1. Show that the cost of the minimized multiple-output circuit constructed from Eqs. (1.6.4), (1.6.5), and (1.6.6) is lower than that of the circuit obtained by applying a single-function minimization method to the functions of Eqs. (1.6.1), (1.6.2), and (1.6.3), individually.
2. Minimize the following sets of functions using the Quine–McCluskey method.

(a) $f_1(x_1, x_2, x_3) = x_1x_2 + x'_2x'_4 + x_3x'_4 + x_4$
$f_2(x_1, x_2, x_3) = x'_2x'_4 + x_3x'_4 + x_1x_3 + x'_2x'_3x'_4$.

(b) $f_1(x_1, x_2, x_3, x_4) = x_1 + x_4 + x'_1x'_3x_4$
$f_2(x_1, x_2, x_3, x_4) = x'_2x'_4 + x_1x_2x_4 + x_2x'_3x_4$
$f_3(x_1, x_2, x_3, x_4) = x'_2x'_4 + x_1x_2x_3x_4 + x'_1x'_2x'_3$

(c) $f_1(x_1, x_2, x_3, x_4) = x_1 + x'_1x'_3x_4 + x_4$
$f_2(x_1, x_2, x_3, x_4) = x_1x_2x_4 + x'_2x'_4 + x_2x'_3x_4$
$f_3(x_1, x_2, x_3, x_4) = x_1x_2x_3x_4 + x'_2x'_4 + x'_1x'_2x'_3$

(d) $f_1(x_1, x_2, x_3, x_4) = \sum (2, 3, 4, 5, 6, 7, 11, 14) + \sum_d (9, 10, 13, 15)$
$f_2(x_1, x_2, x_3, x_4) = \sum (0, 1, 3, 4, 5, 7, 11, 14) + \sum_d (8, 10, 12, 13)$

(e) $f_1(x_1, x_2, x_3, x_4) = \prod (3, 4, 5, 7, 11, 13, 15) \cdot \prod_d (6, 8, 10, 12)$
$f_2(x_1, x_2, x_3, x_4) = \prod (2, 7, 9, 10, 11, 12, 14, 15) \cdot \prod_d (0, 4, 6, 8)$

(f) $f_1(x_1, \ldots, x_5) = \sum (0, 2, 4, 6, 9, 10, 13, 14, 15, 16, 17, 21, 25, 28, 30, 31)$
$f_2(x_1, \ldots, x_5) = \sum (0, 1, 3, 8, 9, 13, 14, 15, 16, 17, 19, 25, 27, 31)$

3. Minimize the set of three incompletely specified functions using the Quine-McCluskey method.

$$f_1(x_1, x_2, x_3, x_4) = \sum (1, 2, 3, 5, 7, 8, 9) + \sum_d (12, 14)$$
$$f_2(x_1, x_2, x_3, x_4) = \sum (0, 1, 2, 3, 4, 6, 8, 9) + \sum_d (10, 11)$$
$$f_3(x_1, x_2, x_3, x_4) = \sum (1, 3, 5, 7, 8, 9, 12, 13) + \sum_d (14, 15)$$

1.7 Iterative Consensus Method

In the Quine–McCluskey tabular method, if a function to be minimized is not in a canonical form, we must first convert it into a canonical form before we apply the minimization procedure. This expanded form usually contains a large number of terms that have to be handled in the process. The iterative consensus method for obtaining the prime implicants overcomes these disadvantages. It can begin with a function that is not in a canonical form. This method consists of the following two basic processes:

1. *Term generation.* This is based on the equality

$$XY + \bar{X}Z = XY + \bar{X}Z + YZ \tag{1.7.1}$$

Whenever two terms in an expression can be represented by XY and $\bar{X}Z$, a new term YZ (if it is not zero) is added to the expression. For example, suppose that in an expression we have two terms, $x'_2x'_3x_4$ and $x'_1x_2x'_3$. Let $X = x_2$. Then $Y = x_1x'_3$ and

Representation of a Term Each term of the function is represented by a row consisting of 1's, 0's, and dashes. A 1, 0, and dash (–) denote that the corresponding variable is in true form, complemented form, or missing, respectively. For example, the term $x_2'x_3'x_4$ of Eq. (1.7.3) is represented by –001.

All pairs of rows—original rows, and rows that may be added to the table—are systematically compared for consensus [Eq. (1.7.1)] and subsumption [Eq. (1.7.2)]. The rules for term generation and term elimination are described below.

Rule for Term Generation Two rows generate a consensus row if, *in one column only*, one row has a 1 and the other has a 0. The consensus row has a dash in that column. In the remaining columns, the entries are determined by the following table:

¢	1	0	–
1	1	x	1
0	x	0	0
–	1	0	–

The entries marked by x mean that they should not occur if the consensus row is nonzero. For example, in the example above, the consensus term of $x_2'x_3'x_4$ and $x_1'x_2x_3'$ can be found by applying this rule as

	x_1	x_2	x_3	x_4	
A	–	0	0	1	
B	0	1	0	–	
A ¢ B	0	–	0	1	$(x_1'x_3'x_4)$

The consensus row is added to the table only if it does not subsume a row already in the table.

