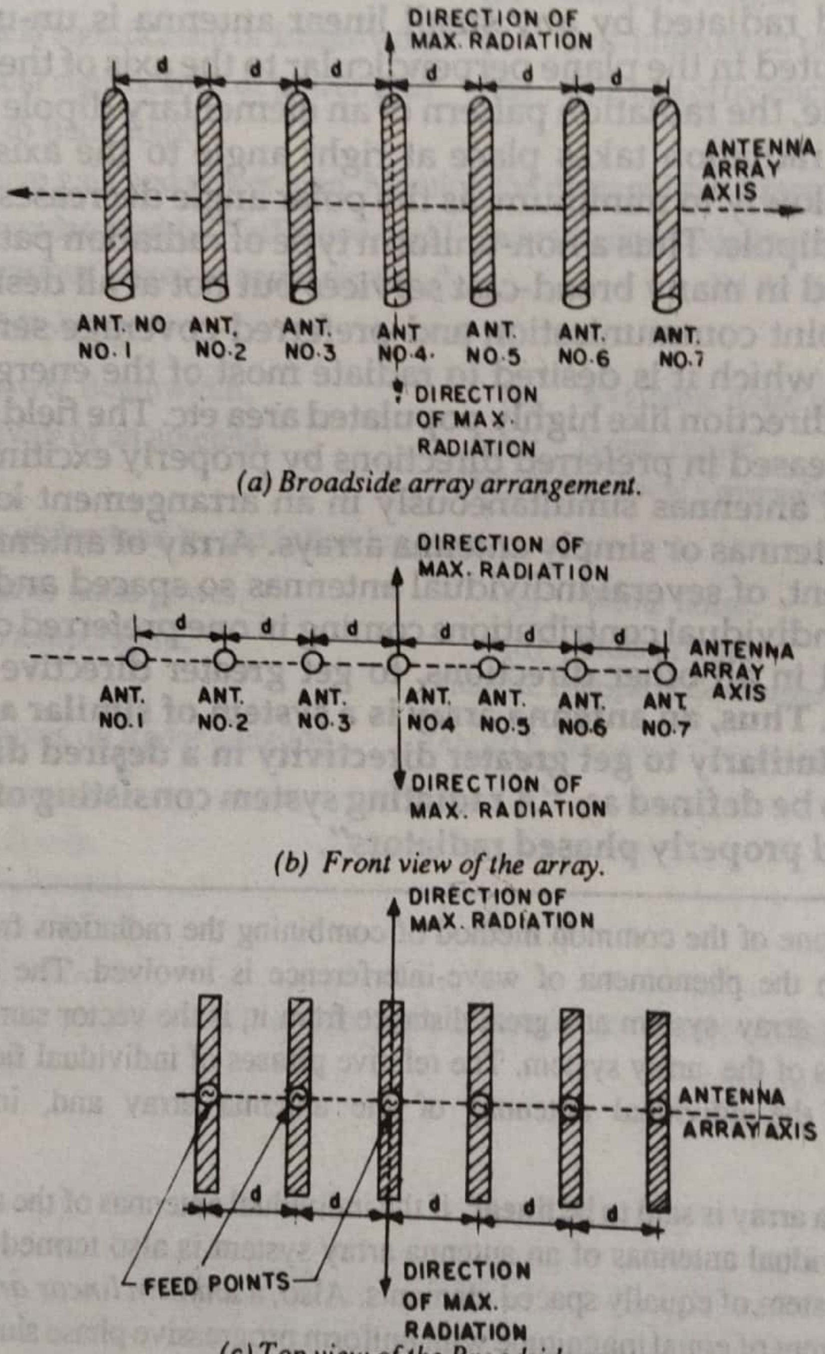
7.2. VARIOUS FORMS OF ANTENNA ARRAYS

Various antenna arrays used in practice are the following :

7.2.1. *Broadside Array* (AMIETE, Nov. 1968, 69, 70, 71, 77, 78, Dec. 1981)

This is one of the important antenna arrays used in practice. Broad-side array is one in which a number of identical parallel antennas are set up along a line drawn perpendicular to their respective axes as shown in Fig 7.1 In the broad-side array, individual antennas (or elements) are equally spaced along a line and each element is fed with current of equal magnitude, all in the same phase. By doing so, this arrangement fires in



(a) Broadside array arrangement.

