$$\alpha(r)=\alpha_1+(\alpha_2-\alpha_1)\frac{n^2(0)-n^2(r)}{n^2(0)-n_2^2} \tag{3.11}$$

where α_1 and α_2 are the axial and cladding attenuation coefficients, respectively, and the n terms are defined by Eq. (2.78). The loss encountered by a given mode is then

$$\alpha_{gi}=\frac{\int_0^\infty \alpha(r)p(r)r\,dr}{\int_0^\infty p(r)r\,dr} \tag{3.12}$$

where $p(r)$ is the power density of that mode at r. The complexity of the multimode waveguide has prevented an experimental correlation with a model. However, it has generally been observed that the loss increases with increasing mode number.[26,33]

3.2 SIGNAL DISPERSION IN FIBERS

As shown in Fig. 3.9, an optical signal weakens from attenuation mechanisms and broadens due to dispersion effects as it travels along a fiber. Eventually these two factors will cause neighboring pulses to overlap. After a certain amount of overlap occurs, the receiver can no longer distinguish the individual adjacent pulses and errors arise when interpreting the received signal.

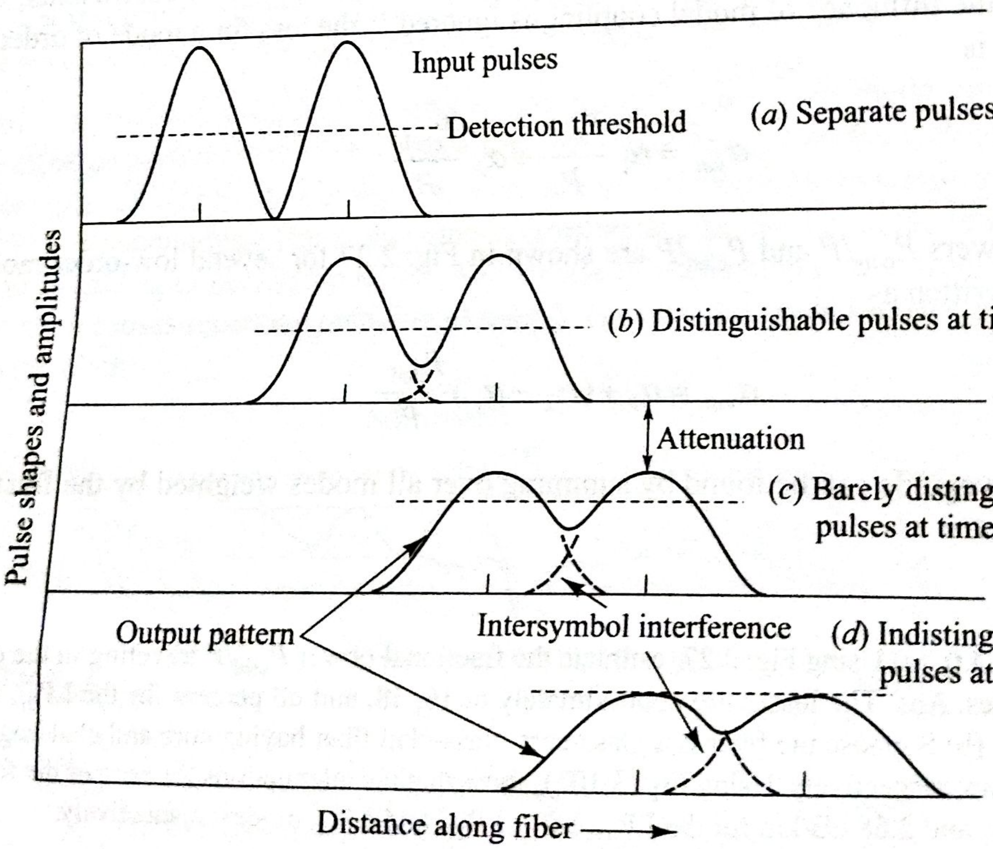

Fig. 3.9 *Broadening and attenuation of two adjacent pulses as they travel along a fiber. (a) Originally the pulses are separate; (b) the pulses overlap slightly and are clearly distinguishable; (c) the pulses overlap significantly and are barely distinguishable; (d) eventually the pulses strongly overlap and are indistinguishable*

This section first discusses the general factors that cause signal dispersion and then examines the various dispersion mechanisms in more detail. Section 3.2.2 addresses modal delay and shows how this delay is related to the information-carrying capacity of a multimode fiber in terms of a transmitted bit rate B. Section 3.2.3 examines the various factors contributing to dispersion in terms of the frequency dependence of the propagation constant β. The next topics include a discussion of group velocity in Sec. 3.2.4 and details of the various dispersion mechanisms in Sec. 3.2.5 through 3.2.8.

3.2.1 Overview of Dispersion Origins

Signal dispersion is a consequence of factors such as intermodal delay (also called intermodal dispersion), intramodal dispersion, polarization-mode dispersion, and higher-order dispersion effects. These distortions can be explained by examining the behavior of the group velocities of the guided modes, where the *group velocity* is the speed at which energy in a particular mode travels along the fiber (see Sec. 3.2.4).

Intermodal Delay (or simply *modal delay*) appears only in multimode fibers. Modal delay is a result of each mode having a different value of the group velocity at a single frequency. From this effect one can derive an intuitive picture of the information-carrying capacity of a multimode fiber.

Intramodal Dispersion or Chromatic Dispersion is pulse spreading that takes place within a single mode. This spreading arises from the finite spectral emission width of an optical source. The phenomenon also is known as *group velocity dispersion*, since the dispersion is a result of the group velocity being a function of the wavelength. Because intramodal dispersion depends on the wavelength, its effect on signal distortion increases with the spectral width of the light source. The spectral width is the band of wavelengths over which the source emits light. This wavelength band normally is characterized by the root-mean-square (rms) spectral width σ_λ. Depending on the device structure of a light-emitting diode (LED), the spectral width is approximately 4 to 9 percent of a central wavelength. For example, as Fig. 3.10 illustrates, if the peak wavelength of an LED is 850 nm, a typical source spectral width would be 36 nm; that is, such an LED emits most of

