Experiment 1

Free Vibration Analysis of SDOF system through initial displacement and using Impact Hammer

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1 Code to plot Graphs

The code to plot all the graphs shown in this document can be found at my Github Repository: https://github.com/Prathamesh001/Dynamics_of_Structures

2 Initial Displacement Test

2.1 Graphs

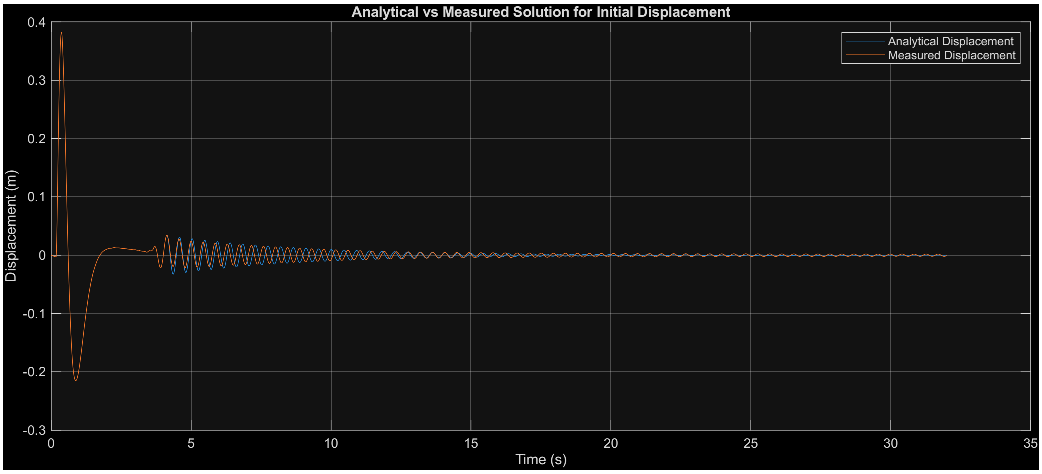

Figure 1: Comparison of Analytical vs Measured Displacement Response

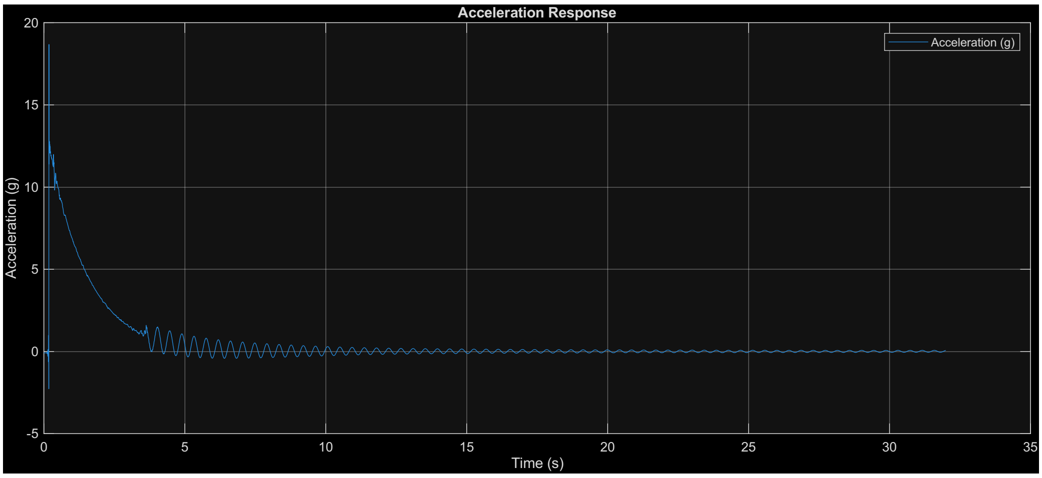

Figure 2: Acceleration Response

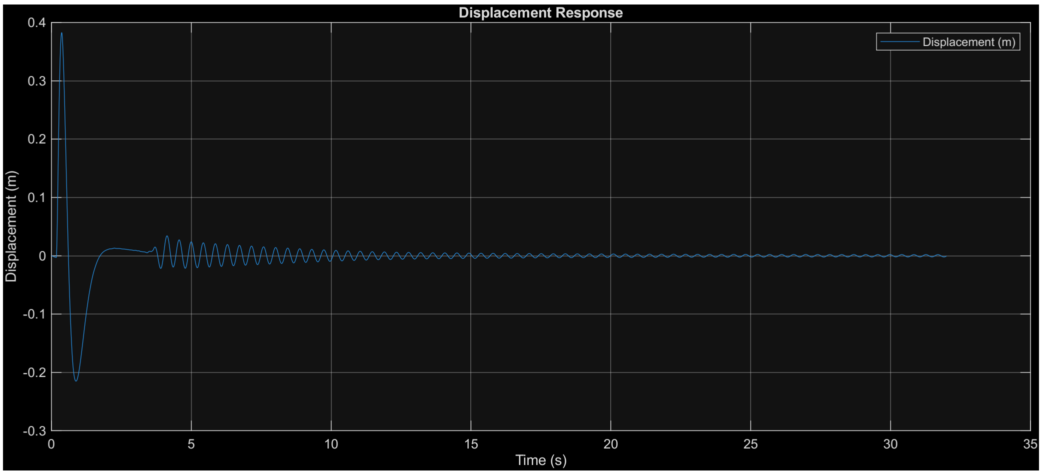

Figure 3: Displacement Response)

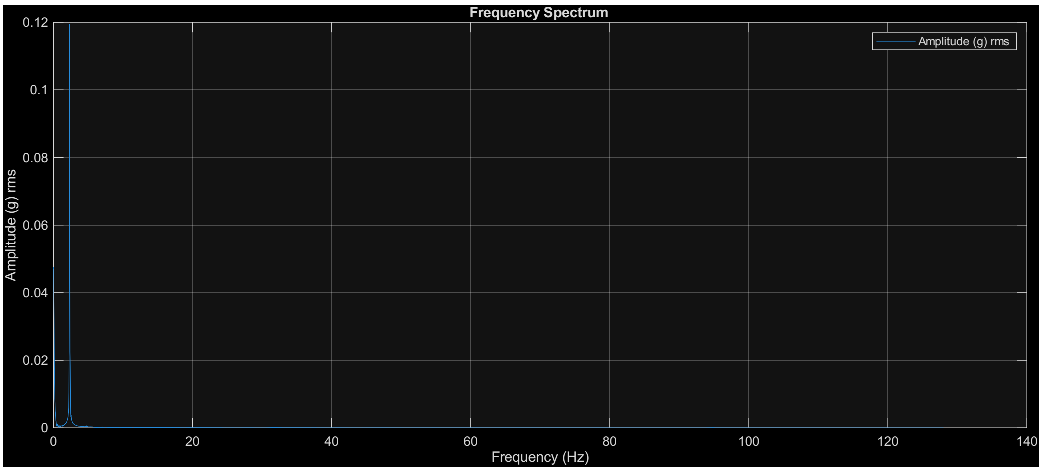

Figure 4: Frequency Spectrum

2.2 Calculations

Experiment-1

Lumped mass (m) = 0.2 kg

Modulus of Elasticity (E) = 200 GPa = 200×10^3 MPa

Diameter of bar (D) = 4 mm

Length of SDOF (assuming that says length of bar) = 580 mm

Q1) $$\omega_n = \sqrt{\frac{K}{m}}$$

The bar can be seen as a cantilever.

$$\delta = \frac{PL^3}{3EI}$$

$$\therefore K = \frac{3EI}{L^3} \quad \ldots \text{Units: } 3 \times \frac{N}{mm^2} \cdot \frac{mm^4}{mm^3} = \frac{N}{mm}$$

$$I = \frac{\pi D^4}{64} = \frac{\pi \times (4)^4}{64} = 4\pi \text{ mm}^4$$

To convert m/s² to mm/s²

$$\therefore \omega_n = \sqrt{\frac{3 \times 200 \times 10^3 \times 4\pi \times 10^3}{0.2 \times (580)^3}} = \cancel{6139.96}$$

$$= \sqrt{\frac{0.0386435 \times 10^3}{0.2}} = \cancel{0.4395}$$

13.90026 rad/sec

1> Initial displacement:

$\cancel{u(0) = 0.0342m}$ $\cancel{0.38m}$

$\cancel{0.382696m}$

$$\boxed{u(0) = 0.0342m}$$ ... from Graph

$$\boxed{t_0 = 4.133 \text{ sec}}$$

Next peak occurs at

$u_{10} = \cancel{4.56}$ $\cancel{0.0273m}$ $0.0131\,m$ .. After 10 cycles

$t_{10} = \cancel{4.566 sec}$ $8.445 sec$

$\cancel{T_D}$ $t_{10} - t_0 = \cancel{0.433 sec}$ 4.312 sec

$$T_D = t_1 - t_0 = 4.566 - 4.133 = 0.433 \text{sec}$$

Logrithmic decrement:

$$\delta = \frac{1}{m} \ln \frac{u_n}{u_{n+m}}$$

$$= \frac{1}{10} \times \ln \left\{ \frac{0.0342}{0.0131} \right\}$$

$$\delta = 0.09596 = 2\pi\xi$$

$$\therefore \boxed{\xi = 0.01527}$$

$$\omega_D = \frac{2\pi}{T_D} = \frac{2\pi}{0.433} = 14.51 \text{ rad/sec}$$

Also, as a check:

$$\omega_D = \omega_n \sqrt{1-\xi^2}$$

$$\omega_D = 13.90026 \sqrt{1-(0.01527)^2}$$

$$= 13.8986 \text{ rad/sec}$$

$\therefore$ ~~Error b/w $\omega_{D_{measured}}$ vs $\omega_{D_{th}}$~~

Or, other way around, finding natural frequency from experimental ω_D, we get,

$$\omega_n = \frac{\omega_D}{\sqrt{1-\xi^2}} = \frac{14.51}{\sqrt{1-(0.01527)^2}}$$

$$\Rightarrow \boxed{\omega_n = 14.511 \text{ rad/sec}}$$

$$K = \omega_n^2 \cdot m$$

$$= (14.511)^2 \times 0.2 = 42.11 \quad > 38.6 \text{ - theoretical}$$

$$\% \text{ error} = \frac{\omega_{n_{measured}} - \omega_{n_{theory}}}{\omega_{n_{theory}}} \times 100 = \underline{\underline{4.394\%}}$$

