CHAPTER

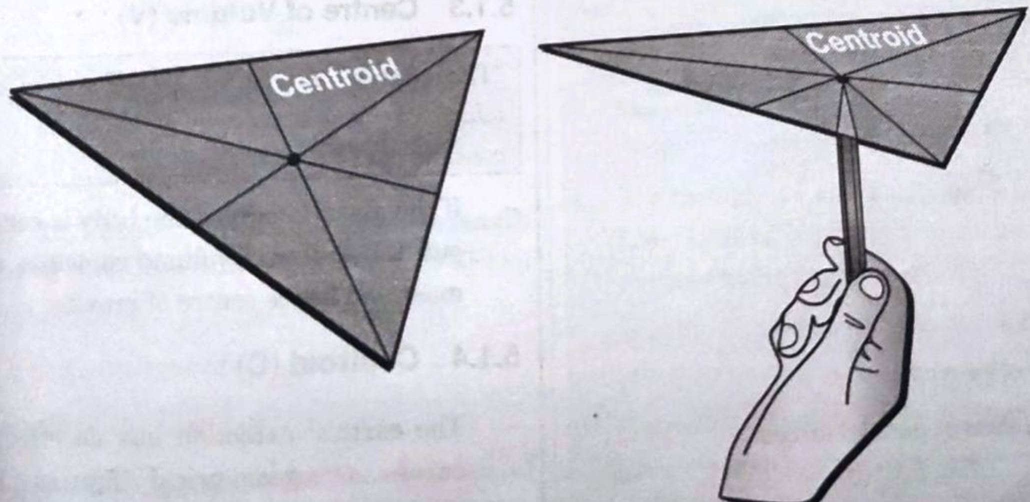
5

UNIT - II

Centroid of Lines and Areas

Introduction : In this chapter, we shall understand the basic difference between centre of gravity, centre of mass, centre of volume and centroid. We shall determine the position of centre of gravity or centroid for 1-Dimentional bent up bars and 2-Dimentional plane laminas.

- Type 1 : Based on Centroid of Lines
- Type 2 : Based on Centroid of Areas



5.1 Important Definitions

5.1.1 Centre of Gravity (G) :

- A body is composed of number of particles. These particles are attracted towards the centre of the earth because of gravitational force. This gravitational force exerted by the earth on the particle is nothing but weight of the particle. All the weights of these particles will form a system of parallel forces as the centre of the earth is far away from the body.

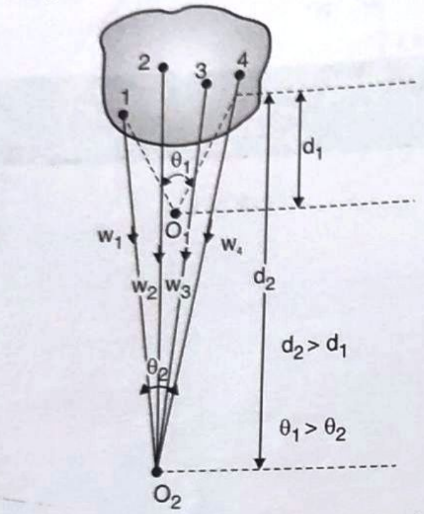
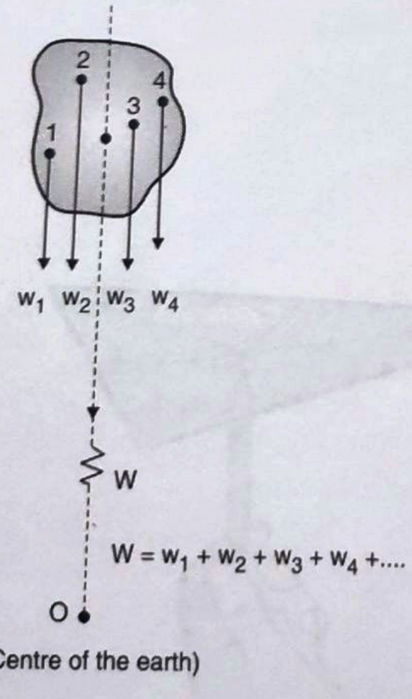


Fig. 5.1.1(a) : System of converging forces



System of parallel forces
Fig. 5.1.1(b)

- In Fig. 5.1.1(a), we can observed that as the distance of point of convergence from the body increases the angle of convergence decreases. Angle of convergence is the angle formed by the lines of action of weight of particles at the point of convergence.
- As $d_2 > d_1$, $\theta_2 < \theta_1$

  d = Distance of point of convergence from the body.

  θ = Angle of convergence

- In Fig. 5.1.1(b), as the centre of the earth is located far away from the body, the angle of convergence is almost nill and the weights of the particles will tend to form a system of parallel forces.

> *"The point through which the resultant of all these parallel forces passes, in all directions of the body is called* ***centre of gravity*** *of the body."*

- Thus, "the centre of gravity of a body is that point at which the whole weight of the body appears to be concentrated."

5.1.2 Centre of Mass (M) :

> *"The* ***centre of mass*** *is defined as the point at which the entire mass of the body appears to be concentrated."*

- It is independent of gravity.
- If the mass density of the body is constant throughout i.e., if the body is made up of homogeneous material, the centre of mass clearly coincides with the centre of gravity of the body.

5.1.3 Centre of Volume (V) :

> *"The* ***centre of volume*** *is defined as the point at which the whole volume of the body appears to be concentrated."*

- If the mass density of the body is constant through out, the centre of volume coincides with centre of mass and hence centre of gravity.

5.1.4 Centroid (C) :

- The earth's attraction has no effect on the lines, curves or geometrical figures having areas because they do not possess mass or volume.

- Therefore, the centre of gravity, centre of mass and centre of volume does not apply to lines, curves or such others geometrical figures.
- But these figures also have a point similar to the centre of gravity of the body, known as "centroid" of the line curve or area.

5.1.5 Centroid of Area (C) :

"Centroid of a plane lamina or figure is defined as the point, at which the whole area of the lamina or figure may be assumed to be concentrated."

5.1.6 Centroid of Line (C) :

"Centroid of line is defined as the point, at which the whole length of the line may be assumed to be concentrated."

Note : Centroid applies to 2-Dimensional plane laminas or figures and 1-Dimensional lines, wires or rods, where , as centre of gravity, centre of mass and centre of volume applies to 3-Dimensional bodies.

5.2 Location of Centroid of a Line

- A line may be either a straight line or a curved line which is simply called a curve.
- The centroid of the line can be located by using principle of moments.

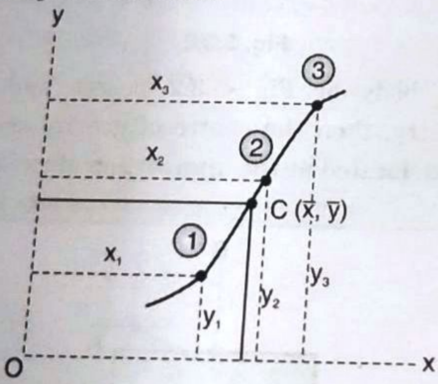


Fig. 5.2.1

- In the Fig. 5.2.1 above, Let the line of whole length 'L' be divided into three line segments of length l_1, l_2 and l_3 respectively.
- Here, 'L' is the resultant of l_1, l_2 and l_3.

  $\therefore$ Total length of the line, $L = l_1 + l_2 + l_3$
- Let x_1, x_2, x_3 and y_1, y_2, y_3 be the distance of centroid of line segments l_1, l_2 and l_3 from y – axes and x – axes respectively.
- Let $\bar{x}$ and $\bar{y}$ be the distance of centroid of the whole line from y and x axes respectively.
- Taking moments of line segments about y-axis and using Varignon's theorem;

$$l_1 x_1 + l_2 x_2 + l_3 x_3 = L.\bar{x}$$

$$\therefore \bar{x} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3}{L} = \frac{\Sigma l_x}{\Sigma l}$$

$$\Sigma l = l_1 + l_2 + l_3 = L$$

- Moment of line at an axis is the product of length of line and perpendicular distance of centroid of line from the axis at which moment is to be taken.
- As per varignon's theorem, the sum of the moments of all line segments at y-axis = moment of resultant is, whole line at y-axis.
- Taking moments of line segments about x – axis and using varignon's theorem.

$$l_1 y_1 + l_2 y_2 + l_3 y_3 = L.\bar{y}$$

$$\bar{y} = \frac{l_1 y_1 + l_2 y_2 + l_3 y_3}{L} = \frac{\sum l_y}{\sum l}$$

$$\Sigma l = l_1 + l_2 + l_3 = L$$

- Therefore the coordinates of centroid of line are :

$$\bar{x} = \frac{\sum l_x}{\sum l} \quad \text{and} \quad \bar{y} = \frac{\sum l_y}{\sum l}$$

Important notes :

1. $\bar{x}$ is the distance of centroid from y-axis and $\bar{y}$ is the distance from x - axis.
2. To find $\bar{x}$, moments are to be taken about y-axis and to find $\bar{y}$, moments are to be taken about x-axis.
3. Moment of line = length of line × perpendicular distance of centroid of the line.
4. The centroid of a line is not necessarily located on the line. However, if the line straight, the centroid is at the midpoint of the line.
5. *l*x is the first moment of line and the unit of moment of line is m^2 or mm^2 is, length^2.
6. The length of line is always positive but the distance of centroid from x or y – axes is positive, negative or zero depending on the position of centroid of the line.

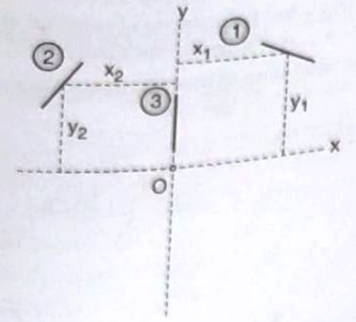


Fig. 5.2.2

Moment of line ① at y-axis $= l_1 x_1$

Moment of line ② at y-axis $= -l_2 x_2$ ($\because x_2$ is –ve)

Moment of line ③ at y-axis $= 0$ ($\because x_3 = 0$)

(Centroid of line ③ is located on y-axis)

5.3 Axis of Symmetry

Axis of symmetry is defined as the line which divides the body or figure in two parts, so that the moments of these parts about this line are equal and opposite.

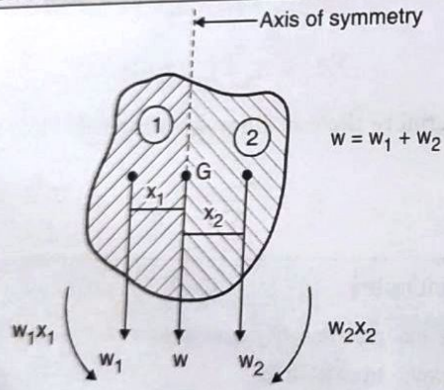


Fig. 5.3.1

$$w = w_1 + w_2$$

- Here, about axis of symmetry,

$$w_1 x_1 = w_2 x_2$$

- As per Varignon's theorem, Algebraic sum of the moments of weights at axis of symmetry = moments of resultant is, w at the axis of symmetry.

$$w_1 x_1 - w_2 x_2 = w_x$$

But x = 0 as C. G. is located on axis of symmetry

$$\therefore w_1 x_1 - w_2 x_2 = 0$$

$$\therefore w_1 x_1 = w_2 x_2$$

- Which means when the moments of weights both sides of axis of symmetry are balanced, the C. G. will be located on the axis of symmetry. C. G. is the point through which the resultant of w_1 and w_2 is, w passes.
- Centre of gravity or centorid is always located on the axis of symmetry.
- If the Fig. is symmetrical at x – axis, then distance of centroid from x – axis is, $\bar{y} = 0$ and vice-versa.
- If the position of C. G. or centroid is known then we can draw any number of axes of symmetry through the C. G. or centroid.
- If we draw any line passing through C. G. or centroid, we can call that line as an axis of symmetry, because C. G. or centroid is always located on axis of symmetry.

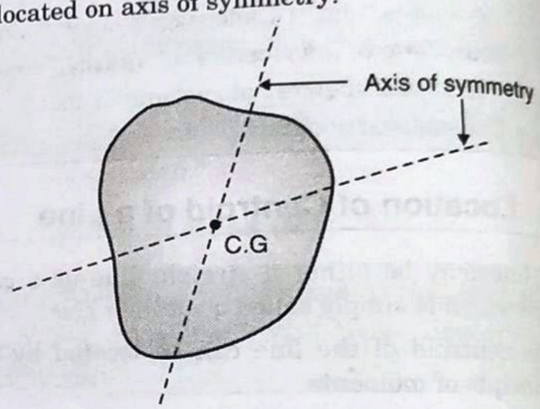


Fig. 5.3.2

- If the body or Fig. 5.3.2 passes two axis of symmetry, then the centre of gravity or centroid must be located at the intersection of two axes.

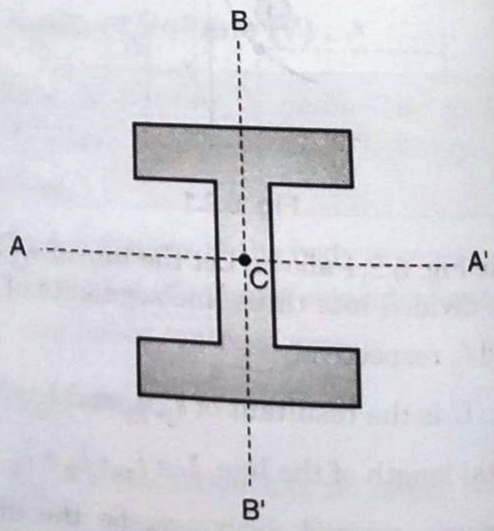


Fig. 5.3.3

- The above Fig. 5.3.3 is symmetrical at A-A′ and B-B′. The intersection point of the lines A-A′ and B-B′ is the centroid (c) of the Fig. 5.3.3.
- Centre of gravity or centroid shifts towards the addition which means that it shifts away from the deduction.

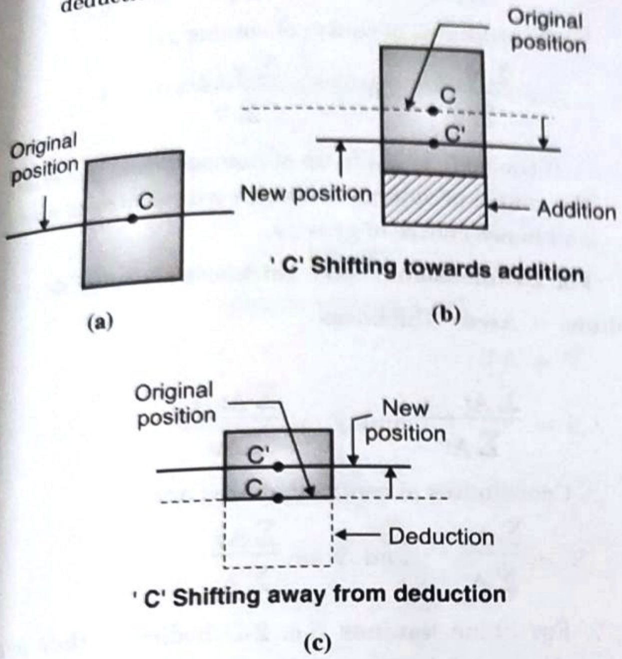


Fig. 5.3.4

Important Notes :

When a body is freely suspended from any point, the centre of gravity of the body is always located on the vertical line passing through point of suspension.

The vertical line passing through point of suspension will act as an axis of symmetry. Because C.G. is always located on axis of symmetry.

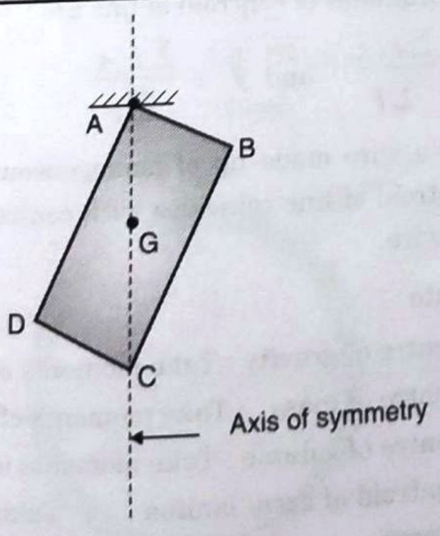


Fig. 5.3.5

- To locate position of C.G., experimentally, suspend the body freely from any two points and draw the vertical lines passing through point of suspension. The intersection point of these two lines gives the position of C. G.

5.4 General procedure to determine the position of centroid of line or linear member is, bar, rod etc. bent up to any shape

(i) Select the reference point and axes if not given.

(ii) See whether the given Fig. or bent is symmetrical at x or y - axes. If it is symmetrical at x-axis, then $\bar{y} = 0$ and vice versa.

(iii) Divide or split the given line or bent in to simple line segments.

(iv) Obtain (and tabulate) the resultants of

a) Length of line segments (l)

b) Position centroid of line segments wrt the reference point is, x and y.

c) Moments of lengths of line segments at x and y axes is, $l \cdot y$ and $l \cdot x$.

(v) Take the sum of

a) Lengths (Σl)

b) Moments of lengths $\Sigma l \cdot x$ and $\Sigma l \cdot y$.

(vi) The position coordinates of centroid are given by

$$\bar{x} = \frac{\Sigma lx}{\Sigma l} \quad \text{and} \quad \bar{y} = \frac{\Sigma l y}{\Sigma l}$$

Shape	Length (l) (mm)	Distance of centroid from y-axis, x (mm)	Distance of centroid from x – axis, y (mm)	$l \cdot x$ $(mm)^2$	$l \cdot y$ $(mm)^2$
	Σl			$\Sigma l \cdot x$	$\Sigma l \cdot y$

l x = moment of line segment at y-axis

l y = moment of line segment at x-axis.

5.5 To Locate Position of Centroid of Area

(i) Select and fix the reference axes if not given.

(ii) Check whether the given figure is symmetrical at either x or y-axes. If symmetrical at x-axis, then $\overline{y} = 0$ and vice versa.

(iii) Divide the figure into different areas of simple shapes.

(iv) Take moments of areas at x and y axes.

(v) Co-ordinates of centroid are given by :

$$\overline{x} = \frac{\Sigma Ax}{\Sigma A} \quad \text{and } \overline{y} = \frac{\Sigma Ay}{\Sigma A}$$

Shape	Area (A) $(mm)^2$	Distance of centroid from y-axis, x (mm)	Distance of centroid from x - axis, y (mm)	A · x (mm^3)	A · y (mm^3)
$\sum A$				$\sum A \cdot x$	$\sum A \cdot y$

$A \cdot x$ = moment of area at y-axis

$A \cdot y$ = moment of area at x-axis.

Important notes :

1. Co-ordinates of centre of gravity are :

$$\overline{x} = \frac{\Sigma w \cdot x}{\Sigma w} \quad \text{and } \overline{y} = \frac{\Sigma w \cdot y}{\Sigma w}$$

2. At a given place, 'g' is always constant.

$$w = m \cdot g$$

$$\overline{x} = \frac{\Sigma mg \cdot x}{\Sigma mg} \quad \text{and } \overline{y} = \frac{\Sigma mg \cdot y}{\Sigma mg}$$

∴ The co-ordinates of centre of mass are :

$$\overline{x} = \frac{\Sigma mx}{\Sigma m} \quad \text{and } \overline{y} = \frac{\Sigma my}{\Sigma m}$$

At a given place, centre of mass coincides with centre of gravity.

3. If the body is made up of homogeneous material, i.e., ρ = constant

Mass density,

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{v}$$

$$\therefore m = \rho \cdot v$$

$$\therefore \overline{x} = \frac{\Sigma \rho \cdot V \cdot x}{\Sigma \rho \cdot V} \quad \text{and } \overline{y} = \frac{\Sigma \rho V \cdot y}{\Sigma \rho V}$$

∴ Co-ordinates of centre of volume are :

$$\therefore \overline{x} = \frac{\Sigma V \cdot x}{\Sigma V} \quad \text{and } \overline{y} = \frac{\Sigma V \cdot y}{\Sigma V}$$

∴ If the body is made up of homogeneous material, the centre of volume coincides with centre of mass and hence centre of gravity.

4. For 2-Dimensional body, thickness is negligible.

Volume = Area × thickness

$$V = A \cdot t$$

$$\therefore \overline{x} = \frac{\Sigma At \cdot x}{\Sigma At} \quad \text{and } \overline{y} = \frac{\Sigma At \cdot y}{\Sigma At}$$

∴ Coordinates of centroid of area are :

$$\overline{x} = \frac{\Sigma Ax}{\Sigma A} \quad \text{and } \overline{y} = \frac{\Sigma Ay}{\Sigma A}$$

∴ For plane laminas (i.e. 2-D bodies) if they are made up of homogeneous material, centroid of area coincides with centre of gravity.

5 For 1-D body, width 'b' is negligible

Area = Length × breadth

$$A = l \times b$$

$$\therefore \overline{x} = \frac{\Sigma lbx}{\Sigma lb} \quad \text{and } \overline{y} = \frac{\Sigma lby}{\Sigma lb}$$

∴ Co-ordinates of centroid of line are :

$$\therefore \overline{x} = \frac{\Sigma l \cdot x}{\Sigma l} \quad \text{and } \overline{y} = \frac{\Sigma l \cdot y}{\Sigma l}$$

∴ For a wire made up of homogeneous material, its centroid of line coincides with centre of gravity of the wire.

6. To locate

(i) Centre of gravity : Take moments of weights

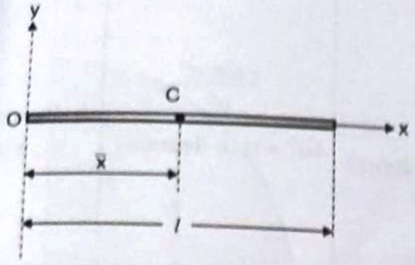
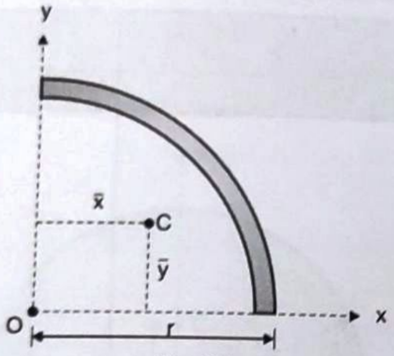
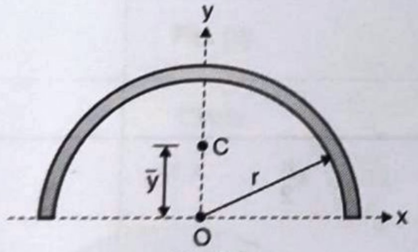
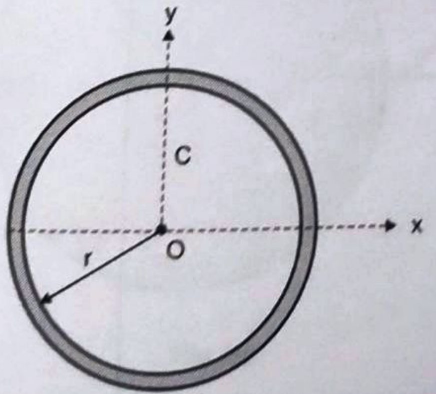
(ii) Centre of mass : Take moments of mass

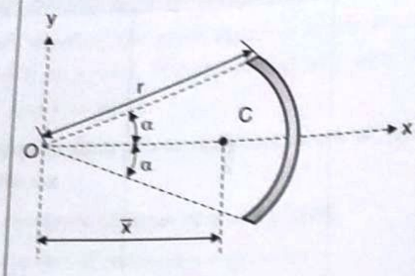
(iii) Centre of volume : Take moments of volume

(iv) Centroid of area/ lamina : Take moments of area

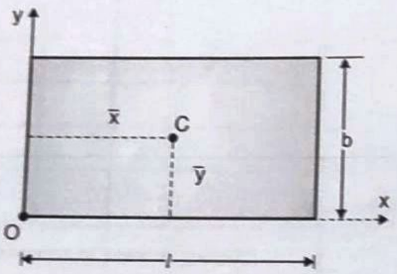
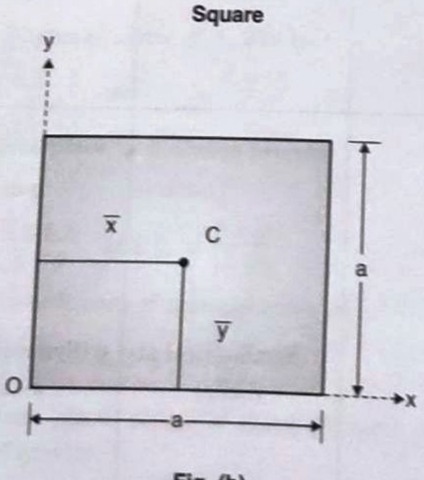
(v) Centroid of line : Take moments of length

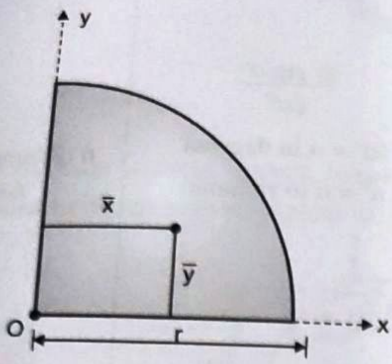
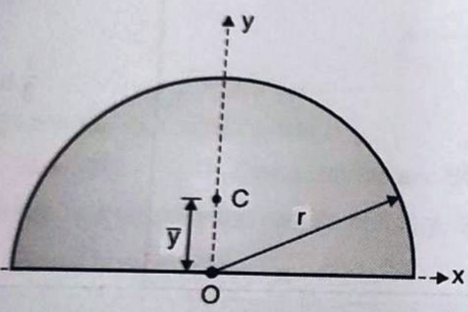
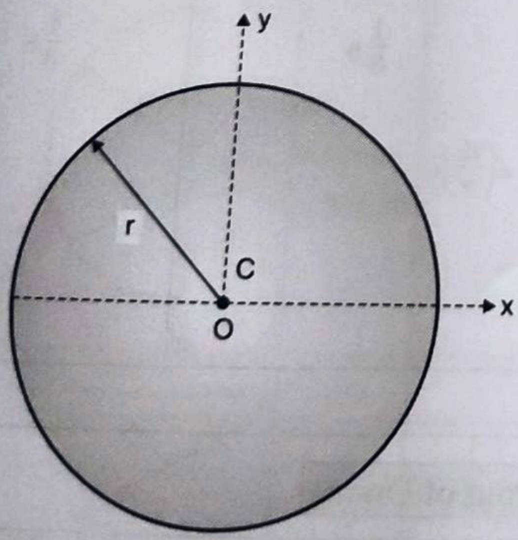
5.6 Centroid of Common Geometrical Shapes of Lines :

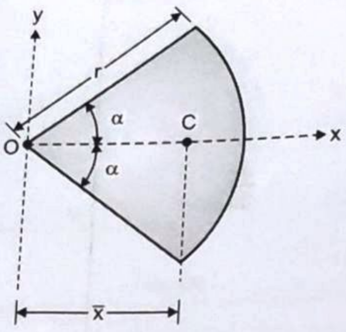
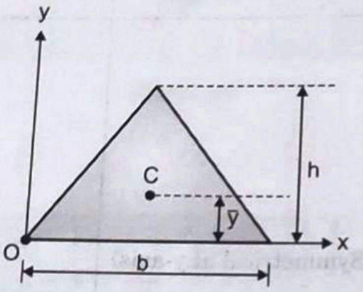
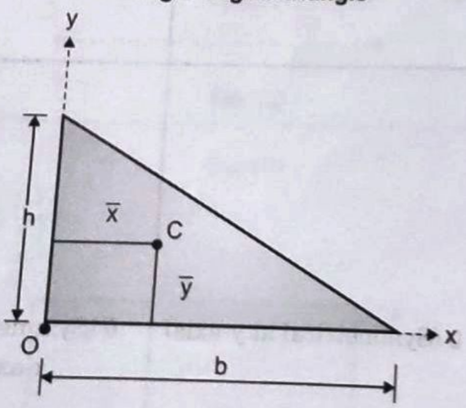
Sr. No.	Shape	Length (l)	$\bar{x}$	$\bar{y}$
1	**Straight line**  Fig. (a)	l	$\frac{l}{2}$	0 (Symmetrical at x-axis)
2	**Quarter Circular Arc**  Fig. (b)	$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$
3	**Semi circular arc**  Fig. (c)	πr	0 (Symmetrical at y-axis)	$\frac{2r}{\pi}$
4	**Circle**  Fig. (d)	$2\pi r$	0 (Symmetrical at y-axis)	0 (Symmetrical at x-axis)

Sr. No.	Shape	Length (l)	$\bar{x}$	$\bar{y}$
5	**Arc of a circle**  **Fig. (e)**	$2r\alpha^c$ ($\alpha^c = \alpha$ in radians)	$\frac{r \sin \alpha^\circ}{\alpha^c}$ ($\alpha^\circ = \alpha$ in degrees)	0 (Symmetrical about x-axis)

5.7 Centroids of Common Geometrical Shapes of Areas :

Sr. No.	Shape	Area (A)	$\bar{x}$	$\bar{y}$
1	**Rectangle**  **Fig. (a)**	lb	$\frac{l}{2}$	$\frac{b}{2}$
2	**Square**  **Fig. (b)**	a^2	$\frac{a}{2}$	$\frac{a}{2}$

Shape	Area (A)	$\bar{x}$	$\bar{y}$
Quarter circle  **Fig. (c)**	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semi-circle  **Fig. (d)**	$\frac{\pi r^2}{2}$	0 (Symmetrical at y-axis)	$\frac{4r}{3\pi}$
Circle  **Fig. (e)**	πr^2	0 (Symmetrical at y-axis)	0 (Symmetrical at x-axis)

Sr. No.	Shape	Area (A)	$\bar{x}$	$\bar{y}$
6	**Circular sector**  Fig. (f)	$\alpha^c r^2$	$\dfrac{2r \sin \alpha^\circ}{3\alpha^c}$ ($\alpha^\circ = \alpha$ in degrees, $\alpha^c = \alpha$ in radians)	0 (Symmetrical at x-axis)
7	**Triangle**  Fig. (g)	$\frac{1}{2}$ bh	-	$\frac{1}{3}$h
8	**Right angled triangle**  Fig. (h)	$\frac{1}{2}$ bh	$\frac{1}{3}$b	$\frac{1}{3}$h

5.8 Solved Examples :

Type 1 : Based on Centroid of Lines

Ex. 5.8.1: A uniform wire is bent into the shape shown in Fig. P. 5.8.1(a). Determine the portion coordinates of centre of gravity of the bent w.r.t. point 'C'.

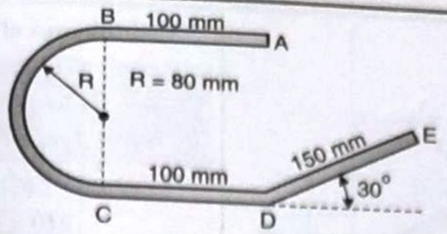


Fig. P. 5.8.1(a)

Soln. :

Wire is 1-D body and it is uniform, hence C.G. coincides with centroid of line.

Step 1 : Select reference axes w.r.t. point 'C' .

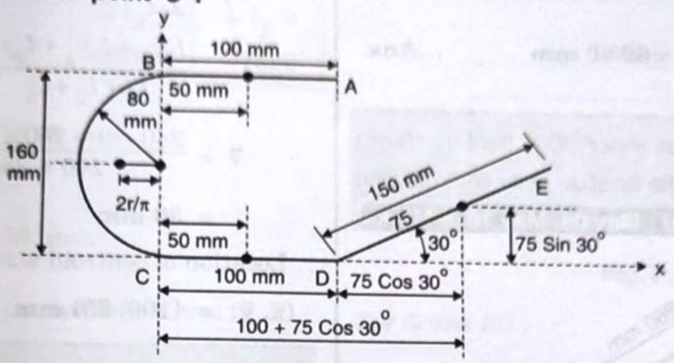


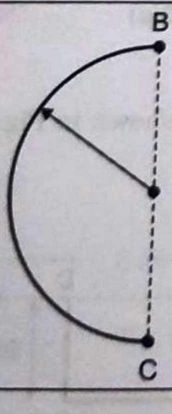
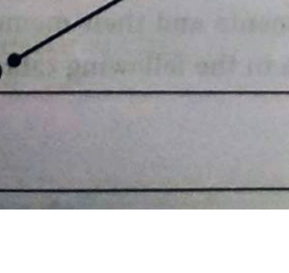
Fig. P. 5.8.1(b)

Step 2 : Divide the bent into 4 parts :

(1) Line AB (2) Semicircular arc BC (3) Line CD (4) Line DE

Step 3 : Obtain and tabulate the results of l, x, y, lx and ly.

Table P. 5.8.1

Line	l (mm)	x (mm)	y (mm)	l.x (mm^2)	l.y (mm^2)
B —— A	100	50	160	5000	16000
 B, C	πr $= \pi \times 80$ $= 251.33$	$-\left(\frac{2r}{\pi}\right) = -\left(\frac{2 \times 80}{\pi}\right) = -50.93$	80	−12800.13	20106.40
C —— D	100	50	0	5000	0
 D, E	150	$100 + 75 \cos 30° = 164.95$	$75 \sin 30° = 37.50$	24742.5	5625
	601.33			21942.37	41731.4

Step 4 : Take Summation of l, lx and ly.

$$\Sigma l = 601.33 \text{ mm}$$

$$\Sigma lx = 21942.37 \text{ mm}^2$$

$$\Sigma ly = 41731.4 \text{ mm}^2$$

Step 5 : Co-ordinates of centre of gravity w.r.t. point 'C' are;

$$\bar{x} = \frac{\Sigma l \cdot x}{\Sigma l} = \frac{21942.37}{601.33} = \mathbf{36.49 \text{ mm}}$$ ...Ans.

$$\bar{y} = \frac{\Sigma l \cdot y}{\Sigma l} = \frac{41731.4}{601.33} = \mathbf{69.40 \text{ mm}}$$ ...Ans.

Ex. 5.8.2 : A thin homogeneous wire ABC is bent as shown in Fig. P. 5.8.2(a). Determine the location of its centroid with respect to A. **SPPU : May 08, May 16, 6 Marks**

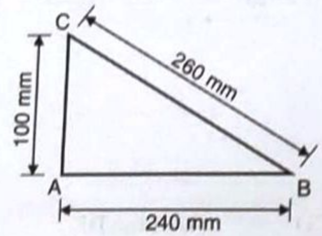


Fig. P. 5.8.2(a)

Soln. :

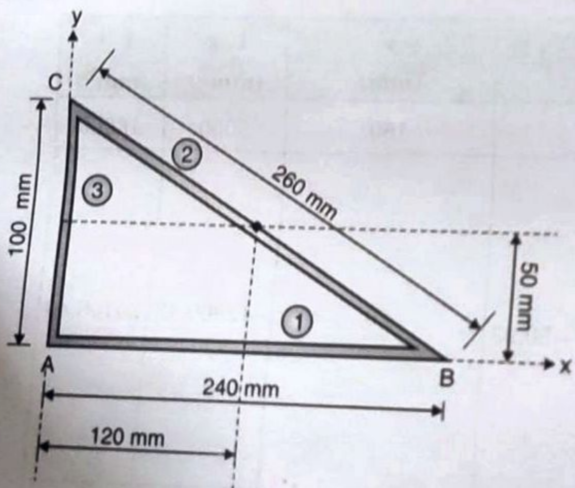


Fig. P. 5.8.2(b)

Dividing the bent into three parts, AB, BC and CA as (1), (2) and (3) respectively.

$l_1 = 240$ mm, $x_1 = 120$ mm, $y_1 = 0$

$l_2 = 260$ mm, $x_2 = 120$ mm, $y_2 = 50$ mm

$l_3 = 100$ mm, $x_3 = 0$, $y_3 = 50$ mm

$\therefore$ Co-ordinates of centroid are given by,

$$\bar{x} = \frac{\Sigma l_x}{\Sigma l} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3}{l_1 + l_2 + l_3} = \frac{240 \times 120 + 260 \times 120 + 100 \times 0}{240 + 260 + 100}$$

$$= \mathbf{100 \text{ mm}}$$ ...Ans.

$$\bar{y} = \frac{\Sigma l_y}{\Sigma l} = \frac{l_1 y_1 + l_2 y_2 + l_3 y_3}{l_1 + l_2 + l_3}$$

$$\bar{y} = \frac{240 \times 0 + 260 \times 50 + 100 \times 50}{240 + 260 + 100}$$

$$= \mathbf{30 \text{ mm}}$$ ...Ans.

$\therefore$ Location of centroid w.r.t. point A is

$$(\bar{x}, \bar{y}) = \mathbf{(100, 30) \text{ mm}}$$ ...Ans.

Ex. 5.8.3 : A thin rod is bend into a shape OABCD as shown in Fig. P. 5.8.3(a). Determine the centroid of the bend with respect to origin O. **SPPU : May 14, 4 Marks**

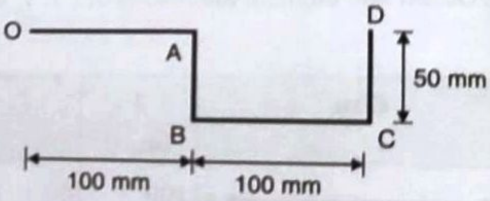


Fig. P. 5.8.3(a)

Soln. :

Selecting x and y axes as shown in Fig. P. 5.8.3(b) w.r.t. 'O'.

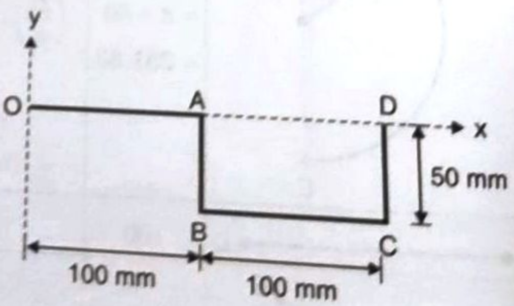


Fig. P. 5.8.3(b)

The length of line segments and their moments about x-axis and y-axis are shown in the following table.

Table P. 5.8.3

Line	l (mm)	x (mm)	y (mm)	$l.x$ (mm^2)	$l.y$ (mm^2)
OA	100	50	0	5000	0
AB	50	100	–25	5000	–1250
BC	100	150	–50	15000	–5000
CD	50	200	–25	10000	–1250
	$\Sigma l = 300$			$\Sigma l_x = 35000$	$\Sigma l_y = -7500$

$$\therefore \bar{x} = \frac{\Sigma l \cdot x}{\Sigma l} = \frac{35000}{300} = \mathbf{116.67\ mm}$$

$$\bar{y} = \frac{\Sigma l \cdot y}{\Sigma l} = \frac{-7500}{300} = \mathbf{-25\ mm}$$

$\therefore$ With respect to point 'O', the centroid of the bent is

$(\bar{x}, \bar{y})$ = **(116.67, –25) mm** ...Ans.

Ex. 5.8.4 : Determine the centroidal coordinates of the line shown in Fig. P. 5.8.4(a) that consist of straight line AB, AD and circular arc BC. **SPPU : May 05, 9 Marks**

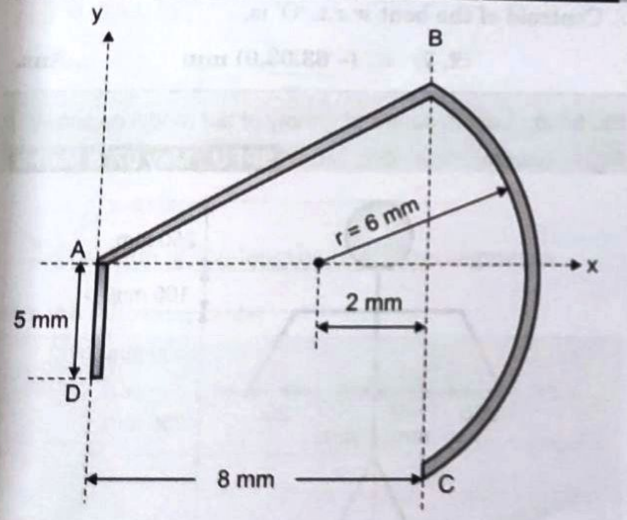


Fig. P. 5.8.4(a)

Soln. :

Dividing Fig. P. 5.8.4(a) into 3 parts :

① Line AD

② Line AB

③ Circular arc BC

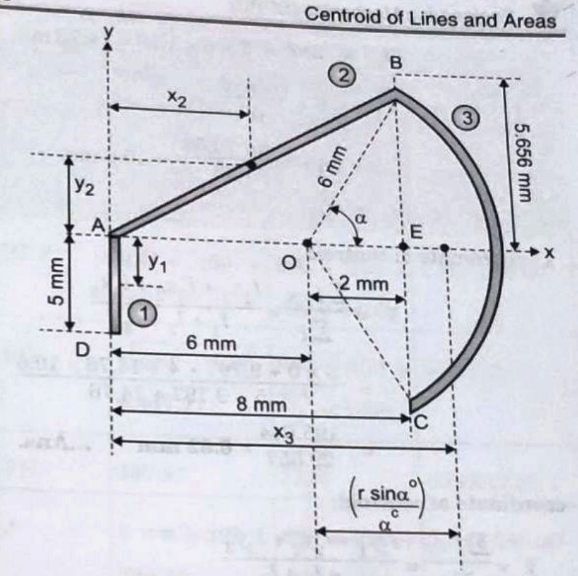


Fig. P. 5.8.4(b)

For ① line AD :

$$l_1 = 5 \text{ mm}$$

$$x_2 = 0$$

$$y_1 = -\frac{5}{2} = -2.5 \text{ mm}$$

For ② line AB :

$$\text{From } \Delta\text{ OBE, } \cos\alpha = \frac{OE}{OB} = \frac{2}{6}$$

$$\therefore \alpha = 70.53^{\circ}$$

$$\sin\alpha = \frac{BE}{OB}$$

$$\therefore BE = OB \sin\alpha = 6 \sin 70.53^{\circ}$$

$$= 5.656 \text{ mm}$$

$$\text{From } \Delta\text{ ABE ; } AB^2 = AE^2 + BE^2$$

$$= 8^2 + 5.656^2$$

$$\therefore AB = 9.797 \text{ mm}$$

$$\therefore l_2 = AB = 9.797 \text{ mm}$$

$$x_2 = \frac{8}{2} = 4 \text{ mm}$$

$$y_2 = -\frac{5.656}{2} = 2.828 \text{ mm}$$

For ③ circular arc 'BC' :

$$\alpha = 70.53^{\circ} = \frac{70.53 \times \pi}{180^{\circ}}$$

$$= 1.23^{c} \text{ (radians)}$$

$$l_3 = 2r\alpha^c = 2 \times 6 \times 1.23 = 14.76 \text{ mm}$$

$$x_3 = 6 + \frac{r \sin \alpha^\circ}{\alpha^c}$$

$$= 6 + \frac{6 \sin 70.53^\circ}{1.23} = 10.6 \text{ mm}$$

$$y_3 = 0$$

$\therefore$ x - coordinate of centroid,

$$\bar{x} = \frac{\Sigma l \cdot x}{\Sigma l} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3}{l_1 + l_2 + l_3}$$

$$= \frac{5 \times 0 + 9.797 \times 4 + 14.76 \times 10.6}{5 + 9.797 + 14.76}$$

$$= \frac{195.644}{29.557} = \mathbf{6.62 \text{ mm}}$$ ...Ans.

y - coordinate of centroid,

$$\bar{y} = \frac{\Sigma l \cdot y}{\Sigma l} = \frac{l_1 y_1 + l_2 y_2 + l_3 y_3}{l_1 + l_2 + l_3}$$

$$= \frac{5 \times (-2.5) + 9.797 \times 2.828 + 14.76 \times 0}{5 + 9.797 + 14.76}$$

$$= \frac{15.206}{29.557} = \mathbf{0.514 \text{ mm}}$$ ...Ans.

$\therefore$ Centroidal coordinates of the line are:

$(\bar{x}, \bar{y})$ = **(6.62,0.514) mm** ...Ans.

Ex. 5.8.5 : A homogeneous wire AB is bent into the shape shown in Fig. P. 5.8.5(a). Determine the centroid of bent up wire. The radius of circle is 200 mm.

SPPU : Dec. 09, 8 Marks

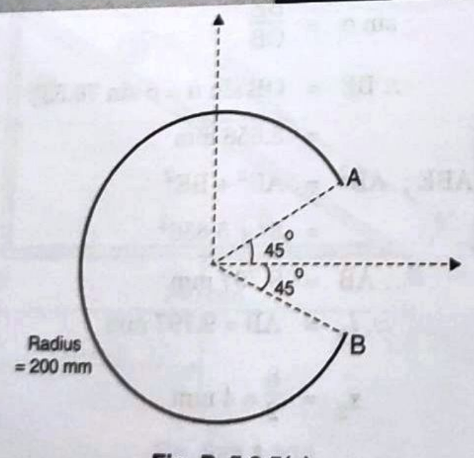


Fig. P. 5.8.5(a)

Soln. :

The bent AB is symmetrical about x – axis.

$$\therefore \bar{y} = 0$$

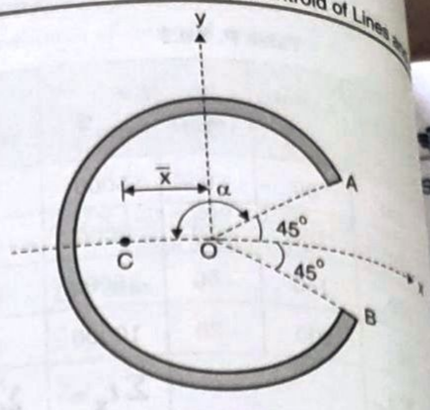


Fig. P. 5.8.5(b)

$\bar{x}$ is given by,

$$\bar{x} = \frac{r \sin \alpha^\circ}{\alpha^c}$$

Radius of circle, $r = 200$ mm

$$\alpha = 180^\circ - 45^\circ$$

$$= \frac{135^\circ \times \pi}{180^\circ} = 2.356^c$$

$$\therefore \bar{x} = \frac{200 \times \sin 135^\circ}{2.356}$$

$$= \mathbf{60.03 \text{ mm}}$$

$\therefore$ Centroid of the bent w.r.t. 'O' is,

$(\bar{x}, \bar{y})$ = **(– 63.03,0) mm**

Ex. 5.8.6 : Locate centre of gravity of bar model as shown in Fig. P. 5.8.6(a).

SPPU : May 07, 8 ...

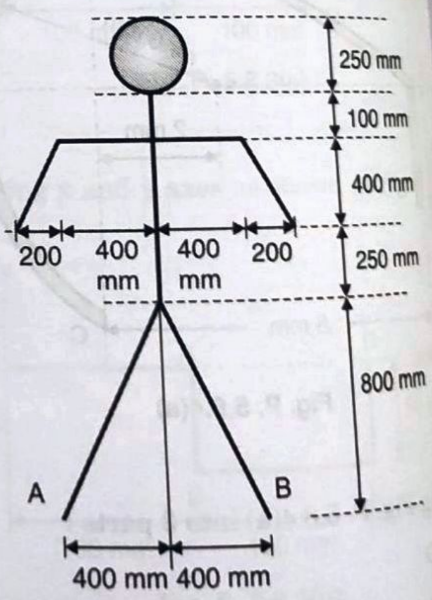


Fig. P. 5.8.6(a)

Soln. :

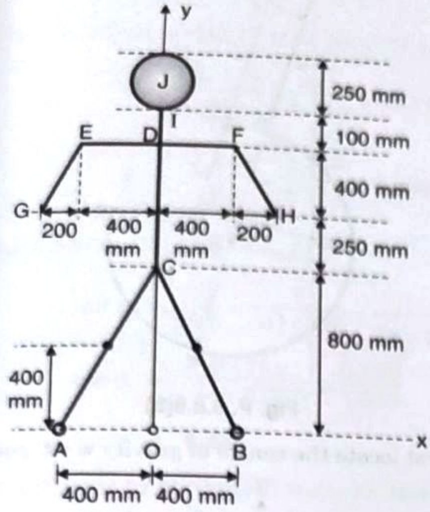


Fig. P. 5.8.6(b)

The bar model is symmetrical about y-axis.

$$\therefore \bar{x} = 0$$

To determine $\bar{y}$, divide the bar model into different line segments as shown in Fig. P. 5.8.6(b).

$\bar{y}$ is given by,

$$\bar{y} = \frac{\Sigma l \cdot y}{\Sigma l}$$

$\Sigma l \cdot y$ = Sum of the moments of line segments at x-axis.

Σl = Sum of the lengths of all line segments.

Refer the following table,

Line Segment	Length l, mm	Distance of centroid from x-axis, 'y' mm	Moment of line segment at x – axis, $(l \cdot y)$ mm^2
AC	$\sqrt{800^2 + 400^2}$ = 894.43	$\frac{800}{2} = 400$	357,772
BC	894.43	400	357,772
CDI	250 + 400 + 100 = 750	$800 + \left(\frac{750}{2}\right)$ = 1,175	881,250.00
EDF	400 + 400 = 800	800 + 250 + 400 = 1450	1,160,000.00
EG	$\sqrt{200^2 + 400^2}$ = 447.21	$800 + 250 + \frac{400}{2} = 1250$	559,012.50
FH	447.21	1250	559,012.50
J	$2 \times \pi \times 125$ = 785.40	800 + 250 + 400 + 100 + 125 = 1675	1,315,545.00
	Σl = 5,018.68	$\Sigma l \cdot y$ =	5,190,364

Note : Segments AC and BC & EG and FH are similar.

$$\therefore \bar{y} = \frac{\Sigma l_y}{\Sigma l} = \frac{5,190,364}{5,018.68} = \mathbf{1034.208\ mm} \quad \text{...Ans.}$$

$\therefore$ The distance of centre of gravity of the model from base AB is **1034.208 mm.**

Ex. 5.8.7 : A slender rod is welded into the shape as shown in Fig. P. 5.8.7(a). Locate the position of centroid of the rod with respect to origin O if AO = BO = CO = 50 mm.

SPPU : Dec. 18, 4 Marks

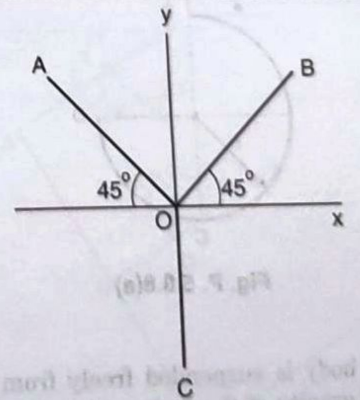


Fig. P. 5.8.7(a)

Soln. :

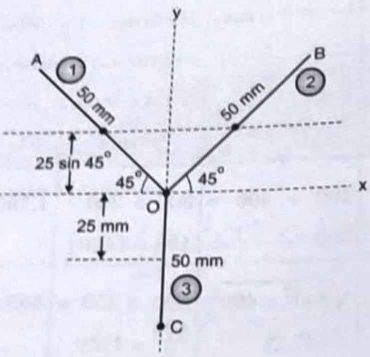


Fig. P. 5.8.7(b)

The rod bent is symmetrical at y-axis.

$$\therefore \bar{x} = 0$$

$$\bar{y} = \frac{\Sigma\, l \cdot y}{\Sigma\, l}$$

$$\bar{y} = \frac{l_1 y_1 + l_2 y_2 + l_3 y_3}{l_1 + l_2 + l_3}$$

$$= \frac{50 \times 25 \sin 45° + 50 \times 25 \sin 45° + 50(-25)}{50 + 50 + 50}$$

$$= \frac{517.776}{150} = \mathbf{3.45\ mm} \qquad \text{...Ans.}$$

With respect to origin 'O', the position of centroid of the rod is, $(\bar{x}, \bar{y})$ = **(0,3.45) mm** ...**Ans.**

Ex. 5.8.8 : A slender homogeneous wire is bent into a shape shown and suspended from point 'A'. Find the angle made by portion AB with the horizontal.

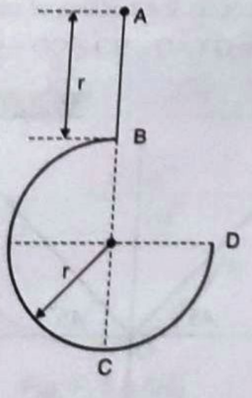


Fig. P. 5.8.8(a)

Soln. :

When a body is suspended freely from any point the centre of gravity of the body will be located on the vertical line passing through point of suspension.

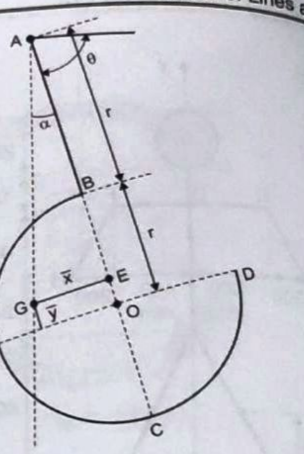


Fig. P. 5.8.8(b)

Let us first locate the centre of gravity w.r.t. point 'O'.

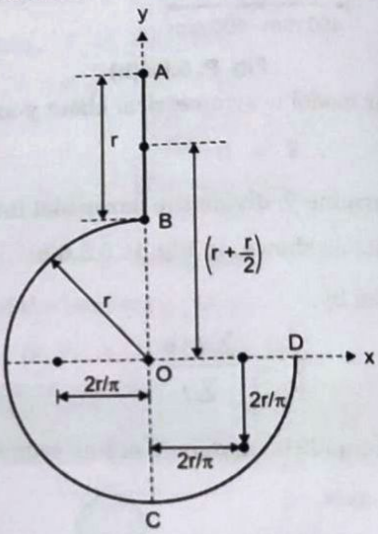


Fig. P. 5.8.8(c)

Divide the bent into 3 parts.

Line	l	x	y	$l \cdot x$	$l \cdot y$
AB	r	0	$r + \frac{r}{2}$ = 1.5 r	0	$1.5\,r^2$
Semicircular arc BC	πr	$-\left(\frac{2r}{\pi}\right)$	0	$-2r^2$	0
Quarter circular arc CD	$\frac{\pi r}{2}$	$\left(\frac{2r}{\pi}\right)$	$-\left(\frac{2r}{\pi}\right)$	r^2	$-r^2$

$$\Sigma\, l = r + \pi r + \frac{\pi r}{2} = 5.71r$$

$$\Sigma\, l_x = -2r^2 + r^2 = -r^2$$

$$\Sigma l_y = 1.5\,r^2 - r^2 = 0.5r^2$$

∴ Position coordinates of C. G. w.r.t point 'O' are :

$$\bar{x} = \frac{\Sigma l \cdot x}{\Sigma l} = \frac{-r^2}{5.71\,r} = \mathbf{-0.175r}$$

$$\bar{y} = \frac{\Sigma l \cdot y}{\Sigma l} = \frac{0.5\,r^2}{5.71\,r} = \mathbf{0.087r}$$

From the Δ AEG,

$$\tan\alpha = \frac{GE}{AE} = \frac{\bar{x}}{AO - OE} = \frac{\bar{x}}{2r - \bar{y}}$$

$$\tan\alpha = \frac{0.175\,r}{2r - 0.087r} = \frac{0.175}{1.913}$$

$$\therefore \alpha = \mathbf{5.23^\circ}$$

∴ The angle made by portion AB w.r.t. horizontal,

$$\theta = 90^\circ - 5.23^\circ = 84.77^\circ \quad \text{...Ans.}$$

Ex. 5.8.9 : A thin homogeneous wire of length 350 mm is bent into the shape of letter C having height 150 mm as shown in Fig. P. 5.8.9(a). This bent is then suspended from the end 'A'. Determine the angle made by portion AB w.r.t. horizontal.

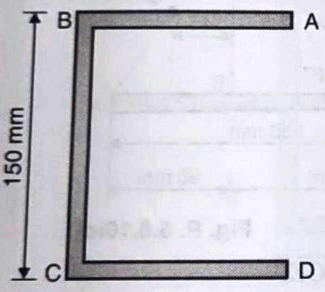


Fig. P. 5.8.9(a)

Soln. :

Total length of the wire is 350 mm

Height of letter C = 150 mm

$$\therefore \text{Width of each horizontal portion} = \frac{350 - 150}{2} = 100 \text{ mm}$$

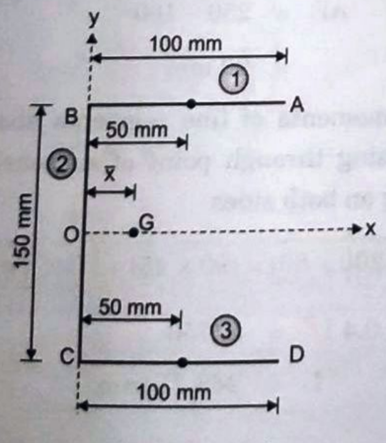


Fig. P. 5.8.9(b)

Let us first determine the position of C. G. of letter 'C' w.r.t. point 'O'.

Bent is symmetrical at x-axis.

$$\therefore \bar{y} = 0.$$

To find $\bar{x}$, divide the bent into 3 parts

$$\bar{x} = \frac{\Sigma l_x}{\Sigma l} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3}{l_1 + l_2 + l_3}$$

$$= \frac{100 \times 50 + 150 \times 0 + 100 \times 50}{100 + 150 + 100} = \mathbf{28.57\ mm}$$

When it is suspended from end 'A', the C.G. of the bent will be located on the vertical line passing through point 'A'. Let 'α' be the angle made by portion, AB w.r.t horizontal.