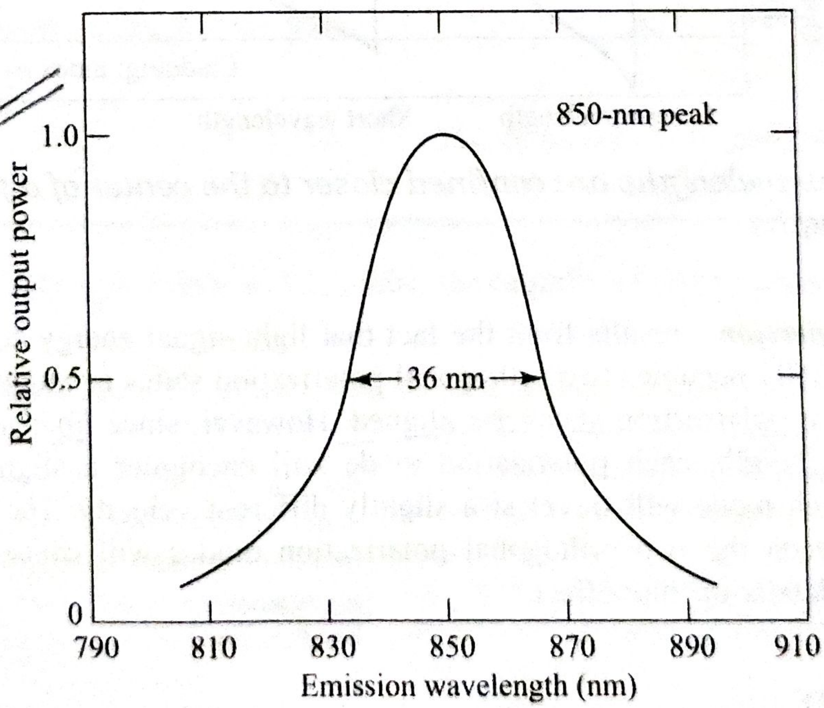

Fig. 3.10 *Spectral emission pattern of a representative $Ga_{1-x}Al_xAs$ LED with a peak emission at 850 nm. The width of the spectral pattern at its half-power point is 36 nm*

its light in the 832-to-868-nm wavelength band. Laser diode optical sources exhibit much narrower spectral widths, with typical values being 1–2 nm for multimode lasers and 10^{-4} nm for single-mode lasers (see Chapter 4).

The two main causes of intramodal dispersion are as follows:

1. *Material dispersion* arises due to the variations of the refractive index of the core material as a function of wavelength. Material dispersion also is referred to as *chromatic dispersion*, since this is the same effect by which a prism spreads out a spectrum. This refractive index property causes a wavelength dependence of the group velocity of a given mode; that is, pulse spreading occurs even when different wavelengths follow the same path.
2. *Waveguide dispersion* causes pulse spreading because only part of the optical power propagation along a fiber is confined to the core. Within a single propagating mode, the cross-sectional distribution of light in the optical fiber varies for different wavelengths. Shorter wavelengths are more completely confined to the fiber core, whereas a larger portion of the optical power at longer wavelengths propagates in the cladding, as shown in Fig. 3.11. The refractive index is lower in the cladding than in the core, so the fraction of light power propagating in the cladding travels faster than the light confined to the core. In addition, we note that the index of refraction depends on the wavelength (see Sec. 3.2.5) so that different spectral components within a single mode have different propagation speeds. Dispersion thus arises because the difference in core-cladding spatial power distributions, together with the speed variations of the various wavelengths, causes a change in propagation velocity for each spectral component. The degree of waveguide dispersion depends on the fiber design (see Sec. 3.3.1). Waveguide dispersion usually can be ignored in multimode fibers, but its effect is significant in single-mode fibers.

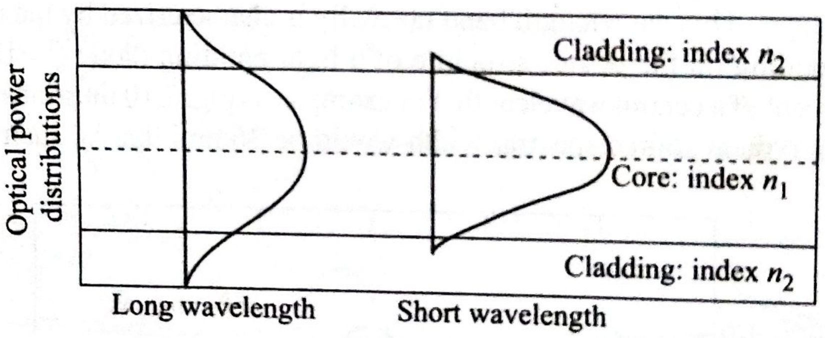

Fig. 3.11 *Shorter wavelengths are confined closer to the center of a fiber core than longer wavelengths*

Polarization-mode Dispersion results from the fact that light-signal energy at a given wavelength in a single-mode fiber actually occupies two orthogonal polarization states or modes (see Sec. 2.5). At the start of the fiber the two polarization states are aligned. However, since fiber material is not perfectly uniform throughout its length, each polarization mode will encounter a slightly different refractive index. Consequently each mode will travel at a slightly different velocity. The resulting difference in propagation times between the two orthogonal polarization modes will cause pulse spreading. Section 3.2.8 gives more details on this effect.

3.2.2 Modal Delay

Intermodal dispersion or *modal delay* appears only in multimode fibers. This signal-distorting mechanism is a result of each mode having a different value of the group velocity at a single frequency. To see why the delay arises, consider the meridional ray picture given in Fig. 2.17 for a multimode step-index fiber. The steeper the angle of propagation of the ray congruence, the higher is the mode number and, consequently, the slower the axial group velocity. This variation in the group velocities of the different modes results in a group delay spread, which is the intermodal dispersion. This dispersion mechanism is eliminated by single-mode operation but is important in multimode fibers. The maximum pulse broadening arising from the modal delay is the difference between the travel time T_{max} of the longest ray congruence paths (the highest-order mode) and the travel time T_{min} of the shortest ray congruence paths (the fundamental mode). This broadening is simply obtained from ray tracing and for a fiber of length L is given by

$$\Delta T = T_{max} - T_{min} = \frac{n_1}{c}\left(\frac{L}{\sin\varphi_c} - L\right) = \frac{Ln_1^2}{cn_2}\Delta \approx \frac{Ln_1\Delta}{c} \tag{3.13}$$

where from Eq. (2.21) $\sin\varphi_c = n_2/n_1$ and Δ is the index difference.

The question now arises as to what maximum bit rate B can be sent over a multimode step-index fiber. Typically the fiber capacity is specified in terms of the *bit rate-distance product BL*, that is, the bit rate times the possible transmission distance L. In order for neighboring signal pulses to remain distinguishable at the receiver, the pulse spread should be less than $1/B$, which is the width of a bit period. For example, a stringent requirement for a high-performance link might be $\Delta T \le 0.1/B$. In general, we need to have $\Delta T < 1/B$. Using Eq. (3.13) this inequality gives the bit rate-distance product

$$BL < \frac{n_2}{n_1^2}\frac{c}{\Delta}$$

Example 3.7

Consider a 1-km long multimode step-index fiber in which $n_1 = 1.480$ and $\Delta = 0.01$, so that $n_2 = 1.465$. What is the modal delay per length in this fiber?

