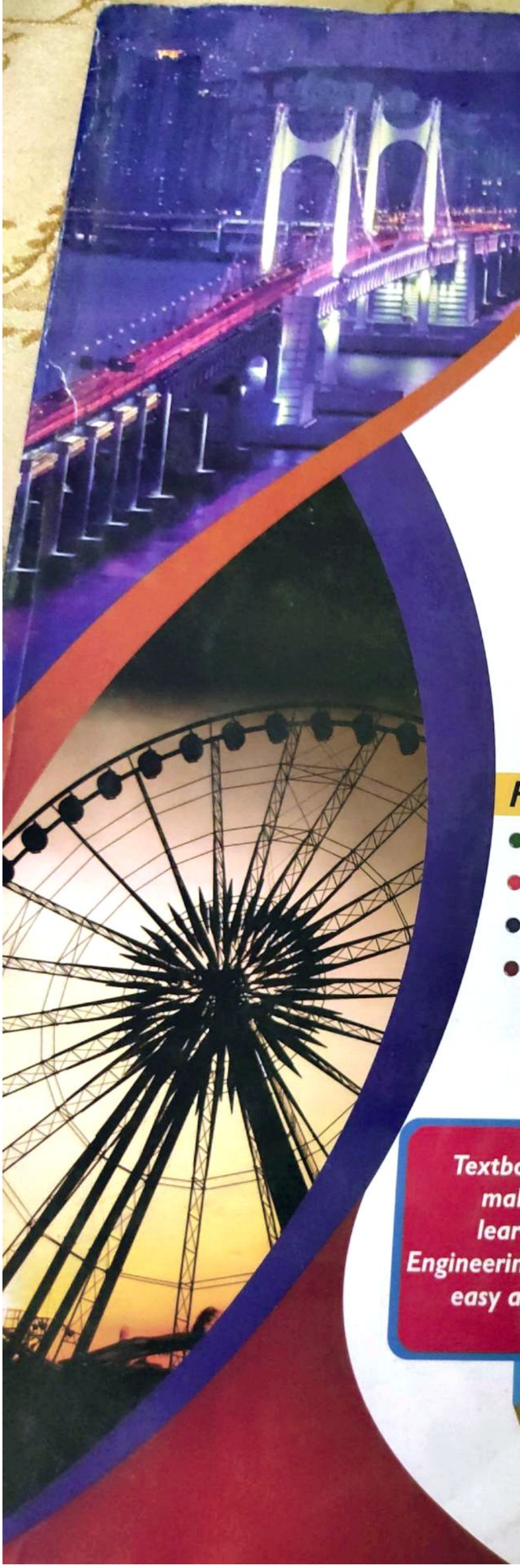

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(Code : 101011)

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M.E. (Civil-Structure)

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Pune Institute of Computer Technology (PICT), Pune - 43

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Syllabus

Engineering Mechanics (101011)

Teaching Scheme:	Credits	Examination Scheme:
TH : 3 Hrs./week PR : 2 Hrs./Week	04	In-Semester :30 Marks End-Semester :70 Marks PR :25 Marks

Course Objectives:

1. To impart knowledge about force systems and methods to determine resultant, centroid and moment of inertia
2. To teach methods to calculate force of friction
3. To impart knowledge to determine reaction of beams, calculate member forces in trusses, cables and frames using principles of equilibrium
4. To teach space force systems
5. To train students to solve problems related to particle mechanics using principles of kinematics, kinetics and work power energy

Course Outcomes

On completion of the course, learner will be able to

CO1 : Determine resultant of various force systems

CO2 : Determine centroid, moment of inertia and solve problems related to friction

CO3 : Determine reactions of beams, calculate forces in cables using principles of equilibrium

CO4 : Solve trusses, frames for finding member forces and apply principles of equilibrium to forces in space

CO5 : Calculate position, velocity and acceleration of particle using principles of kinematics

CO6 : Calculate position, velocity and acceleration of particle using principles of kinetics and Work, Power, Energy

Course Content

Unit I : Resolution and Composition of Forces **(07 Hrs)**

Principle of statics, Force system, Resolution and composition of forces, Resultant of concurrent forces. Moment of a force, Varignon's theorem, resultant of parallel force system, Couple, Equivalent force couple system, Resultant of general force system **(Refer Chapters 1,2,3 & 4)**

Unit II : Distributed Forces and Friction **(06 Hrs)**

Moment of area, Centroid of plane lamina and wire bends, Moment of Inertia. Friction- Laws of friction, application of friction on inclined planes, Wedges and ladders friction. Application to flat belt **(Refer Chapters 5,6 & 7)**

Unit III : Equilibrium **(06 Hrs)**

Free body diagram, Equilibrium of concurrent, parallel forces in a plane, Equilibrium of general forces in a plane, Equilibrium of three forces in a plane, Types of beams, simple and compound beams, Type of supports and reaction, Forces in space, Resultant of concurrent and parallel forces in a space, Equilibrium of concurrent and parallel forces in a space **(Refer Chapter 8 & 9)**

(New Code : PO112A)

Course Content

Unit IV : Analysis of Structures **(06 Hrs)**

Two force member, Analysis of plane trusses by Method of joints, Analysis of plane trusses by method of section, Analysis of plane frames, Cables subjected to point load multi force member.

(Refer Chapters 10, 11 & 12)

Unit V : Kinematics of Particle **(06 Hrs)**

Kinematics of linear motion- Basic concepts Equation of motion for constant acceleration, Motion under gravity, Variable acceleration, motion curves. Kinematics of curvilinear motion- Basic Concepts, Equation of motion in Cartesian, coordinates Equation of motion in path coordinates, Equation of motion in polar coordinates, Motion of projectile.

(Refer Chapters 13,14, 15, 16 & 17)

Unit VI : Kinetics of Particle **(06 Hrs)**

Kinetics- Newton's Second Law of motion, Application of Newton's Second Law. Work, power, energy, conservative and non-conservative forces, Conservation of energy for motion of particle, Impulse, Momentum, Direct central impact. Coefficient of restitution, Impulse Momentum principle of particle

(Refer Chapters 18, 19, 20 & 21)

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Unit I

Unit II

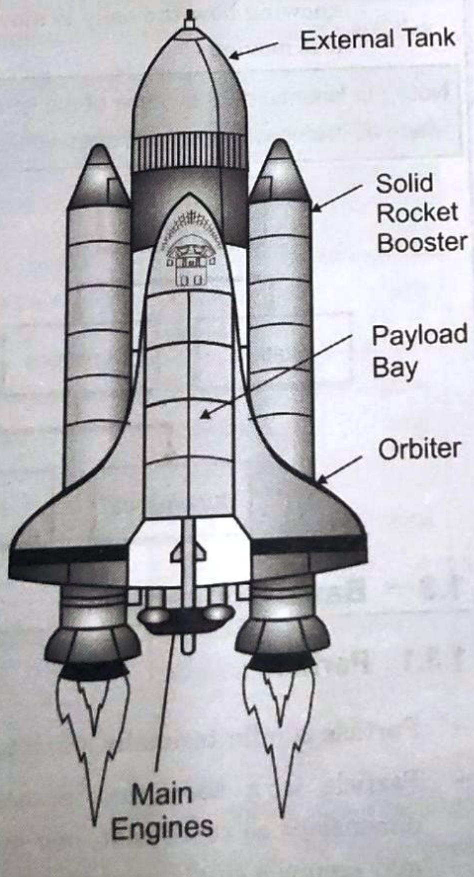
CHAPTER

1

UNIT - I

Basic Concepts of Engineering Mechanics

Introduction : In this chapter, we shall understand the basic concepts and fundamental principles of mechanics. These concepts will help in dealing the subject at ease. This chapter will be the foundation for the subject of Engineering Mechanics.

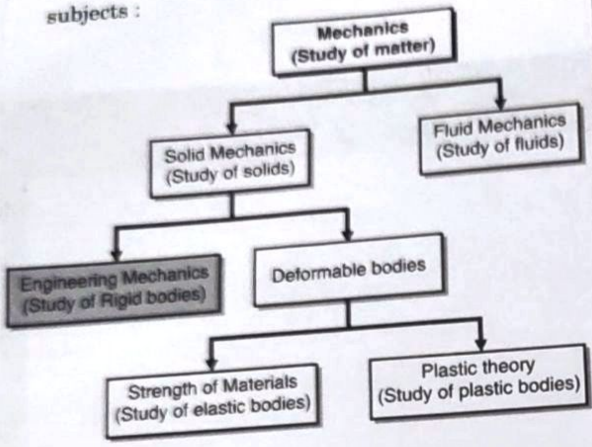


1.1 Mechanics

- Matter is available in two forms i.e. solids and fluids.
- Fluids are further classified into liquids and gases.

The study of behaviour of matter under the action of applied forces is known as ***"Mechanics."***

- Mechanics is a physical science that deals with the study of physical phenomena.
- Mechanics is divided into the following different subjects :



1.2 Engineering Mechanics

- It is the branch of mechanics which deals with the study of behaviour of rigid bodies under the action of applied forces when they are at rest or in motion.
- Engineering mechanics is divided into two parts :

(1) Statics and
(2) Dynamics

(1) Statics :

It is the branch of Engineering mechanics which deals with the study of behaviour of rigid bodies under the action of applied forces when they are at rest.

(2) Dynamics :

It is the branch of Engineering mechanics which deals with the study of behaviour of rigid bodies under the action of applied forces when they are in motion.

- o Dynamics is again subdivided into :

(a) Kinematics and
(b) Kinetics

(a) Kinematics :

Kinematics is the branch of dynamics which deals with the behaviour of bodies without considering the forces causing the motion.

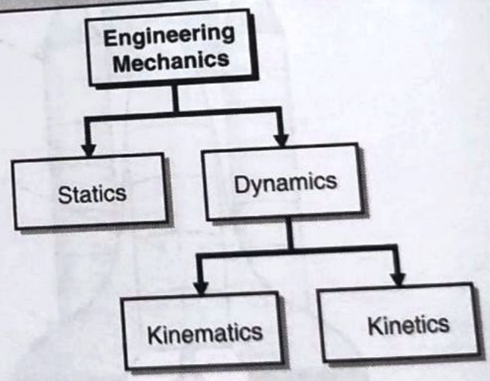
- o Kinematics deals with the motion parameters like displacement, velocity, acceleration, time etc.
- o In kinematics we are interested in knowing how the body is moving but not why it is moving.

(b) Kinetics :

It is the branch of dynamics which deals with the study of behaviour of bodies by considering the forces causing the motion.

- o Kinetics deals with the motion parameters like displacement, velocity, acceleration, time and force.
- o In kinetics we are not only interested in knowing how the body is moving but also why it is moving.

Note : In kinematics, the cause of motion is not considered where as in kinetics, the cause of motion is considered.



1.3 Basic Concepts

1.3.1 Particle :

- Particle is infinitesimally small part of the body.
- Particle is a body that posses mass, but has dimensions so small that, one may assume that it may occupy a single point in space.
- Particle has no size and shape.

- The concept of particle is similar to that of a point in geometry. Point has location but no dimensions. Similarly particle has mass but no dimensions.
- A particle is a body whose size does not influence its response to the forces acting on it. In other words, the body may be modelled as a point of concentrated mass, and the rotational motion of the body can be ignored.

Note :

(i) When the applied forces have no tendency to rotate the body on which they act, the body may be considered as a particle.

(ii) Forces acting on the particle are concurrent, the point through which they pass being the point representing the particle.

(iii) In engineering mechanics, particle is an idealized concept.

1.3.2 Body :

***A body** is defined as a matter which is limited in all directions.*

- A body is composed of infinite number of particles.
- It has definite size and shape.

1.3.3 Differentiate between Particle and Body

Sr. No.	Particle	Body
1	Dimensions are negligible.	Dimensions are considered.
2	It undergoes only translation.	It undergoes translation and rotation.
3	No size and shape.	Definite size and shape.
4	It occupies point in space.	It occupies volume in space.

1.3.4 Rigid Body :

- A rigid body is the body in which the distance between the particles constituting the body does not change under the action of applied forces.
- There is no body or material in the universe which is 100 % rigid. All materials used in engineering (steel, concrete, etc.) do suffer some deformation but too small to change the overall dimensions of the body.
- A body can be considered as rigid, if the deformation of the body is very small and has no effect on the equilibrium or motion of the body.
- Rigid body is again an idealized concept.

1.3.5 Deformable Body :

- Deformable body is the body in which there is a considerable change in the distance between the particles constituting the body under the action of applied forces.
- Because of this, the overall dimensions of the body are affected or it has significant effect on the motion or equilibrium of the body.

Note :

- Depending on the physical situation, a given body may be treated as a particle or a rigid body or a deformable body.
- For example, while studying the motion of the earth about the sun, the earth can be modelled as a particle, since the rotation of earth has as a negligible effect on its motion around the sun.
- With respect to the rotation of earth about its own axis, the earth can be considered as a rigid body, since the deformation of the earth has very small effect on its rotation.
- While in the determination of shape of the earth, the earth's elastic and plastic deformations may be considered and hence the earth can no longer be assumed as a rigid body.

1.3.6 One Dimensional (1-D) body :

- A body is said to be 1-D, if any one of its dimensions out of three is considerably large as compared to other two dimensions.

*If a body undergoes changes considerably in one direction (preferably along its length), then that is known as **1-D body**.*

- **Example :** Thin metal rods, wires, ropes, cables etc.

1.3.7 Two Dimensional (2-D) body :

*A body is said to be **2-D**, if any one dimension is considerably small as compared to other two dimensions.*

- This body undergoes change considerably in two directions i.e. along length and breadth.
- **Example :** Thin Metal sheets, plates, Lamina, etc.

Note :
- 2-D body is also known as lamina.
- Lamina is the body having an area and mass but negligible thickness.

1.3.8 Three Dimensional (3 -D) body :

*A body is said to be **3-D**, if all three dimensions are considerable.*

- The effects are considered in all 3 directions i.e. along length, breadth and depth or height under the action of applied forces.
- **Example :** Machines, buildings, etc.

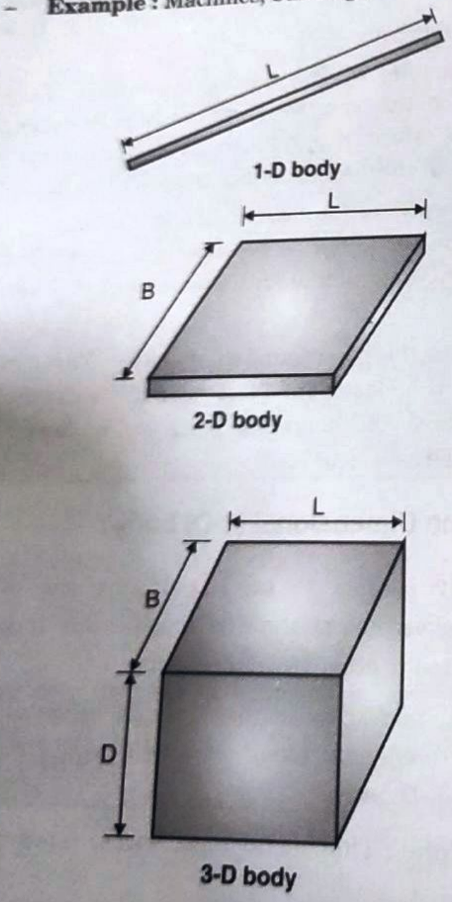


Fig. 1.3.1

1.3.9 Mass :

- Mass is the quantity of matter contained in a body.
- It is measured in kg.
- For a given body it is always constant. It will not change from place to place.

1.3.10 Weight :

*All bodies in the proximity of the earth's surface are subjected to a gravitation force exerted by the earth on the body, known as "**weight**" of the body.*

- It is measured in "Newton (N)."
- It will change from place to place as 'g' changes.

$$\text{Weight} = \text{Mass} \times \text{Acceleration due to gravity}$$

$$W = m \cdot g$$

Where, W = weight in Newton (N)

m = mass in kg.

g = Acceleration due to gravity $= 9.81\ \text{m/s}^2$

1.3.11 Force :

- Force is an external agency which produces motion in the body or changes the motion of the body or checks the motion of the body.
- Force is created when there is an interaction between two bodies.
- When two bodies interact, one of the body is assumed to exert the force and the other body to resist the force. One force of the pair is called action and the other is called reaction.
- Force tends to change the state of rest or motion of the body on which it acts.
- It is a vector quantity and has magnitude and direction.
- It is measured in Newton (N).

1.3.11.1 Characteristics of Force :

To define the force completely, the following characteristics or particulars must be known :

1. Magnitude
2. Direction
3. Point of application and
4. Sense.

These are also known as "specifications" of a force.

1. **Magnitude :** The magnitude of force is obtained by comparing it with some standard unit i.e. Newton.
2. **Direction of force :** It is defined as the direction of straight line in which the force tends to move the body on which it acts.
3. **Point of application :** It is the point on the body at which the force can be assumed to be concentrated.
4. **Sense :** The sense of the force will give its nature i.e. whether the force is acting towards the point of application (push) or away from the point of application (pull). It is indicated by an arrow head.

Illustrative Example :

Let us consider a force as shown in Fig. 1.3.2

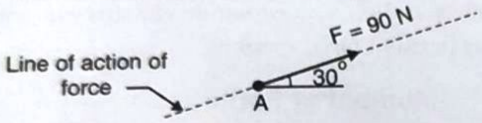


Fig. 1.3.2

The specifications of the above force are as follows :

1. Magnitude of force = 90 N
2. Direction of force = 30° with respect to horizontal
3. Point of application = A
4. Sense = Away from point of application i.e. pull

> **Note :** The line in which the force tends to move the body is known as "Line of action of force."

1.3.12 Force system

> *When number of forces acting on a body at a point, the collection or group of all forces is known as "force system."*

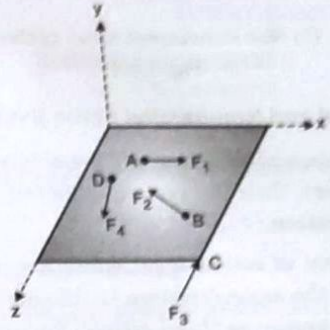
1.3.12.1 Types of Force Systems :

There are mainly four types of force systems :

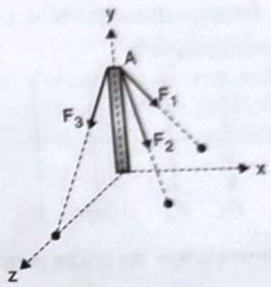
1. Coplanar and non-coplanar force system
2. Concurrent and non-concurrent force system
3. Parallel and non-parallel force system
4. Collinear and Non-collinear force system

1. **Coplanar and non-coplanar force system :**

- If two or more forces acting in the same plane, then it is known as **coplanar force system.**
- If forces are acting in different planes, then the system is known as **non-coplanar force system.**



(a) Coplanar force system

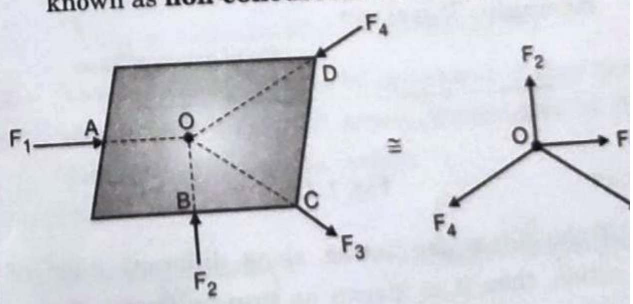


(b) Non-coplanar force system

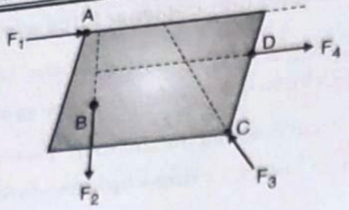
Fig. 1.3.3

2. **Concurrent and Non-concurrent force system :**

- If the lines of action of two or more forces are passing through a common point, then the system is known as **concurrent force system.**
- If the lines of action of all forces are not passing through the common point, then the system is known as **non-concurrent force system.**



(a) Concurrent force system

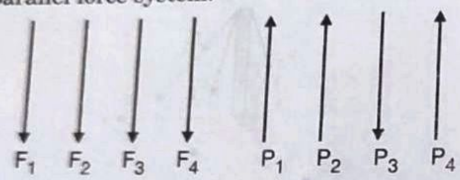


(b) Non-concurrent force system

Fig. 1.3.4

3. Parallel and Non-parallel Force System :

- If the lines of action of all forces are parallel to each other, then the system is known as **parallel force system**.
- If the lines of action of all forces are parallel and acting in the same direction i.e. having same sense then it is known as "Like parallel force system."
- If the sense of the parallel forces is different for different forces, then it is known as "Unlike parallel force system."



(a) Like parallel force system (b) Unlike parallel force system

Fig. 1.3.5

- If the lines of action of all forces are not parallel to each other, then the system is known as **non-parallel force system .**

4. Collinear and Non-collinear force system :

- If the lines of action of all forces are acting along the same line of action, then it is known as **collinear force system**.
- **Example :** Tug of war

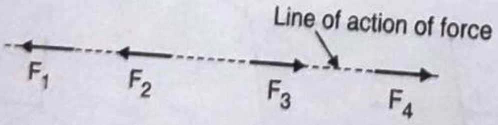


Fig. 1.3.6

- If the forces are acting along different lines of action, then it is known as **non-collinear force system**.

Combination of Force Systems :

In engineering we come across, various types of combination of force systems :

- Coplanar concurrent force system
- Coplanar non-concurrent force system
- Coplanar parallel force system
- Coplanar non- parallel force system
- Coplanar collinear force system
- Coplanar non-concurrent and non parallel force system
- Non coplanar concurrent force system
- Non coplanar parallel force system
- Non coplanar noncurrent or non-parallel force system

In this subject, we study mainly coplanar concurrent, coplanar parallel, coplanar non-concurrent and non- parallel, non-coplanar concurrent and Non-coplanar parallel force systems.

1.3.13 Moment of Force

- The effect of force on the body is two fold.
  (i) To produce translation and
  (ii) To produce rotation
- The moment of force is its tendency to produce rotation of the body upon which it acts about some axis.
- The turning effect produced by the force on the body is called moment of force.
- Moment of force is measured by taking the produ of magnitude of force (acting on the body) and t perpendicular distance between the line of acti of the force and the point about which the mom is to be taken.

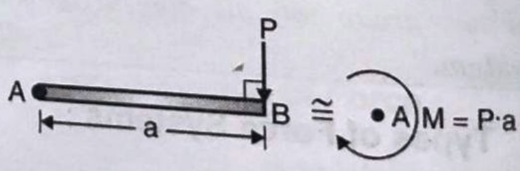


Fig. 1.3.6

- The moment of force 'P' about point 'a' is given $M_A = Pa$ (clockwise)
- The moment of force about a point could be C ACW depending upon the tendency of forc rotate the body in CW or ACW direction.

- In our studies CW moments are considered as – ve and ACW moments +ve.

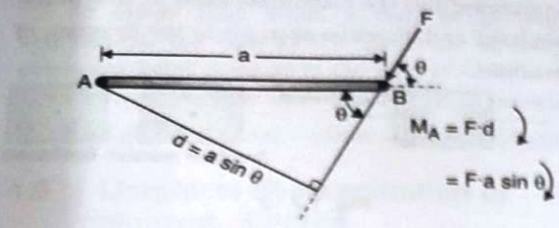


Fig. 1.3.7

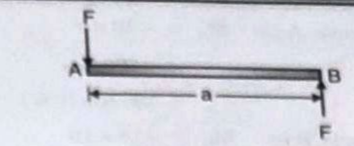
- The perpendicular distance from the line of action of force to the point about which moment is taken is known as "moment arm."
- The point about which moment is taken is known as "moment centre."

Note :

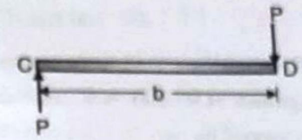
- The moment of the force is zero when either the force is zero or when the perpendicular distance is zero.
- S.I. unit of moment of force = Nm

1.3.14 Couple

- A couple is a pair of two equal and opposite parallel forces acting on the body.
- A couple is formed by two forces equal in magnitude, opposite in direction and parallel in action.
- The effect of couple on the body is to produce only rotation.
- The perpendicular distance between the two lines of action of forces forming the couple is called the "moment arm" of the couple.
- The magnitude of the couple is defined as the product of the magnitude of the force and the moment arm.
- The sense of the couple could be CW or ACW
- CW couples are taken as – ve and ACW couples +ve.
- Couple is measured in "Nm."



(a) ACW couple of magnitude, $M = F \cdot a$



(b) CW couple of magnitude, $M = P \cdot b$

Fig. 1.3.8

Characteristics of couple :

1. Couple does not produce translation but tends to rotate it.
2. The resultant force of the couple in any direction is zero. But the resultant moment of the couple about any point is not equal to zero.
3. The moment of couple is always constant, i.e. the moment of couple is independent of the distance of the point about which the moments are taken.

Example :

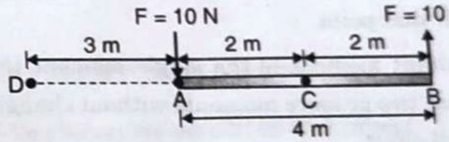


Fig. 1.3.9

$$M = F \cdot d = 10 \times 4 = 40 \text{ Nm (ACW)}$$

Moment of couple, $M = F \cdot d$

$= 10 \times 4 = 40$ Nm (ACW)

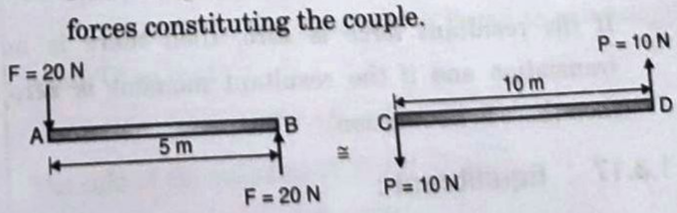
$$\Sigma M_A = 10 \times 0 + 10 \times 4 = 40 \text{ Nm (ACW)}$$

$$\Sigma M_B = 10 \times 4 + 10 \times 0 = 40 \text{ Nm (ACW)}$$

$$\Sigma M_C = 10 \times 2 + 10 \times 2 = 40 \text{ Nm (ACW)}$$

$$\Sigma M_D = -10 \times 3 + 10 \times 7 = 40 \text{ Nm (ACW)}$$

4. Two couples are said to be equivalent, if they have same magnitude and sense, irrespective of the forces constituting the couple.



Couple 'A' Couple 'B'

Fig. 1.3.10

Moment of couple, A i.e. $M_A = -20 \times 5$

$= -100$ Nm

$= 100$ Nm (CW)

Moment of couple, B i.e. $M_B = -10 \times 10$

$= -100$ Nm

$= 100$ Nm (CW)

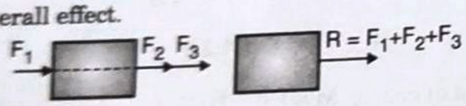
∴ Couples 'A' and 'B' are said to be equivalent, as they have same magnitude i.e. 100 Nm and sense i.e. CW through they are formed by two 20 N and 10 N Forces.

1.3.15 Resultant of Force System :

- Resultant of force system can be a force or a moment.
- Resultant force is the algebraic sum of the all the forces acting on the body or at a point.

If number of forces acting on a body are replaced by a single force which has the same effect as that of all the forces together, then the single force is known as "resultant force."

- Similarly, the algebraic sum of the moments of all forces about a point is known as resultant moment about that point.
- Resultant moment is the single moment that can replace two or more moments without changing the overall effect.



R = Resultant force

Fig. 1.3.11

1.3.16 Equilibrium :

- A body is said to be in equilibrium if the resultant of the force system acting on the body is zero.
- When the resultant is zero, the resultant force and resultant moment both will be zero.
- If the resultant force is zero, then there is no translation and if the resultant moment is zero, then there is no rotation.

1.3.17 Equilibrant :

- It is the force or moment which when applied to the given system of forces or moments, the resultant of the force system would become zero.
- Equilibrant has the magnitude same as that of the resultant and direction opposite to the direction of resultant.

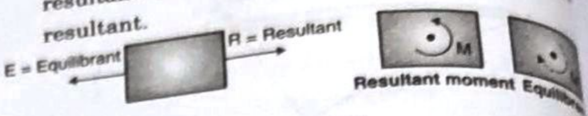


Fig. 1.3.12

1.4 Principles of Statics/Mechanics

1.4.1 Principle of Transmissibility of Forces :

"If a force acts at a point on a rigid body, then it is assumed to act at any other point on the line of action of force within the body." This is principle of transmissibility.

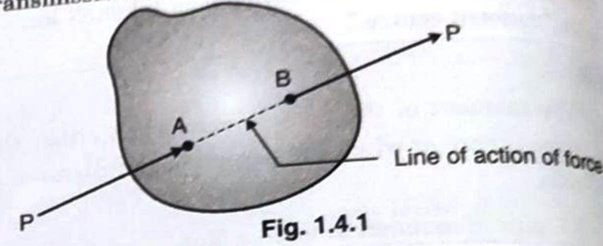


Fig. 1.4.1

1.4.2 Principle of Superposition :

"It states that the action of a given system of forces on a rigid body is unaltered if another system of forces in equilibrium is added or substracted from the given system."

1.4.3 Newton's laws of motion :

(a) **First Law :** A particle or body maintains its original state of rest or motion unless it is acted upon by an unbalanced or resultant force.

(b) **Second Law :** The unbalanced or resultant force acts on a body or particle is proportional to the time rate of change of velocity i.e. acceleration.

$F = ma$, where m = mass of the particle or body.

(c) **Third Law :** Forces of action and reaction between any two particles or bodies are equal in magnitude, opposite in direction and collinear in action.

1.4.4 Newton's Law of Gravitation :

Two particles of masses m_1 and m_2 are mutually attracted with equal and opposite forces of magnitude 'F' and is given by $F = G\dfrac{m_1 m_2}{d^2}$

where, d = distance between two particles

G = gravitational constant

1.4.5 Principle of moments :

"It states that, when a body is in rotational equilibrium, the sum of CW moments of the forces about any point is equal to the sum of ACW moments of the forces about the same point."

Sum of CW moments = Sum of ACW moments.

1.5 Graphical Representation of Moment of Force

Consider a force 'P', which can be represented in magnitude and direction by the line AB. Let 'O' be the point about which the moment is to be taken.

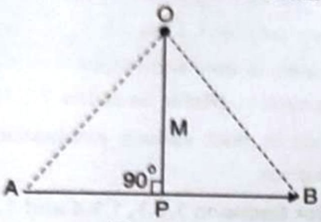


Fig. 1.5.1

Draw OM perpendicular to AB and join OA and OB

Now, Moment of force 'P' about point 'O' = $P \times OM$

$= AB \times OM$

$\therefore$ Area of the $\Delta OAB = \frac{1}{2} \times AB \times OM$ (Geometrically)

$\therefore$ Moment of 'p' at 'O' i.e. $(AB \times OM) = 2$. Area of ΔOAB

$\therefore$ Moment of force at any point = Twice the area of the triangle formed by that force with respect to the point about which moment is to be taken.

1.6 Varignon's Theorem

It states that, "the algebraic sum of the moments of all forces about any point in their plane is equal to the moment of their resultant about the same point."

OR

"Moment of force about any point is equal to the sum of the moments of its components about the same point."

Case 1 : Concurrent Force System :

When the two forces meet at a point :

Let two forces P and Q represented by AB and AD are acting at point A. Complete parallelogram ABCD. AC represents the resultant 'R' of P and Q.

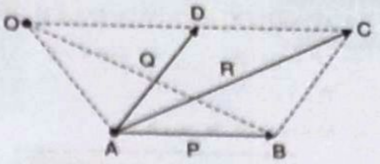


Fig. 1.6.1

Take any point 'O' in the plane of the forces P and Q and on the line CD produced as shown in Fig. 1.6.1

Join OB and OA.

Graphically, the moment of P at O = $2. \Delta OAB$

moment of Q at O = $2 \ \Delta OAD$

moment of R at O = $2 \ \Delta OAC$

But area of ΔOAB = area of Δ ABC

= area of Δ ACD

(Triangles standing on the same base and lying between the same parallel lines are equal in area)

$\therefore$ Moment of P at 'O' + moment of 'Q' at 'O'

$= 2 \ \Delta OAB + 2 \ \Delta OAD$

$= 2 \ \Delta ACD + 2 \ \Delta OAD$

$= 2 \ \Delta OAC$

= Moment of 'R' at 'O'

Case 2 : Parallel force system :

When two forces are parallel to each other :

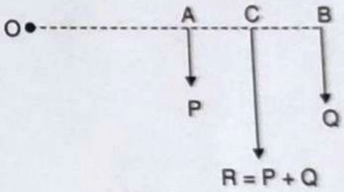


Fig. 1.6.2

Let two parallel forces P and Q be acting at points A and B respectively. Draw the line AB perpendicular to the forces to meet their lines of action at A and B.

Locate any Point 'O' in the plane of two forces on the line AB produced.

Let 'R' be the resultant of P and Q and acts at point 'C' so that

$$P \times AC = Q \times CB$$

The sum of the moments of 'P' and 'Q' at 'O'

$$= P \times OA + Q \times OB$$

$$= P (OC - AC) + Q (OC + CB)$$

$$= P \times OC - P \times AC + Q \times OC + Q \times CB$$
$$= (P + Q)\, OC - P \times AC + Q \times CB$$
$$= (P + Q)\, OC \qquad (\because -P \times AC + Q \times CB = 0)$$
$$= R \times OC$$
= Moment of 'R' at OC

Theory Questions

Q.1 State and prove Varignon's Theorem .
(Refer Section 1.6) **May 03, May 09**

Q.2 State and explain "Properties of couple"
(Refer Section 1.3.14) **May 09**

Q.3 Explain "Principle of Transmissibility of forces"
(Refer Section 1.4.1) **Dec. 92, May 07, Nov. 08, May 10**

Q.4 Explain Varignon's theorem and its use.
(Refer Section 1.6) **Dec. 92, Dec. 99, Dec. 00, Dec. 06, Dec. 09, May 10**

Q.5 Distinguish between moment of force and moment of couple.
(Refer Section 1.3.13 and 1.3.14) **Dec. 00**

Q.6 State the principle of superposition of forces and the Varignon's theorem
(Refer Section 1.4.2 and 1.6) **May 97, Dec. 98**

Q.7 Explain "Classification of force system".
(Refer Section 1.3.1.2) **Dec.**

Q.8 Illustrate with neat sketches different types of for systems.
(Refer Section 1.3.12.1) **Dec.**

Q.9 Define principle of superposition of forces.
(Refer Section 1.4.2) **Dec.**

Q.10 Compare :
(i) Equilibrium and equilibrant
(Refer Sections 1.3.16 and 1.3.17)
(ii) Moment and couple
(Refer Sections 1.3.13 and 1.3.14) **May 0**

Q.11 State the properties of couple.
(Refer Section 1.3.14) **May 9**

Q.12 Differentiate between resultant and equilibrant of force system. **(Refer Sections 1.3.15 and 1.3.17)**

Q.13 Explain in brief various idealizations of bodies mechanics.
(Refer Sections 1.3.1, 1.3.4 and 1.3.6) **May 97**

□□□

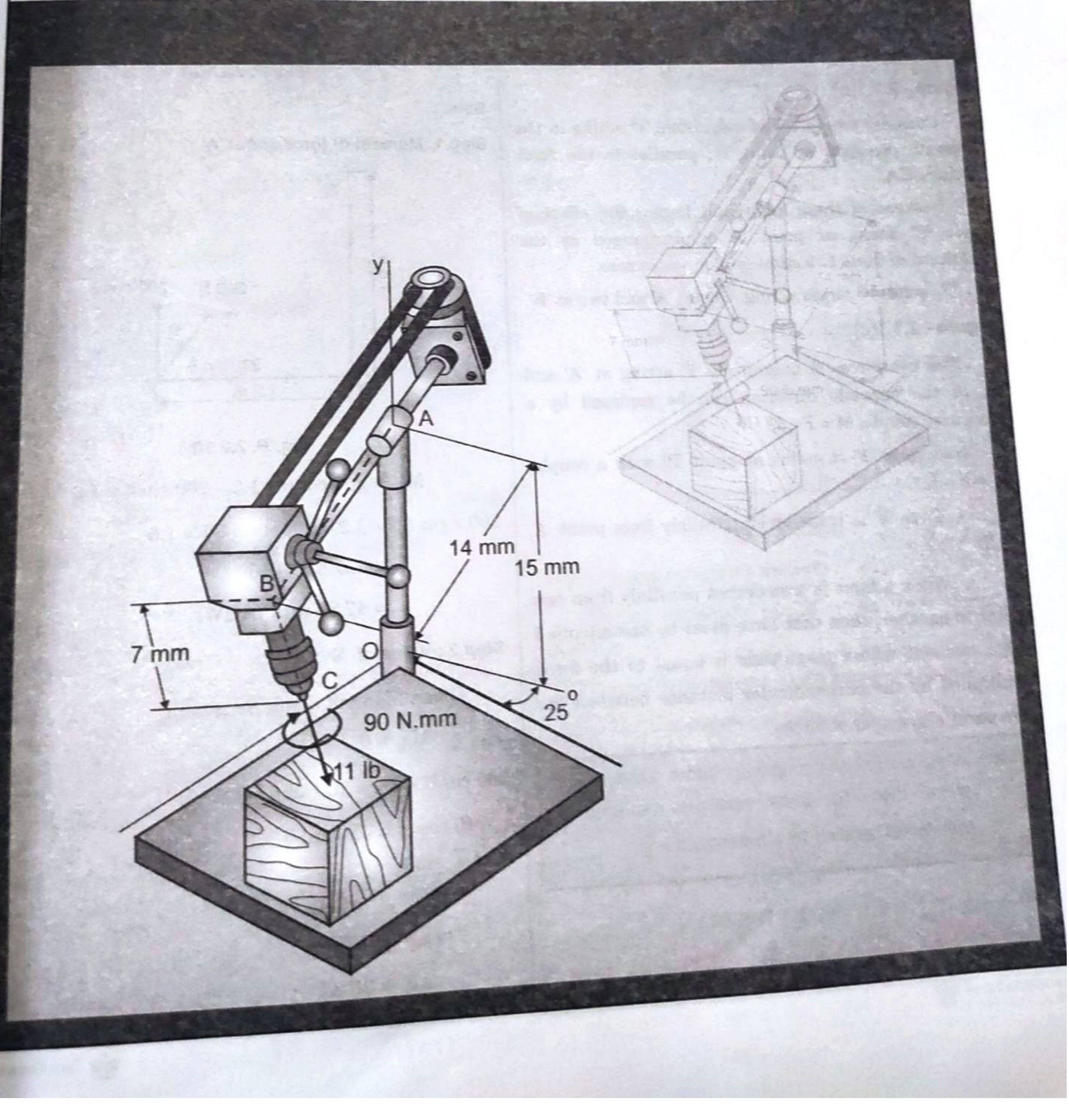
CHAPTER

2

UNIT - I

Force-Couple Systems

Introduction : In this chapter, we shall study the concept of equivalent Force-Couple systems. We shall be able to replace the given force with an equivalent F-C system.