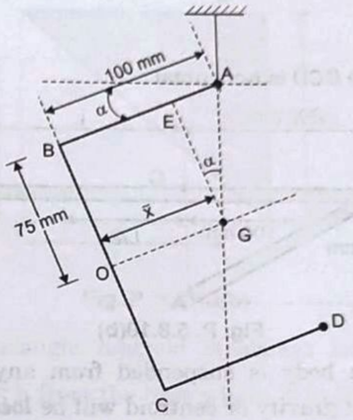


Fig. P. 5.8.9(c)

From Fig. P. 5.8.9(c) considering triangle AEG,

$$\tan\alpha = \frac{AE}{EG} = \frac{100 - \bar{x}}{75} = \frac{100 - 28.57}{75}$$

$$\tan\alpha = \left(\frac{71.43}{75}\right)$$

$$\therefore \alpha = \mathbf{43.60^\circ}$$

∴ The angle made by portion AB w.r.t. horizontal,

$$\alpha = \mathbf{43.60^\circ} \quad \text{...Ans.}$$

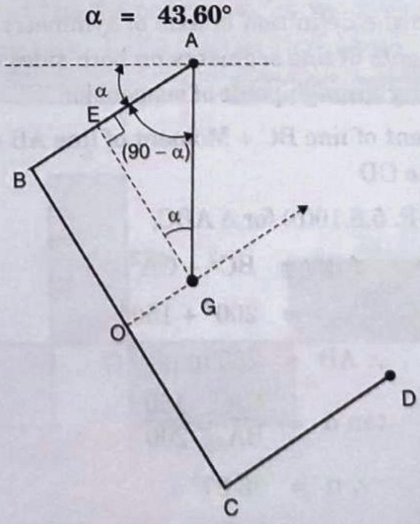


Fig. P. 5.8.9(d)

Ex. 5.8.10 : The homogeneous wire ABCD is bent as shown and is attached to a hinge at C. Determine the length 'L' for which (i) Portion BCD is horizontal

(ii) Portion AB is horizontal

SPPU : May 98, 8 Marks

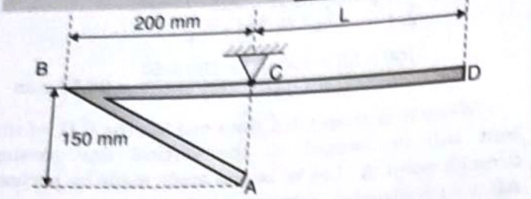


Fig. P. 5.8.10(a)

Soln. :

(i) Portion BCD is horizontal :

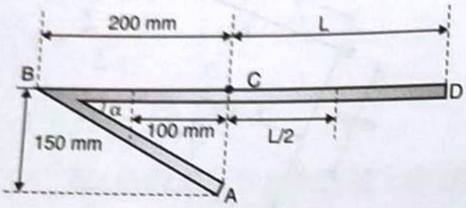


Fig. P. 5.8.10(b)

When a body is suspended from any point the centre of gravity or centroid will be located on the vertical line passing through the point of suspension.

Here the wire is suspended from point 'C'. Therefore, the centroid of the bent will be located on the vertical line passing through point 'C'.

We can consider this vertical line passing through 'C' as an axis of symmetry, because centroid is always located on axis of symmetry.

From the definition of axis of symmetry, equating moments of line segments on both sides of the line passing through point of suspension 'C'.

Moment of line BC + Moment of line AB = Moment of line CD

From Fig. P. 5.8.10(b) for Δ ABC,

$$AB^2 = BC^2 + CA^2$$

$$= 200^2 + 150^2$$

$$\therefore AB = 250 \text{ mm}$$

$$\tan \alpha = \frac{CA}{BA} = \frac{150}{200}$$

$$\therefore \alpha = 36.87°$$

Note : Moment of line at an axis = Length of line × perpendicular distance of centroid of line from axis of symmetry)

$$\therefore 250 \times 100 + 200 \times 100 = L \times \frac{L}{2}$$

$$\therefore \frac{L^2}{2} = 45000$$

$$\therefore \mathbf{L = 300 \text{ mm}}$$ ...Ans.

(ii) Portion 'AB' is horizontal :

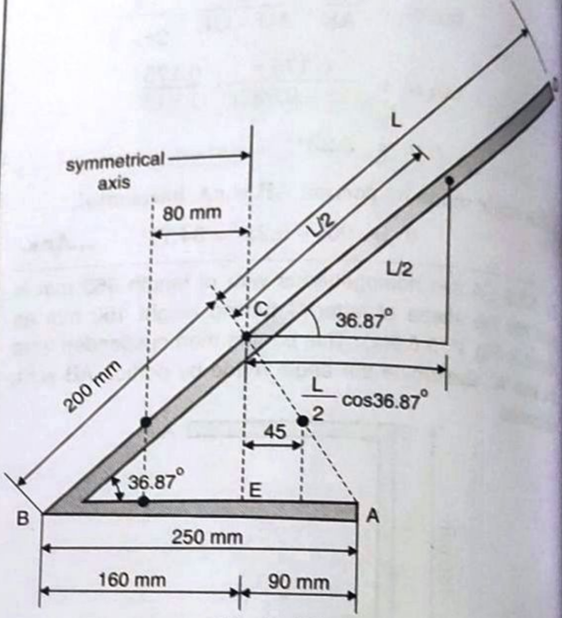


Fig. P. 5.8.10(c)

From Δ BCE,

$$\cos 36.87° = \frac{BE}{BC}$$

$$= \frac{BE}{200}$$

$$\therefore BE = 160 \text{ mm}$$

$$AE = 250 - 160$$

$$= 90 \text{ mm}$$

Taking moments of line segments about vertical line passing through point of suspension 'C' and equating on both sides.

$$(160 \times 80) + (200 \times 80) = (90 \times 45) + \left(L \times \frac{L}{2} \cos 36.87°\right)$$

$$0.4 L^2 = 24750$$

$$\mathbf{L = 248.75 \text{ mm}}$$ ...Ans.

Ex. 5.8.11 : Find the length 'a' of wire such that the centroid will be at centre 'O' of semicircular arc of wire.

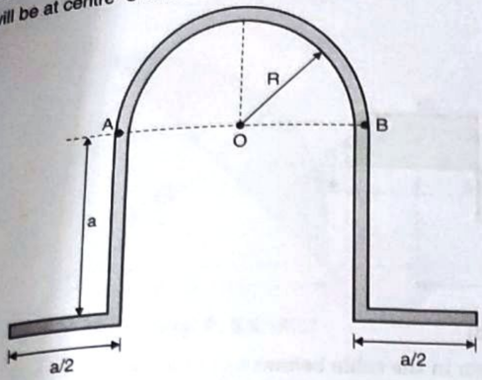


Fig. P. 5.8.11(a)

Soln. :

Give that the centroid is at point 'O'.

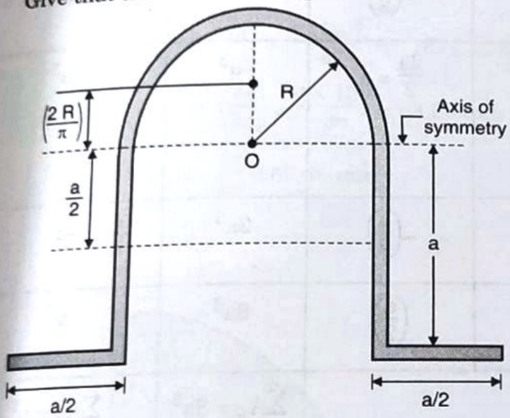


Fig. P. 5.8.11(b)

Therefore considering horizontal line passing through point 'O' as an axis of summetry and equating moments above and below the axis of symmetry,

$$(\pi R) \times \frac{2R}{\pi} = 2\left[\left(\frac{a}{2}\right)(a)\right] + 2\left[(a)\left(\frac{a}{2}\right)\right]$$

$$2R^2 = 2\left[\frac{a^2}{2} + \frac{a^2}{2}\right]$$

$$R^2 = a^2$$

$$\therefore \mathbf{a = R}$$ **...Ans.**

Type 2 : Based on Centroid of Areas

Ex. 5.8.12 : Determine the position of centroid of the shaded area as shown in Fig. P. 5.8.12(a) with respect to origin O.

SPPU : May 13, 6 Marks

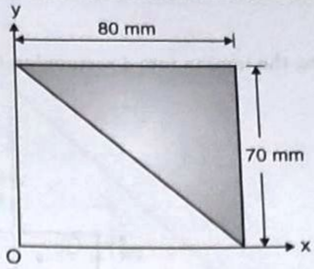


Fig. P. 5.8.12(a)

Soln. :

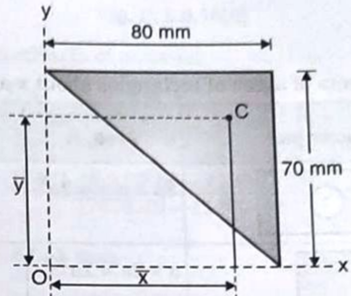


Fig. P. 5.8.12(b)

For a triangle centroid is always located at a distance of $\frac{2}{3}$h from the base where, h is the height of the triangle.

$$\therefore \bar{x} = \frac{2}{3} \times 80 = 53.33 \text{ mm}$$

$$\bar{y} = \frac{2}{3} \times 70 = 46.67 \text{ mm}$$

$\therefore$ The position of centroid of the shaded area wrt origin 'O' is $(\bar{x}, \bar{y})$ **= (53.33,46.67) mm** **...Ans.**

Ex. 5.8.13 : Locate the co-ordinates of the centroid of the lamina, shown in Fig. P. 5.8.13(a) with reference to the axes shown.

SPPU : May 04, 6 Marks

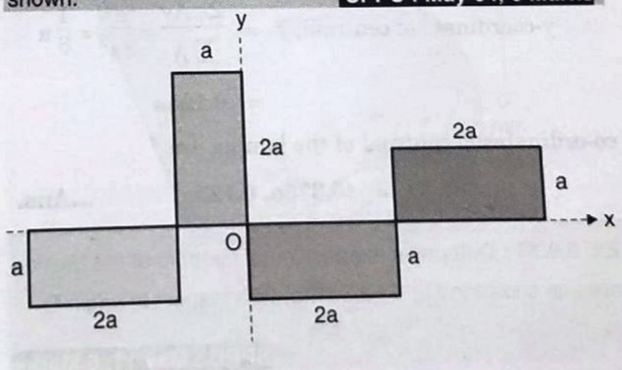


Fig. P. 5.8.13(a)

Soln. :

Dividing the lamina into 4 rectangles.

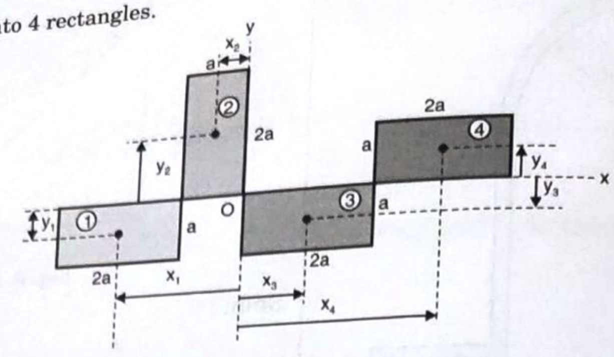


Fig. P. 5.8.13(b)

Moments of areas of rectangles about x and y axes are shown in the table below :

Rectangle	Area, A	x	y	Ax	Ay
1	$2a \times a = = 2a^2$	$-(a+a) = -2a$	$-\left(\frac{a}{2}\right)$	$-4a^3$	$-a^3$
2	$a \times 2a = 2a^2$	$-\left(\frac{a}{2}\right)$	$\frac{2a}{2} = a$	$-a^3$	$2a^3$
3	$2a \times a = 2a^2$	$\frac{2a}{2} = a$	$-\left(\frac{a}{2}\right)$	$2a^3$	$-a^3$
4	$2a \times a = 2a^2$	$2a + \left(\frac{2a}{2}\right) = 3a$	$\left(\frac{a}{2}\right)$	$6a^3$	a^3
	$\Sigma A = 8a^2$			$\Sigma A_x = 3a^3$	$\Sigma A_y = a^3$

x-coordinate of centroid, $\bar{x} = \frac{S\,Ax}{S\,A} = \frac{3a3}{8a2} = \frac{3}{8}a$

$= 0.375a$

y-coordinate of centroid, $\bar{y} = \frac{\Sigma\,Ay}{\Sigma\,A} = \frac{a^3}{8a^2} = \frac{1}{8}a$

$= \mathbf{0.125a}$

co-ordinates of centroid of the lamina are :

$(\bar{x}, \bar{y}) = (0.375a, 0.125a)$ **...Ans.**

Ex. 5.8.14 : Determine the position of centroid of the shaded area as shown in Fig. P. 5.8.14(a) with respect to origin O.

SPPU : May 11, 6 Marks

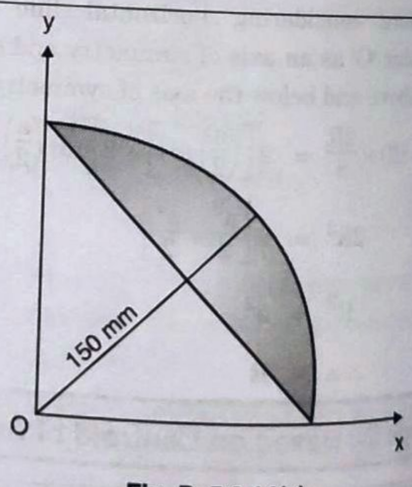


Fig. P. 5.8.14(a)

Soln. :

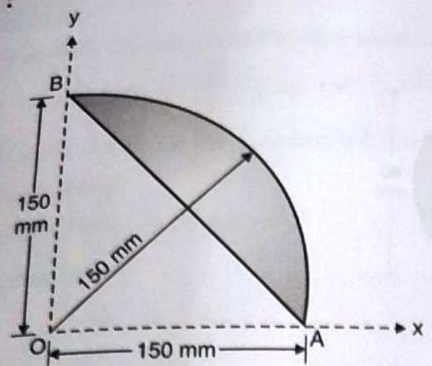


Fig. P. 5.8.14(b)

Dividing the area in to 2 parts :

① Quarter circle OAB

② Triangle OAB (to be removed)

For ① Quarter circle OAB :

$$A_1 = \frac{\pi r^2}{4} = \frac{\pi(150)^2}{4} = 17671.46 \text{ mm}^2$$

$$x_1 = \left(\frac{4r}{3\pi}\right) = \frac{4 \times 150}{3\pi} = 63.66 \text{ mm}$$

$$y_1 = \frac{4r}{3\pi} = 63.66 \text{ mm}$$

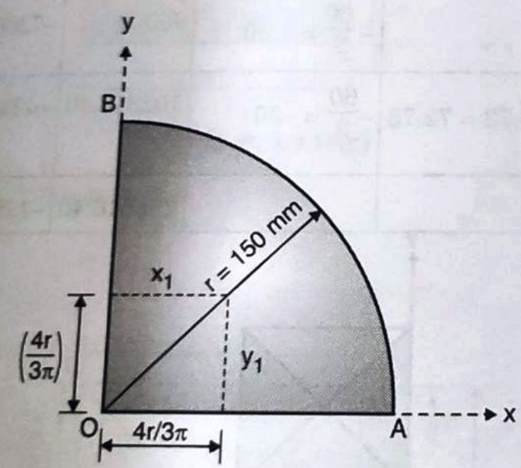


Fig. P. 5.8.14(c)

For ② Triangle OAB :(Cut out area)

$$A_2 = \frac{1}{2} \times 150 \times 150 = 11250 \text{ mm}^2$$

$$x_2 = \frac{1}{3} \times 150 = 50 \text{ mm}$$

$$y_2 = \frac{1}{3} \times 150 = 50 \text{ mm}$$

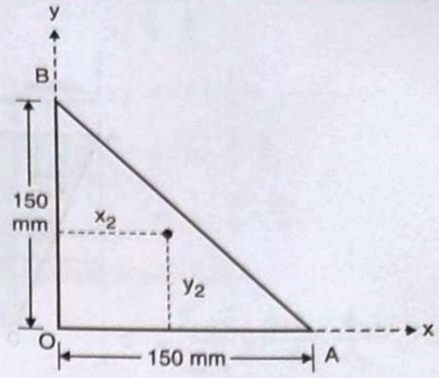


Fig. P. 5.8.14(d)

∴ x-coordinate of centroid,

$$\bar{x} = \frac{\Sigma A_x}{\Sigma A} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

$$= \frac{17671.46 \times 63.66 - 11250 \times 50}{17671.46 - 11250}$$

$$= \mathbf{87.59 \text{ mm}}$$ **...Ans.**

y-coordinate of centroid,

$$\bar{y} = \frac{\Sigma A_y}{\Sigma A} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$= \frac{17671.46 \times 63.66 - 11250 \times 50}{17671.46 - 11250}$$

$$= \mathbf{87.59 \text{ mm}}$$ **...Ans.**

∴ The position of centroid of shaded area w.r.t. of 'O' is

$$(\bar{x}, \bar{y}) = (87.59, 87.59) \text{ mm}$$ **...Ans.**

Ex. 5.8.15 : Locate the centroid of the plane lamina shown in Fig. P. 5.8.15(a).

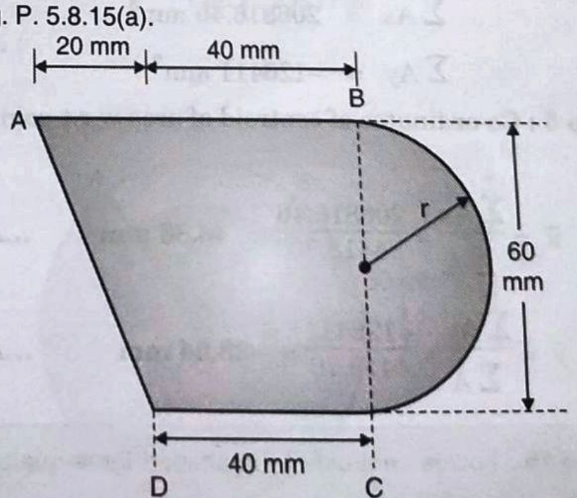


Fig. P. 5.8.15(a)

Soln. :

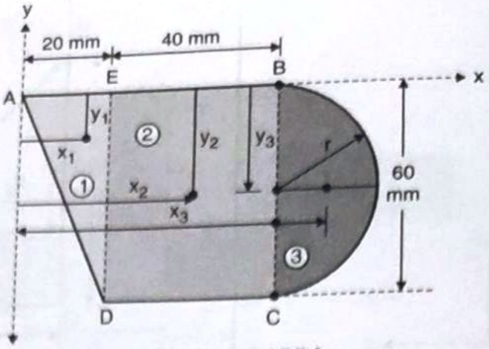


Fig. P. 5.8.15(b)

Step 1 : Select reference point 'A' and axes x and y as shown in Fig.

Step 2 : Divide the Fig. in to 3 parts.

① Triangle AED

② Rectangle EBCD

③ Semicircle BC

Step 3 : Obtain and tabulate the values of A, A_x and A_y.

Fig.	A (mm^2)	x (mm)	y mm)	A_x mm^3	A_y mm
① Triangle AED	$\frac{1}{2} \times 20 \times 60 = 600$	$\frac{2}{3} \times 20 = 13.33$	$-\frac{1}{3} \times 60 = -20$	7998	-1200
② Rectangle EBCD	$40 \times 60 = 2400$	$20 + \frac{40}{2} = 40$	$-\frac{60}{2} = -30$	96000	-7200
③ Semicircle BC	$\frac{\pi(30)^2}{2} = 1413.7$ 1413.7	$20 + 40 + \left(\frac{4 \times 30}{3\pi}\right) = 72.73 = 72.73$	$-\frac{60}{2} = -30$	102818.40	-4241
	4413.70			206816.40	-12641

Step 4 : Get the sum of

$$\sum A = 4413.70 \text{ mm}^2$$

$$\sum Ax = 206816.40 \text{ mm}^3$$

$$\sum Ay = -126411 \text{ mm}^2$$

Step 5 : Co-ordinates of centroid of area w.r.t point 'A' are :

$$\bar{x} = \frac{\sum Ax}{\sum A} = \frac{206816.40}{4413.70} = \mathbf{46.86 \text{ mm}} \quad \text{...Ans.}$$

and $$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{-126411}{4413.70} = \mathbf{-28.64 \text{ mm}} \quad \text{...Ans.}$$

Ex. 5.8.16 : Locate centroid of the shaded three-quarters of the area of a square of dimension 'a' as shown in Fig. P. 5.8.16(a). **SPPU : May 10, 8 Marks**

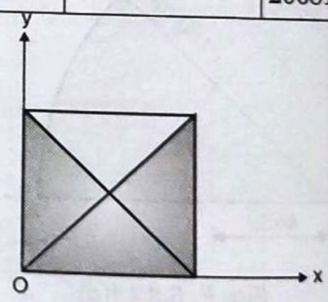


Fig. P. 5.8.16(a)

Soln. :

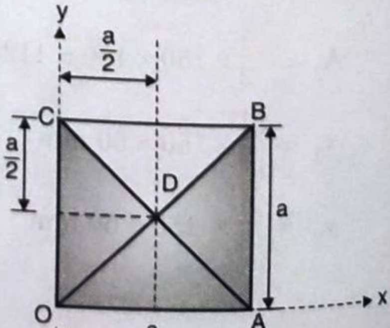


Fig. P. 5.8.16(b)

The given Fig. P. 5.8.16(b) is symmetrical at rtical axis.

Centroid is always located on symmetrical axis.

$$\therefore \bar{x} = \left(\frac{a}{2}\right) = 0.5a$$

To find $\bar{y}$, we divide the area into a 2 parts

Square OABC

Triangle CBD)(to be removed)

r ①Square OABC :

$$A_1 = a \times a = a^2$$

$$x_1 = \frac{a}{2}$$

$$y_1 = \frac{a}{2}$$

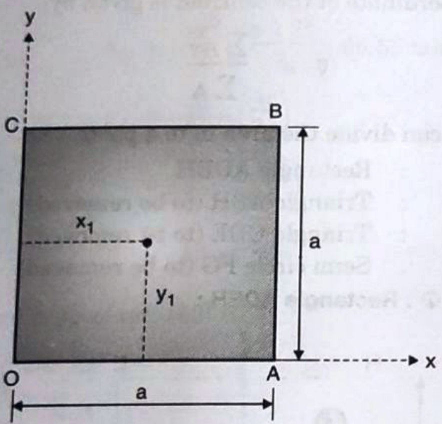


Fig. P. 5.8.16(c)

or ②Triangle CBD :

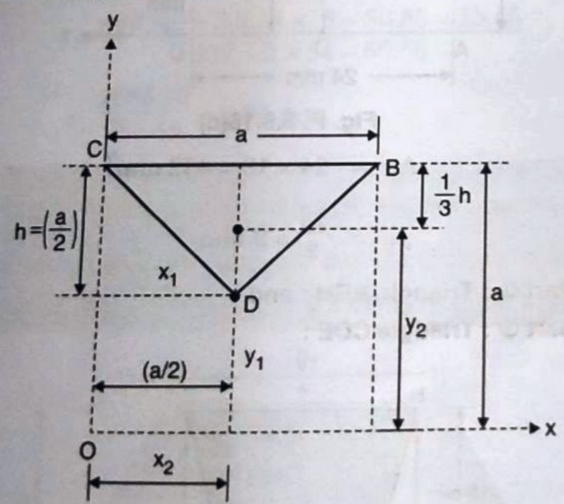


Fig. P. 5.8.16(b)

$$A_2 = \frac{1}{2} \times a \times h$$

$$= \frac{1}{2} \times a \times \frac{a}{2} = \frac{a^2}{4}$$

$$x_2 = \frac{a}{2}$$

$$y_2 = a - \frac{1}{3}h$$

$$= a - \frac{1}{3} \cdot \frac{a}{2}$$

$$= a - \frac{a}{6} = \frac{5a}{6}$$

y-coordinate of centroid,

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$= \frac{\left(a^2 \times \frac{a}{2} - \frac{a^2}{4} \times \frac{5a}{6}\right)}{\left(a^2 - \frac{a^2}{4}\right)} = \frac{\left(\frac{a^3}{2} - \frac{5a^3}{24}\right)}{\left(\frac{3a^2}{4}\right)} = \frac{\left(\frac{12a^3 - 5a^3}{24}\right)}{\left(\frac{3a^2}{4}\right)}$$

$$= \frac{7a^3}{24} \times \frac{4}{3a^2} = \frac{7a}{18} = \mathbf{0.389a} \quad \text{...Ans.}$$

$\therefore$ The position of centroid of shaded area is

$$(\bar{x}, \bar{y}) = \left(\frac{a}{2}, \frac{7a}{18}\right) = \mathbf{(0.5a, 0.389a)} \quad \text{...Ans.}$$

Ex. 5.8.17 : Locate the centroid of remaining lamina with respect to O, if shaded part is removed. Refer Fig. P. 5.8.17(a). **SPPU : May 08, 8 Marks**

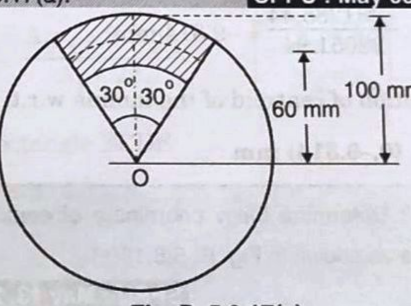


Fig. P. 5.8.17(a)

Soln. :

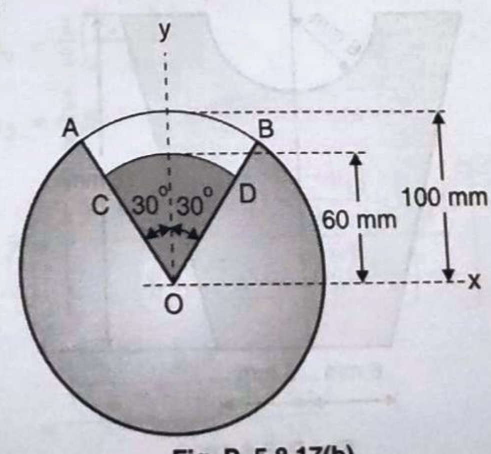


Fig. P. 5.8.17(b)

Divide the lamina into 3 parts.

① Whole circle of radius 100 mm.

② Sector of a circle of radius 100 mm (to be removed)

③ Sector of a circle of radius 60 mm (to be added)

The given lamina is symmetrical at y - axis.

$\therefore \bar{x} = 0$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{A_1y_1 - A_2y_2 + A_3y_3}{A_1 - A_2 + A_3}$$

$$A_1 = \pi R^2 = \pi \times 100^2 = 31415.92 \text{ mm}^2$$

$$y_1 = 0$$

$$A_2 = \alpha^c r^2, \qquad \alpha = 30° \times \frac{\pi}{180°} = 0.52^c$$

$$= 0.52\,(100)^2 = 5200 \text{ mm}^2$$

$$y_2 = \frac{2\,r \sin\alpha°}{3\alpha^c} = \frac{2 \times 100 \times \sin 30°}{3 \times 0.52} = 64.10 \text{ mm}$$

$$A_3 = \alpha^c r^2 = 0.52\,(60)^2 = 1872 \text{ mm}^2$$

$$y_3 = \frac{2\,r \sin\alpha°}{3\alpha^c} = \frac{2 \times 60 \times \sin 30°}{3 \times 0.52}$$

$$= 38.46 \text{ mm}$$

$$\therefore \bar{y} = \frac{31415.92 \times 0 - 5235.98 \times 64.10 + 1872 \times 38.48}{31415.92 - 5235.98 + 1872}$$

$$= \frac{-261285.44}{28051.94} = -9.314 \text{ mm}$$

$\therefore$ The position of centroid of the lamina w.r.t. 'O' is,

$(\bar{x}, \bar{y}) = (0, -9.314)$ mm ...Ans.

Ex. 5.8.18 : Determine the y coordinate of centroid of the shaded area as shown in Fig. P. 5.8.18(a).

SPPU : May 13, 2 Marks

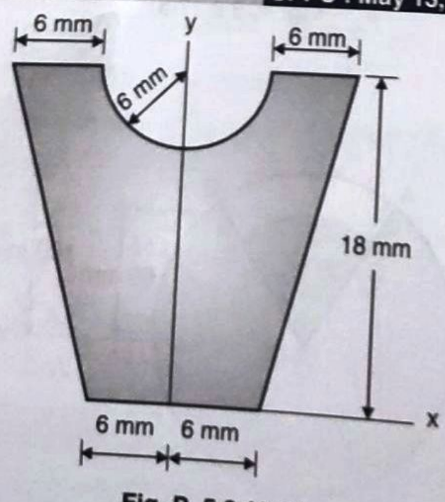


Fig. P. 5.8.18(a)

Soln. :

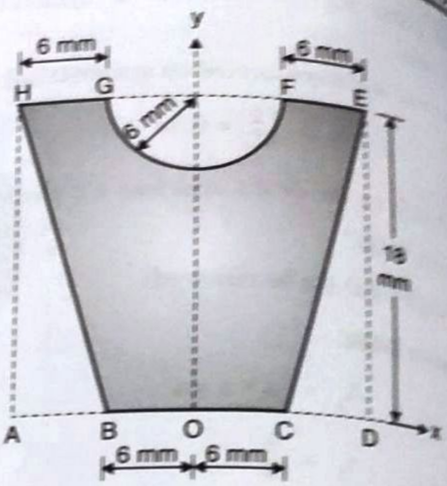


Fig. P. 5.8.18(b)

y-coordinate of the centroid is given by,

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A}$$

We can divide the area in to 4 parts

Part ① : Rectangle ADEH

Part ② : Triangle ABH (to be removed)

Part ③ : Triangle CDE (to be removed)

Part ④ : Semi circle FG (to be removed)

For part ① : Rectangle ADEH :

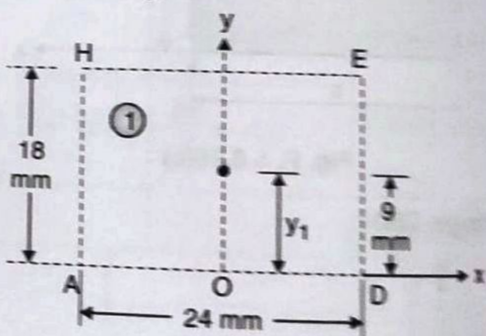


Fig. P. 5.8.18(c)

$$A_1 = 24 \times 18 = 432 \text{ mm}^2$$

$$y_1 = \frac{18}{2} = 9 \text{ mm}$$

For Part ② : Triangle ABH : and

For part ③ : Triangle CDE :

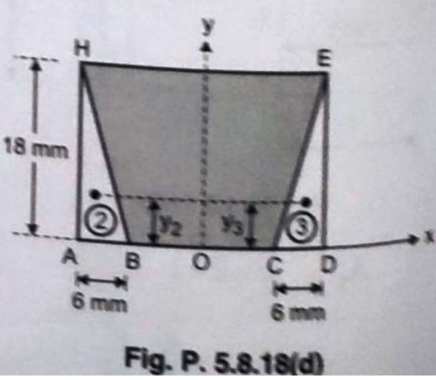


Fig. P. 5.8.18(d)

$$A_2 = A_3 = \frac{1}{2} \times 6 \times 18 = 54 \text{ mm}^2$$

$$y_2 = y_3 = \frac{1}{3} \times 18 = 6 \text{ mm}$$

For part ④: Semicircle FG :

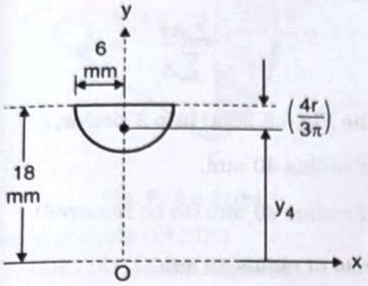


Fig. P. 5.8.18(e)

$$A_4 = \frac{\pi r^2}{2} = \frac{\pi \times 6^2}{2} = 56.55 \text{ mm}^2$$

$$y_4 = 18 - \left(\frac{4r}{3\pi}\right)$$

$$= 18 - \left(\frac{4 \times 6}{3\pi}\right)$$

$$= 15.45 \text{ mm}$$

∴ y-coordinate of centroid is,

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A}$$

$$= \frac{A_1 y_1 - 2 A_2 y_2 - A_4 y_4}{A_1 - 2A_2 - A_4} \qquad \begin{pmatrix} A_2 = A_3 \\ y_2 = y_3 \end{pmatrix}$$

$$= \frac{432 \times 9 - 2 \times 54 \times 6 - 56.55 \times 15.45}{432 - 2 \times 54 - 56.55}$$

$$= \frac{2366.30}{267.45} = \mathbf{8.85 \text{ mm}} \qquad \text{...Ans.}$$

Ex. 5.8.19 : Determine the distance y to the centroid of the trapezoidal area in terms of the dimensions shown in Fig. P. 5.8.19(a). **SPPU : May 12, 6 Marks**

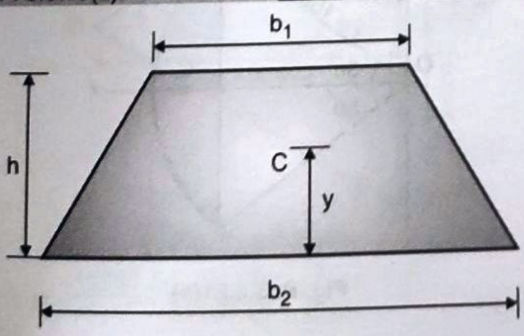


Fig. P. 5.8.19(a)

Soln. :

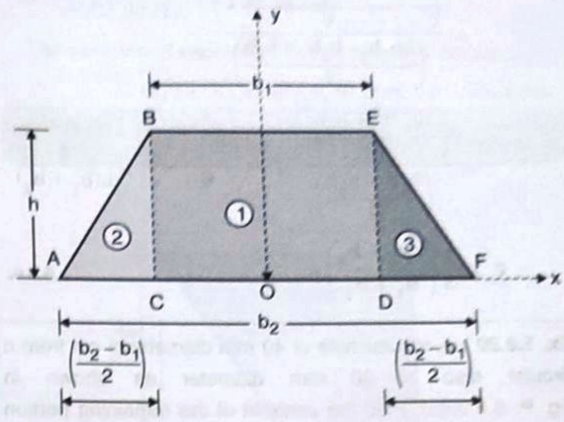


Fig. P. 5.8.19(b)

Part ① : Rectangle BCDE

Part ② : Triangle ABC

Part ③ : Triangle DEF

Dividing the trapezoidal area into three parts, the distance $\bar{y}$ of the centroid is given by :

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A}$$

$$= \frac{A_1 y_1 + 2 A_2 y_2}{A_1 + 2A_2} \qquad \left[\because \begin{matrix} A_2 = A_3 \\ y_2 = y_3 \end{matrix}\right]$$

Part ①Rectangle BCDE

$$A_1 = b_1 \cdot h$$

$$y_1 = \frac{h}{2}$$

Part ② And ③ Triangle ABC And DEF

$$A_2 = \frac{1}{2}\left(\frac{b_2 - b_1}{2}\right) h$$

$$y_2 = \frac{1}{3} h$$

$$\therefore \quad \bar{y} = \left[\frac{b_1 h \cdot \frac{h}{2} + 2 \times \frac{1}{2}\left(\frac{b_2 - b_1}{2}\right) h \times \frac{1}{3} h}{\left(b_1 h + \frac{2 \times 1}{2}\left(\frac{b_2 - b_1}{2}\right) h\right)}\right]$$

$$= \frac{\left(\frac{b_1 h^2}{2} + \frac{b_2 h^2}{6} - \frac{b_1 h^2}{2}\right)}{\left(b_1 h + \frac{b_2 h}{2} - \frac{b_1 h}{2}\right)}$$

$$= \frac{\left(\frac{3b_1 h^2 + b_2 h^2 - b_1 h^2}{6}\right)}{\left(\frac{2b_1 h + b_2 h - b_1 h}{2}\right)}$$

$$= \frac{\left(\frac{2b_1 h^2 + b_2 h^2}{6}\right)}{\left(\frac{b_1 h + b_2 h}{2}\right)} = \frac{h^2(2b_1 + b_2)}{6} \times \frac{2}{h(b_1 + b_2)}$$

$$\therefore \bar{y} = \frac{1}{3}\left(\frac{2b_1 + b_2}{b_1 + b_2}\right) h \qquad \text{...Ans.}$$

Ex. 5.8.20 : A circular hole of 40 mm diameter is cut from a circular disc of 80 mm diameter as shown in Fig. P. 5.8.20(a). Find the centroid of the remaining portion w.r.t. point 'A'.

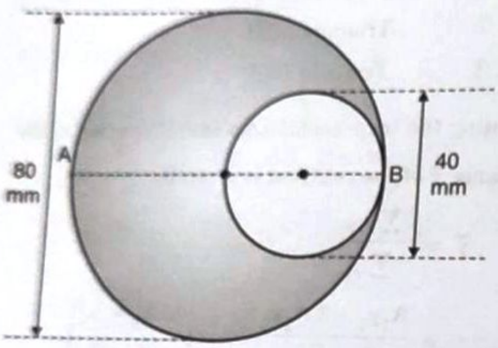


Fig. P. 5.8.20(a)

Soln. :

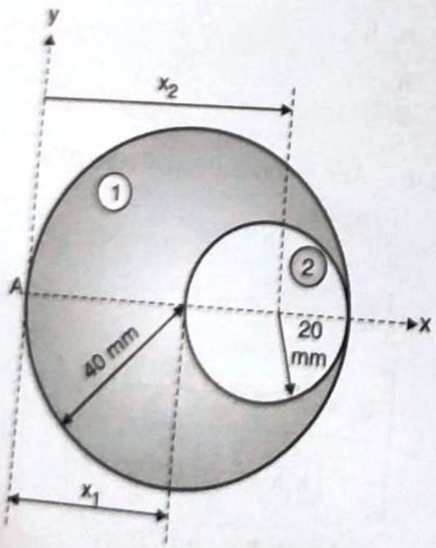


Fig. P. 5.8.20(b)

Selecting x and y axes w.r.t. point 'A' as shown in Fig. P. 5.8.20(b).

Fig. P. 5.8.20(b) is symmetrical at x-axis.

$\therefore$ **y-coordinate of centroid,**

$$\bar{y} = 0$$

x-coordinate is given by

$$\bar{x} = \frac{\sum Ax}{\sum A}$$

Dividing the Fig.5.8.20(a) into 2 circles,

① Circle of radius 40 mm.

② Circle of radius 20 mm (to be removed)

For ① – circle of radius 40 mm :

$$A_1 = \pi(40)^2 = \mathbf{5026.55\ mm^2}$$

$$x_1 = 40 = \mathbf{40\ mm}$$

For ② – circle of radius 20 mm :

$$A_2 = \pi(20)^2 = \mathbf{1256.64\ mm^2}$$

$$x_2 = 40 + 20 = \mathbf{60\ mm}$$

$\therefore$ x-coordinate of centroid,

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

$$\therefore \bar{x} = \frac{5026.55 \times 40 - 1256.64 \times 60}{5026.55 - 1256.64}$$

$$= \mathbf{33.33\ mm}$$

$\therefore$ The position of centroid of the remaining area w.r.t. point 'A' is

$$(\bar{x}, \bar{y}) = \mathbf{(33.33, 0)\ mm} \qquad \text{...Ans.}$$

Ex. 5.8.21 : Determine position of the centroid C of the shaded area which is part of the circle having a radius. Refer Fig. P. 5.8.21(a).

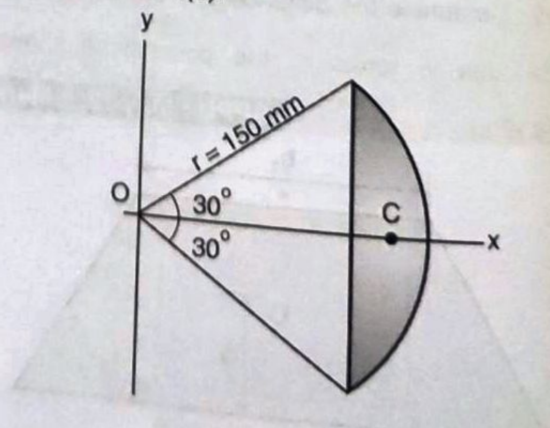


Fig. P. 5.8.21(a)

Soln. :

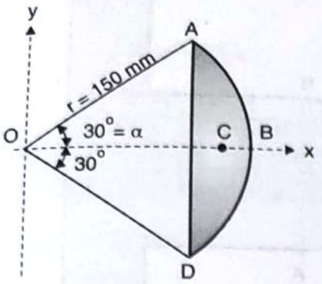


Fig. P. 5.8.21(b)

Part- ① Sector of circle OABDO.

Part- ② Triangle OAD (to be removed).

The given area is symmetrical at x-axis.

$$\therefore \bar{y} = 0$$

$$\bar{x} = \frac{\sum Ax}{\sum A}$$

Part- ① Sector of circle OABDO.

A_1 = Area of whole sector

$$= \alpha^c r^2 \qquad \alpha = 30° = \frac{30° \times \pi}{180°} = 0.523^c$$

$$= 0.523 \times 150^2 = \mathbf{11767.50\ mm^2}$$

x_1 = Distance of centroid of sector from y-axis

$$= \frac{2\,r \sin \alpha°}{3\alpha^c} = \frac{2 \times 150 \times \sin 30°}{3 \times 0.523}$$

$$= \mathbf{95.60\ mm}$$

Part- ② Triangle OAD

$$A_2 = \text{Area of triangle} = \frac{1}{2} \times 150 \times 130 = \mathbf{9750\ mm^2}$$

$$x_2 = \frac{2}{3} \times 130 = \mathbf{86.67\ mm}$$

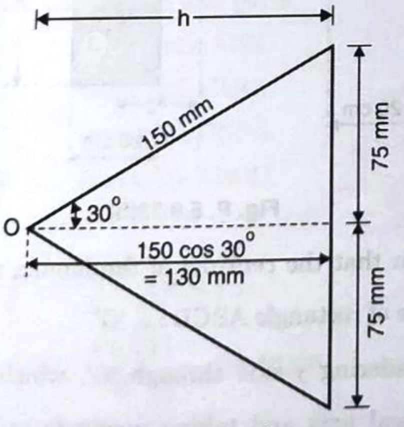


Fig. P. 5.8.21(c)

$$\therefore \bar{x} = \frac{(11767.50)\,(95.60) - (9750)\,(86.67)}{11767.50 - 9750}$$

$$= \frac{279940.5}{2017.5} = \mathbf{138.76\ mm}$$

∴ The position of centroid 'C' of the shaded area is,

$$(\bar{x}, \bar{y}) = (138.76, 0)\ \text{mm} \qquad \text{...Ans.}$$

Ex. 5.8.22 : Locate the centroid of the plane lamina as shown in Fig. P. 5.8.22(a). **SPPU : Dec. 18, 6 Marks**

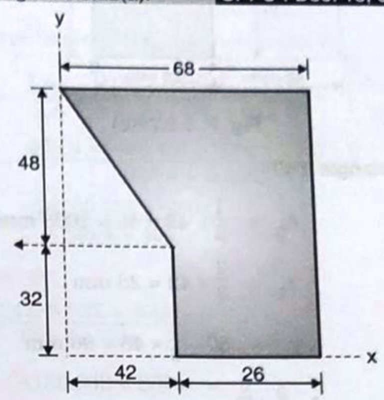


Fig. P. 5.8.22(a)

Soln. :

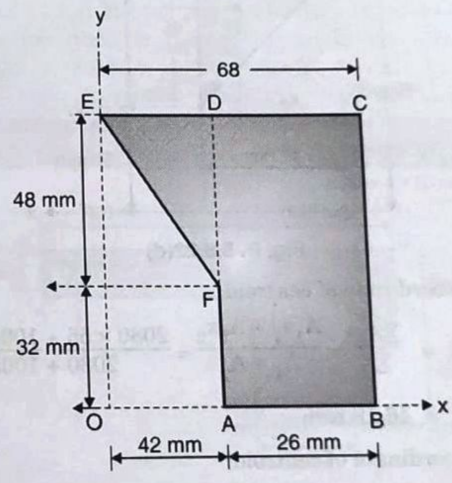


Fig. P. 5.8.22(b)

Divide the Fig. in to two parts.

① Rectangle ABCD ② Triangle DEF

For ① Rectangle ABCD :

$$A_1 = 80 \times 26 = 2080\ \text{mm}^2$$

$$x_1 = 42 + \frac{26}{2} = 55\ \text{mm}$$

$$y_1 = \frac{80}{2} = 40\ \text{mm}$$

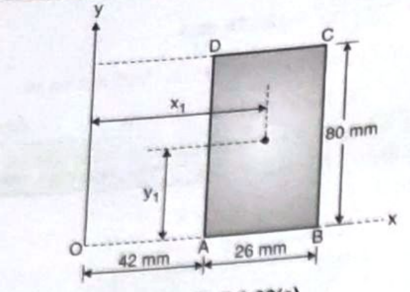


Fig. P. 5.8.22(c)

For ② Triangle 'DEF' :

$$A_2 = \frac{1}{2} \times 42 \times 48 = 1008\ mm^2$$

$$x_2 = \frac{2}{3} \times 42 = 28\ mm$$

$$y_2 = 80 - \frac{1}{3} \times 48 = 96\ mm$$

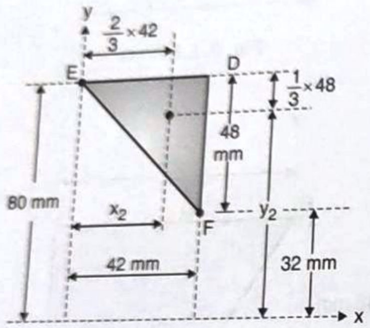


Fig. P. 5.8.22(d)

x-coordinate of centroid,

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{2080 \times 55 + 1008 \times 28}{2080 + 1008}$$

$$= \mathbf{46.18\ mm}$$

y-coordinate of centroid,

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

$$= \frac{2080 \times 40 + 1008 \times 96}{2080 + 1008} = \mathbf{58.28\ mm}$$

∴ Co-ordinate of centroid are :

$$(\bar{x}, \bar{y}) = \mathbf{(46.18, 58.28)\ mm}$$

...Ans.

Ex. 5.8.23 : Find the value of distance 'a' so that the centroid of the uniform lamina shown in Fig. P. 5.8.23(a) remains at the centre of rectangle ABCD.

SPPU : Dec. 02, 6 Marks

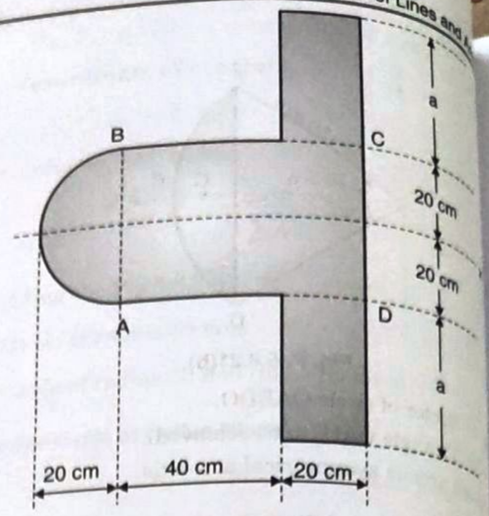


Fig. P. 5.8.23(a)

Soln. :

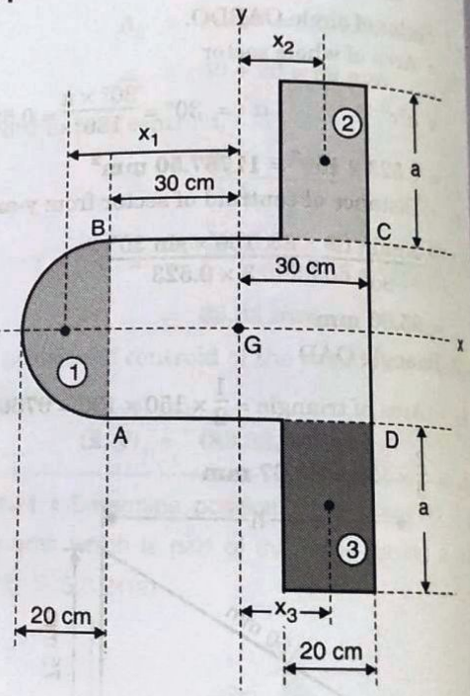


Fig. P. 5.8.23(b)

Given that the centroid of the lamina remains at the centre of rectangle ABCD i.e. 'G'.

Considering y-axis through 'G', which acts as symmetrical axis and taking moments at y-axis and equating on both sides.

$$A_1 \times x_1 = A_2x_2 + A_3x_3$$

Engineering Mechanics (SPPU)

$$\left(\frac{\pi \times 20^2}{2}\right)\left[30 + \left(\frac{4 \times 20}{3\pi}\right)\right] = 20 \times a \times (3$$

$$628.32\ (38.48) = 400a + 4$$

$$\therefore a = \mathbf{30.22\ c}$$

Note : Symmetrical axis is a moments of area of lamin

∴ The distance, **a = 30.22 cm**

Alternate method :

Given that the centroid i

$$\therefore \bar{x} = 0$$

$$\bar{y} = 0$$

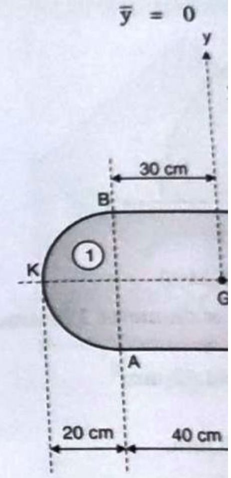


Fig. F

Dividing the Fig-int

Part ① : Semi ci

Part ② : Rectan

Part ③ : Rectan

Part ④ : Rectan

Part ① Semi circle

$$A_1 = \frac{\pi r^2}{2} = \frac{\pi (20)^2}{2}$$

$$x_1 = -\left[30 + \frac{4 \times 2}{3\pi}\right.$$

Part ② Rectangle

$$A_2 = 20a$$

$$x_2 = 20cm$$

Part ③ Rectangle

$$\left(\frac{\pi \times 20^2}{2}\right)\left[30 + \left(\frac{4 \times 20}{3\pi}\right)\right] = 20 \times a \times (30 - 10) + 20 \times a \times (30 - 10)$$

$$628.32\,(38.48) = 400a + 400a$$

$$\therefore a = \mathbf{30.22\ cm}$$

Note : Symmetrical axis is an axis about which the moments of area of lamina are equal on both sides.

$\therefore$ The distance, **a = 30.22 cm** ...**Ans.**

Alternate method :

Given that the centroid is located at 'G'.

$$\therefore \bar{x} = 0$$

$$\bar{y} = 0$$

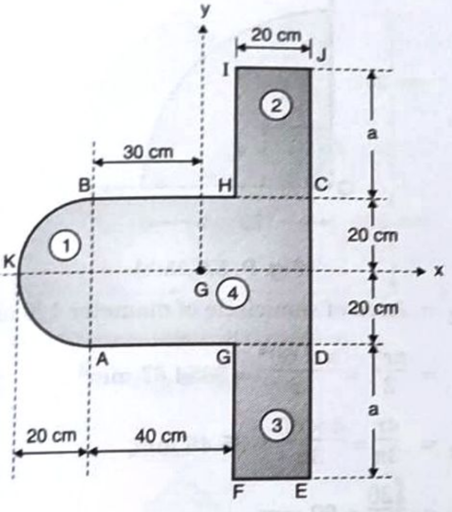


Fig. P. 5.8.23(c)

Dividing the Fig-into four parts.

Part ① : Semi circle ABK.

Part ② : Rectangle HIJC.

Part ③ : Rectangle DEFG.

Part ④ : Rectangle ABCD.

Part ① Semi circle

$$A_1 = \frac{\pi r^2}{2} = \frac{\pi (20)^2}{2} = 628.4\ cm^2$$

$$x_1 = -\left[30 + \frac{4 \times 20}{3\pi}\right] = -38.48\ cm$$

Part ② Rectangle

$$A_2 = 20a$$

$$x_2 = 20cm$$

Part ③ Rectangle

$$A_3 = 20a$$

$$x_3 = 20cm$$

Part ④ Rectangle

$$A_4 = 60 \times 40 = 2400\ cm^2$$

$$x_4 = 0$$

x – coordinate of the lamina is given by,

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{A_1 x_1 + (A_2 x_2)^2 + A_4 x_4}{A_1 + 2A_2 + A_4} \qquad \begin{bmatrix} A_2 = A_3 \\ x_2 = x_3 \end{bmatrix}$$

$$= \frac{628.4 \times (-38.48) + (20a \times 20)2 + 0}{628.4 + 2(20a)}$$

$$= \frac{-24180.832 + 800a}{(628.4 + 40a)}$$

But $\bar{x} = 0$

$$\therefore \frac{-24180.832 + 800a}{628.4 + 40a} = 0$$

$$-24180.832 + 800a = 0$$

$$\therefore a = \mathbf{30.22\ cm} \qquad \textbf{... Ans.}$$

Ex. 5.8.24 : A metal piece of uniform thickness is shown in Fig. P. 5.8.24(a). A hole of diameter 50mm is to be drilled through the piece, as shown by dotted line. Find the maximum distance 'd' of the centre of the hole from the vertical face, such that, when the piece is placed on horizontal floor, as shown in Fig. tipping will not occur.

SPPU : Nov. 04. 8 Marks

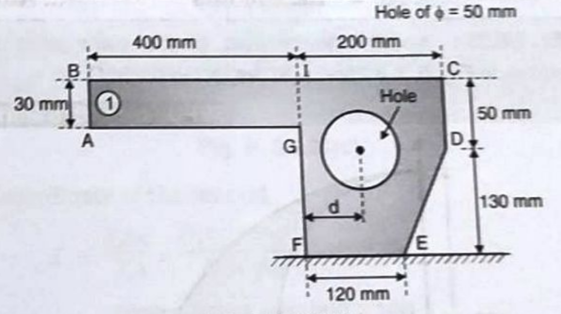


Fig. P. 5.8.24(a)

Soln. :

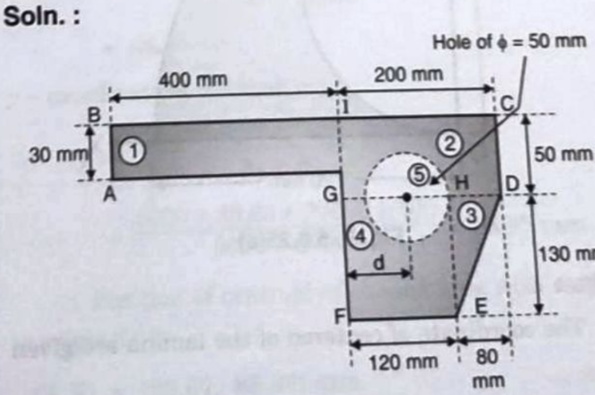


Fig. P. 5.8.24(b)

Part ① : Rectangle ABIG.

Part ② : Rectangle GICD.

Part ③ : Triangle DEH.

Part ④ : Rectangle HEFG.

Part ⑤ : circle of centre O (to be removed)

For tipping not to occur, the moments of areas about the line passing through point 'F' must be equal on both sides, so that they are balanced.

∴ Dividing the given Fig. into different parts and taking moments at line FG and equating on both sides,

moment of ① = moment of ② + moment of ③ + moment of ④ – moment of ⑤

(Hole ⑤ is to be removed)

$$\therefore (400 \times 30) \times 200 = [(200 \times 50) \times 100] + \left[\left(\frac{1}{2} \times 80 \times 130\right)\left(120 + \frac{1}{3} \times 80\right)\right] + [(120 \times 130) \times 60] - \left[\frac{\pi(50)^2}{4} \times d\right]$$

$$24 \times 10^5 = 10 \times 10^5 + 7.626 \times 10^5 + 9.36 \times 10^5 - 1963.49d$$

$$\therefore d = 152.076 \text{ mm} \qquad \text{... Ans.}$$

Ex. 5.8.25 : Locate the centroid of the plane area as shown in Fig. P. 5.8.25(a) with respect to origin O.

SPPU : May 18, 6 Marks

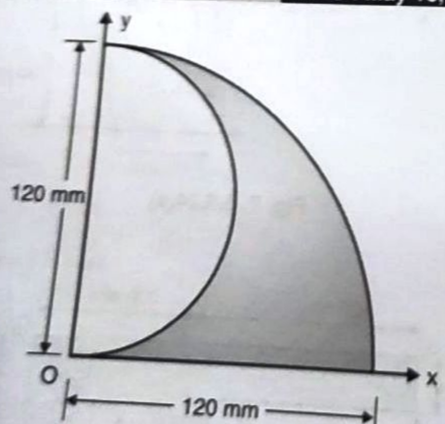


Fig. P. 5.8.25(a)

Soln. :

The coordinate of centered of the lamina are given by,

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} \quad \text{and} \quad \bar{y} = \frac{\Sigma Ay}{\Sigma A}$$

Let us divide the area into two parts.

Part-1 : Quarter circle of radius 120 mm and **Part-2 :** semicircle of diameter 120 mm.

A_1 = Area of quarter circle of radius 120 mm

$$= \frac{\pi r^2}{4} = \frac{\pi \times 120^2}{4} = \mathbf{11309.73\ mm^2}$$

$$x_1 = \frac{4r}{3\pi} = \frac{4 \times 120}{3\pi} = \mathbf{50.93\ mm}$$

$$y_1 = \frac{4r}{3\pi} = \frac{4 \times 120}{3\pi} = \mathbf{50.93\ mm}$$

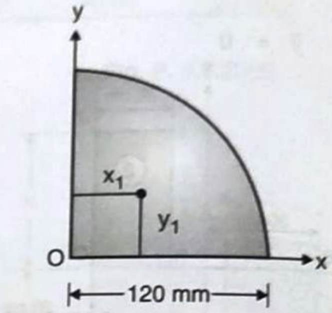


Fig. P. 5.8.25(b)

A_2 = Area of semicircle of diameter 120 mm.

$$= \frac{\pi r^2}{2} = \frac{\pi \times 60^2}{2} = 5654.87 \text{ mm}^2$$

$$x_2 = \frac{4r}{3\pi} = \frac{4 \times 60}{3\pi} = 25.46 \text{ mm.}$$

$$y_2 = \frac{120}{2} = 60 \text{ mm}$$

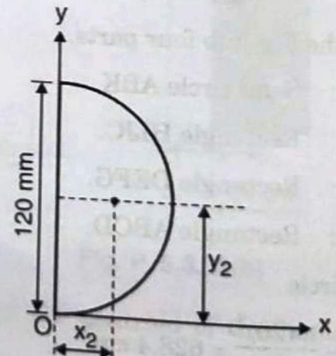


Fig. P. 5.8.25(c)

$$\therefore \bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

$$= \frac{11309.73 \times 50.93 - 5654.87 \times 25.46}{11309.73 - 5654.87}$$

$$= \frac{432031.55}{5654.86} = \mathbf{76.40\ mm}$$

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$= \frac{11309.73 \times 50.93 - 5654.87 \times 60}{11309.73 - 5654.87}$$

$$= \frac{236712.35}{5654.86}$$

$$= \mathbf{41.86\ mm}$$

$\therefore$ The centroid of the area w.r.t. origin 'O' is,

$$(\bar{x}, \bar{y}) = (76.40, 41.86)\ \text{mm} \qquad \textbf{... Ans.}$$

Ex. 5.8.26 : Determine the position of centroid of the shaded area with respect to origin. O as shown in Fig. P. 5.8.26(a).

