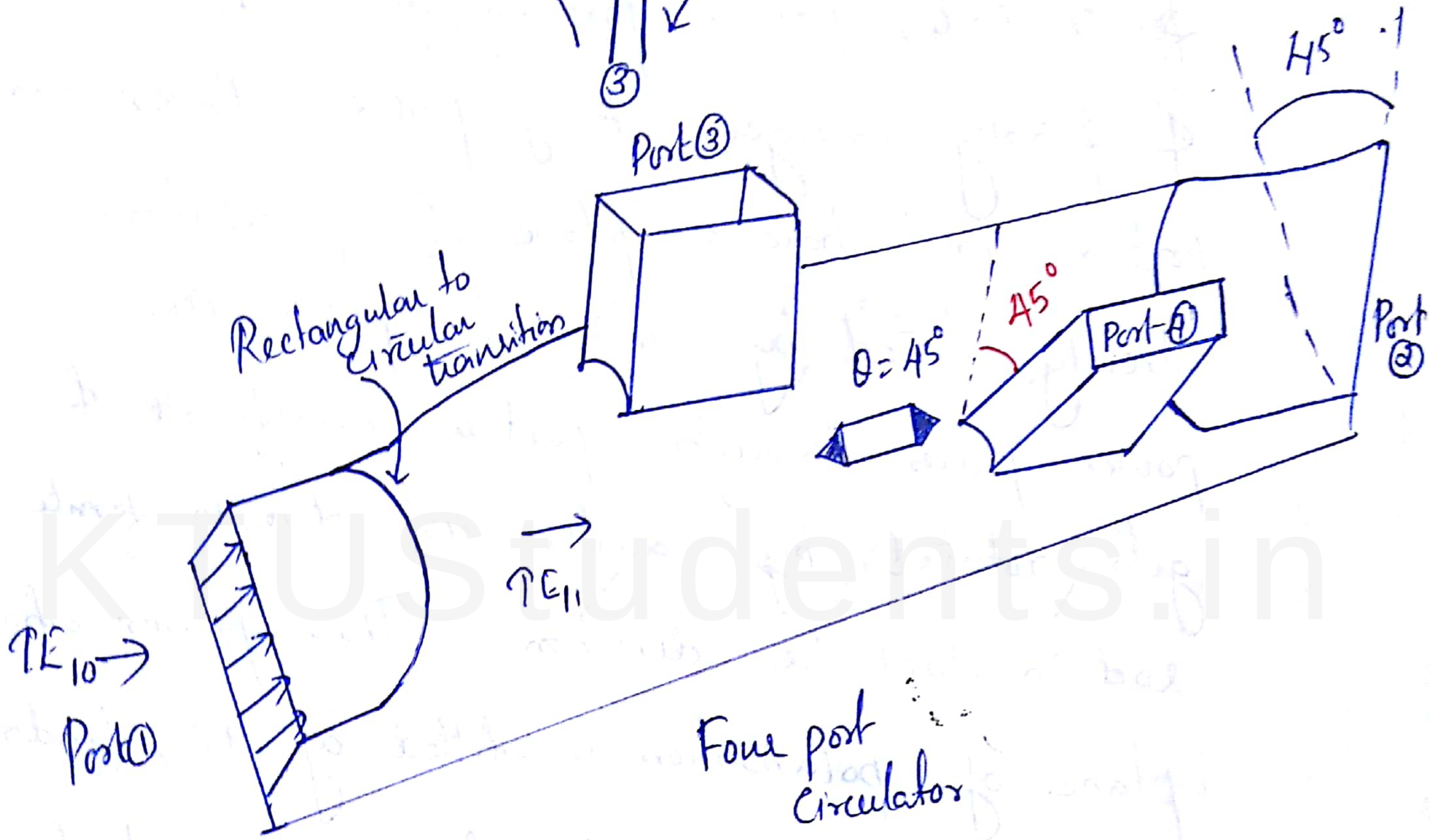


A TE_{10} wave passing from port 1 through the resistive card is not attenuated. After coming out of the card, the wave gets phase shifted by 45° because of twist in anticlockwise direction, and then by another 45° in clockwise direction, because of ferrite rod & hence come out of port 2 with the same polarisation as port 1 without any alternuation.

When a TE_{10} wave is fed from port ② get a pass from the resistive card placed near port 2, since the plane of polarisation of wave is ⊥r to the plane of the resistive card. Then the wave gets rotated by 45° due to faraday rotation in clock wise direction & further gets rotated by 45° in clock wise direction due to twist in the waveguide. Now the plane of polarisation of wave will be ‖el with that of resistive card & hence the wave will be completely absorbed by the resistive card & the output at port 1 will be zero. The power is dissipated in the card as heat. In practice 20 to 30dB attenuation is obtained for transmission from port 2 to port 1.

