nd

$$r_0\left(\frac{d^2\phi_0}{dt^2}\right) + 2\left(\frac{dr_0}{dt}\right)\left(\frac{d\phi_0}{dt}\right) = 0 \tag{2.14}$$

Using standard mathematical procedures, we can develop an equation for the radius of the satellite's orbit, r_0, namely

$$r_0 = \frac{p}{1 + e\cos(\phi_0 - \theta_0)} \tag{2.15}$$

Where θ_0 is a constant and e is the eccentricity of an ellipse whose semilatus rectum p is given by

$$p = (h^2)/\mu \tag{2.16}$$

and h is magnitude of the orbital angular momentum of the satellite. That the equation of the orbit is an ellipse is Kepler's first law of planetary motion.

Kepler's Three Laws of Planetary Motion

Johannes Kepler (1571–1630) was a German astronomer and scientist who developed his three laws of planetary motion by careful observations of the behavior of the planets in the solar system over many years, with help from some detailed planetary observations by the Hungarian astronomer Tycho Brahe. Kepler's three laws are

1. The orbit of any smaller body about a larger body is always an ellipse, with the center of mass of the larger body as one of the two foci.
2. The orbit of the smaller body sweeps out equal areas in equal time (see Figure 2.5).

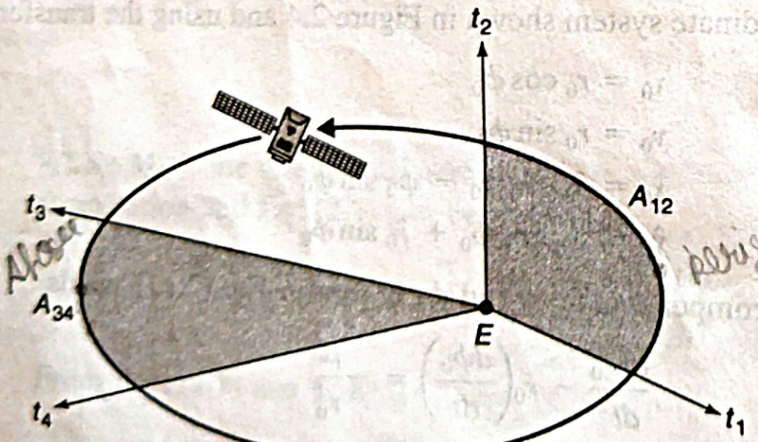

FIGURE 2.5 Illustration of Kepler's second law of planetary motion. A satellite is in orbit about the planet earth, E. The orbit is an ellipse with a relatively high eccentricity, that is, it is far from being circular. The figure shows two shaded portions of the elliptical plane in which the orbit moves, one is close to the earth and encloses the perigee while the other is far from the earth and encloses the apogee. The perigee is the point of closest approach to the earth while the apogee is the point in the orbit that is furthest from the earth. While close to perigee, the satellite moves in the orbit between times t_1 and t_2 and sweeps out an area denoted by A_{12}. While close to apogee, the satellite moves in the orbit between times t_3 and t_4 and sweeps out an area denoted by A_{34}. If $t_1 - t_2 = t_3 - t_4$ then $A_{12} = A_{34}$.

2

SIDEBAR

Kepler's laws were subsequently confirmed, about 50 years later, by Isaac Newton, who developed a mathematical model for the motion of the planets. Newton was one of the first people to make use of differential calculus, and with his understanding of gravity, was able to describe the motion of planets from a mathematical model based on his laws of motion and the concept of gravitational attraction. The work was published in the *Philosophiae Naturalis Principia Mathematica* in 1687. At that time, Latin was the international language of formally educated people, much in the way English has become the international language of e-mail and business today, so Newton's *Principia* was written in Latin.

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3. The square of the period of revolution of the smaller body about the larger body equals a constant multiplied by the third power of the semimajor axis of the orbital ellipse. That is, $T^2 = (4\pi^2 a^3)/\mu$ where T is the orbital period, a is the semimajor axis of the orbital ellipse, and μ is Kepler's constant. If the orbit is circular, then a becomes distance r, defined as before, and we have Eq. (2.6).

Describing the orbit of a satellite enables us to develop Kepler's second two laws.

Describing the Orbit of a Satellite

The quantity θ_0 in Eq. (2.15) serves to orient the ellipse with respect to the orbital plane axes x_0 and y_0. Now that we know that the orbit is an ellipse, we can always choose x_0 and y_0 so that θ_0 is zero. We will assume that this has been done for the rest of this discussion. This now gives the equation of the orbit as

$$r_0 = \frac{p}{1 + e\cos\phi_0} \tag{2.17}$$

The path of the satellite in the orbital plane is shown in Figure 2.6. The lengths a and b of the semimajor and semiminor axes are given by

$$a = p/(1 - e^2) \tag{2.18}$$

$$b = a(1 - e^2)^{1/2} \tag{2.19}$$

The point in the orbit where the satellite is closest to the earth is called the *perigee* and the point where the satellite is farthest from the earth is called the *apogee*. The perigee and apogee are always exactly opposite each other. To make θ_0 equal to zero, we have chosen the x_0 axis so that both the apogee and the perigee lie along it and the x_0 axis is therefore the major axis of the ellipse.

