$$x_1 = ax_0 + bx_1 \qquad \text{...(4.1)}$$

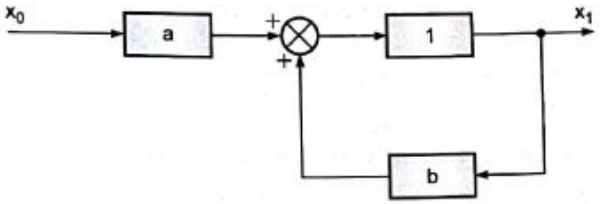


Fig. 4.0.3. Block diagram representation of equation $x_1 = ax_0 + bx_1$.

The signal flow graph representation for the above equation is shown in Fig. 4.0.4.

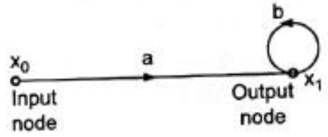


Fig. 4.0.4. Signal flow graph representation of equation $x_1 = ax_0 + bx_1$.

The variables are represented by input and output nodes respectively, a is the transmittance between x_1 and x_0 called forward path transmittance, b is the transmittance between x_1 and x_1 called loop transmittance.

Above two examples form the basis of framing rules for drawing signal flow graphs representing an equation.

4.1 RULES FOR DRAWING SIGNAL FLOW GRAPHS

1. The signal travels along a branch in the direction of an arrow.
2. The input signal is multiplied by the transmittance to obtain the output signal.
3. Input signal at a node is the sum of all signals entering at that node.
4. A node transmits signals in all branches leaving that node.

Signal flow graphs shown in the following examples illustrate the application of the above rules to obtain overall transmittance (also called gain) expressed as the ratio of the output (effect) to input (cause).

Illustrative Example 4.1.1. *The signal flow graph shown in Fig. 4.1.1 has one forward path and single isolated loop. Determine overall transmittance relating x_3 and x_1.*

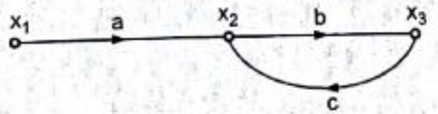


Fig. 4.1.1. Signal flow graph with one forward path and single isolated loop.

Solution.

x_1 is the input node (independent variable node)

x_2 is the intermediate node (intermediate variable node)

x_3 is the output node (dependent variable node)

ab is the forward path transmittance

bc is the loop transmittance.

The equations represented by the signal flow graph are expressed below

(*i*) at node x_2, $x_2 = ax_1 + cx_3$ ...(4.2)

and (*ii*) at node x_3, $x_3 = bx_2$ ...(4.3)

By eliminating the intermediate variable x_2 from equations (4.2) and (4.3), the overall transmittance is determined as

$$\frac{x_3}{x_1} = \frac{ab}{1-bc}. \quad \textbf{Ans.} \qquad \text{...(4.4)}$$

In equation (4.4), the product ab is the forward path transmittance and denoted by a symbol P.

As bc forms a closed-loop on itself, the product bc is termed as loop transmittance and denoted by a symbol L. While traversing the closed-loop each node is encountered only once.

The overall transmittance is denoted by a symbol T.

Therefore, in view of above notations, the overall transmittance for the signal flow graph shown in Fig. 4.1.1 is expressed as

$$T = \frac{P}{1-L} \qquad \text{...(4.5)}$$

Illustrative Example 4.1.5. *The signal flow graph shown in Fig. 4.1.5 has two parallel paths and two isolated (non-touching) loops. Determine the overall transmittance relating x_2 and x_1.*

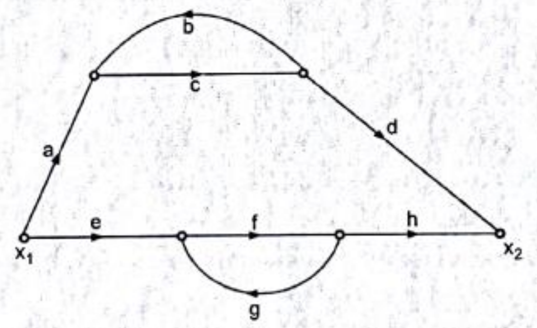


Fig. 4.1.5. Signal flow graph with two forward paths and two isolated (non-touching) loops.

