CHAPTER

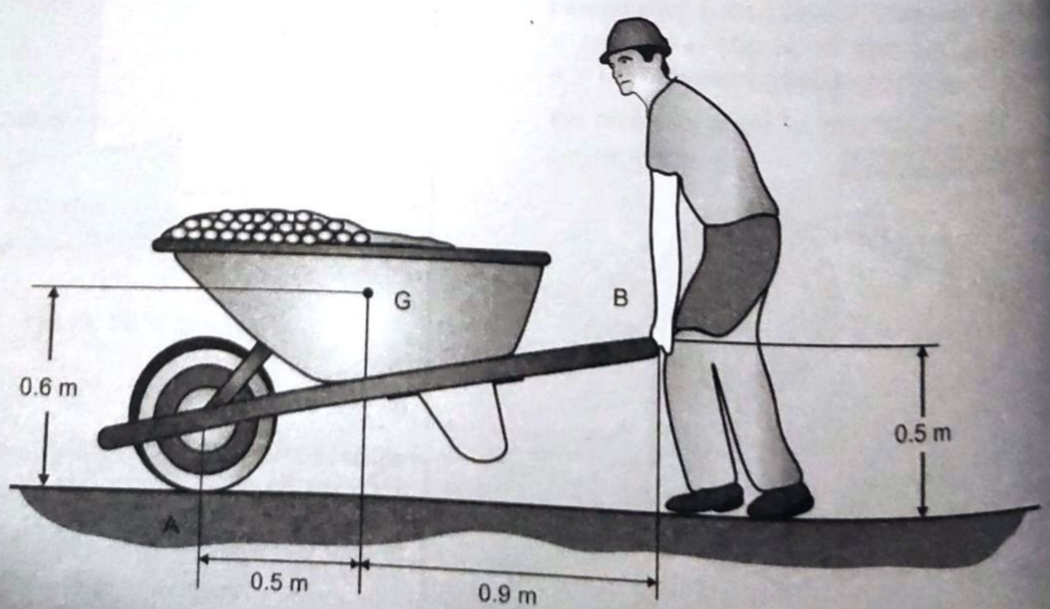
8

UNIT - III

Equilibrium of Force Systems in Plane

Introduction : In this chapter, we shall derive the conditions of equilibrium for coplanar force systems. We shall also study different types of supports, various types of beams and types of loads etc. Most importantly, we shall learn the concept of free body diagram (FBD) and shall determine the support reactions for various examples of engineering significance.

- Type 1 : Equilibrium of concurrent force system
- Type 2 : Equilibrium of parallel force system
- Type 3 : Equilibrium of general force system
- Type 4 : Equilibrium of beams



8.1 Equilibrium of Force Systems in Plane

8.1.1 Equilibrium :

- A rigid body is said to be in equilibrium, when the resultant of the force system on it is zero.
- The resultant of the force system can be a force or a moment.
- If the resultant is zero, it implies that the resultant force ($\sum$ F) and resultant moment ($\sum$ M) both are zero.
- If the resultant force i.e. $\sum$ F i.e. zero, there is no translation and if the resultant moment i.e. $\sum M = 0$. There is no rotation. Then the body is said to be in complete static equilibrium.

8.1.2 Conditions of Equilibrium :

When the body is in equilibrium;

(i) Resultant force, $\sum F = 0$ and

(ii) Resultant moment, $\sum M = 0$

Resultant force $\sum$ F can be resolved into two perpendicular components in x and y directions.

If $\sum F = 0$, then its x-component ($\sum F_x$) and y-component ($\sum F_y$) both are zero.

$\therefore$ The necessary conditions or equations of equilibrium for coplanar force system are :

$$\sum F_x = 0 \quad \text{...(1)}$$

$$\sum F_y = 0 \quad \text{...(2)}$$

And

$$\sum M = 0 \quad \text{...(3)}$$

($\sum$ M about any point must be zero)

8.2 Particular Cases of Equilibrium

(i) Equilibrium with a Single Force :

Let a force 'F' is acting on a body at 'A' at an angle 'θ' w.r.t. horizontal.

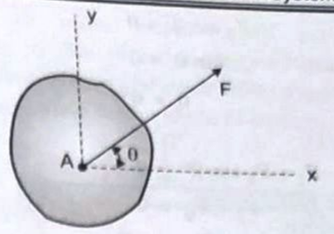


Fig. 8.2.1

To maintain equilibrium ;

$$\sum F_x = 0$$

$$\therefore \quad F \cos\theta = 0 \quad \text{...(1)}$$

$$\sum F_y = 0$$

$$F \sin\theta = 0 \quad \text{...(2)}$$

Suppose, $F \neq 0$, then $\sin\theta = 0$

$$\therefore \quad \theta = 0^\circ$$

Putting in Eq^n (1), $F \cos 0^\circ = 0$

$$F\,(1) = 0$$

$$\therefore \quad \mathbf{F = 0}$$

$\therefore$ **When one force is acting on a body, equilibrium is not possible unless that force itself is zero.**

(ii) Equilibrium with two forces :

Let two non parallel F_1 and F_2 are acting on the body as shown in Fig. 8.2.2.

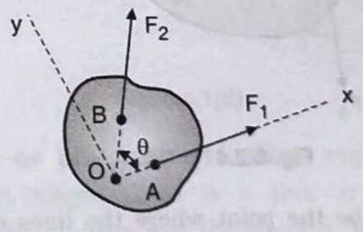


Fig. 8.2.2

Let the lines of action of two forces F_1 and F_2 intersect at point 'O' and 'θ' be the angle between two forces.

Considering x-axis along F_1 and y-axis perpendicular to it and resolving F_1 and F_2 into x and y components.

For equilibrium,

$$\sum F_x = 0$$

$$F_1 + F_2 \cos\theta = 0 \quad \text{...(1)}$$

$$\sum F_y = 0$$

$$F_2 \sin\theta = 0 \quad \ldots(2)$$

But $F_2 \neq 0$, $\therefore \sin\theta = 0$

$$\theta = 0^\circ$$

From Eq^n (1),

$$F_1 + F_2 \cos 0^\circ = 0$$

$$F_1 + F_2 = 0$$

$$F_1 = -F_2$$

$\therefore$ **When two non parallel force are acting to maintain equilibrium, those two forces must be equal in magnitude, opposite in direction and collinear in action.**

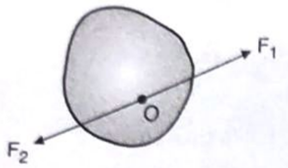


Fig. 8.2.3

(iii) Equilibrium under three forces :

Let F_1, F_2 and F_3 be the three non parallel forces acting on the body as shown in Fig. 8.2.4.

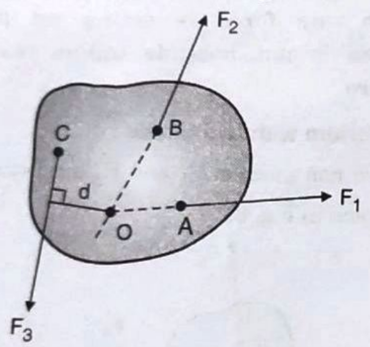


Fig. 8.2.4

Let 'O' be the point where the lines of action of forces F_1 and F_2 intersect.

'd' is the perpendicular distance from the line of action of force F_3 to the point 'O'.

For equilibrium,

ΣM about any point must be zero.

Taking moments about point 'O'

$$\Sigma M_o = 0$$

$$F_3 \times d = 0$$

(Moments of F_1 and F_2 at 'O' are zero as their lines of action passing through point 'O').

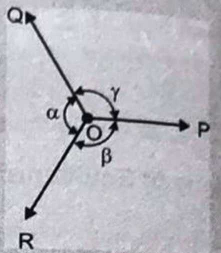
but, $F_3 \neq 0$

$$\therefore \quad d = 0$$

Which means that the line of action of force F_3 also passing through point 'O'.

$\therefore$ **When three non parallel forces are acting on the body, to maintain equilibrium, these three forces must be concurrent.**

Lami's Theorem : It states that when three concurrent forces are acting at a point which is in equilibrium, then the magnitude of each force is proportional to the sine of angle between other two forces.



As per Lami's theorem :

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

8.3 Support and Support Reaction :

***Support** is defined as the structure which tends to maintain equilibrium of the body.*

*While maintaining equilibrium under the action of applied forces or self weight of the body the support exerts a force or moment on the body known as **"support reaction".***

- When a particular displacement of the body is prevented corresponding reaction will develop.
- If the linear displacement is prevented, support will offer a reaction force and if the angular displacement or rotation of the body is prevented, support will offer a reaction moment.

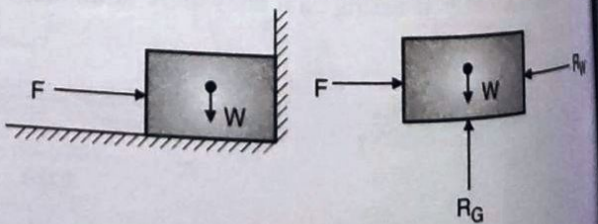


Fig. 8.3.1

Because of weight, W body will have vertical downward displacement which is prevented by the ground surface by offering a support reaction R_G in the opposite direction.

Similarly, applied force 'F' will cause displacement of the body towards right which is prevented by vertical wall by offering a reaction force R_W in the opposite direction.

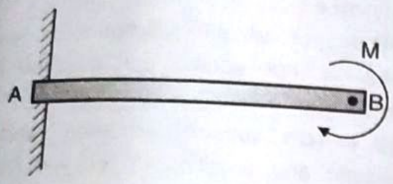


Fig. 8.3.2

Let a clockwise moment 'M' is applied to member 'AB' which tends to rotate the member in CW direction. To maintain equilibrium support at 'A' will offer reaction moment (M_R) in ACW direction i.e. opposite to the direction of rotation.

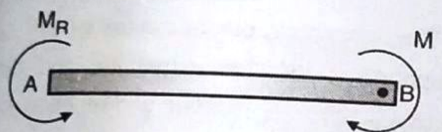


Fig. 8.3.3

8.4 Types of Supports

In engineering, basically three types of supports are used :

(i) Simple support
   a) Sliding b) Roller
(ii) Hinge or pin support
(iii) Fixed support

(i) **Simple supports :** Simple support is the support in which there is no link or connection between the body and the support.

There are two types of simple support :

a) **Sliding support :** If the body rests against a surface whether it is horizontal, vertical or inclined, then it is known as sliding support.

b) **Roller support :** If the body rolls over the surface instead of sliding then it is known as roller support.

Sliding or roller support will prevent the displacement in the direction perpendicular to the plane of surface. Hence sliding or roller support always offer reaction normal to the plane of surface.

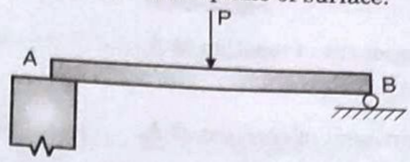


Fig. 8.4.1(a)

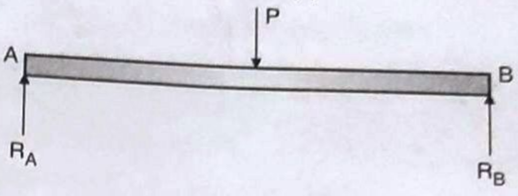


Fig. 8.4.1(b)

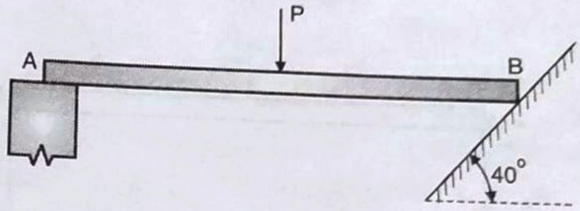


Fig. 8.4.1(c)

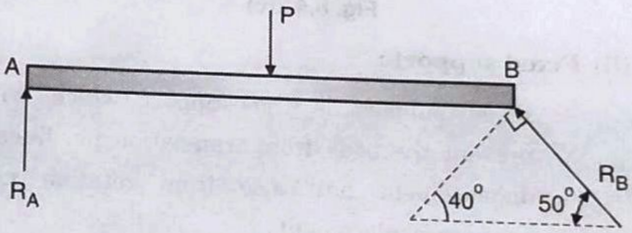


Fig. 8.4.1(d)

(ii) **Hinge or pin support :** Hinge support is the support where there is a link or connection between the body and the support, but it allows the rotation of the body.

Hinge support will prevent linear displacement and allows angular displacement.

$\therefore$ Hinge support will offer reaction force by preventing linear displacement but the reaction moment is zero.

The reaction force will have magnitude 'R' and direction 'θ'. It can be resolved in two reaction components R_x and R_y along horizontal and vertical directions.

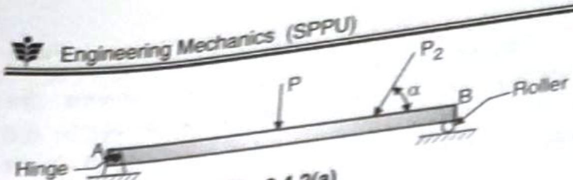


Fig. 8.4.2(a)

x-component of reaction at A,

$$A_x = R_A \cos\theta$$

y-component of reaction at A,

$$A_y = R_A \sin\theta$$

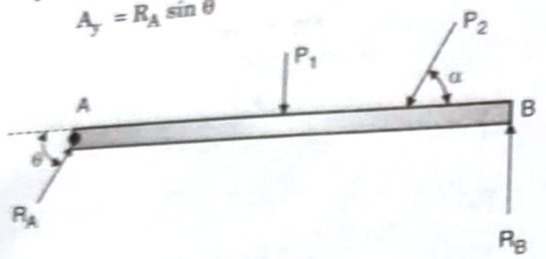


Fig. 8.4.2(b)

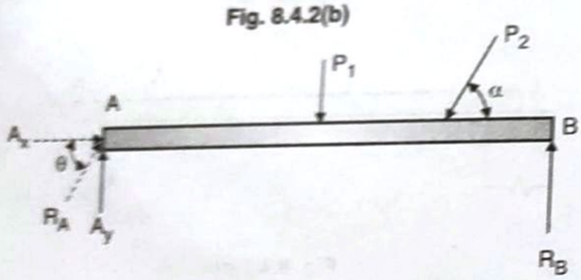


Fig. 8.4.2(c)

(iii) Fixed support :

- Fixed support is the support which will prevent the body from translation i.e. linear displacement and also from rotation i.e. angular displacement.
- Hence fixed support will offer reaction force and reaction moment.
- In fixed support, there is a rigid connection between the body and the support.

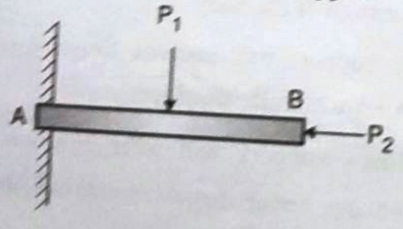


Fig. 8.4.3(a)

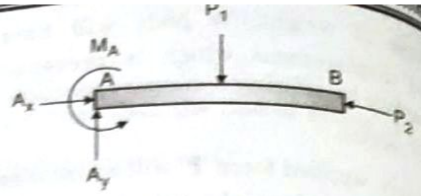


Fig. 8.4.3(b)

Important notes :

(i) Simple support will offer reaction force. Magnitude of reaction is not known but direction is always perpendicular to the plane of surface.

(ii) Hinge or pin support will offer reaction force. Magnitude and direction of reaction both are not known. It can be resolved into x and y components.

(iii) Fixed support will offer reaction force and reaction moment.

(iv) The number of unknowns at

Simple support :	1.	i.e. magnitude of reaction
Hinge/pin support :	2.	i.e. magnitude and direction of reaction or x and y components of reaction force
Fixed support :	3.	i.e. magnitude and direction of reaction and reaction moment. OR x and y components of reaction force and reaction moment.

8.5 Beam

A beam is defined as a structural member which is subjected to transverse loading. i.e. the load acting perpendicular to the longitudinal axis of the member.

Due to applied loads, reactions will develop at the supports and the system of forces consisting of applied loads and reactions keep the beam of equilibrium.

The nature of reaction depends on the type of supports.

8.5.1 Types of beams

Depending on the type of supports, beams are classified into the following types :

(i) **Simply supported beam :** One end is supported by hinge and the other end roller.

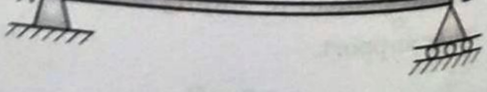


Fig. 8.5.1

(ii) **Cantilever beam :** One end of the beam is fixed and the other end free.

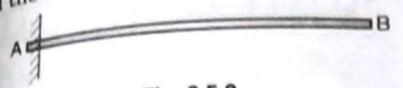


Fig. 8.5.2

(iii) **Fixed beam :** Both ends of the beam are fixed.

Fig. 8.5.3

(iv) **Propped cantilever :** One end of the beam is fixed and the other end is simply supported.

Fig. 8.5.4

(v) **Overhanging beam :** Beam is having projection beyond the support.

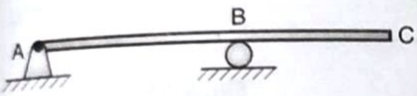


Fig. 8.5.5 : Singly overhanging beam (Having projection on one side)

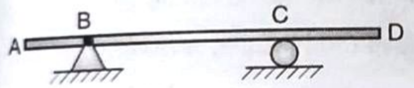


Fig. 8.5.6 : Doubly overhanging beam

(Having projection on both sides)

(vi) **Continuous beam** : Beam supported by more than two supports.

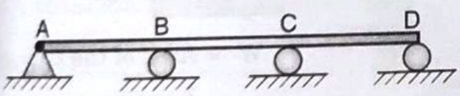


Fig. 8.5.7

(vii) **Compound beam :** It is the combination of two simple beams.

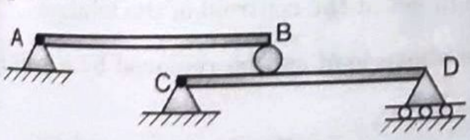


Fig. 8.5.8

Beam AB rests on beam CD.

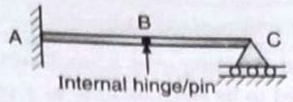


Fig. 8.5.9

Beam AB is connected to beam BC by an internal hinge.

8.6 Statically Determinate and Indeterminate Beams

8.6.1 Statically Determinate Beam :

A beam which can be analysed using three equations of equilibrium is known as ***statically determinate beam.***

The equilibrium equations are :

$$\sum F_x = 0,$$

$$\sum F_y = 0$$

$$\text{and } \sum M = 0$$

$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma M = 0$

In these beams, the number of unknown reaction components are less than or equal to three.

E.g. (i) Simply supported beam

(ii) Cantilever beam, etc.

8.6.2 Statically Indeterminate Beam :

The beam which cannot be analysed using 3 equations of equilibrium is called ***statically indeterminate beam.***

In this type of beam, the number of unknown reaction components are more than three equilibrium equations.

E.g. (i) Fixed beam

Fixed beam

(ii) Propped cantilever

(iii) continuous beam, etc.

8.7 Types of Loads

Beam is subjected to the following types of loads:

(i) Point or concentrated load

(ii) Uniformly Distributed Load (UDL)

(iii) Uniformly Varying Load (UVL)

(iv) Variation is nonlinear

(i) Point or concentrated load :

If the load is acting over a small area or length, it can be assumed to be concentrated at a point known as point load or concentrated load.

It is represented by an arrow head.

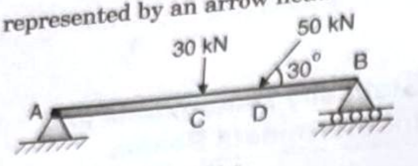


Fig. 8.7.1

Point load of 30 kN is acting at point 'C' vertically and 50 kN load is acting at point 'D' and having inclination of 30° w.r.t. horizontal.

(ii) Uniformly Distributed Load (UDL) :

If the load is spread or occupied over a considerable length of the beam it is known as distributed load. If the intensity of load i.e. load per unit length is uniform throughout, then it is known as Uniformly Distributed Load (UDL).

It is represented as below :

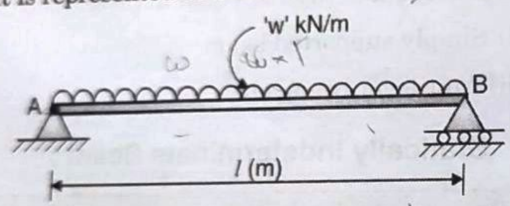


Fig. 8.7.2

Magnitude of UDL,

W = (Intensity of load) × (Length)

Total load, W = w.l kN

Total load will act the midpoint of the length i.e. AB.

UDL can be replaced by the point load as below :

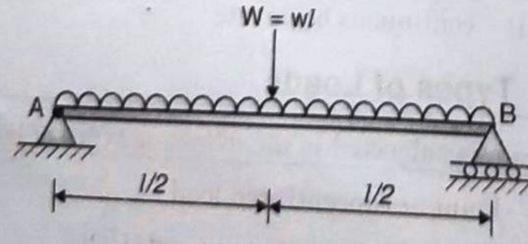


Fig. 8.7.3

Uniformly distributed load is also called rectangular load as it can be represented as a rectangle shown below :

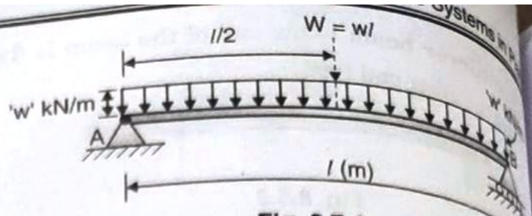


Fig. 8.7.4

Area of rectangle = Total magnitude of load

$\therefore$ W = 'wl' kN

It will act at the centroid of the rectangle.

(iii) Uniformly varying load (UVL) :

If the intensity of load is varying uniformly over length of the beam, then it is known as uniformly varying load or triangular load.

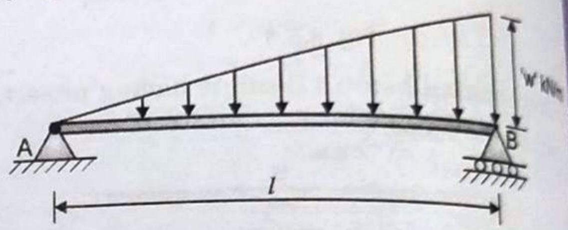


Fig. 8.7.5

Here intensity of load is varying uniformly from zero at end A to 'w' kN/m at end B.

Magnitude of total load, W = (Average intensity load) × Length

$$\therefore \quad W = \left(\frac{0 + w}{2}\right) l$$

$$= \left(\frac{wl}{2}\right) \text{kN}$$

OR

W = Area of the triangle

$$= \frac{1}{2} \times l \times w$$

$$= \left(\frac{wl}{2}\right) \text{kN}$$

It will act at the centroid of the triangle.

The above load can be replaced by a point load as shown below :

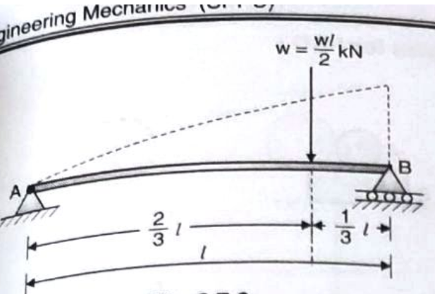


Fig. 8.7.6

(iii) Variation is nonlinear :

If the load is distributed in such a way that it is varying non-linearly from one point to another point, then it is known as nonlinearly varying load.

The magnitude of non-linear load is determined by integration.

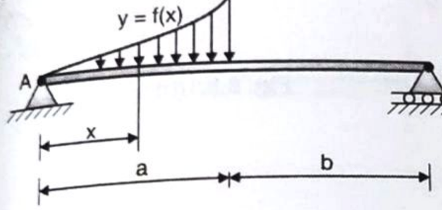


Fig. 8.7.7

Note : If the load is distributed in such a way that the intensity of load is changing linearly from w_1 and w_2, then it is known as trapezoidal load.

Here the load distribution diagram is like a trapezium.

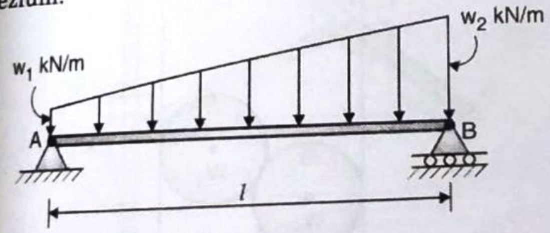


Fig. 8.7.8

Total magnitude of load,

$$W = \text{Area of trapezium}$$

$$= \left(\frac{w_1 + w_2}{2}\right) l$$

$$\text{Position of load} = \text{centroid of trapezium}$$

$$= \left(\frac{w_1 + 2w_2}{w_1 + w_2}\right)\frac{l}{3} \text{ from A}$$

Alternative Method :

We can also divide this load into two parts:

1. Rectangular load of height w_1
2. Triangular load of height $(w_2 - w_1)$

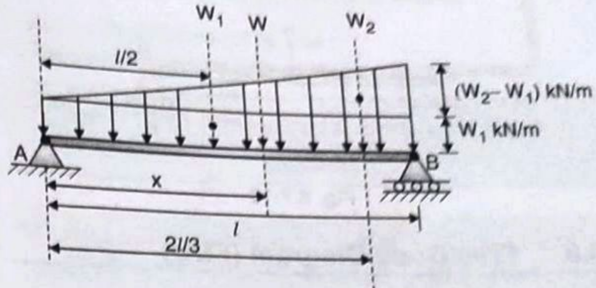


Fig. 8.7.9

Magnitude of rectangular load, $\quad w_1 = l\,W_1$

Position of rectangular load, $\quad = \frac{l}{2}$ from 'A'

Magnitude of triangular load, $\quad W_2 = \frac{1}{2} l\,(w_2 - w_1)$

Position of triangular load, $= \frac{2}{3} l$ from 'A'

Let 'W' be the total trapezoidal load.

$$\therefore \quad W = W_1 + W_2$$

$$= l\,w_1 + \frac{l w_2}{2} - \frac{l\,w_1}{2} = \frac{l\,w_1}{2} + \frac{l\,w_2}{2}$$

$$= \left(\frac{w_1 + w_2}{2}\right) l$$

To find position of 'W', taking moments at 'A' and using Varignon's theorem ;

$$-W_1 \times \frac{l}{2} - W_2 \times \frac{2l}{3} = -W \cdot x$$

$$\therefore \quad l\,w_1 \times \frac{l}{2} + \frac{l}{2}(w_2 - w_1) \cdot \frac{2l}{3} = \left(\frac{w_1 + w_2}{2}\right) l \cdot x$$

$$\frac{w_1 l}{2} + \frac{w_2 l}{3} - \frac{w_1 l}{3} = \left(\frac{w_1 + w_2}{2}\right) x$$

$$\frac{3\,w_1 l + 2\,w_2 l - 2\,w_1 l}{6} = \left(\frac{w_1 + w_2}{2}\right) x$$

$$\frac{w_1 l + 2\,w_2 l}{6} = \left(\frac{w_1 + w_2}{2}\right) x$$

$$\frac{l}{6}(w_1 + 2\,w_2) = \left(\frac{w_1 + w_2}{2}\right) x$$

$$x = \frac{l\,(w_1 + 2\,w_2)}{6} \times \left(\frac{2}{w_1 + w_2}\right)$$

∴

∴ Position of total load from end 'A' is

$$x = \left(\frac{w_1 + 2w_2}{w_1 + w_2}\right)\frac{l}{3}$$

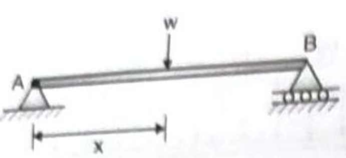


Fig. 8.7.10

8.8 Free Body Diagram (FBD)

"Free body diagram is the diagram of the body under consideration of equilibrium showing all the forces acting on it which is isolated from the surroundings."

These forces include :

(i) Applied forces

(ii) Reactions offered by supports or other bodies which are in contact with this body

(iii) Self weight of the body

All dimensions and distances are also indicated in FBD.

FBD is a very important component in problem solving.

Advantages of FBD

Following are the advantages of FBD:

(i) It helps in reducing the given system into an idealized system that may be analysed mathematically.

(ii) It helps in identifying known forces and unknown forces in the given problem.

(iii) It gives an overall idea about the problem from which you can approach the solution systematically.

(iv) It helps in solving the problem step by step in an effective way.

Examples for FBD :

(i)

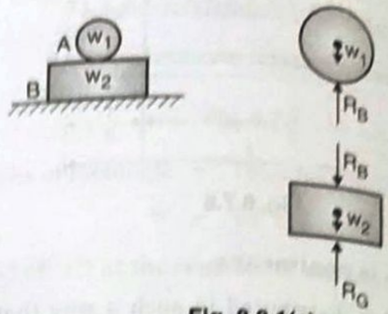


Fig. 8.8.1(a)

(ii)

Fig. 8.8.1(b)

(iii)

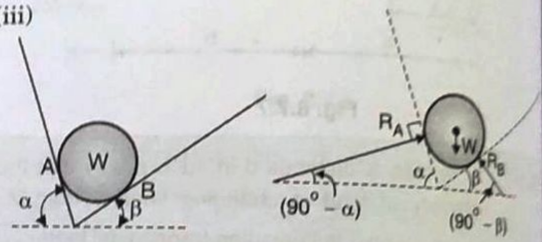


Fig. 8.8.1(c)

(iv)

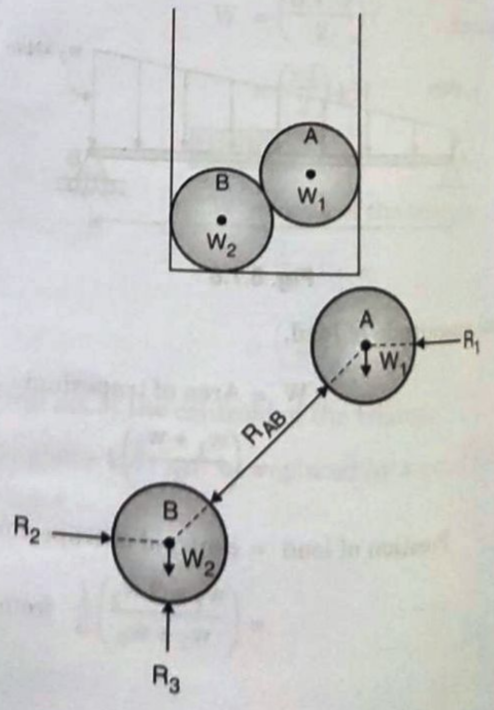
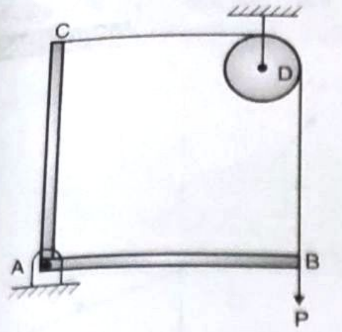


Fig. 8.8.1(d)

(v)



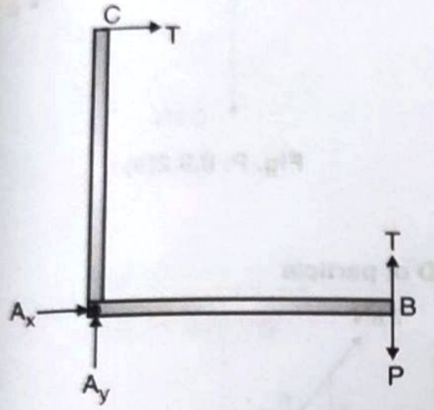
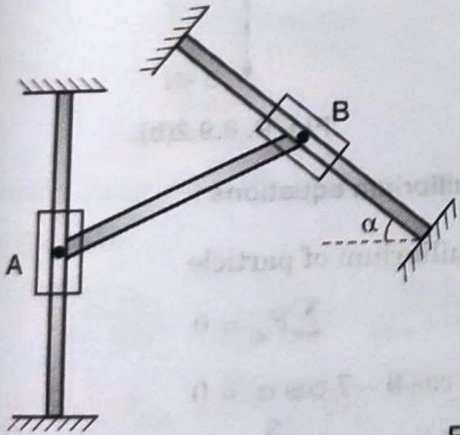


Fig. 8.8.1(e)



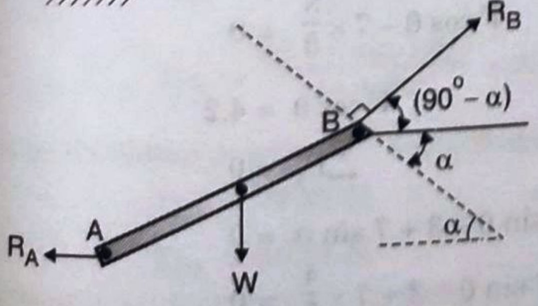


Fig. 8.8.1(f)

8.9 Solved Examples

Type 1 : Equilibrium of concurrent force system

Type 2 : Equilibrium of parallel force system

Type 3: Equilibrium of general force system

Type 4 : Equilibrium of beams.

Type 1 : Equilibrium of Concurrent Forces System

Ex. 8.9.1 : Two cables tied together at 'C' are loaded as shown Fig.P.8.9.1 Knowing that W = 200 N, determine the tension in cable AC and BC.

SPPU : May 09, 6 Marks

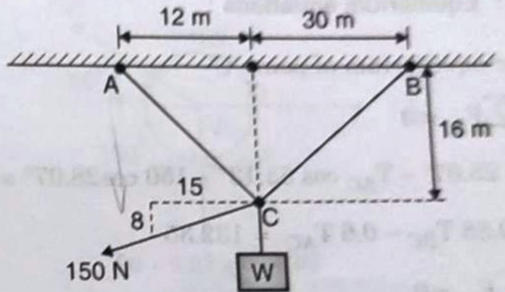


Fig. P. 8.9.1(a)

Soln. :

Step 1 : Forest acting at point 'C' :

(i) Weight, W = 200 N

(ii) Tension in the cable AC = T_{AC}

(iii) Tension in the cable BC = T_{BC}

(iv) Applied force = 150 N

Step 2 : Finding direction of forces from geometry

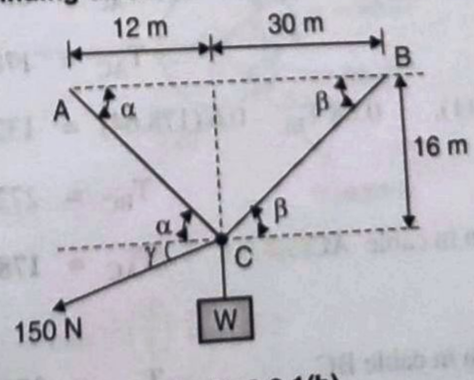
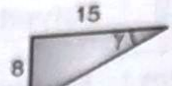


Fig. P. 8.9.1(b)

$$\alpha = \tan^{-1}\left(\frac{16}{12}\right) = 53.13°$$

$$\beta = \tan^{-1}\left(\frac{16}{30}\right) = 28.07^\circ$$



$$\gamma = \tan^{-1}\left(\frac{8}{15}\right) = 28.07^\circ$$

Step 3 : FBD of point 'C'

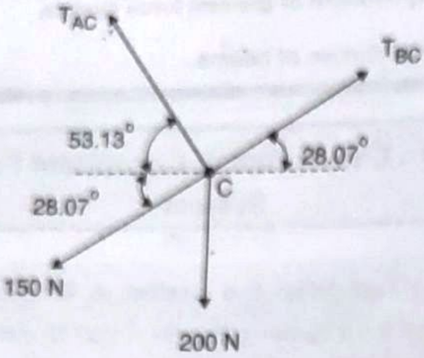


Fig. P. 8.9.1(c)

Step 4 : Equilibrium equations :

For equilibrium of point 'C'

$$\sum F_x = 0 \quad \Sigma F_n = 0$$

$$T_{BC} \cos 28.07^\circ - T_{AC} \cos 53.13^\circ - 150 \cos 28.07^\circ = 0$$

$$0.88\, T_{BC} - 0.6\, T_{AC} = 132.35 \quad \ldots(1)$$

$$\sum F_y = 0$$

$$T_{BC} \sin 28.07^\circ + T_{AC} \sin 53.13^\circ - 150 \sin 28.07^\circ - 200 = 0$$

$$0.47\, T_{BC} + 0.8\, T_{AC} = 270.58 \quad \ldots(2)$$

From Eq^n (1),

$$T_{BC} - 0.68\, T_{AC} = 150.4$$

From Eq^n (2), $T_{BC} = 0.68\, T_{AC} + 150.4$

$$0.47\,(0.68\, T_{AC} + 150) + 0.8\, T_{AC} = 270.58$$

$$0.32\, T_{AC} + 70.5 + 0.8\, T_{AC} = 270.58$$

$$1.12\, T_{AC} = 200.08$$

$$\therefore \quad T_{AC} = 178.64 \text{ N}$$

From Eq^n (1), $0.88\, T_{BC} - 0.6\,(178.64) = 132.35$

$$\therefore \quad T_{BC} = 272.2 \text{ N}$$

(i) Tension in cable AC, $\mathbf{T_{AC} = 178.64\ N}$

...Ans.

(ii) Tension in cable BC, $\mathbf{T_{BC} = 272.20\ N}$

...Ans.

Ex. 8.9.2 : Determine the magnitude and direction of F so that the particle is in equilibrium. Fig. P. 8.9.2(a). **SPPU : Dec. 14, 5 M...**

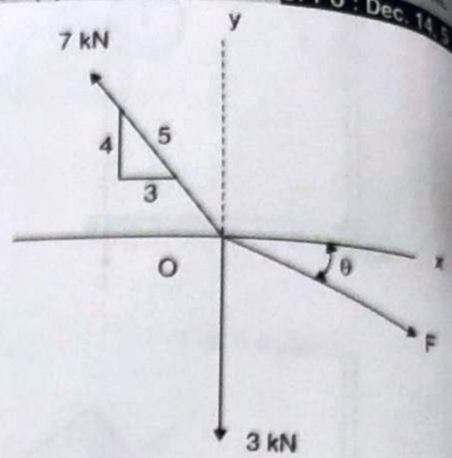


Fig. P. 8.9.2(a)

Soln. :

Step 1 : FBD of particle

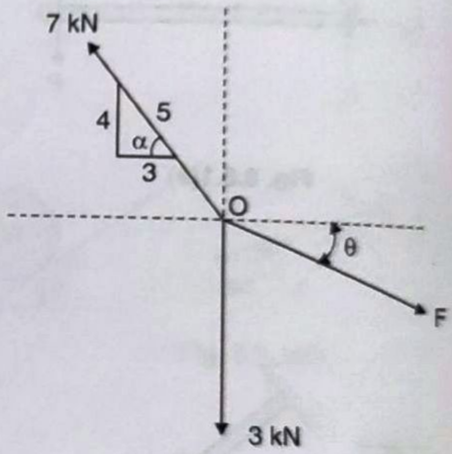


Fig. P. 8.9.2(b)

Step 2 : Equilibrium equations :

For equilibrium of particle

$$\sum F_x = 0$$

$$F \cos \theta - 7 \cos \alpha = 0$$

$$F \cos \theta - 7 \times \frac{3}{5} = 0$$

$$F \cos \theta = 4.2$$

$$\sum F_y = 0$$

$$-F \sin \theta - 3 + 7 \sin \alpha = 0$$

$$-F \sin \theta - 3 + 7 \times \frac{4}{5} = 0$$

$$F \sin \theta = 2.6$$

$$\frac{F \sin \theta}{F \cos \theta} = \left(\frac{2.6}{4.2}\right)$$

$$\theta = 31.76°$$

$\therefore$ From Eq^n (1),

$$F = \frac{4.2}{\cos 31.76°} = 4.94 \text{ kN}$$

(i) Magnitude of force, F = **4.94 kN** ...**Ans.**

(ii) Direction of force, θ = **31.76°** ...**Ans.**

Ex. 8.9.3 : Determine the tension developed in wires CA and CB required for equilibrium of the 10 kg cylinder as shown in Fig. P. 8.9.3(a). **SPPU : May 18, 6 Marks**

Soln. :

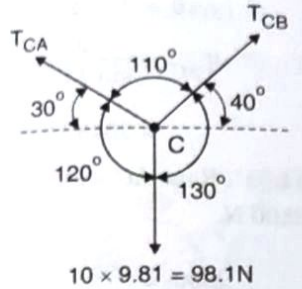


Fig. P. 8.9.3(a)

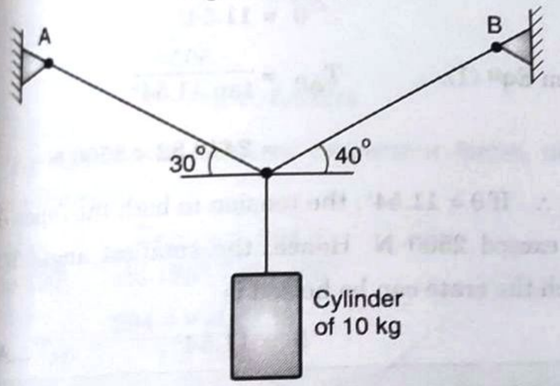


Fig. P. 8.9.3(b)

For equilibrium of three concurrent forces.

Using Lami's theorem :

$$\frac{TCA}{\sin 130°} = \frac{T_{CA}}{\sin 120°} = \frac{98.1}{\sin 110°}$$

$$\therefore \quad T_{CA} = \frac{98.1 \times \sin 130°}{\sin 110°} = 79.97 \text{ N}$$

$$T_{CB} = \frac{98.1 \times \sin 120°}{\sin 110°} = 90.41 \text{ N}$$

$\therefore$ The tension developed in wires CA and CB are :

$\mathbf{T_{CA} = 79.97\ N}$ ...**Ans.**

$\mathbf{T_{CB} = 90.41\ N}$ ...**Ans.**

Ex. 8.9.4 : The 30 kg pipe is supported at A by a system of five cards as shown in Fig. P. 8.9.4(a). Determine the force in each cord for equilibrium. **SPPU : May 16, 6 Marks**

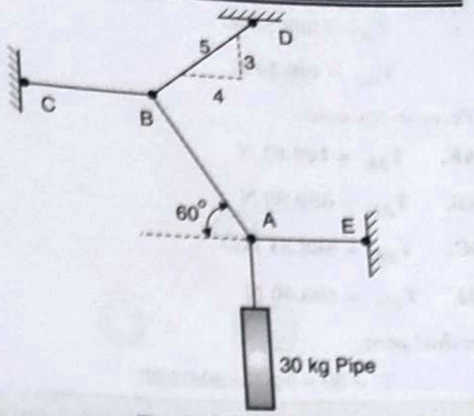


Fig. P. 8.9.4(a)

Soln. :

FBD of point 'A'

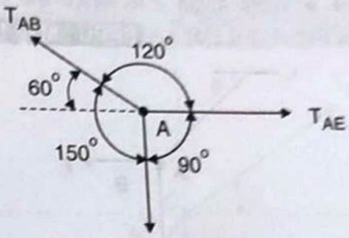


Fig. P. 8.9.4(b)

For equilibrium of three concurrent forces, using Lami's theorem ;

$$\frac{T_{AB}}{\sin 90°} = \frac{T_{AE}}{\sin 150°} = \frac{294.3}{\sin 120°}$$

$$\therefore \quad T_{AB} = 339.83 \text{ N}$$

$$T_{AE} = 169.91 \text{ N}$$

FBD of point 'B'

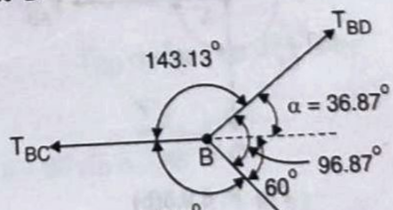


Fig. P. 8.9.4(c)

$$\alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$$

Again using Lami's theorem;

$$\frac{T_{BC}}{\sin 96.87°} = \frac{T_{BD}}{\sin 120°} = \frac{339.83}{\sin 143.13°}$$

$\therefore \quad T_{BC} = 562.31$ N

$T_{BD} = 490.50$ N

Force in the cords :

AE, $\quad T_{AE} = \mathbf{169.91\ N}$

AB, $\quad T_{AB} = \mathbf{339.83\ N}$

BC, $\quad T_{BC} = \mathbf{562.31\ N}$

BD, $\quad T_{BD} = \mathbf{490.50\ N}$

Vertical cord,

$$T = 30 \times 9.81 = 294.3 \text{ N}$$

Ex. 8.9.5 : The 500 N crate is hoisted using the ropes AB and AC. Each rope can withstand a maximum tension of 2500 N before it breaks. If AB always remains horizontal, determine the smallest angle θ to which the crate can be hoisted. Refer Fig. P. 8.9.5(a). **SPPU : May 12, 6 Marks**

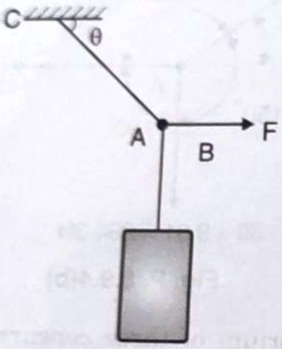


Fig. P. 8.9.5(a)

Soln. :

Considering FBD (free body diagram) of point 'A'

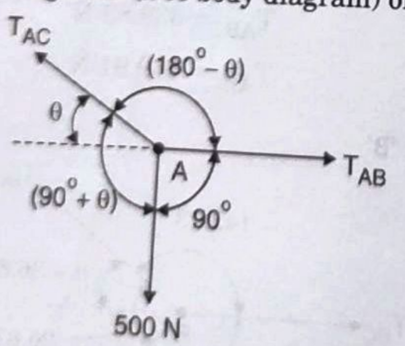


Fig. P. 8.9.5(b)

For equilibrium of three concurrent forces,

Using Lami's theorem,

$$\frac{T_{AB}}{\sin(90° + \theta)} = \frac{T_{AC}}{\sin 90°} = \frac{500}{\sin(180° - \theta)}$$

$$\therefore \quad \frac{T_{AB}}{\cos\theta} = \frac{T_{AC}}{1} = \frac{500}{\sin\theta}$$

$$\therefore \quad T_{AB} = \frac{500 \cos\theta}{\sin\theta}$$

and $\quad T_{AC} = \dfrac{500}{\sin\theta}$

Given that the maximum tension in each ... 2500 N.

$\therefore \quad$ If $T_{AB} = 2500$ N

From Eqn (1), $\quad 2500 = \dfrac{500}{\tan\theta}$

$\therefore \quad \theta = 11.31°$

From Eqn (2), $\quad T_{AC} = \dfrac{500}{\sin 11.31°}$

$= 2549.51 \text{ N} > 2500$ N

$\therefore \quad$ If $\theta = 11.31°$, Rope AC will break

If $T_{AC} = 2500$ N,

From Eqn (2), $\quad 2500 = \dfrac{500}{\sin\theta}$

$\therefore \quad \mathbf{\theta = 11.54°}$

From Eqn (1), $\quad T_{AB} = \dfrac{500}{\tan 11.54°}$

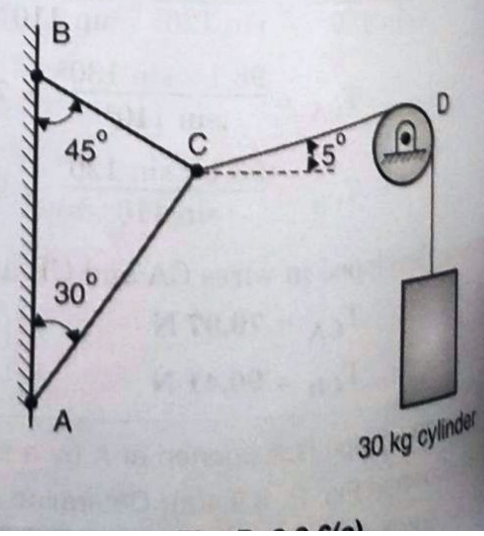
$= \mathbf{2448.82 < 2500\ N}$

$\therefore$ If θ = 11.54°, the tension in both the ropes ... not exceed 2500 N. Hence, the smallest angle ... which the crate can be hoised is

$$\theta = \mathbf{11.54°}$$

Ex. 8.9.6 : Three cables are joined at the junction ... shown in Fig P. 8.9.6(a). Determine the tension in ... and BC caused by the weight of the 30 kg cylinder.

SPPU : May 14, 5 Marks, Dec. 17, 6 ...



Soln. :

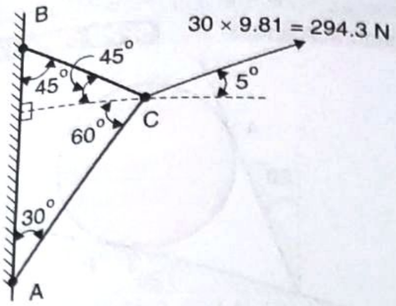


Fig. P. 8.9.6(b)

The tension in the cable CD is $30 \times 9.81 = 294.3$ N

FBD of point 'C'

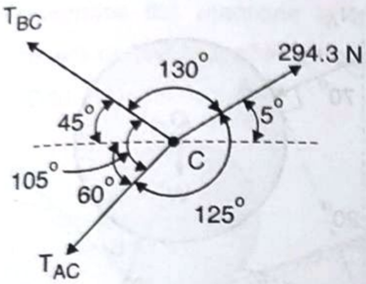


Fig. P. 8.9.6(c)

For equilibrium of three concurrent forces, using Lami's theorem;

$$\frac{T_{AC}}{\sin 130^\circ} = \frac{T_{BC}}{\sin 125^\circ} = \frac{294.3}{\sin 105^\circ}$$

$$\therefore \quad T_{AC} = \frac{294.3 \times \sin 130^\circ}{\sin 105^\circ}$$

$$= 233.40 \text{ N}$$

$$T_{BC} = \frac{294.3 \times \sin 125^\circ}{\sin 105^\circ}$$

$$= 249.58 \text{ N}$$

$\therefore$ Tension in cable AC,

T_{AC} **= 233.40 N** ...**Ans.**

Tension in cable BC,

T_{BC} **= 249.58 N** ...**Ans.**

Ex. 8.9.7 : The motor at B winds up the cord attached to the 65 N crate with a constant speed as shown in Fig. P. 8.9.7(a). Determine the force in cord CD supporting the pulley and the angle θ for equilibrium. Neglect the size pulley at C. **SPPU : May 13, 4 Marks**

Soln. :

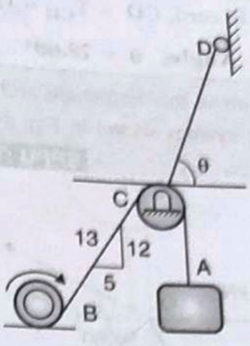


Fig. P. 8.9.7(a)

FBD of point 'C'

Tension in CB and CA is same as the cord is passing over a pulley of negligible size.

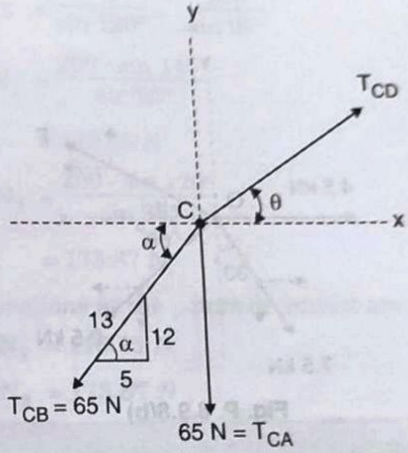


Fig. P. 8.9.7(b)

For equilibrium of the system;

$$\sum F_x = 0$$

$$T_{CD} \cos\theta - 65 \cos\alpha = 0$$

$$T_{CD} \cos\theta = 65 \times \frac{5}{13} = 25 \quad \text{...(1)}$$

$$\sum F_y = 0$$

$$T_{CD} \sin\theta - 65 \sin\alpha - 65 = 0$$

$$T_{CD} \sin\theta = 65 \times \frac{12}{13} + 65 = 125 \quad \text{...(2)}$$

From Eq^n (1) and Eq^n (2)

$$\frac{T_{CD} \sin\theta}{T_{CD} \cos\theta} = \frac{125}{25}$$

$$\tan\theta = 5$$

$$\therefore \quad \theta = 78.69^\circ$$

$\therefore$

From Eq^n (1), $$T_{CD} = \frac{25}{\cos 78.69^\circ}$$

$= 127.47\ N$

$\therefore$ Force in cord, CD $= T_{CD} = 127.47\ N$ ...Ans.

and Angle, $\theta = 78.69°$ ...Ans.

Ex. 8.9.8 : Determine the magnitude and position of force F so that the force system shown in Fig. P. 8.9.8(a) maintain equilibrium. **SPPU : Dec. 15, 6 Marks**

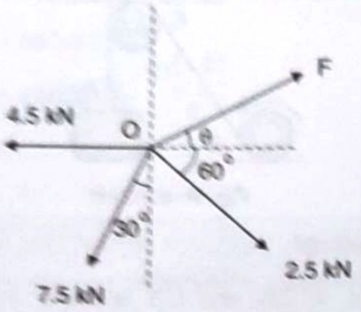


Fig. P. 8.9.8(a)

Soln. :

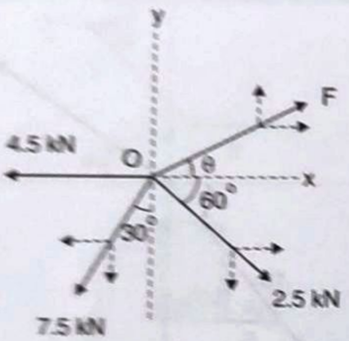


Fig. P. 8.9.8(b)

For equilibrium ;

$$\sum F_x = 0$$

$$F\cos\theta - 4.5 - 7.5\sin 30° + 2.5\cos 60° = 0$$

$$F\cos\theta = 7 \quad ...(1)$$

$$\sum F_y = 0$$

$$F\sin\theta - 7.5\cos 30° - 2.5\sin 60° = 0$$

$$\therefore \quad F\sin\theta = 8.66 \quad ...(2)$$

$$\therefore \quad \frac{F\sin\theta}{F\cos\theta} = \frac{8.66}{7}$$

$$\tan\theta = 1.237$$

$$\therefore \quad \theta = 51.05°$$

From Eq^n (1), $F\cos 51.05° = 7$

$$\therefore \quad F = 11.13\ KN$$

$\therefore$ **Magnitude of force, F = 11.13 KN** ...Ans.