(b) Front view of the array.

(c) Top view of the Broadside array.

Fig. 7.1.

broad-side directions (i.e. perpendicular to the line of array axis) where there are maximum radiations and relatively a little radiations in other directions and hence the radiation pattern broadside array is bidirectional. The broadside array is bidirectional which radiates equally well in either direction of maximum radiations. Therefore, *broadside array may be defined as "An arrangement in which the principal direction radiation is perpendicular to the array axis and also to the plane containing the array element"*.

It may, however, be noted that the bidirectional pattern of a broadside array can be converted into unidirectional by installing an identical array behind this array at distance $\lambda/4$ and exciting it by current leading in phase by 90° or $\pi/2$ radian. Further, broadside arrays may also be arranged in vertical in which case the radiation pattern would be horizontal. In the Fig. 7.1 elements are arranged horizontal and its radiation pattern is vertical normal to the plane of element as in Fig. 7.2 Lastly a **broad-side couplet** is said to form if two isotropic radiators operate in phase thereby they reinforce each other most strongly in the plane right angles to the line joining them.

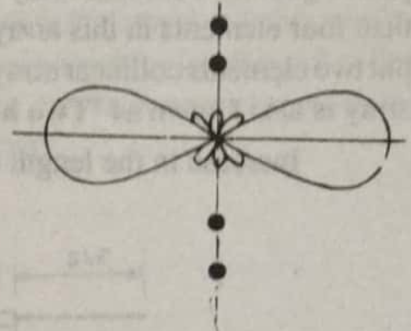


Fig. 7.2. Radiation pattern of Broadside array.

7.2.2. *End Fire Arrays* (AMIETE. Nov. 1969, 70, 77, 78)

The end-fire array is nothing but broadside array except that individual elements are fed in, out of phase (**usually 180^0**). Thus in the end-fire array, a number of identical antennas are spaced equally along a line and individual elements are fed with currents of equal magnitude but their phases varies progressively along the line in such a way as to make the entire arrangement substantially unidirectional. In otherwords, individual elements are excited in such a manner that a progressive phase difference between adjacent elements (in cycles) becomes equal to the spacing (in wavelength) between the elements. Therefore, end fire array may be defined as "*The arrangement in which the principal direction of radiation coincides with the direction of the array axis*". This is illustrated in Fig. 7.3.

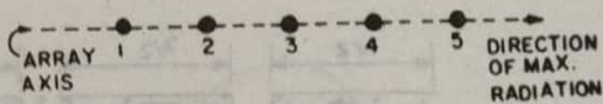


Fig. 7.3. Front view of an end fire array.

It may be noted, however, that an end fire array may be bidirectional also. One such example is a two elements array, fed with equal current, 180° out of phase.

Lastly an end-fire couplet is said to form, if two equal radiators are operated in phase quadrature at a distance of $\lambda/4$ apart.

7.2.3. *Collinear Arrays* (AMIETE, Nov. 1969)

In collinear array, the antennas are arranged co-axially i.e. antennas are mounted end to end in a single line. In otherwords, one antenna is stacked over another antenna as shown in Fig. 7.4 and 7.5.

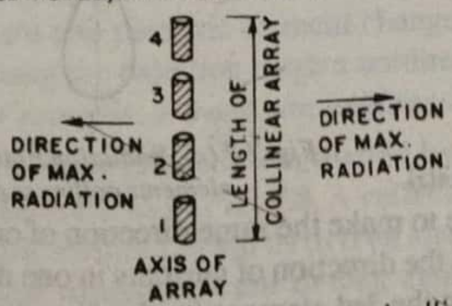


Fig. 7.4. 4 antennas (vertical) arranged collinearly.

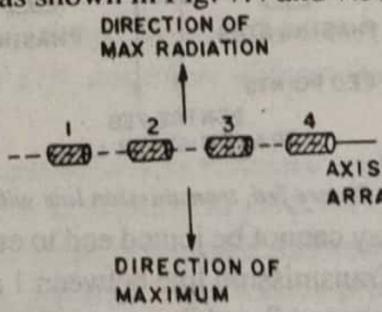


Fig. 7.5. 4 horizontal antennas arranged collinearly.

