# BOOLEAN ALGEBRA

[1] $x + 0 = x$

[2] $x \bullet 0 = 0$

[3] $x + 1 = 1$

[4] $x \bullet 1 = x$

[5] $x + x = x$

[6] $x \bullet x = x$

[7] $x + x' = 1$

[8] $x \bullet X' = 0$

[9] $x + y = y + x$

[10] $xy = yx$

[11] $x + (y + z) = (x + y) + z$

[12] $x(yz) = (xy)z$

[13] $x(y + z) = xy + xz$

[14] $x + yz = (x + y)(x + z)$

**[15] $(x + y)' = x'y'$**

**[16] $(xy)' = x' + y'$**

[17] $(x')' = x$

[15] and [16] : De Morgan's theorem

# BOOLEAN ALGEBRA

**Absorption Law:**

**<u>Law 1:</u>** $A + A.\ B = A$

**<u>Proof:</u>** $A + A.B = A\ (1 + B) = A = RHS$

**<u>Law 2:</u>** $A\ (A + B) = A$

**<u>Proof:</u>** $A\ (A + B) = A.\ A + A.B = A + A.\ B = A\ (1 + B) = A = RHS$

**Another Law:** (Uses Distributive Law)

**<u>Law 1:</u>** $A + A'B = A + B$

**<u>Proof:</u>** $A + A'B = (A + A').\ (A + B) = 1.\ (A + B) = A + B = RHS$

**<u>Law 2:</u>** $A.\ (A' + B) = A.B$

**<u>Proof:</u>** $A\ .\ (A' + B) = A.A' + A.B = 0 + AB = AB = RHS$

# BOOLEAN ALGEBRA

**Consensus Theorem:**

**Law 1:** $AB + A'C + BC = AB + A'C$

**Proof:** $AB + A'C + BC$

$= AB + A'C + BC\,(A + A')$

$= A.B + A'C + ABC + A'BC$

$= AB\,(1 + C) + A'C\,(1 + B)$

$= AB + A'C = RHS$

**Law 2:** $(A + B) . (A' + C) . (B + C) = (A + B) . (A' + C)$

**Proof:** **Proof left as an exercise.**

# BOOLEAN ALGEBRA

**Transposition Theorem:**

$$AB + A'C = (A + C).(A' + B)$$

**<u>Proof:</u>**

$$(A + C).(A' + B)$$

$$= A.A' + A.B + A'C + BC$$

$$= 0 + AB + A'C + BC.1$$

$$= AB + A'C + BC(A + A')$$

$$= AB + A'C + ABC + A'BC$$

$$= (AB + ABC) + (A'C + A'BC)$$

$$= AB(1 + C) + A'C(1 + B)$$

$$= AB + A'C = LHS$$

## Duals

| Given Expression | Dual |
| --- | --- |
| 1. $\overline{0} = 1$ | $\overline{1} = 0$ |
| 2. $0 \cdot 1 = 0$ | $1 + 0 = 1$ |
| 3. $0 \cdot 0 = 0$ | $1 + 1 = 1$ |
| 4. $1 \cdot 1 = 1$ | $0 + 0 = 0$ |
| 5. $A \cdot 0 = 0$ | $A + 1 = 1$ |
| 6. $A \cdot 1 = A$ | $A + 0 = A$ |
| 7. $A \cdot A = A$ | $A + A = A$ |
| 8. $A \cdot \overline{A} = 0$ | $A + \overline{A} = 1$ |
| 9. $A \cdot B = B \cdot A$ | $A + B = B + A$ |
| 10. $A \cdot (B \cdot C) = (A \cdot B) \cdot C$ | $A + (B + C) = (A + B) + C$ |
| 11. $A \cdot (B + C) = AB + AC$ | $A + BC = (A + B)(A + C)$ |
| 12. $A(A + B) = A$ | $A + AB = A$ |
| 13. $A \cdot (A \cdot B) = A \cdot B$ | $A + A + B = A + B$ |
| 14. $\overline{AB} = \overline{A} + \overline{B}$ | $\overline{A + B} = \overline{A}\ \overline{B}$ |
| 15. $(A + B)(\overline{A} + C)(B + C) = A + B(\overline{A} + C)$ | $AB + \overline{A}C + BC = AB + \overline{A}C$ |
| 16. $(A + C)(\overline{A} + B) = AB + \overline{A}C$ | $AC + \overline{A}B = (A + B)(\overline{A} + C)$ |
| 17. $A + \overline{B}C = (A + \overline{B})(A + C)$ | $A(\overline{B} + C) = (A\overline{B} + AC)$ |
| 18. $(A + B)(C + D) = AC + AD + BC + BD$ | $(AB + CD) = (A + C)(A + D)(B + C)(B + D)$ |
| 19. $A + B = AB + \overline{A}B + A\overline{B}$ | $AB = (A + B)(\overline{A} + B)(A + \overline{B})$ |
| 20. $A + B\overline{(C + \overline{DE})} = A + B\overline{C}DE$ | $A[B + \overline{(C \cdot \overline{D + E})}] = A \cdot (B + \overline{C} + D + E)$ |
| 21. $\overline{\overline{AB} + \overline{A} + AB} = 0$ | $\overline{\overline{A + B} \cdot \overline{A} \cdot (A + B)} = 1$ |
| 22. $AB + \overline{AC} + A\overline{B}C(AB + C) = 1$ | $(A + B)\overline{(A + C)} \cdot [(A + \overline{B} + C) + (A + B)C] = 0$ |

