CHAPTER 2

ORBITAL MECHANICS AND LAUNCHERS

2.1 ORBITAL MECHANICS

Developing the Equations of the Orbit

This chapter is about how earth orbit is achieved, the laws that describe the motion of an object orbiting another body, how satellites maneuver in space, and the determination of the look angle to a satellite from the earth using ephemeris data that describe the orbital trajectory of the satellite.

To achieve a stable orbit around the earth, a spacecraft must first be beyond the bulk of the earth's atmosphere, i.e., in what is popularly called space. There are many definitions of space. U.S. astronauts are awarded their "space wings" if they fly at an altitude that exceeds 50 miles (~80 km); some international treaties hold that the space frontier above a given country begins at a height of 100 miles (~160 km). Below 100 miles, permission must be sought to over-fly any portion of the country in question. On reentry, atmospheric drag starts to be felt at a height of about 400,000 ft (~76 miles ≈ 122 km). Most satellites, for any mission of more than a few months, are placed into orbits of at least 250 miles (≈400 km) above the earth. Even at this height, atmospheric drag is significant. As an example, the initial payload elements of the International Space Station (ISS) were injected into orbit at an altitude of 397 km when the shuttle mission left those modules on 9 June 1999. By the end of 1999, the orbital height had decayed to about 360 km, necessitating a maneuver to raise the orbit. Without onboard thrusters and sufficient orbital maneuvering fuel, the ISS would not last more than a few years at most in such a low orbit. To appreciate the basic laws that govern celestial mechanics, we will begin first with the fundamental Newtonian equations that describe the motion of a body. We will then give some coordinate axes within which the orbit of the satellite can be set and determine the various forces on the earth satellite.

Newton's laws of motion can be encapsulated into four equations:

$$s = ut + (\tfrac{1}{2})at^2 \tag{2.1a}$$

$$v^2 = u^2 + 2at \tag{2.1b}$$

$$v = u + at \tag{2.1c}$$

$$P = ma \tag{2.1d}$$

where s is the distance traveled from time $t = 0$; u is the initial velocity of the object at time $t = 0$ and v the final velocity of the object at time t; a is the acceleration of the object; P is the force acting on the object; and m is the mass of the object. Note that the acceleration can be positive or negative, depending on the direction it is acting with respect to the velocity vector. Of these four equations, it is the last one that helps us understand the motion of a satellite in a stable orbit (neglecting any drag or other perturbing forces). Put into words, Eq. (2.1d) states that the force acting on a body is equal to the mass of the body multiplied by the resulting acceleration of the body. Alternatively, the resulting acceleration is the ratio of the force acting on the body to the mass of the body. Thus, for a given force, the lighter the mass of the body, the higher the acceleration will be. When in a stable orbit, there are two main forces acting on a satellite: a centrifugal force due to the kinetic energy of the satellite, which attempts to fling the satellite into a higher orbit, and a centripetal force due to the gravitational attraction of the planet about which the satellite is orbiting, which attempts to pull the satellite down toward the planet. If these two forces are equal, the satellite will remain in a stable orbit. It will continually fall toward the planet's surface as it moves forward in its orbit but, by virtue of its orbital velocity, it will have moved forward just far enough to compensate for the "fall" toward the planet and so it will remain at the same orbital height. This is why an object in a stable orbit is sometimes described as being in "free fall." Figure 2.1 shows the two opposing forces on a satellite in a stable orbit[1].

Force = mass × acceleration and the unit of force is a Newton, with the notation *N*. A Newton is the force required to accelerate a mass of 1 kg with an acceleration of 1 m/s^2. The underlying units of a Newton are therefore (kg) × m/s^2. In Imperial Units, one Newton = 0.2248 ft lb. The standard acceleration due to gravity at the earth's surface is 9.80665×10^{-3} km/s^2, which is often quoted as 981 cm/s^2. This value decreases

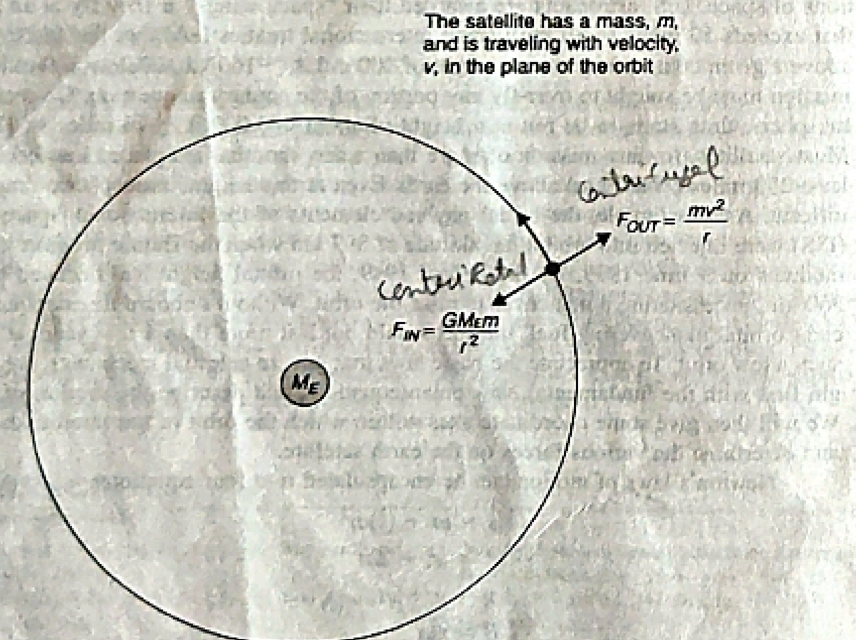


FIGURE 2.1 Forces acting on a satellite in a stable orbit around the earth (from Fig. 3.4 of reference 1). Gravitational force is inversely proportional to the square of the distance between the centers of gravity of the satellite and the planet the satellite is orbiting, in this case the earth. The gravitational force inward (F_{IN}, the centripetal force) is directed toward the center of gravity of the earth. The kinetic energy of the satellite (F_{OUT}, the centrifugal force) is directed diametrically opposite to the gravitational force. Kinetic energy is proportional to the square of the velocity of the satellite. When these inward and outward forces are balanced, the satellite moves around the earth in a "free fall" trajectory: the satellite's orbit. For a description of the units, please see the text.

with height above the earth's surface. The acceleration, a, due to gravity at a distance r from the center of the earth is[1]

$$a = \mu/r^2 \text{ km/s}^2 \tag{2.1}$$

where the constant μ is the product of the universal gravitational constant G and the mass of the earth M_E.

The product GM_E is called Kepler's constant and has the value 3.986004418×10^5 km^3/s^2. The universal gravitational constant is $G = 6.672 \times 10^{-11}$ Nm^2/kg^2 or 6.672×10^{-20} km^3/kg s^2 in the older units. Since force = mass × acceleration, the centripetal force acting on the satellite, F_{IN}, is given by

$$F_{IN} = m \times (\mu/r^2) \tag{2.2a}$$
$$= m \times (GM_E/r^2) \tag{2.2b}$$

In a similar fashion, the centrifugal acceleration is given by[1]

$$a = v^2/r \tag{2.3}$$

which will give the centrifugal force, F_{OUT}, as

$$F_{OUT} = m \times (v^2/r) \tag{2.4}$$

If the forces on the satellite are balanced, $F_{IN} = F_{OUT}$ and, using Eqs. (2.2a) and (2.4),

$$m \times \mu/r^2 = m \times v^2/r$$

hence the velocity v of a satellite in a circular orbit is given by

$$v = (\mu/r)^{1/2} = \left(\frac{GM_E}{r}\right)^{\frac{1}{2}} \tag{2.5}$$

If the orbit is circular, the distance traveled by a satellite in one orbit around a planet is $2\pi r$, where r is the radius of the orbit from the satellite to the center of the planet. Since distance divided by velocity equals time to travel that distance, the period of the satellite's orbit, T, will be

$$T = (2\pi r)/v = (2\pi r)/[(\mu/r)^{1/2}]$$

Giving

$$T = (2\pi r^{3/2})/(\mu^{1/2}) \tag{2.6}$$

Table 2.1 gives the velocity, v, and orbital period, T, for four satellite systems that occupy typical LEO, MEO, and GEO orbits around the earth. In each case, the orbits are

TABLE 2.1 Orbital Velocity, Height, and Period of Four Satellite Systems

Satellite system	Orbital height (km)	Orbital velocity (km/s)	Orbital period (h	min	s)
Intelsat (GEO)	35,786.03	3.0747	23	56	4.1
New-ICO (MEO)	10,255	4.8954	5	55	48.4
Skybridge (LEO)	1,469	7.1272	1	55	17.8
Iridium (LEO)	780	7.4624	1	40	27.0

Mean earth radius is 6378.137 km and GEO radius from the center of the earth is 42,164.17 km.

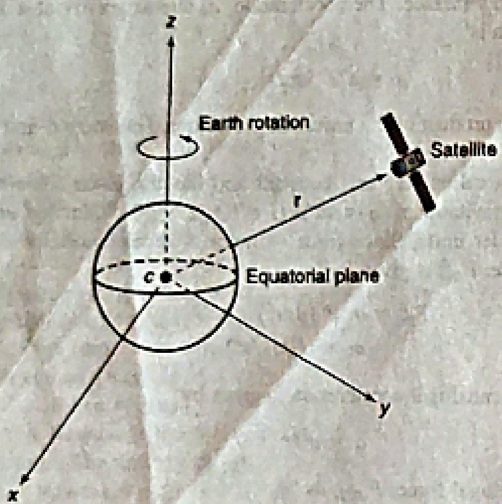


FIGURE 2.2 The initial coordinate system that could be used to describe the relationship between the earth and a satellite. A Cartesian coordinate system with the geographical axes of the earth as the principal axes is the simplest coordinate system to set up. The rotational axis of the earth is about the axis *cz*, where *c* is the center of the earth and *cz* passes through the geographic north pole. Axes *cx*, *cy*, and *cz* are mutually orthogonal axes, with *cx* and *cy* passing through the earth's geographic equator. The vector *r* locates the moving satellite with respect to the center of the earth.

circular and the average radius of the earth is taken as 6378.137 km[1]. A number of coordinate systems and reference planes can be used to describe the orbit of a satellite around a planet. Figure 2.2 illustrates one of these using a Cartesian coordinate system with the earth at the center and the reference planes coinciding with the equator and the polar axis. This is referred to as a geocentric coordinate system.

With the coordinate system set up as in Figure 2.2, and with the satellite mass m located at a vector distance r from the center of the earth, the gravitational force F on the satellite is given by

$$\bar{F} = -\frac{GM_E m\bar{r}}{r^3} \tag{2.7}$$

Where M_E is the mass of the earth and $G = 6.672 \times 10^{-11}$ Nm²/kg². But force = mass × acceleration and Eq. (2.7) can be written as

$$\bar{F} = m\frac{d^2\bar{r}}{dt^2} \tag{2.8}$$

From Eqs. (2.7) and (2.8) we have

$$-\frac{\bar{r}}{r^3}\mu = \frac{d^2\bar{r}}{dt^2} \tag{2.9}$$

Which yields

$$\frac{d^2\bar{r}}{dt^2} + \frac{\bar{r}}{r^3}\mu = 0 \tag{2.10}$$

This is a second-order linear differential equation and its solution will involve six undetermined constants called the *orbital elements*. The orbit described by these orbital elements can be shown to lie in a plane and to have a constant angular momentum. The solution to Eq. (2.10) is difficult since the second derivative of r involves the second derivative of the unit vector r. To remove this dependence, a different set of coordinates can be chosen to describe the location of the satellite such that the unit vectors in the three axes are constant. This coordinate system uses the plane of the satellite's orbit as the reference plane. This is shown in Figure 2.3.

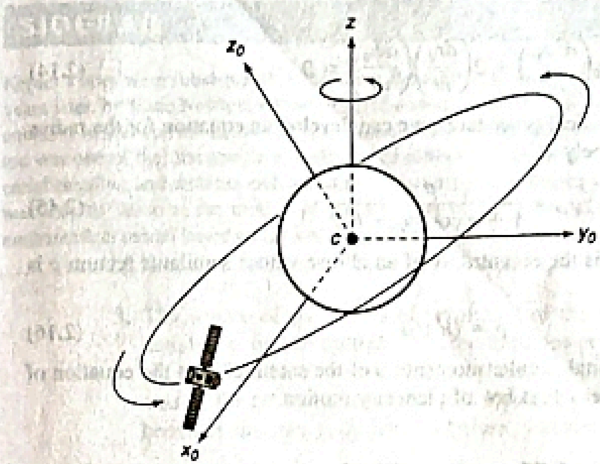


FIGURE 2.3 The orbital plane coordinate system. In this coordinate system, the orbital plane of the satellite is used as the reference plane. The orthogonal axes x_0 and y_0 lie in the orbital plane. The third axis, z_0, is perpendicular to the orbital plane. The geographical *z*-axis of the earth (which passes through the true North Pole and the center of the earth, c) does not lie in the same direction as the z_0 axis except for satellite orbits that are exactly in the plane of the geographical equator.

Expressing Eq. (2.10) in terms of the new coordinate axes x_0, y_0, and z_0 gives

$$\hat{x}_0\left(\frac{d^2x_0}{dt^2}\right) + \hat{y}_0\left(\frac{d^2y_0}{dt^2}\right) + \frac{\mu(x_0\hat{x}_0 + y_0\hat{y}_0)}{(x_0^2 + y_0^2)^{3/2}} = 0 \tag{2.11}$$

Equation (2.11) is easier to solve if it is expressed in a polar coordinate system rather than a Cartesian coordinate system. The polar coordinate system is shown in Figure 2.4.