2. Circulator -





Circulator is a 4 port microwave device which has the peculiar property that each terminal is connected only to the next clock wise terminal. ie port 1 is connected to port 2 only & not to port 3 & port 4. Power entering port 1 is TE_{10} mode & is converted to TE_{11} mode, because of rectangular to circular transition. This power passes through port 3 unaffected & is rotated through 45° due to ferrite rod, passes port to port unaffected & finally emerges out of port 2. Power from port 2 will have a plane of polarisation already tilted by 45° w.r.to port 1. This power passes through port 4 unaffected & gets rotated by another 45° due to ferrite rod in clockwise direction. This power whose plane of polarisation is tilted by 90°, finds port 3 suitably alinged & emerges out of it.

Similarly port 3 is coupled only to port 4 & port 4 to port 1.

Applications

1) Circulator can be used as a duplexer for a radar antenna system.
2) They are used in tunnel diode & parametric amplifiers.
3) They are used as low power devices as they can handle low powers only.

WAVEGUIDE BENDS, CORNERS & TWISTS.

→ These components are used for changing the direction of the guide by a desired angle.

Types of bends.

① <u>H bend</u>

→ bend is in the direction of the wide dimension, the H lines are affected.

② E bend.

→ bend is in the direction of the narrower dimension, the E lines are affected.

The bending radius must be greater than or equal to $2\lambda_g$ to avoid SWR greater than 1.05 & mean length as long as possible

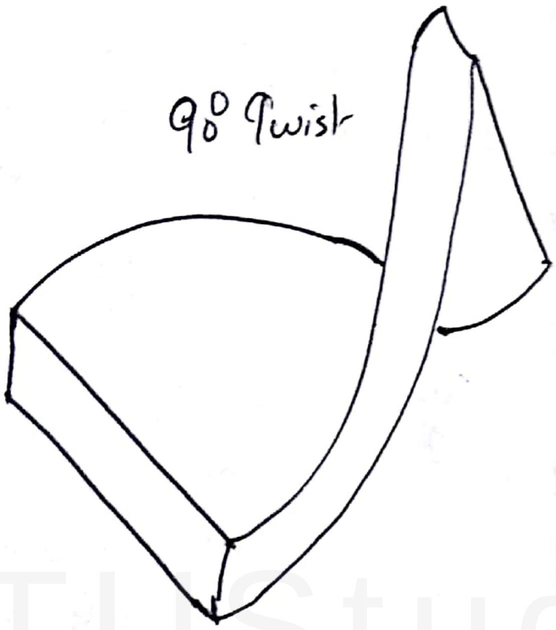
Sharp 90° bends create total reflections resulting in infinite SWR.

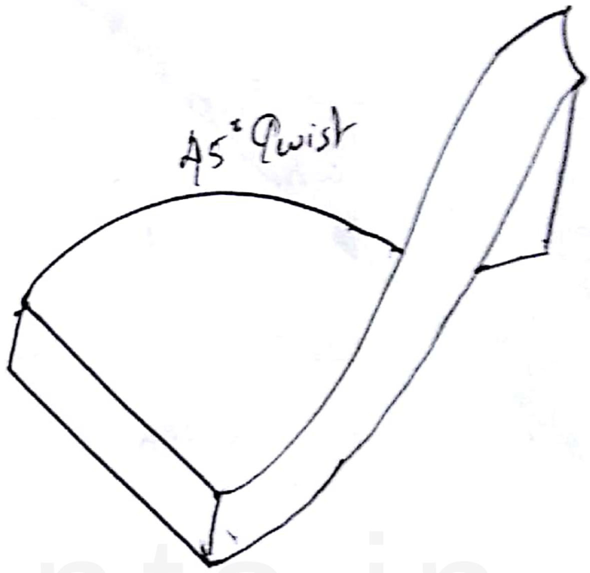
For lower frequencies corners are prefered.

A 90° bend is called Corner. Inorder to minimise the reflections, the mean length L must be an odd multiple of $\lambda/4$, so that reflected wave from both ends of WG are completely cancelled.

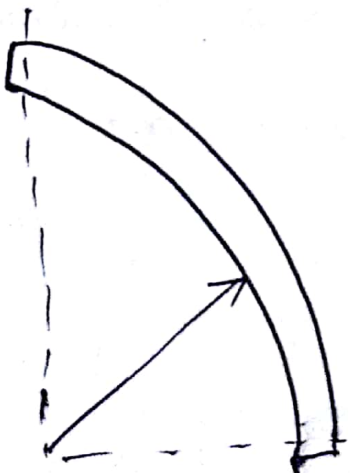
$L = (2n+1)\frac{\lambda_g}{4}$, $n = 0, 1, 2 \ldots$

Waveguide twists 90°, 45° are helpful in converting vertical to horizontal polarisation & vice versa.





H Bend



TRAVELLING WAVE TUBE

Klystrons are used for narrow band operation. For wide band operations twt and magnetrons are used.