Direction of force, $\theta = 51.05°$ ...Ans.

Ex. 8.9.9 : A cylinder of mass 100 kg rest between inclined plane as shown in Fig. P. 8.9.9(a). Determine normal reaction at A and B. **SPPU : May 17, 6 Marks**

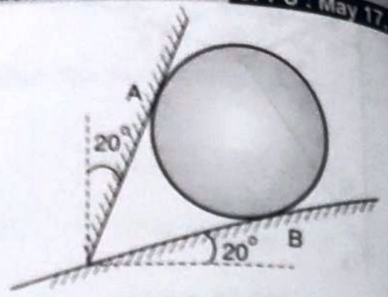


Fig. P. 8.9.9(a)

Soln. :

FBD of cylinder :

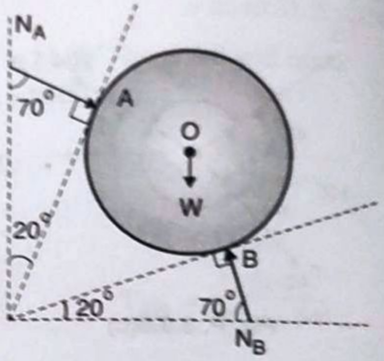


Fig. P. 8.9.9(b)

Let N_A and N_B be the normal reactions at A and B respectively. Three forces are acting on the cylinder i.e. N_A, N_B and W.

When three non parallel forces are acting, to maintain equilibrium, these three forces must be concurrent.

$\therefore$ The line of action of N_A and N_B must pass through the centre of the cylinder 'O'.

$\therefore$ Considering three forces at point 'O' i.e., centre of the cylinder.

For three concurrent forces in equilibrium, using Lami's theorem :

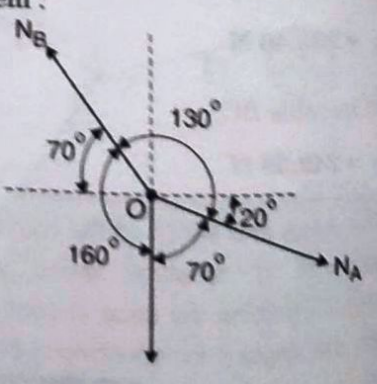


Fig. P. 8.9.9(c)

$$\frac{N_A}{\sin 160^\circ} = \frac{N_B}{\sin 70^\circ} = \frac{981}{\sin 130^\circ}$$

$$\therefore \quad N_A = \frac{981 \times \sin 160^\circ}{\sin 130^\circ}$$

$$= 438 \text{ N}$$

$$N_B = \frac{981 \times \sin 70^\circ}{\sin 130^\circ}$$

$$= 1203.37 \text{ N}$$

$\therefore$ Normal reactions at A and B are :

N_A = 438 N ...Ans.

N_B = 1203.37 N ...Ans.

Ex. 8.9.10 : Determine the reactions at all the point of contacts for a sphere of 200 N kept in a trough as shown in the Fig. P. 8.9.10(a).

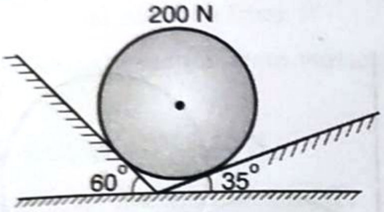


Fig. P. 8.9.10(a)

Soln. :

Three forces are acting on the sphere.

(i) Weight of sphere, W

(ii) Normal reaction from plane making 60° with horizontal, N_1

(iii) Normal reaction from plane making 35° with horizontal, N_2

When three forces are acting on a body, to maintain equilibrium, those three forces must be concurrent.

FBD of sphere :

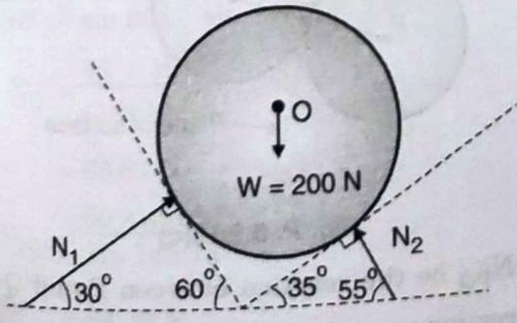


Fig. P. 8.9.10(b)

Considering 3 forces at the centre 'O' of the sphere

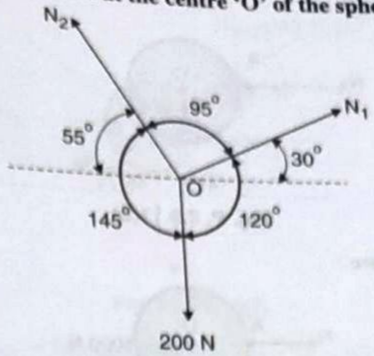


Fig. P. 8.9.10(c)

Using Lami's theorem for three concurrent forces in equilibrium ;

$$\frac{N_1}{\sin 145^\circ} = \frac{N_2}{\sin 120^\circ} = \frac{200}{\sin 95^\circ}$$

$$\therefore \quad N_1 = \frac{200 \cdot \sin 145^\circ}{\sin 95^\circ}$$

$$= 115.15 \text{ N}$$

$$N_2 = \frac{200 \cdot \sin 120^\circ}{\sin 95^\circ}$$

$$= 173.87 \text{ N}$$

$\therefore$ The reactions at the points of contact are :

N_1 = 115.15 N ...Ans.

N_2 = 173.87 N ...Ans.

Ex. 8.9.11 : A sphere weighting 1000 N is placed in a wrench as shown in Fig. P. 8.9.11(a). Find the reactions at the point of contacts. **SPPU : Dec. 15, 5 Marks**

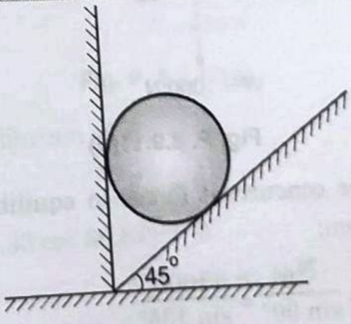


Fig. P. 8.9.11(a)

Soln. :

Here both the surfaces or planes act as simple supports. Therefore, the reactions offered by these planes are normal to the plane.

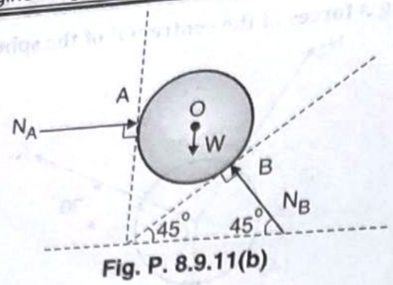


Fig. P. 8.9.11(b)

FBD of sphere :

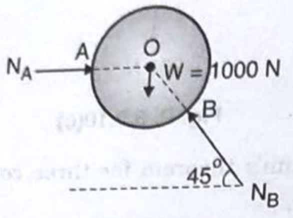


Fig. P. 8.9.11(c)

Three forces are acting on the sphere :

(i) Weight of sphere, W = 1000 N

(ii) Normal reaction from vertical plane at point A = N_A

(iii) Normal reaction from inclined plane at point B = N_B

To maintain equilibrium, these three forces must be concurrent. Considering all forces at the centre of sphere.

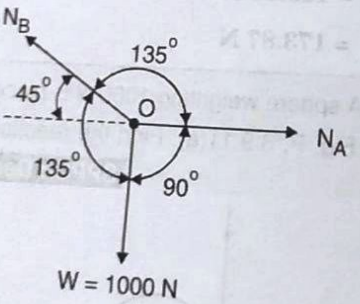


Fig. P. 8.9.11(d)

For three concurrent forces in equilibrium, using Lami's theorem;

$$\frac{N_A}{\sin 135^\circ} = \frac{N_B}{\sin 90^\circ} = \frac{1000}{\sin 135^\circ}$$

$\therefore$ $N_A = 1000$ N

$$N_B = \frac{1000 \sin 90^\circ}{\sin 135^\circ}$$

$= 1414.21$ N

$\therefore$ The reactions at points of contact are :

N_A = 1000 N ...Ans.

N_B = 1414.21 N ...Ans.

Ex. 8.9.12 : Two sphere P and Q each of weight 50 N radius of 100 mm rest in horizontal channel of width 360 as shown in Fig. P. 8.9.12(a). Determine the reactions point of contact A, B and C.

SPPU : Dec. 12, 7

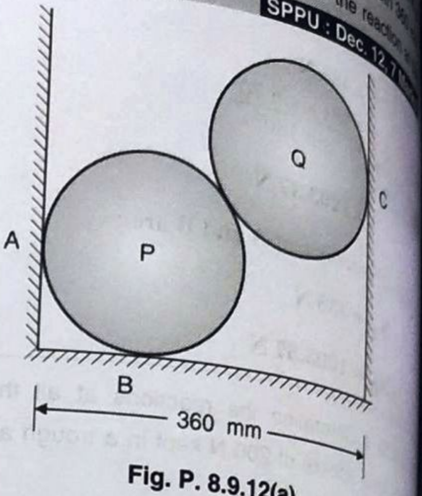


Fig. P. 8.9.12(a)

Soln. :

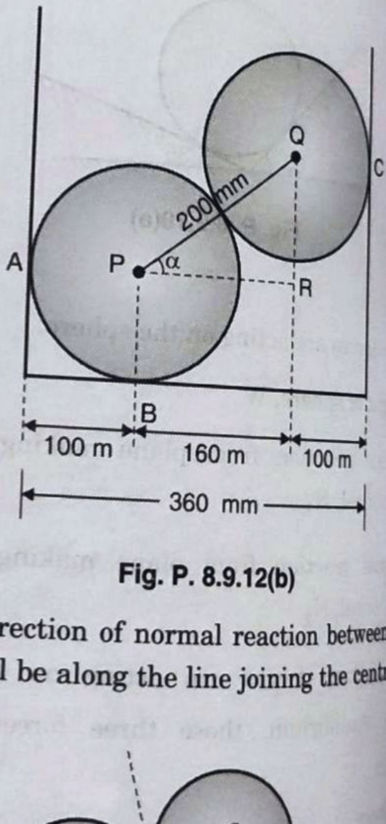


Fig. P. 8.9.12(b)

The direction of normal reaction between the spheres will be along the line joining the centres of spheres.

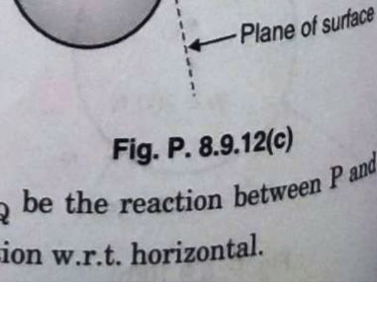


Fig. P. 8.9.12(c)

Let N_{PQ} be the reaction between P and Q and α be the direction w.r.t. horizontal.

From Fig. P.8.9.12 (b) $\triangle$ PQR, $\cos\alpha = \frac{PR}{PQ} = \left(\frac{160}{200}\right)$

$\alpha = 36.87°$

$\therefore$ The supports A, B and C are simple supports.

Hence the reactions are perpendicular to the plane of surface.

FBD of sphere 'Q'

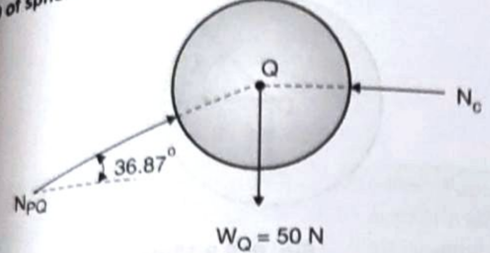


Fig. P. 8.9.12(d)

N_{PQ} = Normal reaction from 'P'

N_C = Normal reaction from vertical surface at 'C'

W_Q = Weight of 'Q' = 50 N

As three forces are acting on 'Q', to maintain equilibrium, they must be concurrent.

Considering all forces at the centre of sphere, 'Q'

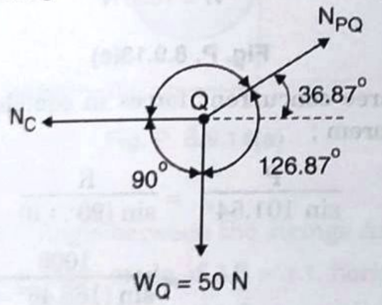


Fig. P. 8.9.12(e)

For equilibrium of three concurrent forces, using Lami's theorem,

$$\frac{N_C}{\sin 126.87°} = \frac{N_{PQ}}{\sin 90°} = \frac{50}{\sin 143.13°}$$

$$\therefore \quad N_C = \frac{50 \times \sin 126.87°}{\sin 143.13°}$$

$$= 66.67 \text{ N}$$

$$N_{PQ} = \frac{50 \times \sin 90°}{\sin 143.13°}$$

$$= 83.33 \text{ N}$$

FBD of sphere 'P'

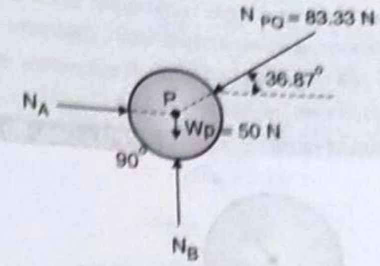


Fig. P. 8.9.12(f)

N_{PQ} = Normal reaction from 'Q' = 83.33 N

N_A = Normal reaction from vertical surface at A

N_B = Normal reaction from horizontal surface at B

W_P = Weight of sphere 'P' = 50 N

Note : The magnitude of normal reaction between P and Q is same but opposite in direction.

Considering all forces at the centre of 'P'.

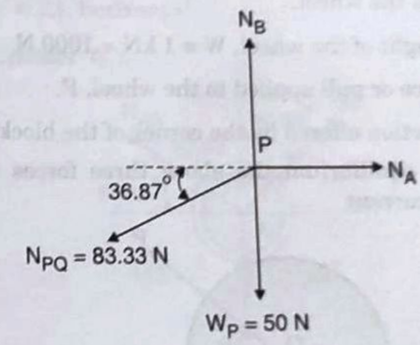


Fig. P. 8.9.12(g)

For equilibrium of 'P';

$$\sum F_x = 0$$

$$N_A - 83.33 \cos 36.87° = 0$$

$$\therefore \quad N_A = 66.67 \text{ N}$$

$$\sum F_y = 0$$

$$N_B - 83.33 \sin 36.87° - 50 = 0$$

$$\therefore \quad N_B = 100 \text{ N}$$

Reaction at point A, $N_A = 66.67 \text{ N} \rightarrow$ ...Ans.

Reaction at point B, $N_B = 100 \text{ N} \uparrow$ ...Ans.

Reaction at point C, $N_C = 66.67 \text{ N} \leftarrow$ ...Ans.

Ex. 8.9.13 : A uniform wheel of 50 cm diameter and 1 kN weight rests against a rigid rectangular block of thickness 20 cm. Considering all surfaces smooth, determine :

(i) Least pull to be applied through the centre of wheel to just turn it over the corner of block.

(ii) Reaction of block. **SPPU : Dec. 09, 6 Marks**

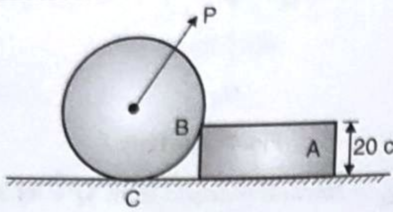


Fig. P. 8.9.13(a)

Soln. :

When the wheel is about to turn over the corner of the block, it loses contact with the ground surface, making normal reaction between the wheel and the ground is zero.

When the wheel is in limiting equilibrium. i.e. when it is about to turn over the block, three forces are acting on the wheel.

(i) Weight of the wheel, W = 1 kN = 1000 N

(ii) Force or pull applied to the wheel, P.

(iii) Reaction offered by the corner of the block R.

For equilibrium the above three forces must be concurrent.

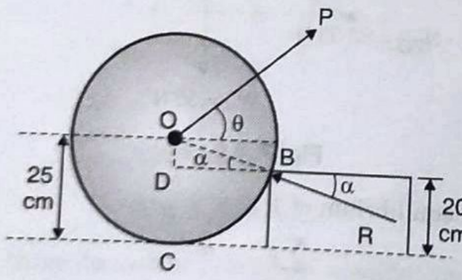


Fig. P. 8.9.13(b)

Let 'θ' be the direction of P w.r.t. horizontal.

'α' be the direction of R w.r.t. horizontal

Form ⊿ OBD,

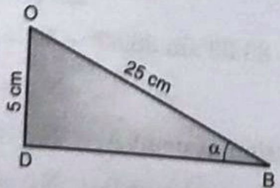


Fig. P. 8.9.13(c)

$$OD = 25 - 20 = 5 \text{ cm}$$

$$OB = 25 \text{ cm}$$

$$\sin \alpha = \left(\frac{5}{25}\right)$$

$$\therefore \quad \alpha = 11.54°$$

FBD of wheel

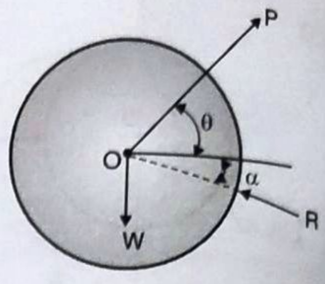


Fig. P. 8.9.13(d)

Considering all forces at 'O'

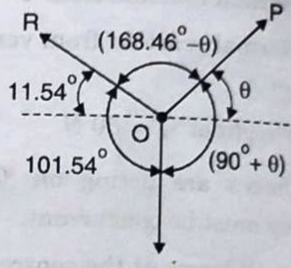


Fig. P. 8.9.13(e)

For three concurrent forces in equilibrium, using Lami's theorem ;

$$\frac{P}{\sin 101.54°} = \frac{R}{\sin (90° + \theta)} = \frac{1000}{\sin (168.46° - \theta)}$$

$$\therefore \quad P = \frac{1000 \cdot \sin 101.54°}{\sin (168.46° - \theta)} = \frac{979.78}{\sin (168.46° - \theta)}$$

For least pull, P the denominator sin(168.46° – θ) should be maximum.

$$\therefore \quad \sin (168.46° - \theta) = 1$$

$$168.46° - \theta = 90°$$

$$\boldsymbol{\theta = 78.46°}$$

∴ When θ = 78.46°, 'P' is least or minimum

$$\therefore \quad \text{Least pull, } P = \frac{979.78}{\sin (168.46° - 78.46°)}$$

$$= \mathbf{979.78\ N}$$

Reaction of block, $R = \dfrac{1000 \sin(90° + \theta)}{\sin(168.46° - \theta)}$

$= \dfrac{1000 \sin(90° + 78.46°)}{1}$

$= \mathbf{200.05\ N}$

) Least pull, $\mathbf{P = 979.78\ N}$ ...**Ans.**

i) Reaction of block,

$\mathbf{R = 200.05\ N,\ \alpha = 11.54°}$ ...**Ans.**

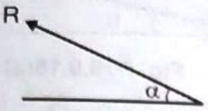


Fig. P. 8.9.13(f)

x. 8.9.14 : Two cylinders A and B of diameters 8 cm and 12 m respectively are held in equilibrium by separate strings as hown in Fig. P. 8.9.14(a). Cylinder B rests against vertical all. If weights of cylinder A and B are 20 N and 40N espectively, determine tension in strings and reactions at all oints of contacts. **SPPU : May 07, 7 Marks**

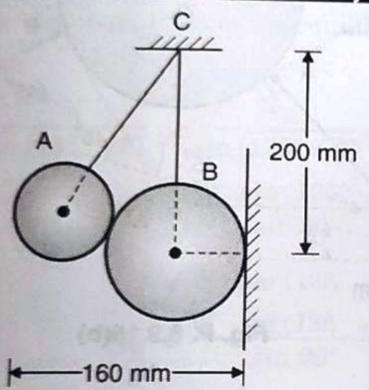


Fig. P. 8.9.14(a)

oln. :

et θ = Angle between the strings AB and AC

α = Angle made of AB w.r.t. horizontal

N_1 = Normal reaction from vertical plane

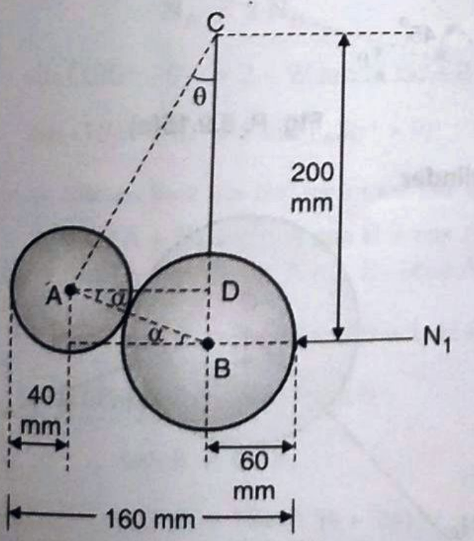


Fig. P. 8.9.14(b)

From Fig. P. 8.9.14(b)

$$AB = 40 + 60 = 100 \text{ mm}$$

$$AD = 160 - 40 - 60 = 60 \text{ mm}$$

From $\triangle$ ABD, $\cos\alpha = \dfrac{AD}{AB} = \dfrac{60}{100}$

$\therefore \quad \alpha = 53.13°$

$$\sin\alpha = \frac{BD}{AB}$$

$$\sin 53.13° = \frac{BD}{100}$$

$\therefore \quad BD = 80 \text{ mm}$

From $\triangle$ ACD, $\tan\theta = \dfrac{AD}{CD} = \dfrac{AD}{CB - BD}$

$$= \frac{60}{200 - 80}$$

$$= \frac{60}{120}$$

$\therefore \quad \theta = 26.56°$

The direction of normal reaction between A and B will be along the line joining centres of cylinder A and B i.e. 'α' w.r.t. horizontal.

FBD of cylinder 'A' :

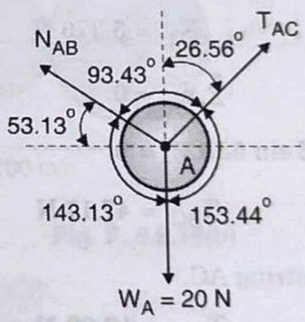


Fig. P. 8.9.14(c)

N_{AB} = Normal reaction between the cylinders A and B

W_A = Weight of cylinder A = 20 N

T_{AC} = Tension in the string AC

For three concurrent forces which are in equilibrium, using Lami's theorem :

$$\frac{T_{AC}}{\sin 143.13°} = \frac{N_{AB}}{\sin 153.44°} = \frac{20}{\sin 93.43°}$$

$$\therefore \quad T_{AC} = \frac{20 \cdot \sin 143.13°}{\sin 93.43°} = 12.02 \text{ N}$$

$$N_{AB} = \frac{20 \times \sin 153.44°}{\sin 93.43°} = 8.96 \text{ N}$$

FBD of cylinder 'B' :

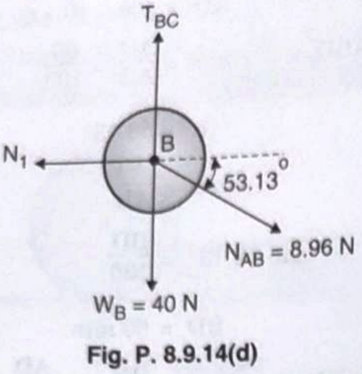


Fig. P. 8.9.14(d)

N_{AB} = Normal reaction between A and B = 8.96 N

W_B = Weight of cylinder B = 40 N

N_1 = Normal reaction from vertical wall

T_{BC} = Tension in the string BC

For equilibrium ;

$$\sum F_x = 0$$

$$8.96 \cos 53.13° - N_1 = 0$$

$$\therefore \quad N_1 = 5.376 \text{ N}$$

$$\sum F_y = 0$$

$$T_{BC} - 40 - 8.96 \sin 53.13° = 0$$

$$\therefore \quad T_{BC} = 47.17 \text{ N}$$

Tension in the string AC,

$$\mathbf{T_{AC} = 12.02 \text{ N}} \quad \text{...Ans.}$$

Tension in the string BC,

$$\mathbf{T_{BC} = 47.17 \text{ N}} \quad \text{...Ans.}$$

Normal reaction between A and B,

$$\mathbf{N_{AB} = 8.96 \text{ N}} \quad \text{...Ans.}$$

Normal reaction between B and wall, $\mathbf{N_1 = 5.376 \text{ N}}$ **...Ans.**

Ex. 8.9.15 : Find the angle of tilt θ with the horizontal so that the contact force at B will be one-half that at A for the smooth cylinder. Refer Fig. P. 8.9.15(a).

SPPU : May 10, 8 Marks, May 14, 6 Marks

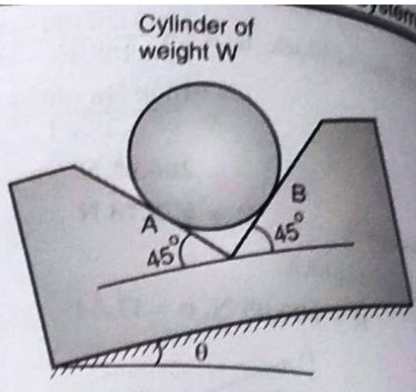


Fig. P. 8.9.15(a)

Soln. :

The contact forces at A and B are normal to planes.

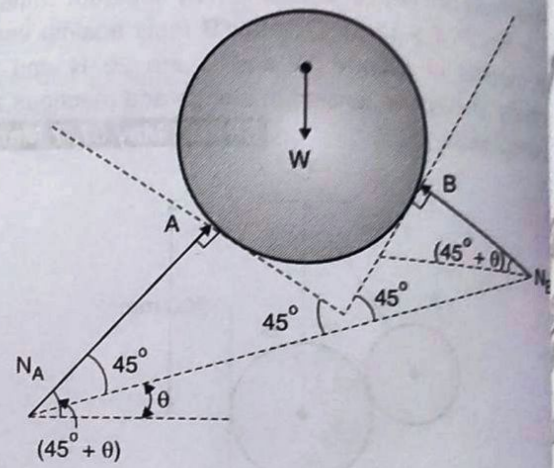


Fig. P. 8.9.15(b)

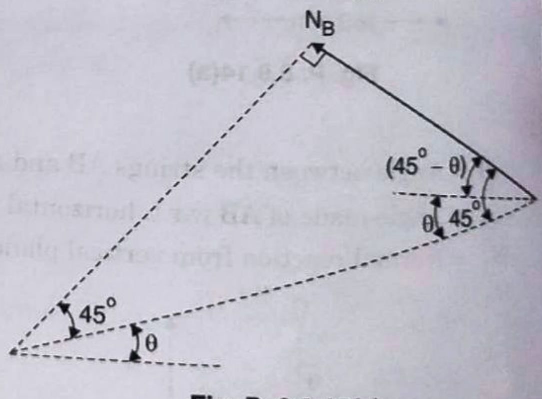


Fig. P. 8.9.15(c)

FBD of cylinder

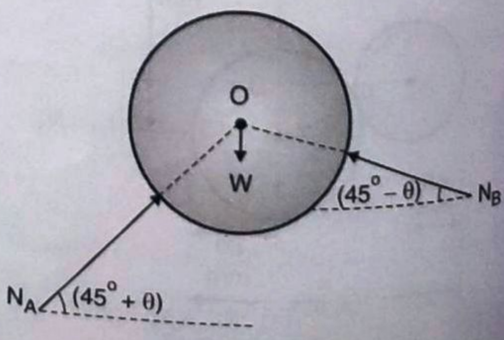


Fig. P. 8.9.15(d)

Three forces are acting on the cylinder :

) Weight of cylinder = W

i) Normal reaction at A = N_A

ii) Normal reaction at B = N_B

To maintain equilibrium, these three forces must e concurrent considering all forces at the centre of the ylinder.

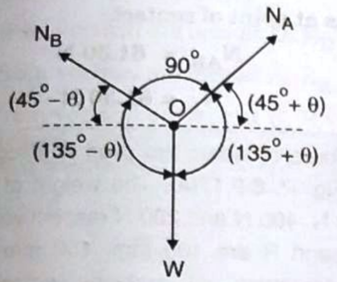


Fig. P. 8.9.15(e)

For three concurrent forces in equilibrium using ami's theorem

$$\frac{N_A}{\sin(135^\circ - \theta)} = \frac{N_B}{\sin(135^\circ + \theta)} = \frac{W}{\sin 90^\circ}$$

$$N_A = \frac{W \cdot \sin(135^\circ - \theta)}{\sin 90^\circ}$$

$$= W \cdot \sin(135^\circ - \theta)$$

$$N_B = \frac{W \cdot \sin(135^\circ + \theta)}{\sin 90^\circ}$$

$$= W \cdot \sin(135^\circ + \theta)$$

Given that the contact force at B is one-half that t 'A'.

$$N_B = \frac{1}{2} N_A$$

r

$$N_A = 2 N_B$$

$$W \cdot \sin(135^\circ - \theta) = 2 \cdot W \sin(135^\circ + \theta)$$

$$\sin(135^\circ - \theta) = 2 \sin(135^\circ + \theta)$$

in $135^\circ \cos\theta - \cos 130^\circ \sin\theta = 2 [\sin 135^\circ \cos\theta + \cos 135^\circ \sin\theta]$

$$\begin{bmatrix} \sin(A+B) = \sin A \cos B + \cos A \sin B \\ \sin(A-B) = \sin A \cos B - \cos A \sin B \end{bmatrix}$$

$$0.707 \cos\theta + 0.707 \sin\theta = 1.414 \cos\theta - 1.414 \sin\theta$$

$$2.121 \sin\theta = 0.707 \cos\theta$$

$$\tan\theta = 0.33$$

$$\theta = \mathbf{18.43^\circ}$$

Angle of tilt, θ = **18.43°** ...**Ans.**

Ex. 8.9.16 : Two spheres A and B of diameter 80 mm and 120 mm respectively are held in equilibrium by separate strings as shown in Fig. P. 8.9.16(a). Sphere 'B' rests against vertical wall. If masses of spheres A and B are 10 kg and 20 kg, determine the tension in the string and reactions at point of contact. **SPPU : Dec. 16, 6 Marks**

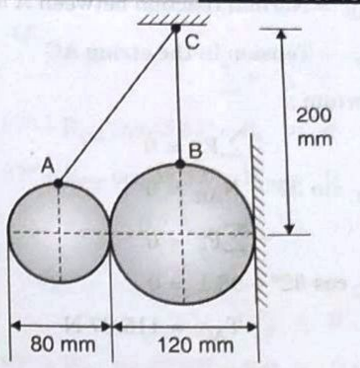


Fig. P. 8.9.16(a)

Soln. :

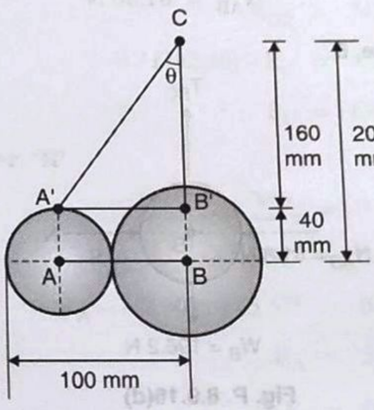


Fig. P. 8.9.16(b)

From ⊿ A'CB',

$$\tan\theta = \frac{A'B'}{CB'} = \left(\frac{100}{160}\right)$$

$$\therefore \quad \theta = 32^\circ$$

FBD of sphere 'A'

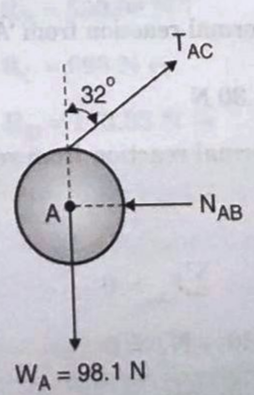


Fig. P. 8.9.16(c)

W_A = Weight of sphere A

$= 10 \text{ kg} = 10 \times 9.81$

$= 98.1 \text{ N}$

N_{AB} = Normal reaction between A and B

T_{AC} = Tension in the string AC

For equilibrium ;

$$\sum F_x = 0$$

$$T_{AC} \sin 32° - N_{AB} = 0 \quad ...(1)$$

$$\sum F_y = 0$$

$$T_{AC} \cos 32° - 98.1 = 0 \quad ...(2)$$

$$\therefore \quad T_{AC} = 115.67 \text{ N}$$

From Eq^n (1),

$$115.67 \sin 32° - N_{AB} = 0$$

$$\therefore \quad N_{AB} = 61.30 \text{ N}$$

FBD of sphere, B

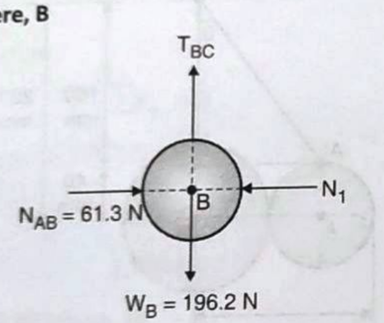


Fig. P. 8.9.16(d)

W_B = Weight of sphere, B

$= 20 \text{ kg} = 20 \times 9.81$

$= 196.2 \text{ N}$

T_{BC} = Tension in the string BC

N_{AB} = Normal reaction from 'A'

$= 61.30 \text{ N}$

N_1 = Normal reaction from vertical wall

For equilibrium ;

$$\sum F_x = 0$$

$$61.30 - N_1 = 0 \quad ...(3)$$

$$\therefore \quad N_1 = 61.30 \text{ N}$$

$$\sum F_y = 0$$

$$T_{BC} - 196.2 = 0$$

$$\therefore \quad T_{BC} = 196.2 \text{ N}$$

(i) Tension in the strings,

$$T_{AC} = 115.67 \text{ N} \quad ...Ans.$$

$$T_{AC} = 196.20 \text{ N} \quad ...Ans.$$

(ii) Reactions at point of contact,

$$N_{AB} = 61.30 \text{ N} \quad ...Ans.$$

$$N_1 = 61.30 \text{ N} \quad ...Ans.$$

Ex. 8.9.17 : Three cylinders are piled in a rectangular ditch as shown in Fig. P. 8.9.17(a). The weight of cylinder P, Q and R are 130 N, 400 N and 200 N respectively. The radii of cylinder P, Q and R are 100 mm, 150 mm and 125 mm respectively. Assuming all surfaces smooth, determine reactions at all points of contact 'A', 'B', 'C' and 'D'.

SPPU : Dec. 11, 7 Marks

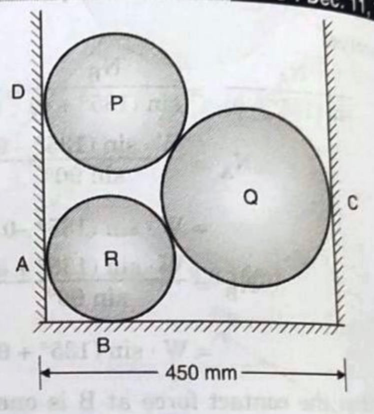


Fig. P. 8.9.17(a)

Soln. :

Step 1 : Geometry of the system

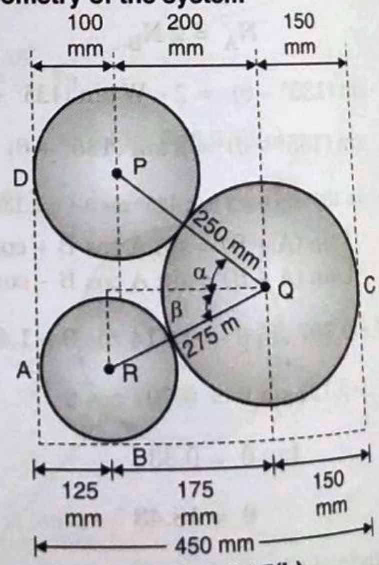


Fig. P. 8.9.17(b)

$$\cos \alpha = \left(\frac{200}{250}\right) \Rightarrow \alpha = 36.87°$$

$$\cos \beta = \left(\frac{175}{250}\right) \Rightarrow \beta = 45.57°$$

Step 2 : Forces acting in the system :

(i) Weights of cylinders P, Q and R

(ii) Reaction between P and Q, R_{PQ}

(iii) Reaction between Q and R, R_{PQ}

(iv) Reaction from vertical surface at D, R_D

(v) Reaction from vertical surface at A, R_A

(vi) Reaction from vertical surface at C, R_C

Step 3 : Free body diagrams :

FBD of cylinder 'P'

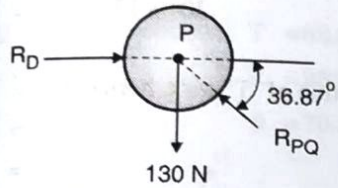


Fig. P. 8.9.17(c)

FBD of cylinder 'Q'

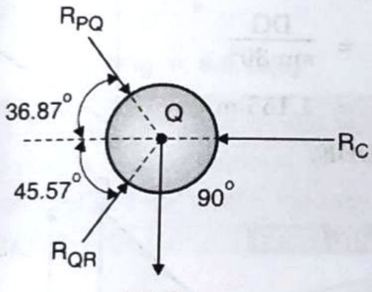


Fig. P. 8.9.17(d)

FBD of cylinder 'R'

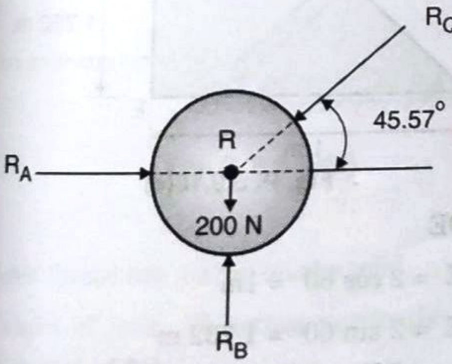


Fig. P. 8.9.17(e)

Step 4 : Equilibrium equations :

For cylinder 'P'

$$\sum F_x = 0$$

$$R_D - R_{PQ} \cos 36.87° = 0 \quad ...(1)$$

$$\sum F_y = 0$$

$$-130 + R_{PQ} \sin 36.87° = 0 \quad ...(2)$$

$$\therefore R_{PQ} = 216.67 \text{ N}$$

From Eq^n (1),

$$R_D = 173.33 \text{ N}$$

For cylinder 'Q'

$$\sum F_x = 0$$

$$R_{PQ} \cos 36.87° + R_{QR} \cos 45.57° - R_C = 0$$

$$216.67 \cos 36.87° + R_{QR} \cos 45.57° - R_C = 0$$

$$0.7\, R_{QR} - R_C = -173.33 \quad ...(3)$$

$$\sum F_y = 0$$

$$-R_{PQ} \sin 36.87° + R_{QR} \sin 45.57° - 400 = 0$$

$$-130 + 0.714\, R_{QR} - 400 = 0 \quad ...(4)$$

$$\therefore R_{QR} = 742.30 \text{ N}$$

From Eq^n (3),

$$0.7\,(742.30) - R_C = -173.33$$

$$\therefore R_C = 693 \text{ N}$$

For cylinder, 'R'

$$\sum F_x = 0$$

$$R_A - R_{QR} \cos 45.57° = 0$$

$$R_A - 742.30 \cos 45.57° = 0 \quad ...(5)$$

$$\therefore R_A = 519.64 \text{ N}$$

$$\sum F_y = 0$$

$$-R_{QR} \sin 45.57° + R_B = 0 \quad ...(6)$$

$$-742.3 \sin 45.57° + R_B = 0$$

$$R_B = 530.08 \text{ N}$$

(i) Reaction at A, $\mathbf{R_A = 519.64\ N \rightarrow}$ ...**Ans.**

(ii) Reaction at B, $\mathbf{R_B = 530.08\ N \uparrow}$ ...**Ans.**

(iii) Reaction at C, $\mathbf{R_C = 693\ N \leftarrow}$ ...**Ans.**

(iv) Reaction at D, $\mathbf{R_D = 173.33\ N \rightarrow}$ ...**Ans.**

Ex. 8.9.18 : The uniform 18 kg bar OA is held in position shown in Fig. P. 8.9.18(a) by the smooth pin at O and the cable AB. Determine the tension T in the cable and the magnitude and direction of the external pin reaction at O.

SPPU : May 14, Dec. 17, 6 Marks

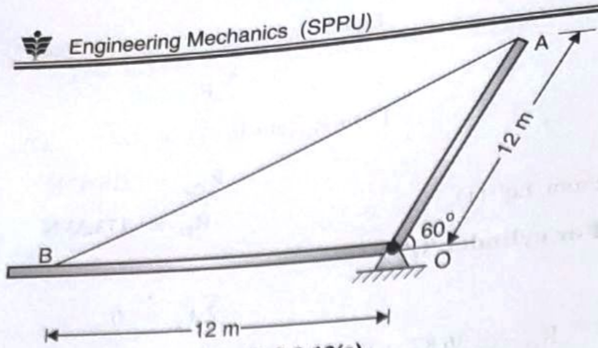


Fig. P. 8.9.18(a)

Soln. :

For FBD of bar OA, 3 forces are acting

(i) Weight of bar, W acting vertically down at the midpoint bar OA, $W = 18 \times 9.81 = 176.58$ N

(ii) Tension in the cable AB, T at A

(iii) Reaction at pin support, R at 'O'

When 3 forces are acting, to maintain equilibrium these three forces must be concurrent.

The direction of forces W and T are known. Hence the direction of reaction at 'O' should be such that the line of action of reaction must pass through the intersection point of W and T.

Let 'θ' be the direction of reaction at O w.r.t. horizontal

FBD of bar 'OA'

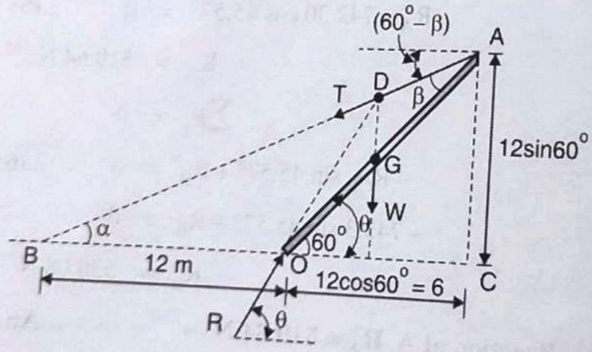


Fig. P. 8.9.18(b)

From ⊿ AOC,

$OC = 12 \cos 60° = 6$ m

$AC = 12 \sin 60° = 10.40$ m

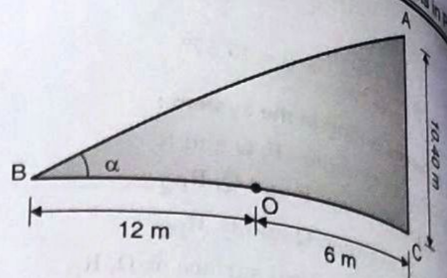
From ⊿ ABC,

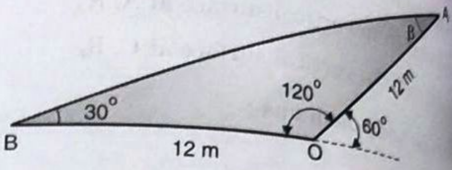
$$\tan \alpha = \frac{AC}{BC} = \frac{10.40}{(12+6)}$$

$$\tan \alpha = \frac{10.40}{18}$$

$\therefore \quad \alpha = 30°$

From Δ OAB,





$$30° + 120° + \beta = 180°$$

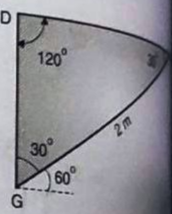
$\therefore \quad \beta = 30°$

$\therefore$ Direction of 'T' w.r.t. horizontal $= 60° - \beta$

$= 60° - 30°$

$= 30°$

From Δ ADG,



Using sine rule,

$$\frac{2}{\sin 120°} = \frac{DG}{\sin 30°}$$

$\therefore \quad DG = 1.155$ m

From Δ ODE,

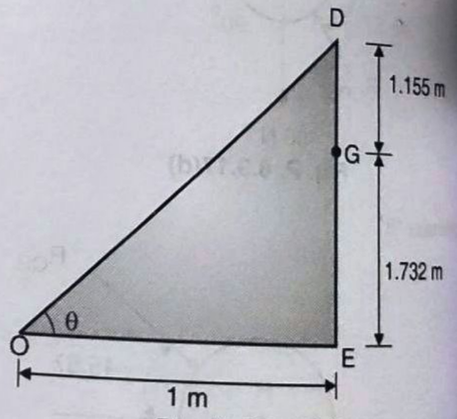


Fig. P. 8.9.18(c)

From Δ GOE

$OE = 2 \cos 60° = 1$m

$GE = 2 \sin 60° = 1.732$ m

$$\tan \theta = \frac{DE}{OE} = \frac{DG + GE}{OE}$$

$$= \frac{1.155 + 1.732}{1}$$

$= 2.887$

$\therefore \quad \theta = 70.89°$

Considering all three forces at point 'D'

For equilibrium of 3 concurrent forces, using Lami's theorem ;

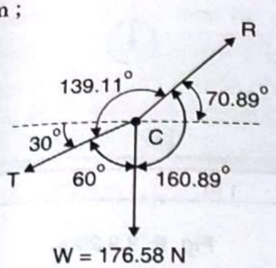


Fig. P. 8.9.18(d)

$$\frac{T}{\sin 160.89^\circ} = \frac{R}{\sin 60^\circ} = \frac{176.58}{\sin 139.11^\circ}$$

$\therefore$ $T = 88.31$ N

$R = 233.61$ N

Tension in the cable, **T = 88.31 N** **...Ans.**

Magnitude of reaction, **R = 233.61 N** **...Ans.**

Direction of reaction, **θ = 70.89°** **...Ans.**

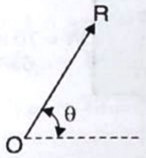


Fig. P. 8.9.18(e)

Ex. 8.9.19 : A joist of length 4 m and weighing 200 N is raised by pulling a rope as shown in Fig. P. 8.9.19(a). Determine the tension T induced in the rope and reaction at end A of joist. **SPPU : Dec. 09, 6 Marks**

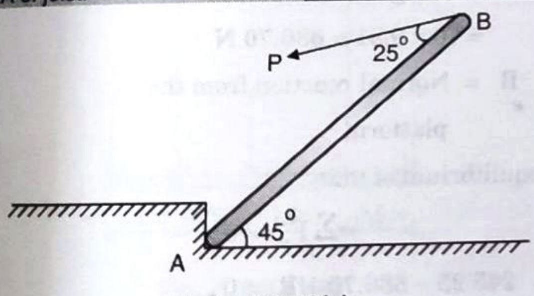


Fig. P. 8.9.19(a)

Soln. :

Three forces are acting on the joist.

(i) Weight of joist, W acting vertically down at the midpoint of AB

(ii) Tension in the string T at B

(iii) Reaction at A

When three forces are acting on the joist, to maintain equilibrium, these three forces must be concurrent.

$\therefore$ The line of action of reaction at A must pass through the intersection point of W and T.

Let 'θ' be the direction of reaction at A.

FBD of joist :

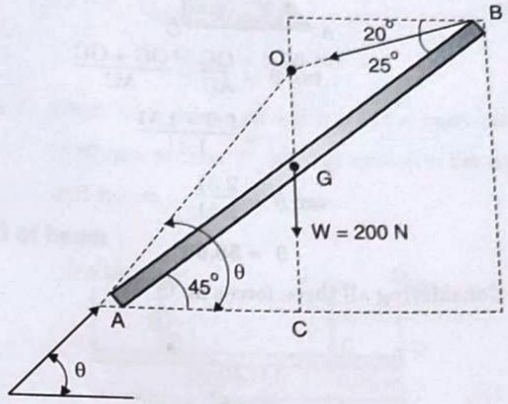


Fig. P. 8.9.19(b)

From $\triangle$ OGB,

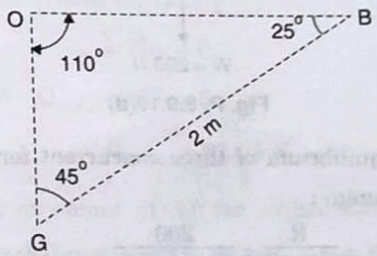
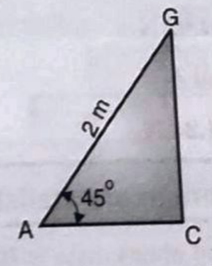


Fig. P. 8.9.19(c)

$$\frac{2}{\sin 110^\circ} = \frac{OG}{\sin 25^\circ}$$

$\therefore$ $OG = 0.9$ m

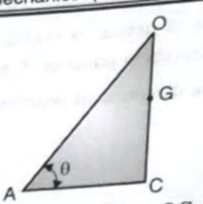
From $\triangle$ AGC



$AC = 2 \cos 45^\circ = 1.41$ m

$GC = 2 \sin 45^\circ = 1.42$ m

From $\triangle$ OAC,



$$\tan\theta = \frac{OC}{AC} = \frac{OG + GC}{AC}$$

$$= \frac{0.9 + 1.41}{1.41}$$

$$\tan\theta = \frac{2.31}{1.41}$$

$$\therefore \quad \theta = 58.60^\circ$$

Considering all three forces at 'O'

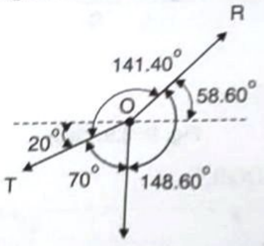


Fig. P. 8.9.19(d)

For equilibrium of three concurrent forces, using Lami's theorem :

$$\frac{T}{\sin 148.60^\circ} = \frac{R}{\sin 70^\circ} = \frac{200}{\sin 141.40^\circ}$$

$$\therefore \quad T = \frac{200 \times \sin 148.60^\circ}{\sin 141.40^\circ} = 167.02 \text{ N}$$

$$R = \frac{200 \times \sin 70^\circ}{\sin 141.40^\circ} = 301.24 \text{ N}$$

∴ Tension in the rope,

$$T = 167.02 \text{ N}$$ ...Ans.

Reaction at end A,

$$R = 301.24 \text{ N}$$ ...Ans.

Type 2 : Equilibrium of Parallel Force System

Ex. 8.9.20 : A person whose mass is 70 kg, represented by 'M', holds 25 kg mass as shown in Fig. P. 8.9.20(a). The pulley is assumed frictionless. The platform on which the person is standing is suspended by two ropes at 'A' and two ropes at 'B'. What is the tension in one rope at points A and B?

SPPU : Dec. 10, 8 Marks

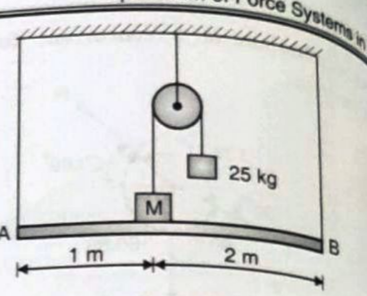


Fig. P. 8.9.20(a)

Soln. :

As pulley is friction less, the tension in the rope passing over the pulley remains same on both sides.

FBD of man

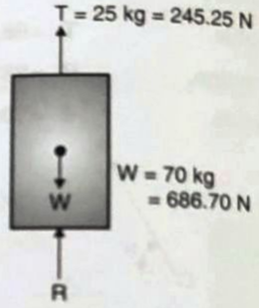


Fig. P. 8.9.20(b)

T = Tension in the rope

$= 25 \times 9.81 = \mathbf{245.25\ N}$

W = Weight of the person

$= 70 \times 9.81 = \mathbf{686.70\ N}$

R = Normal reaction from the platform

For equilibrium of man,

$$\Sigma F_y = 0$$

$$245.25 - 686.70 + R = 0$$

$$\therefore \quad R = 441.45 \text{ N}$$

FBD of platform :

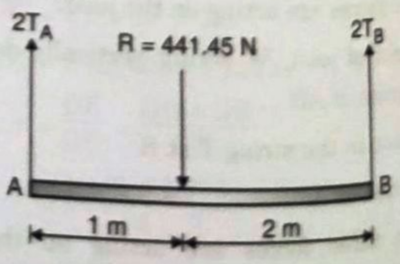


Fig. P. 8.9.20(c)

T_A = Tension in the rope at A

T_B = Tension in the rope at B

R = Normal reaction from the person

Note : (i) There are 2 ropes at A and 2 ropes at B.

(ii) The magnitude of normal reaction between the person and the platform is same but opposite in direction.

For equilibrium of platform ;

$$\Sigma F_y = 0$$

$$2T_A + 2T_B - 441.45 = 0$$

$$\therefore \quad T_A + T_B = 220.725 \text{ N} \quad \ldots (1)$$

Taking moments at 'A',

$$\Sigma M_A = 0$$

$$-441.45 \times 1 + 2\,T_B \times 3 = 0$$

$$\therefore \quad T_B = 73.575 \text{ N}$$

From Eq^n (1), $\quad T_A = 147.15$ N

$\therefore$ Tension in one rope at A and B are :

$$T_A = 147.15 \text{ N} \quad \ldots \text{Ans.}$$

$$T_B = 73.575 \text{ N} \quad \ldots \text{Ans.}$$

Ex. 8.9.21 : Three loads are applied as shown in Fig. P. 8.9.21(a) to a light beam supported by cables attached at B and C. Neglecting the weight of beam, determine the range of values of 'Q' for which neither cable becomes slack when P = 0. **SPPU : Dec. 12, 7 Marks**

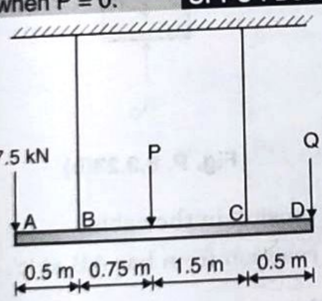


Fig. P. 8.9.21(a)

Soln. :

Case 1 : When 'Q' is minimum and P = 0 because of 7.5 kN, the beam tends to tilt at point B, making tension in the cable at C is zero.

FBD of beam

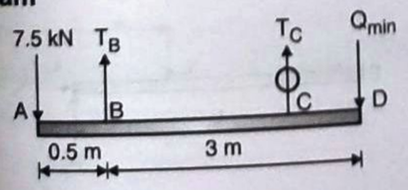


Fig. P. 8.9.21(b)

The beam is about to tilt at B, for limiting equilibrium, taking moments at 'B'.

$$\Sigma M_B = 0$$

$$7.5 \times 0.5 - Q_{min} \times 3 = 0$$

$$\therefore \quad Q_{min} = 1.25 \text{ kN}$$

Case 2 : When 'Q' is maximum and P = 0, the beam tends to tilt now at point 'C', making tension in the cable at B is zero.