SPPU : May 17, 4 Marks

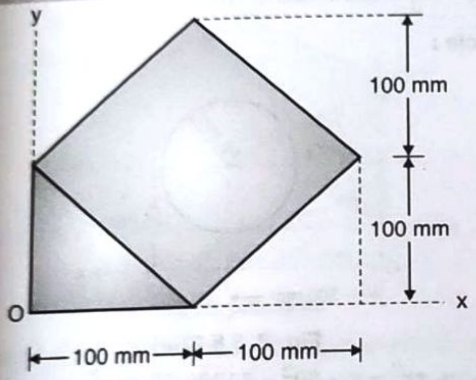


Fig. P. 5.8.26(a)

Soln. :

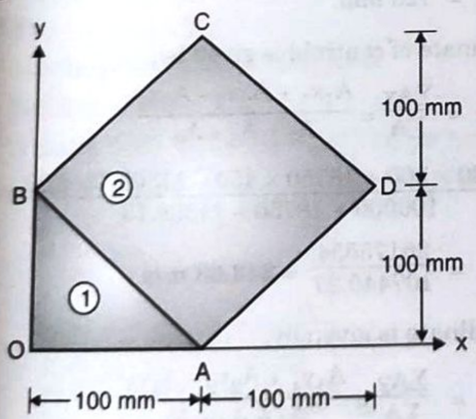


Fig. P. 5.8.26(b)

Let us divide the given area into two parts.

Part ① : Triangle OAB

Part ② : square ABCD

A_1 = Area of triangle OAB

$$= \frac{1}{2} \times 100 \times 100 = 5000\ \text{mm}^2$$

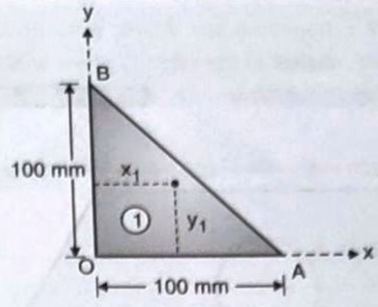


Fig. P. 5.8.26(c)

$$x_1 = \frac{1}{3} \times 100 = 33.33\ \text{mm.}$$

$$y_1 = \frac{1}{3} \times 100 = 33.33\ \text{mm.}$$

A_2 = Area of square ABCD

$$a^2 = 100^2 + 100^2 = 20{,}000\ \text{mm}^2$$

a = side of square

$$x_2 = 100\ \text{mm}$$

$$y_2 = 100\ \text{mm}$$

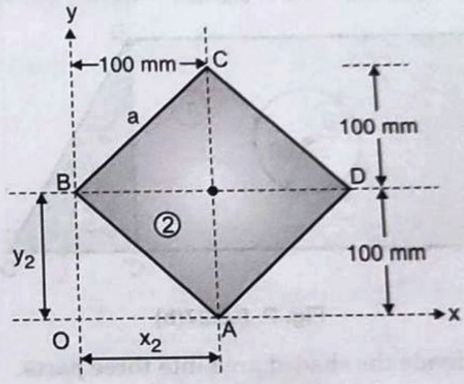


Fig. P. 5.8.26(d)

x-coordinate of the centoid,

$$\bar{x} = \frac{\Sigma A x}{\Sigma A} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$\therefore \bar{x} = \frac{5000 \times 33.33 + 20{,}000 \times 100}{5000 + 20000}$$

$$= \mathbf{86.67\ mm}$$

y – coordinate of the centriod,

$$\bar{y} = \frac{\Sigma A y}{\Sigma A} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{5000 \times 33.33 + 20000 \times 100}{5000 + 20000} = \mathbf{86.67\ mm.}$$

$\therefore$ Position of centroid of shaded area with respec to origin 'O' is

$$(\bar{x}, \bar{y}) = (86.67,\ 86.67)\ \text{mm} \qquad \textbf{... Ans}$$

Ex. 5.8.27 : Determine the x and y coordinates of the centroid with respect to the origin O of the shaded area as shown in Fig. P. 5.8.27(a). **SPPU : May 15, 4 Marks**

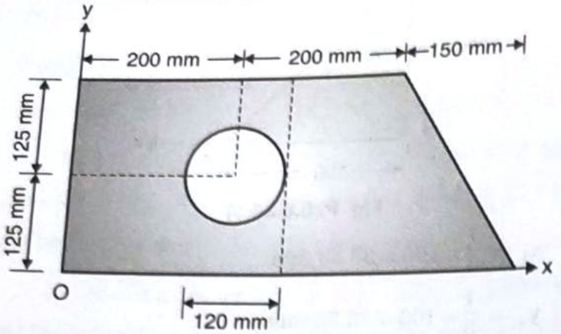


Fig. P. 5.8.27(a)

Soln. :

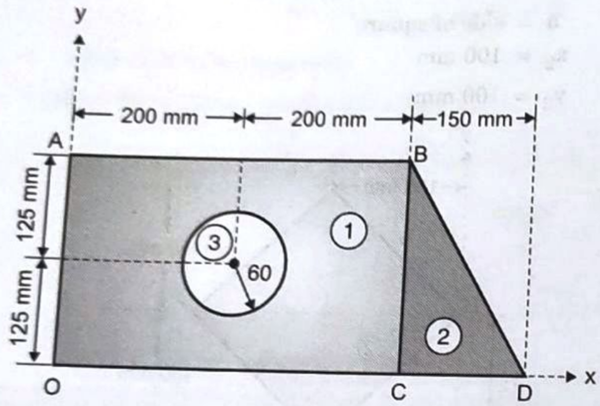


Fig. P. 5.8.27(b)

We can divide the shaded area into three parts.

① → Rectangle OABC .

② → Triangle BCD

③ → Circle (to be removed)

For rectangle :

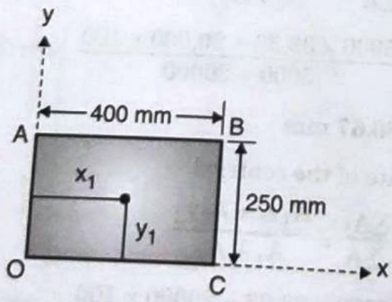


Fig. P. 5.8.27(c)

$$A_1 = 400 \times 250 = 100000 \text{ mm}^2$$

$$x_1 = 200 \text{ mm}$$

$$y_1 = 125 \text{ mm}$$

For triangle :

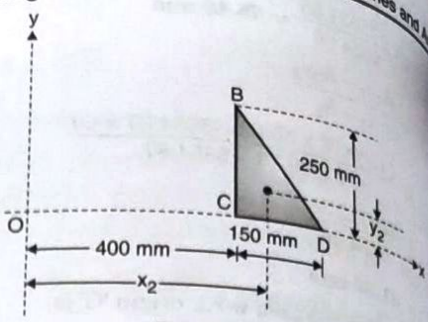


Fig. P. 5.8.27(d)

$$A_2 = \frac{1}{2} \times 150 \times 250 = 18750 \text{ mm}^2$$

$$x_2 = 400 + \frac{1}{3} \times 150 = 450 \text{ mm}$$

$$y_2 = \frac{1}{3} \times 250 = 83.33 \text{ mm.}$$

For circle :

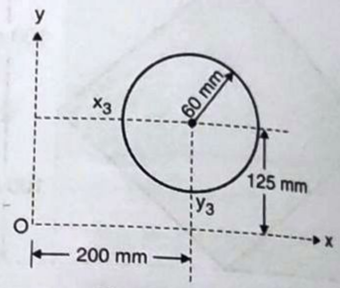


Fig. P. 5.8.27(e)

$$A_3 = \pi r^2 = \pi \times 60^2 = 11309.73 \text{ mm}^2$$

$$x_3 = 200 \text{ mm}$$

$$y_3 = 125 \text{ mm.}$$

x-coordinate of centroid is given by,

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{A_1 x_1 + A_2 x_2 - A_3 x_3}{A_1 + A_2 - A_3}$$

$$= \frac{100000 \times 200 + 18750 \times 450 - 11309.73 \times 200}{100000 + 18750 - 11309.73}$$

$$= \frac{26175554}{107440.27} = \mathbf{243.63 \text{ mm}}$$

y - coordinate is given by,

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3}$$

$$= \frac{100000 \times 125 + 18750 \times 83.33 - 11309.73 \times 125}{107440.27}$$

$$= \frac{12648721.25}{107440.27} = \mathbf{117.73 \text{ mm}}$$

∴ x and y coordinates of the centroid of the shaded area w.r.t. 'O' are :

$$(\bar{x}, \bar{y}) = (243.63, 117.73) \quad \text{... Ans.}$$

Ex. 5.8.28 : Determine the coordinate of centroid of the shaded area as shown in Fig. P. 5.8.28(a).

SPPU : Dec. 14, 4 Marks

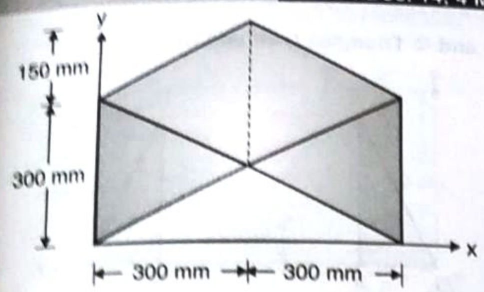


Fig. P. 5.8.28(a)

Soln. :

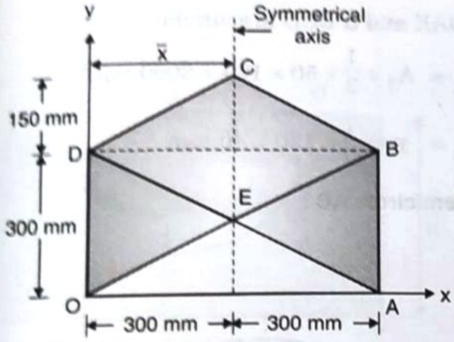


Fig. P. 5.8.28(b)

The Fig. P. 5.8.28(b) is symmetrical at vertical axis.

Ceteroid will be located an symmetrical axis.

$\therefore \bar{x} = 300$ mm

To find $\bar{y}$:

Divide the given figure into 3 parts.

① Rectangle OABD

② Triangle DBC ③ Triangle OAE (to be removed)

For ① rectangle OABD :

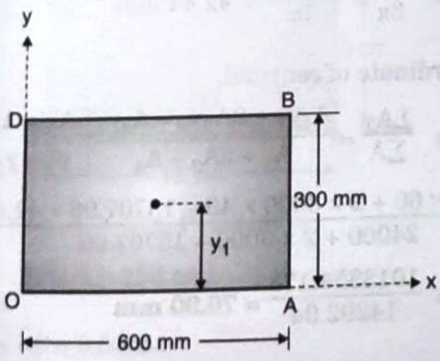


Fig. P. 5.8.28(c)

$$A_1 = 600 \times 300 = 180000 \text{ mm}^2$$

$$y_1 = \frac{300}{2} = 150 \text{ mm.}$$

For ② Δ DBC :

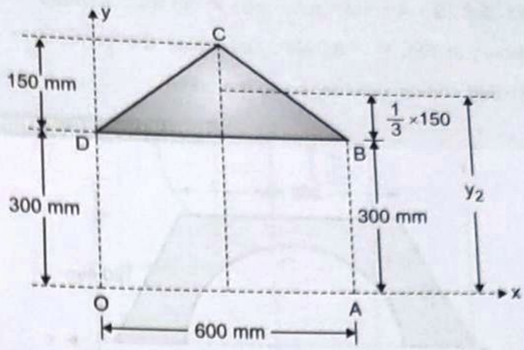


Fig. P. 5.8.28(d)

$$A_2 = \frac{1}{2} \times 600 \times 150 = 45000 \text{ mm}^2$$

$$y_2 = 300 + \frac{1}{3} \times 150$$

$$= 300 + 50$$

$$= 350 \text{ mm.}$$

For ③ Δ OAE :

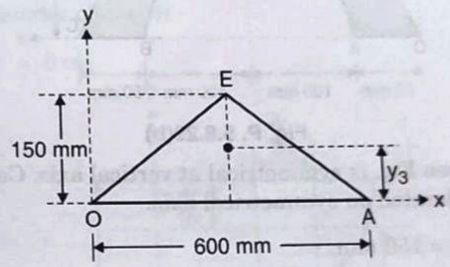


Fig. P. 5.8.28(e)

$$A_3 = \frac{1}{2} \times 600 \times 150 = 45000 \text{ mm}^2$$

$$y_3 = \frac{1}{3} \times 150 = 50 \text{ mm.}$$

$\therefore$ y – coordinate of centroid,

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3}$$

$$= \frac{180000 \times 150 + 45000 \times 350 - 45000 \times 50}{180000 + 45000 - 45000}$$

$$= \mathbf{225 \text{ mm}}$$

$\therefore$ The coordinates of centroid of the shaded area are :

$(\bar{x}, \bar{y})$ = **(300, 225) mm** ... **Ans.**

Ex. 5.8.29 : A semicircular area is cut from a trapezium as shown in Fig. P. 5.8.29(a). Determine the centroid of the shaded portion with respect to the origin.

SPPU : Dec. 15, 4 Marks

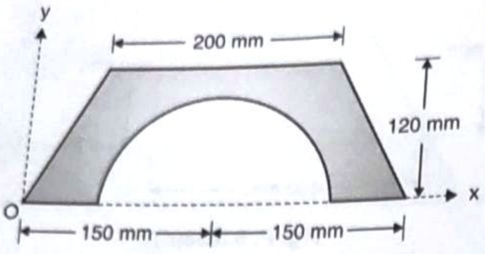


Fig. P. 5.8.29(a)

Soln. :

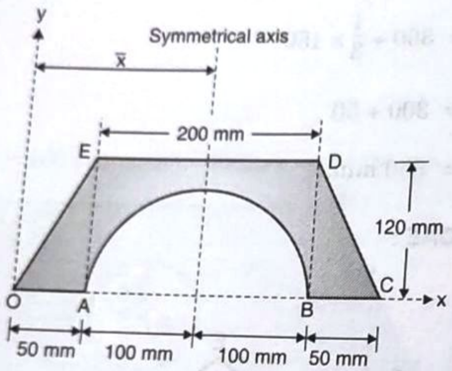


Fig. P. 5.8.29(b)

The given Fig. is symmetrical at vertical axis. Centroid will be located on symmetrical axis.

$\therefore \bar{x} = 150$ mm.

To find $\bar{y}$:

Divide the figure into 4 parts :

① Rectangle ABDE

② Triangle OAE

③ Triangle BCD

④ Semicircle AB (to be removed)

For ① Rectangle ABDE :

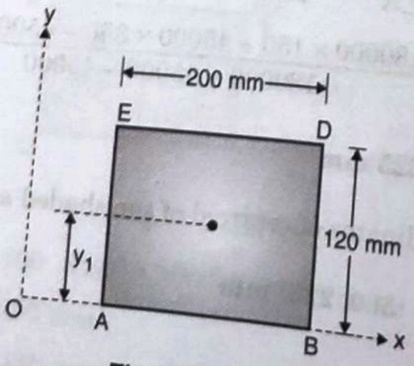


Fig. P. 5.8.29(c)

$$A_1 = 200 \times 120 = 24000 \text{ mm}^2$$

$$y_1 = \frac{120}{2} = 60 \text{ mm}$$

For ② and ③ Triangles OAE and BCD :

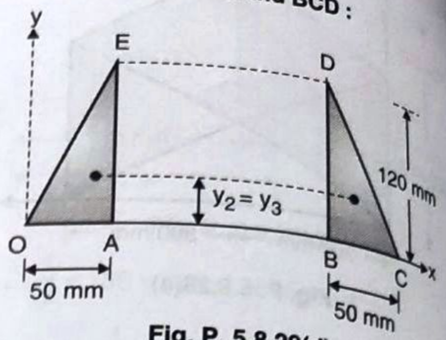


Fig. P. 5.8.29(d)

Δ OAE and Δ BCD are similar.

$$\therefore A_2 = A_3 = \frac{1}{2} \times 50 \times 120 = 3000 \text{ mm}^2$$

$$y_2 = y_3 = \frac{1}{3} \times 120 = 40 \text{ mm}$$

For ④ Semicircle AB :

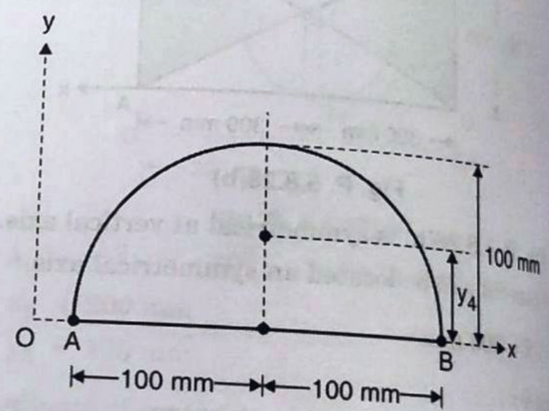


Fig. P. 5.8.29(e)

$$A_4 = \frac{\pi r^2}{2} = \frac{\pi \times 100^2}{2} = 15707.96 \text{ mm}^2$$

$$y_4 = \frac{4r}{3\pi} = \frac{4 \times 100}{3\pi} = 42.44 \text{ mm}$$

$\therefore$ y – coordinate of centroid,

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{A_1 y_1 + 2A_2 y_2 - A_4 y_4}{A_1 + 2A_2 - A_4} \begin{bmatrix} A_2 = A_3 \\ y_2 = y_3 \end{bmatrix}$$

$$= \frac{24000 \times 60 + 2 \times 3000 \times 40 - 15707.96 \times 42.44}{24000 + 2 \times 3000 - 15707.96}$$

$$= \frac{1013354.178}{14292.04} = \mathbf{70.90 \text{ mm}}$$

$\therefore$ The position of centroid of shaded portion with respect to origin is :

$$(\bar{x}, \bar{y}) = (150, 70.90) \text{ mm} \quad \text{...Ans.}$$

Ex. 5.8.30 : Two quarter circular areas are removed form a rectangular plate AEFG as shown in Fig. P. 5.8.30(a). Locate the centroid of the remaining area.

SPPU : Dec. 11, 9 Marks

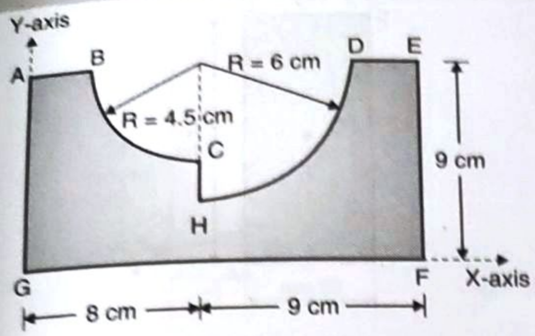


Fig. P. 5.8.30(a)

Soln. :

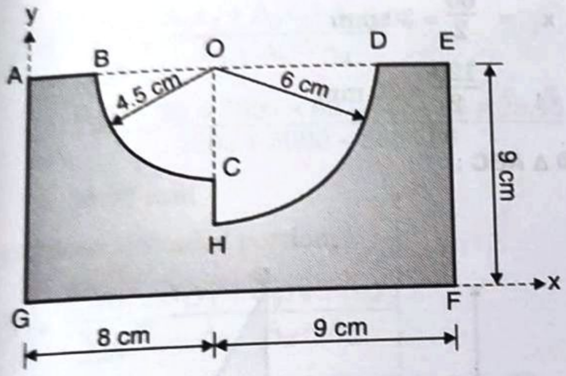


Fig. P. 5.8.30(b)

Divide the figure into 3 parts.

① Rectangle AGFE

② Quarter circle BC of r = 4.5 cm (to be removed)

③ Quarter circle DH of r = 6 cm (to be removed)

For ① Rectangle AGFE :

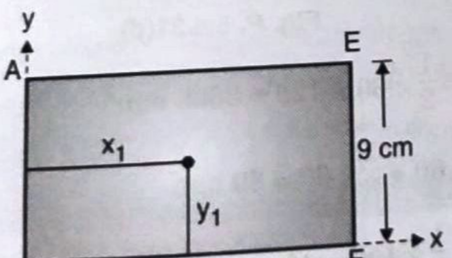


Fig. P. 5.8.30(c)

$$A_1 = 17 \times 9 = 153 \text{ cm}^2$$

$$x_1 = \frac{17}{2} = 8.5 \text{ cm}$$

$$y_1 = \frac{9}{2} = 4.5 \text{ cm}$$

For ② Quarter circle BC :

r = 4.5 cm

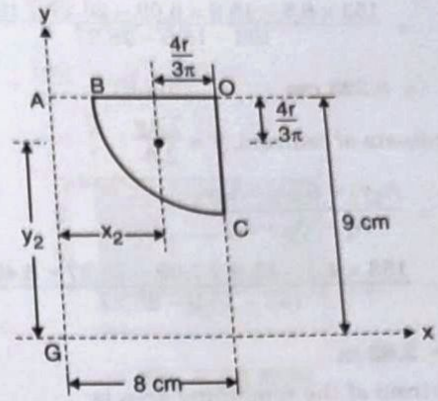


Fig. P. 5.8.30(d)

$$A_2 = \frac{\pi r^2}{4} = \frac{\pi \times 4.5^2}{4} = \mathbf{15.90\ cm^2}$$

$$x_2 = 8 - \frac{4r}{3\pi} = 8 - \frac{4 \times 4.5}{3\pi} = \mathbf{6.09\ cm}$$

$$y_2 = 9 - \frac{4r}{3\pi} = 9 - \frac{4 \times 4.5}{3\pi} = \mathbf{7.09\ cm}$$

For ③ Quarter circle DH :

r = 6 cm

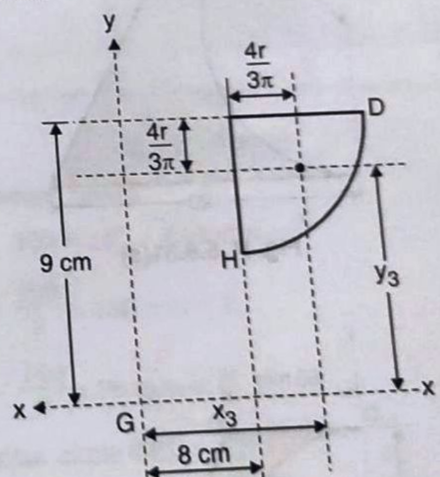


Fig. P. 5.8.30(e)

$$A_3 = \frac{\pi r^2}{4} = \frac{\pi \times 6^2}{4} = \mathbf{28.27\ cm^2}$$

$$x_3 = 8 + \frac{4r}{3\pi} = 8 + \frac{4 \times 6}{3\pi} = \mathbf{10.54\ cm}$$

$$y_3 = 9 - \frac{4r}{3\pi} = 9 - \frac{4 \times 6}{3\pi} = \mathbf{6.45\ cm}$$

with respect to point 'G'

x – coordinate of centroid, $\bar{x} = \frac{\Sigma Ax}{\Sigma A}$

$$\therefore \bar{x} = \frac{A_1x_1 - A_2x_2 - A_3x_3}{A_1 - A_2 - A_3}$$

$$= \frac{153 \times 8.5 - 15.9 \times 6.09 - 28.27 \times 10.54}{153 - 15.9 - 28.27}$$

$$= \mathbf{8.323\ cm}$$

y – coordinate of centroid, $\bar{y} = \frac{\Sigma Ay}{\Sigma A}$

$$\therefore \bar{y} = \frac{A_1y_1 - A_2y_2 - A_3y_3}{A_1 - A_2 - A_3}$$

$$= \frac{153 \times 4.5 - 15.9 \times 7.09 - 28.27 \times 6.45}{153 - 15.9 - 28.27}$$

$$= \mathbf{3.62\ m}$$

$\therefore$ The centroid of the remaining area is

$(\bar{x}, \bar{y})$ = (8.323, 3.62) cm **... Ans.**

Ex. 5.8.31 : A semicircle of radius 60 mm is removed from a trapezium. Locate centroid of the shaded portion that remained. Refer Fig. P. 5.8.31(a). All dimensions are in mm.

SPPU : Dec. 10, 8 Marks

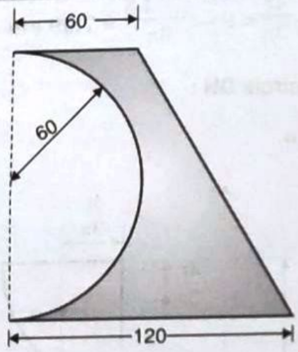


Fig. P. 5.8.31(a)

Soln. :

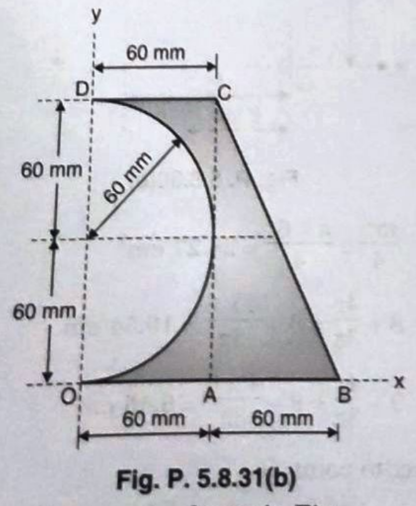


Fig. P. 5.8.31(b)

Selecting x and y axes as shown in Fig.

Dividing the Fig. into 3 parts,

① Rectangle OACD ② Triangle ABC

③ Semicircle OD (to be removed)

For ① Rectangle OACD :

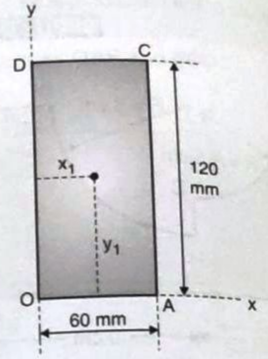


Fig. P. 5.8.31(c)

$$A_1 = 120 \times 60 = 7200\ mm^2$$

$$x_1 = \frac{60}{2} = 30\ mm$$

$$y_1 = \frac{120}{2} = 60\ mm$$

For ② Δ ABC :

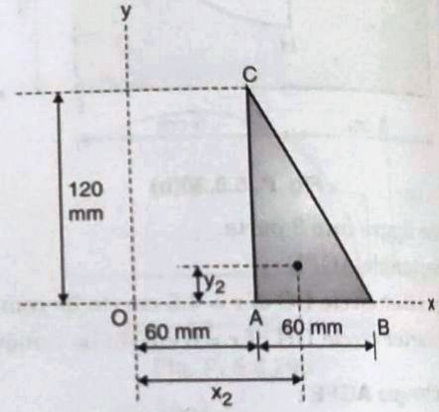


Fig. P. 5.8.31(d)

$$A_2 = \frac{1}{2} \times 60 \times 120 = 3600\ mm^2$$

$$x_2 = 60 + \frac{1}{3} \times 60 = 80\ mm$$

$$y_2 = \frac{1}{3} \times 120 = 40\ mm$$

For ③ Semicircle OD :

$$r = 60\ mm$$

$$A_3 = \frac{\pi r^2}{2} = \frac{\pi \times 60^2}{2} = 5654.86\ mm^2$$

$$x_3 = \frac{4r}{3\pi} = \frac{4 \times 60}{3\pi} = 25.46\ mm^2$$

$$y_3 = 60\ mm$$

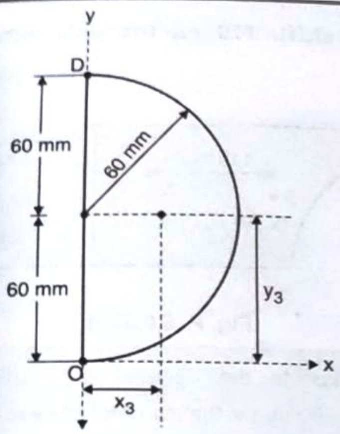


Fig. P. 5.8.31(e)

x – coordinate of shaded portion,

$$\bar{x} = \frac{\sum Ax}{\sum A} = \frac{A_1 x_1 + A_2 x_2 - A_3 x_3}{A_1 + A_2 - A_3}$$

$$= \frac{7200 \times 30 + 3600 \times 80 - 5654.86 \times 25.46}{7200 + 3600 - 5654.86}$$

$$= \mathbf{69.97\ mm}$$

y – coordinate of shaded portion,

$$\bar{x} = \frac{\sum Ay}{\sum A} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3}$$

$$= \frac{7200 \times 60 + 3600 \times 40 - 5654.86 \times 60}{7200 + 3600 - 5654.86}$$

$$= \mathbf{46\ mm}$$

∴ The position of centroid of the shaded portion is

$(\bar{x}, \bar{y})$ = **(69.97, 46) mm** ... **Ans.**

Ex. 5.8.32 : A rectangle ABCE is of very small thickness and has dimensions 140mm × 300 mm. Quarter circular portion ABF is cut and attached in a position CED as shown in Fig. P. 5.8.32(a). Determine the shift of centroid.

SPPU : Dec. 07, 9 Marks

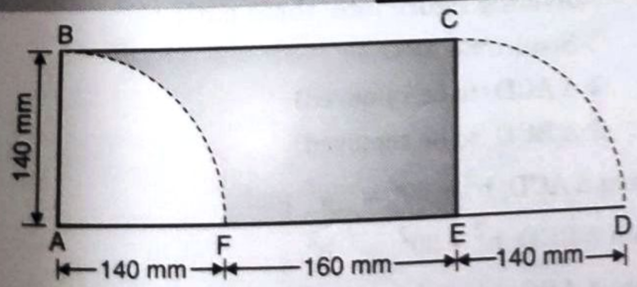


Fig. P. 5.8.32(a)

Soln. :

Consider x and y axes as shown in figure

The coordinates of centroid of a rectangle ABCE are :

$$\bar{x} = \frac{300}{2} = 150\ mm$$

$$\bar{y} = \frac{140}{2} = 70\ mm$$

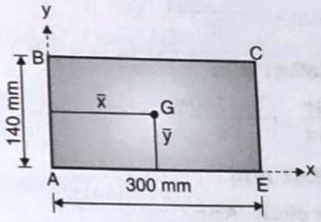


Fig. P. 5.8.32(b)

Now quarter circle ABF is cut and attached in a position CED.

Dividing Fig. into three parts :

① Rectangle ABCE ② Quarter Circle CED

③ Quarter Circle ABF (to be removed)

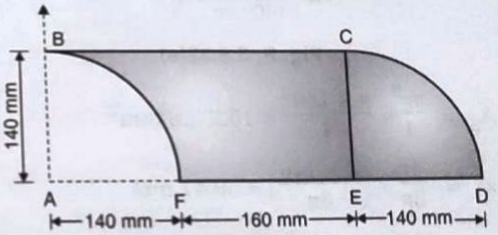


Fig. P. 5.8.32(c)

For ① Rectangle ABCE :

$$A_1 = 300 \times 140 = \mathbf{42000\ mm^2}$$

$$x_1 = \frac{300}{2} = 150\ mm = \bar{x}$$

$$y_1 = \frac{140}{2} = 70\ mm = \bar{y}$$

For ② Quarter circle CED :

$$r = 140\ mm$$

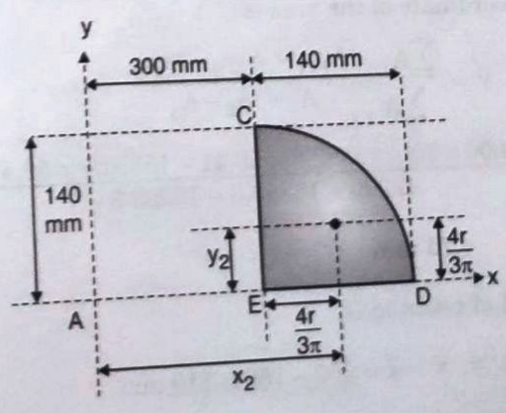


Fig. P. 5.8.32(d)

$$A_2 = \frac{\pi r^2}{4} = \frac{\pi \times 140^2}{4}$$

$$= 15393.8 \text{ mm}^2$$

$$x_2 = 300 + \left(\frac{4r}{3\pi}\right)$$

$$= 300 + \left(\frac{4 \times 140}{3\pi}\right)$$

$$= 359.41 \text{ mm}$$

$$y_2 = \frac{4r}{3\pi} = \left(\frac{4 \times 140}{3\pi}\right)$$

$$= 59.41 \text{ mm}$$

For ③ Quarter circle 'ABF'

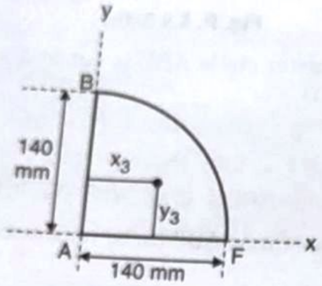


Fig. P. 5.8.32(e)

$$A_3 = \frac{\pi r^2}{4} = \frac{\pi \times 140^2}{4} = 15393.8 \text{ mm}^2$$

$$x_3 = \frac{4r}{3\pi} = \left(\frac{4 \times 140}{3\pi}\right) = 59.41 \text{ mm}$$

$$y_3 = \frac{4r}{3\pi} = \left(\frac{4 \times 140}{3\pi}\right) = 59.41 \text{ mm}$$

$\therefore$ x – coordinate the area is

$$\bar{x}' = \frac{\sum Ax}{\sum A} = \frac{A_1x_1 + A_2x_2 - A_3x_3}{A_1 + A_2 - A_3}$$

$$= \frac{42000 \times 150 + 15393.8 \times 359.41 - 15393.8 \times 59.41}{42000 + 15393.8 - 15393.8}$$

$$= 260 \text{ mm}$$

y – coordinate of the area is

$$\bar{y}' = \frac{\sum Ay}{\sum A} = \frac{A_1y_1 + A_2y_2 - A_3y_3}{A_1 + A_2 - A_3}$$

$$= \frac{42000 \times 70 + 15393.8 \times 59.41 - 15393.8 \times 59.41}{42000 + 15393.8 - 15393.8}$$

$$= 70 \text{ mm}$$

$\therefore$ shift of centroid is,

$$x = \bar{x}' - \bar{x} = 260 - 150 = 110 \text{ mm}$$

$$y = \bar{y}' - \bar{y} = 70 - 70 = 0$$

$\therefore$ **Centroid shifts 110 mm towards right.**

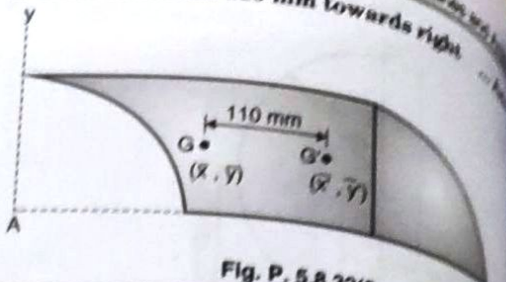


Fig. P. 5.8.32(f)

Ex. 5.8.33 : Locate the centroid of shaded portion of lamina, if AB = 90 mm is the diameter of semi-circle.

SPPU : May

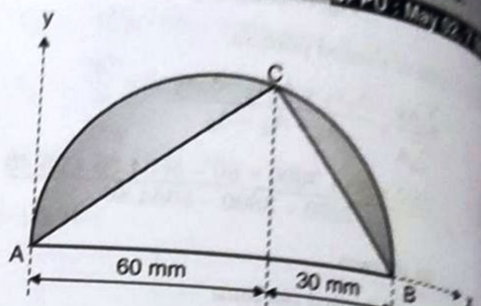


Fig. P. 5.8.33(a)

Soln. :

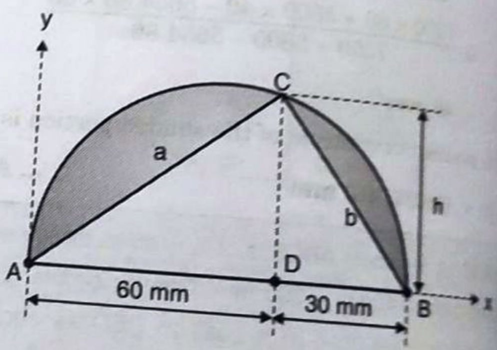


Fig. P. 5.8.33(b)

Dividing figure into three parts :

① Semicircle ABC

② Δ ACD (to be removed)

③ Δ BCD (to be removed)

From Δ ACD, $h^2 + 60^2 = a^2$ and

from Δ BCD, $h^2 + 30^2 = b^2$

from Δ ABC, $a^2 + b^2 = 90^2$

$$(h^2 + 60^2) + (h^2 + 30^2) = 90^2$$

$$h^2 + 3600 + h^2 + 900 = 8100$$

$$2h^2 = 3600$$

$\therefore h = 42.43 \text{ mm}$

For ① Semicircle ABC :

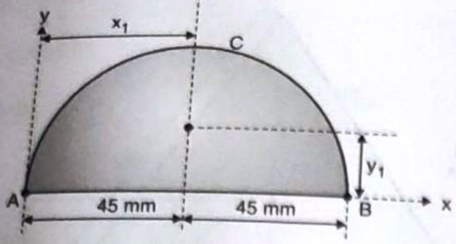


Fig. P. 5.8.33(c)

$$A_1 = \frac{\pi r^2}{2} = \frac{\pi (45)^2}{2} = 3180.86 \text{ mm}^2$$

$$x_1 = 45 \text{ mm}$$

$$y_1 = \frac{4r}{3\pi} = \frac{4 \times 45}{3\pi} = 19.1 \text{ mm}$$

For ② Δ ACD :

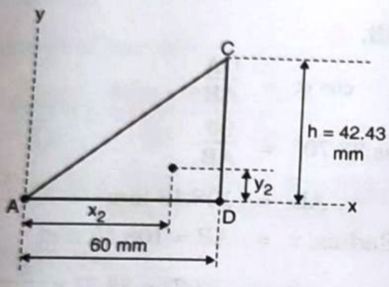


Fig. P. 5.8.33(d)

$$A_2 = \frac{1}{2} \times 60 \times 42.43 = \mathbf{1272.90 \text{ mm}^2}$$

$$x_2 = \frac{2}{3} \times 60 = \mathbf{40 \text{ mm}}$$

$$y_2 = \frac{1}{3} \times 42.43 = \mathbf{14.14 \text{ mm}}$$

For ③ Δ BCD :

$$A_3 = \frac{1}{2} \times 30 \times 42.43 = \mathbf{636.45 \text{ mm}^2}$$

$$x_3 = 60 + \frac{1}{3} \times 30 = \mathbf{70 \text{ mm}}$$

$$y_3 = \frac{1}{3} \times 42.43 = \mathbf{14.14 \text{ mm}}$$

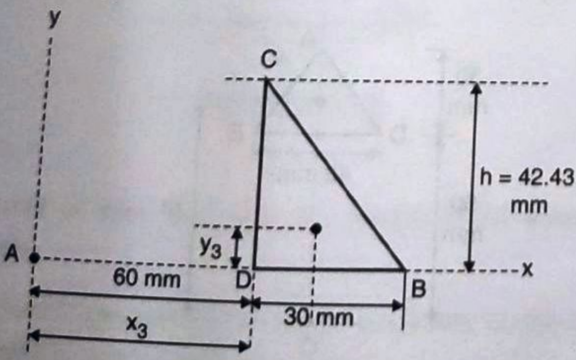


Fig. P. 5.8.33(e)

x – coordinate of shaded portion,

$$\bar{x} = \frac{\sum Ax}{\sum A}$$

$$= \frac{A_1 x_1 - A_2 x_2 - A_3 x_3}{A_1 - A_2 - A_3}$$

$$= \frac{3180.86 \times 45 - 1272.90 \times 40 - 636.45 \times 70}{3180.86 - 1272.90 - 636.45}$$

$$= \frac{47671.2}{1271.51}$$

$$= \mathbf{37.49 \text{ mm}}$$

y – coordinate of shaded portion,

$$\bar{y} = \frac{\sum Ay}{\sum A}$$

$$= \frac{A_1 y_1 - A_2 y_2 - A_3 y_3}{A_1 - A_2 - A_3}$$

$$= \frac{3180.86 \times 19.1 - 1272.90 \times 14.14 - 636.45 \times 14.14}{3180.86 - 1272.90 - 636.45}$$

$$= \frac{33756.217}{1271.51}$$

$$= \mathbf{26.55 \text{ mm}}$$

$\therefore$ The centroid of the shaded portion is

$(\bar{x}, \bar{y}) = (37.49, 26.55)$ **mm** ...**Ans.**

Ex. 5.8.34 : If ABC represents sector of a circle , locate the centroid of shaded portion with reference to given 'x' and 'y' axes. **SPPU : Dec. 03, 8 Marks**

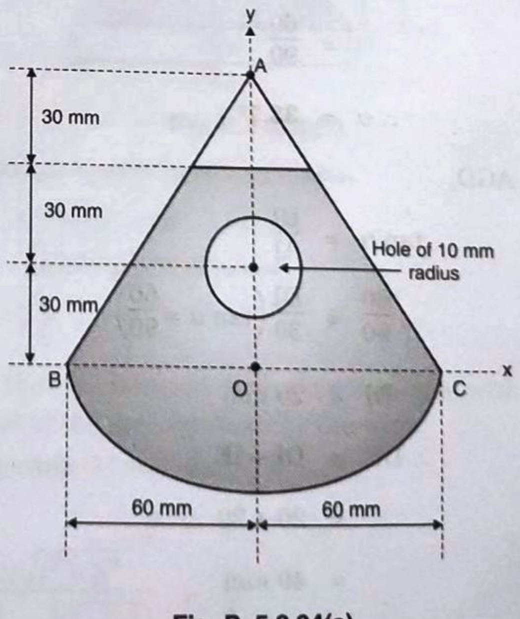


Fig. P. 5.8.34(a)

Soln. :

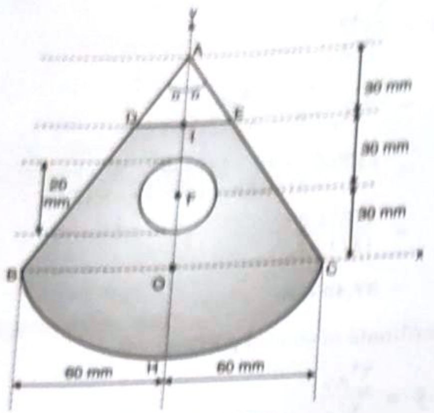


Fig. P. 5.8.34(b)

The given Fig. P. 5.8.34(b) is symmetrical about y-axis.

∴ x-coordinate of centroid,

$$\bar{x} = 0$$

To find $\bar{y}$, divide the figure into 3 parts :

① Sector of a circle ABHC

② Δ ADE (to be removed)

③ Circle 'F' (to be removed)

From Δ AOB,

$$\tan\alpha = \frac{OB}{OA}$$

$$= \frac{60}{90}$$

$$\therefore \alpha = \mathbf{33.7^\circ}$$

For Δ AGD,

$$\tan\alpha = \frac{DI}{AI}$$

$$\therefore \frac{60}{90} = \frac{DI}{30}\left(\tan\alpha = \frac{60}{90}\right)$$

$$\therefore DI = 20 \text{ mm}$$

$$\therefore DE = DI + IE$$

$$= 20 + 20$$

$$= 40 \text{ mm}$$

(∵ Δ AID and Δ AIE are symmetric)

For ① Sector of a circle 'ABC' :

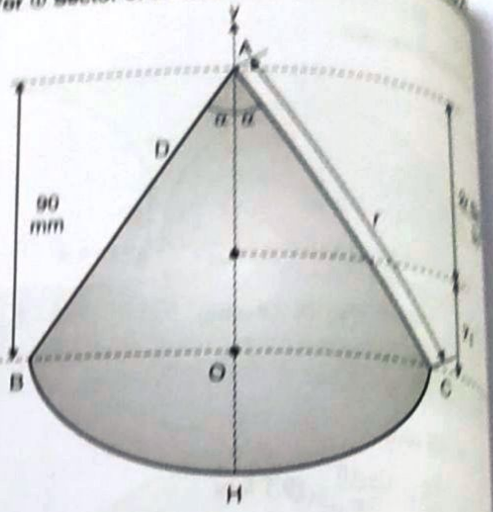


Fig. P. 5.8.34(c)

From Δ AOB,

$$\cos\alpha = \frac{OA}{AB}$$

$$\cos 33.70^\circ = \frac{90}{AB}$$

$$\therefore AB = 108.18 \text{ mm}$$

$$\therefore \text{Radius, } r = AB = 108.18 \text{ mm}$$

$$\alpha = 33.7^\circ = 33.7^\circ \times \frac{\pi}{180^\circ} = 0.588$$

For ① Sector of a circle ABHC,

$$\therefore A1 = \alpha^c r^2$$

$$= 0.588 \times 108.18^2$$

$$= 6881.31 \text{ mm}^2$$

$$y_1 = 90 - \frac{2\, r \sin\alpha^\circ}{3\alpha^c}$$

$$= 90 - \frac{2 \times 108.18 \times \sin 33.7^\circ}{3 \times 0.588}$$

$$= 21.95 \text{ mm}$$

For ② Δ ADE :

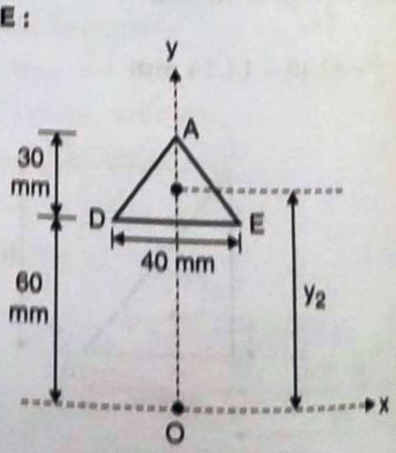


Fig. P. 5.8.34(d)

$$A_2 = \frac{1}{2} \times 40 \times 30 = 600 \text{ mm}^2$$

$$y_2 = 60 + \frac{1}{3} \times 30 = 70 \text{ mm}$$

For ③ Circle 'F'

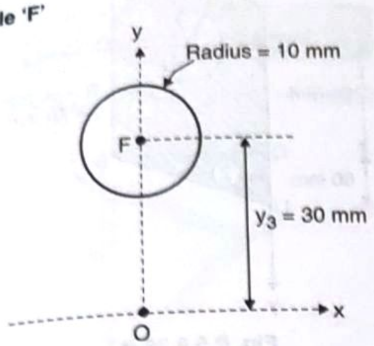


Fig. P. 5.8.34(e)

$$A_3 = \pi r^2 = \pi \times 10^2 = 314.16 \text{ mm}^2$$

$$y_3 = 30 \text{ mm}$$

∴ y – coordinate of centroid,

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{A_1y_1 - A_2y_2 - A_3y_3}{A_1 - A_2 - A_3}$$

$$= \frac{6881.31 \times 21.95 - 600 \times 70 - 314.16 \times 30}{6881.31 - 600 - 314.16}$$

$$= \mathbf{16.69 \text{ mm}}$$

∴ The position of cetroid of the shaded portion w.r.t. x and y axes is :

$$(\bar{x}, \bar{y}) = (0, 16.69) \text{ mm}$$ ... **Ans.**

Ex. 5.8.35 : A thin wire of homogeneous material is best to form an isosceles triangle as shown in Fig. P. 5.8.35(a). Determine the height of the triangle for which the cetroid of the wire coincides with the centroid of area enclosed by the wire. **SPPU : May 97, 8 Marks**

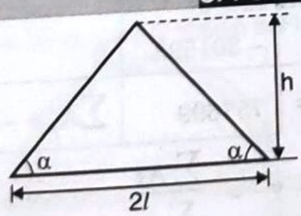


Fig. P. 5.8.35(a)

Soln. :

Centroid of wire bent into the shape of an isosceles triangle

Selecting x and y axes as shown in Fig. P. 5.8.35(b)

The bent is symmetrical at y - axis

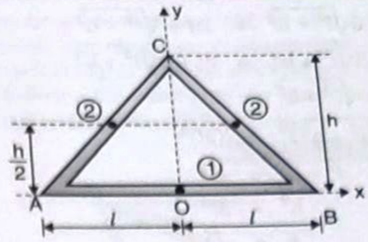


Fig. P. 5.8.35(b)

∴ x – coordinate centroid, $\bar{x} = 0$.

y – coordinate is given by,

$$\bar{y} = \frac{l_1y_1 + 2l_2y_2}{l_1 + 2l_2}$$

$$l_1 = 2l$$

$$y_1 = 0$$

$$l_2 = \sqrt{l^2 + h^2}$$

$$y_2 = \frac{h}{2}$$

$$\therefore \bar{y} = \frac{2l \times 0 + 2 \times \sqrt{l^2 + h^2} \times \frac{h}{2}}{2l + 2\sqrt{l^2 + h^2}}$$

$$= \frac{h\sqrt{l^2 + h^2}}{2\left(l + \sqrt{l^2 + h^2}\right)} \quad \text{... (1)}$$

Centroid of the area enclosed by the wire :

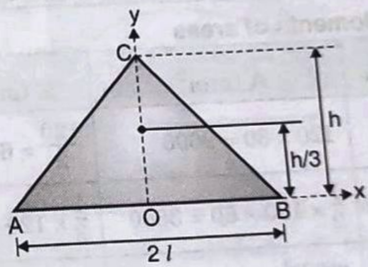


Fig. P. 5.8.35(c)

Area is symmetrical at y – axis.

∴ x – coordinate ; $\bar{x}' = 0$

y – coordinate is given by

$$\bar{y}' = \frac{h}{3} \quad \text{... (2)}$$

Given that the centroid of the wire coincides with the centroid of the area enclosed by the wire.

∴ Equating (1) and (2)

$$\bar{y} = \bar{y}'$$

$$\frac{h\sqrt{l^2 + h^2}}{2\left(l + \sqrt{l^2 + h^2}\right)} = \frac{h}{3}$$

$$\therefore \quad 3\sqrt{l^2 + h^2} = 2(l + \sqrt{l^2 + h^2})$$

$$3\sqrt{l^2 + h^2} = 2l + 2\sqrt{l^2 + h^2}$$

$$\sqrt{l^2 + h^2} = 2l$$

$$(\sqrt{l^2 + h^2})^2 = (2l)^2$$

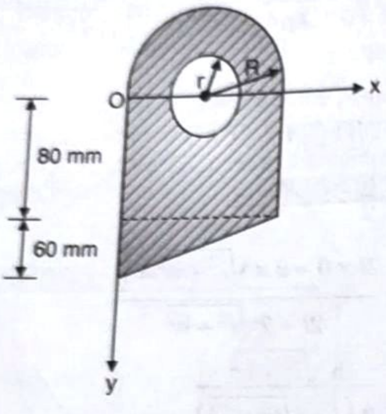
$$l^2 + h^2 = 4l^2$$

$$h^2 = 3l^2$$

$$h = \sqrt{3}\, l$$

$\therefore$ Height of the triangle, $\mathbf{h = \sqrt{3}\, l}$ ... **Ans.**

Ex. 5.8.36 : Locate the centroid of plane lamina as shown in Fig.P.5.8.36 **SPPU : May 19,6 Marks**



R = 60 mm; r = 40 mm

Fig. P. 5.8.36(a)

Soln. :

Step 1 : Dividing the lamina into different parts :

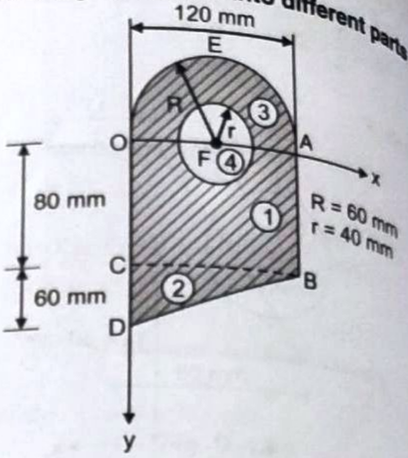


Fig. P.5.8.36 (b)

① Rectangle OABC

② Right angled triangle BCD

③ Semi circle OEA of radius R = 60 mm

④ Circle 'F' of radius, r = 40 mm (to be removed)

Step 2 : Moments of areas

Figure	A (mm²)	X (mm)	Y (mm)	Ax (mm²)	Ay (mm³)
①	$120 \times 80 = 9600$	$\frac{120}{2} = 60$	$-\frac{80}{2} = -40$	576000	– 384000
②	$\frac{1}{2} \times 120 \times 60 = 3600$	$\frac{1}{3} \times 120 = 40$	$-\left(80 + \frac{1}{3} \times 60\right) = -100$	14400	– 360000
③	$\frac{\pi(60)^2}{2} = 5654.86$	60	$\frac{4\,(60)}{3\pi} = 25.46$	339292.00	143999.82
④	$-\pi\,(40)^2 = -5026.55$	60	0	– 301593	0
ΣA	13828.31			ΣA_x = 757699	ΣA_y = – 600000.18

Step 3 : x and y coordinates of centroid :

$$\bar{x} = \frac{\sum Ax}{\sum A} = \frac{757699}{13828.31} = \mathbf{54.79\ mm} \quad \text{... Ans.}$$

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{-600000.18}{13828.31} = \mathbf{-43.39\ mm} \quad \text{... Ans.}$$

Coordinate of centroid of the lamina w.r.t. origin are (54.79, – 43.39) mm

Theory Question

Define centroid of plane area.

(Refer Section 5.1.5) **Dec. 02**

Practice Problems

Q. 1 Locate the centre of gravity of the bent shown in Fig. Q. 1.

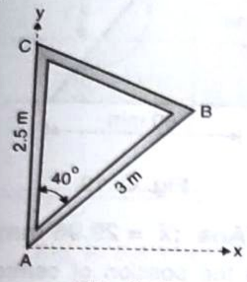


Fig. Q. 1

[Ans. : $\bar{x}$ = 0.595 m, $\bar{y}$ = 1.50 m]

Q. 2 A uniform rod bent into a configuration OABC as shown in Fig. Q. 2. Find position of centre of gravity of this bent with respect to 'O'.

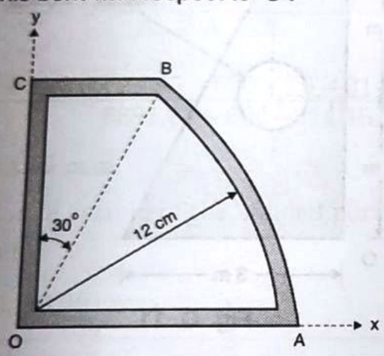


Fig. Q. 2

[Ans. : $\bar{x}$ = 5.24 cm, $\bar{y}$ = 4.6 cm]

Q. 3 Determine the dimension 'b' if centroid of a bent wire is to be at point C shown in Fig. Q. 3.

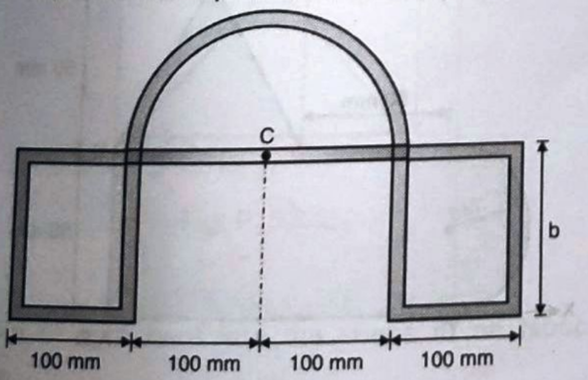


Fig. Q. 3

[Ans. : b = 6.18 mm]

Q. 4 The homogeneous wire ABC is bent into a semi circular arc of radius 'r' and straight section as shown in Fig. Q. 4 and is attached to a hinge at 'A'. Determine the value of 'θ' for which the wire is in equilibrium for the indicated position.

SPPU : Dec. 04, 9 Marks

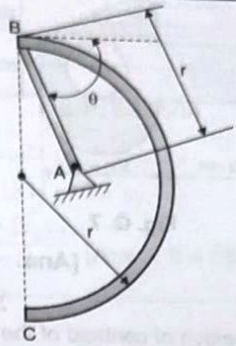


Fig. Q. 4

[Ans. : θ = 56.7°]

Q. 5 Locate the centroid C of the shaded area obtained by cutting a semicircle of diameter a from the quadrant of a circle of radius a as shown in Fig. Q. 5 with respect to origin O.

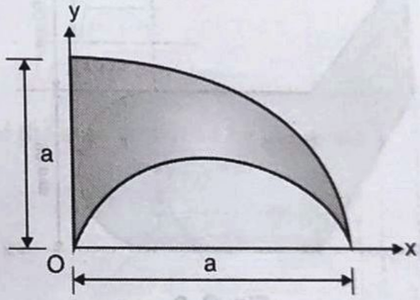


Fig. Q. 5

[Ans. : $\bar{x}$ = 0.35a, $\bar{y}$ = 0.63a]

Q. 6 Determine the y-coordinate of centroid of the shaded area as shown in Fig. Q. 6

SPPU : Dec. 16, 4 Marks

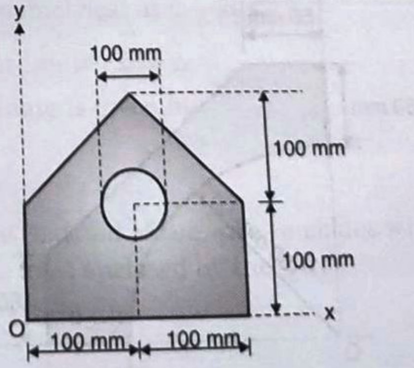


Fig. Q. 6

[Ans. : $\bar{x}$ = 100 mm $\bar{y}$ = 69.89 mm]

Q. 7 Determine the co-ordinates of the centroid of the lamina shown in Fig. Q. 7 w.r.t origin 'o'. Note that the circle of diameter 50 mm is cut out from the plane lamina, with centre at (150, 25).

SPPU : April 94, 8 Marks

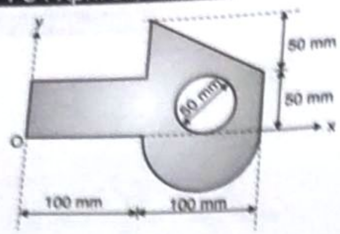


Fig. Q. 7

[Ans. : $\bar{x} = 112.55$ mm

$\bar{y} = 19.66$ mm]

Q. 8 Find the position of centroid of the plane of uniform thickness, shown in Fig. Q. 8 with reference to the origin 'O'. Find the distance of the centroid from the origin 'O'.