Difference between TWT and Klystron

Klystron	TWT.
• Narrow band.	• Wide band.
• Utilize cavity resonators	• No cavity resonators.
• Electron beams are velocity modulated in a narrow gap.	• Interraction space in TWT is extended.
• Election beam travels and RF wave remains stationary.	• Distributed interraction b/w election beam & travelling wave (Both travels)
• Field is stationary & ē beam travels	• Ensure that both travelling with same velocity.
• Low power o/p	• High power o/p.
• Short life	• Long life.

TRAVELLING WAVE TUBE (TWT)

A travelling wave tube, TWT is a specialised vaccum tube used to amplify RF signals in the microwave range. It belongs to the category of linear beam tubes.

Two major categories of TWT are

① Helix TWT:- Here radio waves interract with electron beam while travelling down a wire helix which surrounds the beam.

→ wide BW

→ o/p power low.

② Coupled Cavity TWT :- Here radio waves interracts with beam in a series of cavity resonators through which the beam passes.

→ narrow BW

→ o/p power medium.

HELIX TWT.

- Used for broad band applications such as wide band ground & satellite communications s/ms & antijamming and radar systems.

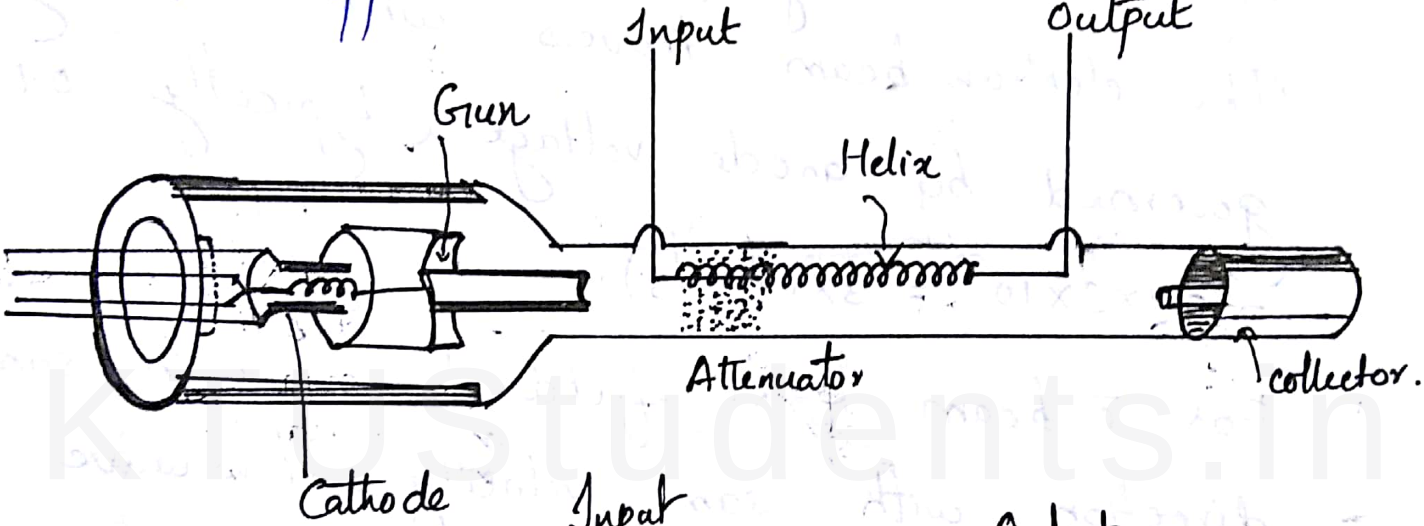
The RF field propogates with a velocity equal to velocity of light (3×10^8 m/s). The electron beam travels with a velocity governed by anode voltage (typically $0.1 (\frac{1}{10} v_c)$ $= 0.1 \times 3 \times 10^8 = 3 \times 10^7$ m/s).