FBD of beam

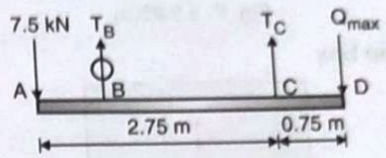


Fig. P. 8.9.21(c)

For limiting equilibrium, taking moments at 'C'

$$\Sigma M_C = 0$$

$$7.5 \times 2.75 - Q_{max} \times 0.75 = 0$$

$$\therefore \quad Q_{max} = 27.5 \text{ kN}$$

$\therefore$ Range of values of 'Q' for which neither cable becomes slack (tension not to be zero) when P = 0 are :

$$1.25 \text{ kN} < Q < 27.5 \text{ kN} \quad \ldots \text{Ans.}$$

Ex. 8.9.22 : Three identical boxes, each having length l and weigh W are placed as shown in Fig. P. 8.9.22(a). Find out the maximum possible distance 'm' through which the top box can extended out from the bottom so that there is no possibility of topling the stack. **SPPU : Dec. 09, 6 Marks**

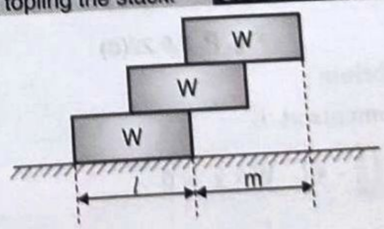


Fig. P. 8.9.22(a)

Soln. :

The C.G. of the top box where the whole weight 'W' acts will be at the centre of the box. i.e. at a distance $\left(\frac{l}{2}\right)$ from either end.

Hence maximum extension of top box over middle box is $\left(\frac{l}{2}\right)$.

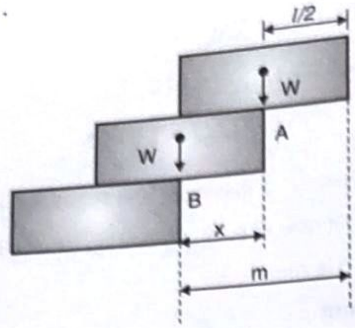


Fig. P. 8.9.22(b)

FBD of top box

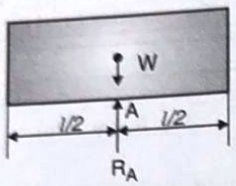


Fig. P. 8.9.22(c)

For equilibrium ;

$$\Sigma F_y = 0$$
$$-W + R_A = 0$$
$$\therefore \ R_A = W$$

FBD of middle box

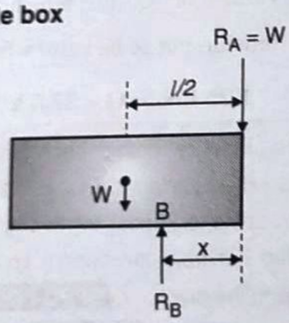


Fig. P. 8.9.22(d)

For equilibrium ;

Taking moments at 'B'

$$+W\left(\frac{l}{2} - x\right) - W \times x = 0$$
$$\therefore \ \frac{l}{2} - x = x$$
$$2x = \frac{l}{2}$$
$$x = \frac{l}{4}$$

$\therefore$ Maximum possible distance,

$$m = x + \frac{l}{2}$$
$$= \frac{l}{4} + \frac{l}{2}$$
$$= \frac{3l}{4} = 0.75\,l$$

Ex. 8.9.23 : A weight w rest on the bar AB as show... Fig. P. 8.9.23(a) The cable connecting W and B passes ... frictionless pulley. If bar AB has negligible weight, show ... the reaction at A is $W\frac{(L-a)}{(L+a)}$

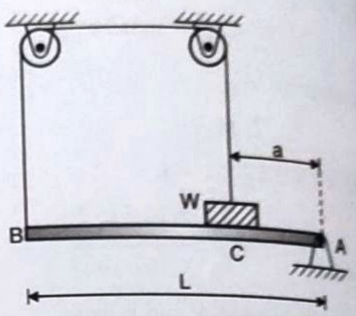


Fig. P. 8.9.23(a)

Soln. :

FBD of weight 'W'

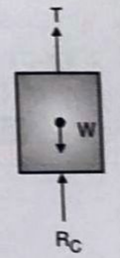


Fig. P. 8.9.23(b)

Let 'T' be the tension in the cable.

R_C = Normal reaction from bar AB at 'c'.

For equilibrium ;

$$\Sigma F_y = 0$$
$$T - W + R_C = 0$$
$$\therefore \ R_C = W - T$$

FBD of bar 'AB'

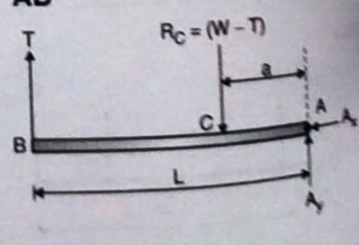


Fig. P. 8.9.23(c)

'A' is hinge support

R_C = Reaction from weight W

$= (W - T)$

A_x = x-component of reaction at A

A_y = y-component of reaction at A

For equilibrium ;

$$\Sigma F_x = 0$$

$$A_x = 0 \quad \dots (2)$$

$$\Sigma F_y = 0$$

$$T - R_C + A_y = 0$$

$$A_y = R_C - T$$

$$\therefore \quad A_y = W - T - T = W - 2T \quad \dots (3)$$

Taking moments at 'A',

$$\Sigma M_A = 0$$

$$-T \times L + R_C \times a = 0 \quad \dots (4)$$

$$\therefore \quad R_C \times a = T \times L$$

$$(W - T)a = T.L$$

$$Wa - Ta = TL$$

$$Wa = TL + Ta = T(L + a)$$

$$\therefore \quad T = \frac{Wa}{L + a}$$

Putting in Eq^n (3),

$$A_y = W - 2\left(\frac{Wa}{L + a}\right)$$

$$= \frac{W(L + a) - 2Wa}{(L + a)}$$

$$= \frac{WL + Wa - 2Wa}{L + a}$$

$$= \frac{WL - Wa}{L + a} = \frac{W(L - a)}{(L + a)}$$

$\therefore$ Reaction at A, $\quad R_A = \sqrt{A_x^2 + A_y^2} = A_y$

$$\therefore \quad R_A = W\left[\frac{L - a}{L + a}\right] \quad \dots \textbf{Ans.}$$

Ex. 8.9.24 : For the given loading of the beam AB, determine the range of values of the mass 'm' of the crate for which the system will be in equilibrium, knowing that the maximum allowable value of the reactions at each support is 2.5 kN and the reaction at E must be directed downward. Refer Fig. P. 8.9.24(a). **SPPU : May 11, 6 Marks**

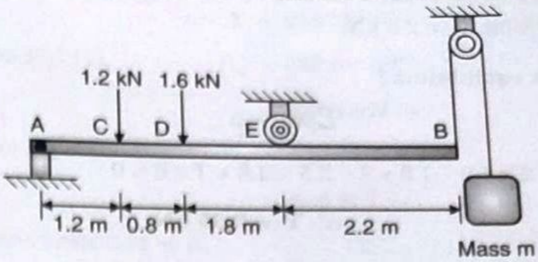


Fig. P. 8.9.24(a)

Soln. :

Step 1 : Forces acting on the beam AB :

(i) Point loads 1.2 kN and 1.6 kN

(ii) Tension in the string 'T' at B

$$T = mg$$

(iii) Support reaction at hinge A,

$$R_A = A_y \quad (\because A_x = 0)$$

(iv) Support reaction at roller E,

R_E acting downwards

Step 2 : FBD of the beam 'AB'

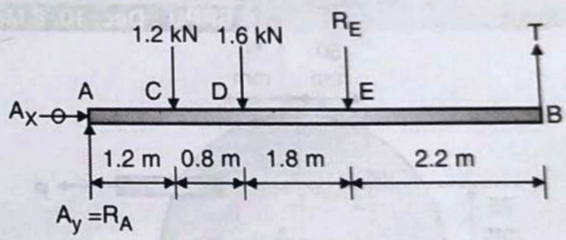


Fig. P. 8.9.24(b)

Step 3 : Equilibrium equations.

Given that the maximum allowable value of the reactions at each support is 2.5 kN.

Taking R_A = 2.5 kN

For equilibrium;

$$\Sigma M_E = 0$$

$$-R_A \times 3.8 + 1.2 \times 2.6 + 1.6 \times 1.8 + T \times 2.2 = 0$$

$$-2.5 \times 3.8 + 3.12 + 2.88 + 2.2T = 0$$

$$\therefore \mathbf{T = 1.6\ kN}$$

$$T = mg$$

$$1.6 \times 10^3 = m \times 9.81$$

$$\therefore \text{Mass, } \mathbf{m = 163.1\ kg}$$

Taking $R_E = 2.5$ kN,

For equilibrium ;

$$\Sigma M_A = 0$$

$$-1.2 \times 1.2 - 1.6 \times 2 - 2.5 \times 3.8 + T \times 6 = 0$$

$$\therefore \quad \mathbf{T = 2.35\ kN}$$

$$T = mg$$

$$2.35 \times 10^3 = m \times 9.81$$

$$\therefore \text{ Mass, } \mathbf{m = 240.23\ kg}$$

Step 4 :

The range of values of mass 'm' for the maximum allowable value of reaction at each support are :

$$\mathbf{163.10\ kg \le m \le 240.23\ kg} \quad \text{...Ans.}$$

Type 3 : Equilibrium of General Force System

Ex. 8.9.25 : Horizontal and vertical links are hinged to a wheel and force 'P' is applied to the link as shown in Fig. P. 8.9.25(a)Determine value of 'P' and reaction at 'A' for equilibrium. **SPPU : Dec. 10, 8 Marks**

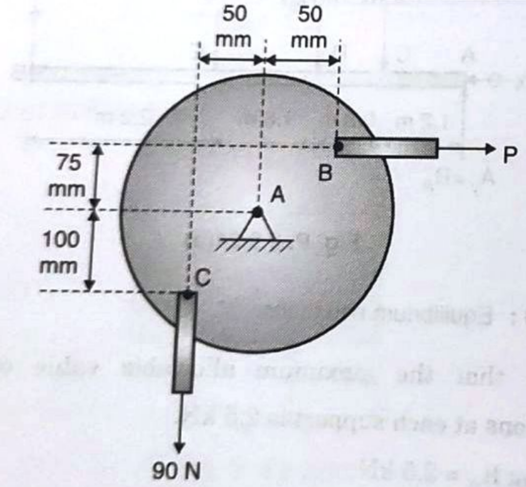


Fig. P. 8.9.25(a)

Soln. :

Step 1 : Forces acting on the wheel :

(i) Force in the link at C = 90 N

(ii) Force in the link at B = P

(iii) Components of reaction at A = A_x and A_y

Step 2 : Free body diagram of wheel :

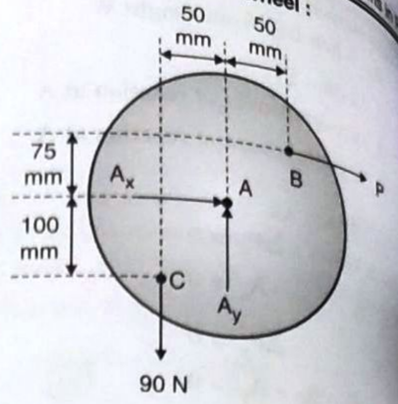


Fig. P. 8.9.25(b)

Step 3 : Equilibrium equations :

For equilibrium of wheel,

$$\Sigma F_x = 0$$

$$P + A_x = 0$$

$$\Sigma F_y = 0$$

$$A_y - 90 = 0$$

$$\therefore \quad \mathbf{A_y = 90\ N \uparrow}$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$-P \times 75 + 90 \times 50 = 0$$

$$\therefore \quad \mathbf{P = 60\ N}$$

From Eq^n (1), $\quad 60 + A_x = 0$

$$\therefore \quad A_x = -60\ \text{N} = \mathbf{60\ N \leftarrow}$$

Reaction at A, $\quad R_A = \sqrt{A_x^2 + A_y^2}$

$$= \sqrt{(60)^2 + (90)^2}$$

$$= \mathbf{108.17\ N}$$

Direction, $\quad \theta_A = \tan^{-1}\left|\frac{A_y}{A_x}\right| = \tan^{-1}\left(\frac{90}{60}\right)$

$$= \mathbf{56.31^\circ} \text{ w.r.t. x-axis}$$

...Ans.

(i) Value of P $= 60$ N

(ii) Reaction at A, $R_A = 108.17$ N

$\theta_A = 56.31^\circ$ w.r.t. x-axis...Ans.

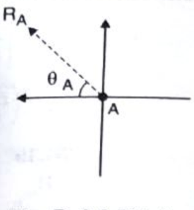


Fig. P. 8.9.25(c)

Ex. 8.9.26 : The rail AB of foundry crane is horizontal and is 20 m long. End A is hinged to vertical wall and end B is tied to a tie rod BC as shown in Fig. P. 8.9.26(a). Find the tension in the tie rod and reaction developed at end A for the loading applied. **SPPU : Dec. 09, 6 Marks**

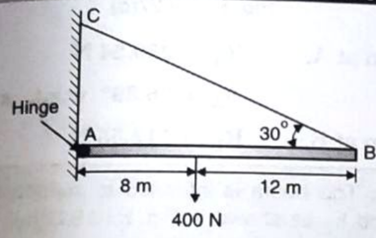


Fig. P. 8.9.26(a)

Soln. :

FBD of rail 'AB'

Let 'T' be the tension in the tie rod.

A_x and A_y = Horizontal and vertical components of reaction at hinge.

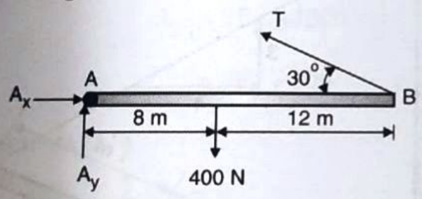


Fig. P. 8.9.26(b)

For equilibrium ;

$$\Sigma F_x = 0$$

$$A_x - T\cos 30° = 0$$

$$A_x = T\cos 30° \qquad \text{... (1)}$$

$$\Sigma F_y = 0$$

$$A_y - 400 + T\sin 30° = 0$$

$$\therefore \quad A_y = 400 - 0.5\,T \qquad \text{... (2)}$$

Taking moments at 'A',

$$\Sigma M_A = 0$$

$$-400 \times 8 + T\sin 30° \times 20 = 0 \qquad \text{... (3)}$$

$$\therefore \quad \mathbf{T = 320\ N}$$

From Eq^n (1), $\quad A_x = 320\cos 30°$

$$= \mathbf{277.13\ N \rightarrow}$$

From Eq^n (2), $\quad A_y = 400 - 0.5 \times 320$

$$= \mathbf{240\ N \uparrow}$$

Support reaction at A,

$$R_A = \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{(277.13)^2 + (240)^2}$$

$$= \mathbf{366.61\ N}$$

Direction, $\quad \theta_A = \tan^{-1}\left|\frac{A_y}{A_x}\right|$

$$= \tan^{-1}\left(\frac{240}{277.13}\right)$$

$$= \mathbf{40.89°\ \ w.r.t.\ x}$$

(i) Tension in the tie rod,

$$\mathbf{T = 320\ N} \qquad \textbf{...Ans.}$$

(ii) Reaction at end A,

$$\mathbf{R_A = 366.61\ N}$$

$$\mathbf{\theta_A = 40.89°\ w.r.t.\ x} \qquad \textbf{...Ans.}$$

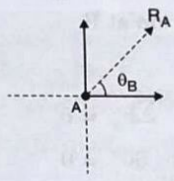


Fig. P. 8.9.26(c)

Ex. 8.9.27 : Determine reactions at support 'A' and 'B' for the bracket ACB supporting 330 N force as shown in Fig. P. 8.9.27(a). **SPPU : Dec. 09, 6 Marks**

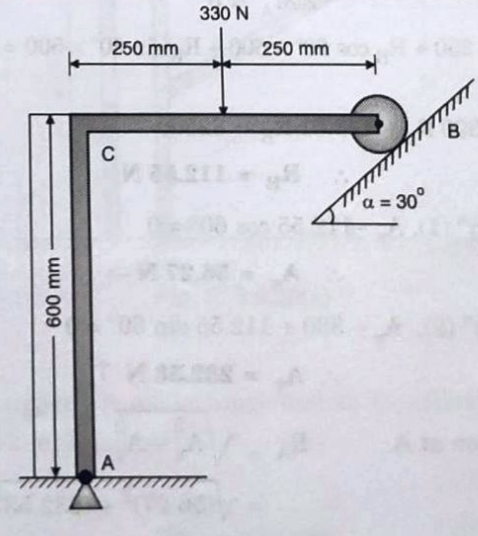


Fig. P. 8.9.27(a)

Soln. :

'A' is hinge support and 'B' is roller support.

Reaction at 'B' is perpendicular to the inclined plane.

FBD of bracket 'ACB'

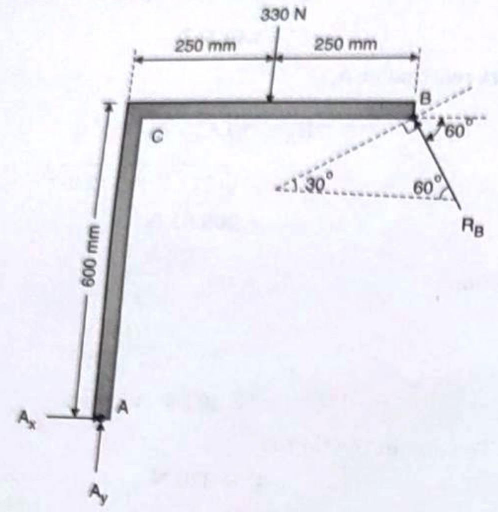


Fig. P. 8.9.27(b)

A_x and A_y = x and y components of reaction at A.

R_B = Reaction at B

For equilibrium ;

$$\Sigma F_x = 0$$

$$A_x - R_B \cos 60° = 0 \quad \dots (1)$$

$$\Sigma F_y = 0$$

$$A_y - 330 + R_B \sin 60° = 0 \quad \dots (2)$$

Taking moments at 'A',

$$\Sigma M_A = 0$$

$$-330 \times 250 + R_B \cos 60° \times 600 + R_B \sin 60° \times 500 = 0$$

$$\therefore 300\, R_B + 433.01\, R_B = 82500 \quad \dots (3)$$

$$\therefore \mathbf{R_B = 112.55\ N}$$

From Eq^n (1), $A_x - 112.55 \cos 60° = 0$

$$\therefore \mathbf{A_x = 56.27\ N \rightarrow}$$

From Eq^n (2), $A_y - 330 + 112.55 \sin 60° = 0$

$$\therefore \mathbf{A_y = 232.53\ N \uparrow}$$

$\therefore$ Reaction at A,

$$R_A = \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{(56.27)^2 + (232.53)^2}$$

$$= \mathbf{239.24\ N}$$

Direction,

$$\theta_A = \tan^{-1}\left|\frac{A_y}{A_x}\right|$$

$$= \tan^{-1}\left(\frac{232.53}{56.27}\right)$$

$$= \mathbf{76.39°\ w.r.t.\ x}$$

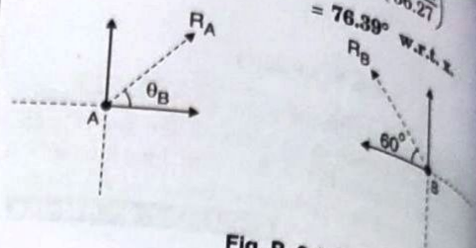


Fig. P. 8.9.27(c)

(i) Reaction at A, $\mathbf{R_A = 239.24\ N}$

$\mathbf{\theta_A = 76.39°\ w.r.t.\ x}$

(ii) Reaction at B, $\mathbf{R_B = 112.55\ N}$

Ex. 8.9.28 : The boom is intended to support two vertical loads, F_1 and F_2 as shown in Fig. P. 8.9.28(a). If the cable CB can sustain a maximum load of 1500 N before it fails, determine the critical loads F_1 and F_2 if $F_1 = 2F_2$. Also determine the reaction at A :

SPPU : May 15, May 16

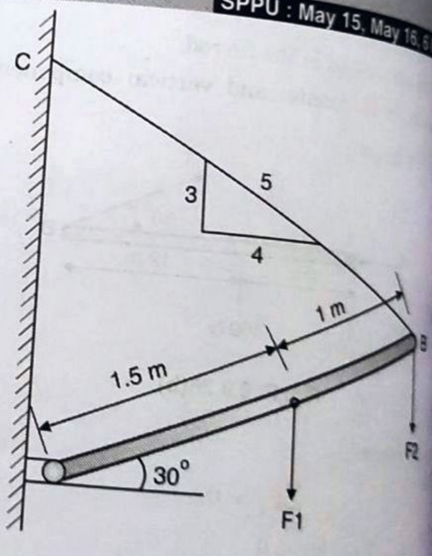


Fig. P. 8.9.28(a)

Soln. :

'A' is hinge support.

Tension in the cable, T = 1500 N

FBD of boom 'AB'

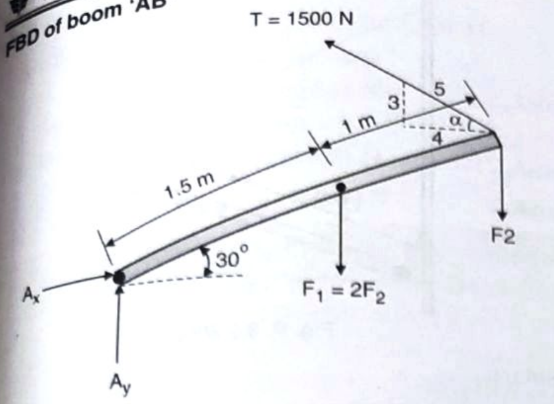


Fig. P. 8.9.28(b)

$\sin \alpha = \frac{3}{5}$ and $\cos \alpha = \frac{4}{5}$

For equilibrium ;

$$\Sigma F_x = 0$$

$$A_x - 1500 \cos \alpha = 0 \quad \dots (1)$$

$$A_x = 1500 \times \frac{4}{5} = \mathbf{1200\ N} \rightarrow$$

$$\Sigma F_y = 0$$

$$A_y - 2F_2 - F_2 + 1500 \sin \alpha = 0$$

$$A_y = 3F_2 + 1500 \times \frac{3}{5}$$

$$= 3F_2 + 900 \quad \dots (2)$$

Taking moments at 'A',

$$\Sigma M_A = 0$$

$$-2F_2 \times 1.5 \cos 30° - F_2 \times 2.5 \cos 30° + 1500 \cos \alpha$$

$$\times 2.5 \sin 30° + 1500 \sin \alpha \times 2.5 \cos 30° = 0 \quad \dots (3)$$

$$-2F_2 \times 1.5 \cos 30° - F_2 \times 2.5 \cos 30° + 1500 \times \frac{4}{5}$$

$$\times 2.5 \sin 30° + 1500 \frac{3}{5} \times 2.5 \cos 30° = 0$$

$$-2.598 F_2 - 2.165 F_2 + 1500 \times \frac{4}{5} \times 1.25 + 1500 \times \frac{3}{5} \times 2.16 = 0$$

$$4.763 F_2 = 3448.557$$

$$\therefore \quad \mathbf{F_2 = 724.03\ N}$$

$$F_1 = 2F_2 = 2 \times 724.03 = \mathbf{1448.06\ N}$$

From Eq^n (2),

$$A_y = 3 \times 724.03 + 900 = \mathbf{3072.09\ N} \uparrow$$

$\therefore$ Reaction at A,

$$R_A = \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{(1200)^2 + (3072.09)^2}$$

$$= \mathbf{3298.14\ N}$$

$$\text{Direction}, \theta_A = \tan^{-1}\left|\frac{A_y}{A_x}\right| = \tan^{-1}\left(\frac{3072.09}{1200}\right)$$

$$= 68.66° \text{ w.r.t. x}$$

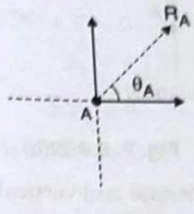


Fig. P. 8.9.28(c)

(i) Critical loads, $\mathbf{F_1 = 1448.06\ N}$ ...Ans.

$\mathbf{F_2 = 724.03\ N}$ ...Ans.

(ii) Reaction at A $\mathbf{R_A = 3298.14\ N}$ ...Ans.

$\mathbf{Q_A = 68.66° \text{ w.r.t. x}}$ ...Ans.

Ex. 8.9.29 : The wall crane is supported by smooth collar at B and pin at A as shown in Fig. P. 8.9.29(a). If the vertical component of reaction at A is 10 kN, determine force P, normal reaction at B and tangential component of reaction at 'A'.

SPPU : May 17, 6 Marks

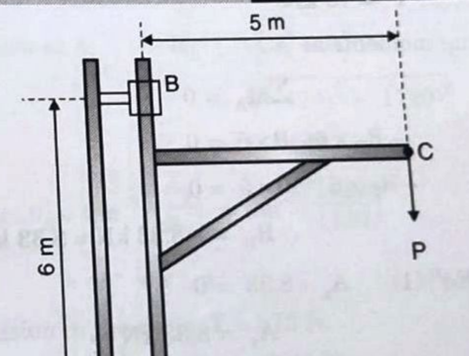


Fig. P. 8.9.29(a)

Soln. :

At 'A' support is pin or hinge and at 'B', collar support acts as simple support.

FBD of the crane

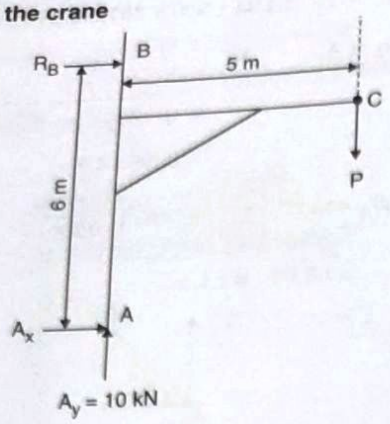


Fig. P. 8.9.29(b)

A_x and A_y = Horizontal and vertical components of reaction at 'A'.

R_B = Reaction offered by collar at B

Given, $A_y = 10$ kN

For equilibrium of crane,

$$\Sigma F_x = 0$$

$$A_x - R_B = 0 \quad \dots (1)$$

$$\Sigma F_y = 0$$

$$A_y - P = 0 \quad \dots (2)$$

$$10 - P = 0$$

$$\therefore \ \mathbf{P = 10\ kN}$$

Taking moments at 'A',

$$\Sigma M_A = 0$$

$$-R_B \times 6 - P \times 5 = 0 \quad \dots (3)$$

$$-R_B \times 6 - 10 \times 5 = 0$$

$$\therefore R_B = -8.33 \text{ kN} = \mathbf{8.33\ kN} \leftarrow$$

From Eq^n (1), $A_x - 8.33 = 0$

$$A_x = 8.33 \text{ kN} \rightarrow$$

(i) Force, $\mathbf{P = 10\ kN}$ ...Ans.

(i) Normal reaction at B, $\mathbf{R_B = 8.33\ kN} \leftarrow$ ...Ans.

(iii) Tangential component of reaction at A,

i.e., $\mathbf{A_x = 8.33\ kN} \rightarrow$ ...Ans.

Ex. 8.9.30 : The boom supports the two vertical loads $P_1 = 800$ N and $P_2 = 350$ N as shown in Fig. P. 8.9.30(a). Determine the tension in cable BC and component of reaction at A. **SPPU : May 13, 6 Marks**

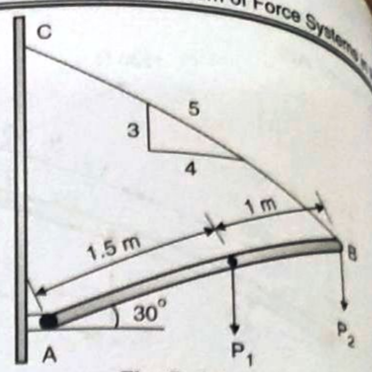


Fig. P. 8.9.30(a)

Soln. :

'A' is hinge support.

FBD of boom AB :

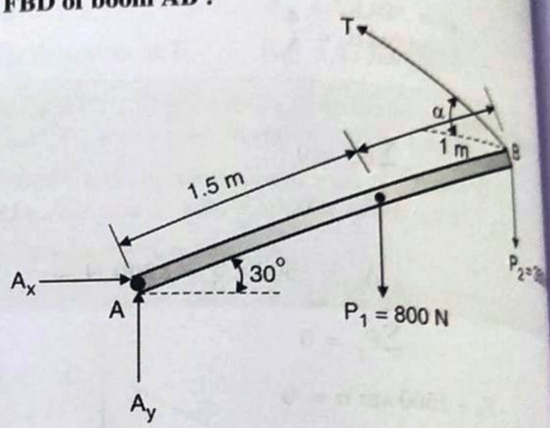


Fig. P. 8.9.30(b)

Let 'T' be the tension in the cable BC.

'α' be the direction of 'T' w.r.t. horizontal

$$\therefore \alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$$

For equilibrium of boom AB,

$$\Sigma F_x = 0$$

$$A_x - T\cos\alpha = 0$$

$$A_x = T\cos 36.87° = 0.8\,T \ \dots (1)$$

$$\Sigma F_y = 0$$

$$A_y - 800 - 350 + T\sin 36.87° = 0$$

$$A_y + 0.6T = 1150$$

Taking moments at 'A',

$$\Sigma M_A = 0$$

$$-800 \times 1.5\cos 30° - 350 \times 2.5\cos 30° + T\cos 36.87° \times 2.5\sin 60° + T\sin 36.87° \times 2.5\cos 60° = 0$$

$$\therefore 1.732T + 0.75T = 1797$$

$$\therefore \mathbf{T = 724.01\ N}$$

From Eq^n (1), $A_x = 0.8 \times 724.01$

$= 579.21 \text{ N} \rightarrow$

From Eq^n (2), $A_y = 1150 - 0.6 \times 724.01$

$= 715.59 \text{ N} \uparrow$

(i) Tension in cable BC, **T = 724.01 N** ...Ans.

(ii) Components of reaction at A,

$\mathbf{A_x = 579.21 \text{ N} \rightarrow}$ ...Ans.

and $\mathbf{A_y = 715.59 \text{ N} \uparrow}$ ...Ans.

Ex. 8.9.31 : A force of magnitude 280 N is applied to member ABCD, which is supported by a frictionless pin at A and by the cable CED. The cable passes over a small and smooth pulley at E. If a = 60 mm, determine the tension in the cable and the reaction at 'A'. Refer Fig. P. 8.9.31(a).

SPPU : Nov. 08, 8 Marks

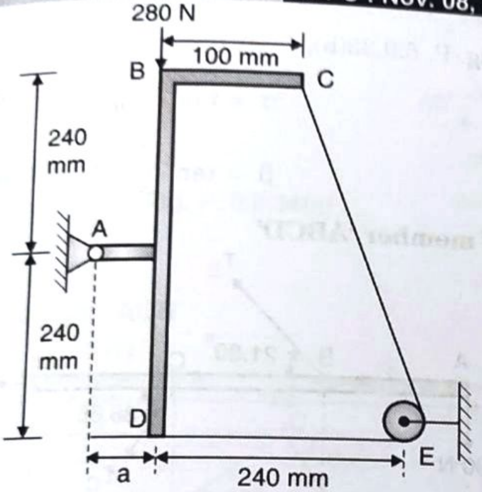


Fig. P. 8.9.31(a)

Soln. :

The tension in the cable remains same at C and D as it is passing over a smooth pulley.

FBD of member 'ABCD'

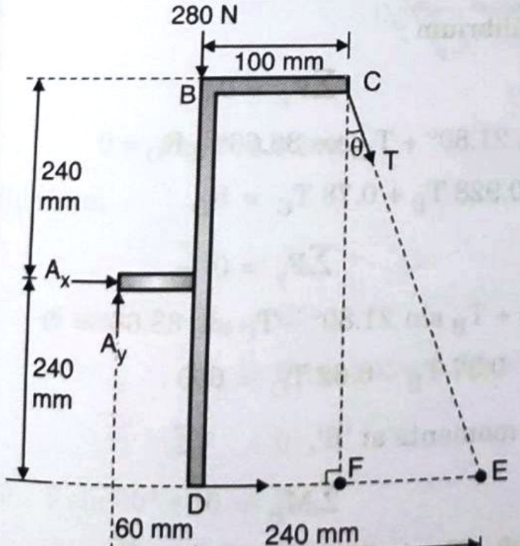


Fig. P. 8.9.31(b)

Let 'T' be the tension in the cable CED.

'θ' be the direction of T at 'C' w.r.t. vertical.

From Δ CFE, $\tan\theta = \frac{FE}{DF}$

$\tan\theta = \left(\frac{140}{480}\right)$

$\theta = \mathbf{16.26°}$

Considering equilibrium of member

$\Sigma F_x = 0$

$A_x + T \sin 16.26° + T = 0$

$A_x = -1.28\,T$ ... (1)

$\Sigma F_y = 0$

$A_y - 280 - T\cos 16.26° = 0$

$A_y = 280 + 0.96\,T$ ... (2)

Taking moments at 'A',

$\Sigma M_A = 0$

$-280 \times 60 - T\cos 16.26° \times 160 - T\sin 16.26° \times 240 + T \times 240 = 0$ ... (3)

$\therefore\ 19.2T = 16800$

$\therefore\ \mathbf{T = 875 \text{ N}}$

From Eq^n (1), $A_x = -1.28 \times 875 = -1120 \text{ N}$

$= \mathbf{1120 \text{ N} \leftarrow}$

From Eq^n (2), $A_y = 280 + 0.96 \times 875$

$= \mathbf{1120 \text{ N} \uparrow}$

$\therefore$ Reaction at A, $R_A = \sqrt{A_x^2 + A_y^2}$

$= \sqrt{(1120)^2 + (1120)^2}$

$= \mathbf{1584 \text{ N}}$

Direction, $\theta_A = \tan^{-1}\left|\frac{A_y}{A_x}\right| = \tan^{-1}\left(\frac{1120}{1120}\right)$

$= 45°$ w.r.t. x

(i) Tension in the cable, **T = 875 N** ...Ans.

(ii) Reaction at A, $\mathbf{R_A = 1584 \text{ N} \rightarrow}$ ...Ans.

$\mathbf{\theta_A = 45°}$ **w.r.t. 'x'** ...Ans.

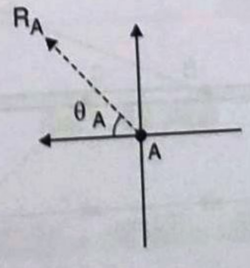


Fig. P. 8.9.31(c)

Ex. 8.9.32 : A beam AB of 5 m length is supported as shown in Fig. P. 8.9.32(a) If AD, AE and BF carry equal axial force, determine the location x of load 1000 N.

SPPU : May 08, 8 Marks

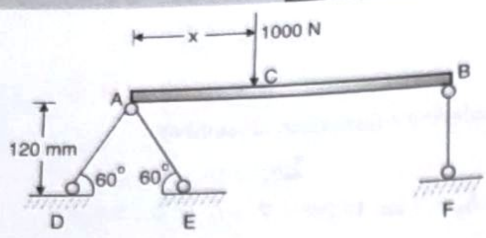


Fig. P. 8.9.32(a)

Soln. :

FBD of beam :

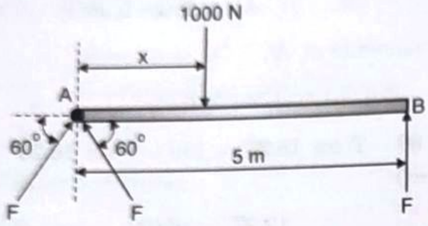


Fig. P. 8.9.32(b)

Let 'F' be the axial force in AD, AE and BF.

For equilibrium;

$$\Sigma F_y = 0$$

$$F \sin 60° + F \sin 60° - 1000 + F = 0 \qquad \ldots (1)$$

$$\therefore \mathbf{F = 366.025\ N}$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$-1000 \times x + F \times 5 = 0 \qquad \ldots (2)$$

$$\therefore \quad x = \frac{366.025 \times 5}{1000} = \mathbf{1.83\ m}$$

$\therefore$ Location of load, $\mathbf{x = 1.83\ m}$ **... Ans.**

Ex. 8.9.33 : Determine the forces in each cable and reaction at roller D. Refer Fig. P. 8.9.33(a) **SPPU : May 08, 6 Marks**

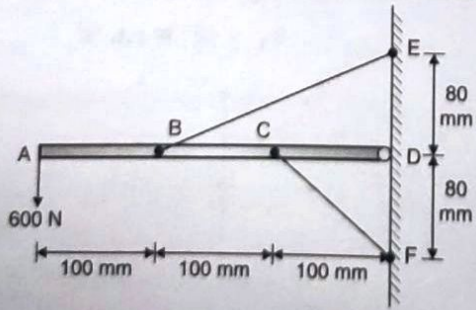


Fig. P. 8.9.33(a)

Soln. :

Let T_B and T_c be the forces in the cables BE and CF.

Let α and β be the angles made by the cables BE and CF w.r.t. horizontal.

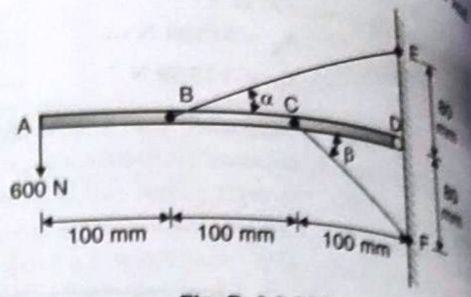


Fig. P. 8.9.33(b)

From Fig. P. 8.9.33(b),

$$\alpha = \tan^{-1}\left(\frac{80}{200}\right) = 21.80°$$

$$\beta = \tan^{-1}\left(\frac{80}{100}\right) = 38.66°$$

FBD of member 'ABCD'

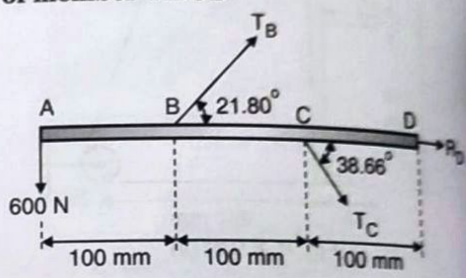


Fig. P. 8.9.33(c)

R_D = Reaction of roller.

(Normal to the plane of roller)

Plane of roller is vertical.

For equilibrium ;

$$\Sigma F_x = 0$$

$$T_B \cos 21.80° + T_C \cos 38.66° - R_D = 0$$

$$0.928\ T_B + 0.78\ T_C = R_D$$

$$\Sigma F_y = 0$$

$$-600 + T_B \sin 21.80° - T_C \sin 38.66° = 0$$

$$0.37\ T_B - 0.62\ T_C = 600$$

Taking moments at 'B',

$$\Sigma M_B = 0$$

$$600 \times 100 - T_C \sin 38.66° \times 100 = 0$$

$$\mathbf{T_C = 960.46\ N}$$

From Eq^n (2), $0.37\ T_B - 0.62 \times 960.46 = 600\ N$

$\therefore \; T_B = 3231 \text{ N}$ ... **Ans.**

From Eq^n (1), $0.928 \times 3231 + 0.78 \times 960.46 = R_D$

$\therefore \; R_D = 3747.57 \text{ N}$ ... **Ans.**

Ex. 8.9.34 : Determine the reactions at roller A and pin B for equilibrium of the member ACB as shown in Fig. P. 8.9.34(a).

SPPU : Dec. 15, 5 Marks

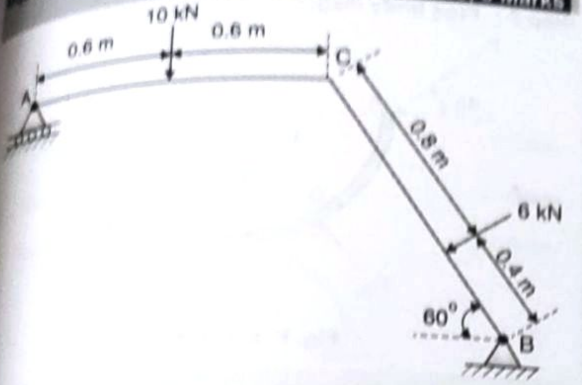


Fig. P. 8.9.34(a)

Soln. :

FBD of member 'ACB'

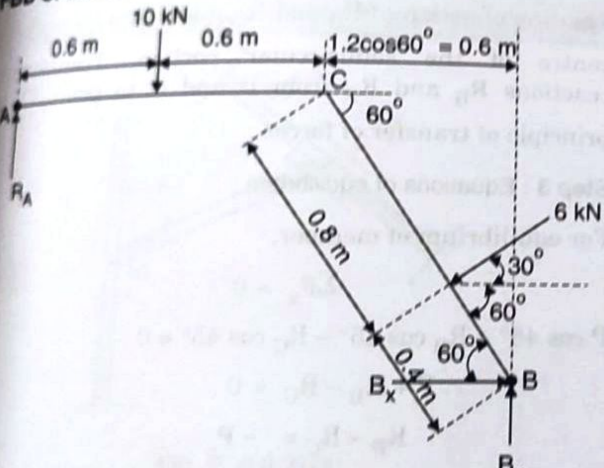


Fig. P. 8.9.34(b)

For equilibrium;

$$\Sigma F_x = 0$$

$$-6 \cos 30° + B_x = 0 \quad \text{... (1)}$$

$$\therefore \; B_x = 6 \cos 30° = 5.2 \text{ kN} \rightarrow$$

$$\Sigma F_y = 0$$

$$R_A - 10 - 6 \sin 30° + B_y = 0$$

$$R_A + B_y = 13 \quad \text{... (2)}$$

Taking moments at 'B'

$$\Sigma M_B = 0$$

$$-R_A \times 1.8 + 10 \times 1.2 + 6 \times 0.4 = 0 \quad \text{... (3)}$$

$$\therefore \; R_A = 8 \text{ kN} \uparrow$$

From Eq^n (2), $8 + B_y = 13$

$$B_y = 5 \text{ kN} \uparrow$$

$\therefore$ Reaction at B,

$$R_B = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.2)^2 + (5)^2} = 7.21 \text{ kN}$$

Direction,

$$\theta_B = \tan^{-1}\left|\frac{B_y}{B_x}\right| = \tan^{-1}\left(\frac{5}{5.2}\right) = 43.87° \text{ w.r.t. 'x'}$$

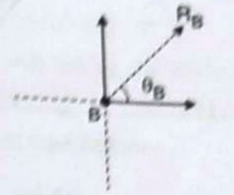


Fig. P. 8.9.34(c)

(i) Reaction at roller A,

$$R_A = 8 \text{ kN} \uparrow \quad \text{...Ans.}$$

(ii) Reaction at pin B,

$$R_B = 7.21 \text{ kN} \quad \text{...Ans.}$$

$$\theta_B = 43.87° \text{ w.r.t. 'x'}. \quad \text{...Ans.}$$

Ex. 8.9.35 : A force of 150 N acts on the end of beam ABD as shown in Fig. P. 8.9.35(a). Determine the magnitude of tension in cable BC to maintain equilibrium.

SPPU : Dec. 14, 6 Marks

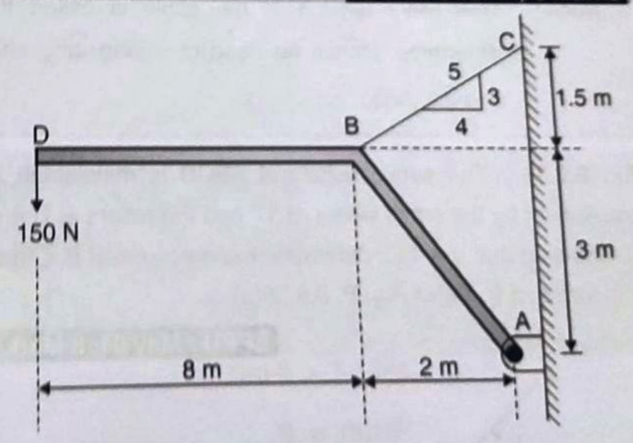


Fig. P. 8.9.35(a)

Soln. :

Step 1 : Forces acting on the beam ABD.

i) Applied force 150 N at D

ii) Tension T, in the cable BC

iii) Reaction components A_x and A_y at hinge support 'A'

Step 2 : FBD of beam ABD

From Fig. P. 8.9.35(b),

$$\tan \alpha = \left(\frac{3}{4}\right),$$

$$\alpha = 36.87^\circ$$

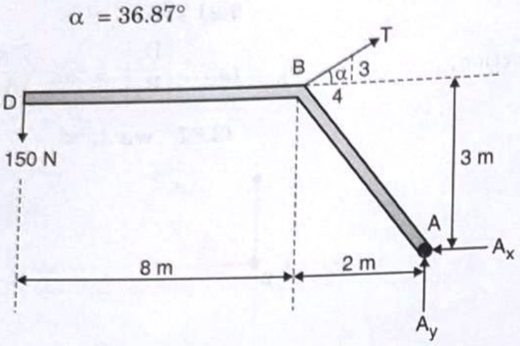


Fig. P. 8.9.35(b)

Step 3 : Equations of equilibrium :

For equilibrium ;

Taking moments at A,

$$\Sigma M_A = 0$$

$$150 \times 10 - T \cos \alpha \times 3 - T \sin \alpha \times 2 = 0$$

$$1500 - T \cos 36.87^\circ \times 3 - T \sin 36.87^\circ \times 2 = 0$$

$$\therefore \quad \mathbf{T = 416.67\ N}$$

The magnitude of tension in cable BC is

$$\mathbf{T = 416.67\ N}$$ ...Ans.

Note : Here only tension in the cable is asked to determine. Hence no need of writing $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

Ex. 8.9.36 : The semicircular rod ABCD is maintained in equilibrium by the small wheel at 'D' and the rollers at B and C. Knowing that $\alpha = 45°$, determine the reactions at B, C and D in terms of P. Refer Fig. P. 8.9.36(a).

SPPU : May 04, 8 Marks

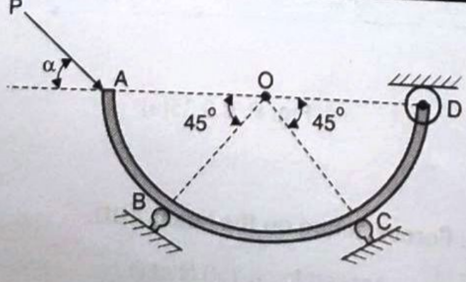


Fig. P. 8.9.36(a)

Soln. :

Step 1 : Forces acting on the member ABCD:

(i) Applied force 'P' at an angle $\alpha = 45°$ at A

(ii) Reactions at rollers B and C i.e., R_B and R_C

(iii) Reaction at wheel 'D' i.e., R_D.

Step 2 : Free body diagram of member 'ABCD'

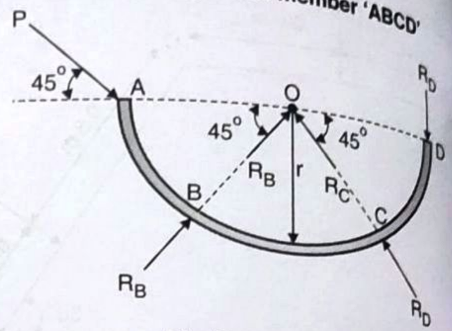


Fig. P. 8.9.36(b)

Let 'r' be the radius of the semicircular rod. Reactions at rollers B and C and at wheel are normal to the plane of surface. At 'D' surface is horizontal.

$\therefore$ R_D is vertical.

The line of action of R_B and R_D must pass through the centre of the semicircular portion. Transferring reactions R_B and R_C from B and C to point 'O' by principle of transfer of forces.

Step 3 : Equations of equilibrium ;

For equilibrium of member,

$$\Sigma F_x = 0$$

$$P \cos 45^\circ + R_B \cos 45^\circ - R_C \cos 45^\circ = 0$$

$$\therefore P + R_B - R_C = 0$$

$$R_B - R_C = -P \quad \text{...(1)}$$

$$\Sigma F_y = 0$$

$$-P \sin 45^\circ + R_B \sin 45^\circ + R_C \sin 45^\circ - R_D = 0$$

$$\therefore \quad R_B + R_C = P + 1.414\, R_D \quad \text{...(2)}$$

Taking moments at point 'O',

$$\Sigma M_o = 0$$

$$P \sin 45^\circ \times r - R_D \times r = 0 \quad \text{...(3)}$$

$$\therefore \quad \mathbf{R_D = 1.414\ P}$$

From Eq^n (2), $$R_B + R_C = P + 1.414\,(1.414P) \quad \text{...(4)}$$

$$\therefore \quad R_B + R_C = 3P$$

Eqⁿ (4), $R_B + \cancel{R_C} = 3P$

Eqⁿ (1), $R_B - \cancel{R_C} = -P$

$$2R_B = 2P$$

$$R_B = P$$

From Eq^n (1), $R_B - R_C = -P$

$$P - R_C = -P$$

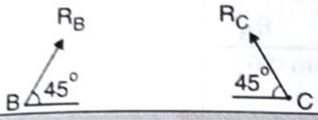
$$R_C = 2P$$

Step 4 :

(i) Reaction at B, $R_B = P$ ...**Ans.**

(ii) Reaction at C, $R_C =$ 2P ...**Ans.**

(iii) Reaction at D, $R_D = 1.414P \downarrow$ ...**Ans.**



Ex. 8.9.37 : An 8 kg slender rod AB is attached to collars which may slide freely along the guides as shown in Fig. P. 8.9.37(a). If the rod is in equilibrium, determine

(i) The angle 'θ' that the rod makes with the vertical, and

(ii) The reactions at A and B. **SPPU : May 04, 8 Marks**

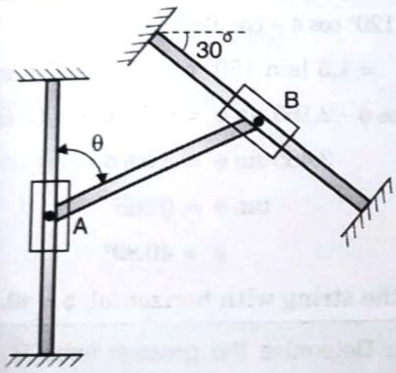


Fig. P. 8.9.37(a)

Soln. :

Step 1 : Forces acting on the rod AB

(i) Weight of the rod AB, W = 8 kg = 8 × 9.81 = 78.48 N

(ii) Reaction at collar A, R_A perpendicular to vertical rod

(iii) Reaction at collar B, R_B normal to the inclined rod.

Step 2 : FBD at rod

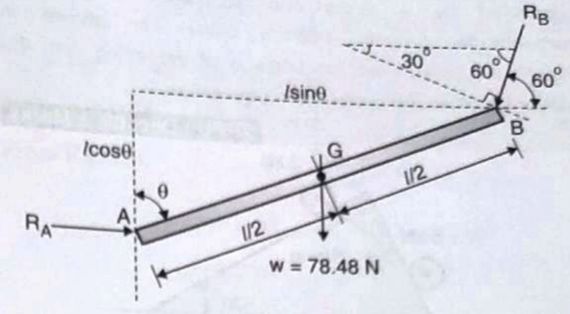


Fig. P. 8.9.37(b)

Let 'l' be the length of the rod AB.

Weight of rod 'W' will act at the midpoint i.e. C.G. of the rod.

Step 3 : Equations of equilibrium :

For equilibrium of rod AB,

$$\Sigma F_x = 0$$

$$R_A - R_B \cos 60° = 0$$

$$R_A = 0.5\, R_B \quad \text{... (1)}$$

$$\Sigma F_y = 0$$

$$-78.48 - R_B \sin 60° = 0 \quad \text{... (2)}$$

$$\therefore \mathbf{R_B = -90.62\ N}$$

From Eqⁿ (1), $R_A = 0.5\,(-90.62)$

$$\therefore \mathbf{R_A = -45.31\ N}$$

Taking moment at B,

$$\Sigma M_B = 0$$

$$R_A \times l \cos\theta + 78.48 \times \frac{l}{2} \sin\theta = 0$$

$$(-45.31) \cos\theta + 39.24 \sin\theta = 0$$

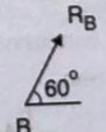
$$\therefore 39.24 \sin\theta = 45.31 \cos\theta$$

$$\tan\theta = 1.1546$$

$$\therefore \mathbf{\theta = 49.10°}$$

(i) Angle, θ = 49.10° w.r.t. vertical ...**Ans.**

(ii) Reactions at A and B



$\mathbf{R_A =}$ **45.31 N ←** ...**Ans.**

$\mathbf{R_B =}$ **90.62 N** ...**Ans.**

Ex. 8.9.38 : Two rollers weighing 5 kN and 3 kN are connected by a string and supported on mutually perpendicular smooth inclined planes as shown in Fig. P. 8.9.38(a). Find the angle ϕ of the string with the horizontal, when the system is in equilibrium.

SPPU : Dec. 02, 8 Marks

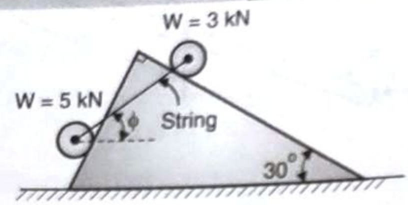


Fig. P. 8.9.38(a)

Soln. :

Step 1 : Forces acting in the system :

(i) Weight of roller 5 kN and 3 kN

(ii) Tension in the string, T

(iii) Normal reactions at the rollers, perpendicular to the inclined surfaces.

