Radar Based Vehicle Localization

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Problem Statement

- **Objective:** Estimate the position of a moving vehicle using data from two RADAR sensors.
- **Trajectory:** Non-linear, involving turns.
- **RADAR Locations:**
  - RADAR 1: $(0, 0)$
  - RADAR 2: $(0, -1000)$
- **Data:** Radar-derived cartesian coordinates (x, y).
- **Road Map Context:** Information about the road intersections and turns.
- **Speed:** Nominal speed is known with increment standard deviation.
- **Goal:** Fuse the radar data along with road dynamics and speed for enhanced tracking.

System Dynamics

- The agent is assumed to be a point mass. Here, we have used two state space representations for the in-depth analysis of the problem.
- **State Space 1:**

$$\begin{bmatrix} x_{k+1} \\ v^x_{k+1} \\ y_{k+1} \\ v^y_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ v^x_k \\ y_k \\ v^y_k \end{bmatrix}$$

- **State Space 2:**

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \end{bmatrix} + \begin{bmatrix} \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} v^x_k \\ v^y_k \end{bmatrix}$$

System Dynamics

- In one case velocity is used as the state variable and in the other case it is used as an input.
- We have used Constant Velocity Motion model with white noise acceleration. The standard deviations for the white noise accelerations were different for North-South path and East-West path.
- Standard Deviations for North-South path:

$$\sigma_{\Delta v_x} = 1.18$$
$$\sigma_{\Delta v_y} = 1.67$$

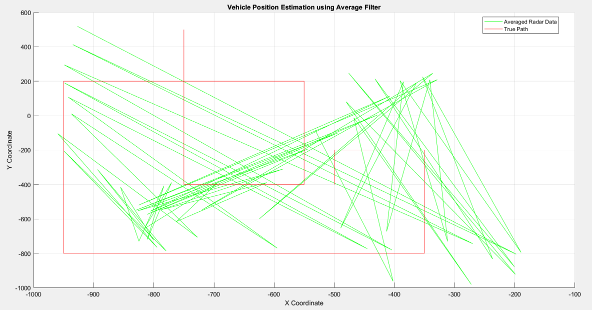
- Standard Deviations for East-West path:

$$\sigma_{\Delta v_x} = 1.18$$
$$\sigma_{\Delta v_y} = 0.83$$

- The Radar errors are known to be:

$$r = 95 \ m$$
$$\theta = 0.0035 \ mr$$

Averaging Filter



The RADAR measurements representing the position of the agent are given in cartesian coordinates $((x_t^1, y_t^1)$ and $(x_t^2, y_t^2))$. Resulting average filter estimate is given by:

$$x_t^{avg} = \frac{x_t^1 + x_t^2}{2} \qquad y_t^{avg} = \frac{y_t^1 + y_t^2}{2}$$

Alpha-Beta Filter

- Using the state space representation 2 and the system dynamics, we can write the state space equations as:

$$(\hat{x})_k \leftarrow A(\hat{x})_{k-1} + B(u)_k + \frac{\Delta t^2}{2}\mathrm{w}_k$$

where $(\hat{x})_k$ is the state estimate at time k, $(u)_k$ is control input and A and B are the state transition and control matrices respectively. w_k is the process noise which is $\mathrm{w}_k \sim \mathcal{N}(0, Q)$. Q (white noise acc$^{\mathrm{n}}$) is given by:

$$Q = \begin{bmatrix} \sigma^2_{\Delta v_x} & 0 \\ 0 & \sigma^2_{\Delta v_y} \end{bmatrix}$$

Alpha-Beta Filter

- The output measurement is expected to deviate form prediction due to noise. So, the predcition error (residual) is given by:

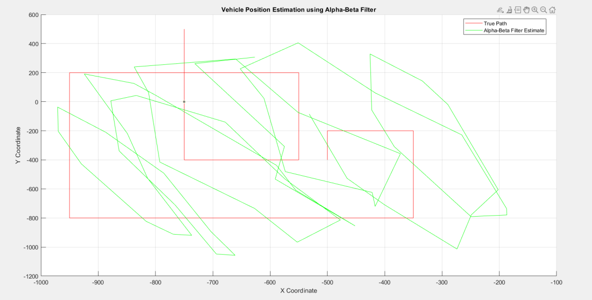
$$\mathsf{r}_k = \mathsf{z}_k - \hat{\mathsf{x}}_k$$

  where z_k is the measurement vector from sensor which here is the RADAR measurement (x_k^{avg}).

