Concept:

- Percentage
- Decimal to Fraction Conversion
- Magic Cycle

Percentage

A percentage is a way to express a number as a fraction of 100. The term

`percent`

–>

`per centum`

, which means

`by the hundred`

. The symbol

`%`

is used to denote a percentage.
When we want to calculate the percentage of a specific number, we divide that number by the total amount and then multiply the result by 100.This calculation converts the fraction to a percentage.

`Or`

To determine the percentage, we have to divide the actual value by the total value and then multiply the resultant by 100.

Note

Percentage formula = (Actual Value/Total value) × 100

Percentages can also be represented in decimal or fraction form, such as 0.6%, 0.25%, etc

Example: If we have 50 apples out of 100,then the percentage of apples we have? Actual apple = 50 Total apple = 100

Percentage formula = (Actual Value/Total value) × 100 Percentage of apples = ((50/100)*100)% = 50% This means we have 50 percent of the total apples.

Examples on basics of percentage:

Question: If 40% of a number is 20, then find the number.

Solution: Let X be the required number. 40% of a number is 20 (given) (40/100) × X = 20 So, X = (20 × 100) / (40) X = 50

Question: Which number is 50% less than 90?

Solution: Required number = 50% of 90 = (50 x 90)/100 = 45 Therefore, the number 45 is 50% less than 90.

Decimal to Fraction Conversion

Let see how to convert Decimal to Fraction with few examples.

Question: What is 50% of 108?

Solution: 50% = ½ (50% = 50/100 = ½) 50% of 108 = ½ *108 = 108/2 = 54.

Question: What is 33.33% of 99?

Solution: 33.33% = ⅓
33.33% of 99 = ⅓ *99 = 99/3 = 33.

Question: What is 16.66% of 120?

Solution: 16.66% = ⅙ 16.66% of 120 = ⅙ *120 = 120/6 = 20.

Question: What is 37.5% of 160?

Solution: 37.5% = ⅜ 37.5% of 160 = (⅜)x160 = (160x3)/8 = 60.

Magic Cycle

- **Magic Circle of (1/7):**

The

`magic circle of 1/7`

refers to the repeating decimal pattern that occurs when we divide 1 by 7. The result is a recurring decimal with a specific sequence that repeats indefinitely.
When 1 is divided by 7, the result is approximately

`0.142857`

. This decimal repeats indefinitely, so it can be written as 0.142857142857..., and so on.
The magic aspect of this cycle is that the sequence of numbers (142857) keeps repeating in the same order. Additionally, if you multiply this repeating decimal by any integer from 1 to 6, the product will be a rearrangement of the same digits (142857).

For example:

- 2/7 = 28.5714%
- 3/7 = 42.8571%

- 4/7 = 57.1428%
- 5/7 = 71.4285%
- 6/7 = 85.7142%

Each result is a rearrangement of the same digits (

`142857`

), which is a unique property of the fraction

`1/7`

.

Question: What is 28.5714% of 140?

Solution: 28.5714% = 2/7 28.5714% of 140 = (2/7) x140 = 140x(2/7) = 40.

Concepts:

- Percentage Change
- Successive Percentage Change

Percentage Change:

`Percentage change`

is a way to describe how much a value has

`increased`

or

`decreased`

in comparison to its

`original value`

.

It's commonly used to represent changes in prices, costs, or values over time.

Percentage Change comprises into two parts:

- Percentage Increase
- Percentage Decrease

Percentage Increase:

`Percentage increase`

is a measure of how much a quantity has grown compared to its original value, expressed as a Percentage Increase.

where,

Increase in number = New number − original number

Example: Let's say the population of a town was 2,000 people in 2010 and it increased to 2,500 people in 2020.

Original number (2010 population): 2,000 people New number (2020 population): 2,500 people % Increase= [(New number − Original number)/Original number] x 100

% Increase = [(2500-2000)/2000]*100 = 25 % So, the population of the town increased by 25% from 2010 to 2020.