The individual elements are *fed with equal in phase currents as is the case in the broad side arrays. A collinear array is a broad-side radiators, in which the direction of maximum radiation is perpendicular to the line of antenna*. This arrangement gives radiation pattern which, when viewed through the major axis, closely resembles with the radiation pattern of a broadside array. But the radiation pattern of a collinear array has circular symmetry with its main lobe everywhere perpendicular to the principal axis. That is why, *a collinear array is also sometimes called as broad cast or omnidirectional arrays.*

The gain of collinear arrays is maximum when the spacing between elements is of order of

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7.7. ARRAY OF *n* ISOTROPIC SOURCES OF EQUAL AMPLITUDE AND SPACING (BROADSIDE CASE)

An array is said to be broadside array, if phase angle is such that it makes maximum radiation perpendicular to the line of array *i.e.* 90° and 270°. In broadside array sources are in phase i.e. $\alpha = 0$ and $\psi = 0$ for max. must be satisfied.

$$\therefore \quad \psi = \beta d \cos\theta + \alpha = \beta d \cos\theta + 0$$

or $$\beta d \cos\theta = 0$$

or $$\cos\theta = 0$$

$$\theta = 90^\circ \text{ or } 270^\circ$$

The principal maxima occurs in these directions.

The other pattern maxima, pattern minima and Beam width of major lobes will be calculated.

7.7.1. ***Directions of pattern maxima.*** For array of *n* isotropic point sources of equal amplitude and spacing S.A. Schelkunoff procedure using the eqn. 7.30 (b) may be used as below. The other minor lobes maxima occur between first nulls and high order nulls. From eqn. 7.30 (b)

$$E_t = E_0 \frac{\sin n\ \psi/2}{\sin \psi/2}$$

This is maximum when numerator is maximum *i.e.* $\sin\ n\ \psi/2$ is maximum provided $\sin \psi/2 \neq 0$

$$\therefore \quad \sin n\,\psi/2 = 1$$

or $$n\,\psi/2 = \pm\,(2N+1)\,\pi/2$$

where $$N = 1, 2, 3, 4 \ldots\ldots\ldots\ldots$$

while $$N = 0 \text{ corresponds to Major lobe maxima}$$

or $$\psi/2 = \pm\,(2N+1)\,\pi/2 \cdot 1/n$$

or $$\psi = \pm(2N+1)\,\pi/n$$

or $$\beta d \cos(\theta_{max})_{min} + \alpha = \pm\,(2N+1)\,\frac{\pi}{n}$$

or $$\beta d \cos(\theta_{max})_{min} = \pm\,(2N+1)\,\frac{\pi}{n} - \alpha$$

or $$\cos\,(\theta_{max})_{minor} = \left\{\frac{1}{\beta d}\left[\pm\,\frac{(2N+1)\,\pi}{n} - \alpha\right]\right\}$$

or $$\boxed{(\theta_{max})_{minor} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm\,\frac{(2N+1)\,\pi}{n} - \alpha\right]\right\}} \qquad \ldots (7.34)$$

where $\theta_{m \cdot max}$ = minor lobe maxima.

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For a broadside array $\alpha = 0$

$$\therefore \quad (\theta_{minor})_{max} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm\frac{(2N+1)\pi}{n}\right]\right\} = \cos^{-1}\left\{\frac{\lambda}{2\pi d}\left[\pm\left(\frac{2n+1}{n}\right)\pi\right]\right\}$$

$$\boxed{(\theta_{max})_{minor} = \cos^{-1}\left\{\pm\frac{(2N+1)\lambda}{2nd}\right\}} \qquad \text{... 7.35 (a)}$$

For example,

Let $n = 4$; $d = \lambda/2$; $\alpha = 0$

Then $$(\theta_{max})_{min} = \cos^{-1}\left\{\pm\frac{(2N+1)\lambda}{2.4\,\lambda/2}\right\} = \cos^{-1}\left\{\pm\left(\frac{2N+1}{4}\right)\right\}$$

or $$(\theta_{max})_{min} = \cos^{-1}\left(\pm\frac{3}{4}\right) \qquad \text{if } N = 1$$

$$= \pm\, 0.7500 \qquad \text{from table}$$

$$\boxed{(\theta_{max})_{min} = \pm\, 41.4^\circ \text{ or } \pm\, 138.6^\circ}$$