2.1 Principle of Parallel Transfer of Forces OR Resolution of Force into a Force and a Couple

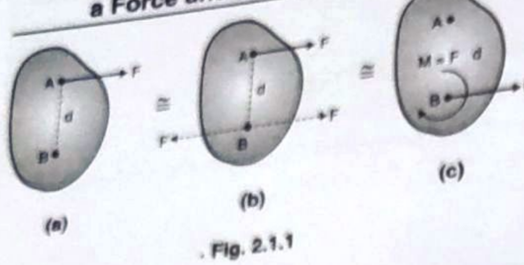


Fig. 2.1.1

Figure - 2.1.1(a) :

Let us consider a force 'F' acting at point 'A'. This force is to be transferred parallely from point 'A' to 'B'.

Figure - 2.1.1(b) :

Consider two forces of magnitude 'F' acting in the opposite direction at point 'B', parallel to the force acting at 'A'.

Because of these additional forces, the effect of force 'F' acting at point 'A' is unchanged as the resultant of these two additional forces is zero.

Now total 3 forces acting. One at 'A' and two at 'B'.

Figure - 2.1.1(c) :

The two forces of magnitude 'F' acting at 'A' and 'B' in the opposite direction can be replaced by a clockwise couple, M = F · d.

Now force 'F' is acting at point 'B' with a couple, M = F · d.

i.e. force 'F' is transferred parallely from point 'A' to 'B'.

∴ "When a force is transferred parallaly from one point to another, then that force must be accompanied by a moment whose magnitude is equal to the force multiplied by the perpendicular distance between the two parallel transfer of forces."

> **Note :** When a force and couple are added together in a plane, then that force must be displaced or transferred parallely by a distance, $d = \frac{M}{F}$.

2.2 Solved Examples

Ex. 2.2.1 : The lever ABC fixed at A shown in Fig. P. 2.2.1(a) is subjected to a 200 N force at C at θ = 30°. Find the moment of this force about A. Also find the value of θ for which the moment about A is Zero.

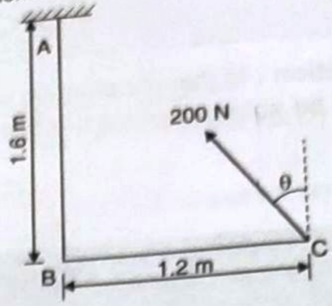


Fig. P. 2.2.1(a)

Soln. :

Step 1 :Moment of force about 'A' :

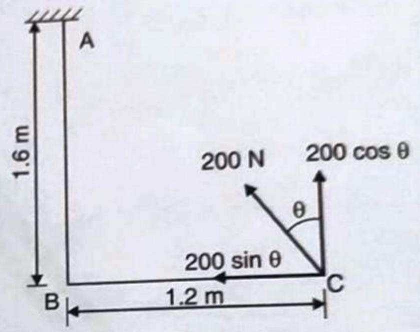


Fig. P. 2.2.1(b)

$$M_A = 200 \cos\theta \times 1.2 - 200 \sin\theta \times 1.6$$

$$200 \times \cos 30° \times 1.2 - 200 \times \sin 30° \times 1.6$$

$$= 207.846 - 160$$

$$= \mathbf{47.85\ mm\ (ACW)}$$ ...Ans.

Step 2 : Value of 'θ' :

Given Moment about 'A' is zero.

$$M_A = 0$$

$$200 \cos\theta \times 1.2 - 200 \sin\theta \times 1.6 = 0$$

$$240 \cos\theta = 320 \sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = 0.75$$

$$\tan\theta = 0.75$$

$$\therefore \theta = \mathbf{36.87°}$$ ... Ans.

Ex. 2.2.2 : A 50 N force acts on the rod at point 'A'. Determine its moment about 'O'.

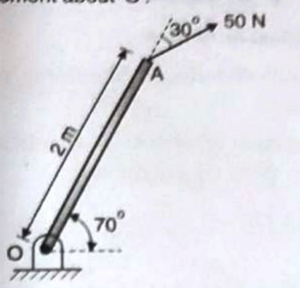


Fig. P. 2.2.2(a)

Soln. :

Method 1 : By finding perpendicular distance :

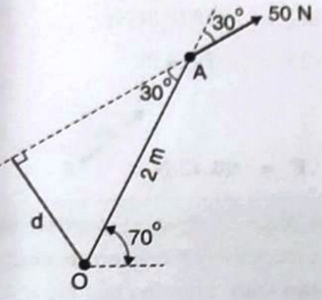


Fig. P. 2.2.2(b)

$$\sin 30^\circ = \frac{d}{2}$$

$$\therefore\ d = 1\text{ m.}$$

$$M_O = -50 \times d$$

$$= -50 \times 1$$

$$= \mathbf{50\ Nm} \circlearrowright \qquad \text{...Ans.}$$

Method 2 : By resolving force into its components :

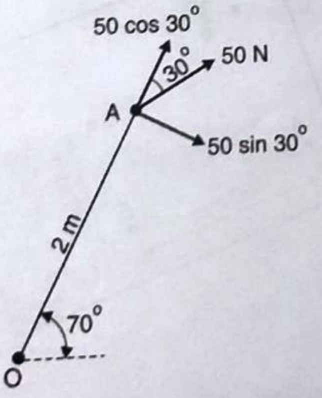


Fig. P. 2.2.2(c)

Taking moments at 'O' and using varignon's theorem.

$$M_O = -50\sin 30^\circ \times 2$$

$$= -50\text{ Nm}$$

$$= \mathbf{50\ Nm} \circlearrowright \qquad \text{... Ans.}$$

Note : Moment of 50cos30° at O is zero as its line of action is passing through point 'O'.

Ex. 2.2.3 : A force of 600N acts on a bracket as shown. In Fig. P. 2.2.3(a). Determine the moment of force about 'B'.

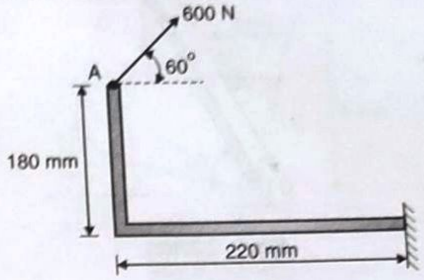


Fig. P. 2.2.3(a)

Soln. :

Step 1 : Resolving force into its components :

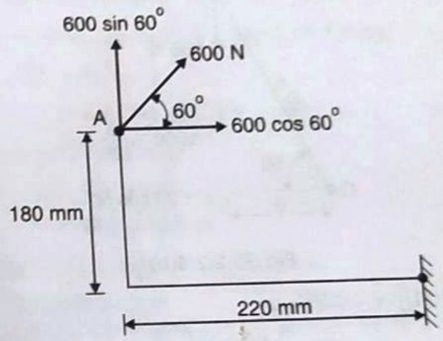


Fig. P. 2.2.3(b)

Step 2 :Using varignon's theorem :

Taking moments at 'B' ; As per varignon's theorem ;

Sum of the moments of the components at B

= moment of force at B.

$$M_B = -600\cos 60^\circ \times 180 - 600\sin 60^\circ \times 220$$

$$= -54000 - 114315.35$$

$$= -168315.35\text{ Nmm}$$

$$= -168.315\text{ Nm}$$

$$= \mathbf{168.315\ \ Nm} \circlearrowright \qquad \text{... Ans.}$$

Ex. 2.2.4 : A 150N force is applied to the rod as shown in Fig. P. 2.2.4(a).

Determine :

(i) The moment of force about point 'O'

(ii) The horizontal force applied at 'A' which creates same moment about point 'O'.

(iii) The smallest force applied at 'A' which creates the same moment about point 'O'.

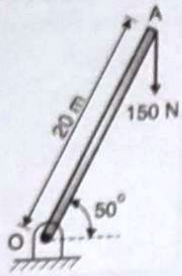


Fig. P. 2.2.4(a)

Soln. :

Step 1 : Moment of force about 'O' :

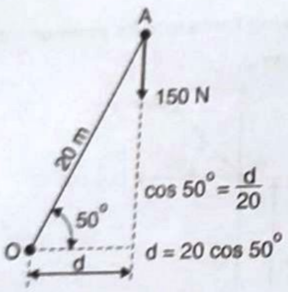


Fig. P. 2.2.4(b)

$M_O = -150 \times d$

$= -150 \times 20 \cos 50°$

$= -1928.36$ Nm

$= \mathbf{1928.36}$ **Nm (CW)** ...Ans.

Step 2 : Horizontal force at 'A' :

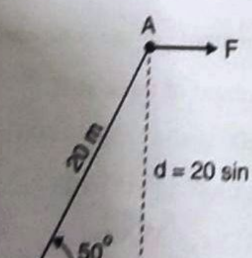


Fig. P. 2.2.4(c)

$M_A = -1928.36$Nm

$-F \times 20 \sin 50° = -1928.36$

$\mathbf{F = 125.86\ N \rightarrow}$ ...Ans.

Step 3 : Smallest force at 'A' :

For smallest force, 'd' should be maximum for given moment.

The direction of force at 'A' should be such that the distance, d = 20 m i.e. maximum.

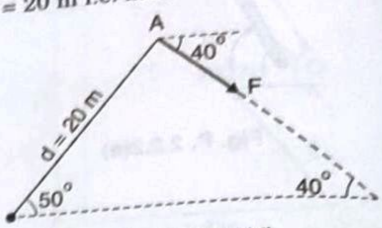
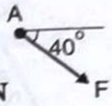


Fig. P. 2.2.4(d)

$M_A = -1928.36$Nm

$-F \times 20 = -1928.36$



$\therefore \mathbf{F = 96.42\ N}$ ...Ans.

Ex. 2.2.5 : A piece of plywood in which several holes are being drilled successively has been secured to a work bench by means of two nails. Knowing that the drill exerts a 12 N.m couple on the piece of plywood, determine the magnitude of the resulting forces applied to the nails if they are located

(a) at A and B (b) at B and C

(c) at A and C

SPPU - May 09, 8 Marks

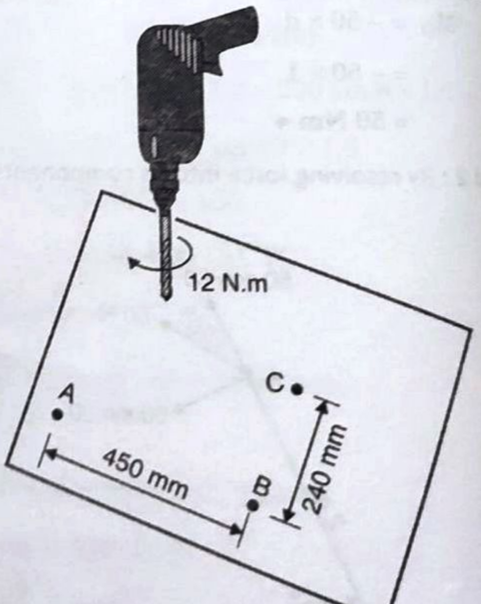


Fig. P. 2.2.5(a)

Soln. :

(a) At A and B :

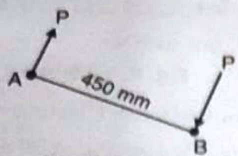


Fig. P. 2.2.5(b)

Let two forces of magnitude 'P' acting at A and B perpendicular to the line AB.

Given that the couple exerted by drilling machine is 12Nm (C.W).

$$\therefore \; P \times 0.45 = 12$$

$$\therefore \; \mathbf{P = 26.67 \text{ N}}$$ **...Ans.**

(b) At B and C :

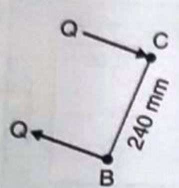


Fig. P. 2.2.5(c)

$$Q \times 0.24 = 12$$

$$\therefore \; \mathbf{Q = 50 \text{ N}}$$ **... Ans.**

(c) At A and C :

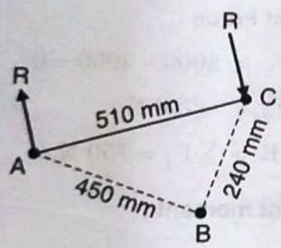


Fig. P. 2.2.5(d)

$$AC = \sqrt{450^2 + 240^2} = 510 \text{ mm}$$

$$R \times 0.510 = 12$$

$$\mathbf{R = 23.53 \text{ N}}$$ **...Ans.**

Ex. 2.2.6 : A plate in the shape of a parallelogram is acted up on by two couples, as shown in Fig. P. 2.2.6(a). Determine :

(a) The moment of couple formed by two 21 N forces.

(b) The perpendicular distance between the 12N forces, if the resultant of the two couples is zero.

(c) The value of 'α' if the resultant couple is 1.5 Nm clockwise and d = 1.05 m

SPPU - May 10, 8 Marks

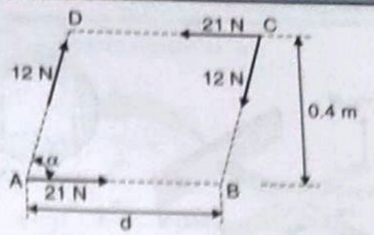


Fig. P. 2.2.6(a)

Soln. :

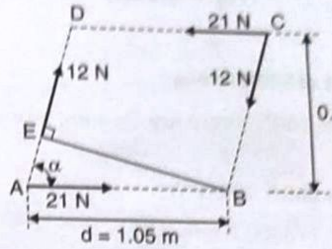


Fig. P. 2.2.6(b)

From Δ AEB ;

$$\sin\alpha = \frac{BE}{1.05}$$

$$BE = 1.05 \sin\alpha$$

(a) Moment of couple formed by 21N forces,

$$M = 21 \times 0.4$$

$$\mathbf{= 8.4 \text{ Nm (ACW)}}$$ **...Ans.**

(b) Resultant couple = 0

$$8.4 - 12 \times BE = 0$$

$$\therefore \; \mathbf{BE = 0.7 \text{ m}}$$ **...Ans.**

(c) Resultant couple = 1.5 Nm C.W.

$$\therefore \begin{bmatrix} \text{Moment couple} \\ \text{formed by 21 N forces} \end{bmatrix} + \begin{bmatrix} \text{Moment of couple} \\ \text{formed by 12 N forces} \end{bmatrix} = -1.5$$

$$8.4 + (-12 \times BE) = -1.5$$

$$8.4 - 12 \times 1.05 \sin\alpha = -1.5$$

$$12.6 \sin\alpha = 9.9$$

$$\sin\alpha = 0.7857$$

$$\therefore \mathbf{\alpha = 51.78^\circ}$$ **...Ans.**

Ex. 2.2.7 : A lug wrench is used to tighten a square head bolt. If 250N force is applied to wrench as shown in Fig. P. 2.2.7(a) determine the magnitude 'F' of the equal force exerted on the four contact points on the 25 mm bolt head so that their external effect on the bolt is equivalent to that of the two 250N forces. Assume that the forces are perpendicular to the flats of the bolt head.

SPPU - June06, 6 Marks

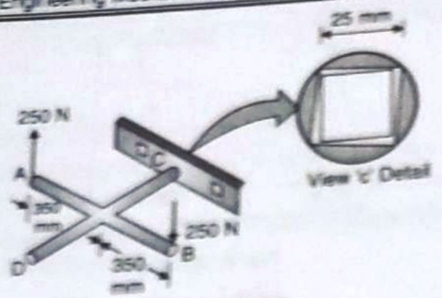


Fig. P. 2.2.7(a)

Soln. :

Step 1 : Effect of 250N forces :

These two 250N forces are forming a couple, whose magnitude is,

$$M = 250 \times 700 \text{ (CW)}$$

$$= 175000 \text{ Nmm (CW)}$$

Step 2 : Equivalent system :

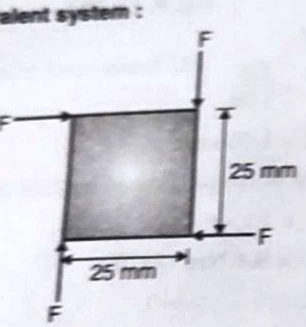


Fig. P. 2.2.7(b)

Two couples are acting in CW direction.

Magnitude of each couple is $(F \times 25)$ Nmm

$$\therefore 2 \times (F \times 25) = 175000$$

$$\therefore F = 3500 \text{ N}$$ ... Ans.

Ex. 2.2.8 : Resolve force F = 1000 N acting at B into a couple and a force at point A. Refer Fig. P. 2.2.8(a)

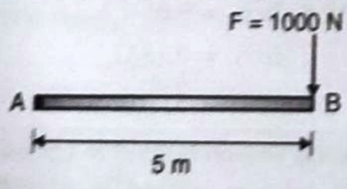


Fig. P. 2.2.8(a)

Soln. :

By principle of parallel transfer of forces, F = 1000 N is transferred parallely from point B to A, that force is accompanied by a moment,

$$M = -1000 \times 5 = -5000 \text{ Nm}$$

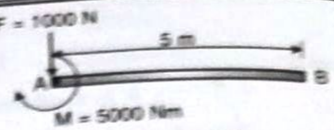


Fig. P. 2.2.8(b)

$\therefore$ Components of force 'F' acting at 'B' are :

(i) $F = 1000 \text{ N} \downarrow$ acting at 'A' and ...Ans.

(ii) $M = 5000$ Nm at 'A'. ...Ans.

Ex. 2.2.9 : Forces exerted on the cross-section of a steel channel are shown in Fig. P. 2.2.9(a) replace this system of forces by equivalent single force at point 'G' and determine the distance 'x' of point G from point 'C'.

SPPU - Dec. 2000, 8 Marks

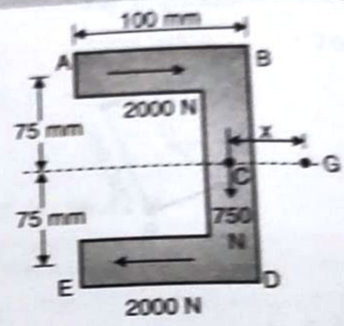


Fig. P. 2.2.9(a)

Soln. :

Step 1 :Resultant Force :

$$\Sigma F_x = 2000 - 2000 = 0$$

$$\Sigma F_y = -750 \text{ N}$$

$$\therefore \text{Resultant, } R = \Sigma F_y = 750 \text{ N} \downarrow$$

Step 2 :Resultant moment :

Taking moments about point 'C'.

$$\Sigma M_C = -2000 \times 75 - 2000 \times 75 = -3$$ Nm

Step 3 :Equivalent single force at 'G' :

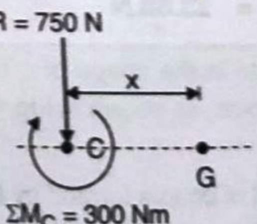


Fig. P. 2.2.9(b)

Add 750N force and couple 300 Nm.

When force and couple are added, force will be transferred parallely by a distance, $x = \frac{\Sigma M_C}{R}$

$$\therefore x = \frac{300}{750} = 0.4 \text{ m} \qquad \text{...Ans.}$$

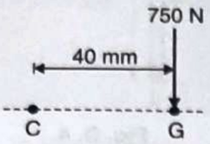


Fig. P. 2.2.9(c)

Ex. 2.2.10 : Replace 750N force acting at 'C' with equivalent force couple system at 'A'. Refer Fig. P. 2.2.10(a).

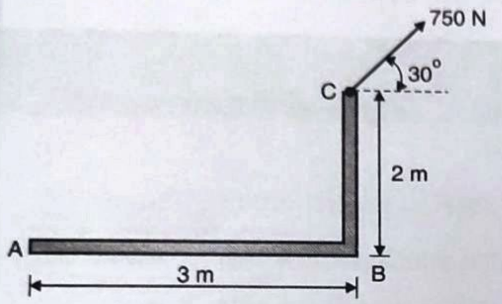


Fig. P. 2.2.10(a)

Soln. :

Step 1 : Moment of force at 'A' :

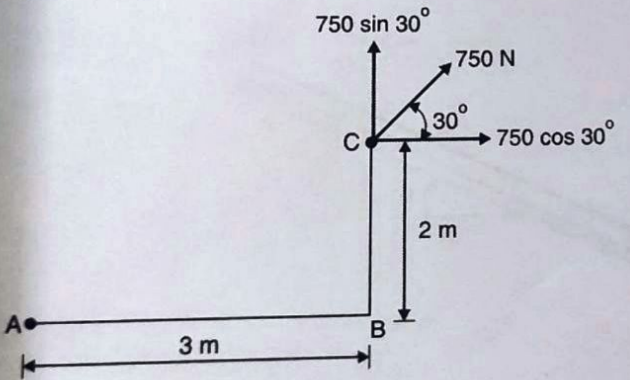


Fig. P. 2.2.10(b)

Moment of force about A,

$$M_A = -750 \cos 30° \times 2 + 750 \sin 30° \times 3$$

$$= -1299.04 + 1125$$

$$= -174.04 \text{ Nm}$$

$$= \mathbf{174.04 \text{ Nm}} \qquad \text{...Ans.}$$

Step 2 : Equivalent F-C system at 'A' :

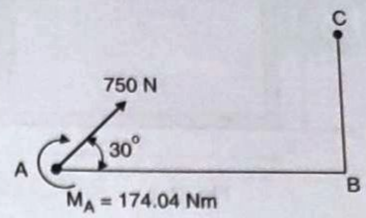


Fig. P. 2.2.10(c)

(i) Force, F = 750 N ...Ans.

(ii) Couple, M_A = 174.04 Nm ...Ans.

Practice Problems

Q. 1 A Force of 15 N is acting on the rod as show in Fig. Q. 1. Find its moment about point 'O'.

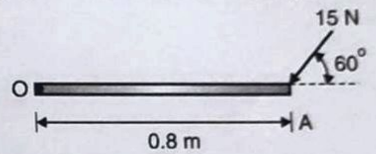


Fig. Q. 1

Ans. : 10.4 Nm

Q. 2 Find the moment of 1000 N force about point 'O' shown in Fig. Q. 2

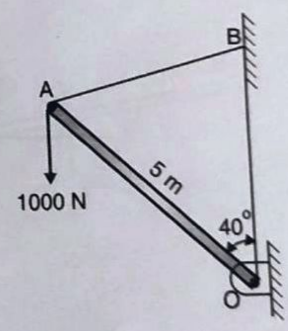


Fig. Q. 2

Ans. : 3214 Nm

Q. 3 A couple of 100 N forces acting at A and B as shown in Fig. Q. 3. Determine the equivalent forces at B and C that has the same effect of couple formed by 100N forces at A and B.

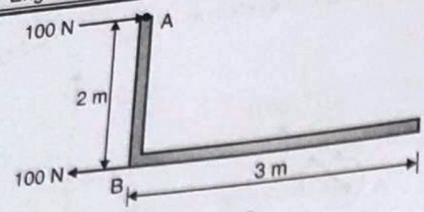


Fig. Q. 3

Ans. : $F_B = 66.67 \text{ N} \uparrow$

$F_C = 66.67 \text{ N} \downarrow$

Q. 4 Replace the 500 N force from point A by equiva force couple system at B.

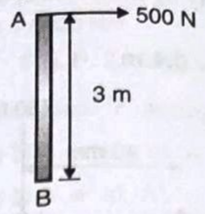


Fig. Q. 4

Ans. : $F = 500 \text{ N} \rightarrow$

$M_B = 1500 \text{ Nm}$

Introductio and shall b oblique co

CHAPTER

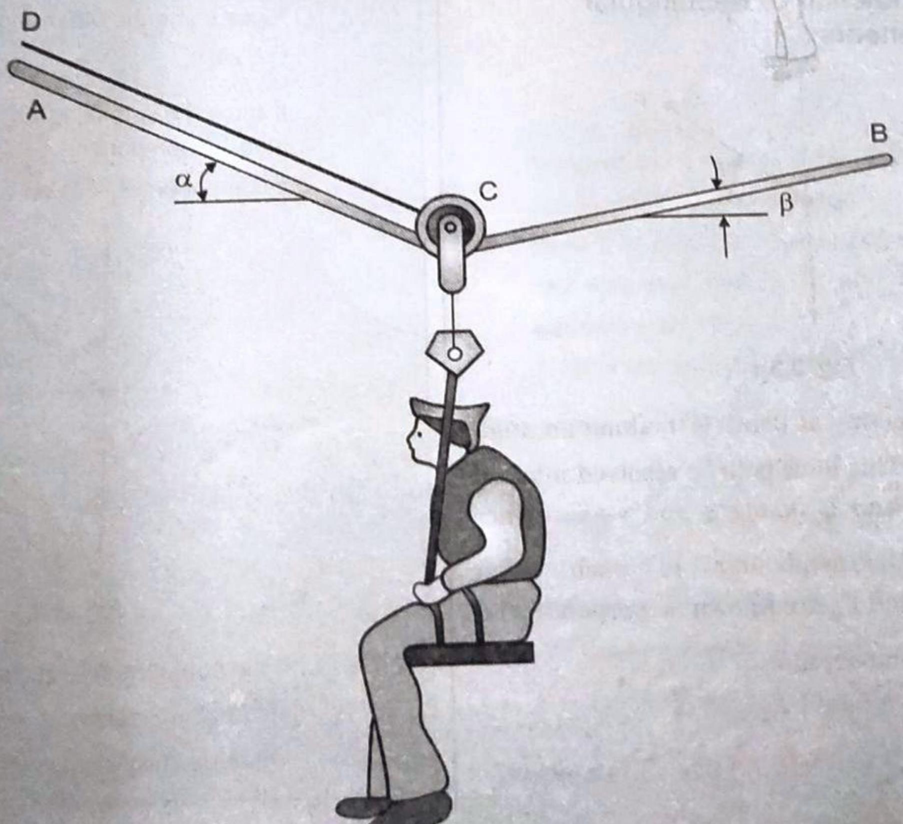
3

UNIT - I

Resolution of Forces

Introduction : In this chapter, we shall study the concept of resolution of force in to its components and shall be able to resolve the given force in to two perpendicular components and also in to oblique components i.e. the components along any two given lines.

- Type 1 : Perpendicular or Rectangular Components
- Type 2 : Non Perpendicular (Oblique) Components
- Type 3 : Non Perpendicular (Parallel) Components



3.1 Resolution of a force

The process of splitting a given force into two or more parts without changing its effect is called ***"resolution of forces"***.

- The force which is splitted into different parts is called the **"resolved force"** and the parts in to which the force is splitted are called "components of force" or **"resolutes"**.

3.2 Components of a force

- Generally force is resolved in to two types of components.

i) Perpendicular (or rectangular) components.

ii) Non perpendicular components.

a) Components along two lines which are not parallel or perpendicular. (Oblique components)

b) Components along two parallel lines (parallel components)

3.2.1 Perpendicular or Rectangular components :

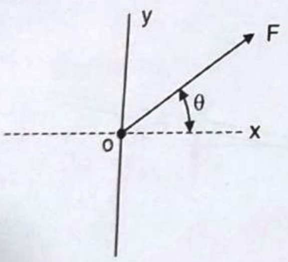


Fig. 3.2.1

- Let force 'F' is acting at point 'O' making an angle 'θ' w.r.t. x-axis. This force is to be resolved into two components F_x and F_y along x and y-axis which are mutually perpendicular to each other components F_x and F_y are known as perpendicular or rectangular components.

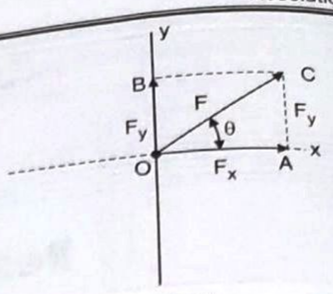


Fig. 3.2.2

- Draw lines parallel to x and y-axis from the tip of the force to form a rectangle OACB.
- Side of rectangle OA represents component of force F along x-axis i.e., F_x and the side OB represents component along y-axis i.e., F_y.

From Δ OAC ; $\cos\theta = \dfrac{OA}{OC} = \dfrac{F_x}{F}$

∴ Component along x-axis, $\boxed{F_x = F\cos\theta}$

Similarly, $\sin\theta = \dfrac{AC}{OC} = \dfrac{OB}{OC} = \dfrac{F_y}{F}$

∴ Component along y-axis ; $\boxed{F_y = F\sin\theta}$

Note :

(i) The component adjacent to the angle is always F cos θ and the other perpendicular component is F sin θ.

(ii) If force 'P' is making an angle 'α' w.r.t. y-axis then y-component, $P_y = P\cos\alpha$ and x-component $P_x = P\sin\alpha$.

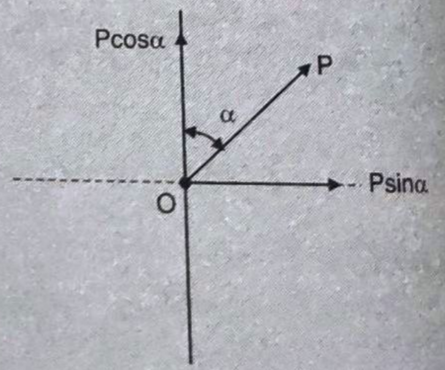


Fig. 3.2.3

(iii) If the component is acting towards right or upwards it is taken as +ve and acting towards left or downwards it is taken as –ve.

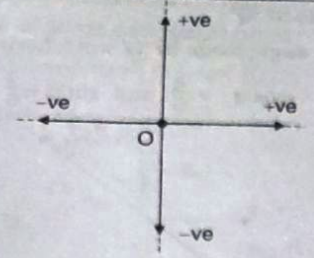


Fig. 3.2.4

3.2.2 Non Perpendicular (Oblique) components :

- Let force 'F' is acting at point 'O' is to be resolved in to two components along the lines ① and ②which are not perpendicular. The components along these line ① and ② i.e., F_1 and F_2 are known as non perpendicular (oblique) components.

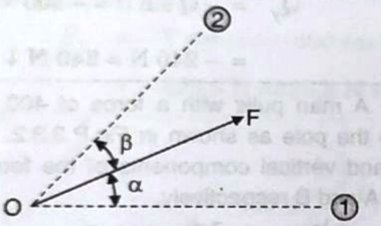


Fig. 3.2.5

- 'α' is the angle made by F with line ① and 'β' is the angle made by F w.r.t. line ②.
- Draw lines parallel to the lines ① and ② from the tip of the force to form a parallelogram OACB.

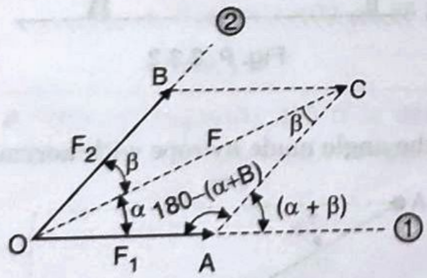


Fig. 3.2.6

- The side OA of parallelogram represents the component of F along line ① i.e., F_1 and the side OB represents component along line ② i.e., F_2.

For the Δ OAC, using sine rule,

$$\frac{OA}{\sin\beta} = \frac{AC}{\sin\alpha} = \frac{OC}{\sin[180-(\alpha+\beta)]}$$

$$\therefore \quad \frac{F_1}{\sin\beta} = \frac{F_2}{\sin\alpha} = \frac{F}{\sin(\alpha+\beta)}$$

∴ Component along line ①,

$$F_1 = F\left[\frac{\sin\beta}{\sin(\alpha+\beta)}\right]$$

Component along line ②,

$$F_2 = F\left[\frac{\sin\alpha}{\sin(\alpha+\beta)}\right]$$

Note :

(i) To resolve a force in to non perpendicular components, we must know the angle made by the force with respect to both the lines i.e., α and β.

(ii) to resolve a force into perpendicular components, any one angle made by the force with respect to either x-axis or y-axis is sufficient.

3.2.3 Non Perpendicular (parallel) components :

- Let a force 'F' is to be resolved into two components along the lines ① – ① and ② – ② which are parallel to each other. The components along these lines F_1 and F_2 are known as non perpendicular (parallel) components.

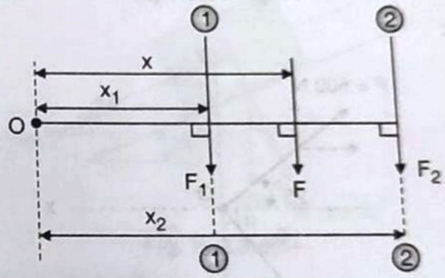


Fig. 3.2.7

- Let us consider a point 'O' at a perpendicular distance x_1, x and x_2 from the line ① – ①,force F and line ② – ②respectively.
- Here F is resolved into components F_1 and F_2. We can also say that if F_1 and F_2 are combined or added we get force F.

∴ F is the resultant of F_1 and F_2.

Hence, we can write

$$F_1 + F_2 = F \qquad \dots (1)$$

Using varignon's theorem, i.e., Algebraic sum of the moments of components at any point = moment of force at the same point.

Taking moments at point 'O',

$$\Sigma M_O = \text{Moment of F at O.}$$

Moment of F_1 at 'O' + Moment of F_2 at 'O' = Moment of F at 'O'

$-F_1x_1 - F_2x_2 = -F \cdot x$

(–ve because CW moments) ... (2)

$\therefore F_1x_1 + F_2x_2 = F \cdot x$

Knowing x_1, x_2, x and F and solving the above two equations we get F_1 and F_2.

3.3 Solved Examples

Type 1: Perpendicular or Rectangular components

Ex. 3.3.1 : Find the components of forces P and Q in x and y direction.

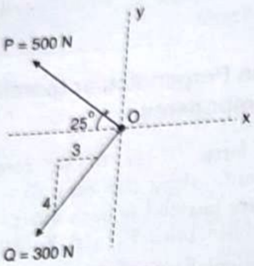


Fig. P. 3.3.1 (a)

Soln. :

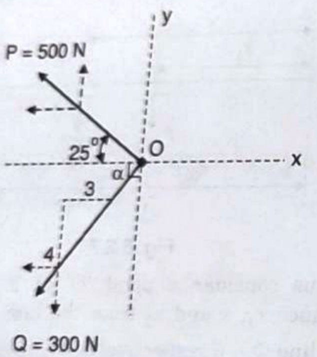


Fig. P. 3.3.1(b)

Components of 'P' :

'P' is making an angle 25° w.r.t. x-axis.

