Example 1.1. Find the transfer function of the given network

Solution : Step 1 : Apply *KVL* in mesh (1)

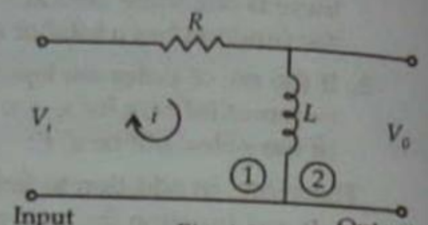


Fig. 1.9

$$V_i = Ri + L\frac{di}{dt} \qquad ...(1.13)$$

Apply *KVL* in mesh (2)

$$V_0 = L\frac{di}{dt} \qquad ...(1.14)$$

Step 2 : Take laplace transform of equation 1.13 & 1.14 with assumption that all initial conditions are zero.

$$V_i(s) = RI(s) + sLI(s) \qquad ...(1.15)$$

$$V_0(s) = sLI(s) \qquad ...(1.16)$$

Step 3 : Calculation of transfer function

$$\frac{V_0(s)}{V_i(s)} = \frac{sLI(s)}{(R+sL)I(s)}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{sL}{R+sL} \qquad ...(1.17)$$

Eq. 1.17 is the required transfer function.

Example 1.2. Determine the transfer function of the electrical network shown in fig. 1.10.

Solution : Step 1 : Apply *KVL* in both meshes

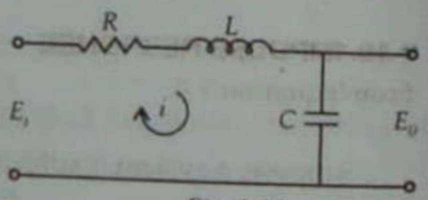


Fig. 1.10.

$$E_i = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \qquad ...(1.18)$$

$$E_0 = \frac{1}{C}\int i\,dt \qquad ...(1.19)$$

Step 2 : Take laplace transform of eqn 1.18 & 1.19

$$E_i(s) = RI(s) + sLI(s) + \frac{1}{Cs}I(s)$$

$$= I(s)\left[R + sL + \frac{1}{Cs}\right]$$

$$E_i(s) = I(s)\left[\frac{RCs + s^2LC + 1}{Cs}\right] \qquad ...(1.20)$$

$$E_0(s) = \frac{1}{Cs}I(s) \qquad ...(1.21)$$

Step 3 : Determination of transfer function

$$\frac{E_0(s)}{E_i(s)} = \frac{I(s)}{Cs} \cdot \frac{Cs}{I(s)\left[s^2Lc + SRC + 1\right]}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{1}{S^2LC + SRC + 1} \qquad \textbf{Ans.} \qquad ...(1.22)$$

Example 1.3. Obtain the transfer function $\frac{V_2(s)}{V_1(s)}$ for fig. 1.11.

Solution : Step 1 : *KCL* at node '*a*'

$$i = i_1 + i_2 \qquad ...(1.23)$$

$$i_1 = \frac{V_1 - V_2}{R_1}$$

$$i_2 = C\frac{d}{dt}(V_1 - V_2)$$

$$i = i_3 = \frac{V_2}{R_2}$$

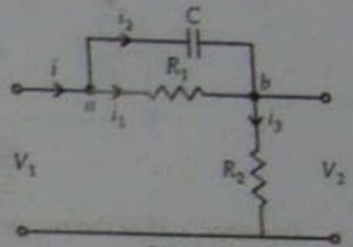


Fig. 1.11

Put all these values in eq^n 1.23

$$\frac{V_2}{R_2} = \frac{V_1 - V_2}{R_1} + C\frac{d}{dt}(V_1 - V_2) \qquad ...(1.24)$$

Step 2 : Take laplace transform of *eq*^{n} 1.24

$$\frac{V_2(s)}{R_2} = \frac{1}{R_1}V_1(s) - \frac{1}{R_1}V_2(s) + Cs\,V_1(s) - Cs\,V_2(s)$$

$$\frac{V_2(s)}{R_2} + \frac{1}{R_1}V_2(S) + Cs\,V_2(s) = \frac{1}{R_1}V_1(s) + Cs\,V_1(s)$$

$$V_2(s)\left[\frac{1}{R_1} + \frac{1}{R_2} + Cs\right] = V_1(s)\left[\frac{1}{R_1} + Cs\right]$$

Step 3 : Determination of transfer function

$$V_2(s)\left[\frac{R_1 + R_2 + R_1R_2Cs}{R_1R_2}\right] = V_1(s)\left[\frac{1 + R_1Cs}{R_1}\right]$$

$$\frac{V_2(s)}{V_1(s)} = \frac{R_2 + R_1R_2Cs}{R_1 + R_2 + R_1R_2Cs} \qquad \textbf{Ans.} \qquad ...(1.25)$$

Example 1.5. Determine the transfer function of fig 1.13.

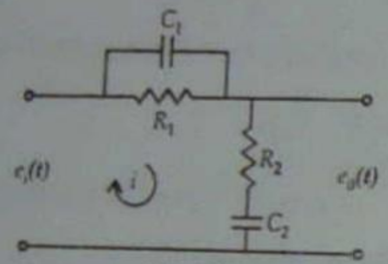


Fig. 1.13.

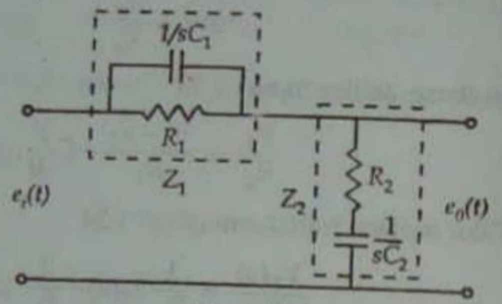


Fig. 1.14.