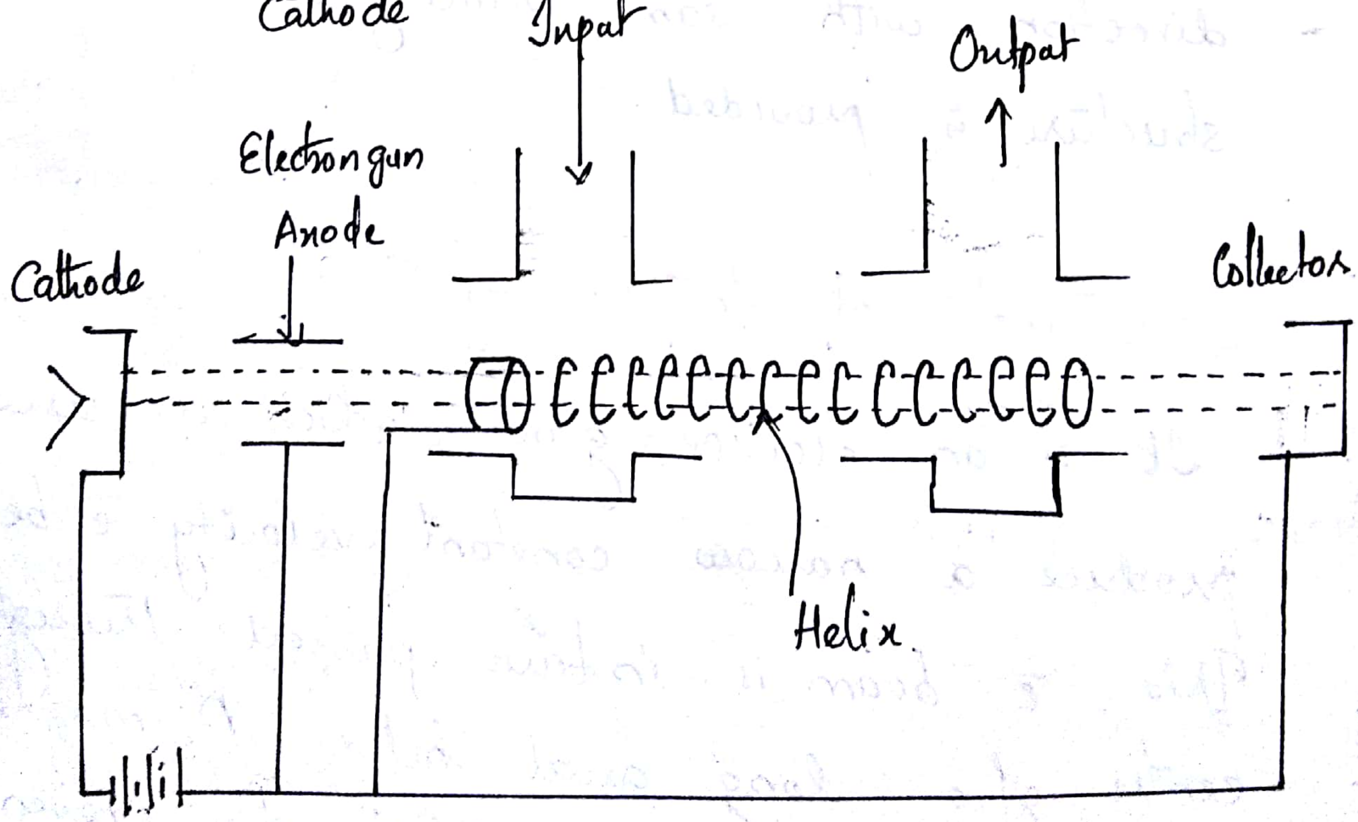
For ē beam & RF field to move in same direction with same velocity slow wave structure is provided.

Constructional features of TWT.

It is an electron gun which is used to produce a narrow constant velocity ē beam. This ē beam is in turn passed through the centre of a long axial helix. A magnetic focussing field is provided to prevent the beam from spreading & to guide it through the centre of helix. Helix is a loosely wound thin conducting helical wire which act as a slow wave structure.

The signal to be amplified is applied on one end of helix and amplified s/l appears at the other end of helix.





Operation of TWT.

The electron gun produces $\bar{e}$ beam with velocity (3×10^8 m/s). when the applied RF s/e travelling with a velocity 3×10^8 m/s propogates around the turns of the helix, it produces an EF at the centre of helix. The axial EF due to RF s/e travels with the velocity of light multiplied by $\frac{\text{helix pitch}}{\text{helix circumference}}$

When the velocity of $\bar{e}$ beam & RF field approximates then interaction takesplace b/w them. Thus the s/e wave grows and amplified o/p is obtained at the o/p of TWT. The axial phase velocity V_p is given by

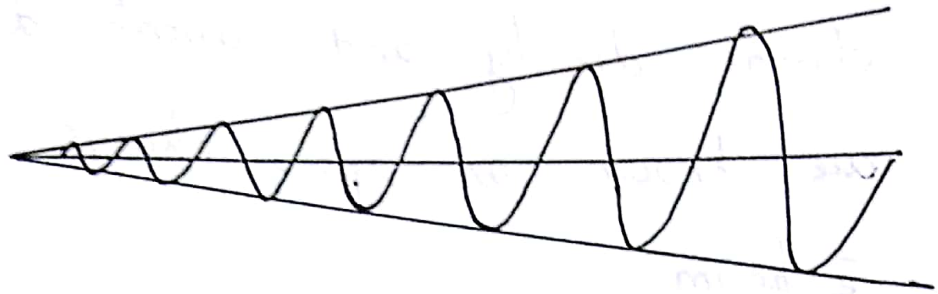
$$v_p = V_c \cdot \frac{(\text{pitch})}{2\pi r}.$$

where $r \Rightarrow$ radius of helix

$V_c = 3 \times 10^8$ m/s.

Helix provides least changes in Vp with frequency is preferred over other slow wave structures of TWT. When the axial field is zero, electron velocity is unaffected.