$\therefore$ x-component of P,

$$P_x = -P \cos 25° = -500 \cos 25° = -453.15 \text{ N}$$

$$= \mathbf{453.15\ N \leftarrow} \qquad \text{... Ans.}$$

$\therefore$ y-component of P,

$$P_y = P \sin 25° = 500 \sin 25°$$

$$= \mathbf{211.31\ N \uparrow} \qquad \text{... Ans.}$$

Components of 'Q' :

Let 'α' is the angle made by 'Q' w.r.t. horizontal

$$\cos \alpha = \frac{3}{5} \text{ and } \sin \alpha = \frac{4}{5}$$

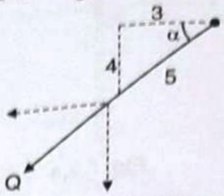


Fig. P. 3.3.1(c)

$\therefore$ x-component of Q,

$$Q_x = -Q \cos \alpha = -300 \times \frac{3}{5}$$

$$= 180 \text{ N} = \mathbf{180\ N \leftarrow} \qquad \text{... Ans.}$$

y-component of Q,

$$Q_y = -Q \sin \alpha = -300 \times \frac{4}{5}$$

$$= -240 \text{ N} = \mathbf{240\ N \downarrow} \qquad \text{... Ans.}$$

Ex. 3.3.2 : A man pulls with a force of 400 N on a rope attached to the pole as shown in Fig.P.3.3.2. What are the horizontal and vertical components of the force exerted by the rope at A and B respectively.

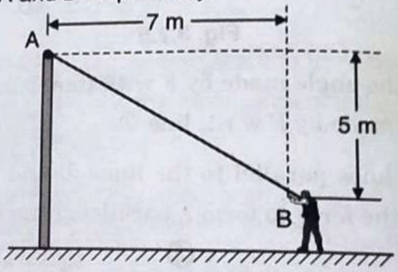


Fig. P. 3.3.2

Soln. :

Let 'α' be the angle made by rope with horizontal.

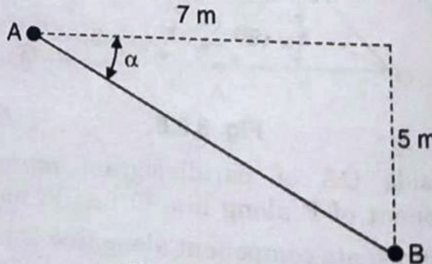


Fig. P. 3.3.2(a)

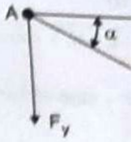
From Fig P.3.3.2(a),

$$\tan \alpha = \left(\frac{5}{7}\right)$$

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{5}{7}\right) = \mathbf{35.54°}$$

Components of force e

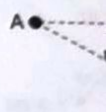
Horizontal or x-compone



Vertical or y-compone

Components of for

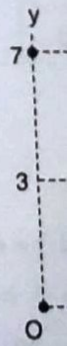
Horizontal compone



Vertical componer

Ex. 3.3.3 : A forc point A(1,3) to poi force.

Soln. :



Components of force exerted at point 'A' :

Horizontal or x-component of force, F

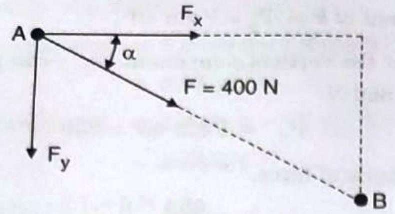


Fig. P. 3.3.2(b)

$$F_x = F \cos \alpha = 400 \cos 35.54°$$

$$= \mathbf{325.5\ N \rightarrow} \qquad \textbf{... Ans.}$$

Vertical or y-component of force, F

$$F_y = -F \sin \alpha = -400 \sin 35.54°$$

$$= -232.51\ N = \mathbf{232.51\ N \downarrow}$$

... Ans.

Components of force exerted at point 'B' :

Horizontal component,

$$F_x = -F \cos \alpha = -400 \cos 35.54°$$

$$= -325.5\ N = \mathbf{325.5\ N \leftarrow} \textbf{ ... Ans.}$$

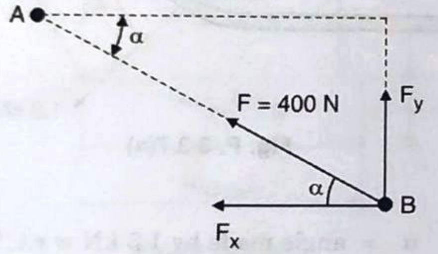


Fig. P.3.3.2(c)

Vertical component,

$$F_y = F \sin \alpha = 400 \sin 35.54°$$

$$= \mathbf{232.5\ N \uparrow} \qquad \textbf{... Ans.}$$

Ex. 3.3.3 : A force of magnitude 200 N is directed from point A(1,3) to point B (4, 7). Find x and y components of the force.

Soln. :

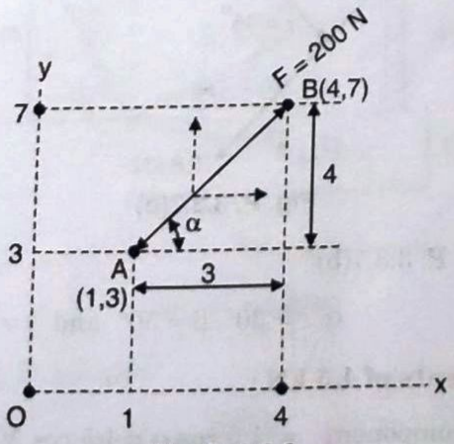


Fig. P. 3.3.3

From Fig. P. 3.3.3

$$\cos \alpha = \frac{3}{5} \text{ and } \sin \alpha = \frac{4}{5}$$

Force is making an angle α w.r.t. x-axis.

$$\therefore \text{ x-component, } F_x = F \cos \alpha = 200 \times \frac{3}{5}$$

$$= \mathbf{120\ N \rightarrow} \qquad \textbf{... Ans.}$$

$$\text{y-component, } F_y = F \sin \alpha = 200 \times \frac{4}{5}$$

$$= \mathbf{160\ N \uparrow} \qquad \textbf{... Ans.}$$

Ex. 3.3.4 : A block weighing 15 kN rests on an inclined plane making an angle of inclination 30° with horizontal. Determine the magnitude of the components of weight parallel and perpendicular the plane.

Soln. :

Let us consider x-axis parallel to the plane and y-axis perpendicular to it.

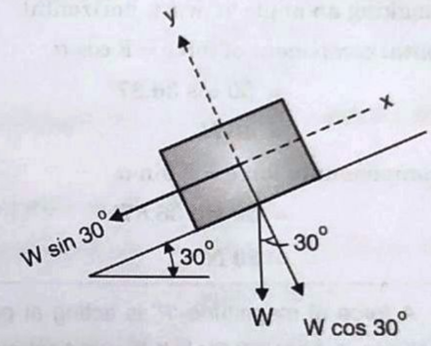


Fig. P. 3.3.4(a)

The angle made by W w.r.t. y-axis is 30°

$$\therefore \text{ y-component of weight} = W \cos 30°$$

$$= 15 \cos 30°$$

$$= 13\ kN$$

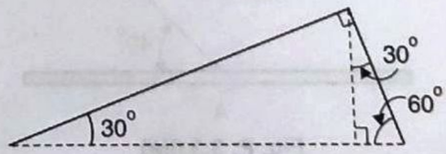


Fig. P. 3.3.4(b)

$$\text{x-component of weight} = W \sin 30°$$

$$= 15 \sin 30° = 7.5\ kN.$$

∴ The components of weight,

Parallel to the plane i.e.,

$$\text{x-component} = \mathbf{7.5\ kN} \qquad \textbf{... Ans.}$$

Perpendicular to the plane i.e.,

$$\text{y-component} = \mathbf{13\ kN} \qquad \textbf{...Ans.}$$

Note : Here magnitude of components are found. Hence sense is omitted i.e., +ve or –ve.

Ex. 3.3.5 : A force of magnitude 50 N is acting along the line of slope $\frac{15}{20}$. Determine the magnitude of horizontal and vertical components of force.

Soln. :

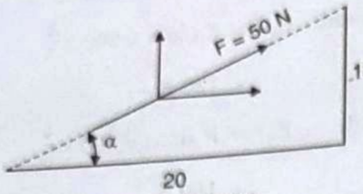


Fig. P. 3.3.5(a)

Slope of the line,

$$\tan\alpha = \left(\frac{15}{20}\right)$$

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{15}{20}\right) = 36.87^\circ$$

Force is making an angle 'α' w.r.t. horizontal.

∴ Horizontal component of force = F cos α

$$= 50\cos 36.87^\circ = 40\text{ N} \quad \text{... Ans.}$$

Vertical component of force = F sin α

$$= 50\sin 36.87^\circ = 30\text{ N} \quad \text{... Ans.}$$

Ex. 3.3.6 : A force of magnitude 'P' is acting at point A on the rod as shown in fig.P.3.3.6(a). If the vertical component of force is 850 N, determine a) the magnitude of force b) horizontal component.

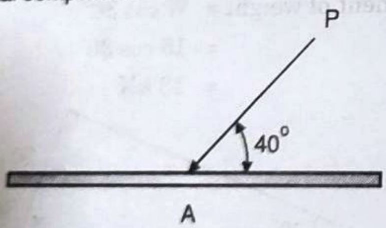


Fig. P. 3.3.6(a)

Soln. :

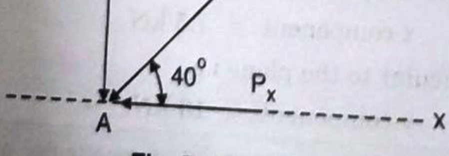


Fig. P. 3.3.6(b)

Let P_x and P_y be the horizontal and vertical components of force P.

Force 'P' is making an angle 40° w.r.t. x-axis.

∴ x-component of P is $P_x = P\cos 40^\circ$ and

y-component of P is $P_y = P\sin 40^\circ$.

Given that the vertical component i.e., y-component of force P is 850 N.

$$\therefore \quad P_y = P\sin 40^\circ = 850$$

∴ Magnitude of force,

$$P = \frac{850}{\sin 40^\circ} = \mathbf{1322.36\text{ N}} \quad \text{... Ans.}$$

Horizontal or x-component of P is $P_x = P\cos 40^\circ$

$$= 1322.36\cos 40^\circ$$

$$= \mathbf{1013\text{ N}} \quad \text{... Ans.}$$

Ex. 3.3.7 : Determine x and y components of each of the forces shown in Fig.P.3.3.7(a).

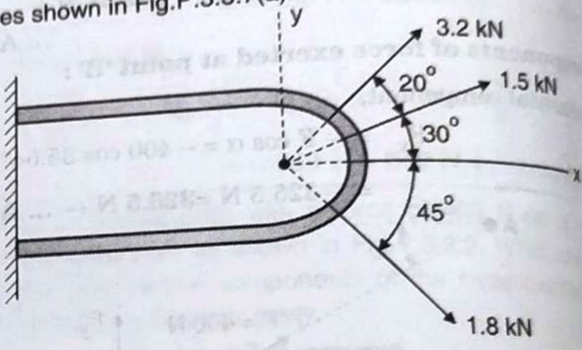


Fig. P. 3.3.7(a)

Soln. :

Let α = angle made by 1.5 kN w.r.t. 'x'.

β = angle made by 3.2 kN w.r.t. 'x'

γ = angle made by 1.8 kN w.r.t. 'x'.

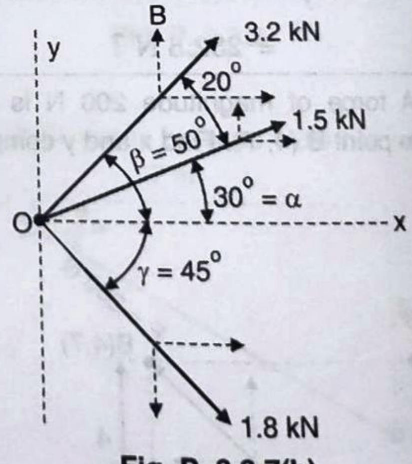


Fig. P. 3.3.7(b)

From Fig. P. 3.3.7(b)

$$\alpha = 30^\circ, \beta = 50^\circ \text{ and } \gamma = 45^\circ$$

Components of 1.5 kN :

$$\text{x-component} = 1.5\cos\alpha = 1.5\cos 30^\circ$$

$$= 1.3\text{ kN} \rightarrow \quad \text{... Ans.}$$

y-component $= 1.5 \sin \alpha = 1.5 \sin 30°$

$= \mathbf{0.75\ kN} \uparrow$ ... **Ans.**

Components of 3.2 kN :

x-component $= 3.2 \cos \beta = 3.2 \cos 50°$

$= \mathbf{2.05\ kN} \rightarrow$ ... **Ans.**

y-component $= 3.2 \sin \beta = 3.2 \sin 50°$

$= \mathbf{2.45\ kN} \uparrow$ ... **Ans.**

Components of 1.8 kN :

x-component $= 1.8 \cos \gamma = 1.8 \cos 45°$

$= \mathbf{1.27\ kN} \rightarrow$ ... **Ans.**

y-component $= -1.8 \sin \gamma = -1.8 \sin 45°$

$= -1.27\ kN = \mathbf{1.27\ kN} \downarrow$ ... **Ans.**

Ex. 3.3.8 : Determine x and y components of each of the forces shown in Fig. P. 3.3.8(a).

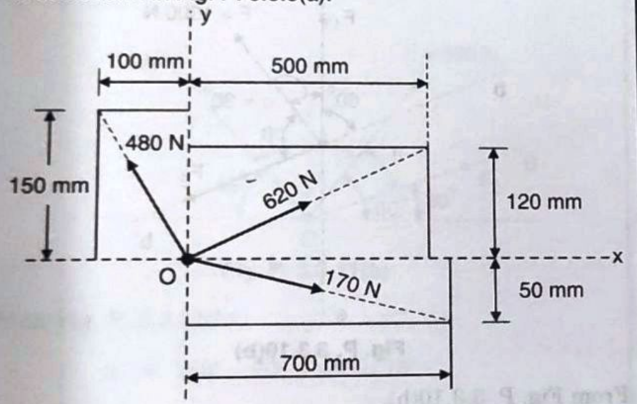


Fig. P. 3.3.8(a)

Soln. : Let α, β and γ be the angles made by the forces w.r.t. x-axis as shown in Fig.P.3.3.8(b)

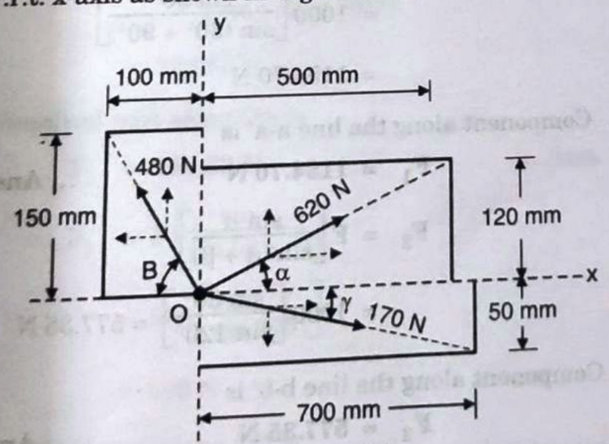


Fig. P. 3.3.8(b)

From Fig. P. 3.3.8(b),

$$\tan \alpha = \left(\frac{120}{500}\right)$$

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{120}{500}\right) = 13.5°$$

$$\tan \beta = \left(\frac{150}{100}\right)$$

$$\therefore \quad \beta = \tan^{-1}\left(\frac{150}{100}\right) = 56.31°$$

$$\tan \gamma = \left(\frac{50}{700}\right)$$

$$\therefore \quad \gamma = \tan^{-1}\left(\frac{50}{700}\right) = 4.08°$$

Components of 620 N force :

x-component $= 620 \cos \alpha = 620 \cos 13.5°$

$= \mathbf{602.87\ N} \rightarrow$ ... **Ans.**

y-component $= 620 \sin \alpha = 620 \sin 13.5°$

$= \mathbf{144.74\ N} \uparrow$ ... **Ans.**

Components of 480 N force :

x-component $= -480 \cos \beta = -480 \cos 56.31°$

$= -266.25$ N

$= \mathbf{266.25\ N} \leftarrow$ ... **Ans.**

y-component $= 480 \sin \beta = 480 \sin 56.31°$

$= \mathbf{399.38\ N} \uparrow$ ... **Ans.**

Components of 170 N force :

x-component $= 170 \cos \gamma = 170 \cos 4.08°$

$= \mathbf{169.57\ N} \rightarrow$ ... **Ans.**

y-component $= -170 \sin \gamma = -170 \sin 4.08°$

$= -12.09\ N = \mathbf{12.09\ N} \downarrow$ ... **Ans.**

Ex. 3.3.9 : In fig.P.3.3.9(a) the x-component of P is 850 N. Determine the magnitude of force P and its y-component.

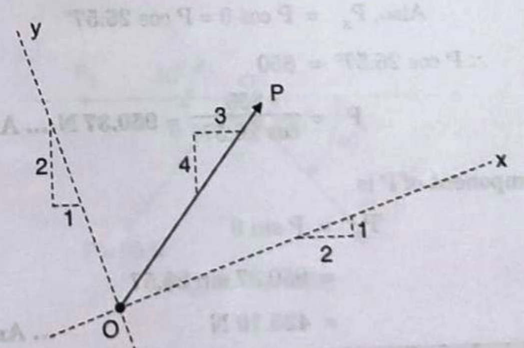


Fig. P. 3.3.9(a)

Soln. :

Let us find out the angle made by force 'P' with respect to x-axis.

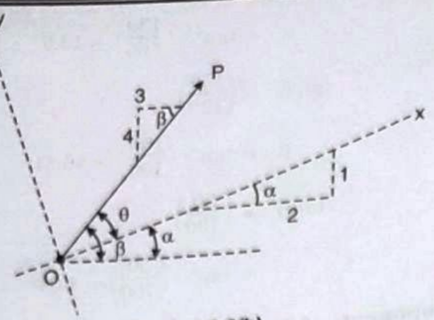


Fig. P. 3.3.9(b)

From Fig. P. 3.3.9(b),

$$\theta = \beta - \alpha$$

$$\alpha = \tan^{-1}\left(\frac{1}{2}\right) = 26.56^\circ$$

$$\beta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

$\therefore$ Angle made of P w.r.t. x-axis, $\theta = 53.13^\circ - 26.56^\circ = 26.57^\circ$

Given, x-component of force P,

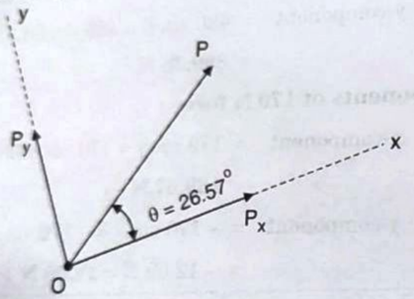


Fig. P. 3.3.9(c)

$$P_x = 850 \text{ N}$$

Also, $P_x = P\cos\theta = P\cos 26.57^\circ$

$\therefore P\cos 26.57^\circ = 850$

$$P = \frac{850}{\cos 26.57^\circ} = \mathbf{950.37\ N} \text{ ... Ans.}$$

y-component of P is

$$P_y = P\sin\theta$$

$$= 950.37 \sin 26.57^\circ$$

$$= \mathbf{425.10\ N} \quad \text{... Ans.}$$

Type 2 : Non Perpendicular (Oblique) Components

Ex. 3.3.10 : Determine components of the 1000 N force along the lines aa′ and bb′ axis shown in Fig. P. 3.3.10(a).

SPPU : May 10, 8 Marks

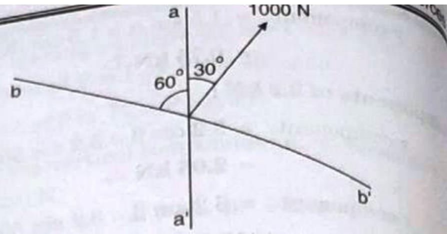


Fig. P. 3.3.10(a)

Soln. :

Let us consider the component of force F along the line a–a′ be F_1 and along the line b–b′ F_2.

'α' is the angle between F and F_1 and

'β' is the angle between F and F_2.

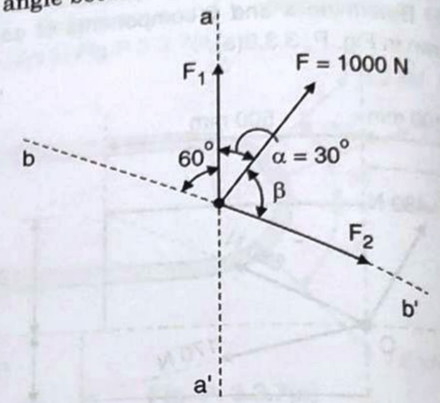


Fig. P. 3.3.10(b)

From Fig. P. 3.3.10(b),

$$\beta = 180 - 60^\circ - 30^\circ = 90^\circ$$

$$F_1 = F\left[\frac{\sin\beta}{\sin(\alpha+\beta)}\right]$$

$$= 1000\left[\frac{\sin 90^\circ}{\sin(30^\circ + 90^\circ)}\right]$$

$$= 1154.70 \text{ N}$$

$\therefore$ Component along the line a-a′ is

$$\mathbf{F_1 = 1154.70\ N} \quad \text{... Ans.}$$

$$F_2 = F\left[\frac{\sin\alpha}{\sin(\alpha+\beta)}\right]$$

$$= 1000\left[\frac{\sin 30^\circ}{\sin 120^\circ}\right] = 577.35 \text{ N}$$

$\therefore$ Component along the line b-b′ is

$$\mathbf{F_2 = 577.35\ N} \quad \text{... Ans.}$$

Ex. 3.3.11 : Find the magnitudes of resolved parts of force shown in fig. P.3.3.11(a). along the directions OA and O and also along axes x and y.

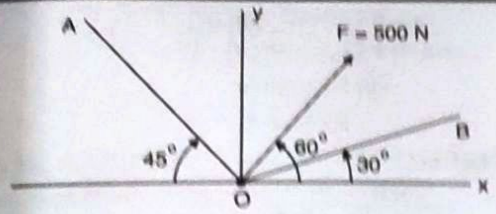


Fig. P. 3.3.11(a)

Soln. :

Along the directions OA and OB :

Let, F_1 = Component along OA

F_2 = Component along OB

α = Angle between F_1 and F

β = Angle between F_2 and F

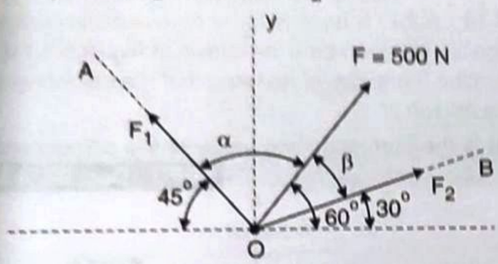


Fig. P. 3.3.11(b)

From Fig. P. 3.3.11(b),

$$\alpha = 180° - 60° - 45° = 75°$$

$$\beta = 60° - 30° = 30°$$

$$F_1 = F\left[\frac{\sin\beta}{\sin(\alpha+\beta)}\right]$$

$$= 500\left[\frac{\sin 30°}{\sin(75° + 30°)}\right]$$

$$= 258.82 \text{ N}$$

$\therefore$ Resolved part along OA is

$$\mathbf{F_1 = 258.82\ N} \qquad \text{... Ans.}$$

$$F_2 = F\left[\frac{\sin\alpha}{\sin(\alpha+\beta)}\right]$$

$$= 500\left[\frac{\sin 75°}{\sin 105°}\right]$$

$$= 500 \text{ N}$$

$\therefore$ Resolved part along OB is

$$\mathbf{F_2 = 500\ N} \qquad \text{... Ans.}$$

Along x and y axes :

Resolved parts i.e., components along x and y axes are :

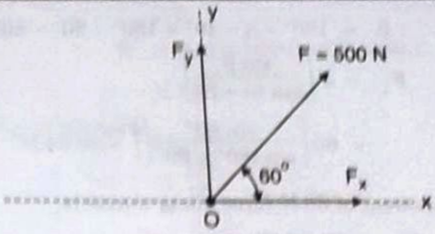


Fig. P. 3.3.11(c)

$$F_x = F\cos 60° = 500\cos 60°$$

$$= \mathbf{250\ N} \rightarrow \qquad \text{... Ans.}$$

$$F_y = F\sin 60° = 500\sin 60°$$

$$= \mathbf{433\ N} \uparrow \qquad \text{... Ans.}$$

Ex. 3.3.12 : Resolve the 60 N force in to components acting along the u and v axes and determine the magnitudes of the components. Refer Fig. P. 3.3.12(a).

SPPU : June 12, 6 Marks

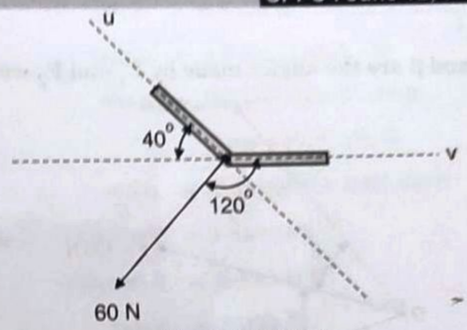


Fig. P. 3.3.12(a)

Soln. :

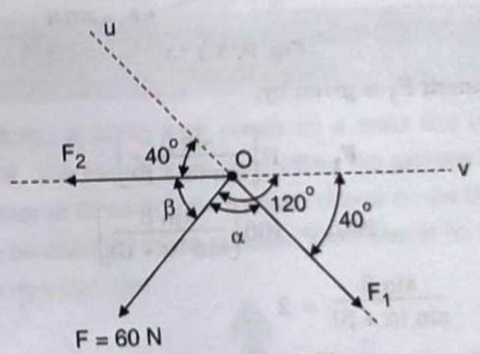


Fig. P. 3.3.12(b)

Let the component of force F along the axis u be F_1 and along the axis v be F_2.

'α' is the angle between F_1 and F

'β' is the angle between F_2 and F

From Fig.P.3.3.12(b), $\alpha = 120° - 40° = 80°$

$$\beta = 180^\circ - \alpha - 40^\circ = 180^\circ - 80^\circ - 40^\circ = 60^\circ$$

$$F_1 = F\left[\frac{\sin\beta}{\sin(\alpha+\beta)}\right]$$

$$= 60\left[\frac{\sin 60^\circ}{\sin(80^\circ + 60^\circ)}\right] = 80.84 \text{ N}$$

$\therefore$ Component of 60 N force along u-axis is,

$$F_1 = 80.84 \text{ N} \qquad \text{... Ans.}$$

$$F_2 = F\left[\frac{\sin\alpha}{\sin(\alpha+\beta)}\right]$$

$$= 60\left[\frac{\sin 80^\circ}{\sin 140^\circ}\right] = 91.92 \text{ N}$$

$\therefore$ Component of 60 N force v-axis is

$$F_2 = 91.92 \text{ N} \qquad \text{... Ans.}$$

Ex. 3.3.13 : A force of 100 N is resolved in to two components of magnitude 200 N each. Determine the angle between the components and the force.

Soln. :

Let α and β are the angles made by F_1 and F_2 with F.

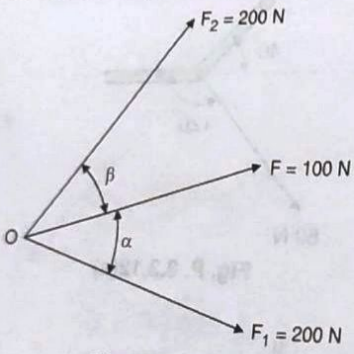


Fig. P. 3.3.13

Component F_1 is given by,

$$F_1 = F\left[\frac{\sin\beta}{\sin(\alpha+\beta)}\right]$$

$$200 = 100\left[\frac{\sin\beta}{\sin(\alpha+\beta)}\right]$$

$$\therefore \frac{\sin\beta}{\sin(\alpha+\beta)} = 2 \qquad \text{... (1)}$$

Similarly component F_2 is given by,

$$F_2 = F\left[\frac{\sin\alpha}{\sin(\alpha+\beta)}\right]$$

$$200 = 100\left(\frac{\sin\alpha}{\sin(\alpha+\beta)}\right)$$

$$\therefore \frac{\sin\alpha}{\sin(\alpha+\beta)} = 2 \qquad \text{... (2)}$$

Solving Eq^n (1) and Eq^n (2),

$$\frac{\sin\beta}{\sin(\alpha+\beta)} = \frac{\sin\alpha}{\sin(\alpha+\beta)}$$

$$\therefore \sin\beta = \sin\alpha$$

$$\beta = \alpha$$

From Eq^n (1),

$$\frac{\sin\beta}{\sin(\beta+\beta)} = 2$$

$$\sin\beta = 2\sin 2\beta$$

$$\sin\beta = 2(2\sin\beta\cos\beta)$$

$$\sin\beta = 4\sin\beta\cos\beta$$

$$\cos\beta = \left(\frac{1}{4}\right)$$

$$\therefore \beta = 75.52^\circ$$

$$\therefore \alpha = \beta = 75.52^\circ$$

Ex. 3.3.14 : A 200 N force is to be resolved into components along the lines a-a' and b-b' as shown in Fig. P.3.3.14(a).
(i) Determine the angle 'α' knowing that the component along a-a' is to be 150 N.
(ii) what is the corresponding value of the component along b-b'.

SPPU : Dec. 03, 4 M

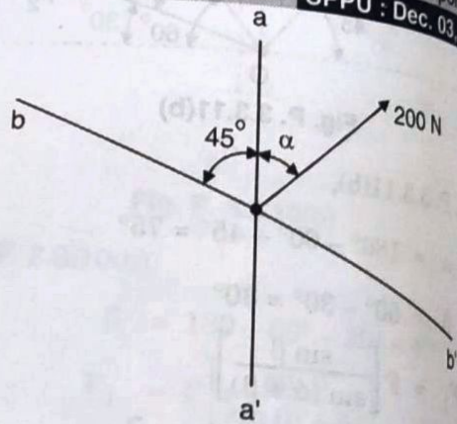
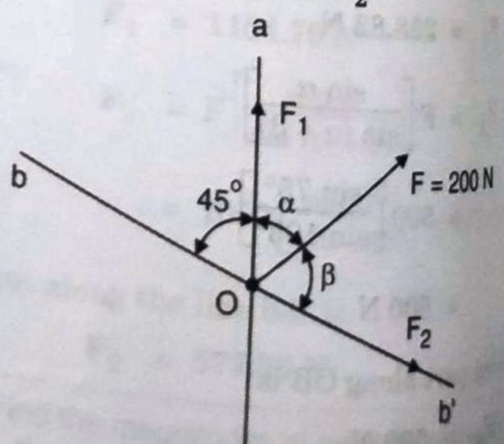


Fig. P. 3.3.14(a)

Soln. :

Let the given force be 'F' and the component along a-a' is F_1 and component along b-b' F_2.



$$\alpha = \text{angle between } F_1 \text{ and } F$$

$$\beta = \text{angle between } F_2 \text{ and } F.$$

$$F_1 = F\left[\frac{\sin\alpha}{\sin(\alpha+\beta)}\right]$$

From Fig. P.3.3.14(b) $\beta = 180° - 45° - \alpha = (135° - \alpha)$

Given, component along a-a′ i.e., $F_1 = 150$ N

$$\therefore \quad 150 = 200\left[\frac{\sin(135° - \alpha)}{\sin(\alpha + 135° - \alpha)}\right]$$

$$= 200\left[\frac{\sin(135° - \alpha)}{\sin 135°}\right]$$

$$\therefore \quad \sin(135° - \alpha) = \frac{150 \times \sin 135°}{200} = 0.53$$

$$\therefore \quad \sin(135° - \alpha) = 0.53$$

$$135° - \alpha = \sin^{-1}(0.53) = 32°$$

$$\therefore \quad \alpha = 103° \qquad \text{... Ans.}$$

The component along b-b′ i.e.,

$$F_2 = F\left[\frac{\sin\alpha}{\sin(\alpha+\beta)}\right]$$

$$= 200\left[\frac{\sin 103°}{\sin 135°}\right]$$

$$= 27.59 \text{ N} \qquad \text{... Ans.}$$

275.59 N

Ex. 3.3.15 : At what angle 'θ' should the force 'F' be directed so that the magnitude of its component along CA does not exceed 80 percent of the magnitude of its component along BC ?

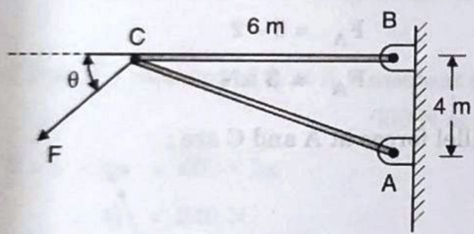


Fig. P. 3.3.15(a)

Soln. :

Let the component of F along BC is F_1 and along CA is F_2.

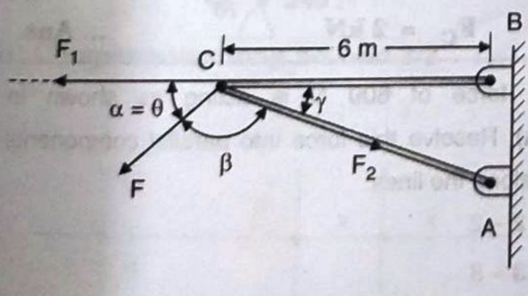


Fig. P. 3.3.15(b)

$$\alpha = \text{Angle between } F_1 \text{ and } F$$

$$= \theta$$

$$\beta = \text{Angle between } F_2 \text{ and } F$$

$$= (180 - \theta - \gamma).$$

From Fig. P.3.3.15(b)

$$\tan\gamma = \left(\frac{4}{6}\right)$$

$$\therefore \quad \gamma = 33.7°$$

$$\therefore \quad \beta = 180 - \theta - 33.7°$$

$$= (146.3 - \theta)$$

Given that, component along CA does not exceed 80 % of the magnitude of its component along BC.

$$\text{i.e.,} \quad F_2 = \left(\frac{80}{100}\right) F_1$$

$$F_2 = 0.8\, F_1$$

$$F\left[\frac{\sin\alpha}{\sin(\alpha+\beta)}\right] = 0.8\, F\left[\frac{\sin\beta}{\sin(\alpha+\beta)}\right]$$

$$\therefore \quad \sin\alpha = 0.8 \sin\beta$$

$$\sin\alpha = 0.8 \sin(146.3 - \theta)$$

$$\sin\alpha = 0.8\,[\sin 146.3° \cos\theta - \cos 146.3° \sin\theta]$$

$$\sin\alpha = 0.44\cos\theta + 0.66\sin\theta$$

$$\sin\theta - 0.66\sin\theta = 0.44\cos\theta \qquad (\because \alpha = \theta)$$

$$0.34\sin\theta = 0.44\cos\theta$$

$$\tan\theta = 1.294$$

$$\therefore \quad \theta = 52.30° \qquad \text{... Ans.}$$

Type 3 : Non Perpendicular (Parallel) components

Ex. 3.3.16 : A 80 kg man stands on a small foot bridge at point 'B'. The man is to be replaced by two persons one at A and other at C, so that the external effects on the bridge are not to be changed in the process. What should be the mass of the new persons.

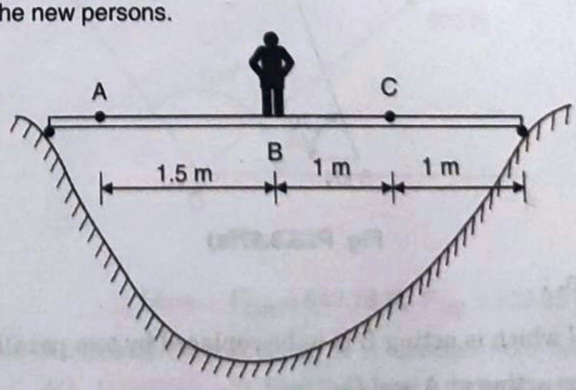


Fig. P. 3.3.16(a)

Soln. :

80 kg man is to be replaced by two persons at A and C without changing the external effects on the bridge. That means, 80 kg weight is to be resolved in to two parallel components at A and C.

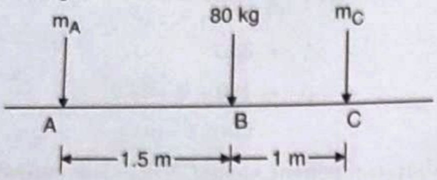


Fig. P. 3.3.16(b)

Here 80 kg is the resultant of m_A and m_B.

Let m_A and m_B be the mass of the persons at A and C.

$$\therefore \quad \Sigma F_y = R$$

$$-m_A - m_C = -80$$

$$\therefore \quad m_A + m_C = 80 \qquad \text{... (1)}$$

Taking moments at A and using varignon's theorem,

$$\Sigma M \text{ at Point A} = \text{Moment of resultant at A.}$$

$$m_A \times 0 - m_C \times 2.5 = -80 \times 1.5$$

$$\therefore \quad m_C = 48 \text{ kg.}$$

From (1), $\quad m_A = 32$ kg.

$\therefore$ Mass of the new persons at A and C are,

$$\mathbf{m_A = 32 \text{ kg}} \qquad \text{... Ans.}$$

$$\mathbf{m_C = 48 \text{ kg}} \qquad \text{... Ans.}$$

Ex. 3.3.17 : A force of magnitude 5 kN is acting at point B as shown in Fig.P.3.3.17(a). Replace this force with an equivalent system formed by two parallel forces at A and C.

