3

ATTENUATION AND DISPERSION

In Chapter 2 we showed the structure of optical fibers and examined the concepts of how light propagates along a cylindrical dielectric optical waveguide. Here, we shall continue the discussion of optical fibers by answering two very important questions:

1. What are the loss or signal attenuation mechanisms in a fiber?
2. Why and to what degree do optical signals get distorted as they propagate along a fiber?

Signal attenuation (also known as *fiber loss* or *signal loss*) is one of the most important properties of an optical fiber because it largely determines the maximum unamplified or repeaterless separation between a transmitter and a receiver. Since amplifiers and repeaters are expensive to fabricate, install, and maintain, the degree of attenuation in a fiber has a large influence on system cost. Of equal importance is signal dispersion. The dispersion mechanisms in a fiber cause optical signal pulses to broaden as they travel along a fiber. If these pulses travel sufficiently far, they will eventually overlap with neighboring pulses, thereby creating errors in the receiver output. The signal dispersion mechanisms thus limit the information-carrying capacity of a fiber.

3.1 ATTENUATION

Attenuation of a light signal as it propagates along a fiber is an important consideration in the design of an optical communication system; the degree of attenuation plays a major role in determining the maximum transmission distance between a transmitter and a receiver or an in-line amplifier. The basic attenuation mechanisms in a fiber are absorption, scattering, and radiative losses of the optical energy.[1–5] Absorption is related to the fiber material, whereas scattering is associated both with the fiber material and with structural imperfections in the optical waveguide. Attenuation owing to radiative effects originates from perturbations (both microscopic and macroscopic) of the fiber geometry.

This section first discusses the units in which fiber losses are measured and then presents the physical phenomena giving rise to attenuation.

3.1.1 Attenuation Units

As light travels along a fiber, its power decreases exponentially with distance. If $P(0)$ is the optical power in a fiber at the origin (at $z = 0$), then the power $P(z)$ at a distance z farther down the fiber is

$$P(z) = P(0)e^{-\alpha_p z} \tag{3.1a}$$

where

$$\alpha_p = \frac{1}{z}\ln\left[\frac{P(0)}{P(z)}\right] \tag{3.1b}$$

is the fiber *attenuation coefficient* given in units of, for example, km^{-1}. Note that the units for $2z\alpha_p$ can also be designated by *nepers* (see App. D).

For simplicity in calculating optical signal attenuation in a fiber, the common procedure is to express the attenuation coefficient in units of *decibels per kilometer,* denoted by dB/km. Designating this parameter by α, we have

$$\alpha\,(\text{dB/km}) = \frac{10}{z}\log\left[\frac{P(0)}{P(z)}\right] = 4.343\ \alpha_p\,(\text{km}^{-1}) \tag{3.1c}$$

This parameter is generally referred to as the *fiber loss* or the *fiber attenuation.* It depends on several variables, as is shown in the following sections, and it is a function of the wavelength.

Example 3.1

An ideal fiber would have no loss so that $P_{out} = P_{in}$. This corresponds to a 0-dB/km attenuation, which, in practice, is impossible. An actual low-loss fiber may have a 3-dB/km average loss at 900 nm, for example. This means that the optical signal power would decrease by 50 percent over a 1-km length and would decrease by 75 percent (a 6-dB loss) over a 2-km length, since loss contributions expressed in decibels are additive.

Example 3.2

As Sec. 1.3 describes, optical powers are commonly expressed in units of *dBm,* which is the decibel power level referred to 1 mW. Consider a 30-km long optical fiber that has an attenuation of 0.4 dB/km at 1310 nm. Suppose we want to find the optical output power P_{out} if 200 μW of optical power is launched into the fiber. We first express the input power in dBm units:

$$P_{in}\,(\text{dBm}) = 10\log\left[\frac{P_{in}\,(\text{W})}{1\text{mW}}\right]$$

$$= 10\log\left[\frac{200\times10^{-6}\ \text{W}}{1\times10^{-3}\ \text{W}}\right] = -7.0\ \text{dBm}$$

From Eq. (3.1c) with $P(0) = P_{in}$ and $P(z) = P_{out}$ the output power level (in dBm) at $z = 30$ km is

$$P_{out}\,(\text{dBm}) = 10\log\left[\frac{P_{out}\,(\text{W})}{1\text{mW}}\right]$$

$$= 10\log\left[\frac{P_{in}\,(\text{W})}{1\text{mW}}\right] - \alpha z$$

$$= -7.0\ \text{dBm} - (0.4\ \text{dB/km})(30\ \text{km})$$

$$= -19.0\ \text{dBm}$$

In unit of watts, the output power is

$$P(30\ \text{km}) = 10^{-19.0/10}\ (1\ \text{mW}) = 12.6\times10^{-3}\ \text{mW}$$

$$= 12.6\ \mu\text{W}$$

Drill Problem 3.1

A 50-km long optical fiber has an attenuation of 0.25 dB/km at 1550 nm. If 100 μW of optical power is launched into the fiber, show that the power emerging at the fiber output is –32.5 dBm or 0.56 μW.

Drill Problem 3.2

An optical fiber loses 75 percent of the optical power traversing the fiber after 25 km. Using the left-hand side of Eq. (3.1c) with $z = 25$ km and $P(z) = 0.25P(0)$, show that the attenuation is $\alpha = 0.25$ dB/km.

3.1.2 Absorption

Absorption is caused by three different mechanisms:

1. Absorption by atomic defects in the glass composition.
2. Extrinsic absorption by impurity atoms in the glass material.
3. Intrinsic absorption by the basic constituent atoms of the fiber material.

Atomic defects are imperfections in the atomic structure of the fiber material. Examples of these defects include missing molecules, high-density clusters of atom groups, or oxygen defects in the glass structure. Usually, absorption losses arising from these defects are negligible compared with intrinsic and impurity absorption effects. However, they can be significant if the fiber is exposed to ionizing radiation, as might occur in a nuclear reactor environment, in medical radiation therapies, in space missions that pass through the earth's Van Allen belts, or in accelerator instrumentation.[6–9] In such applications, high radiation doses may be accumulated over several years.