With the polar coordinate system shown in Figure 2.4 and using the transformations

$$x_0 = r_0 \cos\phi_0 \tag{2.12a}$$
$$y_0 = r_0 \sin\phi_0 \tag{2.12b}$$
$$\hat{x}_0 = \hat{r}_0 \cos\phi_0 - \hat{\phi}_0 \sin\phi_0 \tag{2.12c}$$
$$\hat{y}_0 = \hat{\phi}_0 \cos\phi_0 + \hat{r}_0 \sin\phi_0 \tag{2.12d}$$

and equating the vector components of r_0 and ϕ_0 in turn in Eq. (2.11) yields

$$\frac{d^2r_0}{dt^2} - r_0\left(\frac{d\phi_0}{dt}\right) = -\frac{\mu}{r_0^2} \tag{2.13}$$

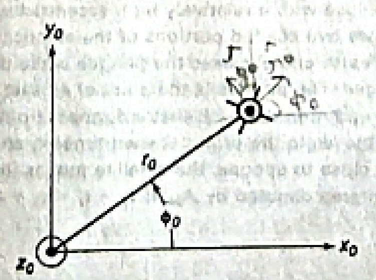


FIGURE 2.4 Polar coordinate system in the plane of the satellite's orbit. The plane of the orbit coincides with the plane of the paper. The axis z_0 is straight out of the paper from the center of the earth, and is normal to the plane of the satellite's orbit. The satellite's position is described in terms of the radius from the center of the earth r_0 and the angle this radius makes with the x_0 axis, ϕ_0.

1

and

$$r_0\left(\frac{d^2\phi_0}{dt^2}\right) + 2\left(\frac{dr_0}{dt}\right)\left(\frac{d\phi_0}{dt}\right) = 0 \tag{2.14}$$

Using standard mathematical procedures, we can develop an equation for the radius of the satellite's orbit, r_0, namely

$$r_0 = \frac{p}{1 + e\cos(\phi_0 - \theta_0)} \tag{2.15}$$

Where θ_0 is a constant and e is the eccentricity of an ellipse whose semilatus rectum p is given by

$$p = (h^2)/\mu \tag{2.16}$$

and h is magnitude of the orbital angular momentum of the satellite. That the equation of the orbit is an ellipse is Kepler's first law of planetary motion.

Kepler's Three Laws of Planetary Motion

Johannes Kepler (1571–1630) was a German astronomer and scientist who developed his three laws of planetary motion by careful observations of the behavior of the planets in the solar system over many years, with help from some detailed planetary observations by the Hungarian astronomer Tycho Brahe. Kepler's three laws are

1. The orbit of any smaller body about a larger body is always an ellipse, with the center of mass of the larger body as one of the two foci.
2. The orbit of the smaller body sweeps out equal areas in equal time (see Figure 2.5).

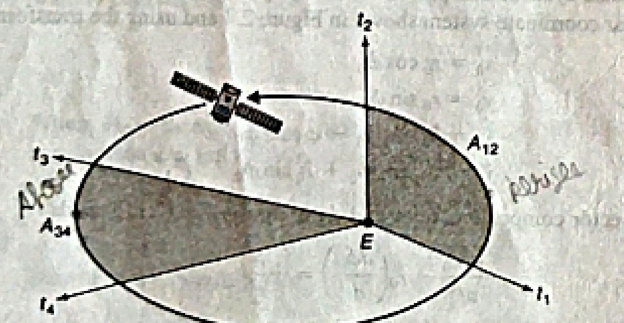


FIGURE 2.5 Illustration of Kepler's second law of planetary motion. A satellite is in orbit about the planet earth, E. The orbit is an ellipse with a relatively high eccentricity, that is, it is far from being circular. The figure shows two shaded portions of the elliptical plane in which the orbit moves, one is close to the earth and encloses the perigee while the other is far from the earth and encloses the apogee. The perigee is the point of closest approach to the earth while the apogee is the point in the orbit that is furthest from the earth. While close to perigee, the satellite moves in the orbit between times t_1 and t_2 and sweeps out an area denoted by A_{12}. While close to apogee, the satellite moves in the orbit between times t_3 and t_4 and sweeps out an area denoted by A_{34}. If $t_1 - t_2 = t_3 - t_4$ then $A_{12} = A_{34}$.

2

SIDEBAR

Kepler's laws were subsequently confirmed, about 50 years later, by Isaac Newton, who developed a mathematical model for the motion of the planets. Newton was one of the first people to make use of differential calculus, and with his understanding of gravity, was able to describe the motion of planets from a mathematical model based on his laws of motion and the concept of gravitational attraction. The work was published in the *Philosophiae Naturalis Principia Mathematica* in 1687. At that time, Latin was the international language of formally educated people, much in the way English has become the international language of e-mail and business today, so Newton's *Principia* was written in Latin.

3. The square of the period of revolution of the smaller body about the larger body equals a constant multiplied by the third power of the semimajor axis of the orbital ellipse. That is, $T^2 = (4\pi^2a^3)/\mu$ where T is the orbital period, a is the semimajor axis of the orbital ellipse, and μ is Kepler's constant. If the orbit is circular, then a becomes distance r, defined as before, and we have Eq. (2.6).

Describing the orbit of a satellite enables us to develop Kepler's second two laws.

Describing the Orbit of a Satellite

The quantity θ_0 in Eq. (2.15) serves to orient the ellipse with respect to the orbital plane axes x_0 and y_0. Now that we know that the orbit is an ellipse, we can always choose x_0 and y_0 so that θ_0 is zero. We will assume that this has been done for the rest of this discussion. This now gives the equation of the orbit as

$$r_0 = \frac{p}{1 + e\cos\phi_0} \tag{2.17}$$

The path of the satellite in the orbital plane is shown in Figure 2.6. The lengths a and b of the semimajor and semiminor axes are given by

$$a = p/(1 - e^2) \tag{2.18}$$

$$b = a(1 - e^2)^{1/2} \tag{2.19}$$

The point in the orbit where the satellite is closest to the earth is called the *perigee* and the point where the satellite is farthest from the earth is called the *apogee*. The perigee and apogee are always exactly opposite each other. To make θ_0 equal to zero, we have chosen the x_0 axis so that both the apogee and the perigee lie along it and the x_0 axis is therefore the major axis of the ellipse.

The differential area swept out by the vector r_0 from the origin to the satellite in time dt is given by

$$dA = 0.5r_0^2\left(\frac{d\phi_0}{dt}\right)dt = 0.5h\,dt \tag{2.20}$$

Remembering that h is the magnitude of the orbital angular momentum of the satellite, the radius vector of the satellite can be seen to sweep out equal areas in equal times. This is Kepler's second law of planetary motion. By equating the area of the ellipse (πab) to the area swept out in one orbital revolution, we can derive an expression for the orbital period T as

$$T^2 = (4\pi^2a^3)/\mu \tag{2.21}$$

$2\pi\sqrt{\frac{a^3}{GM}}$

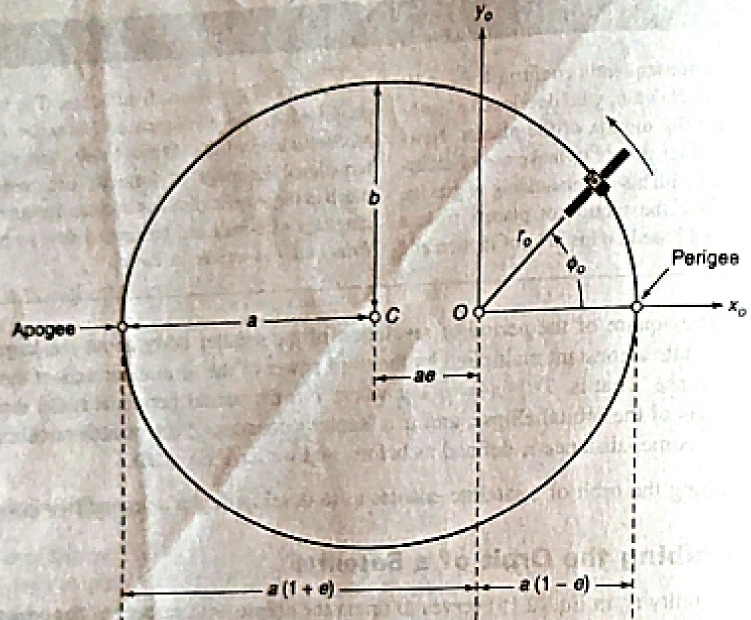


FIGURE 2.6 The orbit as it appears in the orbital plane. The point *O* is the center of the earth and the point *C* is the center of the ellipse. The two centers do not coincide unless the eccentricity, *e*, of the ellipse is zero (i.e., the ellipse becomes a circle and *a* = *b*). The dimensions *a* and *b* are the semimajor and semiminor axes of the orbital ellipse, respectively.

This equation is the mathematical expression of Kepler's third law of planetary motion: the square of the period of revolution is proportional to the cube of the semimajor axis. Note that this is the square of Eq. (2.6) and that in Eq. (2.6) the orbit was assumed to be circular such that semimajor axis a = semiminor axis b = circular orbit radius from the center of the earth r.) *Kepler's third law extends the result from Eq. (2.6), which was derived for a circular orbit, to the more general case of an elliptical orbit.* Equation (2.21) is extremely important in satellite communications systems. This equation determines the period of the orbit of any satellite, and it is used in every GPS receiver in the calculation of the positions of GPS satellites. Equation (2.21) is also used to find the orbital radius of a GEO satellite, for which the period T must be made exactly equal to the period of one revolution of the earth for the satellite to remain stationary over a point on the equator.

An important point to remember is that the period of revolution, T, is referenced to inertial space, namely, to the galactic background. The orbital period is the time the orbiting body takes to return to the same reference point in space with respect to the galactic background. Nearly always, the primary body will also be rotating and so the period of revolution of the satellite may be different from that perceived by an observer who is standing still on the surface of the primary body. This is most obvious with a geostationary earth orbit (GEO) satellite (see Table 2.1). The orbital period of a GEO satellite is exactly equal to the period of rotation of the earth, 23 h 56 min 4.1 s, but, to an observer on the ground, the satellite appears to have an infinite orbital period: it always stays in the same place in the sky.

To be perfectly geostationary, the orbit of a satellite needs to have three features: (a) it must be exactly circular (i.e., have an eccentricity of zero); (b) it must be at the correct altitude (i.e., have the correct period); and (c) it must be in the plane of the equator (i.e., have a zero inclination with respect to the equator). If the inclination of the satellite is not zero and/or if the eccentricity is not zero, but the orbital period is correct, then the satellite will be in a *geosynchronous* orbit. The position of a geosynchronous satellite will appear to oscillate about a mean look angle in the sky with respect to a stationary observer on the earth's surface. The orbital period of a GEO satellite, 23 h 56 min 4.1 s, is one sidereal day. A sidereal day is the time between consecutive crossings of any particular longitude on the earth by any star, other than the sun[1]. The mean solar day of 24 h is the time between any consecutive crossings of any particular longitude by the sun, and is the time between successive sunrises (or sunsets) observed at one location on earth, averaged over an entire year. Because the earth moves round the sun once per 365 ¼ days, the solar day is 1440/365.25 = 3.94 min longer than a sidereal day.

Locating the Satellite in the Orbit

Consider now the problem of locating the satellite in its orbit. The equation of the orbit may be rewritten by combining Eqs. (2.15) and (2.18) to obtain

$$r_0 = \frac{a(1 - e^2)}{1 + e\cos\phi_0} \tag{2.22}$$

R

The angle ϕ_0 (see Figure 2.6) is measured from the x_0 axis and is called the *true anomaly*. [*Anomaly* was a measure used by astronomers to mean a planet's angular distance from its perihelion (closest approach to the sun), measured as if viewed from the sun. The term was adopted in celestial mechanics for all orbiting bodies.] Since we defined the positive x_0 axis so that it passes through the perigee, ϕ_0 measures the angle from the perigee to the instantaneous position of the satellite. The rectangular coordinates of the satellite are given by

$$x_0 = r_0\cos\phi_0 \tag{2.23}$$

$$y_0 = r_0\sin\phi_0 \tag{2.24}$$

As noted earlier, the orbital period T is the time for the satellite to complete a revolution in inertial space, traveling a total of 2π radians. The average angular velocity η is thus

$$\eta = (2\pi)/T = (\mu^{1/2})/(a^{3/2}) \tag{2.25}$$

If the orbit is an ellipse, the instantaneous angular velocity will vary with the position of the satellite around the orbit. If we enclose the elliptical orbit with a *circumscribed circle* of radius a (see Figure 2.7), then an object going around the circumscribed circle with a constant angular velocity η would complete one revolution in exactly the same period T as the satellite requires to complete one (elliptical) orbital revolution.

Consider the geometry of the circumscribed circle as shown in Figure 2.7. Locate the point (indicated as A) where a vertical line drawn through the position of the satellite intersects the circumscribed circle. A line from the center of the ellipse (C) to this point (A) makes an angle E with the x_0 axis; E is called the *eccentric anomaly* of the satellite.

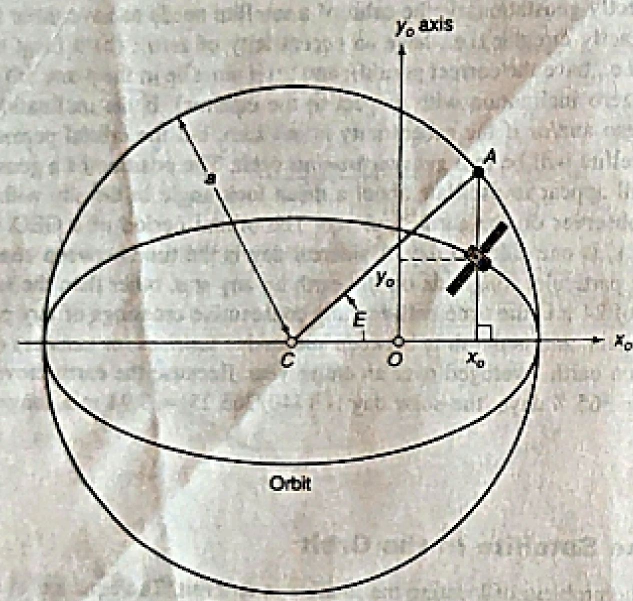


FIGURE 2.7 The circumscribed circle and the eccentric anomaly E. Point O is the center of the earth and point C is both the center of the orbital ellipse and the center of the circumscribed circle. The satellite location in the orbital plane coordinate system is specified by (x_o, y_o). A vertical line through the satellite intersects the circumscribed circle at point A. The eccentric anomaly E is the angle from the x_o axis to the line joining C and A.