Thus $+ 41.4$, $+ 138.6^\circ$, $- 41.4^\circ$ and $- 138.6^\circ$ are the 4 minor lobe maxima of the array of 4 isotropic sources, in phase, spaced $\lambda/2$ apart. No other maxima exist because for $N = 2$, $\cos(\theta_{max})_{min} = +\frac{5}{4}$ i.e. > 1 where as cosine function is always < 1. Major lobe maxima occur when $\psi = 0$ which gives $\theta = 90^\circ$ or 270°. Thus the pattern can now be drawn as Fig. 7.23.

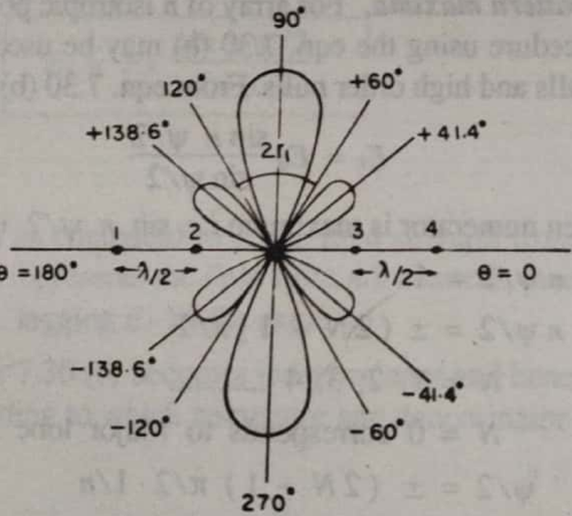

Fig. 7.23. Four isotropic sources of equal amplitude and phase, field pattern in broadside case.

7.7.2. *Direction of pattern minima.* According to S.A. Schelkunoff the directions of minima of minor lobes in the array of n isotropic sources of equal amplitude and phase is given when

$$E_t = E_0\,\frac{\sin n\,\psi/2}{\sin \psi/2} = 0$$

or $$\sin n\,\psi/2 = 0 \qquad \Big|\ \text{provided } \sin\psi/2 \neq 0 \text{ where } N = 1, 2, 3$$

or $$n\,\psi/2 = \pm\, N\pi$$

or $$\psi = \pm\,\frac{2N\pi}{n}$$

or $$\beta d\,(\cos\theta_{min})_{minor} + \alpha = \pm\,\frac{2N\pi}{n} \qquad \text{... 7.35 (b)}$$

or $$(\cos\theta_{min})_{minor} = \frac{1}{\beta d}\left\{\pm\,\frac{2N\pi}{n} - \alpha\right\}$$

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or
$$\boxed{(\theta_{min})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n} - \alpha\right\}\right]} \quad \text{... 7.35 (c)}$$

This is the general eqn. which gives the direction of minor lobe minima.

For broad side $\alpha = 0$

$$(\theta_{min})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left(\pm\frac{2N\pi}{n}\right)\right]$$

$$= \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(\pm\frac{2N\pi}{n}\right)\right] = \cos^{-1}\left[\pm\frac{2N\lambda}{2nd}\right]$$

$$\boxed{(\theta_{min})_{minor} = \cos^{-1}\left[\pm\frac{N\lambda}{nd}\right]} \quad \text{... (7.36)}$$

For example, if $n = 4$, $d = \lambda/2$, $\alpha = 0$

$$(\theta_{min})_{minor} = \cos^{-1}\left[\pm\frac{1\cdot\lambda}{4\cdot\lambda/2}\right] = \cos^{-1}\left[\pm\frac{1}{2}\right] \quad [\text{if } N = 1]$$

$$(\theta_{min})_{minor} = \pm 60^\circ, \pm 120^\circ \quad \text{... (7.37)}$$

Also
$$(\theta_{min})_{minor} = \cos^{-1}\left[\pm\frac{2\cdot\lambda}{4.\lambda/2}\right] \quad \text{if } N = 2$$

$$= \cos^{-1}[\pm 1] = \pm 0^\circ, \pm 180^\circ = 0^\circ, 180^\circ \quad \text{... (7.38)}$$

Thus 0°, 60°, 120°, $+180^\circ$, -60°, -120° are six minor lobe minima of the array of 4 isotropic sources in phase spaced $\lambda/2$ apart (Fig. 7.23). No other minima exist because cosine functions becomes more than one which is not possible.