Solution. The overall transmittance relating x_2 and x_1 is determined as follows :

There are two forward paths as mentioned below :

$$P_1 = acd \quad \text{and} \quad P_2 = efh$$

The two loop transmittances are :

$$L_1 = bc \quad \text{and} \quad L_2 = gf$$

Therefore, the corresponding individual transmittances are :

$$T_1 = \frac{P_1}{1 - L_1} \qquad \text{...(4.25)}$$

and

$$T_2 = \frac{P_2}{1 - L_2} \qquad \text{...(4.26)}$$

The overall transmittance is obtained by adding T_1 and T_2.

Hence,

$$\frac{x_2}{x_1} = T_1 + T_2 \qquad \text{...(4.27)}$$

Substituting for T_1 and T_2 from (4.25) and (4.26) respectively in (4.27) following relation for the overall transmittance is obtained.

$$\frac{x_2}{x_1} = \frac{P_1}{1 - L_1} + \frac{P_2}{1 - L_2} \qquad \text{...(4.28)}$$

$$= \frac{P_1(1 - L_2) + P_2(1 - L_1)}{1 - (L_1 + L_2) + L_1 L_2} \qquad \text{...(4.29)}$$

Substituting the values of P_1, P_2, L_1 and L_2 in (4.29), the overall transmittance is determined below :

$$\frac{x_2}{x_1} = \frac{acd\,(1 - gf) + efh\,(1 - bc)}{1 - (bc + gf) + bcgf}. \quad \textbf{Ans.} \qquad \text{...(4.30)}$$

4.2 MASON'S GAIN FORMULA

The overall transmittance (gain) can be determined by Mason's gain formula given below

$$T = \sum_{k=1}^{k} \frac{P_k \Delta_k}{\Delta} \qquad \text{...(4.31)}$$

The terms in the Mason's gain formula are explained below :

P_k is the forward path transmittance of k_{th} path from a specified input node to an output node. In asserting P_k no node should be encountered more than once.

Δ is the graph determinant which involves closed-loop transmittances and mutual interactions between non-touching loops.

Δ = 1 – [Sum of all individual loop transmittances]
+ [Sum of loop transmittance products of all possible pairs of NON-TOUCHING loops]
– [Sum of loop transmittance products of all possible triplets of NON-TOUCHING loops]
+ [.........] – [.........]

Δ_k is the path factor associated with the concerned path and involves all closed loops in the graph which are isolated from the forward path under consideration.

The path factor Δ_k for the k_{th} path is equal to the value of the graph determinant of a signal flow graph which exists after ERASING the k_{th} path from the graph.

Illustrative Example 4.3.1. *Represent the block diagram given in Fig. 4.3.1 by signal flow graph and determine the overall transmittance relating C and R.*

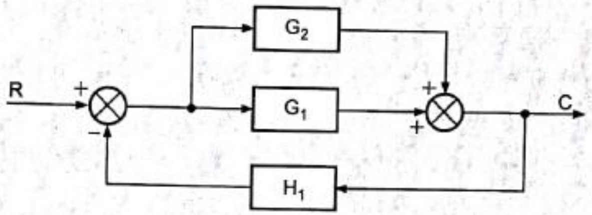


Fig. 4.3.1. Block diagram for example 4.3.1.

Example 4.4.2. *Represent the following set of equations by a signal flow graph and determine the overall gain relating x_5 and x_1.*

$$x_2 = ax_1 + fx_2\ ; \qquad x_3 = bx_2 + ex_4$$

$$x_4 = cx_3 + hx_5\ ; \qquad x_5 = dx_4 + gx_2.$$

Example 4.4.3. *Obtain signal flow graph representation for a system whose block diagram is given in Fig. 4.4.3. Specify (a) forward path, (b) individual loops, (c) path factors, (d) non-touching loops, (e) determine the graph determinant.*