SPPU : Oct. 99, 8 Marks

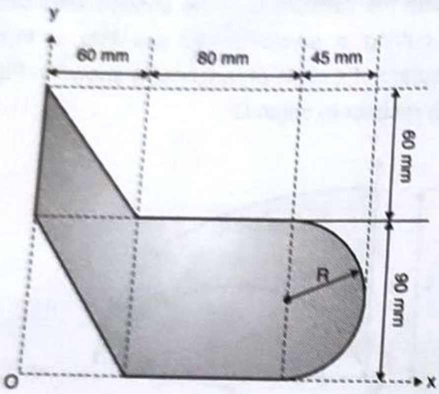


Fig. Q. 8

[Ans. : $\bar{x} = 92.07$ mm, $\bar{y} = 55.58$ mm]

Q. 9 Locate the centroid of the shaded area as shown in Fig. Q. 9.

SPPU : Dec. 99, 8 Marks

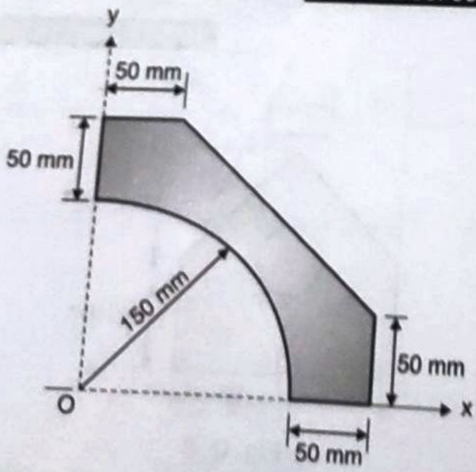


Fig. Q. 9

[Ans. : $\bar{x} = 107.28$ mm $\bar{y} = 107.28$ mm]

Q.10 Determine the coordinates of the centroid of the plane lamina shown in Fig. Q. 10.

SPPU : Dec. 98, 8 Marks

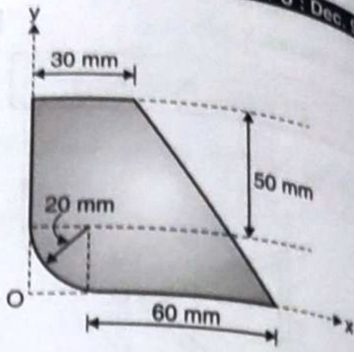


Fig. Q. 10

[Ans. : $\bar{x} = 29.96$ mm $\bar{y} = 30.27$ mm]

Q. 11 Determine the position of centroid of the shaded area as shown in Fig. Q. 11 with respect to origin O.

SPPU : Dec. 12, 6 Marks

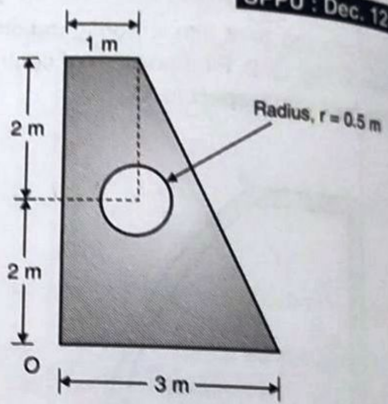


Fig. Q. 11

[Ans. : $\bar{x} = 1.094$ m, $\bar{y} = 1.628$ m]

Q.12 Find the centre of gravity of the plane, uniform lamina shown in Fig. Q. 12 w.r.t AB and AC.

SPPU : April 93, 8 Marks

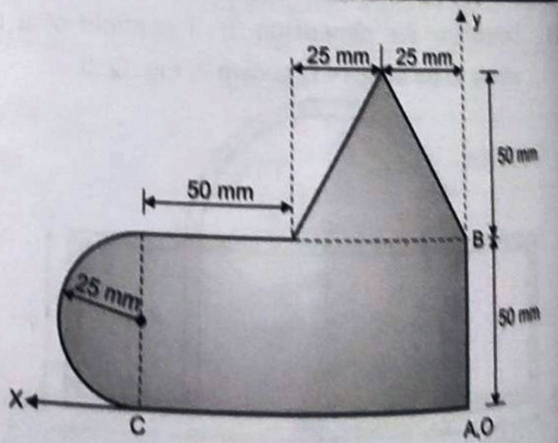


Fig. Q. 12

[Ans. : $\bar{x} = 71.1$ mm from AB $\bar{y} = 32.20$ mm from AC]

Q. 13 Find the centroid of the lamina shown in Fig. Q. 13.

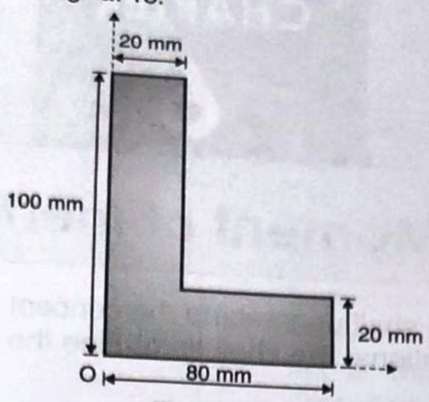


Fig. Q. 13

[Ans. : $\bar{x}$ = 35 mm, $\bar{y}$ = 25 mm

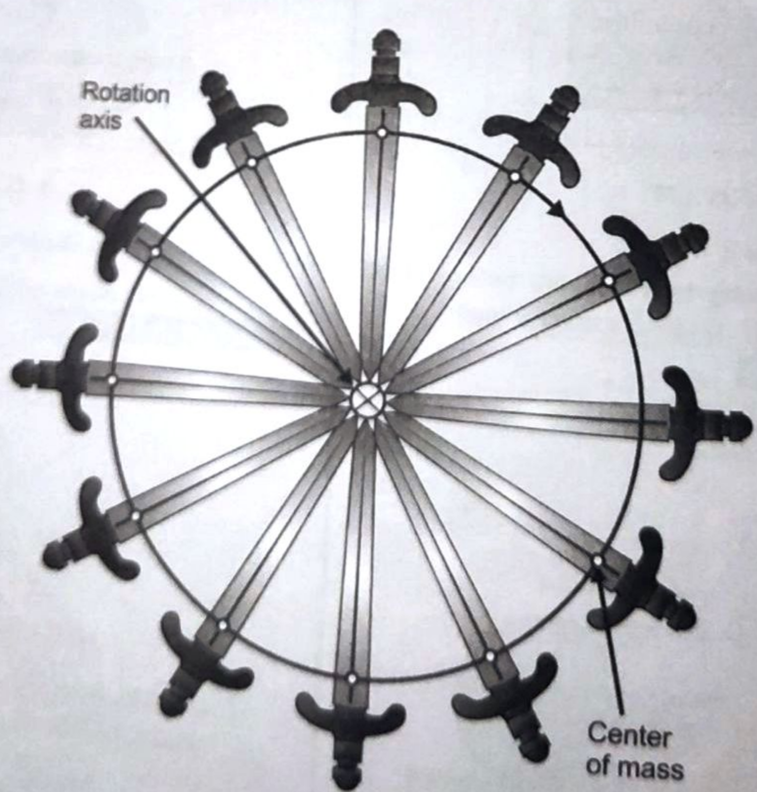
CHAPTER

6

UNIT - II

Moment of Inertia

Introduction : In this chapter, we shall understand the concept of Area Moment of Inertia and its importance in engineering applications. We shall determine the moment of inertia for various cross sections of different shape



6.1 Introduction

- We know that the moment of force about any point is the product of magnitude of force (F) and the perpendicular distance (d) from the line of action of the force to the point.

  $M_o = F \cdot d$

- It is known as first moment of force.

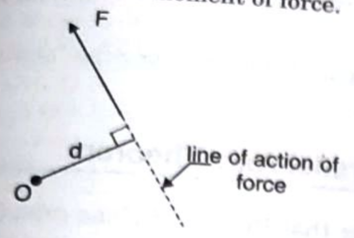


Fig. 6.1.1

- If $(F \cdot d)$ is again multiplied with distance, 'd' then it is known as second moment of force i.e. $F \cdot d^2$, also called as "Moment of Inertia" of force. It is represented by letter (I).
- Sometimes force is replaced by area of the figure or mass of the body. Then the second moment is known as area moment of inertia or mass moment of inertia.

6.2 Types of moment of inertia

(A) Area Moment of Inertia :

The second moment of area about a particular axis is known as ***"area moment of inertia"*** *about that axis.*

- **Unit :** mm^4 or m^4

First moment of area = Area × distance

$= mm^2 \times mm = mm^3$

Second moment of area = moment of area × distance

$= mm^3 \times mm = mm^4$

(B) Mass Moment of Inertia :

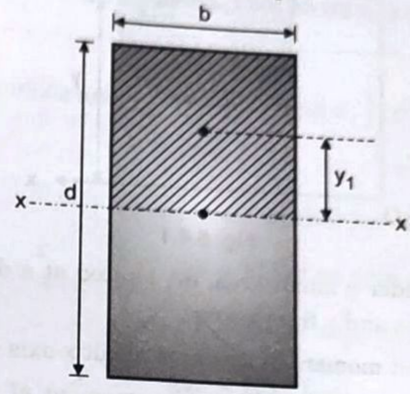
The second moment of mass of the body about a particular axis is known as ***"mass moment of inertia"*** *about that axis.*

Unit : kg-mm^2 or kg-m^2

In this chapter we discuss about area moment of inertia.

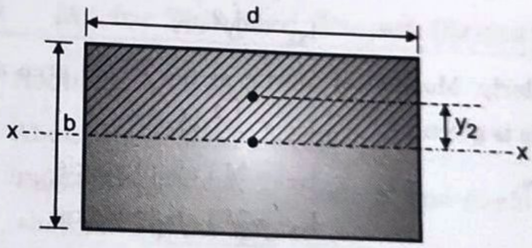
6.3 Importance of Area Moment of Inertia

- It is the quantitative estimate of the relative distribution of area with respect to some reference axis.
- The area moment of inertia (M.I) of a beam's cross-sectional area measures the beam's ability to resist bending.
- Larger the moment of inertia, less the beam will bend.
- M.I is the geometrical property of beam and depends on the reference axis.
- Consider the following example :



Section - I

Fig. 6.3.1



Section - II

Fig. 6.3.2

There are two cross-sections I and II having same area i.e. $(b \cdot d)$.

Taking moments about x-x axis ;

M.I about x-x axis = Second moment of area about x-x axis

For section I : $I_{xx} = \left\{\left(\frac{A}{2}\right).Y_1^2\right\} 2$ (Refer Fig. 6.3.1)

For section II : $I_{xx} = \left\{\left(\frac{A}{2}\right).Y_2^2\right\} 2$ (Refer Fig. 6.3.2)

where, A = Total area of section

As $y_1 > y_2$, the M.I. of section I is greater than M. I. of section II about x-x axis.

$\therefore$ Section I will have more resistance against bending at x-x axis as area is distributed over large distance from the x-x axis.

6.4 Moment of inertia by Integration

Let us consider a plane Fig. 6.4.1 whose moment of inertia is required to be found about x and y axes.

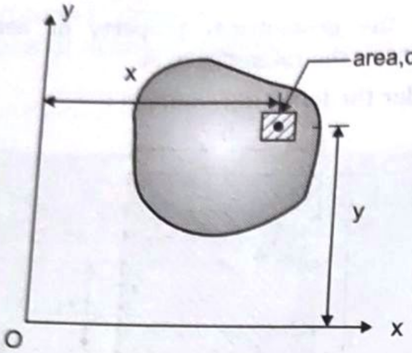


Fig. 6.4.1

Consider a small area, dA located at a distance x from y-axis and y from x-axis.

The first moment of this area about x-axis = $y \cdot dA$.

Second moment of area i.e. moment of inertia of this area about x-axis is $I_{xx} = y^2 \cdot dA$.

$\therefore$ M.I for the whole area about x-axis is

$$I_{xx} = \int y^2 \cdot dA$$

...imilarly, Moment of inertia of the whole area about ... axis is given by, $I_{yy} = \int x^2 \cdot dA$

...ere, I_{xx} = M.I about x-axis

I_{yy} = M.I about y-axis

...5 Neutral Axis

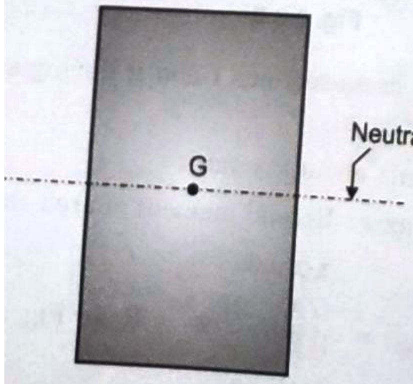


Fig. 6.5.1

- The line passing through the centroid or centre of gravity of the section is known as "neutral axis".
- It is also known as "centroidal axis".
- The portion above neutral axis subjected to compressive forces and below the neutral axis to tensile forces when the beam bends in "∪" shape.
- If it bends in "∩" shape, top portion is subjected to tensile forces and the lower portion compressive forces.
- The forces or stresses along the "neutral axis" will be zero.

6.6 Parallel Axis Theorem

It states that the M.I of plane area (A) about any axis AB, which is parallel to the centroidal axis located at a distance 'h' is given by,

$$I_{AB} = I_G + Ah^2$$

where, I_{AB} = M.I of an area about the line 'AB' parallel to centroidal axis

I_G = M.I of an area about centroidal axis

A = Total area of the figure

h = Distance between centroidal axis and parallel axis, AB.

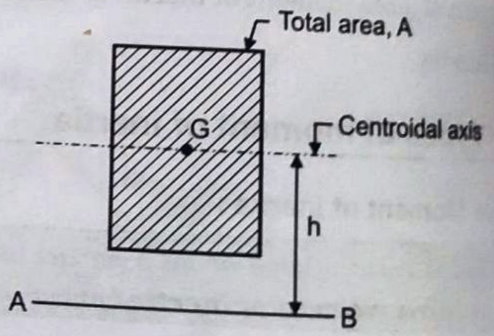


Fig. 6.6.1

Proof :

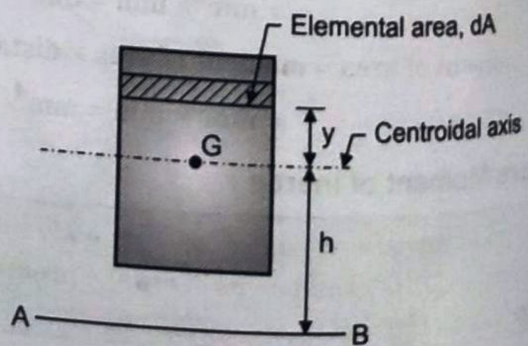


Fig. 6.6.2

Consider ...

Let, y = distance of this strip from centroidal axis.

AB = Axis about which M.I is to be found.

h = Distance between the centroidal axis and parallel axis.

A = Total area of the figure

Second moment of area of an elemental strip about axis AB, i.e., M.I of elemental area an axis AB,

$$dI_{AB} = (y+h)^2 dA$$

For whole area,

$$I_{AB} = \int (y+h)^2 dA = \int (y^2 + h^2 + 2yh) dA$$

$$= \int y^2 \cdot dA + h^2 \int dA + 2h \int y \cdot dA$$

$\int y^2 \cdot dA$ = M.I of an area about centroidal axis, I_G.

$h^2 \int dA = h^2 \cdot A \qquad (\because \int dA = A)$

$2h \int y \cdot dA = 2h \times 0 = 0$

$\int y \cdot dA = \bar{y} \cdot A = 0 \qquad (\because \bar{y} = 0)$

Centroid is located on centroidal axis.

$$\therefore \boxed{I_{AB} = I_G + Ah^2}$$

Note : Parallel axis theorem is used to find M.I. about an axis which is not passing through the C.G. of the section.

6.7 Perpendicular Axis Theorem

– It states that, the M.I of plane area about an axis perpendicular or normal to the x-y plane i.e. z -axis is equal to the sum of M.I of the plane area about x and y axes.

Proof :

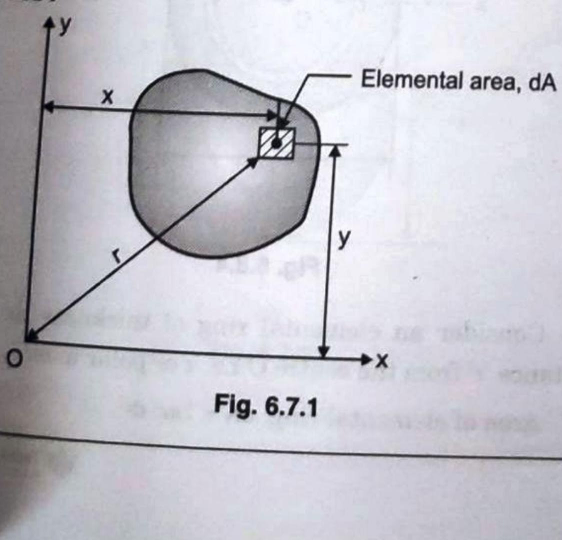


Fig. 6.7.1

$$I_{zz} = I_{xx} + I_{yy}$$

where,

I_{zz} = M.I of an area about z-axis. (also called as polar M.I.) z-axis is known as polar axis.

I_{xx} = M.I of an area about x-axis.

and I_{yy} = M.I of an area about y-axis.

– Let 'r' be the distance of elemental area from z-axis i.e. passing through the origin and normal to x-y plane.

– Second moment of elemental area about z-axis, i.e. M.I of an elemental area about z-axis,

$$dI_{zz} = r^2 \cdot dA.$$

For the whole area ; $I_{zz} = \int r^2 \cdot dA = \int (x^2 + y^2) dA \qquad (\therefore r^2 = x^2 + y^2)$

$$= \int x^2 \cdot dA + \int y^2 \cdot dA$$

$\int x^2 \cdot dA$ = M.I of an area at y-axis

and $\int y^2 \cdot dA$ = M.I of an area at x-axis

$$\therefore \boxed{I_{zz} = I_{xx} + I_{yy}}$$

Note : I_{zz} is also known as polar M.I.

6.8 M.I for Standard Shapes (Areas)

1. Rectangular Area :

(A) About Centroidal x-axis :

– Consider rectangle of width 'b' and depth 'd' as shown in Fig. 6.8.1.

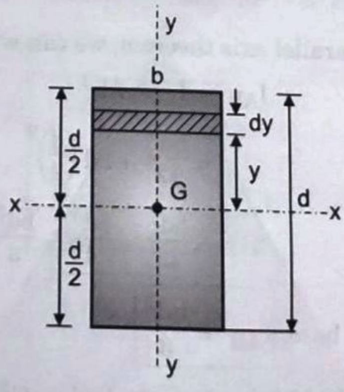


Fig. 6.8.1

– Consider a small strip of width 'b' and thickness dy at a distance 'y' from the centroidal x-axis.

– Area of elemental strip, $dA = b \cdot dy$

Moment of inertia of elemental strip at centroidal x-axis,

$$dI_{xx} = y^2 \cdot dA$$

Total M.I about x-axis,

$$I_{xx} = \int y^2 \cdot dA = b \int_{-d/2}^{+d/2} y^2 \cdot dy$$

$$= b\left[\frac{y^3}{3}\right]_{-d/2}^{+d/2} = \frac{b}{3}\left[\frac{d^3}{8} + \frac{d^3}{8}\right]$$

$$= \frac{b}{24}(2d^3) = \frac{bd^3}{12}$$

$$\therefore \boxed{I_{xx} = \frac{bd^3}{12}}$$

Similarly, $\boxed{I_{yy} = \frac{b^3d}{12}}$

(B) About Base of the Rectangle :

M.I at the base i.e AB can be found by using parallel axis theorem.

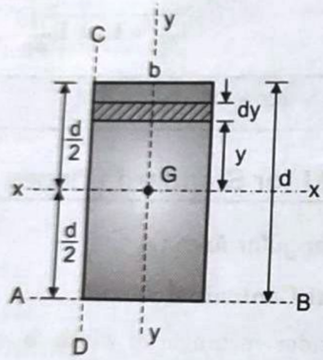


Fig. 6.8.2

From parallel axis theorem, we can write,

$$I_{AB} = I_{xx} + Ah^2$$

$$= \frac{bd^3}{12} + bd\left(\frac{d}{2}\right)^2$$

$$= \frac{bd^3}{12} + \frac{bd^3}{4} = \frac{bd^3}{3}$$

$\therefore$ M.I about base, $\boxed{I_{AB} = \frac{bd^3}{3}}$

Similarly, M.I about vertical edge, CD is

$$\boxed{I_{CD} = \frac{b^3d}{3}}$$

2. Hollow Rectangular area :

$$I_{xx} = \left(\frac{BD^3}{12} - \frac{bd^3}{12}\right)$$

and

$$I_{yy} = \left(\frac{B^3D}{12} - \frac{b^3d}{12}\right)$$

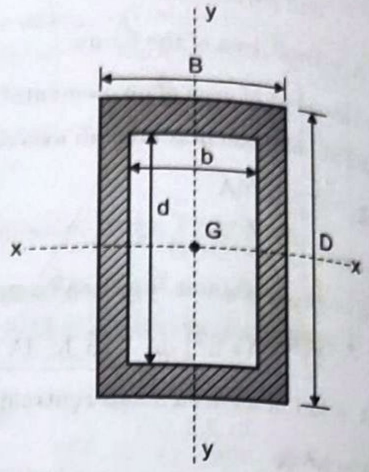


Fig. 6.8.3

3. Square :

Here, $b = d = a$

$$\therefore \boxed{I_{xx} = I_{yy} = \frac{a^4}{12}}$$

For rectangle, $I_{xx} = \frac{bd^3}{12}$

$I_{yy} = \frac{b^3d}{12}$

4. Circular area :

Consider circular area of radius, R.

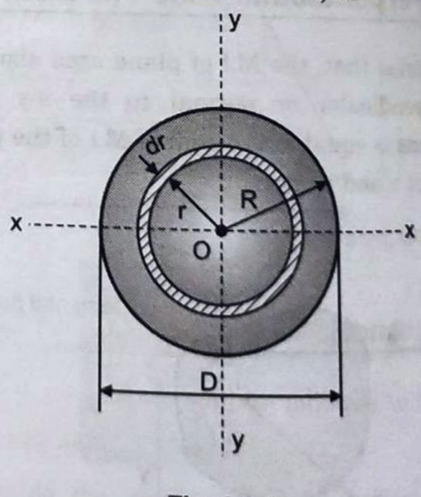


Fig. 6.8.4

Consider an elemental ring of thickness dr at a distance 'r' from the centre O.i.e. z or polar axis.

Area of elemental ring, $dA = 2\pi r \cdot dr$

M.I of an elemental ring about z-axis is

$$d\,I_{zz} = r^2 \cdot dA$$

$$= r^2 \cdot 2\pi\, r\, dr$$

M.I of the whole area,

$$I_{zz} = \int_0^R r^2 2\pi r dr = 2\pi \int_0^R r^3 \cdot dr = 2\pi \left[\frac{r^4}{4}\right]_0^R$$

$$= 2\pi \frac{R^4}{4} = \frac{\pi R^4}{2}$$

$$\boxed{I_{zz} = \frac{\pi R^4}{2}}$$

Using perpendicular axis theorem;

$$I_{zz} = I_{xx} + I_{yy}$$

But $I_{xx} = I_{yy}$ due to symmetry

$$\therefore I_{zz} = 2I_{xx} \text{ or } 2I_{yy}$$

$$\therefore I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{1}{2}\left(\frac{\pi R^4}{2}\right)$$

$$\therefore \boxed{I_{xx} = I_{yy} = \frac{\pi R^4}{4}}$$

$$R = \frac{D}{2} \qquad \text{(D = Diameter)}$$

$$\therefore I_{zz} = \frac{\pi}{2}\left(\frac{D}{2}\right)^4 = \frac{\pi D^4}{32}$$

$$\therefore \boxed{I_{zz} = \frac{\pi D^4}{32}}$$

and $\boxed{I_{xx} = I_{yy} = \frac{\pi D^4}{64}}$

5. Hollow Circular Area :

Due to symmetry,

$$\boxed{I_{xx} = I_{yy} = \frac{\pi}{64}(D^4 - d^4)}$$

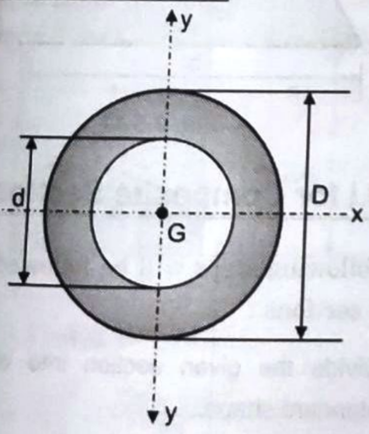


Fig. 6.8.5

6. Semi-circle

$$I_{zz} = \frac{\pi R^4}{4}$$

$$I_{xx} = I_{yy} = I_{base} = \frac{\pi R^4}{8}$$

$$= \frac{\pi D^4}{128}$$

$$I_G = 0.11R^4$$

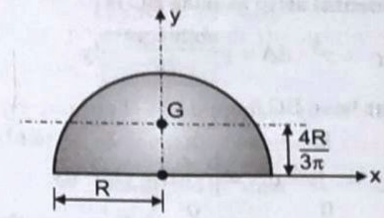


Fig. 6.8.6

7. Quarter circle :

$$I_{xx} = I_{yy} = \frac{\pi R^4}{16} = \frac{\pi D^4}{256}$$

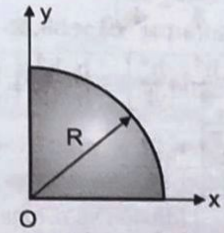


Fig. 6.8.7

8. Triangle

(A) About base 'BC'

Consider a triangular area with base 'b' and height 'h'.

Consider an elemental strip of width x and thickness dy at a distance y from the base BC.

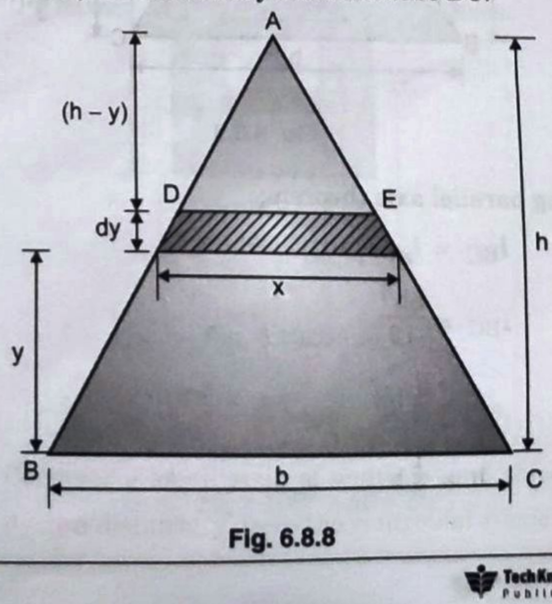


Fig. 6.8.8

From similar Δ^{les} ABC and ADE ;

$$\frac{b}{h} = \frac{x}{h-y}$$

$$\therefore x = \frac{b(h-y)}{h}$$

Area of elemental strip, $dA = x \cdot dy$

$$= \frac{b(h-y)}{h} dy$$

M.I of elemental strip at base BC is

$$dI_{BC} = y^2 \cdot dA = y^2 \frac{b(h-y)}{h} \cdot dy$$

Total M.I at base BC,

$$I_{BC} = \int_0^h y^2 \cdot dA = \frac{b}{h} \int_0^h (h-y) y^2 \cdot dy$$

$$= \frac{b}{h} \int_0^h (hy^2 - y^3) dy$$

$$= \frac{b}{h} \left[\frac{hy^3}{3} - \frac{y^4}{4} \right]_0^h$$

$$= \frac{b}{h} \left[\frac{h^4}{3} - \frac{h^4}{4} \right] = \frac{b}{h} \cdot \frac{h^4}{12} = \frac{bh^3}{12}$$

$$\therefore \quad \boxed{I_{BC} = \frac{bh^3}{12}}$$

(B) About Centroidal x-axis :

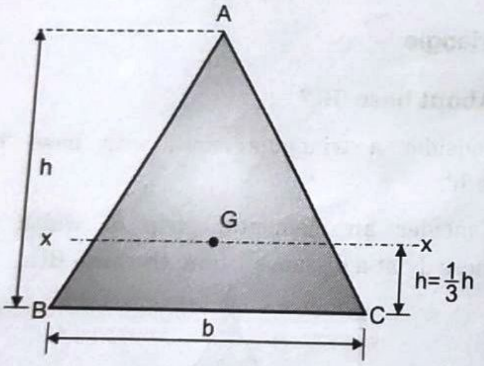


Fig. 6.8.9

Using parallel axis theorem;

$$I_{BC} = I_{xx} + Ah^2$$

$$I_{BC} = \frac{bh^3}{12}$$

$$A = \frac{1}{2} bh$$

$$h = \frac{1}{3} h$$

$$\frac{bh^3}{12} = I_{xx} + \frac{bh}{2} \cdot \left(\frac{h}{3}\right)^2$$

$$I_{xx} = \frac{bh^3}{12} - \frac{bh^3}{18} = \frac{bh^3}{36}$$

$$\therefore \boxed{I_{xx} = \frac{bh^3}{36}}$$

(C) About Vertex 'A'

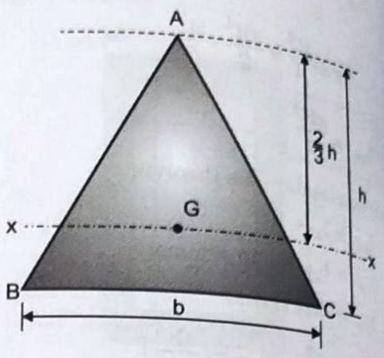


Fig. 6.8.10

Again using parallel axis theorem ;

$$I_A = I_{xx} + Ah^2 = \frac{bh^3}{36} + \frac{bh}{2} \left(\frac{2h}{3}\right)^2$$

$$= \frac{bh^3}{36} + \frac{4bh^3}{18} = \frac{bh^3}{36} + \frac{2bh^3}{9} = \frac{bh^3}{4}$$

$$\therefore \boxed{I_A = \frac{bh^3}{4}}$$

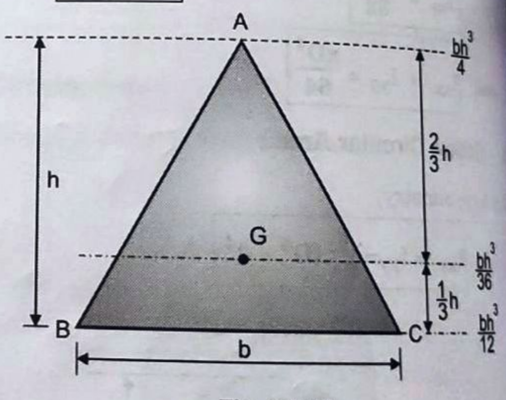


Fig. 6.8.11

6.9 M.I for Composite Sections

The following steps will be followed to find M.I of composite sections :

Step 1 : Divide the given section into different parts of standard shape.

Step 2 : Determine the centre of gravity of the section.

Step 3 : Find M.I of each part using standard formulae.

Step 4 : Using parallel axis theorem, find the M.I of each part about the centroidal axes of the composite section.

Step 5 : Addition of M.I of each part will give the M.I of whole section.

Step 6 : If hollow section is given, M.I of inner portion is to be subtracted from the M.I of external portion.

6.10 Solved Examples

Ex. 6.10.1 : For the rectangular section shown in Fig. P. 6.10.1 compute the values of I_{xx}, I_{yy}, I_{AB} and I_{PQ}.

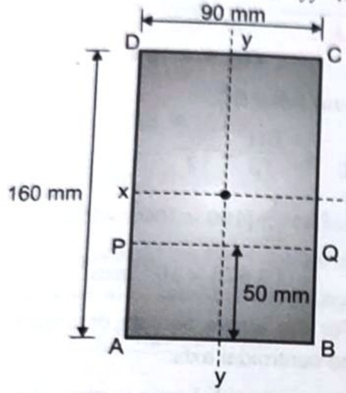


Fig. P. 6.10.1

Soln. :

Step 1 : Area of the section,

$$A = 90 \times 160 = 14400 \text{mm}^2$$

$$b = 90 \text{ mm}$$

$$d = 160 \text{ mm}$$

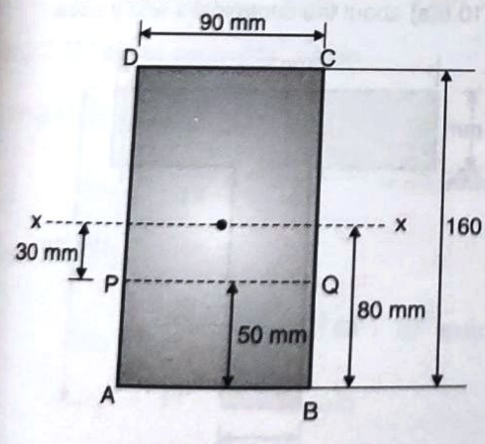


Fig. P. 6.10.2

Step 2 : $I_{xx} = \dfrac{bd^3}{12} = \dfrac{90 \times 160^3}{12}$

$= \mathbf{30.72 \times 10^6 \text{ mm}^4}$ **...Ans.**

Step 3 : $I_{yy} = \dfrac{b^3 d}{12} = \dfrac{90^3 \times 160}{12}$

$= \mathbf{9.72 \times 10^6 \text{ mm}^4}$ **...Ans.**

Step 4 : Using parallel axis theorem :

$I_{AB} = I_{xx} + Ah^2$ (h = 80 mm)

$= 30.72 \times 10^6 + 14400 \times 80^2$

$= 30.72 \times 10^6 + 92.16 \times 10^6$

$= \mathbf{122.88 \times 10^6 \text{ mm}^4}$

OR

$I_{AB} = \dfrac{bd^3}{3} = \dfrac{90 \times 160^3}{3}$

$= \mathbf{122.88 \times 10^6 \text{ mm}^4}$

Step 5 : Using parallel axis theorem ;

$I_{PQ} = I_{xx} + Ah^2$ (h = 30 mm)

$= 30.72 \times 10^6 + 14400 \times 30^2$

$= 30.72 \times 10^6 + 12.96 \times 10^6$

$= \mathbf{43.68 \times 10^6 \text{ mm}^4}$ **...Ans.**

Ex. 6.10.2 : Find the moment of inertia of a hollow rectangular section about the centroidal x and y-axes whose dimensions are external breadth 80 mm, depth 100 mm and internal dimensions are breadth 40 mm and depth 50 mm respectively.

Soln. :

Step 1 : Given section :

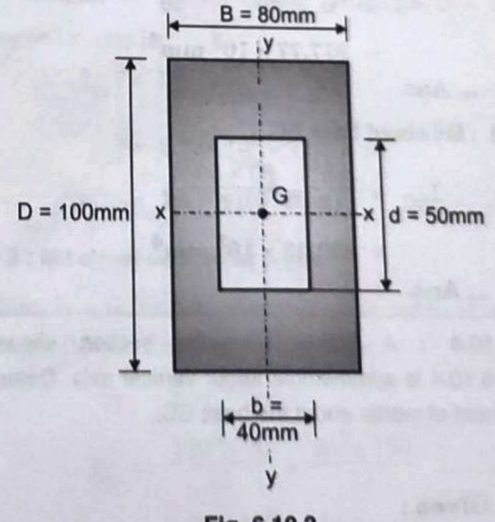


Fig. 6.10.2

Step 2 : $I_{xx} = \dfrac{BD^3}{12} - \dfrac{bd^3}{12} = \dfrac{1}{12}(BD^3 - bd^3)$

$= \dfrac{1}{12}[80 \times 100^3 - 40 \times 50^3]$

Step 4 : Using parallel axis theorem, find the M.I of each part about the centroidal axes of the composite section.

Step 5 : Addition of M.I of each part will give the M.I of whole section.

Step 6 : If hollow section is given, M.I of inner portion is to be subtracted from the M.I of external portion.

6.10 Solved Examples

Ex. 6.10.1 : For the rectangular section shown in Fig. P. 6.10.1 compute the values of I_{xx}, I_{yy}, I_{AB} and I_{PQ}.

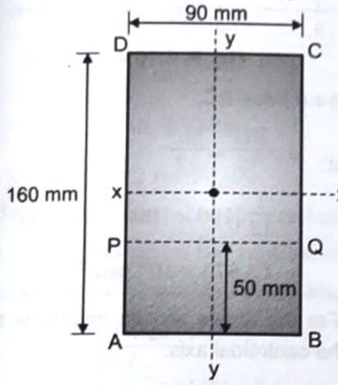


Fig. P. 6.10.1

Soln. :

Step 1 : Area of the section,

$$A = 90 \times 160 = 14400 \text{mm}^2$$

$$b = 90 \text{ mm}$$

$$d = 160 \text{ mm}$$

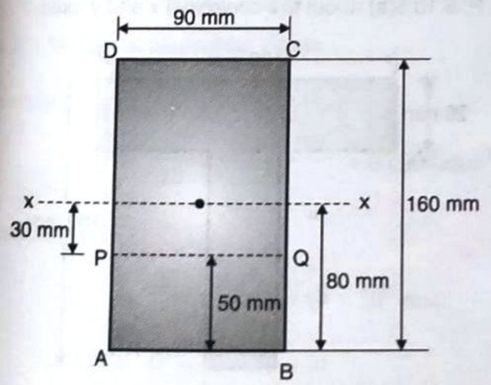


Fig. P. 6.10.2

Step 2 : $I_{xx} = \dfrac{bd^3}{12} = \dfrac{90 \times 160^3}{12}$

$$= \mathbf{30.72 \times 10^6 \text{ mm}^4} \quad \text{...Ans.}$$

Step 3 : $I_{yy} = \dfrac{b^3 d}{12} = \dfrac{90^3 \times 160}{12}$

$$= \mathbf{9.72 \times 10^6 \text{ mm}^4} \quad \text{...Ans.}$$

Step 4 : Using parallel axis theorem :

$$I_{AB} = I_{xx} + Ah^2 \qquad (h = 80 \text{ mm})$$

$$= 30.72 \times 10^6 + 14400 \times 80^2$$

$$= 30.72 \times 10^6 + 92.16 \times 10^6$$

$$= \mathbf{122.88 \times 10^6 \text{ mm}^4}$$

OR

$$I_{AB} = \frac{bd^3}{3} = \frac{90 \times 160^3}{3}$$

$$= \mathbf{122.88 \times 10^6 \text{ mm}^4}$$

Step 5 : Using parallel axis theorem ;

$$I_{PQ} = I_{xx} + Ah^2 \qquad (h = 30 \text{ mm})$$

$$= 30.72 \times 10^6 + 14400 \times 30^2$$

$$= 30.72 \times 10^6 + 12.96 \times 10^6$$

$$= \mathbf{43.68 \times 10^6 \text{ mm}^4} \quad \text{...Ans.}$$

Ex. 6.10.2 : Find the moment of inertia of a hollow rectangular section about the centroidal x and y-axes whose dimensions are external breadth 80 mm, depth 100 mm and internal dimensions are breadth 40 mm and depth 50 mm respectively.

Soln. :

Step 1 : Given section :

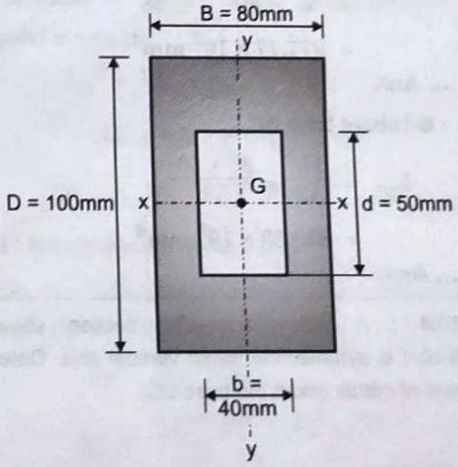


Fig. 6.10.2

Step 2 : $I_{xx} = \dfrac{BD^3}{12} - \dfrac{bd^3}{12} = \dfrac{1}{12}(BD^3 - bd^3)$

$$= \frac{1}{12}[80 \times 100^3 - 40 \times 50^3]$$

$$= 6.25 \times 10^6 \text{ mm}^4 \quad \text{...Ans.}$$

Step 3 :

$$I_{yy} = \frac{B^3D}{12} - \frac{b^3d}{12}$$

$$= \frac{1}{12}[B^3D - b^3D]$$

$$= \frac{1}{12}[80^3 \times 100 - 40^3 \times 50]$$

$$= 4 \times 10^6 \text{ mm}^4 \quad \text{... Ans.}$$

Ex. 6.10.3 : An isosceles triangular section ABC has base width 80 mm and height 50 mm. Determine the moment of inertia of the section about centre of gravity and the base BC.

Soln. :

Step 1 : Given section :

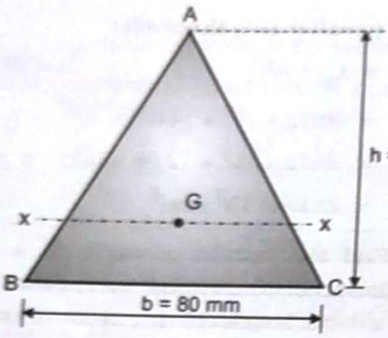


Fig. 6.10.3

Step 2 : M-I about centre of gravity,

$$I_G = I_{xx} = \frac{bh^3}{36} = \frac{80 \times 50^3}{36}$$

$$= 277.77 \times 10^3 \text{ mm}^4$$

... Ans.

Step 3 : M-I about base BC,

$$I_{BC} = \frac{bh^3}{12} = \frac{80 \times 50^3}{12}$$

$$= 833.33 \times 10^3 \text{ mm}^4$$

... Ans.

Ex. 6.10.4 : A hollow triangular section shown in Fig. P. 6.10.4 is symmetrical about vertical axis. Determine the moment of inertia about the base BC.

Soln. :

Step 1 : Given :

B = 190 mm ; H = 100 mm

b = 120 mm ; h = 60 mm

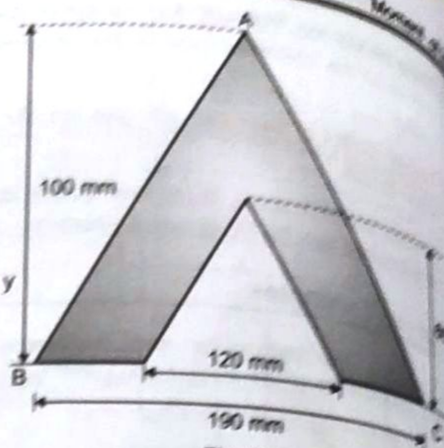


Fig. P. 6.10.4

Step 2 : M-I about base BC,

$$I_{BC} = \frac{BH^3}{12} - \frac{bh^3}{12}$$

$$= \frac{1}{12}[190 \times 100^3 - 120 \times 60^3]$$

$$= 13.673 \times 10^6 \text{ mm}^4 \quad \text{... Ans.}$$

Ex. 6.10.5 : For the above section determine the moment of inertia about the centroidal axis.

Soln. : M-I about centroidal axis is given by,

$$I_G = \frac{BH^3}{36} - \frac{bh^3}{36}$$

$$= \frac{1}{36}[190 \times 100^3 - 120 \times 60^3]$$

$$= 4.558 \times 10^6 \text{ mm}^4 \quad \text{... Ans.}$$

Ex. 6.10.6 : Find the moment of inertia of 'T' section shown in Fig. P. 6.10.6(a) about the centroidal x and y axes.

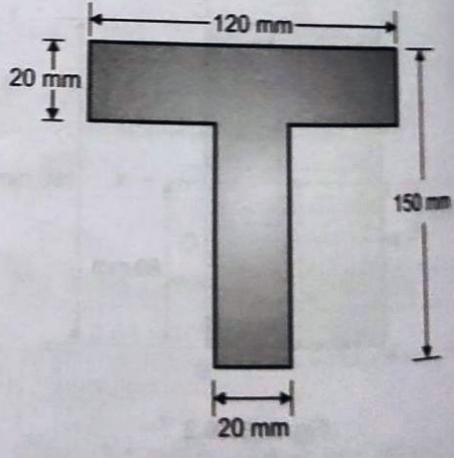


Fig. P. 6.10.6(a)

Soln. :

Step 1 : Locate centroid of the section :

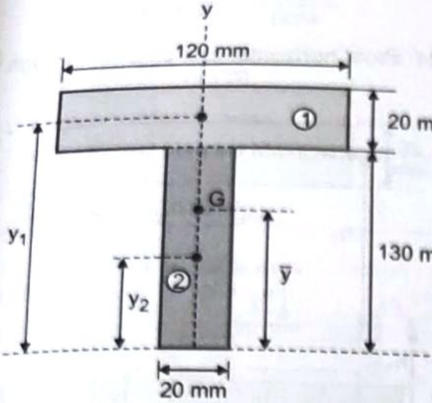


Fig. 6.10.6(b)

Divide the section into 2 rectangles.

Given section is symmetrical about vertical axis.

∴ Centroid will be located on the vertical axis.

From base, position of centroid is given by,

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$A_1 = 120 \times 20 = 2400 \text{ mm}^2$$

$$y_1 = 130 + 10 = 140 \text{ mm}$$

$$A_2 = 130 \times 20 = 2600 \text{ mm}^2$$

$$y_2 = \frac{130}{2} = 65 \text{ mm}$$

$$\therefore \bar{y} = \frac{2400 \times 140 + 2600 \times 65}{2400 + 2600} = \mathbf{101\ mm}$$

Step 2 : M-I about centroidal x-axis

For rectangle 1;

$$I_{G_1} = \frac{bd^3}{12} = \frac{120 \times 20^3}{12} = 80{,}000 \text{ mm}^4$$

For rectangle 2;

$$I_{G_2} = \frac{20 \times 130^3}{12} = 3.66 \times 10^6 \text{ mm}^4$$

$$h_1 = 49 - 10 = 39 \text{ mm}$$

$$h_2 = 101 - 65 = 36 \text{ mm}$$

Using parallel axis theorem;

$$I_{xx} = I_G + Ah^2$$

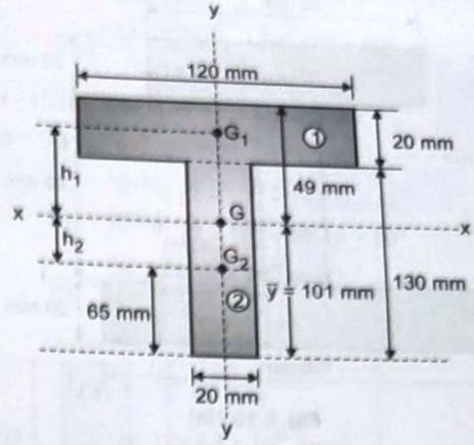


Fig. 6.10.6(c)

For rectangle 1;

$$I_{xx_1} = I_{G_1} + A_1 h_1^2$$

$$I_{xx_1} = 80 \times 10^3 + 2400 \times 39^2$$

$$= \mathbf{3.73 \times 10^6\ mm^4}$$

For rectangle 2;

$$I_{xx_2} = I_{G_2} + A_2 h_2^2$$

$$I_{xx_2} = 3.66 \times 10^6 + 2600 \times 36^2$$

$$= \mathbf{7.03 \times 10^6\ mm^4}$$

∴ Moment of inertia of the whole section about centroidal x-axis is;

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$= 3.73 \times 10^6 + 7.03 \times 10^6$$

$$= \mathbf{10.76 \times 10^6\ mm^4} \quad \text{...Ans.}$$

Step 3 : M.I about centroidal y-axis

Here, h = 0 for both rectangles as centroid of each rectangle lies on the centroidal y-axis.

$$I_{yy} = I_{yy_1} + I_{yy_2}$$

$$I_G = \frac{120^3 \times 20}{12} + \frac{20^3 \times 130}{12}$$

$$= 2.88 \times 10^6 + 0.087 \times 10^6$$

$$= \mathbf{2.967 \times 10^6\ mm^4} \quad \text{...Ans.}$$

Ex. 6.10.7 : Determine the moment of inertia of an I section shown in Fig. P. 6.10.7(a) about the horizontal axis passing through the centre of gravity of the section.

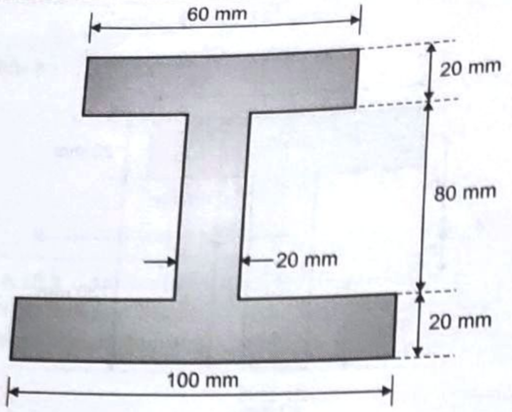


Fig. 6.10.7(a)

Soln. :

Step 1 : Location of centroid of the section :

Section is symmetrical about vertical axis i.e. y-axis.

$\therefore \ \bar{x} = 0$

To find $\bar{y}$, divide the section into 3 rectangles.

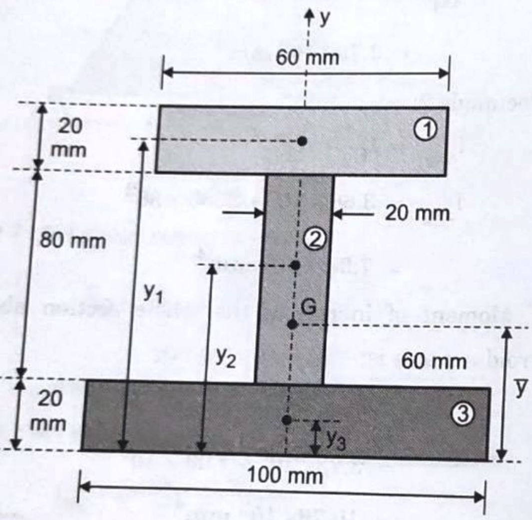


Fig. 6.10.7(b)

$$A_1 = 60 \times 20 = 1200 \text{ mm}^2$$

$$y_1 = 120 - 10 = 110 \text{ mm}$$

$$A_2 = 80 \times 20 = 1600 \text{ mm}^2$$

$$y_2 = 20 + 40 = 60 \text{ mm}$$

$$A_3 = 100 \times 20 = 2000 \text{ mm}^2$$

$$y_3 = 10 \text{ mm}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$\therefore \bar{y} = \frac{1200 \times 110 + 1600 \times 60 + 2000 \times 10}{1200 + 1600 + 2000}$$

$$= \frac{248000}{4800} = \mathbf{51.67 \text{ mm}}$$

Step 2 : M·I about horizontal axis passing through C.G.

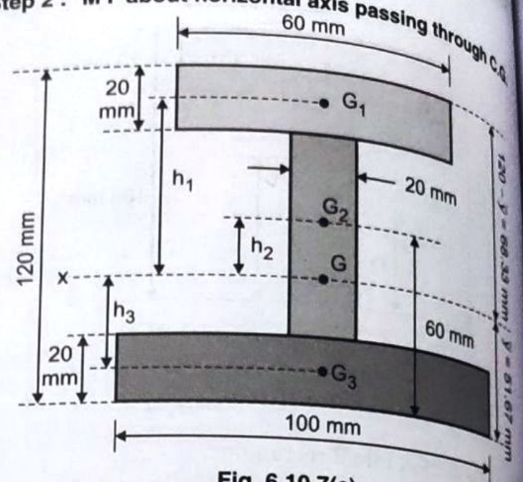


Fig. 6.10.7(c)

$$h_1 = 68.33 - 10 = 58.33 \text{ mm}$$

$$h_2 = 60 - 51.67 = 8.33 \text{ mm}$$

$$h_3 = 51.67 - 10 = 41.67 \text{ mm}$$

Using parallel axis theorem;

$$I_{G_1} = \frac{60 \times 20^3}{12} = 40{,}000 \text{ mm}^4$$

$$I_{xx1} = I_{G_1} + A_1 h_1^2$$

$$= 40{,}000 + 1200 \times 58.33^2$$

$$= 4.123 \times 10^6 \text{ mm}^4.$$

$$I_{G_2} = \frac{20 \times 80^3}{12} = 853333.33 \text{ mm}^4$$

$$I_{xx_2} = I_{G_2} + A_2 h_2^2$$

$$= 853333.33 + 1600 \times 8.33^2$$

$$= 0.964 \times 10^6 \text{ mm}^4$$

$$I_{G_3} = \frac{100 \times 20^3}{12} = 66666.67 \text{ mm}^4$$

$$I_{xx_3} = I_{G_3} + A_3 h_3^2 = 66666.67 + 2000 \times 41.67^2$$

$$= 3.54 \times 10^6 \text{ mm}^4$$

M.I of the whole section,

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$\therefore I_{xx} = 4.123 \times 10^6 + 0.964 \times 10^6 + 3.54 \times 10^6$$

$$= \mathbf{8.627 \times 10^6 \text{ mm}^4} \quad \text{...Ans.}$$

Ex. 6.10.8 : Find the moment of inertia about the centroidal x and y- axes of the angle section shown in Fig. P. 6.10.8(a).

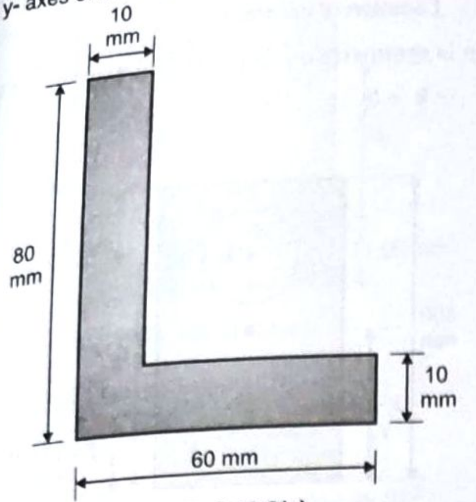


Fig. P. 6.10.8(a)

Soln. :

Step 1 : Location of centroid

Divide the section into two rectangles.

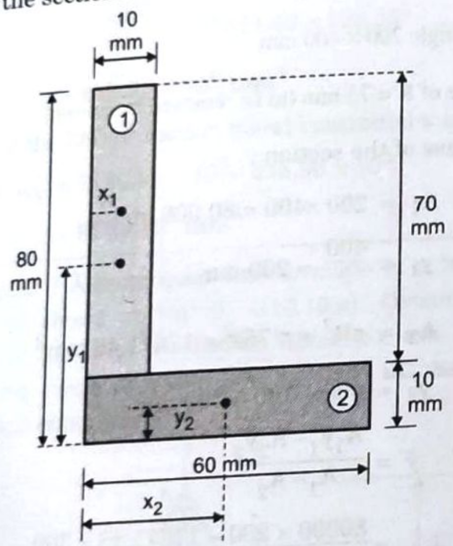


Fig. P. 6.10.8(a)

$$A_1 = 10 \times 70 = 700 \text{ mm}^2$$

$$y_1 = 10 + 35 = 45 \text{ mm}$$

$$x_1 = 5 \text{ mm}$$

$$A_2 = 60 \times 10 = 600 \text{ mm}^2$$

$$x_2 = 30 \text{ mm}$$

$$y_2 = 5 \text{ mm}$$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{700 \times 5 + 600 \times 30}{700 + 600}$$

$$= \mathbf{16.54 \text{ mm}}$$

$$\bar{y} = \frac{A_2 y_2 + A_2 y_2}{A_1 + A_2} = \frac{700 \times 45 + 600 \times 5}{700 + 600}$$

$$= \mathbf{26.54 \text{ mm}}$$

Step 2 : M.I. about centroidal x-axis :

$$h_1 = 53.46 - 35 = 18.46 \text{ mm}$$

$$h_2 = 26.54 - 5 = 21.54 \text{ mm}$$

$$I_{G_1} = \frac{10 \times 70^3}{12} = 285.83 \times 10^3 \text{ mm}^4$$

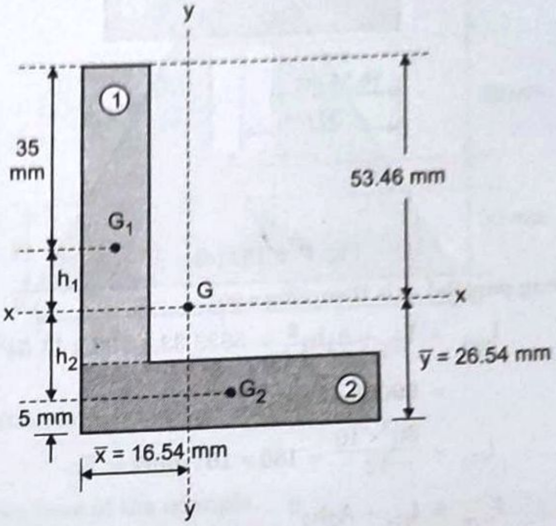


Fig. P. 6.10.8(c)

Using parallel axis theorem :

$$I_{xx1} = I_{G_1} + A_1 h_1^2$$

$$= 285.83 \times 10^3 + 700 \times 18.46^2$$

$$= \mathbf{524.373 \times 10^3 \text{ mm}^4}$$

$$I_{G_2} = \frac{60 \times 10^3}{12} = 5000 \text{ mm}^4$$

$$I_{xx2} = I_{G_2} + A_2 h_2^2 = 5000 + 600 \times 21.54^2$$

$$= \mathbf{283.383 \times 10^3 \text{ mm}^4}$$

$\therefore$ M.I for the entire section, $I_{xx} = I_{xx1} + I_{xx2}$

$$\therefore \quad I_{xx} = 524.373 \times 10^3 + 283.383 \times 10^3$$

$$= \mathbf{807.756 \times 10^3 \text{ mm}^4} \quad \text{... Ans.}$$

Step 3 : M.I. about centroidal y-axis :

$$h_1 = 16.54 - 5 = 11.54 \text{ mm}$$

$$h_2 = 30 - 16.54 = 13.46 \text{ mm}$$

$$I_{G_1} = \frac{10^3 \times 70}{12} = 5833.33 \text{ mm}^4$$

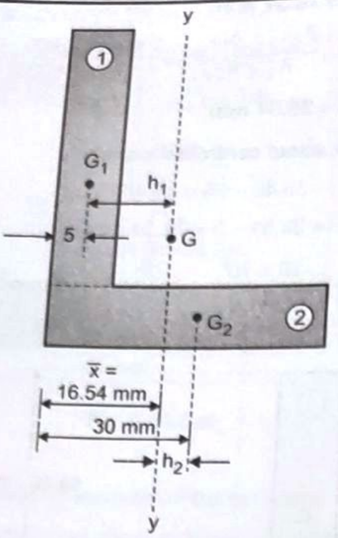


Fig. P. 6.10.8(d)

Using parallel axis theorem ;

$$I_{yy1} = I_{G1} + A_1 h_1^2 = 5833.33 + 700 \times 11.54^2$$

$$= \mathbf{99053.45 mm^4}$$

$$I_{G2} = \frac{60^3 \times 10}{12} = 180 \times 10^3 \ mm^4$$

$$I_{yy2} = I_{G1} + A_2 h_2^2$$

$$= 180 \times 10^3 + 600 \times 13.46^2$$

$$= \mathbf{288.703 \times 10^3 \ mm^4}$$

$\therefore$ M.I for the entire section,

$$I_{yy} = I_{yy1} + I_{yy2}$$

$$= 99053.45 + 288.703 \times 10^3$$

$$= \mathbf{387.756 \times 10^3 \ mm^4} \quad \text{...Ans.}$$

Ex. 6.10.9 : Find the moment of inertia of a hollow section shown in Fig. P. 6.10.9(a)about an x-axis passing through its centre of gravity.