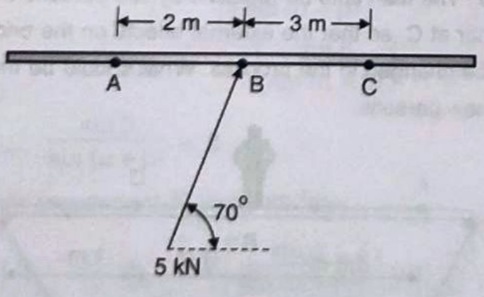


Fig. P. 3.3.17(a)

Soln. :

5 kN which is acting B is to be replaced by two parallel forces acting at A and C.

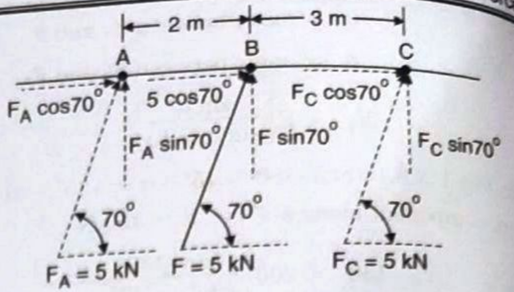


Fig. P. 3.3.17(b)

Let us consider two parallel forces F_A and F_C acting at A and C in the direction of the given force F.

Now, 'F' is the resultant of F_A and F_C.

$$\therefore \quad F_A + F_C = F$$

$$F_A + F_C = 5 \qquad \text{... (1)}$$

Taking moments at point 'A' and using varignon's theorem;

Moment of F_A at A + Moment of F_C at A = Moment of F at A

$$0 + F_C \sin 70° \times 5 = 5 \sin 70° \times 2$$

$$\therefore \quad \mathbf{F_C = 2 \text{ kN}}$$

From Eq^n (1),

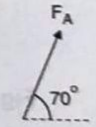
$$F_A + 2 = 5$$

$$F_A = 5 - 2$$

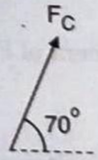
$$\mathbf{F_A = 3 \text{ kN}}$$

$\therefore$ Parallel forces at A and C are ;

$$\mathbf{F_A = 3 \text{ kN}} \qquad \text{... Ans.}$$



$$\mathbf{F_C = 2 \text{ kN}} \qquad \text{... Ans.}$$



Ex. 3.3.18 : A force of 600 N is acting as shown in Fig.P.3.3.18(a). Resolve this force into parallel components and P and Q along the lines.

(i) 1 – 1 and 2 – 2

(ii) 2 – 2 and 3 – 3

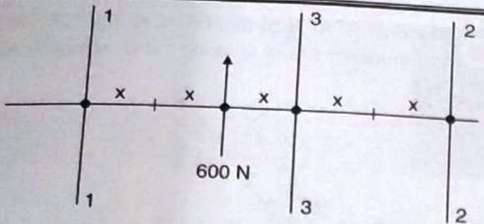


Fig. P.3.3.18(a)

Soln. :

(i) Components along 1-1 and 2-2

Let P and Q be the two parallel components along the lines 1-1 and 2-2.

Assuming both upwards.

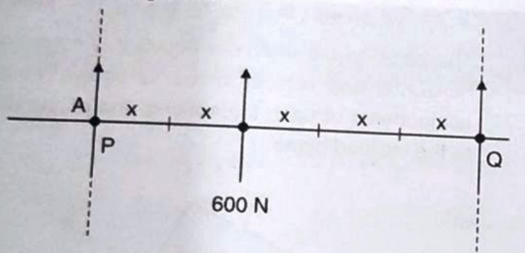


Fig. P. 3.3.18(b)

600 N is the resultant of P and Q.

$$\therefore \; P + Q = 600 \qquad \ldots (1)$$

Taking moments at point 'A' and using varignon's theorem.

moment of P at A + moment of Q at A = moment of 600 N at A.

$$0 + Q \times 5x = 600 \times 2x$$

$$\therefore \; Q = 240 \text{ N}$$

From Eqn (1), $P = 600 - 240 = 360$ N

$\therefore$ Components along the lines 1-1 and 2-2 are :

$$\mathbf{P = 360\ N \uparrow} \qquad \textbf{... Ans.}$$

$$\mathbf{Q = 240\ N \uparrow} \qquad \textbf{... Ans.}$$

(ii) Components along 2-2 and 3-3

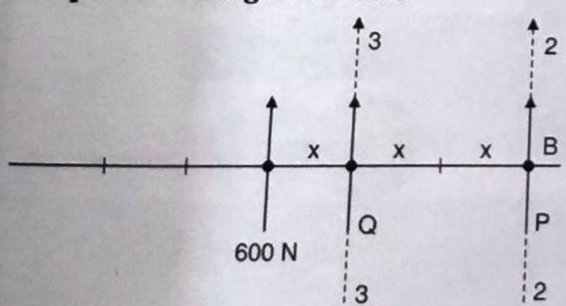


Fig. P. 3.3.18(c)

Let 'P' and 'Q' be the two parallel components along the line 2-2 and 3-3.

Assuming both upwards.

$$\therefore \; P + Q = 600 \qquad \ldots (2)$$

Taking moments at point 'B' and using varignon's theorem :

$$0 - Q \times 2x = -600 \times 3x$$

$$\therefore \; Q = 900 \text{ N} \uparrow$$

From Eqn (2), $P = 600 - 900 = -300$ N

$$= 300 \text{ N} \downarrow$$

$\therefore$ The components along the lines 2-2 and 3-3 are :

$$\mathbf{P = 300\ N \downarrow} \qquad \textbf{... Ans.}$$

$$\mathbf{Q = 900\ N \uparrow} \qquad \textbf{... Ans.}$$

Practice Problems

Q.1 . A force of 750 N is exerted on a bolt at A as shown in Fig. Q.1. Determine the horizontal and vertical components of the force. SPPU : May 08, 5 Marks

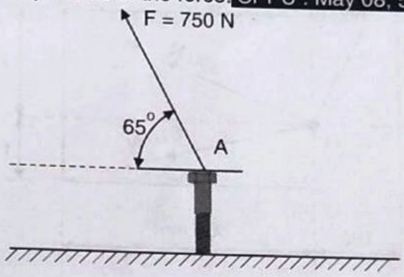


Fig. Q. 1

[Ans. : F_x = 316.96 N ← F_y = 679.73 N ↑]

Q.2 . Resolve the force of 500 kN into its components along the directions OA and OB shown in Fig. Q.2.

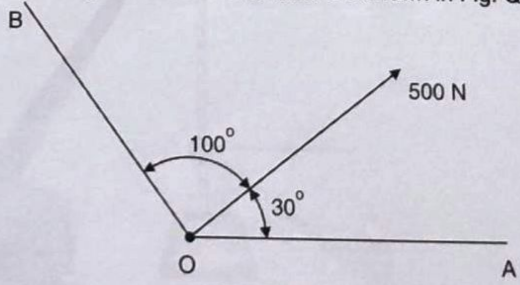


Fig. Q.2

[Ans. : F_{OA} = 642.78 N, F_{OB} = 326.35 N]

Q.3 . A force of magnitude 50 N is directed from point A(1, 1) to B(–3, –2). Find the x and y components of the force. **[Ans. : F_x = – 40 N and F_y = – 30 N]**

forces shown in Fig. Q. 4.

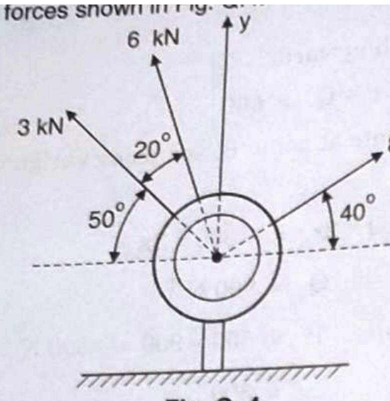


Fig. Q. 4

[Ans. : 8 kN : F_x = 6.128 kN, F_y = 5.14 kN
6 kN : F_x = − 2.05 kN, F_y = 5.64 kN
3 kN : F_x = − 1.93 kN, F_y = 2.30 kN]

Q.5. Determine x and y components of each of the forces shown in Fig. Q. 5.

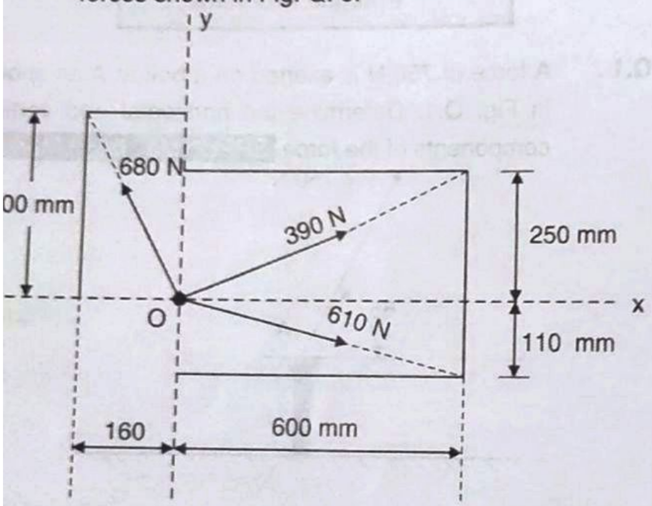


Fig. Q. 5

[Ans. :610 N : F_x = 600 N, F_y = − 110 N
390 N : F_x = 300 N, F_y = 150 N
680 N : F_x = − 320 N, F_y = 600 N]

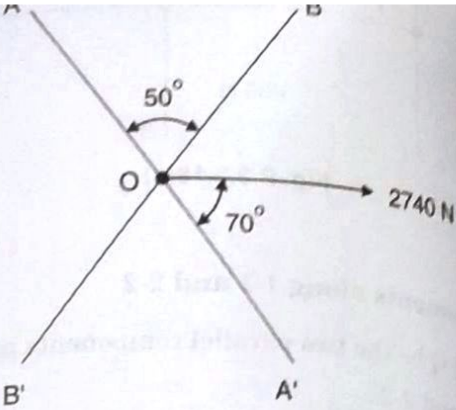


Fig. Q.6

[Ans. : P = 3097.6 N and Q = 3360.6 N]

Q.7. The body on the incline in Fig. Q.7 is subjected to horizontal and vertical forces as shown. Find the components of each force along and perpendicular to the inclined plane.

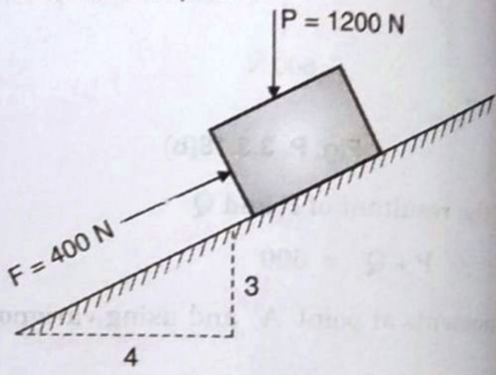


Fig. Q.7

[Ans. : For 'P' : 720 N and 960 N
For 'F' : 320 N and 240 N]

CHAPTER

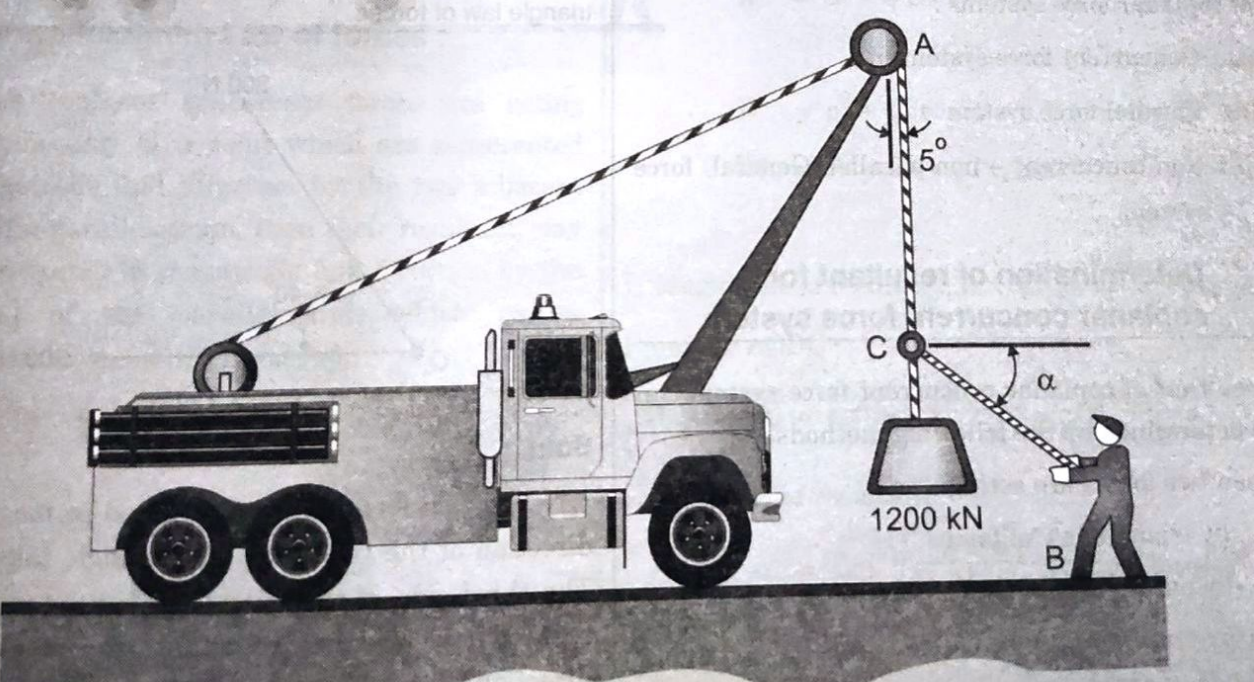
4

UNIT - I

Composition of Forces

Introduction : In this chapter, we shall be able to understand the concept of resultant. We shall also determine the resultant for coplanar concurrent force system, parallel force system and general force systems.

- Type 1 : Resultant of Concurent Force System
- Type 2 : Resultant of Parallel Force System
- Type 3 : Resultant of Non Concurent and Non parallel (General) Force System



4.1 Composition of forces

- The process of combining or adding all forces to get a single force which is equivalent to all forces is known as "composition of forces." The single force obtained is called the "resultant force."
- Simply it is the reduction of given system of forces to the simplest system that is equivalent to the given system. In other words, the process of determining the resultant of number of forces is called "composition of forces".

4.2 Resultant

- The resultant of a system of forces is the simplest form of an equivalent system which can replace the given system without changing the effects produced.
- The equivalent system may have a single force (resultant force) or single moment (resultant moment) or both.
- Determination of resultant force includes its
  (i) Magnitude
  (ii) Direction
  (iii) Sense and
  (iv) Print of application

4.3 Determination of resultant

- We determine the resultant of the following types of coplanar force systems :
  (i) Concurrent force system
  (ii) Parallel force system
  (iii) Non concurrent – non parallel (General) force system.

4.4 Determination of resultant for coplanar concurrent force system

- Resultant of coplanar concurrent force system can be determined by the following methods :

a) when two forces are acting :
  (i) by triangle law of forces
  (ii) by parallelogram law of forces
  (iii) by resolving each force into its x and y components.

b) When more than two forces are acting
  (i) by polygon law of forces
  (ii) by resolving each force into its x and y components.

4.4.1 Triangle law of forces

- "If two coplanar concurrent forces are acting simultaneously at a point which are represented in magnitude and direction by the two sides of a triangle taken in order, then the resultant of these two forces is represented in magnitude and direction by the third side of the triangle in opposite order."

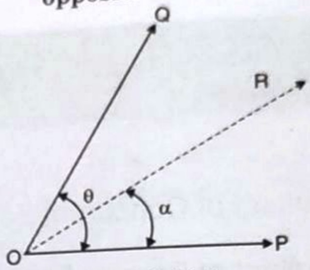


Fig. 4.4.1(a)

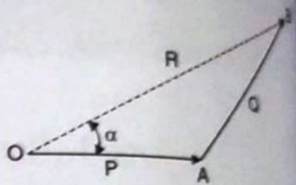


Fig. 4.4.1(b)

From the vector triangle OAB ;

$$\overline{OA} + \overline{AB} = \overline{OB}$$

$$\therefore \bar{P} + \bar{Q} = \bar{R}$$

'R' is the resultant of two concurrent forces P and Q.

Illustrative Example –1: Two forces of magnitude 300 N and 200 N are acting at a point 'O' as shown in Fig.4.4.2(a). Determine the magnitude and direction of resultant using triangle law of forces.

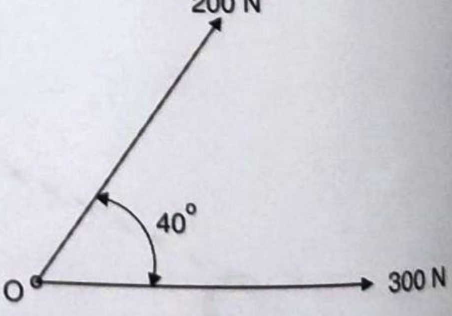


Fig. 4.4.2(a)

Soln. :

The two forces are represented in magnitude and direction of the two sides of a triangle taken in order. The third side of the triangle taken in opposite order will represent the resultant in magnitude and direction.

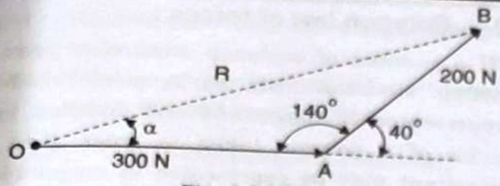


Fig. 4.4.2(b)

For the Δ OAB, using cosine rule,

$$OB^2 = OA^2 + AB^2 - 2 \times OA \times AB \times \cos A$$

$$R^2 = 300^2 + 200^2 - 2 \times 300 \times 200 \times \cos 140°$$

∴ Magnitude of resultant, R = 471.089 N

For the Δ OAB, using sine rule,

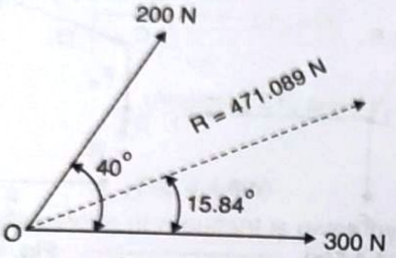


Fig. 4.4.2(c)

$$\frac{AB}{\sin \alpha} = \frac{OB}{\sin 140°}$$

$$\frac{200}{\sin \alpha} = \frac{R}{\sin 140°} = \frac{471.089}{\sin 140°}$$

$$\sin\alpha = 0.2789$$

$$\alpha = \mathbf{15.84°}$$

∴ Direction of resultant, α = 15.84° w.r.t. force 300 N.

4.4.2 Parallelogram Law of forces :

- "If two coplanar concurrent forces are acting simultaneously at a point which are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant may be represented in magnitude and direction by the diagonal of the parallelogram which passes through their point of intersection."

Expression for resultant :

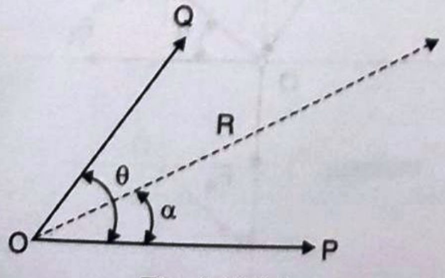


Fig. 4.4.3(a)

- Let the forces P and Q are acting at point 'O' at angle 'θ' between them. These forces are represented by the two adjacent sides OA and OB of a parallelogram OACB.

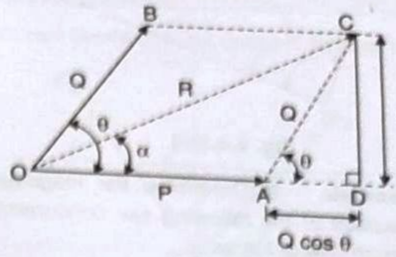


Fig. 4.4.3(b)

- Draw perpendicular CD on the line OA produced. The diagonal of the parallelogram OC represents the resultant of P and Q in magnitude and direction.
- 'α' is the direction of resultant w.r.t. force 'P'.

From the ΔADC ;

$$\sin\theta = \frac{CD}{AC} = \frac{CD}{Q}$$

$$\therefore \quad CD = Q \sin \theta$$

$$\cos \theta = \frac{AD}{AC} = \frac{AD}{Q}$$

$$\therefore \quad AD = Q \cos \theta$$

From the ΔODC ;

$$OC^2 = OD^2 + CD^2$$

$$R^2 = (P + Q \cos \theta)^2 + (Q \sin \theta)^2$$

$$(OD = OA + AD = P + Q \cos \theta)$$

$$\therefore \quad R^2 = p^2 + Q^2 \cos^2 \theta + 2\, PQ \cos \theta + Q^2 \sin^2\theta$$

$$R2 = P^2 + Q^2 + 2\, PQ \cos \theta$$

$$(\because \sin^2 \theta + \cos^2 \theta = 1)$$

∴ Magnitude of resultant, $R = \sqrt{P^2 + Q^2 + 2\, PQ \cos \theta}$

From the ΔODC ;

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{Q \sin \theta}{P + Q \cos \theta}$$

∴ Direction of resultant,

$$\alpha = \tan^{-1}\left(\frac{Q \sin \theta}{P + Q \cos \theta}\right) \text{ w.r.t. force 'P'.}$$

> **Note :** Force 'P is not necessarily always horizontal. It can have any direction. Butα is the direction of resultant w.r.t. force 'P'.

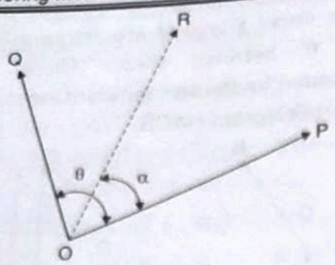


Fig. 4.4.3(c)

Illustrative Example – 2 : Determine the magnitude and direction of resultant of the following two concurrent forces using parallelogram law of forces.

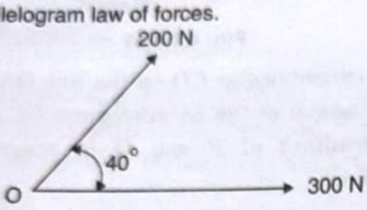


Fig. 4.4.4(a)

Soln. :

Let $P = 300\text{ N}$

$Q = 200\text{ N}$

$\theta = 40°$ (angle between P and Q)

Using parallelogram law of forces,

Magnitude of resultant is given by,

$$R = \sqrt{P^2 + Q^2 + 2\,PQ\cos\theta}$$

$$= \sqrt{300^2 + 200^2 + 2 \times 300 \times 200 \times \cos 40°}$$

$$= \mathbf{471.089\ N}$$

Direction of resultant is given by,

$$\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right) \quad \text{w.r.t. force 'P'}$$

$$= \tan^{-1}\left(\frac{200\sin 40°}{300 + 200\cos 40°}\right)$$

$$= \mathbf{15.84°} \quad \textbf{w.r.t. force 'P'}$$

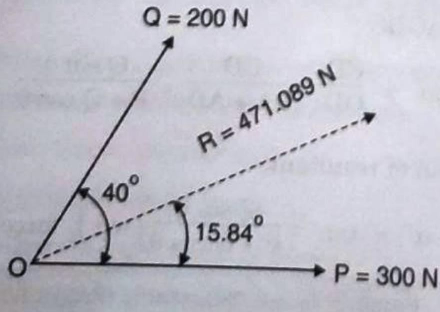


Fig. 4.4.4(b)

4.4.3 Polygon law of forces :

– "If a number of coplanar concurrent forces acting simultaneously at a point which are represented in magnitude and direction by the sides of a polygon taken in order, then the resultant may be represented in magnitude and direction by the closing side of a polygon taken in the opposite order."

Note : Polygon law is an extension of triangle law.

Let us consider four concurrent forces acting at point 'O' as shown in Fig. 4.4.5

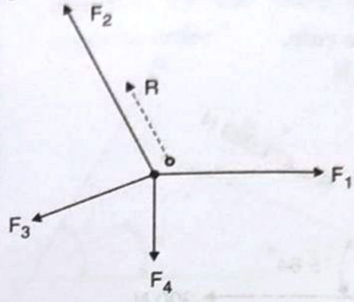


Fig. 4.4.5(a)

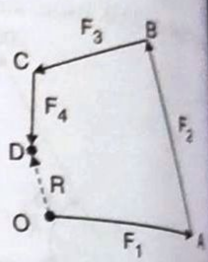


Fig. 4.4.5(b)

– These four forces are represented in magnitude and direction by the sides of a polygon OA, AB, BC and CD taken in order (ACW). The closing side of the polygon OD taken in the opposite order (CW) represents in magnitude and direction of resultant of four forces F_1, F_2, F_3 and F_4.

Note : If the polygon is closed i.e. point D coincides with 'O', the resultant of the system R = 0, then the system is said to be in equilibrium.

4.4.4 Determination of resultant by resolving each force into its x and y components :

Consider the following coplanar concurrent force system.

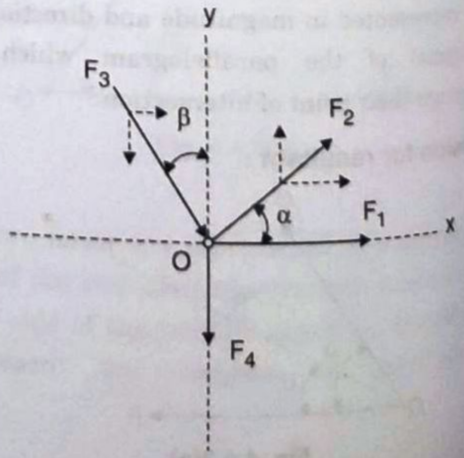


Fig. 4.4.6(a)

Step-1 : Resolve each force into components along x and y axes.

Step-2 : Take algebraic sum of all components in x-direction and in y-direction.

$$\Sigma F_x = R_x \text{ (x-component of resultant)}$$
$$= F_1 + F_2 \cos \alpha + F_3 \sin \beta + 0$$

$$\Sigma F_y = R_y \text{ (y-component of resultant)}$$
$$= 0 + F_2 \sin \alpha - F_3 \cos \beta - F_4$$

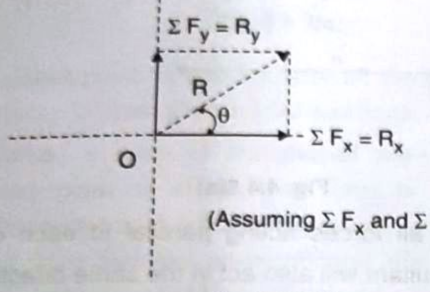


Fig. 4.4.6(b)

Step-3 : Magnitude of resultant is given by,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

Step-4 : Direction of resultant is given by,

$$\theta = \tan^{-1} \left| \frac{\Sigma F_y}{\Sigma F_x} \right| \text{ w.r.t. x-axis.}$$

Step-5 : Sense of the resultant is obtained from the signs of ΣF_x and ΣF_y .

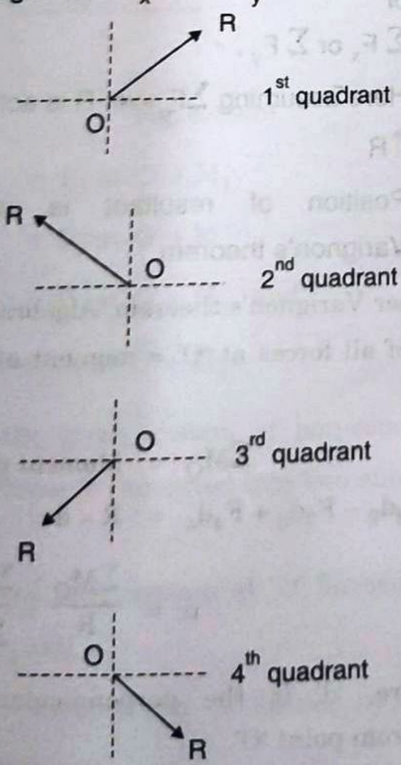


Fig. 4.4.6(c)

Step-6 : Point of application of resultant = Point of concurrence 'O'

> **Note :** For concurrent force system, the resultant will always act at the point of concurrence.

We can prove this by using Varignon's theorem.

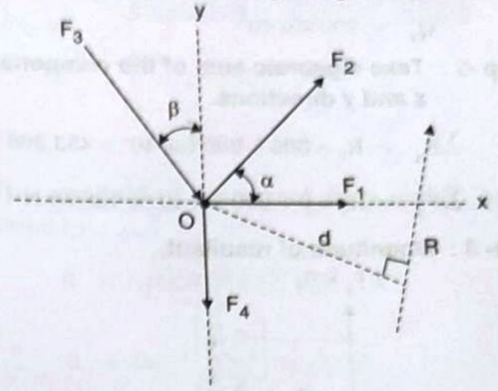


Fig. 4.4.6(d)

Let the resultant of four forces be act at the position shown in Fig. 4.4.6(d), i.e., at a perpendicular distance 'd' from the point of concurrence.

Taking moments at 'O' and using Varignon's theorem.

Algebraic sum of the moments of all four forces at point 'O' = Moment of resultant at the same point 'O'.

$$\Sigma M_O = R \times d$$

$$O = R \times d \; (\because \Sigma M_O = O \text{ as the lines of action of all forces passing through point 'O'})$$

But, $R \neq O$

$\therefore \; d = 0$

Which means that the line of action of resultant is passing through point 'O' i.e., point of concurrence.

Illustrative Example – 3 : Determine the magnitude and direction of resultant of the following concurrent forces by resolving them into x and y components.

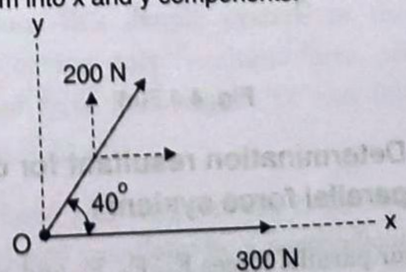


Fig. 4.4.7(a)

Let P = 300 N and Q = 200 N

Soln. :

Step- 1 : Resolve forces into x and y components.

$$P_x = 300 \text{ N}$$
$$P_y = 0$$
$$Q_x = 200 \cos 40°$$
$$Q_y = 200 \sin 40°$$

Step -2 : Take algebraic sum of the components in x and y directions.

$$\Sigma F_x = R_x = 300 + 200 \cos 40° = 453.208 \text{ N} \rightarrow$$

$$\Sigma F_y = R_y = 0 + 200 \sin 40° = 128.55 \text{ N} \uparrow$$

Step- 3 : Magnitude of resultant,

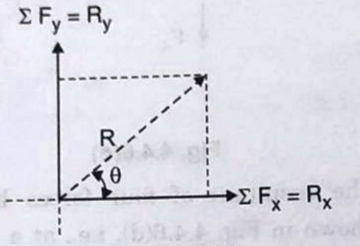


Fig. 4.4.7(b)

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(453.208)^2 + (128.55)^2}$$

$$= \mathbf{471.087\ N}$$

Step- 4 : Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \tan^{-1}\left|\frac{128.55}{453.208}\right|$$

$$= \textbf{15.84° w.r.t. x-axis i.e., w.r.t. force 'p'}$$

Step- 5 :

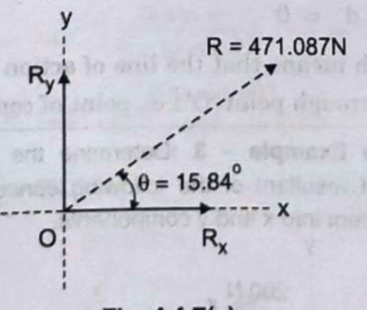


Fig. 4.4.7(c)

4.4.5 Determination resultant for coplanar parallel force system :

- Let four parallel forces F_1, F_2, F_3 and F_4 are acting as shown in Fig. 4.4.8(a) at a distance d_1, d_2 d_3 and d_4 respectively from point 'O'.
- Let 'R' be the resultant of these four forces at a distance d from point 'O'.

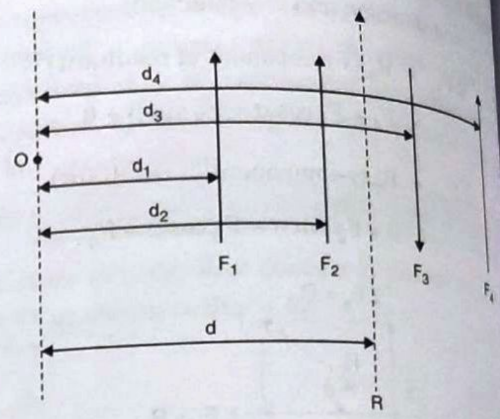


Fig. 4.4.8(a)

Step-1 : As all forces acting parallel to each other resultant will also act in the same direction. Here all forces are acting in y-direction.

∴ Magnitude of resultant,

$$R = \Sigma F = \Sigma F_y$$

(∵ x-components of all forces zero)

Step-2 : Direction of resultant is same as that of the parallel forces.

Step-3 : Sense of the resultant depends on the sign of ΣF_x or ΣF_y.

Here assuming ΣF_y+ve, R is acting upwards ↑R

Step-4 : Position of resultant is obtained by Varignon's theorem.

As per Varignon's theorem, Algebraic sum of the moments of all forces at 'O' = moment of resultant at point 'O'.

$$\Sigma M_O = \text{Moment of R at 'O'}$$

$$F_1 d_1 + F_2 d_2 - F_3 d_3 + F_4 d_4 = R \times d$$

$$\therefore \quad d = \frac{\Sigma M_O}{R} = \frac{\Sigma M_O}{\Sigma F_y}$$

Where, 'd' is the perpendicular distance of resultant from point 'O'.

4.4.6 Determination of Resultant for Non-concurrent and Non-parallel (General) Coplanar Force System :

– Let us consider the following general force system :

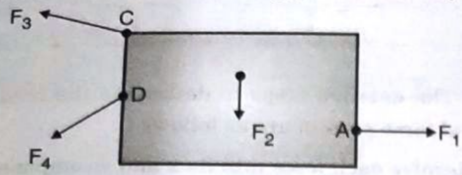


Fig. 4.4.9(a)

– Select point 'O' and transfer all forces parallelly to point 'O' from their initial positions.

– When a force is transferred parallelly from one point to other, that force is accompanied by a moment whose magnitude is the product of force and the perpendicular distance between the two parallel lines of action.

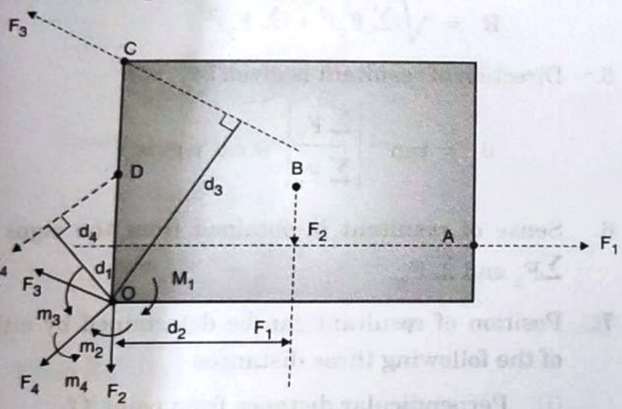


Fig. 4.4.9(b)

$$\therefore\ F_1 \text{ at A} = F_1 \text{ at 'O'} + M_1$$

$$F_2 \text{ at B} = F_2 \text{ at 'O'} + M_2$$

$$F_3 \text{ at C} = F_3 \text{ at 'O'} + M_3 \text{ and}$$

$$F_4 \text{ at D} = F_4 \text{ at 'O'} + M_4$$

Thus the given system of non-concurrent and nonparallel forces is converted into two simple systems i.e.,

(i) Concurrent force system at 'O' formed by P_1, P_2, P_3 and P_4 and

(ii) System of moments at 'O' formed by M_1, M_2, M_3 and M_4.

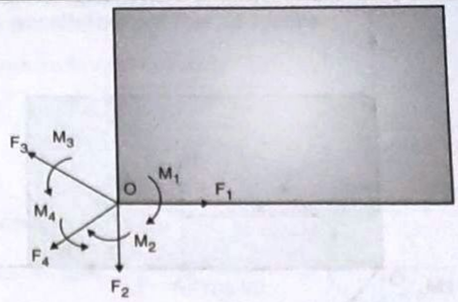


Fig. 4.4.9(c)

– The resultant of concurrent force system can be found by,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| \text{ w.r.t. 'x'}$$

– The resultant of system of moments is found by taking algebraic sum of the moments at 'O' = ΣM_O

$$\Sigma M_O = -M_1 - M_2 + M_3 + M_4$$

(ACW moment +ve and CW moment –ve)

– Further the system is reduced to a simple system with resultant force and resultant moment.

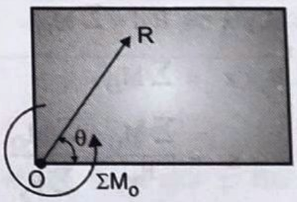


Fig. 4.4.9(d)

– Here ΣFx and ΣFy are assumed +ve, hence 'R' is acting in 1st quadrant.