Percentage Decrease:

Percentage decrease

is a measure that shows how much a quantity has reduced compared to its original value, expressed as a Percentage Decrease.

where,

Decrease in number = Original number − New number

Example: Let's consider a scenario where a company's profits decreased from 8000 in one year to 7000 the next year.

Original number (previous year's profits): 8000 New number (current year's profits): 7000

% Decrease = [(Original number - New number)]*100 % Decrease = [(8000 - 7000)/8000]*100 = 12.5%

Successive Percentage Change:

When two or more percentage changes are applied to a quantity consecutively,then the percentage change is called a ''successive percentage change.''

Example: Let's say a product's price is initially 100. First, it increases by 20%, and then it decreases by 10%.

Initial Price: 100 First Change (20% Increase): New Price = Original Price + (Percentage Increase × Original Price) New Price = 100 + (20% × 100) = 100 + 20 = 120 Second Change (10% Decrease on the New Price): New Price = Previous New Price - (Percentage Decrease × Previous New Price) New Price = 120 - (10% × 120) = 120 - 12 = 108 So, after a 20% increase and a 10% decrease, the final price of the product is 108.

Successive Percentage Change for two consecutive intervals:

Successive change for two consecutive intervals involves applying one percentage change after another to a value. The final value is determined by the compounded effect of these changes.

Net % Change or Overall Percentage Change = x + y + (xy/100) Where,

`x,y`

→ Increment or Decrement Percentage

`+ve`

→ Increment

`-ve`

→ Decrement

Question: Price of gold increased by 10% in the first year and decreased 20% in the second year. Find the overall percentage change after 2 years.

Solution: Let, x = +10% (increment) y = -20% (decrement) Overall Percentage change = 10 + (-20) + [10*(-20)]/100 Overall Percentage change = 10 - 20 + (-2) = -12 Hence, After 2 year overall percent change = -12%(Decrement)

Question: Price of petrol increased by 20% in the first year and increased by 20% in the second year. Find the overall percentage change after 2 years.

Solution: Let, x = 20% (increment) y = 20% (increment) Overall Percent change = 20 + (20) + [20*20]/100 Overall Percent change = 20 + 20 + 4 = 44 Hence, After 2 year overall percent change = 44%

Concepts:

- Cost Price
- Profit
- Loss
- Profit Percent
- Loss Percent

Cost Price

This is the amount seller has to pay to buy a product or commodity. It's often abbreviated as

`CP`

.

Selling Price

This is the amount at which a product is sold to a customer. It's often abbreviated as

`SP`

. or This price is what the buyer pays for the item, and it can be different from the cost price.

Difference between selling price and cost price:

Cost Price is the amount at which the

retailer/seller

has bought the product.
Selling Price is the amount at which the

buyer/customer

is willing to purchase that product.

Profit(P)

The amount gained by selling a product for more than its cost price.

Note

Profit = Selling Price(SP) - Cost Price(CP)

Question: Suppose a shopkeeper has bought 1 kg of apples for Rs. 400. And sold it for Rs. 550. How much is the profit gained by him?

Solution: Cost Price for apples is 400. Selling Price for apples is 550. Profit(P) gained by shopkeeper is P = SP – CP P = 550 – 400 = 150

Loss(L)

The amount lost by selling a product for less than its cost price.

Note

Loss = Cost Price(CP) - Selling Price(SP)

Question: John bought a bicycle for 5000 (Cost Price, CP). He later sold the bicycle for 4000 (Selling Price, SP). Calculate the loss John incurred on this sale.

Solution: Cost Price for Bicycle 5000. Selling Price for Bicycle is 4000. loss(L) gained by shopkeeper is L = CP – SP L = 5000 - 4000 = 1000

Profit Percent

Profit percentage is a measure of how much profit you make on a sale relative to the cost price.

Note

Profit Percent(P%) =[(SP - CP)/CP]*100

Question: Emma runs a small jewellery business. She bought a necklace for Rs. 12,000 (Cost Price, CP) and sold it for Rs. 18,000 (Selling Price, SP). Calculate the profit percentage Emma made on this sale.

