phasing conditions are taken and then the idea is extended for more and finally for n isotropic point sources. Further, the case of non-isotropic but similar to point sources will also be taken which will lead to the principle of multiplication of pattern.

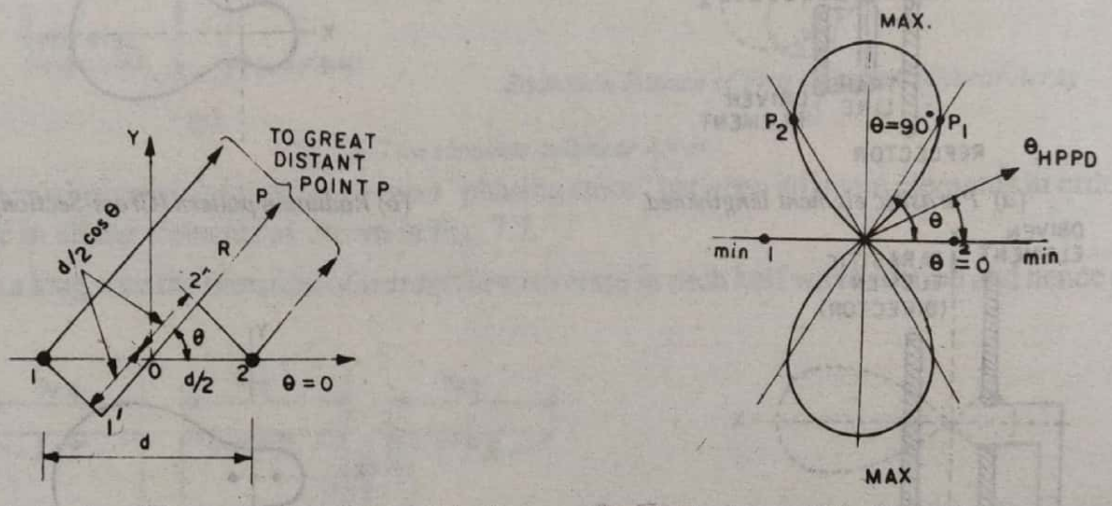
7.3.1. *Arrays of two point sources.* This is the simplest situation in the arrays of isotropic point sources in which it is assumed that the two point sources are separated by a distance (say d) and have the same polarization. Since in array theory of antennas, the superposition or addition of fields from the various sources at a great distance with due regard to phases, is involved and hence the following cases (out of many) will be dealt with Arrays of two isotropic point sources with

(1) Equal amplitude and phase. (2) Equal amplitude and opposite phase.

(3) Unequal amplitude any opposite phase.

Let us proceed now one by one.

(1) ***Arrays of two point sources with equal amplitude and phase :*** Two isotropic point sources symmetrically situated w.r.t. the origin in the cartesian coordinate system is shown in Fig. 7.9.



(a) Two isotropic point sources situated symmetrically w.r.t. origin O same amplitude and phase. *(b). Field pattern of Fig. 7.9 (a) i.e. same amplitude and phase with d = λ/2.*

Fig. 7.9.

We are to calculate fields at a great distant point, at distance (say R) from the origin O and the origin is taken as reference point for phase calculation. Obviously, waves from source 1 reaches the point P at a latter time than the waves from source 2 because of path difference ($1'2'$) involved between the two waves. Thus the fields due to source 1 lags while that due to source 2 leads. Path difference between the two waves is ($1'2'$) and is given by

$$\text{Path difference} = (1'2') \text{ metres} = \left(\frac{d}{2}\cos\theta + \frac{d}{2}\cos\theta\right) \text{ metres}$$

$$= d\cos\theta \text{ metres} \quad \ldots \; 7.1\,(a)$$

$$= \frac{d}{\lambda}\cos\theta \text{ wavelengths} \quad \ldots \; 7.1\,(b)$$

Then from optics, it is known as

$$\boxed{\text{Phase angle } (\psi) = 2\pi \text{ (Path difference)}} \quad \ldots \; (7.2)$$

$$\therefore \quad \psi = 2\pi\left(\frac{d}{\lambda}\cos\theta\right) \text{ radians} = \frac{2\pi}{\lambda} d\cos\theta \text{ radians} \quad \ldots \; 7.2\,(a)$$

or

$$\boxed{\psi = \beta d \cos\theta \text{ radians}} \quad \text{if } \beta = \frac{2\pi}{\lambda} \quad \ldots \; 7.2\,(b)$$

Also let, E_1 = Far electric field at distance point P, due to source 1

E_2 = Far electric field at distant point P, due to source 2

E = Total electric field at distant point,

and $\psi = \beta d \cos\theta$ radians

= Phase angle difference between the fields of the two sources measured at angle θ along radius vector line.