# Duality principle:

Every theorem has an equivalent theorem by performing the following operation:

1. Change every AND operation to OR operation (Change every dot sign to plus sign) and vice-versa.

1. Change every ‘0’ to ‘1’ and vice-versa.

# PROBLEMS BASED ON BOOLEAN ALGEBRA

Q1. Reduce the expression:

(i) $[A + (B.C)' ]'. (A.B' + ABC)$

(ii) $AB'C + B + B.D' + ABD' + A'C$

# ANSWERS:

A1. Reduce the expression:

(i) 0

(ii) $B + C$

# MINIMIZATION OF BOOLEAN EXPRESSIONS

- Minimization will result in reduction of:

  - Number of gates (resulting from less number of terms)

  - Number of inputs per gate (resulting from less number of variables per term or literals)

  $Y = A'BC + BCD + A'C$

- The minimization will reduce cost, complexity and power consumption.

# MINIMIZATION TECHNIQUES

- **Boolean Algebra:**
  - Used maximum upto 4-variables or 5-variable (beyond it, very becomes very cumbersome)
- **Karnaugh maps (K-maps):**
  - Used maximum upto 6-variables.
- **Variable Entered mapping (VEM technique):**
  - Used upto 7 or 8-variables.
- **Quine Mc-Cluskey method (QM method):**
  - Used for any number of variables.
- **Iterative Consensus method:**
  - Method can be applied for function in standard or non-standard form.

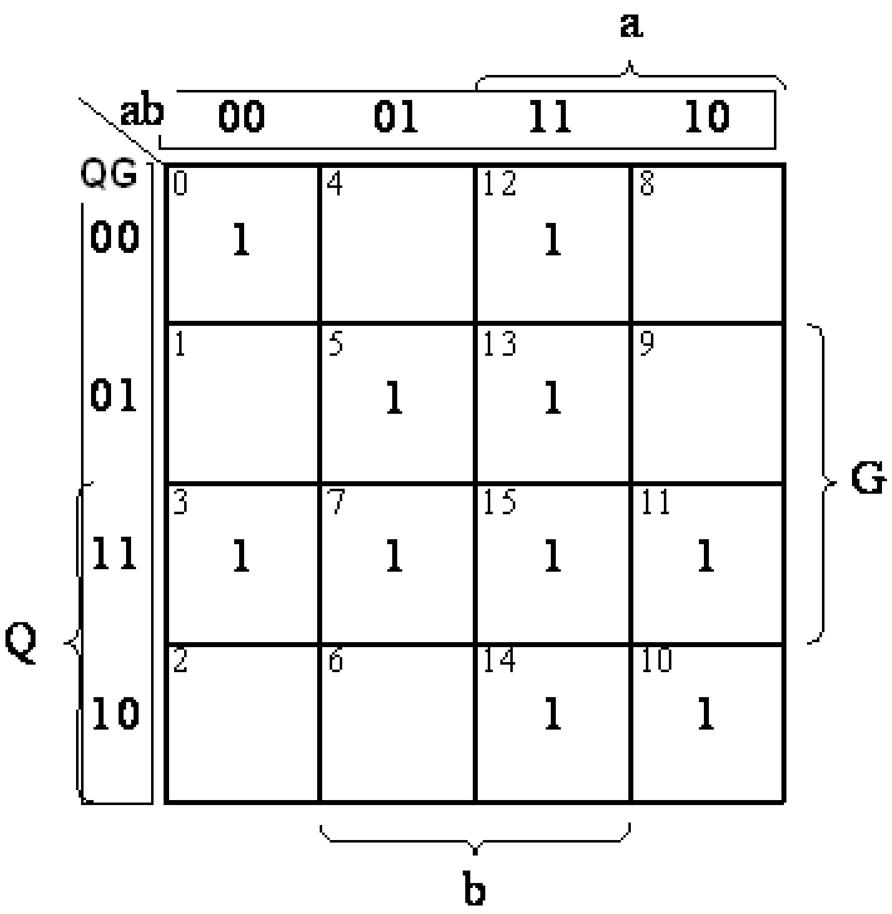
# KARNAUGH MAPS (K-MAPS)