When the axial field is positive (+90°), the electrons get acclerated & when field is negative (-90°) the $\bar{e}$ gets deccelarated. Thus the $\bar{e}$ gets velocity modulated. As a result of energy transfer from the $\bar{e}$ to the RF field in phase with the RF field at the axis a second wave is induced on helix. This produces an axial EF that lags behind the orginal EF by 1/4. Bunching continues to take place. The $\bar{e}$ in the bunch encounter retarting field & deliver energy to the wave on helix. The o/p becomes larger than i/p and amplification results. Thus the energy ↑ in RF is a continous process.



Mathematical / Small signal Analysis of TWT.

The helix supports a slow wave with an axial phase velocity $v_p = V_c \times \sin\psi$.

where $\psi \Rightarrow$ helix angle. $= \tan^{-1}\left(\frac{P}{2\pi r}\right)$.

$P \Rightarrow$ pitch of helix.

$r \Rightarrow$ radius of helix.

$v_c \Rightarrow$ velocity of wave propogating along the helix.

The wave travelling along the helix has a longitudinal component of EF which causes velocity modulation of $\bar{e}$ beam.

In addition to velocity modulation, the beam will also experience a fluctuation in charge density and current density. These are known as space charge waves on $\bar{e}$ beam.

Let v_0, ρ_0, J_{z0} represents the static RF velocity, charge density and current density and v_1, ρ_1, J_{z1} represents their corresponding time varying equivalents.

$$v = v_0 + v_1 e^{j(\omega t - \gamma z)} \quad \text{——— (1)}$$

$$\rho = \rho_0 + \rho_1 e^{j(\omega t - \gamma z)} \quad \text{——— (2)}$$

$$J_z = J_{z0} + J_{z1} e^{j(\omega t - \gamma z)} \quad \text{——— (3)}$$

The electric field in z direction is given by

$$E_z = E_{z1} e^{j(\omega t - \gamma z)} \quad \text{——— (4)}$$

The current density is given by

$$J_z = \rho v.$$

$$J_z = \rho_0 + \rho_1 e^{j(\omega t - \gamma z)} \left[\left(v_0 + v_1 e^{j(\omega t - \gamma z)} \right) \right]$$

$$= \rho_0 v_0 + (\rho_1 v_0 + \rho_0 v_1) e^{j(\omega t - \gamma z)} + \rho_1 v_1 e^{2j(\omega t - \gamma z)} \quad \text{——— (5)}$$

The last term is very small and neglected.

$$J_z = \rho_0 v_0 + (\rho_1 v_0 + \rho_0 v_1) e^{j(\omega t - \gamma z)}$$

$$J_z = J_{z_0} + J_{z_1} e^{j(\omega t - \gamma z)} \quad \text{——— (6)}.$$

The beam current I_z is the product of J_z and beam area A.

$$I_z = I_0 + I_1 e^{j(\omega t - \gamma z)} \quad \text{——— (7)}$$

$$= J_z \times A.$$

$$I_z = \left[\rho_0 v_0 + (\rho_1 v_0 + \rho_0 v_1) e^{j(\omega t - \gamma z)}\right] A$$

$$\text{——— (8)}$$

Comparing eqns (7) and (8)

$$i_0 = \rho_0 v_0$$

$$i_1 = (\rho_1 v_0 + \rho_0 v_1) A$$

From current continuity eqn

$$\nabla \cdot J = \frac{-\partial \rho}{\partial t}.$$

$$\nabla \cdot \frac{i_1}{A} = \frac{-\partial \rho_1}{\partial t}$$

$$\frac{1}{A}\left[\frac{\partial}{\partial z}\left(\mathring{i}_1 e^{j(\omega t - \gamma z)}\right)\right] = -\frac{\partial}{\partial t}\left(\rho_1 e^{j(\omega t - \gamma z)}\right)$$

$$\frac{1}{A}\,\mathring{i}_1\left(e^{j(\omega t - \gamma z)}\right) \times -\gamma = -\rho_1\left(e^{j(\omega t - \gamma z)}\right) \times j\omega. \quad \text{———— (9)}$$

$$\frac{-\gamma \mathring{i}_1}{A} = -j\omega\rho_1$$

$$\frac{1}{-j} = +j$$

$$\frac{-1}{j} = \underline{\underline{-j}}$$

$$\therefore \rho_1 = \frac{-\gamma \mathring{i}_1}{-j\omega A}$$

$$\rho_1 = \frac{-\gamma j \mathring{i}_1}{\omega A} \quad \text{———— (10)}$$

Since $\mathring{i}_1 = (\rho_0 v_1 + \rho_1 v_0)\, A.$

$$= \left(\rho_0 v_1 - \frac{\gamma j \mathring{i}_1}{\omega A} \cdot v_0\right) A$$

$$\mathring{i}_1 = \rho_0 v_1 A - \frac{\gamma j \mathring{i}_1 v_0}{\omega}$$

$$\mathring{i}_1 + \frac{\gamma j \mathring{i}_1 v_0}{\omega} = \rho_0 v_1 A.$$

$$\mathring{i}_1\left[1 + \frac{\gamma j v_0}{\omega}\right] = \rho_0 v_1 A$$

$$\mathring{i}_1 = \frac{\omega \rho_0 v_1 A}{\omega + \gamma j v_0} \quad \text{———— (11)}$$