Rule for Term Elimination A row subsumes another row if it has a 1 in every column in which other row has a 1, and if it has a 0 in every column in which the other row has a 0. Any row that subsumes another is eliminated. For example, the first three rows in the following table subsume the fourth row and therefore can all be eliminated.

	x_1	x_2	x_3	x_4	
A	0	1	0	–	$(x_1'x_2x_3')$
B	1	1	0	–	$(x_1x_2x_3')$
C	–	1	0	1	$(x_2x_3'x_4)$
D	–	1	0	–	(x_2x_3')

(see the second cycle of the minimization procedure in Example 1.7.1)

This tabular method for obtaining prime implicants of a function is illustrated by the following example.

Example 1.7.2

Minimize the function of Eq. (1.7.3) by use of the iterative consensus tabular method.

Solution:

		x_1	x_2	x_3	x_4	
Initial sum-of-products	A	–	0	0	1	
	B	0	1	0	–	
	C	1	1	0	–	
	D	1	1	1	–	
First cycle	A	–	0	0	1	
	B	0	1	0	–	
	C	1	1	0	–	
	D	1	1	1	–	
	$E = A ¢ B$	0	–	0	1	
	$F = A ¢ C$	1	–	0	1	
Second cycle	A	–	0	0	1	
	B	0	1	0	–	✓
	C	1	1	0	–	✓
	D	1	1	1	–	
	E	0	–	0	1	
	F	1	–	0	1	
	$G = B ¢ C$	–	1	0	–	
	$H = B ¢ F$	–	1	0	1	✓
Third cycle	A	–	0	0	1	
	D	1	1	1	–	✓
	E	0	–	0	1	
	F	1	–	0	1	
	G	–	1	0	–	
	$I = D ¢ F$	1	1	–	1	✓
	$J = D ¢ G$	1	1	–	–	
Fourth cycle	A	–	0	0	1	✓
	E	0	–	0	1	✓
	F	1	–	0	1	✓
	G	–	1	0	–	(x_2x_3')
	J	1	1	–	–	(x_1x_2)
	$K = E ¢ F$	–	–	0	1	$(x_3'x_4)$

A' – 0 0 1
B 0 1 0 –
E 0 – 0 1

C + A

ABC + ABD

BCD + ACD + A

Note that a check mark placed at the right of a row means that the row is deleted.

After the prime implicants of a function are obtained by the iterative consensus method, the selection of an optimum set of these prime implicants is then accomplished as described earlier in the chapter.

With some modification, the iterative consensus method can be applied to the obtaining of prime implicants of (1) product-of-sums functions and (2) functions of a multiple-output circuit.

Exercise 1.7

Find the prime implicants of the following functions. Use the iterative consensus tabular method.

1. $f(x_1, x_2, x_3, x_4) = x_1x_2x_3' + x_1'x_2x_3 + x_1x_2'x_3' + x_1x_3x_4$

2. $f(x_1, x_2, x_3, x_4) = x_1'x_3'x_4 + x_1x_2'x_4 + x_1x_2x_3'x_4 + x_1'x_2'x_3x_4 + x_1x_2x_3'x_4'$

3. The set of functions of a multiple-output circuit:

$$f_1(x_1, x_2, x_3, x_4) = x_2x_3'x_4 + x_1'x_2'x_3' + x_2x_3x_4 + x_1'x_2x_3'x_4' + x_1x_2'x_3'x_4$$

$$f_2(x_1, x_2, x_3, x_4) = x_1'x_2x_4 + x_2'x_3x_4' + x_1x_2x_4 + x_1'x_2'x_3' + x_1x_2'x_3'x_4'$$

Combinational

1. Show that the cost of the minimized multiple-output circuit constructed from Eqs. (1.6.4), (1.6.5), and (1.6.6) is lower than that of the circuit obtained by applying a single-function minimization method to the functions of Eqs. (1.6.1), (1.6.2), and (1.6.3), individually.
2. Minimize the following sets of functions using the Quine–McCluskey method.

(a) $f_1(x_1, x_2, x_3) = x_1x_2 + x_2'x_4' + x_3x_4' + x_4$
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(b) $f_1(x_1, x_2, x_3, x_4) = x_1 + x_4 + x_1'x_3'x_4$
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(c) $f_1(x_1, x_2, x_3, x_4) = x_1 + x_1'x_3'x_4 + x_4$
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(d) $f_1(x_1, x_2, x_3, x_4) = \sum (2, 3, 4, 5, 6, 7, 11, 14) + \sum_d (9, 10, 13, 15)$
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