- Karnaugh maps -- A tool for representing Boolean functions of up to six variables.
- K-maps are tables of rows and columns with entries represent 1`s or 0`s of SOP and POS representations.
- An n-variable K-map has $(2)^n$ cells with each cell corresponding to an n-variable truth table value.
- K-map cells are arranged such that adjacent cells correspond to truth rows that differ in only one bit position (logical adjacency).

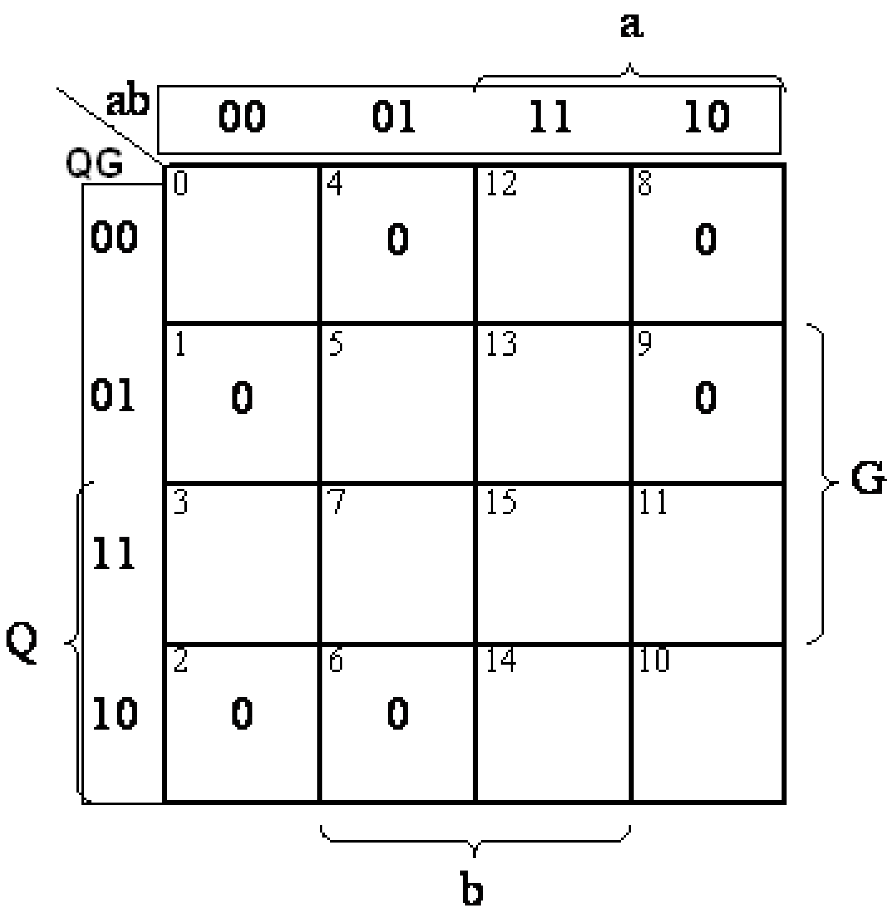
**Karnaugh Maps (K-maps)**

- **If mi is a minterm of f, then place a 1 in cell i of the K-map.**
- **If Mi is a maxterm of f, then place a 0 in cell i.**
- **If di is a don’t care of f, then place a d or x in cell i.**