Solution:

Eq. (3.13) yields

$$\frac{\Delta T}{L} = \frac{n_1^2\Delta}{cn_2} = 50 \text{ ns/km}$$

This means that a pulse broadens by 50 ns after traveling a distance of 1 km in this type of fiber.

Taking values of $n_1 = 1.480$, $n_2 = 1.465$, and $\Delta = 0.01$, the capacity of this multimode step-index fiber is $BL = 20$ Mb/s-km.

Example 3.8

Viewed alternatively, as illustrated in Example 3.7, for a multimode step-index fiber with a bandwidth-distance value of $BL = 20$ Mb/s-km the pulse spreading is 50 ns/km. As an example, suppose the pulse width in a transmission system is allowed to widen by at most 25 percent. Then for a 10-Mb/s data rate, in which one pulse is transmitted every 100 ns, this limitation allows a spread of at most 25 ns, which occurs in a transmission distance of 500 m. Now, suppose the data rate is increased to 100 Mb/s, which means that one pulse is transmitted every 10 ns. In this case the 50-ns/km allowable spreading factor will limit the transmission distance to only 50 m in such a multimode step-index fiber.

The root-mean-square (rms) value of the time delay is a useful parameter for assessing the effect of modal delay in a multimode fiber. If it is assumed that the light rays are uniformly distributed over the acceptance angles of the fiber, then the rms impulse response σ_s due to intermodal dispersion in a step-index multimode fiber can be estimated from the expression

$$\sigma_s \approx \frac{Ln_1 \Delta}{2\sqrt{3}c} \approx \frac{L(NA)^2}{4\sqrt{3}n_1 c} \tag{3.14a}$$

Here L is the fiber length and NA is the numerical aperture. Equation (3.14a) shows that the pulse broadening is directly proportional to the core-cladding index difference and the length of the fiber.

Drill Problem 3.8

A 10-km transmission link consists of a step-index multimode fiber that has a core index $n_1 = 1.480$ and a core-cladding refractive index difference $\Delta = 0.01$. (a) Using the approximation on the right-hand side of Eq. (3.13), show that the delay difference between the fastest and slowest modes is 493 ns. (b) Using Eq. (3.14a), show that the rms pulse broadening resulting from intermodal delay is 142 ns. (c) Using Eq. (3.13) and the condition that the maximum bit rate B should satisfy the condition $B < 0.1/\Delta T$, show that the maximum bit rate-distance product is BL = (2.03 Mb/s)•km.

A successful technique for reducing modal delay in multimode fibers is through the use of a graded refractive index in the fiber core, as shown in Fig. 2.15. In any multimode fiber the ray paths associated with higher-order modes are concentrated near the edge of the core and thus follow a longer path through the fiber than lower-order modes (which are concentrated near the fiber axis). However, if the core has a graded index, then the higher-order modes encounter a lower refractive index near the core edge. Since the speed of light in a material depends on the refractive index value, the higher-order modes travel faster in the outer core region than those modes that propagate through a higher refractive index along the fiber center. Consequently this reduces the delay difference between the fastest and slowest modes. A detailed analysis using electromagnetic mode theory gives the following absolute modal delay at the output of a graded-index fiber that has a parabolic ($\alpha = 2$) core index profile (see Sec. 2.6):

$$\sigma_s \approx \frac{Ln_1 \Delta^2}{20\sqrt{3}c} \tag{3.14b}$$

Thus for an index difference of $\Delta = 0.01$, the theoretical improvement factor for intermodal rms pulse broadening in a graded-index fiber is 1000.

Example 3.9

Consider the following two multimode fibers: (a) a step-index fiber with a core index $n_1 = 1.458$ and a core-cladding index difference $\Delta = 0.01$; (b) a parabolic-profile graded-index fiber with the same values of n_1 and Δ. Compare the rms pulse broadening per kilometer for these two fibers.

Solution:

(a) From Eq. (3.14a) we have

$$\frac{\sigma_s}{L} = \frac{n_1 \Delta}{2\sqrt{3}c} = \frac{1.458(0.01)}{2\sqrt{3} \times 3 \times 10^8 \text{ m/s}} = 14.0 \text{ ns/km}$$

(b) From Eq. (3.14b) we have

$$\frac{\sigma_s}{L} \approx \frac{n_1 \Delta^2}{20\sqrt{3}c} = \frac{1.458(.01)^2}{20\sqrt{3} \times 3 \times 10^8 \text{ m/s}} = 14.0 \text{ ps/km}$$

In graded-index fibers, careful selection of the radial refractive-index profile can lead to bit rate-distance products of up to 1 Gb/s-km.

3.2.3 Factors Contributing to Dispersion

This section briefly examines the various factors contributing to dispersion. Sections 3.2.4 through 3.2.8 and Sec. 3.3 describe these factors in more detail.

As Sec. 2.4 notes, the wave propagation constant β is a function of the wavelength, or, equivalently, of the angular frequency ω. Since β is a slowly varying function of this angular frequency, one can see where various dispersion effects arise by expanding β in a Taylor series about a central frequency ω_0. Inserting such an expansion into the waveform equation, for example Eq. (2.1), then shows the effects of variations in β due to modal dispersion and delay effects on the frequency components of a pulse during propagation along a fiber. Expanding β to third order in a Taylor series yields

$$\beta(\omega) \approx \beta_0(\omega_0) + \beta_1(\omega_0)(\omega - \omega_0) + \frac{1}{2}\beta_2(\omega_0)(\omega-\omega_0)^2 + \frac{1}{6}\beta_3(\omega_0)(\omega-\omega_0)^3 \tag{3.15}$$

where $\beta_m(\omega_0)$ denotes the m^{th} derivative of β with respect to ω evaluated at $\omega = \omega_0$; that is,

$$\beta_m = \left(\frac{\partial^m \beta}{\partial \omega^m}\right)_{\omega=\omega_0} \tag{3.16}$$

Now let us examine the different components of the product βz, where z is the distance traveled along the fiber. The resulting first term $\beta_0 z$ describes a phase shift of the propagating optical wave. From the second term of Eq. (3.15), the factor $\beta_1(\omega_0)z$ produces a group delay $\tau_g = z/V_g$, where z is the distance traveled by the pulse and $V_g = 1/\beta_1$ is the group velocity [see Eqs. (3.20) and (3.21)]. Assume β_{1x} and β_{1y} are the propagation constants of the polarization components along the x-axis and y-axis, respectively, of a particular mode. If the corresponding group delays of these two polarization components are $\tau_{gx} = z\beta_{1x}$ and $\tau_{gy} = z\beta_{1y}$ in a distance z, then the difference in the propagation times of these two modes

$$\Delta\tau_{\text{PMD}} = z\,|\beta_{1x} - \beta_{1y}| \tag{3.17}$$

is called the *polarization-mode dispersion* (PMD) of the ideal uniform fiber. Note that in a real fiber the PMD varies statistically (see Sec. 3.2.8).