Solution : Step 1 : calculation *of* Z_1 :

$$Z_1 = \frac{R_1 \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{R_1 C_1 s + 1} \qquad \text{...(1.29)}$$

Step 2 : Calculation of Z_2 :

$$Z_2 = R_2 + \frac{1}{sC_2} = \frac{R_2 C_2 S + 1}{sC_2} \qquad \text{...(1.30)}$$

Step 3 : Calculation of transfer function in terms of Z_1 & Z_2

$$\frac{E_0(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)} \qquad \text{...(1.31)}$$

Step 4 : Calculation of transfer function in terms of R_1, R_2, C_1 & C_2 Put the values of Z_1 (s) &
from eq^n 1.29 & 1.30 in eq^n 1.31

$$\frac{E_0(s)}{E_i(s)} = \frac{(1 + R_2 C_2 S)/Sc_2}{\frac{R_1}{C_1 R_1 S + 1} + \frac{R_2 C_2 S + 1}{SC_2}}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{(1+R_1C_1S)(1+R_2C_2S)}{(1+R_1C_1S)(1+R_2C_2S)+R_1C_2S} \quad ...(1.32)$$

The above eq^n is the required transfer function of the given circuit.

Example 1.6. Determine the transfer function of given transformer coupled circuit (fig. 2.9).

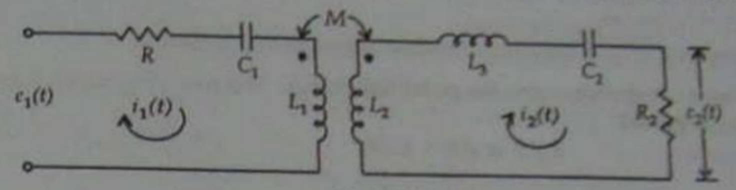


Fig. 1.15.

Solution : Apply *KVL* in both meshes

$$e_1(t) = Ri_1(t) + \frac{1}{C_1}\int i_1\, dt + L_1\frac{di_1(t)}{dt} - M\frac{di_2(t)}{dt} \quad ...(1.33)$$

$$e_2(t) = R_2i_2(t) \quad ...(1.34)$$

$$0 = R_2i_2(t) + (L_3+L_2)\frac{d}{dt}i_2(t) + \frac{1}{C_2}\int i_2(t)dt - M\frac{d}{dt}i_1(t) \quad ...(1.35)$$

Take Laplace transform of eq^n 1.33, 1.34 & 1.35

$$E_1(s) = I_1(s)\left[R + \frac{1}{C_1S} + SL_1\right] - SMI_2(s) \quad ...(1.36)$$

$$E_2(s) = R_2I_2(s) \quad ...(1.37)$$

$$0 = I_2(s)[R_2 + s(L_2+L_3) + 1/sC_2] - sMI_1(s) \quad ...(1.38)$$

Solving the equations 1.36, 1.37 & 1.38. the required transfer function.

$$G(s) = \frac{S^3R_2C_1C_2M}{\left[SR_2C_2 + S^2C_2(L_2+L_3)+1\right]\left[S^2L_1C_1 + SC_1R_1 + 1\right] - M^2S^4C_1C_2} \quad \textbf{Ans.}$$

MECHANICAL SYSTEM

1.11. TRANSLATIONAL SYSTEMS

The motion takes place along a straight line is known as translational motion. There are three types of forces that resists motion.

1. Intertia Force : Consider a body of mass 'M' & acceleration 'a', then according to Newton's second law of motion the inertia force will be equal to the product of mass 'M' & acceleration 'a'.

$$F_M(t) = Ma(t) \quad ...(1.39)$$

In terms of velocity the eq^n (1.39) becomes

$$F_M(t) = M\frac{dv(t)}{dt} \quad ...(1.40)$$

$\mapsto x(t)$

In terms of displacement the eq^n (1.39) can be expressed as

2. Damping Force : … to the velocity.

$$F_D(t) = Bv(t) = B\frac{d}{dt}x(t) \qquad ...(1.42)$$

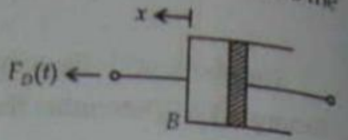


Fig. 1.17.

Where, B = Damping coefficient

unit of B = N/m/sec.

We can represent 'B' by a dashpot, consists of piston and cylinder.

3. Spring Force : A spring stores the potential energy. The restoring force of a spring is proportional to the displacement.

$$F_K(t) \;\alpha\; x(t) = Kx(t)$$

$$F_K(t) = K\int v(t)\,dt \qquad ...(1.43)$$

Where 'K' = spring constant or stiffness

unit of $K = N/m$

The stiffness of a spring can be defined as restoring force per unit displacement.

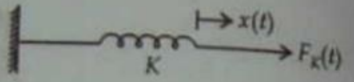


Fig. 1.18

1.12. ROTATIONAL SYSTEM

The rotational motion of a body can be defined as the motion of a body about a fixed axis. There are three types of torques resists the rotational motion.

1. Inertia Torque : Inertia (J) is the property of an element that stores the kinetic energy of rotational motion. The inertia Torque T_I is the product of moment of Inertia J and angular acceleration $\alpha(t)$

$$T_I(t) = J\,\alpha(t)$$

$$T_I(t) = J\frac{d}{dt}\omega(t)$$

$$T_I(t) = J\frac{d^2}{dt^2}\theta(t) \qquad ...(1.44)$$

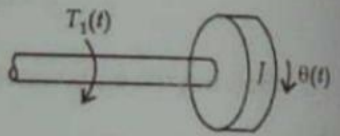


Fig. 1.19.

Where $\omega(t)$ = angular velocity

$\theta(t)$ = angular displacement

unit of Torque = N-m

2. Damping Torque : The damping Torque $T_D(t)$ is the product of damping coefficient B and angular velocity ω.

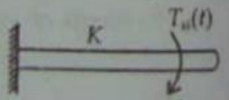
$$T_D(t) = B\omega(t)$$

$$T_D(t) = B\frac{d}{dt}\theta(t) \qquad ...(1.45)$$

3. Spring Torque : Spring torque $T_\theta(t)$ is the product of torsional stiffness and angular displacement.

$$T_\theta(t) = K\theta(t)$$

unit of K = N-m/rad. ...(1.46)



If we compare the equations

Table 1.2.