# FOUR VARIABLE EXAMPLE
# (A) MINTERM FORM. (B) MAXTERM FORM.




(a)




(b)

# SIMPLIFICATION GUIDELINES FOR K-MAPS

- Always combine as many cells in a group as possible. This will result in the fewest number of literals in the term that represents the group. (Maximize the number of elements in each grouping)
- Make as few groupings as possible to cover all minterms. This will result in the fewest product terms. (Minimize the number of groupings)

# 5-VAR

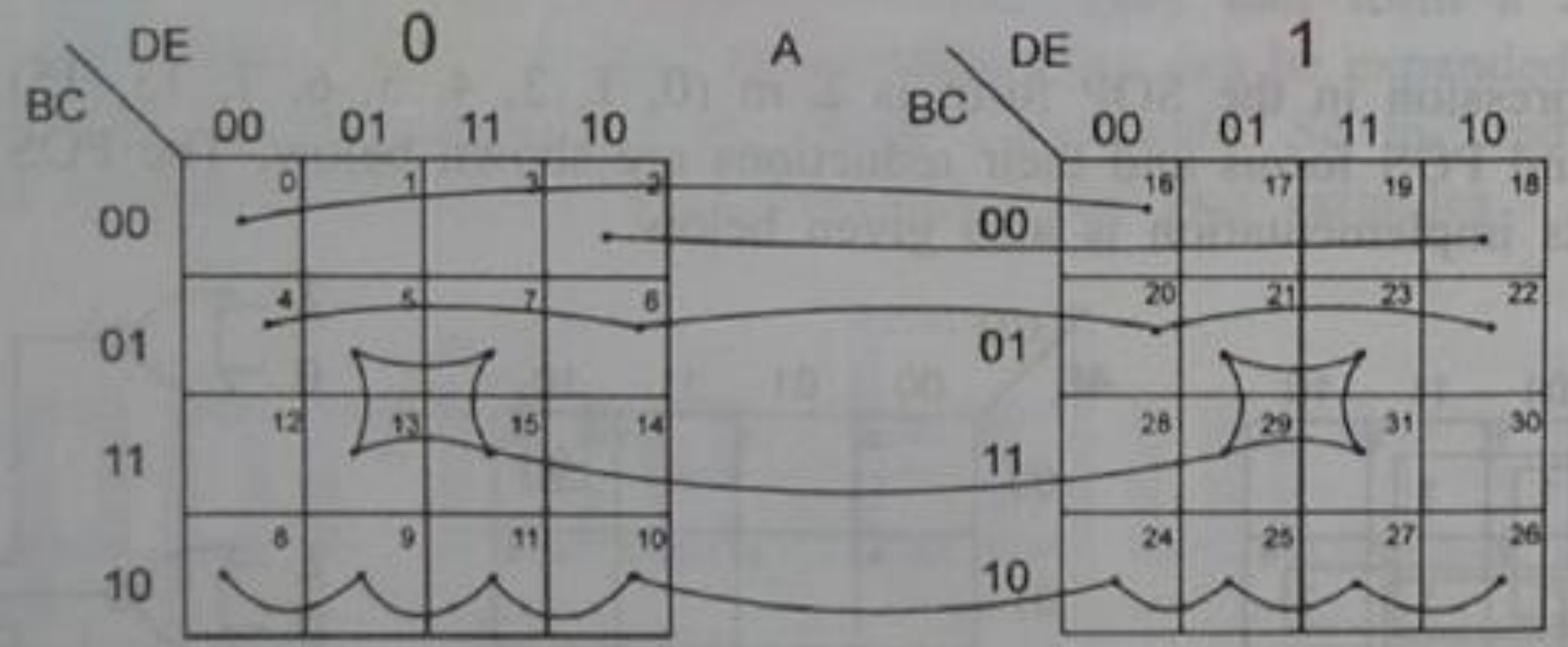



Fig. 6.10 Some possible groupings in a five-variable K-map.