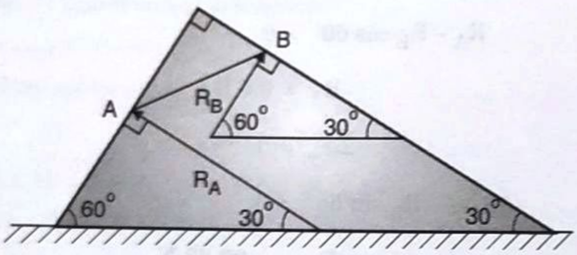


Fig. P. 8.9.38(b)

Step 2 : Free body diagrams

FBD of roller 'A'

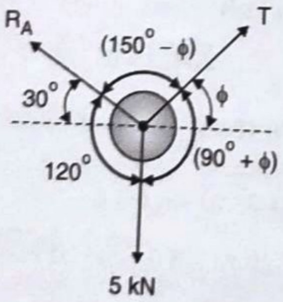


Fig. P. 8.9.38(c)

Step 3 : Equilibrium equations :

For three concurrent forces in equilibrium, using Lami's theorem.

$$\frac{R_A}{\sin(90° + \phi)} = \frac{T}{\sin 120°} = \frac{5}{\sin(150° - \phi)}$$

$$\therefore \quad T = \frac{5 \sin 120°}{\sin(150° - \phi)}$$

FBD of roller B

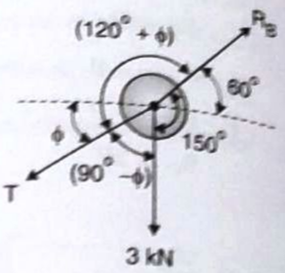


Fig. P. 8.9.38(d)

For equilibrium of three concurrent forces, using Lami's theorem,

$$\frac{R_B}{\sin(90° - \phi)} = \frac{T}{\sin 150°} = \frac{3}{\sin(120° + \phi)}$$

$$\therefore \quad T = \frac{3 \sin 150°}{\sin(120° - \phi)}$$

Equating Eqn (1) and Eqn (2) for T

$$\frac{5 \sin 120°}{\sin(150° - \phi)} = \frac{3 \sin 150°}{\sin(120° - \phi)}$$

$$\therefore \quad 4.33 \sin(120° + \phi) = 1.5 \sin(150° - \phi)$$

$$\therefore \quad 4.33[\sin 120° \cos\phi + \cos 120° \sin\phi]$$

$$= 1.5\,[\sin 150° \cos\phi + \cos 150° \sin\phi]$$

$$3.75 \cos\phi - 2.165 \sin\phi = 0.75 \cos\phi + 1.3 \sin\phi$$

$$3.465 \sin\phi = 3 \cos\phi$$

$$\tan\phi = 0.866$$

$$\phi = 40.89°$$

Angle ϕ of the string with horizontal, $\phi = 40.89°$ ...Ans.

Ex. 8.9.39 : Determine the greatest force 'P' that can be applied to the frame, if the largest force resultant acting at A can have a magnitude of 2 kN.

SPPU : May 02, 8 Marks

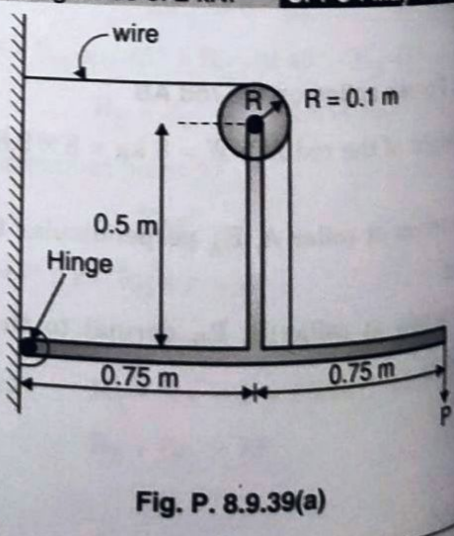


Fig. P. 8.9.39(a)

Soln. :

Step 1 : Forces acting on frame :

i) Applied force P

ii) Tension in the wire, T

iii) Support reaction at hinge, A, $R_A = 2$ kN (given)

there will be horizontal and vertical components of reaction at A i.e., A_x and A_y.

Step 2 : FBD of frame :

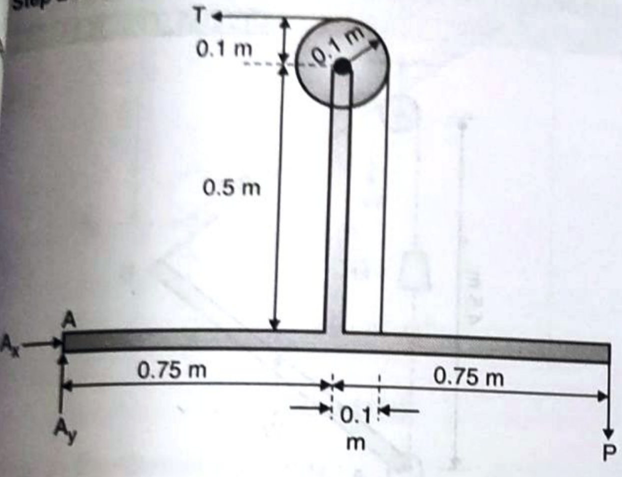


Fig. P. 8.9.39(b)

Step 3 : Equilibrium equations :

For equilibrium of frame,

$$\Sigma F_x = 0$$

$$A_x - T = 0$$

$$A_x = T \quad \dots (1)$$

$$\Sigma F_y = 0$$

$$A_y - P = 0$$

$$A_y = P \quad \dots (2)$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$T \times 0.6 - P \times 1.5 = 0$$

$$\therefore 0.6T - 1.5P = 0 \quad \dots (3)$$

Given that the largest force resultant at A i.e., the support reaction at A, $R_A = 2$kN

$$R_A = \sqrt{A_x^2 + A_y^2} = 2 \text{ kN}$$

$$\therefore A_x^2 + A_y^2 = 4$$

$$(T)^2 + (P)^2 = 4 \quad (\because A_x = T \text{ and } A_y = P)$$

From Eqn (1) and Eqn (2)

$$T^2 + P^2 = 4 \quad \dots (4)$$

From Eqn (3),

$$T = \left(\frac{1.5}{0.6}\right) P = 2.5P$$

$$\therefore (2.5P)^2 + P^2 = 4$$

$$6.25P^2 + P^2 = 4$$

$$7.25P^2 = 4$$

$$\therefore \mathbf{P = 0.743 \text{ kN}}$$

From Eqn (4), $T^2 + 0.743^2 = 4$

$$\therefore \mathbf{T = 1.857 \text{ kN}}$$

(i) Greatest force, **P = 0.743 kN** ...**Ans.**

(ii) Tension in the wire, **T = 1.857 kN** ...**Ans.**

Ex. 8.9.40 : Determine the force in the member BD of the frame shown in Fig. P. 8.9.40(a). Also find the components of reaction at 'C'. **SPPU : Dec. 98, 8 Marks**

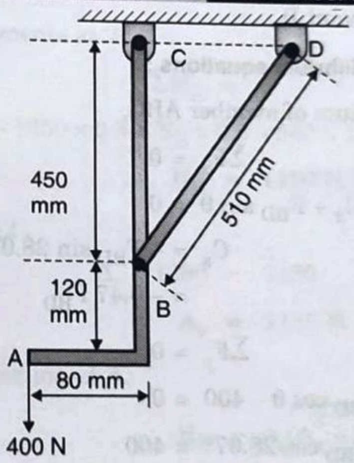


Fig. P. 8.9.40(a)

Soln. :

Step 1 : Forces acting on the member ABC :

i) Applied force 400 N at A

ii) Force in the member BD, F_{BD}

iii) Components of reaction at C i.e., C_x and C_y

Step 2 : FBD of member ABC

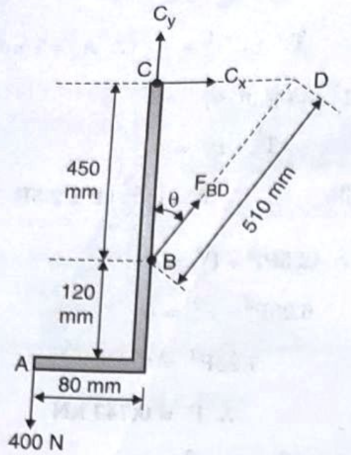


Fig. P. 8.9.40(b)

Let 'θ' be the angle made by BD w.r.t. vertical.

From Fig. P. 8.10.40(b),

$$\cos\theta = \frac{450}{510}$$

$$\therefore \quad \theta = 28.07^\circ$$

Force in member BD is assumed tensile. Hence shown away from point B.

Step 3 : Equilibrium equations ;

For equilibrium of member ABC,

$$\Sigma F_x = 0$$

$$C_x + F_{BD}\sin\theta = 0$$

$$\therefore \quad C_x = -F_{BD}\sin 28.07^\circ$$

$$= -0.47\,F_{BD} \quad \ldots (1)$$

$$\Sigma F_y = 0$$

$$C_y + F_{BD}\cos\theta - 400 = 0$$

$$C_y + F_{BD}\cos 28.07^\circ = 400$$

$$\therefore \quad C_y + 0.882\,F_{BD} = 400 \quad \ldots (2)$$

Taking moments at 'C',

$$\Sigma M_C = 0$$

$$F_{BD}\sin 28.07^\circ \times 450 + 400 \times 80 = 0 \quad \ldots (3)$$

$$\therefore \quad F_{BD} = -151.12\text{ N}$$

$$\mathbf{= 151.12\ N\ (compressive)}$$

From Eq^n (1), $C_x = -0.47(-151.12)$

$$\mathbf{= 71.026\ N \rightarrow}$$

From Eq^n (2), $C_y + 0.882\,(-151.12) = 400$

$$\therefore \quad C_y = 533.28\text{ N}\uparrow$$

i) Force in the member BD,

$$F_{BD} = 151.12\text{ N (compressive)}$$

...Ans.

ii) Components of reaction at C,

$$C_x = 71.026\text{ N}\rightarrow$$

...Ans.

$$C_y = 533.28\text{ N}\uparrow$$

...Ans.

Ex. 8.9.41 : A prismatic bar AB of weight 'W' kN and a length 3m is hinged at A and supported at B by a string that passes over a pulley D and carries a load of 1 kN at its free end. For equilibrium of the system, find the weight 'W' of the bar.

SPPU : May 98, 6 Marks

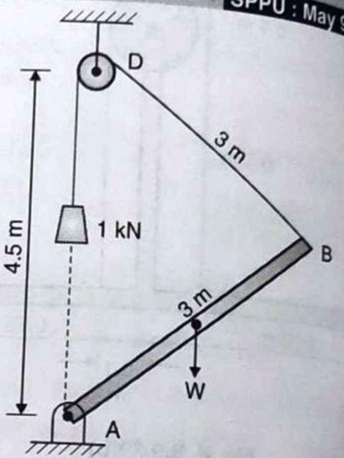


Fig. P. 8.9.41(a)

Soln. :

Step 1 : Forces acting on the bar :

i) Weight of bar 'W' at the midpoint of AB.

ii) Tension in the string, T = 1 kN at B.

iii) Reaction components at A_x and A_y at hinge support 'A'.

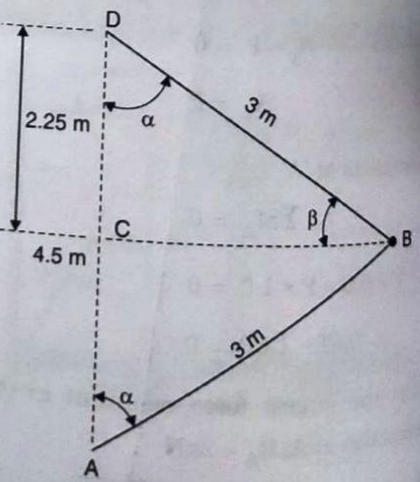


Fig. P. 8.9.41(b)

Δ ABD is an isosceles triangle.

From Δ BCD,

$$\cos\alpha = \left(\frac{2.25}{3}\right)$$

$$\therefore \quad \alpha = 41.41^\circ$$

$$\beta = 90^\circ - \alpha = 90^\circ - 41.41^\circ = 48.59^\circ$$

Step 2 : FBD of member AB :

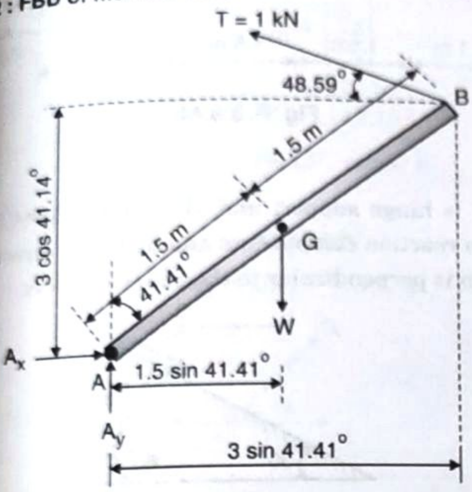


Fig. P. 8.10.41(c)

Step 3 : Equilibrium equations :

For equilibrium of member AB,

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$-W \times 1.5 \sin 41.41^\circ + 1 \times \cos 48.59^\circ \times 3 \cos 41.41^\circ$$

$$+ 1 \sin 48.59^\circ \times 3 \sin 41.41^\circ = 0$$

$$\therefore \quad W = 3\ \text{Kn}$$

The weight of the bar, $\quad W = 3\ \text{kN}$ ...Ans.

Type 4 : Equilibrium of Beams

Ex. 8.9.42 : Find the support reactions at hing 'A' and roller 'B' for the beam shown in Fig. P. 8.9.42(a)

SPPU : Nov. 08, 8 Marks

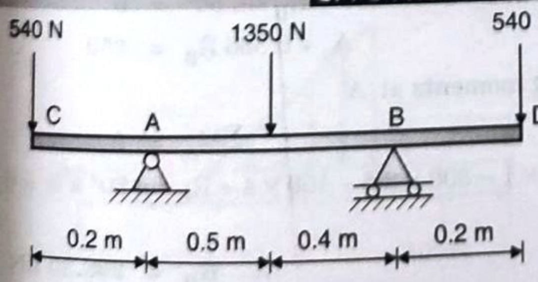


Fig. P. 8.9.42(a)

Soln. :

Hinge support 'A' will have two reaction components horizontal and vertical where as at roller support B, the reaction is normal to the plane of rollers.

FBD of beam

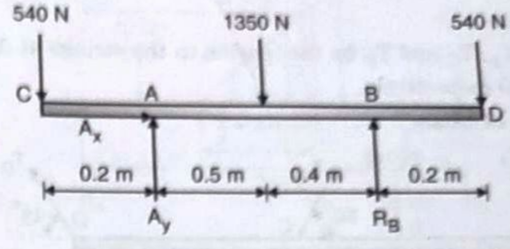


Fig. P. 8.9.42(b)

For equilibrium of beam;

$$\Sigma F_x = 0$$

$$A_x = 0 \quad \text{... (1)}$$

$$\Sigma F_y = 0$$

$$-540 + A_y - 1350 + R_B - 540 = 0$$

$$A_y + R_B = 2430 \quad \text{... (2)}$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$540 \times 0.2 - 1350 \times 0.5 + R_B \times 0.9 - 540 \times 1.1 = 0 \quad \text{... (3)}$$

$$\therefore \quad R_B = 1290\ \text{N} \uparrow$$

From Eq^n (2),

$$A_y + 1290 = 2430$$

$$\therefore \quad A_y = 1140\ \text{N} \uparrow$$

Support reaction at A,

$$R_A = \sqrt{A_x^2 + A_y^2} = A_y \quad (\because A_x = 0)$$

(i) Reaction at A, $\quad R_A = A_y = 1140\ \text{N} \uparrow$...Ans.

(ii) Reaction at B, $\quad R_B = 1290\ \text{N} \uparrow$ ...Ans.

Ex. 8.9.43 : A light beam ABCD carries a load 250 N at B as shown in Fig. P. 8.9.43(a) It is supported by three strings at A, C and D. Determine tension in the strings.

SPPU : May 07, 8 Marks

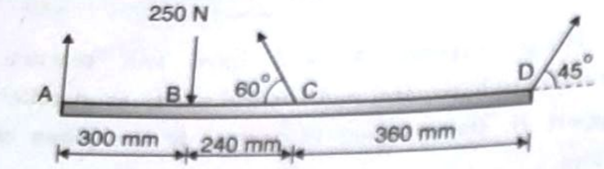


Fig. P. 8.9.43(a)

Soln. :

Let T_A, T_C and T_D be the tension in the strings at A, C and D respectively.

FBD of beam

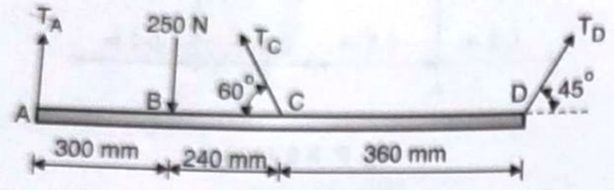


Fig. P. 8.9.43(b)

For equilibrium ;

$$\Sigma F_x = 0$$

$$-T_C \cos 60° + T_D \cos 45° = 0$$

$$0.5\ T_C = 0.707\ T_D \qquad \dots (1)$$

$$\Sigma F_y = 0$$

$$T_A - 250 + T_C \sin 60° + T_D \sin 45° = 0$$

$$T_A + 0.866\ T_C + 0.707\ T_D = 250 \qquad \dots (2)$$

Taking moments at 'A'.

$$\Sigma M_A = 0$$

$$-250 \times 300 + T_C \sin 60° \times 540 + T_D \sin 45° \times 900 = 0$$

$$467.65\ T_C + 636.4\ T_D = 75000 \qquad \dots (3)$$

From Eq^n (1), $\quad T_C = 1.414\ T_D$

Putting in Eq^n (3),

$$467.65\ (1.414\ T_D) + 636.4\ T_D = 75000$$

$$1297.65\ T_D = 75000$$

$$\therefore\ \mathbf{T_D = 57.8\ N}$$

... Ans.

From Eq^n (1), $\quad 0.5\ T_C = 0.707 \times 57.8$

$$\therefore\ \mathbf{T_C = 81.72\ N}$$

... Ans.

From Eq^n (2), $T_A + 0.866\ (81.72) + 0.707\ (57.8) = 250$

$$\therefore\ \mathbf{T_A = 138.36\ N}$$

... Ans.

Ex. 8.9.44 : Determine reactions R_A and R_B at support A and B of horizontal beam AB due to action of vertical loads applied as shown in Fig. P. 8.9.44(a).

SPPU : May 10, 8 Marks

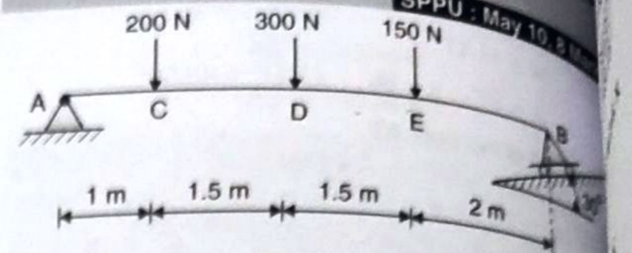


Fig. P. 8.9.44(a)

Soln. :

'A' is hinge support and 'B' is roller. At 'A', there are two reaction components and at B, the direction of reaction is perpendicular to the plane of rollers.

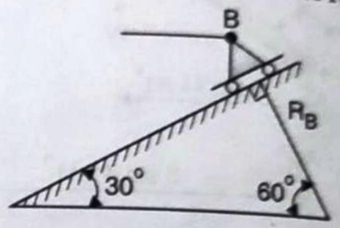


Fig. P. 8.9.44(b)

FBD of beam 'AB'

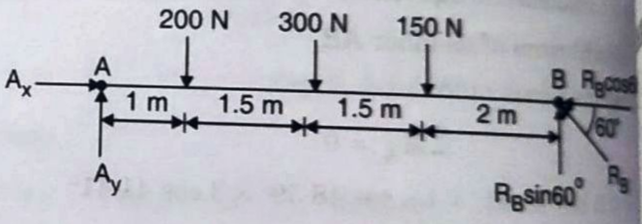


Fig. P. 8.9.44(c)

For equilibrium of beam AB,

$$\Sigma F_x = 0$$

$$A_x - R_B \cos 60° = 0$$

$$\therefore\ A_x = 0.5\ R_B \qquad \dots$$

$$\Sigma F_y = 0$$

$$A_y - 200 - 300 - 150 + R_B \sin 60° = 0$$

$$A_y + 0.866\ R_B = 650 \qquad \dots$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$-200 \times 1 - 300 \times 2.5 - 150 \times 4 + R_B \sin 60° \times 6 = 0$$

...

$$\therefore\ \mathbf{R_B = 298.30\ kN}$$

From Eq^n (1), $\quad A_x = 0.5 \times 298.3$

From Eq^n (2), $A_y + 0.866 \times 298.3 = 650$

$\therefore$ $\mathbf{A_y = 391.67\ N \uparrow}$

$\therefore$ Support reaction at A,

$$R_A = \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{(149.15)^2 + (391.67)^2}$$

$$= \mathbf{419.11\ N}$$

Direction,

$$\theta_A = \tan^{-1}\left|\frac{A_y}{A_x}\right|$$

$$= \tan^{-1}\left(\frac{391.67}{149.15}\right)$$

$$= \mathbf{69.15^\circ\ w.r.t.\ x}$$

(i) Support reaction at A,

$R_A = \mathbf{419.11\ N}$ ...Ans.

$\theta_A = 69.15^\circ$ w.r.t. 'x' ...Ans.

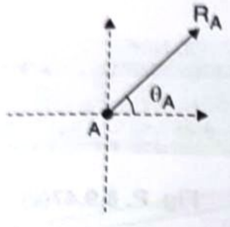


Fig. P. 8.9.44(d)

(ii) Support reaction at B,

$R_B = 298.30$ N ...Ans.

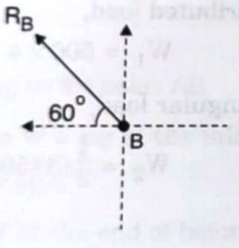


Fig. P. 8.9.44(e)

Ex. 8.9.45 : A simply supported beam AB of span 6 m is loaded and supported as shown in Fig. P. 8.9.45(a) Find the reactions at support A and B.

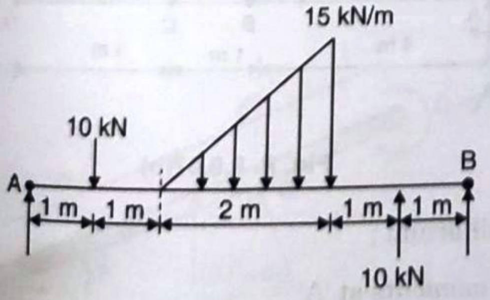


Fig. P. 8.9.45(a)

Soln. :

Magnitude of triangular load $= \frac{1}{2} \times 2 \times 15 = 15$ kN.

FBD of beam :

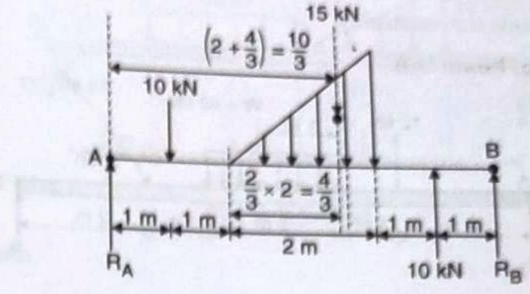


Fig. P. 8.9.45(b)

For equilibrium

$$\Sigma F_y = 0$$

$$R_A - 10 - 15 + 10 + R_B = 0$$

$$\therefore \quad R_A + R_B = 15 \qquad \text{... (1)}$$

Taking moments at 'A'.

$$\Sigma M_A = 0$$

$$-10 \times 1 - 15 \times \frac{10}{3} + 10 \times 5 + R_B \times 6 = 0 \qquad \text{... (2)}$$

$$\therefore \quad R_B = \frac{10}{6} = \mathbf{1.67\ kN \uparrow}$$

From Eq^n (1), $R_A + 1.67 = 15$

$\therefore$ $\mathbf{R_A = 13.33\ kN \uparrow}$

$\therefore$ Support reactions at A and B are ;

$\mathbf{R_A = 13.33\ kN \uparrow}$ ... Ans.

$\mathbf{R_B = 1.67\ kN \uparrow}$ ... Ans.

Ex. 8.9.46 : Determine the support reaction for the beam loaded and supported at shown in Fig. P. 8.9.46(a)

SPPU : May 13, 6 Marks

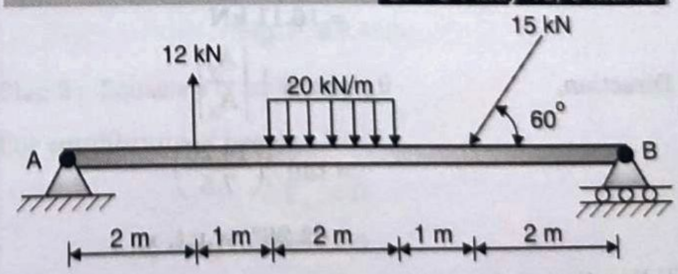


Fig. P. 8.9.46(a)

Soln. :

'A' is hinge or pin support and B is roller support.

At 'A' there are two reaction components and at B, the reaction is perpendicular to the plane of rollers.

Magnitude of distributed (rectangular) load,

$W = 20 \times 2 = 40$ kN will act at the midpoint of rectangle (i.e. centroid)

FBD of beam 'AB'

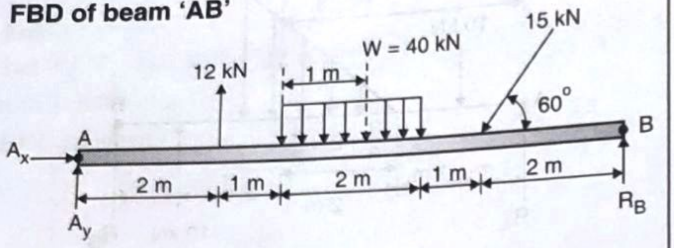


Fig. P. 8.9.46(b)

For equilibrium of beam ;

$$\Sigma F_x = 0$$

$$A_x - 15 \cos 60° = 0 \quad \text{... (1)}$$

$$\therefore \mathbf{A_x = 7.5\ kN \rightarrow}$$

$$\Sigma F_y = 0$$

$$A_y + 12 - 40 - 15 \sin 60° + R_B = 0$$

$$A_y + R_B = 41 \quad \text{... (2)}$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$12 \times 2 - 40 \times 4 - 15 \sin 60° \times 6 + R_B \times 8 = 0 \quad \text{... (3)}$$

$$\therefore \mathbf{R_B = 26.74\ kN \uparrow}$$

From Eq^n (2), $A_y + 26.74 = 41$

$$\therefore \mathbf{A_y = 14.26\ kN \uparrow}$$

$\therefore$ Reaction at support A,

$$R_A = \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{(7.5)^2 + (14.26)^2}$$

$$= \mathbf{16.11\ kN}$$

Direction,

$$\theta_A = \tan^{-1}\left|\frac{A_y}{A_x}\right|$$

$$= \tan^{-1}\left(\frac{14.26}{7.5}\right)$$

$$= \mathbf{62.26°\ w.r.t.\ x.}$$

i) Reaction at A,

$$\mathbf{R_A = 16.11\ kN} \quad \text{...Ans.}$$

$$\mathbf{\theta_A = 62.26°\ w.r.t.\ 'x'} \quad \text{...Ans.}$$

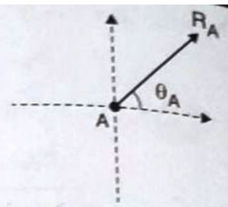


Fig. P. 8.9.46(c)

(ii) Reaction at B,

$$\mathbf{R_B = 26.74\ kN \uparrow}$$

Ex. 8.9.47 : For the cantilever, determine range of values of force 'P' for which the magnitude of the fixing moment at does not exceed 5000 Nm. Refer Fig. P. 8.9.47(a).

SPPU : Dec. 09, 8 Ma

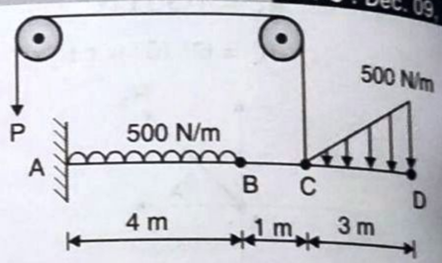


Fig. P. 8.9.47(a)

Soln. :

'A' is fixed support. Hence at 'A' there are two react components A_x and A_y and a moment M_A.

Magnitude of distributed load,

$$W_1 = 500 \times 4 = 2000\ \text{N}$$

Magnitude of triangular load,

$$W_2 = \frac{1}{2}(3)(500) = 750\ \text{N}$$

FBD of beam

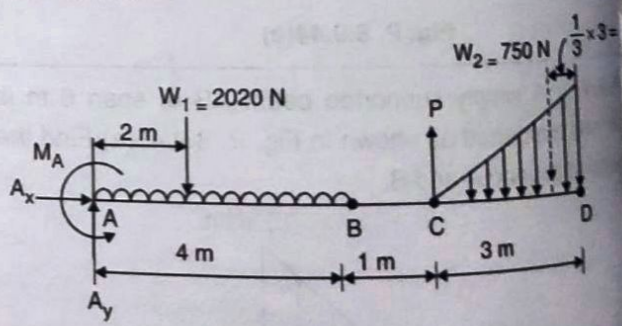


Fig. P. 8.9.47(b)

For equilibrium ;

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$M_A - W_1 \times 2 + P \times 5 - W_2 \times 7 = 0$$

$$M_A - 2000 \times 2 + P \times 5 - 750 \times 7 = 0$$

$$M_A + 5P = 9250 \quad \ldots (1)$$

Given that the magnitude of fixing moment does not exceed 5000 Nm.

Case 1 : Taking, M_A = 5000 Nm (ACW)

From Eq^n (1), $5000 + 5P = 9250$

$$\therefore \quad \mathbf{P = 850\ N}$$

Case 2 : Taking, MA = – 5000 Nm (CW)

From Eq^n (1), $-5000 + 5P = 9250$

$$\therefore \quad \mathbf{P = 2850\ N}$$

$\therefore$ The range of values of P for which the magnitude of M_A does not exceed 5000 Nm are : **850 N ≤ P ≤ 2850 N**

... Ans.

Ex. 8.9.48 : Draw the free body diagram of the beam of mass 'm' shown in Fig. P. 8.9.48(a). Clearly indicating the direction of all forces. **SPPU : May 04, 4 Marks**

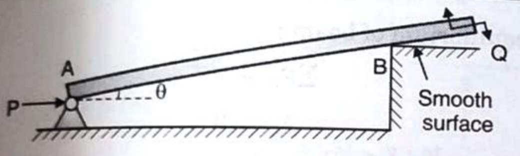


Fig. P. 8.9.48(a)

Soln. :

Step 1 : Forces acting on the beam AB.

i) Weight of beam W = mg at the midpoint of beam

ii) Applied force P at A.

iii) C. W. couple 'Q' at the end of beam

iv) Reaction components A_x and A_y at hinge support 'A'.

v) Normal reaction R_B from the corner at B

Step 2 : FBD of beam

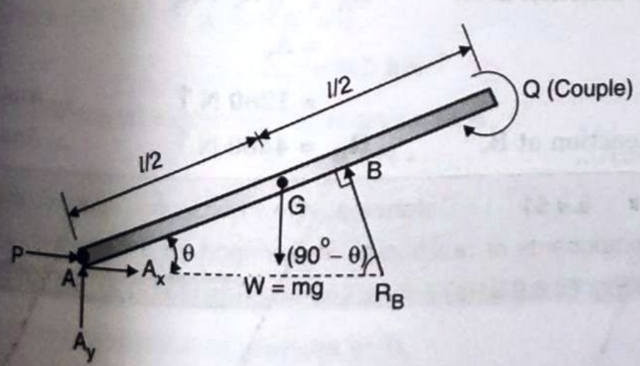


Fig. P. 8.9.48(b)

Ex. 8.9.49 : Find the reactions for the beam AB, shown in Fig. P. 8.9.49(a), at the hinged support at A and the roller support at B. **SPPU : May 04, 8 Marks**

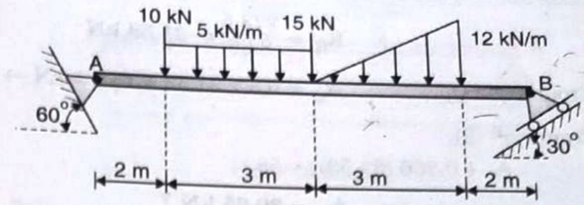


Fig. P. 8.9.49(a)

Soln.

Step 1 : Forces acting on the beam.

i) Point loads 10 kN and 15 kN

ii) Uniformly distributed or rectangular load,

$$W_1 = 5 \times 3 = 15\ \text{kN}$$

iii) Uniformly varying or triangular load,

$$W_2 = \frac{1}{2} \times 3 \times 12 = 18\ \text{kN}$$

iv) Reaction components A_x and A_y at hinge support 'A'.

v) Reaction at roller support B, R_B normal to the plane.

Step 2 : FBD of beam AB

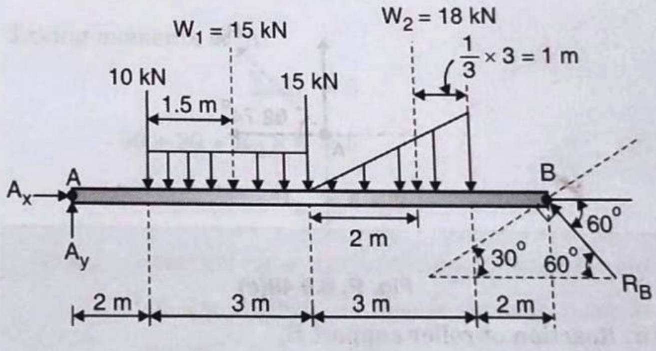


Fig. P. 8.9.49(b)

Step 3 : Equations of equilibrium :

For equilibrium of beam ;

$$\Sigma F_x = 0$$

$$A_x - R_B \cos 60° = 0$$

$$A_x = 0.5\, R_B \quad \ldots (1)$$

$$\Sigma F_y = 0$$

$$A_y - 10 - 15 - 15 - 18 + R_B \sin 60° = 0$$

$$\therefore A_y + 0.866\, R_B = 58 \quad \ldots (2)$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$-10 \times 2 - 15 \times 3.5 - 15 \times 5 - 18 \times 7 + R_B \sin 60° \times 10 = 0$$

$$\therefore \quad R_B = \frac{273.5}{8.66} = \mathbf{31.58\ kN}$$

From Eq^n (1), $\quad A_x = 0.5 \times 31.58 = \mathbf{15.79\ kN} \rightarrow$

From Eq^n (2),

$$A_y + 0.866\,(31.58) = 58$$

$$\therefore \quad \mathbf{A_y = 30.65\ kN} \uparrow$$

$\therefore$ Support reaction at A,

$$R_A = \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{(15.79)^2 + (30.65)^2}$$

$$= \mathbf{34.48\ kN}$$

Direction, $$\theta_A = \tan^{-1}\left|\frac{A_y}{A_x}\right|$$

$$= \tan^{-1}\left(\frac{30.65}{15.79}\right)$$

$$= \mathbf{62.74°\ \ w.r.t.\ x}$$

(i) Reaction at hinge support A,

$$\mathbf{R_A = 34.48\ kN} \quad \textbf{...Ans.}$$

$$\mathbf{\theta_A = 62.74°\ w.r.t.\ 'x'} \quad \textbf{...Ans.}$$

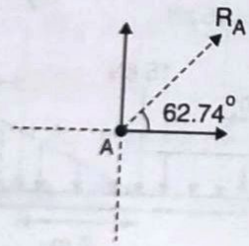


Fig. P. 8.9.49(c)

(ii) Reaction at roller support B,

$$R_B = 31.58\ kN$$

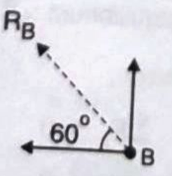


Fig. P. 8.9.49(d)

Ex. 8.9.50 : The beam AB with pin at 'A' and roller at 'B' loaded as shown in the Fig. P. 8.9.50(a). Determine the reactions at the supports A and B.

SPPU : Dec. 18, 6 Marks

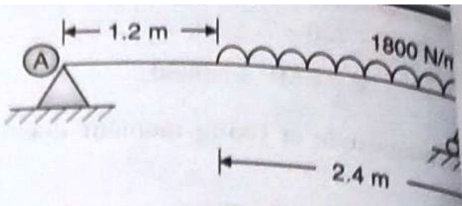


Fig. P. 8.9.50(a)

Soln. :

FBD of beam

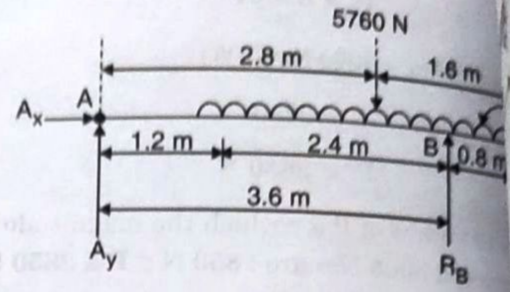


Fig. P. 8.9.50(b)

Magnitude of UDL = 1800 (2.4 + 0.8

It will act at midpoint of 3.2 m i.e. 1.6 m fr m from A.

For equilibrium of beam ;

$$\Sigma F_x = 0$$

$$A_x = 0$$

$$\Sigma F_y = 0$$

$$A_y + R_B - 5760 = 0$$

$$A_y + R_B = 5760 \quad ...(2)$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$-5760 \times 2.8 + R_B \times 3.6 = 0$$

$$\therefore \quad \mathbf{R_B = 4480\ N} \uparrow$$

From Eq^n (2), $\quad \mathbf{A_y = 1280\ N} \uparrow$

$$A_x = 0$$

$\therefore$ Reaction at A, $$R_A = \sqrt{A_x^2 + A_y^2}$$

$$= A_y$$

$$= \mathbf{1280\ N} \uparrow$$

Reaction at B, $\quad \mathbf{R_B = 4480\ N} \uparrow$

Ex. 8.9.51 : Determine the horizontal and components of reaction at the support for the bea in Fig. P. 8.9.51(a)

SPPU : May 18,

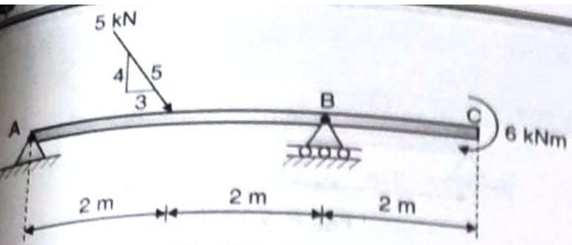


Fig. P. 8.9.51(a)

Soln. : 'A' is hinge support and 'B' is roller.

FBD of beam :

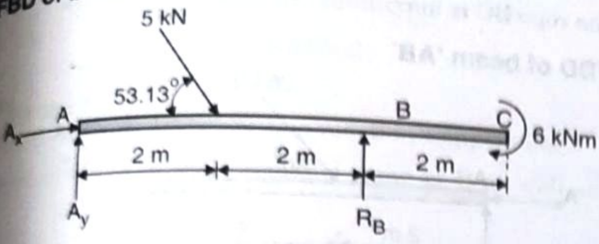
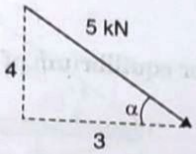


Fig. P. 8.9.51(b)

For equilibrium ;



$$\Sigma F_x = 0$$

$$A_x + 5 \cos 53.13° = 0 \quad \ldots (1)$$

$$\therefore \; A_x = -3 \text{ kN} = 3 \text{ kN} \leftarrow$$

$$\Sigma F_y = 0$$

$$A_y - 5 \sin 53.13° + R_B = 0$$

$$A_y + R_B = 4 \quad \ldots (2)$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$-5 \sin 53.13° \times 2 + R_B \times 4 - 6 = 0 \ldots (3)$$

$$\therefore \; \mathbf{R_B = 3.5 \text{ kN} \uparrow}$$

> **Note :** Moment of couple 6 kNm at A is 6 kNm C.w. ($\because$ moment of couple at any point is always constant)

From Eq^n (2), $A_y + 3.5 = 4$

$$\therefore \; \mathbf{A_y = 0.5 \text{ kN} \uparrow}$$

(i) Horizontal component of reaction at A,

$$\mathbf{A_x = 3 \text{ kN} \leftarrow} \quad \textbf{...Ans.}$$

Vertical component of reaction at A,

$$\mathbf{A_y = 0.5 \text{ kN} \uparrow} \quad \textbf{...Ans.}$$

(ii) Horizontal component of reaction at B, $B_x = 0$

Vertical component of reaction at B,

$$\mathbf{B_y = R_B = 3.5 \text{ kN} \uparrow} \quad \textbf{...Ans.}$$

Ex. 8.9.52 : Determine the support reactions for a beam loaded and supported as shown in Fig. P. 8.9.52(a).

SPPU : May 16, May 18, 6 Marks

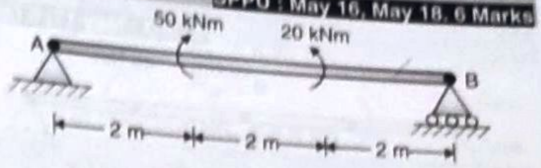


Fig. P. 8.9.52(a)

Soln. :

'A' is hinge support and 'B' is roller support.

FBD of beam

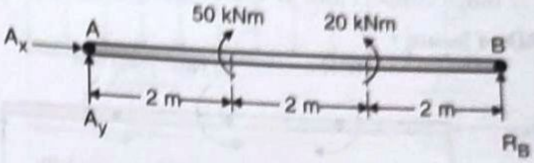


Fig. P. 8.9.52(b)

For equilibrium of beam ;

$$\Sigma F_x = 0$$

$$A_x = 0 \quad \ldots (1)$$

$$\Sigma F_y = 0$$

$$A_y + R_B = 0 \quad \ldots (2)$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$-50 + 20 + R_B \times 6 = 0 \quad \ldots (3)$$

$$\therefore \; \mathbf{R_B = 5 \text{ kN} \uparrow}$$

> **Note :** Resultant force of couple in any direction is zero but the moment of couple at any point is constant.

From Eq^n (2), $A_y + 5 = 0$

$$A_y = -5 \text{ kN}$$

$$= \mathbf{5 \text{ kN} \downarrow}$$

$\therefore$ Reaction at A, $R_A = \sqrt{A_x^2 + A_y^2}$

$$= A_y$$

$$= \mathbf{5 \text{ kN} \downarrow} \quad \textbf{... Ans.}$$

Reaction at B, $\mathbf{R_B = 5 \text{ kN} \uparrow}$ **... Ans.**

Ex. 8.9.53 : Determine reaction at A and B for the beam loaded and supported as shown in Fig. P. 8.9.53(a). Moments are acting at point C, D and E.

SPPU : May 17, 5 Marks

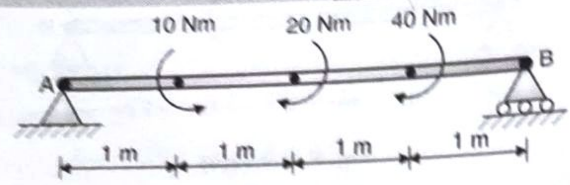


Fig. P. 8.9.53(a)

Soln. :

'A' is hinge support and 'B' is roller.

FBD of beam :

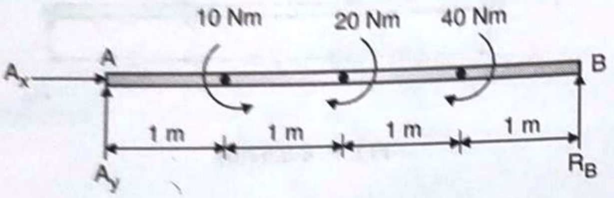


Fig. P. 8.9.53(b)

For equilibrium of beam AB,

$$\Sigma F_x = 0$$

$$A_x = 0 \quad \text{... (1)}$$

$$\Sigma F_y = 0$$

$$A_y + R_B = 0 \quad \text{... (2)}$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$10 - 20 - 40 + R_B \times 4 = 0 \quad \text{... (3)}$$

$$\therefore \quad \mathbf{R_B = 12.5\ N \uparrow}$$

From Eq^n (2), $A_y + 12.5 = 0$

$$A_y = -12.5\ N = \mathbf{12.5\ N \downarrow}$$

$\therefore$ Reaction at A, $R_A = \sqrt{A_x^2 + A_y^2}$

$$= A_y = \mathbf{12.5\ N\downarrow} \quad \textbf{... Ans.}$$

(i) Reaction at B, $\mathbf{R_A = 12.5\ N \downarrow}$ **... Ans.**

(ii)Reaction at B, $\mathbf{R_B = 12.5\ N \uparrow}$ **... Ans.**

Ex. 8.9.54 : If rope BC fails when the tension becomes 50 kN, determine greatest vertical load F that can be applied to the beam AB. Also determine reaction components at A. Refer Fig. P. 8.9.54(a).

SPPU : May 13, 6 Marks

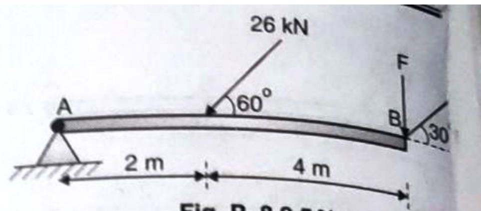


Fig. P. 8.9.54(a)

Soln. :

'A' is hinge support.

Let 'F' be the greatest vertical load, when the rope BC is maximum i.e., 50 kN.

FBD of beam 'AB'

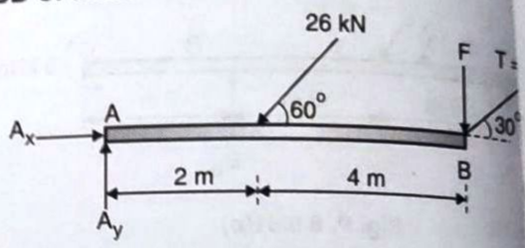


Fig. P. 8.9.54(b)

For equilibrium of beam ;

$$\Sigma F_x = 0$$

$$A_x - 26 \cos 60° + 50 \cos 30° = 0$$

$$A_x = -30.30\ kN$$

$$= \mathbf{30.30\ kN \leftarrow}$$

$$\Sigma F_y = 0$$

$$A_y - 26 \sin 60° - F + 50 \sin 30° = 0$$

$$\therefore \quad A_y - F = -2.48$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$-26 \sin 60° \times 2 - F \times 6 + 50 \sin 30° \times 6 = 0$$

$$\therefore \quad \mathbf{F = 17.49\ kN \downarrow}$$

From Eq^n (2), $A_y - 17.49 = -2.48$

$$\therefore \quad \mathbf{A_y = 15\ kN \uparrow}$$

(i) Greatest vertical load,

$$\mathbf{F = 17.49\ kN \downarrow}$$

(ii) Reaction component at A,

$$A_x = 30.30\ kN \leftarrow$$

$$A_y = 15\ kN \uparrow$$

Ex. 8.9.55 : Determine the component of reaction at hinge A and tension in the cable BC as shown in Fig. P. 8.10.55(a).

SPPU : May 17, 5 Marks

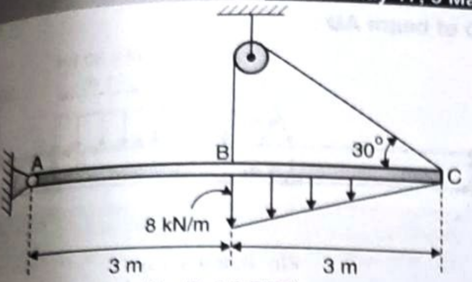


Fig. P. 8.9.55(a)

Soln. :

Being A is hinge, there are two reaction components.

The tension in the cable remains same at B as well as C.

FBD of beam ABC :

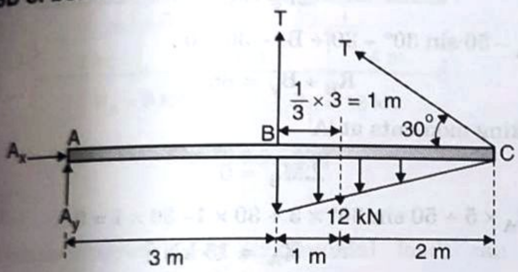


Fig. P. 8.9.55(b)

Magnitude of triangular load,

$$= \frac{1}{2} \times 8 \times 3 = 12 \text{ kN}$$

OR

$$= \left(\frac{8+0}{2}\right) 3 = 12 \text{ kN}$$

It will act the centroid of the triangle

For equilibrium;

$$\Sigma F_x = 0$$

$$A_x - T \cos 30° = 0 \quad \ldots (1)$$

$$\Sigma F_y = 0$$

$$A_y + T - 12 + T \sin 30° = 0$$

$$A_y + 1.5\,T = 12 \quad \ldots (2)$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$T \times 3 - 12 \times 4 + T \sin 30° \times 6 = 0 \ldots \quad (3)$$

$$\therefore \quad \mathbf{T = 8 \text{ kN}}$$

From Eq^n (1), $\quad A_x = 8 \cos 30°$

$$= \mathbf{6.93 kN \rightarrow}$$

From Eq^n (2), $\quad \therefore \quad \mathbf{A_y = 12 - 1.5 \times 8 = 0}$

(i) Components of reaction at hinge A,

$$\mathbf{A_x = 6.93 \text{ kN} \rightarrow} \quad \ldots\textbf{Ans.}$$

$$\mathbf{A_y = 0} \quad \ldots\textbf{Ans.}$$

(ii) Tension in the cable BC, **T = 8 kN** ...**Ans.**

Ex. 8.9.56 : simply supported beam loaded and supported is as shown in Fig. P. 8.9.56(a). If the reactions at supports are equal in magnitude, determine the overhang 'a'.

SPPU : May 16, 5 Marks

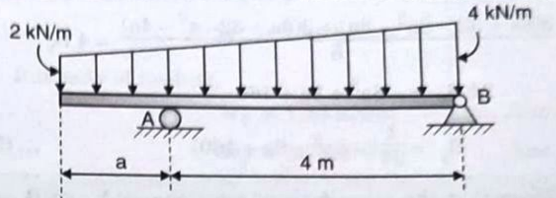


Fig. P. 8.9.56(a)

Soln. :

Dividing the trapezoidal load i.e. distributed load into two parts ;

(i) Rectangular load,

$$W_1 = 2(a+4) \text{ kN} = (2a+8) \text{ kN}$$

(ii) Triangular load,

$$W_2 = \frac{1}{2}(a+4)\,2 = (a+4) \text{ kN}$$

FBD of the beam :

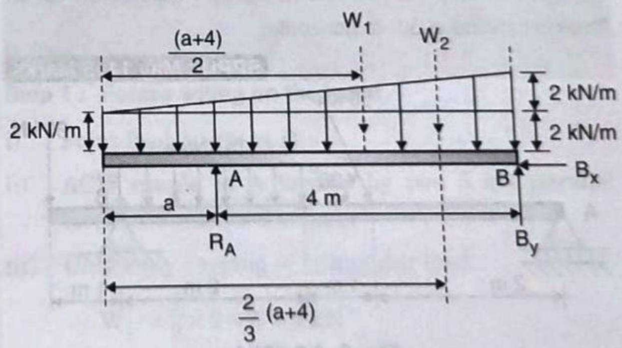


Fig. P. 8.9.56(b)

For equilibrium;

$$\Sigma F_x = 0$$

$$B_x = 0 \quad \ldots (1)$$

$$\Sigma F_y = 0$$

$$R_A - W_1 - W_2 + B_y = 0$$

$$R_A + B_y = W_1 + W_2$$

$$= (2a + 8) + (a + 4) \quad \dots (2)$$

$$R_A + B_y = 3a + 12$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$-W_1\left(\frac{a+4}{2} - a\right) - W_2\left[\frac{2}{3}(a+4) - a\right] + B_y \times 4 = 0$$

$$(2a+8)\left(\frac{a+4-2a}{2}\right) + (a+4)\left[\frac{2(a+4)-3a}{3}\right] = 4\,B_y$$

$$\frac{(2a+8)(4-a)}{2} + \frac{(a+4)(8-a)}{3} = 4\,B_y$$

$$\frac{3(8a+32-2a^2-8a)+2(8a+32-a^2-4a)}{6} = 4\,B_y$$

$$24\,B_y = -8a^2 + 8a + 160$$

$$\therefore \quad B_y = \frac{1}{24}(-8a^2 + 8a + 160) \quad \dots (3)$$

Given that the magnitude of reactions at A and B are equal.

$$\therefore \quad R_A = R_B = B_y$$

From Eqⁿ (2), $\quad B_y + B_y = 3a + 12$

$$2\,B_y = 3a + 12$$

From Eqⁿ (3), $2\left[\frac{1}{24}(-8a^2 + 8a + 160)\right] = 3a + 12$

$$8a^2 - 28a - 16 = 0$$

Solving for a,

$$a = 4\text{ m} \quad \dots \text{Ans.}$$

Ex. 8.9.57 : Determine the support reaction for the beam loaded and supported as shown in Fig. P. 8.9.57(a). 50 kN force is inclined at 30° to horizontal.

SPPU : May 14, 5 Marks

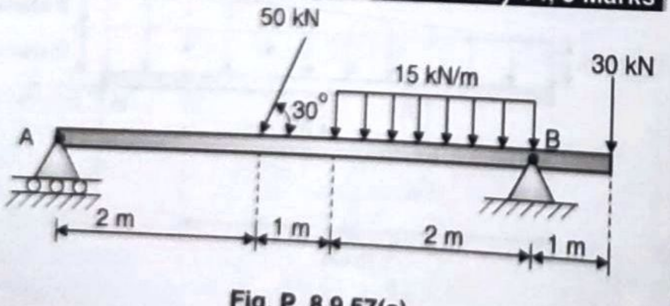


Fig. P. 8.9.57(a)

Soln. :

'A' is roller support and 'B' is hinge or pin support.

At 'A', reaction is perpendicular to the plane of rollers where as at support B, there are two reaction components.

Magnitude of ... (rectangular) load, W = 30 kN, will act at the midpoint of rectangular distributed load.