In the third term of Eq. (3.15), the factor β_2 shows that the group velocity of a monochromatic wave depends on the wave frequency. This means that the different group velocities of the frequency components of a pulse cause it to broaden as it travels along a fiber. This spreading of the group velocities is known as *chromatic dispersion* or *group velocity dispersion* (GVD). The factor β_2 is called the *GVD parameter* (see Sec. 3.2.4), and the *dispersion D* is related to β_2 through the expression

$$D = -\frac{2\pi c}{\lambda^2}\beta_2 \tag{3.18}$$

In the fourth term of Eq. (3.15), the factor β_3 is known as the *third-order dispersion*. This term is important around the wavelength at which β_2 equals zero. The third-order dispersion can be related to the dispersion D and the *dispersion slope* $S_0 = \partial D/\partial\lambda$ (the variation in the dispersion D with wavelength) by transforming the derivative with respect to ω into a derivative with respect to λ. Thus we have

$$\begin{aligned}\beta_3 &= \frac{\partial \beta_2}{\partial \omega} = -\frac{\lambda^2}{2\pi c}\frac{\partial \beta_2}{\partial \lambda} = -\frac{\lambda^2}{2\pi c}\frac{\partial}{\partial \lambda}\left[-\frac{\lambda^2}{2\pi c}D\right] \\ &= \frac{\lambda^2}{(2\pi c)^2}(\lambda^2 S_0 + 2\lambda D)\end{aligned} \tag{3.19}$$

Section 3.5.3 illustrates how the factors in Eq. (3.19) are specified for commercial fibers.

3.2.4 Group Delay

As Example 3.8 mentions, the information-carrying capacity of a fiber link can be determined by examining the deformation of short light pulses propagating along the fiber. The following discussion on signal dispersion thus is carried out primarily from the viewpoint of pulse broadening, which is representative of digital transmission.

First consider an electrical signal that modulates an optical source. For this case, assume that the modulated optical signal excites all modes equally at the input of the fiber. Each waveguide mode thus carries an equal amount of energy through the fiber. Furthermore, each mode contains all the spectral components in the wavelength band over which the source emits. In addition, assume that each of these spectral components is modulated in the same way. As the signal propagates along the fiber, each spectral component can be assumed to travel independently and to undergo a time delay or *group delay* per unit length τ_g/L in the direction of the propagation given by[34]

$$\boxed{\frac{\tau_g}{L} = \frac{1}{V_g} = \frac{1}{c}\frac{d\beta}{dk} = -\frac{\lambda^2}{2\pi c}\frac{d\beta}{d\lambda}} \tag{3.20}$$

Here, L is the distance traveled by the pulse, ~~β is the propagation~~ constant along the fiber axis, $k = 2\pi/\lambda$, and the *group velocity*

$$V_g = c\left(\frac{d\beta}{dk}\right)^{-1} = \left(\frac{\partial\beta}{\partial\omega}\right)^{-1} \tag{3.21}$$

is the velocity at which the energy in a pulse travels along a fiber.

Since the group delay depends on the wavelength, each spectral component of any particular mode takes a different amount of time to travel a certain distance. As a result of this difference in time delays, the optical signal pulse spreads out with time as it is transmitted over the fiber. Thus the quantity we are interested in is the amount of pulse spreading that arises from the group delay variation.

If the spectral width of the optical source is not too wide, the delay difference per unit wavelength along the propagation path is approximately $d\tau_g/d\lambda$. For spectral components that are $\delta\lambda$ apart and which lie $\delta\lambda/2$ above and below a central wavelength λ_0, the total delay difference $\delta\tau$ over a distance L is

$$\boxed{\delta\tau = \frac{d\tau_g}{d\lambda}\delta\lambda = -\frac{L}{2\pi c}\left(2\lambda\frac{d\beta}{d\lambda} + \lambda^2\frac{d^2\beta}{d\lambda^2}\right)\delta\lambda} \tag{3.22}$$

In terms of the angular frequency ω, this is written as

$$\delta\tau = \frac{d\tau_g}{d\omega}\delta\omega = \frac{d}{d\omega}\left(\frac{L}{V_g}\right)\delta\omega = L\left(\frac{d^2\beta}{d\omega^2}\right)\delta\omega \tag{3.23}$$

The factor $\beta_2 \equiv d^2\beta/d\omega^2$ is the GVD parameter, which determines how much a light pulse broadens as it travels along an optical fiber.

If the spectral width $\delta\lambda$ of an optical source is characterized by its rms value σ_λ (see Fig. 3.10 for a typical LED), then the pulse spreading can be approximated by the rms pulse width,

$$\sigma_g \approx \left|\frac{d\tau_g}{d\lambda}\right|\sigma_\lambda = \frac{L\sigma_\lambda}{2\pi c}\left|2\lambda\frac{d\beta}{d\lambda} + \lambda^2\frac{d^2\beta}{d\lambda^2}\right| \tag{3.24}$$

The factor

$$D = \frac{1}{L}\frac{d\tau_g}{d\lambda} = \frac{d}{d\lambda}\left(\frac{1}{V_g}\right) = -\frac{2\pi c}{\lambda^2}\beta_2 \tag{3.25}$$

is designated as the *dispersion.* It defines the pulse spread as a function of wavelength and is measured in picoseconds per kilometer per nanometer [ps/(nm · km)]. It is a result of material and waveguide dispersion. In many theoretical treatments of intramodal dispersion it is assumed, for simplicity, that material dispersion and waveguide dispersion can be calculated separately and then added to give the total dispersion of the mode. In reality, these two mechanisms are intricately related, since the dispersive properties of the refractive index (which gives rise to material dispersion) also affects the waveguide dispersion. However, an examination[35] of the interdependence of material and waveguide dispersion has shown that, unless a very precise value to a fraction of a percent is desired, a good estimate of the total intramodal dispersion can be obtained by calculating the effect of signal distortion arising from one type of dispersion in the absence of the other. Thus, to a very good approximation, D can be written as the sum of the material dispersion D_{mat} and the waveguide dispersion D_{wg}. Material dispersion and waveguide dispersion are therefore considered separately in the next two sections.