7.7.3. *Beam width of Major lobe.* It is defined as,

(i) The angle between first nulls or

(ii) Double the angle between first null and major lobe maximum directions.

It is denoted by the complementary angle $\gamma = 90 - \theta$ because the beam width of major lobe is usually small and hence the complementary angle γ will be a better deal. Thus

Beam width of major lobe = 2 × Angle between first null and maximum of major lobe

$$BWFN = 2 \times \gamma \quad \text{... (7.39)}$$

From eqn. 7.36 $(\theta_{min}) = \cos^{-1}\left[\pm\frac{N\lambda}{nd}\right]$ | $\because\ 90 - \theta_m = \gamma$ or $90 - \gamma = \theta_m$

or
$$(90 - \gamma) = \pm\cos^{-1}\left[\pm\frac{N\lambda}{nd}\right]$$

or
$$\cos(90 - \gamma) = \sin\gamma = \pm\frac{N\lambda}{nd} \quad \text{... (7.40)}$$

$\because$ γ is very small $\quad\therefore\quad \sin\gamma \equiv \gamma$

$$\therefore \quad \gamma = \pm\frac{N\lambda}{nd}$$

$$\boxed{\gamma \simeq \pm\frac{N\lambda}{nd}}$$

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First null occurs when $N = 1$

$$\therefore \quad \gamma_1 = \frac{+\lambda}{n\,d}$$

$$\therefore \quad \boxed{BWFN = 2 \times \gamma_1 = \frac{2 \times \lambda}{n\,d} = \frac{2\lambda}{n\,d}} \quad \text{... (7.41)}$$

If the broadside array is large, so that $N\lambda >> nd$, then

$$2\gamma_1 = \frac{2\lambda}{n\,d} = \frac{2\lambda}{L}$$

where L = Total length of the array in metre $= (n-1)\,d \simeq nd$ [if n is large]

$$\therefore \quad 2\gamma_1 = \frac{2\lambda}{L} = \frac{2}{L/\lambda} \text{ radian} = \frac{2 \times 57.3}{L/\lambda} \text{ degree}$$

or

$$\boxed{2\gamma_1 = \frac{114.6^\circ}{L/\lambda} = BWFN} \quad \text{... (7.42)}$$

Still another commonly used parameter is half power beam width ($HPBW$) which is approximately half of $BWFN$

$$\therefore \quad \boxed{HPBW = \frac{BWFN}{2} = \frac{1}{L/\lambda} \text{ radian} = \frac{57.3}{L/\lambda} \text{ degree}} \quad \text{... (7.43)}$$

7.8. ARRAY OF *n* SOURCES OF EQUAL AMPLITUDE AND SPACING (END-FIRE CASE)

For an array to be end fire, the phase angles is such that makes the maximum radiation in the line of array i.e. $\theta = 0$ or 180°. Thus for an array to be end fire $\psi = 0$ and $\theta = 0^\circ$ or 180°. This requires

$$\psi = \beta\,d\cos\theta + \alpha$$

i.e.

$$0 = \beta\,d\cos 0^\circ + \alpha$$

or

$$\alpha = -\beta\,d = -\frac{2\pi d}{\lambda} \quad \text{... (7.44)}$$

This indicates that the phase difference between the sources of an end fire is retarded progressively by same amount as the spacing between the sources in radians. For example, if spacing between two sources is $\lambda/2$ or $\lambda/4$, then the phase angle by which angle by which source 2 lags behind source is $\frac{2\pi}{\lambda} \times \frac{\lambda}{2}$ or $\frac{2\pi}{\lambda} \times \frac{\lambda}{4}$ i.e. π or $\pi/2$ radians respectively.