- The alpha-beta filter corrects the estimated state using the residual as:

$$\hat{\mathsf{x}}_k \leftarrow \hat{\mathsf{x}_k} + \alpha \mathsf{r}_k$$
$$\hat{\mathsf{v}}_k \leftarrow \hat{\mathsf{v}_k} + \frac{\beta}{\Delta t}\mathsf{r}_k$$

Alpha-Beta Filter: Results and Analysis



In general, larger α and β gains tend to produce faster response for tracking transient changes, while smaller α and β gains reduce the level of noise in the state estimate.

$$\alpha = 0.5 \qquad \beta = 0.1$$

Kalman Filter

- Kalman filter is a bayesian filter based on the fact that, system dynamics is linear and Markovian.
- The state of the system is represented by a Gaussian distribution with mean μ_k and covariance P_k.
- Here, we are following the state space representation 2 and the corresponding system dynamics. The motion model is given by:

$$\hat{\mathrm{x}}_k \leftarrow A\hat{\mathrm{x}}_{k-1} + C\mathrm{w}_k$$

where w_k is the process noise which is $\mathrm{w}_k \sim \mathcal{N}(0, Q)$. A and C are given by:

$$A = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} \frac{\Delta t^2}{2} & 0 \\ \Delta t & 0 \\ 0 & \frac{\Delta t^2}{2} \\ 0 & \Delta t \end{bmatrix}$$

Kalman Filter

- Observation model is given by:

$$\mathrm{z}_k = H\hat{\mathrm{x}}_k + \mathrm{v}_k$$

where v_k is the measurement noise which is $\mathrm{v}_k \sim \mathcal{N}(0, R)$. H and R are given by:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad R = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix}$$

where σ_x and σ_y are the standard deviations of the RADAR measurements in cartesian coodinates calculated from range and bearing angle errors.

Kalman Filter

- The Kalman Filter is a two step process:
  1. **Prediction:** Predict the state of the system at time k using the motion model.

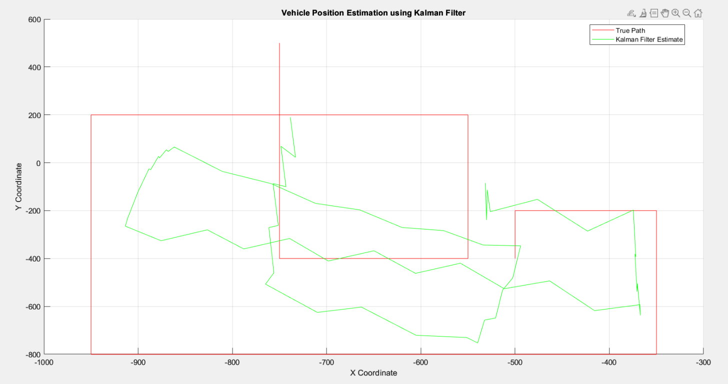
$$\mu_k^- = A\mu_{k-1}$$
$$P_k^- = AP_{k-1}A^T + Q$$

  2. **Correction:** Correct the predicted state using the measurement model.

$$\mathrm{K}_k = P_k^- H^T (HP_k^- H^T + R)^{-1}$$
$$\mu_k = \mu_k^- + \mathrm{K}_k(\mathrm{z}_k - H\mu_k^-)$$
$$P_k = (I - \mathrm{K}_k H)P_k^-$$

  where K_k is the Kalman Gain.