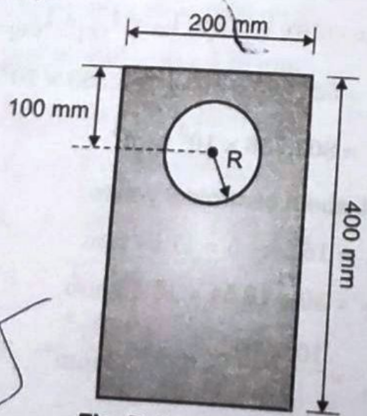


Fig. P. 6.10.9(a)

Soln. :

Step 1 : Location of centroid :

Section is symmetrical about vertical axis i.e. y-axis

$$\therefore \bar{x} = 0$$

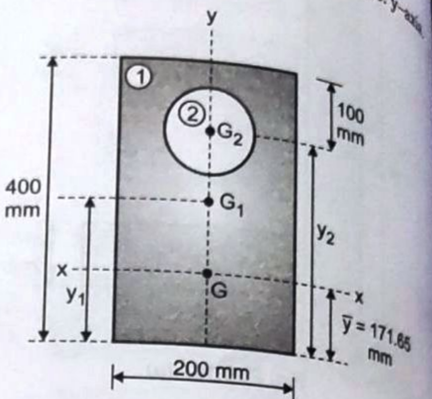


Fig. P. 6.10.9(b)

Dividing section in two parts :

1. Rectangle 200 × 400 mm
2. Circle of R = 75 mm (to be removed or cut-out)

From base of the section ;

$$A_1 = 200 \times 400 = 80{,}000 \ mm^2$$

$$y_1 = \frac{400}{2} = 200 \ mm$$

$$A_2 = \pi R^2 = \pi (75)^2 = 17671.46 \ mm^2$$

$$y_2 = 400 - 100 = 300 \ mm$$

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$= \frac{80000 \times 200 - 17671.46 \times 300}{80000 - 17671.46}$$

$$= \frac{10698562.4}{62328.54} = \mathbf{171.65 \ mm}$$

Step 2 : M.I about centroidal x-axis :

$$I_{xx} = I_{xx_1} - I_{xx_2}$$

$$h_1 = 200 - 171.65$$

$$= 28.35 \ mm$$

$$h_2 = 300 - 171.65$$

$$= 128.35 \ mm$$

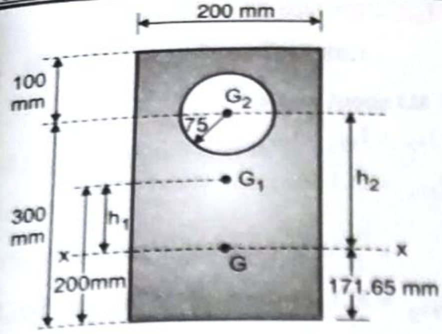


Fig. P. 6.10.9(c)

Using parallel axis theorem ;

$$I_{xx_1} = I_{G_1} + A_1 h_1^2$$

$$= \left(\frac{200 \times 400^3}{12}\right) + 80{,}000 \times 28.35^2$$

$$= \mathbf{1130.96 \times 10^6\ mm^4}$$

$$I_{xx_2} = I_{G_2} + A_2 h_2^2$$

$$= \frac{\pi(150)^4}{64} + 17671.46 \times 128.35^2$$

$$= \mathbf{315.96 \times 10^6\ mm^4}$$

∴ M.I of the hollow section about centroidal x-axis is,

i.e. $I_{xx} = 1130.96 \times 10^6 - 315.96 \times 10^6$

$= \mathbf{815 \times 10^6\ mm^4}$ **...Ans.**

Ex. 6.10.10 : A rectangular hole is made in a triangular section as shown in Fig. P. 6.10.10(a). Determine the moment of inertia of the section about x-x axis passing through the centre of gravity and the base BC. Section is symmetrical about y-axis.

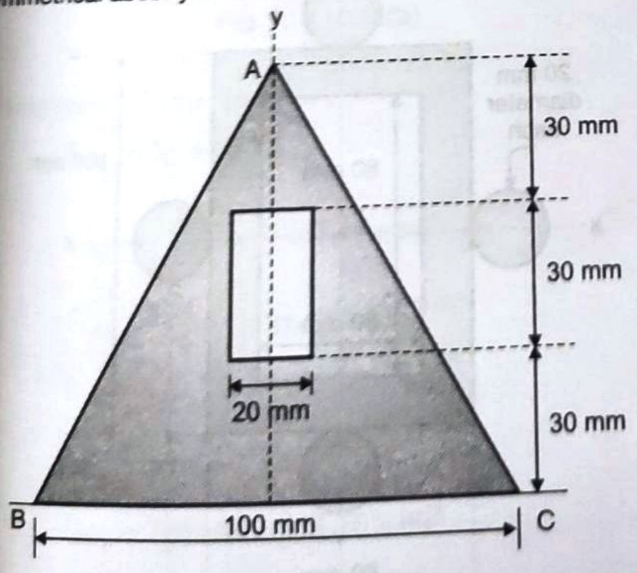


Fig. P. 6.10.10(a)

Soln. :

Step 1 : Location of centre of gravity :

Section can be divided in to two parts.

1. Δ ABC
2. Rectangle (to be removed)

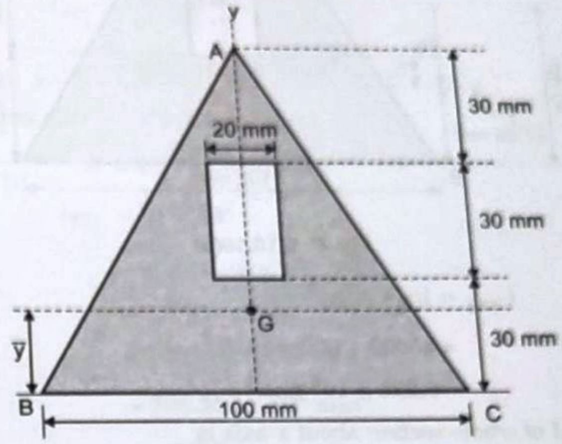


Fig. P. 6.10.10(b)

Section is symmetrical about y-axis.

$$\therefore \bar{x} = 0$$

From base of the triangle,

$$\therefore \bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$A_1 = \frac{1}{2} \times 100 \times 90 = 4500\ mm^2$$

$$y_1 = \frac{1}{3} \times 90 = 30\ mm$$

$$A_2 = 20 \times 30 = 600\ mm^2$$

$$y_2 = 30 + 15 = 45\ mm$$

$$\bar{y} = \frac{4500 \times 30 - 600 \times 45}{4500 - 600} = \mathbf{27.69\ mm}$$

Step 2 : M.I about centroidal x-axis :

$$h_1 = 30 - 27.69 = 2.31\ mm$$

$$h_2 = 45 - 27.69 = 17.31\ mm$$

$$I_{G_1} = \frac{bh^3}{36} = \frac{100 \times 90^3}{36} = 2.025 \times 10^6 mm^4$$

$$I_{xx_1} = I_{G_1} + A_1 h_1^2$$

$$= 2.025 \times 10^6 + 4500 \times 2.31^2$$

$$= \mathbf{2.049 \times 10^6\ mm^4}$$

$$I_{G_1} = \frac{bd^3}{12} = \frac{20 \times 30^3}{12} = 45000\ mm^4$$

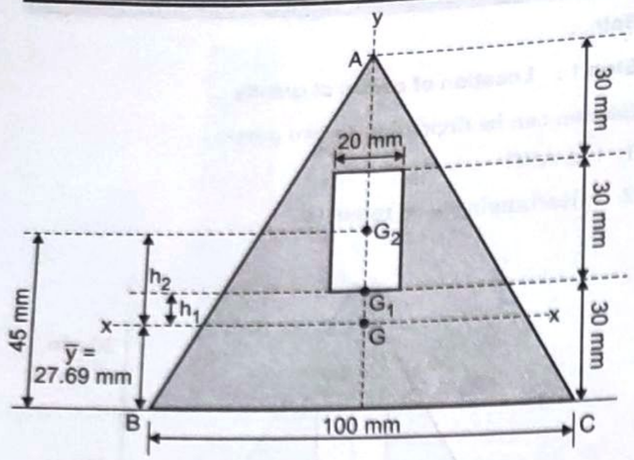


Fig. P. 6.10.10(c)

$$I_{xx_2} = I_{G_2} + A_2 h_2^2$$

$$= 45000 + 600 \times 17.31^2$$

$$= \mathbf{0.225 \times 10^6\ mm^4}$$

$\because$ M.I of whole section about x-axis is

$$I_{xx} = I_{xx_1} - I_{xx_2}$$

$$= 2.049 \times 10^6 - 0.225 \times 10^6$$

$$= \mathbf{1.824 \times 10^6\ mm^4} \quad \text{... Ans.}$$

Ex. 6.10.11 : Determine the M.I of the shaded area shown in Fig. P. 6.10.11(a)about x and y axes.

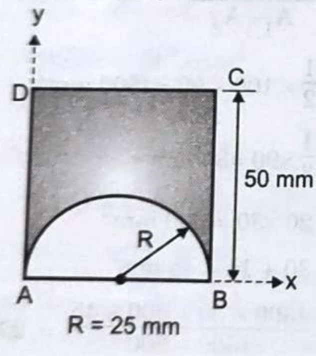


Fig. P. 6.10.11(a)

Soln. :

Step 1 : M.I about x-axis :

$$I_{xx} = I_{xx_1} - I_{xx_2}$$

I_{xx_1} = M.I of square ABCD about base

$$= \frac{b^4}{3} = \frac{50^4}{3} = 2083333.33\ mm^4$$

I_{xx_2} = M.I of semi-circle about base AB

$$= \frac{\pi D^4}{128} = \frac{\pi (50)^4}{128} = 153398.08\ mm^4$$

$$\therefore I_{xx} = 2083333.33 - 153398.08$$

$$= \mathbf{1.93 \times 10^6\ mm^4} \quad \text{...Ans.}$$

Step 2 : M.I about y-axis :

$$I_{yy} = I_{yy_1} - I_{yy_2}$$

I_{yy_1} = M.I of square about edge

$$= \frac{b^4}{3} = \frac{50^4}{3} = 2.083 \times 10^6\ mm^4$$

I_{yy_2} = M.I of Semicircle about the line AD parallel to its centroidal axis.

$$h_2 = 25\ mm$$

$$A_2 = \frac{\pi (25)^2}{2}$$

$$= 981.75\ mm^2$$

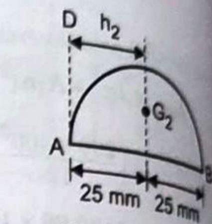


Fig. P. 6.10.11(b)

Using parallel axis theorem ;

$$I_{yy_2} = I_{G_2} + A_2 h_2^2$$

$$= \frac{\pi (50)^4}{128} + 981.75 \times 25^2$$

$$= 153.39 \times 10^3 + 613.592 \times 10^3$$

$$= 766.982 \times 10^3\ mm^4$$

$$\therefore I_{yy} = 2083 \times 10^3 - 766.982 \times 10^3$$

$$= \mathbf{1316.018 \times 10^3 mm^4} \quad \text{... Ans.}$$

Ex. 6.10.12 : Find the M.I of the section shown in Fig. P. 6.10.12(a) about centroidal x and y axes.

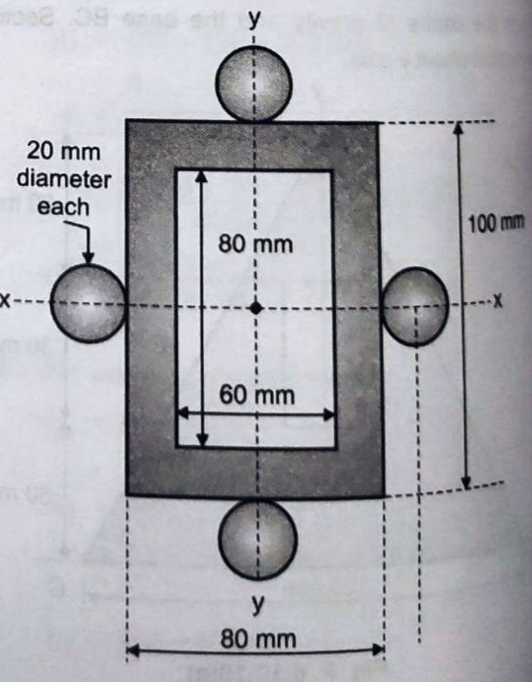


Fig. P. 6.10.12(a)

Soln. :

Step 1 : M.I about x-x axis :

$$I_{xx} = I_{xx_1} + 2\, I_{xx_2} + 2\, I_{xx_3}$$

1. Hollow rectangular section
2. Horizontal circles
3. Vertical circles

$$I_{xx_1} = \frac{1}{12}[BD^3 - bd^3]$$

$$= \frac{1}{12}[80 \times 100^3 - 60 \times 80^3]$$

$$= 4.107 \times 10^6 \text{ mm}^4$$

$$I_{xx_2} = \frac{\pi d^4}{64} = \frac{\pi(20)^4}{64}$$

$$= 7853.98 \text{ mm}^4$$

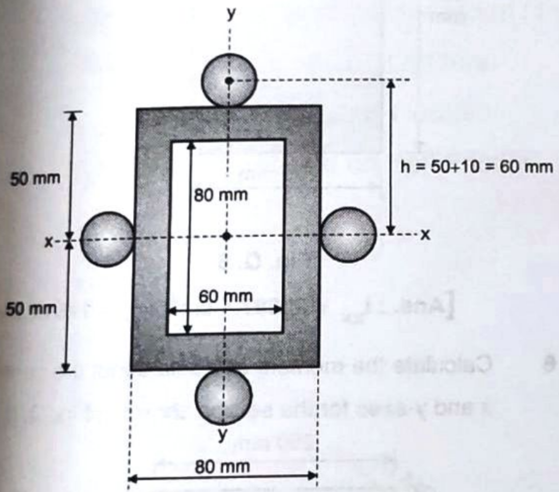


Fig. P. 6.10.12(b)

Using parallel axis theorem ;

$$I_{xx_3} = I_G + Ah^2$$

$$I_G = \frac{\pi(20)^4}{64} = 7853.98 \text{ mm}^4$$

$$A = \frac{\pi(20)^2}{4} = 314.16 \text{ mm}^2$$

$$h = 60 \text{ mm}$$

$$I_{xx_3} = I_G + Ah^2$$

$$I_{xx_3} = 7853.98 + 314.16 \times 60^2$$

$$= 1.139 \times 10^6 \text{ mm}^4$$

$\therefore$ M.I of the whole section about x-axis is

$$I_{xx} = 4.107 \times 10^6 + 2\,(7853.98) + 2\,(1.139 \times 10^6)$$

$$= \mathbf{6.4 \times 10^6 \text{ mm}^4}$$ ... **Ans.**

Step 2 : M.I about y-axis :

$$I_{yy} = I_{yy_1} + 2\, I_{yy_2} + 2\, I_{yy_3}$$

$$I_{yy_1} = \left(\frac{80^3 \times 100}{12} - \frac{60^3 \times 80}{12}\right)$$

$$= 2826.67 \times 10^3 \text{ mm}^4$$

Using parallel axis theorem;

$$h = 40 + 10 = 50\text{mm}$$

$$I_{yy_2} = I_G + Ah^2$$

$$= \frac{\pi(20)^4}{64} + \frac{\pi(20)^2}{4} \times 50^2$$

$$= 7853.98 + 314.16 \times 50^2$$

$$= \mathbf{793.254 \times 10^3 \text{ mm}^4}$$

$$\mathbf{I_{yy_3} = \frac{\pi(20)^4}{64} = 7853.98 \text{ mm}^4}$$

M.I. of whole section about centroidal y-axis is :

$$I_{yy} = 2826.67 \times 10^3 + 2(793.254 \times 10^3) + 2(7853.98)$$

$$= \mathbf{4.429 \times 10^6 \text{mm}^4}$$ ... **Ans.**

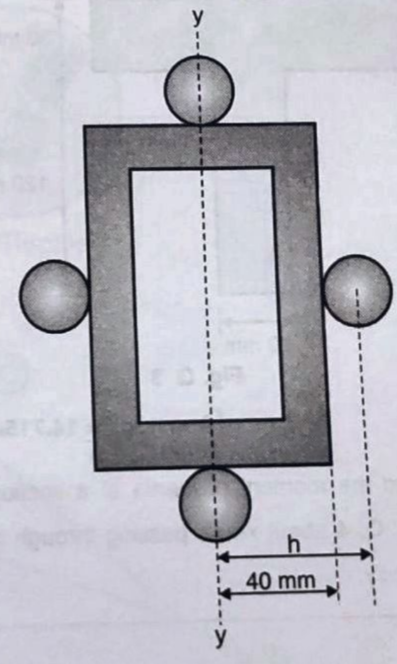


Fig. P. 6.10.12(c)

Practice Problems

Q. 1 Determine the moment of inertia of the hollow rectangular section about x-x and y-y axis passing through its centre of gravity.

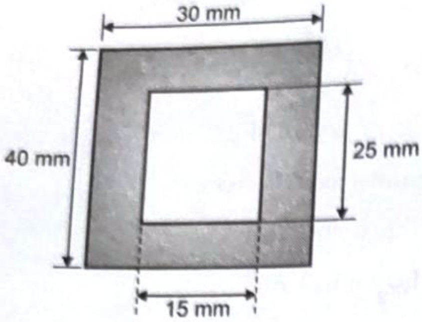


Fig. Q. 1

[Ans. : $I_{xx} = 140.47 \times 10^3$ mm^4

$I_{yy} = 82.97 \times 10^3$ mm^4]

Q. 2 Find the moment of inertia of a triangular section having 50 mm base and 60 mm height about an axis an axis passing through its centre of gravity and its base.

[Ans. : 300×10^3 mm^4 and 900×10^3 mm^4]

Q. 3 Find the moment of inertia for the section shown in Fig. Q. 3 about an x-axis passing through its centre of gravity.

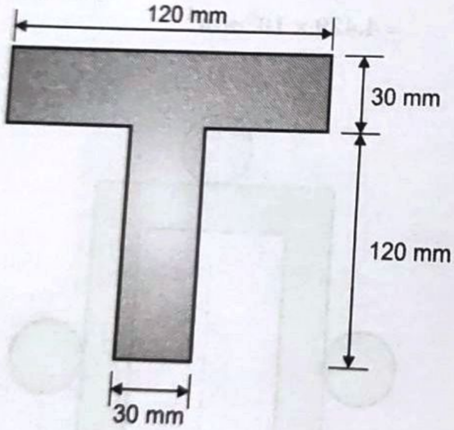


Fig. Q. 3

[Ans. : $I_{xx} = 14.715 \times 10^3$ mm^4]

Q. 4 Find the moment of inertia of a section shown in Fig. Q. 4 about x-axis passing through its centre of gravity.

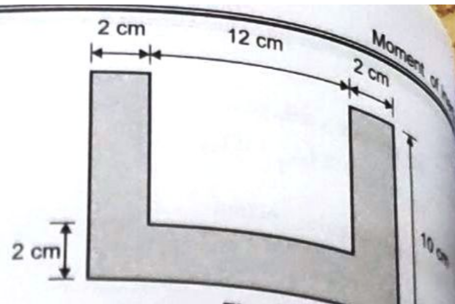


Fig. Q. 4

[Ans. : I_{xx} = 581.3...

Q. 5 Find the moment of inertia of a hollow rectang... section shown in Fig. Q. 5 about centroidal x... y-axes.

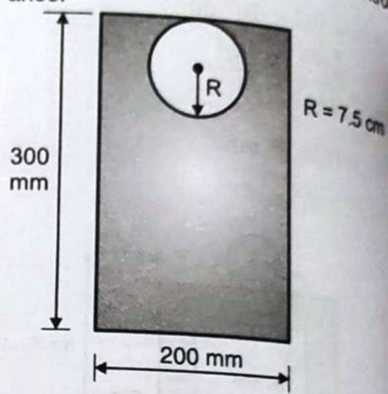


Fig. Q. 5

[Ans. : I_{xx} = 28387.4 cm^4, I_{yy} = 17514.9 cm...

Q. 6 Calculate the moment of inertia about the centro... x and y-axes for the section shown in Fig. Q. 6.

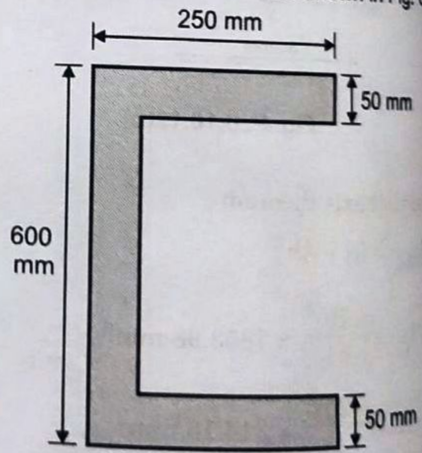


Fig. Q. 6

[Ans. : $I_{xx} = 2417 \times 10^6$ mm^4, $I_{yy} = 260.41 \times 10^6$ ...

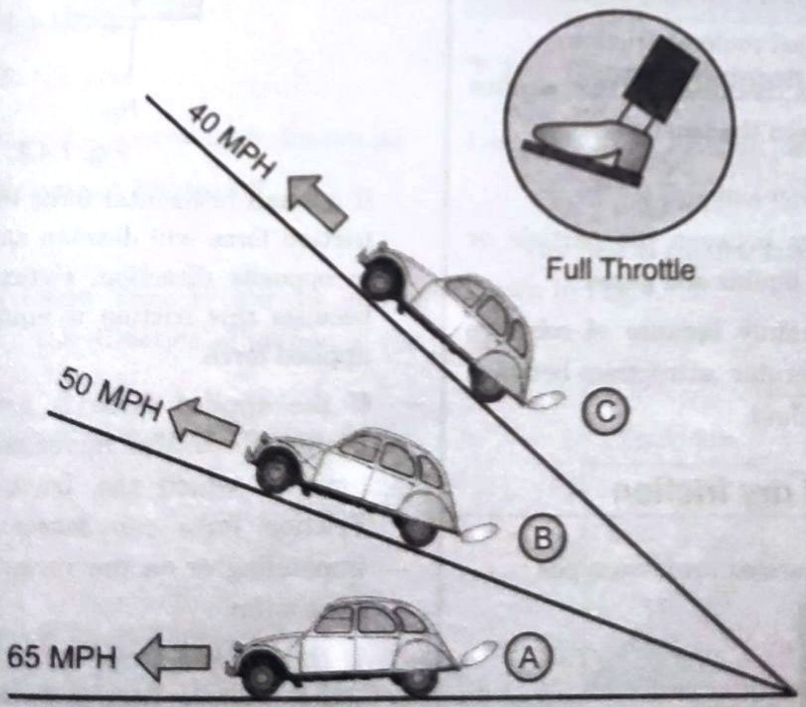
CHAPTER
7

UNIT - II

Friction

Introduction : In this chapter, we shall study the concept of friction, types of friction, laws of friction etc. We shall study the limiting equilibrium, limiting friction and application of friction for inclined planes, ladders, wedges and belt drives.

- Type 1 : Based on Horizontal / Inclined Planes
- Type 2 : Based on Ladders
- Type 3 : Based on Wedges
- Type 4 : Based on Belt Friction



7.1 Friction

*Friction is the property of two surfaces in contact by virtue of which a resisting force is developed so as to oppose the sliding motion between two surfaces. This resisting force is known as "**friction force**".*

- Friction force will act in the direction opposite to the direction of motion or impending motion.
- Impending motion is the state of the body in which the body is on the verge of motion or about to move.

*The condition in which the body is in impending motion is known as "**Limiting equilibrium.**"*

- When there is a relative motion between the two surfaces, each surface exerts friction force on the other surface. These two forces will form an action and reaction pair.

7.2 Types of Friction

There are two types of friction :

(i) Dry friction
(ii) Fluid friction

(i) Dry friction :

- It is the friction between two dry surfaces.
- Sometimes it is called coulomb friction.
- Dry friction exists because of the surface irregularities between the two bodies.

(ii) Fluid friction :

- Fluid friction exists between the particle or layers of fluids like liquids and gases.
- Fluid friction is mainly because of cohesive forces or intermolecular attraction between the particles of the fluid.

7.3 Classification of dry friction

Dry friction can be categorized into two types :

(i) Static friction
(ii) Kinetic friction

(i) Static friction :

It is the friction experienced by the surface of the body when the body is at rest or in equilibrium.

(ii) Kinetic friction :

It is the friction experienced by the surface of the body when the body is

a) Sliding over the surface (sliding friction)
b) Rolling over the surface (rolling friction)

7.4 Limiting friction (F_L)

- It is the maximum friction force that can be developed between the two surfaces in contact under the given situation before the motion starts.
- Friction force will be maximum when the body is in impending motion i.e. in limiting equilibrium.

Consider a block resting on the rough horizontal surface.

Fig. 7.4.1

Forces acting on the block are :

(i) Weight of the block, W = mg
(ii) Normal reaction from the surface.

- These two forces do not cause any motion along the surface. Hence there is no friction force in this position.

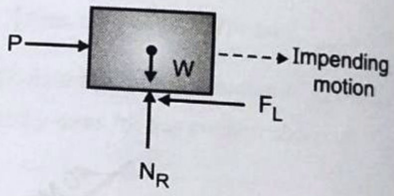


Fig. 7.4.2

- If a small horizontal force is applied on the block, friction force will develop at the surface of contact in opposite direction. Hence block will not move, because this friction is equal and opposite to the applied force.
- If the applied force is gradually increased, the friction force also increases. However, there is a limit to which the friction force can increase. Friction force can increase till the body is in impending or on the verge of motion and becomes maximum.
- If the applied force is further increased, the body moves and the friction force gets reducted.
- Hence the friction force is maximum when the body is in impending motion or limiting equilibrium, known as "limiting friction".

Note : The force required to maintain motion with uniform velocity is less than the limiting static friction force.

7.5 Coulomb's Laws of Friction

For Static Friction :

These laws are applicable for sliding or rolling friction between two dry surfaces :

(i) The direction of friction force is opposite to the direction of motion of the body or the direction in which the body tends to move.

(ii) Friction force between the two surfaces is independent of the area of contact of two surfaces.

(iii) Friction force depends on the irregularities i.e. nature of the contact surface.

(iv) The limiting friction i.e. the friction that can be developed when the body is impending motion is directly proportional to the normal reaction (N_R).

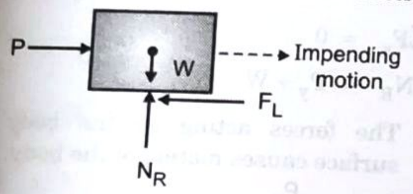


Fig. 7.5.1

$$F_L \propto N_R$$

$$F_L = \mu N_R$$

Where, F_L = Limiting friction

N_R = Normal reaction

μ = Constant of proportionality known as **"coefficient of friction."**

For Kinetic Friction :

(i) The direction of friction force is always in the direction opposite to the direction of motion of the body.

(ii) The ratio of magnitude of kinetic friction to the normal reaction is

$$\frac{F_k}{N_R} = \mu_k$$

μ_k = coefficient of kinetic friction

(iii) μ_k is less than that of μ_s.

(iv) For moderate speeds, the force of friction remains constant. But it decreases slightly with increase of speed.

7.6 Types of coefficient of friction (μ)

There are two types of coefficient of friction :

(i) Coefficient of static friction (μ_s) and

(ii) Coefficient of kinetic friction (μ_k)

(i) Coefficient of static friction (μ_s) :

– When the body is in impending motion i.e. in limiting equilibrium (at rest), then the friction force is maximum, given by, $F_L = \mu_s N_R$.

Where, μ_s = Coefficient of static friction

(ii) Coefficient of kinetic friction (μ_k) :

– When the body is in motion, the friction is given by,

$$F = \mu_k \cdot N_R$$

Where, μ_k = coefficient of kinetic friction

Important Notes :

- The friction force is less when the body is in dynamic condition (in motion) than the body is in limiting condition of equilibrium (at rest).
- $\mu_k < \mu_s$
- When the motion is about to start, $F_s = \mu_s N_R$ and when the motion starts, $F_k = \mu_k N_R$.
- $F_k < F_s$ because when the body is in motion, there is less interpretation between the surface irregularities.

7.7 Graph between F and P

Let F = Friction force and

P = Applied force

The plot of friction force against applied force is as shown in Fig. 7.7.1.

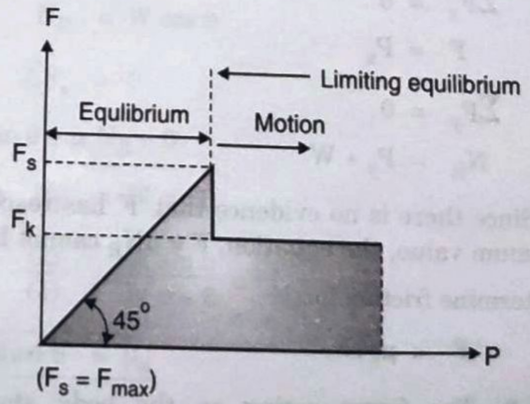
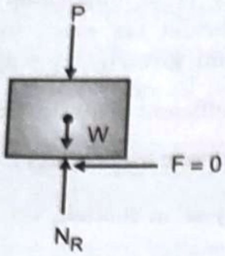


Fig. 7.7.1

7.8 Different Cases

There are four different situations where the rigid body is in contact with a horizontal surface.

Case 1 : The forces acting on the body do not tend to move the body along the surface of the contact.



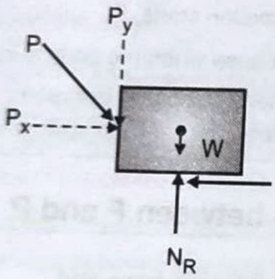
Block is in equilibrium Friction force, F = 0

Fig. 7.8.1

As there is no force acting along the surface of the contact, friction force, F = 0.

$$N_R = P + W$$

Case 2 : The forces acting on the body along the surface of contact tend to move the body, but are not large enough to set the body in motion.



Block is in equilibrium Friction force, $F = P_x$, $F < F_L$

Fig. 7.8.2

Friction force 'F' can be found by using equations of equilibrium.

$$\Sigma F_x = 0$$
$$F = P_x$$
$$\Sigma F_y = 0$$
$$N_R = P_y + W$$

Since there is no evidence that 'F' has reached its maximum value, the equation, $F = \mu N_R$ cannot be used to determine friction force.

$$F < \mu_s N$$

Case 3 : The forces acting on the body along the surface are just sufficient to start the motion of the body.

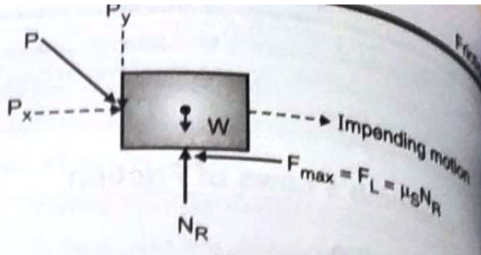


Fig. 7.8.3

The block is in impending motion. The friction force F reaches its maximum value F_{max} and together with normal force N_R, balances the applied forces.

Equations of equilibrium and $F_{max} = \mu_s N$ can be used.

$$\Sigma F_x = 0$$
$$P_x - F_{max} = 0$$
$$\therefore F_{max} = F_L = P_x$$
$$F_L = \mu_s N_R$$
$$\Sigma F_y = 0$$
$$N_R = P_y + W$$

Case 4 : The forces acting on the body along the surface causes motion of the body.

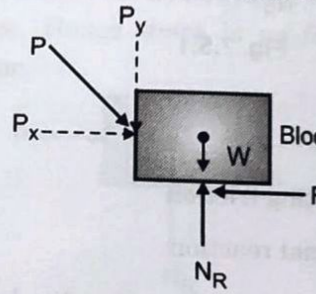


Fig. 7.8.4

Equations of equilibrium do not apply any more. However,

$$F = F_k = \mu_k N_R$$
$$F_k < P_x$$
$$N_R = P_y + W$$

Note that the body is in equilibrium in y-direction but not in x-direction i.e. along the surface.

Important notes :

Let F = Force required to maintain equilibrium

F_L = Limiting friction

If $F < F_L \rightarrow$ then the body is in equilibrium

$F = F_L \rightarrow$ Limiting equilibrium

$F > F_L \rightarrow$ body is in motion.

7.9 Angle of friction (ϕ)

When the body is in limiting equilibrium, the maximum angle between the resulting reaction and the normal reaction is called as ***"angle of friction".***

- It is also called as limiting angle of friction and denoted by "ϕ".

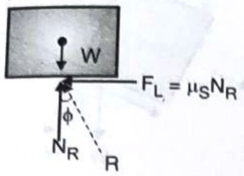


Fig. 7.9.1

$$\tan\phi = \frac{F_L}{N_R} = \frac{\mu_s N_R}{N_R} = \mu_s$$

$$\boxed{\tan\phi = \mu_s}$$

Tangent of angle of friction = coefficient of friction.

$$R^2 = F_L^2 + N_R^2$$

$$= \mu_s^2 N_R^2 + N_R^2$$

$$= N_R^2\left(\mu_s^2 + 1\right)$$

$$\therefore\ R = N_R\sqrt{\mu_s^2 + 1}$$

7.10 Cone of friction

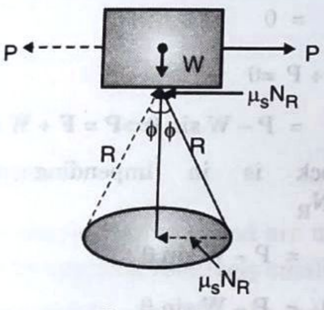


Fig. 7.10.1

In the limiting state of equilibrium of the block, if the line of action of applied force 'P' rotated in a horizontal plane, then the line of action of resultant 'R' will generate a cone, known as "cone of friction".

Properties of cone of friction :

(i) Semi-vertical (vertex) angle = angle of friction (ϕ)

(ii) Altitude or height of cone = Normal reaction (N_R)

(iii) Base radius = Limiting friction force (F_L)

(iv) Generator of the cone = Resultant reaction (R)

7.11 Angle of repose (θ)

- It is the angle made by an inclined plane with horizontal at which the blocks tends to slide down the plane under its own weight without application of any other external force.

Angle of repose (θ) is the minimum angle of inclination of plane at which the block impends its motion under its own weight.

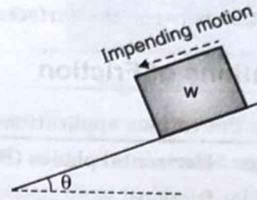


Fig. 7.11.1

FBD of Block

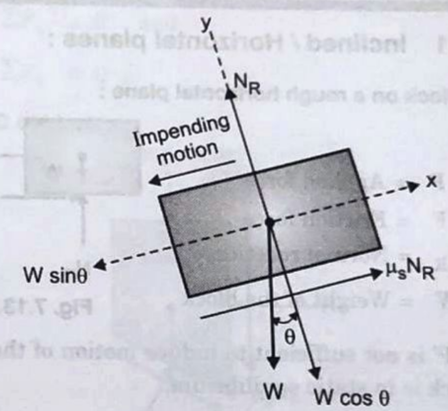


Fig. 7.11.2

Considering x-axis along the plane, for equilibrium of block :

$$\Sigma F_y = 0$$

$$N_R = W\cos\theta \quad \text{... (1)}$$

$$\Sigma F_x = 0$$

$$-W\sin\theta + \mu_s N_R = 0$$

$$\mu_s N_R = W\sin\theta \quad \text{... (2)}$$

$$\frac{(2)}{(1)} = \frac{W\sin\theta}{W\cos\theta} = \frac{\mu_s N_R}{N_R}$$

$$\therefore \boxed{\tan\theta = \mu_s}$$

$\therefore$ Tangent of angle of repose = coefficient of friction

But we know that, $\tan\phi = \mu_s$

$\therefore \quad \tan\theta = \tan\phi$

$$\boxed{\theta = \phi}$$

Angle of repose = Angle of friction

7.12 Factors on which Friction Depends

Friction mainly depends on two factors :

(i) Nature of the surfaces in contact.

(ii) Normal reaction between the surfaces of contact.

7.13 Applications of Friction

Following are the various applications of friction :

(i) Inclined planes / Horizontal planes (Block friction)
(ii) Ladders (Ladder friction)
(iii) Wedges (Wedge friction)
(iv) Flat belts (Belt friction)

7.13.1 Inclined / Horizontal planes :

(A) Block on a rough horizontal plane :

Let,

P = Applied force

F = Friction force

N_R = Normal reaction

W = Weight of the block

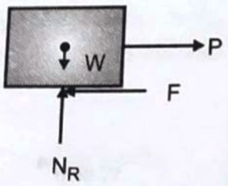


Fig. 7.13.1

– If 'P' is not sufficient to induce motion of the block, block is in static equilibrium.

Writing equations of equilibrium;

$\Sigma F_x = 0 \qquad F = P$

$\Sigma F_y = 0 \qquad N_R = W$

If the block is on the verge of motion, i.e. about to move or in impending motion under the action of force, then the friction force will be maximum i.e. limiting friction.

Now the block is in limiting static equilibrium.

Writing equations of equilibrium :

$\Sigma F_x = 0 \qquad F = F_L = P$

$F_L = \mu_s N_R \qquad \Sigma F_y = 0$

$N_R = W$

(B) Block on a rough inclined plane :

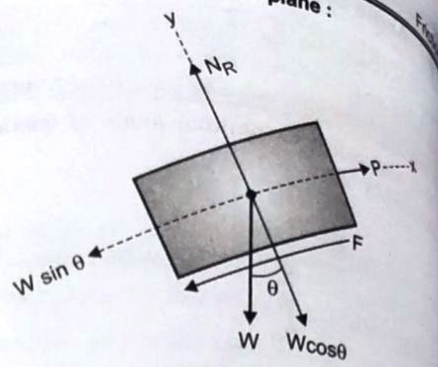


Fig. 7.13.2

Let, P = Applied force

W = Weight of the block

N_R = Normal reaction from the plane

θ = Angle of inclination of the plane.

Friction force can be found by resolving forces along and normal to the planes.

If P tends to induce motion upwards, then the friction force acts down the plane.

$\Sigma F_y = 0$

$N_R = W\cos\theta$

$\Sigma F_x = 0$

$-W\sin - F + P = 0$

$\therefore \quad F = P - W\sin\theta \Rightarrow \mathbf{P = F + W\sin\theta}$

If block is in impending motion, then $F = F_L = \mu_s N_R$

$\therefore \mu_s N_R = P - W\sin\theta$

$\mu_s W\cos\theta = P - W\sin\theta$

$$\mathbf{P = W(\mu_s\cos\theta + \sin\theta)}$$

7.13.2 Ladders :

– Ladder is a structure used for climbing or scaling on the walls or roofs.

– In the absence of friction, ladder will slide down and fall on the floor.

– The upper end tends to slide down and lower end moves away from the wall.

– Friction force at end B will be upwards and at A towards right.

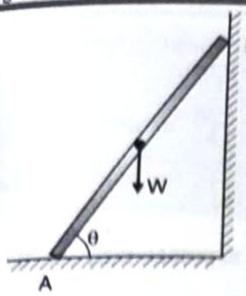


Fig. 7.13.3

☞ **FBD of ladder**

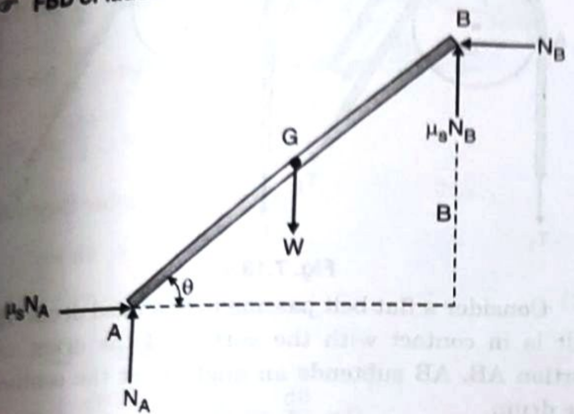


Fig. 7.13.4

- The problems on ladder can be solved by using three equations of equilibrium.

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma M = 0$$

7.13.3 Wedges :

- Wedges are simple machines and are used to raise heavy loads by applying relatively small forces.
- They are invariably used for the adjustment of supports to the framework in RCC construction.
- Due to frictional force developed between the surfaces of contact between the wedge and the load, the wedge may become "self locking".
- We analyse the problem on wedges, by using two equations of equilibrium as the site of the wedges is neglected.

$$\Sigma F_x = 0 \quad \text{and}$$

$$\Sigma F_y = 0$$

☞ **Steps to be followed for the analysis of problems on wedges :**

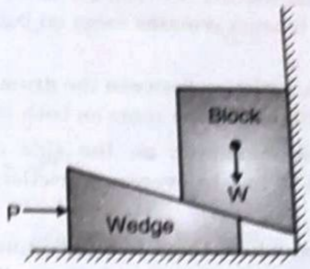


Fig. 7.13.5

1. Observe the direction of impending motion.
2. Draw FBD of block and wedge separately.
3. Apply friction force ($\mu_s N_R$) at all contact surfaces, in the opposite direction of impending motion.
4. Write equations of equilibrium ;

$$\Sigma F_x = 0 \quad \text{and}$$

$$\Sigma F_y = 0$$

☞ **FBD of block**

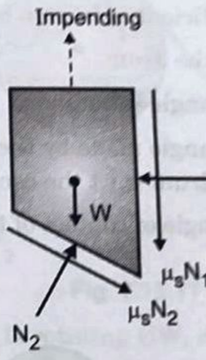


Fig. 7.13.6

☞ **FBD of wedge**

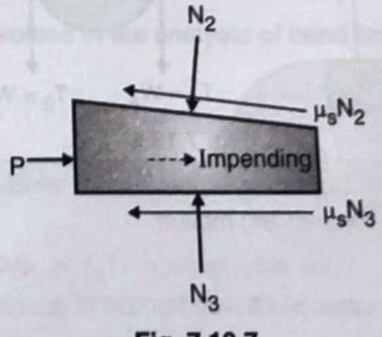


Fig. 7.13.7

7.13.4 Belt Friction :

- A flat belt or rope along with pulley or drum is used to raise the load, transmitting power from one shaft to another shaft.

- When a belt is passing over a fixed drum and if there is no friction between the drum and the belt, then the tension remains same on both sides of the drum.
- If there is a friction between the drum and belt, the tension will not be the same on both sides.
- Tension is maximum on the side of impending motion as it has to overcome friction to start the motion.
- The side on which there is a maximum tension i.e. the side in which there is an impending motion is called "tight side" and the tension is represented by 'T_2'. The other side is known as "slack side" and the tension is represented by 'T_1'.
- These two tensions T_2 and T_1 are related by an expression :

$$\boxed{\frac{T_2}{T_1} = e^{\mu\beta}}$$

Where, T_2 = Tight side tension

T_1 = Slack side tension

μ = Coefficient of friction between the belt and the drum.

β = Lap angle in radians.

- Lap angle is the angle made by the belt which is in contact with the drum w.r.t the centre of the drum.
- Simply it is the angle of contact of the belt.

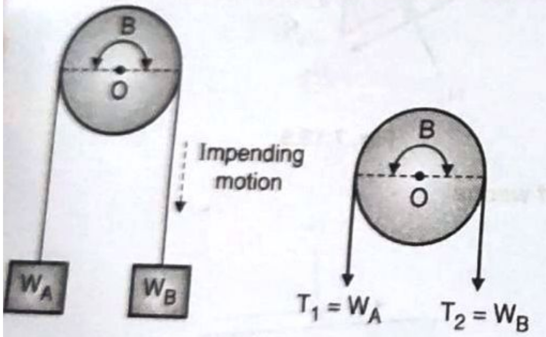


Fig. 7.13.8

Note :

i) Tight side is the side in which there is an impending motion.

ii) Tight side tension (T_2) is always greater than slack side tension (T_1).

iii) Lap angle 'β' does not depend on the radius of the drum.

iv) If the belt is in limiting equilibrium i.e., in impending motion, $\mu = \mu_s$.

$$\frac{T_2}{T_1} = e^{\mu_s\beta}$$

v) If the drum is rotating or the belt is ... then $\mu = \mu_k$.

$$\frac{T_2}{T_1} = e^{\mu_k\beta}$$

μ_k = coefficient of kinetic fri...

Derivation for expression $\frac{T_2}{T_1} = e^{\mu\beta}$:

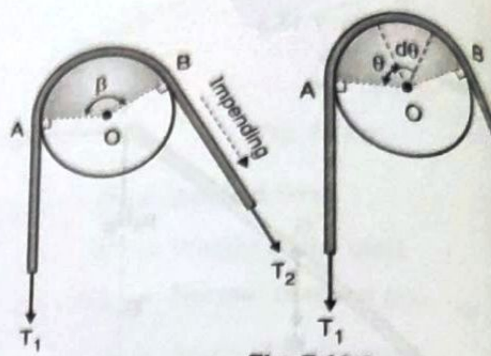


Fig. 7.13.9

Consider a flat belt passing over a fixed drum ... belt is in contact with the surface of the drum ... portion AB. AB subtends an angle 'β' at the centre ... the drum.

$\therefore \beta$ = Lap angle or angle of contact

The belt is tangential to the drum at points A and B.

Let the belt is impending its motion on right hand ...

Let, T_2 = Tension on tight side

T_1 = Tension on slack side

$T_2 > T_1$ as T_2 has to overcome friction to impend motion

Consider an elemental strip of the belt w... makes an angle 'dθ' with the centre of the drum a... angle 'θ' from point 'A'.

FBD of elemental strip

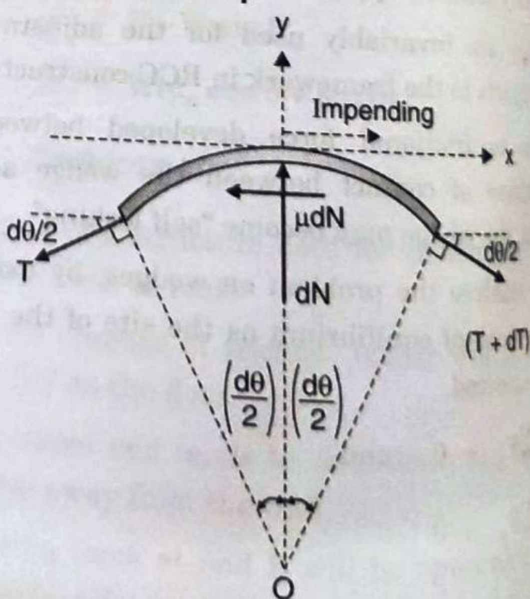


Fig. 7.13.10

- When a belt is passing over a fixed drum and if there is no friction between the drum and the belt, then the tension remains same on both sides of the drum.
- If there is a friction between the drum and belt, the tension will not be the same on both sides.
- Tension is maximum on the side of impending motion as it has to overcome friction to start the motion.
- The side on which there is a maximum tension i.e. the side in which there is an impending motion is called "tight side" and the tension is represented by 'T_2'. The other side is known as "slack side" and the tension is represented by 'T_1'.
- These two tensions T_2 and T_1 are related by an expression :

$$\boxed{\frac{T_2}{T_1} = e^{\mu\beta}}$$

Where, T_2 = Tight side tension

T_1 = Slack side tension

μ = Coefficient of friction between the belt and the drum.

β = Lap angle in radians.

- Lap angle is the angle made by the belt which is in contact with the drum w.r.t the centre of the drum.
- Simply it is the angle of contact of the belt.

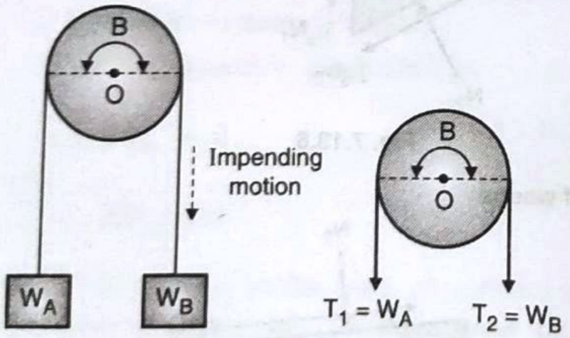


Fig. 7.13.8

Note :

i) Tight side is the side in which there is an impending motion.

ii) Tight side tension (T_2) is always greater than slack side tension (T_1).

iii) Lap angle 'β' does not depend on the radius of the drum.

iv) If the belt is in limiting equilibrium i.e., in impending motion, $\mu = \mu_s$.

$$\frac{T_2}{T_1} = e^{\mu_s\beta}$$

v) If the drum is rotating or the belt is m... then $\mu = \mu_k$.

$$\frac{T_2}{T_1} = e^{\mu_k\beta}$$

μ_k = coefficient of kinetic fric...

Derivation for expression $\frac{T_2}{T_1} = e^{\mu\beta}$:

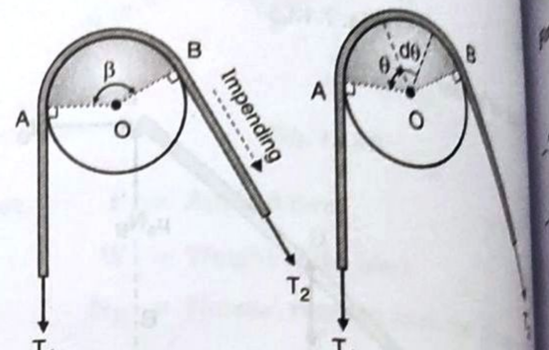


Fig. 7.13.9

Consider a flat belt passing over a fixed drum. The belt is in contact with the surface of the drum in portion AB. AB subtends an angle 'β' at the centre of the drum.

$\therefore \beta$ = Lap angle or angle of contact

The belt is tangential to the drum at points A and B.

Let the belt is impending its motion on right hand side.

Let, T_2 = Tension on tight side

T_1 = Tension on slack side

$T_2 > T_1$ as T_2 has to overcome friction to impend motion

Consider an elemental strip of the belt which makes an angle 'dθ' with the centre of the drum at angle 'θ' from point 'A'.

FBD of elemental strip

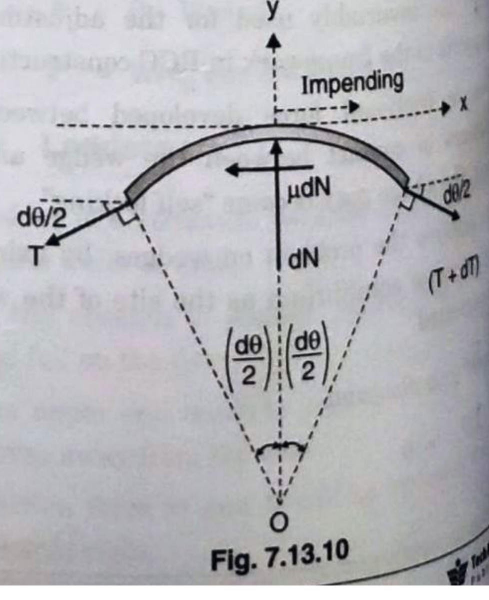


Fig. 7.13.10

The tension is changing from T to (T + dT) over an angle θ to (θ + dθ).

Let, dN = Normal reaction from drum on the belt

μdN = Limiting friction acting in the opposite direction of impending motion.

For equilibrium of the strip;

$$\Sigma F_x = 0$$

$$-T\cos\frac{d\theta}{2} - \mu dN + (T + dT)\cos\frac{d\theta}{2} = 0$$

$$-T\cos\frac{d\theta}{2} - \mu dN + T\cos\frac{d\theta}{2} + dT\cos\frac{d\theta}{2} = 0$$

$$dT\cos\frac{d\theta}{2} = \mu dN$$

For small values of θ, cos θ ≅ 1

$$\therefore \cos d\theta \cong 1$$

$$\therefore dT = \mu dN \quad \ldots (1)$$

$$\Sigma F_y = 0$$

$$dN - T\sin\frac{d\theta}{2} - (T + dT)\sin\frac{d\theta}{2} = 0$$

$$dN - T\sin\frac{d\theta}{2} - T\sin\frac{d\theta}{2} - dT\sin\frac{d\theta}{2} = 0$$

Product of two small quantities i.e. $dT.\sin\frac{d\theta}{2}$ can be neglected.

$$\therefore dN = 2T\sin\frac{d\theta}{2}$$

For small values of θ, sin θ ≅ θ

i.e., $\sin\left(\frac{d\theta}{2}\right) \cong \left(\frac{d\theta}{2}\right)$

$$\therefore dN = 2T.\left(\frac{d\theta}{2}\right)$$

$$\therefore dN = T.d\theta \quad \ldots (2)$$

From Eq^n (1),

$$dT = \mu T.d\theta \quad (\because dN = T\,d\theta)$$

$$\frac{dT}{T} = \mu.d\theta$$

Integrating on both sides with the limits,

When, θ = 0, T = T_1 and θ = β, T = T_2

$$\therefore \int_{T_1}^{T_2}\frac{dT}{T} = \mu\int_0^{\beta} d\theta$$

$$[\ln T]_{T_1}^{T_2} = \mu\,[\theta]_0^{\beta}$$

$$\ln(T_2 - T_1) = \mu(\beta - 0)$$

$$\ln\left(\frac{T_2}{T_1}\right) = \mu.\beta$$

$$\boxed{\therefore \frac{T_2}{T_1} = e^{\mu\beta}}$$

β = Lap angle in radians

Band Brakes :

- Band brakes are used to apply braking forces by operating lever.
- Band brake consists of lever, drum and belt.
- The friction between the belt and the drum will help in applying brakes on the rotating drum.
- Braking torque i.e. $(T_2 - T_1)r$ always acts in the opposite direction of rotation of the drum.

Where, r = radius of the drum

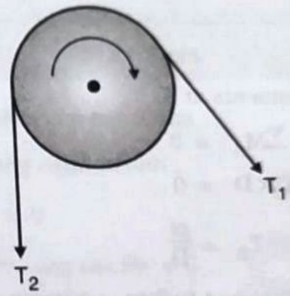


Fig. 7.13.11

- If the drum is rotating CW, maximum tension will be in the opposite direction i.e. in ACW direction.

∴ Braking torque $(T_2 - T_1)r$ is in ACW direction.

Steps involved in the analysis of band brakes :

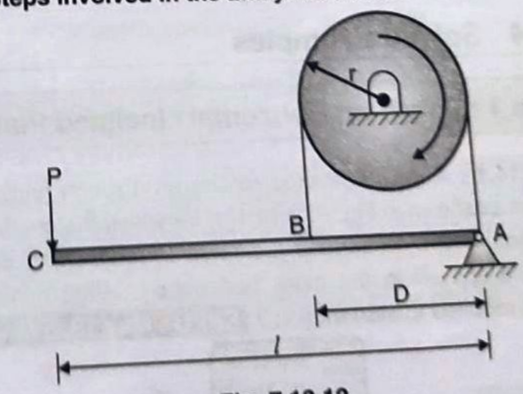


Fig. 7.13.12

Let the drum is rotating in CW direction maximum tension will be at B.

The tension is changing from T to (T + dT) over an angle θ to (θ + dθ).

Let, dN = Normal reaction from drum on the belt

μdN = Limiting friction acting in the opposite direction of impending motion.

For equilibrium of the strip;

$$\Sigma F_x = 0$$

$$-T\cos\frac{d\theta}{2} - \mu dN + (T + dT)\cos\frac{d\theta}{2} = 0$$

$$-T\cos\frac{d\theta}{2} - \mu dN + T\cos\frac{d\theta}{2} + dT\cos\frac{d\theta}{2} = 0$$

$$dT\cos\frac{d\theta}{2} = \mu dN$$

For small values of θ, $\cos\theta \cong 1$

$$\therefore \cos d\theta \cong 1$$

$$\therefore dT = \mu dN \quad \text{... (1)}$$

$$\Sigma F_y = 0$$

$$dN - T\sin\frac{d\theta}{2} - (T + dT)\sin\frac{d\theta}{2} = 0$$

$$dN - T\sin\frac{d\theta}{2} - T\sin\frac{d\theta}{2} - dT\sin\frac{d\theta}{2} = 0$$

Product of two small quantities i.e. $dT.\sin\frac{d\theta}{2}$ can be neglected.

$$\therefore dN = 2T\sin\frac{d\theta}{2}$$

For small values of θ, $\sin\theta \cong \theta$

i.e., $\sin\left(\frac{d\theta}{2}\right) \cong \left(\frac{d\theta}{2}\right)$

$$\therefore dN = 2T.\left(\frac{d\theta}{2}\right)$$

$$\therefore dN = T.d\theta \quad \text{... (2)}$$

From Eq^n (1),

$$dT = \mu T.d\theta \qquad (\because dN = T\,d\theta)$$

$$\frac{dT}{T} = \mu.d\theta$$

Integrating on both sides with the limits,

When, θ = 0, $T = T_1$ and θ = β, $T = T_2$

$$\therefore \int_{T_1}^{T_2}\frac{dT}{T} = \mu\int_0^{\beta} d\theta$$

$$[\ln T]_{T_1}^{T_2} = \mu\,[\theta]_0^{\beta}$$

$$\ln(T_2 - T_1) = \mu(\beta - 0)$$

$$\ln\left(\frac{T_2}{T_1}\right) = \mu.\beta$$

$$\boxed{\therefore \frac{T_2}{T_1} = e^{\mu\beta}}$$

β = Lap angle in radians

Band Brakes :

- Band brakes are used to apply braking forces by operating lever.
- Band brake consists of lever, drum and belt.
- The friction between the belt and the drum will help in applying brakes on the rotating drum.
- Braking torque i.e. $(T_2 - T_1)r$ always acts in the opposite direction of rotation of the drum.