Then, total far field at distant point P, in the direction of θ is given by

$$\boxed{E = E_1 e^{-j\psi/2} + E_2 e^{+j\psi/2}} \quad \text{... (7.3)}$$

where $E_1 e^{-j\psi/2}$ ≡ field component due to source 1 ;

$E_2 e^{+j\psi/2}$ ≡ field component due to source 2;

But, in this case it is assumed that amplitudes are same, hence

$$E_1 = E_2 \equiv E_0 \text{ (say)} \quad \text{... (7.4)}$$

$$\therefore \quad E = E_0 \left(e^{-j\psi/2} + e^{+j\psi/2} \right)$$

$$= 2\, E_0 \left(\frac{e^{-j\psi/2} + e^{+j\psi/2}}{2} \right) \quad \Bigg| \quad \because \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \text{ from trigonometry}$$

$$\boxed{E = 2\, E_0 \cos\psi/2} \quad 7.4\text{ (a)}$$

$$\boxed{E = \underset{\text{(Amp.)}}{2\, E_0} \underset{\text{(Phase)}}{\cos\left(\frac{\beta d \cos\theta}{2} \right)}} \quad \text{... 7.4 (b)}$$

This is the equation of far field pattern of two isotropic point sources of same amplitude and phase. Here the total amplitude is $2\,E_0$ whose maximum value may be 1. By putting $2\,E_0 = 1$ or $E_0 = \frac{1}{2}$, the pattern is said to be normalized. Thus Eqn. 7.4 (b) becomes

$$E = \cos\left(\frac{\beta d \cos\theta}{2} \right) = \cos\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \frac{\cos\theta}{2} \right) \quad \Big| \text{ if } d = \lambda/2 \text{ taken}$$

$$\boxed{E = \cos\left(\frac{\pi}{2} \cos\theta \right)} \quad \text{... (7.5)}$$

In order to draw the field pattern, the directions of maxima, minima and half power points must be known, which can be calculated with the help of eqn. 7.5 as follows :

Maxima direction. E is maximum, when $\cos(\pi/2 \cos\theta)$ is maximum, and its maximum value is ± 1.

$\therefore E$ is max. when $\quad \cos(\pi/2 \cos\theta) = \pm 1$

or $\quad \pi/2 \cos\theta_{max} = \pm n\pi \quad$ where $n = 0, 1, 2, ...$

or $\quad \pi/2 \cos\theta_{max} = 0 \quad$ if $n = 0$

or $\quad \cos\theta_{max} = 0$

$$\boxed{\theta_{max} = 90^0 \text{ and } 270^0} \quad 7.5\text{ (a)}$$

Minima directions. E is minimum when $\cos(\pi/2 \cos\theta)$ is minimum and its minimum value is 0.

$\therefore$ E is minimum. When $\cos(\pi/2 \cos\theta) = 0$

or $(\pi/2 \cos\theta_{min}) = \pm(2n+1)\pi/2$ where $n = 0, 1, 2, \ldots$

or $(\pi/2 \cos\theta_{min}) = \pi/2$

or $\cos\theta_{min} = \pm 1$

$$\boxed{\theta_{min} = 0^\circ \text{ and } 180^\circ} \quad \ldots 7.5\text{ (b)}$$

Half power point direction. At half power points power is $\frac{1}{2}$ or voltage or current is $1/\sqrt{2}$ times the maximum value of voltage or current.

$\therefore$ $\cos(\pi/2 \cos\theta) = \pm 1/\sqrt{2}$

or $\pi/2 \cos\theta_{HPPD} = \pm(2n+1)\pi/4.$ where $n = 0, 1, 2, \ldots$

or $\pi/2 \cos\theta_{HPPD} = \pm\pi/4$

or $\cos\theta_{HPPD} = \pm\frac{1}{2}$

$$\boxed{\theta_{HPPD} = 60^\circ, 120^\circ} \quad \ldots 7.5\text{ (c)}$$

If now the field patterm bet. E versus θ is drawn for the case $d = \lambda/2$, then the Fig. 7.9 (b) is obtained which is a bidirectional, figure of eight. 360^0 rotation of this figure around x-axis will generate the 3-dimensional space pattern — **a doughnut shape.**

This is the simplest type of "**broad-side array**" and is also known as "**broad-side couplet**" as two isotropic radiators radiates in phase.