It is related to the radius r_0 by

$$r_0 = a(1 - e\cos E) \tag{2.26}$$

Thus

$$a - r_0 = ae\cos E \tag{2.27}$$

We can also develop an expression that relates eccentric anomaly E to the average angular velocity η, which yields

$$\eta\, dt = (1 - e\cos E)dE \tag{2.28}$$

Let t_p be the *time of perigee*. This is simultaneously the time of closest approach to the earth; the time when the satellite is crossing the x_0 axis; and the time when E is zero. If we integrate both sides of Eq. (2.28), we obtain

$$\eta(t - t_p) = E - e\sin E \tag{2.29}$$

The left side of Eq. (2.29) is called the *mean anomaly*, M. Thus

$$M = \eta(t - t_p) = E - e\sin E \tag{2.30}$$

The mean anomaly M is the arc length (in radians) that the satellite would have traversed since the perigee passage if it were moving on the circumscribed circle at the mean angular velocity η.

If we know the time of perigee, t_p, the eccentricity, e, and the length of the semimajor axis, a, we now have the necessary equations to determine the coordinates (r_0, ϕ_0)

and (x_0, y_0) of the satellite in the orbital plane. The process is as follows

1. Calculate η using Eq. (2.25).
2. Calculate M using Eq. (2.30).
3. Solve Eq. (2.30) for E.
4. Find r_0 from E using Eq. (2.27).
5. Solve Eq. (2.22) for ϕ_0.
6. Use Eqs. (2.23) and (2.24) to calculate x_0 and y_0.

Now we must locate the orbital plane with respect to the earth.

Locating the Satellite with Respect to the Earth

At the end of the last section, we summarized the process for locating the satellite at the point (x_0, y_0, z_0) in the rectangular coordinate system of the orbital plane. The location was with respect to the center of the earth. In most cases, we need to know where the satellite is from an observation point that is not at the center of the earth. We will therefore develop the transformations that permit the satellite to be located from a point on the rotating surface of the earth. We will begin with a *geocentric equatorial coordinate system* as shown in Figure 2.8. The rotational axis of the earth is the z_i axis, which is through the geographic North Pole. The x_i axis is from the center of the earth toward a fixed location in space called the *first point of Aries* (see Figure 2.8). This coordinate system moves through space; it translates as the earth moves in its orbit around the sun, but it does not rotate as the earth rotates. The x_i direction is always the same, whatever the earth's position around the sun and is in the direction of the first point of Aries. The (x_i, y_i) plane contains the earth's equator and is called the *equatorial plane*.

Angular distance measured eastward in the equatorial plane from the x_i axis is called *right ascension* and given the symbol RA. The two points at which the orbit

elevation angle

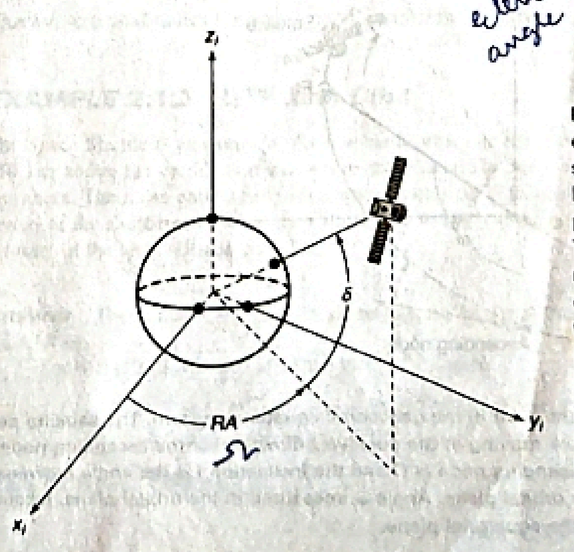


FIGURE 2.8 The geocentric equatorial system. This geocentric system differs from that shown in Figure 2.1 only in that the x_i axis points to the first point of Aries. The first point of Aries is the direction of a line from the center of the earth through the center of the sun at the vernal equinox (about March 21 in the Northern Hemisphere), the instant when the subsolar point crosses the equator from south to north. In the above system, an object may be located by its right ascension RA and its declination δ.

penetrates the equatorial plane are called nodes; the satellite moves upward through the equatorial plane at the *ascending node* and downward through the equatorial plane at the *descending node*, given the conventional picture of the earth, with north at the top, which is in the direction of the positive z axis for the earth centered coordinate set. Remember that in space there is no *up* or *down*; that is a concept we are familiar with because of gravity at the earth's surface. For a weightless body in space, such as an orbiting spacecraft, up and down have no meaning unless they are defined with respect to a reference point. The *right ascension of the ascending node* is called Ω. The angle that the orbital plane makes with the equatorial plane (the planes intersect at the line joining the nodes) is called the *inclination*, i. Figure 2.9 illustrates these quantities.

The variables Ω and i together locate the orbital plane with respect to the equatorial plane. To locate the orbital coordinate system with respect to the equatorial coordinate system we need ω, the *argument of perigee west*. This is the angle measured along the orbit from the ascending node to the perigee.

Standard time for space operations and most other scientific and engineering purposes is *universal time* (UT), also known as *zulu time* (z). This is essentially the mean solar time at the Greenwich Observatory near London, England. Universal time is measured in hours, minutes, and seconds or in fractions of a day. It is 5 h later than Eastern Standard Time, so that 07:00 EST is 12:00:00 h UT. The civil or calendar day begins at 00:00:00 hours UT, frequently written as 0 h. This is, of course, midnight (24:00:00) on the previous day. Astronomers employ a second dating system involving *Julian days* and *Julian dates*. Julian days start at noon UT in a counting system whereby noon on December 31, 1899, was the beginning of Julian day 2415020, usually written 241 5020. These are extensively tabulated in reference 2 and additional information is in reference 14. As an example, noon on December 31, 2000, the eve of the twenty-first century, is the start of Julian day 245 1909. Julian dates can be used to indicate time by appending a decimal fraction; 00:00:00 h UT on January 1, 2001—zero hour, minute, and

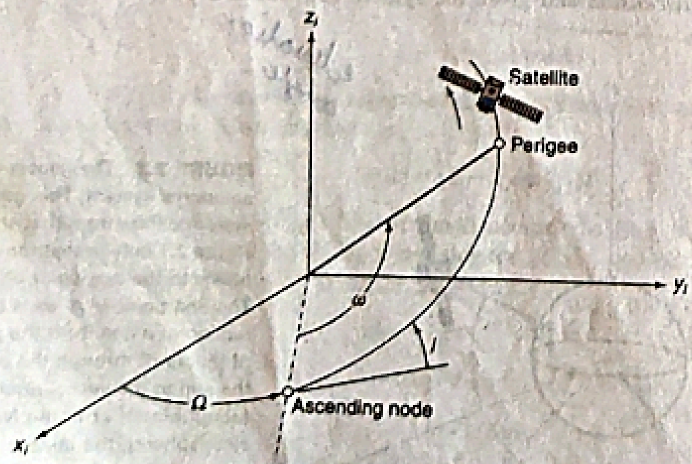


FIGURE 2.9 Locating the orbit in the geocentric equatorial system. The satellite penetrates the equatorial plane (while moving in the positive z direction) at the ascending node. The right ascension of the ascending node is Ω and the inclination i is the angle between the equatorial plane and the orbital plane. Angle ω, measured in the orbital plane, locates the perigee with respect to the equatorial plane.

second for the third millenium A.D.—is given by Julian date 245 1909.5. To find the exact position of an orbiting satellite at a given instant in time requires knowledge of the orbital elements.

Orbital Elements

To specify the absolute (i.e., the inertial) coordinates of a satellite at time t, we need to know six quantities. (This was evident earlier when we determined that a satellite's equation of motion was a second order vector linear differential equation.) These quantities are called the orbital elements. More than six quantities can be used to describe a unique orbital path and there is some arbitrariness in exactly which six quantities are used. We have chosen to adopt a set that is commonly used in satellite communications: eccentricity (e), semimajor axis (a), time of perigee (t_p), right ascension of ascending node (Ω), inclination (i), and argument of perigee (ω). Frequently, the mean anomaly (M) at a given time is substituted for t_p.

$e, a, t_p, \Omega, i, \omega$

EXAMPLE 2.1.1 *Geostationary Satellite Orbit Radius*

The earth rotates once per sidereal day of 23 h 56 min 4.09 s. Use Eq. (2.21) to show that the radius of the GEO is 42,164.17 km as given in Table 2.1.

Answer Equation (2.21) gives the square of the orbital period in seconds

$$T^2 = (4\pi^2 a^3)/\mu$$

Rearranging the equation, the orbital radius a is given by

$$a^3 = T^2\mu/(4\pi^2)$$

For one sidereal day, T = 86,164.09 s. Hence

$$a^3 = (86{,}164.1)^2 \times 3.986004418 \times 10^5/(4\pi^2) = 7.496020251 \times 10^{13}\ \text{km}^3$$
$$a = 42{,}164.17\ \text{km}$$

This is the orbital radius for a geostationary satellite, as given in Table 2.1. ■

EXAMPLE 2.1.2 *Low Earth Orbit*

The Space Shuttle is an example of a low earth orbit satellite. Sometimes, it orbits at an altitude of 250 km above the earth's surface, where there is still a finite number of molecules from the atmosphere. The mean earth's radius is approximately 6378.14 km. Using these figures, calculate the period of the shuttle orbit when the altitude is 250 km and the orbit is circular. Find also the linear velocity of the shuttle along its orbit.

Answer The radius of the 250-km altitude Space Shuttle orbit is $(r_e + h)$ = 6378.14 + 250.0 = 6628.14 km

From Eq. 2.21, the period of the orbit is T where

$$T^2 = (4\pi^2 a^3)/\mu = 4\pi^2 \times (6628.14)^3/3.986004418 \times 10^5\ \text{s}^2$$
$$= 2.88401145 \times 10^7\ \text{s}^2$$

Hence the period of the orbit is

$$T = 5370.30\ \text{s} = 89\ \text{min}\ 30.3\ \text{s}.$$

This orbit period is about as small as possible. At a lower altitude, friction with the earth's atmosphere will quickly slow the Shuttle down and it will return to earth. Thus, all spacecraft in stable earth orbit have orbital periods exceeding 89 min 30 s.

The circumference of the orbit is $2\pi a = 41{,}645.83$ km.

Hence the velocity of the Shuttle in orbit is

$$2\pi a/T = 41{,}645.83/5370.13 = 7.755 \text{ km/s}$$

Alternatively, you could use Eq. (2.5): $v = (\mu/r)^{1/2}$. The term $\mu = 3.986004418 \times 10^5$ km^3/s^2 and the term $r = (6378.14 + 250.0)$ km, yielding $v = 7.755$ km/s.

Note: If μ and r had been quoted in units of m^3/s^2 and m, respectively, the answer would have been in meters/second. Be sure to keep the units the same during a calculation procedure.

A velocity of about 7.8 km/s is a typical velocity for a low earth orbit satellite. As the altitude of a satellite increases, its velocity becomes smaller. ■

EXAMPLE 2.1.3 *Elliptical orbit*

A satellite is in an elliptical orbit with a perigee of 1000 km and an apogee of 4000 km. Using a mean earth radius of 6378.14 km, find the period of the orbit in hours, minutes, and seconds, and the eccentricity of the orbit.

Answer The major axis of the elliptical orbit is a straight line between the apogee and perigee, as seen in Figure 2.7. Hence, for a semimajor axis length a, earth radius r_e, perigee height h_p, and apogee height h_a,

$$2a = 2r_e + h_p + h_a = 2 \times 6378.14 + 1000.0 + 4000.0 = 17{,}756.28 \text{ km}$$

Thus the semimajor axis of the orbit has a length $a = 8878.14$ km. Using this value of a in Eq. (2.21) gives an orbital period T seconds where

$$T^2 = (4\pi^2 a^3)/\mu = 4\pi^2 \times (8878.07)^3/3.986004418 \times 10^5 \text{ s}^2$$
$$= 6.930872802 \times 10^7 \text{ s}^2$$
$$T = 8325.1864 \text{ s} = 138 \text{ min } 45.19 \text{ s} = 2 \text{ h } 18 \text{ min } 45.19 \text{ s}$$

The eccentricity of the orbit is given by e, which can be found from Eq. (2.27) by considering the instant at which the satellite is at perigee. Referring to Figure 2.7, when the satellite is at perigee, the eccentric anomaly $E = 0$ and $r_0 = r_e + h_p$. From Eq. (2.27), at perigee

$$r_0 = a(1 - e\cos E) \quad \text{and} \quad \cos E = 1$$

Hence

$$r_e + h_p = a(1 - e)$$
$$e = 1 - (r_e + h_p)/a = 1 - 7{,}378.14/8878.14 = 0.169$$ ■

2.2 LOOK ANGLE DETERMINATION

FL & Az

Navigation around the earth's oceans became more precise when the surface of the globe was divided up into a gridlike structure of orthogonal lines: latitude and longitude. Latitude is the angular distance, measured in degrees, north or south of the equator and longitude is the angular distance, measured in degrees, from a given reference longitudinal line. At the time that this grid reference became popular, there were two major seafaring nations vying for dominance: England and France. England drew its reference zero longitude through Greenwich, a town close to London, England, and France, not surprisingly, drew its reference longitude through Paris, France. Since the British Admiralty chose to give away their maps and the French decided to charge a fee for theirs, it was not surprising that the use of Greenwich as the zero reference longitude became dominant within a few years. [It was the start of .com market dominance through giveaways three centuries before E-commerce!] Geometry was a much older science than navigation and so 90° per quadrant on the map was an obvious selection to make. Thus, there are 360° of longitude (measured from 0° at the *Greenwich Meridian*, the line drawn from the North Pole to the South Pole through Greenwich, England) and ±90° of latitude, plus being measured north of the equator and minus south of the equator. Latitude 90° N (or +90°) is the North Pole and latitude 90° S (or −90°) is the South Pole. When GEO satellite systems are registered in Geneva, their (subsatellite) location over the equator is given in degrees east to avoid confusion. Thus, the INTELSAT primary location in the Indian Ocean is registered at 60° E and the primary location in the Atlantic Ocean is at 335.5° E (not 24.5° W). Earth stations that communicate with satellites are described in terms of their geographic latitude and longitude when developing the pointing coordinates that the earth station must use to track the apparent motion of the satellite.

longi
lati

The coordinates to which an earth station antenna must be pointed to communicate with a satellite are called the *look angles*. These are most commonly expressed as *azimuth* (Az) and *elevation* (El), although other pairs exist. For example, right ascension and declination are standard for radio astronomy antennas. Azimuth is measured eastward (clockwise) from geographic north to the projection of the satellite path on a (locally) horizontal plane at the earth station. Elevation is the angle measured upward from the local horizontal plane at the earth station to the satellite path. Figure 2.10 illustrates these look angles. In all look angle determinations, the precise location of the satellite is critical. A key location in many instances is the subsatellite point.