FBD of beam AB :

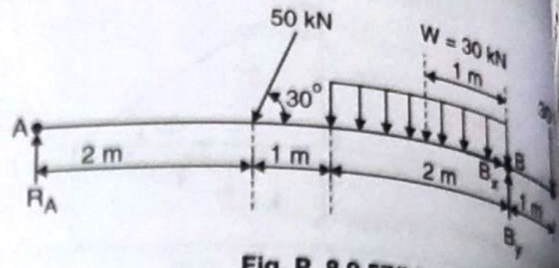


Fig. P. 8.9.57(b)

For equilibrium of beam AB,

$$\Sigma F_x = 0$$

$$-50\cos 30° + B_x = 0$$

$$\therefore \quad \mathbf{B_x = 43.3\ kN \rightarrow}$$

$$\Sigma F_y = 0$$

$$R_A - 50\sin 30° - 30 + B_y - 30 = 0$$

$$R_B + B_y = 85$$

Taking moments at 'A'

$$\Sigma M_B = 0$$

$$-R_A \times 5 + 50\sin 30° \times 3 + 30 \times 1 - 30 \times 1 = 0 \dots$$

$$\therefore \quad \mathbf{R_A = 15\ kN \uparrow}$$

From Eqⁿ (1), $\quad 15 + B_y = 85$

$$\therefore \quad \mathbf{B_y = 60\ kN \uparrow}$$

Reaction at B,

$$R_B = \sqrt{B_x^2 + B_y^2}$$

$$= \sqrt{(43.3)^2 + (60)^2}$$

$$= \mathbf{74\ kN}$$

Direction, $\quad \theta_B = \tan^{-1}\left|\frac{B_y}{B_x}\right|$

$$= \tan^{-1}\left(\frac{60}{43.3}\right)$$

$$= 54.18° \text{ w.r.t. x}$$

(i) Support reaction at B,

$$\mathbf{R_B = 74\ kN}$$

$$\mathbf{\theta_B = 54.18° \text{ w.r.t. 'x'}}$$

$$R_A + B_y = W_1 + W_2$$

$$= (2a + 8) + (a + 4)$$

$$R_A + B_y = 3a + 12 \quad \text{... (2)}$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$-W_1\left(\frac{a+4}{2} - a\right) - W_2\left[\frac{2}{3}(a+4) - a\right] + B_y \times 4 = 0$$

$$(2a+8)\left(\frac{a+4-2a}{2}\right) + (a+4)\left[\frac{2(a+4)-3a}{3}\right] = 4\,B_y$$

$$\frac{(2a+8)(4-a)}{2} + \frac{(a+4)(8-a)}{3} = 4\,B_y$$

$$\frac{3(8a+32-2a^2-8a)+2(8a+32-a^2-4a)}{6} = 4\,B_y$$

$$24\,B_y = -8a^2 + 8a + 160$$

$$\therefore \quad B_y = \frac{1}{24}(-8a^2 + 8a + 160) \quad \text{... (3)}$$

Given that the magnitude of reactions at A and B are equal.

$$\therefore \quad R_A = R_B = B_y$$

From Eq^n (2), $\quad B_y + B_y = 3a + 12$

$$2\,B_y = 3a + 12$$

From Eq^n (3), $2\left[\frac{1}{24}(-8a^2 + 8a + 160)\right] = 3a + 12$

$$8a^2 - 28a - 16 = 0$$

Solving for a,

$$a = 4\text{ m} \quad \text{... Ans.}$$

Ex. 8.9.57 : Determine the support reaction for the beam loaded and supported as shown in Fig. P. 8.9.57(a). 50 kN force is inclined at 30° to horizontal.

SPPU : May 14, 5 Marks

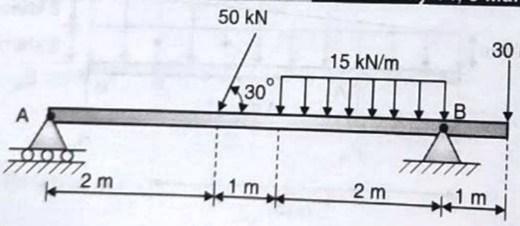


Fig. P. 8.9.57(a)

Soln. :

'A' is roller support and 'B' is hinge or pin support.

At 'A', reaction is perpendicular to the plane of rollers where as at support B, there are two reaction components.

Magnitude of distributed (rectangular) load, = 30 kN, will act at the midpoint of re[ctangular] distributed load.

FBD of beam AB :

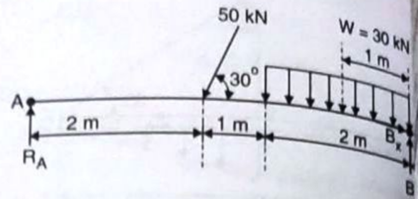


Fig. P. 8.9.57(b)

For equilibrium of beam AB,

$$\Sigma F_x = 0$$

$$-50\cos 30° + B_x = 0$$

$$\therefore \quad \mathbf{B_x = 43.3\text{ kN} \rightarrow}$$

$$\Sigma F_y = 0$$

$$R_A - 50\sin 30° - 30 + B_y - 30 = 0$$

$$R_B + B_y = 85$$

Taking moments at 'A'

$$\Sigma M_B = 0$$

$$-R_A \times 5 + 50\sin 30° \times 3 + 30 \times 1 - 30 \times 1 = 0$$

$$\therefore \quad \mathbf{R_A = 15\text{ kN}\uparrow}$$

From Eq^n (1), $\quad 15 + B_y = 85$

$$\therefore \quad \mathbf{B_y = 60\text{ kN}\uparrow}$$

Reaction at B,

$$R_B = \sqrt{B_x^2 + B_y^2}$$

$$= \sqrt{(43.3)^2 + (60)^2}$$

$$= \mathbf{74\text{ kN}}$$

Direction, $\quad \theta_B = \tan^{-1}\left|\frac{B_y}{B_x}\right|$

$$= \tan^{-1}\left(\frac{60}{43.3}\right)$$

$$= 54.18° \text{ w.r.t. x}$$

(i) Support reaction at B,

$$\mathbf{R_B = 74\text{ kN}}$$

$$\mathbf{\theta_B = 54.18° \text{ w.r.t. 'x'}}$$

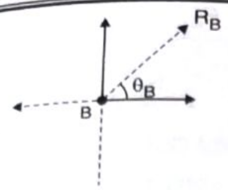


Fig. P. 8.9.57(c)

(ii) Support Reaction at A,

$$R_A = 15 \text{ kN} \uparrow \quad \text{...Ans.}$$

Ex. 8.9.58 : A beam supports a load varying uniformly from an intensity w_1 kN/m at left end to w_2 kN/m at the right end as shown in Fig. P. 8.9.58(a). If the reactions R_L = 6 kN and R_R = 12 kN, determine the intensity of loading w_1 and w_2.

SPPU : May 15, 5 Marks

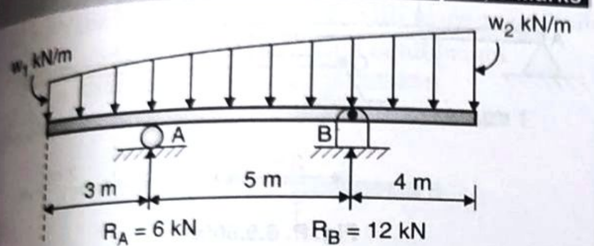


Fig. P. 8.9.58(a)

Soln. :

Uniformly varying load (trapezoidal load) can be divided into two parts :

(i) Rectangular load, magnitude,

$$W_1 = (w_1 \times 12) \text{ kN}$$

(ii) Triangular load, magnitude,

$$W_2 = \frac{1}{2}(12)(w_2 - w_1) = 6(w_2 - w_1) \text{ kN}$$

FBD of beam :

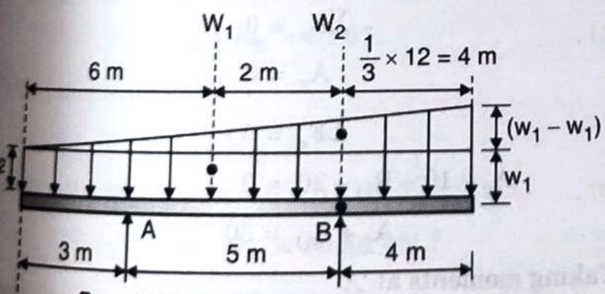


Fig. P. 8.9.58(b)

For equilibrium;

$$\Sigma F_y = 0$$

$$-W_1 - W_2 + R_A + R_B = 0$$

$$-12w_1 - 6w_2 + 6w_1 + 6 + 12 = 0$$

$$6 w_1 + 6w_2 = 18$$

$$w_1 + w_2 = 3 \quad \text{... (1)}$$

Taking moments at 'B'

$$\Sigma M_B = 0$$

$$-6 \times 5 + w_1 \times 2 = 0 \quad \text{... (2)}$$

$$-30 + 12w_1 \times 2 = 0$$

$$24 w_1 = 30$$

$$\therefore \quad w_1 = 1.25 \text{ kN/m}$$

From Eq^n (1), $\quad 1.25 + w_2 = 3$

$$\therefore \quad w_2 = 1.75 \text{ kN/m}$$

$\therefore$ Intensity of loading,

$$w_1 = 1.25 \text{ kN/m} \quad \text{... Ans.}$$

$$w_2 = 1.75 \text{ kN/m} \quad \text{... Ans.}$$

Ex. 8.9.59 : A beam ABCD having self weight 2 kN/m is subjected to additional load as shown in Fig. P. 8.9.59(a) Find the support reactions at B and C.

SPPU : May 02, 8 Marks

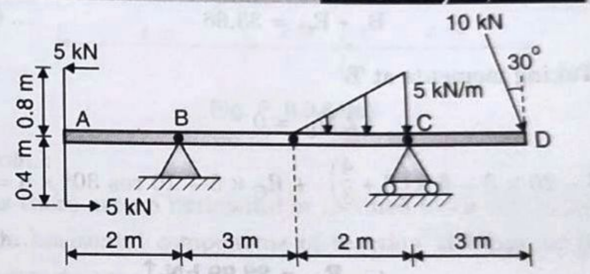


Fig. P. 8.9.59(a)

Soln. :

Step 1 : Forces acting on the beam

i) Point load 10 kN at D

ii) ACW couple at A formed by two 5 kN parallel forces = 5 × 1.2 = 6 kNm

iii) Uniformly varying or triangular load,

$$W_1 = \frac{1}{2} \times 2 \times 5 = 5 \text{ kN}$$

iv) Self weight of beam, W_2 = 2 × 10 = 20 kN which will act at C.G i.e., midpoint of beam.

v) Support reactions B_x and B_y at hinge B

vi) Support reaction at roller 'C'.

Step 2 : Free body diagram of beam

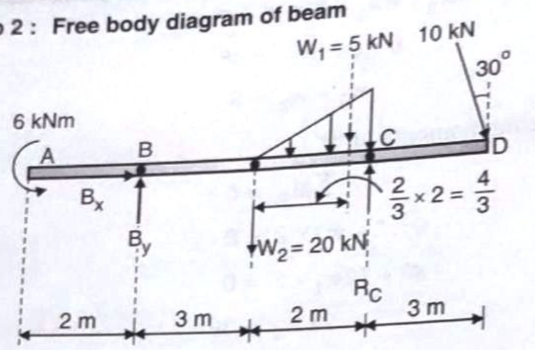


Fig. P. 8.9.59(b)

Step 3 : Equilibrium equations :

For equilibrium of beam;

$$\Sigma F_x = 0$$

$$B_x + 10 \sin 30° = 0 \qquad \text{... (1)}$$

$$\therefore \quad B_x = -5 \text{ kN} = \mathbf{5 \text{ kN} \leftarrow}$$

$$\Sigma F_y = 0$$

$$B_y - 20 - 5 + R_C - 10 \cos 30° = 0$$

$$B_y + R_C = 33.66 \qquad \text{... (2)}$$

Taking moments at 'B'

$$\Sigma M_B = 0$$

$$6 - 20 \times 3 - 5 \times \left(3 + \frac{4}{3}\right) + R_C \times 5 - 10 \cos 30° \times 8 = 0 \qquad \text{...(3)}$$

$$\therefore \quad \mathbf{R_C = 28.99 \text{ kN} \uparrow}$$

From Eq^n (2), $\quad B_y = 33.66 - 28.99$

$$= \mathbf{4.67 \text{ kN} \uparrow}$$

Reaction at B,

$$R_B = \sqrt{B_x^2 + B_y^2}$$

$$= \sqrt{5^2 + 4.67^2}$$

$$= \mathbf{6.84 \text{ kN}}$$

Direction, $\quad \theta_B = \tan^{-1} \left| \frac{B_y}{B_x} \right|$

$$= \tan^{-1}\left(\frac{4.67}{5}\right)$$

$$= 43.04° \text{ w.r.t. x}$$

(i) Support reaction at B,

$$\mathbf{R_B = 6.84 \text{ kN}} \qquad \text{...Ans.}$$

$$\mathbf{\theta_B = 43.04° \text{ w.r.t. x}} \qquad \text{...Ans.}$$

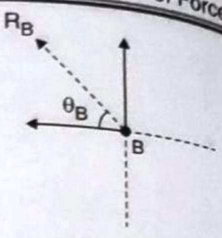


Fig. P. 8.9.59(c)

(ii) Support Reaction at A,

$$\mathbf{R_C = 28.99 \text{ kN} \uparrow} \qquad \text{...Ans.}$$

Ex. 8.9.60 : A compound beam is loaded as shown in Fig. P. 8.9.60(a). Find the reactions at the supports A, D and E.

SPPU : Dec. 11, 6 Marks

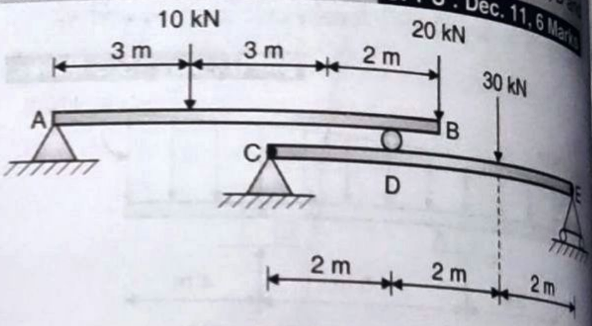


Fig. P. 8.9.60(a)

Soln. :

Supports A and C are hinges and D and E are rollers.

Consider **FBD of beam 'AB'**

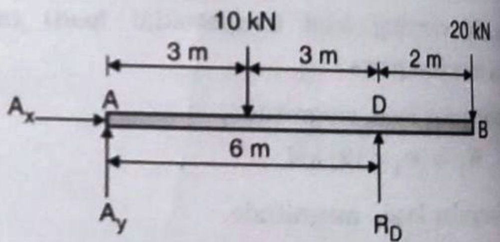


Fig. P. 8.9.60(b)

For equilibrium ;

$$\Sigma F_x = 0$$

$$A_x = 0 \qquad \text{...(1)}$$

$$\Sigma F_y = 0$$

$$A_y - 10 + R_D - 20 = 0$$

$$A_y + R_D = 30 \qquad \text{...(2)}$$

Taking moments at 'A'

$$\Sigma M_A = 0 \qquad \text{...(3)}$$

$$-10 \times 3 + R_D \times 6 - 20 \times 8 = 0$$

$$\therefore \quad \mathbf{R_D = 31.67 \text{ kN} \uparrow}$$

From Eq^n (2), $\quad A_y + 31.67 = 30$

$$A_y = -1.67 \text{ kN} = \mathbf{1.67 \text{ kN} \downarrow}$$

∴ Support reaction at A,

$$R_A = \sqrt{A_x^2 + A_y^2}$$

$$= A_y \qquad (\because A_x = 0)$$

$$\therefore \; \mathbf{R_A = 1.67 \; kN \downarrow}$$

Now consider **FBD of beam 'CDE'**

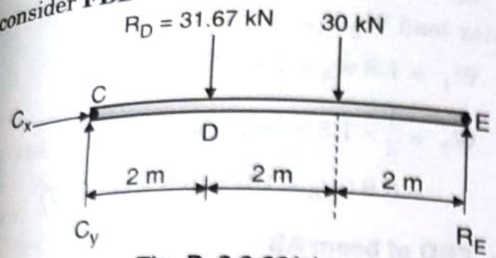


Fig. P. 8.9.60(c)

Note that the support reaction D i.e., R_D will act on both the beams AB and CDE with same magnitude but opposite direction, to maintain equilibrium.

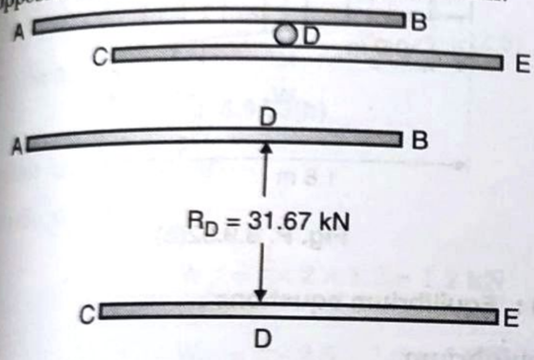


Fig. P. 8.9.60(d)

For equilibrium of beam 'CDE'

$$\Sigma F_x = 0$$

$$C_x = 0 \qquad \ldots (4)$$

$$\Sigma F_y = 0$$

$$C_y - 31.67 - 30 + R_E = 0$$

$$C_y + R_E = 61.67 \qquad \ldots (5)$$

Taking moments at 'C'

$$\Sigma M_C = 0$$

$$-31.67 \times 2 - 30 \times 4 + R_E \times 6 = 0 \qquad \ldots (6)$$

$$\therefore \; \mathbf{R_E = 30.55 \; kN \uparrow}$$

From Eq^n (5), $C_y + 30.55 = 61.67$

$$\therefore \; \mathbf{C_y = 31.11 \; kN \uparrow}$$

∴ Support reaction at C,

$$R_C = \sqrt{C_x^2 + C_y^2}$$

$$= C_y \qquad (\because C_x = 0)$$

$$\therefore \; \mathbf{R_C = 31.11 \; kN \uparrow}$$

Support reactions at

i) A, $\quad \mathbf{R_A = 1.67 \; kN \downarrow}$ ...Ans.

ii) D, $\quad \mathbf{R_D = 31.67 \; kN}$ ...Ans.

iii) E, $\quad \mathbf{R_E = 30.55 \; kN \uparrow}$ ...Ans.

iv) C, $\quad \mathbf{R_C = 31.11 \; kN \uparrow}$ ...Ans.

Note : Check for whole beam :

$$\Sigma F_y = 0$$

$$R_A + R_C + R_E - 10 - 20 - 30 = -1.67 + 31.11 + 30.55 - 60$$

$$= 0$$

Ex. 8.9.61 : Two beams ABC and CD are pinned together at C as shown in Fig. P. 8.9.61(a). Beam CD carries uniformly distributed load w kN/m which produces reaction at B as 60 kN. Determine w and find reaction at A and D.

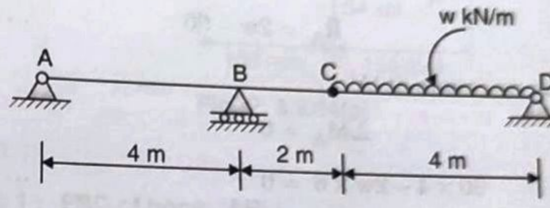


Fig. P. 8.9.61(a)

Soln. :

As there are no horizontal or inclined loads are acting, the horizontal components of reaction at hinge or pin supports are zero.

Given that $R_B = 60$ kN

Considering **FBD of beam 'CD'**

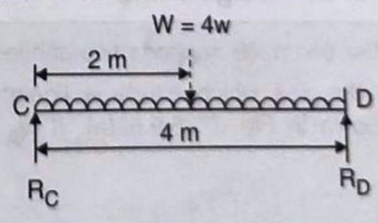


Fig. P. 8.9.61(b)

For equilibrium ;

$$\Sigma F_y = 0$$

$$R_C + R_D - 4w = 0$$

$$R_C + R_D = 4w \qquad \ldots (1)$$

Taking moments at 'C'

$$\Sigma M_C = 0$$

$$-4w \times 2 - R_D \times 4 = 0$$

$$\therefore \quad R_D = 2w \uparrow \qquad \ldots (2)$$

From Eq^n (1), $\quad R_C + 2w = 4w$

$$\therefore \quad R_D = 2w \uparrow \qquad \ldots (3)$$

Now, considering **FBD of beam 'ABC'**

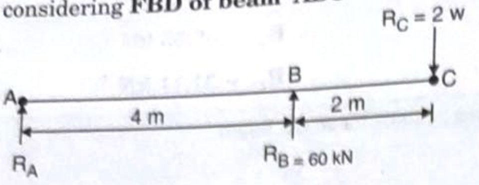


Fig. P. 8.9.61(c)

At internal hinge 'C', the magnitude of reaction for beam CD and beam ABC is same but direction is opposite. This is to maintain equilibrium.

For equilibrium ;

$$\Sigma F_y = 0$$

$$R_A + 60 - 2w = 0$$

$$R_A = 2w - 60 \qquad \ldots (4)$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$60 \times 4 - 2w \times 6 = 0 \qquad \ldots (5)$$

$$\therefore \quad \mathbf{w = 20\ kN/m}$$

From Eq^n (4), $\quad R_A = 2 \times 20 - 60$

$$\mathbf{= -20\ kN = 20\ kN \downarrow}$$

From Eq^n (2), $\quad \mathbf{R_D = 2 \times 20 = 40\ kN \uparrow}$

(i) UDL, $\quad$ **w = 20 kN/m** ...**Ans.**

(ii) Reaction at A, $\quad \mathbf{R_A = 20\ kN \downarrow}$ ...**Ans.**

(iii) Reaction at D, $\quad \mathbf{R_D = 40\ kN \uparrow}$ ...**Ans.**

Ex. 8.9.62 : The beam AB supports two concentrated loads and rests on the soil which exerts a linearly distributed reaction as shown in Fig. P. 8.9.62(a). If w_A = 18 kN/m, determine :

(i) Distance, a

(ii) The corresponding value of 'w_B' in kN/m.

SPPU : Dec. 98, 8 Marks

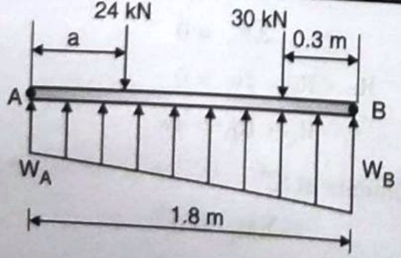


Fig. P. 8.9.62(a)

Soln. :

Step 1 : Forces acting on the beam AB :

(i) Point loads 24 kN and 30 kN

(ii) Linearly distributed load

It can be divided into rectangular load, W_1 and triangular load W_2.

$$W_1 = 1.8\ w_A = 1.8 \times 18 = 32.4\ kN$$

$$W_2 = \frac{1}{2} \times 1.8 \times (w_B - w_A)$$

$$= 0.9\ (w_B - 18) = (0.9\ w_B - 16.2)$$

Step 2 : FBD of beam AB

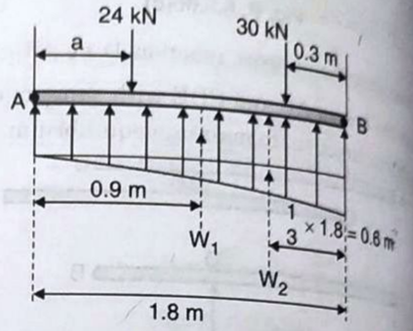


Fig. P. 8.9.62(b)

Step 3 : Equilibrium equations :

For equilibrium ;

$$\Sigma F_y = 0$$

$$-24 - 30 + W_1 + W_2 = 0$$

$$W_1 + W_2 = 54$$

$$32.4 + 0.9\ w_B - 16.2 = 54$$

$$\therefore \quad \mathbf{w_B = 42\ kN/m}$$

Taking moments at 'A'

$$\Sigma M_A = 0$$

$$-24 \times a - 30 \times 1.5 + W_1 \times 0.9 + W_2 \times 1.2 = 0$$

$$-24a - 45 + 32.4 \times 0.9 + (0.9 \times 42 - 16.2) \times 1.2 = 0$$

$$\mathbf{a = 0.42\ m}$$

(i) Distance, $\quad \mathbf{a = 0.42\ m}$ ...**Ans.**

(ii) Value of $\quad \mathbf{w_B = 42\ kN/m}$ ...**Ans.**

Ex. 8.9.63 : The beam AB with pin at 'B' and roller at 'A' is loaded as shown in Fig. P. 8.9.63(a). Determine the reactions at the supports A and B.

SPPU : May 19, 6 Marks

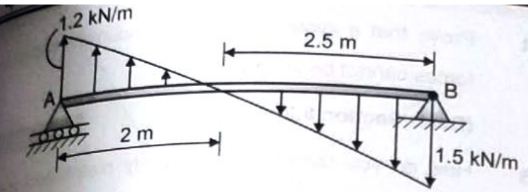


Fig. P. 8.9.63(a)

Soln. :

Step 1 : FBD of beam 'AB' :

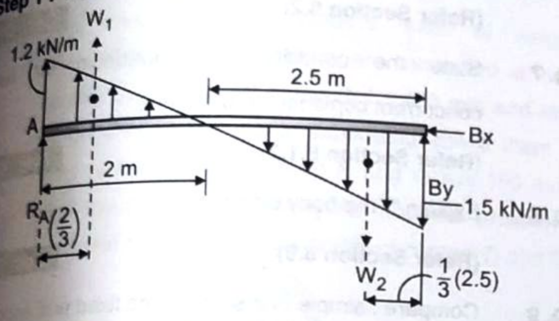


Fig. 8.9.63(b)

'A' is roller and 'B' is hinge.

Magnitude of UDL,

$$W_1 = \frac{1}{2} \times 2 \times 1.2 = 1.2 \text{ kN}$$

$$W_2 = \frac{1}{2} \times 2.5 \times 1.5 = 1.875 \text{ kN}$$

Step 2 : Equilibrium equations :

For equilibrium;

$$\Sigma F_x = 0$$

$$B_x = 0 \quad \text{...(1)}$$

$$\Sigma F_y = 0$$

$$R_A + W_1 - W_2 + B_y = 0$$

$$R_A + B_y = -W_1 + W_2 = -1.2 + 1.875$$

$$\therefore R_A + B_y = 0.675 \quad \text{...(2)}$$

Taking moments at 'B',

$$\Sigma M_B = 0$$

$$-R_A \times 4.5 - W_1 \times \left(4.5 - \frac{2}{3}\right) + W_2 \times \left(\frac{2.5}{3}\right) = 0$$

$$-4.5 R_A - 1.2\,(3.83) + 1.875\,(0.83) = 0$$

$$R_A = -0.675 \text{ kN}$$

$$= \mathbf{0.675 \text{ kN}\downarrow} \quad \text{...Ans.}$$

Step 3 : Support reaction at 'B' :

From Eq^n (2),

$$-0.675 + B_y = 0.675$$

$$\therefore B_y = 1.35 \text{ kN}\uparrow$$

$\therefore$ Reaction at B,

$$R_B = \sqrt{B_x^{\,2} + B_y^{\,2}}$$

$$= B_y \quad (\because B_x = 0)$$

$$\therefore \mathbf{R_B = 1.35 \text{ kN}\uparrow} \quad \text{...Ans.}$$

Ex. 8.9.64 : Boom AB is supported by a pin at 'A' and cable BC as shown in Fig. P.8.9.64. Determine the reactions at pin 'A' and the tension in the cable BC.

SPPU : May 19, 6 Marks

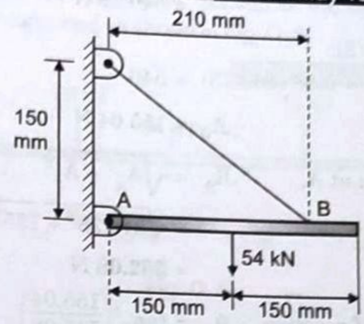


Fig. P. 8.9.64(a)

Soln. :

Step 1 : FBD of boom 'AB' :

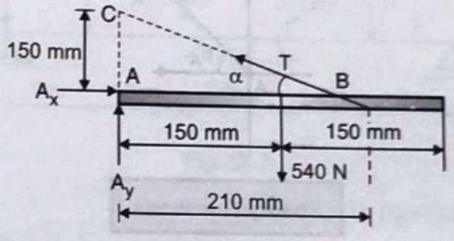


Fig. P. 8.9.64(b)

Let 'T' be the tension in the string.

'A' is hinge / pin support

From Fig. P. 8.9.64(b),

$$\tan \alpha = \left(\frac{150}{210}\right)$$

$$\therefore \alpha = 35.53^\circ$$

Step 2 : Equilibrium equations :

For equilibrium of boom AB;

$$\Sigma F_x = 0$$

$$A_x - T \cos 35.53^\circ = 0$$

$$A_x = 0.814\, T \quad \text{...(1)}$$

$$\Sigma F_y = 0$$

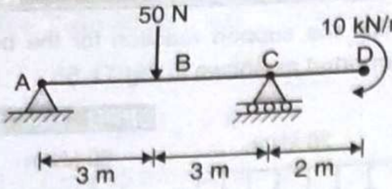


Fig. Q. 9

[Ans. : R_A = 23.33 kN (↑) ; R_C = 26.67 kN (↑)]

Q. 10 A cylinder of weight 1000 N is rest on the stair as shown in Fig. Q. 10. Determine the minimum magnitude of force P to raise the cylinder over the step. **SPPU : Dec. 14, 6 Marks**

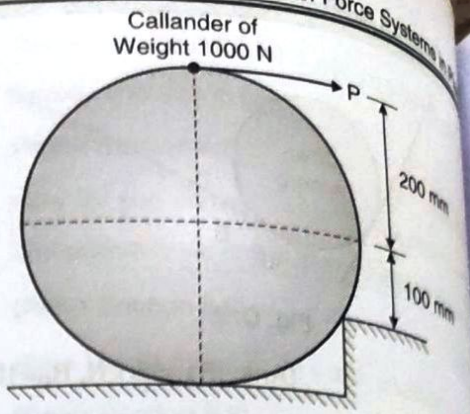


Fig. Q. 10

[Ans. : P = 577

CHAPTER

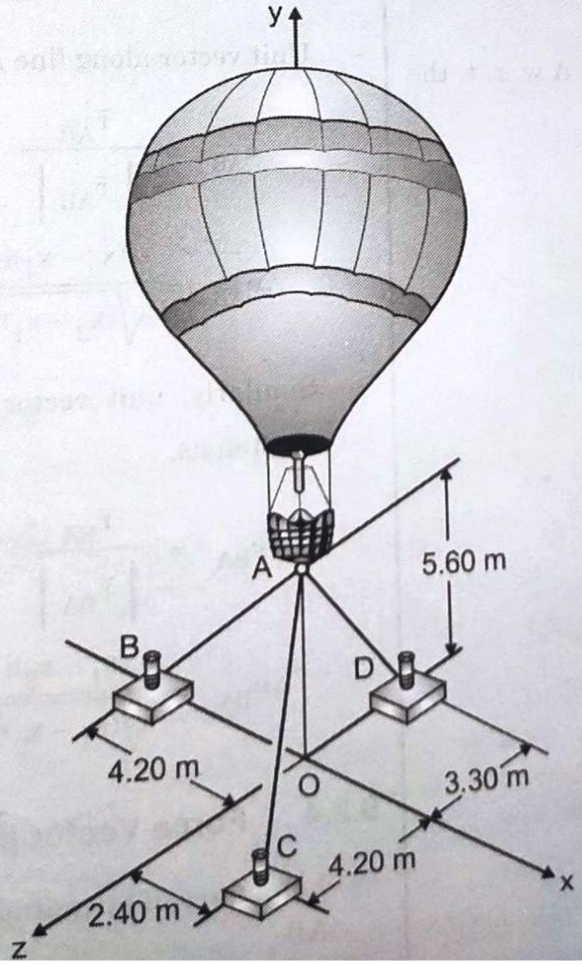
9

UNIT - III

Space Force Systems

Introduction : Till now, we have studied the resultant and equilibrium of different types of force systems in 2-Dimentional plane. In this chapter, we shall study the resolution of force and equilibrium of concurrent and parallel force systems in 3-Dimentional plane or non coplanar force systems.

- Type 1 : Based on Components of Force
- Type 2 : Based on Resultant of Concurrent Force System in Space
- Type 3 : Based on Equilibrium of Concurrent Force System in Space
- Type 4 : Based on Resultant of Parallel Force System in Space



9.1 Introduction to Space Force Systems

If the force systems are acting in differeSnt planes, then it is known as ***non-coplanar force systems.***

Non-coplanar force system is also known as ***"space force system"*** *or* ***"3-Dimensional force system".***

In this type of system, to analyse the forces we resolve them into 3 components along x, y and z directions.

9.2.1 Unit Vector ($\bar{e}$) :

- A unit vector is a vector having magnitude equal to one unit.
- A unit vector can be in any direction.
- Unit vector gives the direction of line along which it is acting.
- Unit vector is also called as "directional vector"
- It is represented by $\bar{e}$.

9.2.2 Position or Radius Vector ($\bar{r}$) :

- Let i, j and k be the unit vectors along x, y and z axes respectively.
- (x_1, y_1, z_1) be the coordinates of point A w. r. t. the origin.

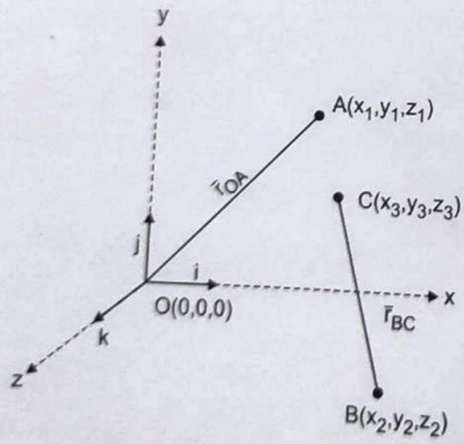


Fig. 9.2.1

- Position vector $\bar{r}_{OA}$ can be written as

$$\bar{r}_{OA} = (x_1 - 0)i + (y_1 - 0)j + (z_1 - 0)k$$

$$\bar{r}_{OA} = x_1 i + y_1 j + z_1 k$$

(This is the position of A, w. r. t. 'O')

- Position Vector

$$\bar{r}_{BC} = (x_3 - x_2)i + (y_3 - y_2)j + (z_3 - z_2)k$$

(Position of C w. r. t. B)

OR

Position Vector

$$\bar{r}_{CB} = (x_2 - x_3)i + (y_2 - y_3)j + (z_2 - z_3)k$$

(Position of B w. r. t. C)

9.2.3 Unit Vector along a Given Line

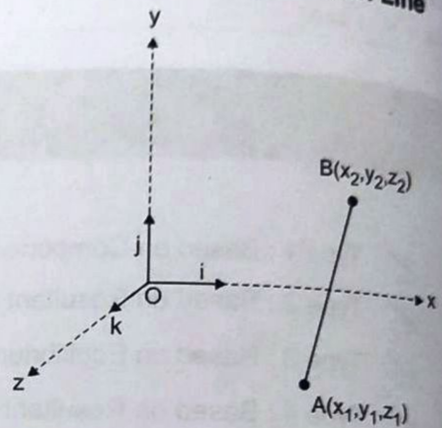


Fig. 9.2.2

- Unit vector along line AB can be written as,

$$\bar{e}_{AB} = \frac{\bar{r}_{AB}}{|\bar{r}_{AB}|}$$

$$\therefore \bar{e}_{AB} = \frac{(x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

- Similarly, unit vector along, the line BA can be written as,

$$\bar{e}_{BA} = \frac{\bar{r}_{BA}}{|\bar{r}_{BA}|}$$

$$\therefore \bar{e}_{BA} = \frac{(x_1 - x_2)i + (y_1 - y_2)j + (z_1 - z_2)k}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}}$$

9.2.4 Force Vector $|\bar{F}|$:

- Let a force of magnitude 'F' be acting along the line AB.

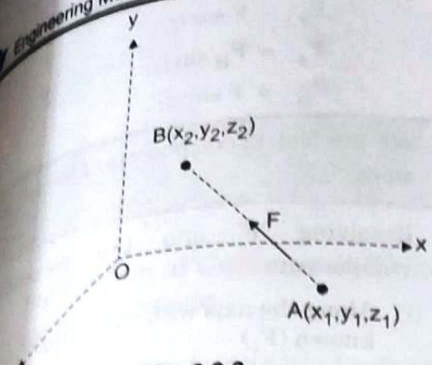


Fig. 9.2.3

Force vector $\bar{F}$ can be written as

$$\bar{F} = F \cdot \bar{e}_{AB}$$

F = Magnitude of force

$\bar{e}_{AB}$ = Unit vector along the line AB (It gives the direction of line AB)

9.2.5 Direction Cosines :

Let a force 'F' be acting at the origin in the direction shown in Fig. 9.2.4.

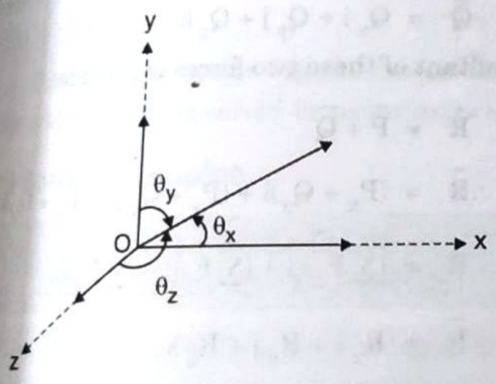


Fig. 9.2.4

Let θ_x, θ_y and θ be the components of force 'F' along x, y and z axes respectively.

$$F_x = F \cos \theta_x$$

$$F_y = F \cos \theta_y$$

$$F_z = F \cos \theta_z$$

Force vector $\bar{F}$ can be written as,

$$\bar{F} = F_x i + F_y j + F_z k \qquad ...(1)$$

Where i, j and k are the unit vectors along x, y and z axes respectively.

$$\therefore \bar{F} = (F \cos \theta_x) i + (F \cos \theta_y) j + (F \cos \theta_z) k$$

Also, $$\bar{F} = F \cdot \bar{e} \qquad ...(2)$$

$\therefore$ Unit vector,

$$\bar{e} = \cos \theta_x i + \cos \theta_y j + \cos \theta_z k$$

Magnitude of unit vector,

$$|\bar{e}| = 1$$

$$\therefore \quad |\bar{e}| = \sqrt{\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z} = 1$$

Squaring on both sides

$$\boxed{\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1}$$

→ Rule of direction cosines

Where as, $\cos \theta_x$, $\cos \theta_y$ and $\cos \theta_z$ are known as direction – cosines.

From Eq^n (1) and (2),

$$F_x = F \cos \theta_x$$

$$F_y = F \cos \theta_y$$

$$F_z = F \cos \theta_z$$

Direction cosines can be calculated as,

$$\cos \theta_x = \left(\frac{F_x}{F}\right)$$

$$\cos \theta_y = \left(\frac{F_y}{F}\right) \quad \text{and}$$

$$\cos \theta_z = \left(\frac{F_z}{F}\right)$$

Note : In 2-D plane, to define the direction of force any one angle made by force either w. r. t. x-axis or y-axis is sufficient.

In 3-D system, to define the direction of force, we must know the angle made by force w. r. t. x, y and z axes i.e. θ_x, θ_y and θ_z.

9.2.6 Components of Force in Space :

Type 1 : If the coordinates of points are given

- Write unit vector along the line in which force is acting.
- Force vector,

$$\bar{F} = F \cdot \bar{e}$$

Where, F = Magnitude of force

– We get,

$$\bar{F} = F_x i + F_y j + F_z k$$

Where, F_x, F_y and F_z are the components of force along x, y and z axes.

Type 2: If the direction of force along the axes is given

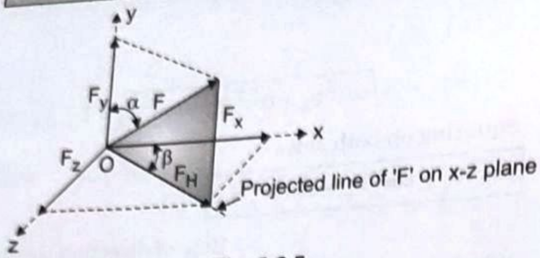


Fig. 9.2.5

Let a force 'F' be acting at angle 'α' w. r. t. y-axis

Let force 'F' be projected on x-z plane.

β = Angle made by projected line w. r. t. x-axis.

Components of 'F' :

– 'F' is making angle α w. r. t. y-axis ∴ y-component of F is given by $\mathbf{F_y = F\cos\alpha}$.

– Angle between y-axis and x-z plane (projected line) is 90°.

∴ Component of force 'F' along the projected line is

$$F_H = F\sin\alpha.$$

– Resolving F_H further into two perpendicular components,

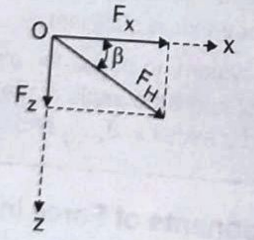


Fig.9.2.6

– F_H is making angle 'β' w. r. t. x-axis.

∴ x-component of F_H,

$$\mathbf{F_x = F_H\cos\beta}$$

z– component, $\mathbf{F_z = F_H\sin\beta}$

– The components of force 'F' are :

$$F_x = F_H\cos\beta$$

$$F_y = F\cos\alpha$$

$$F_z = F_H\sin\beta$$

Where, $F_H = F\sin\alpha$

Note : We are finding components of force F in two steps.

Step 1: Resolving 'F' into two perpendicular components.

(i) Along the axis with which angle is known (F_y)

(ii) Perpendicular to the axis (F_H)

Step 2 : Resolving F_H further into two perpendicular components

(i) Along the 2nd axis (F_x)

(ii) Along the 3rd axis (F_z)

9.2.7 Resultant of Concurrent Force System in Space :

Let $\bar{P}$ and $\bar{Q}$ be the two concurrent forces acting at a point in space.

$$\bar{P} = P_x i + P_y j + P_z k \text{ and}$$

$$\bar{Q} = Q_x i + Q_y j + Q_z k$$

Resultant of these two forces is given by,

$$\bar{R} = \bar{P} + \bar{Q}$$

$$\therefore \bar{R} = (P_x + Q_x)i + (P_y + Q_y)j + (P_z + Q_z)k$$

$$\bar{R} = (\sum F_x)\, i + (\sum F_y) j + (\sum F_z) k$$

$$\bar{R} = R_x i + R_y j + R_z k$$

Where, $R_x = \sum F_x$ = x –component of resultant

$R_y = \sum F_y$ = y - component of resultant

$R_z = \sum F_z$ = z-component of resultant

Magnitude of resultant,

$$R = \sqrt{R_x^{\,2} + R_y^{\,2} + R_t^{\,2}}$$

$$= \sqrt{(\sum F_x)^2 + (\sum F_y)^2 + (\sum F_z)^2}$$

To find direction of resultant;

$$R_x = R\cos\theta_x$$

$$R_y = R\cos\theta_y \text{ and}$$

$$R_z = R\cos\theta_z$$

$$\therefore \theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{\sum F_x}{R}\right) \text{ w.r.t x-axis}$$

$$\therefore \theta_y = \cos^{-1}\left(\frac{R_y}{R}\right) = \cos^{-1}\left(\frac{\sum F_y}{R}\right) \text{ w.r.t. y-axis}$$

$$\therefore \theta_z = \cos^{-1}\left(\frac{R_z}{R}\right) = \cos^{-1}\left(\frac{\sum F_z}{R}\right) \text{ w.r.t. z-axis}$$

9.2.8 Equilibrium of Concurrent Force system in space :

If the concurrent force system in space is in equilibrium, then the resultant of the force system should be zero.

$$\therefore \bar{R} = R_x\, i + R_y\, j + R_z\, k = 0$$

$$\bar{R} = (\sum F_x)\, i + (\sum F_y)\, j + (\sum F_z)\, k = 0$$

$$\therefore \sum F_x = 0$$

$$\sum F_y = 0 \text{ and}$$

$$\sum F_z = 0$$

9.2.9 Parallel Force System in Space :

In parallel force system, all the forces are acting in the same direction i.e, the direction in which they are parallel to each other.

$\therefore$ Directional or vector aspects can be eliminated and the problems can be solved in scalar aspects.

9.2.10 Solved Examples

Type 1 : BASED ON COMPONENTS OF FORCE

Ex. 9.2.1: A force acts at the origin in a direction defined by the angles $\theta_y = 65°$ and $\theta_z = 40°$. If the x-component of the force is –750 N, determine the other components and magnitude of the force. **SPPU : May 98, 4 Marks**

Soln. :

Step 1 : Rule of direction cosines

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos^2\theta_x + \cos^2 65° + \cos^2 40° = 1$$

$$\therefore \cos\theta_x = -0.484 \quad (\because F_x \text{ is negative})$$

$$\theta_x = 119°$$

Step 2 : Magnitude of force (F) :

We know that, $F_x = F\cos\theta_x$

$$-750 = F \times -0.484$$

$$\therefore F = 1550 \text{ N} \qquad \text{... Ans.}$$

Step 3 : Components F_y and F_z

$$F_y = F\cos\theta_y = 1550\cos 65° = 654.88 \text{ N} \qquad \text{...Ans.}$$

$$F_z = F\cos\theta_z = 1550\cos 40° = 1187.37 \text{ N} \qquad \text{...Ans.}$$

Ex. 9.2.2 : A tower guy is anchored by means of a bolt at A. The tension in the wire is 2500 N.

Determine :

a) The components F_x, F_y and F_z of the force acting on the bolt.

b) The angle θ_x, θ_y and θ_z defining direction of force

SPPU : May 10, 8 Marks

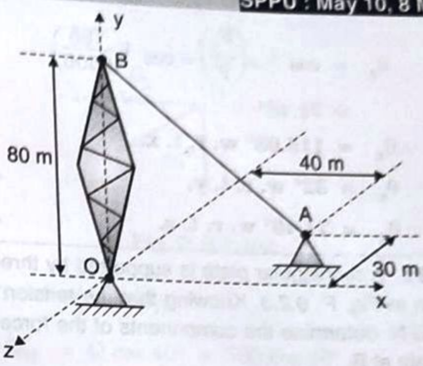


Fig. P. 9.2.2

Soln. :

Step 1 : Co-ordinates :

A (40, 0, – 30)

B (0, 80, 0)

Step 2 : Force vector $(\bar{F})$:

We have to find out the components of force acting on the bolt.

$\therefore$ The direction of force is from A to B.

$$\therefore (\bar{F}) = F.\,\bar{e}_{AB}$$

$$\bar{F} = 2500\left[\frac{(0-40)\, i + (80-0)\, j + (0+30)\, k}{\sqrt{40^2 + 80^2 + 30^2}}\right]$$

$$\therefore \bar{F} = -1060\, i + 2120\, j + 795\, k$$

Step 3 : Components of force :

$$\bar{F} = F_x\, i + F_y\, j + F_z\, k$$

$\therefore$ x-component,

$$F_x = -1060 \text{ N} \qquad \text{... Ans.}$$

... Ans.

y-component,

$F_y = 2120\ N$ ... Ans.

and z-component,

$F_z = 795\ N$

Step 4 : Angles $\theta_x, \theta_y, \theta_z$:

We know that,

$$F_x = F \cos \theta_x$$

$$\therefore \theta_x = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{-1060}{2500}\right) = 115.08^\circ$$

Similarly,

$$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right) = \cos^{-1}\left(\frac{2120}{2500}\right) = 32^\circ$$

$$\theta_z = \cos^{-1} = \left(\frac{F_y}{F}\right) = \cos^{-1}\left(\frac{795}{2500}\right)$$

$$= 71.46^\circ$$

$\therefore \theta_x = 115.08^\circ$ **w. r. t. x.** ... Ans.

$\theta_y = 32^\circ$ **w. r. t. y.** ... Ans.

$\therefore \theta_z = 71.46^\circ$ **w. r. t. z.** ... Ans.

Ex. 9.2.3 : A rectangular plate is supported by three cables as shown in Fig. P. 9.2.3. Knowing that the tension in cable AB is 450 N, determine the components of the force exerted on the plate at B.

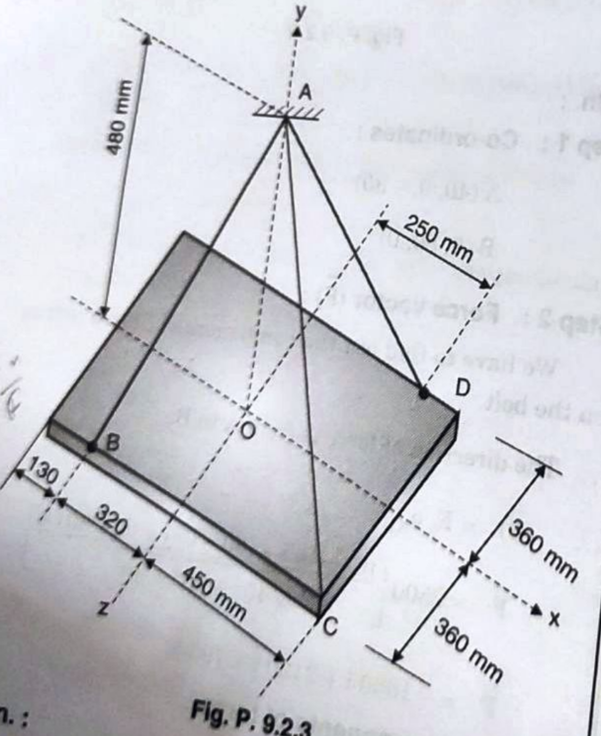


Fig. P. 9.2.3

$\theta = \frac{F_x}{F}$

Soln. :

Step 1 : Co-ordinates

A (0, 480, 0)

B (–320, 0, 360)

Step 2 : Force vector ($\bar{F}$) :

Force of magnitude 450 acting at B from B...

$$\therefore (\bar{F}) = F \cdot \bar{e}_{BA}$$

$$= 450\left[\frac{(0+320)\,i + (480-0)\,j + (0-360)\,k}{\sqrt{320^2 + 480^2 + 360^2}}\right]$$

$$\therefore \bar{F} = 211.76\, i + 317.65\, j - 238.23\, k$$

Step 3 : Components of force :

$$\bar{F} = F_x\, i + F_y\, j + F_z\, k$$

$$\therefore F_x = \mathbf{211.76\ N}$$

$$F_y = \mathbf{317.65\ N}$$

$$F_z = \mathbf{-238.23\ N}$$

Step 4 : Direction of force w. r. t. components :

$$\therefore \theta_x = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{211.76}{450}\right)$$

$$= \mathbf{61.93^\circ}$$

$$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right) = \cos^{-1}\left(\frac{317.65}{450}\right)$$

$$= \mathbf{45.10^\circ}$$

$$\theta_z = \cos^{-1}\left(\frac{F_z}{F}\right) = \cos^{-1}\left(\frac{-238.23}{450}\right)$$

$$= \mathbf{121.96^\circ}$$

Ex. 9.2.4 : Determine

(i) the x, y, z components of 600 N and 500 N forces.

(ii) the angles θ_x, θ_y, θ_z that the forces form with the coordinate axes.

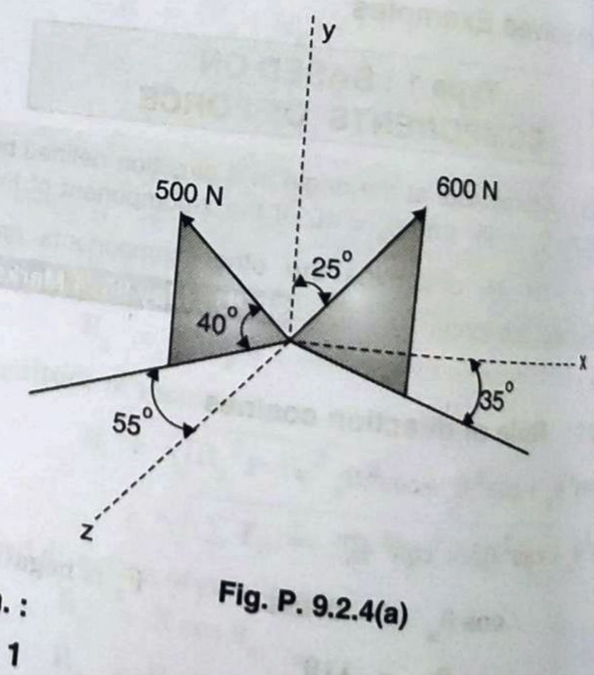


Fig. P. 9.2.4(a)

Soln. :

Step 1

Let 600 N = P

500 N = Q

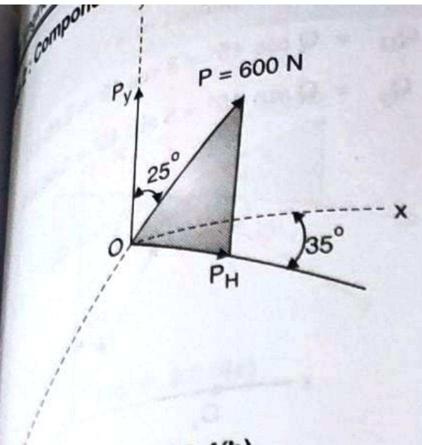


Fig. P. 9.2.4(b)

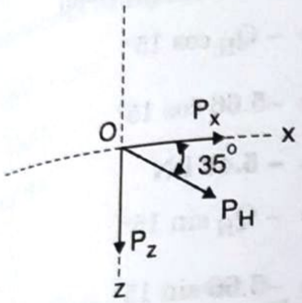


Fig. P. 9.2.4(c)

P_y is perpendicular to P_H

$P_y = P\cos 25° = 600\cos 25°$

$= 543.78$ N ...Ans.

$P_H = P\sin 25° = 600\sin 25°$

$= 253.57$ N ...Ans.

$P_x = P_H\cos 35° = 253.57\cos 35°$

$= 207.71$ N ...Ans.

$P_z = P_H\sin 35° = 253.57\sin 35°$

$= 145.44$ N ...Ans.

$\theta_x = \cos^{-1}\left(\frac{P_x}{P}\right) = \cos^{-1}\left(\frac{207.71}{600}\right)$

$= 69.74°$ ...Ans.

$\theta_y = \cos^{-1}\left(\frac{P_y}{P}\right) = \cos^{-1}\left(\frac{543.78}{600}\right)$

$= 25°$ ...Ans.

$\theta_z = \cos^{-1}\left(\frac{P_z}{P}\right) = \cos^{-1}\left(\frac{145.44}{600}\right) = \mathbf{75.97°}$ ...Ans.

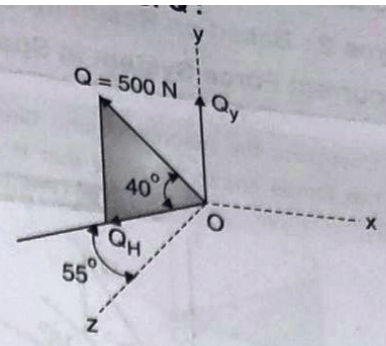


Fig. P. 9.2.4(d)

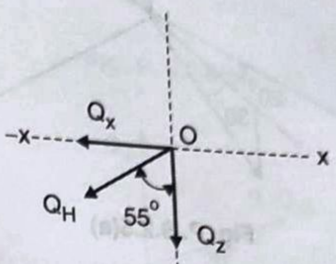


Fig. P. 9.2.4(e)

Q_y is perpendicular to Q_H

$Q_H = Q\cos 40° = 500\cos 40°$

$= \mathbf{383.02\ N}$ ...Ans.