Radiation damages a material by changing its internal structure. The damage effects depend on the energy of the ionizing particles or rays (e.g., electrons, neutrons, or gamma rays), the radiation flux (dose rate), and the fluence (particles per square centimeter). The total dose a material receives is expressed in units of rad(Si), which is a measure of radiation absorbed in bulk silicon. This unit is defined as

$$1 \text{ rad(Si)} = 100 \text{ erg/g} = 0.01 \text{ J/kg}$$

The basic response of a fiber to ionizing radiation is an increase in attenuation owing to the creation of atomic defects, or attenuation centers, that absorb optical energy. The higher the radiation level, the larger the attenuation, as Fig. 3.1*a* illustrates. However, the attenuation centers will relax or anneal out with time, as shown in Fig. 3.1*b*. The degree of the radiation effects depends on the dopant materials used in the fiber. Pure silica fibers or fibers with a low Ge doping and no other dopants have the lowest radiation-induced losses.

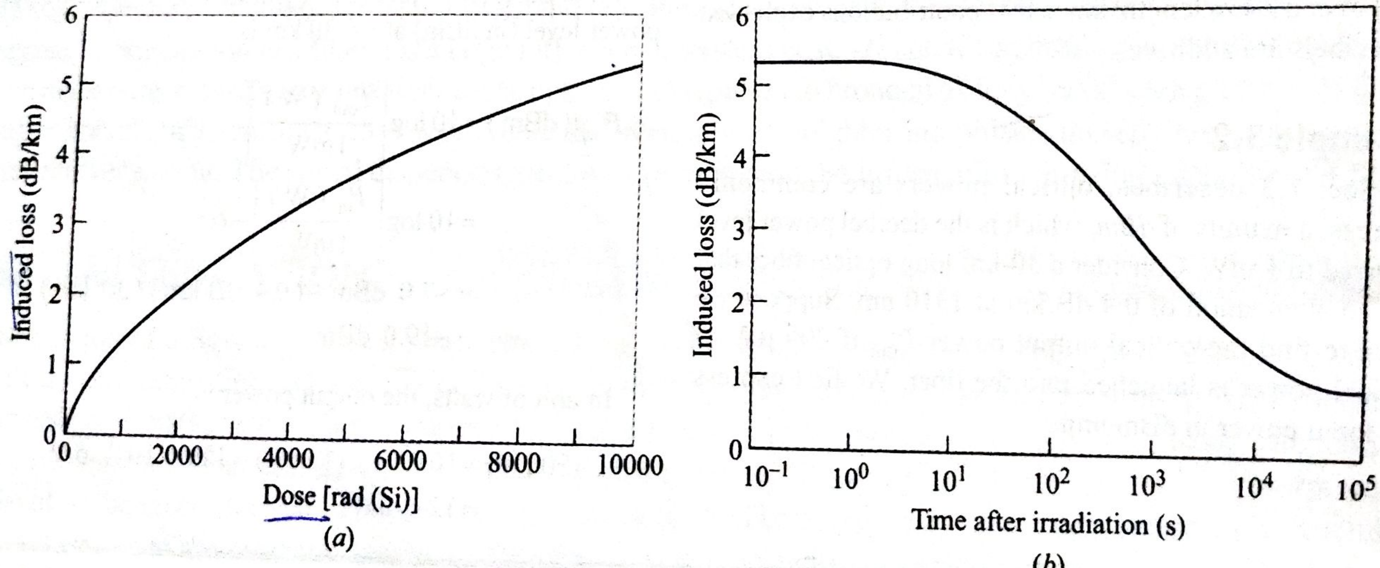

Fig. 3.1 *General trend of the effects of ionizing radiation on optical fiber attenuation. (a) Loss increase during steady irradiation to a total dose of 10^4 rad (SiO_2). (b) Subsequent recovery as a function of time after radiation has stopped. (Modified with permission from West et al.,[7] © 1994, IEEE)*

The dominant absorption factor in silica fibers is the presence of minute quantities of impurities in the fiber material. These impurities include OH^- (water) ions that are dissolved in the glass and transition metal ions such as iron, copper, chromium, and vanadium. Transition metal impurity levels were around 1 part

(3)

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per million (ppm) in glass fibers made in the 1970s, which resulted in losses ranging from 1 to 4 dB/km, as Table 3.1 shows. Impurity absorption losses occur either because of electron transitions between the energy levels within these ions or because of charge transitions between ions. The absorption peaks of the various transition metal impurities tend to be broad, and several peaks may overlap, which further broadens the absorption in a specific region. Modern vapor-phase fiber techniques for prodncing a fiber preform (see Sec. 2.9) have reduced the transition-metal impurity levels by several orders of magnitude. Such low impurity levels allow the fabrication of low-loss fibers.

Table 3.1 *Examples of absorption loss in silica glass at different wavelengths due to 1 ppm of water-ions and various transition-metal impurities*

Impurity	Loss due to 1 ppm of impurity (dB/km)	Absorption peak (nm)
Iron: Fe^{2+}	0.68	1100
Iron: Fe^{3+}	0.15	400
Copper: Cu^{2+}	1.1	850
Chromium: Cr^{2+}	1.6	625
Vanadium: V^{4+}	2.7	725
Water: OH^-	1.0	950
Water: OH^-	2.0	1240
Water: OH^-	4.0	1380

The presence of OH ion impurities in a fiber preform results mainly from the oxyhydrogen flame used in the hydrolysis reaction of the $SiCl_4$, $GeCl_4$, and $POCl_3$ starting materials. Water impurity concentrations of less than a few parts per billion (ppb) are required if the attenuation is to be less than 20 dB/km. The high levels of OH ions in early fibers resulted in large absorption peaks at 725, 950, 1240, and 1380 nm. Regions of low attention lie between these absorption peaks.

The peaks and valleys in the attenuation curves resulted in the designation of the various *transmission windows* shown in Fig. 3.2. By reducing the residual OH content of fibers to below 1 ppb, standard commercially available single-mode fibers have nominal attenuations of 0.4 dB/km at 1310 nm (in the O-band) and less than 0.25 dB/km at 1550 nm (in the C-band). Further elimination of water ions diminishes the absorption peak around 1440 nm and thus opens up the E-band for data transmission, as indicated by the dashed line in Fig. 3.2. Optical fibers that can be used in the E-band are known by names such as *low-water-peak* or *full-spectrum fibers*.

Intrinsic absorption is associated with the basic fiber material (e.g., pure SiO_2) and is the principal physical factor that defines the transparency window of a material over a specified spectral region. Intrinsic absorption sets the fundamental lower limit on absorption for any particular material; it is defined as the absorption that occurs when the material is in a perfect state with no density variations, impurities, or material inhomogeneities.