N
E
S

SIDEBAR

Frequencies and orbital slots for new satellites are registered with the International Frequency Registration Board (IFRB), part of the ITU located in Geneva. The initial application by an organization or company that wants to orbit a new satellite is made to the national body that controls the allocation and use of radio frequencies—the FCC in the United States, for example—which must first approve the application and then forward it to the IFRB. The first organization to file with the IFRB for a particular service is deemed to have protection from newcomers. Any other organization filing to carry the same service at, or close to, that orbital location (within 2°) must coordinate their use of the frequency bands with the first organization. The first user may cause interference into subsequent filer's satellite systems, since they were the first to be awarded the orbital slot and frequencies, but the later filers' satellites must not cause interference with the first user's system.

The Subsatellite Point

The subsatellite point is the location on the surface of the earth that lies directly between the satellite and the center of the earth. It is the *nadir* pointing direction from the satellite and, for a satellite in an equatorial orbit, it will always be located on the equator. Since geostationary satellites are in equatorial orbits and are designed to stay "stationary" over

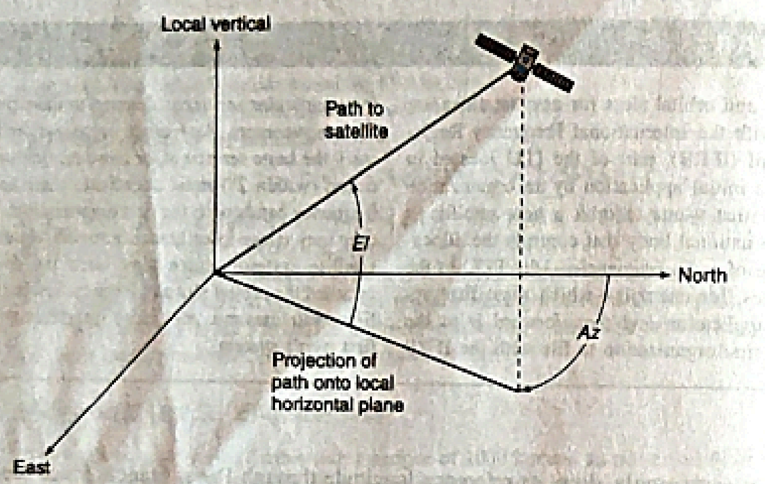


FIGURE 2.10 The definition of elevation (*El*) and azimuth (*Az*). The elevation angle is measured upward from the local horizontal at the earth station and the azimuth angle is measured from true north in an eastward direction to the projection of the satellite path onto the local horizontal plane.

the earth, it is usual to give their orbital location in terms of their subsatellite point. As noted in the example given earlier, the Intelsat primary satellite in the Atlantic Ocean Region (AOR) is at 335.5° E longitude. Operators of international geostationary satellite systems that have satellites in all three ocean regions (Atlantic, Indian, and Pacific) tend to use longitude east to describe the subsatellite points to avoid confusion between using both east and west longitude descriptors. For U.S. geostationary satellite operators, all of the satellites are located west of the Greenwich meridian and so it has become accepted practice for regional systems over the United States to describe their geostationary satellite locations in terms of degrees W.

To an observer of a satellite standing at the subsatellite point, the satellite will appear to be directly overhead, in the *zenith* direction from the observing location. The zenith and nadir paths are therefore in opposite directions along the same path (see Figure 2.11). Designers of satellite antennas reference the pointing direction of the satellite's antenna beams to the nadir direction. The communications coverage region on the earth ~~from a satellite~~ is defined by angles measured from nadir at the satellite to the edges of the coverage. Earth station antenna designers, however, do not reference their pointing direction to zenith. As noted earlier, they use the local horizontal plane at the earth station to define elevation angle and geographical compass points to define azimuth angle, thus giving the two look angles for the earth station antenna toward the satellite (*Az*, *El*).

Elevation Angle Calculation

Figure 2.12 shows the geometry of the elevation angle calculation. In Figure 2.12, r_s is the vector from the center of the earth to the satellite; r_e is the vector from the center of the earth to the earth station; and d is the vector from the earth station to the satellite. These three vectors lie in the same plane and form a triangle. The central angle γ measured between r_e and r_s is the angle between the earth station and the

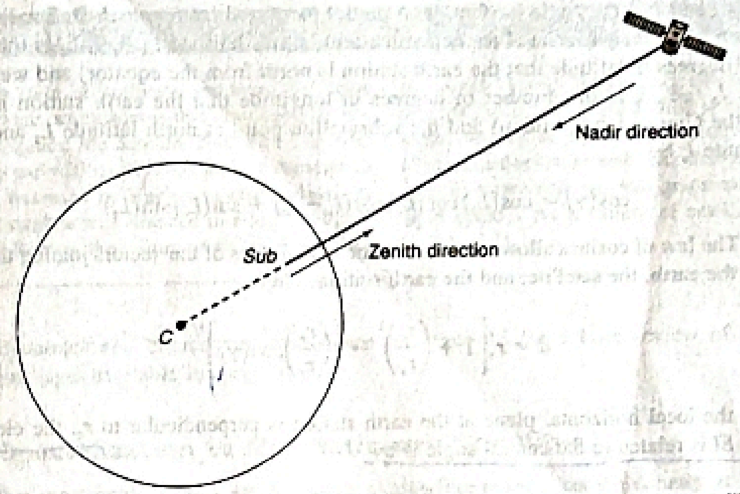
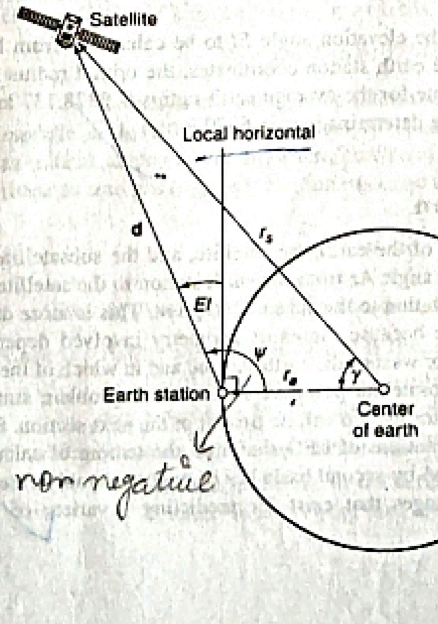


FIGURE 2.11 Zenith and nadir pointing directions. The line joining the satellite and the center of the earth, *C*, passes through the surface of the earth at point *Sub*, the subsatellite point. The satellite is directly overhead at this point and so an observer at the subsatellite point would see the satellite at zenith (i.e., at an elevation angle of 90°). The pointing direction from the satellite to the subsatellite point is the nadir direction from the satellite. If the beam from the satellite antenna is to be pointed at a location on the earth that is not at the subsatellite point, the pointing direction is defined by the angle away from nadir. In general, two off-nadir angles are given: the number of degrees north (or south) from nadir; and the number of degrees east (or west) from nadir. East, west, north, and south directions are those defined by the geography of the earth.



$\gamma \gg$ central angle

non negative

FIGURE 2.12 The geometry of elevation angle calculation. The plane of the paper is the plane defined by the center of the earth, the satellite, and the earth station. The central angle is γ. The elevation angle *El* is measured upward from the local horizontal at the earth station.

satellite, and ψ is the angle (within the triangle) measured from r_e to d. Defined so that it is nonnegative, γ is related to the earth station north latitude L_e (i.e., L_e is the number of degrees in latitude that the earth station is north from the equator) and west longitude l_e (i.e., l_e is the number of degrees in longitude that the earth station is west from the Greenwich meridian) and the subsatellite point at north latitude L_s and west longitude l_s by

$$\cos(\gamma) = \cos(L_e)\cos(L_s)\cos(l_s - l_e) + \sin(L_e)\sin(L_s) \qquad (2.31)$$

The law of cosines allows us to relate the magnitudes of the vectors joining the center of the earth, the satellite, and the earth station. Thus

$$d = r_s\left[1 + \left(\frac{r_e}{r_s}\right)^2 - 2\left(\frac{r_e}{r_s}\right)\cos(\gamma)\right]^{1/2} \qquad (2.32)$$

Since the local horizontal plane at the earth station is perpendicular to r_e, the elevation angle El is related to the central angle ψ by

$$El = \psi - 90° \qquad (2.33)$$

By the law of sines we have

$$\frac{r_s}{\sin(\psi)} = \frac{d}{\sin(\gamma)} \qquad (2.34)$$

Combining the last three equations yields

$$\cos(El) = \frac{r_s\sin(\gamma)}{d} = \frac{\sin(\gamma)}{\left[1 + \left(\frac{r_e}{r_s}\right)^2 - 2\left(\frac{r_e}{r_s}\right)\cos(\gamma)\right]^{1/2}} \qquad (2.35)$$

Equations (2.35) and (2.31) permit the elevation angle El to be calculated from knowledge of the subsatellite point and the earth station coordinates, the orbital radius r_s, and the earth's radius r_e. An accurate value for the average earth radius is 6378.137 km[1] but a common value used in approximate determinations is 6370 km.

Azimuth Angle Calculation

Because the earth station, the center of the earth, the satellite, and the subsatellite point all lie in the same plane, the azimuth angle Az from the earth station to the satellite is the same as the azimuth from the earth station to the subsatellite point. This is more difficult to compute than the elevation angle because the exact geometry involved depends on whether the subsatellite point is east or west of the earth station, and in which of the hemispheres the earth station and the subsatellite point are located. The problem simplifies somewhat for geosynchronous satellites, which will be treated in the next section. For the general case, in particular for constellations of LEO satellites, the tedium of calculating the individual look angles on a second-by-second basis has been considerably eased by a range of commercial software packages that exist for predicting a variety of orbital

SIDEBAR

A popular suite of software employed by many launch service contractors is that developed by Analytical Graphics: the *Satellite Tool Kit*[3]. The core program in early 2001, STK 4.0, and the subsequent subseries, was used by Hughes to rescue *AsiaSat3* when that satellite was stranded in a highly elliptical orbit following the failure of an upper stage in the launch vehicle. Hughes used two *lunar* flybys to provide the necessary additional velocity to circularize the orbit at geostationary altitude. A number of organizations offer web sites that provide orbital plots in a three-dimensional graphical format with rapid updates for a variety of satellites (e.g., the NASA site[4]).

dynamics and intercept solutions (see reference 13 for a brief review of 10 software packages available in early 2001).

Specialization to Geostationary Satellites

For most geostationary satellites, the subsatellite point is on the equator at longitude l_s, and the latitude L_s is 0. The geosynchronous radius r_s is 42,164.17 km[1]. Since L_s is zero, Eq. (2.31) simplifies to

$$\cos(\gamma) = \cos(L_e)\cos(l_s - l_e) \qquad (2.36)$$

Substituting r_s = 42,164.17 km and r_e = 6,378.137 km in Eqs. (2.32) and (2.35) gives the following expressions for the distance d from the earth station to the satellite and the elevation angle El at the earth station

$$d = 42{,}164.17[1.02288235 - 0.30253825\cos(\gamma)]^{1/2}\ \text{km} \qquad (2.37)$$

$$\cos(El) = \frac{\sin(\gamma)}{[1.02288235 - 0.30253825\cos(\gamma)]^{1/2}} \qquad (2.38)$$

For a geostationary satellite with an orbital radius of 42,164.17 km and a mean earth radius of 6378.137 km, the ratio r_s/r_e = 6.6107345 giving

$$El = \tan^{-1}[(6.6107345 - \cos\gamma)/\sin\gamma] - \gamma \qquad (2.39)$$

To find the azimuth angle, an intermediate angle α must first be found. The intermediate angle α permits the correct 90° quadrant to be found for the azimuth since the azimuthal angle can lie anywhere between 0° (true north) and clockwise through 360° (back to true north again). The intermediate angle is found from

$$\alpha = \tan^{-1}\left[\frac{\tan|(l_s - l_e)|}{\sin(L_e)}\right] \qquad (2.40)$$

Having found the intermediate angle α, the azimuth look angle Az can be found from:

Case 1: Earth station in the Northern Hemisphere with

(a) Satellite to the SE of the earth station: $Az = 180° - \alpha$ (2.41a)

(b) Satellite to the SW of the earth station: $Az = 180° + \alpha$ (2.41b)

Case 2: Earth station in the Southern Hemisphere with

(c) Satellite to the NE of the earth station: $Az = \alpha$ (2.41c)

(d) Satellite to the NW of the earth station: $Az = 360° - \alpha$ (2.41d)

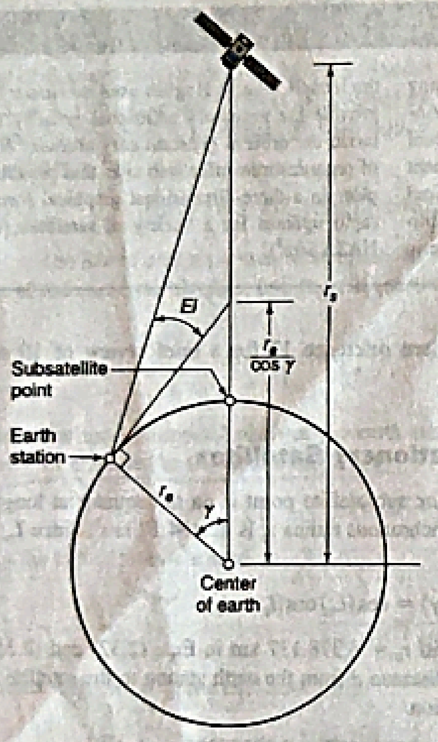


FIGURE 2.13 The geometry of the visibility calculation. The satellite is said to be visible from the earth station if the elevation angle El is positive. This requires that the orbital radius r_s be greater than the ratio $r_e/\cos(\gamma)$ where r_e is the radius of the earth and γ is the central angle.