$Q_y = Q\sin 40° = 500\sin 40°$

$= \mathbf{321.40\ N}$ ...Ans.

$Q_x = -Q_H\sin 55° = -383.025\sin 55°$

$= \mathbf{-313.75N}$ ...Ans.

$Q_z = Q_H\cos 55° = 383.02\cos 55°$

$= \mathbf{219.69\ N}$ ...Ans.

$\theta_x = \cos^{-1}\left(\frac{Q_x}{Q}\right) = \cos^{-1}\left(\frac{-313.75}{500}\right)$

$= \mathbf{128.86°}$ ...Ans.

$\theta_y = \cos^{-1}\left(\frac{Q_y}{Q}\right) = \cos^{-1}\left(\frac{321.40}{500}\right)$

$= \mathbf{50°}$ ...Ans.

$\theta_z = \cos^{-1}\left(\frac{Q_z}{Q}\right) = \cos^{-1}\left(\frac{219.69}{500}\right)$

$= \mathbf{63.93°}$ ...Ans.

Type 2 : Based on Resultant of Concurrent Force System in Space

Ex. 9.2.5 : Determine the magnitude and direction of the resultant of two forces shown knowing that P = 4 kN and Q = 8 kN. **SPPU : May 09, 6 Marks**

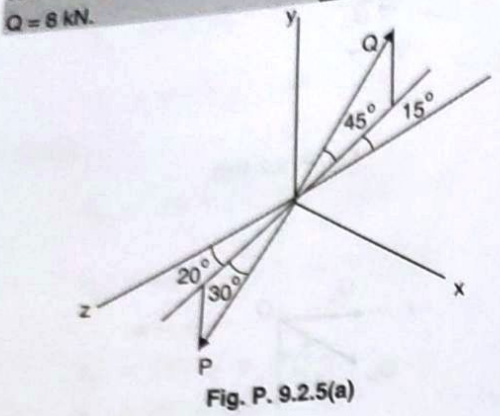


Fig. P. 9.2.5(a)

Soln. :

Step 1 : Components of force :

Components of 'P'

Let P_H be the component in x - z plane

$$P_H = P\cos 30° = 4\cos 30° = \mathbf{3.46\ kN}$$

$$P_y = -P\sin 30° = -4\sin 30° = \mathbf{-2\ kN}$$

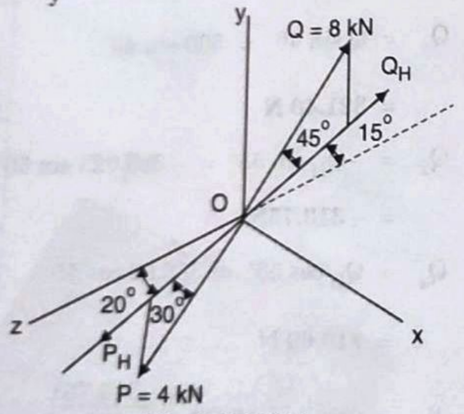


Fig. P. 9.2.5(b)

$$P_x = P_H\sin 20° = 3.46\sin 20° = \mathbf{1.183\ kN}$$

$$P_z = P_H\cos 20° = 3.46\cos 20° = \mathbf{3.25\ kN}$$

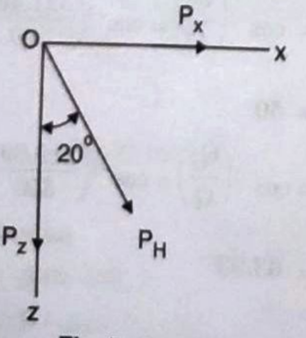


Fig. P. 9.2.5(c)

Component of 'Q'

Let Q_H be the component in 'x - z' plane

$$Q_H = Q\cos 45° = 8\cos 45° = \mathbf{5.66\ kN}$$

$$Q_y = Q\sin 45° = 8\sin 45° = \mathbf{5.66\ kN}$$

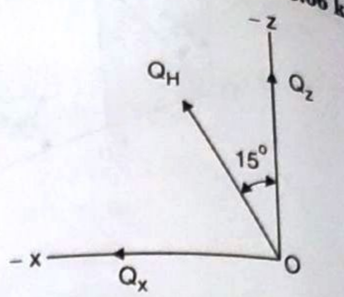


Fig. P. 9.2.5(d)

$$Q_z = -Q_H\cos 15°$$

$$= -5.66\cos 15°$$

$$= \mathbf{-5.46\ kN}$$

$$Q_x = -Q_H\sin 15°$$

$$= -5.66\sin 15°$$

$$= \mathbf{-1.46\ kN}$$

Step 2 : Resultant force :

$$\bar{R} = \bar{P} + \bar{Q}$$

$$= (P_x + Q_x)\,i + (P_y + Q_y)\,j + (P_z + Q_z)k$$

$$= (1.183 - 1.46)\,i + (-2 + 5.66)\,j + (3.25 - 5.46)k$$

$$= -(0.277)\,i + (3.66)\,j + -(2.21)k$$

Step 3 : Magnitude of resultant :

$$R = \sqrt{(0.273)^2 + (3.66)^2 + (2.21)^2}$$

$$= \mathbf{4.28\ kN} \quad \text{...Ans.}$$

Direction of resultant,

$$\theta_x = \tan^{-1}\left(\frac{-0.277}{4.28}\right)$$

$$= \mathbf{-3.70°\ w.\ r.\ t.\ x\text{-}axis} \quad \text{... Ans.}$$

$$\theta_y = \tan^{-1}\left(\frac{3.66}{4.28}\right)$$

$$= \mathbf{40.53°\ w.\ r.\ t.\ y\text{-}axis} \quad \text{... Ans.}$$

$$\theta_z = \tan^{-1}\left(\frac{-2.21}{4.28}\right)$$

$$= \mathbf{-27.31°\ w.\ r.\ t.\ z\text{-}axis} \quad \text{... Ans.}$$

Ex. 9.2.6: Find the magnitude and direction of the resultant of the two forces shown in Fig. P. 9.2.6(a). Lines oa and ob are in x-y plane.

SPPU : May 04,8 Mark

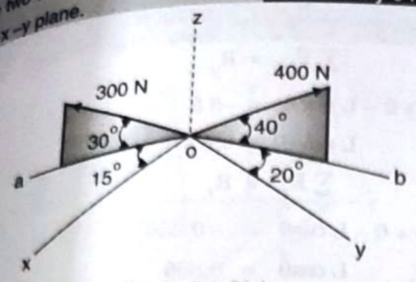


Fig. P. 9.2.6(a)

Soln. :

Step 1 : Components of forces :

Let $P = 300$ N and

$Q = 400$ N

Components of 'P'

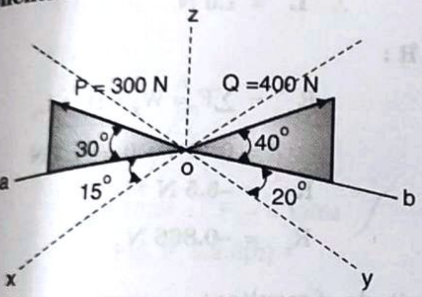


Fig. P. 9.2.6(b)

'P' is making 30° with line line 'oa'.

$$\therefore \; P_{oa} = P\cos 30° = 300\cos 30° = \mathbf{259.8\ N}$$

$$P_z = P\sin 30° = 300\sin 30° = \mathbf{150\ N}$$

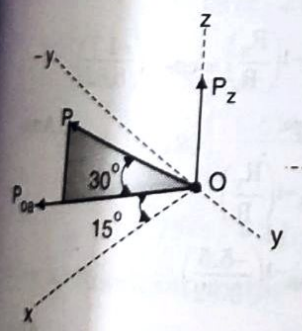


Fig. P. 9.2.6(c)

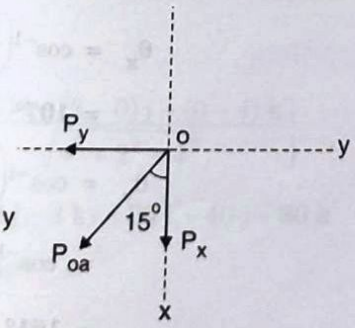


Fig. P. 9.2.6(d)

Resolving P_{oa} to two perpendicular components

$$P_x = P_{oa}\cos 15° = 259.8\cos 15° = \mathbf{250.94\ N}$$

$$P_y = -P_{oa}\sin 15° = -259.8\sin 15° = \mathbf{-67.24\ N}$$

Components of 'Q'

'Q' is making 40° with line ob.

$$\therefore \; Q_{ob} = Q\cos 40° = 400\cos 40° = \mathbf{306.42\ N}$$

$$Q_z = Q\sin 40° = 400\sin 40° = \mathbf{257.11\ N}$$

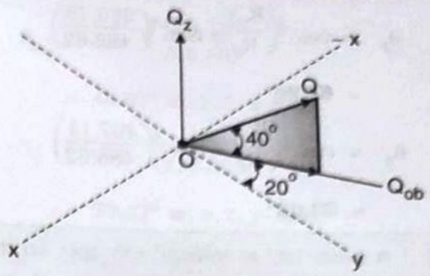


Fig. P. 9.2.6(e)

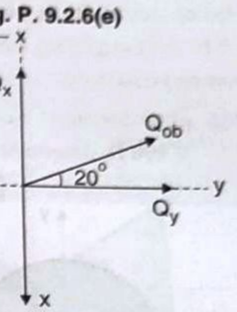


Fig. P. 9.2.6(f)

$$Q_y = Q_{ob}\cos 20° = 306.42\cos 20° = \mathbf{288\ N}$$

$$Q_x = Q_{ob}\sin 20° = -306.42\sin 20°$$

$$= \mathbf{-104.80\ N}$$

Step 2 : Resultant (R) :

$$\bar{R} = \bar{P} + \bar{Q}$$

$$\bar{R} = (P_x + Q_x)\,i + (P_y + Q_y)\,j + (P_z + Q_z)k$$

$$\bar{R} = (\sum F_x)\,i + (\sum F_y)j + (\sum F_z)k$$

$$= R_x\,i + R_y\,j + R_z\,k$$

$$\therefore \; R_x = \Sigma F_x = 250.94 - 104.80$$

$$= \mathbf{146.14\ N}$$

$$R_y = \Sigma F_y = -67.24 + 288 = \mathbf{220.76\ N}$$

$$R_z = \Sigma F_z = 150 + 257.11 = \mathbf{407.11\ N}$$

$\therefore$ Magnitude of resultant,

$$R = \sqrt{R_x^{\,2} + R_y^{\,2} + R_z^{\,2}}$$

$$= \sqrt{146.14^2 + 220.76^2 + 407.11^2}$$

$$= \mathbf{485.62\ N}$$ ... Ans.

Direction of resultant,

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{146.14}{485.62}\right)$$

$$= 72.48° \quad \text{...Ans.}$$

$$\theta_y = \cos^{-1}\left(\frac{R_y}{R}\right) = \cos^{-1}\left(\frac{220.76}{485.62}\right)$$

$$= 62.96° \quad \text{...Ans.}$$

$$\theta_z = \cos^{-1}\left(\frac{R_z}{R}\right) = \cos^{-1}\left(\frac{407.11}{485.62}\right)$$

$$= 33.03° \quad \text{...Ans.}$$

Ex. 9.2.7 : A base ball is thrown with spin so that three concurrent forces act on it, as shown in Fig. P. 9.2.7. The weight W is 5 N, the drag D is 1.7 N and Lift L, which is in y-z plane and perpendicular to x.

It is known that y-component of the resultant is – 5.5 N and z -component is – 0.866 N, determine L, θ and resultant R.

SPPU : May 05, 8 Marks

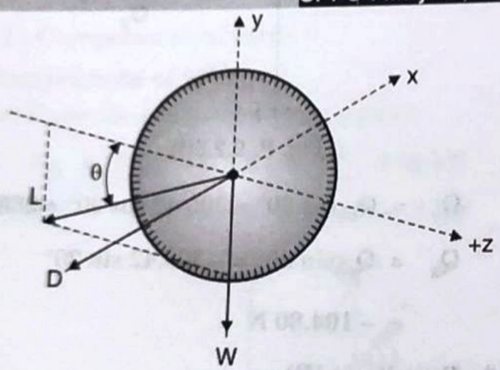


Fig. P. 9.2.7

Soln. :

Step 1 : Forces acting on the ball :

Weight, $W = 5$ N acting along –ve y - direction

Drag, $D = 1.7$ N acting along –ve x-axis

Lift, L acting in y-z plane.

Step 2 : Components of forces :

For W : $W_y = -5$ N

$W_x = W_z = 0$

For D : $D_x = -1.7$ N

$D_y = D_z = 0$

For L : $L_x = 0$

$L_z = -L\cos\theta$

$L_y = -L\sin\theta$

Step 3 : To determine L, θ and R :

Given, $R_y = -5.5$ N

$R_z = -0.866$ N

$$\therefore \sum F_y = R_y$$

$$-5 + 0 - L\sin\theta = -5.5$$

$$\therefore L\sin\theta = 0.5 \quad \text{...(1)}$$

$$\therefore \sum F_z = R_z$$

$$0 + 0 - L\cos\theta = -0.866$$

$$\therefore L\cos\theta = 0.866 \quad \text{...(2)}$$

$$\frac{L\sin\theta}{L\cos\theta} = \frac{0.5}{0.866}$$

$$\tan\theta = 0.577$$

$$\therefore \theta = 30° \quad \text{...Ans.}$$

From Eq^n (2),

$$L\cos 30° = 0.866$$

$$\therefore L = 1.0 \text{ N} \quad \text{...Ans.}$$

To find R :

$$R_x = \Sigma F_x = W_x + D_x + L_x$$

$$= 0 - 1.7 + 0 = -1.7 \text{ N}$$

$$R_y = -5.5 \text{ N}$$

$$R_z = -0.866 \text{ N}$$

∴ Magnitude of resultant,

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$= \sqrt{1.7^2 + 5.5^2 + 0.866^2}$$

$$= 5.82 \text{ N} \quad \text{...Ans.}$$

Direction of resultant,

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{-1.7}{5.82}\right)$$

$$= 107° \quad \text{...Ans.}$$

$$\theta_y = \cos^{-1}\left(\frac{R_y}{R}\right)$$

$$= \cos^{-1}\left(\frac{-5.5}{5.82}\right)$$

$$= 161° \quad \text{...Ans.}$$

$$\theta_z = \cos^{-1}\left(\frac{R_z}{R}\right)$$

$$= \cos^{-1}\left(\frac{-0.866}{5.82}\right) \quad \text{...Ans.}$$

$$= 98.55°$$

Ex. 9.2.8 : The cable exert forces F_{AB} = 100 N and F_{AC} = 120 N on the ring at A as shown in Fig. P. 9.2.8(a). Determine the magnitude of the resultant force acting at A.

SPPU : May 14, Dec 17, 6 Marks

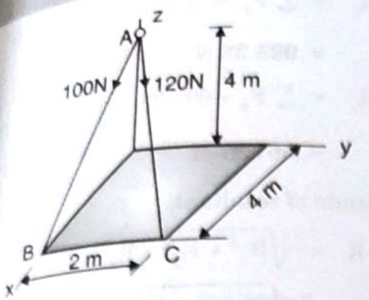


Fig. P. 9.2.8(a)

Soln. :

Step 1 : Coordinates of points :

A (0,0,4)

B (4,0,0)

C (4,2,0)

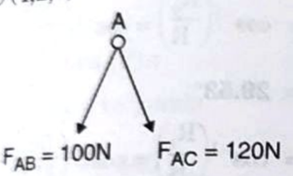


Fig. P. 9.2.8(b)

Step 2 : Force vectors :

$$\therefore \bar{F}_{AB} = F_{AB} \cdot \bar{e}_{AB}$$

$$= 100\left[\frac{(4-0)\,i + (0-0)\,j + (0-4)\,k}{\sqrt{4^2 + 0 + 4^2}}\right]$$

$$= 17.68\,(4\,j + 0\,j - 4\,k) = 70.71\,i - 70.71\,k$$

$$\therefore \bar{F}_{AC} = F_{AC} \cdot \bar{e}_{AC}$$

$$= 120\left[\frac{(4-0)\,i + (2-0)\,j + (0-4)\,k}{\sqrt{4^2 + 2^2 + 4^2}}\right]$$

$$= 20\,(4\,i + 2\,j - 4\,k) = 80\,i + 40\,j - 80\,k$$

Step 3 : Resultant vector :

$$\therefore \bar{R} = \bar{F}_{AB} + \bar{F}_{AC}$$

$$= (70.71 + 80)\,i + (0 + 40)\,j + (-70.71 - 80)\,k$$

$$= 150.71\,i + 40\,j - 150.71\,k$$

Step 4 : Magnitude of resultant,

$$R = \sqrt{(150.71)^2 + (40)^2 + (150.71)^2}$$

$$= \mathbf{216.86\ N}$$ ... **Ans.**

Direction of Resultant,

$$\theta_x = \cos^{-1}\left(\frac{150.71}{216.86}\right)$$

$$= \mathbf{45.97^\circ\ w.\ r.\ t.\ x\text{-}axis}$$ ... **Ans.**

$$\theta_y = \cos^{-1}\left(\frac{40}{216.86}\right)$$

$$= \mathbf{79.37^\circ\ w.\ r.\ t.\ y\text{-}axis}$$ ... **Ans.**

$$\theta_z = \cos^{-1}\left(\frac{-150.71}{216.86}\right)$$

$$= \mathbf{-134.02^\circ\ w.\ r.\ t.\ z\text{-}axis}$$ ... **Ans.**

Ex. 9.2.9 : Find the resultant of force system shown in Fig. P. 9.2.9(a).

SPPU : May 02, 8 Marks

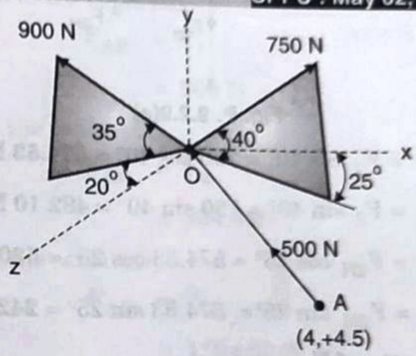


Fig. P. 9.2.9(a)

Soln. :

Step 1 : Components of forces :

Let F_1 = 900 N

F_2 = 750 N

F_3 = 500 N

Components of 'F_1' :

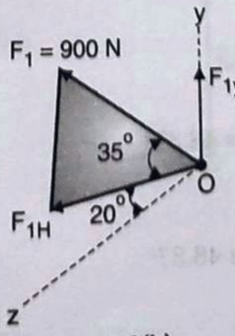


Fig. P. 9.2.9(b)

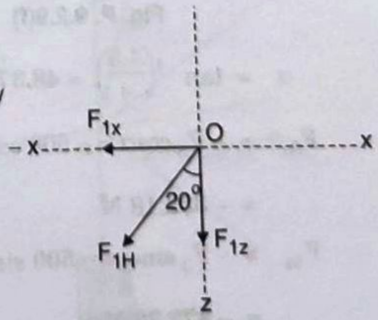


Fig. P. 9.2.9(c)

F_{1H} = F cos 35° = 900 cos 35° = **737.24 N**

F_{1y} = F sin 35° = 900 sin 35° = **516.22 N**

$$F_{1x} = -F_{1H} \sin 20° = -737.24 \sin 20°$$

$$= \mathbf{-252.15\ N}$$

$$F_{1z} = F_{1H} \cos 20° = 737.24 \cos 20° = \mathbf{692.78\ N}$$

Components of 'F_2' :

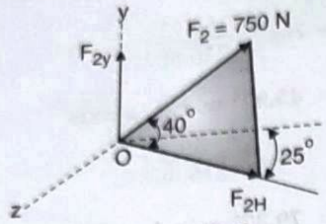


Fig. P. 9.2.9(d)

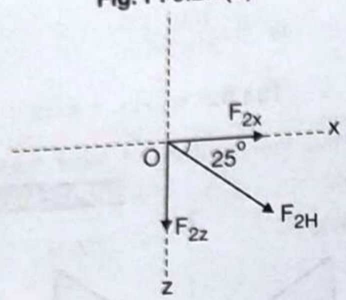


Fig. P. 9.2.9(e)

$$F_{2H} = F_2 \cos 40° = 750 \cos 40° = \mathbf{574.53\ N}$$

$$F_y = F_2 \sin 40° = 750 \sin 40° = \mathbf{482.10\ N}$$

$$F_{2x} = F_{2H} \cos 25° = 574.53 \cos 25° = \mathbf{520.70\ N}$$

$$F_{2z} = F_{2H} \sin 25° = 574.53 \sin 25° = \mathbf{242.81\ N}$$

Components of F_3 :

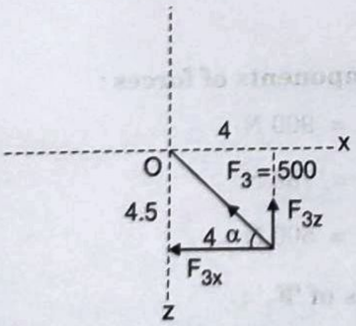


Fig. P. 9.2.9(f)

$$\alpha = \tan^{-1}\left(\frac{4.5}{4}\right) = \mathbf{48.37°}$$

$$F_{3x} = -F_3 \cos\alpha = -500 \cos 48.37°$$

$$= \mathbf{-332.18\ N}$$

$$F_{3z} = -F_3 \sin\alpha = -500 \sin 48.37°$$

$$= \mathbf{-373.72\ N}$$

$$F_{3y} = \mathbf{0} \qquad (\because \text{ in } x-z \text{ plane})$$

Step 2 : Resultant (R) :

$$R_x = \Sigma F_x = -252.15 + 520.70 - 332.18$$

$$= \mathbf{-63.63\ N}$$

$$R_y = \Sigma F_y = 516.22 + 482.10 + 0$$

$$= \mathbf{998.32\ N}$$

$$R_z = \Sigma F_z = 692.78 + 242.81 - 373.72$$

$$= \mathbf{561.87\ N}$$

$\therefore$ Magnitude of resultant,

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$= \sqrt{63.63^2 + 998.32^2 + 561.87^2}$$

$$= \mathbf{1147.34\ N} \qquad \text{...Ans.}$$

Direction of resultant,

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{-63.63}{1147.34}\right)$$

$$= \mathbf{93.18°} \qquad \text{...Ans.}$$

$$\theta_y = \cos^{-1}\left(\frac{R_y}{R}\right) = \cos^{-1}\left(\frac{998.32}{1147.34}\right)$$

$$= \mathbf{29.53°} \qquad \text{...Ans.}$$

$$\theta_z = \cos^{-1}\left(\frac{R_z}{R}\right) = \cos^{-1}\left(\frac{561.87}{1147.34}\right)$$

$$= \mathbf{60.68°} \qquad \text{...Ans.}$$

Type 3 : Based on Equilibrium of Concurrent Force System in Space

Ex. 9.2.10: The Three cables are used to support the 800 N lamp as shown in Fig. P. 9.2.10(a). Determine the force developed in each cable for equilibrium.

SPPU : May 13, 6 Marks

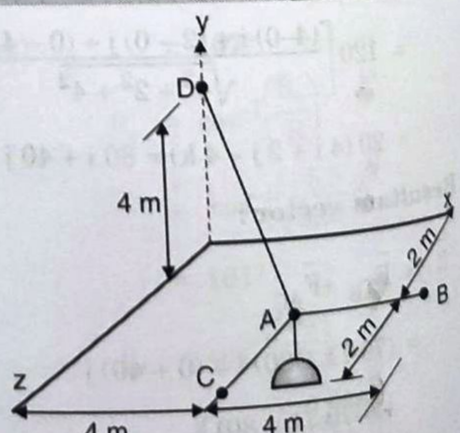


Fig. P. 9.2.10(a)

Soln. :

Step 1 : Coordinates:

A (– 2, 4, 0)

B (2, 4, 0)

C (0, 0, 3)

Step 2 : FBD of the point 'C' :

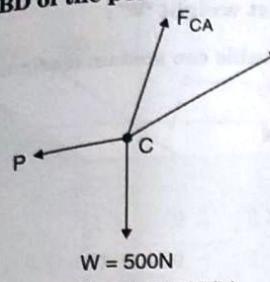


Fig. P. 9.2.12(b)

Let $F_{CA} = F_1$

$F_{CB} = F_2$

Step 3 : Force vectors ($\bar{F}$) :

'W' is acting along –ve y-axis

$$\bar{W} = -Wj = -500j \quad \text{... (1)}$$

'P' is acting along +ve z-axis

$$\bar{P} = Pk \quad \text{... (2)}$$

$\bar{F}_1$ is acting along CA

$$\bar{F}_1 = F_1 . \bar{e}_{CA}$$

$$\therefore \bar{F}_1 = F_1\left[\frac{(-2-0)\,i + (4-0)j + (0-3)\,k}{\sqrt{2^2+4^2+3^2}}\right]$$

$$= F_1\,(-0.37\,i + 0.74\,j - 0.56\,k) \quad \text{... (3)}$$

F_2 is acting along CB

$$\bar{F}_2 = F_2 . \bar{e}_{CB}$$

$$= F_2\left[\frac{(2-0)\,i + (4-0)j + (0-3)\,k}{\sqrt{2^2+4^2+3^2}}\right]$$

$$= F_2\,(0.372\,i + 0.74\,j - 0.56\,k) \quad \text{... (4)}$$

Step 4 : Equilibrium equations :

For equilibrium; $\bar{R} = \sum \bar{F} = 0$

$$\bar{W} + \bar{P} + \bar{F}_1 + \bar{F}_2 = 0$$

$$\sum F_x = 0$$

$$-0.37\,F_1 + 0.37\,F_2 = 0$$

$$\therefore F_1 = F_2 \quad \text{... (I)}$$

$$\sum F_y = 0$$

$$-500 + 0.74\,F_1 + 0.74\,F_2 = 0 \quad \text{... (II)}$$

$$\sum F_z = 0$$

$$P - 0.56\,F_1 - 0.56\,F_2 = 0 \quad \text{... (III)}$$

Step 5 : Tension in Strings :

From Eq^n (I), $F_1 = F_2$

From Eq^n (II),

$$0.74\,F_1 + 0.74\,F_2 = 500$$

$$\therefore \mathbf{F_1 = 337.84\ N}$$

$$\mathbf{F_2 = 337.84\ N}$$

From Eq^n (III),

$$P - 0.56\,(337.84) - 0.56\,(337.84) = 0$$

$$\therefore \mathbf{P = 378.38\ N}$$

$\therefore$ Tension in the strings are

$$T_{CA} = F_1$$

$$= \mathbf{337.84\ N} \quad \text{... Ans.}$$

$$T_{CB} = F_2$$

$$= \mathbf{337.84\ N} \quad \text{... Ans.}$$

Magnitude of $\mathbf{P = 378.38\ N}$ **... Ans.**

Ex. 9.2.13 : If each cable can sustain a maximum tension of 600 N, determine the greatest weight of the bucket and its contents that can be supported. Refer Fig. P. 9.2.13(a).

SPPU : Dec.15, 6 Marks

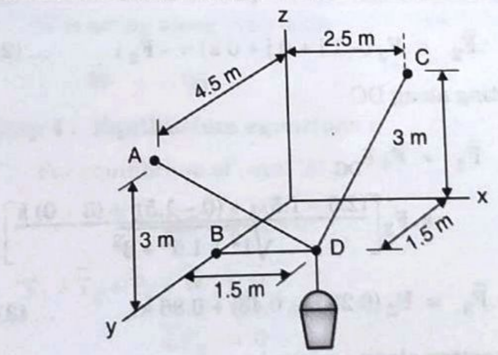


Fig. P. 9.2.13(a)

Soln. :

Step 1: Coordinates :

A (0, 4.5, 3)

B (0, 1.5, 0)

C (2.5, 0, 3)

D (1.5, 1.5, 0)

Step 2 : FBD of point 'D' :

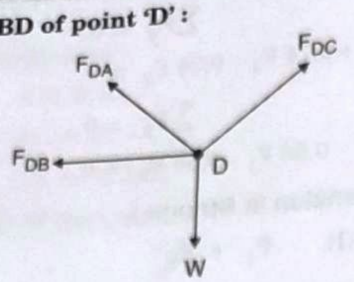


Fig. P. 9.2.13(b)

Let $F_{DA} = F_1$

$F_{DB} = F_2$

$F_{DC} = F_3$

Step 3 : Force vectors ($\bar{F}$)

F_1 is acting along DA

$$\bar{F}_1 = F_1 \cdot \bar{e}_{DA}$$

$$= F_1\left[\frac{(0-1.5)\,i + (4.5-1.5)j + (3-0)\,k}{\sqrt{1.5^2+3^2+3^2}}\right]$$

$$\therefore \bar{F}_1 = F_1(-0.33\,i + 0.67\,j + 0.67\,k) \quad \text{... (1)}$$

$\bar{F}_2$ is acting along DB

$$\bar{F}_2 = F_2 \cdot \bar{e}_{DB}$$

$$= F_2\left[\frac{(0-1.5)\,i + (1.5-1.5)j + (0-0)\,k}{\sqrt{1.5^2+0^2+0^2}}\right]$$

$$\therefore \bar{F}_2 = F_2(-1\,i + 0\,j + 0\,k) = -F_2\,i \quad \text{... (2)}$$

F_3 is acting along DC

$$\bar{F}_3 = F_3 \cdot \bar{e}_{DC}$$

$$= F_3\left[\frac{(2.5-1.5)\,i + (0-1.5)j + (3-0)\,k}{\sqrt{1^2+1.5^2+3^2}}\right]$$

$$\therefore \bar{F}_3 = F_3(0.28\,i - 0.43j + 0.86\,k) \quad \text{... (3)}$$

W is acting along –ve z-axis

$$\therefore \bar{W} = -Wk \quad \text{... (4)}$$

Step 4 : Equilibrium equations :

For equilibrium of point D,

$$\bar{R} = \sum \bar{F} = 0$$

$$\bar{F}_1 + \bar{F}_2 + \bar{F}_3 + + \bar{W} = 0$$

$$\sum F_x = 0$$

$$-0.33\,F_1 - F_2 + 0.28\,F_3 = 0$$

$$\sum F_y = 0$$

$$0.67\,F_1 - 0.43\,F_3 = 0$$

$$\sum F_z = 0$$

$$0.67\,F_1 + 0.86\,F_3 - W = 0$$

Step 5 : Greatest weight 'W' :

Given that each cable can sustain maximum tension of 600 N.

Take F_1 = 600 N

From Eq^n (2),

$$0.67\,(600) - 0.43\,F_3 = 0$$

$$F_3 = 934.88\text{ N} > 600\text{ N}$$

(Hence not safe)

Take F_2 = 600 N

From Eq^n (I),

$$-0.33\,F_1 - 600 + 0.28\,F_3 = 0$$

$$-0.33\,F_1 + 0.28\,F_3 = 600 \quad \text{... (IV)}$$

Solving Eq^n (II) and Eq^n (IV)

From Eq^n (II), $F_1 = 0.64\,F_3$

Putting in Eq^n (IV)

$$-0.33\,(0.64\,F_3) + 0.28\,F_3 = 600$$

$$-0.21\,F_3 + 0.28\,F_3 = 600$$

$$\therefore F_3 = 8571.42 > 600\text{ N}$$

(Hence not safe)

Take F_3 = 600 N

From Eq^n (II),

$$0.67\,F_1 - 0.43\,(600) = 0$$

$$F_1 = 385.07 < 600\text{ N} (\because \text{safe})$$

From Eq^n (I),

$$-0.33\,(385.07) - F_2 + 0.28\,(600) = 0$$

$$\therefore F_2 = 40.93\text{ N} < 600\text{ N} (\therefore \text{Safe})$$

From Eq^n (III),

$$0.67\,(385.07) + 0.86\,(600) - W = 0$$

$$\therefore W = 774\text{ N}$$

Greatest weight of bucket,

$$W = 774\text{ N} \quad \text{...Ans.}$$

Force in the cables,

$$F_{DA} = F_1 = 385.07\text{ N} \quad \text{...Ans.}$$

$F_{DB} = F_2 = 40.93$ N ...Ans.

$F_{DC} = F_3 = 600$ N ...Ans.

P. 9.2.14 : A container is supported by three cables that are attached to a ceiling as shown Fig. P. 9.2.14(a). Determine the weight W of the container knowing that the tension in cable AD is 4.3 kN. **SPPU : May 09, 6 Marks**

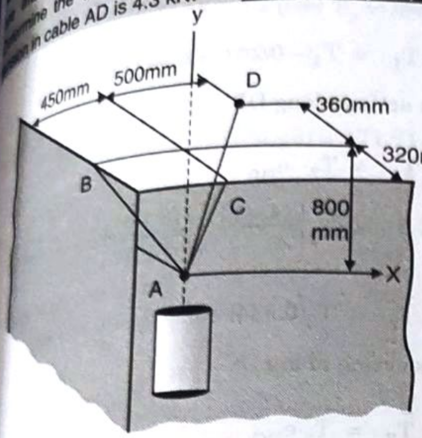


Fig. P. 9.2.14(a)

Soln. :

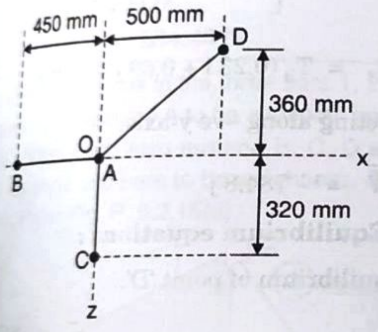


Fig. P. 9.2.14(b)

Step 1: Coordinates :

A (0, – 800, 0)

B (– 450, 0, 0)

C (0, 0, 320)

D (500, 0, – 360)

Step 2: FBD of point 'A' :

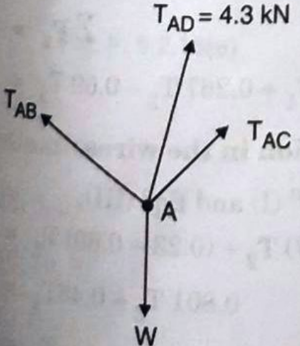


Fig. P. 9.2.14(c)

Let
$$T_{AB} = T_1$$
$$T_{AC} = T_2$$
$$T_{AD} = T_3 = 4.3 \text{ kN}$$

Step 3 : Force vectors ($\bar{F}$) :

T_1 is acting along AB

$$\therefore \bar{T}_1 = T_1 \cdot \bar{e}_{AB}$$
$$= T_1\left[\frac{(-450-0)i + (0+800)j + (0-0)k}{\sqrt{450^2 + 800^2 + 0}}\right]$$
$$\therefore \bar{T}_1 = T_1(-0.49i + 0.87j) \quad ...(1)$$

T_2 is acting along AC

$$\therefore \bar{T}_2 = T_2 \cdot \bar{e}_{AC}$$
$$= T_2\left[\frac{(0-0)i + (0+800)j + (320-0)k}{\sqrt{0 + 800^2 + 320^2}}\right]$$
$$\therefore \bar{T}_2 = T_2(0.93j + 0.37k) \quad ...(2)$$

T_3 is acting along AD.

$$\therefore \bar{T}_3 = T_3 \cdot \bar{e}_{AD}$$
$$= 4.3\left[\frac{(500-0)i + (0+800)j + (-360-0)k}{\sqrt{500^2 + 800^2 + 360^2}}\right]$$
$$\therefore \bar{T}_3 = 2.13i + 3.41j - 1.53k \quad ...(3)$$

W is acting along –ve y-axis.

$$\therefore \bar{W} = -Wj \quad ...(4)$$

Step 4 : Equilibrium equations :

For equilibrium of point 'A' ;

$$\Sigma\bar{F} = \bar{R} = 0$$
$$\bar{T}_1 + \bar{T}_2 + \bar{T}_4 + \bar{W} = 0$$
$$\Sigma F_x = 0$$
$$0.49\,T_1 + 2.13 = 0 \quad ...(I)$$
$$\therefore \quad T_1 = 4.35 \text{ kN}$$
$$\Sigma F_y = 0$$
$$0.87T_1 + 0.93T_2 + 3.41 - W = 0 \quad ...(II)$$
$$\Sigma F_z = 0$$
$$0.37T_2 - 1.53 = 0 \quad ...(III)$$
$$\therefore \quad T_2 = 4.135 \text{ kN}$$

Step 5 : Weight 'W' of the container :

From Eq^n (I), $T_1 = 4.35$ kN

From Eq^n (II), $T_2 = 4.135$ kN

From Eq^n (II), $0.87(4.35) + 0.93\,(4.135) + 3.41 - W = 0$

$\therefore W = 11.04$ kN ...Ans.

Ex. 9.2.15 : A 80 kg mass as shown in Fig. P. 9.2.15(a) is supported by three wires concurrent at D(2, 0, –1). The wires are attached to the point A (1, 3, 0), B(3, 3, 4) and C(– 4, 3, 0). Determine tension in each wire.

SPPU : Dec. 10, 10 Marks

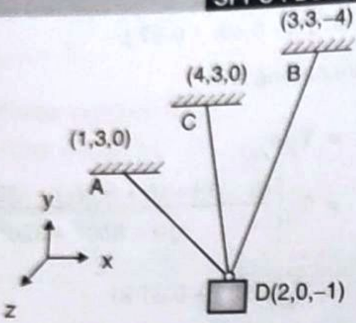


Fig. P. 9.2.15(a)

Soln. :

Step 1 : Coordinates :

Given : A(1, 3, 0)

B(4, 3 0)

C(3, 3, –4)

D(2, 0, –1)

Step 2 : FBD of point 'D'

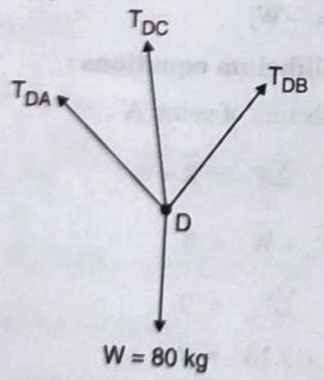


Fig. P. 9.2.15(b)

Let tension in the wires be,

$$T_{DA} = T_1$$

$$T_{DB} = T_2$$

$$T_{DC} = T_3$$

$$W = 80 \times 9.81 = 784.8 \text{ N}$$

Step 3 : Force vectors $(\bar{F})$:

T_1 is acting along DA.

$$\bar{T}_1 = T_1 \cdot \bar{e}_{DA}$$

$$= T_1\left[\frac{(1-2)\,i + (3-0)\,j + (0+1)\,k}{\sqrt{2^2+3^2+1^2}}\right]$$

$$\therefore \bar{T}_1 = T_1(-\,0.267i + 0.802\,j + 0.267\,k) \quad ...(1)$$

T_2 is acting along DB

$$\bar{T}_2 = T_2 \cdot \bar{e}_{DB}$$

$$= T_2\left[\frac{(4-2)\,i + (3-0)\,j + (0+1)\,k}{\sqrt{2^2+3^2+1^2}}\right]$$

$$\therefore \bar{T}_2 = T_2(0.534i + 0.802\,j + 0.267\,k) \quad ...(2)$$

T_3 is acting along DC

$$\bar{T}_3 = T_3 \cdot \bar{e}_{DC}$$

$$= T_3\left[\frac{(3-2)\,i + (3-0)\,j + (-4+1)\,k}{\sqrt{1^2+3^2+3^2}}\right]$$

$$\therefore \bar{T}_3 = T_3\,(0.23\,i + 0.69\,j - 0.69\,k) \quad ...(3)$$

W is acting along –ve y-axis.

$$\therefore \overline{W} = -\,784.8\,j \quad ...(4)$$

Step 4 : Equilibrium equations :

For equilibrium of point 'D'.

$$\bar{R} = \Sigma\bar{F} = 0$$

$$\bar{T}_1 + \bar{T}_2 + \bar{T}_3 + \overline{W} = 0$$

$$\Sigma F_x = 0$$

$$-\,0.267\,T_1 + 0.534\,T_2 + 0.23\,T_3 = 0 \quad ...(I)$$

$$\Sigma F_y = 0$$

$$0.802\,T_1 + 0.802\,T_2 + 0.69\,T_3 - 784.8 = 0 \quad ...(II)$$

$$\Sigma F_z = 0$$

$$0.267\,T_1 + 0.267\,T_2 - 0.69\,T_3 = 0 \quad ...(III)$$

Step 5 : Tension in the wires :

Adding Eq^n (I) and Eq^n (III);

$$(0.534 + 0.267)\,T_2 + (0.23 - 0.69)\,T_3 = 0$$

$$0.801\,T_2 - 0.46T_3 = 0$$

$$\therefore T_2 = 0.57T_3 \quad ...(IV)$$

From Eq^n (I),

$$-0.267\,T_1 + 0.534(0.577\,T_3) + 0.23\,T_3 = 0$$
$$-0.267\,T_1 + 0.538\,T_3 = 0$$
$$\therefore T_1 = 2.01\,T_3 \quad ...(V)$$

From Eq^n (II),

$$0.802\,(2.01\,T_3) + 0.802(0.57\,T_3) + 0.69\,T_3 = 784.8$$
$$\therefore \mathbf{T_3 = 284.43\ N}$$

From Eq^n (V),

$$T_1 = 2.01\,(284.43) = \mathbf{571.71\ N}$$

From Eq^n (IV),

$$T_2 = 0.57\,(284.43) = \mathbf{162.12\ N}$$

∴ **Tension in the wires :**

$$T_{DA} = T_1 = \mathbf{571.71\ N} \quad ...\textbf{Ans.}$$
$$T_{DB} = T_2 = \mathbf{162.12\ N} \quad ...\textbf{Ans.}$$
$$T_{DC} = T_3 = \mathbf{284.43\ N} \quad ...\textbf{Ans.}$$

Ex. 9.2.16 : Find the forces in the three bars 1, 2, 3 in terms of W for equilibrium. Assume A to be a smooth spherical joint i.e. ball and socket joint. Also assume B, C, D as ideal ball and socket supports and bars to be weightless. Support C is in x - z plane. Refer Fig. P. 9.2.16(a).

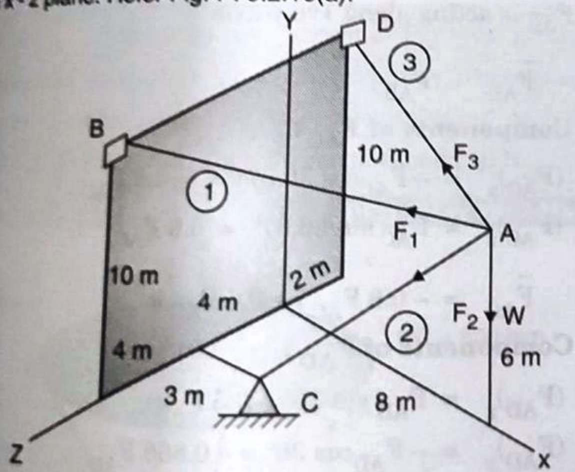


Fig. P. 9.2.16(a)

Soln. :

Step 1 : Coordinates :

A(8, 6, 0)

B(0, 10, 8)

C(3, 0, 4)

D(0, 10, −2)

Step 2 : FBD of point 'A' :

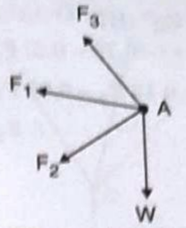


Fig. P. 9.2.16(b)

Step 3 : Force vectors ($\bar{F}$) :

F_1 is acting along AB

$$\bar{F}_1 = F_1 \cdot \bar{e}_{AB}$$
$$= F_1\left[\frac{(0-8)\,i + (10-6)\,j + (8-0)k}{\sqrt{8^2+6^2+8^2}}\right]$$
$$\therefore \bar{F}_1 = F_1(-0.62\,i + 0.31\,j + 0.62\,k) \quad ...(1)$$

F_2 is acting along AC

$$\bar{F}_2 = F_2 \cdot \bar{e}_{AC}$$
$$= F_2\left[\frac{(3-8)i + (0-6)\,j + (4-0)k}{\sqrt{5^2+6^2+4^2}}\right]$$
$$\bar{F}_2 = F_2\,(-0.57\,i - 0.68\,j + 0.45\,k) \quad ...(2)$$

F_3 is acting along AD

$$\bar{F}_3 = F_3 \cdot \bar{e}_{AD}$$
$$= F_3\left[\frac{(0-8)i + (10-6)\,j + (-2-0)k}{\sqrt{8^2+6^2+2^2}}\right]$$
$$\therefore \bar{F}_3 = F_3\,(-0.78\,i + 0.39\,j - 0.20\,k) \quad ...(3)$$

'W' is acting along –ve y-axis

$$\therefore \bar{W} = -Wj \quad ...(4)$$

Step 4 : Equilibrium equations :

For equilibrium of point 'A' :

$$\bar{R} = \Sigma\bar{F} = 0$$
$$\bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{W} = 0$$
$$\Sigma F_x = 0$$
$$-0.62\,F_1 - 0.57\,F_2 - 0.78\,F_3 = 0 \quad ...(I)$$
$$\Sigma F_y = 0$$
$$0.31\,F_1 - 0.68\,F_2 + 0.39\,F_3 - W = 0 \quad ...(II)$$
$$\Sigma F_z = 0$$
$$0.62\,F_1 + 0.45\,F_2 - 0.2\,F_3 = 0 \quad ...(III)$$

Step 5 : Forces in the bars :

Adding Eq^n (I) and Eq^n (III)

$$(-0.57 + 0.45)\,F_2 + (-0.78 - 0.2)\,F_3 = 0$$

$$-0.12\,F_2 - 0.98\,F_3 = 0$$

$$\therefore F_2 = -8.17\,F_3 \quad ...(IV)$$

From Eq^n (I)

$$-0.62\,F_1 - 0.57\,(-8.17\,F_3) - 0.78\,F_3 = 0$$

$$-0.62\,F_1 + 3.87\,F_3 = 0$$

$$F_1 = 6.25\,F_3 \quad ...(V)$$

From Eq^n (V) and Eq^n (IV) putting F_1 and F_2 in Eq^n (II)

$$0.31(6.25\,F_3) - 0.68(-8.17\,F_3) + 0.39\,F_3 = W$$

$$7.88\,F_3 = W$$

$$F_3 = 0.127\,W$$

From Eq^n (IV),

$$F_2 = -8.17(0.127\,W) = -1.038\,W$$

From Eq^n (V),

$$F_1 = 6.25\,(0.127\,W) = 0.794\,W$$

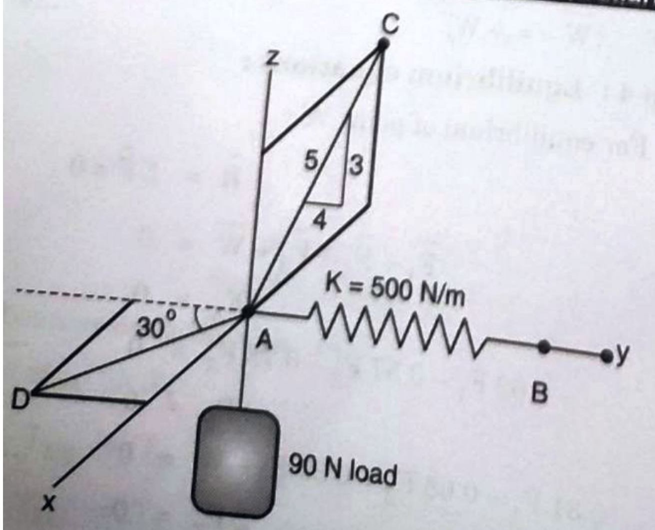
Forces in bars;

F_1 = **0.794 W (tensile)** ...Ans.

F_2 = **– 1.038 W = 1.038 W (compressive)** ...Ans.

F_3 = **0.127 W (Tensile)** ...Ans.

Ex. 9.2.17 : A 90 N load is suspended from the hook shown in Fig. P. 9.2.17(a). The load is supported by two cables and a spring having stiffness K = 500 N/m. Determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x-y plane and cable AC lies in the x-z plane. **SPPU : May 16, Dec. 17, 7 Marks**



Soln. :

Step 1 : FBD of point 'A' :

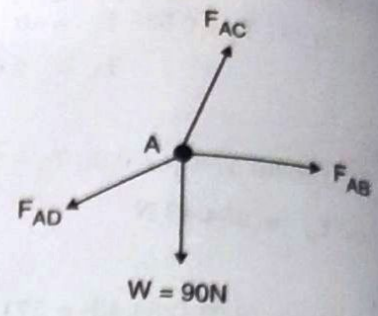


Fig. P. 9.2.17(b)

Step 2 : Force vectors :

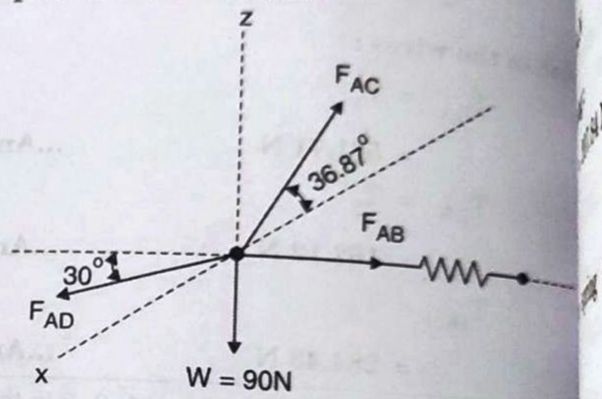


Fig. P. 9.2.17(c)

W is acting along –ve z - axis.

$$\therefore \overline{W} = -Wk = -90\,k$$

F_{AB} is acting along +ve y-axis

$$\overline{F}_{AB} = F_{AB}\,j$$

Components of F_{AC} :

$$(F_{AC})_x = -F_{AC}\cos 36.87° = -0.8\,F_{AC}$$

$$(F_{AC})_z = F_{AC}\sin 36.87° = 0.6\,F_{AC}$$

$$\overline{F}_{AC} = -0.8\,F_{AC}\,i + 0.6\,F_{AC}\,k$$

Components of F_{AD} :

$$(F_{AD})_x = F_{AD}\sin 30° = 0.5\,F_{AD}$$

$$(F_{AD})_y = -F_{AD}\cos 30° = -0.866\,F_{AD}$$

$$\therefore \overline{F}_{AD} = 0.5\,F_{AD}\,i - 0.866\,F_{AD}\,j$$

Step 3 : Equilibrium equations :

For equilibrium of point 'A'

$F_{AC} + 0.5\,F_{AD} = 0$ ...(I)

$\Sigma F_y = 0$

$_{AB} - 0.866\,F_{AD} = 0$ ...(II)

$\Sigma F_z = 0$

$-90 + 0.6\,F_{AC} = 0$ ...(iii)

4 : Forces in cables :

Eq^n (III), $F_{AC} = 150$ N ...Ans.

Eq^n (I),

$(150) + 0.5\,F_{AD} = 0$

$F_{AD} = 240$ N ...Ans.

Eq^n (II),

$_{AB} - 0.866\,(240) = 0$

$F_{AB} = 207.84$ N ...Ans.

5 : Stretch of the spring :

Spring force $= F_{AB} = 207.84$ N

$F_{AB} = K \cdot x$

K = stiffness of the spring and 'x' is the rmation of spring.

$207.84 = 500 \times x$

$\therefore$ Stretch / deformation of the spring,

$x = 0.416$ m ...Ans.

9.2.18 : The 10 kg lamp shown in Fig. P. 9.2.18(a) is pended from three equal length cords. Determine its llest vertical distances from the ceiling if the force eloped in any cord is not allowed to exceed 50 N.

SPPU : May 18, 7 Marks

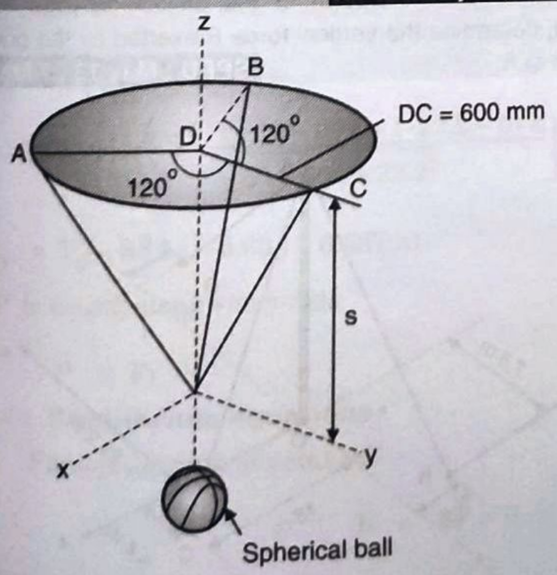


Fig. P. 9.2.18(a)

Soln. :

Step 1 : FBD of point 'O' :

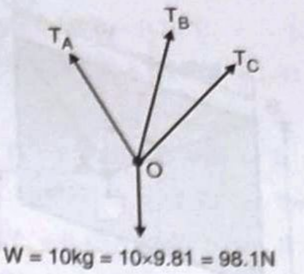


Fig. P. 9.2.18(b)

$W = 10 \text{ kg} = 10 \times 9.81 = 98.1$ N

Let 'θ' be the angle made by each cord w.r.t. vertical i.e., z-axis.

Step 2 : Equilibrium equation :

For equilibrium of point 'O',

$\Sigma F_y = 0$

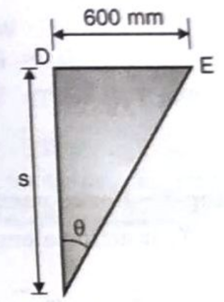
y-component of tension in each cord, $= T \cos\theta$

$= 50 \cos\theta$

$\therefore 3 \times 50 \cos\theta - 98.1 = 0$

$\theta = 49.16°$

Step 3 : Vertical distance 's' :



For any cord

$$\tan\theta = \frac{600}{s}$$

$$\therefore s = \frac{600}{\tan 49.16°}$$

$\therefore$ **s = 518.64 mm** ...Ans.

Ex. 9.2.19 : A 200 kg cylinder is hung by means of two cables AB and AC, which is attached to the top of vertical wall. A horizontal force 'P' perpendicular to the wall holds the cylinder in the position shown in Fig. P. 9.2.19(a). Determine the magnitude of P and the tension in each cable.