Intrinsic absorption results from electronic absorption bands in the ultraviolet region and from atomic vibration bands in the near-infrared region. The electronic absorption bands are associated with the band gaps of the amorphous glass materials. Absorption occurs when a photon interacts with an electron in the valence band and excites it to a higher energy level, as is described in Sec. 2.1. The ultraviolet edge of the electron absorption bands of both amorphous and crystalline materials follow the empirical relationship[1,3]

$$\alpha_{uv} = Ce^{E/E_0} \tag{3.2a}$$

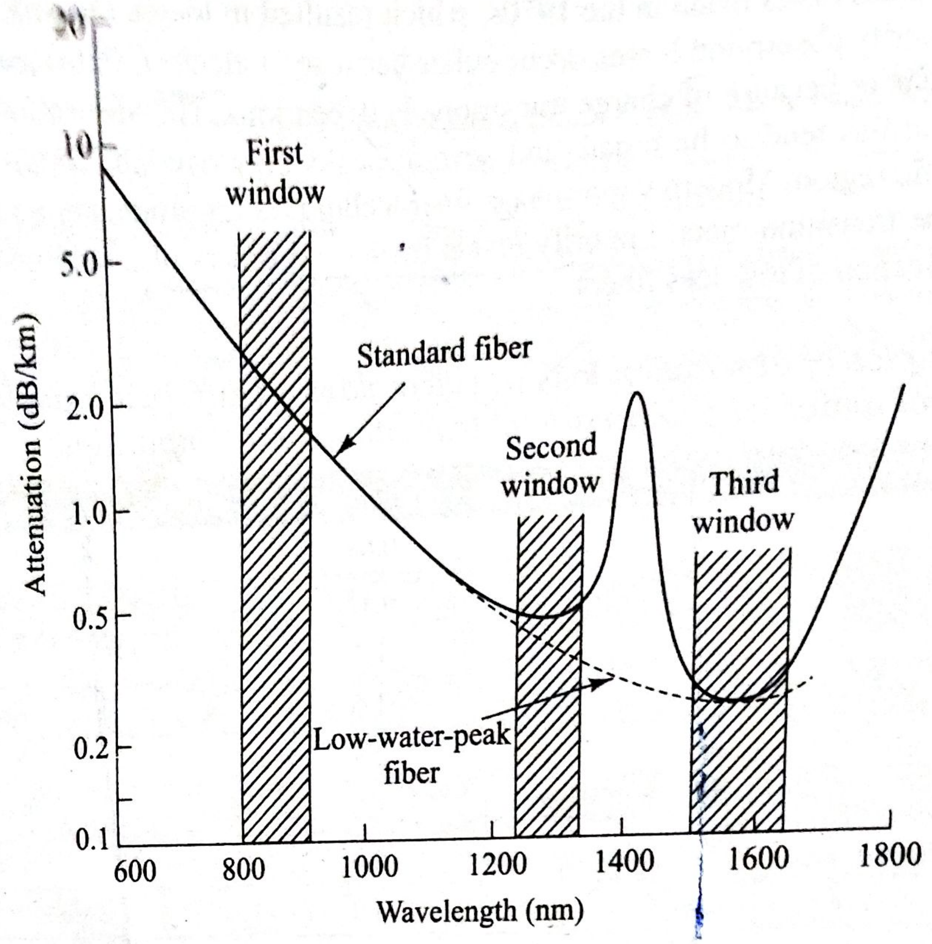

Fig. 3.2 *Optical fiber attenuation as a function of wavelength yields nominal values of 0.40 dB/km at 1310 nm and 0.25 dB/km at 1550 nm for standard single-mode fiber. Absorption by water molecules causes the attenuation peak around 1400 nm for standard fiber. The dashed curve is the attenuation for low-water-peak fiber*

which is known as Urbach's rule. Here, C and E_0 are empirical constants and E is the photon energy. The magnitude and characteristic exponential decay of the ultraviolet absorption are shown in Fig. 3.3. Since E is inversely proportional to the wavelength λ, ultraviolet absorption decays exponentially with increasing wavelength. In particular, the ultraviolet loss contribution in dB/km at any wavelength (given in μm) can be expressed empirically (derived from observation or experiment) as a function of the mole fraction x of GeO_2 as[10]

$$\alpha_{uv} = \frac{154.2x}{46.6x+60} \times 10^{-2} \exp\left(\frac{4.63}{\lambda}\right) \tag{3.2b}$$

Example 3.3

Consider two silica fibers that are doped with 6 percent and 18 percent mole fractions of GeO_2, respectively. Compare the ultraviolet absorptions at wavelengths of 0.7 μm and 1.3 μm.

Solution:

Using Eq. (3.2*b*) for the ultraviolet absorption, we have the following:

(*a*) For the fiber with $x = 0.06$ and $\lambda = 0.7$ μm

$$\alpha_{uv} = \frac{1.542(0.06)}{46.6(0.06)+60} \exp\left(\frac{4.63}{0.7}\right) = 1.10 \text{ dB/km}$$

(*b*) For the fiber with $x = 0.06$ and $\lambda = 1.3$ μm

$$\alpha_{uv} = \frac{1.542(0.06)}{46.6(0.06)+60} \exp\left(\frac{4.63}{1.3}\right) = 0.07 \text{ dB/km}$$

(*c*) For the fiber with $x = 0.18$ and $\lambda = 0.7$ μm

$$\alpha_{uv} = \frac{1.542(0.18)}{46.6(0.18)+60} \exp\left(\frac{4.63}{0.7}\right) = 3.03 \text{ dB/km}$$

(*d*) For the fiber with $x = 0.18$ and $\lambda = 1.3$ μm

$$\alpha_{uv} = \frac{1.542(0.18)}{46.6(0.18)+60} \exp\left(\frac{4.63}{1.3}\right) = 0.19 \text{ dB/km}$$

Drill Problem 3.3

A silica fiber is doped with a fifteen percent mole fraction of GeO_2. Compare the ultraviolet absorption at 860 and 1550 nm. Answer: 0.75 dB/km at 860 nm; 0.068 dB/km at 1550 nm.

As shown in Fig. 3.3, the ultraviolet loss is small compared with scattering loss in the near-infrared region.

