Problem 5

Problem 5:
Problem 5:
a) S= {1, (n+1), (n+2)2;, (n+i)1}. (n+i)1}
- Tet (x+ia)ia be a vector in S that is a linear
Combination of all ofter vectors: (xtia)ia = (xxi(xti')i)
if ia $< b$, $(x+in)in = x_n^{-1}(x+ia)ia + x_n^{-1} + x_n^{-1}(x+i)i$
thus LHS win be then vector with the
thus LHS win be then vector with the highest degree. without los of generality; we can say:
ia>n, [no vector exists less than n (size of s)
which will be a linear combination of all others.
. Sis linearly independent.

b) C°(0, 1] is a set of continuous functions f: [0, 1] = R

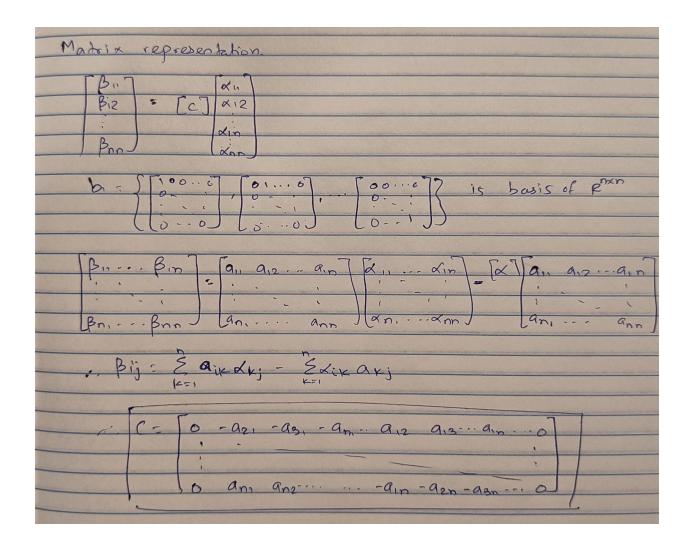
: C°[0, 1] = |x-0.5|

= |sinx| = |cosx|
= e^x.

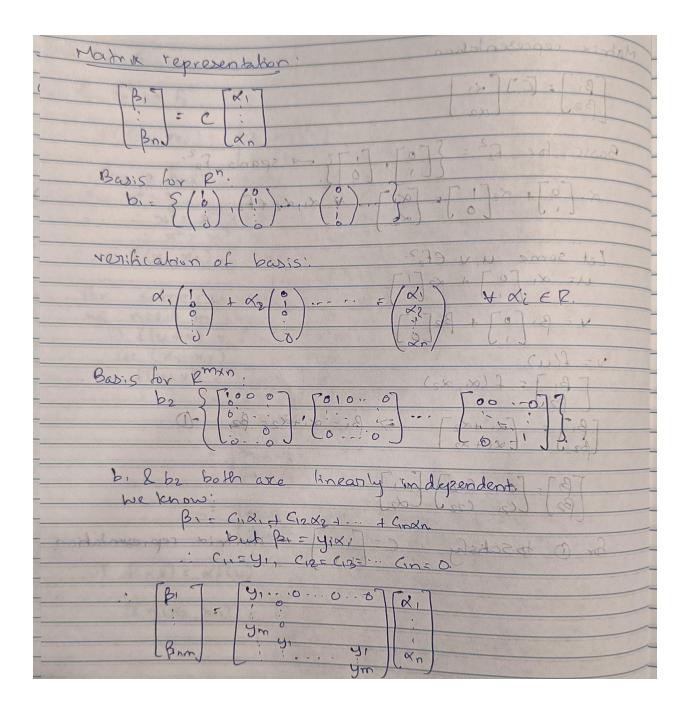
suppose e^x = \(\frac{2}{3} \) \(\frac{2}{3} \