Where, r = radius of the drum

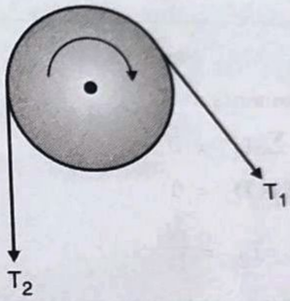


Fig. 7.13.11

- If the drum is rotating CW, maximum tension will be in the opposite direction i.e. in ACW direction.

∴ Braking torque $(T_2 - T_1)r$ is in ACW direction.

Steps involved in the analysis of band brakes :

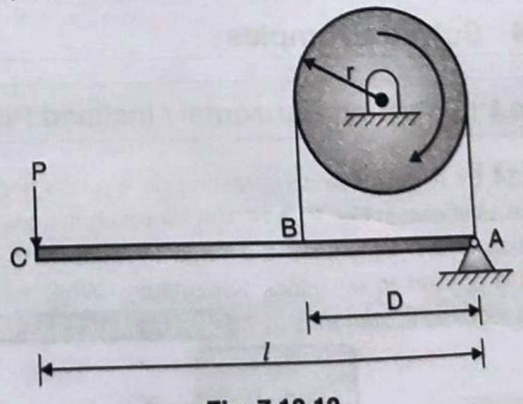


Fig. 7.13.12

Let the drum is rotating in CW direction maximum tension will be at B.

1. Draw FBD of drum.
2. Get the relation between T_1 and T_2.

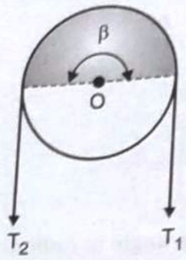


Fig. 7.13.13

$$\frac{T_2}{T_1} = e^{\mu\beta} \quad \text{... (1)}$$

3. Draw FBD of lever AC.

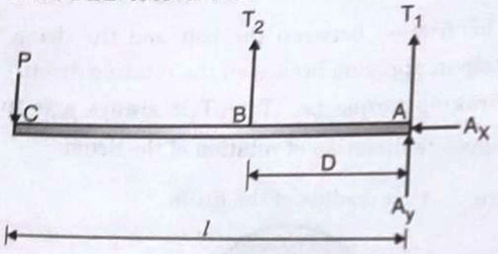


Fig. 7.13.14

4. Taking moments at A.

$$\Sigma M_A = 0$$

$$-P \times l + T_2 \times D = 0$$

$$T_2 = \frac{pl}{D} \quad \text{... (2)}$$

5. Solving Eq^n(1) and Eq^n(2), get the values of T_1 and T_2.
6. Braking Torque,

$$T = (T_2 - T_1)r \text{ ACW}$$

$$r = \text{radius of drum.}$$

7.14 Solved Examples

Type 1 : Based on Horizontal / Inclined Planes

Ex. 7.14.1 : A 400 N block is resting on a rough horizontal surface as shown in Fig. P. 7.14.1(a) for which the coefficient of friction is 0.4. Determine the force P required to cause motion if applied to the block horizontally. What minimum force is required to start motion ? **SPPU : May 13, 5 Marks**

Fig. P. 7.14.1(a)

Soln. :

Step 1 : FBD of the block :

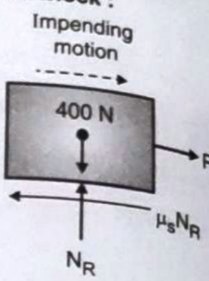


Fig. P. 7.14.1(b)

N_R = Normal reaction from the horizontal surface.

$\mu_s = 0.4$

Step 2 : Equilibrium equations :

For limiting equilibrium;

$$\Sigma F_y = 0$$

$$-400 + N_R = 0$$

$$\therefore \quad N_R = 400 \text{ N}$$

$$\Sigma F_x = 0$$

$$P - \mu_s N_R = 0$$

$$P = 0.4 \times 400 = 160 \text{ N}$$

Force required to cause the motion,

$$P = 160 \text{ N} \quad \text{...Ans.}$$

The minimum force required to cause the motion is 160N. ...Ans.

Ex. 7.14.2 : A body of weight 300 N is kept on a rough horizontal plane and a force P is applied to just move the body horizontally as shown in Fig. P. 7.14.2(a). Find the magnitude of force P required if coefficient of static friction is $\mu_s = 0.4$. **SPPU : Dec. 12, 7 Marks**

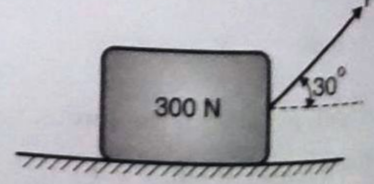


Fig. P. 7.14.2(a)

Soln. :

Step 1 : **FBD of the block :**

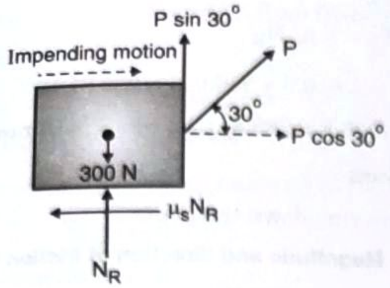


Fig. P. 7.14.2(b)

N_R = Normal reaction from horizontal plane

$\mu_s = 0.4$

Step 2 : **Equilibrium equations :**

For limiting equilibrium ;

$$\Sigma F_y = 0$$

$$P \sin 30° - 300 + N_R = 0$$

$$\therefore \quad N_R = 300 - P \sin 30° \quad \text{... (1)}$$

$$\Sigma F_x = 0$$

$$P \cos 30° - \mu_s N_R = 0$$

$$\therefore \; P \cos 30° = \mu_s N_R$$

$$P \cos 30° = 0.4\, N_R \quad \text{... (2)}$$

Step 3 : **Magnitude of force P**

From Eq^n (1), $N_R = 300 - P \sin 30°$

From Eq^n(2), $P \cos 30° = 0.4[300 - P \sin 30°]$

$$\therefore \quad 0.866\,P = 120 - 0.2\,P$$

$$1.066\,P = 120$$

$$\therefore P = \mathbf{112.57\ N} \quad \text{... Ans.}$$

Ex. 7.14.3 : Determine the horizontal force P needed to just start moving the 30 kg block up the plane as shown in Fig. P. 7.14.3(a). Take $\mu_s = 0.25$ and $\mu_k = 0.2$.

SPPU : May 17, 5 Marks

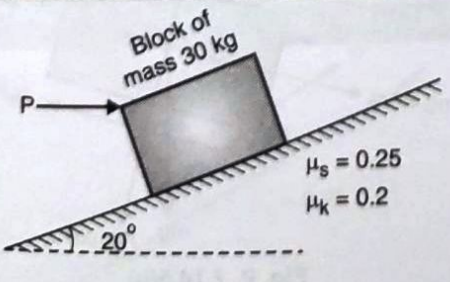


Fig. P. 7.14.3(a)

Soln. :

Step 1 : **FBD of the block**

Taking x-axis along the plane and y-axis normal to it.

$$W = mg = 30 \times 9.81 = 294.3\ N$$

$$\mu_s = 0.25$$

N_R = Normal reaction from inclined plane.

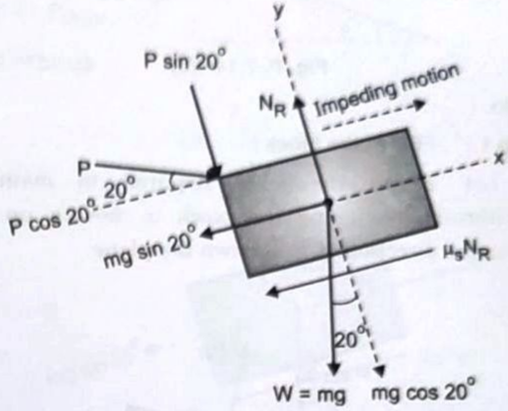


Fig. P. 7.14.3(b)

Note : Here block is not in motion. Hence μ_k is not be used.

Step 2 : **Equilibrium equations**

For limiting equilibrium ;

$$\Sigma F_y = 0$$

$$N_R - P \sin 20° - mg \cos 20° = 0$$

$$N_R = P \sin 20° - 294.3 \cos 20°$$

$$\therefore \quad N_R = 0.342\,P - 276.55 \quad \text{... (1)}$$

$$\Sigma F_x = 0$$

$$P \cos 20° - mg \sin 20° - \mu_s N_R = 0$$

$$0.94P - 294.3 \sin 20° = 0.25\, N_R$$

$$0.94P - 100.66 = 0.25\, N_R \quad \text{... (2)}$$

Step 3 : **Horizontal force 'P'**

From Eq^n (1), $N_R = 0.342P - 276.55$

From Eq^n(2),

$$0.94P - 100.66 = 0.25(0.342P - 276.55)$$

$$0.94P - 100.66 = 0.085P - 69.14$$

$$0.855\,P = 31.52$$

$$\therefore \quad P = \mathbf{36.865\ N} \quad \text{... Ans.}$$

Ex. 7.14.4 : Determine whether the block shown in Fig. P. 7.14.4(a) is in equilibrium, and find the magnitude and direction of the friction force when $\theta = 30°$ and $P = 200$ N.

SPPU : May 11, 7 Marks

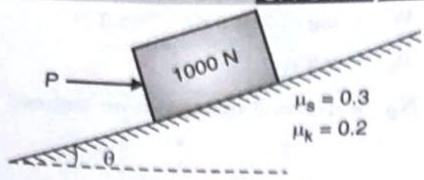


Fig. P. 7.14.4(a)

Soln. :

Step 1 : FBD of the block :

Let 'F' be the force required to maintain equilibrium. Assuming the block is moving up the plane, the direction of 'F' is down the plane.

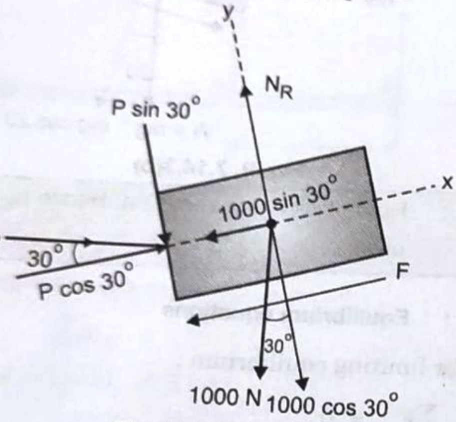


Fig. P. 7.14.4(b)

$P = 200$ N

N_R = Normal reaction from inclined plane

Step 2 : Equilibrium equations :

Taking x-axis along the plane and y-axis normal to the plane ;

$$\Sigma F_y = 0$$

$$N_R - P \sin 30° - 1000 \cos 30° = 0$$

$$N_R = 200 \sin 30° + 1000 \cos 30°$$

$$\therefore \quad N_R = 966.025 \text{ N} \quad \ldots (1)$$

$$\Sigma F_x = 0$$

$$P \cos 30° - 1000 \sin 30° - F = 0$$

$$\therefore \quad F = 200 \cos 30° - 1000 \sin 30°$$

$$\therefore \quad F = -326.8 \text{ N} \quad \ldots (2)$$

As 'F' is –ve, it is acting up the plane and block is nding to move down the plane.

Step 3 : To check whether the block is in equilibrium :

Let F_L = Limiting friction

$$F_L = \mu_s . N_R$$

$$= 0.3 \times 966.025 = 289.81 \text{ N}$$

As $F > F_L (326.8 > 289.81)$ the **block will not be equilibrium.**

It is moving down the plane. ...Ans.

Step 4 : Magnitude and direction of friction force :

As the block is moving, friction force,

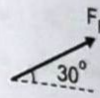
$$F_r = \mu_k . N_R$$

Where, μ_k = coefficient of kinetic friction = 0.2

$$\therefore \quad F_r = 0.2 \times 966.025$$

$$= \mathbf{193.205 \text{ N}} \quad \text{...Ans.}$$

As the block is moving down the plane, the friction force will act up the plane.



...Ans.

Ex. 7.14.5 : A block of mass m rest on a frictional plane which makes an angle α with the horizontal as shown in Fig. P. 7.14.5(a). If the coefficient of friction between the block and the frictional plane is 0.2, determine the angle α for limiting condition.

SPPU : Dec. 14, 5 Marks

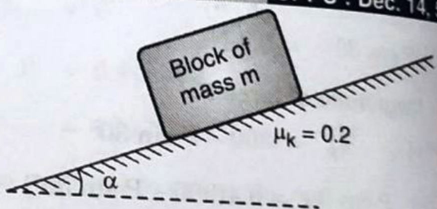


Fig. P. 7.14.5(a)

Soln. :

Step 1 : FBD of the block

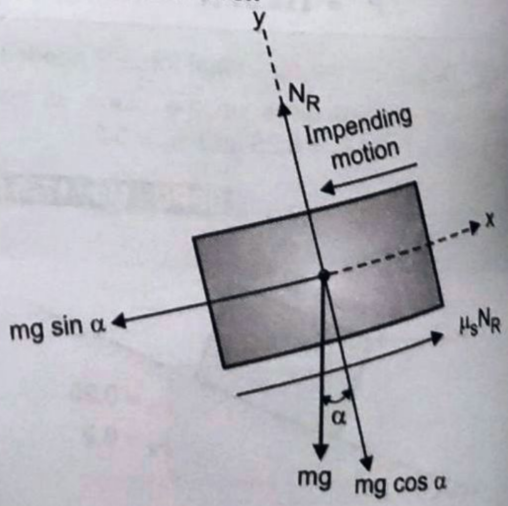


Fig. P. 7.14.5(b)

Let the block is in impending motion down the plane.

N_R = Normal reaction from frictional plane

$\mu_s = 0.2$

mg = weight of the block

Step 2 : Equilibrium equations :

Taking x and y axes along and normal to the plane,

$$\Sigma F_y = 0$$

$$N_R - mg \cos \alpha = 0$$

$$\therefore N_R = mg \cos \alpha \quad \text{... (1)}$$

$$\Sigma F_x = 0$$

$$- mg \sin \alpha + \mu_s N_R = 0$$

$$\therefore \mu_s N_R = mg \sin \alpha$$

$$0.2 \, N_R = mg \sin \alpha \quad \text{... (2)}$$

Step 3 : Angle 'α'

From Eq^n (1), $N_R = mg \cos \alpha$

From Eq^n (2), $0.2[mg \cos \alpha] = mg \sin \alpha$

$$0.2 \cos \alpha = \sin \alpha$$

$$\tan \alpha = 0.2$$

$$\therefore \alpha = \mathbf{11.31°} \quad \text{... Ans.}$$

Alternate method :

For limiting condition, angle of inclination of plane is the angle of repose.

$$\alpha = \theta$$

$$\tan \theta = \mu_s$$

$$\tan \theta = 0.2$$

$$\theta = 11.31°$$

$$\therefore \alpha = \mathbf{11.31°} \quad \text{... Ans.}$$

Ex. 7.14.6 : A block of mass 150 kg is resting on a plane inclined at 30° with horizontal as shown in Fig. P. 7.14.6(a). Determine range of an external force 'P' to maintain equilibrium. Assume $\mu_s = 0.25$. **SPPU : May 10, 7 Marks**

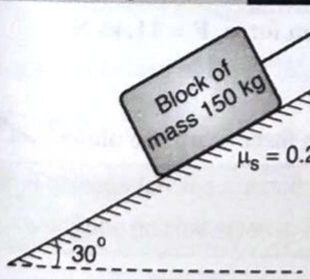


Fig. P. 7.14.6(a)

Soln.

Step 1 : Impending motion :

For maximum force 'P', the impending motion of block is in the direction of force i.e. upwards.

For minimum force 'P', impending motion of block is down the plane.

Step 2 : P_{max}

FBD of block :

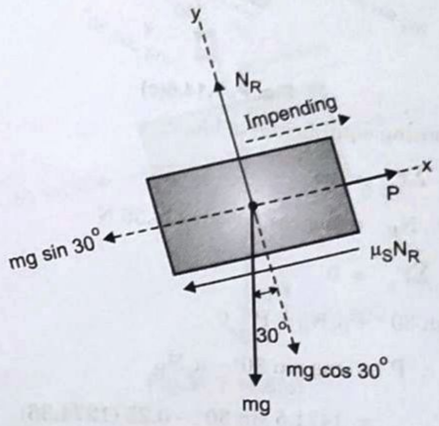


Fig. P. 7.14.6(b)

Let N_R = Normal reaction from the inclined plane

$mg = 150 \times 9.81 = 1471.5$ N

$\mu_s = 0.25$

For limiting equilibrium of block;

$$\Sigma F_y = 0$$

$$N_R - mg \cos 30° = 0$$

$$N_R = mg \cos 30°$$

$$\therefore N_R = 1471.5 \cos 30°$$

$$= 1274.36 \text{ N} \quad \text{... (1)}$$

$$\Sigma F_x = 0$$

$$- mg \cos 30° - \mu_s N_R + P = 0$$

$$\therefore P = mg \sin 30° + \mu_s N_R$$

$$= 1471.5 \sin 30° + 0.25\,(1274.36)$$

$$\mathbf{P = 1054.34 \text{ N}}$$

Step 3 : P_{min}

FBD of block :

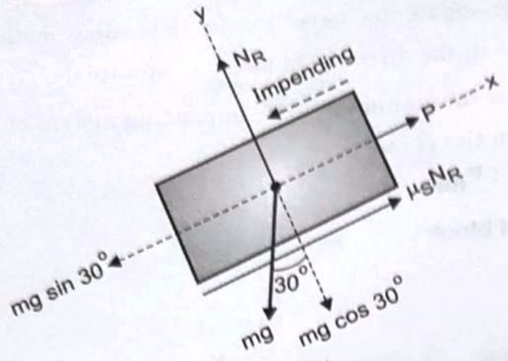


Fig. P. 7.14.6(c)

For limiting equilibrium of block;

$$\Sigma F_y = 0$$

$$N_R = mg\cos 30° = 1274.36 \text{ N}$$

$$\Sigma F_x = 0$$

$$-mg\sin 30° + \mu_s N_R + P = 0$$

$$\therefore P = mg\sin 30° - \mu_s N_R$$

$$= 1471.5 \sin 30° - 0.25\,(1274.36)$$

$$\therefore \mathbf{P = 417.16\ N}$$

$\therefore$ Range of values of 'P' to maintain equilibrium;

417.16 N ≤ P ≤ 1054.34 N ... **Ans.**

Ex. 7.14.7 : Determine whether the 10 kg block shown in Fig. P. 7.14.7(a) is in equilibrium and find the magnitude and direction of the friction force when P = 40 N and θ = 20°.

SPPU : May 12, 6 Marks

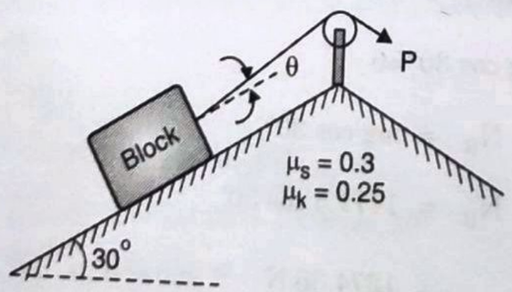


Fig. P. 7.14.7(a)

Soln. :

Step 1 : FBD of block

Let 'F' be the force required to maintain equilibrium. Assuming the block is moving up the plane, the direction of 'F' is down the plane.

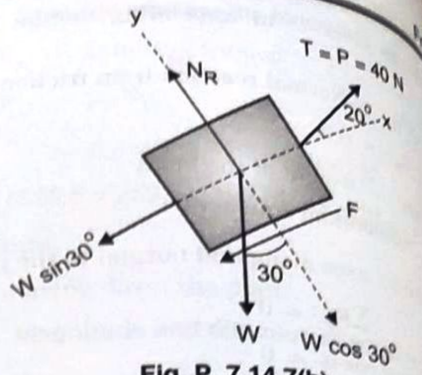


Fig. P. 7.14.7(b)

Let, N_R = Normal reaction from inclined plane

W = Weight of the block

$= 10 \times 9.81 = 98.1$ N

T = Tension in the string

$= P = 40$ N

Step 2 : Equations of equilibrium :

(Taking x-axis along the plane)

$$\Sigma F_y = 0$$

$$N_R + 40\sin 20° - W\cos 30° = 0$$

$$N_R = -40\sin 20° + 98.1\cos 30°$$

$$\therefore \mathbf{N_R = 71.27\ N}$$

$$\Sigma F_x = 0$$

$$-W\sin 30° - F + 40\cos 20° = 0$$

$$\therefore F = -98.1\sin 30° + 40\cos 20°$$

$$\therefore \mathbf{F = -11.46\ N}$$

Block is tending to move down the plane.

Step 3 : Limiting friction

$$F_L = \mu_s . N_R$$

$$= 0.3 \times 71.27 = \mathbf{21.38\ N}$$

As $F < F_L$, block is in equilibrium. ...Ans.

Magnitude of friction force, **F = 11.46 N** ...Ans.

Direction of friction force is up the plane.

Note : Here, friction force is not equal to $F_L = \mu_s N_R$ as the block is not in limiting equilibrium.

Ex. 7.14.8 : Two blocks A and B are connected by a string passing over a smooth pulley as shown in Fig. P. 7.14.8(a). The block A weighs 50 N. If the coefficient of friction between the plane and the block is 0.40, find the maximum and minimum values of mass of block B for equilibrium of the system. **SPPU : Dec. 04, 8 Marks**

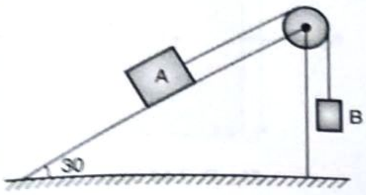


Fig. P. 7.14.8(a)

Soln. :

Step 1 : Impending motion :

For maximum value of m_B, impending motion of block B is downwards and block A, up the plane.

For minimum value of m_B, impending motion of B is upwards and block A, down the plane.

Step 2 : Maximum value of 'm_B'

FBD of block 'A'

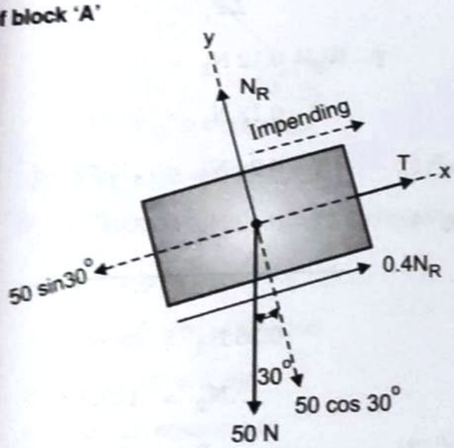


Fig. P. 7.14.8(b)

Let, T = Tension in the string

$T = m_B \cdot g$

N_R = Normal reaction from the plane

For limiting equilibrium of block 'A',

$$\Sigma F_y = 0$$

$$N_R = 50 \cos 30°$$

$$\Sigma F_x = 0$$

$$-50 \sin 30° - 0.4\, N_R + T = 0$$

$$\therefore T = 50 \sin 30° + 0.4\,(50 \cos 30°)$$

$$\therefore \mathbf{T = 42.32\ N}$$

But, $T = m_B \cdot g$

$$\therefore m_B = \frac{42.32}{9.81} = \mathbf{4.31\ kg}$$

Step 3 : Minimum value of 'm_B'

FBD of block 'A'

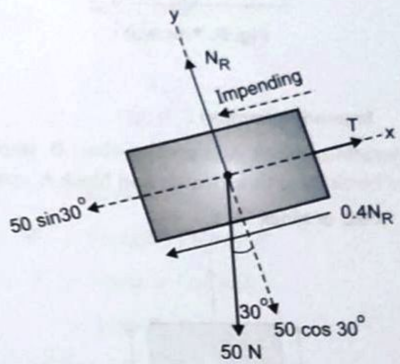


Fig. P. 7.14.8(c)

For limiting equilibrium of block 'A' ;

$$\Sigma F_y = 0$$

$$N_R = 50 \cos 30°$$

$$\Sigma F_x = 0$$

$$-50 \sin 30° + T + 0.4\, N_R = 0$$

$$\therefore T = 50 \sin 30° - 0.4\,(50 \cos 30°)$$

$$\therefore \mathbf{T = 7.68\ N}$$

But, $T = m_B \cdot g$

$$\therefore m_B = \frac{7.68}{9.81} = \mathbf{0.783\ kg}$$

$\therefore$ Maximum and minimum value of B for equilibrium of the system are :

Maximum, $\mathbf{m_B = 4.31\ kg}$ **... Ans.**

Minimum, $\mathbf{m_B = 0.783\ kg}$ **... Ans.**

Ex. 7.14.9 : Block 'A' of mass 12 kg and block B of mass 6 kg are connected by a cable that passes over frictionless pulley 'C', which can rotate freely. If coefficient of static friction at all contact surfaces is 0.12, determine the smallest value of P, for which equilibrium is maintained. Refer Fig. P. 7.14.9(a). **SPPU : May 04, 8 Marks**

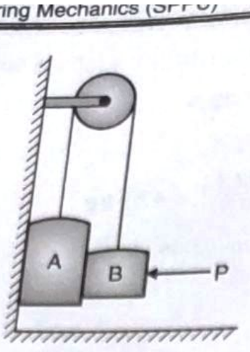


Fig. P. 7.14.9(a)

Soln. :

Step 1 : Impending motion :

As weight of block A is greater than B, impending motion of block 'A' is downwards and block B upwards.

Step 2 : FBD of block 'A' :

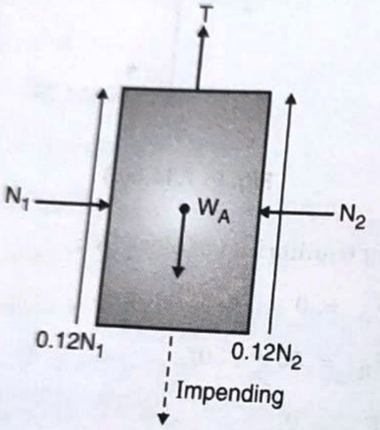


Fig. P. 7.14.9(b)

Let T = Tension in the cable

N_1 = Normal reaction from wall

N_2 = Normal reaction from block 'B'

$W_A = 12 \times 9.81 = 117.72$ N

Step 3 : Equilibrium Equations :

$$\Sigma F_x = 0$$

$$N_1 - N_2 = 0$$

$$\therefore N_1 = N_2 \quad \text{...(1)}$$

$$\Sigma F_y = 0$$

$$+ 0.12\, N_1 + 0.12\, N_2 - W_A = 0$$

$$+ 0.24\, N_2 = 117.72 \quad \text{... (2)} \qquad (W_A = 117.72 \text{ N})$$

Step 4 : FBD of block 'B' :

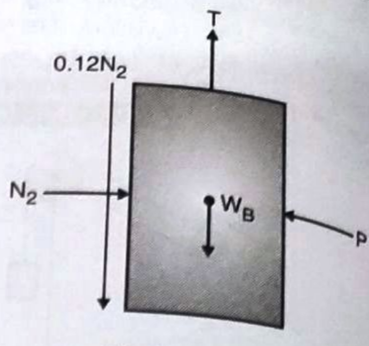


Fig. P. 7.14.9(c)

N_2 = Normal reaction between A and B

$W_B = 6 \times 9.81 = 58.86$ N

Step 5 : Equilibrium Equations :

$$\Sigma F_x = 0$$

$$N_2 - P = 0$$

$$\therefore N_2 = P \quad \text{...(3)}$$

$$\Sigma F_y = 0$$

$$T - W_B - 0.12\, N_2 = 0$$

$$T - 0.12\, N_2 = 58.86 \quad \text{...(4)}$$

From Eq^n(1), $\quad T + 0.24\, N_2 = 117.72$

From Eq^n(4), $\quad T - 0.12\, N_2 = 58.86$

$\quad - \quad + \qquad -$

$$0.36\, N_2 = 58.86$$

$$\therefore \mathbf{N_2 = 163.5\ N}$$

From Eq^n (3), $\quad P = N_2$

$$\therefore \mathbf{P = 163.5\ N} \quad \text{... Ans.}$$

Ex. 7.14.10: Two blocks connected by a horizontal link AB are supported on two rough planes as shown in Fig. P. 7.14.10(a) coefficient of static friction for block A and horizontal is 0.4. The angle of friction for block B on inclined plane is 15º what is the smallest weight 'W' of the block 'A' for which the equilibrium of the system can exist.

SPPU : Dec. 98, 10 Marks

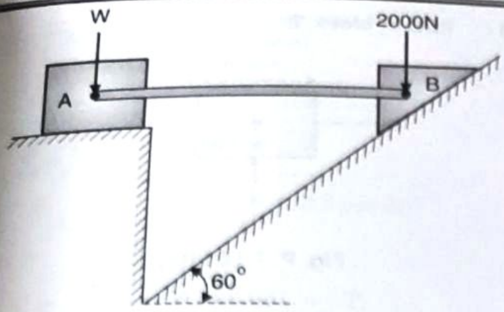


Fig. P. 7.14.10(a)

Soln.:

Step 1: Impending motion:

As block 'B' tends to move down the plane, the impending motion of block B is down the plane and block 'A' towards left.

Step 2: FBD of block 'B'

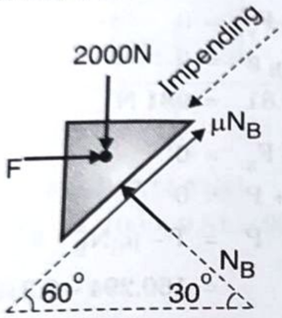


Fig. P. 7.14.10(b)

Let F = Force in the link AB

N_B = Normal reaction from inclined plane.

θ = Angle of friction = 15°

μ = coefficient of friction

$\mu = \tan 15° = 0.268$

Step 3: Equilibrium equations:

For limiting equilibrium of block 'B':

$$\Sigma F_x = 0$$

$$F + \mu N_B \cos 60° - N_B \cos 30° = 0$$

$$F = N_B \cos 30° - \mu N_B \cos 60°$$

$$= N_B \cos 30° - 0.268\, N_B \cos 60°$$

$$\therefore \quad \mathbf{F = 0.732\, N_B} \qquad \ldots(1)$$

$$\Sigma F_y = 0$$

$$-2000 + \mu N_B \sin 60° + N_B \sin 30° = 0$$

$$-2000 + 0.268\, N_B \sin 60° + N_B \sin 30° = 0$$

$$0.732\, N_B = 2000$$

$$\therefore \quad NB = 2731.88 \text{ N}$$

From Eq^n(1), $F = 0.732\,(2731.88) = \mathbf{1999.74\ N}$

Step 4: FBD of block 'A'

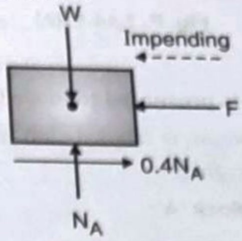


Fig. P. 7.14.10(c)

Let, N_A = Normal reaction from horizontal surface

W = Weight of block 'A'

F = Force in the link

= 1999.74 N

Force in the link AB at A and B is equal in magnitude and opposite in direction.

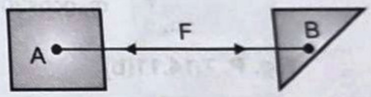


Fig. P. 7.14.10(d)

Step 5: Equilibrium equations:

For limiting equilibrium of block 'A';

$$\Sigma F_x = 0$$

$$0.4\, N_A - F = 0$$

$$0.4\, N_A = F$$

$$0.4\, N_A = 1999.74$$

$$\therefore \quad \mathbf{N_A = 4999.35\ N}$$

$$\Sigma F_y = 0$$

$$-W + N_A = 0$$

$$\mathbf{W = N_A = 4999.35\ N} \qquad \ldots\text{An}$$

Ex. 7.14.11: Two blocks A and B having masses 50 kg a 100 kg respectively are connected by a string which pas over frictionless pulley as shown in Fig. P. 7.14.11 Coefficient of friction between block and surface is 0.2 both A and B. Determine force P if,

(1) The system is prevented to move towards left.

(2) The system is just on the point of moving towards r

SPPU : Dec. 06, 9 M

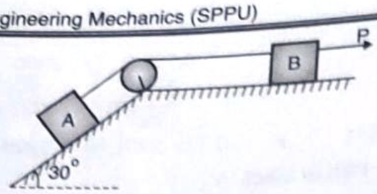


Fig. P. 7.14.11(a)

Soln.:

Case I : System is prevented to move towards left

Impending motion is towards left and friction force acts towards right.

Step 1: **FBD of block 'A':**

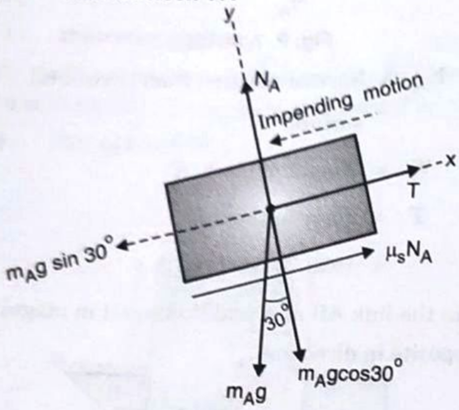


Fig. P. 7.14.11(b)

Let 'T' be the tension in the string

N_A = Normal reaction from inclined plane

m_A = 50 kg

N_s = 0.20

Step 2 : **Equilibrium equations:**

Taking x and y axes along and normal to the plane.

For limiting equilibrium:

$$\Sigma F_y = 0$$

$$N_A - m_A g \cos 30° = 0$$

$$\therefore \quad N_A = 50 \times 9.81 \cos 30°$$

$$= 424.78 \text{ N} \quad \ldots(1)$$

$$\Sigma F_x = 0$$

$$-m_A g \sin 30° + \mu_S N_A + T = 0$$

$$\therefore \quad T = m_A g \sin 30° - \mu_S N_A$$

$$= 50 \times 9.81 \sin 30° - 0.2\,(424.78)$$

$$(\because N_A = 424.78 \text{ N})$$

$$\therefore \quad \mathbf{T = 160.294 \text{ N}} \quad \ldots(2)$$

Step 3 : **FBD of block 'B'**

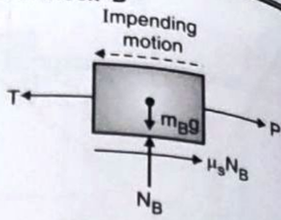


Fig. P. 7.14.11(c)

T = 160.294 N

N_B = Normal reaction from horizontal plane

μ_S = 0.2

m_B = 100kg

Step 4: **Equilibrium equations:**

For limiting equilibrium;

$$\Sigma F_y = 0$$

$$N_B - m_B g = 0$$

$$\therefore \quad N_B = 100 \times 9.81 = \mathbf{981 \text{ N}}$$

$$\Sigma F_x = 0$$

$$-T + \mu_S N_B + P = 0$$

$$\therefore \quad P = T - \mu_S N_B$$

$$= 160.294 - 0.2\,(981)$$

$$(\because N_B = 981 \text{ N})$$

$$\therefore \quad \mathbf{P = -36 \text{ N}}$$

∴ The force required to prevent the system to move toward left, **P = 0** ...Ans.

Case 2 : **System is just on the point of moving towards right:**

Impending motion is towards right and hence friction force is towards left.

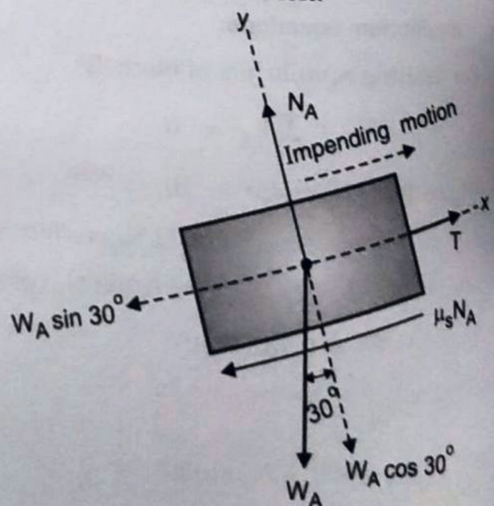


Fig. P. 7.14.11(d)

$W_A = 50 \times 9.81 = \mathbf{490.5\ N}$

$\mu_s = 0.2$

For equilibrium:

$\Sigma F_y = 0$

$N_A = W_A \cos 30° = 490.5 \cos 30°$

$= \mathbf{424.78\ N}$

$\Sigma F_x = 0$

$-W_A \sin 30° - \mu_S N_A + T = 0$

$T = 490.5 \sin 30° + 0.2\,(424.78)$

$= \mathbf{330.206\ N}$

FBD of block 'B'

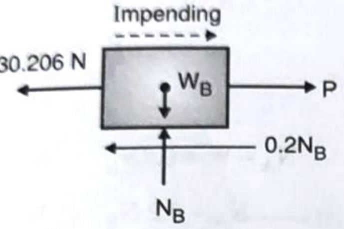


Fig. P. 7.14.11(e)

$W_B = 100 \times 9.81 = 981\ N$

For equilibrium:

$\Sigma F_y = 0$

$N_B - W_B = 0$

$N_B = \mathbf{981\ N}$

$\Sigma F_x = 0$

$-330.206 - 0.2\, N_B + P = 0$

$P = 330.206 + 0.2\,(981)$

$= \mathbf{526.406\ N}$ **...Ans.**

Ex. 7.14.12: The homogeneous semi-cylinder has a mass m and mass center at G as shown in Fig. P. 7.14.12. Determine the largest angle θ of the inclined plane upon which it rest so that it does not slip down the plane. The coefficient of static friction between the plane and the cylinder is 0.3.

SPPU : May 16, 5 Marks

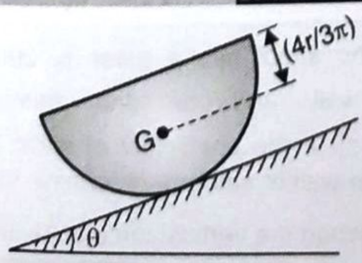


Fig. P. 7.14.12

Soln.:

When the semi-cylinder is about to slide down the plane, under its own weight the angle made by the plane w.r.t. horizontal i.e. 'θ' is known as angle of repose.

We know that, $\tan\theta = \mu_s$ But $\mu_s = 0.3$

$\therefore \tan\theta = 0.3$

$\therefore \theta = \mathbf{16.7°}$ **...Ans.**

Ex. 7.14.13: A steel shelf 1.5 m high × 1.0 m wide and weighing 400 N is mounted on bushes A and B. These bushes do not rotate when the shelf is moved along the floor. Assuming that the coefficient of friction between the bushes and floor is 0.75, work out force required just to cause the shelf to move. If the shelf is not to tip over, determine the maximum height at which the force can be applied.

SPPU : Dec. 09, 6 Marks

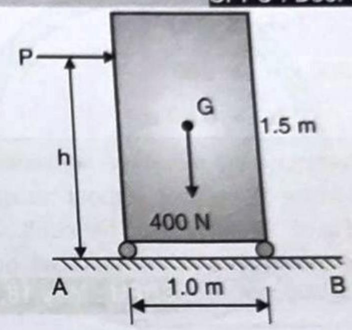


Fig. P. 7.14.13(a)

Soln.:

Step 1: FBD of shelf:

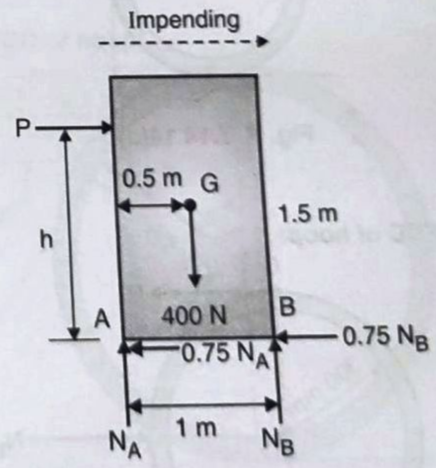


Fig. P. 7.14.13(b)

Let N_A and N_B = Normal reactions from rollers and B.

Step 2: Equilibrium equations:

$\Sigma F_y = 0$

$$W_A = 50 \times 9.81 = \mathbf{490.5\ N}$$

$$\mu_s = 0.2$$

For equilibrium:

$$\Sigma F_y = 0$$

$$N_A = W_A \cos 30° = 490.5 \cos 30°$$

$$= \mathbf{424.78\ N}$$

$$\Sigma F_x = 0$$

$$-W_A \sin 30° - \mu_S N_A + T = 0$$

$$T = 490.5 \sin 30° + 0.2\ (424.78)$$

$$= \mathbf{330.206\ N}$$

FBD of block 'B'

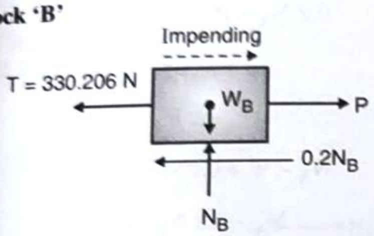


Fig. P. 7.14.11(e)

$$W_B = 100 \times 9.81 = 981\ N$$

For equilibrium:

$$\Sigma F_y = 0$$

$$N_B - W_B = 0$$

$$N_B = \mathbf{981\ N}$$

$$\Sigma F_x = 0$$

$$-330.206 - 0.2\ N_B + P = 0$$

$$P = 330.206 + 0.2\ (981)$$

$$= \mathbf{526.406\ N}$$ **...Ans.**

Ex. 7.14.12: The homogeneous semi-cylinder has a mass m and mass center at G as shown in Fig. P. 7.14.12. Determine the largest angle θ of the inclined plane upon which it rest so that it does not slip down the plane. The coefficient of static friction between the plane and the cylinder is 0.3.

SPPU : May 16, 5 Marks

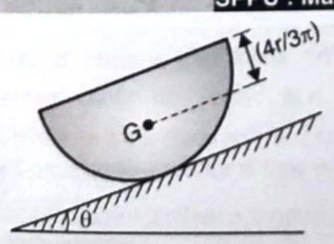


Fig. P. 7.14.12

Soln.:

When the semi-cylinder is about to slide down the plane, under its own weight the angle made by the plane w.r.t. horizontal i.e. 'θ' is known as angle of repose.

We know that, $\tan\theta = \mu_s$ But $\mu_s = 0.3$

$\therefore$ $\tan\theta = 0.3$

$\therefore$ $\theta = \mathbf{16.7°}$ **...Ans.**

Ex. 7.14.13: A steel shelf 1.5 m high × 1.0 m wide and weighing 400 N is mounted on bushes A and B. These bushes do not rotate when the shelf is moved along the floor. Assuming that the coefficient of friction between the bushes and floor is 0.75, work out force required just to cause the shelf to move. If the shelf is not to tip over, determine the maximum height at which the force can be applied.

SPPU : Dec. 09, 6 Marks

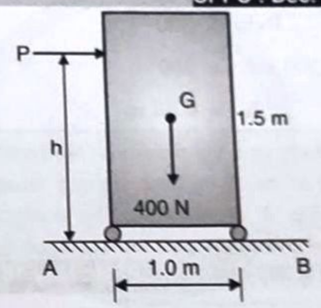


Fig. P. 7.14.13(a)

Soln.:

Step 1: FBD of shelf:

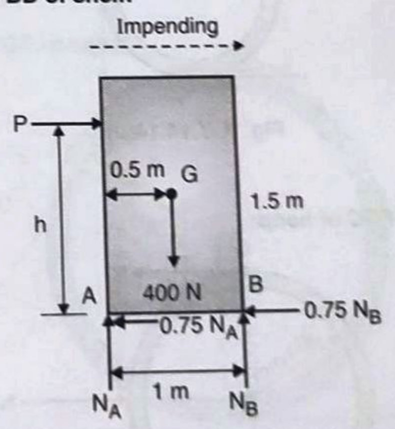


Fig. P. 7.14.13(b)

Let N_A and N_B = Normal reactions from rollers A and B.

Step 2: Equilibrium equations:

$$\Sigma F_y = 0$$

$$N_A + N_B - 400 = 0 \quad ...(1)$$

$$N_A + N_B = 400$$

$$\Sigma F_x = 0$$

$$P - 0.75\, N_A - 0.75\, N_B = 0$$

$$P = 0.75\, N_A + 0.75\, N_B \quad ...(2)$$

As the shelf is about to tip at roller B, it loses contact with the surface at A, making normal reaction at 'A' is zero.

$$\therefore \quad N_A = 0$$

From Eq^n (1), $N_B = 400$ N

From Eq^n (2), $P = 0.75\,(400) = \mathbf{300\ N}$ ...**Ans.**

Taking moments at 'B',

$$\Sigma M_B = 0$$

$$-P \times h - N_A \times 1 + 400 \times 0.5 = 0$$

$$P \times h = 200 \quad (\because N_A = 0)$$

$$300 \times h = 200 \quad (\because P = 300\ N)$$

$$\mathbf{h = 0.66\ m}$$ ...**Ans.**

Ex. 7.14.14: Determine the maximum horizontal force P that can be applied to the 12 kg hoop without causing it to rotate as shown in Fig. P. 7.14.14(a). The coefficient of static friction between the hoop and the surfaces at A and B is $\mu_s = 0.2$. Take r = 300 mm. **SPPU : May 18, 6 Marks**

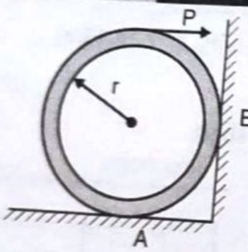


Fig. P. 7.14.14(a)

Soln.:

Step 1: FBD of hoop:

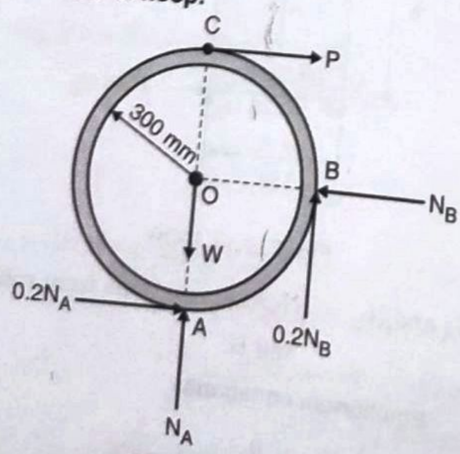


Fig. P. 7.14.14(b)

The hoop tends to rotate in CW direction

∴ Friction force at A is towards right and ... upwards.

Let, N_A = Normal reaction at A

N_B = Normal reaction at B

W = Weight of hoop = 12 × 9.81 = 117.72 N

μ_s = 0.2

Step 2: Equations of equilibrium :

Let, For limiting equilibrium of hoop,

$$\Sigma F_x = 0$$

$$0.2\, N_A + P - N_B = 0$$

$$\therefore \quad P = N_B - 0.2\, N_A \quad ...(1)$$

$$\Sigma F_y = 0$$

$$N_A - W + 0.2 N_B = 0$$

$$N_A + 0.2\, N_B = W$$

$$N_A + 0.2\, N_B = 117.72 \quad ...(2)$$

$$\Sigma M_C = 0$$

$$0.2\, N_A \times 300 + 0.2\, N_B \times 150 - N_B \times 150 = 0$$

$$60\, N_A + 30\, N_B - 150\, N_B = 0$$

$$60\, N_A = 120\, N_B$$

$$N_A = 2\, N_B \quad ...(3)$$

Step 3: Value of 'P'

From Eq^n (2), $2N_B + 0.2\, N_B = 117.72$ $(\because N_A = 2N_B)$

$$\therefore \quad \mathbf{N_B = 53.51\ N}$$

From Eq^n (3), $N_A = 2 \times 53.51 = \mathbf{107.02\ N}$

From Eq^n (1), $P = 53.51 - 0.2 \times 107.02 = 32.106$

$$\mathbf{P = 32.106\ N}$$ ...**Ans.**

Ex. 7.14.15: The spool has a mass of 200 kg and rests against the wall and on the beam shown in Fig. P. 7.14.15(a). If the coefficient of static friction at B is $\mu_B = 0.3$ and the wall is smooth, determine the friction force developed at B when the vertical force applied to the cable is P = 800 N. **SPPU : Dec. 17, 6 Marks**

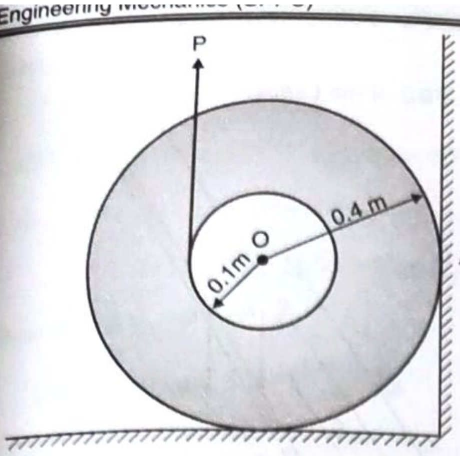


Fig. P. 7.14.15(a)

Soln.:

Step 1: FBD of spool:

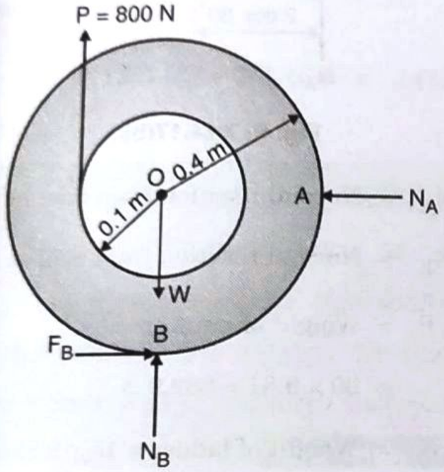


Fig. P. 7.14.15(b)

Let N_A = Normal reaction from wall

Friction force at wall = 0 ($\because$ smooth wall)

N_B = Normal reaction from beam

F_B = Friction force at B

W = Weight of spool = 200×9.81

= 1962 N

Step 2: Equilibrium equations :

For equilibrium;

$$\sum F_x = 0$$

$$F_B - N_A = 0$$

$$\therefore \quad F_B = N_A \quad \text{...(1)}$$

$$\sum F_y = 0$$

$$N_B + 800 - W = 0$$

$$N_B = W - 800 = 1962 - 800 = \mathbf{1162\ N} \quad \text{...(2)}$$

Taking moments at 'O',

$$\sum M_o = 0$$

$$-800 \times 0.1 + F_B \times 0.4 = 0 \quad \text{...(3)}$$

$$\mathbf{F_B = 200\ N} \quad \text{...Ans.}$$

Note :

1) System is not in limiting equilibrium.

$\therefore$ Friction force at B, $F_B \neq \mu N_B$

2) To find F_B, only equation (3) is required.

Ex. 7.14.16: A uniform hoop of weight W is suspended from the peg at A and a horizontal force P is slowly applied at B as shown in Fig. P. 7.14.16(a). If the hoop begins to slip at A when θ = 30°, determine the coefficient of static friction between the hoop and the peg. **SPPU : May 16, 6 Marks**

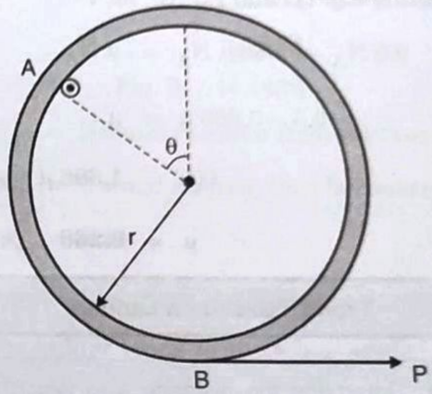


Fig. P. 7.14.16(a)

Soln.:

Step 1: FBD of hoop:

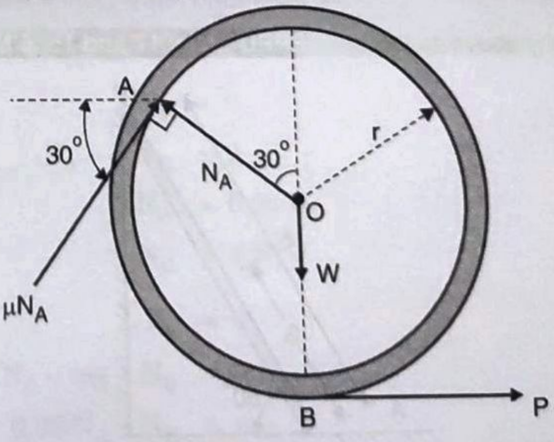


Fig. P. 7.14.16(b)

Let N_A = Normal reaction from peg.

W = Weight of hoop

Impending motion of hoop is ACW.

Step 2 : Equilibrium equations:

For limiting equilibrium of hoop,

$$\Sigma F_x = 0$$

$$\mu N_A \cos 30° - N_A \sin 30° + P = 0$$

$$\therefore \quad \mathbf{P = 0.5\, N_A - 0.866\, \mu\, N_A} \qquad ...(1)$$

Taking moments at 'O',

$$\Sigma M_o = 0$$

$$-\mu N_A \times r + P \times r = 0$$

$$\mathbf{P = \mu N_A} \qquad ...(2)$$

Step 3 : Coefficient of static friction 'μ'

Equating Eq^n (1) and Eq^n (2) for P

$$0.5\, N_A - 0.866\, \mu\, N_A = \mu N_A$$

$$0.5 - 0.866\,\mu = \mu$$

$$0.5 = 1.866\,\mu$$

$$\therefore \quad \mu = \mathbf{0.268} \qquad \text{...Ans.}$$

Type 2 : Based on Ladders

Ex. 7.14.17: Determine the distance s to which the 90 kg painter can climb without causing the 4 m ladder to slip at its lower end A as shown in Fig. P. 7.14.17(a) The top of the 15 kg ladder has a small roller and at the ground the coefficient of static friction $\mu_s = 0.25$. The mass center of the painter is directly above its feet. **SPPU : May 17 and 15, 6 Marks**

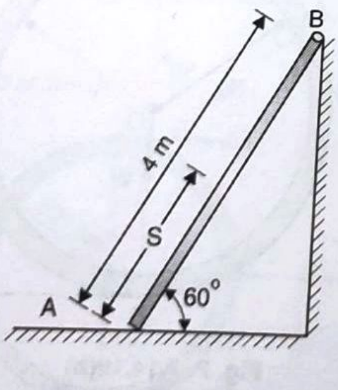


Fig. P. 7.14.17(a)

Soln.:

Step 1: FBD of the Ladder:

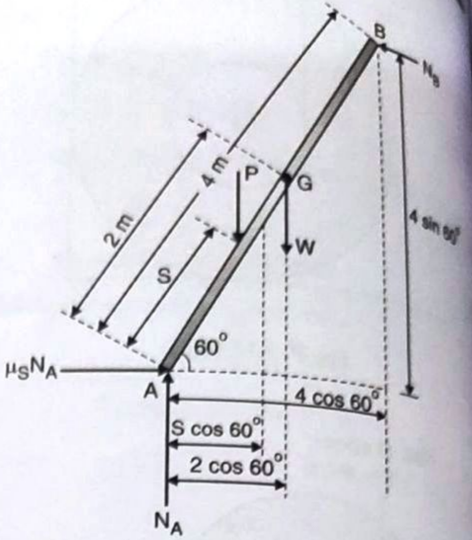


Fig. P. 7.14.17(b)

Let N_A = Normal reaction from floor at A

N_B = Normal reaction from wall at B

P = Weight of painter

$= 90 \times 9.81 = 882.9$ N

W = Weight of ladder $= 15 \times 9.81$

= 147.15 N

Let the ladder is on the verge of slipping when painter is at a distance 's' from A. While slipping end A tends to move towards left. Hence friction force will act towards right.

Step 2: Equilibrium equations:

For limiting equilibrium of ladder;

$$\Sigma F_x = 0$$

$$\mu_S N_A - N_B = 0$$

$$\mathbf{0.25\, N_A = N_B} \qquad ...(1)$$

$$\Sigma F_y = 0$$

$$N_A - P - w = 0$$

$$N_A = P + W = 882.9 + 147.15$$

$$\mathbf{N_A = 1030.05\ N} \qquad ...(2)$$

Taking moments at 'A',

$$\Sigma M_A = 0$$

$$-P \times S \cos 60° - W \times 2 \cos 60° + N_B \times 4 \sin 60° = 0$$

$$-882.9 \times S \cos 60° - 147.15 \times 2 \cos 60° + N_B\, 4 \sin 60° = 0$$

$$-441.45\, S - 147.15 + 3.46\, N_B = 0$$

$$3.46\, N_B - 441.45\, S = 147.15 \quad ...(3)$$

Step 3: Distance 'S':

From Eq^n (2), $N_A = 1030.05$ N

From Eq^n (1), $0.25\,(1030.05) = N_B$

$$N_B = 257.51 \text{ N}$$

From Eq^n (2),

$$3.46\,(257.51) - 441.45\, S = 147.15$$

$$\therefore \quad S = 1.68 \text{ m} \quad ...\text{Ans.}$$

Ex. 7.14.18 : The uniform pole of length l and mass m is leaned against the vertical wall as shown in Fig. P. 7.14.18(a). If the coefficient of static friction between supporting surfaces and the ends of the pole is 0.25, calculate the maximum angle θ at which the pole may place before it starts to slip. **SPPU : May 16, 6 Marks**

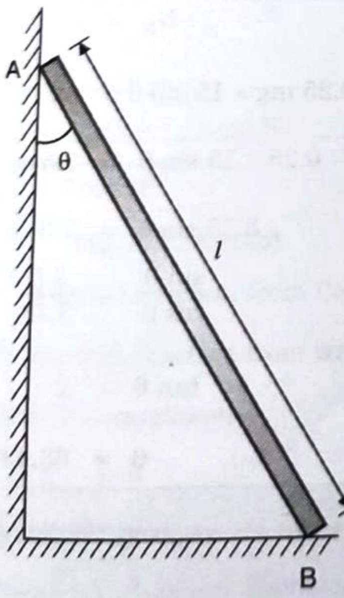


Fig. P. 7.14.18(a)

Soln.:

Step 1: FBD of pole:

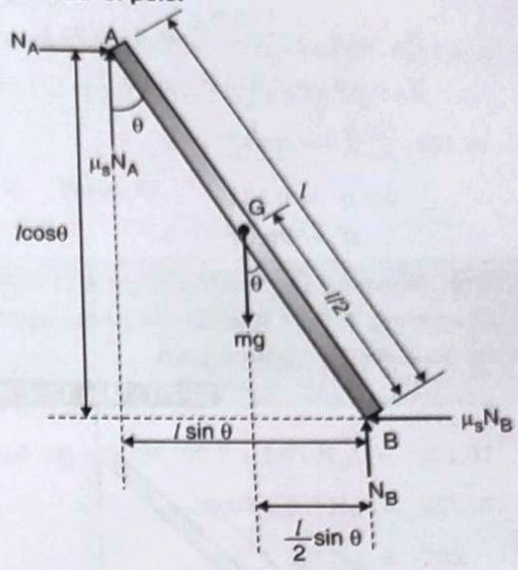


Fig. P. 7.14.18(b)

Let N_A = Normal reaction from vertical wall

N_B = Normal reaction from horizontal surface

μ_s = 0.25

mg = Weight of the ladder

Let the ladder tends to slip when angle is 'θ'; w.r.t. vertical.