Visibility Test

For a satellite to be visible from an earth station, its elevation angle El must be above some minimum value, which is at least 0°. A positive or zero elevation angle requires that (see Figure 2.13)

$$r_s \geq \frac{r_e}{\cos(\gamma)} \quad (2.42)$$

This means that the maximum central angular separation between the earth station and the subsatellite point is limited by

$$\gamma \leq \cos^{-1}\left(\frac{r_e}{r_s}\right) \quad (2.43)$$

For a nominal geostationary orbit, the last equation reduces to $\gamma \leq 81.3°$ for the satellite to be visible.

EXAMPLE 2.2.1 *Geostationary Satellite Look Angles*

An earth station situated in the Docklands of London, England, needs to calculate the look angle to a geostationary satellite in the Indian Ocean operated by Intelsat. The details of the earth station site and the satellite are as follows:

Earth station latitude and longitude are 52.0° N and 0°.

Satellite longitude (subsatellite point) is 66.0° E.

Step 1: Find the central angle γ

$$\cos(\gamma) = \cos(L_e)\cos(l_s - l_e)$$
$$= \cos(52.0)\cos(66.0) = 0.2504$$

yielding $\gamma = 75.4981°$

The central angle γ is less than 81.3° so the satellite is visible from the earth station.

Step 2: Find the elevation angle El

$$El = \tan^{-1}[(6.6107345 - \cos\gamma)/\sin\gamma] - \gamma$$
$$= \tan^{-1}[(6.6107345 - 0.2504)/\sin(75.4981)] - 75.4981$$
$$= 5.847°$$

Step 3: Find the intermediate angle α

$$\alpha = \tan^{-1}\left[\frac{\tan|(l_s - l_e)|}{\sin(L_e)}\right]$$
$$= \tan^{-1}[(\tan(66.0 - 0))/\sin(52.0)]$$
$$= 70.667°$$

Step 4: Find the azimuth angle

The earth station is in the Northern Hemisphere and the satellite is to the southeast of the earth station. From Eq. (2.41a), this gives

$$Az = 180° - \alpha = 180 - 70.667 = 109.333°\text{(clockwise from true north)}$$ ■

Note that, in the example above, the elevation angle is relatively low (5.85°). Refractive effects in the atmosphere will cause the mean ray path to the satellite to bend in the elevation plane (making the satellite appear to be higher in the sky than it actually is) and to cause the amplitude of the signal to fluctuate with time. These aspects are discussed more fully in the propagation effects chapter. While it is unusual to operate to a satellite below established elevation angle minima (typically 5° at C band, 10° at Ku band, and in most cases, 20° at Ka band and above), many times it is not possible to do this. Such cases exist for high latitude regions and for satellites attempting to reach extreme east and west coverages from their given geostationary equatorial location. To establish whether a particular satellite location can provide service into a given region, a simple visibility test can be carried out, as shown earlier in Eqs. (2.42) and (2.43).

A number of geosynchronous orbit satellites have inclinations that are much larger than the nominal 0.05° inclination maximum for current geosynchronous satellites. (In general, a geosynchronous satellite with an inclination of <0.1° may be considered to be geostationary.) In extreme cases, the inclination can be several degrees, particularly if the orbit maneuvering fuel of the satellite is almost exhausted and the satellite's position in the nominal location is only controlled in longitude and not in inclination. This happens with most geostationary communications satellites toward the end of their operational lifetime since the reliability of the payload, or a large part of the payload, generally exceeds that of the lifetime of the maneuvering fuel. Those satellites that can no longer be maintained in a fully geostationary orbit, but are still used for communications services, are referred to as *inclined orbit* satellites. While they now need to have tracking antennas at the earth terminals once the inclination becomes too large to allow the satellite to remain within the 1-dB beamwidth of the earth station antennas, substantial additional revenue can be earned beyond the normal lifetime of the satellite. Those satellites that eventually reach significantly inclined orbits can also be used to communicate to parts of the high latitude regions that were once beyond reach, but only satellite ceases to be useful). Note that, due to the nonsphericity of the earth, etc., the stable points are neither exactly 180° apart, nor are the stable and unstable points precisely 90° apart.

Inclination Changes: Effects of the Sun and the Moon

The plane of the earth's orbit around the sun—the *ecliptic*—is at an inclination of 7.3° to the equatorial plane of the sun (Figure 2.14). The earth is titled about 23° away from the normal to the ecliptic. The moon circles the earth with an inclination of around 5° to the equatorial plane of the earth. Due to the fact that the various planes—the sun's equator, the ecliptic, the earth's equator (a plane normal to the earth's rotational axis), and the moon's orbital plane around the earth—are all different, a satellite in orbit around the earth will be subjected to a variety of out-of-plane forces. That is, there will generally be a net acceleration force that is not in the plane of the satellite's orbit, and this will tend to try to change the inclination of the satellite's orbit from its initial inclination. Under these conditions, the orbit will precess and its inclination will change.

The mass of the sun is significantly larger than that of the moon but the moon is considerably closer to the earth than the sun (see Table 2.2). For this reason, the acceleration force induced by the moon on a geostationary satellite is about twice as large as that of the sun. The net effect of the acceleration forces induced by the moon and the sun on a

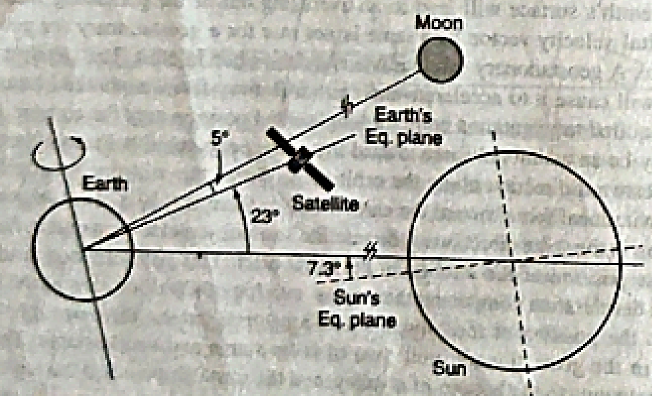


FIGURE 2.14 Relationship between the orbital planes of the sun, moon, and earth. The plane of the earth's orbit around the sun is the *ecliptic*. The geostationary orbit plane (the earth's equatorial plane) is about 23° out of the ecliptic, and leads to maximum out-of-geostationary-orbit-plane forces at the solstice periods (approximately June 21 and December 21). The orbit of the moon is inclined about 6° to the earth's equatorial plane. The moon revolves around the earth in 27.3 days, the earth (and the geostationary satellite) rotates once about 24 h, and the earth revolves around the sun every 365.25 days. In addition, the sun—which has a greater girth at the equator than at the poles—has its equator inclined about 7.3° to the ecliptic. All of these various angular differences and orbital periods lead to conditions where all of the out-of-plane gravitational forces are in one direction with respect to the equatorial (geostationary orbital) plane at a given time as well as to conditions where the various gravitational out-of-plane forces partially cancel each other out. The precessional forces that cause the inclination of the geostationary satellite's orbit to move away from the equatorial plane therefore vary with time.

TABLE 2.2 Comparative Data for the Sun, Moon, and Earth

	Mean radius	Mass	Mean orbit radius	Spin period
Sun	696,000 km	333,432 units	30,000 light years	25.04 earth days
Moon	3,476 km	0.012 units	384,500 km	27.3 earth days
Earth	6,378.14 km	1.0 units	149,597,870 km	1 earth day

The orbit radius refers to the center of the home galaxy (Milky Way) for the sun, center of earth for the moon, and center of the sun for the earth, respectively.

geostationary satellite is to change the plane of the orbit at an initial average rate of change of 0.85°/year from the equatorial plane[1].

When both the sun and moon are acting on the same side of the satellite's orbit, the rate of change of the plane of the geostationary satellite's orbit will be higher than average. When they are on opposite sides of the orbit, the rate of change of the plane of the satellite's orbit will be less than average. Examples of maximum years are 1988 and 2006 (0.94°/year) and examples of minimum years are 1997 and 2015 (0.75°/year)[1]. These rates of change are neither constant with time nor with inclination. They are at a maximum when the inclination is zero and they are zero when the inclination is 14.67°. From an initial zero inclination, the plane of the geostationary orbit will change to a maximum inclination of 14.67° over 26.6 years. The acceleration forces will then change direction at this maximum inclination and the orbit inclination will move back to zero in another 26.6 years and out to −14.67° over a further 26.6 years, and so on.

In some cases, to increase the orbital maneuver lifetime of a satellite for a given fuel load, mission planners deliberately place a satellite planned for geostationary orbit into an initial orbit with an inclination that is substantially larger than the nominal 0.05° for a geostationary satellite. The launch is specifically timed, however, so as to set up the necessary precessional forces that will automatically reduce the inclination "error" to close to zero over the required period without the use of any thruster firings on the spacecraft. This will increase the maneuvering lifetime of the satellite at the expense of requiring greater tracking by the larger earth terminals accessing the satellite for the first year or so of the satellite's operational life.

Under normal operations, ground controllers command spacecraft maneuvers to correct for both the in-plane changes (longitudinal drifts) and out-of-plane changes (inclination changes) of a satellite so that it remains in the correct orbit. For a geostationary satellite, this means that the inclination, ellipticity, and longitudinal position are controlled so that the satellite appears to stay within a "box" in the sky that is bounded by ±0.05° in latitude and longitude over the subsatellite point. Some maneuvers are designed to correct for both inclination and longitude drifts simultaneously in the one burn of the maneuvering rockets on the satellite. In others, the two maneuvers are kept separate: one burn will correct for ellipticity and longitude drift; another will correct for inclination changes. The latter situation of separated maneuvers is becoming more common for two reasons. The first is due to the much larger velocity increment needed to change the plane of an orbit (the so-called north–south maneuver) as compared with the longitude/ellipticity of an orbit (the so-called east–west maneuver). The difference in energy requirement is about 10:1. By alternately correcting for inclination changes and in-plane changes, the attitude of the satellite can be held constant and different sets of thrusters exercised for the required maneuver.

The second reason is the increasing use of two completely different types of thrusters to control N–S maneuvers on the one hand and E–W maneuvers on the other. In the

Scanned By Scanner Go

mid-1990s, one of the heaviest items that was carried into orbit on a large satellite was the fuel to raise and control the orbit. About 90% of this fuel load, once on orbit, was to control the inclination of the satellite. Newer rocket motors, particularly arc jets and ion thrusters, offer increased efficiency with lighter mass. In general, these low thrust, high efficiency rocket motors are used for N–S maneuvers leaving the liquid propellant thrusters, with their inherently higher thrust (but lower efficiency) for orbit raising and in-plane changes. In order to be able to calculate the required orbit maneuver for a given satellite, the controllers must have an accurate knowledge of the satellite's orbit. Orbit determination is a major aspect of satellite control.

EXAMPLE 2.3.1 *Drift with a Geostationary Satellite*

A quasi-GEO satellite is in a circular equatorial orbit close to geosynchronous altitude. The quasi GEO satellite, however, does not have a period of one sidereal day: its orbital period is exactly 24 h—one solar day. Calculate

(i) the radius of the orbit

(ii) the rate of drift around the equator of the subsatellite point in degrees per (solar) day. An observer on the earth sees that the satellite is drifting across the sky.

(iii) Is the satellite moving toward the east or toward the west?

Answer *Part (i)* The orbital radius is found from Eq. (2.21), as in worked Example 2.2.1. Equation (2.21) gives the square of the orbital period in seconds (remembering that T here is one solar day)

$$T^2 = (4\pi^2 a^3)/\mu$$

Rearranging the equation, the orbital radius a is given by

$$a^3 = T^2\mu/(4\pi^2) = (86{,}400)^2 \times 3.986004418 \times 10^5/4\pi^2$$
$$= 7.5371216 \times 10^{13}\ \text{km}^3$$
$$a = 42{,}241.095\ \text{km}$$

Part (ii) The orbital period of the satellite (one solar day) is longer than a sidereal day by 3 min 55.9 s = 235.9 s. This will cause the subsatellite point to drift at a rate of 360° × 235.9/86400 per day or 0.983° per day.

Part (iii) The earth moves toward the east at a faster rate than the satellite, so the drift will appear to an observer on the earth to be toward the west. ∎

2.4 ORBIT DETERMINATION

Orbit determination requires that sufficient measurements be made to determine uniquely the six orbital elements needed to calculate the future orbit of the satellite, and hence calculate the required changes that need to be made to the orbit to keep it within the nominal orbital location. Three angular position measurements are needed because there are six unknowns and each measurement will provide two equations. Conceptually, these can be thought of as one equation giving the azimuth and the other the elevation as a function of the six (as yet unknown) orbital elements.

The control earth stations used to measure the angular position of the satellites also carry out range measurements using unique time stamps in the telemetry stream or communications carrier. These earth stations are generally referred to as the TTC&M (telemetry tracking command and monitoring) stations of the satellite network. Major satellite networks maintain their own TTC&M stations around the world. Smaller satellite systems generally contract for such TTC&M functions from the spacecraft manufacturer or from the larger satellite system operators, as it is generally uneconomic to build advanced TTC&M stations with fewer than three satellites to control. Chapter 3 discusses TTC&M systems.

2.5 LAUNCHES AND LAUNCH VEHICLES

A satellite cannot be placed into a stable orbit unless two parameters that are uniquely coupled together—the velocity vector and the orbital height—are simultaneously correct. There is little point in obtaining the correct height and not having the appropriate velocity component in the correct direction to achieve the desired orbit. A geostationary satellite, for example, must be in an orbit at a height of 35,786.03 km above the surface of the earth (42,164.17-km radius from the center of the earth) with an inclination of zero degrees, an ellipticity of zero, and a velocity of 3074.7 m/s tangential to the earth in the plane of the orbit, which is the earth's equatorial plane. The further out from the earth the orbit is, the greater the energy required from the launch vehicle to reach that orbit. In any earth satellite launch, the largest fraction of the energy expended by the rocket is used to accelerate the vehicle from rest until it is about 20 miles (32 km) above the earth. To make the most efficient use of the fuel, it is common to shed excess mass from the launcher as it moves upward on launch: this is called staging. Figure 2.15 gives a schematic of a Proton launch from the Russian Baikonur complex at Kazakhstan, near Tyuratam.