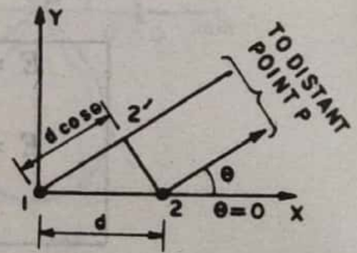


Fig. 7.10. Two point source of equal amplitude and phase, separated by a distance d.

As an alternative, if reference point in Fig. 7.9 (a) is shifted to say source 1 (instead midway of the array then amplitude of the field pattern remains the same (e.g. $2E_0$) but the phase pattern changes as shown below.

The resultant far field pattern, in this case, is the vector sum of the fields of individual sources at the distant point P,

$$\therefore \quad E = E_1 e^{j0} + E_2 e^{j\psi} = E_1 + E_2 e^{j\psi} \quad \Big| \because e^{j0} = e^0 = 1 \quad \ldots 7.6\text{ (a)}$$

where $\psi = \beta d \cos\theta$

Applying the condition 7.4, we have

$$E = E_0(1 + e^{j\psi}) = E_0 e^{j\psi/2}(e^{-j\psi/2} + e^{j\psi/2}) \quad \Big| \text{ Taking } e^{+j\psi/2} \text{ common}$$

$$\text{or} \quad = 2E_0 e^{j\psi/2}\left(\frac{e^{-j\psi/2} + e^{j\psi/2}}{2}\right) = 2E_0 e^{j\psi/2}\cos\psi/2$$

$$\boxed{\begin{array}{l} E = \underset{\text{Amp.}}{(2E_0)}\ \underset{\text{Phase}}{(\cos\psi/2\, e^{j\psi/2})} \\ E_{norm} = (\cos\psi/2\, e^{j\psi/2}) \end{array}} \quad \begin{array}{r} \ldots 7.7\text{ (a)} \\ \Big| \because 2E_0 = 1 \quad \ldots 7.7\text{ (b)} \end{array}$$

Thus, comparison of 7.7. (b) and 7.4 (a) indicates that phases are not the same. Eqn. 7.7 (b) may be rewritten as

$$E_{norm} = \cos\psi\left\{\cos\frac{\psi}{2} + j\sin\frac{\psi}{2}\right\} \quad \ldots 7.8\text{ (a)}$$

$$= \cos\psi\left\{\angle\frac{\psi}{2}\right\} \quad \ldots 7.8\text{ (b)}$$

Here $e^{j\psi/2}$ or $\angle\ \psi/2$ represents the variation of phase w.r.t. reference (source 1).

(2) ***Arrays of two point sources with equal amplitude and opposite phase.*** This is exactly similar to above except that point source 1 is out of phase or opposite phase (180°) to source 2 i.e. when there is maximum in source 1 at one particular instant, then there is minimum in source 2 at that instant and vice-versa. Referring to Fig. 7.9 (a), the total far field at distant point P, is given by

$$E = (-E_1 e^{-j\psi/2}) + (+E_2 e^{+j\psi/2})$$

because phase of source 1 and source 2 at distant point P is $-\psi/2$ and $+\psi/2$, since the reference being at between midway of two sources.

But $$E_1 = E_2 = E_0 \text{ (say)}$$

Then $$E = E_0\, 2j\left(\frac{e^{j\psi/2} - e^{-j\psi/2}}{2j}\right)$$

$$\boxed{E = 2jE_0 \sin\psi/2} \qquad \text{... 7.9 (a)}$$

$$\boxed{E = 2jE_0 \sin\left(\frac{\beta d}{2}\cos\theta\right)} \qquad \text{... 7.9 (b)}$$

The Eqn. (7.9) of total far field is similar to that of Eqn. 7.4 but for that it is sine function instead of cosine and the operator j is involved. Presence of operator j simply means that opposite phase brings a phase shift of 90° in the total field. This will be evident by drawing the field pattern. for let $d = \lambda/2$ and and $2E_0 j = 1$

$\therefore$ $$\boxed{E_{norm} = \sin(\pi/2 \cos\theta)} \qquad \text{... (7.10)}$$