Problem 6

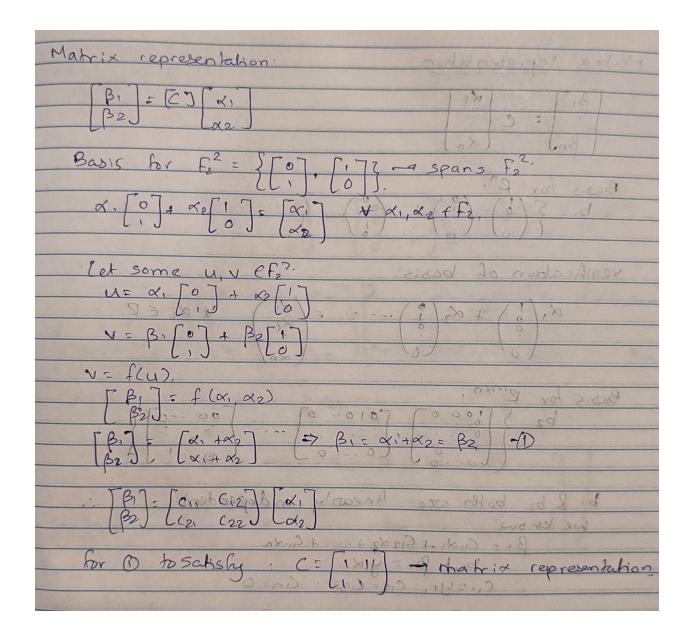
- landaubai as an total and other an arest
Problem 6: linear maps
Sold B. Cittee 11.4
a) f. Phan > phan f(x)= Ax - xA where Alis given my
- has the a trace a mar it much extist, have exceented
in the se a mean that the same of the series
- for f to be a linear map it must satisfy two properties i) Momogeneity and ii) Additivity
i) f(xx) = &f(x) &C F, x & Enxn
1) f(XX) = XXCX)
$f(\alpha x) = A(\alpha x) - (\alpha x)A = \alpha Ax - \alpha xA$
$= \alpha(A \times) - \alpha(X + A) = \alpha(A \times - X + A)$
= X + (X)
$ \begin{aligned} f(\alpha x) &= A(\alpha x) - (\alpha x)A = \alpha Axn - \alpha xA & \text{TMH} \\ &= \alpha (Ax) - \alpha (xA) = \alpha (Ax - xA) \\ &= \alpha f(x) \end{aligned} $
· f(xx) = xf(x) - Satisfied.
ii) + (x, + x2) = + (x,) + + (x2) x1, x2 E E
$f(x_1 + x_2) = f(x_1) + f(x_2) \qquad x_1, x_2 \in \mathbb{R}^{n \times n}.$
$f(x, +x_2) = A(x, +x_2) - (x(+x_2)A)$ $= Ax + Ax_2 - x + A - x_2A$ $= (Ax - x + A) + (Ax_2 - x_2A)$ $= f(x, x) + f(x_2)$
$= A \times_1 + A \times_2 - X_1 A - X_2 A$
$(A\times_1 - \times_1 A) + (A\times_2 - \times_2 A)$
= f(x) of f(x2)
to a total of the same of the
f(x, +x2) = f(x,) + f(x2) - satisfied
.: f(x) is a linear map



(431, 4 42/2) [431, 4 4312]
b) f: P"=> Pmm f(x) = xy where y & P" is given.
- for f to be a linear map, it should salish: i) f(dx) = df(x)
i) f(dx) = df(x)
and Dight tourson to high.
$f(\alpha x) = (\alpha x)y^{T} = \alpha xy^{T} = \alpha(xy^{T})$ $= \alpha f(x)$ $= \alpha f(x)$
= x f(n) [shring]
if(xx) plat(x) blos satisfied gen reserve a set of 1
$(i) f(x_1 + x_2) = f(x_1) + f(x_2)$
ii) $f(x_1 + x_2) = f(x_1) + f(x_2)$ $f(x_1 + x_2) = (x_1 + x_2)y^T = x_1y^T + x_2y^T$
[f(n,+n2) = f(n,) + f(n2) = satisfied.
: both properties hold true, f(n) is a lineour map



Problem 6 smaller
c) f: F2 -> F2 (84) 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
f((x1,1x2)) = x2 + x2)
for a finite while the same of state of the
$\frac{(2) \cdot (2)}{x^2 - x} = \frac{x^2 - x}{x^2 - x}$ $\frac{(2) \cdot (2)}{x^2 - x^2 - x} = \frac{x^2 - x}{x^2 - x}$ $\frac{(2) \cdot (2)}{x^2 - x^2 - x} = \frac{x^2 - x}{x^2 - x}$ $\frac{(2) \cdot (2) \cdot (2)}{x^2 - x^2 - x} = \frac{x^2 - x}{x^2 - x}$
For some r.u. v EF2 & & & EF2 have be we need to show: i) f(u+v) = f(u) + f(v)
f(u+v) = f(x, xe) + f(xe xe)
Mat do t Ma Las
$= \frac{(x_1 + x_2)}{(x_1 + x_2)} + \frac{(x_3 + x_4)}{(x_3 + x_4)}$
$\frac{1}{2} = \frac{1}{2} = \frac{1}$
f(x(x3, x4)) = f(xx3, xx4) = [xx3+xx4]
$= \left[\frac{1}{2} \times \frac{1}{2} \times$
fi F2 -> F22 is a linear mgp



d) f: P" -> R , f(x) = { x1, if x1, + x1 n = 0 } moldon)
L xn otherwise
Given ABEFIX ABI
Troof by contradiction:
stogsly: Journal ser Mertunal and seoggue A xirom and
woodness will sto far sp. of:
SI WHYDO SIE VIANATO
O=MA . J.C. Care so me so robsu cros-non E.
S.t. $u_1+u_1>0$ $0=xA$ $f(u+v)=f([u,]+[v,])=f([u, +v,])=8A$ $\lim_{x\to \infty} \frac{1}{x} 1$
Sin de restaunt you Test tod
- Land Cons
we have u. + v, + un + vn = (un + un) + (v, + vo)
The mase with the to
which contradicts Assessed
that means that and formal that (unit van) = (A) the means to the (unit van) = (A) the means to the (unit van) = (A) the means to th
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(Unit Va)) - (A) All Miles
B 27 Hove mak all columns of
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f(u+v) & f(u) + f(v) - a progenty not Anor IIII and how of the saleshied
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