The differential area swept out by the vector $\mathbf{r_0}$ from the origin to the satellite in time dt is given by

$$dA = 0.5r_0^2\left(\frac{d\phi_0}{dt}\right)dt = 0.5h\,dt \tag{2.20}$$

Remembering that h is the magnitude of the orbital angular momentum of the satellite, the radius vector of the satellite can be seen to sweep out equal areas in equal times. This is Kepler's second law of planetary motion. By equating the area of the ellipse (πab) to the area swept out in one orbital revolution, we can derive an expression for the orbital period T as

$$T^2 = (4\pi^2 a^3)/\mu \quad \approx 2\pi\sqrt{\frac{a^3}{GM}} \tag{2.21}$$

centripetal force = centrifugal force

m = mass of body in kg
v = velocity of body in km/s
r = radius of circle in km
a = acceleration km/s²

$$\boxed{F_{out} = \frac{mv^2}{r}}$$ ✓

$F = ma$

$$F_{in} = m\frac{\mu}{r^2}$$

$$\boxed{\mu = GM}$$

$$\boxed{F_{in} = m\frac{GM}{r^2}}$$ ✓

$F_{in} = F_{out}$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}\frac{GM}{r^2}$$

$$\boxed{v^2 = \frac{GM}{r}} = \frac{\mu}{r}$$

$$\boxed{v^2 = \frac{\mu}{r}}$$

✓

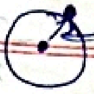
$$\because T = \frac{2\pi r}{v}$$

$$\therefore v = \frac{2\pi r}{T}$$

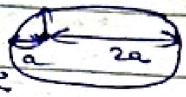
$$\left(\frac{2\pi r}{T}\right)^2 = \frac{\mu}{r} \Rightarrow \frac{4\pi^2 r^2}{T^2} = \frac{\mu}{r}$$

$$\Rightarrow T^2 = \frac{4\pi^2 r^3}{\mu}$$

$$\boxed{T^2 = r^3}$$

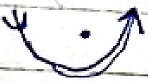
circular orbit  constant distance b/w center & object

2) Elliptical orbit



Periapse apoapse

③



Parabolic

Unbound orbits

④

Hyperbolic

Laws of orbital mechanics

we are more intrested in Elliptical orbits

① Kepler's 1st law:- All Plants follows the eleptical orbit around the sun.

② Kepler's 3rd law:- The square of the period of a planet is proportional to the cube of the mean distance from sun.

$T^2 = a^3$ ← semimajor axis

(Time) Period it takes for a planet to go around the sun is (in years)

AU → Astronomical ~~years~~ units.

Distance b/w earth & sun.

Important thing in orbital mechanics

~~Mars = a = 1.5236 AU [mean is 1.5236 AU for mean earth~~

~~P = 1.88 years.~~

~~687 days~~

~~More to revolve around the sun~~

Conservation of En

① E = PE + KE

$E = \frac{1}{2}mv^2 - \frac{GmM}{r}$

gravitational constant

mass of the object

mass of object in the center (Sun)

its velocity

distance (object in the center from place)

Mass of plant around

② Angular momentum:- $L = m r . V$ ← velocity

distant

mass of the object

distance b/w center & position of the object

E & L

They will always be constant

when the comic goes close to the sun it goes faster & when it is away it is slower.

$$E = \frac{1}{2} m v^2 - \frac{GmM}{r}$$

If r small

then $\frac{GmM}{r}$ is ↑es

so to remain E constant $\frac{1}{2} MV^2$ ~~has~~ to be ↑es M is constant so that means V will ↑es.

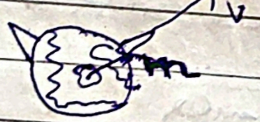
~~How to get into orbit?~~

Escape velocity:-

$$E = \frac{1}{2} MV^2 - \frac{GmM}{r}$$

How far

Initial velocity



$$E_{surface} = \left[\frac{1}{2} mv^2 - \frac{GmM_E}{R_E} \right] = E_\infty$$

Radius of earth

$$V_{ESC} = \sqrt{\frac{2GM_E}{R_E}}$$

$G = 6.67 \times 10^{-11}$ $M^3 kg^{-1}$

$$\boxed{V_{ESC} = 11.2 \, km/s}$$

$M_E = 5.972 \times 10^{24}$ kg

$R_E = 6371 \, KM$