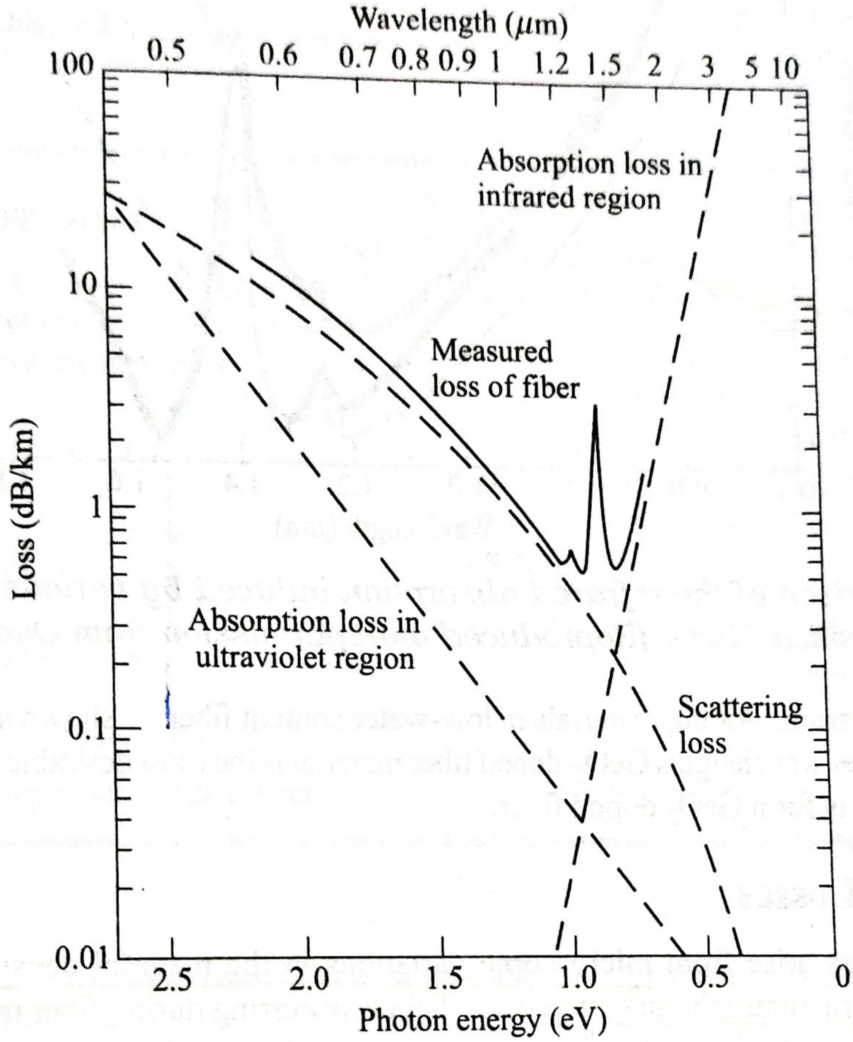

Fig. 3.3 *Optical fiber attenuation characteristics and their limiting mechanisms for a GeO_2-doped low-loss low-water-content silica fiber. (Reproduced with permission from Osanai et al[13])*

In the near-infrared region above 1.2 μm, the optical waveguide loss is predominantly determined by the presence of OH ions and the inherent infrared absorption of the constituent material. The inherent infrared absorption is associated with the characteristic vibration frequency of the particular chemical bond between the atoms of which the fiber is composed. An interaction between the vibrating bond and the electromagnetic field of the optical signal results in a transfer of energy from the field to the bond, thereby giving rise to absorption. This absorption is quite strong because of the many bonds present in the fiber. An empirical expression for the infrared absorption in dB/km for GeO_2-SiO_2 glass with λ given in μm is[10]

$$\alpha_{IR} = 7.81 \times 10^{11} \times \exp\left(\frac{-48.48}{\lambda}\right) \tag{3.3}$$

These mechanisms result in a wedge-shaped spectral-loss characteristic. Within this wedge, losses as low as 0.148 dB/km at 1.57 μm in a single-mode fiber have been measured.[11,12] A comparison[13] of the infrared

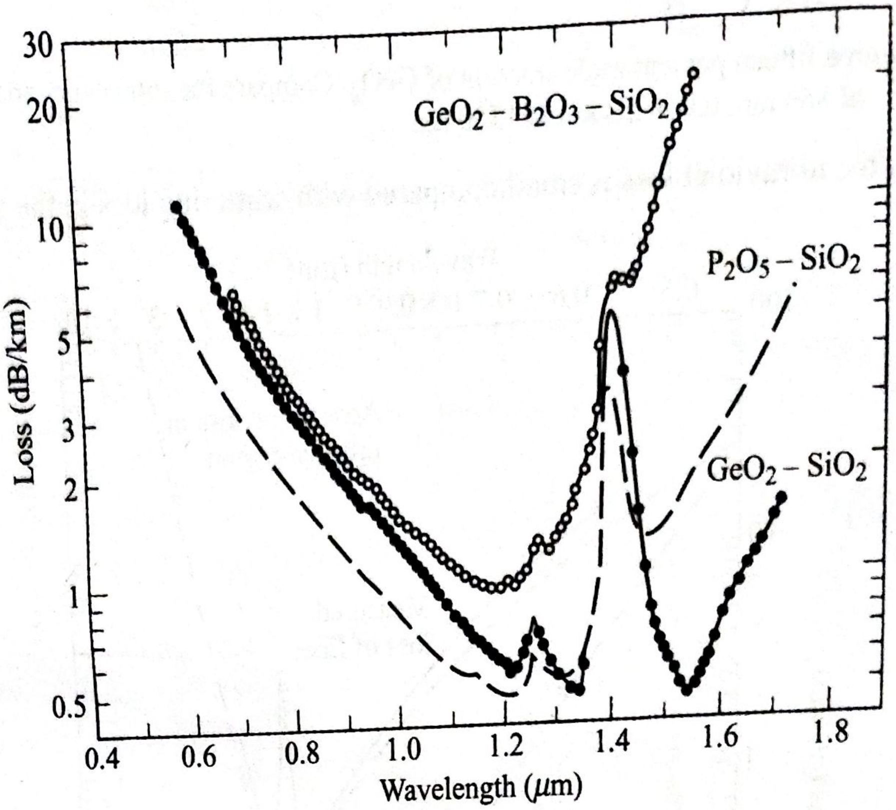

Fig. 3.4 *A comparison of the infrared absorption induced by various doping materials in low-loss silica fibers. (Reproduced with permission from Osanai et al[13])*

absorption induced by various doping materials in low-water content fibers is shown in Fig. 3.4. This indicates that for operation at longer wavelengths GeO_2-doped fiber material is the most desirable. Note that the absorption curve shown in Fig. 3.3 is for a GeO_2 doped fiber.