3.2.5 Material Dispersion

Material dispersion occurs because the index of refraction varies as a function of the optical wavelength.[36] This is exemplified in Fig. 3.12 for silica. As a consequence, since the group velocity V_g of a mode is a function of the index of refraction, the various spectral components of a given mode will travel at different speeds, depending on the wavelength. Material dispersion is, therefore, an intramodal dispersion effect and is of particular importance for single-mode waveguides and for LED systems (since an LED has a broader output spectrum than a laser diode).

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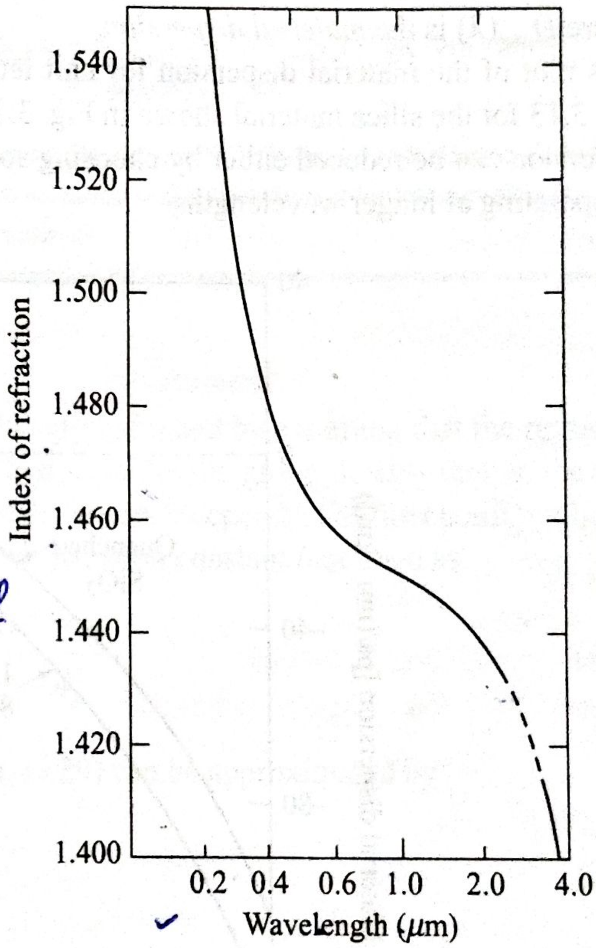

Fig. 3.12 *Variations in the index of refraction as a function of the optical wavelength for silica. (Reproduced with permission from I. H. Malitson, J. Opt. Soc. Amer., vol. 55, pp. 1205–1209, Oct. 1965)*

To calculate material-induced dispersion, we consider a plane wave propagating in an infinitely extended dielectric medium that has a refractive index $n(\lambda)$ equal to that of the fiber core. The propagation constant β is thus given as

$$\beta = \frac{2\pi n(\lambda)}{\lambda} \tag{3.26}$$

n(λ) = f

note

Substituting this expression for β into Eq. (3.20) with $k = 2\pi/\lambda$ yields the group delay τ_{mat} resulting from material dispersion.

$\frac{2\pi n(\lambda)}{\lambda}$

$$\tau_{mat} = \frac{L}{c}\left(n - \lambda\frac{dn}{d\lambda}\right) \tag{3.27}$$

$\frac{L}{c}\left(n - \lambda\frac{dn}{d\lambda}\right)$

Using Eq. (3.24), the pulse spread σ_{mat} for a source of spectral width σ_λ is found by differentiating this group delay with respect to wavelength and multiplying by σ_λ to yield

$$\sigma_{mat} \approx \left|\frac{d\tau_{mat}}{d\lambda}\right|\sigma_\lambda = \frac{\sigma_\lambda L}{c}\left|\lambda\frac{d^2 n}{d\lambda^2}\right| = \sigma_\lambda L\,|D_{mat}(\lambda)| \tag{3.28}$$

where $D_{mat}(\lambda)$ is the *material dispersion*.

A plot of the material dispersion for unit length L and unit optical source spectral width σ_λ is given in Fig. 3.13 for the silica material shown in Fig. 3.12. From Eq. (3.28) and Fig. 3.13 it can be seen that material dispersion can be reduced either by choosing sources with narrower spectral output widths (reducing σ_λ) or by operating at longer wavelengths.