S.No.	Translational	Rotational
1.	Force, F	Torque, T
2.	Acceleration, a	angular acceleration, α
3.	Velocity, v	angular velocity, ω
4.	Displacement, x	angular displacement, θ
5.	Mass, M	Moment of inertia, J
6.	Damping coefficient B	Rotational damping coeff., B
7.	Stiffness	torsional stiffness

1.13. D'ALEMBERT'S PRINCIPLE

This principle states that "for any body, the algebraic sum of externally applied forces and the forces resisting motion in any given direction is zero".

D' Alembert principle is useful in writing the equation of motion of mechanical system. Consider, a system shown in fig. 1.21. consisting of a mass M, spring & dashpot.

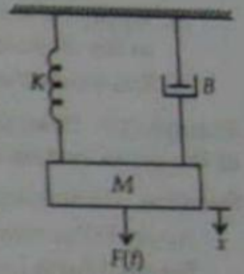


Fig. 1.21.

First choose a reference direction. All the forces in the direction of reference direction considered as positive & the forces opposite to the reference direction taken as negative.

External Force : $F(t)$

Resisting Forces : *a.* Inertia Force $F_m(t) = -M\frac{d^2}{dt^2}x(t)$

b. Damping Force $F_D(t) = -B\frac{d}{dt}x(t)$

c. spring Force $F_K(t) = -Kx(t)$

According to D' Alembert's principle

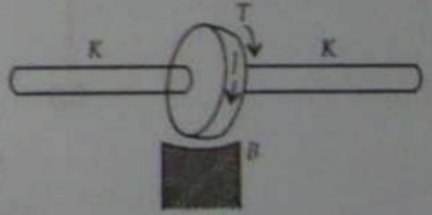
$$F(t) + F_m(t) + F_D(t) + F_K(t) = 0$$

$$F(t) - M\frac{d^2}{dt^2}x(t) - B\frac{d}{dt}x(t) - Kx(t) = 0$$

or, $$F(t) = M\frac{d^2}{dt^2}x(t) + B\frac{d}{dt}x(t) + Kx(t) \quad ...(1.47)$$

Consider a rotational system shown in fig. 1.22.

External Torque : $T(t)$



Resisting Torque : *a.* Inertia Torque $T_J(t) = -J\frac{d\omega(t)}{dt}$

b. Damping Torque $T_D(t) = -B\frac{d}{dt}\theta(t)$

c. spring Torque $T_K(t) = -K\theta$

or, $$T(t) = J\frac{d}{dt}\omega(t) + B\frac{d}{dt}\theta(t) + K\theta(t) \quad ...(1.48)$$

So, D'Alembert principle for rotational motion is

"For any body, the algebaric sum of externally applied torques and the torques resisting rotation about any axis is zero".

1.14. PROCEDURE OF WRITING THE MODELS OF MECHANICAL SYSTEM

1. Assume system is in equilibrium.
2. Assume that the system is given same arbitrary displacement if no. of distributing forces are present.
3. Draw the free body diagram of forces exerted on each mass in the system.
4. Apply Newton's law of motion to each diagram, using the convention that any force acting in the direction of assume displacement is positive.
5. Rearrange the equations in suitable form to be solved by any means.

Example 1.7. Draw the free body diagram and write the diffrential equation of the given system shown in fig. 1.23.

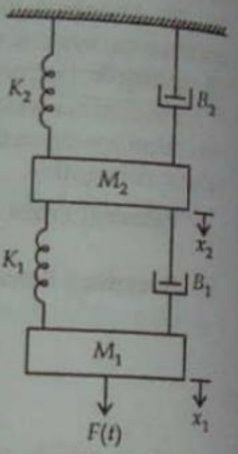


Fig. 1.23

Solution : Differential equation For M_1 :

Apply D'Alembert principle

External force : $F(t)$

Resisting forces :

1. Inertia force $\quad F_M = -M_1\frac{d^2}{dt^2}x_1$
2. Damping force

$$F_D = -B_1\frac{d}{dt}(x_1 - x_2)$$

3. Spring force $\quad F_K = K_1(x_1 - x_2)$

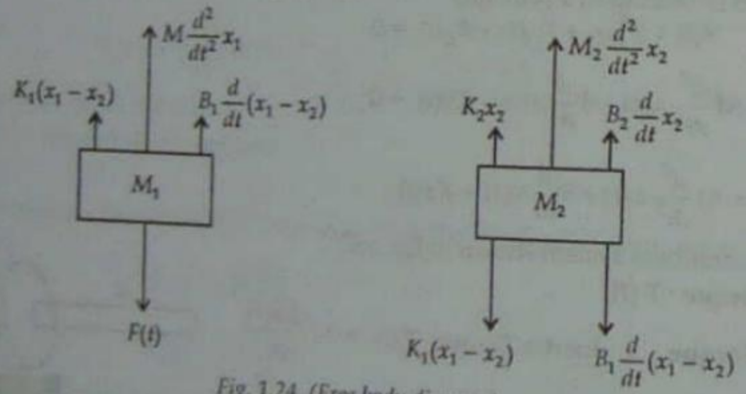


Fig. 1.24. (Free body diagram)

$\therefore$ $$F(t) = M_1\frac{d^2}{dt^2}x_1 + B_1\frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x_2) \quad ...(1.49)$$

Similarly for mass M_2 :

Example 1.10. Drive the system equations & find the value of $X_2(s)/F(s)$ for the system shown in fig. 1.29.