Most launch vehicles have multiple stages and, as each stage is completed, that portion of the launcher is expended until the final stage places the satellite into the desired trajectory. Hence the term: *expendable launch vehicle (ELV)*. The Space Shuttle, called the *Space Transportation System (STS)* by NASA, is partially reusable. The solid rocket boosters are recovered and refurbished for future missions and the shuttle vehicle itself is flown back to earth for refurbishment and reuse. Hence the term: *reusable launch vehicle (RLV)* for such launchers. More advanced launch vehicles are being developed that would provide both *single stage to orbit (SSTO)* and RLV capabilities. The NASA series of X-33 and X-34 test vehicles form the public portion of this quest (see the NASA home page[4]).

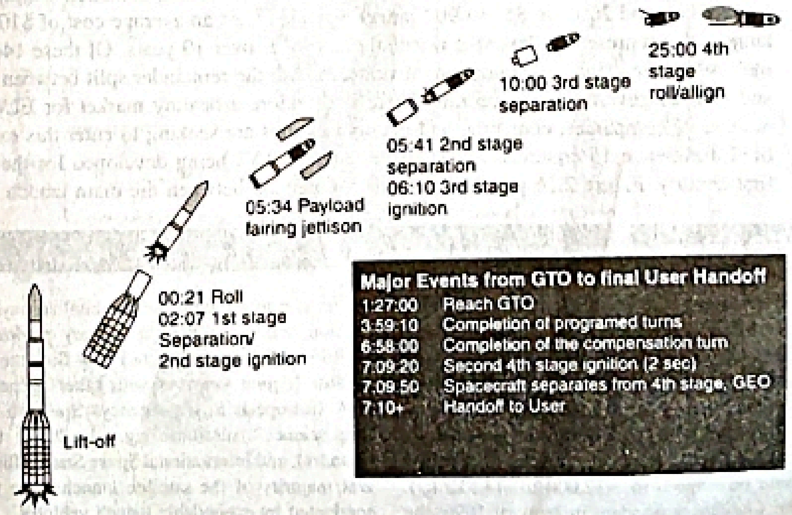


FIGURE 2.15 Schematic of a Proton launch (after reference 5).

There are also a number of private ventures that aim to achieve RLV capabilities in the first decade of the twenty-first century. Two excellent web sites to keep abreast of these, and related space issues, are those maintained by Spaceviews[6] and Orbreport[7]. Of equal importance to the orbital height the satellite is intended for is the inclination of the orbit that the spacecraft needs to be launched into.

The earth spins toward the east. At the equator, the rotational velocity of a sea level site in the plane of the equator is ($2\pi \times$ radius of the earth)/(one sidereal day) = 0.4651 km/s. This velocity increment is approximately 1000 mph (~1610 km/h). An easterly launch from the equator has a velocity increment of 0.465 km/s imparted by the rotation of the earth. A satellite in a circular, equatorial orbit at an altitude of 900 km requires an orbital velocity of about 7.4 km/s tangential to the surface of the earth. A rocket launched from the equator needs to impart an additional velocity of (7.4 − 0.47) km/s = 6.93 km/s; in other words, the equatorial launch has reduced the energy required by about 6%. This equatorial launch "bonus" led to the concept of a sea launch by Hughes and Boeing[8]. If the launch is not to be into an equatorial orbit, the payload capabilities of any given rocket will reduce as the inclination increases.

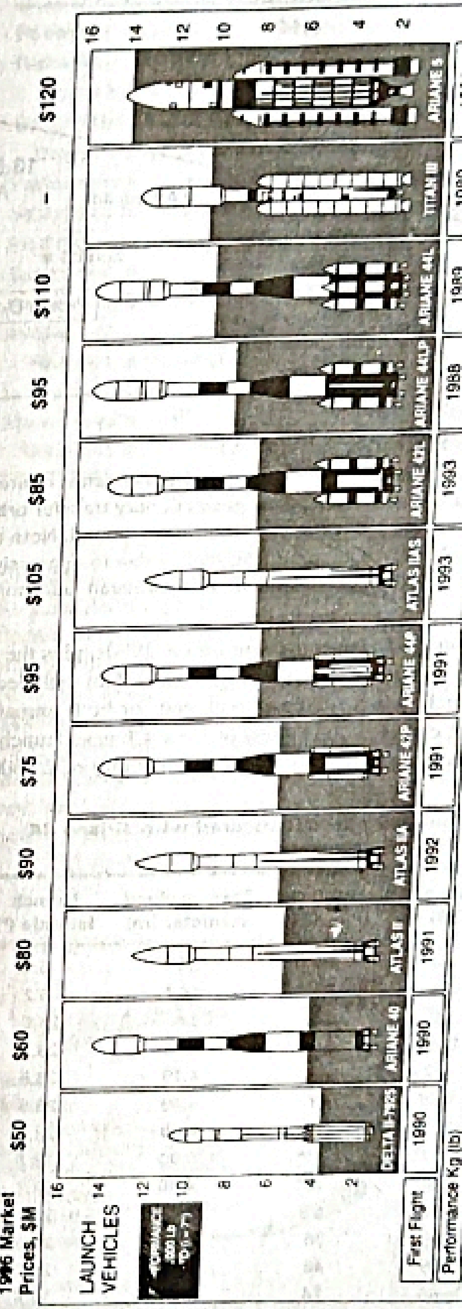
A satellite launched into a prograde orbit from a latitude of Φ degrees will enter an orbit with an inclination of Φ degrees to the equator. If the satellite is intended for geostationary orbit, the satellite must be given a significant velocity increment to reorient the orbit into the earth's equatorial plane. For example, a satellite launched from Cape Canaveral at 28.5° N latitude requires a velocity increment of 366 m/s to attain an equatorial orbit from a geosynchronous orbit plane of 28.5°. Ariane is launched from the Guiana Space Center in French Guiana, located at latitude 5° S in South America, and SeaLaunch can launch from the equator. The lower latitude of these launch sites results in significant savings in the fuel used by the apogee kick motor (AKM).

Expendable Launch Vehicles (ELVs)

1998 was an important year for ELVs: it was the year when the number of commercial launches in the United States surpassed the number of government launches for the first time[9]. The gap between commercial and government launches will continue to grow. The Teal Group estimated in mid-1999 that 1447 satellites would be launched worldwide between 2000 and 2009 on 850 to 900 launch vehicles[10]. At an average cost of $100 M per launch, this represents a business worth about $ 90 B over 10 years. Of these 1447 satellites, 893 were considered commercial ventures with the remainder split between military and civilian government spacecraft. There is therefore a healthy market for ELVs and a number of companies, consortia, and national entities are seeking to enter this expanding field. Reference 15 contains a good survey of the ELVs being developed for the twenty-first century. Figure 2.16 presents a rough comparison between the main launch vehicles

SIDEBAR

The STS can launch approximately 65,000 lb (29,478 kg) into a standard 28.5° orbital inclination at an orbital height of about 200 km from the Kennedy Space Flight Center in Cape Canaveral. If the Vandenburg Air Force Base launch site in California still had the capability of launching the Shuttle, the payload capability of the Shuttle for a polar launch (inclination 90°) would be reduced to ~32,000 lb (14,512 kg). Since the *Challenger* accident in January 1996, the shuttle is rarely used to launch civilian payloads, its mission being confined to military payloads (e.g., TDRSS (Tracking and Data Relay Satellite System satellites), joint ventures with other agencies (e.g., ESA (European Space Agency) Spacelab facility), "big science" missions (e.g., the X-ray telescope Chandra), and International Space Station flights. The vast majority of the satellite launches are therefore conducted by expendable launch vehicles.



1996 Market Prices, $M	Launch Vehicle	First Flight	Performance Kg (lb): GTO, i = 7 deg (200 × 35,786 km) (108 × 19,323 NM)	Payload Max Diameter m (ft)	Launch Site (Projected)
$50	DELTA II-7925	1990	1,820 (1) (4,010)	2.54 (8.3)	CCAS (VAFB)
$60	ARIANE 40	1990	2,050 (4,520)	3.65 (12)	KOUROU
$80	ATLAS II	1991	2,810 (2) (6,200)	3.65 (12)	CCAS (VAFB)
$90	ATLAS IIA	1992	3,050 (2) (6,710)	3.65 (12)	CCAS (VAFB)
$75	ARIANE 42P	1991	2,840 (6,260)	3.65 (12)	KOUROU
$95	ARIANE 44P	1991	3,320 (7,320)	3.65 (12)	KOUROU
$105	ATLAS IIAS	1993	3,700 (2) (8,150)	3.65 (12)	CCAS (VAFB)
$85	ARIANE 42L	1993	3,380 (7,450)	3.65 (12)	KOUROU
$95	ARIANE 44LP	1988	4,060 (8,950)	3.65 (12)	KOUROU
$110	ARIANE 44L	1989	4,520 (9,965)	3.65 (12)	KOUROU
–	TITAN III	1989	5,000 (1) (11,000)	3.65 (12)	CCAS
$120	ARIANE 5	1996	5,000-6,700 (11-13,900)	4.57 (15)	KOUROU

(1) DELTA & TITAN @ i = 28 deg
(2) ATLAS GTO REFERENCE ORBIT IS 167 × 35,788 KM (90 × 19,324 NM), i = 27 deg

FIGURE 2.16 Representative ELVs (after reference 5). CCAS, Cape Canaveral Air Station; VAFB, Vandenburg Air Force Base.

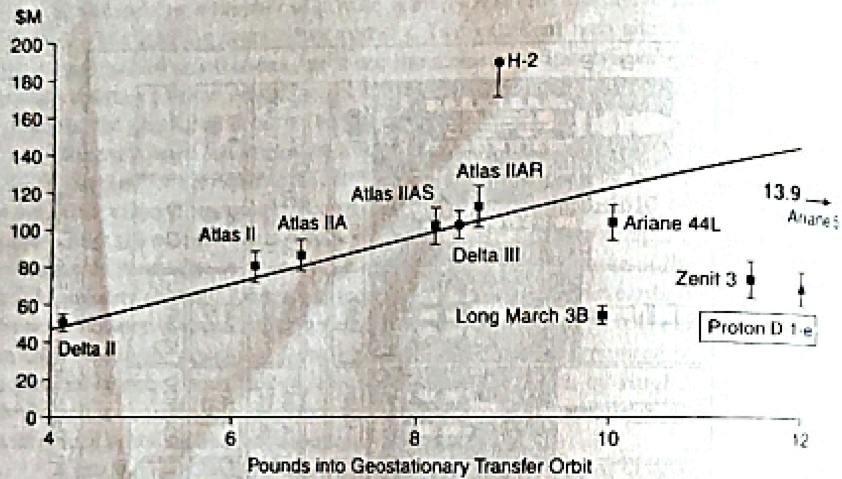


FIGURE 2.17 Launch vehicle market price vs performance, 1996 prices (after reference 5). The launch vehicles have been normalized to a launch into geostationary transfer orbit at an inclination of 28°. The trend line for launchers is shown as $12,000 per pound. Note that Long March, Zenit, and Proton are well below this trend line, mainly due to aggressive pricing objectives to break into a market long dominated by U.S. and European launchers.

used for Geostationary Transfer Orbit (GTO) injection during the 1990s, plus the Ariane 5 launcher. The 1996 pricing of these vehicles is shown in Figure 2.17. Not included in these data are the advanced Chinese launch vehicles being developed for both unmanned and manned missions in the twenty-first century. The largest of these Chinese launch vehicles rivals the Ariane 5 vehicle with a geostationary transfer orbit capability of 26,000 lb.

TABLE 2.3 Some Next Generation Launchers Compared with Ariane 44 and Atlas IIAS Baseline Vehicles (1999 Prices)

Launcher	Weight to orbit (kg)	Total cost ($M)	Lead time (months)	Max. payload diameter (m)	Launch latitude (°)
Ariane 44	4000	130	36	3.65	5.2
Ariane 5	6800	120	36	4.57	5.2
Atlas IIAS	3700	100	36	3.45	28.5
Atlas IIIA	4120	125	36	4.19	28.5
Atlas IIIB	4500	135	48	4.19	28.5
Atlas V	6500	150*	48	5.40	28.5
Delta III	3800	130	36	4.00	28.7
Delta IV(small)	2177	60*	36*	3.00	28.7
Delta IV(med.)	4173	120*	36*	4.00	28.7
Delta IV(heavy)	13200	400*	48*	5.00	28.7
Titan III	4500	260	36	3.65	28.6
Titan IV	5700	435	48	4.57	28.6
Proton M	4800	80	24	3.68	51.6

* These data are estimated values.

The Atlas V and Proton M vehicles are planned for operational flights beginning in 2002 or 2003. The Delta IV family of launch vehicles will become operational from 2002 to 2004.

TABLE 2.4 Some Launch Vehicle Selection Factors

Price/cost
Reliability
Recent launch success/failure history
Dependable launch schedule
Urgency of your launch requirement
Performance
Spacecraft fit to launcher (size, acoustic, and vibration environment)
Flight proven (see recent launch history)
Safety issues
Launch site location
Availability
What is the launcher backlog of orders?
What is the launch site backlog of launches?
Market issues
What will the market bear at this particular time?

It can be seen from Figure 2.17 that there was a well-established trend line of about $25,000 per kg into GTO prior to the introduction of the Chinese Long March and the Russian Zenit and Proton vehicles. The pricing of the Chinese and Russian launchers reflected an aggressive marketing strategy to break into the launch services field. Ariane 5 was the first of the next-generation launchers aimed at both large, single payloads into GTO and multiple payload injection into LEO and MEO. Some more next-generation launchers are shown in Table 2.3 on the previous page. It is anticipated that the bulk of the large satellite launches will be conducted with Atlas V, Delta IV, and Ariane vehicles and their Russian and Chinese equivalents over the first 2 decades of the twenty-first century. The decision on which particular rocket to use in a given situation will depend on a variety of factors. Some of these are set out in Table 2.4.

The decision-making routine using the above criteria is shown in Figure 2.18.