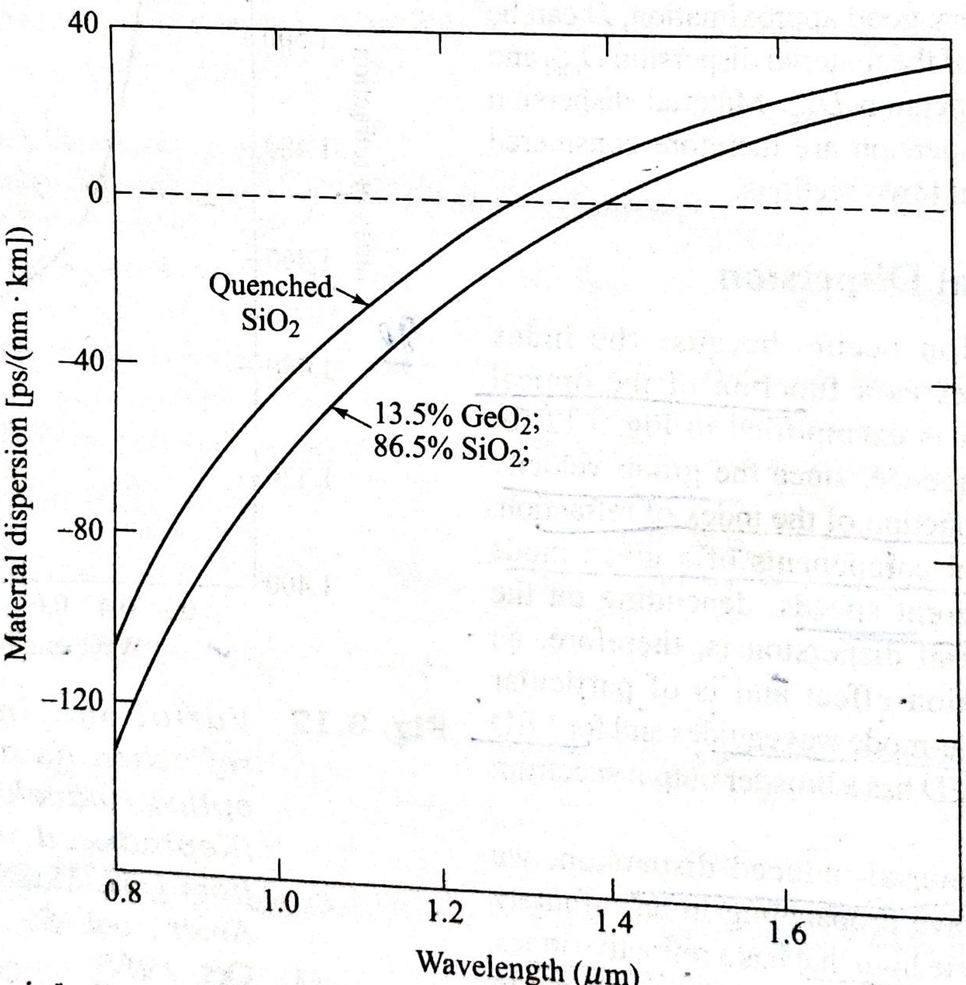

Fig. 3.13 *Material dispersion as a function of optical wavelength for pure silica and 13.5-percent GeO_2/86.5-percent SiO_2. (Reproduced with permission from J. W. Fleming, Electron. Lett., vol. 14, pp. 326–328, May 1978)*

Example 3.10

A manufacturer's data sheet lists the material dispersion D_{mat} of a GeO_2-doped fiber to be 110 ps/(nm · km) at a wavelength of 860 nm. Find the rms pulse broadening per kilometer due to material dispersion if the optical source is a GaAlAs LED that has a spectral width σ_λ of 40 nm at an output wavelength of 860 nm.

Solution:

From Eq. (3.28) we find that the rms material dispersion is

$$\sigma_{mat}/L = \sigma_\lambda D_{mat} = (40 \text{ nm}) \times [110 \text{ ps/(nm} \cdot \text{km)}]$$
$$= 4.4 \text{ ns/km}$$

Example 3.11

The manufacturer's data shows that the same fiber as in Example 3.10 has a material dispersion D_{mat} of 15 ps/(nm · km) at a wavelength of 1550 nm. However, now suppose we use a laser source with a spectral width σ_λ of 0.2 nm at an operating wavelength of 1550 nm. What is the rms pulse broadening per kilometer due to material dispersion in this case?

Solution:

From Eq. (3.28) we find that the rms material dispersion is

$$\sigma_{mat}/L = \sigma_\lambda D_{mat} = (0.2 \text{ nm}) \times [15 \text{ ps/(nm} \cdot \text{km)}]$$
$$= 7.5 \text{ ps/km}$$

This example shows that a dramatic reduction in dispersion can be achieved when operating at longer wavelengths with laser sources.

3.2.6 Waveguide Dispersion

The effect of waveguide dispersion on pulse spreading can be approximated by assuming that the refractive index of the material is independent of wavelength. Let us first consider the group delay—that is, the time required for a mode to travel along a fiber of length L. To make the results independent of fiber configuration,[37] we shall express the group delay in terms of the normalized propagation constant b defined as

$$b = 1 - \left(\frac{ua}{V}\right)^2 = \frac{\beta^2/k^2 - n_2^2}{n_1^2 - n_2^2} \tag{3.29}$$

For small values of the index difference $\Delta = (n_1 - n_2)/n_1$, Eq. (3.29) can be approximated by

$$b \approx \frac{\beta/k - n_2}{n_1 - n_2} \tag{3.30}$$

Solving Eq. (3.30) for β, we have

$$\beta \approx n_2 k(b\Delta + 1) \tag{3.31}$$

With this expression for β and using the assumption that n_2 is not a function of wavelength, we find that the group delay τ_{wg} arising from waveguide dispersion is

$$\tau_{wg} = \frac{L}{c}\frac{d\beta}{dk} = \frac{L}{c}\left[n_2 + n_2\Delta\frac{d(kb)}{dk}\right] \tag{3.32}$$

The modal propagation constant β is obtained from the eigenvalue equation expressed by Eq. (2.54) and is generally given in terms of the normalized frequency V defined by Eq. (2.57). We shall therefore use the approximation

$$V = ka\left(n_1^2 - n_2^2\right)^{1/2} \approx kan_1\sqrt{2\Delta} \tag{3.33}$$

which is valid for small values of Δ, to write the group delay in Eq. (3.32) in terms of V instead of k, yielding

$$\tau_{wg} = \frac{L}{c}\left[n_2 + n_2\Delta\frac{d(Vb)}{dV}\right] \tag{3.34}$$

The first term in Eq. (3.34) is a constant and the second term represents the group delay arising from waveguide dispersion. The factor $d(Vb)/dV$ can be expressed as[37]

$$\frac{d(Vb)}{dV} = b\left[1 - \frac{2J_\nu^2(ua)}{J_{\nu+1}(ua)J_{\nu-1}(ua)}\right] \tag{3.35}$$

where u is defined by Eq. (2.48) and a is the fiber radius. This factor is plotted in Fig. 3.14 as a function of V for various LP modes. The plots show that, for a fixed value of V, the group delay is different for every guided mode. When a light pulse is launched into a fiber, it is distributed among many guided modes. These various modes arrive at the fiber end at different times depending on their group delay, so that a pulse spreading results. For multimode fibers the waveguide dispersion is generally very small compared with material dispersion and can therefore be neglected.