While slipping end A moves down and end B towards right. Hence friction forces will act in the opposite directions.

Step 2: Equilibrium equations :

For limiting equilibrium of pole;

$$\Sigma F_x = 0$$

$$N_A - \mu_s N_B = 0$$

$$N_A = 0.25\, N_B$$

$$N_B = 4\, N_A \quad ...(1)$$

$$\Sigma F_y = 0$$

$$\mu_s N_A - mg + N_B = 0$$

$$0.25\, N_A + N_B = mg$$

$$0.25\, N_A + 4\, N_A = mg$$

$$4.25\, N_A = mg \quad ...(2)$$

$$\Sigma M_B = 0$$

$$-N_A \times l \cos\theta + mg \times \frac{l}{2} \sin\theta - \mu_s N_A \times l \sin\theta = 0$$

$$N_A \cos\theta = \frac{mg \sin\theta}{2} - 0.25\, N_A \sin\theta$$

$$N_A \cos\theta = \frac{4.25\, N_A}{2} \sin\theta - 0.25\, N_A \sin\theta$$

$$N_A \cos\theta = 1.875\, N_A \sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{1.875}$$

$$\tan\theta = 0.533$$

$$\theta = 28.07^\circ \quad \text{...Ans.}$$

Ex. 7.14.19: Determine the smallest angle θ at which the ladder shown in Fig. P. 7.14.19(a) can be placed against the side of smooth wall without having it slip.

SPPU : May 13, 6 Marks

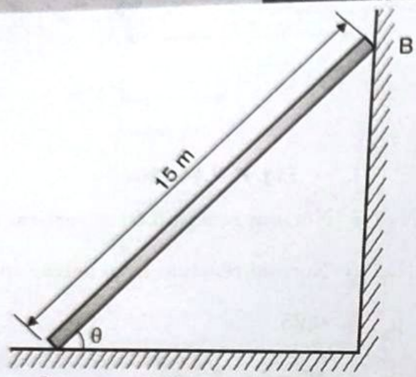


Fig. P. 7.14.19(a)

Soln.:

Step 1: FBD of ladder :

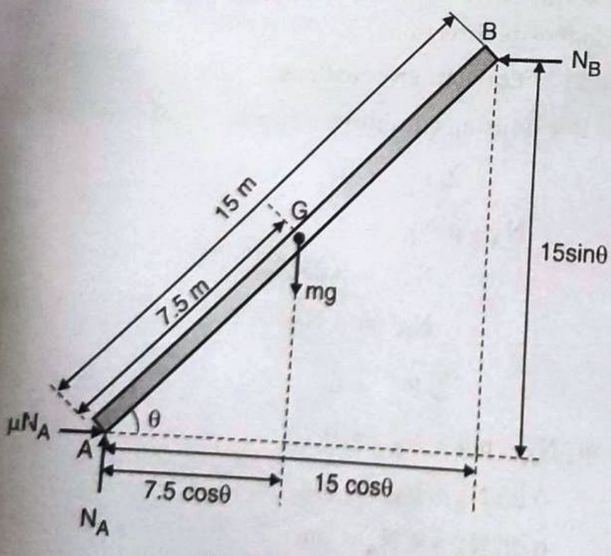


Fig. P. 7.14.19(b)

N_A = Normal reaction from horizontal floor

N_B = Normal reaction from vertical wall

When 'θ' decreases, ladder tends to slip with towards left and end B downwards.

Friction force at A is towards right.

Friction force at B = 0 ($\because$ smooth)

mg = Weight of ladder

$\mu = 0.25$

Step 2: Equilibrium equations :

For limiting equilibrium of ladder;

$$\Sigma F_x = 0$$

$$\mu N_A - N_B = 0$$

$$0.25\, N_A = N_B$$

$$\Sigma F_y = 0$$

$$N_A - mg = 0 \therefore N_A = mg$$

$$\Sigma M_A = 0$$

$$-mg \times 7.5 \cos\theta + N_B \times 15 \sin\theta = 0$$

$$N_B \times 15 \sin\theta = mg \times 7.5 \cos\theta$$

Step 3: Smallest angle 'θ'

From Eq^n (2), $N_A = mg$

From Eq^n (1), $0.25\,(mg) = N_B$

$$N_B = 0.25\, mg$$

From Eq^n (3), $0.25\, mg \times 15 \sin\theta = mg \times 7.5 \cos\theta$

$$0.25 \times 15 \sin\theta = 7.5 \cos\theta$$

$$3.75 \sin\theta = 7.5 \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{7.5}{3.75}$$

$$\tan\theta = 2$$

$$\theta = 63.43^\circ \quad \text{...Ans.}$$

Ex. 7.14.20: A ladder AB weighing 196 N is resting against wall and floor as shown in Fig. P. 7.14.20(a). Calculate the minimum horizontal force 'P' required to be applied at 'C' in order to push the ladder towards the wall.

SPPU : Dec. 02, 8 Marks

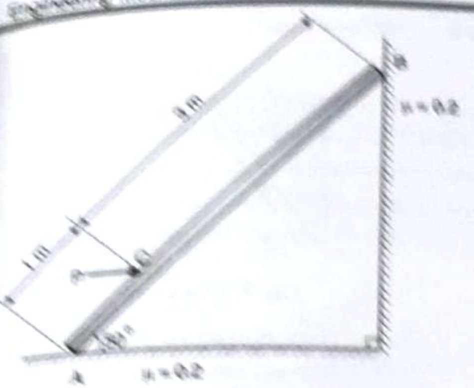


Fig. P. 7.14.20(a)

Soln.:

Step 1: Impending motion

In order to push the ladder towards the wall, the impending motion of end A is towards right and end B upwards.

Step 2: FBD of ladder:

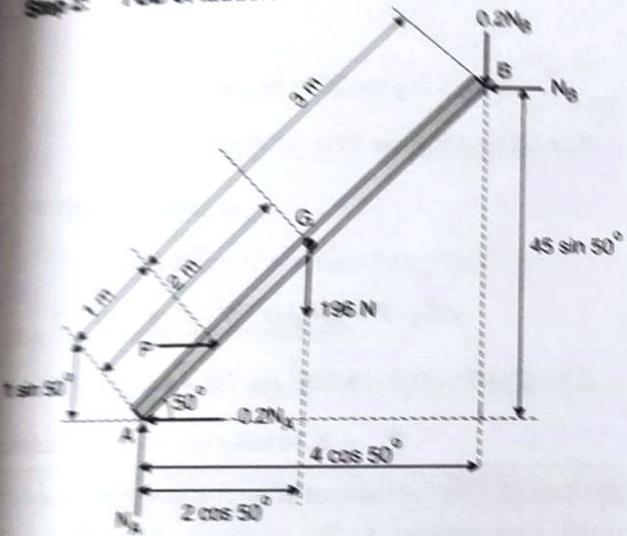


Fig. P. 7.14.20(b)

Let N_A = Normal reaction from floor

N_B = Normal reaction from wall

Step 3: Equilibrium equations:

$$\Sigma F_x = 0$$

$$P - 0.2\,N_A - N_B = 0$$

$$P = 0.2\,N_A + N_B \quad \text{...(1)}$$

$$\Sigma F_y = 0$$

$$N_A - 196 - 0.2\,N_B = 0$$

$$N_A - 0.2\,N_B = 196 \quad \text{...(2)}$$

Taking moments at A

$$\Sigma M_A = 0$$

$$-P \times 1 \sin 50^\circ - 196 \times 2 \cos 50^\circ - 0.2\,N_B \times 4 \cos 50^\circ + N_B \times 4 \sin 50^\circ = 0$$

$$-0.766\,P - 251.97 - 0.51\,\mu_B + 3.06\,N_B = 0$$

$$2.55\,N_B - 0.766\,P = 251.97 \quad \text{...(3)}$$

Step 4: Force, P:

From Eq^n (2), $N_A = 196 + 0.2\,N_B$

From Eq^n (1),

$$P = 0.2\,(196 + 0.2\,N_B) + N_B$$

$$P = 39.2 + 0.04\,N_B + N_B$$

$$P = 39.2 + 1.04\,N_B \quad \text{...(4)}$$

Putting in Eq^n (3),

$$2.55\,N_B - 0.766\,(39.2 + 1.04\,N_B) = 251.97$$

$$2.55\,N_B - 30.03 - 0.8\,N_B = 251.97$$

$$1.753\,N_B = 282$$

$$\therefore \quad N_B = 160.87 \text{ N}$$

From Eq^n (4), $P = 39.2 + 1.04\,(160.87)$

$$P = 206.50\text{N} \quad \text{...Ans.}$$

Type 3 : Based on Wedges

Ex. 7.14.21: A heavy concrete block weighing 10 kN is to be shifted away from the wall with the help of a 15° wedge as shown in Fig. P. 7.14.21(a). Calculate magnitude of the vertical force that has to be applied to the top of the wedge for achieving this objective, if the coefficient of friction between all the rubbing surfaces is 0.25.

SPPU : May 10, 8 Marks

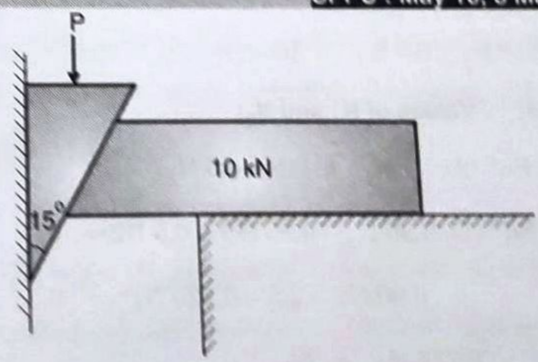


Fig. P. 7.14.21(a)

Soln.:

Step 1: Impending motion

In order to shift 10 kN block away from the wa[ll] the wedge has to move down.

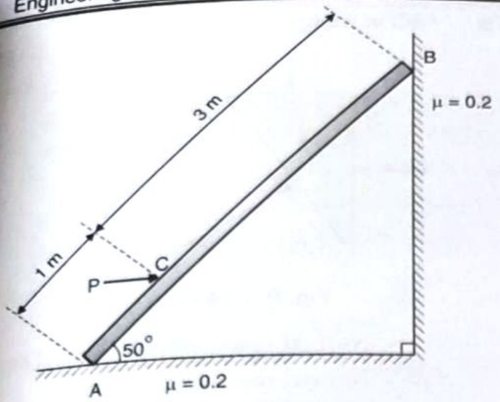


Fig. P. 7.14.20(a)

Soln.:

Step 1: Impending motion

In order to push the ladder towards the wall, the impending motion of end A is towards right and end B upwards.

Step 2: FBD of ladder:

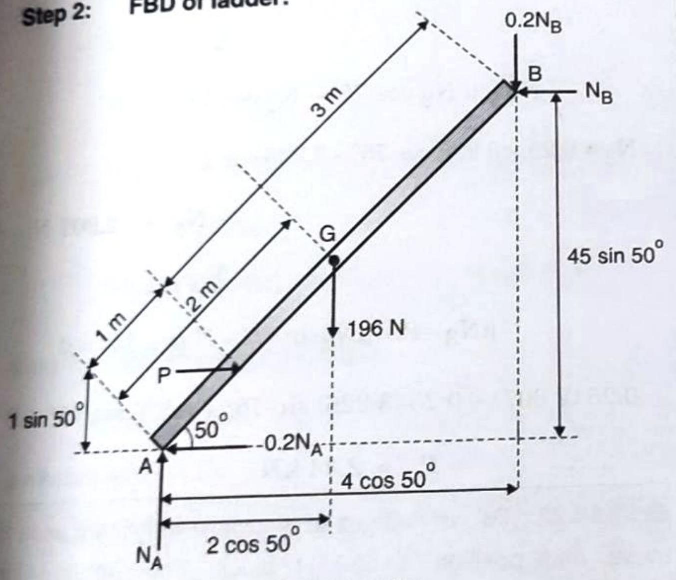


Fig. P. 7.14.20(b)

Let N_A = Normal reaction from floor

N_B = Normal reaction from wall

Step 3: Equilibrium equations:

$$\Sigma F_x = 0$$

$$P - 0.2\,N_A - N_B = 0$$

$$P = 0.2\,N_A + N_B \quad \text{...(1)}$$

$$\Sigma F_y = 0$$

$$N_A - 196 - 0.2\,N_B = 0$$

$$\mathbf{N_A - 0.2\,N_B = 196} \quad \text{...(2)}$$

Taking moments at A

$$\Sigma M_A = 0$$

$$-P \times 1 \sin 50° - 196 \times 2 \cos 50° - 0.2\,N_B \times 4 \cos 50° + N_B \times 4 \sin 50° = 0$$

$$-0.766\,P - 251.97 - 0.51\,\mu_B + 3.06\,N_B = 0$$

$$2.55\,N_B - 0.766\,P = \mathbf{251.97} \quad \text{...(3)}$$

Step 4: Force, P:

From Eq^n (2), $N_A = 196 + 0.2\,N_B$

From Eq^n (1), $P = 0.2\,(196 + 0.2\,N_B) + N_B$

$$P = 39.2 + 0.04\,N_B + N_B$$

$$\mathbf{P = 39.2 + 1.04\,N_B} \quad \text{...(4)}$$

Putting in Eq^n (3),

$$2.55\,N_B - 0.766\,(39.2 + 1.04\,N_B) = 251.97$$

$$2.55\,N_B - 30.03 - 0.8\,N_B = 251.97$$

$$1.753\,N_B = \mathbf{282}$$

$$\therefore \quad \mathbf{N_B = 160.87\ N}$$

From Eq^n (4), $P = 39.2 + 1.04\,(160.87)$

$$\mathbf{P = 206.50N} \quad \text{...Ans.}$$

Type 3 : Based on Wedges

Ex. 7.14.21: A heavy concrete block weighing 10 kN is to be shifted away from the wall with the help of a 15° wedge as shown in Fig. P. 7.14.21(a). Calculate magnitude of the vertical force that has to be applied to the top of the wedge for achieving this objective, if the coefficient of friction between all the rubbing surfaces is 0.25.

SPPU : May 10, 8 Marks

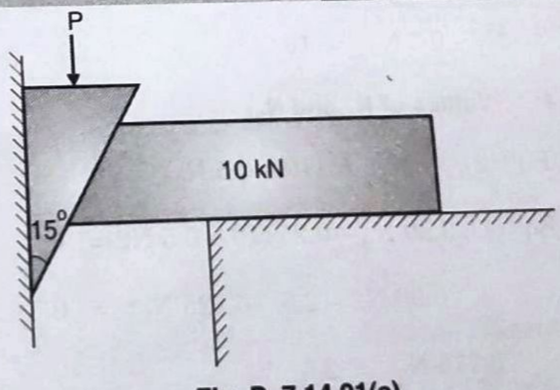


Fig. P. 7.14.21(a)

Soln.:

Step 1: Impending motion

In order to shift 10 kN block away from the wall the wedge has to move down.

$\therefore$ Impending motion of wedge is downwards and block towards right. Accordingly friction forces will act in the opposite direction.

Step 2: **FBD of concrete block:**

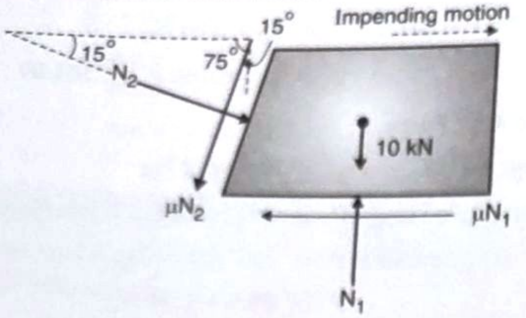


Fig. P. 7.14.21(b)

Let N_1 = Normal reaction from horizontal floor

N_2 = Normal reaction from the wedge

μ = 0.25

Step 3: **Equilibrium equations:**

For limiting equilibrium;

$$\Sigma F_x = 0$$

$$N_2 \cos 15° - \mu N_2 \cos 75° - \mu N_1 = 0$$

$$N_2 \cos 15° - 0.25 N_2 \cos 75° - 0.25 N_1 = 0$$

$$\therefore 0.90 N_2 - 0.25 N_1 = 0 \quad ...(1)$$

$$\Sigma F_y = 0$$

$$-N_2 \sin 15° - \mu N_2 \sin 75° - 10 + N_1 = 0$$

$$-N_2 \sin 15° - 0.25 N_2 \sin 75° + N_1 = 10$$

$$N_1 - 0.5 N_2 = 10 \quad ...(2)$$

Step 4: **Values of N_1 and N_2:**

From Eq^n (2), $N_1 = (10 + 0.5 N_2)$

From Eq^n (1), $0.90 N_2 - 0.25 (10 + 0.5 N2) = 0$

$$0.90 N2 - 2.5 - 0.125 N_2) = 0$$

$$0.775 N_2 = 2.5$$

$$\therefore \mathbf{N_2 = 3.226\ kN}$$

$$N_1 = 10 + 0.5 (3.226)$$

$$\therefore \mathbf{N_1 = 11.61\ kN}$$

Step 5: **FBD of wedge:**

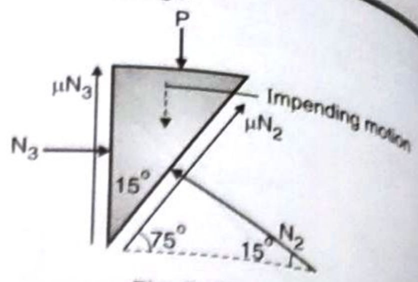


Fig. P. 7.14.21(c)

Let N_3 = Normal reaction from vertical surface

N_2 = Normal reaction from block

= 3.226 kN

> **Note :** Normal reaction between block and wedge will have same magnitude but opposite in direction.

Step 6: **Equilibrium equations :**

For limiting equilibrium of wedge;

$$\Sigma F_x = 0$$

$$N_3 + \mu N_2 \cos 75° - N_2 \cos 15° = 0$$

$$N_3 + 0.25 \times 3.226 \cos 75° - 3.226 \cos 15° = 0$$

$$N_3 = 2.907\ N$$

$$\Sigma F_y = 0$$

$$\mu N_3 - P + \mu N_2 \sin 75° + N_2 \sin 15° = 0$$

$$0.25 (2.907) + 0.25 (3.226) \sin 75° + 3.226 \sin 15° = P$$

$$\therefore \mathbf{P = 2.34\ kN} \quad ...Ans.$$

Ex. 7.14.22 : Two 8° wedges of negligible weight are used to move and position a 530 N block. Knowing that the coefficient of static friction is 0.40 at all surfaces of contact, determine the magnitude of the force P for which motion of the block is impending. **SPPU : May 09, 8 Marks**

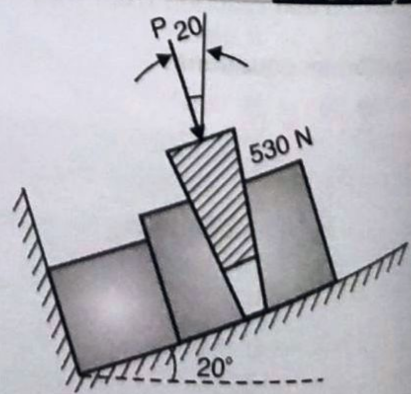


Fig. P. 7.14.22(a)

Soln.:

Step 1: Impending motion :

Because of force 'P', wedge tends to move downwards and hence the block up the plane.

∴ Impending motion of wedge is downwards and the block up the plane. Accordingly friction forces will act in the opposite direction.

Step 2: FBD of block:

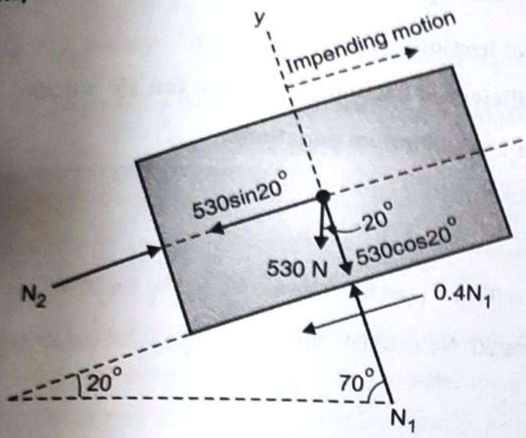


Fig. P. 7.14.22(b)

Let N_1 = Normal reaction from the inclined plane

N_2 = Normal reaction from the wedge

Step 3: Equilibrium equations:

Taking x-axis along the plane and y-axis perpendicular to the plane;

For limiting equilibrium;

$$\sum F_x = 0$$

$$N_2 - 530 \sin 20° - 0.4 N_1 = 0$$

$$N_2 - 0.4 N_1 = 181.27 \quad \ldots(1)$$

$$\sum F_y = 0$$

$$-530 \cos 20° + N_1 = 0$$

$$N_1 = 498.04 \text{ N}$$

From Eqⁿ (1),

$$N_2 - 0.4 (498.04) = 181.27$$

$$\therefore \quad N_2 = 380.48 \text{ N}$$

Step 4: FBD of wedge:

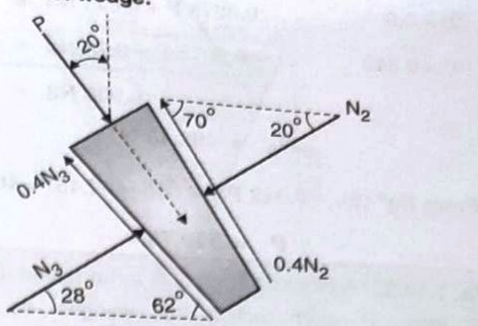


Fig. P. 7.14.22(c)

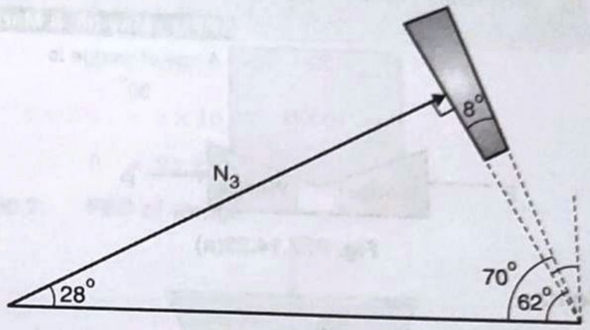


Fig. P. 7.14.22(d)

Let, N_3 = Normal reaction from second wedge.

N_2 = Normal reaction between wedge and block.

Step 5: Equilibrium equations:

For limiting equilibrium of wedge;

$$\sum F_x = 0$$

$$P \sin 20° - N_2 \cos 20° - 0.4 N_2 \cos 70° - 0.4 N_3 \cos 62° + N_3 \cos 28° = 0$$

$$P \sin 20° - (380.48) \cos 20° - 0.4 (380.48) \cos 70° - 0.4 N_3 \cos 62° + N_3 \cos 28° = 0$$

$$0.342 P - 357.53 - 52.05 - 0.188 N_3 + 0.883 N_3 = 0$$

$$0.342 P + 0.695 N_3 = 409.58 \quad \ldots(2)$$

$$\sum F_y = 0$$

$$-P \cos 20° - N_2 \sin 20° + 0.4 N_2 \sin 70° + 0.4 N_3 \sin 62° + N_3 \sin 28° = 0$$

$$-0.9 P - (380.48) \sin 20° + 0.4 (380.48) \sin 70° + 0.4 N_3 \sin 62° + N_3 \sin 28° = 0$$

$$-0.9 P - 130.13 + 143.01 + 0.35 N_3 + 0.47 N_3 = 0$$

$$0.82 N_3 - 0.9 P = -12.88 \quad \ldots(3)$$

Step 6: Magnitude of 'P'

From Eqⁿ (2); $0.342 P + 0.695 N3 = 409.58$

Form Eqⁿ (3); $-0.9 P + 0.82 N3 = -12.88$

$(2) \times 0.9 \qquad 0.3078 P + 0.625 N3 = 368.62$

$(3) \times 0.342 \qquad -0.3078 P + 0.28 N3 = -4.40$

$0.905 N3 = 364.22$

$\therefore N_3 = 402.45$ N

From Eq^n (2), $0.342 P + 0.695 (402.45) = 409.58$

$\therefore \quad P = 379.75$N ...Ans.

Ex. 7.14.23: A block of 1000 N is to be raised up by means of force P each acting on wedges as shown in Fig. P. 7.14.23(a). If angle of friction at all rubbing surfaces is 15°, determine P. Ignore weight of wedge.

SPPU : May 08, 8 Marks

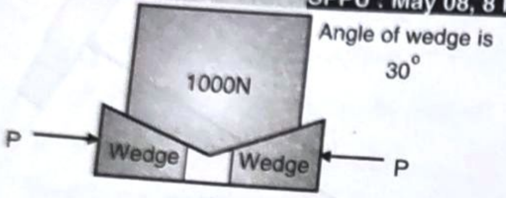


Fig. P. 7.14.23(a)

Soln.:

Step 1: Impending motion :

1000 N block is to be raised .

$\therefore$ Impending motion of block is upwards, left wedge towards right and right wedge towards left.

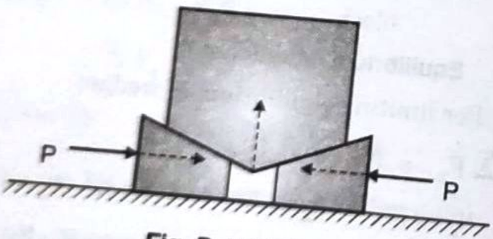


Fig. P. 7.14.23(b)

Step 2: FBD of block:

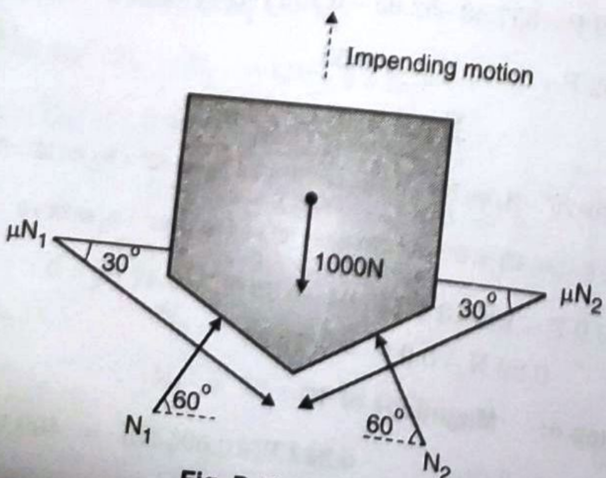


Fig. P. 7.14.23(c)

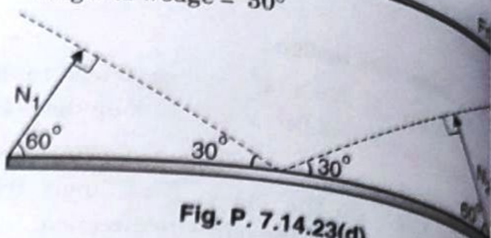


Fig. P. 7.14.23(d)

Let, N_1 and N_2 be the normal reactions from wedg

Angle of friction, $\phi = 15°$

$\therefore$ Coefficient of friction, $\mu = \tan \phi = \tan 15° = 0.268$

Step 3: Equilibrium equations:

For limiting equilibrium;

$$\Sigma F_x = 0$$

$$\mu N_1 \cos 30° + N_1 \cos 60° - N_2 \cos 60° - \mu N_2 \cos 30° = 0$$

$$0.268 \cos 30° N_1 + \cos 60° N_1 = \cos 60° N_2 + 0.268 \cos 30° N_2$$

$$\therefore \quad N_1 = N_2$$

$$\Sigma F_y = 0$$

$$N_1 \sin 60° + N_2 \sin 60° - \mu N_1 \sin 30° - \mu N_2 \sin 30° - 1000 = 0$$

$$N_1 \sin 60° + N_2 \sin 60° - 0.268 N_1 \sin 30° - 0.268 N_2 \sin 30° =$$

$$0.732 N_1 + 0.732 N_2 = 1000$$

$$N_1 + N_2 = 1366.07$$

$$N_1 + N_1 = 1366.07 (\because N_1 =$$

$$\mathbf{N_1 = 683.04\ N}$$

$$\mathbf{N_2 = 683.04\ N}$$

Step 4: FBD of wedge:

NOTE: Being both wedges are similar any one w can be considered.

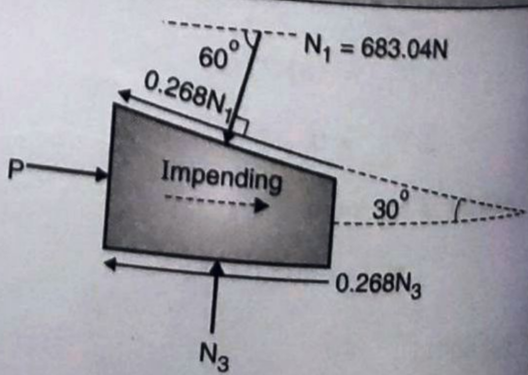


Fig. P. 7.14.23(e)

Step 5: Equilibrium equations:

For limiting equilibrium of wedge;

$$\sum F_x = 0$$

$$-N_1 \sin 60^\circ + 0.268\, N_2 \sin 30^\circ + N_3 = 0$$

$$-683.04 \sin 60^\circ + 0.268 \times 683.04 \sin 30^\circ + N_3 = 0$$

$$\therefore \quad N_3 = 500\ \text{N}$$

$$\sum F_y = 0$$

$$P - N_1 \cos 60^\circ - 0.268\, N_2 \cos 30^\circ - 0.268\, N_3 = 0$$

$$P - 683.04 \cos 60^\circ - 0.268\,(683.04) \cos 30^\circ - 0.268\,(500) = 0$$

$$\therefore \quad P = 634.05\ \text{N} \qquad \text{...Ans.}$$

Ex. 7.14.24: A wedge A of negligible weight is to be driven between two 51 kg blocks B and C. Knowing that coefficient of static friction is 0.35 between blocks and horizontal surface and zero between the wedge and each of blocks, determine the smallest force P required to start moving the wedge if the blocks are equally free to move; and if block C is securely bolted to horizontal surface. Refer Fig. P. 7.14.24(a). **SPPU : May 06, 9 Marks**

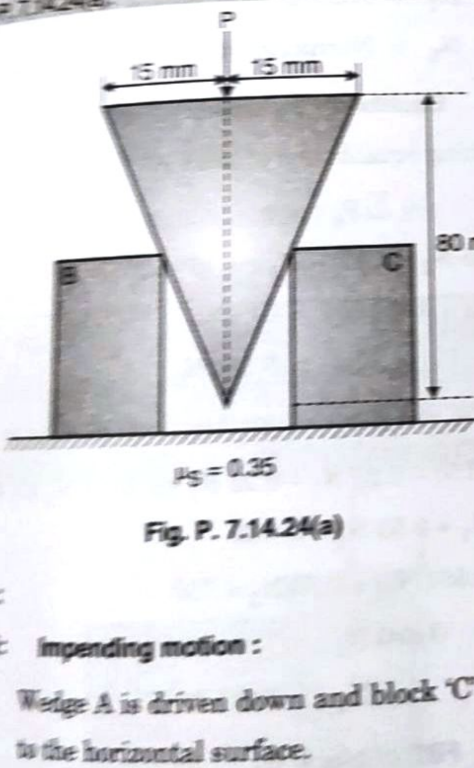


Fig. P. 7.14.24(a)

Soln.:

Step 1: Impending motion :

Wedge A is driven down and block 'C' is bolted to the horizontal surface.

$\therefore$ Impending motion of wedge 'A' is downwards and block 'B' towards left. Accordingly friction forces will act in the opposite direction of impending motion.

Step 2: Angle of wedge 'θ'

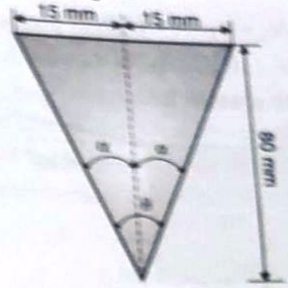


Fig. P. 7.14.24(b)

From Fig. P. 7.14.24(b), $\tan \alpha = \dfrac{15}{80}$

$$\therefore \quad \alpha = 10.62^\circ$$

$$\theta = 2\alpha = 2 \times 10.62^\circ$$

$$\theta = 21.24^\circ$$

Step 3: FBD of wedge:

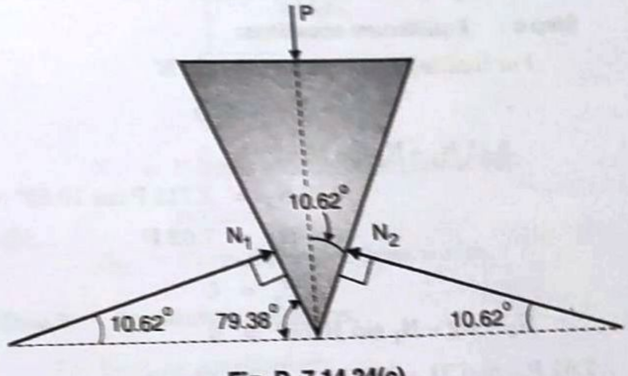


Fig. P. 7.14.24(c)

Let P = force required to drive the wedge.

Let N_1 and N_2 = Normal reactions from the blocks

$\mu = 0$ between wedge and blocks.

$\therefore$ Friction force is zero.

Step 4: Equilibrium equations:

For limiting equilibrium;

$$\sum F_x = 0$$

$$N_1 \cos 10.62^\circ - N_2 \cos 10.62^\circ = 0 \qquad \text{...(1)}$$

$$\therefore \quad N_1 = N_2$$

$$\sum F_y = 0$$

$$-P + N_1 \sin 10.62^\circ + N_2 \sin 10.62^\circ = 0$$

$$\therefore \quad 0.184\, N_1 + 0.184\, N_2 = P$$

Step 5: Equilibrium equations:

For limiting equilibrium of wedge;

$$\sum F_x = 0$$

$$-N_1 \sin 60^\circ + 0.268\, N_1 \sin 30^\circ + N_3 = 0$$

$$-683.04 \sin 60^\circ + 0.268 \times 683.04 \sin 30^\circ + N_3 = 0$$

$$\therefore \quad N_3 = 500\ N$$

$$\sum F_y = 0$$

$$P - N_1 \cos 60^\circ - 0.268\, N_1 \cos 30^\circ - 0.268\, N_3 = 0$$

$$P - 683.04 \cos 60^\circ - 0.268\,(683.04) \cos 30^\circ - 0.268\,(500) = 0$$

$$\therefore \quad P = 634.05\ N \qquad \text{...Ans.}$$

Ex. 7.14.24: A wedge A of negligible weight is to be driven between two 51 kg blocks B and C. Knowing that coefficient of static friction is 0.35 between blocks and horizontal surface and zero between the wedge and each of blocks, determine the smallest force P required to start moving the wedge if the blocks are equally free to move; and if block C is securely bolted to horizontal surface. Refer Fig. P. 7.14.24(a). **SPPU : May 06, 9 Marks**

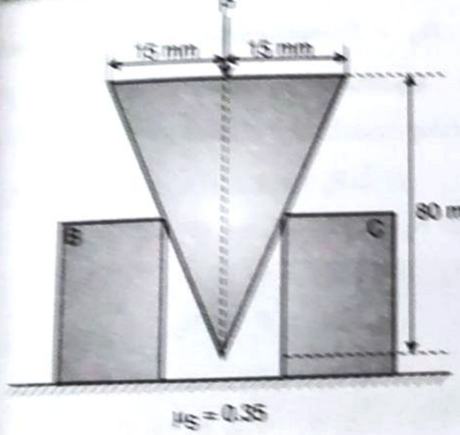


Fig. P. 7.14.24(a)

Soln.:

Step 1: Impending motion :

Wedge A is driven down and block 'C' is bolted to the horizontal surface.

$\therefore$ Impending motion of wedge 'A' is downwards and block 'B' towards left. Accordingly friction forces will act in the opposite direction of impending motion.

Step 2: Angle of wedge 'θ'

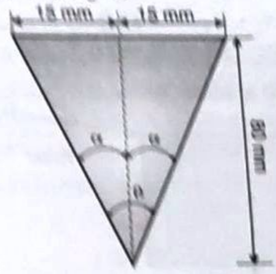


Fig. P. 7.14.24(b)

From Fig. P. 7.14.24(b), $\tan \alpha = \dfrac{15}{80}$

$$\therefore \quad \alpha = 10.62^\circ$$

$$\theta = 2\alpha = 2 \times 10.62^\circ$$

$$\theta = 21.24^\circ$$

Step 3: FBD of wedge:

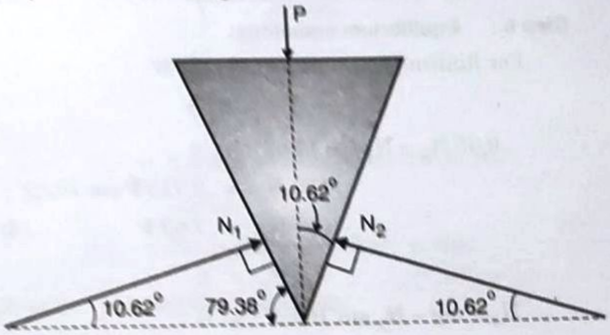


Fig. P. 7.14.24(c)

Let P = force required to drive the wedge.

Let N_1 and N_2 = Normal reactions from the blocks

$\mu = 0$ between wedge and blocks.

$\therefore$ Friction force is zero.

Step 4: Equilibrium equations:

For limiting equilibrium;

$$\sum F_x = 0$$

$$N_1 \cos 10.62^\circ - N_2 \cos 10.62^\circ = 0$$

$$\therefore \quad N_1 = N_2 \qquad \text{...(1)}$$

$$\sum F_y = 0$$

$$-P + N_1 \sin 10.62^\circ + N_2 \sin 10.62^\circ = 0$$

$$\therefore \quad 0.184\, N_1 + 0.184\, N_2 = P$$

Step 5 : Equilibrium equations:

For limiting equilibrium of wedge;

$$\Sigma F_y = 0$$

$$-N_1 \sin 60° + 0.268 N_1 \sin 30° + N_3 = 0$$

$$-683.04 \sin 60° + 0.268 \times 683.04 \sin 30° + N_3 = 0$$

$$\therefore \quad N_3 = \mathbf{500\ N}$$

$$\Sigma F_x = 0$$

$$P - N_1 \cos 60° - 0.268 N_1 \cos 30° - 0.268 N_3 = 0$$

$$P = 683.04 \cos 60° + 0.268 (683.04) \cos 30° + 0.268 (500) = 0$$

$$\therefore \quad P = \mathbf{634.05\ N} \quad \text{...Ans.}$$

Ex. 7.14.24: A wedge A of negligible weight is to be driven between two 51 kg blocks B and C. Knowing that coefficient of static friction is 0.35 between blocks and horizontal surface and zero between the wedge and each of blocks, determine the smallest force P required to start moving the wedge if the blocks are equally free to move; and if block C is securely bolted to horizontal surface. Refer. Fig. P. 7.14.24(a). **SPPU : May 06, 9 Marks**

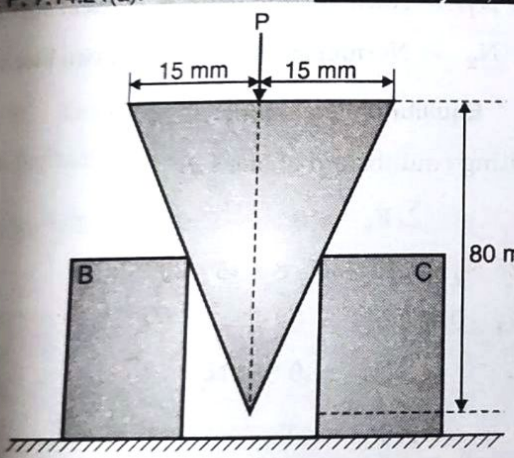


Fig. P. 7.14.24(a)

Soln.:

Step 1: Impending motion :

Wedge A is driven down and block 'C' is bolted to the horizontal surface.

∴ Impending motion of wedge 'A' is downwards and block 'B' towards left. Accordingly friction forces will act in the opposite direction of impending motion.

Step 2: Angle of wedge 'θ'

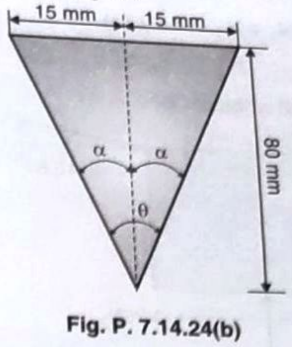


Fig. P. 7.14.24(b)

From Fig. P. 7.14.24(b), $\tan \alpha = \frac{15}{80}$

$$\therefore \quad \alpha = 10.62°$$

$$\theta = 2\alpha = 2 \times 10.62°$$

$$\theta = \mathbf{21.24°}$$

Step 3: FBD of wedge:

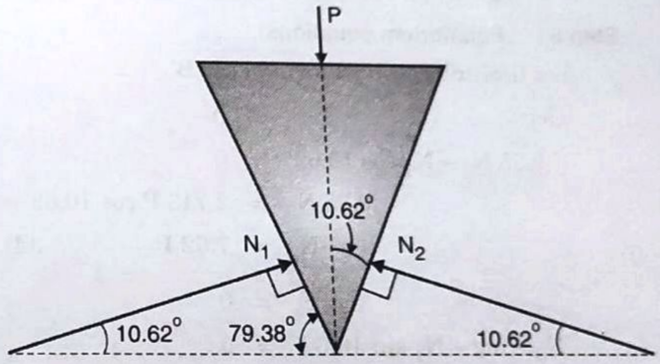


Fig. P. 7.14.24(c)

Let P = force required to drive the wedge.

Let N_1 and N_2 = Normal reactions from the blocks

$\mu = 0$ between wedge and blocks.

∴ Friction force is zero.

Step 4: Equilibrium equations:

For limiting equilibrium;

$$\Sigma F_x = 0$$

$$N_1 \cos 10.62° - N_2 \cos 10.62° = 0$$

$$\therefore \quad N_1 = N_2 \quad \text{...(1)}$$

$$\Sigma F_y = 0$$

$$-P + N_1 \sin 10.62° + N_2 \sin 10.62° = 0$$

$$\therefore \quad 0.184 N_1 + 0.184 N_2 = P$$

$$N_1 + N_2 = 5.43\ P \qquad ...(2)$$

$$N_1 + N_1 = 5.43\ P \qquad (\because N_1 = N_2)$$

$$N_1 = 2.713\ P \qquad ...(3)$$

Step 5: FBD of block 'B':

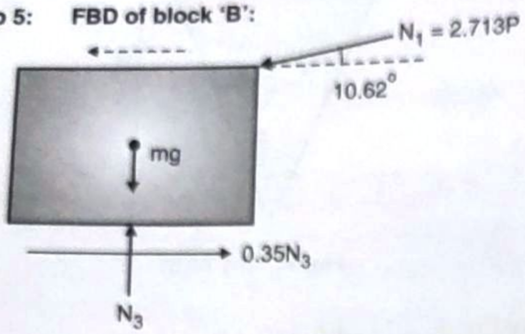


Fig. P. 7.14.24(d)

N_1 = Normal reaction between block and wedge

= 2.713 P

N_3 = Normal reaction from horizontal floor.

mg = 51 × 9.81 = 500.31 N

Step 6 : Equilibrium equations:

For limiting equilibrium of block 'B'

$$\Sigma F_x = 0$$

$$0.35\ N_3 - N_1 \cos 10.62° = 0$$

$$0.35\ N_3 = 2.713\ P \cos 10.62°$$

$$\therefore \quad N_3 = \mathbf{7.62\ P} \qquad ...(4)$$

$$\Sigma F_y = 0$$

$$N_3 - mg - N_1 \sin 10.62° = 0$$

$$\therefore 7.62\ P - 500.31 - 2.713\ P \times \sin 10.62° = 0$$

$$7.12\ P = 500.31$$

$$\mathbf{P = 70.268\ N} \qquad \textbf{...Ans.}$$

Ex. 7.14.25: Block 'A' supports a pipe column and rests on a wedge 'B' as shown in Fig. P. 7.14.25(a). Knowing the coefficient of static friction at all surfaces of contact is 0.25 and that θ = 45°, determine the smallest force 'P' required to raise the block 'A'. **SPPU : Dec. 11, 8 Marks**

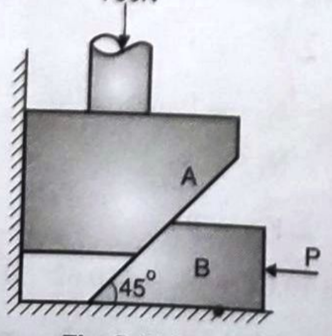


Fig. P. 7.14.25(a)

Soln.:

Step 1: Impending motion :

We have to find smallest force 'P' required to raise the block 'A'.

∴ Impending motion of block A is upwards and block 'B' towards left.

When the force 'P' is smallest to raise the block, block 'B' and 'A' are just on the verge of moving i.e. they are in impending motion.

Step 2: FBD of block 'A'

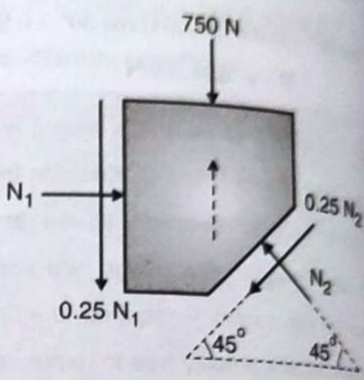


Fig. P. 7.14.25(b)

N_1 = Normal reaction from vertical surface

N_2 = Normal reaction between two blocks

Step 3: Equilibrium equations:

For limiting equilibrium of block 'A'

$$\Sigma F_x = 0$$

$$N_1 - 0.25\ N_2 \cos 45° - N_2 \cos 45° = 0$$

$$N_1 - 0.884\ N_2 = 0$$

$$\therefore \quad N_1 = 0.884\ N_2$$

$$\Sigma F_y = 0$$

$$-750 - 0.25\ N_1 - 0.25\ N_2 \sin 45° + N_2 \sin 45°$$

$$-0.25\ N_1 + 0.53\ N_2 = 750$$

$$-0.25\ (0.884\ N_2) + 0.53\ N_2 = 750$$

$$0.309\ N_2 = 750$$

$$\therefore \quad \mathbf{N_2 = 2427.18\ N}$$

Step 4: FBD of block 'B':

N_3 = Normal reaction from horizontal surface

N_2 = 2427.18 N

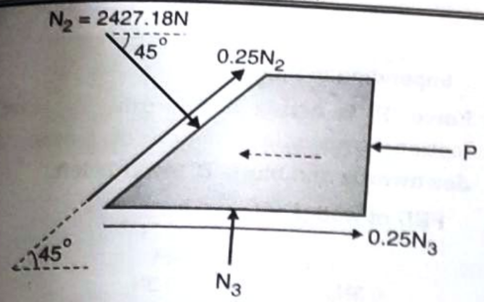


Fig. P. 7.14.25(c)

ep 5: Equilibrium equations:

For limiting equilibrium of block 'B';

$$\Sigma F_x = 0$$

$$N_2 \cos 45° + 0.25\ N_2 \cos 45° + 0.25\ N_3 - P = 0$$

$$427.18 \cos 45° + 0.25 \times 2427.18 \cos 45° + 0.25\ N_3 = P$$

$$2145.34 + 0.25\ N_3 = P \qquad ...(3)$$

$$\Sigma F_y = 0$$

$$-N_2 \sin 45° + 0.25\ N_2 \sin 45° + N_3 = 0$$

$$2427.18 \sin 45° + 0.25 \times 2427.18 \sin 45° + N_3 = 0$$

$$N_3 = 1287.21\ N \qquad ...(4)$$

ep 6: Smallest force 'P':

om Eq^n(4); $N_3 = 1287.21$ N

om Eq^n (3); $2145.34 + 0.25\ (1287.21) = P$

$\therefore$ **P = 2467.14 N** **...Ans.**

7.14.26: A block A weighing 80 kN is to be moved rds left by light wedge B. Find necessary force 'P', if e of friction at all rubbing surfaces is 15°. Refer P. 7.14.26(a). **SPPU : May 07, 10 Marks**

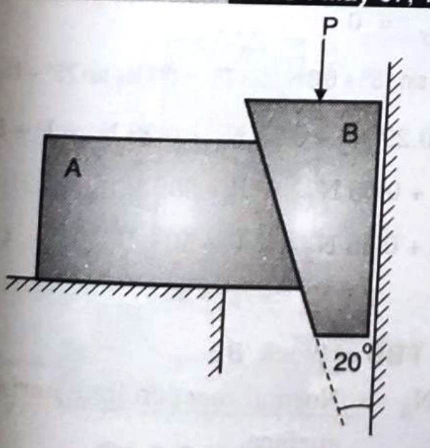


Fig. P. 7.14.26(a)

Soln.:

Step 1: Impending motion :

Block A is to be moved towards left.

$\therefore$ Impending motion of A is towards left and wedge downwards. Accordingly friction forces will act in the opposite direction.

Angle or friction, $\phi = 15°$ coefficient or friction ,

$$\mu = \tan\phi$$

$$= \tan 15°$$

$$= 0.268$$

Step 2: FBD of block 'A'

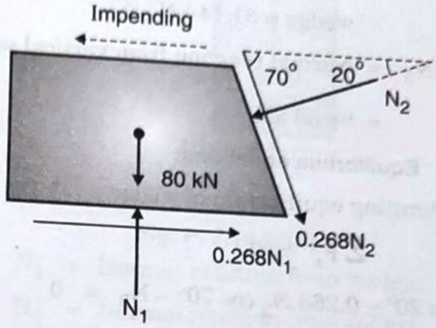


Fig. P. 7.14.26(b)

N_1 = Normal reaction from horizontal surface.

N_2 = Normal reaction from wedge.

Step 3: Equilibrium equations:

For limiting equilibrium;

$$\Sigma F_x = 0$$

$$0.268\ N_1 + 0.268\ N_2 \cos 70° - N_2 \cos 20° = 0$$

$$0.268\ N_1 + 0.092\ N_2 - 0.94\ N_2 = 0$$

$$0.268\ N_1 - 0.85\ N_2 = 0$$

$$\therefore \quad N_1 = 3.163\ N_2 \qquad ...(1)$$

$$\Sigma F_y = 0$$

$$N_1 - 80 - 0.268\ N_2 \sin 70° - N_2 \sin 20° = 0$$

$$N_1 - 0.594\ N_2 = 80$$

$$3.163\ N_2 - 0.594\ N_2 = 80$$

$$\therefore \quad \mathbf{N_2 = 31.14\ kN}$$

$$\therefore \quad \mathbf{N_1 = 3.163\ (31.14)}$$

$$\mathbf{= 98.50\ kN}$$

Step 4: FBD of wedge:

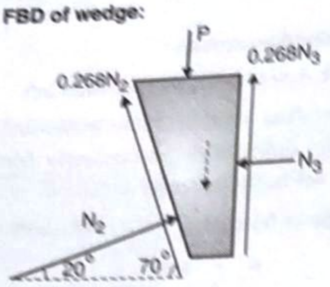


Fig. P. 7.14.26(c)

N_2 = Normal reaction between block and wedge = 31.14 kN

N_3 = Normal reaction from vertical surface = 98.50 kN

Step 5: Equilibrium equations:

For limiting equilibrium of wedge ;

$$\Sigma F_x = 0$$

$$N_2 \cos 20° - 0.268\ N_2 \cos 70° - N_3 = 0$$

$$31.14 \cos 20° - 0.268\ (31.14) \cos 70° = N_3$$

$$\therefore \quad N_3 = 26.41\ kN$$

$$\Sigma F_y = 0$$

$$N_2 \sin 20° + 0.268\ N_2 \sin 70° - P + 0.268\ N_3 = 0$$

$$31.14 \sin 20° + 0.268\ (31.14) \sin 70° + 0.268\ (26.41) = P$$

$$\therefore \quad P = 25.57\ kN \quad \text{...Ans.}$$

Ex. 7.14.27: A wedge A of 50 N is to be driven between inclined plane and block B of 2000 N as shown in Fig. P. 7.14.27(a). The coefficient of friction between all surfaces of contact is 0.30. Determine magnitude of the force 'P' required to start motion of the wedge A.

SPPU : Dec. 10, 9 Marks

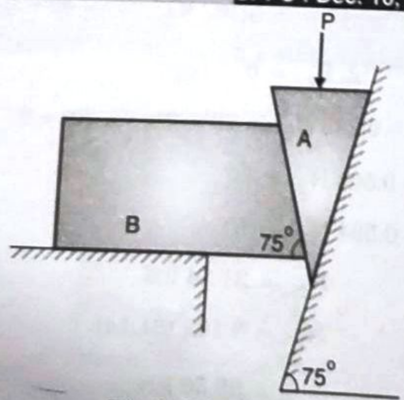


Fig. P. 7.14.27(a)

Soln.:

Step 1: Impending motion :

Force 'P' is acting downwards. To start motion, impending motion of wedge 'A' downwards and block 'B' towards left.

Step 2: FBD of wedge 'A'

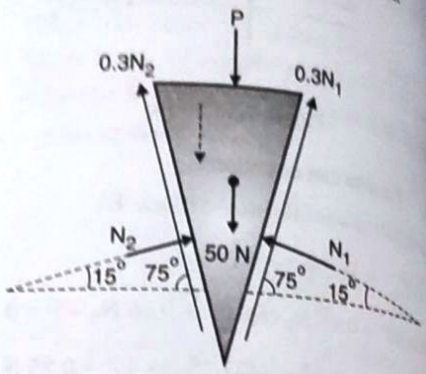


Fig. P. 7.14.27(b)

N_1 = Normal reaction from inclined plane

N_2 = Normal reaction between the wedge 'A' and block 'B'

Weight of wedge = 50 N

μ = 0.3.

Step 3: Equilibrium equations:

For limiting equilibrium of wedge ;

$$\Sigma F_x = 0$$

$$N_2 \cos 15° - 0.3\ N_2 \cos 75° + 0.3\ N_1 \cos 75° - N_1 \cos 15° = 0$$

$$0.96\ N_2 - 0.077\ N_2 + 0.077\ N_1 - 0.96\ N_1 = 0$$

$$0.882\ N_2 - 0.882\ N_1 = 0$$

$$\therefore \quad N_1 = N_2 \quad \text{...(1)}$$

$$\Sigma F_y = 0$$

$$-P - 50 + N_2 \sin 15° + 0.3\ N_2 \sin 75° + 0.3\ N_1 \sin 75° + N_1 \sin 15° = 0$$

$$0.26\ N_2 + 0.29\ N_2 + 0.29\ N_1 + 0.26\ N_1 = P + 50$$

$$0.55\ N_2 + 0.55\ N_1 = P + 50$$

$$0.55\ N_2 + 0.55\ N_2 = P + 50 \quad (\because N_1 = N_2)$$

$$\therefore \quad 1.1\ N_2 = P + 50 \quad \text{...(2)}$$

Step 4: FBD of block 'B':

N_3 = Normal reaction from horizontal surface.

N_2 = Normal reaction between 'A' and 'B'

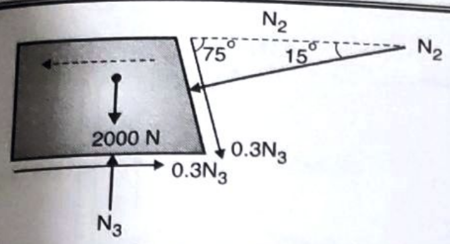


Fig. P. 7.14.27(c)

Step 5: Equilibrium equations:

For limiting equilibrium of wedge

$$\Sigma F_x = 0$$

$$0.3\ N_3 + 0.3\ N_2 \cos 75° - N_2 \cos 15° = 0$$

$$0.3\ N_3 - 0.89\ N_2 = 0 \quad ...(3)$$

$$\Sigma F_y = 0$$

$$-2000 + N_3 - 0.3\ N_2 \sin 75° - N_2 \sin 15° = 0$$

$$N_3 - 0.55\ N_2 = 2000$$

$$N_3 = 0.55\ N_2 + 2000 \quad ...(4)$$

From Eq^n (3);

$$0.3\ [0.55\ N_2 + 2000] - 0.89\ N_2 = 0$$

$$0.165\ N_2 + 600 - 0.89\ N_2 = 0$$

$$0.725\ N_2 = 600$$

$$\therefore \quad \mathbf{N_2 = 827.58\ N} \quad ...(5)$$

Step 6: Force 'P'

From Eq^n (5), $N_2 = 827.58$ N

From Eq^n (2), $1.1\ (827.58) = P + 50$

$$\therefore \quad \mathbf{P = 860.34\ N} \quad \textbf{...Ans.}$$

Ex. 7.14.28: To adjust the vertical position of column supporting a weight of 2000 kN two 5° wedges are used as shown in Fig. P. 7.14.28(a). Determine the force 'P' necessary to start the wedges if the angle of friction at all the surfaces is 25°.(Neglect friction at rollers).

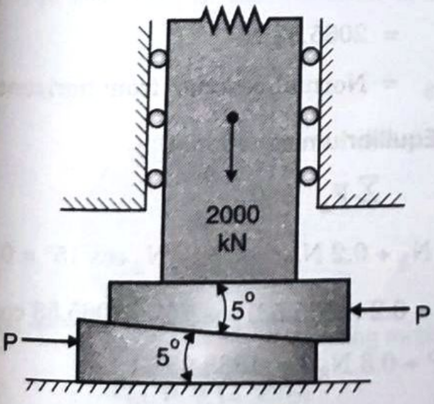


Fig. P. 7.14.28(a)

Soln.:

Step 1: Impending motion

Impending motion of upper wedge is in the direction of for P, i.e. towards left.

$\therefore$ Friction force for wedge is towards right, i.e. in the opposite direction of impending motion. Whereas for the column friction force is towards left.

Step 2: FBD of column

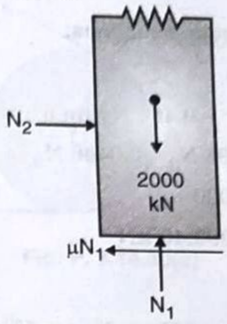


Fig. P. 7.14.28(b)

N_1 = Normal reaction from wedge.