$\beta_e = \frac{\omega}{v_0} \Rightarrow$ phase constant of velocity modulated $\bar{e}$ beam & $v_0 \Rightarrow$ dc velocity of $\bar{e}$ beam.

$$\therefore i_1 = \frac{\beta_e v_0 \cdot \rho_0 v_1 A}{\beta_e v_0 + \gamma j v_0}$$

$$= \frac{\beta_e \cancel{v_0} \rho_0 v_1 A}{\cancel{v_0}(\beta_e + \gamma j)} = \frac{\beta_e \rho_0 v_1 A}{(\beta_e + j\gamma)}$$

Multiplying both num & denominator by j

$$i_1 = \frac{j\beta_e \rho_0 A v_1}{j\beta_e + j^2\gamma} = \frac{j\beta_e \rho_0 A}{(j\beta_e - \gamma)} \cdot v_1$$

$$\therefore i_1 = \frac{j\beta_e \rho_0 A}{(j\beta_e - \gamma)} \cdot v_1 \quad \text{———} (12)$$

(12) is an expression for convection current induced by axial EF.

We know that

$$\frac{dV(z)}{dt} = \frac{dV_1}{dt} = \frac{\partial V_1}{\partial t} + \frac{\partial V_1}{\partial z} \cdot \frac{\partial z}{\partial t}$$

$$\frac{\partial V_1}{\partial t} = \frac{\partial}{\partial t}\left(V_1 e^{j(\omega t - \gamma z)}\right)$$

$$= V_1 e^{j(\omega t - \gamma z)} \times +j\omega.$$

$$= \underline{\underline{+j\omega V_1}}$$

$$\frac{\partial V_1}{\partial z} = \frac{\partial}{\partial z}\left(V_1 e^{j(\omega t - \gamma z)}\right)$$

$$= V_1 e^{j(\omega t - \gamma z)} \times -\gamma$$

$$= \underline{\underline{-\gamma V_1}}$$

$$\therefore \frac{dv(z)}{dt} = j\omega V_1 - \gamma V_1 . v_0 .$$

z is the distance travelled by $\bar{e}$.

$$z = v_0 (t - t_0)$$

$$\therefore \frac{\partial z}{\partial t} = v_0 .$$

$$\therefore \frac{dv(z)}{dt} = j\omega V_1 - \gamma V_1 v_0$$

$$= (j\omega - \gamma v_0) V_1$$

From the eqn of motion of $\bar{e}$

$$\frac{dv(z)}{dt} = \eta E(z)$$

$$= \frac{\eta \partial V}{\partial z} = \eta \frac{\partial V_1}{\partial z}$$

$$-\eta \gamma V_1 = (j\omega - \gamma v_0) V_1$$

$$\therefore \quad v_1 = \frac{-\eta \gamma V_1}{(j\omega - \gamma v_0)} = \frac{-\eta \gamma V_1}{(j\beta_e v_0 - \gamma v_0)}$$

$$= \frac{-\eta \gamma V_1}{v_0 (j\beta_e - \gamma)}.$$

Electric Field produced by rf current in the beam.

This analysis is based on transmission line theory approach. Consider the equivalent ckt of helix. The helix is represented by lossless transmission line with series impedance Z per unit length & shunt admittance Y per unit length.

From transmission line equation, the propogation constant, $\gamma = j\sqrt{ZY}$ —— I

The characteristic impedance of the transmission line equivalent of helix is

$$Z_0 = \sqrt{Z/Y} \quad \text{—— II}$$

$$\gamma = j\sqrt{ZY}$$

$$\frac{\gamma}{j} = \sqrt{ZY}$$

$$-j\gamma = \sqrt{ZY}$$

$$Z_0^2 = Z/Y$$

$$\therefore Z = Z_0^2 Y \text{ ——— } (1)$$

$$-j\gamma_0 = \sqrt{Z_0^2 Y \cdot Y}$$

$$= \sqrt{Z_0^2 Y^2}$$

$$-j\gamma_0 = Z_0 Y$$

$$\therefore Y = \frac{-j\gamma_0}{Z_0} \text{ ——— } (2)$$

$$Z = Z_0^2 \times Y$$

$$= Z_0^2 \times \frac{-j\gamma_0}{Z_0} = -j\gamma_0 Z_0 \text{ ——— } (3)$$

$$\frac{\partial V_1}{\partial z} = -jZ I_1 \text{ ——— } \underline{III}$$

$$\frac{\partial I_1}{\partial z} = -jY V_1 + J \text{ ——— } \underline{IV}$$

where $J = \frac{-\partial i_1}{\partial z}$

$$I_1 = i_1 e^{j(\omega t - \gamma z)} \text{ ——— } (4)$$

$$V_1 = v_1 e^{j(\omega t - \gamma z)} \text{ ——— } (5)$$

Then

$$\frac{\partial V_1}{\partial z} = -\gamma V_1 \quad \text{------} \quad (6)$$

$||^{ly}$

$$\frac{\partial I_1}{\partial z} = -\gamma I_1 \quad \text{------} \quad (7)$$

$$-\gamma I_1 = -jYV_1 + \gamma i_1 \quad \text{------} \quad (8)$$

$$-\gamma V_1 = -jZ I_1 \quad \text{------} \quad (9)$$

$$I_1 = \frac{-\gamma V_1}{-jZ} = \frac{-j\gamma V_1}{Z} \quad \text{------} \quad (10)$$