3.1.3 Scattering Losses

Scattering losses in glass arise from microscopic variations in the material density, from compositional fluctuations, and from structural inhomogeneities or defects occurring during fiber manufacture. As Sec. 2.7 describes, glass is composed of a randomly connected network of molecules. Such a structure naturally contains regions in which the molecular density is either higher or lower than the average density in the glass. In addition, since glass is made up of several oxides, such as SiO_2, GeO_2, and P_2O_5, compositional fluctuations can occur. These two effects give rise to refractive-index variations that occur within the glass over distances that are small compared with the wavelength. These index variations cause a Rayleigh-type scattering of the light. Rayleigh scattering in glass is the same phenomenon that scatters light from the sun in the atmosphere, thereby giving rise to a blue sky.

The expressions for scattering-induced attenuation are fairly complex owing to the random molecular nature and the various oxide constituents of glass. For single-component glass the scattering loss at a wavelength λ (given in μm) resulting from density fluctuations can be approximated by[3,14] (in base *e* units)

$$\alpha_{\text{scat}} = \frac{8\pi^3}{3\lambda^4}(n^2-1)^2\, k_B T_f \beta_T \tag{3.4a}$$

Here, n is the refractive index, k_B is Boltzmann's constant, β_T is the isothermal compressibility of the material, and the fictive temperature T_f is the temperature at which the density fluctuations are frozen into the glass as it solidifies (after having been drawn into a fiber). Alternatively, the relation[3,15] (in base *e* units)

$$\alpha_{\text{scat}} = \frac{8\pi^3}{3\lambda^4} n^8 p^2 k_B T_f \beta_T \tag{3.4b}$$

nepers

has been derived, where p is the photoelastic coefficient. A comparison of Eqs. (3.4*a*) and (3.4*b*) is given in Prob. 3.6. Note that Eqs. (3.4*a*) and (3.4*b*) are given in units of *nepers* (that is, base *e* units). As shown in Eq. (3.1), to change this to decibels for optical power attenuation calculations, multiply these equations by $10 \log e = 4.343$.

Example 3.4

For silica the fictive temperature T_f is 1400 K, the isothermal compressibility β_T is 6.8×10^{-12} cm²/dyn = 6.8×10^{-11} m²/N, and the photoelastic coefficient is 0.286. Estimate the scattering loss at a 1.30-μm wavelength where $n = 1.450$.

Solution:

Using Eq. (3.4*b*),

$$\begin{aligned}\alpha_{\text{scat}} &= \frac{8\pi^3}{3\lambda^4} n^8 p^2 k_B T_f \beta_T \\ &= \frac{8\pi^3}{3(1.3)^4}(1.45)^8\,(0.286)^2 \\ &\quad \times (1.38\times10^{-23})(1400)(6.8\times10^{-12}) \\ &= 6.08\times10^{-2} \text{ nepers/km} = 0.26 \text{ dB/km}\end{aligned}$$

Example 3.5

For pure silica glass an approximate equation for the Rayleigh scattering loss is given by

$$\alpha(\lambda) \simeq \alpha_0 \left(\frac{\lambda_0}{\lambda}\right)^4$$

where $\alpha_0 = 1.64$ dB/km at $\lambda_0 = 850$ nm. This formula predicts scattering losses of 0.291 dB/km at 1310 nm and 0.148 dB/km at 1550 nm.

Drill Problem 3.4

Using Eq. (3.4b) and the parameter values listed in Example 3.4, show that the estimated scattering loss in a silica fiber at 850 nm where $n = 1.455$ is 1.49 dB/km.

For multicomponent glasses the scattering at a wavelength λ (measured in μm) is given by[3]

$$\alpha = \frac{8\pi^3}{3\lambda^4}(\delta n^2)^2\,\delta V \tag{3.5}$$

where the square of the mean-square refractive-index fluctuation $(\delta n^2)^2$ over a volume of δV is

$$(\delta n^2)^2 = \left(\frac{\partial n^2}{\partial \rho}\right)^2 (\delta\rho)^2 + \sum_{i=1}^{m}\left(\frac{\partial n^2}{\partial C_i}\right)^2 (\delta C_i)^2 \tag{3.6}$$

Here, $\delta\rho$ is the density fluctuation and δC_i is the concentration fluctuation of the *i*th glass component. Their magnitudes must be determined from experimental scattering data. The factors $\partial n^2/\partial\rho$ and $\partial n^2/\partial C_i$ are the variations of the square of the index with respect to the density and the *i*th glass component, respectively.

Structural inhomogeneities and defects created during fiber fabrication can also cause scattering of light out of the fiber. These defects may be in the form of trapped gas bubbles, unreacted starting materials, and crystallized regions in the glass. In general, the preform manufacturing methods that have evolved have minimized these extrinsic effects to the point where scattering that results from them is negligible compared with the intrinsic Rayleigh scattering.

Since Rayleigh scattering follows a characteristic λ^{-4} dependence, it decreases dramatically with increasing wavelength, as is shown in Fig. 3.3. For wavelengths below about 1 μm it is the dominant loss mechanism in a fiber and gives the attenuation-versus-wavelength plots their characteristic downward trend with increasing wavelength. At wavelengths longer than 1 μm, infrared absorption effects tend to dominate optical signal attenuation.

Combining the infrared, ultraviolet, and scattering losses, we get the results shown in Fig. 3.5 for multimode fibers and Fig. 3.6 for single-mode fibers.[16] Both of these figures are for typical commercial-grade silica fibers. The losses of multimode fibers are generally higher than those of single-mode fibers. This is a result of higher dopant concentrations and the accompanying larger scattering loss due to greater compositional fluctuation in multimode fibers. In addition, multimode fibers are subject to higher-order-mode losses owing to perturbations at the core-cladding interface.