7.8.1. *Direction of Pattern Maxima.* According to S.A. Schelkunoff, other pattern maxima can be obtained from eqn. 7.30 (b) which is maximum when

$$\sin n\psi/2 = 1 \quad \text{if } \sin\psi/2 \neq 0$$

or

$$n\psi/2 = \pm(2N+1)\pi/2$$

or

$$n\psi/2 = \pm(2N+1)\pi$$

or

$$\psi = \pm\frac{(2N+1)\pi}{n}$$

For end fire case, $\alpha = -\beta d$, $\psi = 0$

$$\therefore \quad \psi = \beta\,d\cos(\theta_{max})_{min} + \alpha = \pm\frac{(2N+1)\pi}{n}$$

or

$$\beta\,d(\cos\theta_{max}) - \beta\,d = \pm\frac{(2N+1)\pi}{n}$$

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$$\beta d(\cos\theta_{max} - 1) = \pm\frac{(2N+1)\pi}{n}$$

$$(\cos\theta_{max} - 1) = \pm\frac{(2N+1)\pi}{\beta n d}$$

or

$$\cos\theta_{max} = \pm\frac{(2N+1)\pi}{\beta n d} + 1$$

or

$$\boxed{(\theta_{max})_{minor} = \cos^{-1}\left[\pm\frac{(2N+1)\pi}{\beta n d} + 1\right]} \quad \text{... (7.45)}$$

If $n = 4,\ d = \lambda/2,\ \alpha = -\pi$

$$(\theta_{max})_1 = \cos^{-1}\left[\frac{(2\cdot 1 + 1)\pi}{2\frac{\pi}{\lambda}\,4\,\lambda/2} + 1\right] \quad \text{if, } N = 1$$

$$= \cos^{-1}\left[\pm\frac{3}{4} + 1\right] = \cos^{-1}\left[\frac{7}{4}, \frac{1}{4}\right] = \cos^{-1}\left[\frac{7}{4}\right],\ \cos^{-1}\left[\frac{1}{4}\right]$$

$$= \cos^{-1}[0\cdot 25] = 75.5^\circ. \qquad \Big|\ \cos^{-1}\left(\frac{7}{4}\right) \text{ does not exist.}$$

$$(\theta_{max})_2 = \cos^{-1}\left[\pm\frac{(2\times 2 + 1)\pi}{\frac{2\pi}{\lambda}\cdot 4\cdot\lambda/2} + 1\right] \quad \text{if } N = 2$$

$$= \cos^{-1}\left[\pm\frac{5}{4} + 1\right] = \cos^{-1}\left[-\frac{1}{4}\right] = \cos^{-1}[-0.25] = -75.5^\circ.$$

7.8.2. ***Directions of Pattern Minima*** is obtained by putting $\alpha = -\beta d$ in eqn. 7.35. (b)

$$\beta d\cos(\theta_{min}) + \alpha = \pm\frac{2N\pi}{n}$$

7.35

∴

$$\beta d\cos(\theta_{min}) - \beta d = \pm\frac{2N\pi}{n}$$

$$\beta d\{(\cos\theta_{min}) - 1\} = \pm\frac{2N\pi}{n}$$

$$(\cos\theta_{min} - 1) = \pm\frac{2N\pi}{\beta n d} = \pm\frac{2N\pi}{\frac{2\pi}{\lambda}\cdot n\cdot d} = \pm\frac{N\lambda}{nd}$$

or

$$\left(1 - 2\sin^2\frac{\theta_{min}}{2} - 1\right) = \pm\frac{N\lambda}{nd} \qquad \Big|\ \because \cos\theta = 1 - 2\sin^2\frac{\theta}{2}$$

or

$$-2\sin^2\frac{\theta_{min}}{2} = \pm\frac{N\lambda}{nd}$$

$$\sin^2\frac{\theta_{min}}{2} = \pm\frac{N\lambda}{2nd}$$

or

$$\sin\frac{\theta_{min}}{2} = \pm\sqrt{\frac{N\lambda}{2nd}}$$

or

$$\frac{\theta_{min}}{2} = \sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right)$$

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or
$$\theta_{min} = 2\sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right) \qquad ...(7.46)$$

Now the pattern minima can be obtained by putting $N = 1, 2\ 3, ...$ etc. For example, $n = 4$, $d = \lambda/2$, $N = 1$.