Sub (10) in (8)

$$-\gamma\left(\frac{-j\gamma V_1}{Z}\right) = -jYV_1 + \gamma i_1$$

$$\frac{j\gamma^2 V_1}{Z} = -j\cancel{Y}V_1 + \gamma i_1$$

$$j\gamma^2 V_1 = -jZYV_1 + Z\gamma i_1$$

$$j\gamma^2 V_1 + jZYV_1 = Z\gamma i_1$$

$$(\gamma^2 + ZY)V_1 = -j\gamma Z i_1$$

$$\left(\gamma_{+}^{2} - j\gamma_0 Z_0 \gamma - \frac{j\gamma_0}{Z_0}\right) V_1 = -j\gamma Z_{f1} i_1$$

$$\therefore V_1 = \frac{-j\gamma Z i_1}{(\gamma^2 - \gamma_0^2)}$$

$$= \frac{-j\gamma \cdot -j\gamma_0 Z_0 i_1}{(\gamma^2 - \gamma_0^2)}$$

$$= \frac{+j^2 \gamma \gamma_0 Z_0 i_1}{(\gamma^2 - \gamma_0^2)}$$

$$V_1 = \frac{-\gamma \gamma_0 Z_0 i_1}{(\gamma^2 - \gamma_0^2)} \quad \text{——— (11)}$$

is the ckt eqn delating rf circuit voltage & rf beam current i_1.

The Gain parameter C and Gain.

$$V_1 = \frac{-\gamma \gamma_0 Z_0 i_1}{(\gamma^2 - \gamma_0^2)}$$

Sub $i_1 = \dfrac{j I_0 \beta_e \gamma V_1}{2V_0 (j\beta_e - \gamma)^2}$.

$$V_1 = \frac{j I_0 \beta_e \gamma V_1 x - \gamma \gamma_0 Z_0}{2V_0 (j\beta_e - \gamma)^2 (\gamma^2 - \gamma_0^2)}$$

$$\frac{V_1}{V_1} = \frac{j I_0 \beta_e \gamma \cancel{V_1} x - \gamma \gamma_0 Z_0}{2V_0 (j\beta_e - \gamma)^2 (\gamma^2 - \gamma_0^2)}$$

$$1 = \frac{-j (Z_0 I_0 / 2V_0) \gamma_0 \cdot \beta_e \cdot \gamma^2}{(j\beta_e - \gamma)^2 (\gamma^2 - \gamma_0^2)}$$

where $V_0 \Rightarrow$ dc voltage equivalent to the beam velocity v_0.

$Z_0 \Rightarrow$ characteristic impedence.

$I_0 \Rightarrow$ beam current.

$$(\gamma^2 - \gamma_0^2)(j\beta_e - \gamma)^2 = -j (Z_0 I_0 / 2V_0) \gamma_0 \cdot \beta_e \cdot \gamma^2.$$

$$-(\gamma_0^2 - \gamma^2)(j\beta_e - \gamma)^2 = \frac{-j \gamma^2 \gamma_0 \cdot \beta_e \cdot Z_0 I_0}{2V_0}$$

here γ has 4 roots. 4 waves are setup on the ckt in the presence of beam.

$$\delta_n = (-j)^{1/3} = e^{-j(\pi/2 + 2n\pi)/3} \quad , n = 0, 1, 2.$$

and the roots are

$$\delta_1 = \frac{\sqrt{3}}{2} - j/2$$

$$\delta_2 = -\frac{\sqrt{3}}{2} - j/2$$

$$\delta_3 = j$$

$$\delta_4 = -jc^2/4.$$

and the value of 4 propogation constants are

$$\gamma_1 = -\beta_e c\sqrt{3}/2 + j\beta_e(1 + c/2)$$

$$\gamma_2 = \beta_e c\sqrt{3}/2 + j\beta_e(1 + c/2)$$

$$\gamma_3 = j\beta_e(1 - c)$$

$$\gamma_4 = -j\beta_e(1 - c^{3/4})$$

$$C = \left(\frac{I_0 Z_0}{4V_0}\right)^{1/3}$$ is the travelling wave gain parameter.

Fourth root δ_4 produces a backward wave set up at o/p & travels towards i/p end with a velocity higher than the velocity of $\bar{e}$ beam.