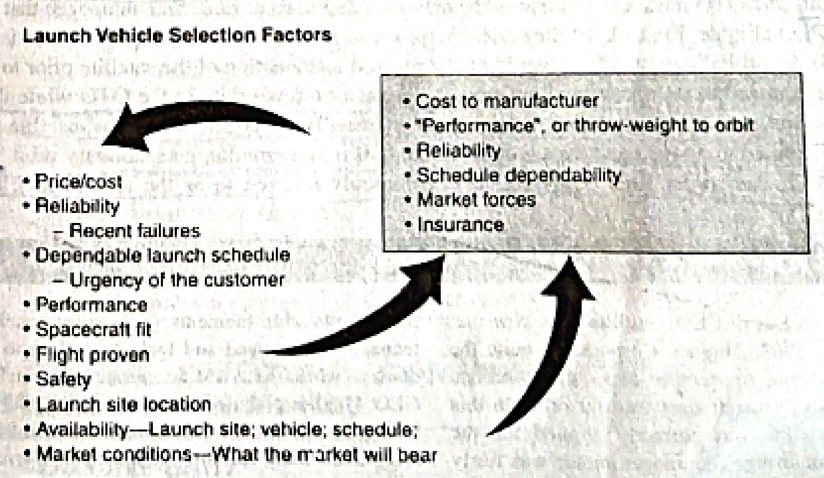


FIGURE 2.18 Schematic of the decision making process to select a rocket for a given satellite requirement (after reference 5).

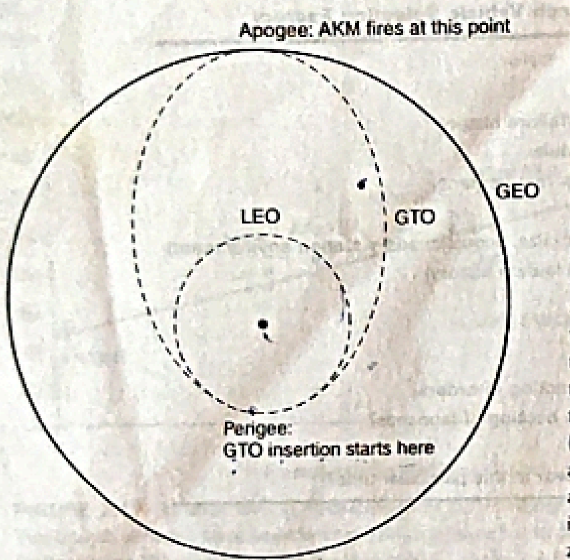


FIGURE 2.19 Illustration of the GTO/AKM approach to geostationary orbit (not to scale). The combined spacecraft and final rocket stage are placed into low earth orbit (LEO) around the earth. After careful orbit determination measurements, the final stage is ignited in LEO and the spacecraft inserted into a transfer orbit that lies between the LEO and the geostationary orbit altitude: the so-called geostationary transfer orbit or GTO. Again, after more careful orbit determination, the apogee kick motor (AKM) is fired on the satellite and the orbit is both circularized at geostationary altitude and the inclination reduced to close to zero. The satellite is then in GEO.

Some of the launch vehicles deliver the spacecraft directly to geostationary orbit (called a direct-insertion launch) while others inject the spacecraft into a geostationary transfer orbit (GTO). Spacecraft launched into GTO must carry additional rocket motors and/or propellant to enable the vehicle to reach the geostationary orbit. There are three basic ways to achieve geostationary orbit.

Placing Satellites into Geostationary Orbit

Geostationary Transfer Orbit and AKM The initial approach to launching geostationary satellites was to place the spacecraft, with the final rocket stage still attached, into low earth orbit. After a couple of orbits, during which the orbital elements are measured, the final stage is reignited and the spacecraft is launched into a geostationary transfer orbit. The GTO has a perigee that is the original LEO orbit altitude and an apogee that is the GEO altitude. Figure 2.19 illustrates the process. The position of the apogee point is close to the orbital longitude that would be the in-orbit test location of the satellite prior to it being moved to its operational position. Again, after a few orbits in the GTO while the orbital elements are measured, a rocket motor (usually contained within the satellite itself) is ignited at apogee and the GTO is raised until it is a circular, geostationary orbit. Since the rocket motor fires at apogee, it is commonly referred to as the apogee kick motor (AKM). The AKM is used both to circularize the orbit at GEO and to remove any inclination error so that the final orbit of the satellite is very close to geostationary.

> **SIDEBAR**
>
> *The first successful GEO satellite was Syncom, launched in 1963. Hughes Corporation built the satellite and the spacecraft was spin-stabilized while it was in geostationary transfer orbit. In this way, the satellite was correctly aligned for the apogee motor firing. The apogee motor was fairly powerful and the apogee burn was only for a few minutes. During this apogee burn, all of the satel*lite's deployable elements (e.g., solar panels, antennas) were stowed and locked in place to avoid damage while the AKM accelerated the satellite to GEO. Hughes patented the technique of spin stabilizing the spacecraft in GTO. To avoid infringing this patent, other satellite manufacturers developed a new way to achieve GEO, known as a *slow orbit raising* technique.

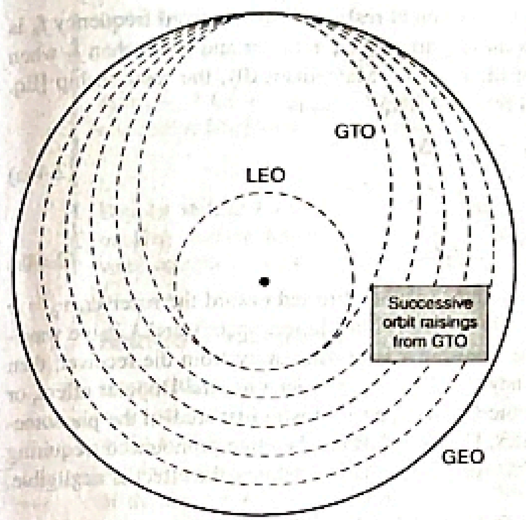


FIGURE 2.20 Illustration of slow orbit raising to geostationary orbit (not to scale). The combined spacecraft and final rocket stage are placed into low earth orbit (LEO) around the earth. As before (see Figure 2.19), the spacecraft is injected into GTO but, in this case, once the satellite is ejected from the final rocket stage, it deploys many of the elements that it will later use in GEO (solar panels, etc.) and stabilizes its attitude using thrusters and momentum wheels, rather than being spin-stabilized. The higher power thrusters are then used around the apogee to raise the perigee of the orbit until the orbit is circular at the GEO altitude. At the same time as the orbit is being raised, the thruster firings will be designed gradually to reduce the inclination to close to zero.

Geostationary Transfer Orbit with Slow Orbit Raising In this procedure, rather than employ an apogee kick motor that imparts a vigorous acceleration over a few minutes, the spacecraft thrusters are used to raise the orbit from GTO to GEO over a number of burns. Since the spacecraft cannot be spin-stabilized during the GTO (so as not to infringe the Hughes patent), many of the satellite elements are deployed while in GTO, including the solar panels. The satellite has two power levels of thrusters: one for more powerful orbit raising maneuvers and one for on-orbit (low thrust) maneuvers. Since the thrusters take many hours of operation to achieve the geostationary orbit, the perigee of the orbit is gradually raised over successive thruster firings. The thruster firings occur symmetrically about the apogee although they could occur at the perigee as well. The burns are typically 60 to 80 min long on successive orbits and up to six orbits can be used. Figure 2.20 illustrates the process.

In the first two cases, AKM and slow orbit raising, the GTO may be a modified orbit with the apogee well above the required altitude for GEO. The excess energy of the orbit due to the higher-than-necessary altitude at apogee can be traded for energy required to raise the perigee. The net energy to circularize the orbit at GEO is therefore less and the satellite can retain more fuel for on-orbit operations.

Direct Insertion to GEO This is similar to the GTO technique but, in this case, the launch service provider contracts to place the satellite into GEO. The final stages of the rocket are used to place the satellite directly into GEO rather than the satellite using its own propulsion system to go from GTO to GEO.

2.6 ORBITAL EFFECTS IN COMMUNICATIONS SYSTEMS PERFORMANCE

Doppler Shift

To a stationary observer, the frequency of a moving radio transmitter varies with the transmitter's velocity relative to the observer. If the true transmitter frequency (i.e., the

Radius of Earth

$\mu = 3.98 \times 10^5$

frequency that the transmitter would send when at rest) is f_T, the received frequency f_R is higher than f_T when the transmitter is moving toward the receiver and lower than f_T when the transmitter is moving away from the receiver. Mathematically, the relationship [Eq. (2.44a)] between the transmitted and received frequencies is

$$\frac{f_R - f_T}{f_T} = \frac{\Delta f}{f_T} = \frac{V_T}{v_p} \tag{2.44a}$$

or

$$\Delta f = V_T f_T / c = V_T / \lambda \tag{2.44b}$$

where V_T is the component of the transmitter velocity directed toward the receiver, $v_p = c$ the phase velocity of light ($2.9979 \times 10^8 \approx 3 \times 10^8$ m/s in free space), and λ is the wavelength of the transmitted signal. If the transmitter is moving away from the receiver, then V_T is negative. This change in frequency is called the *Doppler shift*, the Doppler effect, or more commonly just "Doppler" after the German physicist who first studied the phenomenon in sound waves. For LEO satellites, Doppler shift can be quite pronounced, requiring the use of frequency-tracking receivers. For geostationary satellites, the effect is negligible.

EXAMPLE 2.6.1 *Doppler Shift for a LEO Satellite*

A low earth orbit satellite is in a circular polar orbit with an altitude, h, of 1000 km. A transmitter on the satellite has a frequency of 2.65 GHz. Find

(i) The velocity of the satellite in orbit

(ii) The component of velocity toward an observer at an earth station as the satellite appears over the horizon, for an observer who is in the plane of the satellite orbit.

(iii) Hence, find the Doppler shift of the received signal at the earth station. Use a mean earth radius value, r_e, of 6378 km.

The satellite also carries a Ka-band transmitter at 20.0 GHz.

(iv) Find the Doppler shift for this signal when it is received by the same observer.

Answer *Part (i)* The period of the satellite is found from Eq. (2.21):

$$T^2 = (4\pi^2 a^3)/\mu$$

$$T^2 = 4\pi^2 \times (6378 + 1000)^3 / 3.986004418 \times 10^5$$

$$= 3.977754 \times 10^7 \text{ s}^2$$

$$T = 6306.94 \text{ s}$$

The circumference of the orbit is $2\pi a = 46{,}357.3$ km so the velocity of the satellite in orbit is v_s where

$$v_s = 46{,}357.3/6306.94 = 7.350 \text{ km/s}$$

Part (ii) The component of velocity toward an observer in the plane of the orbit as the satellite appears over the horizon is given by $v_r = v_s \cos\theta$, where θ is the angle between the satellite velocity vector and the direction of the observer at the satellite. The angle θ can be found from simple geometry to be

$$\cos\theta = r_e/(r_e + h) = 6378/7378 = 0.8645$$

Hence the component of satellite velocity toward the observer is

$$v_r = v_s \cos\theta = 6.354 \text{ km/s} = 6354 \text{ m/s}$$

Part (iii) The Doppler shift of the received signal is given by Eq. (2.44b). Hence, for this satellite and observer, with a transmitter frequency of 2.65 GHz, $\lambda = 0.1132$ m, and the Doppler shift in the received signal is

$$\Delta f = V_T/\lambda = 6354/0.1132 = 56{,}130 \text{ Hz} = 56.130 \text{ kHz}$$

Part (iv) A Ka-band transmitter with frequency 20.0 GHz has a wavelength of 0.015 m. The corresponding Doppler shift at the receiver is

$$\Delta f = V_T/\lambda = 6354/0.015 = 423.60 \text{ kHz}$$

Doppler shift at Ka band with a LEO satellite can be very large and requires a fast frequency-tracking receiver. Ka-band LEO satellites are better suited to wideband signals than narrowband voice communications. ■

Range Variations

Even with the best station-keeping systems available for geostationary satellites, the position of a satellite with respect to the earth exhibits a cyclic daily variation. The variation in position will lead to a variation in range between the satellite and user terminals. If time division multiple access (TDMA) is being used, careful attention must be paid to the timing of the frames ~~within the TDMA bursts~~ (see Chapter 6) so that the individual user frames arrive at the satellite in the correct sequence and at the correct time. Range variations on LEO satellites can be significant, as can path loss variations. While guard times between bursts can be increased to help in any range and/or timing inaccuracies, this reduces the capacity of the transponder. The on-board capabilities of some satellites permit both timing control of the burst sequence and power level control of individual user streams.

Solar Eclipse

equinox March 21

A satellite is said to be in *eclipse* when the earth prevents sunlight from reaching it, that is, when the satellite is in the shadow of the earth. For geostationary satellites, eclipses occur during two periods that begin 23 days before the equinoxes (about March 21 and about September 23) and end 23 days after the equinox periods. Figure 2.21 from reference 11 and Figure 2.22 from reference 12 illustrate the geometry and duration of the eclipses. Eclipses occur close to the equinoxes, as these are the times when the sun, the earth, and the satellite are all nearly in the same plane.

During full eclipse, a satellite receives no power from its solar array and it must operate entirely from its batteries. Batteries are designed to operate with a maximum depth of discharge; the better the battery, the lower the percentage depth of discharge can be. If the battery is discharged below its maximum depth of discharge, the battery may not recover ~~to full operational capacity~~ once recharged. The depth of discharge therefore sets the power drain limit during eclipse operations. Nickel–Hydrogen batteries, ~~long the mainstay of communications satellites~~, can operate at about a 70% depth of discharge and recover fully once recharged. Ground controllers perform battery-conditioning routines prior to eclipse operations to ensure the best battery performance during the eclipse. The routines consist of deliberately discharging the batteries until they are close to their maximum depth of discharge, and then fully recharging the batteries just before eclipse season begins.

fully

The eclipse season is a design challenge for spacecraft builders. Not only is the main power source withdrawn (the sun) but also the rapidity with which the satellite enters and exits the shadow can cause extreme changes in both power and heating effects over relatively short periods. Just like a common light bulb is more likely to fail when the current is switched on as opposed to when it is under steady state conditions, satellites can suffer many of their component failures under sudden stress situations. Eclipse periods are

Scanned By Scanner Go

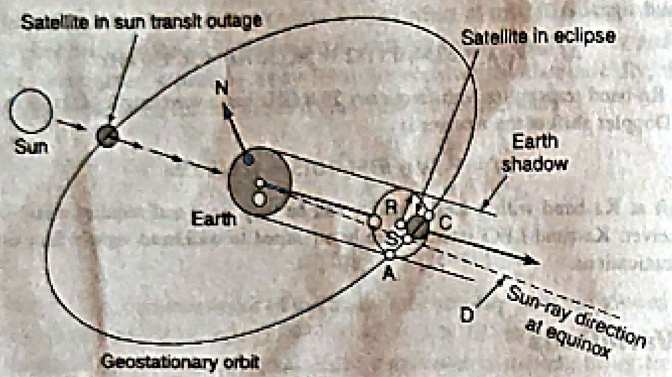


FIGURE 2.21 Eclipse geometry (Source: J. J. Spilker, Jr., *Digital Communications by Satellite*, Prentice Hall, p. 144, copyright © 1977, Pearson Education, Upper Saddle River, NJ, reprinted with permission). During the equinox periods around the March 21 and September 23, the geostationary plane is in the shadow of the earth on the far side of the earth from the sun. As the satellite moves around the geostationary orbit, it will pass through the shadow and undergo an eclipse period. The length of the eclipse period will vary from a few minutes to over an hour (see Figure 2.22), depending on how close the plane of the geostationary orbit is with respect to the center of the shadow thrown by the earth.