Maximum directions. Maximum value of sine function is ± 1

$\therefore$ $$\sin(\pi/2\cos\theta) = \pm 1$$

or $$(\pi/2 \cos\theta_{max}) = \pm(2n+1)\pi/2 \qquad \text{where } n = 0, 1, 2,\ldots\ldots$$

or $$(\cos\theta_{max}) = \pm 1 \qquad \text{if } n = 0$$

$$\boxed{\theta_{max} = 0^\circ \text{ and } 180^\circ} \qquad \text{... 7.11 (a)}$$

Minima directions. Minimum value of a sine function is 0

$\therefore$ $$\sin(\pi/2\cos\theta) = 0$$

or $$\pi/2\cos\theta_{min} = \pm n\pi \qquad \text{where } n = 0, 1, 2,\ldots\ldots$$

or $$\cos\theta_{min} = 0$$

or $$\boxed{\theta_{min} = 90^\circ \text{ and } -90^\circ} \qquad \text{... 7.11 (b)}$$

Half power point directions. For

$\therefore$ $$\sin(\pi/2\cos\theta) = \pm\frac{1}{\sqrt{2}}$$

or $$\pi/2\cos\theta_{HPPD} = \pm(2n+1)\pi/4$$

or $$\pi/2\cos\theta_{HPPD} = \pm\pi/4 \quad \text{if } n = 0$$

or $$\cos\theta_{HPPD} = \pm\frac{1}{2}$$

or $$\boxed{\theta_{HPPD} = 60^\circ,\ \pm 120^\circ} \qquad \text{... 7.11 (a)}$$

From these, it is possible to draw the field pattern which is as shown in Fig. 7.11.

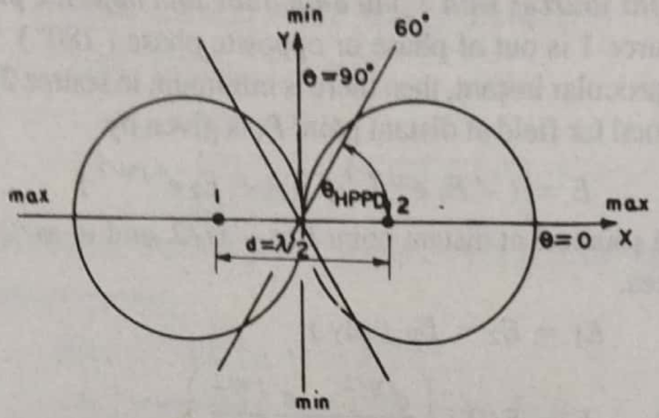


Fig. 7.11. Two point sources with equal amplitude and opposite phase spacing λ/2.

It is seen that maxima have shifted 90° along X-axis in comparison to in-phase field pattern. The figure is horizontal figure of 8 and 3-dimensional space pattern is obtained by rotating it along X-axis. Once the arrangement gives maxima along line joining the two sources and hence this is one of the simplest type of **"End fire" 'Array'**.

(3) *Arrays of two point sources with unequal amplitude and any phase.* Let us now consider a general condition in which the amplitudes of two point sources are not equal and hence any phase difference say α. With reference to Fig. 7.12, let us also assume at source 1 is taken as reference for phase and amplitudes

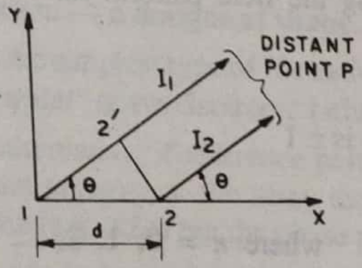


Fig. 7.12 (a). Two point sources with unequal amplitude and any phase difference δ.

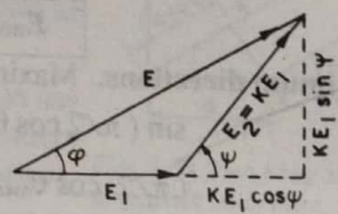


Fig. 7.12 (b). Vector diagram of Fig.. 7.12 (a)

of fields due to source 1 and 2 at a distant point P is E_1 and E_2 in which E_1 is greater than E_2. Then the total-phase difference between the radiations of two sources at point P is given by

$$\boxed{\psi = \frac{2\pi}{\lambda} d \cos\theta + \alpha} \quad \text{... (7.12)}$$