Gain Relation

Total circuit voltage is the sum of of three forward voltages corresponding to three forward travelling waves. The i/p o/l voltage at the i/p end naturally split into three voltages that is

$$V(z) = 0$$

$$V(0) = V_1 + V_2 + V_3$$

$$V_1 = V_2 = V_3 = \frac{V(0)}{3}.$$

The amplitude of the growing wave is

$$V(z) = \frac{V(0)}{3} e^{(\sqrt{3}/2)\beta_e C z}$$

when $z = L$ the circuit length

$$V(L) = \frac{V(0)}{3} e^{(\sqrt{3}/2)\beta_e C L}$$

$$= \frac{V(0)}{3} e^{\sqrt{3}\pi NC}$$

where $\beta_e L = 2\pi N$

$N \Rightarrow$ ckt length in terms of electronic wave length.

$$N = \frac{L}{\lambda_e} \; ; \; \beta_e = \frac{2\pi}{\lambda_e}.$$

$\beta_e = \frac{2\pi}{\lambda}$

$N = \frac{L}{\lambda_e}$

The o/p power gain in decibels is

$$A_p = 10 \log \left| \frac{V(L)}{V(0)} \right|^2 = 10 \log \left[\frac{1}{3} e^{\sqrt{3}\pi NC} \right]^2$$

$$= -9.54 + 47.3\, NC \text{ dB}$$

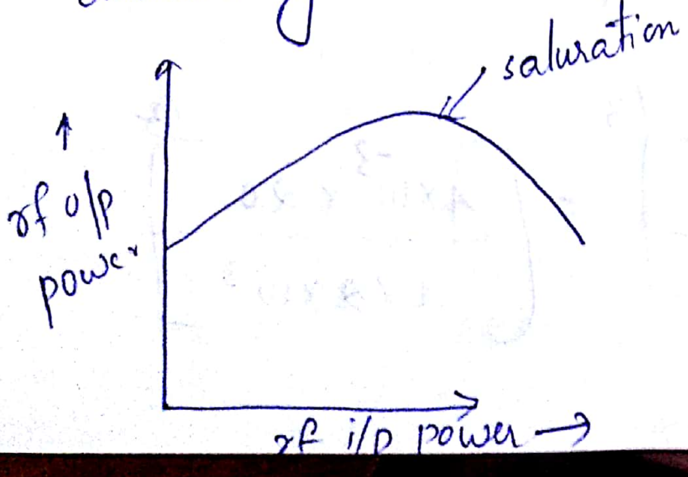
where NC is a numerical number.

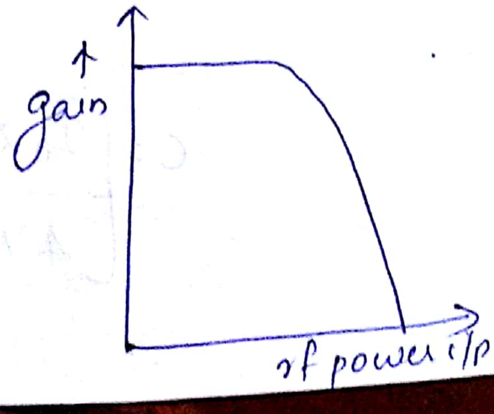
$A_p \propto N$ & C.

For larger ~~gain~~ C gain ↑

There is a initial loss of 9.54dB due to splitting of s/l into three waves.

With the increase in rf i/p power, rf o/p power attains a maximum value & finally starts decreasing.





The point at which rf o/p power attains a maximum value called saturation point & corresponding gain called saturation gain.

Q) A travelling wave tube has the following characteristics.

Beam voltage, $V_0 = 2KV$.

Beam current, $I_0 = 4mA$.

frequency, $f = 8GHz$.

Circuit length, $N = 50$.

Characteristic impedence, $Z_0 = 20\Omega$.

Determine

(a) The gain parameter C

(b) The power gain in decibels.

$$C = \left[\frac{I_0 Z_0}{4V_0}\right]^{1/3} = \left[\frac{4\times10^{-3}\times 20}{4\times2\times10^{3}}\right]^{1/3}$$

(b) The power gain in decibels.

$$A_p = -9.54 + 47.3NC \text{ dB}$$

$$= -9.54 + 47.3 \times 50 \times$$

Q) A helical TWT has diameter of 2mm with 50 turns per cm. Calculate axial phase velocity and the anode voltage at which TWT can be operated for useful gain.

$$V_p = V_c \times \frac{p}{\pi d} = \frac{3\times10^8 \times p}{\pi \times 2\times10^{-3}}$$

$$p = \frac{1}{50\times10^{-2}} = 2\times10^{-4} m$$

$$V_p = \frac{3\times10^8 \times 2\times10^{-4}}{\pi \times 2\times10^{-3}} = \underline{\underline{0.958\times10^7}}$$

$$eV_0 = \tfrac{1}{2} m v_p^2$$

$$V_0 = \frac{1}{2}\frac{m v_p^2}{e} = \frac{\tfrac{1}{2} \times 9.1\times10^{-31} \times (0.958\times10^7)^2}{1.6\times10^{-19}}$$

$$= \underline{\underline{25.92 kV}}$$