N_2 = Normal reaction from rollers.

ϕ = Angle of friction = 25°

μ = Coefficient of friction

$\mu = \tan\phi = \tan 25° = 0.466$

Note: Normal reaction from rollers on right hand side is zero as there is no force to balance.

Normal reaction from rollers on left hand side i.e. N_2 will be balanced by friction force $\mu\ N_1$.

Step 3: Equilibrium equations:

$$\Sigma F_y = 0$$

$$-2000 + N_1 = 0$$

$$N_1 = 2000\ \text{kN}$$

$$\Sigma F_x = 0$$

$$N_2 - \mu\ N_1 = 0$$

$$N_2 = \mu\ N_1 = 0.466\ (2000)$$

$$= \mathbf{932.61\ kN}$$

Step 4: FBD of upper wedge:

N_3 = Normal reaction from lower wedge.

N_1 = Normal reaction from column

= 2000 kN

μ = 0.466

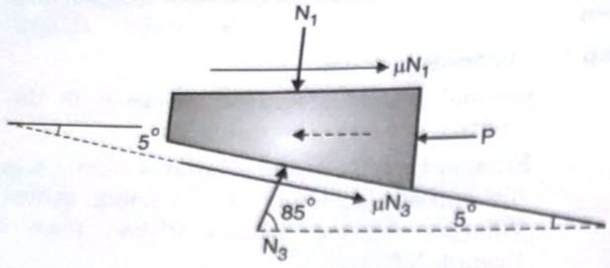


Fig. P. 7.14.28(c)

Step 5: Equilibrium equations:

$$\Sigma F_y = 0$$

$$-N_1 + N_3 \sin 85^\circ - 0.466\, N_3 \sin 5^\circ = 0$$

$$0.996\, N_3 - 0.0406\, N_3 = N_1$$

$$0.955\, N_3 = 2000$$

$$\therefore \quad \mathbf{N_3 = 2093.40\ kN}$$

$$\Sigma F_x = 0$$

$$\mu N_1 - P + N_3 \cos 85^\circ + \mu N_3 \cos 5^\circ = 0$$

$$\therefore P = 0.466\,(2000) + 2093.40 \cos 85^\circ + 0.466\,(2093.40) \cos 5^\circ = 0$$

$$\therefore \quad \mathbf{P = 2086.26\ kN}$$

...**Ans.**

Ex. 7.14.29: Find the minimum horizontal force 'P' to be applied to block A, weighing 500 N so as to keep block B of 1500 N in limiting condition of equilibrium.

SPPU : May 07, 8 Marks

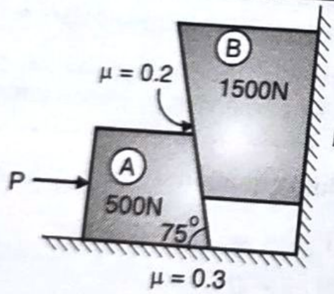


Fig. P. 7.14.29(a)

Soln.:

Step 1: Impending motion

Because of weight of block B, 'B' tends to move downwards and block 'A' towards left.

Step 2: FBD of block 'B'

N_1 = Normal reaction from vertical wall.

N_2 = Normal reaction from blocks 'A'.

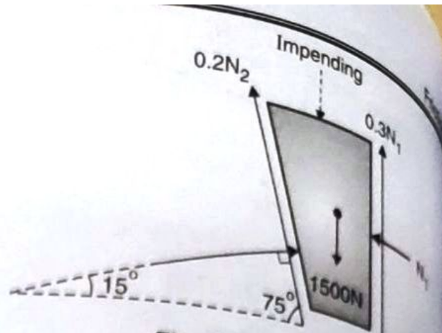


Fig. P. 7.14.29(b)

Step 3: Equilibrium equations:

$$\Sigma F_x = 0$$

$$N_2 \cos 15^\circ - 0.2\, N_2 \cos 75^\circ - N_1 = 0$$

$$\therefore \quad N_1 = 0.914\, N_2$$

$$\Sigma F_y = 0$$

$$N_2 \sin 15^\circ + 0.2\, N_2 \sin 75^\circ - 1500 + 0.3\, N_1 = 0$$

$$0.452\, N_2 + 0.3\, N_1 = 1500$$

$$0.452\, N_2 + 0.3\,(0.914\, N_2) = 1500$$

$$\therefore \quad \mathbf{N_2 = 2065.53\ N}$$

Step 4 : FBD of block 'A':

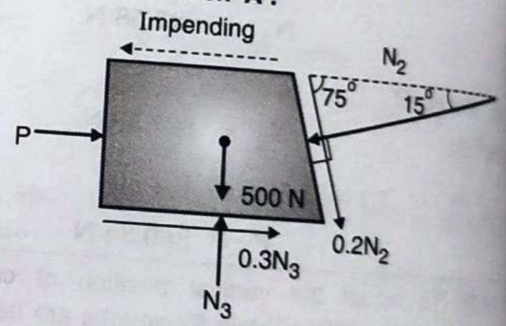


Fig. P. 7.14.29(c)

N_2 = Normal reaction between A and B

$= 2065.53$ N.

N_3 = Normal reaction from horizontal floor

Step 5: Equilibrium equations:

$$\Sigma F_x = 0$$

$$P + 0.3\, N_3 + 0.2\, N_2 \cos 75^\circ - N_2 \cos 15^\circ = 0$$

$$P + 0.3\, N_3 + 0.2\,(2065.53) \cos 75^\circ - 2065.53 \cos 15^\circ = 0$$

$$P + 0.3\, N_3 = 1888.23 \quad ...(3)$$

$$\Sigma F_y = 0$$

$_3 - 500 - 0.2\ N_2 \sin 75^\circ - N_2 \sin 15^\circ = 0$

$_3 = 500 + 0.2\ (2065.53) \sin 75^\circ + 2065.53 \sin 15^\circ$

$N_3 = 1433.63\ N$ ...(4)

n Eq^n (3), $P + 0.3\ (1433.63) = 1888.23$

$P = 1458.14\ N$ ...Ans.

Type 4 : Based on Belt Friction

.14.30 : Determine the maximum tension in the rope at s A and B that is necessary to maintain equilibrium as n in Fig. P. 7.14.30(a). Take $\mu_s = 0.3$ between the rope fixed post D.

SPPU : June 12, April 16, 6 Marks, May 18, 6 Marks

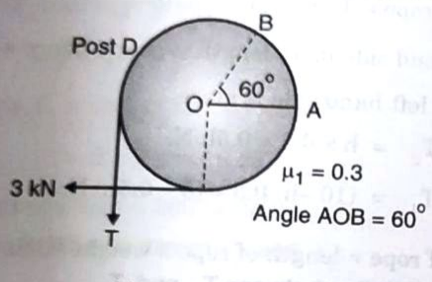


Fig. P. 7.14.30(a)

n. :

p 1 : Impending motion

As maximum tension in the rope is to be found A l B, the impending motion of rope is anticlockwise.

Maximum tension is always in the direction of pending motion of rope.

p 2 : Considering rope up to point 'A'

ght side tension, $T_2 = T_A$

ack side tension, $T_1 = 3$ kN

Tight side is always in the direction of impending tion.

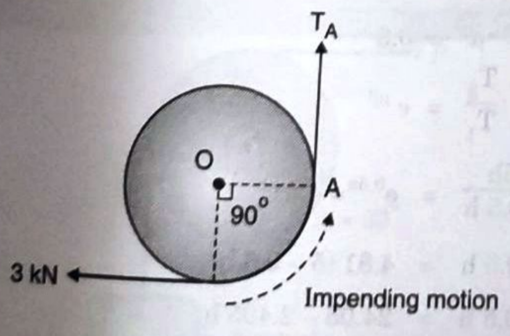


Fig. P. 7.14.30(b)

Lap angle, $\beta = \dfrac{90^\circ \times \pi}{180} = 1.571^c$

Using the relation, $\dfrac{T_2}{T_1} = e^{\mu_s \beta}$ $\mu_s = 0.3$

$$\frac{T_A}{3} = e^{0.3 \times 1.571}$$

$$\therefore T_A = 4.806\ \text{kN}$$ ...Ans.

Step 3 : Considering rope up to point 'B'

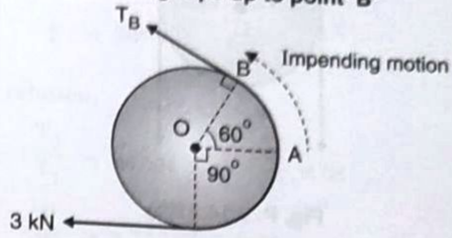


Fig. P. 7.14.30(c)

$$T_2 = T_B$$

$$T_1 = 3\ \text{kN}$$

$$\beta = 90^\circ + 60^\circ = 150^\circ$$

$$= \frac{150^\circ \times \pi}{180^\circ} = 2.618^c$$

$$\mu_s = 0.3$$

Using the relation,

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$$\frac{T_B}{3} = e^{0.3 \times 2.618}$$

$$T_B = 6.58\ \text{kN}$$ ...Ans.

Ex. 7.14.31 : Determine the minimum coefficient of static friction between the rope and the fixed shaft as shown in Fig. P. 7.14.31(a) will prevent the unbalanced cylinder from moving. SPPU : May 15, 5 Marks

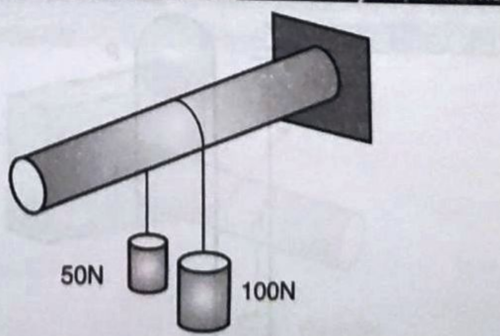


Fig. P. 7.14.31(a)

Soln. :

Step 1 : T_1 and T_2

Tight side tension, $T_2 = 100$ N

Slack side tension, $T_1 = 50$ N

Lap angle, $\beta = 180° = \pi^c$

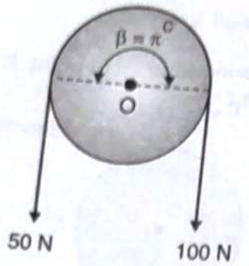


Fig. P. 7.14.31(b)

Step 2 : Relation between T_1 and T_2

Using the relation,

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{100}{50} = e^{\mu\pi}$$

$$e^{\mu\pi} = 2$$

$$\mu\pi = \ln[2] = 0.693$$

$$\therefore \mu = \frac{0.693}{\pi} = 0.22$$

$\therefore$ Coefficient of friction, **μ = 0.22** ... **Ans.**

Ex. 7.14.32 : A cord having a weight of 0.5 N/m and a total length of 10 m is suspended over a peg P as shown in Fig. P. 7.14.32(a). If the coefficient of static friction between the peg and cord is 0.5, determine the longest length h which one side of the suspended cord can have without causing motion. Neglect the size of peg and length of cord draped over it.

SPPU : May 13, 6 Marks

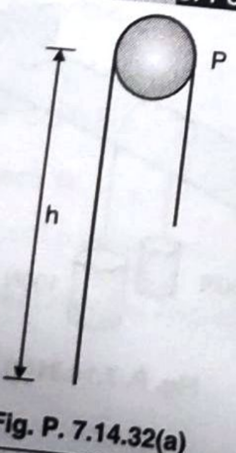


Fig. P. 7.14.32(a)

Soln. :

Step 1 : T_1 and T_2

Total length of the cord = 10 m

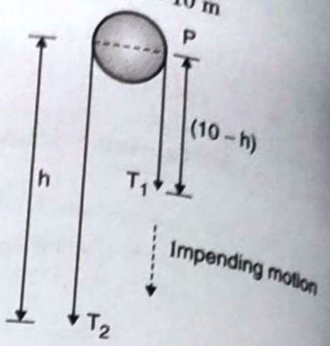


Fig. P. 7.14.32(b)

Weight of rope = Tension in the rope.

On Left hand side more length is there, hence weight of rope on left hand side is more.

$$\therefore T_2 = h \times 0.5 = 0.5h \text{ N.}$$

$$T_1 = (10 - h)\, 0.5 = (5 - 0.5h) \text{ N.}$$

(weight of rope = length of rope × weight / unit length)

Step 2 : Relation between T_1 and T_2

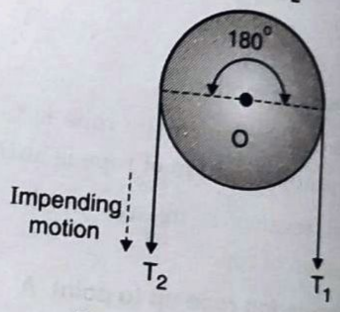


Fig. P. 7.14.32(c)

$$T_2 = 0.5\text{ h}$$

$$T_1 = 5 - 0.5\text{ h}$$

$$\beta = 180° = \pi^C$$

$$\mu = 0.5$$

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{0.5h}{5 - 0.5\text{ h}} = e^{0.5\pi} = 4.81$$

$$0.5\text{ h} = 4.81\,(5 - 0.5\text{ h})$$

$$0.5\text{ h} = 24.05 - 2.405\text{ h}$$

$$2.905\text{ h} = 24.05$$

$$\therefore \textbf{h} = \textbf{8.28 m}$$ ... Ans.

Ex. 7.14.33 : The hawser thrown from ship to a pier is wrapped by two full turns around the capstan as shown in the Fig. P. 7.14.33. If the tension in the hawser is 7500 N and is maintained without slipping by exerting 150 N force on the free end. Determine the coefficient of friction between hawser and capstan. **SPPU : Dec 18, 6 Marks**

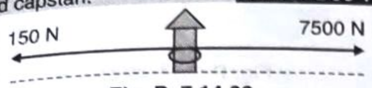


Fig. P. 7.14.33

Soln. :

Step 1 : T_1 **and** T_2

T_1 = 150 N (slack side tension)

T_2 = 7500 N (tight side tension)

T_2 is always greater than T_1

Step 2 : **Lap angle, β :**

$$\text{Lap angle, } \beta = (\text{no. of turns}) \times 2\pi^C = 2 \times 2\pi^C = 12.57^C$$

(One complete turn = 360° = $2\pi^C$)

Step 3 : **Relation between T_1 and T_2:**

Using the relation,

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{7500}{150} = e^{\mu \times 12.57}$$

$$\therefore \; e^{12.57\mu} = 50$$

$$12.57\mu = ln[50] = 3.912$$

$\therefore$ Coefficient of friction, μ = **0.311** ... **Ans.**

Ex. 7.14.34 : Determine the range of P for the equilibrium of block of weight W as shown in Fig. P. 7.14.34(a). The coefficient of friction between rope and pulley is 0.2 **SPPU : Dec 14, 6 Marks**

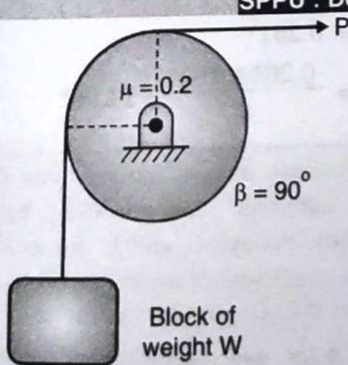


Fig. P. 7.14.34(a)

Soln. :

Step 1 : **For P_{max} :**

For maximum value of P, impending motion of rope is towards left i.e. in the direction of 'P'

$\therefore \; T_2 = P$

T_1 = W (wt. of the block)

$\mu = 0.2$

$$\beta = 90° = \frac{\pi^c}{2} = 1.57^C$$

Using relation,

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{P}{W} = e^{0.2 \times 1.57} = 1.37$$

$\therefore$ **P = 1.37W**

Step 2 : **For P_{min} :**

For minimum value of P, impending motion of rope is in the direction of W.

$\therefore \; T_2 = W \quad T_1 = P$

$\mu = 0.2 \quad \beta = 1.57^C$

Using, $\dfrac{T_2}{T_1} = e^{\mu\beta}$

$$\frac{W}{P} = e^{0.2 \times 1.57} = 1.37$$

$\therefore$ **P = 0.73W**

$\therefore$ Range of values of 'P' for equilibrium of block;

0.73W ≤ P ≤ 1.37W ... **Ans.**

Ex. 7.14.35 : A force $P = \frac{mg}{6}$ is required to lower the cylinder with the cord making 1.25 Turns around the fixed shaft. Determine the coefficient of friction μ_s between the cord and the shaft. Refer Fig. P. 7.14.35(a). **SPPU : Dec 14, 5 Marks**

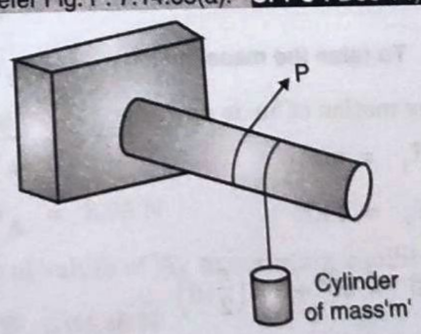


Fig. P. 7.14.35(a)

Soln. :

Step 1 : Impending motion

To lower the cylinder, impending motion of cylinder is down wards

$$\therefore \; T_2 = mg$$

$$T_1 = \frac{mg}{6}$$

$$\text{Lap angle}, \beta = n \times 2\pi^c$$

$$n = \text{no. of turns} = 1.25$$

$$\therefore \beta = 1.25 \times 2\pi = 7.85^c$$

Step 2 : Relation between T_1 and T_2

Using,

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{mg}{\left(\frac{mg}{6}\right)} = e^{\mu 7.85}$$

$$7.85\,\mu = 1.79$$

$$\therefore \mu = \mathbf{0.23}$$

...Ans.

Ex. 7.14.36 : The force 'P' required to raise the mass 'm' is 4 kN and that required to lower it is 1.6 kN. Calculate the mass 'm' and angle 'θ' if $\mu = 0.25$ between the circular pole and the rope.

SPPU : May 04, 8 Marks

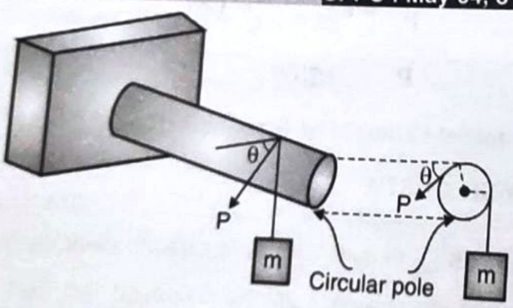


Fig. P. 7.14.36(a)

Soln. :

Step 1 : To raise the mass 'm' :

Impending motion of 'm' is upwards.

$$\therefore \; T_1 = mg$$

$$T_2 = 4 \text{ kN}$$

$$\beta = 90° + \theta = \left(\frac{\pi}{2} + \theta\right)^c$$

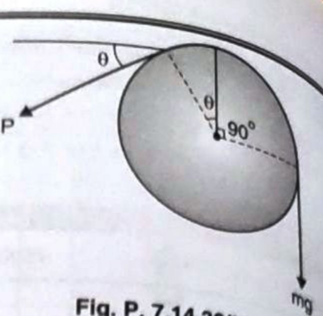


Fig. P. 7.14.36(b)

Using relation,

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{4 \times 10^3}{mg} = e^{\mu\left(\frac{\pi}{2} + \theta\right)}$$

Step 2 : To lower mass 'm'

Impending motion of 'm' is down wards.

$$\therefore \; T_2 = mg$$

$$T_1 = 1.6 \text{ kN}$$

$$\beta = \left(\frac{\pi}{2} + \theta\right)$$

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{mg}{1.6 \times 10^3} = e^{\mu\left(\frac{\pi}{2} + \theta\right)}$$

Step 3: Values of 'm' and 'θ'

$$\frac{\text{Eq}^n\,(1)}{\text{Eq}^n\,(2)} \quad \frac{4 \times 10^3}{mg} \times \frac{1.6 \times 10^3}{mg} = 1$$

$$mg = 2529.82\text{N}$$

$$\mathbf{m = 257.88\ kg}$$

...Ans.

$$\text{From Eq}^n\,(1), \frac{4 \times 10^3}{2529.82} = e^{0.25\left(\frac{\pi}{2} + \theta\right)}$$

$$0.25\left(\frac{\pi}{2} + \theta\right) = 0.458$$

$$\theta = 0.261^C$$

$$\theta = \frac{0.261 \times 180°}{\pi} = \mathbf{14.96°}$$

...Ans.

Ex. 7.14.37 : A cable passes around three 0.05 m radius pulleys and supports two blocks as shown in Fig. P. 7.14.37(a). Pulleys C and E are locked to prevent rotation and the coefficient of friction between the cable and pulleys are $\mu_S = 0.2$. Determine the range of values of the weight of block A for which equilibrium is maintained, if the pulley D is free to rotate.

SPPU : May 11, 7 Marks

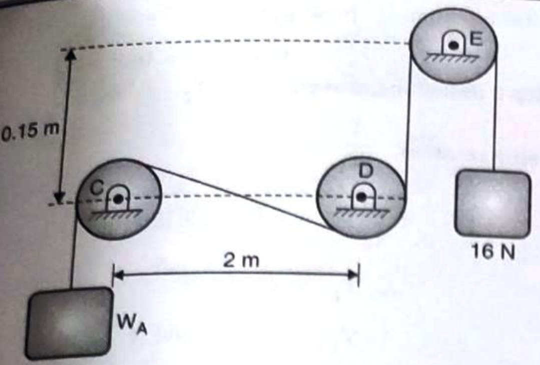


Fig. P. 7.14.37(a)

Soln. :

Step 1 : Lap angle (β) :

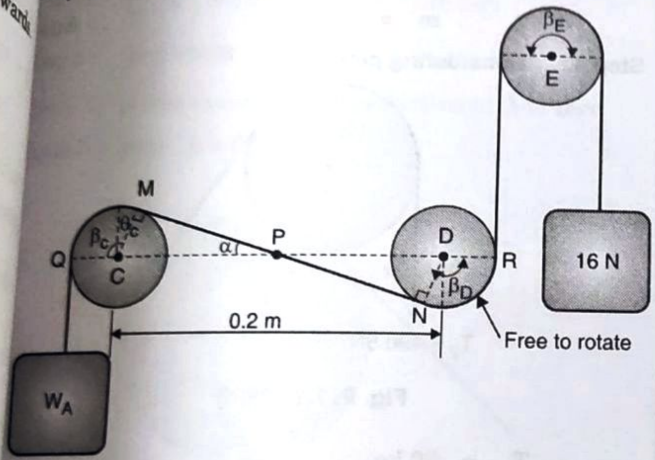


Fig. P. 7.14.37(b)

Total lap angle of the system, $\beta = \beta_c + \beta_E$.

As pulley 'D' is free to rotate, there is no relative motion between the cable and the pulley. Hence friction is zero.

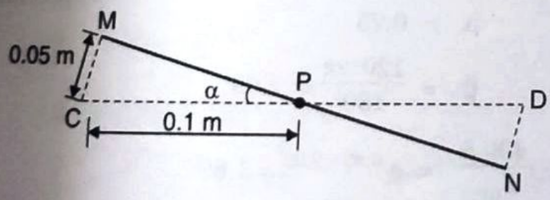


Fig. P. 7.14.37(c)

Considering Δ CMP

$$\tan\alpha = \frac{0.05}{0.1}$$

$$\therefore \alpha = 26.56^\circ$$

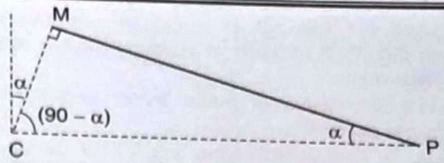


Fig. P. 7.14.37(d)

$$\therefore \alpha = \theta_C = 26.56^\circ$$

$$\therefore \beta_c = 90^\circ + 26.56^\circ = 116.56^\circ$$

$$= \frac{116.56^\circ \times \pi}{180^\circ} = \mathbf{2.034^C}$$

$$\beta_E = 180^\circ = 2\,\pi^C$$

$$\therefore \beta = 2.034 + 2\,\pi = \mathbf{8.32^C}$$

Step 2 : 'W_A' maximum :

For maximum value of 'W_A' impending motion of 'W_A' is downwards.

$$\therefore\ T_2 = W_A T_1 = 16\text{N}$$

$$\beta = 8.32^c \quad \mu = 0.2$$

Using the relation,

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{W_A}{16} = e^{0.2 \times 8.32}$$

$$\therefore\ \mathbf{W_A = 84.48\ N}$$

Step 3: 'W_A' minimum :

For minimum value of 'W_A' impending motion of 16N is downwards.

$$\therefore\ T_2 = 16\ \text{N} T_1 = W_A$$

$$\beta = 8.32^c \mu = 0.2$$

Using the relation,

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{16}{W_A} = e^{0.2 \times 8.32}$$

$$\therefore\ \mathbf{W_A = 3.03\ N}$$

$\therefore$ Range of values of W_A to maintain equilibrium;

$3.03\ N \le W_A \le 84.48\ N$ ...Ans.

Ex. 7.14.38 : A belt ABCD is placed over two pipes as shown in Fig. P. 7.14.38(a) to support a mass of 50 kg at end A. Determine;

(i) The smallest value of mass 'm' at end D, for which equilibrium is possible.

(ii) Tension in portion BC of the belt for the above value of 'm'. **SPPU : Dec 02, 8 Marks**

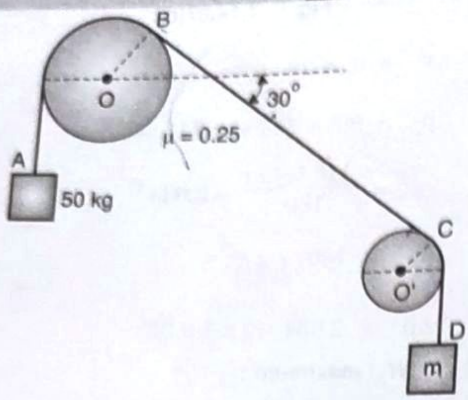


Fig. P. 7.14.38(a)

Soln. :

Step 1 : Impending motion :

For smallest value of 'm', impending motion of 'A' is down wards and 'm' upwards.

$$\therefore T_2 = 50 \times 9.81 = 490.5 \text{ N}$$

$$T_1 = W$$

Step 2 : Lap angle, 'β' (for whole system)

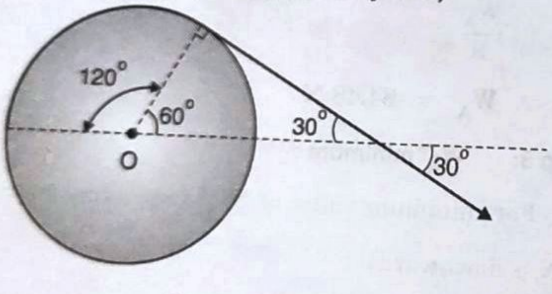


Fig. P. 7.14.38(b)

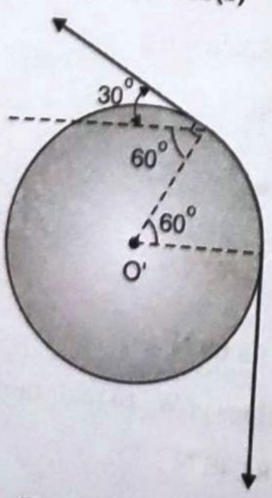


Fig. P. 7.14.38(c)

Total lap angle, $\beta = \beta_B + \beta_C$

$= 120° + 60° = 180° = \pi$

Step 3 : Relation between T_1 and T_2

Using relation, $\frac{T_2}{T_1} = e^{\mu\beta}$

$$\frac{490.5}{T_1} = e^{0.25\pi} = 2.193$$

$$\therefore T_1 = 223.64 \text{ N}$$

$$T_1 = W = 223.64 \text{ N}$$

$$mg = 223.64 \text{ N}$$

$$\therefore m = \frac{223.64}{9.81}$$

$$\therefore \mathbf{m = 22.80 \text{ kg}}$$

Step 4 : Considering pipe 'C':

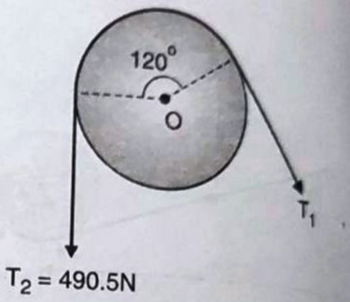


Fig. P. 7.14.38(d)

$$\therefore T_2 = 50 \text{ kg}$$

$$= 50 \times 9.81$$

$$\therefore T_2 = 490.5 \text{ N}$$

$$T_1 = T_{BC}$$

$$\mu = 0.25$$

$$\beta = \frac{120° \times \pi}{180°} = 2.09^c$$

$$\therefore \frac{490.5}{T_{BC}} = e^{0.25 \times 2.09^c} = 1.69$$

$$\mathbf{T_{BC} = 290.56 \text{ N}}$$

Ex. 7.14.39 : The maximum tension 'T' that can be developed in the belt is 500 N. If the pulley at A is free to rotate and coefficient of static friction at the fixed drums B and C is 0.25, determine the largest mass of the cylinder that can be lifted by the belt. **SPPU : Dec. 98, 12 Marks**

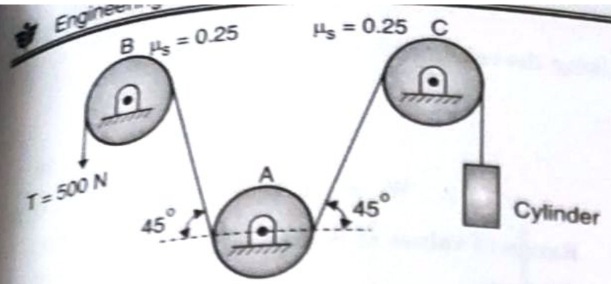


Fig. P. 7.14.39(a)

Soln. :

Step 1 : T_1 and T_2:

Maximum tension in the belt is 500 N

$$T_2 = 500 \text{ N}$$

$$T_1 = W \text{ (weight of the cylinder)}$$

Step 2 : Lap angle, (β) :

Drum 'A' is free to rotate. Hence friction at A is zero

Total lap angle, $\beta = \beta_B + \beta_C$

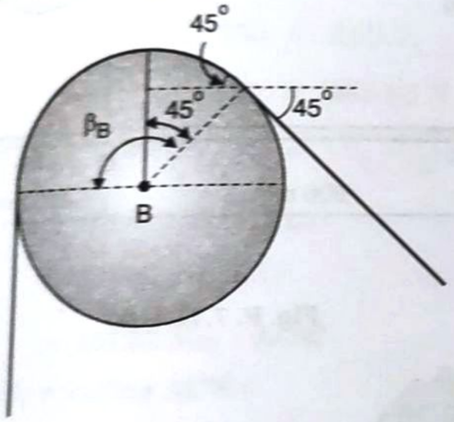


Fig. P. 7.14.39(b)

$$\beta_B = 90° + 45° = 135°$$

Similarly,

$$\beta_c = 135°$$

$$\therefore \beta = 135° + 135° = 270°$$

$$= \frac{270° \times \pi}{180°} = 4.71^c$$

Step 3 : Relation between T_1 and T_2:

Using the relation,

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{500}{W} = e^{0.25 \times 4.71} = 3.25$$

$$\therefore W = \frac{500}{3.25} = 153.93 \text{ N}$$

$\therefore$ Mass of the cylinder, $m = \dfrac{W}{g}$

$$\therefore m = \frac{153.93}{9.81} = \mathbf{15.69\ kg}$$

...Ans.

Ex. 7.14.40 : Determine the range of cylinder weight W as shown in Fig. P. 7.14.40(a) for which the system is in equilibrium. The coefficient of friction between cord and cylindrical support surface is 0.3 and that between 100 N block and the incline surface is zero. **SPPU : May 14, 6 Marks**

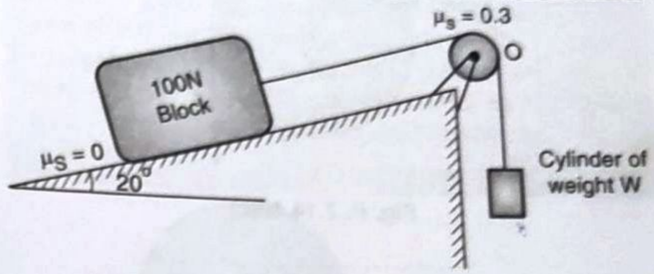


Fig. P. 7.14.40(a)

Soln. :

Step 1 : Impending motion :

For maximum value of 'W', impending motion of W is downwards and block up the plane.

For minimum value of 'W', impending motion of block down the plane and W upwards.

Step 2 : W_{max}

FBD of block :

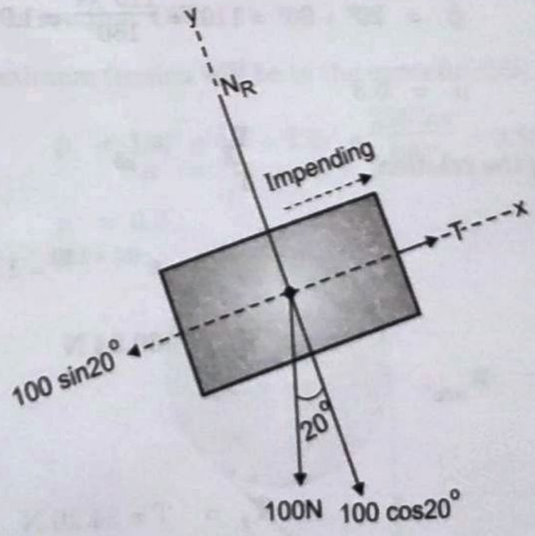


Fig. P. 7.14.40(b)

Let, T = Tension in the cord.

N_R = Normal reaction from the inclined plane.

There is no friction between the block and the plane.

For equilibrium :

$$\sum F_x = 0$$

$$-100 \sin 20° + T = 0$$

$$T = 34.20 \text{ n}$$

FBD of cylindrical support

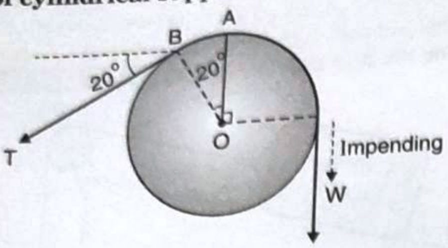


Fig. P. 7.14.40(c)

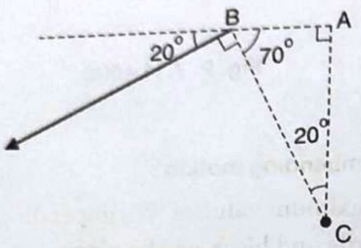


Fig. P. 7.14.40(d)

$$T_2 = W$$

$$T_1 = T = 34.20\text{N}$$

$$\beta = 20° + 90° = 110° = \frac{110° \times \pi}{180°} = 1.92^c$$

$$\mu = 0.3$$

Using the relation, $\dfrac{T_2}{T_1} = e^{\mu\beta}$

$$\frac{W}{34.20} = e^{0.3 \times 1.92} = 1.78$$

$$\therefore W = \mathbf{60.84\ N}$$

Step 3 : W_{min}:

Now,

$$T_2 = T = 34.20 \text{ N}$$

$$T_1 = W$$

$$\beta = 1.92^c$$

$$\mu = 0.3$$

Using the relation, $\dfrac{T_2}{T_1} = e^{\mu\beta}$

$$\frac{34.20}{W} = e^{0.3 \times 1.92} \ldots$$

$$\therefore W = \mathbf{19.21\ N}$$

$\therefore$ Range of values of W for which the system is in equilibrium;

$$\mathbf{19.21\ N \le W \le 60.84\ N}$$

Ex. 7.14.41 : A bord brake passes over drum ... connected to the horizontal lever at points Q and ... shown in fig. The coefficient of friction between the ... the drum is 0.33. A force of 300 N is applied at end ... lever. Calculate the braking torque if the drum is (i) clockwise, (ii) anticlockwise. **SPPU : May 0...**

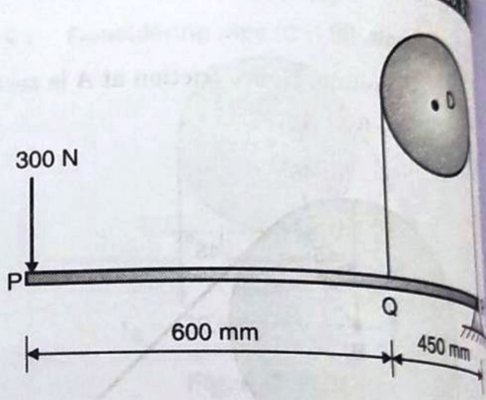


Fig. P. 7.14.41(a)

Soln. :

Step 1 :

(i) Drum is rotating CW:

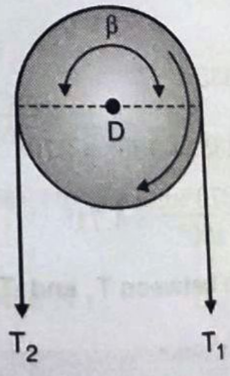


Fig. P. 7.14.41(b)

$$\mu = 0.33 \qquad \beta = \pi^C$$

Using $\dfrac{T_2}{T_1} = e^{\mu\beta}$

$$\frac{T_2}{T_1} = e^{0.33\pi} = 2.82$$

$$\therefore \; T_2 = 2.82\,T_1 \quad ...(1)$$

FBD of lever :

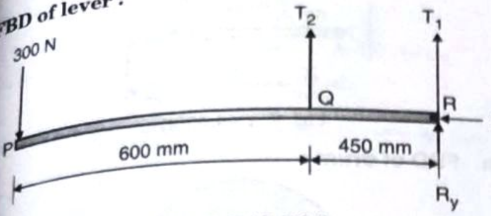


Fig. P. 7.14.41(c)

For equilibrium,

Taking moments at 'R'

$$\sum M_R = 0$$

$$300 \times 1050 - T_2 \times 450 = 0$$

$$\mathbf{T_2 = 700N}$$

$$\text{From Eq}^n\text{ (1)}, 700 = 2.82\,T_1$$

$$\therefore \; \mathbf{T_1 = 248.23\ N}$$

Braking Torque, $T = (T_2 - T_1)\, r$

$= (700 - 248.23)\ 225$

$= 101649$ Nmm ↺

$= \mathbf{101.65\ Nm}$ **ACW** ... **Ans.**

(ii) Drum is rotating ACW :

$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{0.33\pi}$$

$$\therefore \frac{T_2}{T_1} = 2.82$$

$$\therefore \; T_2 = 2.82\,T_1 \quad ...(1)$$

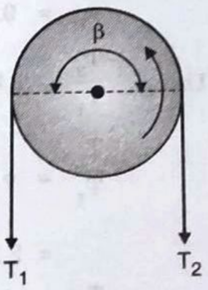


Fig. P. 7.14.41(d)

FBD of lever :

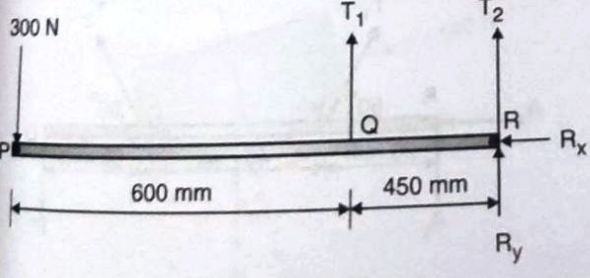


Fig. P. 7.14.41(e)

$$\sum M_R = 0$$

$$300 \times 1050 - T_1 \times 450 = 0$$

$$\mathbf{T_1 = 700\ N}$$

From Eq^n (1), $T_2 = 2.82\,(700) = \mathbf{1974\ N}$

$\therefore$ Braking Torque, $T = (T_2 - T_1)\, r$

$= (1974 - 700)\ 225$

$= 286650$ N mm ↻

$= \mathbf{286.65\ Nm\ CW}$ ... **Ans.**

Ex. 7.14.42 : A pulley of diameter 80 cm is rotating due to torque of 100 Nm clockwise as shown in Fig. P. 7.14.42(a). To apply brake a force P is applied to brake lever ABC at A. The lever is hinged at B. Find force 'P' required it μ between belt and pulley is 0.3. AB = 60 cm and BC = 30 cm.

SPPU : Dec 06, 10 Marks

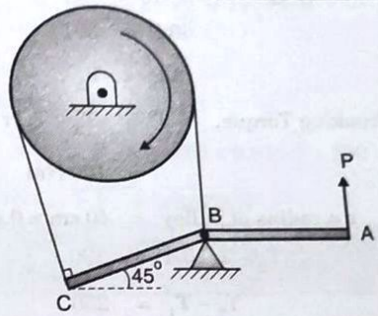


Fig. P. 7.14.42(a)

Soln. :FBD of pulley :

Pulley is rotating CW.

$\therefore$ Maximum tension will be in the opposite side.

$$\beta = 180° + 45° = 225° = \frac{225° \times \pi}{180°} = 3.93^C$$

$$\mu = 0.3$$

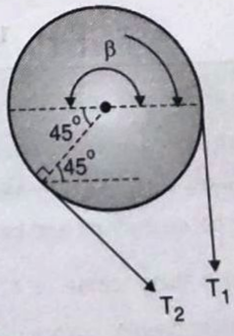


Fig. P. 7.14.42(b)

Using the relation : $\dfrac{T_2}{T_1} = e^{\mu\beta}$

$$\frac{T_2}{T_1} = e^{0.3 \times 3.93} = 3.25$$

$$\therefore \; T_2 = 3.25\, T_1 \qquad \ldots(1)$$

FBD of lever

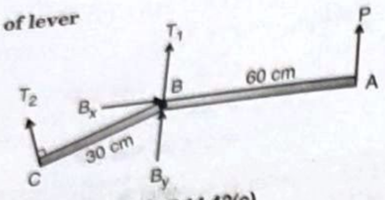


Fig. P. 7.14.42(c)

For equilibrium,

$$\sum M_B = 0$$

$$-T_2 \times 30 + P \times 60 = 0$$

$$60\,P = 30 T_2$$

$$2\,P = T_2 \qquad \ldots(2)$$

Given; Breaking Torque, $T = (T_2 - T_1)\, r$

$$= 100 \text{ Nm}$$

r = radius of pulley $= 40$ cm $= 0.4$m

$$(T_2 - T_1)\, 0.4 = 100$$

$$T_2 - T_1 = 250 \qquad \ldots(3)$$

From Eq^n (1), putting $T_2 = 3.25\, T_1$

$$3.25 T_1 - T_1 = 250$$

$$\therefore \; \mathbf{T_1 = 111.11\ N}$$

From Eq^n (1), $T_2 = 3.25 \times 111.11$

$$= \mathbf{361.11\ N}$$

From Eq^n (2), $2\,P = 361.11$

$$\therefore \; \mathbf{P = 180.55\ N} \quad \textbf{...Ans.}$$

Ex. 7.14.43 : As shown in Fig. P. 7.14.43(a) a flexible and inextensible flat belt placed around a rotating drum of 40 mm radius, acts as a brake when the arm ABCD is pulled down. Assuming $\mu_k = 0.2$ between drum and belt, find the force 'P' that would result in braking torque of 4000 Nmm. Assume that the drum is rotating counter – clock wise.

SPPU : May 05, 8 Marks

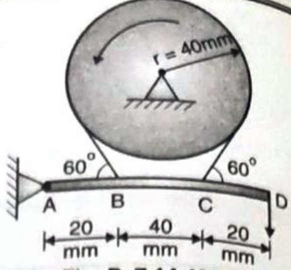


Fig. P. 7.14.43(a)

Soln. :FBD of drum :

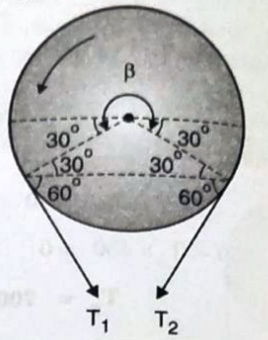


Fig. P. 7.14.43(b)

Drum is rotating ACW direction

$\therefore$ T_2 will be on right hand side.

$$\beta = 180° + 30° + 30° = 240°$$

$$= \frac{240° \times \pi}{180°} = 4.19^C$$

$$\mu_k = 0.2$$

Using, $\dfrac{T_2}{T_1} = e^{\mu_k \beta}$

$$\frac{T_2}{T_1} = e^{0.2 \times 4.19}$$

$$= 2.31$$

$$\therefore \; T_2 = 2.31\, T_1 \qquad \ldots(1)$$

FBD of lever arm :

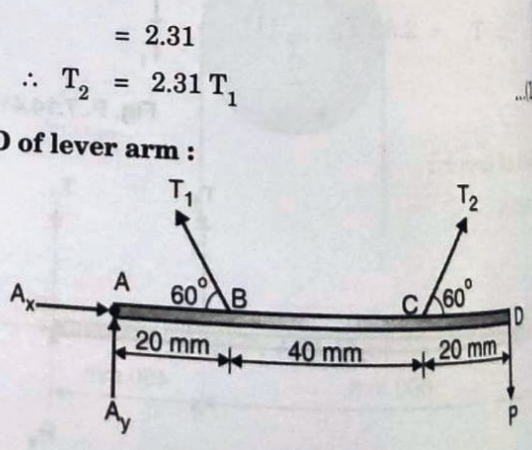


Fig. P. 7.14.43(c)

For equilibrium,

$$\Sigma M_A = 0$$

$$T_1 \sin 60° \times 20 + T_2 \sin 60° \times 60 - P \times 80 = 0$$

$$20\,T_1 + 60 T_2 = 92.37\,P \quad ...(2)$$

Given, that the braking Torque, T = 4000 Nmm

$$\therefore \quad (T_2 - T_1)\, r = 4000$$

$$T_2 - T_1 = \frac{4000}{40} = 100 \; (\because \; r = 40 \text{ mm})$$

$$\therefore \quad T_2 - T_1 = 100 \quad ...(3)$$

From Eq^n (1), $\quad T_2 = 2.31\,T_1$

$$\therefore \quad 2.31 T_1 - T_1 = 100$$

$$\mathbf{T_1 = 76.33\ N}$$

From Eq^n (1), $\quad T_2 = 2.31 \times 76.33 = \mathbf{176.33\ N}$

From Eq^n (2), 20 (76.33) + 60 (176.33) = 92.37 P

$\therefore$ **P = 131.07 N** ...**Ans.**

Ex. 7.14.44 : A 500 N force acting on the 150 kg block resting on the inclination as shown in Fig. P. 7.14.44(a) If the coefficient of static and kinetic friction are 0.25 and 0.20 respectively, state whether the block is in equilibrium or not. Also find the value of the frictional force.

SPPU : May 19, 7 Marks

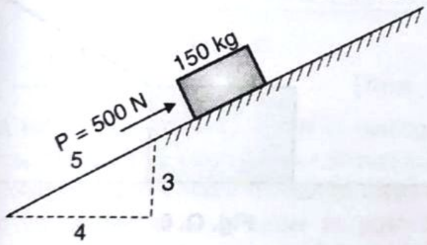


Fig. P. 7.14.44(a)

Soln. :

Step 1 : FBD of block.

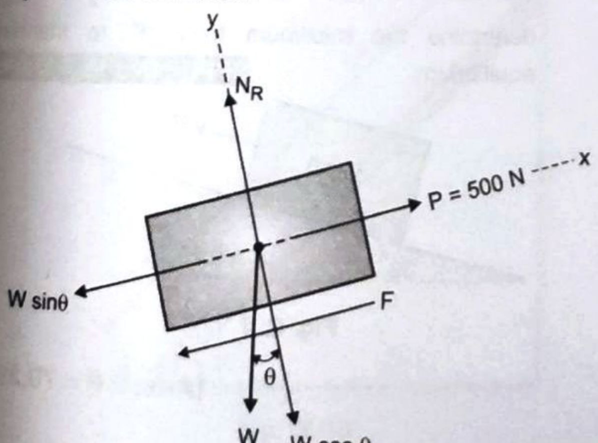


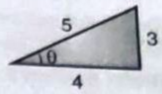
Fig. P. 7.14.44(b)

Let 'F' be the force required to maintain equilibrium.

(Assuming block will move upwards).

$$\cos\theta = \left(\frac{4}{5}\right)$$

$$\sin\theta = \left(\frac{3}{5}\right)$$



Step 2 : To find force 'F'

Writing equations of equilibrium, taking x-axis along the plane and y-axis normal to it.

$$\Sigma F_y = 0$$

$$N_R - W\cos\theta = 0 \quad ...(1)$$

$$N_R = W \cos\theta$$

$$= 150 \times 9.81 \times \frac{4}{5} = 1177.2\text{ N}$$

$$\Sigma F_x = 0$$

$$-W\sin\theta - F + 500 = 0 \quad ...(2)$$

$$\therefore \quad F = -150 \times 9.81 \times \frac{3}{5} + 500$$

$$\therefore \quad F = -382.9\text{ N}$$

Note : As 'F' is negative, the block is moving down the plane.

Step 3 : To check the condition of equilibrium

Limiting friction, $F_L = \mu_s \cdot N_R$

$$\therefore \quad F_L = 0.25 \times 1177.2 = 294.3\text{ N}$$

As $\quad F_L < F \quad$ (i.e. 294.3 N < 382.9 N)

Block is not in equilibrium. ...**Ans.**

Step 4 : Friction force :

As block is in motion, actual friction force is,

$$F = \mu_k N_R$$

$$= 0.2 \times 1177.2$$

$$\mathbf{= 235.44\ N}$$ ...**Ans.**

Theory Questions

Q.1 Explain "Angle of repose" and "Impending motion"
(Refer Sections 7.11 and 7.4) **June 09**

Q.2 Define the terms :
(i) Coefficient of friction (ii) Angle of friction
(iii) Angle of repose (iv) Cone of friction
(Refer Sections 7.5, 7.9, 7.11 and 7.10)
May 97, Dec. 09

Q.3 Prove that for a body placed on rough inclined plane, the angle of repose is same as the angle of limiting friction.
(Refer Section 7.11) **May 02, May 08**

Q.4 Derive the relation between tight side and slack side of the flat belt using usual notations.
(Refer Section 7.13.4) **May 97, Dec 10**

Q.5 Differentiate the terms angle of friction and cone of friction for the friction between two dry surfaces.
(Refer Section 7.10.1) **May 98**

Q.6 Derive the relation $\frac{T_1}{T_2} = e^{\mu\beta}$ used in belt friction theory, symbols having the usual meaning. Draw a neat sketch.
(Refer Section 7.13.4) **May 99**

Q.7 Explain the following with the help of neat sketches
(i) Cone of friction
(ii) Resultant reaction
(iii) Angle of lap
(iv) Angle of limiting friction
(Refer Sections 7.10, 7.9, 7.13.4 and 7.11)
May 94

Practice Problems

Q. 1 Determine the horizontal force P needed to just start moving the 300 N crate up the plane as shown in Fig. Q. 1. Take $\mu_S = 0.1$.
SPPU : May 15, 6 Marks

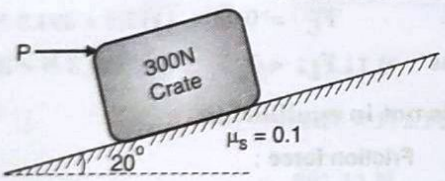


Fig. Q. 1

[Ans : P = 144.12 N]

Q. 2 Determine the range of 'P' for the limiting equilibrium of block of mass 150 kg rest on an inclined plane as shown in Fig. Q. 2.
SPPU : Dec 14, 6 Marks

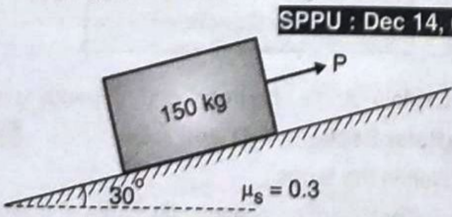


Fig. Q. 2

[Ans. : 353.44 N ≤P ≤1118 N]

Q. 3 Two blocks A and B of weights 1000 N and 20[...] respectively are in equilibrium as show[n in] Fig. Q. 3. If $\mu = 0.3$ between all contact su[rfaces] find the force 'P' required to move the blo[ck ...]

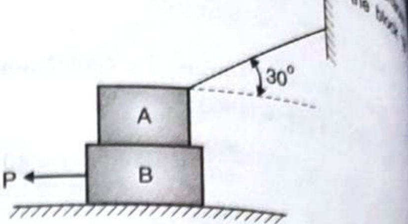


Fig. Q. 3

[Ans. : P = 1.11[...]]

Q. 4 Find the horizontal required to drag a bo[dy of] weight 100 N along a horizontal plane. Angl[e of] repose, θ = 15°.
[Ans. : 26.8[...]]

Q. 5 A body weight 50 N is hauled along a [rough] horizontal plane by a pull of 18 N acting at an a[ngle] of 14° with horizontal. Find the coefficient of fric[tion.]
[Ans. : 0.3[...]]

Q. 6 A body of weight 50 N is travelled along a r[ough] horizontal plane by a pull of 18 N acting at an an[gle] of 14° with the horizontal. Find the coefficien[t of] static friction.
SPPU : May 14, 6 Ma[rks]

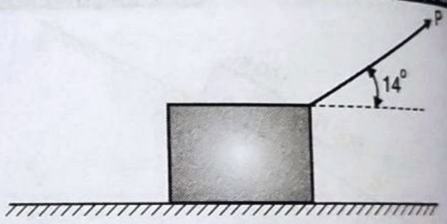


Fig. Q. 6

[Ans. : μ = 0.3[...]]

Q. 7 A block of 10 kg mass rests on an inclined plane as shown in Fig. Q. 7. If the coefficient of static fric[tion] between the block and plane is μ_S = 0.25 determine the maximum force 'P' to main[tain] equilibrium.
SPPU : Dec 16, 6 Ma[rks]

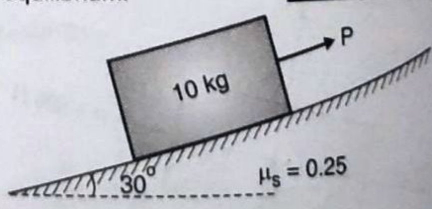


Fig. Q. 7

[Ans. : P = 70.29 N]

Q. 8 Determine the horizontal force P needed to just start moving the 300 N crate up the plane. Take $\mu_S = 0.3$. Refer Fig. Q. 8. **SPPU : Dec. 15, 5 Marks**

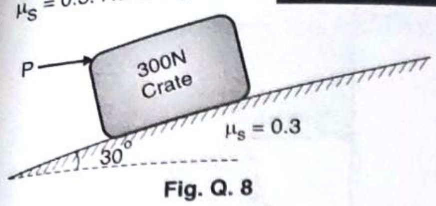


Fig. Q. 8

[Ans. : P = 318.42 N]

Q. 9 The uniform ladder AB has a length of 8 m and a mass 24 kg. End 'A' is on a horizontal floor and end B rests against a vertical wall. A man of mass 60 kg has to climb this ladder. At what position from the base will be induce slipping ? Assume $\mu = 0.34$ at all contact surfaces.

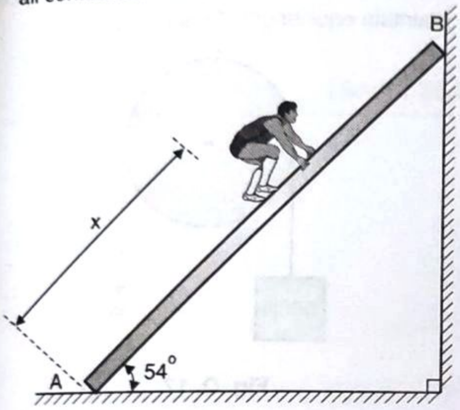


Fig. Q. 9

[Ans. : 4.26 m]

Q. 10 A ladder AB weighing 196 N is resting against a rough wall and a rough floor as shown in Fig. Q. 10. Calculate the minimum horizontal force P required to be applied at C in order to push the ladder towards the wall. Assume coefficient of friction at A $\mu = 0.3$ and at B, $\mu = 0.2$ **SPPU : Dec 16, 6 Marks**

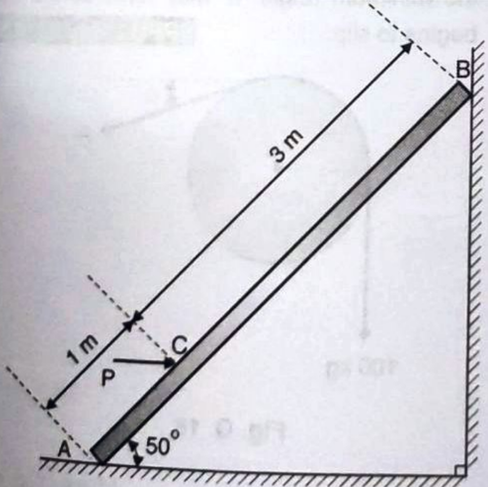


Fig. Q. 10

[Ans. : 238.94 N]

Q. 11 The uniform and rod having a weight 'W' and length 'L' is supported at its ends A and B as shown in Fig. Q. 11. Determine the greatest angle 'θ' so that the rod does not slip. Take $\mu_S = 0.2$.

SPPU : Dec 15, 6 Marks

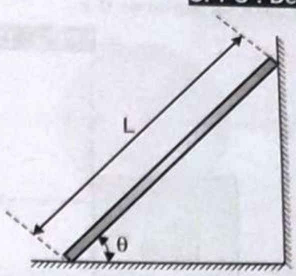


Fig. Q. 11

[Ans. : θ = 67.38°]

Q. 12 The 15 m ladder has uniform weight of 80 N. It rests against smooth vertical wall at B and horizontal floor at A. If $\mu_S = 0.4$ at A, determine the smallest angle θ with vertical wall at which the ladder will slip.

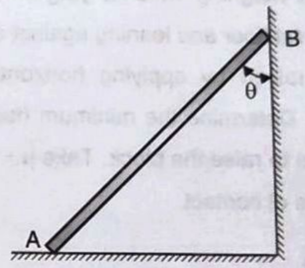


Fig. Q. 12

[Ans. : θ = 38.66°]

Q.13 Determine the distance 'S' to which the 90 kg man can climb without causing the 4 m ladder to slip at its lower end. The top of the 15 kg ladder has a small roller and at the ground the coefficient of static friction is $\mu_s = 0.25$. The mass centre of the man is directly above his feet.

SPPU : May 15, 6 Marks

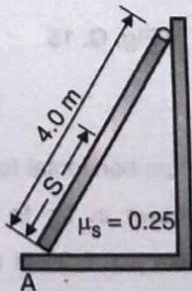


Fig. Q. 13

[Ans. : S = 2.55m]