– ΣM_O is assumed +ve, hence acting in ACW direction.

– To reduce this simple system to the simplest system having only resultant force, combine the resultant force R acting at 'O' and the resultant moment ΣM_O.

– When force and moment are combined, force will be displaced parallelly by a perpendicular distance 'd' given by, $d = \dfrac{\Sigma M_O}{R}$

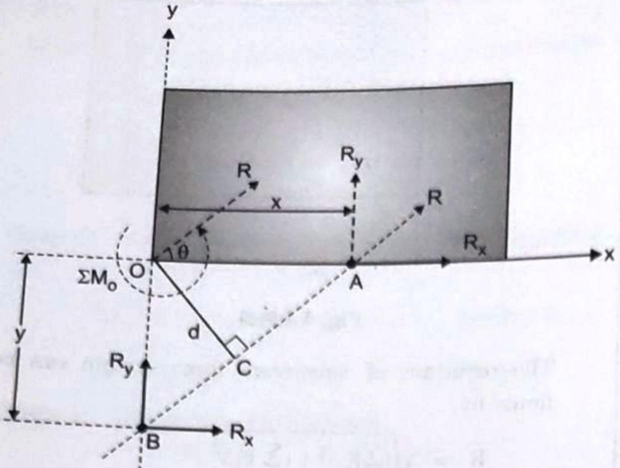


Fig. 4.4.9(e)

– Let the line of action of resultant intersects x and y axes at a distance 'x' and 'y' respectively from point 'O'.

– Resolving resultant 'R' into two components Rx and Ry at point A on x-axis and taking moments at point 'O'.

– As per Varignon's theorem, Algebraic sum of the moments of Rx and Ry at 'O' = moment of resultant 'R' at 'O'.

$$R_x \times 0 + R_y \times x = R \cdot d = \Sigma M_O$$

$$\therefore R_y \times x = \Sigma M_O$$

$$x = \frac{\Sigma M_O}{R_y} = \frac{\Sigma M_O}{\Sigma F_y}$$

– Similarly, by resolving 'R' into Rx and R_y at point 'B' on y-axis and taking moments at point 'O'.

– As per Varignon's theorem;

$$R_x \times y + R_y \times 0 = R \cdot d = \Sigma M_O$$

$$\therefore y = \frac{\Sigma M_O}{R_x} = \frac{\Sigma M_O}{\Sigma F_x}$$

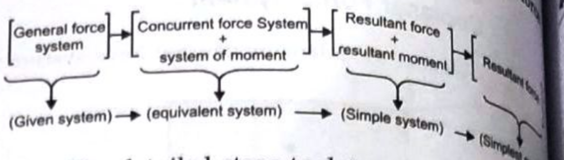
$\therefore$ To determine the position of resultant, we can find out either x, y or d.

Where, x = horizontal distance of 'R' from point 'O'.

y = Vertical distance of 'R' from point 'O'.

d = Perpendicular distance of 'R' from point 'O'

Steps to determine the resultant of general force system are;



The detailed steps to determine the resultant of general force system are as follows :

1. Resolve each force into its x and y components.
2. Take algebraic sum of x and y components.

$$\Sigma F_x = Rx \quad \text{(x - component of resultant)}$$

$$\Sigma F_y = Ry \quad \text{(y - component of resultant)}$$

3. Select any point 'O' and take algebraic sum of the moments of all forces about point 'O' i.e., ΣM_O
4. Magnitude of resultant is given by,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

5. Direction of resultant is given by,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| \text{ w.r.t. x-axis}$$

6. Sense of resultant is obtained from the signs of ΣF_x and ΣF_y.
7. Position of resultant can be determined by either of the following three distances :

(i) Perpendicular distance from point 'O'.

$$d = \frac{\Sigma M_O}{R}$$

(ii) Horizontal distance from point 'O'.

$$x = \frac{\Sigma M_O}{\Sigma F_y}$$

(iii) Vertical distance from point 'O'.

$$y = \frac{\Sigma M_O}{\Sigma F_x}$$

Note :
1. If the resultant is acting in x-direction, then $\Sigma F_y = 0$ and $\Sigma F_x = R$.
2. If the resultant is acting in y-direction, then $\Sigma F_x = 0$ and $\Sigma F_y = R$.

4.5 Solved Examples :

Type 1 : Resultant of concurrent force system

Ex. 4.5.1 : The angle between the two concurrent forces 90° and their resultant is 2500 N. The resultant makes an angle of 46° with one of the force, determine the magnitude of each force. **SPPU : May 14, 4 Marks**

Soln. :

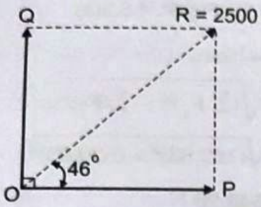


Fig. P. 4.5.1

Let the concurrent forces be 'P' and 'Q'.

When forces are perpendicular, their resultant is

$$R = \sqrt{P^2 + Q^2}$$

$$R^2 = P^2 + Q^2$$

$$2500^2 = P^2 + Q^2 \quad \text{...(1)}$$

Direction of resultant,

$$\tan\theta = \frac{Q}{P}$$

$$\tan 46° = \frac{Q}{P}$$

$$\therefore Q = 1.035\ P \quad \text{...(2)}$$

From Eq^n (1),

$$2500^2 = P^2 + (1.035\ P)^2$$

$$6.25 \times 10^6 = P^2 + 1.071P^2$$

$$= 2.071\ P^2$$

$$\therefore \mathbf{P = 1737.2\ N} \quad \text{...Ans.}$$

From Eq^n (2), $\mathbf{Q = 1798\ N}$ **...Ans.**

Ex. 4.5.2 : The angle between two forces of magnitude 100 N each is 120°. Determine the magnitude and direction of resultant.

Soln. :

We can find out the resultant by 3 methods :

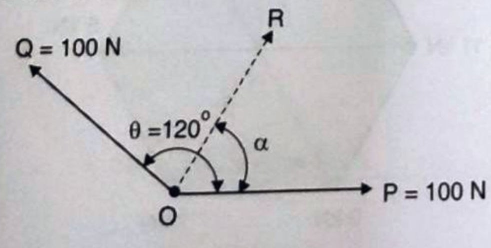


Fig. P. 4.5.2(a)

(i) By parallelogram low of forces :

Magnitude of resultant,

$$R^2 = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{100^2 + 100^2 + 2 \times 100 \times 100 \times \cos 120°}$$

$$= \mathbf{100\ N} \quad \text{...Ans.}$$

Direction of resultant,

$$\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{100\sin 120°}{100 + 100\cos 120°}\right)$$

$$= \mathbf{60\ w.r.t.\ force\ 'P'} \quad \text{...Ans.}$$

OR

(ii) By triangle law of forces :

These two forces are represented by two adjacent sides of a triangle. The third side in opposite order gives the resultant.

The triangle formed is equilateral triangle.

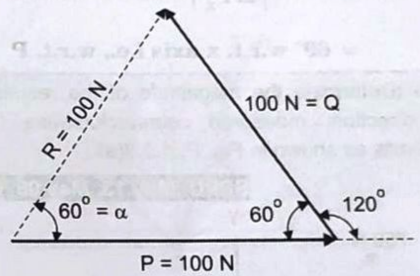


Fig.P. 4.5.2(b)

$\therefore$ Length of the third side is the magnitude of resultant.

$\therefore$ Magnitude of resultant,

$$\mathbf{R = 100\ N} \quad \text{...Ans.}$$

Direction of resultant,

$$\alpha = \mathbf{60°\ w.r.t.\ force\ 'P'} \quad \text{...Ans.}$$

OR

(iii) By resolving forces into x and y components :

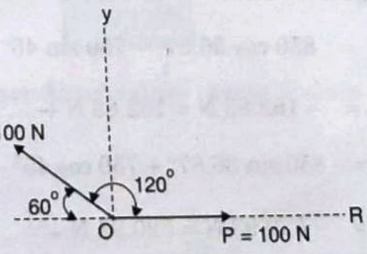


Fig. P. 4.5.2(c)

Taking x-axis along the direction of 'P' and y-axis perpendicular to it.

Resolving forces into x and y components and taking algebraic sum;

$$\Sigma F_x = R_x \quad \text{(x - component of resultant)}$$

$$= 100 - 100\cos 60^\circ = \mathbf{50\ N} \rightarrow$$

$$\Sigma F_y = R_y \quad \text{(y - component of resultant)}$$

$$= 0 + 100\sin 60 = \mathbf{86.6\ N} \uparrow$$

∴ Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(50)^2 + (86.6)^2} = \mathbf{100\ N} \quad \text{...Ans.}$$

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \tan^{-1}\left(\frac{86.6}{50}\right)$$

$$= \mathbf{60^\circ\ w.r.t.\ x\ axis\ i.e.,\ w.r.t.\ P} \quad \text{...Ans.}$$

Ex. 4.5.3 : Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x-axis as shown in Fig. P. 4.5.3(a)

SPPU : May 13, May 08, 6 Marks

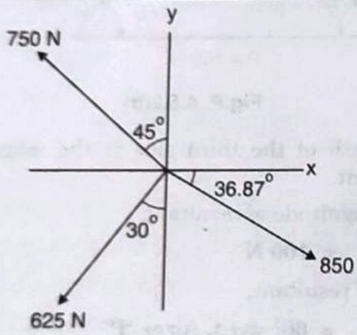


Fig. P. 4.5.3(a)

Soln. :

Resolving each force into its x and y components and taking algebraic sum;

$$\Sigma F_x = 850\cos 36.87^\circ - 750\sin 45^\circ - 625\sin 30^\circ$$

$$= -162.83\ N = 162.83\ N \leftarrow$$

$$\Sigma F_y = -850\sin 36.87^\circ + 750\cos 45^\circ - 625\cos 30^\circ$$

$$= -520.93\ N = 520.93\ N \downarrow$$

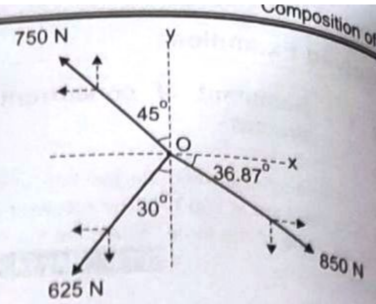


Fig. P. 4.5.3(b)

Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(162.83)^2 + (520.93)^2}$$

$$= \mathbf{545.79\ N} \quad \text{...Ans.}$$

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \tan^{-1}\left(\frac{520.93}{162.83}\right)$$

$$= 72.64^\circ \text{ w.r.t. x-axis}$$

∴ Direction of resultant w.r.t. +ve x-axis,

$$\alpha = 180 + 72.64 = \mathbf{252.64^\circ\ in\ ACW} \quad \text{...Ans.}$$

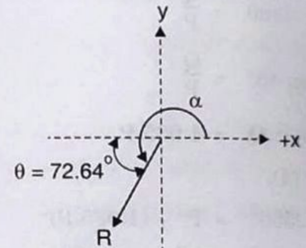


Fig. P. 4.5.3(c)

Ex. 4.5.4 : Concurrent forces of 1, 3, 5, 7, 9 and 11 kN are applied at the centre of a regular hexagon acting towards vertices as shown in Fig. P. 4.5.4(a). Determine the resultant completely.

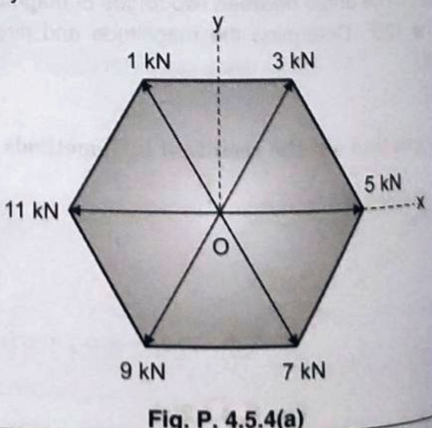


Fig. P. 4.5.4(a)

Soln. :

Being it is regular hexagon, the angle between the forces is

$$\frac{360^\circ}{6} = 60^\circ$$

Resolving each force into x and y components and taking algebraic sum;

$\Sigma F_x = R_x$ (x - component of resultant)

$\Sigma F_x = -1 \cos 60^\circ + 3 \cos 60^\circ + 5 + 7 \cos 60^\circ$

$- 9 \cos 60^\circ - 11$

$= -6 \text{ kN} = \mathbf{6 \text{ kN} \leftarrow}$

$\Sigma F_y = R_y$ (y - component o resultant)

$\Sigma F_x = 1 \sin 60^\circ + 3 \sin 60^\circ - 7 \sin 60^\circ$

$- 9 \sin 60^\circ$

$= -10.39 \text{ kN} = \mathbf{10.39 \text{ kN} \downarrow}$

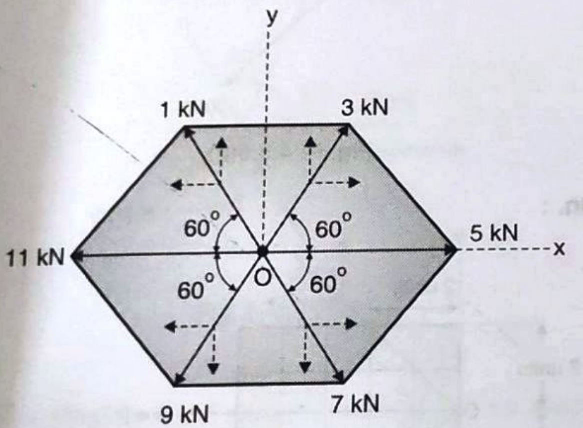


Fig. P. 4.5.4(b)

$\therefore$ Magnitude of resultant ,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(6)^2 + (10.39)^2} = \mathbf{12 \text{ kN}}$$ **...Ans.**

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$

$$= \tan^{-1}\left(\frac{10.39}{6}\right)$$

$$= \mathbf{60^\circ \text{w.r.t. x-axis}}$$ **...Ans.**

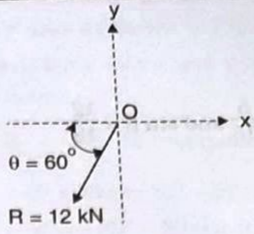


Fig. P. 4.5.4(c)

Ex. 4.5.5 : Find the resultant of the concurrent force system as shown in Fig. P. 4.5.5(a) in magnitude and direction.

SPPU-Dec. 08, 8 Marks

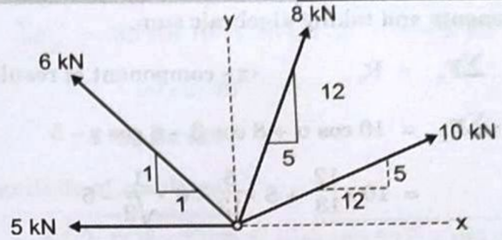


Fig. P. 4.5.5(a)

Soln. :

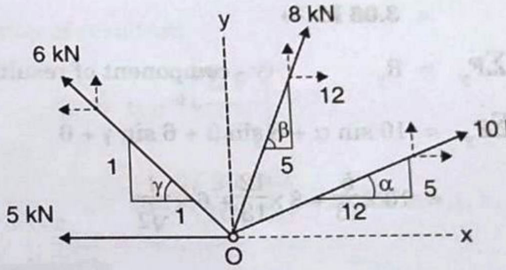


Fig. P. 4.5.5(b)

From Fig. P. 4.5.5(b)

$$\tan\alpha = \left(\frac{5}{12}\right)$$

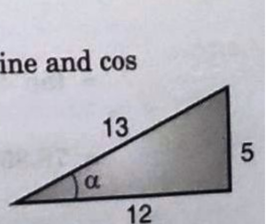
$$\alpha = \tan^{-1}\left(\frac{5}{12}\right) = 22.62^\circ$$

$$\beta = \tan^{-1}\left(\frac{12}{5}\right) = 67.38^\circ$$

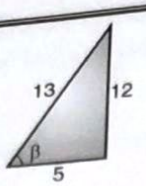
$$\gamma = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ$$

OR

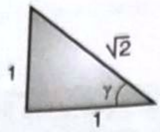
We can have direct values of sine and cos



$$\cos\alpha = \frac{12}{13} \text{ and } \sin = \frac{5}{13}$$



$$\cos\beta = \frac{5}{13} \text{ and } \sin\beta = \frac{12}{13}$$



$$\cos\gamma = \frac{1}{\sqrt{2}} \text{ and } \sin\gamma = \frac{1}{\sqrt{2}}$$

Now resolving each force into its x and y components and taking algebraic sum;

$$\Sigma F_x = R_x \quad \text{(x - component of resultant)}$$

$$\Sigma F_x = 10\cos\alpha + 8\cos\beta - 6\cos\gamma - 5$$

$$= 10\times\frac{12}{13} + 8\times\frac{5}{13} - 6\times\frac{1}{\sqrt{2}} - 5$$

$$= 9.23 + 3.07 - 4.24 - 5$$

$$= \mathbf{3.06\ kN \rightarrow}$$

$$\Sigma F_y = R_y \quad \text{(y - component of resultant)}$$

$$\Sigma F_y = 10\sin\alpha + 8\sin\beta + 6\sin\gamma + 0$$

$$= 10\times\frac{5}{13} + 8\times\frac{12}{13} + 6\times\frac{1}{\sqrt{2}}$$

$$= 3.84 + 7.38 + 4.24$$

$$= \mathbf{15.46\ kN \uparrow}$$

Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(3.06)^2 + (15.46)^2}$$

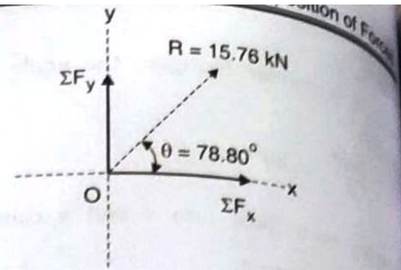
$$\therefore \mathbf{R = 15.76\ kN} \quad \text{...Ans.}$$

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$

$$= \tan^{-1}\left(\frac{15.46}{3.06}\right)$$

$$= \mathbf{78.80^\circ\ w.\ r.\ t.\ x\text{-}axis} \quad \text{...Ans.}$$



Ex. 4.5.6 : **Determine the magnitude of force P so that the resultant of the force system as shown in Fig.P. 4.5.6(a) is vertical and hence find magnitude of resultant force.**

SPPU-Dec. 14, 4 Marks

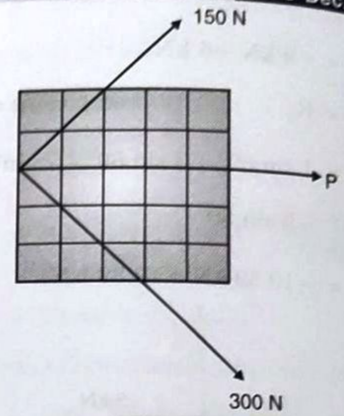


Fig. P. 4.5.6(a)

Soln. :

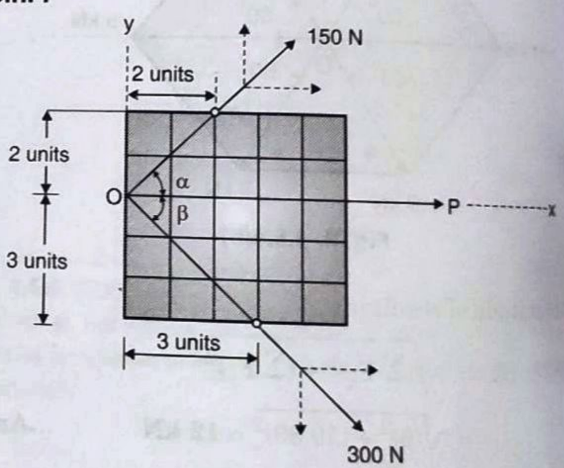


Fig. P. 4.5.6(b)

From Fig. P. 4.5.6(b)

$$\tan\alpha = \left(\frac{2}{2}\right) \Rightarrow \alpha = 45^\circ$$

$$\tan\beta = \left(\frac{3}{3}\right) \Rightarrow \alpha = 45^\circ$$

Given that the resultant of the force system is vertical.

$\therefore \Sigma F_x = 0$ and $\Sigma F_y = R$

Resolving each force into x and y component along the axes shown in Fig. and taking algebraic sum;

$$\Sigma F_x = 0$$

$$P + 150 \cos \alpha + 300 \cos \alpha = 0$$

$$P + 150 \cos 45° + 300 \cos 45° = 0$$

$$\therefore P = -318.2 \text{ N} = 318.2 \text{ N} \leftarrow \quad \text{...Ans.}$$

$$\Sigma F_y = R$$

$$\therefore R = 150 \sin \alpha - 300 \sin \alpha$$

$$= 150 \sin 45° - 300 \sin 45°$$

$$= -106.06 \text{ N}$$

$$\therefore R = 106.06 \text{ N} \downarrow \quad \text{...Ans.}$$

Ex.4.5.7 : Determine the equilibrant of the system of forces shown in Fig.P. 4.5.7(a). **SPPU : May 99, 8 Marks**

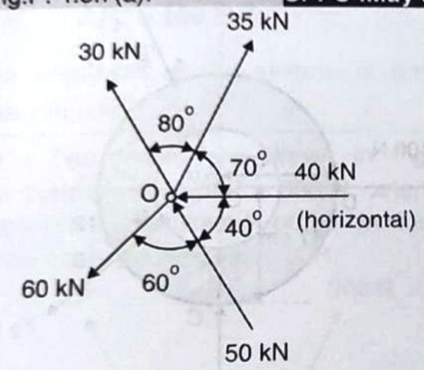


Fig.P. 4.5.7(a)

Soln. :

The magnitude of equilibrant is same as that of resultant but the direction is opposite to that of resultant.

Let us find out the resultant of the given system first.

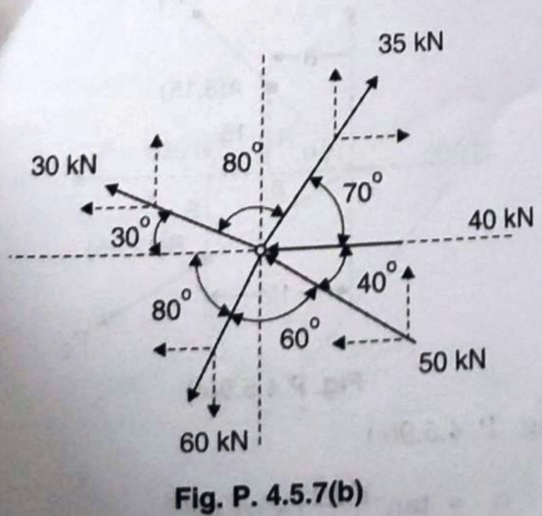


Fig. P. 4.5.7(b)

Taking x an y axes as shown in Fig. P. 4.5.7(b)

Resolving each force into x and y components and taking algebraic sum;

$$\Sigma F_x = R_x \quad \text{(x - component of resultant)}$$

$$= -40 + 35 \cos 70° - 30 \cos 30° - 60 \cos 80° - 50 \cos 40°$$

$$= -102.73 \text{ kN}$$

$$= 102.73 \text{ kN} \leftarrow$$

$$\Sigma F_y = R_y \quad \text{(y - component of resultant)}$$

$$\Sigma F_y = 35 \sin 70° + 30 \sin 30° - 60 \sin 80° + 50 \sin 40°$$

$$= 20.94 \text{ kN} \uparrow$$

$\therefore$ Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(102.73)^2 + (20.94)^2}$$

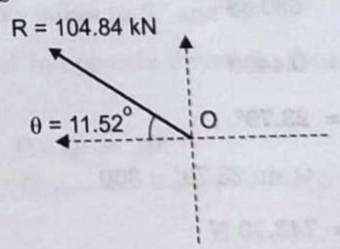
$$= 104.84 \text{ kN} \quad \text{...Ans.}$$

Direction of resultant,

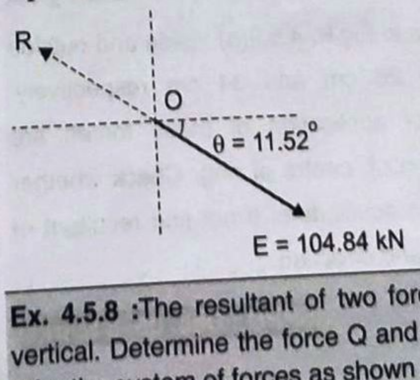
$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$

$$= \tan^{-1}\left(\frac{20.94}{102.73}\right) = 11.52° \text{ w.r.t. x - axis}$$

$\therefore$ Resultant is



Equilibrant is



...Ans.

Ex. 4.5.8 : The resultant of two forces P and Q is 1200 N vertical. Determine the force Q and the corresponding angle θ for the system of forces as shown in Fig. P. 4.5.8(a). **SPPU : May 13, 4 Marks**

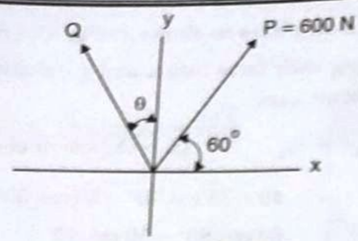


Fig.P. 4.5.8(a)

Soln. :

Given that the resultant of two forces P and Q is 1200 N vertical

$$\therefore \Sigma F_x = 0 \text{ and}$$

$$\Sigma F_y = R = 1200 \text{ N}$$

Resolving 'P' and 'Q' into x and y components and taking algebraic sum,

$$\Sigma F_x = 0$$

$$600 \cos 60° - Q \sin \theta = 0$$

$$\therefore Q \sin \theta = 300 \quad \text{...(1)}$$

$$\Sigma F_y = 1200 \text{ N}$$

$$600 \sin 60° + Q \cos \theta = 1200$$

$$\therefore Q \cos \theta = 680.38 \text{ N} \quad \text{...(2)}$$

From Eq^n (1) Eq^n and (2);

$$\frac{Q \sin \theta}{Q \cos \theta} = \frac{300}{680.38}$$

$$\tan\theta = 0.4409$$

$$\therefore \theta = \mathbf{23.79°} \quad \text{...Ans.}$$

From Eq^n (1), $Q \sin 23.79° = 300$

$$\therefore \mathbf{Q = 743.70 \text{ N}} \quad \text{...Ans.}$$

Ex. 4.5.9 : Four concurrent forces P_1, P_2 and P_3 and P_4 act on annual ring as shown in Fig.P. 4.5.9(a) Inside and outside diameters of ring are 26 cm and 34 cm respectively. Coordinates of point of application of these forces are indicated in the bracket w.r.t. centre of ring. Check whether the system of forces is in equilibrium. If not find resultant of four forces in magnitude and direction.

SPPU : Dec. 06, 8 Marks

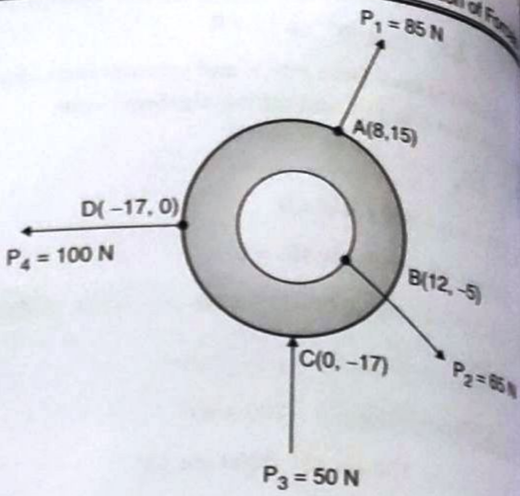


Fig.P.4.5.9(a)

Soln. :

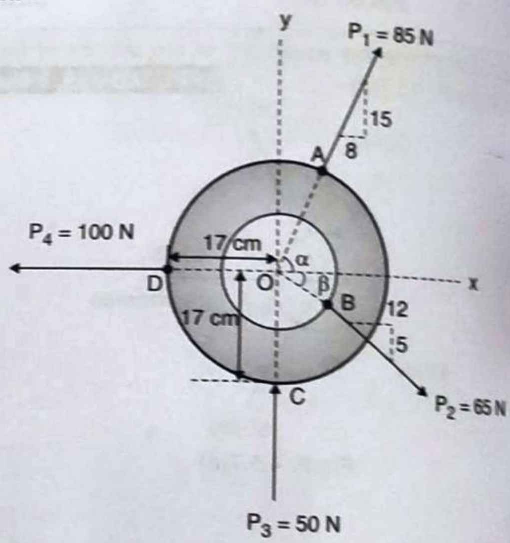


Fig.P.4.5.9(b)

Let α and β be the direction of forces P_1 and P_2 respectively.

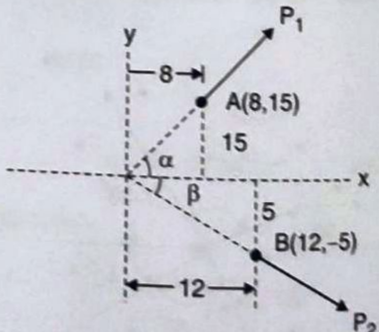


Fig. P.4.5.9(c)

From Fig. P. 4.5.9(c)

$$\alpha = \tan^{-1}\left(\frac{15}{8}\right) = 61.93°$$

$$\beta = \tan^{-1}\left(\frac{5}{12}\right) = 22.62^\circ$$

Resolving each force into x and y components and taking algebraic sum;

$$\Sigma F_x = R_x \quad \text{(x - component of resultant)}$$

$$\Sigma F_x = P_1 \cos\alpha + P_2\cos\beta - P_4$$

$$= 85\cos 61.93^\circ + 65\cos 22.62^\circ - 100 = \mathbf{0}$$

$$\Sigma F_y = R_y \quad \text{(y - component of resultant)}$$

$$\Sigma F_y = P_1 \sin\alpha - P_2\sin\beta + P_3$$

$$= 85\sin 61.93^\circ - 65\sin 22.62^\circ + 50$$

$$= \mathbf{100\ N} \uparrow$$

As $\Sigma F_x = 0$, The magnitude of resultant is

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(0)^2 + (100)^2}$$

$$R = \Sigma F_y = \mathbf{100\ N} \uparrow \qquad \textbf{...Ans.}$$

As the resultant of the system is not zero, the system is not in equilibrium.

Ex. 4.5.10 : Two forces are shown in Fig.P.4.5.10(a) knowing that the magnitude of P is 600 N, determine (a) the required angle θ if the resultant R of the two forces is to be vertical, (b) the corresponding value of R.

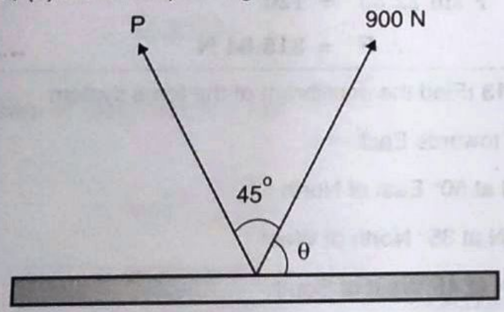


Fig. P. 4.5.10(a)

Soln. :

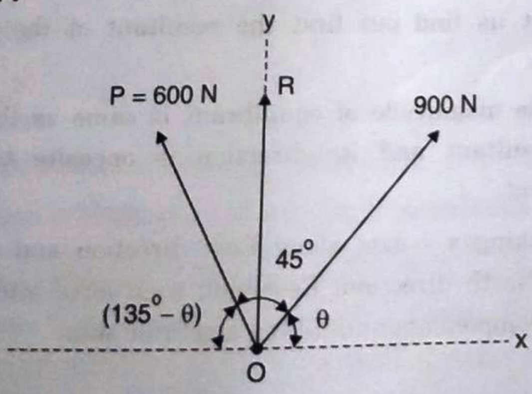


Fig. P. 4.5.10(b)

Given that the resultant 'R' is to be vertical

$$\therefore \Sigma F_x = 0 \text{ and}$$

$$\Sigma F_y = R$$

Resolving each force into x and y components and taking algebraic sum;

$$\Sigma F_x = 0$$

$$900\cos\theta - 600\cos(135^\circ - \theta) = 0$$

$$900\cos\theta = 600\,[\cos 135^\circ \cos\theta + \sin 135^\circ \sin\theta]$$

$$9\cos\theta = -4.24\cos\theta + 4.24\sin\theta$$

$$13.24\cos\theta = 4.24\sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = 3.123$$

$$\therefore \theta = \mathbf{72.25^\circ} \qquad \textbf{...Ans.}$$

$$\Sigma F_y = R$$

$$\therefore R = 900\sin\theta + 600\sin(135^\circ - \theta)$$

$$= 900\sin 72.25^\circ + 600\sin(135^\circ - 72.25^\circ)$$

$$\mathbf{R = 1390.56\ N} \uparrow \qquad \textbf{...Ans.}$$

Ex. 4.5.11 : Two forces acting at a point in opposite direction have a resultant of 10 N. If they act at right angles to each other, their resultant has a magnitude of 75 N. Determine the magnitude of the two forces.

Soln. :

Let the two forces be 'P' and 'Q'.

If they act in opposite direction, their resultant is 10 N.

$$\therefore P - Q = 10 \qquad ...(1)$$

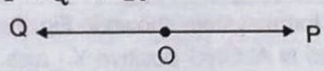
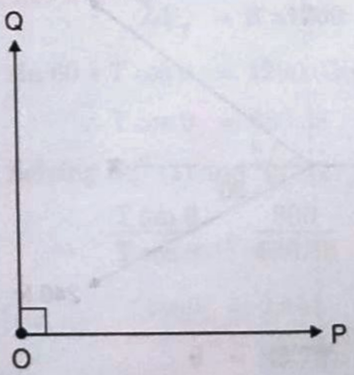


Fig. P. 4.5.11

If they act at right angles to each other, their resultant is 75 N.



$$R = \sqrt{P^2 + Q^2}$$

$$75 = \sqrt{P^2 + Q^2} \quad ...(2)$$

$$P^2 + Q^2 = 75^2$$

From Eq^n (1) $P - Q = 10$

Squaring on both sides,

$$(P - Q)^2 = 10^2$$

$$P^2 + Q^2 - 2PQ = 100$$

$$75^2 - 2PQ = 100 \quad (\because P^2 + Q^2 = 75^2)$$

$$\therefore 2PQ = 5525$$

$$Q = \left(\frac{2762.5}{P}\right)$$

From Eq^n (1), $P - \left(\frac{2762.5}{P}\right) = 10$

$$P^2 - 2762.5 = 10P$$

$$P^2 - 10P - 2762.5 = 0$$

Solving for P,

$$P = \frac{10 \pm \sqrt{10^2 + 4 \times 1 \times 2762.5}}{2 \times 1}$$

$$= \frac{10 \times 105.6}{2}$$

$$= \mathbf{57.8\ N\ or\ -47.8\ N}$$

From Eq^n (1), If $P = 57.8$ N,

$$57.8 - Q = 10$$

$$\therefore Q = \mathbf{47.8\ N}$$

If $P = -47.8$ N,

$$-47.8 - Q = 10$$

$$\therefore Q = \mathbf{-57.8\ N}$$

$\therefore$ Magnitude of two forces are : **57.8 N and 47.8 N**

...Ans.

Ex.4.5.12 : The force system shown in Fig. P. 4.5.12(a) have a resultant of 200 N ALONG positive Y - axis, determine the magnitude and position θ of force F.

SPPU : Dec. 15, 4 Marks

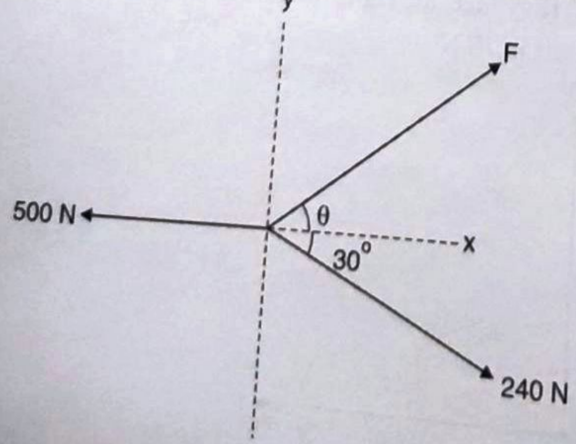


Fig. P. 4.5.12(a)

Soln. :

Given that the resultant of the force system R = 200 N acting upwards.

$$\therefore \Sigma F_x = 0 \text{ and}$$

$$\Sigma F_y = R$$

$$= 200\ N$$

Resolving forces into components along x and y axes and taking algebraic sum;

$$\Sigma F_x = 0$$

$$F \cos\theta - 500 + 240 \cos 30° = 0$$

$$\therefore F \cos\theta = \mathbf{292.15\ N}$$

$$\Sigma F_y = 200$$

$$F \sin\theta - 240 \sin 30° = 0$$

$$\therefore F \sin\theta = \mathbf{120\ N}$$

From Eq^n (1) and Eq^n (2),

$$\frac{F \sin\theta}{F \cos\theta} = \frac{120}{292.15}$$

$$\tan\theta = 0.4107$$

$$\therefore \theta = \mathbf{22.33°}$$

...Ans.