(R.M.L., University, Faizabad, 2003)

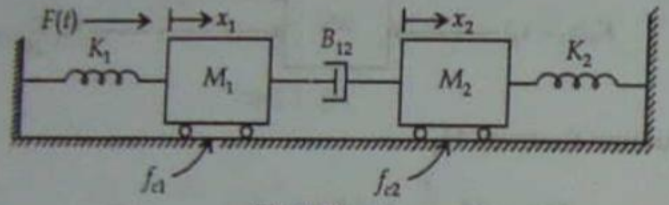
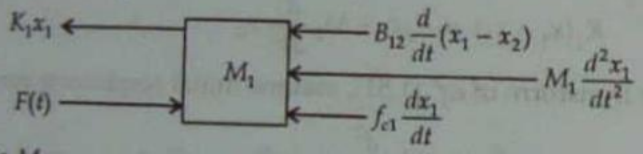
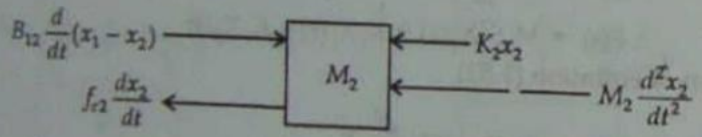


Fig. 1.29.

Solution : Free body diag. for mass M_1:



Free body diag. For mass M_2:



System equation for mass M_1:

$$F(t) = M_1 \frac{d^2x_1}{dt^2} + fc_1 \frac{dx_1}{dt} + B_{12} \frac{d}{dt}(x_1 - x_2) + K_1 x_1 \qquad \text{.....(1.56)}$$

System equation for mass M_2:

$$B_{12} \frac{d}{dt}(x_1 - x_2) = M_2 \frac{d^2x_2}{dt^2} + fc_2 \frac{dx_2}{dt} + K_2 x_2 \qquad \text{....(1.57)}$$

Laplace transform of equation 1.56

$$F(s) = X_1(s)[s^2M_1 + sB_{12} + sfc_1 + K_1] - B_{12}\, sX_2(s) \qquad \text{.....(1.58)}$$

Laplace transform of equation 1.57

$$B_{12}sX_1(s) = (s^2M_2 + B_{12}s + sfc_2 + K_2)\, X_2(s)$$

$$\therefore \quad X_1(s) = \frac{X_2(s)\left(s^2M_2 + B_{12}s + sfc_2 + K_2\right)}{B_{12}s} \qquad \text{....(1.59)}$$

Put the value of $X_1(s)$ for eqn (1.59) in equation (1.58)

$$F(s) = \frac{X_2(s)\left[S^2M_2 + B_{12}S + sfc_2 + K_2\right]\left[S^2M_1 + SB_{12} + sfc_1 + K_1\right]}{B_{12}S} - B_{12}SX_2(s)$$

$$\therefore \quad \frac{X_2(s)}{F(s)} = \frac{SB_{12}}{\left(s^2M_1 + sB_{12} + sfc_1 + K_1\right)\left(s^2M_2 + sB_{12} + sfc_2 + K_2\right) - s^2B_{12}^{\ 2}} \qquad \textbf{Ans.}$$

1.15. ANALOGOUS SYSTEM

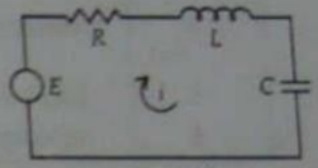


Fig. 1.30

Consider a series *RLC* circuit

Apply kirchhoff's voltage law

$$E = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \qquad \text{.....(1.60)}$$

In terms of charge eq^n 1.60 becomes

$$E = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{1}{C}q \qquad \text{.....(1.61)}$$

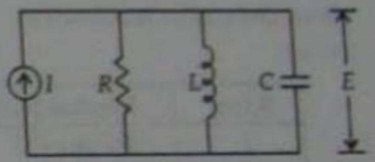


Fig. 1.31.

Now consider a parallal RLC circuit

Now apply Kirchhoff's current law

$$I = \frac{E}{R} + \frac{1}{L}\int E\,dt + C\frac{dE}{dt} \qquad \text{.....(1.62)}$$

In terms of magnetic flux linkage, the eq^n. 1.62 becomes

Since $\qquad \phi = \int E\,dt$

$$I = \frac{1}{R}\left(\frac{d\phi}{dt}\right) + \frac{1}{L}\phi + C\frac{d^2\phi}{dt^2} \qquad \text{.....(1.63)}$$

Now, compare the equation (1.47) with the equation (1.61), both equations are differential equations of same order *i.e.* identical such type of systems whose differential equations are in the same form are called analogous systems.

On comparison the eqn. 1.47 with eqn. 1.61 we can see that in mechanical system the force F is analogous to voltage E in electrical system, such type of analogy is known as force-voltage analogy.

From equation 1.47 & 1.61, mass (M) is analogous to inductance (L), coeff. of viscous friction (B) is analogous to resistance (R), spring stiffness (K) is analogous to $\frac{1}{C}$ & so on.

Table 1.3.

S.No.	Mechanical Translational System	Electrical System
1.	Force (F)	Voltage E
2.	Mass (M)	Inductance (L)
		Reciprocal of capacitance ($1/C$)

Now compare the equation (1.47) with eqn (1.63), since the force F is analogous to the current source, such type of analogy is known as force current analogy. The analogous quantities are tabulated as.

Table 1.4.