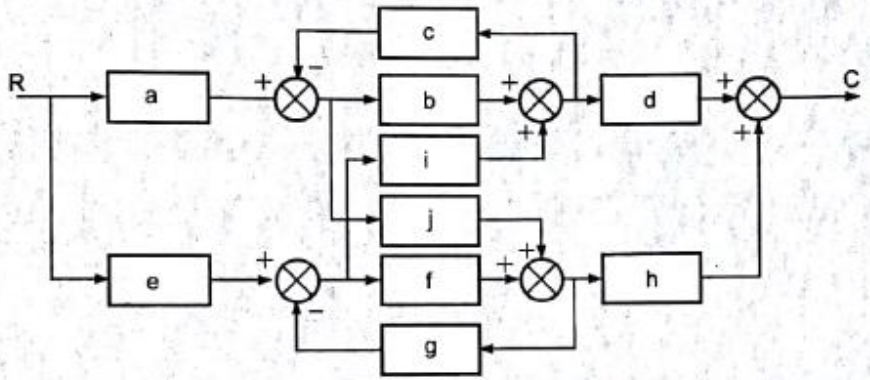


Fig. 4.4.3. Block diagram for example 4.4.3.

Example 4.4.7. *Draw the signal flow graph and determine the overall transfer function. The block diagram is given below:*

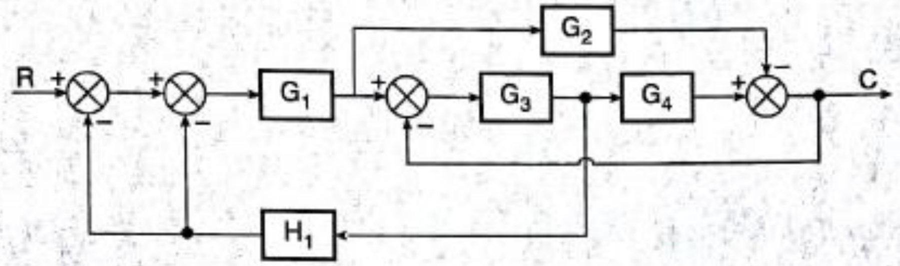


Fig. 4.4.7. Block diagram for example 4.4.7.

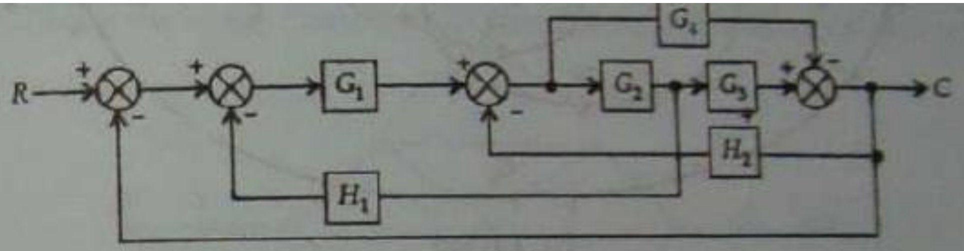
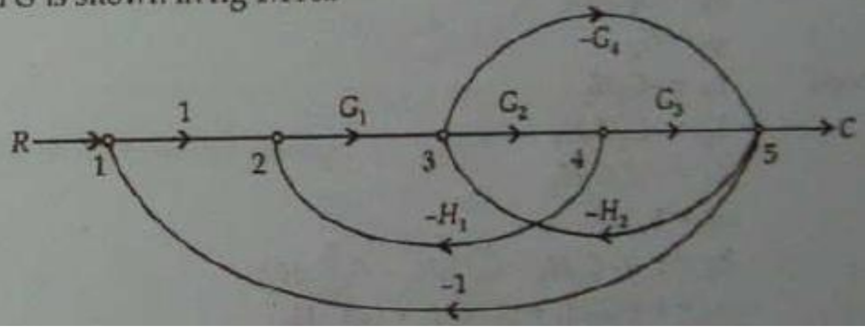


Fig. 1.110.

Solution : The *SFG* is shown in fig 1.110*a*



1.29. BLOCK DIAGRAM FROM SIGNAL

Step 1 : For given signal flow graph, write the system equations.

Step 2 : At each node consider the incoming branches only.

Step 3 : Add all incoming signals algebraically at a node.

Step 4 : For + or – sign in system equations use a summing point

Step 5 : For the gain of each branch of signal flow graph draw the block having the same transfer function as the gain of the branch.

Example 1.49. Draw the block diagram