$$(\theta_{min})_1 = 2\sin^{-1}\left(\pm\sqrt{\frac{1\lambda}{2\cdot 4\lambda/2}}\right)$$
$$= 2\sin^{-1}\left(\pm\frac{1}{2}\right) = 2\times(\pm 30^\circ) = \pm 60^\circ.$$

$$(\theta_{min})_2 = 2\sin^{-1}\left(\pm\sqrt{\frac{2\lambda}{2\cdot 4\lambda/2}}\right)$$
$$= 2\sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = 2\times(\pm 45^\circ) = \pm 90^\circ.$$

$$(\theta_{min})_3 = 2\sin^{-1}\left(\pm\sqrt{\frac{3\cdot\lambda}{2.4\lambda/2}}\right)$$
$$= 2\sin^{-1}\left(\pm\sqrt{\frac{3}{4}}\right) = 2\sin^{-1}\left(\pm\sqrt{\frac{3}{2}}\right) = 2\times(\pm 60^\circ) = \pm 120^\circ.$$

$$(\theta_{min})_4 = 2\sin^{-1}\left(\pm\sqrt{\frac{4\cdot\lambda}{2\cdot 4\lambda/2}}\right) = 2\sin^{-1}(\pm 1) = 2\times(\pm 90^\circ) = \pm 180^\circ$$

Thus the minima are situated at $+60^\circ, +90^\circ, +120^\circ, +180^\circ, -60^\circ, -90^\circ, -120^\circ$ and -180°.

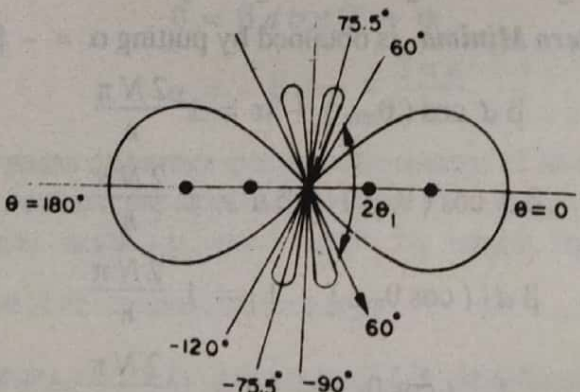

Fig. 7.24. Field pattern of end fire array consisting of 4 isotropic sources of same amplitude, spacing $\lambda/2$ and $\alpha = -\pi$.

7.8.3. *Beam width of Major lobes.* Here the complementary angle γ, as has been used in broad-side case, is not required. Because the beam width of a end fire array is larger than broadside.

$\therefore$ Beam width $= 2\times$ Angle between first nulls and maximum of major lobes

$$\boxed{BW = 2\times\theta_1} \qquad ...(7.47)$$

From eqn. 7.46
$$\theta_{min} = 2\sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right)$$

or
$$\sin\theta_{min} \simeq (\theta_{min}) = 2\cdot\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right) = \left(\pm\sqrt{\frac{4N\lambda}{2nd}}\right)$$

$$\boxed{\theta_{min} = \pm\sqrt{\frac{2N\lambda}{nd}}} \qquad ...(7.48)$$

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If the array is long of length L then

$$L = (n - 1)\,d$$

$$L \cong n\,d$$

$$\therefore \quad \boxed{\theta_{min} = \pm\sqrt{\frac{2N\lambda}{nd}} = \pm\sqrt{\frac{2N\lambda}{L}}}$$

$\therefore$ Beam width between first nulls (*BWFN*) = $2 \times (\theta_{min})$

$$BWFN = 2 \times \left(\pm\sqrt{\frac{2N\lambda}{n\,d}}\right) = \pm\, 2\sqrt{\frac{2N\lambda}{n\,d}}$$

$$2\,\theta_1 = 2 \times \sqrt{\frac{2N}{L/\lambda}} = \pm\sqrt{\frac{2 \times 1}{L/\lambda}} \text{ rad (if } N = 1\text{)}$$

$$BWFN = \pm\, 2\sqrt{\frac{2}{L/\lambda}} = \pm\, 57.3 \times 2\sqrt{\frac{2}{L/\lambda}} \text{ degree}$$

$$\boxed{\boldsymbol{BWFN = \pm\, 114 \cdot 6\sqrt{\frac{2}{L/\lambda}}}}$$

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