From Eq^n (2),

$$F \sin 22.33 = 120$$

$$\therefore F = \mathbf{315.84\ N}$$

...Ans.

Ex. 4.5.13 : Find the equilibrant of the force system

(i) 60 kN towards East

(ii) 45 kN at 50° East of North

(iii) 75 kN at 35° North of West

(iv) 20 kN at 45° West of South

(v) 100 kN towards South

Soln. :

Let us find out first the resultant of the force system.

The magnitude of equilibrant is same as that of the resultant and its direction is opposite to the resultant.

Taking x - axis along East direction and y-axis along North direction; Resolving each force into its x and y components and taking algebraic sum;

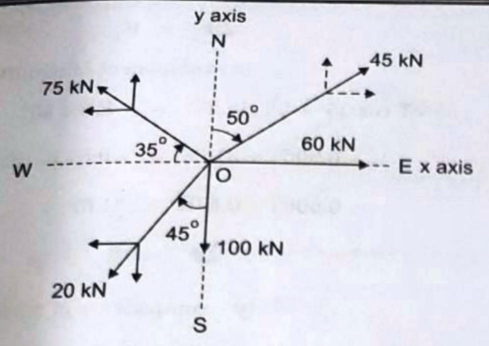


Fig. P. 4.5.13(a)

$$\Sigma F_x = R_x \quad \text{(x - component of resultant)}$$

$$= 60 + 45 \sin 50° - 75 \cos 35° - 20 \sin 45°$$

$$= \mathbf{18.89\ kN} \rightarrow$$

$$\Sigma F_y = R_y \quad \text{(y - component of resultant)}$$

$$= 45 \cos 50° + 75 \sin 35° - 20 \cos 45° - 100$$

$$= -42.2\ kN = \mathbf{42.2\ kN}\downarrow$$

$\therefore$ Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

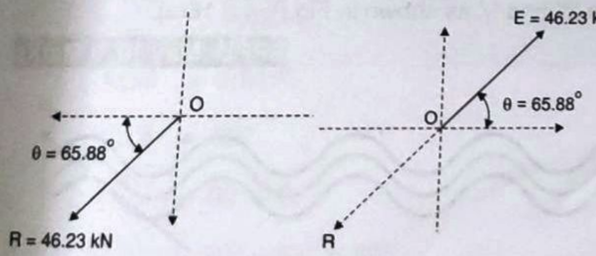
$$= \sqrt{(18.89)^2 + (42.2)^2}$$

$$= \mathbf{46.23 kN}$$

Direction of resultant, $\theta = \tan^{-1}\left|\dfrac{\Sigma F_x}{\Sigma F_y}\right|$

$$= \tan^{-1}\left(\frac{42.2}{18.89}\right)$$

$$= \mathbf{65.88°} \text{ w.r.t. x – axis}$$



$\therefore$ Magnitude of equilibrant = **46.23 kN** ...**Ans.**

Direction is **65.88 w.r.t. x-axis in 1st quadrant...Ans.**

Ex. 4.5.14 : The post is to be pulled out of the ground using two ropes A and B as shown in Fig.P. 4.5.14(a). Rope A is subjected to a force of 600 N and is directed at 60° from the horizontal. If the resultant force acting on the post is be 1200 N vertically upward, determine the force T in rope B and the corresponding angle θ. **SPPU : May 16, 4 Marks**

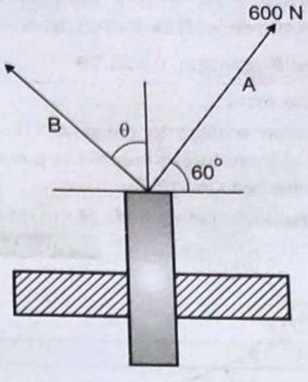


Fig. P. 4.5.14(a)

Soln. :

Given that the resultant force is acting vertically upward i.e., in y-direction.

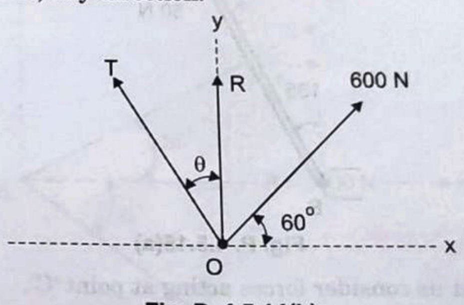


Fig. P. 4.5.14(b)

$$\therefore \Sigma F_x = 0 \text{ and}$$

$$\Sigma F_y = R = 1200N$$

Resolving each force into its x and y components and taking algebraic sum;

$$\therefore \Sigma F_x = 0$$

$$600 \cos 60° - T \sin \theta = 0$$

$$\therefore T \sin \theta = 300 \quad \text{...(1)}$$

$$\Sigma F_y = R = 1200$$

$$600 \sin 60 + T \cos \theta = 1200 \text{ (Given R = 1200 N} \uparrow)$$

$$\therefore T \cos \theta = 680.38 \quad \text{...(2)}$$

Solving Eq^n (1) and Eq^n (2)

$$\frac{T \sin \theta}{T \cos \theta} = \frac{300}{680.38}$$

$$\therefore \tan\theta = 0.441$$

$$\theta = \mathbf{23.79°} \quad \text{...Ans.}$$

From Eq^n (1),

$$T \sin 23.79° = 300$$

$$\therefore T = 743.70 \text{ N} \quad \text{...Ans.}$$

$\therefore$ Tension in the rope, B is **T=743.70 N**

and corresponding angle, **θ =23.79°**

Ex. 4.5.15 :Determine;

(i) The required tension in cable AC, knowing that the resultant of three forces exerted at point C of boom BC must be directed along BC.

(ii) The corresponding magnitude of the resultant.

SPPU :May 04, 8 Marks

Soln. :

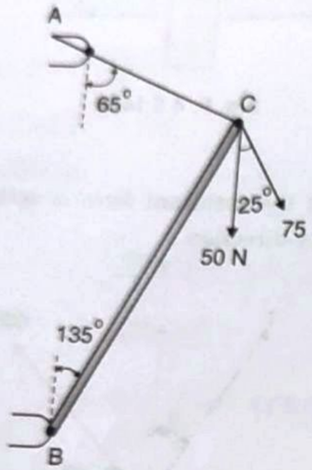


Fig. P. 4.5.15(a)

Let us consider forces acting at point 'C'.

Given that the resultant of the three forces is acting along the line BC.

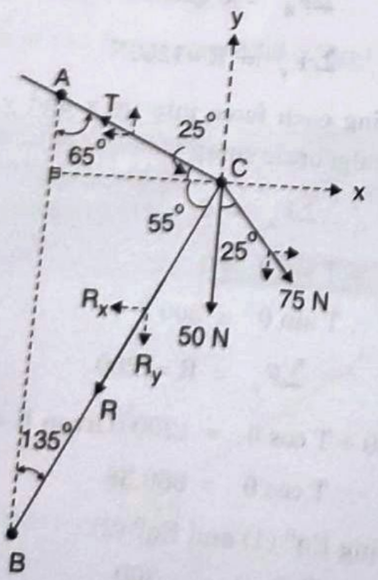


Fig. P. 4.5.15(b)

Considering x and y axes as shown in P. 4.5.15(b) and taking algebraic sum of all x and y ponents;

$$\Sigma F_x = R_x$$

(x - component of result...

$$-T \cos 25° + 75 \sin 25° = -R \cos 55°$$

$$-0.906T + 31.69 = -0.57 R$$

$$\therefore 0.906T - 0.57R = 31.69$$

$$\Sigma F_y = R_y$$

(y – components of result...

$$T \sin 25° - 50 - 75 \cos 25° = -R \sin 55°$$

$$0.42 T + 0.82 R = 118$$

Solving Eq^n (1) and Eq^n (2)

$$(1) \times 0.42 \qquad 0.38 \cancel{T} - 0.24 R = 13.3$$

$$(2) \times 0.906 \qquad 0.\cancel{38} T + 0.74R = 107$$

$$- \qquad - \qquad -$$

$$-0.98R = -93.7$$

$$\therefore \mathbf{R} = \mathbf{95.61 \text{ N}}$$

From Eq^n (1) $0.906 T - 0.57(95.61) = 31.69$

$$\therefore \mathbf{T} = \mathbf{95.13 \text{ N}}$$

$\therefore$ (i) Tension in the cable, **T = 95.13 N ...Ans.**

(ii) Magnitude of resultant, **R = 95.61 N ...Ans.**

Ex. 4.5.16 :A boat is moved uniformly along a canal by two horses pulling with forces P = 890 N and Q = 1068 N acting at an angle α = 60° as shown in Fig.P. 4.5.16(a).

Determine magnitude of the resultant pull on the boat and the angle 'β' and 'γ' as shown in Fig.P. 4.5.16(a).

SPPU :May 10, 8 Marks

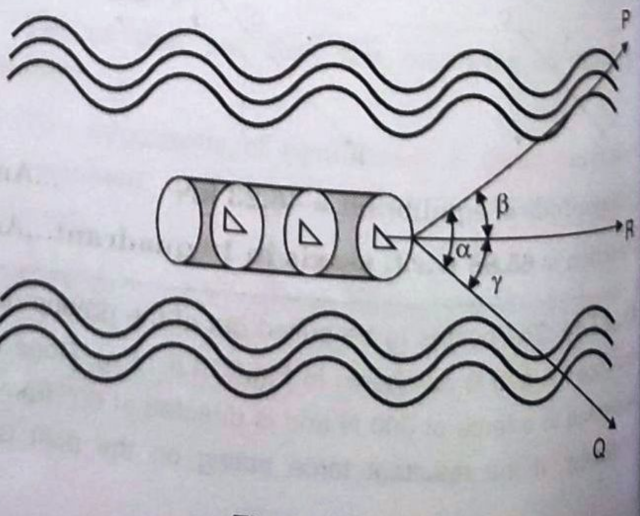


Fig. P. 4.5.16(a)

Soln. :

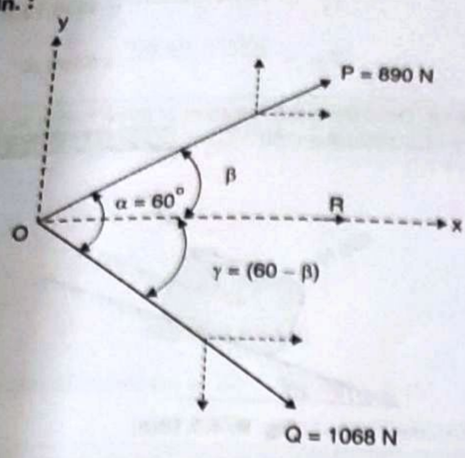


Fig. P. 4.5.16(b)

Here the resultant is acting along x-axis.

$$\therefore \Sigma F_y = 0 \text{ and}$$

$$\Sigma F_x = R$$

Resolving forces P and Q into x and y components and taking algebraic sum ;

$$\Sigma F_y = 0$$

$$890 \sin\beta - 1068 \sin\gamma = 0$$

$$890 \sin\beta = 1068 \sin(60° - \beta)$$

$$890 \sin\beta = 1068[\sin 60° \cos\beta - \cos 60° \sin\beta]$$

$$890 \sin\beta = 924.91 \cos\beta - 534 \sin\beta$$

$$1424 \sin\beta = 924.91 \cos\beta$$

$$\frac{\sin\beta}{\cos\beta} = \frac{924.91}{1424}$$

$$\tan\beta = 0.6495$$

$$\therefore \beta = \mathbf{33°} \qquad \text{...Ans.}$$

$$\gamma = 60° - \beta$$

$$= 60° - 33° = \mathbf{27°} \qquad \text{...Ans.}$$

∴ Magnitude of resultant,

$$R = \Sigma F_x$$

$$= 890 \cos\beta + 1068 \cos\gamma$$

$$= 890 \cos 33° + 1068 \cos 27°$$

$$= \mathbf{1698.01N \rightarrow} \qquad \text{... Ans.}$$

Ex. 4.5.17 : A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 5000 N force directed along the axis of the barge, determine the value of α for which the tension in rope 2 is minimum.

SPPU : May 09, 6 Marks

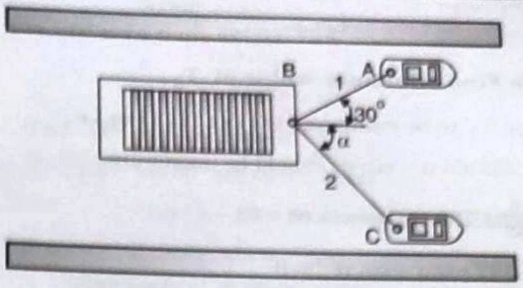


Fig. P. 4.5.17(a)

Soln. :

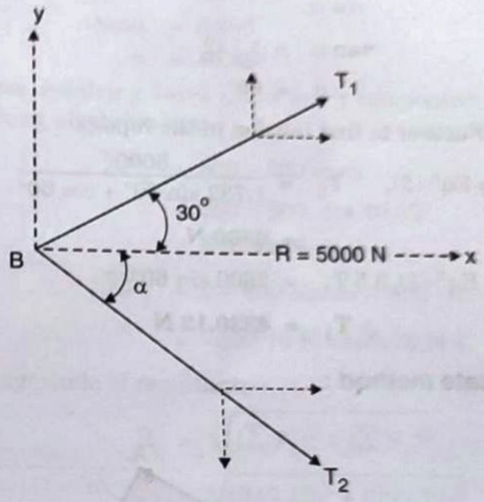


Fig. P. 4.5.17(b)

Here resultant R is acting along x – axis

$$\therefore \Sigma F_y = 0 \text{ and}$$

$$\Sigma F_x = R$$

Resolving T_1 and T_2 into x and y components and taking algebraic sum;

$$\Sigma F_y = 0$$

$$T_1 \sin 30° - T_2 \sin\alpha = 0$$

$$\therefore 0.5\, T_1 = T_2 \sin\alpha \qquad \text{...(1)}$$

$$\Sigma F_x = R = 5000$$

$$T_1 \cos 30° + T_2 \cos\alpha = 5000$$

$$0.866\, T_1 = 5000 - T_2 \cos\alpha \qquad \text{...(2)}$$

From Eq^n (1), $T_1 = 2\, T_2 \sin\alpha$

Putting in Eq^n (2),

$$0.866(2\,T_2 \sin\alpha) = 5000 - T_2 \cos\alpha$$

$$1.732\,T_2 \sin\alpha + T_2 \cos\alpha = 5000$$

$$T_2(1.732 \sin\alpha + \cos\alpha) = 5000$$

$$\therefore T_2 = \frac{5000}{(1.732 \sin\alpha + \cos\alpha)} \qquad ...(3)$$

To Find minimum -value of T_2,

For 'T_2' to be minimum, the denominator i.e., $(1.732 \sin\alpha + \cos\alpha)$ should be maximum.

$$\therefore \frac{d}{dx}(1.732 \sin\alpha + \cos\alpha) = 0$$

$$1.732 \cos\alpha - \sin\alpha = 0$$

$$1.732 \cos\alpha = \sin\alpha$$

$$\frac{\sin\alpha}{\cos\alpha} = 1.732$$

$$\tan\alpha = 1.732$$

$$\therefore \alpha = \mathbf{60^\circ} \qquad \text{...Ans.}$$

Further to find tension in the ropes,

From Eq^n (3), $$T_2 = \frac{5000}{1.732 \sin 60^\circ + \cos 60^\circ}$$

$$= \mathbf{2500\ N}$$

From Eq^n (1), $0.5\,T_1 = 2500 \sin 60^\circ$

$$\therefore \mathbf{T_1 = 4330.12\ N}$$

Alternate method :

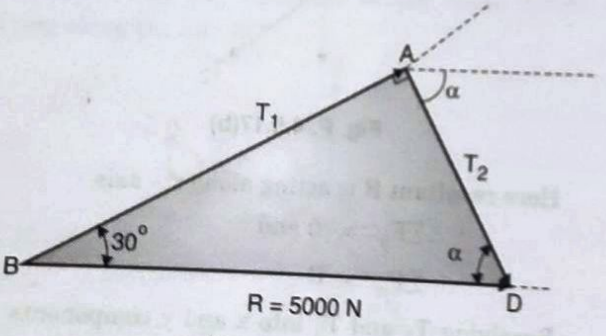


Fig. P. 4.5.17(c)

Using triangle law of forces the direction of T_1 is 30° w.r.t. x. For T_2 to be minimum, direction of T_2 should be perpendicular to T_1, because perpendicular distance is always minimum.

From the Δ ABD, $\alpha = \mathbf{90^\circ - 30^\circ = 60^\circ}$ **...Ans.**

For Δ ABD, using sine rule,

$$\frac{T_1}{\sin 60^\circ} = \frac{T_2}{\sin 30^\circ} = \frac{5000}{\sin 90^\circ}$$

$$\therefore T_1 = \frac{5000 \times \sin 60^\circ}{1} = 4330.12\ N$$

$$T_2 = \frac{5000 \times \sin 30^\circ}{1} = 2500\ N$$

Ex. 4.5.18 : Determine the resultant of three forces as shown in Fig. P. 4.5.18(a) if $\alpha = 50^\circ$. **SPPU-May 16, 6 Marks**

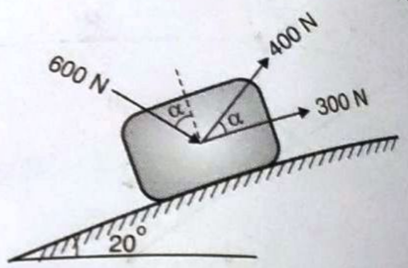


Fig. P. 4.5.18(a)

Soln. :

Let us consider x-axis along the plane and y-axis perpendicular to it.

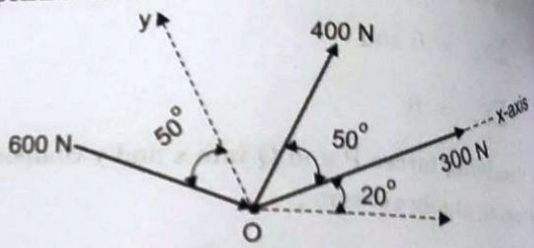


Fig. P. 4.5.18(b)

Resolving each force into x and y components and taking algebraic sum;

$$\Sigma F_x = 300 + 400 \cos 50^\circ + 600 \sin 50^\circ$$

$$= \mathbf{1016.74\ N \rightarrow}$$

$$\Sigma F_y = 0 + 400 \sin 50^\circ - 600 \cos 50^\circ$$

$$= \mathbf{-79.25\ N = 79.25\ N \downarrow}$$

Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(1016.74)^2 + (79.25)^2}$$

$$= \mathbf{1019.82\ N} \qquad \text{...Ans.}$$

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \tan^{-1}\left(\frac{79.25}{1016.74}\right)$$

$$= \mathbf{4.45^\circ}\ \text{w.r.t. x-axis}$$

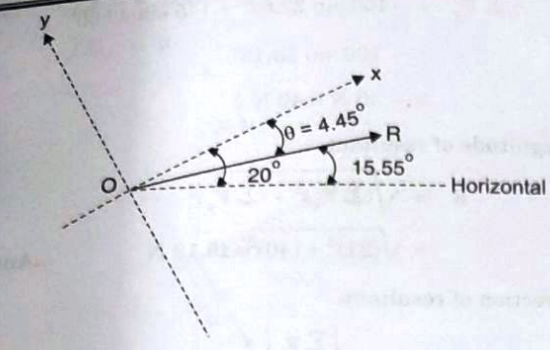


Fig. P. 4.5.18(c)

$\therefore$ Direction of resultant = $20° - \theta = 20° - 4.45°$

$= \mathbf{15.55°}$ **w.r.t. horizontal**

Ex. 4.5.19 : Combine the two forces 800 N and 600 N, which act on the fixed dam structure at B, into a single equivalent force R if AC = 3 m, BC = 6 m and angle BCD = 60°. Refer Fig. P. 4.5.19(a). **SPPU : May 15, 4 Marks**

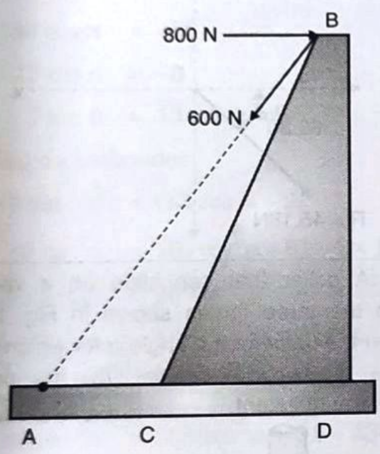


Fig. P. 4.5.19(a)

Soln. :

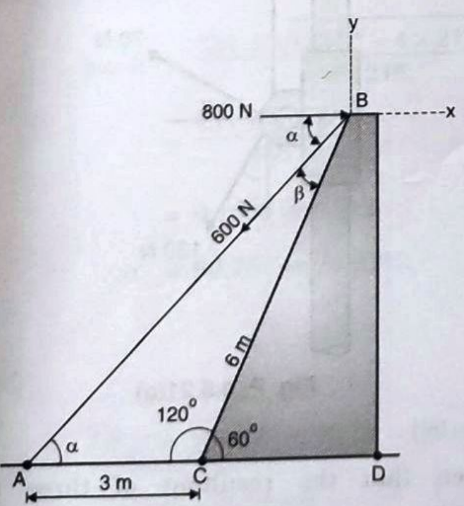


Fig. P. 4.5.19(b)

From Fig. P. 4.5.19(b)

$$\alpha + \beta = 60°$$

$$\therefore \beta = (60° - \alpha)$$

For the Δ ABC; using sine rule,

$$\frac{6}{\sin \alpha} = \frac{3}{\sin \beta}$$

$$\frac{6}{\sin \alpha} = \frac{3}{\sin(60° - \alpha)}$$

$$\therefore 6 \sin (60° - \alpha) = 3 \sin \alpha$$

$$6 [\sin 60° \cos\alpha - \cos 60° \sin \alpha] = 3 \sin \alpha$$

$$[\because \sin (A - B) = \sin A \cos B - \cos A \sin B]$$

$$1.732 \cos\alpha - \sin \alpha = \sin \alpha$$

$$1.732 \cos\alpha = 2 \sin \alpha$$

$$\frac{\sin \alpha}{\cos \alpha} = \frac{1.732}{2}$$

$$\therefore \tan\alpha = 0.866$$

$$\alpha = \mathbf{40.89°}$$

Now, resolving forces into x and y component at 'B' and taking algebraic sum,

$$\Sigma F_x = 800 - 600 \cos\alpha$$

$$= 800 - 600 \cos 40.89°$$

$$= \mathbf{346.42\ N} \rightarrow$$

$$\Sigma F_y = 0 - 600 \sin \alpha = -600 \sin 40.89°$$

$$= -392.76\ N = \mathbf{392.76\ N} \downarrow$$

Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

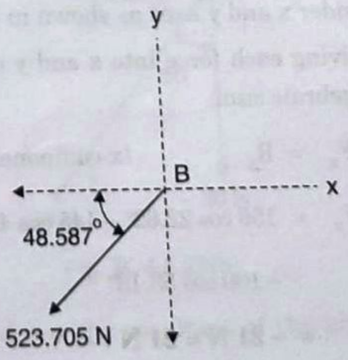
$$= \sqrt{(346.42)^2 + (392.76)^2}$$

$$= \mathbf{523.705\ N} \qquad \text{...Ans.}$$

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \tan^{-1}\left(\frac{392.76}{346.42}\right)$$

$$= \mathbf{48.587° \text{ w.r.t. x-axis}} \qquad \text{...Ans.}$$



Ex.4.5.20 : Knowing that the tension in the cable BC is 145 N, determine the resultant of the three forces exerted at point 'B' of beam AB. Refer Fig. P. 4.5.20(a).

SPPU : Dec. 11, 9 Marks

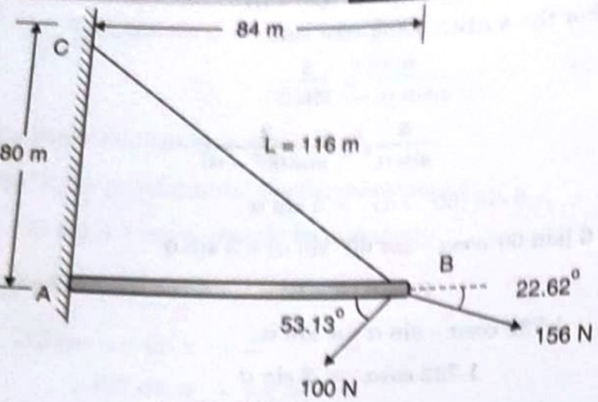


Fig. P. 4.5.20(a)

Soln. :

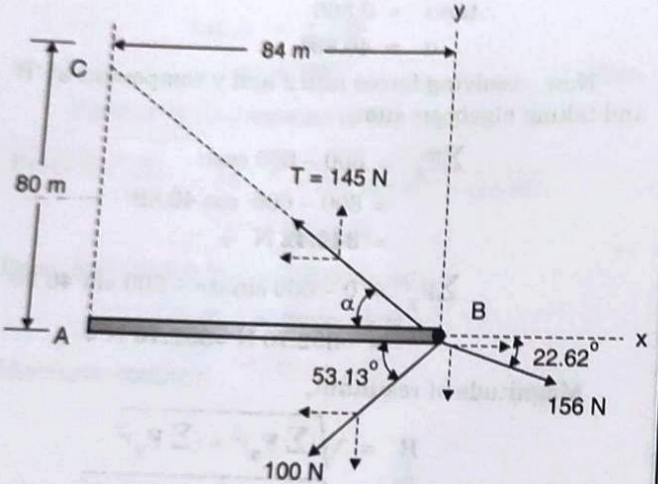


Fig. P. 4.5.20(b)

From Fig. P. 4.5.20(b) for Δ ABC,

$$\tan\alpha = \left(\frac{80}{84}\right)$$

$$\therefore \alpha = \tan^{-1}\left(\frac{80}{84}\right) = 43.60°$$

Consider x and y axes as shown in Fig. P. 4.5.20(b)

Resolving each force into x and y components and taking algebraic sum;

$$\Sigma F_x = R_x \quad \text{(x-component of resultant)}$$

$$\Sigma F_x = 156 \cos 22.62° - 145 \cos 43.60° - 100 \cos 53.13°$$

$$= -21\ N = 21\ N \leftarrow$$

$$\Sigma F_y = R_y \quad \text{(y-component of resultant)}$$

$$\Sigma F_y = -156 \sin 22.62° + 145 \sin 43.60° - 100 \sin 53.13°$$

$$= -40\ N = 40\ N \downarrow$$

Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(21)^2 + (40)^2} = \mathbf{45.18\ N}$$

...Ans.

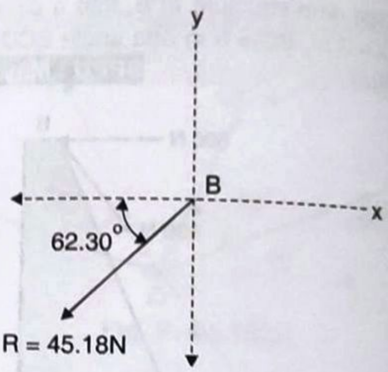
Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$

$$= \tan^{-1}\left(\frac{40}{21}\right)$$

$$= \mathbf{62.30° \text{ w.r.t. x-axis}}$$

...Ans.



Ex. 4.5.21 : A collar that can slide on a vertical rod is subjected to the three forces shown in Fig. P. 4.5.21(a). Determine (a) the value of the angle α for which the resultant of the three forces is horizontal, (b) the corresponding magnitude of the resultant.

SPPU : May 09, 6 Marks

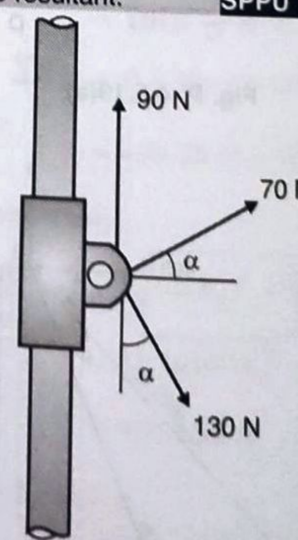


Fig. P. 4.5.21(a)

Soln. :

Given that the resultant of three forces is horizontal

$\therefore \Sigma F_x = R$ and

$\Sigma F_y = 0$

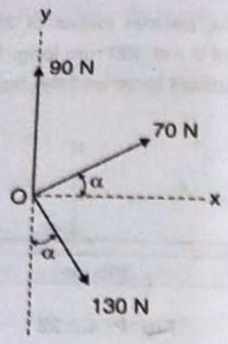


Fig. P. 4.5.21(b)

Resolving each force into x and y components and taking algebraic sum;

$$\Sigma F_y = 0$$

$$90 + 70 \sin \alpha - 130 \cos \alpha = 0$$

$$70 \sin \alpha - 130 \cos \alpha = -90$$

$$7 \sin \alpha - 13 \cos \alpha = -9$$

$$7 \sin \alpha = 13 \cos \alpha - 9$$

Squaring on both sides

$$(7 \sin \alpha)^2 = (13 \cos \alpha - 9)^2$$

$$49 \sin^2\alpha = 169 \cos^2\alpha + 81 - 2 \times 13 \times 9 \cos \alpha$$

$$49 \sin^2\alpha = 169 \cos^2\alpha + 81 - 234 \cos \alpha$$

$$49(1 - \cos^2\alpha) = 169 \cos^2\alpha + 81 - 234 \cos \alpha$$

$$(\because \sin^2\alpha = 1 - \cos^2\alpha)$$

$$49 - 49 \cos^2\alpha = 169 \cos^2\alpha + 81 - 234 \cos \alpha$$

$$\therefore 218 \cos^2\alpha - 234 \cos \alpha + 32 = 0$$

Solving for cos α,

$$\cos \alpha = \frac{234 \pm \sqrt{(234)^2 - 4 \times 218 \times 32}}{2 \times 218}$$

$$= \frac{234 \pm 163.86}{436}$$

$$= 0.1608 \text{ or } 0.912$$

$$\therefore \alpha = \mathbf{80.75° \text{ or } 24.21°}$$

Check :

If α = 80.75°,

$$\Sigma F_y = 90 + 70 \sin 80.75° - 130 \cos 80.75°$$

$$= 138.19 \neq 0$$

If α = 24.14

$$\Sigma F_y = 90 + 70 \sin 24.14° - 130 \cos 24.14° = 0$$

$$\therefore \alpha = \mathbf{24.21°} \quad \text{...Ans.}$$

$$\Sigma F_x = R$$

$$\therefore R = 70 \cos 24.14° + 130 \sin 24.14°$$

$$\therefore \mathbf{R = 117.04 \text{ N} \rightarrow} \quad \text{...Ans.}$$

Ex. 4.5.22 : Three forces are applied to the bracket as shown in Fig. P.4.5.22(a). Determine and show the equilibrant force for α = 40° if the angle between two 30N forces always remain 50°. **SPPU : Dec. 09, 6 Marks**

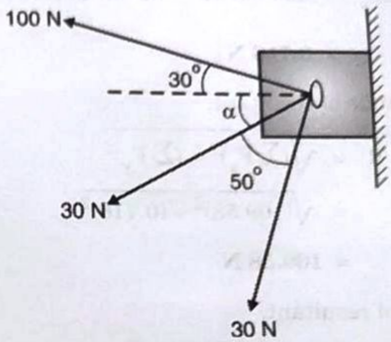


Fig. P. 4.5.22(a)

Soln. :

The magnitude of equilibrant force is same as that of the resultant force but its direction is opposite to the direction of resultant force.

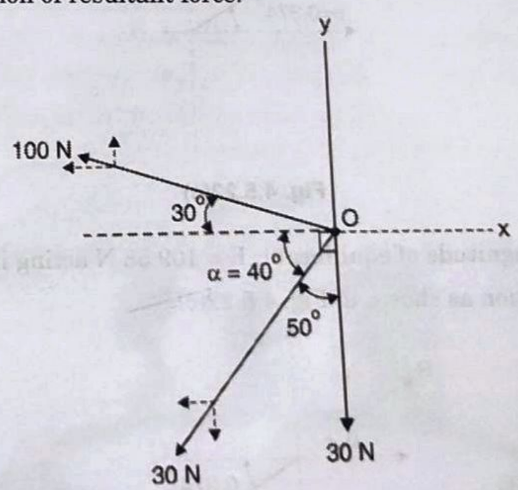


Fig. P. 4.5.22(b)

Let us find out the resultant of the given system first.

Considering x and y axes as shown in Fig. P. 4.5.22(b)

Resolving each force into its x and y components and taking algebraic sum;

$\Sigma F_x = R_x$ (x- component of resultant)

$$\Sigma F_x = -100 \cos 30° - 30 \cos 40°$$
$$= -109.58 \text{ N}$$
$$= \mathbf{109.58 \text{ N}} \leftarrow$$

$\Sigma F_y = R_y$ (y-component of resultant)

$$\Sigma F_y = 100 \sin 30° - 30 \sin 40° - 30$$
$$= \mathbf{0.716 \text{ N}} \uparrow$$

$\therefore$ Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$= \sqrt{(109.58)^2 + (0.716)^2}$$
$$= \mathbf{109.58 \text{ N}}$$

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{0.716}{109.58}\right|$$
$$= \mathbf{0.374° \text{ w.r.t. x-axis}}$$

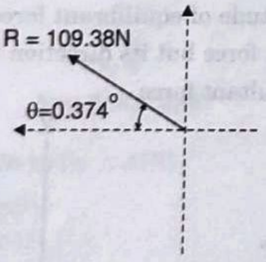


Fig. 4.5.22(c)

$\therefore$ Magnitude of equilibrant, E = 109.58 N acting in the direction as shown in Fig. 4.5.22(c)

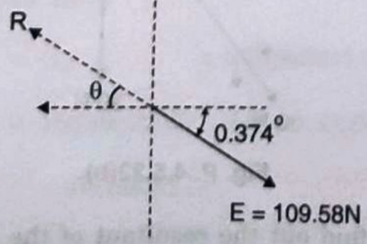


Fig. 4.5.22(d)

Type 2 : Resultant of Parallel Force System

Ex. 4.5.23 : Two like parallel forces of 25 N and 60 N acting at the ends of a rod 300 mm long. Find the magnitude and direction of resultant force and the point where it acts.

Soln. :

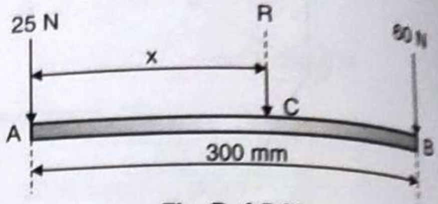


Fig. P. 4.5.23

Let the forces be act in y-direction

$$\Sigma F_y = R$$
$$\therefore R = -25 - 60$$
$$= -85 \text{ N}$$
$$= \mathbf{85 \text{ N}} \downarrow$$

...Ans.

Let the resultant acts at a distance 'x' mm from point 'A'.

Taking moments at 'A' and using varignon's theorem;

$$\Sigma M_A = \text{Moment of R at 'A'}$$
$$25 \times 0 - 60 \times 300 = -R \times x$$
$$-60 \times 300 = -85 \times x$$
$$\therefore \mathbf{x = 211.76 \text{ mm}}$$

...Ans.

Ex. 4.5.24 : Two unlike parallel forces of magnitude 450 N and 150 N are acting in such a way that their lines of action are 250 mm apart. Determine the magnitude of resultant force and the point where it acts.

Soln. :

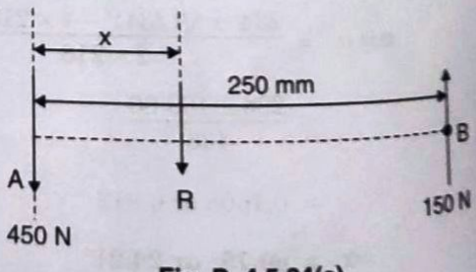


Fig. P. 4.5.24(a)

Let the forces be act in y-direction.

$$\therefore \Sigma F_y = R$$
$$R = -450 + 150$$
$$= -300 \text{ N} = \mathbf{300 \text{ N}} \downarrow$$

...Ans.

Let the position of resultant be at a distance 'x' ...n 450 N force.

Taking moments at 'A' and using varignon's ...orem

ΣM_A = Moment of 'R' at 'A'

$150 \times 250 = -R \times x$

$...50 \times 250 = -300 \times x$

$\therefore x = -125$ mm

= **125 mm to the left of 'A'** ...**Ans.**

$\therefore$ Resultant is acting at a distance of 125 mm to ... left of 'A'.