where α is the phase angle by which the current (I_2) of source-2 leads the current (I_1) of source 1. If $\alpha = 0$ or 180^0 and $E_1 = E_2 = E_0$, then it will correspond to the above two cases resp. just described. The vector diagram is shown in Fig. 7.12 (b). The total fields at P its given by,

$$E = E_1 e^{j0} + E_2 e^{j\psi} = E_1 \left(1 + \frac{E_2}{E_1} e^{j\psi}\right) \quad \Big| \because e^{j0} = e^0 = 1$$

$$\boxed{E = E_1 (1 + K e^{j\psi})} \quad \text{... (7.13)}$$

where
$$K = \frac{E_2}{E_1} \quad \text{... (7.14)}$$

and since
$$E_1 > E_2$$

∴
$$K < 1$$

i.e.
$$0 \le K \le 1$$

From Eqn 7.13 magnitude and phase angle (φ) at point P is given by taking its modulus

$$\therefore \quad E = \left| E_1 \left\{ 1 + K(\cos\psi + j\sin\psi) \right\} \right|$$

or
$$E = E_1 \sqrt{(1 + K\cos\psi)^2 + (K\sin\psi)^2} \angle\varphi \qquad \text{... 7.15 (a)}$$

where φ = Phase angle at P

$$\varphi = \tan^{-1} \frac{K\sin\psi}{1 + K\cos\psi} \qquad \text{... 7.15 (b)}$$

It may be noted that if $E_1 = E_2$,, $K = 1$ then Eqns. 7.14 and 7.13 become Eqns 7.6 (a).

7.4. NON-ISOTROPIC BUT SIMILAR POINT SOURCES

So far, array of two isotropic point sources were considered but this idea may be extended to the sources which are not-isotropic i.e. non-isotropic provided their field patterns are similar to that of isotropic point source. In other words, fields patterns of non-isotropic must have the same shape and orientation. However, it is not necessary that amplitude of individual non-isotropic source is equal. Such a non-isotropic source is given the name non-isotropic but similar point source. In case, the amplitudes of the individual sources are equal, the sources would be non-isotropic but identical.

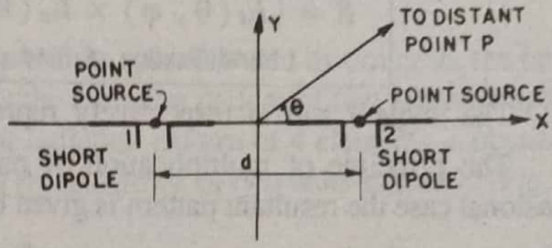


Fig. 7.13. Two non-isotropic (short dipoles and point sources).

Let us now consider two short dipoles which are superimposed over the two isotropic point sources and are separated by a distance.

Let the field pattern of each isotropic point source be given by

$$E_0 = E_1 \sin\theta \qquad \text{... (7.16)}$$

Since it is also possible to obtain such pattern from a short dipole and hence the two types of sources are symmetrically superimposed w.r.t. to origin. From eqn. 7.4 (a), the field pattern of two identical isotropic source is given by

$$E = 2E_0 \cos\psi/2 \qquad \text{... (7.4 a)}$$

where
$$\psi = \beta d\cos\theta + \alpha$$

Combining eqn. 7.4 (a) and 7.16, we have

$$E = 2E_1 \sin\theta \cos\psi/2$$

$$E_{norm} = (\sin\theta) \times (\cos\psi/2) \qquad \text{... (7.17 a)}$$

$$E_{norm} = \left(\begin{array}{c} \textbf{Pattern of Individual} \\ \textbf{isotropic source} \end{array} \right) \times \left(\begin{array}{c} \textbf{Pattern of Array of two} \\ \textbf{isotropic point sources} \end{array} \right)$$

This lead to the **principle of multiplication of pattern** as multiplication pattern of individual point source and pattern of array of isotropic point sources gives the field pattern of non-isotropic but similar point sources.

7.5. MULTIPLICATION OF PATTERN

(AMIE S/1993, 1994 W/1994, AMIETE, Nov. 1977)

Multiplication of pattern or simply pattern multiplication, in general, can be stated as follows :

"The total field pattern of an array of non-isotropic but similar sources is the multiplication of the individual source patterns and the pattern of an array of isotropic point sources each located at the phase centre of individual source and having the relative amplitude and phase, whereas the total phase pattern is the addition of the phase pattern of the individual sources and that of the array of isotropic point sources."

Here the pattern of the individual sources is assumed to be same whether it is in the array or isolated.