S.No.	Mechanical Translational System	Electrical system
1.	Force (*F*)	Current (*I*)
2.	Mass (*M*)	Capacitance (*c*)
3.	Damping coeff. (*B*)	Reciprocal of resistance ($1/R$) *i.e.* conductance *G*
4.	Stiffness (*K*) (Elastance, $1/K$)	Reciprocal of Inductance ($1/L$) Inductance (*L*)
5.	Displacement (*x*)	Flux linkage (ϕ)
6.	Velocity ($\dot{x}$)	Voltage(*E*)

Now consider the rotational system & compare the *eq*ⁿ. 1.48 with 1.61. On comparison the analogous quantities can be tabulated as

Table 1.5 : Torque-voltage analogy

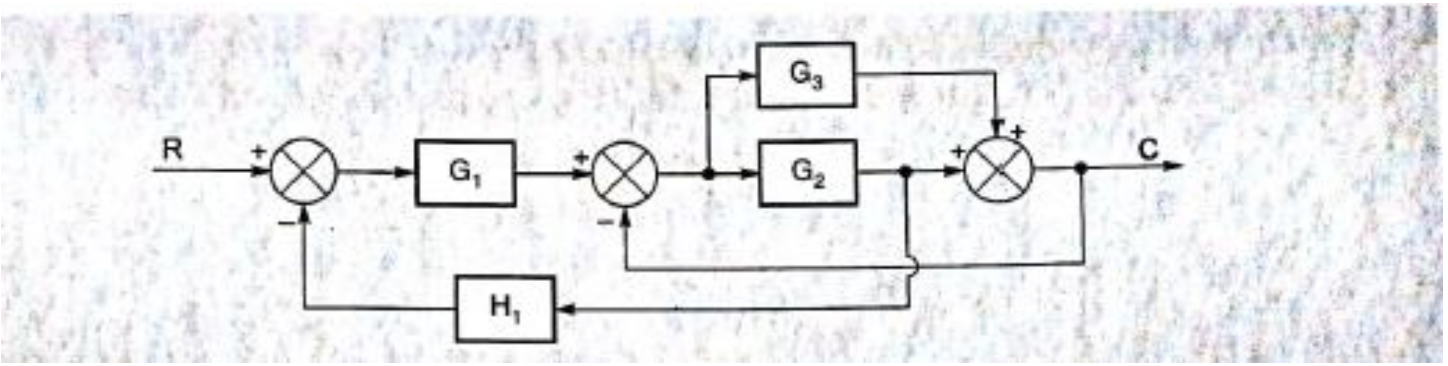
S.No.	Mechanical Rotational System	Electrical system
1.	Torque (*T*)	Voltage (*E*)
2.	Moment of Inertia (*J*)	Inductance (*L*)
3.	Damping coeff. (*B*)	Resistance (R)
4.	Stiffness (*K*) (Elastance, $1/K$)	Reciprocal of capacitance ($^1/_C$) capacitance (*C*)
5.	Angular displacement (θ)	Charge (*q*)
6.	Angular Velocity (*w*)	current(*i*)

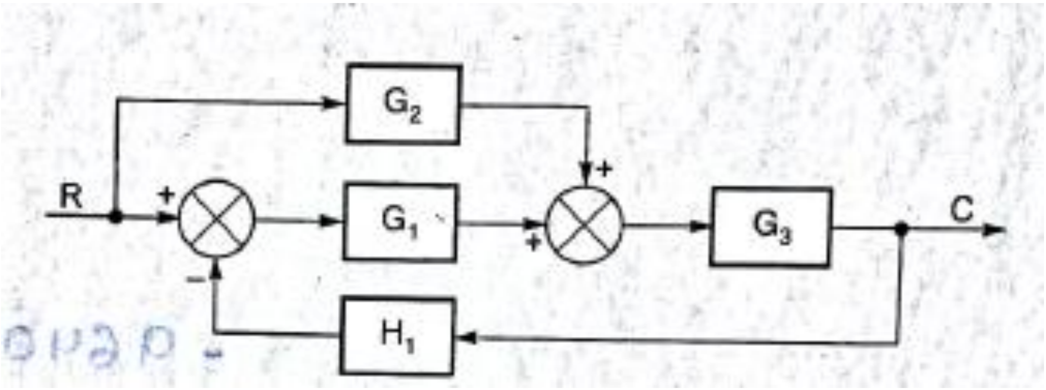
On comparison the eqn. 1.48 with eqn 1.63 we get Torque current analogy and can be tabulated as

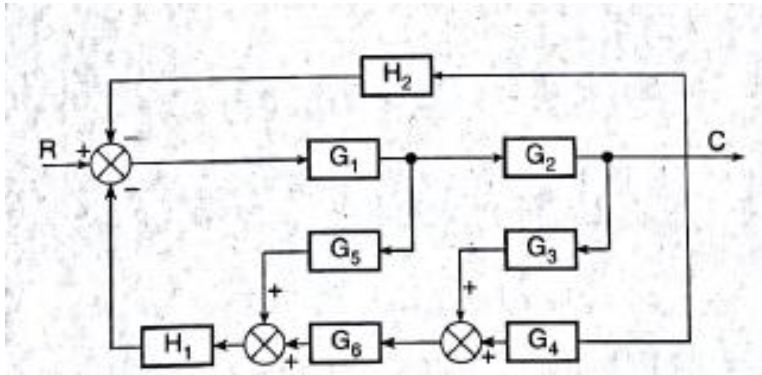
Table 1.6

S.No.	Mechanical Rotational System	Electrical system
1.	Torque (*T*)	current (*I*)
2.	Moment of Inertia (*J*)	capacitance (*c*)
3.	Damping coeff. (*B*)	Reciprocal of Resistance ($^1/_R$) *i.e.* conductance (*G*)
4.	Stiffness (*K*) (Elastance, $1/_K$)	Reciprocal of Inductance ($^1/_L$) Inductance (*L*)
5.	Angular displacement (θ)	Flux linkage (ϕ)

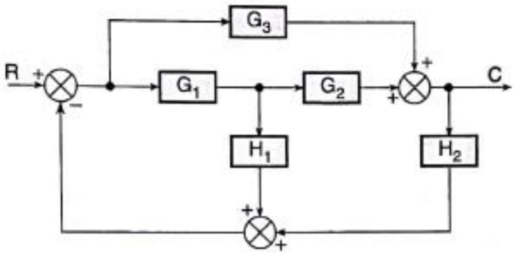
Following Chapt...