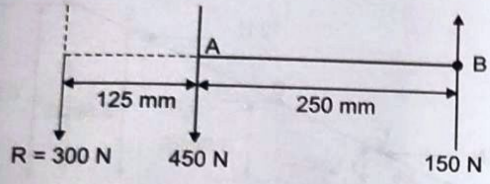


Fig. P. 4.5.24(b)

...4.5.25 : A force and a couple are applied to a beam as ...own in Fig. P. 4.5.25(a). Replace this system with a single ...rce applied at 'G' and determine distance 'd'.

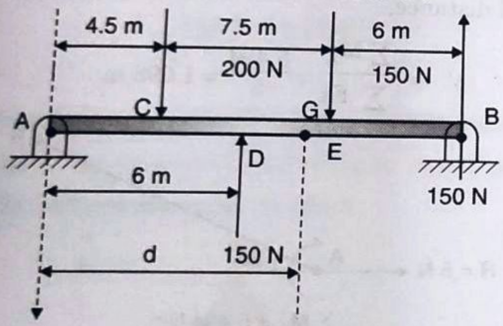


Fig. P. 4.5.25(a)

...oln. :

Let us first replace the given system to a single ...orce and single moment at point 'A'.

Single force is nothing but resultant force and the ...ingle moment is resultant moment.

As all forces are acting in y-direction, the ...esultant force is,

$$R = \Sigma F_y$$

$$= -200 - 150 + 150$$

$$= -200 \text{ N} = \mathbf{200\ N} \downarrow$$

Resultant moment at A is,

$$\Sigma M_A = -200 \times 4.5 - 150 \times 12 + 150 \times 6$$

$$= -1800 \text{ Nm}$$

$$= \mathbf{1800\ Nm} \circlearrowleft \text{ (ACW moment)}$$

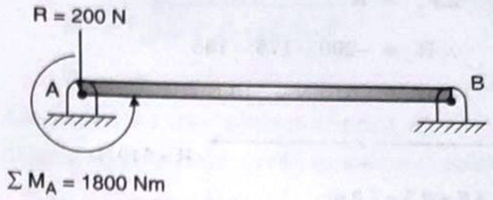


Fig. P. 4.5.25(b)

When force and couple are added, force will displace parallelly by a distance,

$$d = \left| \frac{\Sigma M_A}{R} \right|$$

$$= \frac{1800}{200} = 9 \text{ m}$$ ...**Ans.**

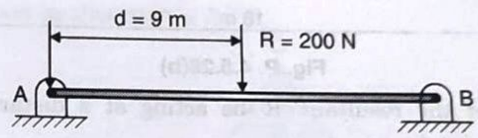


Fig. P. 4.5.25(c)

$\therefore$ The resultant (single) force of the given system is 200 N acting downwards at a distance 9 m from point 'A'.

Ex. 4.5.26 : Three hikers are shown crossing footbridge. Knowing that the weights of the hikers at points C, D and E are 200N, 175N and 135N, respectively, determine the horizontal distance from 'A' to the line of action of the resultant of the three weights when a = 3.3 m.

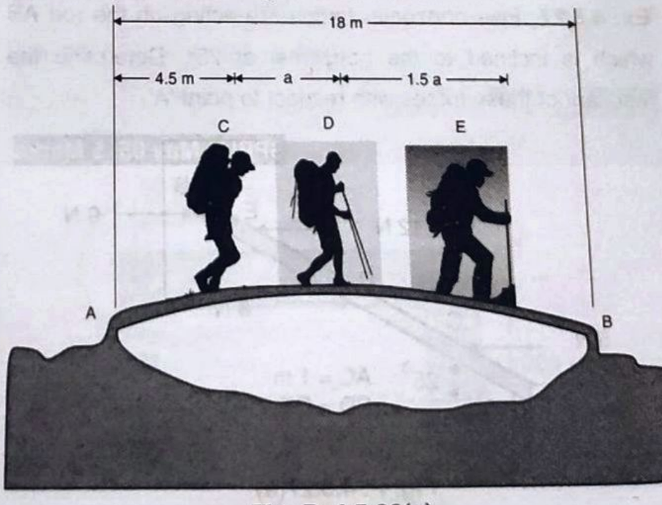


Fig. P. 4.5.26(a)

Soln. :

Being all forces are acting vertically down, i.e. in y-direction.

$$\Sigma F_y = R$$

$$\therefore R = -200 - 175 - 135$$

$$= -510 \text{ N} = 510 \text{ N} \downarrow$$

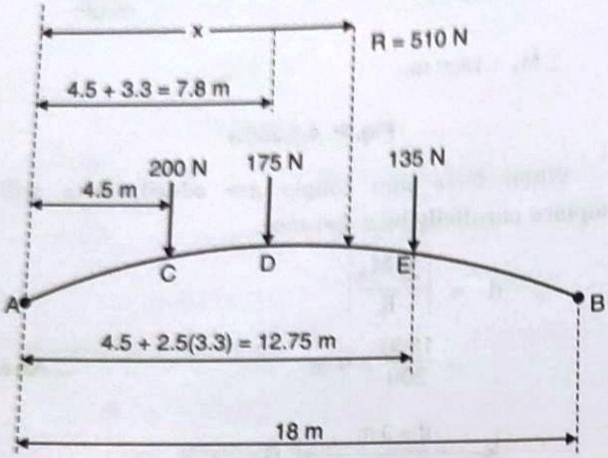


Fig. P. 4.5.26(b)

Let the resultant 'R' be acting at a distance 'x' from point A.

Taking moments at point A and wing varignon's theorem,

$$\Sigma M_A = \text{Moment of 'R' at 'A'}$$

$$-200 \times 4.5 - 175 \times 7.8 - 135 \times 12.75 = -510 \times x$$

$$\therefore \mathbf{x = 7.816 \text{ m}} \quad \text{...Ans.}$$

$\therefore$The line of action of resultant of the three weights act a horizontal distance **x = 7.816** m from 'A'.

Ex. 4.5.27 : Five horizontal forces are acting on the rod AB which is inclined to the horizontal at 25°. Determine the resultant of these forces with respect to point 'A'.

SPPU : May 05, 5 Marks

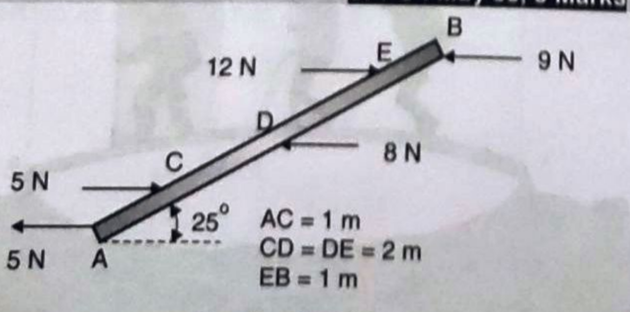


Fig. P. 4.5.27(a)

Soln. :

Here all forces are acting in x - direction.

$$\therefore R = \Sigma F_x$$

$$R = -5 + 5 - 8 + 12 - 9$$

$$\therefore R = -5 \text{ N} = 5 \text{ N} \leftarrow$$

Taking moments at point 'A',

Resultant moment,

$$\Sigma M_A = -5 \times 1 \sin 25^\circ + 8 \times 3 \sin 25^\circ - 12 \times 5 \sin 25^\circ + 9 \times 6 \sin 25^\circ$$

$$= 5.494 \text{ Nm} \circlearrowleft \text{ (ACW moment)}$$

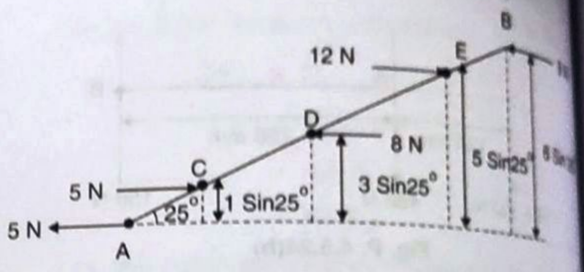


Fig. P. 4.5.27(b)

With respect to point 'A', resultant will act at vertical distance,

$$y = \frac{\Sigma M_A}{\Sigma Fx} = \frac{5.494}{5} = 1.098 \text{ m}$$

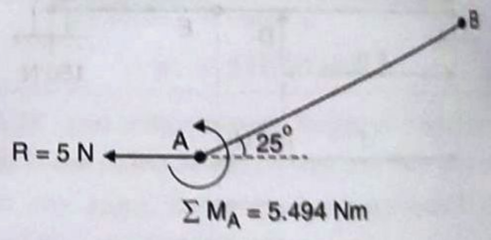


Fig. P. 4.5.27(c)

Let the line of action of resultant intersects rod AB at a distance x from A.

From Δ AFG;

$$\sin 25^\circ = \frac{1.098}{x}$$

$$\therefore \mathbf{x = 2.6 \text{ m}}$$

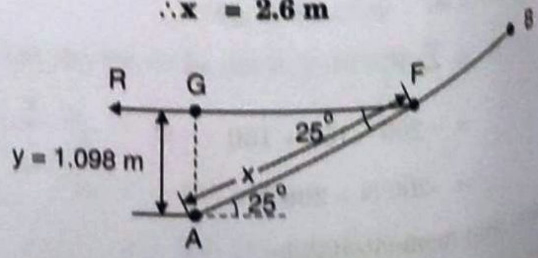


Fig. P. 4.5.27(d)

Resultant will act horizontally at a distance **x = 2.6 m** with respect to point A, along the rod AB

...Ans.

Type 3 : Resultant of non-concurrent and non parallel (General) Force System

Ex. 4.5.28 : Three forces of magnitude 100 N each are acting along the sides of an equilateral triangle as shown in Fig. P. 4.5.28(a). Determine resultant in magnitude and direction with reference to point A.

SPPU : Dec. 09, 6 Marks

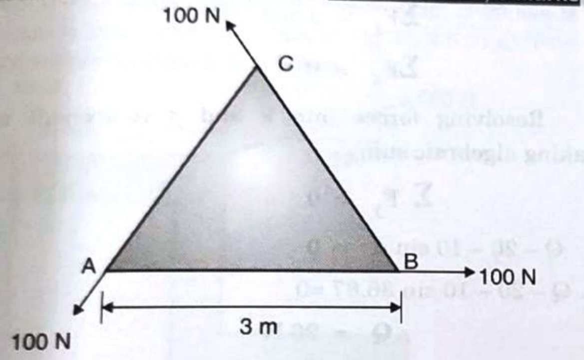


Fig. P. 4.5.28(a)

Soln. :

By principle of transmissibility, 100 N force acting at point 'C' can be transferred along its line of action to point 'B' without changing its effect.

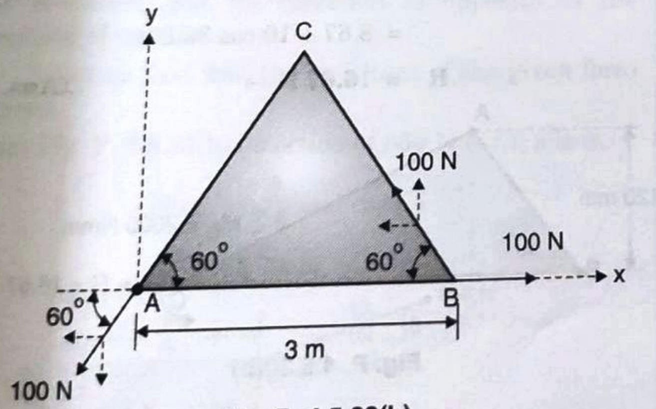


Fig. P. 4.5.28(b)

Taking x and y axes as shown in Fig. P. 4.5.28(b).

Resolving each force into its x and y components and taking algebraic sum;

$$\Sigma F_x = R_x \qquad \text{(x-component of resultant)}$$

$$\Sigma F_x = 100 - 100 \cos 60° - 100 \cos 60° = \mathbf{0}$$

$$\Sigma F_y = R_y \qquad \text{(y-component of resultant)}$$

$$\Sigma F_y = 0 + 100 \sin 60° - 100 \sin 60° = \mathbf{0}$$

∴ Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\therefore \mathbf{R = 0} \qquad \text{...Ans.}$$

Let us find out the moments at point 'A',

Taking algebraic sum of the moments at point 'A',

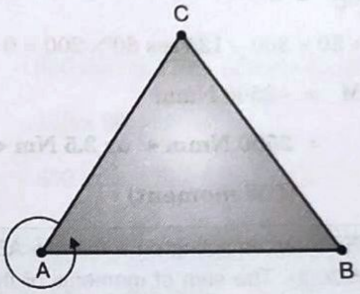
$$\Sigma M_A = 100 \sin 60° \times 3 = \mathbf{259.81 Nm}$$

Note : Moments of 100 N force acting at points A and B and x - component of 100 N acting at B are zero as their lines of action are passing through point 'A'.

∴ The magnitude of resultant w.r.t. 'A' is

$$\Sigma M_A = \mathbf{259.81\ Nm} \qquad \text{...Ans.}$$

and its direction is ACW.



$\Sigma M_A = 259.81$ Nm

Fig. P. 4.5.28(c)

Ex. 4.5.29 : Three forces as shown and a couple 'M' are applied to an angle bracket. Determine the moment of the couple 'M' if the line of action of the resultant of this force system is to pass through point (i) B (ii) C.

SPPU : Dec. 98, 8 Marks

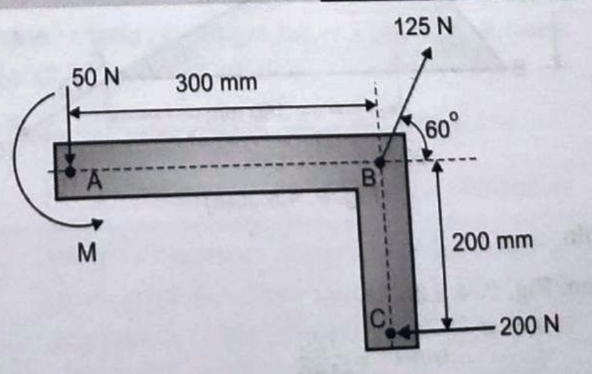


Fig. P. 4.5.29(a)

Soln. :

(i) Line of action of resultant is to pass through point 'B' :

As the line of action of resultant is to pass through point 'B', its moment at B is zero.

$\therefore$ Taking moments at B and equated to zero (as per Varignon's theorem)

$$\Sigma M_B = 0$$

$M + 50 \times 300 - 200 \times 200 = 0$ (moment of 125 N at B = 0)

Note : Moment of couple about any point is always constant

$$M = 25000 \text{ Nmm} = 25 \text{ Nm}$$

(ACW moment) ...**Ans.**

(ii) Line of action of resultant is to pass through point 'C' :

$$\Sigma M_C = 0$$

$$M + 50 \times 300 - 125 \cos 60° \times 200 = 0$$

$$\therefore M = -2500 \text{ Nmm}$$

$$= 2500 \text{ Nmm or } 2.5 \text{ Nm}$$

(CW moment) ...**Ans.**

Ex. 4.5.30 : Four forces acting on a triangle ABC are shown in Fig. P. 4.5.30(a). The sum of moments of these forces at point C is 200 N-mm clockwise. If resultant of force system is in horizontal direction, find its magnitude and point of application. **SPPU : May 08, 10 Marks**

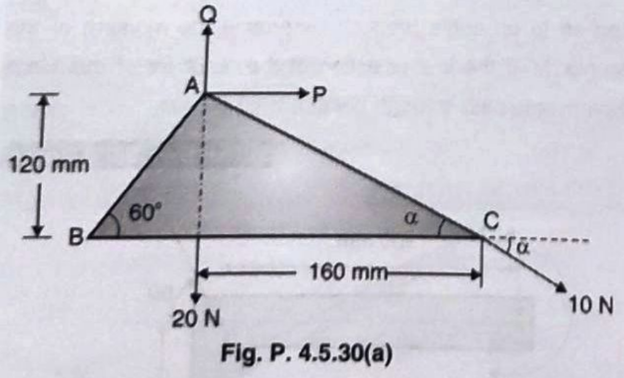


Fig. P. 4.5.30(a)

Soln. :

From Fig. P. 4.5.30(a)

$$\tan\alpha = \frac{120}{160}$$

$$\therefore \alpha = \mathbf{36.87°}$$

Given, sum of moments of four forces about point 'C' is 200 Nmm clockwise.

$$\therefore \Sigma M_C = -2000 \quad \text{(CW moment is -ve)}$$

Taking moment at point 'C'.

$-P \times 120 - Q \times 160 + 20 \times 160 = -2000$ (moment of 10 N force at C is zero)

$$\therefore 120 P + 160 Q = 5200$$

$$\therefore 12 P + 16 Q = 520 \quad ...(1)$$

Also, given that the resultant of force system is in horizontal direction.

$$\therefore \Sigma F_y = 0 \text{ and}$$

$$\Sigma F_x = R$$

Resolving forces into x and y component and taking algebraic sum;

$$\Sigma F_y = 0$$

$$Q - 20 - 10 \sin \alpha = 0$$

$$\therefore Q - 20 - 10 \sin 36.87 = 0$$

$$\therefore \mathbf{Q = 26 \text{ N}}$$

From Eq^n (1),

$$12P + 16(26) = 520$$

$$\therefore \mathbf{P = 8.67 \text{ N}}$$

$$R = \Sigma F_x$$

$$\therefore R = P + 10 \cos \alpha$$

$$= 8.67 + 10 \cos 36.87°$$

$$\therefore \mathbf{R = 16.67 \text{ N} \rightarrow} \quad ...\text{Ans.}$$

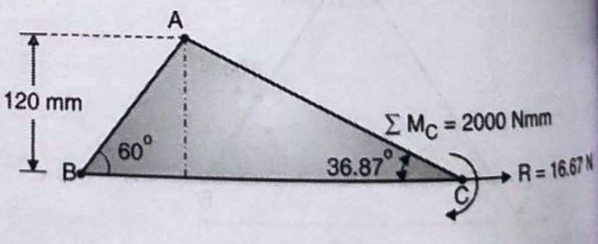


Fig. P. 4.5.30(b)

To find point of application of resultant, add ΣM_C and R when moment and force are added, force will be displaced parallelly by a distance, $d = \frac{\text{Moment}}{\text{Force}}$

$$\therefore d = \frac{\Sigma M_C}{R} = \frac{200}{16.67} = 120 \text{ mm}$$

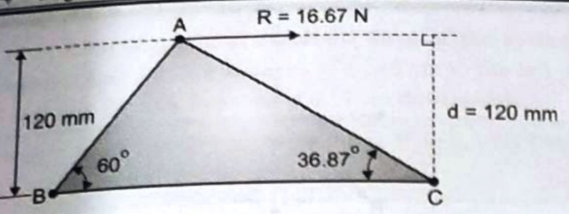


Fig. P. 4.5.30(c)

Resultant will be displaced upwards by 120 mm because its moment at C is having CW sense.

$\therefore$ Point of application of resultant is 'A' acting horizontally towards right. **...Ans.**

Ex. 4.5.31 : A Z-shaped lamina of uniform width of 20 mm is subjected to four forces as shown in Fig. P. 4.5.31(a). Find equilibrant in magnitude and direction.

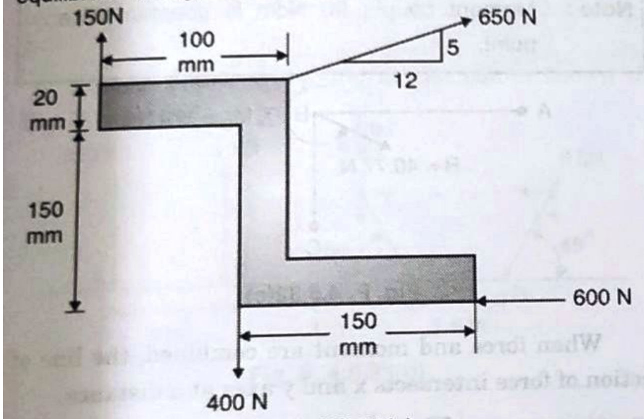


Fig. P. 4.5.31(a)

Soln. : The magnitude of equilibrant is same as that of the resultant, but its direction is opposite to the direction of resultant.

$\therefore$ Let us first find the resultant of the given force system.

From Fig. P. 4.5.31(b) direction of 650 N w.r.t. x is α.

$$\therefore \sin\alpha = \frac{5}{13} \text{ and } \cos\alpha = \frac{12}{13}$$

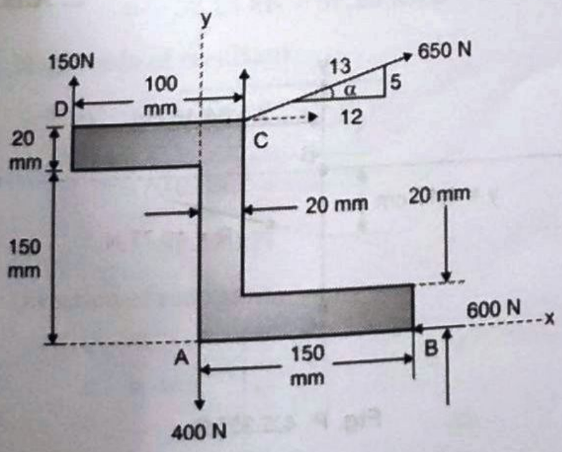


Fig. P. 4.5.31(b)

With respect to point 'A', taking x-y axes as shown in Fig. P. 4.5.31(b) and resolving each force into x and y components.

$$\Sigma F_x = R_x \text{ (x-component of resultant)}$$

$$\Sigma F_x = 650 \cos\alpha - 600$$

$$= 650 \times \frac{12}{13} - 600$$

$$= \mathbf{0}$$

$$\Sigma F_y = R_y \text{ (y-component of resultant)}$$

$$\Sigma F_y = 650 \sin\alpha + 150 - 400$$

$$= 650 \times \frac{5}{13} + 150 - 400 = \mathbf{0}$$

$\therefore$ Magnitude of resultant force,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \mathbf{0}$$

Let us find out resultant moment at point 'A'.

Taking algebraic sum of the moments of all forces at point 'A'.

$$\Sigma M_A = -650 \cos\alpha \times 170 + 650 \sin\alpha \times 20 - 150 \times 80$$

$$= -650 \times \frac{12}{13} \times 170 + 650 \frac{5}{13} \times 20 - 150 \times 80$$

$$= -109000 \text{ Nmm}$$

$\therefore$ Resultant moment = – 109 Nm

= 109 Nm (ACW moment)

Equilibrant is a moment of magnitude **109 Nm** acting in **ACW direction**.

$\therefore$ Equilibrant moment = **109 Nm** (ACW moment)

OR

Taking algebraic sum of the moments of all forces at point 'C'.

$$\Sigma M_C = -150 \times 100 + 400 \times 20 - 600 \times 170$$

$$= -10900 \text{ Nmm} = -109 \text{ Nm} = 109 \text{ Nm}$$

Note : Being the resultant force is zero, the resultant moment of the force system remains same irrespective of the point about which moments are taken.

∴ To find resultant moment, any convenient point can be selected while taking moments.

Ex. 4.5.32 : Three forces and a couple are applied to an angle bracket.

(a) Find the resultant of this system of forces.

(b) Locate the points where the line of action of resultant intersects line AB and line BC

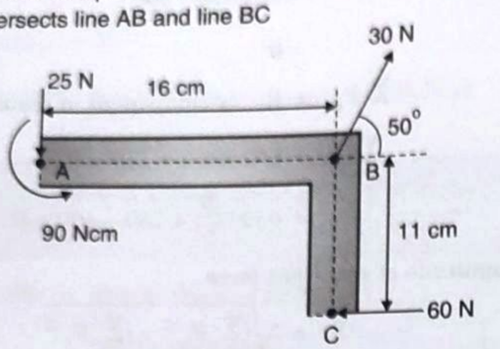


Fig. P. 4.5.32(a)

Soln. :

Resolving forces into x and y components and taking algebraic sum;

$\sum F_x = R_x$ (x - component of resultant)

$= 30\cos 50° - 60 = -40.72N$

$= \mathbf{40.72N \leftarrow}$

$\sum F_y = R_y$ (y - component of resultant)

$= -25 + 30\sin 50°$

$= -2.02$

$= \mathbf{2.02N \downarrow}$

Note : Resultant force of the couple 90 Ncm in any direction is zero

Magnitude of resultant force,

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$= \sqrt{(40.72)^2 + (2.02)^2}$$

$$= \mathbf{40.77N}$$ ...Ans.

Direction of resultant force,

$$\theta = \tan^{-1}\left|\frac{\sum F_y}{\sum F_x}\right|$$

$$= \tan^{-1}\left(\frac{2.02}{40.72}\right)$$

$$= \mathbf{2.84° \text{ w.r.t. x-axis}}$$ ...Ans.

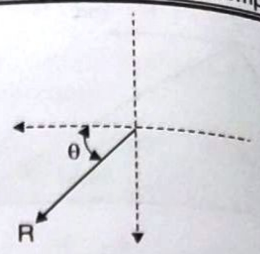


Fig. P. 4.5.32(b)

Taking moments at 'B',

Resultant moment,

$$\sum M_B = 90 + 25 \times 16 - 60 \times 11$$

$$= -170 \text{ Ncm} = \mathbf{170 \text{ Ncm}} \text{ (CW moment)}$$

Note : Moment couple 90 Ncm is constant at any point.

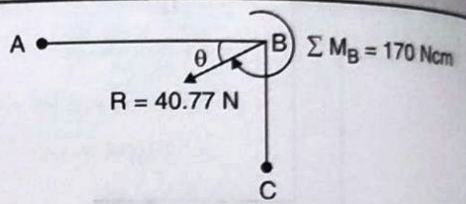


Fig. P. 4.5.32(c)

When force and moment are combined, the line of action of force intersects x and y axes at a distance,

$$x = \left|\frac{\sum M_B}{\sum F_y}\right| = \frac{170}{2.02}$$

$$= \mathbf{84.16 \text{ cm}}$$ ...Ans.

$$y = \left|\frac{\sum M_B}{\sum F_x}\right| = \frac{170}{40.72}$$

$$= \mathbf{4.17 \text{ cm}}$$ ...Ans.

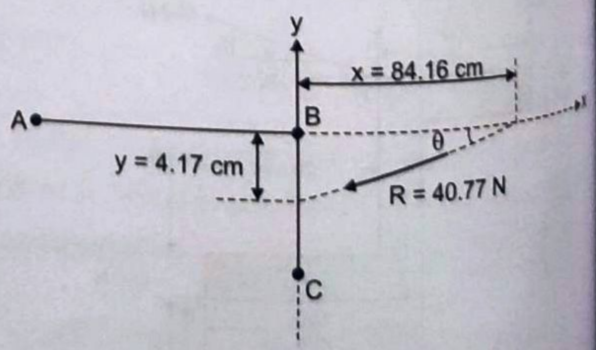


Fig. P. 4.5.32(d)

∴ Line of action of resultant force of the system interests line AB at a distance of 84.16 cm to the left of B and line BC at a distance of 4.17 cm downwards.

Note : The moment of resultant at B is having CW sense.

Ex. 4.5.33 : Find the magnitude, direction and line of action of the resultant of the plane force system shown in Fig. P. 4.5.33(a). **SPPU : Nov. 04, 8 Marks**

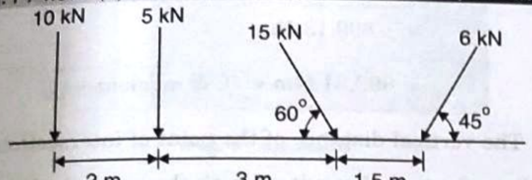


Fig. P. 4.5.33(a)

Soln. :

Let the forces are acting at the points shown in Fig. P. 4.5.33(b)

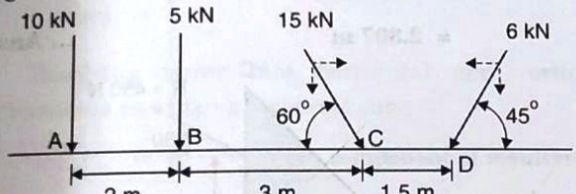


Fig. P. 4.5.33(b)

Resolving each force into x and y components and taking algebraic sum;

ΣF_x = Rx (x - components of resultant)

$\Sigma F_x = 15\cos 60° - 6\cos 45° = \mathbf{3.26\ kN \rightarrow}$

ΣF_y = R_y (y - component of resultant)

$\Sigma F_y = -10 - 5 - 15\sin 60° - 6\sin 45°$

$= -32.23\ kN = \mathbf{32.23\ kN\downarrow}$

Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(3.26)^2 + (32.23)^2}$$

$$= \mathbf{32.40\ kN} \quad \textbf{... Ans.}$$

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$

$$= \tan^{-1}\left(\frac{32.23}{3.26}\right)$$

$$= \mathbf{84.22°\ w.r.t.\ x-axis} \quad \textbf{... Ans.}$$

To find line of action of resultant,

Taking algebraic sum of the moments of all forces at point 'A',

Resultant moment,

$$\Sigma M_A = -5 \times 2 - 15\sin 60° \times 5 - 6\sin 45° \times 6.5$$

$$= -10 - 64.95 - 27.57$$

$$= -102.53\ kNm$$

$$= \mathbf{102.53\ kNm} \text{ (CW moment)}$$

The given system can be replaced with single force (Resultant force) and single moment (Resultant moment) at point 'A' as shown in Fig.

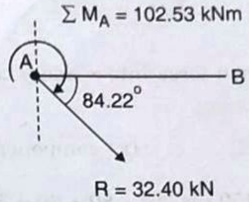


Fig. P. 4.5.33(c)

When the resultant force and resultant moment are combined, resultant force will displaced horizontally by a distance,

$$x = \left|\frac{\Sigma M_A}{\Sigma F_y}\right|$$

$$= \left|\frac{102.53}{32.23}\right|$$

$$= \mathbf{3.18\ m} \quad \textbf{... Ans.}$$

Resultant will be displaced towards right as it forms CW moment about 'A'.

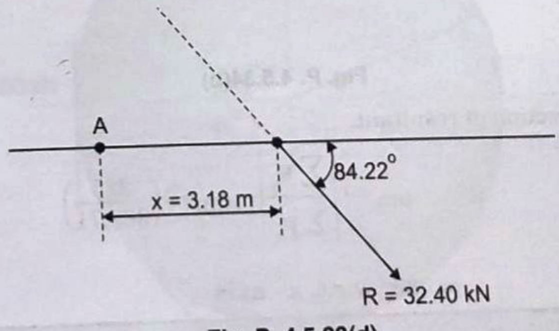


Fig. P. 4.5.33(d)

∴ The line of action of resultant is at a distance x = 3.18 m horizontally towards right from point 'A'.

Ex. 4.5.34 : Replace the force system acting on a triangular plate ABC as shown in Fig. P. 4.5.34(a) by a single resultant force. Give the point of intersection of the line of action of this force with vertical edge BC of the plate.

SPPU : April 92, 6 Marks

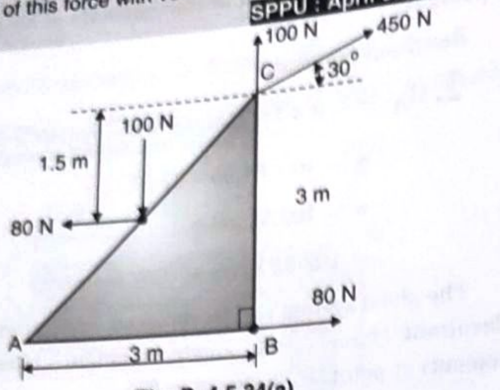


Fig. P. 4.5.34(a)

Soln. :

Resolving each force into x and y components and taking algebraic sum;

$\Sigma Fx = R_x$ (x - component of resultant)

$\Sigma Fx = 450 \cos 30° - 80 + 80 = 389.71 N \rightarrow$

$\Sigma Fy = R_y$ (y - components of resultant)

$\Sigma Fy = 450 \sin 30° + 100 - 100 = 225 N \uparrow$

Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(389.71)^2 + (225)^2} = \mathbf{450\ N}$$

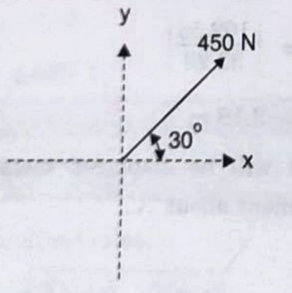


Fig. P. 4.5.34(b)

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \tan^{-1}\left(\frac{225}{389.71}\right)$$

$$= \mathbf{30° \text{ w.r.t. x - axis}}$$

Note : Two vertical 100 N forces and two horizontal forces of 80 N are forming couples. The resultant force of couple in any direction is zero.

∴ The resultant force of the given system is force 450 N itself acting at 'C' at an angle of 30° w.r.t. horizontal.

To find resultant moment about point B,

Taking moments at 'B' and its algebraic sum;

$$\Sigma M_B = -450 \cos 30° \times 3 + 100 \times 1.5 + 80 \times 1.5$$

$$= -899.13 \text{ Nm}$$

$$= \mathbf{899.31\ Nm} \text{ (CW moment)}$$

The vertical distance of the point of intersection of the line of action of resultant with the vertical edge BC of the plate of is,

$$y = \left|\frac{\Sigma M_B}{\Sigma F_x}\right| = \left(\frac{899.31}{389.71}\right)$$

$$= \mathbf{2.307\ m}$$ ...Ans.

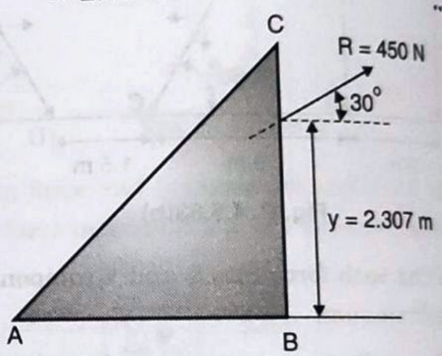


Fig. P. 4.5.34(c)

Ex. 4.5.35 : Find the equilibrant of the coplanar system of four forces acting on the plate shown in Fig. P. 4.5.35(a).

SPPU :Oct. 92, 8 Marks

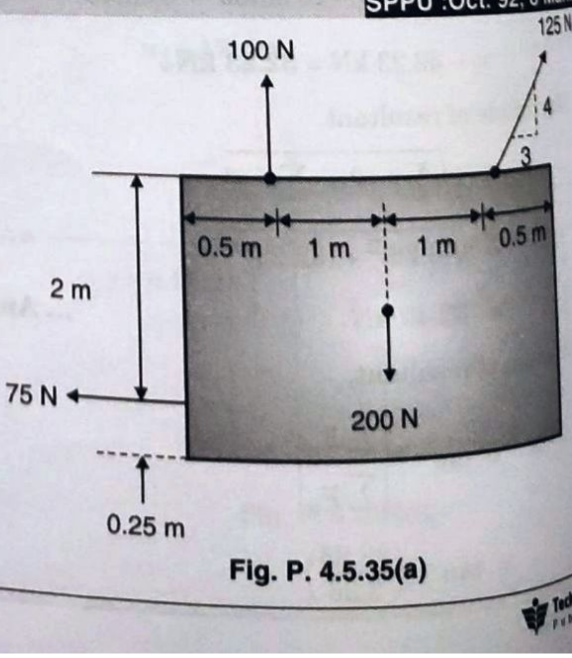


Fig. P. 4.5.35(a)

Soln. :

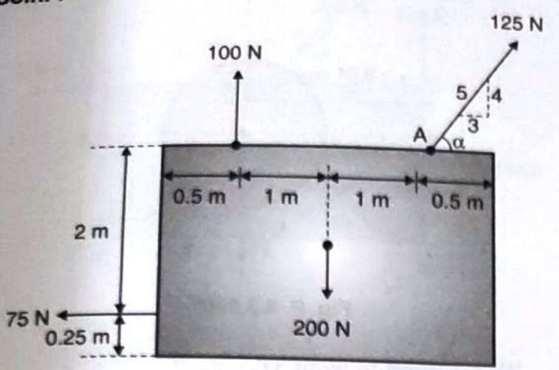


Fig. P. 4.5.35(b)

Direction of 125 N w.r.t. horizontal is α.

From Fig. P. 4.5.35(b)

$$\cos\alpha = \frac{3}{5}$$

$$\sin\alpha = \frac{4}{5}$$

Resolving forces into horizontal and vertical components and taking algebraic sum;

$$\sum F_x = R_x \quad \text{(x - component of resultant)}$$

$$\sum F_x = 125\cos\alpha - 75$$

$$= 125 \times \frac{3}{5} - 75$$

$$= 0$$

$$\sum F_y = R_y \quad \text{(y - component of resultant)}$$

$$\sum F_y = 125\sin\alpha + 100 - 200$$

$$= 125 \times \frac{4}{5} + 100 - 200$$

$$= 0$$

$\therefore$ Magnitude of resultant force,

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$= 0$$

As the resultant force is zero, the resultant of the given system will be a resultant moment.

To find resultant moment, taking moments at point 'A'

$$\sum M_A = -100 \times 2 + 200 \times 1 - 75 \times 2$$

$$= -150 \text{ Nm}$$

$$= 150 \text{ Nm (CW moment)}$$

Equilibrant moment will have the magnitude same as that of resultant but direction is opposite to that of resultant.