Further the reference point for total phase pattern is the phase centre of the array. The principle of multiplication pattern is applicable for two and three dimensional patterns.

Let E = Total field.

$E_i(\theta, \varphi)$ = Field pattern of individual source.

$E_a(\theta, \varphi)$ = Field pattern of array of isotropic point sources.

$E p_i(\theta, \varphi)$ = Phase pattern of individual source.

$E_{pa}(\theta, \varphi)$ = Phase pattern of array of isotropic point source.

Then the total field pattern of an array of non-isotropic but similar source, symbolically, may be written as

$$\boxed{E = \underbrace{\{E_i(\theta, \varphi) \times E_a(\theta, \varphi)\}}_{\text{(Multiplication of field pattern)}} \times \underbrace{\{E p_i(\theta, \varphi) + E p_a(\theta, \varphi)\}}_{\text{(Addition of phase pattern)}}} \quad \text{... (7.18)}$$

The angle θ and φ respectively, represent the 'polar' and 'azimuth' angles.

The principle of multiplication of pattern is true for any number of **similar sources.** For two dimensional case the resultant pattern is given by eqn. 7.4 (a) or 7.17 (a)

$$E = 2E_0 \cos \psi/2 \quad \text{... (7.4 a)}$$

$$E = 2E_1 \sin\theta \cos \psi/2 \quad \text{... (7.17 a)}$$

or

$$\boxed{E = E(\theta) \cdot \cos \psi/2} \quad \text{... (7.19)}$$

Evidently, E_0 is a function of θ say $E(\theta)$. The total field pattern, in this case, is multiplication of field pattern known as primary and $\cos \psi/2$ the secondary pattern or array factor that the principle is equally applicable to 3 dimensional case also.

The principle of multiplication of patterns provides a speedy method for sketching the pattern of complicated arrays just by inspection and thus the principle proves to be a useful tool in design of antenna arrays. The width of the principle lobe (i.e. width between nulls) and the corresponding width of array pattern are same. The secondary lobes are determined from the number of nulls in the resultant pattern. In resultant pattern the number of nulls are the sum of nulls of individual pattern and array pattern. This method is exact and point by point multiplication of patterns provides the exact pattern of the resultant.

Let us now use the principle to some typical cases.

7.5.1. *Radiation Pattern of 4-isotropic elements fed in phase, spaced $\lambda/2$ apart :*

(AMIETE, May 1978)

Let the 4--elements of isotropic (or non-directive) radiators are in a linear arrays (Fig. 7.14) in which elements are placed at a distance of $\lambda/2$ and are fed in phase. i.e. $\alpha = 0$. One of the method to get the radiation pattern of the array is to add the fields of individual four elements at a distance point P vectorially but instead an alternative method, using the principle of multiplicity of pattern, will be shown to get the same.

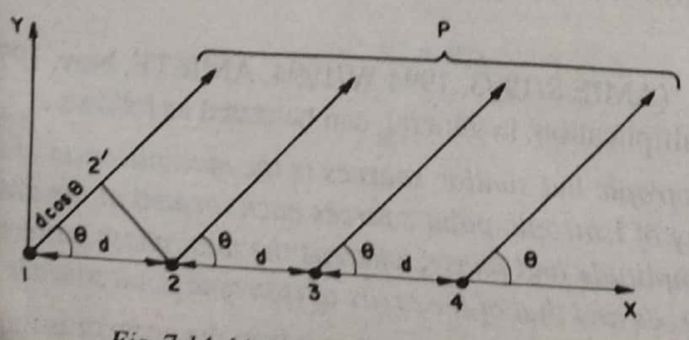


Fig. 7.14. Linear arrays of 4-isotropic elements spaced $\lambda/2$ apart, fed in phase.

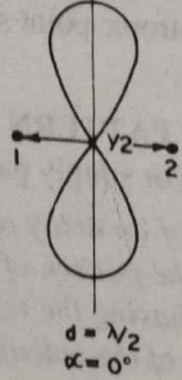


Fig. 7.15. (a)

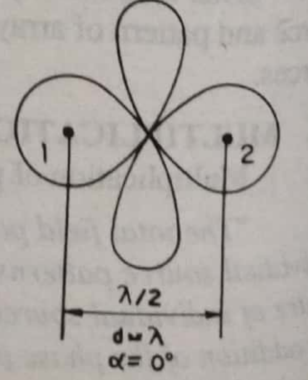


Fig. 7.15. (b)