3 Impact Hammer Test

3.1 Graphs

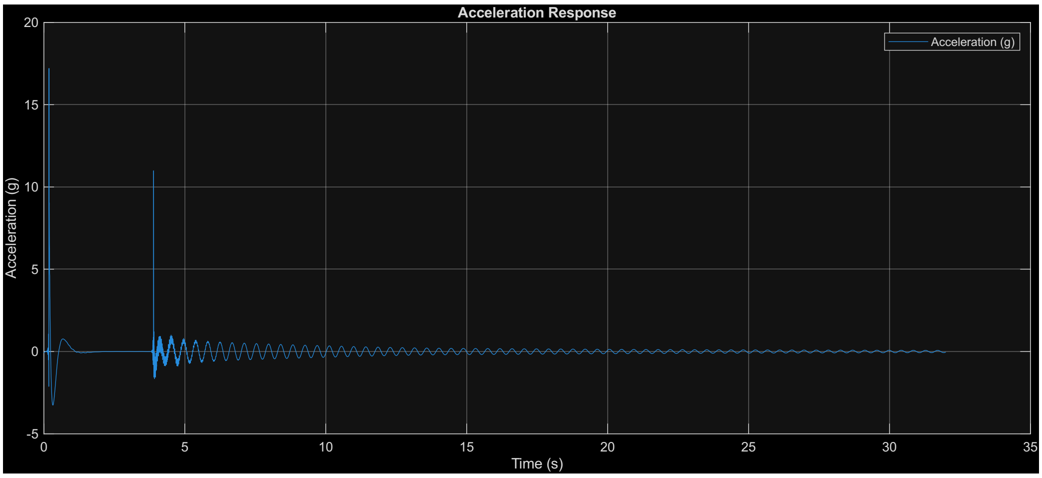

Figure 5: Acceleration Response

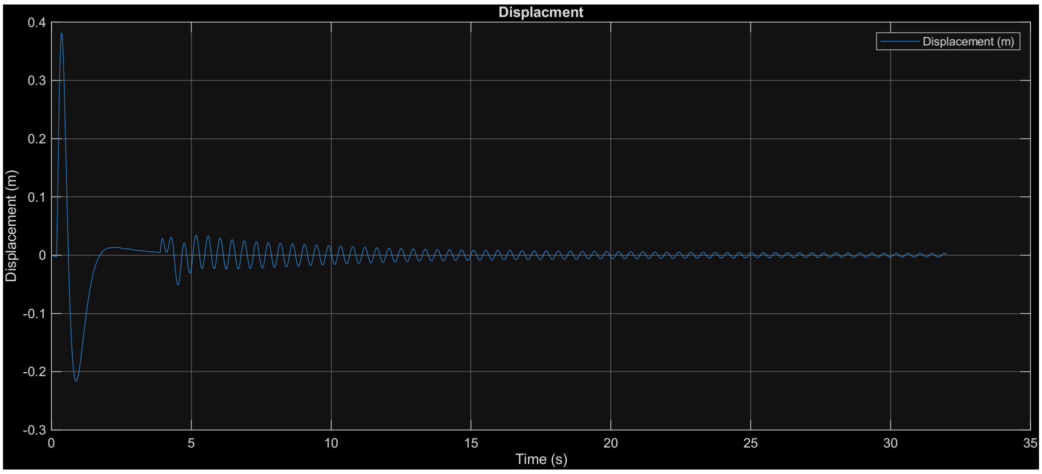

Figure 6: Displacement Response

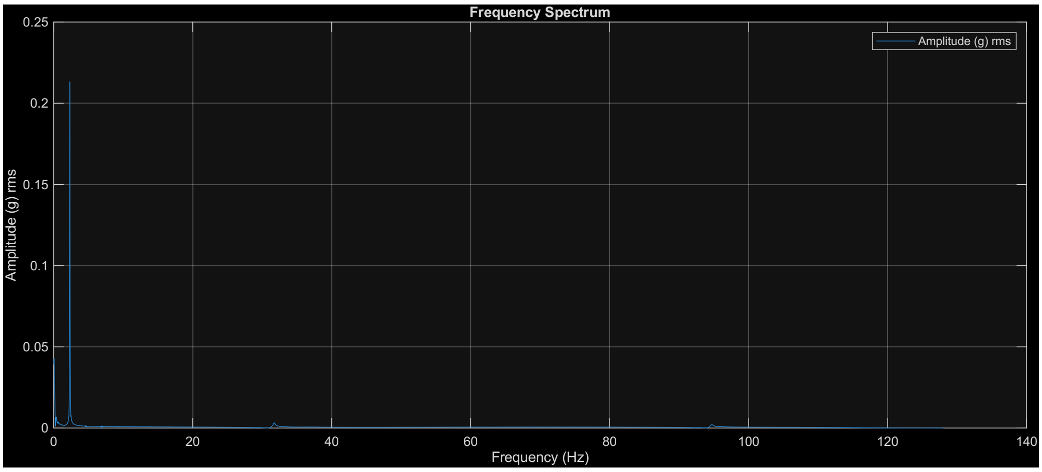

Figure 7: Frequency Spectrum

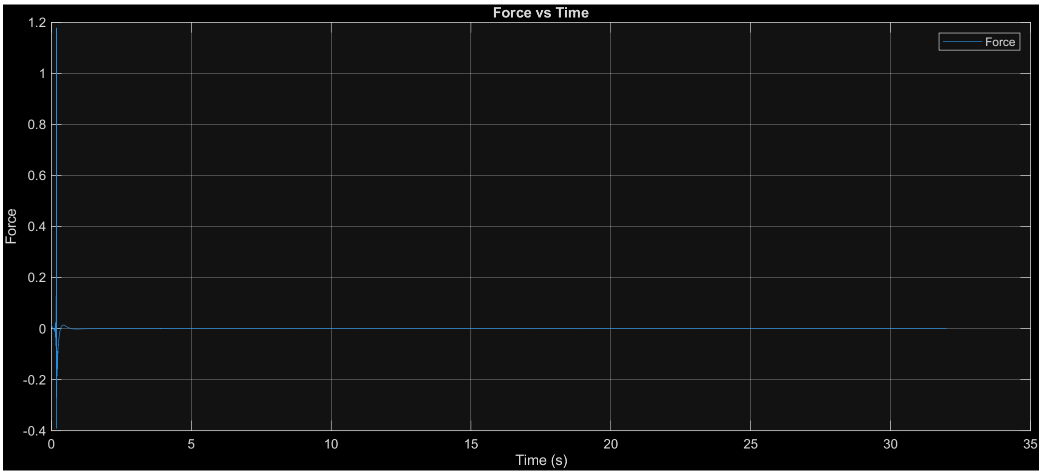

Figure 8: Force Applied

3.2 Calculations

27 Initial velocity / Impact hammer.

~~$t_0 = 5.172$ sec~~ ~~$u_0 =$~~

$t_1 =$

To theoretically solve this problem, we must assume $u(0) = 0$ as we provided an impact which we ~~assume~~ imparts only velocity to the system.