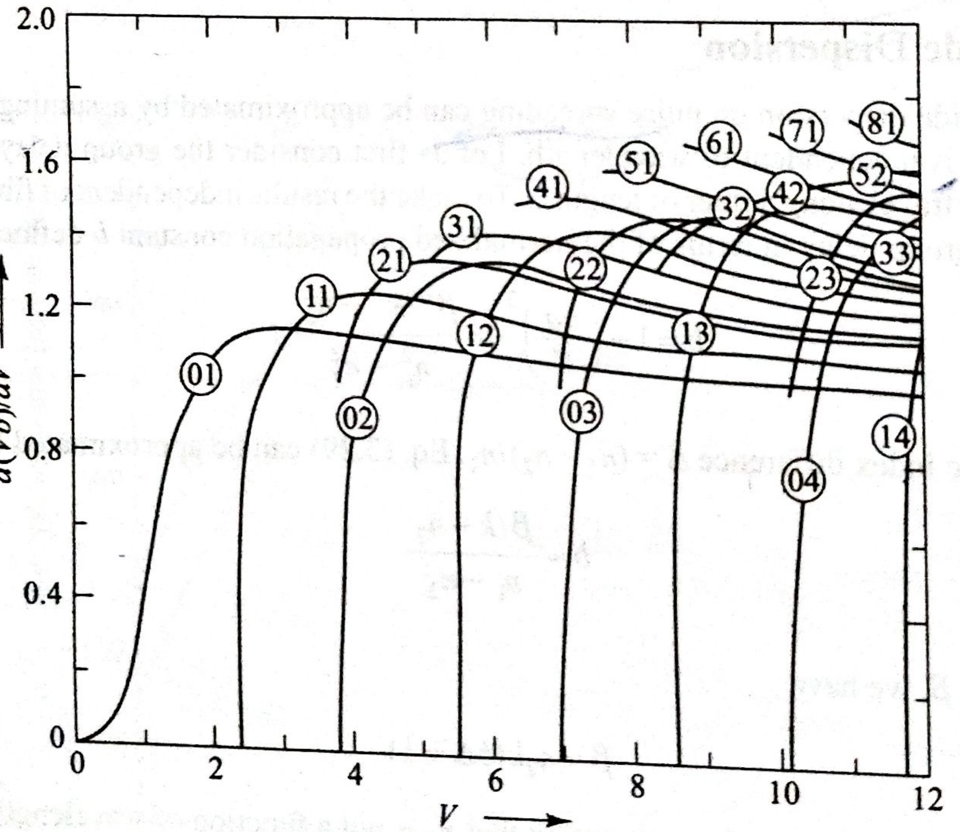

Fig. 3.14 *The group delay arising from waveguide dispersion as a function of the V number for a step-index optical fiber. The curve numbers jm designate the LP_{jm} modes. (Reproduced with permission from Gloge[37])*

3.2.7 Dispersion in Single-Mode Fibers

For single-mode fibers, waveguide dispersion is of importance and can be of the same order of magnitude as material dispersion. To see this, let us compare the two dispersion factors. The pulse spread σ_{wg} occurring over a distribution of wavelengths σ_λ is obtained from the derivative of the group delay with respect to wavelength:[37]

$$\sigma_{wg} = \left|\frac{d\tau_{wg}}{d\lambda}\right|\sigma_\lambda = L|D_{wg}(\lambda)|\sigma_\lambda$$
$$= \frac{V}{\lambda}\left|\frac{d\tau_{wg}}{dV}\right|\sigma_\lambda = \frac{n_2 L\Delta\sigma_\lambda}{c\lambda} V\frac{d^2(Vb)}{dV^2} \tag{3.36}$$

where $D_{wg}(\lambda)$ is the *waveguide dispersion*.

To see the behavior of the waveguide dispersion, consider the expression of the factor ua for the lowest-order mode (i.e., the HE_{11} mode or, equivalently, the LP_{01} mode) in the normalized propagation constant. This can be approximated by[37]

$$ua = \frac{(1+\sqrt{2})V}{1+(4+V^4)^{1/4}} \tag{3.37}$$

Substituting this into Eq. (3.29) yields, for the HE_{11} mode,

$$b(V) = 1 - \frac{(1+\sqrt{2})^2}{[1+(4+V^4)^{1/4}]^2} \tag{3.38}$$

Figure 3.15 shows plots of this expression for b and its derivatives $d(Vb)/dV$ and $Vd^2(Vb)/dV^2$ as functions of V.

Example 3.12

From Eq. (3.36) we have that the waveguide dispersion is

$$D_{wg}(\lambda) = -\frac{n_2\Delta}{c}\frac{1}{\lambda}\left[V\frac{d^2(Vb)}{dV^2}\right]$$

Let $n_2 = 1.48$ and $\Delta = 0.2$ percent. At $V = 2.4$, from Fig. 3.15 the expression in square brackets is 0.26. Choosing $\lambda = 1320$ nm, we then have $D_{wg}(\lambda) = -1.9$ ps/(nm · km).

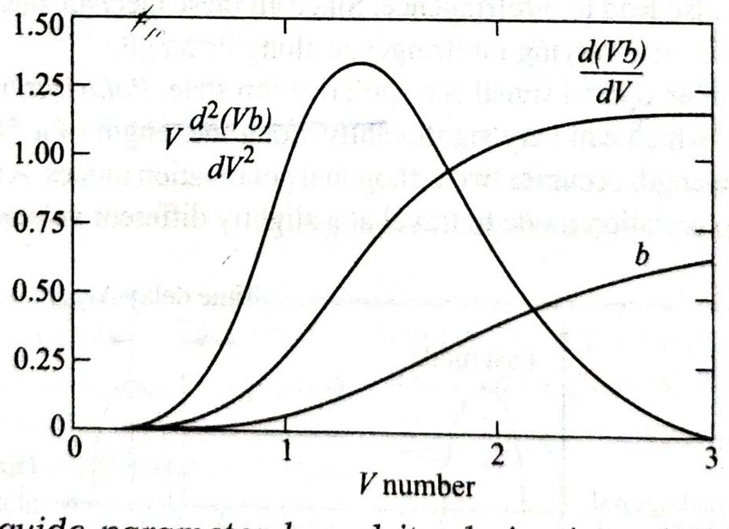

Fig. 3.15 *The waveguide parameter b and its derivatives d(Vb)/dV and Vd²(Vb)/dV² plotted as a function of the V number for the HE_{11} mode*

Figure 3.16 gives examples of the magnitudes of material and waveguide dispersion for a fused-silica-core single-mode fiber having $V = 2.4$. Comparing the waveguide dispersion with the material dispersion, we see that for a standard non-dispersion-shifted fiber, waveguide dispersion is important around 1320 nm. At this

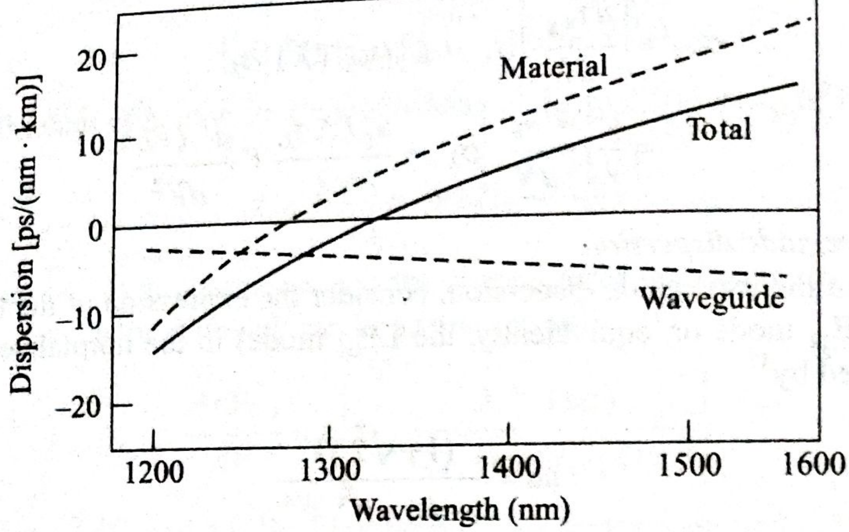

Fig. 3.16 *Examples of the magnitudes of material and waveguide dispersion as a function of optical wavelength for a single-mode fused-silica-core fiber. (Reproduced with permission from Keck,[16] © 1985, IEEE)*

point, the two dispersion factors cancel to give a zero total dispersion. However, material dispersion dominates waveguide dispersion at shorter and longer wavelengths; for example, at 900 nm and 1550 nm. This figure used the approximation that material and waveguide dispersions are additive.

