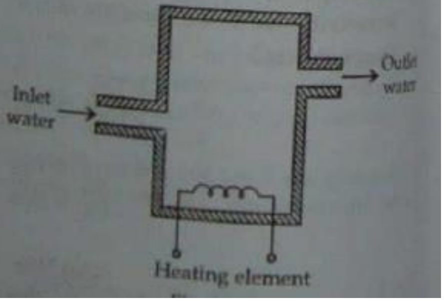
...feedback affects the stability. Feedback ...ility or may be harmful to stability if it is not properly design and apply.

1.31. THERMAL SYSTEMS

In thermal systems, there is transfer of heat from one substance to another substance. The thermal system can be analysed in terms of resistance and capacitance. Let us consider a simple thermal system shown in fig. 1.119.

1.31.1. Heat Transfer System

Suppose, there is no heat store in the insulation and water having uniform temperature.



Let, θ_i = temperature of inlet water (°C)

θ_o = temperature of outlet water(°C)

θ = temperature of the surroundings

q = rate of heat flow from heating elements (J/S)

q_i = rate of heat flow to the water

q_t = rate of heat flow through tank insulation

C = thermal capacitance (J/°C)

R = thermal resistance (°C/J-S^{-1})

Rate of heat flow for the water in tank

$$q_i = C\frac{d\theta_0}{dt}$$

The rate of heat flow from water to the surrounding through insulation

$$q_t = \frac{\theta_0 - \theta}{R}$$

$$q = q_i + q_t$$

$$q = C\frac{d\theta_0}{dt} + \frac{\theta_0 - \theta}{R}$$

neglect the term $\frac{\theta}{R}$

$$q = C\frac{d\theta_0}{dt} + \frac{\theta_0}{R}$$

Take laplace transform of (1.149)

$$Q(s) = sC\,\theta_o(s) + \theta_o(s)$$

$$\text{Transfer function} = \frac{\theta_0(s)}{Q(s)}$$

$$\frac{\theta_0(s)}{Q(s)} = \frac{R}{1+sCR}$$

$\therefore$ Time constant of thermal system is RC.

1.31.2. Thermometer

Consider the fig.1.120 A thermometer is immersed in a tub containing water.

Let θi = temperature of the water in tub.

θ_o = temperature indicated by thermometer

then, rate of heat flow to the thermometer

$$\frac{dq}{dt} = \frac{\theta_i - \theta_0}{R} \quad ...(1.151)$$

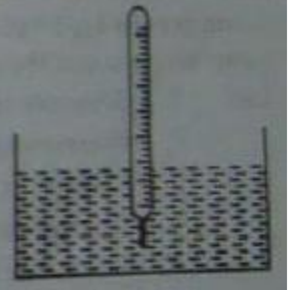


Fig. 1.120.

The indicated temperature rises at the rate

$$\frac{d\theta_0}{dt} = \frac{1}{C}\frac{dq}{dt}$$

$$\therefore \quad \frac{d\theta_0}{dt} = \frac{1}{C}\left(\frac{\theta_i - \theta_0}{R}\right) \quad ...(1.152)$$

Take the laplace transform

$$s\,\theta_o(s) = \frac{1}{RC}[\theta_i(s) - \theta_0(s)]$$

$$\therefore \quad \frac{\theta_0(s)}{\theta_i(s)} = \frac{1}{1+sRC}$$

Let $\theta_i(s)$ = unit step input

$$\theta_o(s) = \frac{1}{s(1+sRC)}$$

Take inverse laplace

$$\therefore \quad \theta_o(t) = 1 - e^{-t/RC}$$

$\therefore$ Time constant = RC

THERMAL SYSTEMS (Heat Transfer Systems & Thermometer)