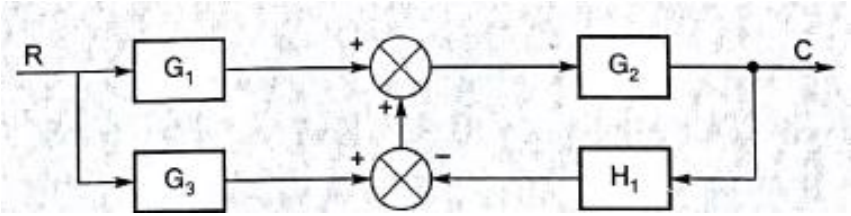

R
+
−
G_1
+
−
G_3
G_2
+
+
C
H_1

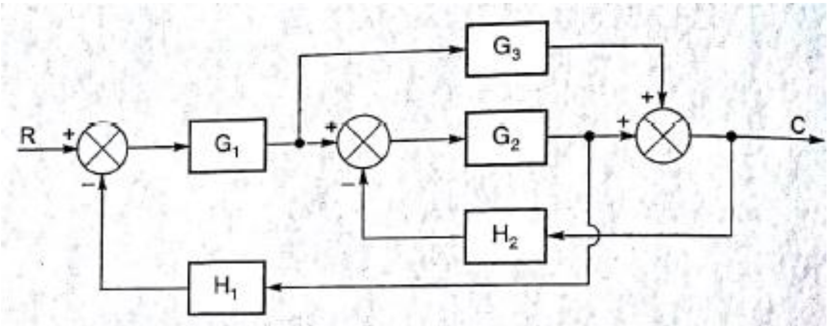

R
C
G_1
G_2
G_3
H_1
+
+
+
−

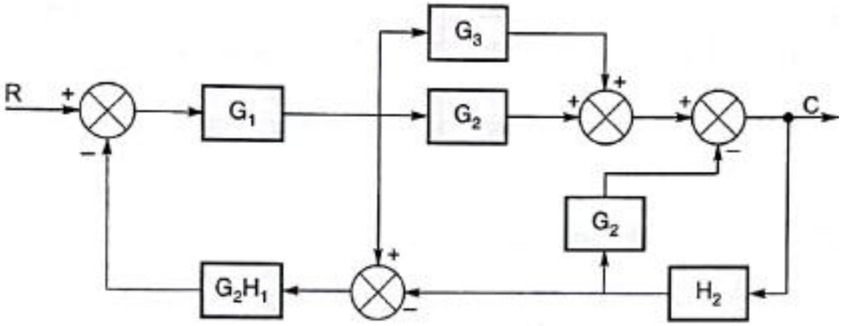


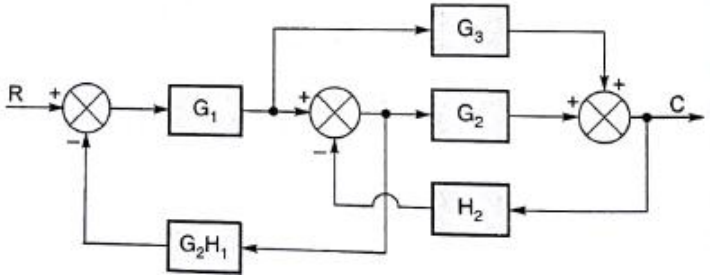
R
C
H2
G1
G2
G3
G4
G5
G6
H1

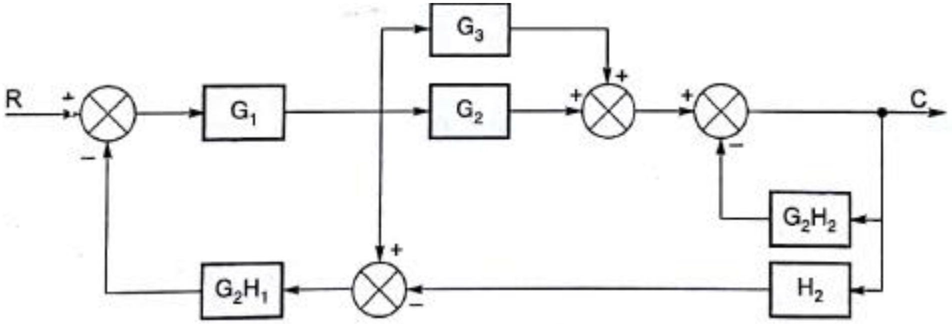

R
+
−
G3
G1
G2
+
+
C
H1
H2
+
+


R
G1
G3
+
+
+
−
G2
H1
C









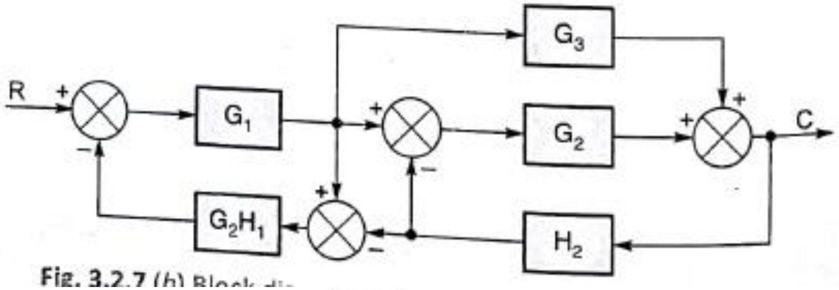
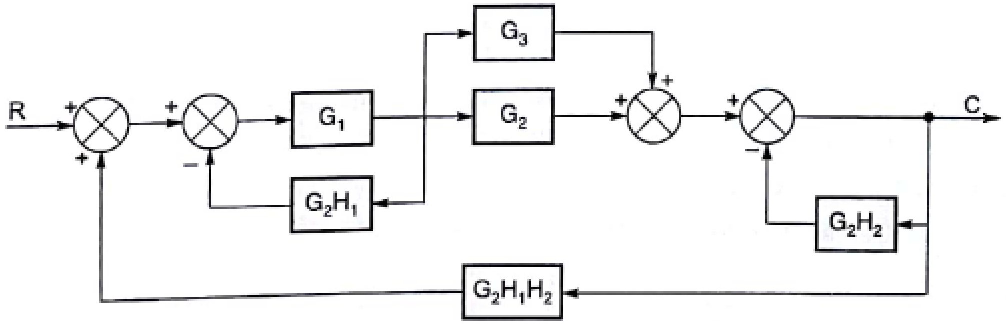
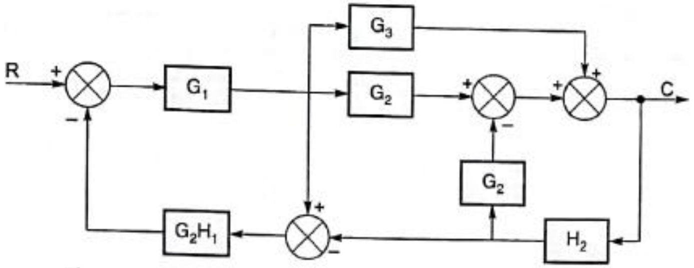