Solution: CP = 12,000 SP = 18,000 Profit Percent(P%) =[(SP - CP)/CP]*100 Profit Percent(P%) = [(18,000-12,000)/12,000]*100 Profit Percent(P%) = 50%

Loss Percent

Loss percent is a measure of how much loss you incur on a sale relative to the cost price.

Note

Loss Percent(L%) = [(CP - SP)/CP)]*100

Question: Mike owns a small electronics store. He purchased a smartphone for Rs.15,000 (Cost Price, CP) but was only able to sell it for Rs. 12,000 (Selling Price, SP) due to a new model being released. Calculate the loss percent Mike incurred on this sale.

Solution: CP = 15,000 SP = 12,000

Loss Percent(L%) = [(CP - SP)/CP)]*100 Loss Percent(L%) = [(15,000 - 12,000)/15,000]*100 Loss Percent(L%) = 20%

Concepts:

- Marked Price
- Discount
- Discount Percent

Marked Price(MP)

The

Marked Price

is the price a shopkeeper puts on a product, usually so they can offer discounts to customers. This makes it seem like customers are getting a good deal when they buy the product at a lower price than the marked price.

Discount

The

`Discount Value`

is the actual amount by which the price of an item is reduced from its original or marked price.

Note

Discount = Marked Price(MP) - Selling Price(SP)

Question: If a book is priced at Rs. 300 and the store is selling it at price Rs. 275, how much discount do shopkeepers offer on book?

Solution: Marked Price = 300 Selling Price = 275

Discount = Marked Price - Selling Price
Discount = 300 - 275 = 25

Discount Percent(D%)

The

`Discount %`

is the percentage by which the price of an item is reduced.
Discount Percent(D%) = (Discount/Marked Price) x 100 = [(MP - SP)/MP] x 100

Question: A bag originally costs Rs.1000 and is being sold for Rs.800, What is a discount %?

Solution: Marked Price(MP) = 1000 Selling Price(SP) = 800 Discount = MP-SP = 1000 - 800 = 200

Discount % = (Discount/Marked Price) x 100 = (200/1000) x 100 Discount % = 20

Discount percent can be categorized into three main types:

- Up to
- Flat
- Successive

Up to Discount

This kind of discount advertises savings of "up to" a certain percentage.

Example: A sale might say "up to 50% off". This means that some items might have a 50% discount, but others might have less, like 10% or 20%. The key point is that the discount varies across different items, and the maximum discount available is the percentage advertised.

Flat Discount

A flat discount is a straightforward reduction on the price by a specific amount or percentage.

Example: A sign might read "Flat 30% off", which means that all applicable items are being sold at 30% less than their marked prices.

Successive Discount

`Successive discounts`

involve applying multiple discounts one after the other. Successive discounts compound and usually offer greater savings than a single discount of the same total percentage.

Method to calculate two successive discount %: Let a shopkeeper offer successive discounts

`x%`

and

`y%`

on a product. Find the overall discount% or effective discount%.

Note

`Effective Discount% = [(-X) + (-Y) + (-X)*(-Y))/100] Effective Discount% = [- X - Y + (XY))/100]`

Where,

`X`

= First discount

`Y`

= Second discount

`-ve`

sign represents reduction in price.

Question: What is the effective discount on two successive discounts of 40% and 50%?

Solution: The successive discounts are 40% and 50%. Successive discount% = - X - Y + (XY)/100 Successive discount% = - 40 - 50 + 40*50/100 = - 90 + 20 = - 70 -ve sign represents reduction in price. Hence, Effective discount = 70%.

Concepts:

- Simple Interest
- Amount

Simple Interest

Simple Interest is a way to calculate how much

extra money

you will pay or earn on a loan or investment. This extra amount is a percentage of the borrowed money, decided upon an agreement, and is paid every year.

The formula used to calculate the simple interest is:

Note

SI = (P x R x T)/100

Where,

SI

→ Simple Interest

P

→ Principal

R

→ Rate of Interest

T

→ Time (in years)

In simple interest,

- The Principal is the amount borrowed.
- The rate of interest is the percentage charged on that amount.
- Time is the duration for which the money is borrowed.

