

QM method:

PI chart \rightarrow for simple fns but for complex fns, it has to be reduced further to get the reduced PI chart & reduced exp.

- ① Two rows/col are said to be equal if they have exactly X's in same cols/rows. ($I=J$)
- ⊛ A col 'I' in a PI chart is said to dominate another col 'J' if col 'I' has ~~every~~ X in every row in which col 'J' has a X.
- ⊛ If col 'I' dominates col 'J', then col 'J' can be deleted from chart without affecting search for minimal expression (remove dominating column)
- ⊛ If row 'I' is said to dominate another row 'J' if row 'I' has a X in every col in which row 'J' has a X.
- ⊛ If row 'I' dominates row 'J', then row 'J' can be deleted from chart, without affecting the search for minimal expression (remove dominated rows).

- Procedure:
- ① Determine Essential PI from PI chart.
 - ② Form Reduced PI chart by removing EPI, and columns covered by them.
 - ③ Remove dominated rows & dominating columns.
 - ④ Identify secondary EPI from reduced chart and continue process until one is obtained.

eg: Dominated & dominating rows & columns

	3	4	9	11	12	13
A	X					
B		X	X		X	X
C	X			X		
D					X	X
E		X		X		

col '12' = col '13'
Equal Column

Row 'A' is dominated by row 'C' (Dominating Row)
Dominated Row

Col '9' is dominated by col '4' & '12' & '13' (Dominating Columns)
Dominated Column

Branching method: If PI chart contains no essential PI, no dominated row/col & no dominating column/row, then branching method is used.
 → Randomly select any one of row (randomly select a PI) and then apply normal reduction procedure.

Use tabular procedure to find reduced expression:-

$$f(A, B, C, D, E) = \sum m(0, 4, 12, 16, 19, 24, 27, 28, 29, 31)$$

Index	Minterms	Pair	¹⁶ _A ⁸ _B ⁴ _C ² _D ¹ _E
0	✓ 0	P (0, 4) (4)	0 0 - 0 0
1	✓ 4	Q (0, 16) (16)	- 0 0 0 0
	✓ 16	R (4, 12) (8)	0 - 1 0 0
2	✓ 12	S (16, 24) (8)	1 - 0 0 0
	✓ 24	T (12, 28) (16)	- 1 1 0 0
3	✓ 19	U (24, 28) (4)	1 1 - 0 0
	✓ 28	V (19, 27) (8)	1 - 0 1 1
4	✓ 27	W (28, 29) (1)	1 1 1 0 -
	✓ 29	X (27, 31) (4)	1 1 - 1 1
5	✓ 31	Y (29, 31) (2)	1 1 1 - 1

No further combinations are possible.

- All are prime ⇒ no essential PI, no dominated & no dominating row/col.

PI chart

	0	4	12	16	19	24	27	28	29	31
P	X	X								
Q	X			X						
R		X	X			X				
S				X				X		
T			X					X		
U						X		X		
V					X		X		X	
W								X	X	
X										X
Y										X

V → Essential PI.

Row Y ⊃ X

Col → none.

Eliminate dominated row i.e. Row X.

Col 29 ⊃ 31

Eliminate col '29'

Y → Secondary Essential PI

Q, R & U are PI.

$$\therefore f = \overline{B}\overline{C}\overline{D}\overline{E} + \overline{A}\overline{C}\overline{D}\overline{E} + A\overline{B}\overline{D}\overline{E} + ABCE + A\overline{C}DE$$

	0	4	12	16	24	28	31
P	X	X					
Q	X			X			
R		X	X				
S				X	X		
T			X			X	
U					X	X	
V						X	
W							X
Y							X

(3)

Branching method

$$f = f(A, B, C, D) = \sum m(2, 3, 4, 6, 9, 11, 12, 13)$$

Index	Minterms
1	2, 4
2	3, 6, 9, 12
3	11, 13

Pair	A	B	C	D
P 2, 3 (1)	0	0	1	-
Q 2, 6 (4)	0	-	1	0
R 4, 6 (2)	0	1	-	0
S 4, 12 (8)	-	1	0	0
T 3, 11 (8)	-	0	1	1
U 9, 11 (2)	1	0	-	1
V 9, 13 (4)	1	-	0	1
W 12, 13 (1)	1	1	0	-

PI chart

	2	3	4	6	9	11	12	13
P	X	X						
Q	X			X				
R			X	X				
S			X				X	
T		X				X		
U					X	X		
V					X			X
W							X	X

No essential PI
No dominating rows/col

No dominated rows/col

So, Branching method is used.

Select any minterm, randomly, say '2'.

'2' is covered by row 'P' & 'Q'. Randomly select 'P'.

Remove row 'P' & minterms covered by it (i.e. 2 & 3)

Remove dominated row 'Q'.

Remove dominated row (Q) LT

Row 'R' dominates row 'Q'.

Remove dominating col (4) L9

col '4' dominates col '6'.

Remove dominating col (4) L9

Reduced PI chart

	1	4	6	9	12	13	11
*R		X	(X)				
S		X			X		
*U	(X)			X			(X)
V				X		X	
W					X	X	

Secondary Essential PI
are R & U.
Minterms '12', '13' & '11'
can be covered by W

$$\therefore f = P + R + \cancel{U} + W$$

$$= \bar{A}\bar{B}C + \bar{A}B\bar{D} + \bar{A}B\bar{C} + \bar{A}B\bar{D} + \bar{A}B\bar{C} + \bar{A}B\bar{D}$$

$$= \bar{A}\bar{B}C + \bar{A}B\bar{D} + \bar{A}B\bar{C} + \bar{A}B\bar{D}$$