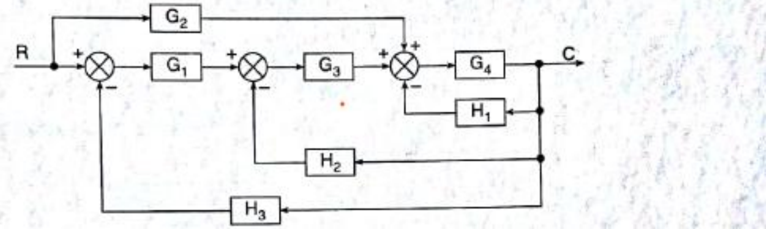
Fig. 3.2.7 (b) Block di

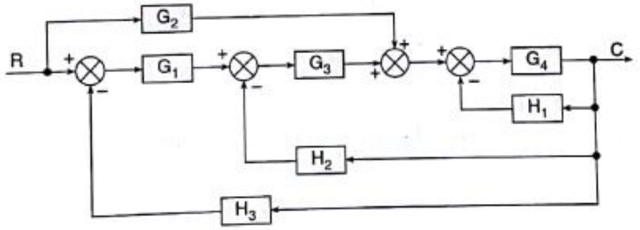


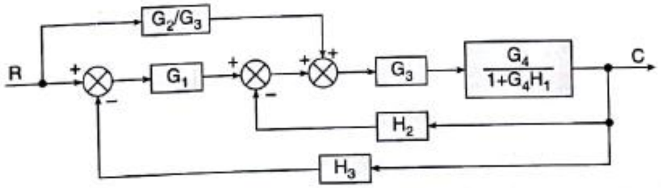


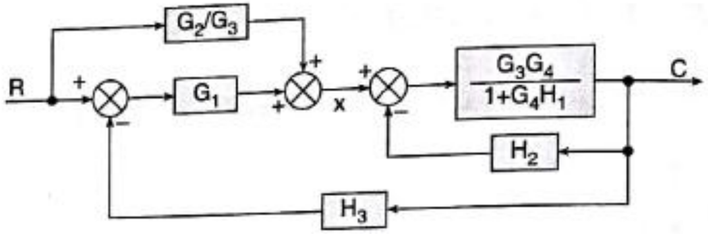
$$\frac{G_1G_2 + G_1G_3}{1 + G_2H_2 + G_1G_2H_1 - G_1G_2G_3H_1H_2}$$

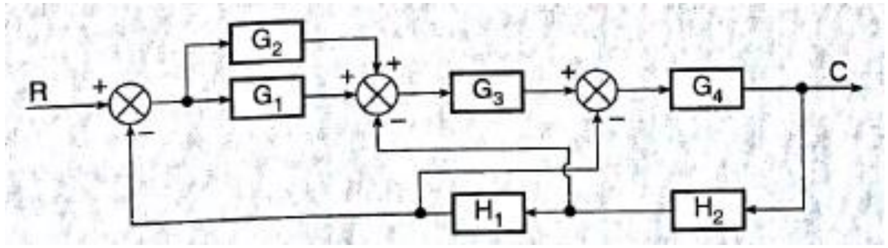
3.2.11. *Reduce the system shown below to a single block representation.*