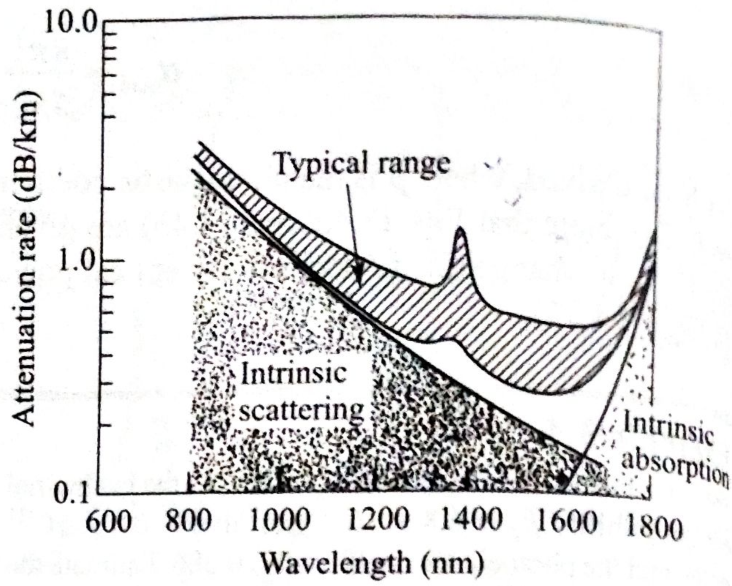

Fig. 3.5 *Typical spectral attenuation range for production-run graded-index multimode fibers. (Reproduced with permission from Keck,[16] © 1985, IEEE)*

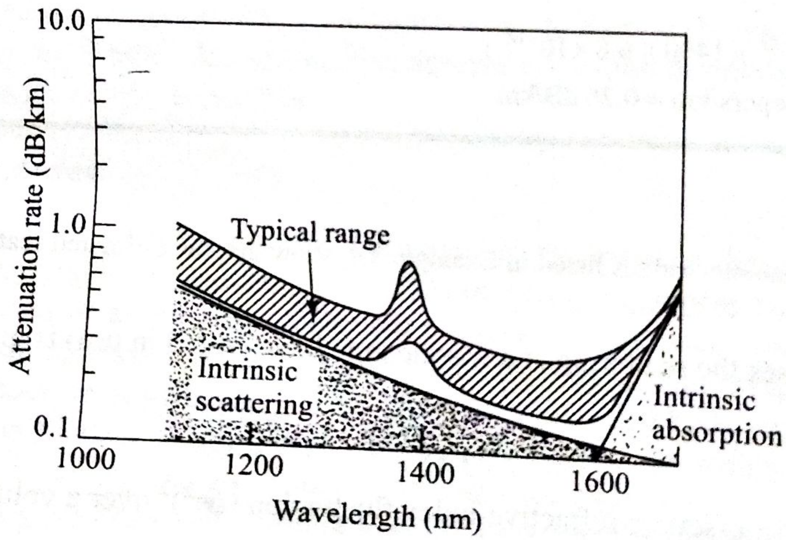

Fig. 3.6 *Typical spectral attenuation range for production-run single-mode fibers. (Reproduced with permission from Keck,[16] © 1985, IEEE)*

3.1.4 Bending Losses

Radiative losses occur whenever an optical fiber undergoes a bend of finite radius of curvature.[17-26] Fibers can be subject to two types of curvatures: (*a*) macroscopic bends having radii that are large compared with the fiber diameter, such as those that occur when a fiber cable turns a corner, and (*b*) random microscopic bends of the fiber axis that can arise when the fibers are incorporated into cables.

Let us first examine large-curvature radiation losses, which are known as *macrobending losses* or simply *bending losses*. For slight bends the excess loss is extremely small and is essentially unobservable. As the radius of curvature decreases, the loss increases exponentially until at a certain critical radius the curvature loss becomes observable. If the bend radius is made a bit smaller once this threshold point has been reached, the losses suddenly become extremely large.

Qualitatively, these curvature loss effects can be explained by examining the modal electric field distributions shown in Fig. 2.19. Recall that this figure shows that any bound core mode has an evanescent field tail in the cladding that decays exponentially as a function of distance from the core. Since this field tail moves along with the field in the core, part of the energy of a propagating mode travels in the fiber cladding. When a fiber is bent, the field tail on the far side of the center of curvature must move faster to keep up with the field in the core, as is shown in Fig. 3.7 for the lowest-order fiber mode. At a certain critical distance x_c from the center of the fiber, the field tail would have to move faster than the speed of light to keep up with the core field. Since this is not possible, the optical energy in the field tail beyond x_c radiates away.

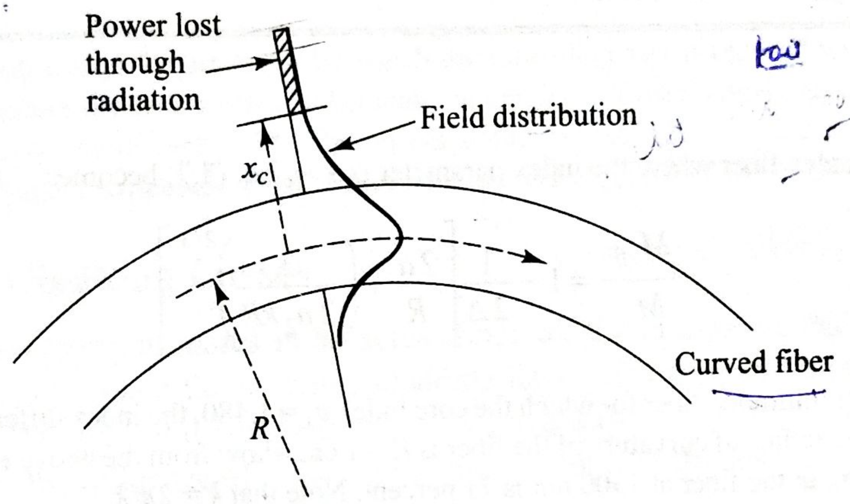

Fig. 3.7 *Sketch of the fundamental mode field in a curved optical waveguide. (Reproduced with permission from E.A.J. Marcatili and S. E. Miller,* Bell Sys. Tech. J., *vol. 48, p. 2161, Sept. 1969, © 1969, AT&T)*

The amount of optical radiation from a bent fiber depends on the field strength at x_c and on the radius of curvature R. Since higher-order modes are bound less tightly to the fiber core than lower-order modes, the higher-order modes will radiate out of the fiber first. Thus the total number of modes that can be supported by a curved fiber is less than in a straight fiber. The following expression[18] has been derived for the effective number of modes M_{eff} that are guided by a curved multimode fiber of radius a:

$$M_{\text{eff}} = M_\infty \left\{1 - \frac{\alpha+2}{2\alpha\Delta}\left[\frac{2a}{R} + \left(\frac{3}{2n_2 kR}\right)^{2/3}\right]\right\} \tag{3.7}$$

where α defines the graded-index profile, Δ is the core-cladding index difference, n_2 is the cladding refractive index, $k = 2\pi/\lambda$ is the wave propagation constant, and

$$M_\infty = \frac{\alpha}{\alpha+2}(n_1 ka)^2\,\Delta \tag{3.8}$$

is the total number of modes in a straight fiber [see Eq. (2.81)].