Kalman Filter: Results



Extended Kalman Filter

- The Extended Kalman Filter is used to deal with non-linear system dynamics. Here we are using the state space representation 1 and the corresponding system dynamics.
- The motion model is given by:

$$\hat{\mathrm{x}}_k \leftarrow A\hat{\mathrm{x}}_{k-1} + B\mathrm{u}_k + \frac{\Delta t^2}{2}\mathrm{w}_k$$

  where, the notations are same as in the case of Kalman Filter and alpha-beta filter.
- The observation model is given by:

$$\mathrm{z}_k = h(\hat{\mathrm{x}}_k, \mathrm{v}_k)$$

  where $h(\hat{\mathrm{x}}_k)$ is the non-linear observation model and v_k is the measurement noise which is $\mathrm{v}_k \sim \mathcal{N}(0, R)$. R is given by:

$$R = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}$$

Extended Kalman Filter

- The range and bearing angle errors are given by:

$$\sigma_r = 95m$$
$$\sigma_\theta = 0.035 radians$$

- Since, the non-linearity arise in the observation model for our case. We can linearize the observation model using Taylor series expansion.
- The linearized observation model is given by:

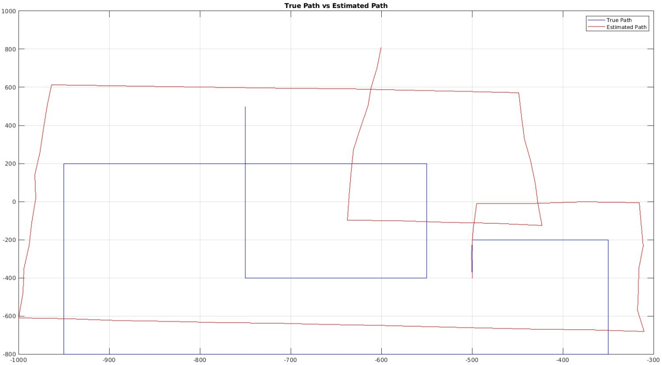
$$z_k = h(\hat{x}_k, v_k) \approx H(\hat{x}_k) + v_k$$

where, H is the Jacobian of the observation model.

- The Jacobian of the observation model is given by:

$$H = \begin{bmatrix} \frac{\hat{x}_k}{\sqrt{\hat{x}_k^2+\hat{y}_k^2}} & \frac{\hat{y}_k}{\sqrt{\hat{x}_k^2+\hat{y}_k^2}} \\ \frac{-\hat{y}_k}{\hat{x}_k^2+\hat{y}_k^2} & \frac{\hat{x}_k}{\hat{x}_k^2+\hat{y}_k^2} \end{bmatrix}$$

Extended Kalman Filter: Results and Analysis



Observed bias in the above plot is due to the error in the initial RADAR measurement.

Particle Filter

- Particle filter is a non-parametric implementation of the Bayes filter. Each particle represents a hypothesis of the state of the system.
- The state of the system is represented by a set of particles $\{\mathrm{x}_k^i\}_{i=1}^N$ with weights $\{w_k^i\}_{i=1}^N$.
- The motion model is given by:

$$\mathrm{x}_k^i \leftarrow f(\mathrm{x}_{k-1}^i, \mathrm{u}_k)$$
$$w_k^i \leftarrow w_{k-1}^i p(\mathrm{z}_k | \mathrm{x}_k^i)$$

- The observation model is given by:

$$\mathrm{z}_k = h(\mathrm{x}_k^i)$$

- The weights are normalized at each time step to take care of dying weights problem.

Particle Filter: Illustration

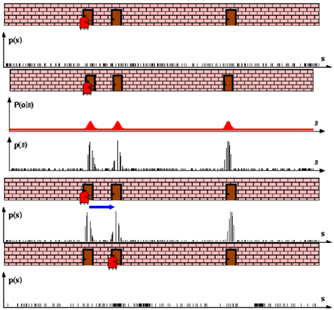
Particle Filter Localization (1-D)

Prior:

Update:

Predict:

Resample:

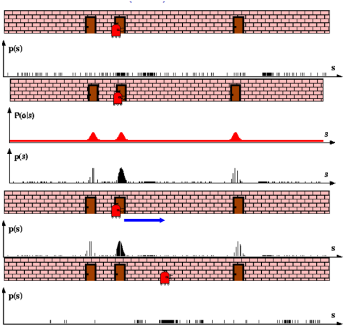


Prior:

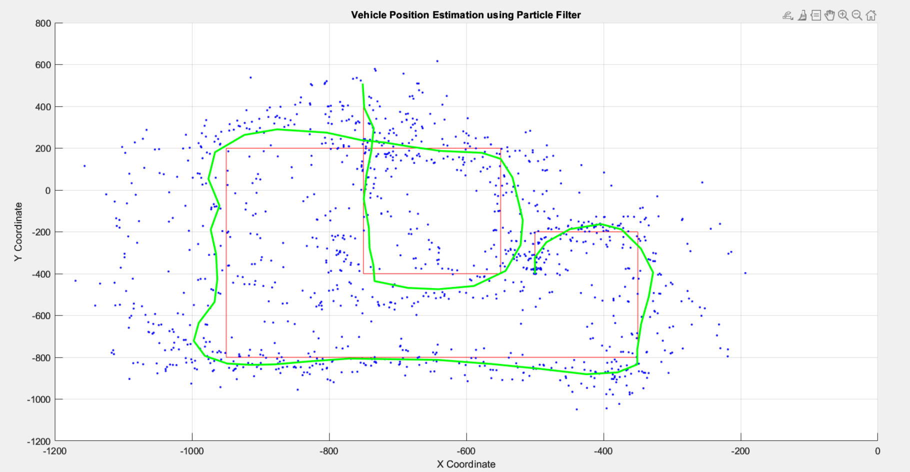
Update:

Predict:

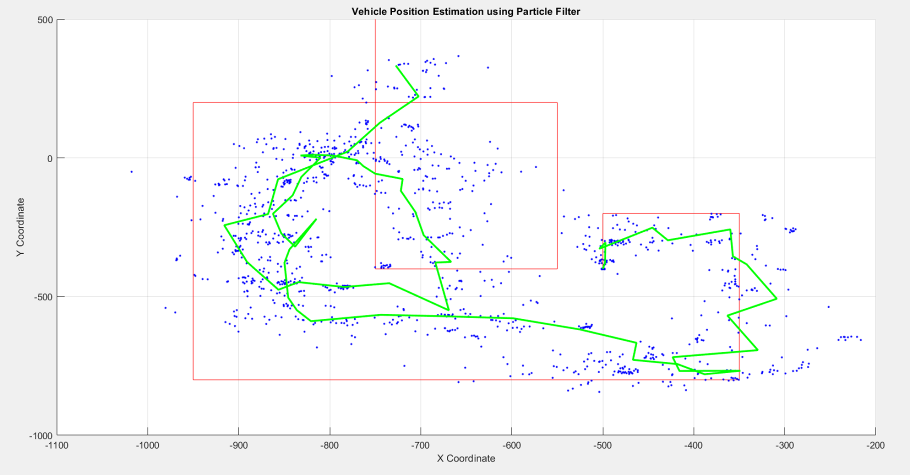
Resample:



Particle Filter: Using True Velocities



Particle Filter: Using RADAR Velocities



Conclusion and Future Work

- The Kalman Filter and Extended Kalman Filter are the most efficient filters for tracking the agent.
- Since, the RADAR measurements are really skewed and noisy, the averaging and the alpha-beta filters are not able to track the agent properly.
- The particle filter is able to provide decent results as shown but not only is it computationally expensive but also we would need to use appropriate observation model to update the likelihood of the particles.
- In the future, we could drastically improve the results if we fuse the data from a sensor measuring velocity and heading of the agent. The heading of the agent could be estimated using the IMU measurements.
- We were also trying to implement a piecewise Kalman Filter which would use the information from the map to reduce the covariance of the state estimate.

References

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Assurance regions in tracking
link

Nathanel L. Baisa (2020)
Derivation of Constant Velocity Motion Model for Visual Tracking
link

Thank You