$$\therefore u(t) = e^{-\xi \omega t} \left\{ u(0) \cos \omega_D t + \frac{\xi \omega (u(0)) + \dot{u}(0)}{\omega_D} \sin \omega_D t \right\}$$

becomes,

$$u(t) = e^{-\xi \omega t} \left\{ \frac{\dot{u}(0)}{\omega_D} \sin \omega_D t \right\}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$= 13.90026$ ... rad/sec as ~~measured~~ calculated in previous question.

$$\omega_D = \omega \sqrt{1 - \xi^2}$$

$$\omega_D = 13.90026 \times \sqrt{1 - \xi^2}$$

Finding ξ:

From graph:

First peak of disp response is:

$$t_1 = 5.173 \text{ sec}$$

$$u_1 = 0.0344 \text{ m}$$

After ~~The~~ 10th cycles, the 11th peak is located at:

$$t_{11} = \cancel{0.0174}\ 9.478 \text{ sec}$$

$$u_{11} = 0.0174 \text{ m}$$

Logarithmic decrement

$$\therefore \quad \delta = \frac{1}{n} \ln\left(\frac{u_1}{u_{11}}\right)$$

$$= \frac{1}{10} \ln\left(\frac{0.0344}{0.0174}\right)$$

$$= 0.06815$$

$$\delta = 2\pi\xi$$

$$\therefore \boxed{\xi = 0.0108}$$

$$\therefore \omega_{D_{theory}} = 13.90026\sqrt{1-(0.0108)^2}$$

$$= 13.8994 \text{ rad/sec}$$

$$\therefore u(t) = e^{-\xi \omega t} \left\{ \frac{\dot{u}(0)}{\omega_0} \sin \omega_0 t \right\}$$

$$= e^{-0.0108 \times 13.90026 \times t} \left\{ \frac{\dot{u}(0)}{13.8994} \times \sin(13.8994 \times t) \right\}$$

which can now be plotted for $t = 0$ to 32 sec.

~~From measured data:~~ data: $\dot{u}(0) =$

$$\cancel{-\xi \omega e^{-\xi \omega t} \{ B \sin \omega_0 t \} + e^{-\xi \omega t} \{ B \cos \omega_0 t \}}$$

$$\cancel{\dot{u}(0) = e^{0} \{ P}$$

From measured data:

$$T_0 = t_2 - t_1 = 5.6 - 5.173$$

$$= \underline{0.427 \text{ sec}}$$

$$\therefore \omega_{0\,measured} = \frac{2\pi}{T_0} = \frac{2\pi}{0.427} = 14.7147 \text{ rad/sec}$$

$$\omega_{measured} = \frac{\omega_0}{\sqrt{1-\xi^2}} = \frac{14.7147}{1 - 0.0108^2} \approx 14.71$$

$$K_{measured} = \omega_{measured}^2 \times m$$

$$= (14.71)^2 \times 0.2 = 43.276 \text{ N/m}$$

$$\therefore \omega\% \text{ error} = \cancel{43.276} \; \frac{14.71 - 13.90026}{13.90026} \times 100$$

% error = 5.825%

Results:

→ Damped time period (measured) for:

1> Initial disp. → 0.433 sec

2> Initial vel. → 0.427 sec

→ Damping Ratio ξ for:

1> Initial displacement → 0.01527

2> Initial Velocity → 0.0108

→ Measured Natural frequency

1> Init. Disp. → 14.511 rad/sec

2> Init. vel. → 14.71 rad/sec

Analytically the value is: 13.9 rad/sec

→ Stiffness

↳ Analytical: 38.6 N/m

↳ Theoretical:

1> Init Disp. → 42.11 N/m

2> Init vel. → 43.276 N/m

Discussion:

The measured natural frequency and measured stiffness are consistently higher than analytically solved values.

The error may be due to the motion of the SDOF system not being in a straight line due to inaccuracy in applying displacement, hence one component of acceleration gets generated in a mutually perpendicular direction (in the same plane) to our desired motion direction which is not being considered at all. Other sources of error include, measurement, human and dimensioning errors.

We see that the natural frequency in both the cases is almost equal which solidifies the belief that natural frequency is intrinsic property of system and does not depend on external loading conditions.

4 Results:

Comparison of Natural frequency in Hz

Sr. No.	Experiment	From analytical equation	% Error
1. Initial Displacement	14.51	13.90	4.394
2. Using impact hammer	14.71	13.90	5.83