SPPU : Dec. 02, 10 Marks, Dec. 12, 6 Marks

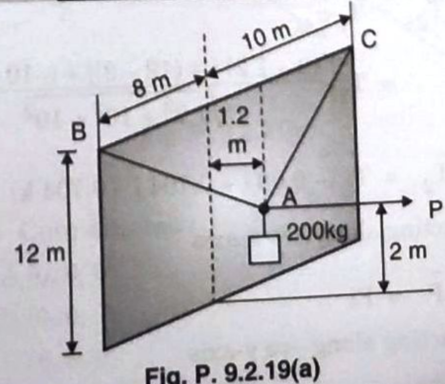


Fig. P. 9.2.19(a)

D (–1, 6, 0)

Step 2 : FBD of joint 'B'

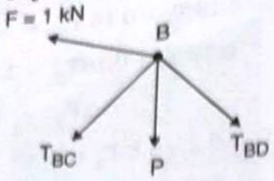


Fig. P. 9.2.21(b)

Let P = Force in the mast

= Reaction at joint A

$T_{BC} = T_1$

$T_{BD} = T_2$

Step 3 : Force vectors ($\bar{F}$) :

F is acting along –ve y-axis

$$\therefore \bar{F} = -Fj = -1j \quad ...(1)$$

'P' is acting along –ve z-axis

$$\bar{P} = -Pk \quad ...(2)$$

T_1 is acting along BC

$$\bar{T}_1 = T_1 \cdot \bar{e}_{BC}$$

$$= T_1\left[\frac{(6-0)\,i + (0-0)j + (0-6)\,k}{\sqrt{6^2+0+6^2}}\right]$$

$$\therefore \bar{T}_1 = T_1(0.707\,i - 0.707\,k) \quad ...(3)$$

T_2 is acting along BD

$$T_2 = T_2 \cdot \bar{e}_{BD}$$

$$= T_2\left[\frac{(-1-0)\,i + (6-0)j + (0-6)\,k}{\sqrt{1^2+6^2+6^2}}\right]$$

$$\bar{T}_2 = T_2(-0.117\,i + 0.702\,j - 0.702\,k) \quad ...(4)$$

Step 4 : Equilibrium equations :

For equilibrium of point 'B'.

$$\bar{R} = \Sigma\bar{F} = 0$$

$$\therefore \bar{F} + \bar{P} + \bar{T}_1 + \bar{T}_2 = 0$$

$$\Sigma F_x = 0$$

$$0.707\,T_1 - 0.117\,T_2 = 0 \quad ...(I)$$

$$\Sigma F_y = 0$$

$$-1 + 0.702\,T_2 = 0 \quad ...(II)$$

$$\Sigma F_z = 0$$

$$-P - 0.707\,T_1 - 0.702\,T_2 = 0 \quad ...(III)$$

Step 5 : Tension in the cables :

From Eq^n (I),

$$T_1 = 0.165\,T_2$$

From Eq^n (II),

$$T_2 = \mathbf{1.424\ kN}$$

$$\therefore T_1 = 0.165\,(1.424)$$

$$= \mathbf{0.235\ kN}$$

∴ Tension in the cables,

$$T_{BC} = \mathbf{0.235\ kN} \quad ...Ans.$$

$$T_{BD} = \mathbf{1.424\ kN} \quad ...Ans.$$

Step 6 : Reaction at the joint 'A'

From Eq^n (III)

$$-P - 0.707\,(0.235) - 0.702\,(1.424) = 0$$

$$\therefore P = -1.165\ \text{kN}$$

∴ Force in the mast,

$$P = 1.165\ \text{kN (compressive)}$$

∴ Reaction at A,

$$\mathbf{R_A = 1.165\ kN} \quad ...Ans.$$

Ex. 9.2.22 : Three cables AB, AC and AD are used to hold down a ballon as shown in Fig. P. 9.2.22(a). Determine vertical force 'P' exerted by the ballon at 'A' knowing that the tension in the cable AB is 259 N.

SPPU : Dec. 03, 10 Marks

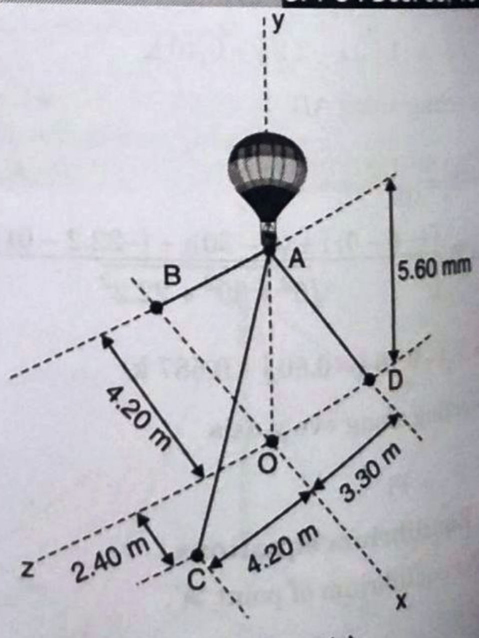


Fig. P. 9.2.22(a)

Soln. :

Step 1 : Coordinates :

A (0, 5.60, 0)

B (– 4.20, 0, 0)

C (2.40, 0, 4.20)

D (0, 0, – 3.30)

Step 2 : FBD of point 'A' :

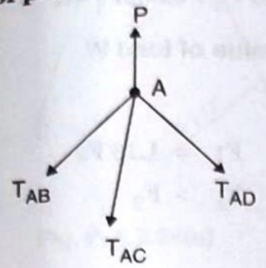


Fig. P. 9.2.22(b)

Let $T_{AB} = T_1 = 259$ N

$T_{AC} = T_2$

$T_{AD} = T_3$

Step 3 : Force vectors ($\bar{F}$) :

T_1 is acting along AB

$$\bar{T}_1 = T_1 . \bar{e}_{AB}$$

$$= 259 \left[\frac{(-4.2-0)\,i + (0-5.6)\,j + (0-0)\,k}{\sqrt{4.2^2 + 5.6^2 + 0}} \right]$$

$$\therefore \bar{T}_1 = -155.4\,i - 207.2\,j \qquad \ldots (1)$$

T_2 is acting along AC

$$\bar{T}_2 = T_2 . \bar{e}_{AC}$$

$$= T_2 \left[\frac{(2.4-0)\,i + (0-5.6)\,j + (4.2-0)\,k}{\sqrt{2.4^2 + 5.6^2 + 4.2^2}} \right]$$

$$\therefore \bar{T}_2 = T_2 (0.324\,i - 0.756\,j + 0.567\,k) \qquad \ldots (2)$$

T_3 is acting along AD

$$\bar{T}_3 = T_3 . \bar{e}_{AD}$$

$$= T_3 \left[\frac{(0-0)\,i + (0-5.6)\,j + (-3.3-0)\,k}{\sqrt{0 + 5.6^2 + 3.3^2}} \right]$$

$$\therefore \bar{T}_3 = T_3 (-0.86\,j - 0.507\,k) \qquad \ldots (3)$$

P is acting along +ve y-axis

$$\therefore \bar{P} = Pj \qquad \ldots (4)$$

Step 4 : Equilibrium equations :

For equilibrium of point 'A'

$$\bar{R} = \sum \bar{F} = 0$$

$$\bar{T}_1 + \bar{T}_2 + \bar{T}_3 + \bar{P} = 0$$

$$\sum F_x = 0$$

$$-155.4 + 0.324\, T_2 = 0 \qquad \ldots (I)$$

$$\sum F_y = 0$$

$$-207.2 - 0.756\, T_2 - 0.86\, T_3 + P = 0 \qquad \ldots (II)$$

$$\sum F_z = 0$$

$$0.567\, T_2 - 0.507 T_3 = 0 \qquad \ldots (III)$$

Step 5 : Vertical force 'P'

From Eq^n (I),

$$T_2 = \frac{155.4}{0.324} = \mathbf{479.63\ N}$$

From Eq^n (III),

$0.567\,(479.63) - 0.507\, T_3 = 0$

$$\therefore T_3 = \mathbf{536.40N}$$

From Eq^n (II),

$-207.2 - 0.756\,(479.63) - 0.86\,(536.40) + P = 0$

$$\therefore P = \mathbf{1031.10\ N} \uparrow \qquad \ldots \textbf{Ans}$$

Ex. 9.2.23 : Determine the safe value of load W that can b[e] supported by the tripod shown in Fig. P. 9.2.23(a) witho[ut] exceeding compressive force of 5 kN in any member.

SPPU : May 03, 10 Mark[s]

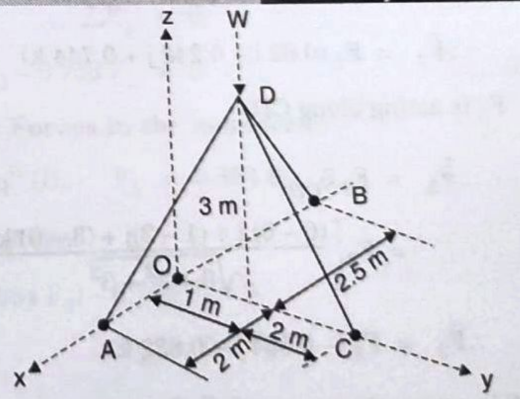


Fig. P. 9.2.23(a)

Soln. :

Step 1 : Coordinates :

A (2, 0, 0)

B (– 2.5, 0, 0)

C (0, 3, 0)

D (0, 1, 3)

Step 2 : FBD of point 'D' :

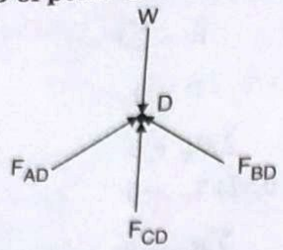


Fig. P. 9.2.23(b)

Let F_{AD}, F_{CD} and F_{BD} are the force in the legs of the tripod AD, CD and BD respectively.

Let, $F_{AD} = F_1$

$F_{BD} = F_2$

$F_{CD} = F_3$

Step 3 : Force vectors ($\bar{F}$) :

F_1 is acting along AD

$$\bar{F}_1 = F_1 \cdot \bar{e}_{AD}$$

$$= F_1\left[\frac{(0-2)\,i + (1-0)j + (3-0)\,k}{\sqrt{2^2 + 1^2 + 3^2}}\right]$$

$$\therefore \bar{F}_1 = F_1(-0.534\,i + 0.267\,j + 0.802\,k) \quad \dots \text{(I)}$$

F_2 is acting along BD

$$\bar{F}_2 = F_2 \cdot \bar{e}_{BD}$$

$$= F_2\left[\frac{(0+2.5)\,i + (1-0)j + (3-0)\,k}{\sqrt{2.5^2 + 1^2 + 3^2}}\right]$$

$$\therefore \bar{F}_2 = F_2(0.62\,i + 0.248\,j + 0.744\,k) \quad \dots \text{(II)}$$

F_3 is acting along CD

$$\bar{F}_3 = F_3 \cdot \bar{e}_{CD}$$

$$= F_3\left[\frac{(0-0)\,i + (1-3)j + (3-0)\,k}{\sqrt{0 + 2^2 + 3^2}}\right]$$

$$\therefore \bar{F}_3 = F_3(-0.554\,j + 0.832\,k) \quad \dots \text{(III)}$$

W is acting along –ve z-axis

$$\therefore \bar{W} = -Wk \quad \dots \text{(IV)}$$

Step 4 : Equilibrium equations :

For equilibrium of point D

$$\sum \bar{F} = \bar{R} = 0$$

$$\bar{F}_1 + \bar{F}_2 + \bar{F}_3 + + \bar{W} = 0$$

$$\sum F_x = 0$$

$$-0.534\,F_1 + 0.62\,F_2 = 0$$

$$\sum F_y = 0$$

$$0.267\,F_1 + 0.248\,F_2 - 0.554F_3 = 0$$

$$\sum F_z = 0$$

$$0.802\,F_1 + 0.744\,F_2 + 0.832F_3 - W = 0$$

Step 5 : Safe value of load W

From Eq^n (I),

$$F1 = 1.16\,F2$$

$$F_1 > F_2$$

From Eq^n (II),

$$0.267\,(1.16F_2) + 0.248\,F_2 = 0.554\,F_3$$

$$\therefore F_3 = 1.007\,F_2$$

$$F_3 > F_2$$

$\therefore F_1$ and F_3 both are greater than F_2

But F_1 is greater than F_3

$\therefore$ Maximum compressive force 5 kN is in the leg A

$$\therefore F_1 = 5\text{ kN}$$

From Eq^n (IV),

$$5 = 1.16\,F_2$$

$$\therefore F_2 = \mathbf{4.31\ kN}$$

From Eq^n (V),

$$F_3 = 1.007\,(4.31) = \mathbf{4.34\ kN}$$

From Eq^n (III),

$$0.802\,(5) + 0.744\,(4.31) + 0.832\,(4.34) = W$$

$$\therefore W = \mathbf{10.83\ kN}$$

Ex. 9.2.24 : The support assembly show Fig. P. 9.2.24(a) is bolted in place B, C and D suppo downward force of 45 N applied at A. Determine the f the members AB, AC and AD. **SPPU : Dec. 11, 7 M**

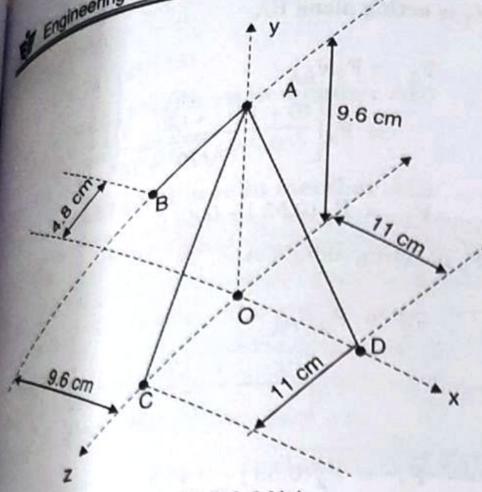


Fig. P. 9.2.24(a)

Soln. :

Step 1 : Coordinates :

A (0, 9.6, 0)

B (– 9.6, 0, – 4.8)

C (0, 0,11)

D (11, 0, 0)

Step 2 : FBD of point 'A' :

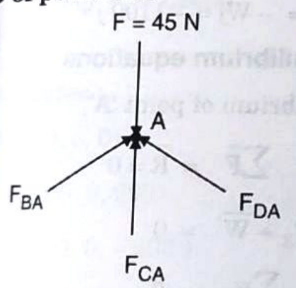


Fig. P. 9.2.24(b)

Let the forces in the members be

$F_{BA} = F_1$

$F_{CA} = F_2$

$F_{DA} = F_3$

Step 3 : Force vectors ($\bar{F}$) :

F_1 is acting along BA

$$\bar{F}_1 = F_1 . \bar{e}_{BA}$$

$$= F_1 \left[\frac{(0 + 9.6)\, i + (6 - 0)\, j + (0 + 4.8)\, k}{\sqrt{9.6^2 + 6^2 + 4.8^2}} \right]$$

$$\therefore \bar{F}_1 = F_1\, (0.781\, i + 0.488 j + 0.39\, k) \qquad \dots (1)$$

F_2 is acting along CA

$$\bar{F}_2 = F_2 . \bar{e}_{CA}$$

$$= F_2 \left[\frac{(0 - 0)\, i + (9.6 - 0)\, j + (0 - 11)\, k}{\sqrt{0 + 9.6^2 + 11^2}} \right]$$

$$\bar{F}_2 = F_2 (0.657\, j - 0.753\, k) \qquad \dots (2)$$

F_3 is acting along DA.

$$\bar{F}_3 = F_3 . \bar{e}_{DA}$$

$$= F_3 \left[\frac{(0 - 11)\, i + (9.6 - 0)\, j + (0 - 0)\, k}{\sqrt{11^2 + 9.6^2 + 0}} \right]$$

$$\therefore \bar{F}_3 = F_3\, (-\, 0.753\, i + 0.657\, j) \qquad \dots (3)$$

F is acting along the –ve y-axis.

$$\therefore \bar{F} = -\, Fj = -\, 45\, j \qquad \dots (4)$$

Step 4 : Equilibrium equations :

For equilibrium of point 'A'

$$\sum \bar{F} = \bar{R} = 0$$

$$\bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F} = 0$$

$$\therefore \sum F_x = 0$$

$$\therefore 0.781\, F_1 - 0.753\, F_3 = 0 \qquad \dots (I)$$

$$\sum F_y = 0$$

$$0.488\, F_1 + 0.657\, F_2 + 0.657\, F_3 - 45 = 0 \qquad \dots (II)$$

$$\sum F_z = 0$$

$$0.39\, F_1 - 0.753\, F_2 = 0 \qquad \dots (III)$$

Step 5 : Forces in the members :

From Eq^n (I), $\quad F_1 = 0.964\, F_3 \qquad \dots (5)$

From Eq^n (II),

$$0.39\, (0.964\, F_3) - 0.753\, F_2 = 0$$

$$\therefore F_2 = 0.5\, F_3 \qquad \dots (6)$$

Putting F_1 and F_2 in Eq^n (II)

$$0.488\, (0.964\, F_3) + 0.657\, (0.5\, F_3) + 0.657\, F_3 = 45$$

$$\therefore \mathbf{F_3 = 30.91\ N}$$

From Eq^n (IV), $\quad F_1 = 0.964\, (30.91)$

$$= \mathbf{29.79\ N}$$

From Eq^n (V), $\quad F_2 = 0.5\, (30.91)$

$$= \mathbf{15.45\ N}$$

$\therefore$ Force in the members :

AB, $\quad F_1 = 29.79$ N ... **Ans.**

AC, $\quad F_2 = 15.45$ N ... **Ans.**

AD, $\quad F_3 = 30.91$ N ... **Ans.**

Ex. 9.2.25 : A vertical load of 1100 kN is supported by a tripod as shown in Fig. P. 9.2.25(a). Find the force in each member. Point C, O and D are in x-z plane while point B is 2 m above this plane. **SPPU : Dec. 11, 8 Marks**

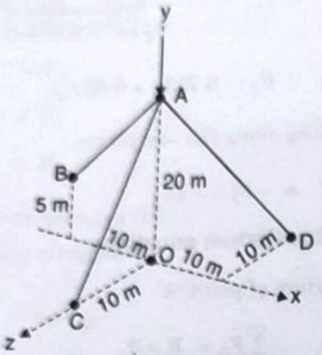


Fig. P. 9.2.25(a)

Soln. :

Step 1 : Coordinates :

A (0, 20, 0)

B (– 10, 5, 0)

C (0, 0, 10)

D (10, 0, – 10)

Step 2 : FBD of point 'A' :

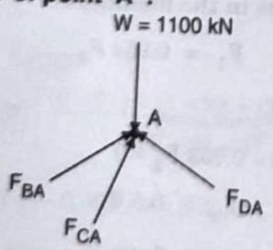


Fig. P. 9.2.25(b)

Let Force in the members,

$F_{BA} = F_1$

$F_{CA} = F_2$

$F_{DA} = F_3$

Step 3 : Force vectors ($\bar{F}$) :

F_1 is acting along BA

$$\bar{F}_1 = F_1 \cdot \bar{e}_{BA}$$

$$= F_1 \left[\frac{(0+10)\,i + (20-5)\,j + (0-0)\,k}{\sqrt{10^2+15^2+0}}\right]$$

$$\therefore \bar{F}_1 = F_1\,(0.55\,i + 0.83\,j)$$

F_2 is acting along CA

$$\bar{F}_2 = F_2 \cdot \bar{e}_{CA}$$

$$= F_2 \left[\frac{(0-0)\,i + (20-0)\,j + (0-10)\,k}{\sqrt{0^2+20^2+10^2}}\right]$$

$$\bar{F}_2 = F_2\,(0.89\,j - 0.45\,k)$$

F_3 is acting along DA

$$\bar{F}_3 = F_3 \cdot \bar{e}_{DA}$$

$$= F_3 \left[\frac{(0-10)\,i + (20-0)\,j + (0+10)\,k}{\sqrt{10^2+20^2+10^2}}\right]$$

$$\therefore \bar{F}_3 = F_3\,(-0.41\,i + 0.82\,j + 0.41\,k)$$

W is acting along the –ve y-axis.

$$\therefore \bar{W} = -Wj = -1100\,j$$

Step 4 : Equilibrium equations

For equilibrium of point 'A',

$$\sum \bar{F} = \bar{R} = 0$$

$$\therefore \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{W} = 0$$

$$\therefore \sum F_x = 0$$

$$0.55\,F_1 - 0.41\,F_3 = 0 \quad \text{...(I)}$$

$$\sum F_y = 0$$

$$0.83\,F_1 + 0.89\,F_2 + 0.82\,F_3 - 1100 = 0 \quad \text{...(II)}$$

$$\sum F_z = 0$$

$$-0.45\,F_2 + 0.41\,F_3 = 0 \quad \text{...(III)}$$

Step 5 : Force in the members :

From Eq^n (I), $\quad F_1 = 0.745\,F_3$

From Eq^n (III), $\quad F_2 = 0.911\,F_3$

Putting F_1 and F_2 in Eq^n (II)

$$0.83\,(0.745\,F_3) + 0.89\,(0.911\,F_3) + 0.82\,F_3 = 1100$$

$\therefore$ **F_3 = 489.05kN (Force in member AD)** ...

From Eq^n (IV),

$F_1 = 0.745\ (489.15)$

$\therefore$ F_1 = **364.34 kN (Force in member AB)** ... **Ans.**

From Eq^n (V), $F_2 = 0.911\ (489.05)$

$\therefore$ F_2 = **445.52 kN (Force in member AC)** ... **Ans.**

Ex. 9.2.26 : A triangular plate ABC weighing 160 N is in horizontal plane and it is supported by three wires connected to A, B and C. All the wires meet at D as shown in Fig. P. 9.2.26(a) which is exactly above the centroid 'E' of the plate at a height of 600 mm. If AE = 400 mm, EF = 200 mm and CF = FB = 200 mm, find tension in each wire.

SPPU : May 16,7 Marks

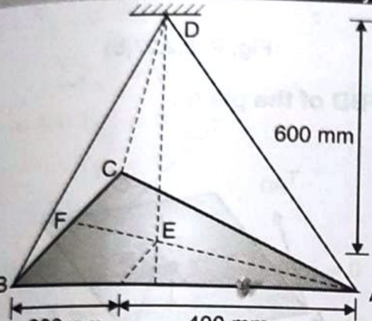


Fig. P. 9.2.26(a)

Soln. :

Step 1 : Coordinates :

A (400, 0, 0)

B (– 200, 0, 200)

C (– 200, 0, – 200)

D (0, 600, 0)

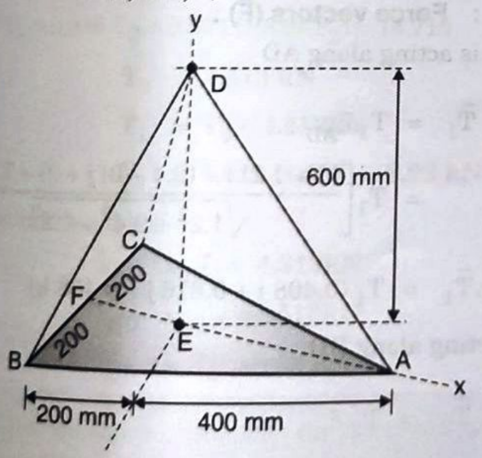


Fig. P. 9.2.26(b)

Step 2 : FBD of plate :

Let

$T_{AD} = T_1$

$T_{BD} = T_2$

$T_{CD} = T_3$

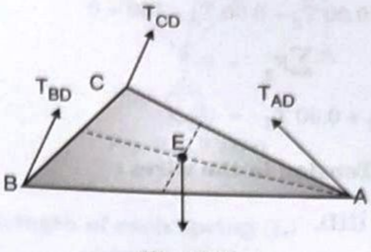


Fig. P. 9.2.26(b)

Step 3 : Force vectors ($\bar{F}$) :

T_1 is acting along AD

$$\bar{T}_1 = T_1 . \bar{e}_{AD}$$

$$= T_1 \left[\frac{(0-400)\,i + (600-0)\,j + (0-0)\,k}{\sqrt{400^2 + 600^2 + 0}} \right]$$

$$\therefore \bar{T}_1 = T_1\,(-0.55\,i + 0.83\,j) \quad \dots (1)$$

T_2 is acting along BD

$$\bar{T}_2 = T_2 . \bar{e}_{BD}$$

$$= T_2 \left[\frac{(0+200)\,i + (600-0)\,j + (0-200)\,k}{\sqrt{200^2 + 600^2 + 200^2}} \right]$$

$$\bar{T}_2 = T_2 . (0.30\,i + 0.90\,j - 0.30\,k) \quad \dots (2)$$

T_3 is acting along CD

$$\bar{T}_3 = T_3 . \bar{e}_{CD}$$

$$= T_3 \left[\frac{(0+200)\,i + (600-0)\,j + (0+200)\,k}{\sqrt{200^2 + 600^2 + 200^2}} \right]$$

$$\therefore \bar{T}_3 = T_3\,(0.30\,i + 0.90\,j + 0.30\,k) \quad \dots (3)$$

W is acting along –ve y-axis.

$$\therefore \bar{W} = -Wj = -160\,j \quad \dots (4)$$

Step 4 : Equilibrium equations :

For equilibrium of plate;

$$\bar{R} = \sum \bar{F} = 0$$

$$\therefore \bar{T}_1 + \bar{T}_2 + \bar{T}_3 + \bar{W} = 0$$

$$\therefore \sum F_x = 0 \qquad \text{... (I)}$$

$$-0.55\,T_1 + 0.30\,T_2 + 0.30\,T_3 = 0$$

$$\sum F_y = 0 \qquad \text{... (II)}$$

$$0.83\,T_1 + 0.90\,T_2 + 0.90\,T_3 - 160 = 0$$

$$\sum F_z = 0 \qquad \text{... (III)}$$

$$-0.30\,T_2 + 0.30\,T_3 = 0$$

Step 5 : Tension in the wires :

From Eq^n (III), ... (IV)

$$T_2 = T_3$$

From Eq^n (I), $-0.55\,T_1 + 0.30\,T_3 + 0.30\,T_3 = 0$

$$\therefore T_1 = 1.09\,T_3$$

From Eq^n (II), $0.83\,(1.09\,T_3) + 0.90\,T_3 + 0.90\,T_3 = 160$

$$\therefore \mathbf{T_3 = 59.14\ N}$$

$$T_1 = 1.09 \times 59.14 = \mathbf{64.46\ N}$$

$$T_2 = T_3 = \mathbf{59.14\ N}$$

T_1 = **64.46 N** ...**Ans.**

T_2 = **59.14 N** ...**Ans.**

T_3 = **59.14 N** ...**Ans.**

Ex. 9.2.27 : The square steel plate has a mass of 1500 kg with mass center at its center G. Calculate the tension in each of the three cables with which the plate is lifted while remaining horizontal. **SPPU : May 15,6 Marks**

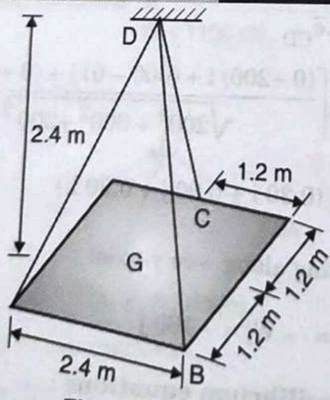


Fig. P. 9.2.27(a)

Soln. :

Step 1 : Coordinates :

Considering coordinate axes as shown in Fig. P. 9.2.27(a)

A (– 1.2, 0, 1.2)

..., 1.2)

C (0, 0, – 1.2)

D (0, 2.4, 0)

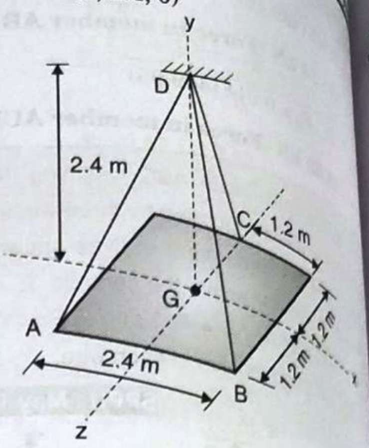


Fig. P. 9.2.27(b)

Step 2 : FBD of the plate :

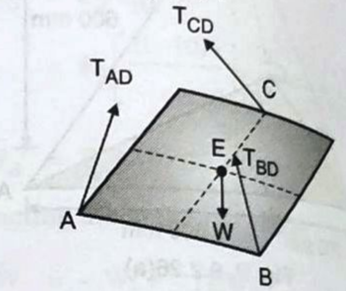


Fig. P. 9.2.27(c)

Let $T_{AD} = T_1$

$T_{BD} = T_2$

$T_{CD} = T_3$

$W = 1500\text{ kg} = 1500 \times 9.81 = 14.715\text{ kN}$

Step 3 : Force vectors ($\bar{F}$) :

T_1 is acting along AD

$$\bar{T}_1 = T_1 \cdot \bar{e}_{AD}$$

$$= T_1 \left[\frac{(0+1.2)\,i + (2.4-0)\,j + (0-1.2)\,k}{\sqrt{1.2^2 + 2.4^2 + 1.2^2}} \right]$$

$$\therefore \bar{T}_1 = T_1\,(0.408\,i + 0.816\,j - 0.408\,k)$$

T_2 is acting along BD

$$\bar{T}_2 = T_2 \cdot \bar{e}_{BD}$$

$$= T_2 \left[\frac{(0-1.2)\,i + (2.4-0)\,j + (0-1.2)\,k}{\sqrt{1.2^2 + 2.4^2 + 1.2^2}} \right]$$

$$\bar{T}_2 = T_2(-0.408\,i + 0.816\,j - 0.408\,k)$$

T_3 is acting along CD.

$$\bar{T}_3 = T_3 \cdot \bar{e}_{CD}$$

$$= T_3\left[\frac{(0-0)\,i + (2.4-0)\,j + (0+1.2)\,k}{\sqrt{0+2.4^2+1.2^2}}\right]$$

$$\therefore \bar{T}_3 = T_3\,(0.894\,j + 0.447\,k) \qquad \dots (3)$$

W is acting along –ve y-axis.

$$\therefore \bar{W} = -Wj = -14.715\,j \qquad \dots (4)$$

Step 4 : Equilibrium equations :

For equilibrium of plate;

$$\bar{R} = \sum \bar{F} = 0$$

$$\therefore \bar{T}_1 + \bar{T}_2 + \bar{T}_3 + \bar{W} = 0$$

$$\sum F_x = 0$$

$$0.408\,T_1 - 0.408\,T_2 = 0$$

$$\therefore T_1 = T_2 \qquad \dots (I)$$

$$\sum F_y = 0$$

$$0.816\,T_1 + 0.816\,T_2 + 0.894\,T_3 - 14.715 = 0 \qquad \dots (II)$$

$$\sum F_z = 0$$

$$-0.408\,T_1 - 0.408\,T_2 + 0.447\,T_3 = 0 \qquad \dots (III)$$

Step 5 : Tension in the cables:

From Eq^n (I), $T_1 = T_2$ ... (IV)

From Eq^n (III),

$$-0.408\,T_2 - 0.408\,T_2 + 0.447\,T_3 = 0$$

$$\therefore T_3 = 1.825\,T_2$$

From Eq^n (II),

$$0.816\,T_2 + 0.816\,T_2 + 0.894\,(1.825\,T_2) = 14.715$$

$$\therefore \mathbf{T_2 = 4.513\ kN}$$

$$\mathbf{T_1 = T_2 = 4.513 kN}$$

$$T_3 = 1.825\,(4.509) = \mathbf{8.23\ kN}$$

Tension in the cables;

$T_{AD} = T_1 =$ **4.513kN** ...**Ans.**

$T_{BD} = T_2 =$ **4.513kN** ...**Ans.**

$T_{CD} = T_3 =$ **8.23 kN** ...**Ans.**

Ex. 9.2.28 : The ball is suspended from the horizontal ring using three spring each having a stiffness of k = 50 N/m and an unstretched length of 1.5 m. If h = 2 m, determine the weight of ball. Refer Fig. P. 9.2.28(a).

SPPU : Dec 14, 6 Marks

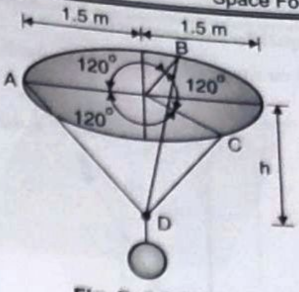


Fig. P. 9.2.28(a)

Soln. :

Step 1 : Length of each spring (L)

$$L = \sqrt{2^2 + 1.5^2}$$

$$= 2.5\ m.$$

Unstretched length of the spring,

$$L_o = 1.5\ m$$

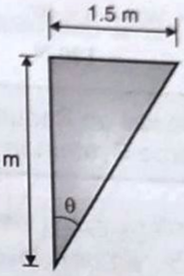


Fig. P. 9.2.28(b)

$$\theta = \tan^{-1}\left(\frac{1.5}{2}\right)$$

$$= 36.87°$$

$\therefore$ Deformation of each spring,

$$x = L - L_0$$

$$= 2.5 - 1.5 = 1.0\ m$$

Step 2 : Force in each spring,

$$F = k.x$$

$$= 50 \times 1 = 50\ N$$

Step 3 : FBD of point 'D' :

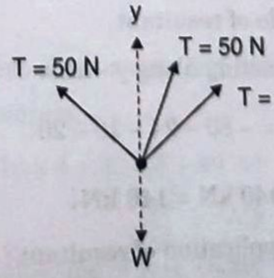


Fig. P. 9.2.28(c)

Step 4 : Equilibrium equations :

Vertical component of tension in each spring.

$$T_y = T\cos\theta$$

$$= 50\cos 36.87^\circ = 40\ N$$

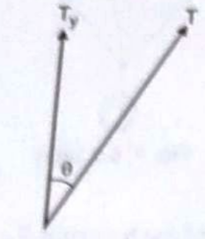


Fig. P. 9.2.28(d)

For equilibrium ;

$$\sum F_y = 0$$

$$3\,T_y - W = 0$$

$$\therefore W = 3\times 40$$

$$= 120\ N \qquad \text{... Ans.}$$

Type 4 : Based on Resultant of Parallel Force System in Space

Ex. 9.2.29 : A square foundation mat supports four columns as shown in Fig. P. 9.2.29(a). Determine magnitude and point of application of resultant of four loads.

SPPU : Dec. 09, 6 Marks

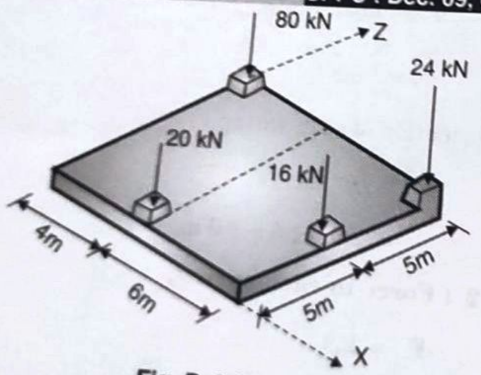


Fig. P. 9.2.29(a)

Soln. :

Step 1 : Magnitude of resultant

All forces are acting along y - axis.

$$\therefore \sum F_y = R = -80 - 24 - 16 - 20$$

$$= -140\ kN = 140\ kN\downarrow \qquad \text{... Ans.}$$

Step 2 : Point of application of resultant :

Let the point of application of resultant be (x, o, z)

i.e. It is acting at a distance 'x' m from z-axis and 'z' m from x-axis.

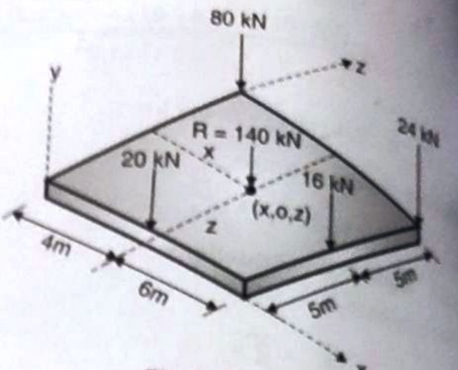


Fig. P. 9.2.29(b)

To find 'x'

Taking moment about z-axis and using Varignon's theorem;

$\sum$M of all force about z-axis = moment of R about z-axis

$$0 - 20\times 4 - 16\times 10 - 24\times 10 = -140\times x$$

$$\therefore x = 3.43\ m$$

To find 'y' :

Taking moments about x-axis and using Varignon's theorem,

$$0 - 80\times 10 - 16\times 5 - 24\times 10 = -140\times z$$

$$\therefore z = 8\ m$$

∴ Point of application of resultant,

$$= (3.43,\ 0,\ 8)\ m \qquad \text{... Ans.}$$

Ex. 9.2.30 : Three forces are acting on a plate of as shown in Fig. P. 9.2.30(a). Find magnitude and point of application of resultant force on the plate.

SPPU : May 13, 7 Marks

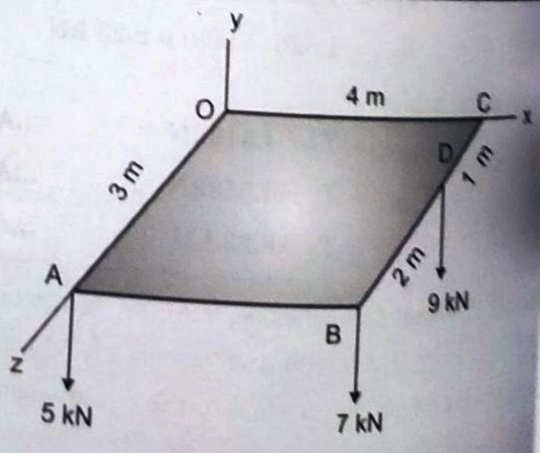


Fig. P. 9.2.30(a)

Soln. :

Step 1 : Magnitude of resultant :

All forces are acting along y–axis.

$$\therefore \sum F_y = R = -5-7-9 = -21 \text{ kN}$$

$$= 21 \text{ kN} \downarrow \qquad \text{... Ans.}$$

Step 2 : Point of application of 'R' :

Let (x, o, z) m be the point of application of resultant.

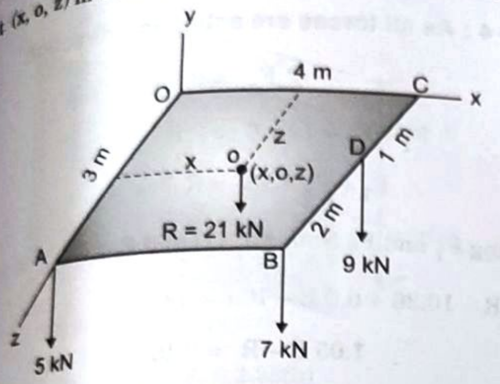


Fig. P. 9.2.30(b)

To find 'x'

Taking moment about z-axis and using Varignon's theorem,

$$-9\times4-7\times4 = -21\times x$$

Moment of 5kN is zero as it is acting on z-axis

$$\therefore x = 3.05 \text{ m}$$

To find 'z'

Taking moment about x-axis and using Varignon's theorem,

$$5\times3+7\times3+9\times1 = 21\times z$$

$$\therefore z = 2.14 \text{ m}$$

$\therefore$ Point of application of resultant is,

$$(3.05, 0, 2.14) \text{ m} \qquad \text{... Ans.}$$

Ex. 9.2.31 : Determine the magnitude of the resultant and its location with respect to origin 'O' as shown in Fig. P. 9.2.31(a). **SPPU : May 11, 6 Marks**

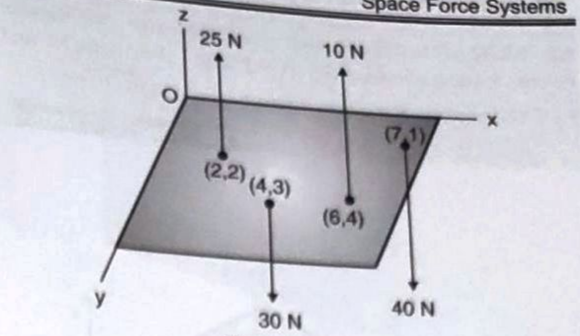


Fig. P. 9.2.31(a)

Soln. :

Step 1 : Magnitude of resultant :

All force are acting along z-axis.

$$\therefore \sum F_z = R = 25+10-40-30 = -35 \text{ N}$$

$$= 35 \text{ N}\downarrow \qquad \text{...Ans.}$$

Step 2 : Point of application of 'R'

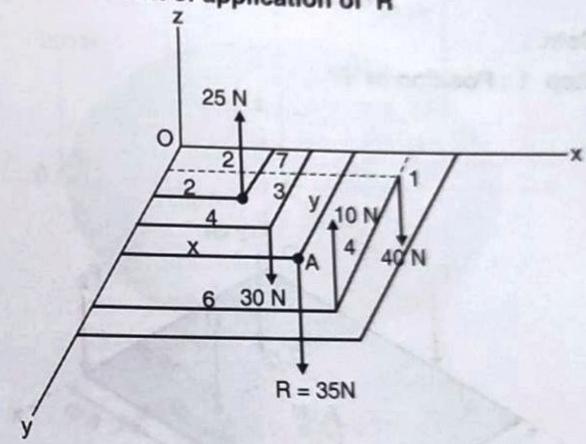


Fig. P. 9.2.31(b)

Let A(x, y, o) be the point of application of R.

To find 'x'

Taking moment about y-axis and using Varignon's theorem,

$$25\times2-40\times7+10\times6-30\times4 = -35\times x$$

$$\therefore x = 8.28$$

To find 'y'

Taking moment about x-axis and using Varignon's theorem,

$$-25\times2-10\times4+30\times3+40\times1 = 35\times y$$

$$\therefore y = 1.143$$

$\therefore$ Point of application of R is, (8.28, 1.143, 0) ...Ans.

Ex. 9.2.32 : The building slab is subjected to four parallel column loading shown in Fig. P. 9.2.32(a). Determine F_1 and F_2 if the resultant force acts through point (12 m, 10 m)

SPPU : May 18, 6 Marks

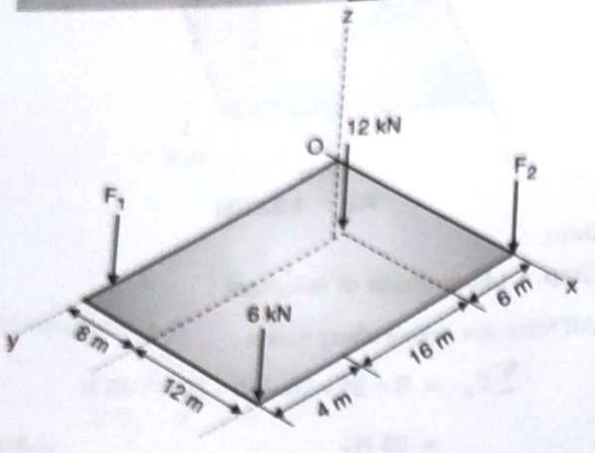


Fig. P. 9.2.32(a)

Soln. :

Step 1 : Position of 'R'

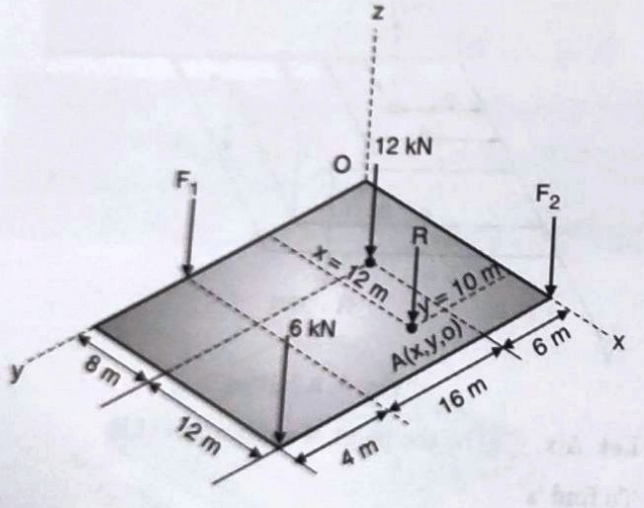


Fig. P. 9.2.32(b)

Step 2 :

Taking moments about y-axis and using Varignon's theorem,

$$F_1 \times 22 + 6 \times 26 + 12 \times 6 = R \times 10$$

Moment of F_2 is zero at x-axis

$$\therefore 22\,F_1 = 10\,R - 228$$

$$\therefore F_1 = 0.45\,R - 10.36 \quad \ldots (1)$$

Step 3 :

Taking moments about y-axis and using Varignon's theorem,

$$-12 \times 8 - 6 \times 20 - F_2 \times 20 = -R \times 12$$

Moment of F_1 at y-axis is zero

$$\therefore 20\,F_2 = 12\,R - 216$$

$$F_2 = 0.6R - 10.8$$

Step 4 : As all forces are acting in z direction.

$$\sum F_z = R$$

$$-F_1 - F_2 - 12 - 6 = -R$$

$$\therefore F_1 + F_2 + 18 = R$$

Putting F_1 and F_2 from Eq^n (1) and Eq^n (2)

$$0.45\,R - 10.36 + 0.6\,R - 10.8 + 18 = R$$

$$1.05\,R - R = 3.16$$

$$0.05\,R = 3.16$$

$$\therefore \mathbf{R = 63.2\ kN}$$

Step 5 : Loads F_1 and F_2

From Eq^n (1), $F_1 = 0.45\,(63.2) - 10.36$

$$\therefore \mathbf{F_1 = 18.08\ kN} \quad \ldots$$

From Eq^n (2), $F_2 = 0.6\,(63.2) - 10.8$

$$\therefore \mathbf{F_2 = 27.12\ kN} \quad \ldots$$

Ex. 9.2.33 : Determine the resultant of the parallel system which act on the plate as shown in Fig. P. 9.2.33

SPPU : May 15, 6 M

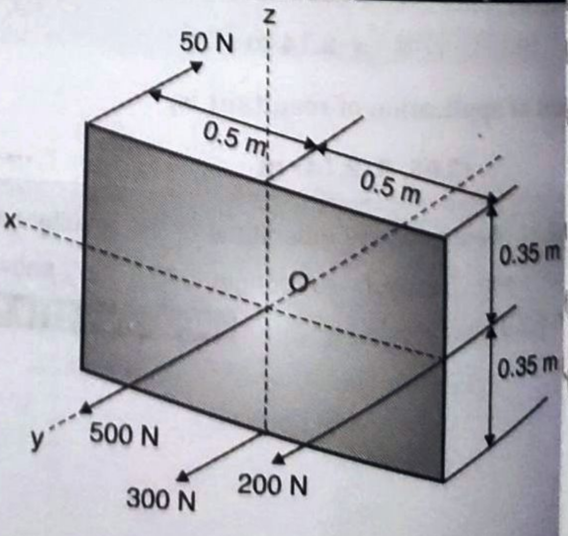


Fig. P. 9.2.33(a)

Soln. :

Step 1 : Magnitude of resultant :

All forces are acting along y-axis

$$\sum F_y = R = 500 + 300 + 200 - 50$$

$$= 950 \text{ N} \swarrow \qquad \text{... Ans.}$$

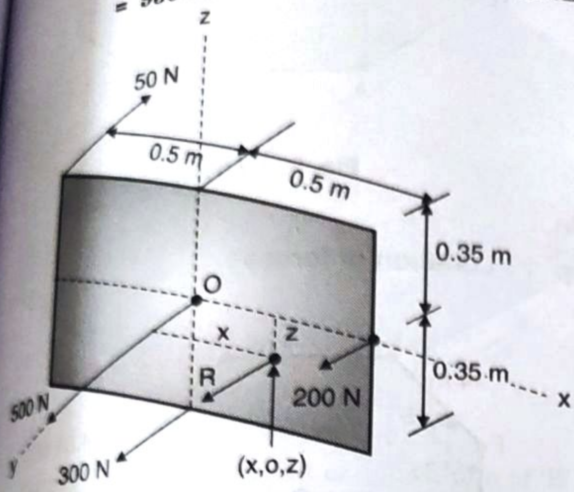


Fig. P. 9.2.33(b)

Step 2 : Point of application of resultant :

Let the resultant be acting at point (x, o, z) as shown in Fig. P. 9.2.33(b).

To find x :

Taking moment about z-axis and using Varignon's theorem,

$$-50 \times 0.5 + 0 - 200 \times 0.5 + 0 = -950 \times x$$

Moments of 500 N and 300 N are zero as they are acting on z-axis

$$\therefore x = 0.132 \text{ m}$$

To find 'z' :

Taking moment about x-axis

$$-50 \times 0.35 - 300 \times 0.35 = -950 \times z$$

Moment of 500 N and 200 N are zero as they are acting on x-axis.

$$\therefore z = 0.13 \text{ m}$$

∴ Point of application of resultant is

$$(0.132, 0, -0.13) \qquad \text{...Ans.}$$

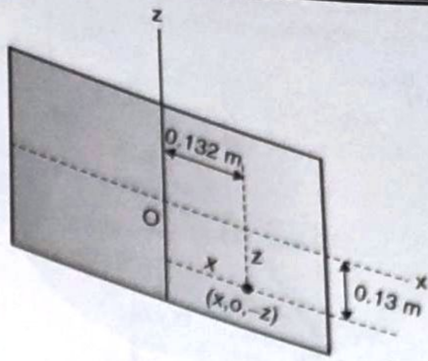


Fig. P. 9.2.33(c)

Ex. 9.2.34 : Three parallel bolting forces act on the rim of the circular cover plate as shown in Fig. P. 9.2.34(a). Determine the magnitude, nature and point of application of the resultant force with respect to origin O.

SPPU :Dec. 14, May 16 , 6 Marks, May 17,7 Marks

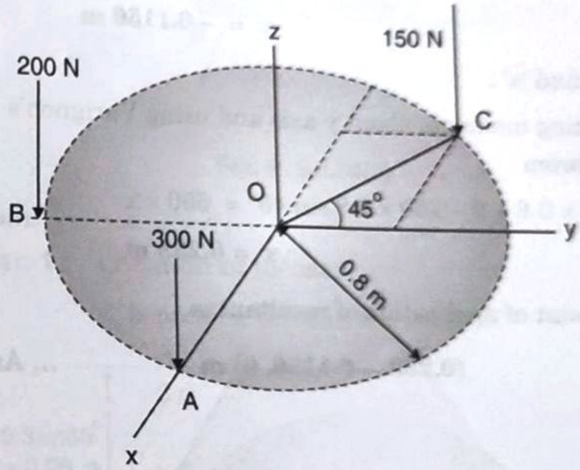


Fig. P. 9.2.34(a)

Soln. :

Step 1 : Magnitude of resultant :

As all force are acting along z-axis,

$$\sum F_z = R = -200 - 300 - 150 = -650 \text{ N}$$

$$= 650 \text{ N} \downarrow \qquad \text{...Ans}$$

Nature is downwards ... Ans

Step 2 : Point of application of resultant :

Let (x, y, o) be the point of application of resultan

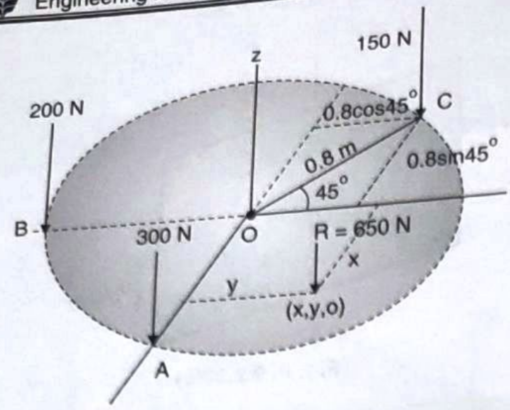


Fig. P. 9.2.34(b)

To find 'y' :

Taking moments about x-axis and using Varignon's theorem,

$$200 \times 0.8 + 0 - 150 \times 0.8 \cos 45° = -650 \times y$$

$$\therefore \quad \mathbf{y = -0.1156\ m}$$

To find 'x' :

Taking moments about y-axis and using Varignon's theorem

$$300 \times 0.8 + 0 - 150 \times 0.8 \sin 45° = 650 \times x$$

$$\therefore \mathbf{x = 0.238\ m}$$

$\therefore$ Point of application of resultant is

(0.238, – 0.1156, 0) m ... **Ans.**

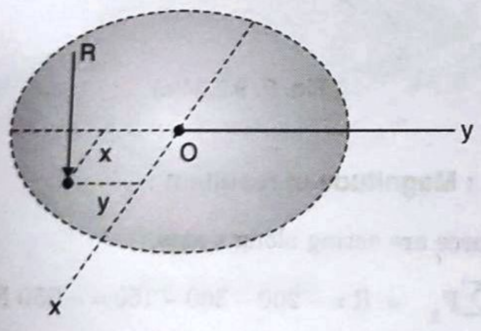


Fig. P. 9.2.34(c)

Ex. 9.2.35 : ABCDEFGH is a regular octagon of side 4 m. Fig. P. 9.2.35(a) shows positions of six columns which carry vertical forces normal to plane of octagon and acting at its six corner points. Locate the resultant of these forces w.r.t. centroid of octagon 'O'. **SPPU : May 07, 8 Marks**

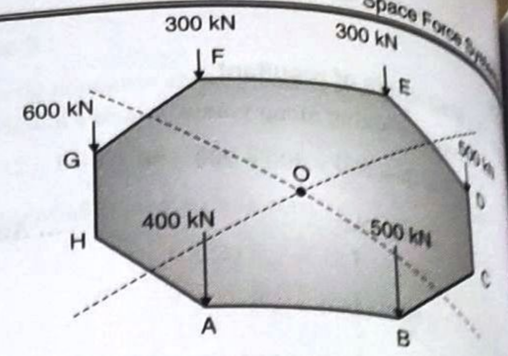


Fig. P. 9.2.35(a)

Soln. :

Step 1 : Location of forces :

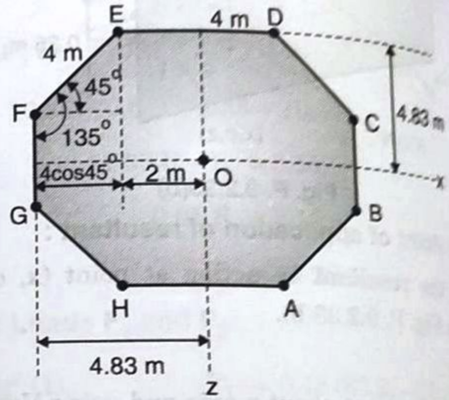


Fig. P. 9.2.35(b)

$$\text{Sum of all angles of octagon} = (2\,N - 4)\,90°$$

$$= (2 \times 8 - 4)\,90°$$

$$= 1080°$$

$$\therefore \text{Each angle} = \frac{1080°}{8}$$

$$= 135°$$

Step 2 : Magnitude of resultant

Taking y-axis vertical,

All forces are acting in y-direction

$$\therefore \sum F_y = R$$

$$= -600 - 300 - 300 - 500 - 500 - 400$$

$$= -2600\ \text{kN}$$

$$= \mathbf{2600\ kN \downarrow}$$

Step 3 : Location of 'R'

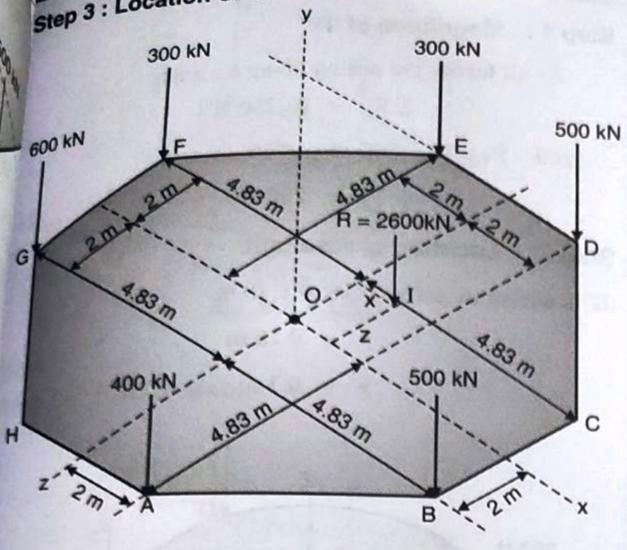


Fig. P. 9.2.35(c)

Let I (x, o, – z) be the point of application of 'R'

To find 'x':

Taking moment about z-axis and using Varignon's theorem;

$600 \times 4.83 + 300 \times 4.83 + 300 \times 2 - 500 \times 2 - 500 \times 4.83 - 400 \times 2 = -2600 \times x$

$\therefore x = -0.281$ m

To find 'z':

Taking moment about x-axis and using Varignon's theorem;

$600 \times 2 + 400 \times 4.83 + 500 \times 2 - 300 \times 2 - 300 \times 4.83 - 500 \times 4.83 = -2600 \times z$

$\therefore z = 0.127$ m

$\therefore$ Position of resultant is

(– 0.281, 0, – 0.127) m ... Ans.