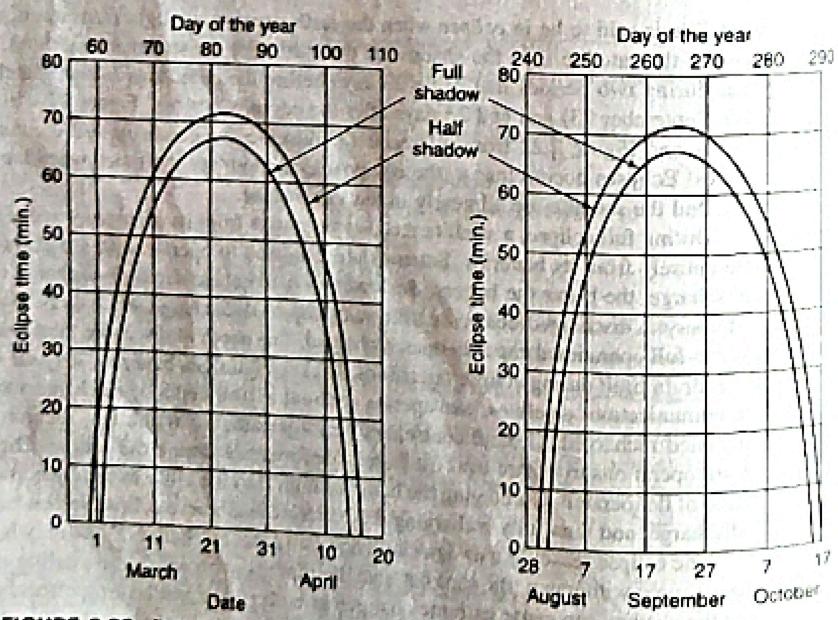


FIGURE 2.22 Dates and duration of eclipses. (Source: James Martin, *Communications Satellite Systems*, Prentice Hall, p. 37, copyright © 1978 Pearson Education, Upper Saddle River, NJ. Reprinted with permission.)

therefore monitored carefully by ground controllers, as this is when most of the equipment failures are likely to occur.

Sun Transit Outage

During the equinox periods, not only does the satellite pass through the earth's shadow on the "dark" side of the earth, but the orbit of the satellite will also pass directly in front of the sun on the sunlit side of the earth (Figure 2.23). The sun is a "hot" microwave source with an equivalent temperature of about 6000 to 10,000 K, depending on the time within the 11-year sunspot cycle, at the frequencies used by communications satellites (4 to 50 GHz). The earth station antenna will therefore receive not only the signal from the satellite but also the noise temperature transmitted by the sun. The added noise temperature will cause the fade margin ~~of the receiver to be exceeded~~ and an outage will occur. These outages may be precisely predicted. For satellite system operators with more

fade Margin

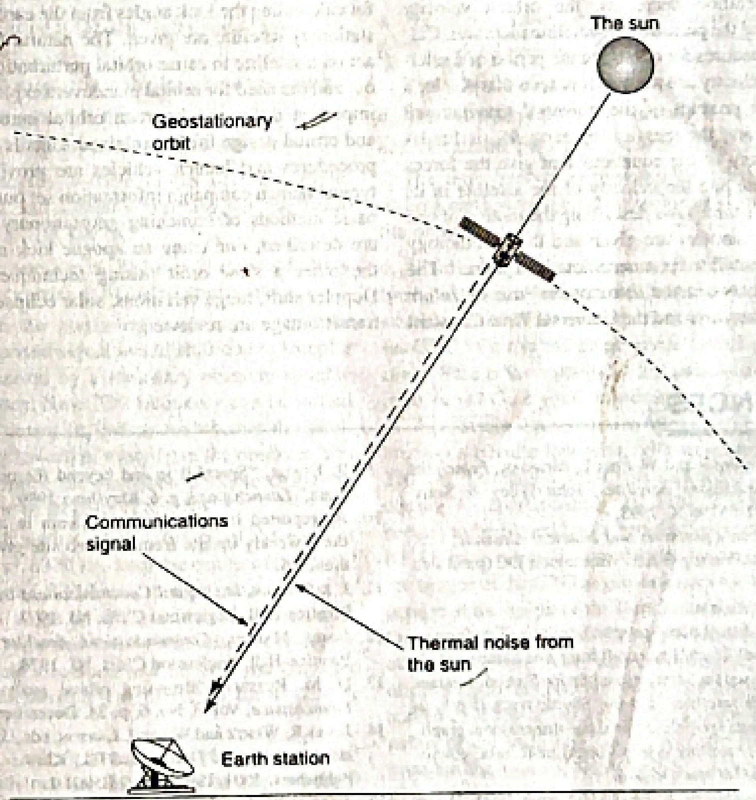


FIGURE 2.23 Schematic of sun outage conditions. During the equinox periods, not only does the earth's shadow cause eclipse periods to occur for geostationary satellites, during the sunlit portion of the orbit, there will be periods when the sun appears to be directly behind the satellite. At the frequencies used by communications satellites (4 to 50 GHz), the sun appears as a hot noise source. The effective temperature of the sun at these frequencies is on the order of 10,000 K. The precise temperature observed by the earth station antenna will depend on whether the beamwidth partially, or completely, encloses the sun.

Scanned By Scanner Go

than one satellite at their disposal, traffic can be off-loaded to satellites that are just out of, or are yet to enter, a sun outage. The outage in this situation can therefore be limited as far as an individual user is concerned. However, the outages can be detrimental to operators committed to operations during daylight hours.

2.7 SUMMARY

Newton's laws of motion explain the forces on a satellite in orbit. The balance between the force pulling a satellite inward to the earth—i.e. gravity—and that trying to fling a satellite away from the earth—kinetic energy—is a fine one. To achieve stable orbit, a satellite must have the correct velocity, be traveling in the right direction, and be at the right height for its velocity. As the orbital height increases, the gravitational acceleration decreases, the orbital velocity decreases, and the period of the satellite increases. Calculation procedures for obtaining the period of a satellite and its velocity are set out. It is seen that Kepler's constant, the product of the universal gravitational constant, G, and the mass of the earth M_E, is fundamental to many of the equations that give the forces on the satellite and the velocity of the satellite in its orbit. Kepler's three laws describing the motion of one body orbiting another are given and the terminology employed in satellite ephemeris data is explained. The relationship between the astronomers' use of *Julian dates* and *Julian days* and the Universal Time Constant (UTC), otherwise referred to as GMT, is given. The use of Julian days, which begin at noon, was introduced by astronomers to allow them to make observations overnight without having the day change on them (as normal UTC days do at midnight).

Locating the satellite in its orbit is a complex process, with a number of possible frames of reference. Different approaches are discussed. Procedures for calculating the look angles from the earth to a geostationary satellite are given. The natural forces that act on a satellite to cause orbital perturbations are set out and the need for orbital maneuvers explained. The important difference between orbital maneuver life and orbital design life is explained. Details on launch procedures and launch vehicles are provided, with typical launch campaign information set out. The two basic methods of launching geostationary satellites are described, one using an apogee kick motor and the other a slow orbit raising technique. Finally, Doppler shift, range variations, solar eclipse, and sun transit outage are reviewed.

REFERENCES

1. Gary D. Gordon and Walter L. Morgan, *Principles of Communications Satellites*, John Wiley & Sons, ISBN 0-471-55796-X, 1993.
2. *The American Ephemeris and Nautical Almanac*, U.S. Government Printing Office, Washington, DC (published annually).
3. http://www.stk.com
4. The NASA liftoff home page is http://liftoff.msfc.nasa.gov/realtime/JTrack/Spacecraft.html The home page allows you to see the International Space Station, weather and research satellites, and the Shuttle track if it is in orbit. The page specializing in three-dimensional graphical views of satellites is http://liftoff.msfc.nasa.gov/realtime/jtrack/3d/Jtrack3d.html
5. Private communication. EE 464 Spring 1997. David Walsh and Cliff Groves.
6. http://www.spaceviews.com
7. http://www.orbreport.com is a site dedicated to the space transportation industry and is an element of ISIR, International Space Industry Report.
8. K. Roundtree, "Launching Payloads by Sea," *Launchspace*, pp. 38–39, May/June 1999; see also in the Hughes web site at http://www.hcisat.com
9. R. Dekok, "Spacelift in and beyond the next millennium," *Launchspace*, p. 6, May/June 1999.
10. As reported in jeff@spaceviews.com in July 1999, the biweekly update from the web site given in reference 6.
11. J. J. Spilker, Jr., *Digital Communications by Satellite*, Prentice-Hall, Englewood Cliffs, NJ, 1977.
12. James Martin, *Communications Satellite Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1978.
13. D. M. Russell, "Browsing orbital analysis tools," *Launchspace*, Vol. 3, No. 6, p. 24, December 1, 1998.
14. James R. Wertz and Wiley J. Larson, eds., *Space Mission Analysis and Design*, 3rd Ed., Kluwer Academic Publishers, 2001. ISBN 0-7923-5901-1.
15. *Aviation Week and Space Technology*, Vol. 151, No. 24, December 13, 1999. Special issue on 21st century launch vehicles.
16. "Chinese Rockets and R&D Advances." *Aviation Week and Space Technology*, Vol. 155, No. 20, pp. 51–55, November 12, 2001.

PROBLEMS

$a = \frac{v^2}{R}$ $\frac{GM}{r^2}$

1. Explain what the terms centrifugal and centripetal mean with regard to a satellite in orbit around the earth.

A satellite is in a circular orbit around the earth. The altitude of the satellite's orbit above the surface of the earth is 1400 km. (i) What are the centripetal and centrifugal accelerations acting on the satellite in its orbit? Give your answer in m/s. (ii) What is the velocity of the satellite in this orbit? Give your answer in km/s. (iii) What is the orbital period of the satellite in this orbit? Give your answer in hours, minutes, and seconds. Note: Assume the average radius of the earth is 6378.137 km and Kepler's constant has the value 3.986004418×10^5 km^3/s^2.

2. A satellite is in a 322-km high circular orbit. Determine:

a. The orbital angular velocity in radians per second;

b. The orbital period in minutes; and

c. The orbital velocity in meters per second.

Note: Assume the average radius of the earth is 6378.137 km and Kepler's constant has the value 3.986004418×10^5 km^3/s^2.

3. The same satellite in Problem 2 above (322-km circular orbit) carries a 300-MHz transmitter.

a. Determine the maximum frequency range over which the received signal would shift due to Doppler effects if received by a stationary observer suitably located in space. Note: The frequency can be shifted both up and down, depending on whether the satellite is moving toward or away from the observer. You need to determine the *maximum* possible change in frequency due to Doppler (i.e., $2\Delta_f$).

b. If an earth station on the surface of the earth at mean sea level, 6370 km from the center of the earth, can receive the 300-MHz transmissions down to an elevation angle of 0°, calculate the maximum Doppler shift that this station will observe. Note: Include the earth's rotation and be sure you consider the *maximum possible* Doppler shift for a 322-km circular orbit.

4. What are Kepler's three laws of planetary motion? Give the mathematical formulation of Kepler's third law of planetary motion. What do the terms perigee and apogee mean when used to describe the orbit of a satellite orbiting the earth?

A satellite in an elliptical orbit around the earth has an apogee of 39,152 km and a perigee of 500 km. What is the orbital period of this satellite? Give your answer in hours. Note: Assume the average radius of the earth is 6378.137 km and Kepler's constant has the value 3.986004418×10^5 km^3/s^2.

5. An observation satellite is to be placed into a circular equatorial orbit so that it moves in the same direction as the earth's rotation. Using a synthetic aperture radar system, the satellite will store data on surface barometric pressure, and other weather related parameters, as it flies overhead. These data will later be played back to a controlling earth station after each trip around the world. The orbit is to be designed so that the satellite is directly above the controlling earth station, which is located on the equator, once every 4 h. The controlling earth station's antenna is unable to operate below an elevation angle of 10° to the horizontal in any direction. Taking the earth's rotational period to be exactly 24 h, find the following quantities:

a. The satellite's angular velocity in radians per second.

b. The orbital period in hours.

c. The orbital radius in kilometers.

d. The orbital height in kilometers.

e. The satellite's linear velocity in meters per second.

f. The time interval in minutes for which the controlling earth station can communicate with the satellite on each pass.

6. What is the difference, or are the differences, between a *geosynchronous* satellite and a *geostationary* satellite orbit? What is the period of a geostationary satellite? What is the name given to this orbital period? What is the velocity of a geostationary satellite in its orbit? Give your answer in km/s.

A particular shuttle mission released a TDRSS satellite into a circular low orbit, with an orbital height of 270 km. The shuttle orbit was inclined to the earth's equator by approximately 28°. The TDRSS satellite needed to be placed into a geostationary transfer orbit (GTO) once released from the shuttle cargo bay, with the apogee of the GTO at geostationary altitude and the perigee at the height of the shuttle's orbit. (i) What was the eccentricity of the GTO? (ii) What was the period of the GTO? (iii) What was the difference in velocity of the satellite in GTO between when it was at apogee and when it was at perigee? Note: Assume the average radius of the earth is 6378.137 km and Kepler's constant has the value 3.986004418×10^5 km^3/s^2.

7. For a variety of reasons, typical minimum elevation angles used by earth stations operating in the commercial fixed services using satellites (FSS) communications bands are as follows: C band 5°; Ku band 10°; and Ka band 20°.

(i) Determine the maximum and minimum range in kilometers from an earth station to a geostationary satellite in the three bands. (ii) To what *round-trip*