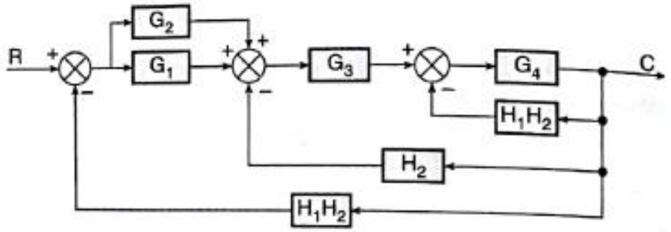


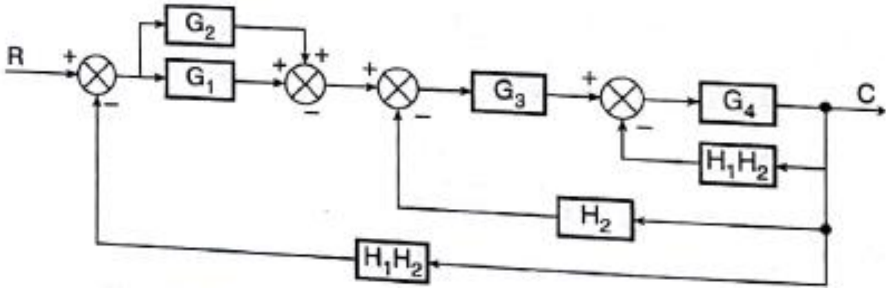




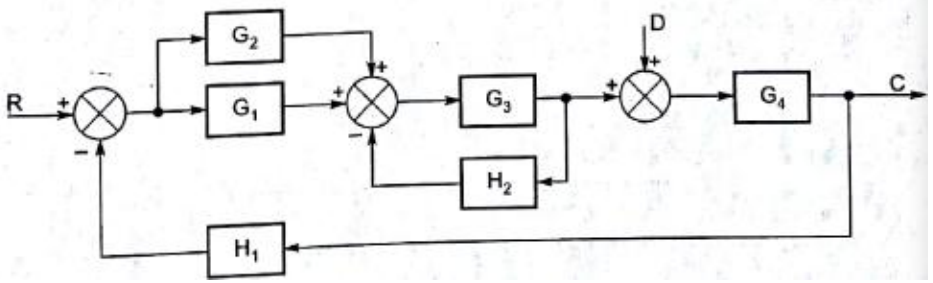


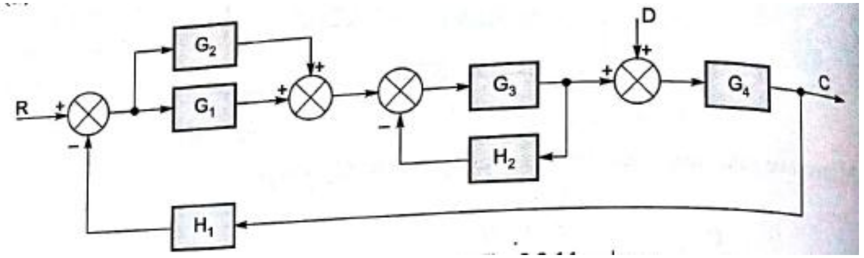

R
C
G₂
G₁
G₃
G₄
H₁
H₂
+
−

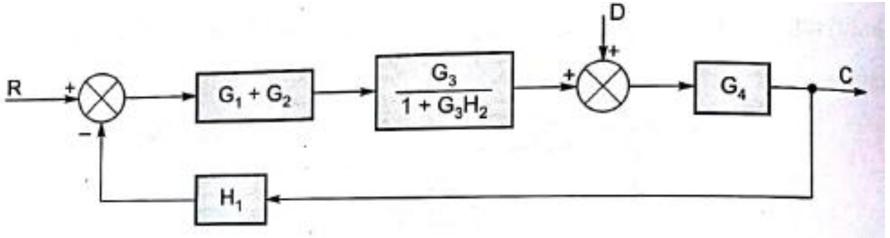

R
C
G₂
G₁
G₃
G₄
H₁H₂
H₂
H₁H₂
+
−


R
C
G₂
G₁
G₃
G₄
H₁H₂
H₂
H₁H₂
+
−

Determine the ratio C/R, C/D and the total output for the system







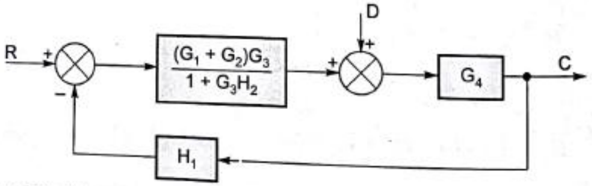
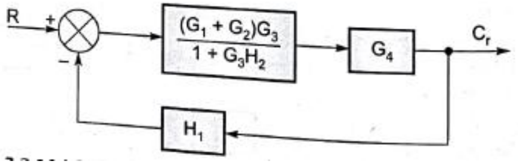


Fig. 3.2.14 (c) Block diagram reduction of system shown in Fig. 3.2.14 (b)

Consider $D = 0$, the corresponding output is denoted as C_r.



$$\frac{C_r}{R} = \frac{\frac{(G_1+G_2)G_3}{(1+G_3H_2)}\cdot G_4}{1+\frac{(G_1+G_2)\,G_3}{(1+G_3H_2)}\cdot G_4\cdot H_1}$$

$$= \frac{G_1G_3G_4 + G_2G_3G_4}{1+G_3H_2+G_1G_3G_4H_1+G_2G_3G_4H_1}$$

Consider $R = 0$, the corresponding output is denoted as C_d, hence the bl shown in Fig. 3.2.14 (c) takes the form as shown in Fig. 3.2.14 (e).

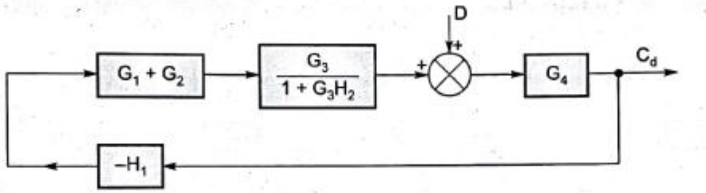
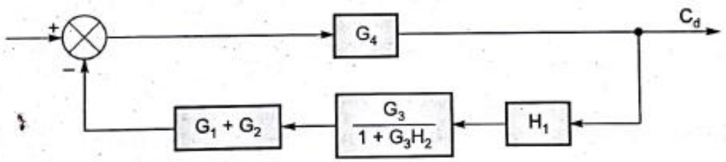


Fig. 3.2.14 (e) Block diagram for example 3.2.14 considering $R = 0$.

block diagram shown in Fig. 3.2.14 (e) is redrawn as shown in Fig. 3



$$\frac{C_d}{D} = \frac{G_4}{1+G_4\cdot\frac{(G_1+G_2)G_3H_1}{(1+G_3H_2)}} = \frac{G_4(1+G_3H_2)}{1+G_3H_2+G_1G_3G_4H_1+G_2G_3G_4H_1} \quad \textbf{Ans.}$$

The total output is given by,

$$C = C_rR + C_dD = \frac{G_1G_3G_4+G_2G_3G_4}{1+G_3H_2+G_1G_3G_4H_1+G_2G_3G_4H_1}\cdot R$$

$$+\frac{G_4(1+G_3H_2)}{1+G_3H_2+G_1G_3G_4H_1+G_2G_3G_4H_1}\cdot D \quad \textbf{Ans.}$$