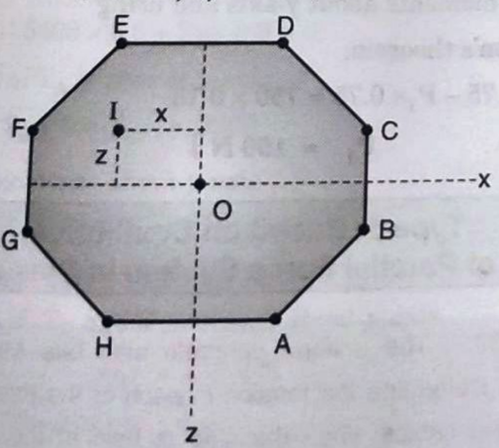


Fig. P. 9.2.35(d)

'I' is the position of resultant.

Ex.9.2.36: A concrete foundation mat in the shape of regular hexagon with 3m side support column loads as shown in Fig. P. 9.2.36(a). Determine the magnitude of the additional loads P_1 and P_2 that must be applied at B and F if resultant of all six loads is to pass through the centre the centre of the mat.

SPPU : May 13, 6 Marks

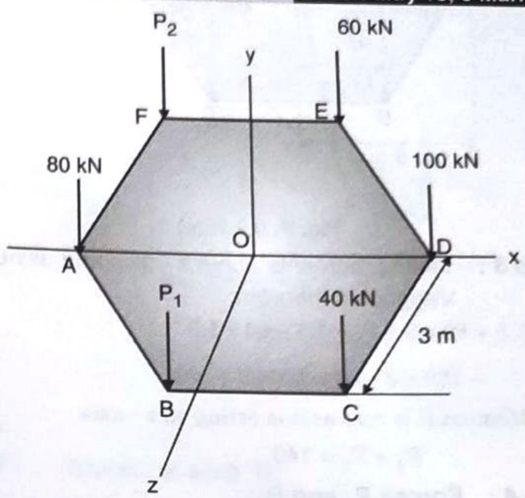


Fig. P. 9.2.36(a)

Soln:

Step 1 : Location of forces:

'R' is passing through the point 'O'

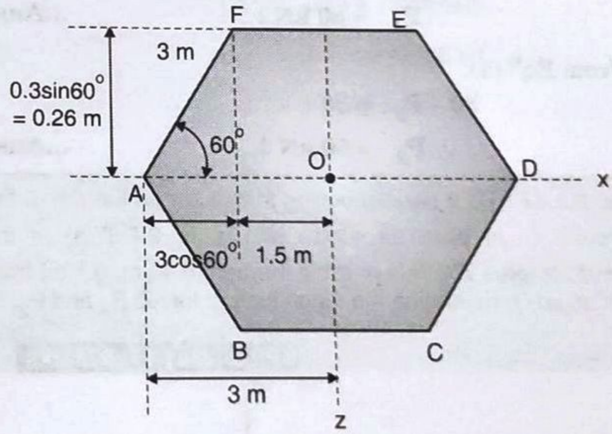


Fig. P. 9.2.36(b)

Step 2 : Taking moments about x-axis and using Varignon's theorem.

$-P_2 \times 0.26 - 60 \times 0.26 + P_1 \times 0.26 + 40 \times 0.26 = 0$

Moments of 80 KN, 100 KN and R are zero as they are acting on x-axis.

$P_1 - P_2 = 20$ ...(1)

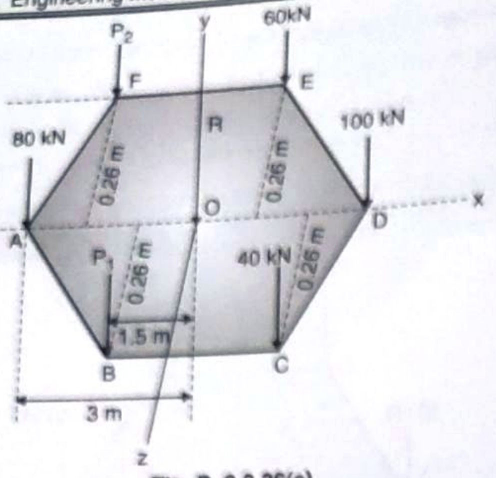


Fig. P. 9.2.36(c)

Step 3 : Taking moments about z - axis and using Varignon's theorem;

$$P_1 \times 1.5 + 80 \times 3 + P_2 \times 1.5 - 60 \times 1.5$$

$$- 100 \times 3 - 40 \times 1.5 = 0$$

Moment R is zero as it is acting on z - axis

$$P_1 + P_2 = 140 \qquad \ldots(2)$$

Step 4 : Forces P_1 and P_2:

From equations (1) and (2);

$$P_1 - P_2 = 20$$

$$P_1 + P_2 = 140$$

$$2P_1 = 160$$

$$\mathbf{P_1 = 80\ kN \downarrow} \qquad \textbf{...Ans.}$$

From Eq^n (1),

$$80 - P_2 = 20$$

$$\therefore \mathbf{P_2 = 60\ kN \downarrow} \qquad \textbf{...Ans.}$$

Ex. 9.2.37 : Four parallel bolting forces act on the rim of the circular cover plate as shown in Fig. P. 9.2.37(a). If the resultant force 750 N is passing through (0.15 m, 0.1 m) from the origin O, determine the magnitude of forces P_1 and P_2.

SPPU : May 17, 6 Marks

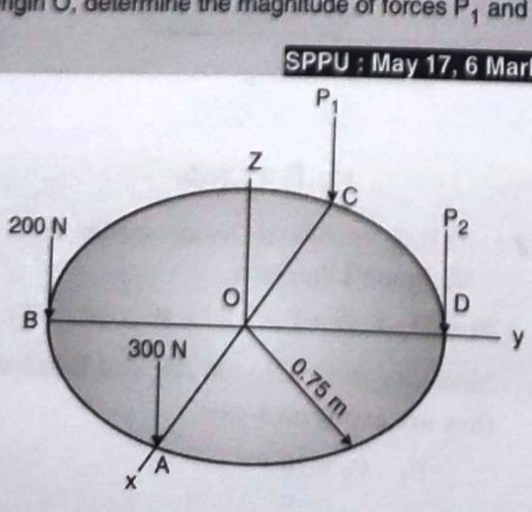


Fig. P. 9.2.37(a)

Soln :

Step 1 : Magnitude of 'R'

As all forces are acting along z – axis;

$$\Sigma F_z = R = 750\ N \downarrow$$

$$\therefore -200 - P_1 - P_2 - 300 = -750$$

$$\therefore P_1 + P_2 = 250\ N$$

Step 2 : Location of resultant:

'R' is acting at point (0.15, 0.1, 0) m.

$$x = 0.15\ m$$

$$y = 0.1\ m$$

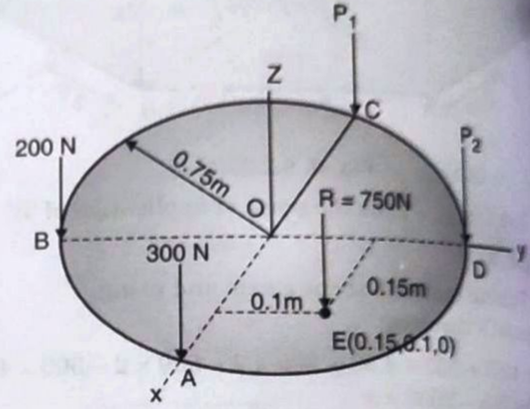


Fig. P. 9.2.37(b)

Taking moments about x-axis and using Varignon's theorem:

$$200 \times 0.75 - P_2 \times 0.75 = -750 \times 0.1$$

Moments of 300 N and P_1 are zero as they are acting on x-axis

$$\therefore \quad \mathbf{P_2 = 300\ N \downarrow} \qquad \textbf{...Ans.}$$

Taking moments about y-axis and using Varignon's theorem;

$$300 \times 0.75 - P_1 \times 0.75 = 750 \times 0.15$$

$$\therefore \quad \mathbf{P_1 = 150\ N \downarrow} \qquad \textbf{...Ans.}$$

Type 5: Based on Equilibrium of Parallel Force System In Space

Ex. 9.2.38: The uniform concrete slab has a weight 5500 N. Determine the tension in each of the three parallel supporting cables when the slab is held in the horizontal plane as shown in Fig. P. 9.2.38(a).

SPPU : May 10, 7 Marks , May 16, 6 Marks

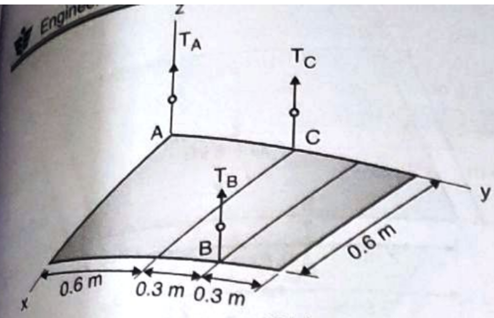


Fig. P. 9.2.38(a)

Soln :

Step 1 : FBD of slab:

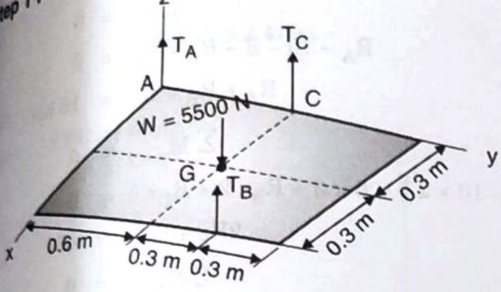


Fig. P. 9.2.38(b)

Weight of the slab is acting at the centre of gravity of the slab.

Step 2 : Equilibrium equations:

For equilibrium of the slab;

$$\Sigma F_z = 0$$

$$T_A + T_B + T_C - 5500 = 0$$

$$\therefore \quad T_A + T_B + T_C = 5500 \text{ N} \qquad \ldots(1)$$

Taking moments about x-axis

$$\Sigma M_{x-x} = 0$$

$$T_C \times 0.6 - 5500 \times 0.6 + T_B \times 0.9 = 0$$

Moment of T_A is zero at x-axis

$$0.9\, T_B + 0.6\, T_C = 3300 \qquad \ldots(2)$$

Taking moments about y-axis

$$\Sigma M_{y-y} = 0$$

$$5500 \times 0.3 - T_B \times 0.6 = 0$$

Moments of T_A and T_c are zero about y-axis

$$\mathbf{T_B = 2750 \text{ N}} \qquad \text{...Ans.}$$

Step 3 : Tension in wires:

From Eq^n (2),

$$0.9\,(2750) + 0.6\, T_C = 3300$$

$$\mathbf{T_C = 1375 \text{ N}} \qquad \text{...Ans.}$$

From Eq^n (1),

$$T_A + 2750 + 1375 = 5500$$

$$\mathbf{T_A = 1375 \text{ N}} \qquad \text{...Ans.}$$

Ex. 9.2.39: A plate ACED weighs 7600 kg/m^3. It is held in horizontal plane by three wires at A, B and C. Find tensions in the wires.

SPPU : Dec. 06, 8 Marks

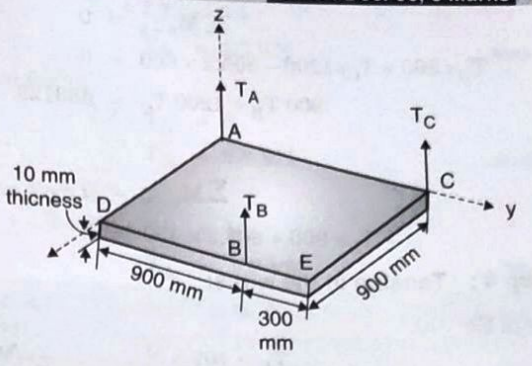


Fig. P. 9.2.39(a)

Soln :

Step 1 : Weight of slab 'W'

Weight, $\quad W = \text{Mass} \times g$

Mass density, $\quad \rho = \dfrac{\text{Mass}}{\text{volume}} = \dfrac{m}{v}$

Volume of the plate,

$$v = \text{Area} \times \text{thickness}$$

$$= (1.2 \times 0.9) \times 0.01$$

$$= 0.0108 \text{ m}^3$$

$$7600 = \frac{\text{mass}}{0.0108}$$

mass, $\quad m = 82.08$ Kg

Weight, $\quad W = 82.08 \times 9.81 = 805.2$ N

Step 2 : FBD of plate:

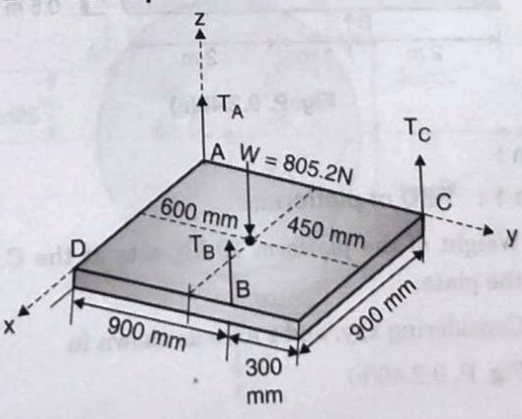


Fig. P. 9.2.39(b)

Weight will act the centre of gravity i. e., mid point of the plate.

Step 3 : Equilibrium equations:

$$\Sigma F_z = 0$$

$$T_A + T_B + T_C = W = 805.2 \text{ N} \qquad ...(1)$$

$$\Sigma M_{x-x} = 0$$

$$T_B \times 900 + T_C \times 1200 - 805.2 \times 600 = 0$$

$$900\, T_B + 1200\, T_C = 483120 \qquad ...(2)$$

$$\Sigma M_{y-y} = 0$$

$$-T_B \times 900 + 805.2 \times 450 = 0 \qquad ...(3)$$

Step 4 : Tension in the wires:

From Eq^n (3),

$$T_B = 402.6 \text{ N} \qquad \text{...Ans.}$$

From Eq^n (2),

$$900\,(402.6) + 1200\, T_C = 483120$$

$$T_C = 100.65 \text{ N} \qquad \text{...Ans.}$$

From Eq^n (1),

$$T_A + 402.6 + 100.65 = 805.20$$

$$T_A = 301.95 \text{ N} \qquad \text{...Ans.}$$

Ex. 9.2.40: A 10 kN platform is supported vertically at A, B and C as shown in Fig. P. 9.2.40(a). If it carries a load of 6 kN, Determine the reaction at each support.

SPPU : May 14, 6 Marks

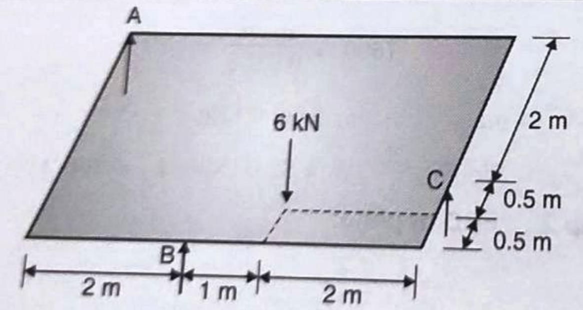


Fig. P. 9.2.40(a)

Soln :

Step 1 : *FBD of platform:*

Weight of the platform 10 kN acts at the C.G. of the plate.

Considering x, y, and z axes as shown in Fig. P. 9.2.40(b)

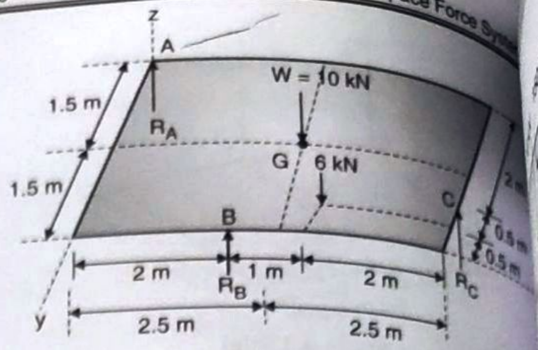


Fig. P. 9.2.40(b)

Step 2 : Equilibrium equations:

$$\Sigma F_z = 0$$

$$R_A - 10 - 6 + R_B + R_C = 0$$

$$R_A + R_B + R_C = 16 \text{ kN}$$

$$\Sigma M_{y-y} = 0$$

$$-10 \times 2.5 - 6 \times 3 + R_B \times 2 + R_C \times 5 = 0$$

$$2R_B + 5R_C = 43$$

$$\Sigma M_{x-x} = 0$$

$$R_A \times 3 - 10 \times 1.5 - 6 \times 0.5 + R_C \times 1 = 0$$

$$3R_A + R_C = 18$$

Step 3 : Reactions at supports:

From Eq^n(2), $\quad 2R_B + \cancel{5R_C} = 43$

$(3) \times 5 \quad 15R_A + \cancel{5R_C} = 90$

$\quad - \quad - \quad -$

$$2R_B - 15\,R_A = -47$$

From Eq^n (1), $\quad R_A + R_B + R_C = 16$

Eq^n (1) × 2 $\quad 2R_A + \cancel{2R_B} + 2R_C = 32$

From Eq^n (4) $\quad -15R_A + \cancel{2R_B} = -47$

$\quad + \quad - \quad +$

$$17R_A + \cancel{2R_C} = 79$$

Eq^n (3) × 2 $\quad 6\,R_A + \cancel{2}\,R_C = 36$

$\quad - \quad - \quad -$

$$11\,R_A = 43$$

$$R_A = 3.91 \text{ kN} \uparrow$$

From Eq^n (3),

$$3\,(3.91) + R_C = 18$$

$$R_C = 6.27 \text{ kN} \uparrow$$

From Eq^n (1),

$$3.91 + R_B + 6.27 = 16$$

$$R_B = 5.82 \text{ kN} \uparrow$$

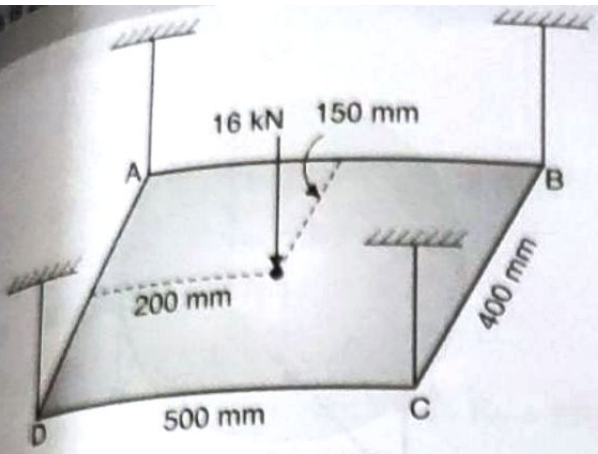


Fig. P. 9.2.41(a)

Soln.:

Step 1: FBD of plate :

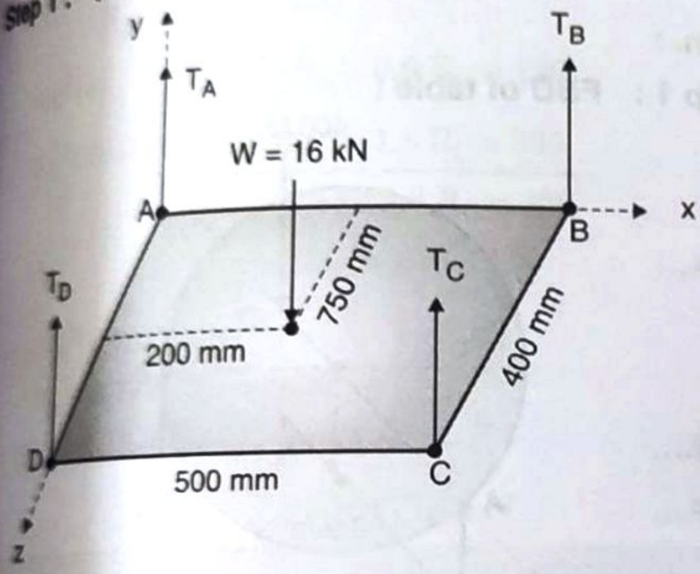


Fig. P. 9.2.41(b)

Let the forces in the wires be T_A, T_B, T_C and T_D.

Step 2: Equilibrium equations :

$$\Sigma F_y = 0$$

$$T_A + T_B + T_C + T_D = 16 \text{ kN} \qquad ...(1)$$

$$\Sigma M_{z-z} = 0$$

$$-16 \times 200 + T_B \times 500 + T_C \times 500 = 0$$

$$500\, T_B + 500\, T_C = 3200$$

$$T_B + T_C = 6.4 \qquad ...(2)$$

$$\Sigma M_{x-x} = 0$$

$$-T_D \times 400 - T_C \times 400 + 16 \times 150 = 0$$

$$400\, T_C + 400\, T_D = 2400$$

$$\therefore T_C + T_D = 6 \qquad ...(3)$$

... away from the po

applied load, force in the wire at C is mini

$$T_C = 2.2 \text{ kN}$$

From Eq^n(2),

$$T_B + 2.2 = 6.4$$

$$T_B = 4.2 \text{ kN}$$

From Eq^n(3),

$$2.2 + T_D = 6$$

$$T_D = 3.8 \text{ kN}$$

From Eq^n(1),

$$T_A + 4.2 + 2.2 + 3.8 = 16$$

$$T_A = 5.8 \text{ kN}$$

Ex. 9.2.42 : A horizontal plate (in xz plane) of 3m 1.5 kN weight is supported by three vertical wires Find tension in each of these wires.

SPPU : May. 0

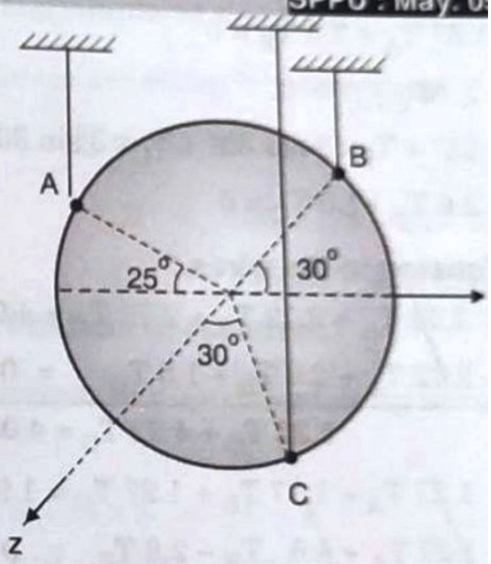


Fig. P. 9.2.42(a)

Soln :

Step 1 : Location of points A, B and C

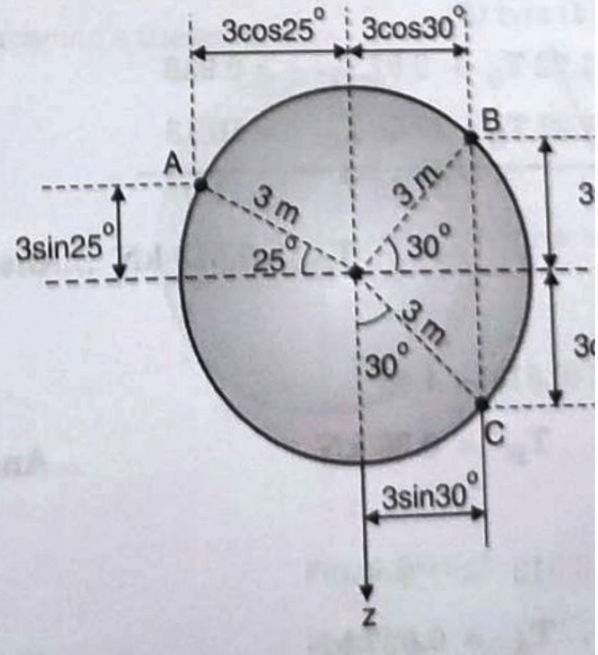


Fig. P. 9.2.42(b)

Step 2 : FBD of the plate :

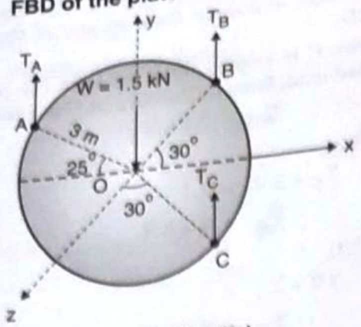


Fig. P. 9.2.42(c)

Step 3 : Equilibrium equations :

$$\Sigma F_y = 0$$

$$T_A + T_B + T_C = 1.5 kN \quad ...(1)$$

$$\Sigma M_{x-x} = 0$$

$$-T_c \times 3 \cos 30° + T_A \times 3 \sin 25° + T_B \times 3 \sin 30° = 0$$

$$-2.6\ T_C + 1.27\ T_A + 1.5\ T_B = 0 \quad ...(2)$$

$$\Sigma M_{z-z} = 0$$

$$-T_A \times 3 \cos 25° + T_B \times 3 \cos 30° + T_C \times 3 \sin 30° = 0$$

$$-2.72\ T_A + 2.6\ T_B + 1.5\ T_C = 0 \quad ...(3)$$

Step 4 : Tension in the wires :

$(1) \times 2.72$, $\quad \cancel{2.72}\ T_A + 2.72\ T_B + 2.72\ T_C = 4.08$

$(3) \quad -\cancel{2.72}\ T_A + 2.6\ T_B + 1.5\ T_C = 0$

$$5.32\ T_B + 4.22\ T_C = 4.08 \quad ...(4)$$

$(1) \times 1.27 \quad \cancel{1.27}\ T_A + \cancel{1.27}\ T_B + 1.27\ T_C = 1.905$

$(2) \quad \cancel{1.27}\ T_A + \cancel{1.5}\ T_B - 2.6\ T_C = 0$

$\quad - \quad - \quad + \quad -$

$$-0.23\ T_B + 3.87\ T_C = 1.905 \quad ...(5)$$

Solving Eq^n (4) and (5),

$(4) \times 0.23, \quad \cancel{1.22}\ T_B + 0.97\ T_C = 0.938$

$(5) \times 5.32, \quad -\cancel{1.22}\ T_B + 20.58\ T_C = 10.13$

$$21.55\ T_C = 11.07$$

$$\mathbf{T_C = 0.513\ kN} \quad \text{...Ans.}$$

From Eq^n (4),

$$5.32\ T_B + 4.22\ (0.513) = 4.08$$

$$\therefore \quad \mathbf{T_B = 0.36\ kN} \quad \text{...Ans.}$$

From Eq^n (1),

$$T_A + 0.36 + 0.513 = 1.5$$

$$\therefore \mathbf{T_A = 0.627\ kN} \quad \text{...Ans.}$$

Ex. 9.2.43 : The circular table, 1.8m diameter shown in Fig. P. 9.2.43(a) supports a load of 400 N, located at point D. The support reactions R_A, R_B, R_C are equally spaced along the circumference. Determine magnitude of reactions.

SPPU : Dec. 10, 9 Marks

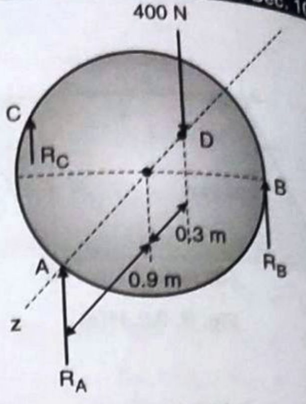


Fig. P. 9.2.43(a)

Soln. :

Step 1 : FBD of table :

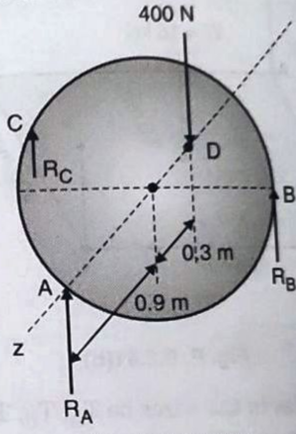


Fig. P. 9.2.43(b)

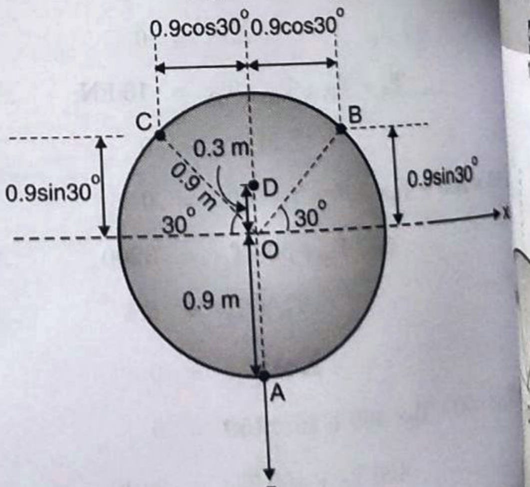


Fig. P. 9.2.43(c)

Step 2 : Equilibrium equations :

$$\Sigma F_y = 0$$

$$R_A + R_B + R_C = 400 \qquad ...(1)$$

$$\Sigma M_{z-z} = 0$$

$$-R_C \times 0.9 \cos 30^\circ + R_B \times 0.9 \cos 30^\circ = 0$$

$$R_B = R_C \qquad ...(2)$$

$$\Sigma M_{x-x} = 0$$

$$-R_A \times 0.9 - 400 \times 0.3 + R_B \times 0.9 \sin 30^\circ + R_C \times 0.9 \sin 30^\circ = 0$$

$$-0.9\, R_A + 0.45\, R_B + 0.45\, R_C = 120 \qquad ...(3)$$

Step 3 : Magnitude of reactions :

From Eq^n (2), $R_B = R_C$

From Eq^n (3), $-0.9\, R_A + 0.45\, R_C + 0.45\, R_C = 120$

$$-0.9\, R_A + 0.9\, R_C = 120 \qquad ...(4)$$

From Eq^n (1),

$$R_A + R_C + R_C = 400$$

$$\therefore R_A + 2R_C = 400 \qquad ...(5)$$

Eq^n(4) $\quad -0.9\cancel{R_A} + 0.9\, R_C = 120$

Eq^n(5) $\times$ 0.9 $\quad 0.9\cancel{R_A} + 1.8\, R_C = 360$

$$2.7\, R_C = 480$$

$$\mathbf{R_C = 177.78\ N} \qquad \textbf{...Ans.}$$

From Eq^n (5),

$$R_A + 2(177.78) = 400$$

$$\therefore \mathbf{R_A = 44.44\ N} \qquad \textbf{...Ans.}$$

$$\mathbf{R_B = R_C = 177.78\ N} \qquad \textbf{...Ans.}$$

Ex. 9.2.44 : A circular mat foundation of radius 5 m is supporting 4 columns at a distance of 4 m from the centre 'O' as shown in Fig. P.9.2.44. Determine the magnitude and position of the resultant force with respect to the origin 'O'.

SPPU : May 19, 7 Marks

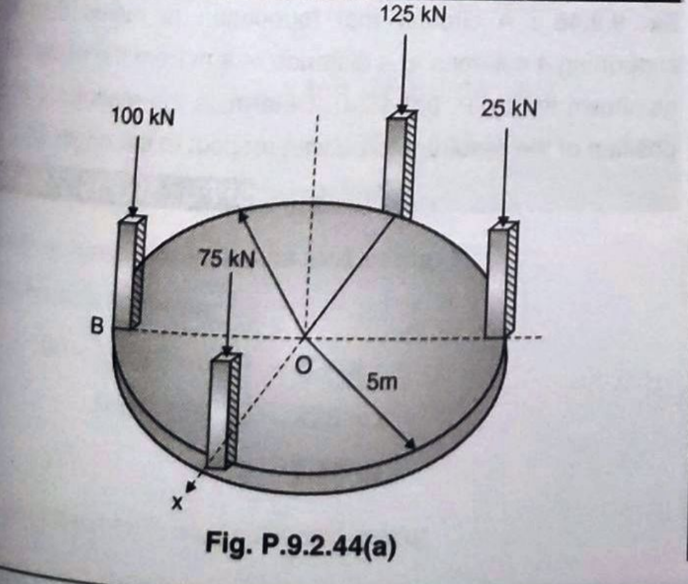


Fig. P.9.2.44(a)

Soln. :

Step 1 : Co-ordinate axes :

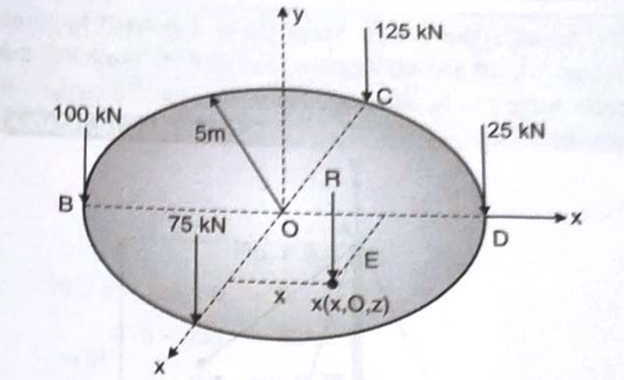


Fig. P.9.2.44(b)

Let 'R' be the resultant and (x, 0, z) be the position of the resultant w.r.t. 'O'.

Step 2 : Magnitude of resultant :

Being all forces are acting in y-direction;

$$\Sigma F_y = R$$

$$\therefore \quad R = -100 - 125 - 25 - 75$$

$$= -325\ kN = \mathbf{325\ kN\downarrow} \qquad \textbf{...Ans.}$$

Step 3 : Position of resultant :

Taking moments at z – axis and using Varignon's theorem;

$$100 \times 5 - 25 \times 5 = -R \times x$$

$$500 - 125 = -325 \times x$$

$$\therefore \quad \mathbf{x = -1.15\ m}$$

Taking moments at x-axis and using Varignon's theorem;

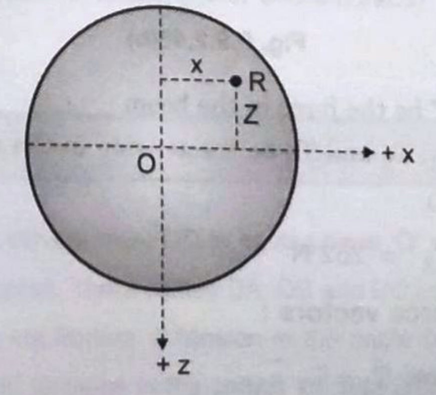


Fig. 9.2.44(c)

$$75 \times 5 - 125 \times 5 = R \times z$$

2.45 : The vertical boom OA is suppo... AC, AB and AD as shown in Fig. If the tension in the ...D is 252 N, determine the tensions in the cables AB ...

SPPU : May 19,7Marks

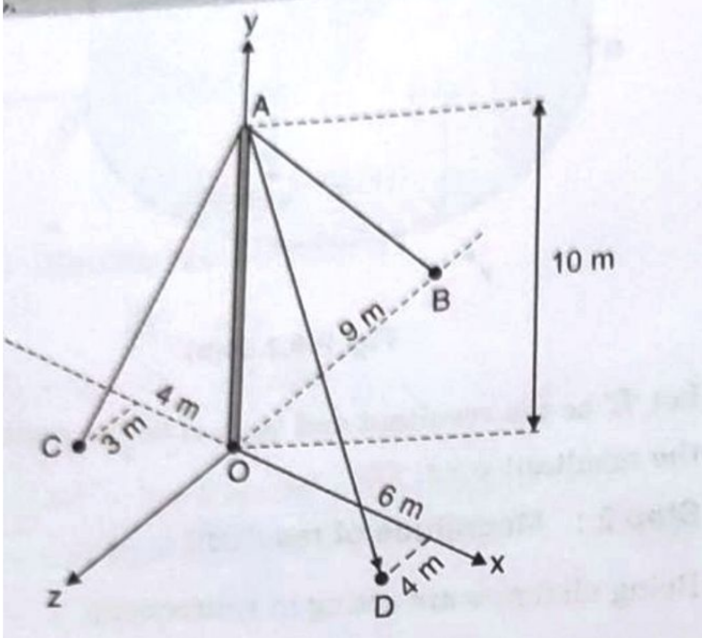


Fig. P.9.2.45(a)

...Co-ordinates :

...10, 0)

...0, – 9)

..., 0, 3) and

...), 4)

...BD of point 'A'

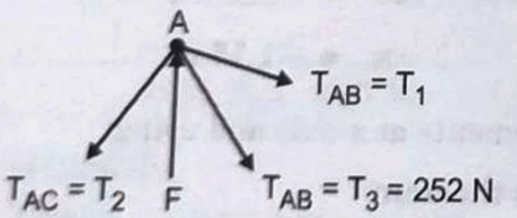


Fig. P.9.2.45(b)

...e the force in the boom

... T_2 and T_3 be the tension in the cables AB,

... = 252 N

...e vectors :

$\bar{T}_1 = T_1 \cdot \bar{e}_{AB}$

$= T_1 \left[\frac{(0-0)i + (0-10)j + (-9-0)k}{\dots} \right]$

$\therefore \bar{T}_2 = T_2(-0.357i - 0.89j + 0.268k)$

Force Vector, $\bar{T}_3 = T_3 \cdot \bar{e}_{AD}$

$= 252 \left[\frac{(6-0)i + (0-10)j + (4 \dots}{\sqrt{6^2 + 10^2 + 4^2}} \right.$

$\therefore \bar{T}_3 = 119.53i - 199.22j + 79.69k$

Force Vector, $\bar{F} = F \cdot j$

Step 4 :Equilibrium equations :

For equilibrium of point 'A',

$$\sum F_x = 0$$

$$-0.357\, T_2 + 119.53 = 0$$

$$\sum F_y = 0$$

$$-0.743\, T_1 - 0.89\, T_2 - 199.22 + F = 0$$

$$\sum F_z = 0$$

$$-0.669\, T_1 + 0.268\, T_2 + 79.69 = 0$$

Step 5 :Equilibrium equations :

From (I), $\quad T_2 = 334.82N$

From (II),$-0.669\, T_1 + 0.268(334.82) + 79.69 \dots$

$\therefore \quad T_1 = 253.24N$

$\therefore$ Tension in the cables are :

$T_{AB} = T_1 = 253.24N$

$T_{AC} = T_2 = 334.82N$

Ex. 9.2.46 : A circular mat foundation of ra... supporting 4 columns at a distance of 4 m from ... as shown in Fig. P. 9.2.46(a). Determine the m... position of the resultant force with respect to the...

SPPU : Ma...

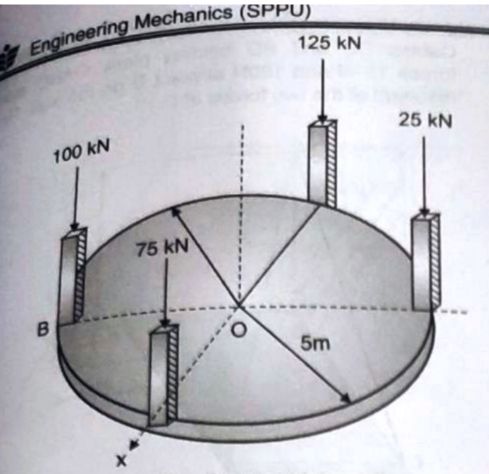


Fig. P. 9.2.46(a)

Soln. :

Step 1 : Co-ordinate axes :

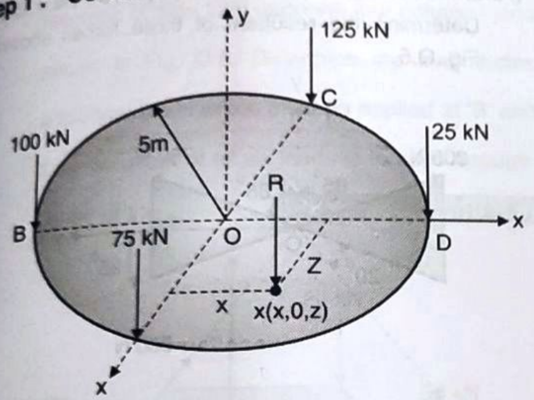


Fig. P. 9.2.46(b)

Let 'R' be the resultant and (x, 0, z) be the position of the resultant w.r.t. 'O'.

Step 2 : Magnitude of resultant :

Being all forces are acting in y-direction;

$$\Sigma F_y = R$$

$$\therefore \quad R = -100 - 125 - 25 - 75$$

$$= -325 \text{ kN} = \mathbf{325 \text{ kN}} \downarrow \quad \text{...Ans.}$$

Step 3 : Position of resultant :

Taking moments at z-axis and using Varignon's theorem;

$$100 \times 5 - 25 \times 5 = -R \times x$$

$$500 - 125 = -325 \times x$$

$$\therefore \quad x = -1.15 \text{ m}$$

Taking moments at x-axis and using Varignon's theorem;

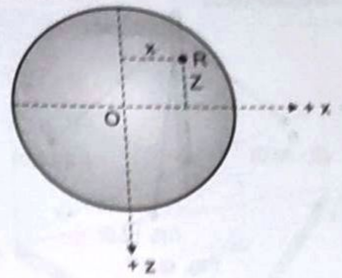


Fig. P.9.2.46(c)

$$75 \times 5 - 125 \times 5 = R \times z$$

$$375 - 625 = 325 \times z$$

$$\therefore \quad z = -0.77 \text{ m}$$

Position of resultant is (– 1.15, 0, – 0.77) m ...Ans.

Theory Questions

Q.1 A force $\bar{F}$ acts from point A towards point B in space. Write a vector expression for the force in terms of coordinates of points A and B. **(Refer Section 9.2.4)** **Dec. 00**

Q.2 Explain with neat sketches the following terms related to space forces :

i) Representation of force in vector form

ii) Condition of equilibrium for concurrent and parallel forces.

(Refer Sections 9.2.4, 9.2.8 and 9.2.9) **May 08**

Q.3 State the conditions of static equilibrium of space system of forces. **(Refer Section 9.2.8)** **Dec. 02**

Practice Problems

Q.1 A vertical mast OD is having base 'O' with ball and socket. Three cables DA, DB and DC keep the mast in equilibrium. If tension in the cable DA is 100kN, find tensions in the cables DB and DC and force in the mast. Refer Fig. Q.1. **SPPU : Dec. 06, 8 Marks**

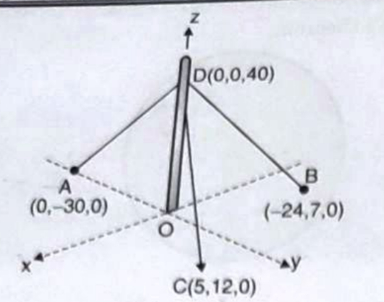


Fig. Q.1

[Ans. : (a) T_{DB} = 43.8kN, (b) T_{DC} = 187.52 kN, (c) F = 295.48 kN]

Q.2 A crate is supported by 3 cables as shown in Fig. Q.2. Determine the weight of the crate if the tension in cable AB is 750N.

SPPU : May 04, 8 Marks

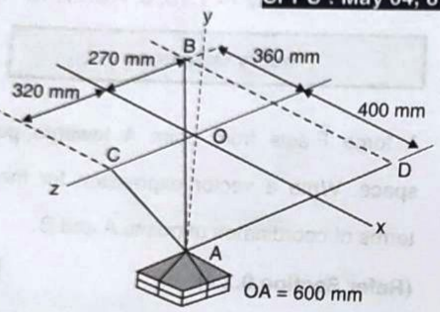


Fig. Q.2

[Ans. : W = 2100N]

Q.3 Fig. Q.3 shows a weight of 1000N supported by two cables AB and AC. Points B and C are in the same vertical plane. The horizontal force F keeps the weight in equilibrium in the position shown. Find the value of F and the tensions in the two cables.

SPPU : May 84, Marks

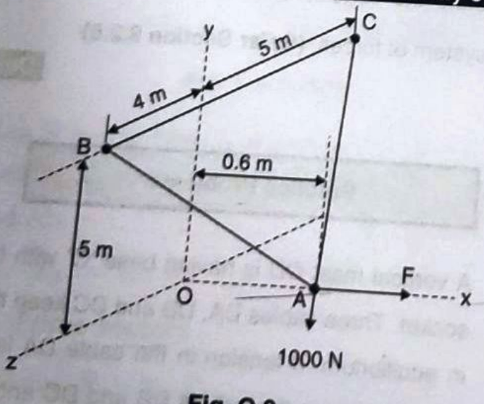


Fig. Q.3

[Ans.: (a)F = 120N, (b)T_{AB} = 714.8N, (c)T_{AC} = 630.96N]

Q.4 Cables CD and BD holding plank OABC exert forces 150N and 180N at point D on the wall. Find resultant of the two forces at D.

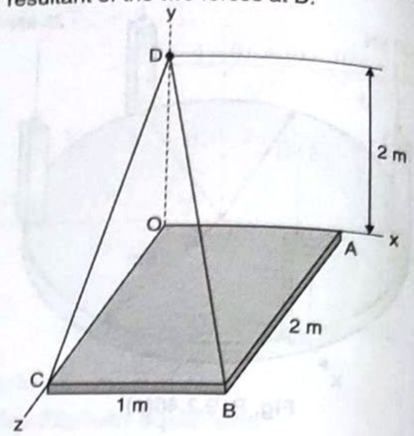


Fig. Q.4

[Ans. : R= 325.287N, θx = 79.37°, θy = 134°, θz = 45.98°]

Q.5 Determine the resultant of three forces shown in Fig. Q.5.

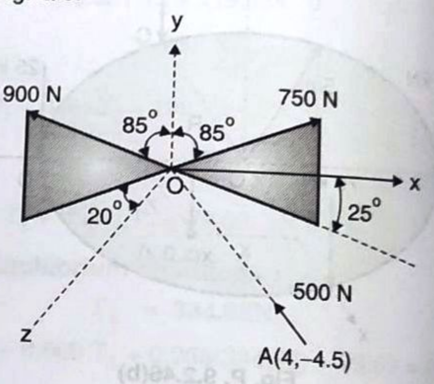


Fig. Q.5

[Ans. : R = 1163.8N, θx = 52.34°, θy = 69.45°, θz = 44.83°]

Q.6 A right angle triangular plate ABC having lengths AB = 400mm, AC = 300mm and BC = 300mm is having weight 50 N. It is held in Equilibrium by means of three parallel wires connected to points A, B and C in such a way that plate ABC remains in horizontal plane. Determine the tension in three wires.

SPPU : May 08, 8 Marks

[Ans. : T_A = 16.67 N, T_B = 16.67 N, T_C = 16.67 N]

Q.7 A square foundation mat supports four columns as shown in Fig. Q.7. Determine magnitude, direction and point of application of resultant of the four loads, as shown in Fig. Q.7.

SPPU : Dec. 09, May 10, 8 Marks

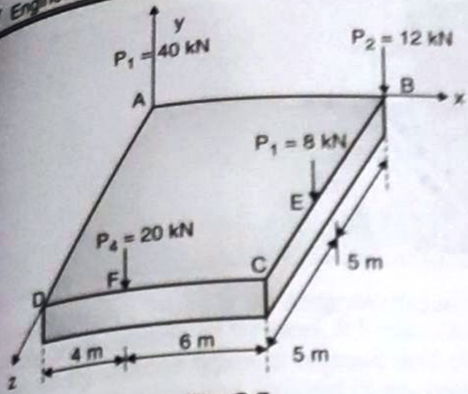


Fig. Q.7

[Ans. :R = 80 kN↓, (3.5, 0, 3.0)m]

Q.8 A concrete foundation mat in the shape of a regular hexagon of side 12m supports four column loads as shown in Fig. Q.8. Determine the magnitudes of additional loads which must be applied at 'B' and 'F' if the resultant of all six loads is to pass through the centre of the mat 'O'. **SPPU : Dec. 03, 10 Marks**

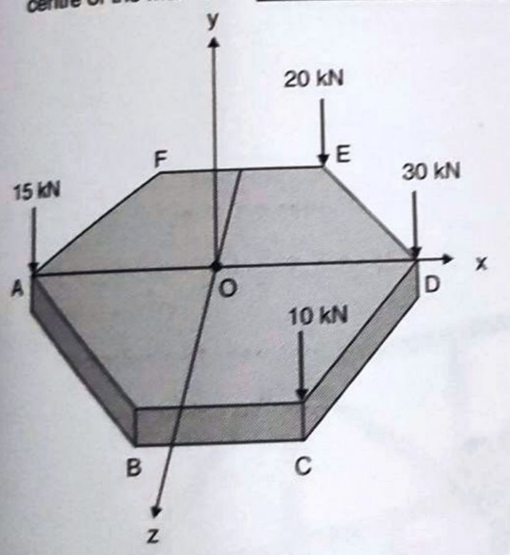


Fig. Q.8

[Ans. : P_B = 35kN↓, and P_F = 25kN↓]

Q.9 A uniform equilateral triangular plate weighing 2.4 kN is held in a horizontal plane by means of 3 cables at A, B and C. An additional load of 1.2 kN acts at the midpoint of edge AC of the plate. Determine the tensions in the 3 cables.

SPPU : May 04, 6 Marks

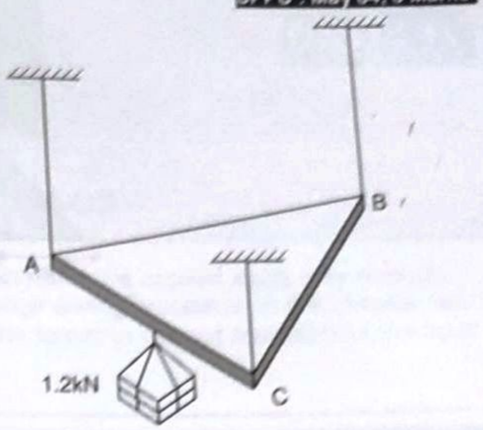


Fig. Q.9

[Ans. :T_A = T_C = 1.4 kN, T_B = 0.8 kN]

Q.10 Determine the resultant of the four parallel forces shown in Fig. Q.10 that act on the 1m × 0.7m wooden board. **SPPU : May 04, 8 Marks**

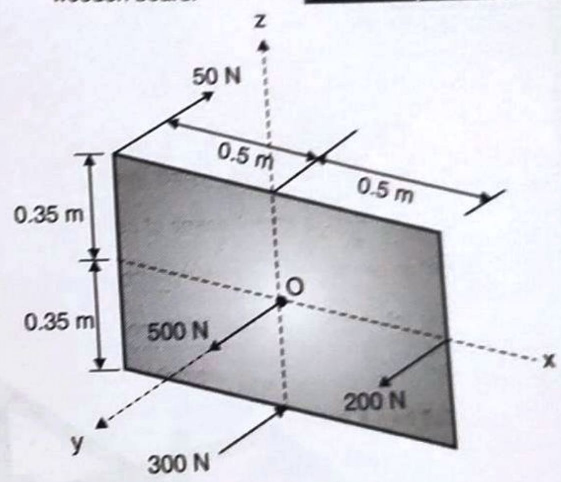


Fig. Q.10

[Ans. : R = 350 N, x = 0.35 m, y = 0.25 m]

Q.11 The square mat foundation supports four columns as shown in the Fig. Q.11. Determine the magnitude and position of the resultant force w.r.t. origin 'O'. **SPPU : Dec. 18, 7 Marks**

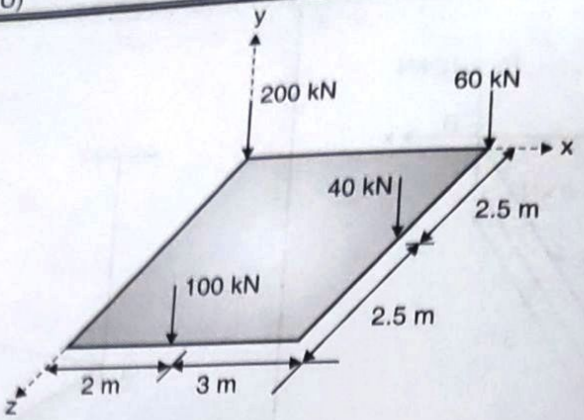


Fig. Q.11

[Ans. : R = 400 kN↓, x = 1.75m, z = 1.5