Example 3.6

Consider a graded-index multimode fiber for which the index profile $\alpha = 2.0$, the core index $n_1 = 1.480$, the core-cladding index difference $\Delta = 0.01$, and the core radius $a = 25$ μm. If the radius of curvature of the fiber is $R = 1.0$ cm, what percentage of the modes remain in the fiber at a 1300-nm wavelength?

Solution:

First, from Eq. (2.20), $n_2 = n_1(1 - \Delta) = 1.480(1 - 0.01) = 1.465$. Then given that $k = 2\pi/\lambda$, from Eq. (3.7) the percentage of modes at a given curvature R is

$$\frac{M_{\text{eff}}}{M_\infty} = 1 - \frac{\alpha + 2}{2\alpha\Delta}\left[\frac{2a}{R} + \left(\frac{3}{2n_2 kR}\right)^{2/3}\right]$$

$$= 1 - \frac{1}{.01}\left[\frac{2(25)}{10000} + \left(\frac{3(1.3)}{2(1.465)\,2\pi(10000)}\right)^{2/3}\right]$$

$$= 0.42$$

Thus 42 percent of the modes remain in this fiber at a 1.0-cm bend radius.

Drill Problem 3.5

(a) Show that for a step-index fiber where the index parameter $\alpha = \infty$, Eq. (3.7) becomes

$$\frac{M_{\text{eff}}}{M_\infty} = 1 - \frac{1}{2\Delta}\left[\frac{2a}{R} + \left(\frac{3}{2n_2 kR}\right)^{2/3}\right]$$

(b) Consider a step-index multimode fiber for which the core index $n_1 = 1.480$, the index difference $\Delta = 0.01$, and the core radius $a = 25$ μm. If the radius of curvature of the fiber is $R = 1$ cm, show from the above equation that the percentage of the modes remaining in the fiber at 1300 nm is 71 percent. Note that $k = 2\pi/\lambda$.

Another form of radiation loss in optical waveguide results from mode coupling caused by random microbends of the optical fiber.[27–30] *Microbends* are repetitive small-scale fluctuations in the radius of curvature of the fiber axis, as is illustrated in Fig. 3.8. They are caused either by nonuniformities in the manufacturing of the fiber or by nonuniform lateral pressures created during the cabling of the fiber. The latter effect is often referred to as *cabling* or *packaging losses.* An increase in attenuation results from microbending because the fiber curvature causes repetitive coupling of energy between the guided modes and the leaky or nonguided modes in the fiber.

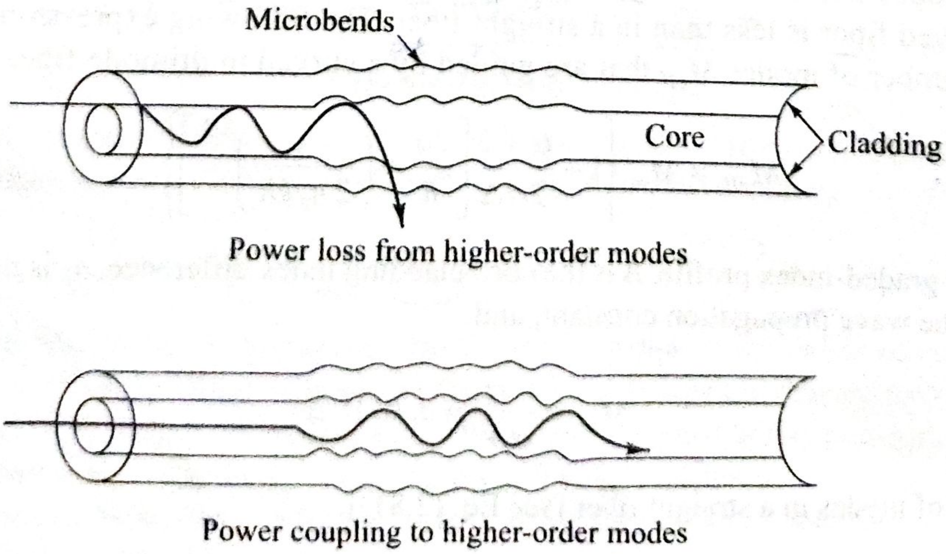

Fig. 3.8 *Small-scale fluctuations in the radius of curvature of the fiber axis lead to microbending losses. Microbends can shed higher-order modes and can cause power from low-order modes to couple to higher-order modes*

One method of minimizing microbending losses is by extruding a compressible jacket over the fiber. When external forces are applied to this configuration, the jacket will be deformed but the fiber will tend to stay relatively straight. For a multimode graded-index fiber having a core radius a, outer radius b (excluding the jacket), and index difference Δ, the microbending loss α_M of a jacketed fiber is reduced from that of an unjacketed fiber by a factor[31]

$$F(\alpha_M) = \left[1 + \pi\Delta^2\left(\frac{b}{a}\right)^4 \frac{E_f}{E_j}\right]^{-2} \tag{3.9}$$

Here, E_j and E_f are the Young's moduli of the jacket and fiber, respectively. The Young's modulus of common jacket materials ranges from 20 to 500 MPa. The Young's modulus of fused silica glass is about 65 GPa.

Drill Problem 3.6

Equation (3.9) gives an expression for the factor by which microbending loss is reduced when a compressible jacket is extruded over a fiber. Consider the case when a jacket material that has a Young's modulus $E_j = 58$ MPa is extruded over a glass fiber that has a Young's modulus $E_j = 64$ GPa and a cladding-to-core ratio $b/a = 2.0$. Show that when the refractive index difference $\Delta = 0.01$, the microbending loss reduction factor is $F(\alpha_M) = 0.0233 = 2.33\%$.