$\therefore$ Equilibrant moment of the given force system is 150 Nm (ACW moment) **...Ans.**

Ex. 4.5.36 : Determine the resultant of four forces tangential to the circle of radius 4 cm as shown in Fig. P. 4.5.36(a). What will be the location of the resultant with respect to the centre of the circle.

SPPU : April 94, 8 Marks

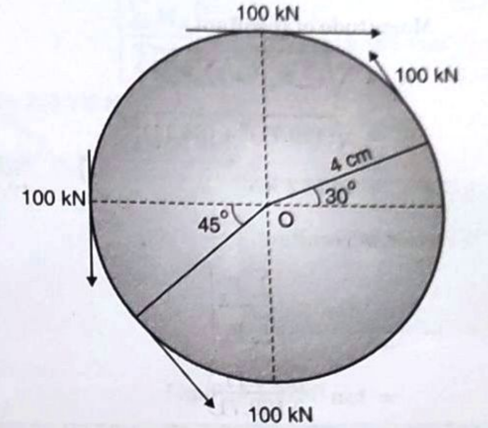


Fig. P. 4.5.36(a)

Soln. :

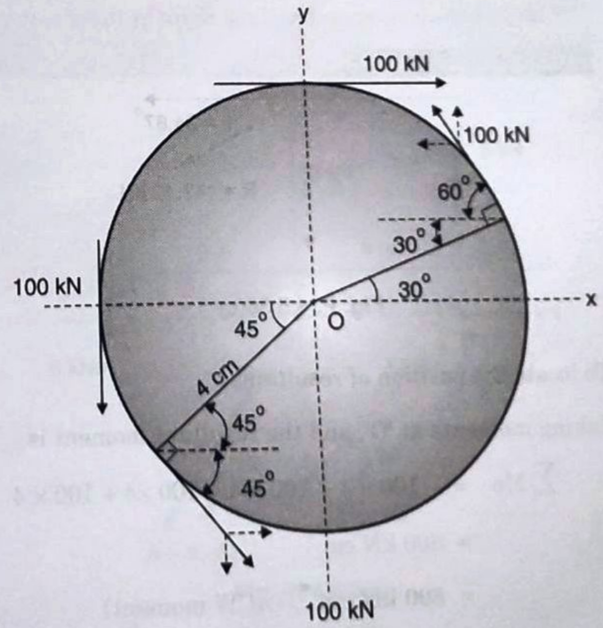


Fig. P. 4.5.36(b)

Resolving forces into x and y components and taking algebraic sum;

$\Sigma F_x = R_x$ (x - component of resultant)

$\Sigma F_x = 100 - 100\cos 60^\circ + 100\cos 45^\circ$

$= \mathbf{120.71\ kN \rightarrow}$

$\Sigma F_y = R_y$ (y - component of resultant)

$\Sigma F_y = 100 \sin 60^\circ - 100 - 100 \sin 45^\circ$

$= -84.11$ kN

$= 84.11$ kN$\downarrow$

$\therefore$ Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(120.71)^2 + (84.11)^2}$$

$$= \mathbf{147.12\ kN}$$ ... Ans.

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$

$$= \tan^{-1}\left(\frac{84.11}{120.71}\right)$$

$= \mathbf{34.87^\circ}$ **w.r.t. x - axis** ... Ans.

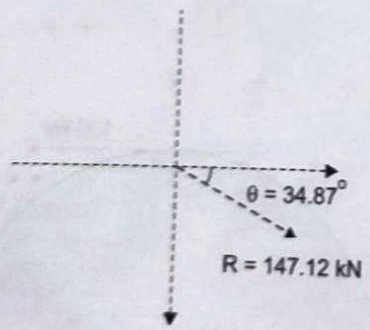


Fig. P. 4.5.36(c)

To locate the position of resultant,

Taking moments at 'O', and the resultant moment is

$$\Sigma Mo = -100 \times 4 + 100 \times 4 + 100 \times 4 + 100 \times 4$$

$$= 800 \text{ kN cm}$$

$= \mathbf{800\ kN\ cm}$ (ACW moment)

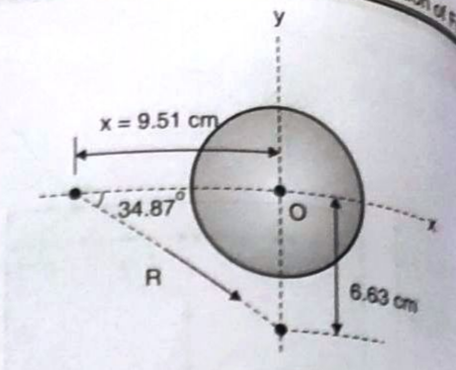


Fig. P. 4.5.36(d)

With respect to point 'O',

$$x = \left|\frac{\Sigma Mo}{\Sigma F_y}\right|$$

$$= \frac{800}{84.11}$$

$$= \mathbf{9.51\ cm}$$

$$y = \left|\frac{\Sigma Mo}{F_x}\right|$$

$$= \frac{800}{120.71}$$

$$= \mathbf{6.63\ cm}$$

Note : 'R' is forming ACW moment at the centre of the circle 'O'.

Ex. 4.5.37 : A 150 × 300 mm plate is subjected to four loads. Find the resultant of the four forces 200 N and the two points at which the line of action of the resultant intersects the edge of the plate.

SPPU : May 98, 8 Marks

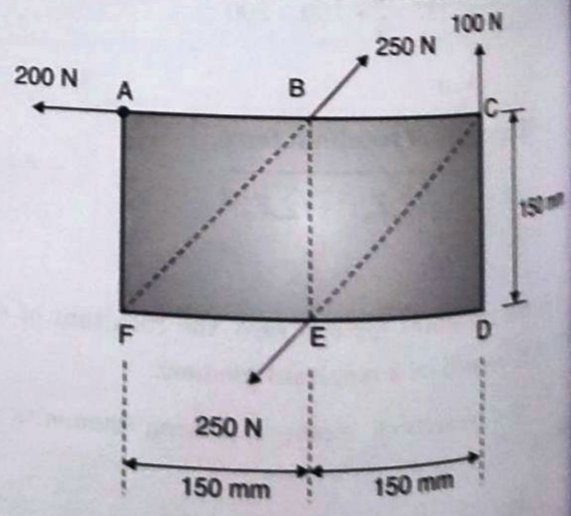


Fig. P. 4.5.37(a)

Soln. :

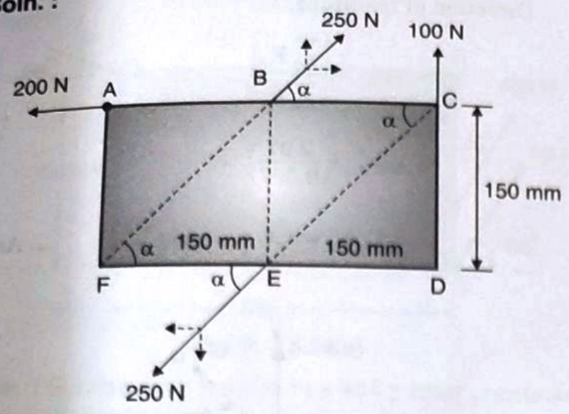


Fig. P. 4.5.37(b)

From Fig. P. 4.5.37(b)

$$\tan\alpha = \left(\frac{150}{150}\right)$$

$$\therefore \alpha = \mathbf{45^\circ}$$

Resolving each force in to x and y components and taking algebraic sum;

$$\sum F_x = R_x \qquad \text{(x - component of resultant)}$$

$$\sum F_x = -200 + 250\cos\alpha - 250\cos\alpha$$

$$= -200\ \text{N} = \mathbf{200\ N \leftarrow}$$

$$\sum F_y = R_y \qquad \text{(y - component of resultant)}$$

$$\sum F_y = 250\sin\alpha + 100 - 250\sin\alpha = \mathbf{100\ N \uparrow}$$

Magnitude of resultant,

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = \sqrt{(200)^2 + (100)^2}$$

$$= \mathbf{223.606\ N} \qquad \text{... Ans.}$$

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\sum F_y}{\sum F_x}\right| = \tan^{-1}\left(\frac{100}{200}\right)$$

$$= \mathbf{26.56^\circ\ w.r.t.\ x-axis} \qquad \text{...Ans.}$$

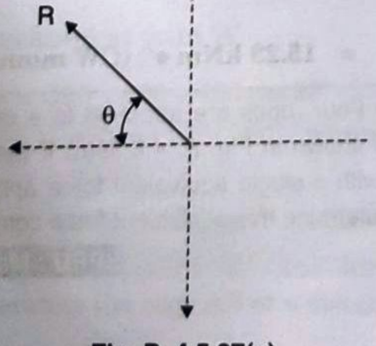


Fig. P. 4.5.37(c)

Taking moments at point 'A'

$$\sum M_A = 250\sin 45^\circ \times 150 + 100 \times 300$$

$$- 250\cos 45^\circ \times 150 - 250\sin 45^\circ \times 300$$

$$= -23033\ \text{Nmm}$$

$$= \mathbf{23.33\ Nm} \curvearrowright \quad \text{(CW moment)}$$

With respect to point 'A', the line of action of resultant intersects the edge of the plate, at a distance,

$$x = \left|\frac{\sum M_A}{\sum F_y}\right| = \frac{23033}{100} = 230.33\ \text{mm}$$

$$y = \left|\frac{\sum M_A}{\sum F_x}\right| = \frac{23033}{200} = 115.165\ \text{mm}$$

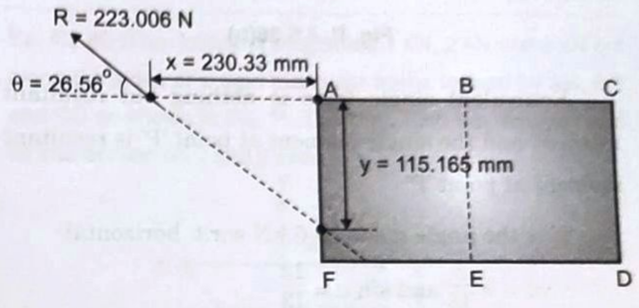


Fig. P. 4.5.37(d)

Note that the resultant moment at A is CW.

Ex. 4.5.38 : Replace the force and couple system shown, by an equivalent single force and single moment at point 'P'.

SPPU : May 08, 7 Marks

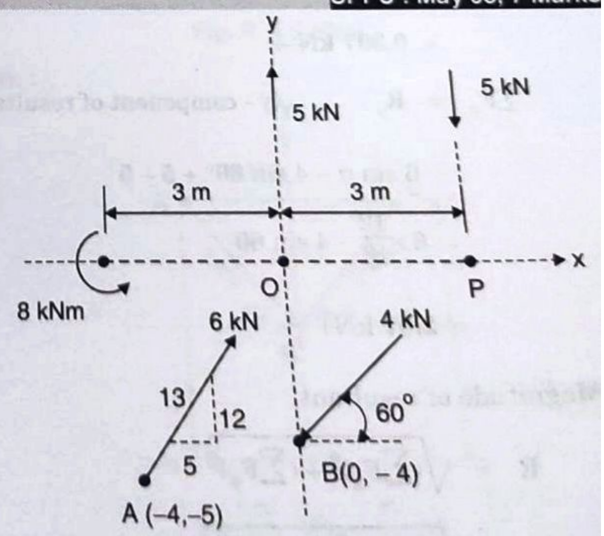


Fig. P. 4.5.38(a)

Soln. :

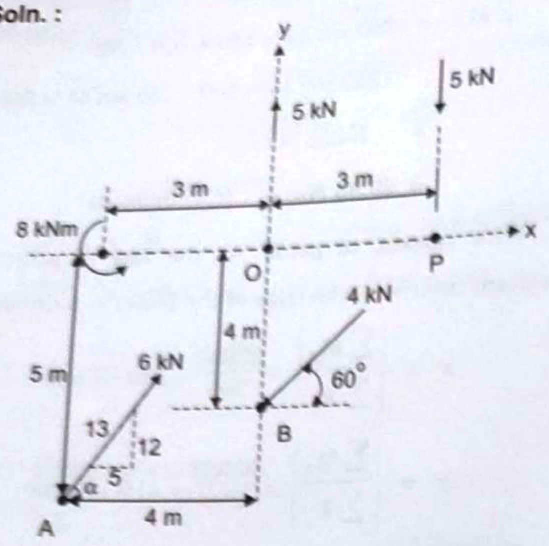


Fig. P. 4.5.38(b)

Equivalent single force is nothing but resultant moment and the single moment at point 'P' is resultant moment at point 'P'.

'α' is the angle made by 6 kN w.r.t. horizontal

$\therefore \cos\alpha = \frac{5}{13}$ and $\sin\alpha = \frac{12}{13}$

Resolving forces in x and y components and taking algebraic sum;

$$\Sigma F_x = R_x \quad \text{(x - component of resultant)}$$

$$= 6\cos\alpha - 4\cos 60° = 6\times\frac{5}{13} - 4\cos 60°$$

$$= \mathbf{0.307\ kN\rightarrow}$$

$$\Sigma F_y = R_y \quad \text{(y - component of resultant)}$$

$$= 6\sin\alpha - 4\sin 60° + 5 - 5$$

$$= 6\times\frac{12}{13} - 4\sin 60°$$

$$= \mathbf{2.07\ kN\uparrow}$$

Magnitude of resultant,

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$= \sqrt{(0.307)^2 + (2.07)^2}$$

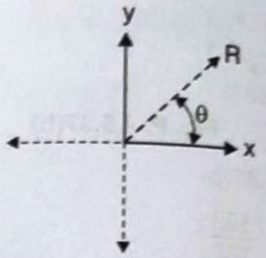
$$= \mathbf{2.093\ kN} \quad \text{... Ans.}$$

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\sum F_y}{\sum F_x}\right|$$

$$= \tan^{-1}\left(\frac{2.07}{0.307}\right)$$

$$= \mathbf{81.56°\ w.r.t.\ x-axis} \quad \text{... Ans.}$$



Taking moment abut point 'P',

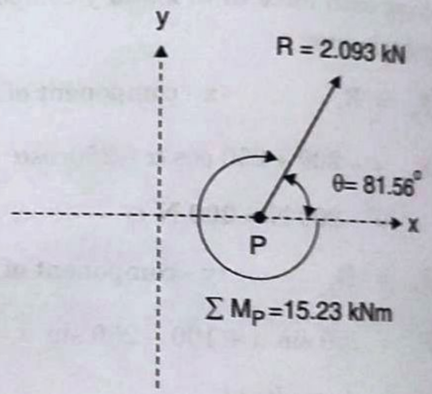


Fig. P. 4.5.38(c)

$$\sum M_P = -5\times 3 - 4\cos 60°\times 4 + 6\cos\alpha\times 5$$
$$- 6\sin\alpha\times 4 + 8 + 4\sin 60°\times 3$$
$$= -15 - 2\times 4 + 6\times\frac{5}{13}\times 5 - 6\times\frac{12}{13}\times 4$$
$$+ 8 + 3.46\times 3$$
$$= -15.23\ \text{kNm}$$
$$= \mathbf{15.23\ kNm} \circlearrowright \ \text{(CW moment)... Ans.}$$

Ex. 4.5.39 : Four ropes are attached to a crate and exert the forces as shown in Fig. P. 4.5.39(a) If the forces are to be replaced with a single equivalent force applied at a point on line AB, determine this equivalent force completely.

SPPU : May 04, 8 Marks

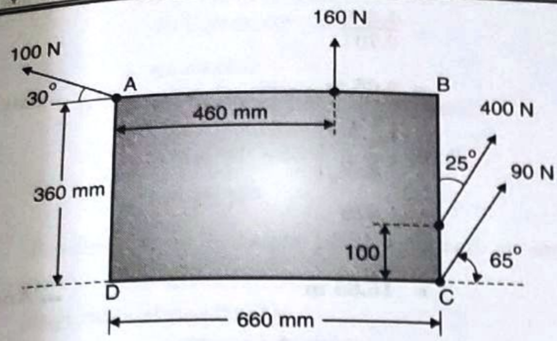


Fig. P. 4.5.39(a)

Soln. : Solving each force in to x and y components and taking algebraic sum;

$$\Sigma F_x = R_x \text{ (x - component of resultant)}$$

$$= 90 \cos 65° + 400 \sin 25° - 100 \cos 30°$$

$$= \mathbf{120.48\ N} \rightarrow$$

$$\Sigma F_y = R_y \text{ (y - component of resultant)}$$

$$= 90 \sin 65° + 400 \cos 25° + 160 + 100 \sin 30° = \mathbf{654.10\ N} \uparrow$$

Magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

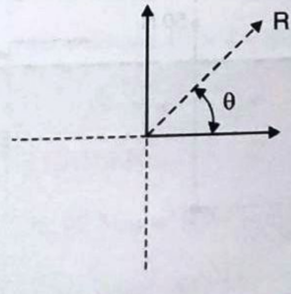
$$= \sqrt{(120.48)^2 + (654.10)^2}$$

$$= \mathbf{665.103\ N}$$ **... Ans.**

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \tan^{-1}\left(\frac{654.10}{120.48}\right)$$

$$= \mathbf{79.56° \text{ w.r.t. x-axis}}$$ **... Ans.**



Taking moment at point 'A'

$$\Sigma M_A = 160 \times 460 + 400 \cos 25° \times 660 + 400 \sin 25° \times 260 + 90 \sin 65° \times 660 + 90 \cos 65° \times 360$$

$$= 4.24 \times 10^5 \text{Nmm} \circlearrowleft \text{ (ACW)}$$

With respect to point 'A', the line of action resultant intersects the edge AB at a distance,

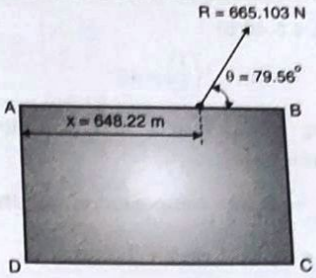


Fig. P. 4.5.39(b)

$$x = \left|\frac{\Sigma M_A}{\Sigma F_y}\right| = \left|\frac{4.24 \times 10^5}{654.10}\right|$$

$$= \mathbf{648.22\ mm}$$ **... Ans.**

Ex. 4.5.40 : The forces of magnitude 1 kN, 2 kN and 2 kN act along the sides of a rigid triangular frame formed by AB, BC and CD as shown in Fig. P. 4.5.40(a). Find the resultant and its intersection on x and y axes.

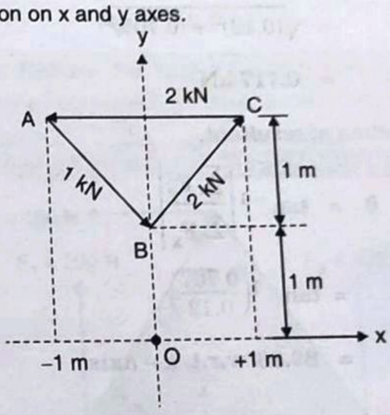


Fig. P. 4.5.40(a)

Soln. :

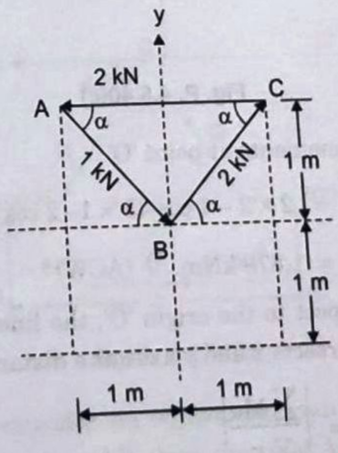


Fig. P. 4.5.40(b)

From Fig. P. 4.5.40(b)

$$\alpha = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ$$

Resolving forces into x and y components and taking algebraic sum;

$$\sum F_x = R_x \text{ (x - component of resultant)}$$
$$= -2 + 1\cos\alpha + 2\cos\alpha$$
$$= -2 + \cos 45^\circ + 2\cos 45^\circ = \mathbf{0.12\ kN}\rightarrow$$

$$\sum F_y = R_y \text{ (y - component of resultant)}$$
$$= -1\sin\alpha + 2\sin\alpha$$
$$= -\sin 45^\circ + 2\sin 45^\circ$$
$$= \mathbf{0.707\ kN}\uparrow$$

$\therefore$ Magnitude of resultant,

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$
$$= \sqrt{(0.12)^2 + (0.707)^2}$$
$$= \mathbf{0.717\ kN} \qquad \text{... Ans.}$$

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\sum F_y}{\sum F_x}\right|$$
$$= \tan^{-1}\left(\frac{0.707}{0.12}\right)$$
$$= \mathbf{80.36^\circ \text{w.r.t. x - axis}} \qquad \text{... Ans.}$$

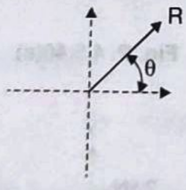


Fig. P. 4.5.40(c)

Taking moments at point 'O'

$$\sum M_o = 2\times 2 - 1\cos 45^\circ \times 1 - 2\cos 45^\circ \times 1$$
$$= 1.878 \text{ kNm. (ACW)}$$

With respect to the origin 'O', the line of action of resultant intersects x and y axes at a distance,

$$x = \left|\frac{\sum Mo}{\sum F_y}\right|$$
$$= \frac{1.878}{0.707}$$
$$= \mathbf{2.65\ m} \qquad \text{... Ans.}$$

$$y = \left|\frac{\sum Mo}{\sum F_x}\right|$$
$$= \frac{1.878}{0.12}$$
$$= \mathbf{15.65\ m} \qquad \text{... Ans.}$$

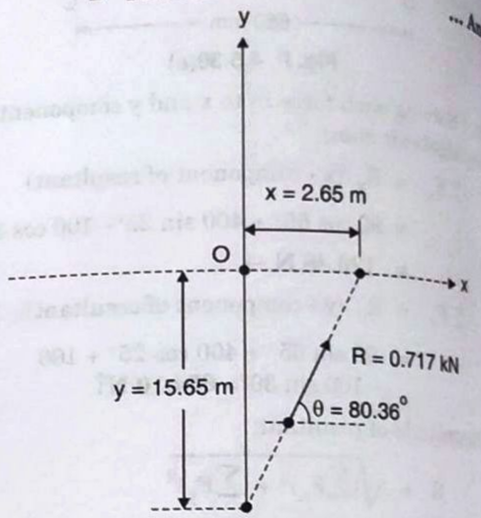


Fig. P. 4.5.40(d)

Note : Resultant is forming ACW moment at 'O'.

Ex. 4.5.41 : Determine the resultant of the four forces and one couple that act on the plate shown in Fig. P. 4.5.41(a). Locate it from point 'O'. **SPPU : May 98, 6 Marks**

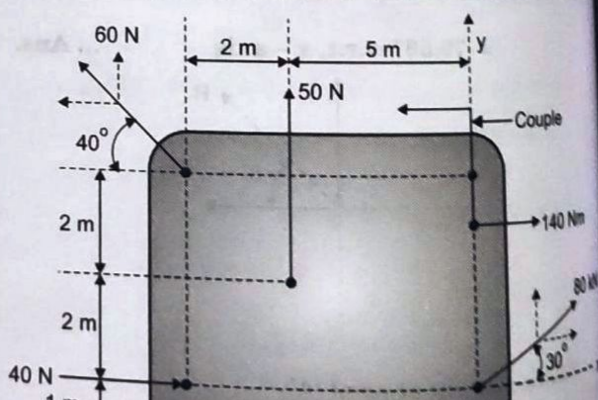


Fig. P. 4.5.41(a)

Soln. :

Resolving forces into x and y components and taking algebraic sum;

$$\sum F_x = R_x \text{ (x - component of resultant)}$$

$$= 40 + 80\cos 30° - 60\cos 40°$$

$$= \mathbf{63.32\ N \rightarrow}$$

$$\Sigma F_y = R_y \text{ (y - component of resultant)}$$

$$= 80 \sin 30° + 50 + 60 \sin 40°$$

$$= \mathbf{128.57\ N \uparrow}$$

(Resultant force a couple 140 Nm is zero in any direction)

Magnitude of resultant,

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$= \sqrt{(63.32)^2 + (128.57)^2}$$

$$= \mathbf{143.31\ N} \qquad \text{...Ans.}$$

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\sum F_y}{\sum F_x}\right|$$

$$= \tan^{-1}\left(\frac{128.57}{63.32}\right)$$

$$= \mathbf{63.78° \text{w.r.t. x - axis}}$$

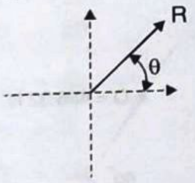


Fig. P. 4.5.41(b)

Taking moment at 'O',

$$\sum Mo = 140 + 60 \cos 40° \times 4 - 60 \sin 40° \times 7$$

$$- 50 \times 5$$

$$= -196.12 \text{ Nm}$$

$$= \mathbf{196.12\ Nm} \curvearrowright \text{ (CW moment)}$$

$\therefore$ with respect to point 'O', the line of action of resultant intersects x and y axes at a distance;

$$x = \left|\frac{\sum Mo}{\sum F_y}\right|$$

$$= \frac{196.12}{128.57}$$

$$= \mathbf{1.525\ m} \qquad \text{... Ans.}$$

$$y = \left|\frac{\sum Mo}{\sum F_x}\right|$$

$$= \frac{196.12}{63.32}$$

$$= \mathbf{3.097\ m} \qquad \text{... Ans.}$$

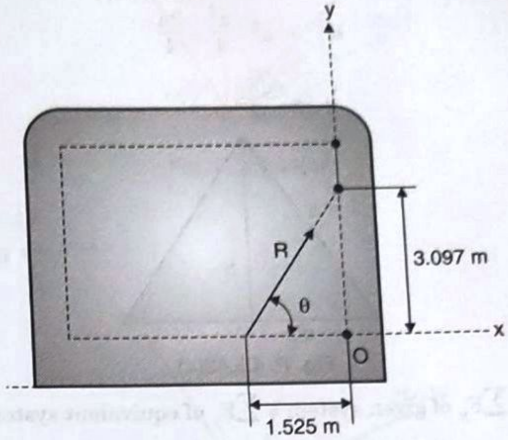


Fig. P. 4.5.41(c)

Ex. 4.5.42 : Replace the force F_1 = 200 N, F_1 = 150 N, F_3 = 300 N by an equivalent system of forces along the sides AB, BC and CA if the equilateral triangle shown in Fig. P. 4.5.42(a). **SPPU : Dec. 98, 6 Marks**

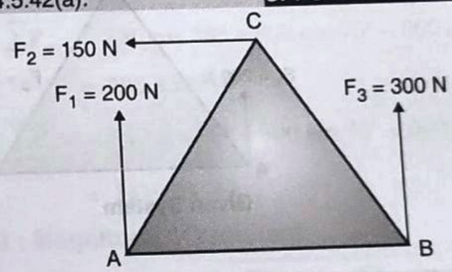


Fig. P. 4.5.42(a)

Soln. :

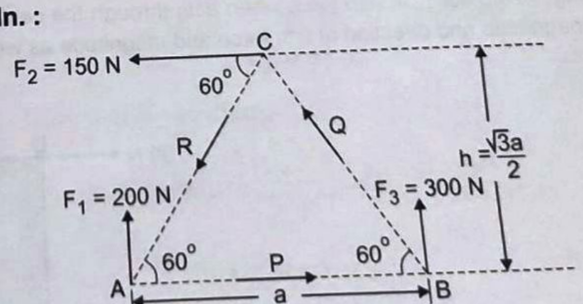


Fig. P. 4.5.42(b)

Let us consider an equivalent system formed by three forces, P, Q and R along the sides AB, BC and CA as shown in Fig. P. 4.5.42(b).

Given system is formed by three forces F_1, F_2 and F_3.

Equivalent system formed by three forces P, Q and R.

$\sum F_x$ of given system $= \sum F_x$ of equivalent system.

$$-150 = P - Q\cos 60° - R\cos 60°$$

$$-150 = P - 0.5Q - 0.5R \quad \dots (1)$$

$$h^2 = a^2 - \frac{a^2}{4} = \frac{3a}{4}$$

$$h = \frac{\sqrt{3}a}{2}$$

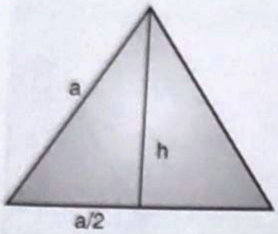


Fig. P. 4.5.42(c)

$\sum F_y$ of given system $= \sum F_y$ of equivalent system.

$$200 + 300 = Q\sin 60° - R\sin 60°$$

$$\therefore 500 = 0.866\,Q - 0.866\,R \quad (2)$$

Taking moment at point 'A';

$\sum M_A$ of given system $= \sum M_A$ of equivalent system

$$-150 \times \frac{\sqrt{3}a}{2} - 300 \times a = -Q\sin 60° \times a$$

$$-130a - 300a = -0.866\,Q{\cdot}a$$

(moments of P and R at A is ...)

$$\therefore 0.866\,Q = 430$$

$$\therefore \mathbf{Q = 496.52\ N}$$

From Eq^n (2),

$$\therefore \mathbf{R = -80.85\ N}$$

From Eq^n (1),

$$-150 = P - 0.5(496.52) - 0.5(-80.85)$$

$$\therefore \mathbf{P = 57.83\ N}$$

$\therefore$ Equivalent system is :

Force along AB,

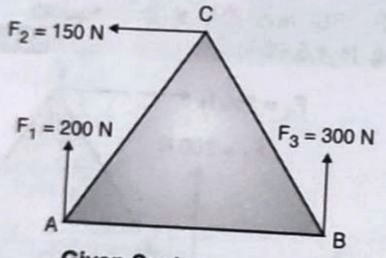
$$\mathbf{P = 57.83N \rightarrow}$$

Force along BC,

$$\mathbf{Q = 496.52N}$$

Force along, CA,

$$\mathbf{R = 80.85\ N}$$



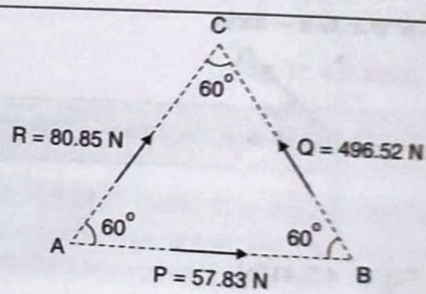


Fig. P. 4.5.42(d)

Ex. 4.5.43 : Four forces 10 N, 20 N, 30 N and 40 N act along the sides of a square (100mm × 100 mm) as shown in Fig. P. 4.5.43(a). A fifth force when acts through the centre of square 'O' reduces the force system in to a couple. Determine magnitude and direction of fifth force and magnitude as well as sense of the resultant couple. **SPPU : May 07, 8 Marks**

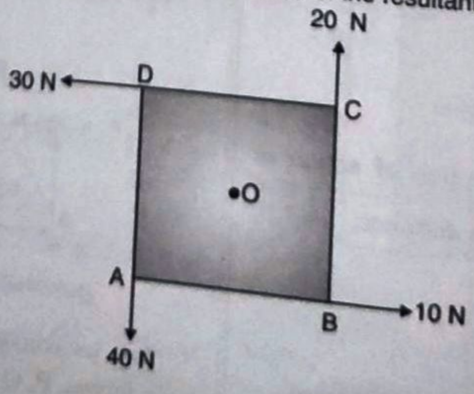


Fig. P. 4.5.43(a)

Soln. :

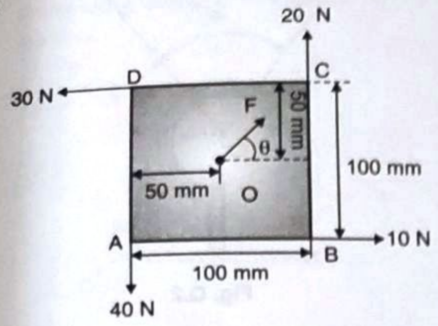


Fig. P. 4.5.43(b)

Let the magnitude of fifth force be 'F' and direction 'θ' w.r.t. horizontal acts at the centre 'O' of the square.

Given that when fifth force acts at point 'O', the system reduces into a couple.

The resultant force of the couple in any direction is zero.

∴ with fifth force acting at point 'O',

$$\sum F_y = 0$$

$$10 - 30 + F\cos\theta = 0$$

$$\therefore F\cos\theta = 20 \quad \ldots (1)$$

$$\sum F_y = 0$$

$$20 - 40 + F\sin\theta = 0$$

$$\therefore F\sin\theta = 20 \quad \ldots (2)$$

$$\frac{F\sin\theta}{F\cos\theta} = \frac{20}{20}$$

$$\tan\theta = 1$$

$$\therefore \theta = \mathbf{45^\circ}$$

From Eq^n (1), $\quad F = \dfrac{20}{\cos 45^\circ} = 28.28\text{ N}$

∴ The magnitude of fifth force,

$$\mathbf{F = 28.28\ N} \quad \textbf{... Ans.}$$

and its direction,

$$\boldsymbol{\theta = 45^\circ}\textbf{ w.r.t. x – axis} \quad \textbf{... Ans.}$$

To find resultant couple taking moments at point 'O',

$$\sum M_o = 10 \times 50 + 20 \times 50 + 30 \times 50 + 40 \times 50$$

$$= \mathbf{5000\ Nmm}\ \textbf{(ACW moment)} \quad \textbf{... Ans.}$$

Ex. 4.5.44 : Determine the magnitude and direction of the resultant of three forces acting on the hook as shown in Fig.P.4.5.44 (a) **SPPU – May 19,6 Marks**

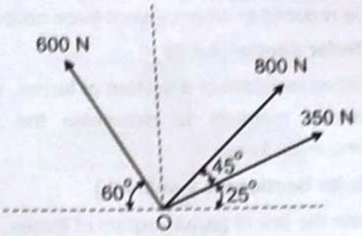


Fig. P. 4.5.44(a)

Soln.:

Step 1 :

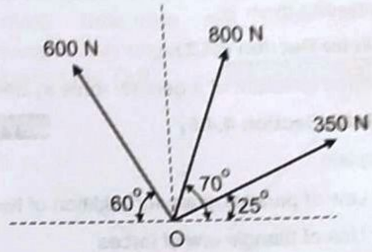


Fig. P. 4.5.44(b)

Step 2 :

$$\sum F_x = 350\cos 25^\circ + 800\cos 70^\circ - 600\cos 60^\circ$$

$$= 290.82\text{ N} \rightarrow$$

$$\sum F_y = 350\sin 25^\circ + 800\sin 70^\circ + 600\sin 60^\circ$$

$$= 1419.28\text{ N}\uparrow$$

Step 3 : Magnitude of resultant,

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$R = \sqrt{(290.82)^2 + (1419.28)^2}$$

$$= \mathbf{1448.77\ N} \quad \textbf{...Ans.}$$

Direction of resultant,

$$\theta = \tan^{-1}\left|\frac{\sum F_y}{\sum F_x}\right| = \tan^{-1}\left(\frac{1419.28}{290.82}\right)$$

$$= \mathbf{78.42^\circ\ w.r.t.\ x} \quad \textbf{... Ans.}$$

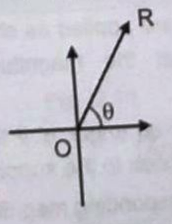


Fig. P. 4.5.44(c)

Q.11 Determine completely the resultant of four co-planar forces shown in Fig.Q.11. Locate the line of action with respect to point 'A'.

[Ans. :

R = 17.937 kN

Q = 21.13°

x = 14.785 m } w.r.t. 'A']
y = 5.713

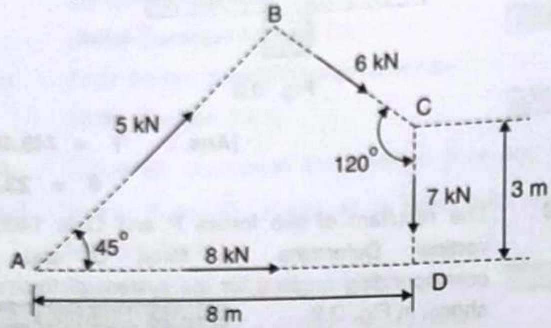


Fig. Q.11

Q.12 Find the value of angle α for which the resultant of the three forces will be along x-axis. Also find the magnitude of resultant. Refer Fig. Q.12.

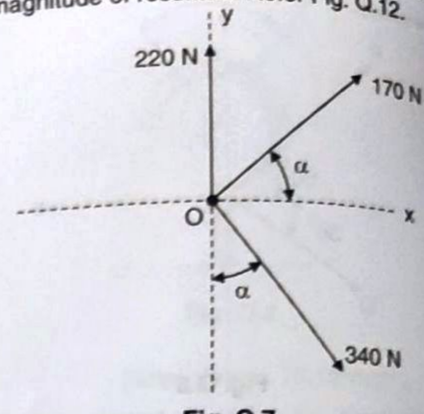


Fig. Q.7

[Ans. : $\alpha = 28.07°$ and
R = 310 N]

□□□