# 6-VAR

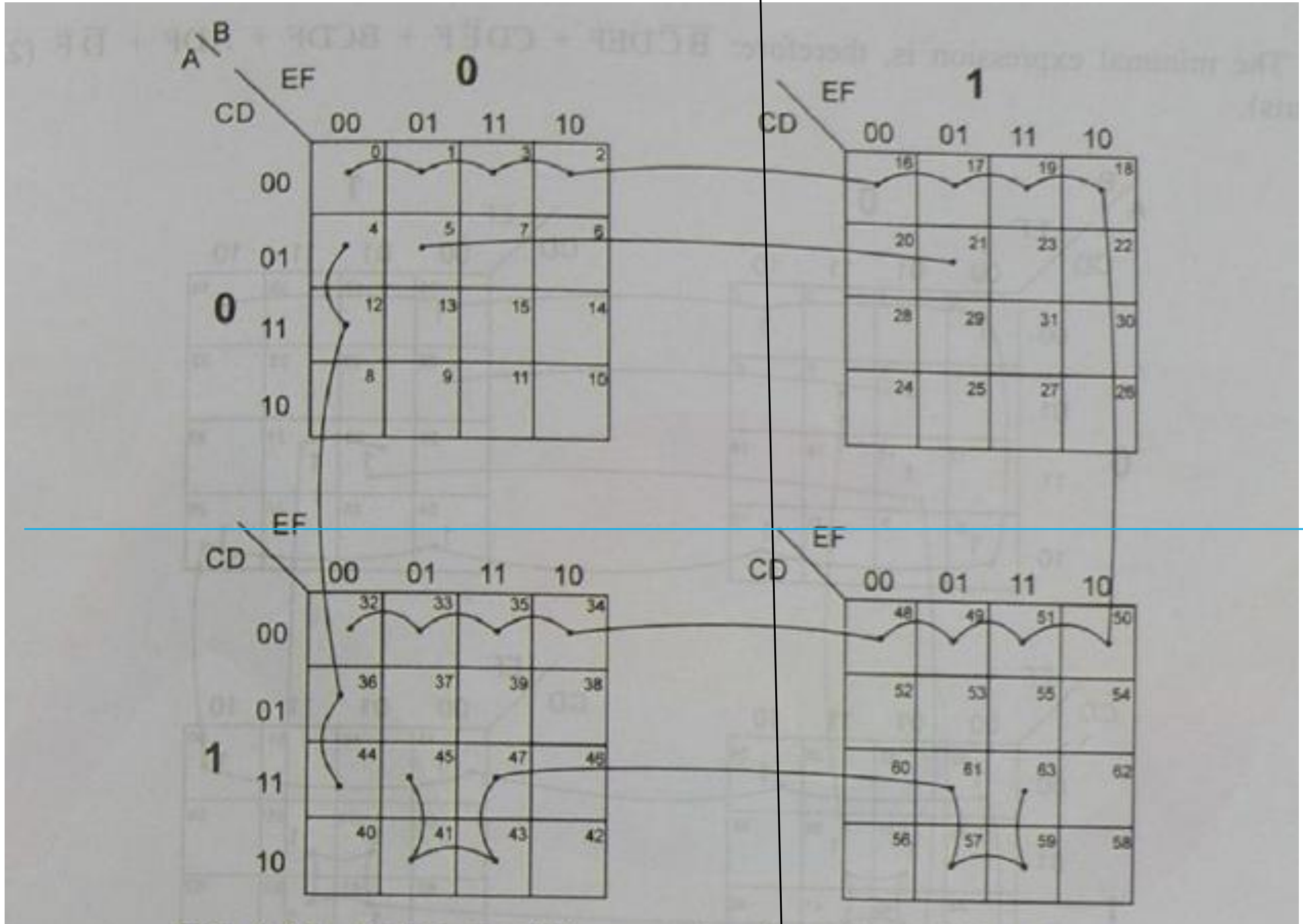



Fig. 6.11 Some possible groupings in a six-variable K-map.

# PROBLEMS BASED ON K-MAPS

Q1. Reduce the expression:

(i) $F = \sum m\,(0,1,2,3,5,7,8,9,10,12,13)$

(ii) $F = \sum m\,(02,3,10,11,12,13,16,17,18,19,20,21,26,27)$

# ANSWERS:

A1. Reduce the expression:

(i) $B'D' + AC' + A'D$

(ii) $A'BCD' + B'C'E' + AB'D' + C'D$