3.1.5 Core and Cladding Losses

Upon measuring the propagation losses in an actual fiber, all the dissipative and scattering losses will be manifested simultaneously. Since the core and cladding have different indices of refraction and therefore differ in composition, the core and cladding generally have different attenuation coefficients, denoted α_1 and α_2, respectively. If the influence of modal coupling is ignored,[32] the loss for a mode of order (ν, m) for a step-index waveguide is

$$\alpha_{\nu m} = \alpha_1 \frac{P_{\text{core}}}{P} + \alpha_2 \frac{P_{\text{clad}}}{P} \tag{3.10a}$$

where the fractional powers P_{core}/P and P_{clad}/P are shown in Fig. 2.27 for several low-order modes. Using Eq. (2.71), this can be written as

$$\alpha_{\nu m} = \alpha_1 + (\alpha_2 - \alpha_1)\frac{P_{\text{clad}}}{P} \tag{3.10b}$$

The total loss of the waveguide can be found by summing over all modes weighted by the fractional power in that mode.

Drill Problem 3.7

A step-index fiber has $V = 4.0$. (a) Using Fig. 2.27, estimate the fractional power P_{clad}/P traveling in the cladding for the four lowest-order LP modes. Ans. The losses are approximately 6, 16, 38, and 68 percent for the LP_{01}, LP_{11}, LP_{21}, and LP_{02}, modes, respectively. (b) Suppose the fiber is a glass-core glass-clad fiber having core and cladding attenuations of $\alpha_1 = 2.0$ and $\alpha_2 = 3.0$ dB/km, respectively. Using Eq. (3.10b), show that the attenuations for each of the four lowest order modes are 2.06, 2.16, 2.38, and 2.68 dB/km for the LP_{01}, LP_{11}, LP_{21}, and LP_{02}, modes, respectively.

For the case of a graded-index fiber the situation is much more complicated. In this case, both the attenuation coefficients and the modal power tend to be functions of the radial coordinate. At a distance r from the core axis the loss is[32]

$$\alpha(r) = \alpha_1 + (\alpha_2 - \alpha_1)\frac{n^2(0) - n^2(r)}{n^2(0) - n_2^2} \tag{3.11}$$

where α_1 and α_2 are the axial and cladding attenuation coefficients, respectively, and the n terms are defined by Eq. (2.78). The loss encountered by a given mode is then

$$\alpha_{gi} = \frac{\int_0^\infty \alpha(r)p(r)r\,dr}{\int_0^\infty p(r)r\,dr} \tag{3.12}$$

where $p(r)$ is the power density of that mode at r. The complexity of the multimode waveguide has prevented an experimental correlation with a model. However, it has generally been observed that the loss increases with increasing mode number.[26,33]

3.2 SIGNAL DISPERSION IN FIBERS

As shown in Fig. 3.9, an optical signal weakens from attenuation mechanisms and broadens due to dispersion effects as it travels along a fiber. Eventually these two factors will cause neighboring pulses to overlap. After a certain amount of overlap occurs, the receiver can no longer distinguish the individual adjacent pulses and errors arise when interpreting the received signal.

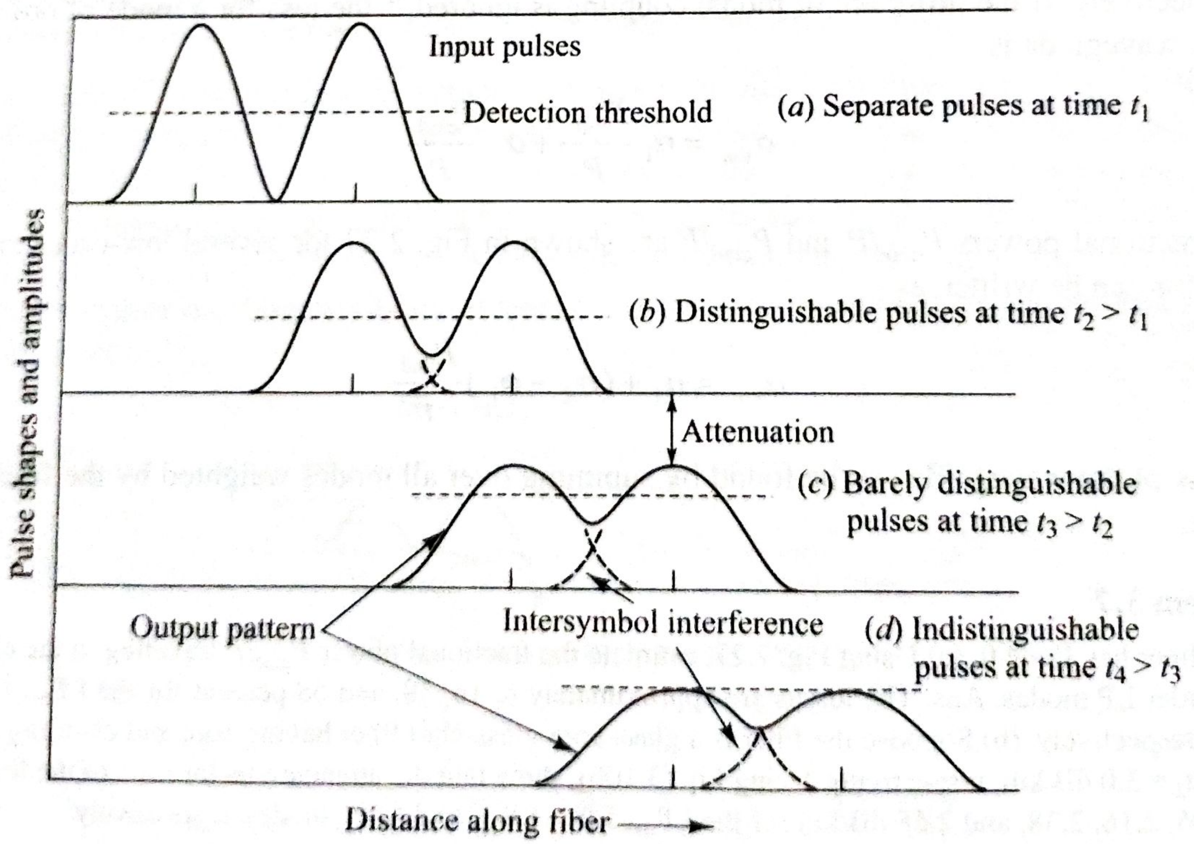

Fig. 3.9 *Broadening and attenuation of two adjacent pulses as they travel along a fiber. (a) Originally the pulses are separate; (b) the pulses overlap slightly and are clearly distinguishable; (c) the pulses overlap significantly and are barely distinguishable; (d) eventually the pulses strongly overlap and are indistinguishable*