3.2.8 Polarization-Mode Dispersion

The effects of fiber birefringence on the polarization states of an optical signal are another source of pulse broadening. This is particularly critical for high-rate long-haul transmission links (e.g., 10 and 40 Gb/s over tens of kilometers). Birefringence can result from intrinsic factors such as geometric irregularities of the fiber core or internal stresses on it. Deviations of less than 1 percent in the circularity of the core can already have a noticeable effect in a high-speed lightwave system. In addition, external factors, such as bending, twisting, or pinching of the fiber, can also lead to birefringence. Since all these mechanisms exist to some extent in any field-installed fiber, there will be a varying birefringence along its length.

A fundamental property of an optical signal is its polarization state. *Polarization* refers to the electric-field orientation of a light signal, which can vary significantly along the length of a fiber. As shown in Fig. 3.17, signal energy at a given wavelength occupies two orthogonal polarization modes. A varying birefringence along its length will cause each polarization mode to travel at a slightly different velocity. The resulting difference

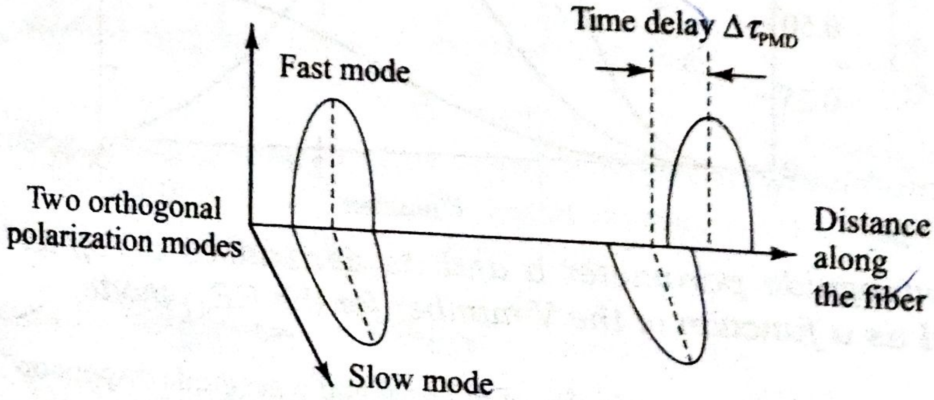

Fig. 3.17 *Differences in the polarization-mode propagation times as an optical pulse passes through a fiber with varying birefringence along its length*

in propagation times $\Delta\tau_{PMD}$ between the two orthogonal polarization modes will result in pulse spreading. This is the *polarization-mode dispersion* (PMD).[38,39] If the group velocities of the two orthogonal polarization modes are V_{gx} and V_{gy}, then the differential time delay $\Delta\tau_{PMD}$ between the two polarization components during propagation of the pulse over a distance L is

$$\Delta\tau_{PMD} = \left| \frac{L}{V_{gx}} - \frac{L}{V_{gy}} \right| \tag{3.39}$$

An important point to note is that, in contrast to chromatic dispersion, which is a relatively stable phenomenon along a fiber, PMD varies randomly along a fiber. A principal reason for this is that the perturbations causing the birefringence effects vary with temperature and stress dynamics. In practice, the effect of these perturbations shows up as a random, time-varying fluctuation in the value of the PMD at the fiber output. Thus $\Delta\tau_{PMD}$ given in Eq. (3.39) cannot be used directly to estimate PMD. Instead, statistical estimations are needed to account for its effects.

A useful means of characterizing PMD for long fiber lengths is in terms of the mean value of the differential group delay (see Chapter 14 for PMD measurement techniques). This can be calculated according to the relationship

$$\Delta\tau_{PMD} \approx D_{PMD}\sqrt{L} \tag{3.40}$$

where D_{PMD}, which is measured in ps/$\sqrt{\text{km}}$, is the average PMD parameter. Typical values of D_{PMD} range from 0.05 to 1.0 ps/$\sqrt{\text{km}}$. As an example, one experiment measured values of PMD for three types of cable installations that were subjected to different environments.[40] The setups were a 36-km spooled fiber in a temperature-controlled chamber, a 48.8-km buried cable, and a 48-km aerial cable. Over a 12- to 15-h period, the average PMD parameters were measured to be 0.028, 0.29, and 1.28 ps/$\sqrt{\text{km}}$, respectively. The larger value of PMD for the aerial cable is caused by both gradual and rapid stress variations in the fiber due to temperature fluctuations or from sudden movements of the fiber due to wind.[41]

To keep the probability of errors due to PMD low, a standard limit on the maximum tolerable value of $\Delta\tau_{PMD}$ ranges between 10 and 20 percent of a bit duration. Thus $\Delta\tau_{PMD}$ should be no more than 10 to 20 ps for 10-Gb/s data rates and 3 ps at 40 Gb/s. For example, taking the lower tolerance limit, this means that for a 10-Gb/s link that has 20 spans of 80 km each, the PMD of the transmission fiber must be less than 0.2 ps/$\sqrt{\text{km}}$. Various optical and electronic means to monitor and mitigate PMD in a fiber have been investigated.[42–48] In addition, fibers with low polarization-mode dispersion are being developed and characterized.[49–50]

Example 3.13

Consider a 160-km fiber link on which data is being sent at a bit rate B = 10 Gb/s. Assume that the maximum tolerable delay due to PMD is 10 percent of a bit period, so that $\Delta\tau_{PMD} < 0.1/B = 0.1/(10 \times 10^9\ \text{s}) = 10^{-11}\ \text{s} = 10\ \text{ps}$. Thus for this link, from Eq. (3.40) the PMD must satisfy the condition

$$D_{PMD} < 0.1\Delta\tau_{PMD}/\sqrt{L} = 10\,\text{ps}/\sqrt{1600\ \text{km}} = 0.25\ \text{ps}/\sqrt{\text{km}}$$

Drill Problem 3.9

Consider a 160-km fiber link on which data is being sent at a bit rate B = 40 Gb/s. Assume that the maximum tolerable delay due to PMD is 20 percent of a bit period. Show that the tolerable PMD of the transmission fiber should be less than 0.13 ps/$\sqrt{\text{km}}$.