Amount

Amount refers to the total sum that the borrower needs to repay, which includes the original borrowed money (principal) plus the interest charged over the borrowing period.

Note

Amount(A) = Principal(P) + Simple Interest(SI)

Question: Find the SI and amount on rupees 1000 deposited for 4 years at 6% per annum.

Solution: Given, P = Rupees 1,000 R = 6% per annum T = 4 years

SI = (P x R x T)/100 SI = (1,000×6×4)/100 = 240

Amount = Principal(P) + SI Amount = 1000+240 = 1240 Thus the interest and amount earned is rupees 240 and 1240 in 4 years.

Method to Calculate SI when time duration is in:

Months:

Convert time duration in year by dividing given months with 12. Now, Use Formulae:

Note

SI = (P x R x T)/12*100

Where,

T

is given a time duration in

months

.

Question: Find the SI on rupees 10,000 deposited for 9 months at 5% per annum.

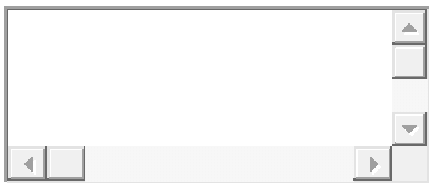
Solution: Given, P = Rupees 10,000 R = 5% per annum T = 9 months

SI = (P x R x T)/12*100 SI = (10,000×5×9)/12*100 = 375

Thus the interest earned is rupees 375 in 9 months.

Days:

Convert time duration in year by dividing given days with 365. Now, Use Formulae:



SI = (P x R x T)/365*100

Where,

T

is given time duration in

days

.

Question: Find the SI on rupees 5,000 deposited for 73 days at 5% per annum.

Solution: Given, P = Rupees 5,000 R = 5% per annum T = 73 days

SI = (P x R x T)/365*100 SI = (5,000×5×73)/365*100 = 50

Thus the interest earned is rupees 50 in 73 days.

Important Concept:

A sum of money becomes

m

times of itself in

a

years at simple interest. In how many years

$"y"$

sum becomes

n

times of itself.

Let , Principal →

P

Rate of Interest →

R

Time →

a

Sum becomes

m

times of itself(P) in

a

years. Means, SI for

a

year = m*P - P = P(m-1) (P x R x a)/100 = P(m-1)

Sum becomes

n

times of itself(P) in

y

years. Means, SI for

y

year = n*P - P = P(n-1) (P x R x y)/100 = P(n-1)

Now,

=>

SI for "a" years

/

SI for "y" years

$= P(m-1) / P(n-1) \Rightarrow \{(P \times R \times a)/100\} / \{(P \times R \times y)/100\} = (m-1) / (n-1) \Rightarrow a/y = (m-1) / (n-1) \Rightarrow$

$y = a(n-1) / (m-1)$

Question: A sum of money becomes 4 times of itself in 9 years at simple interest. In how many years the sum becomes 6 times of itself.

Solution: Let number of years to becomes sum 6 times of itself is "y" A sum of money becomes 4 times of itself in 9 years. m = 4 a = 9 n = 6 y = ?

Formulae used:

a/y = (m-1) / (n-1) 9 / y = (4-1) / (6-1) y = (9*5)/3 = 15 years.

Hence it takes 15 years to become sum 6 times of itself.

Concepts:

- Compound Interest
- Half yearly Compound Interest
- Quarterly Compound Interest
- Difference between Compound Interest and Simple Interest

Compound Interest:

Compound interest is the interest that is calculated against a loan or deposit amount in which interest is calculated for the principal as well as the previous interest earned.

It is denoted by C.I

Note
Compound Interest (C.I) = Amount – Principal

Amount:

Sum of money at the end of the compounding period, which includes both the initial principal and all the interest earned during each compounding interval.

Where, A = Amount P = Principal r = Rate of interest n = Number of times interest is compounded per year t = Time (in years)

We have two method to calculate CI:

Using Simple Interest:

Steps:

- Calculate Simple Interest for One Period: Calculate the Simple Interest for the first period (usually one year) using the formula:

S.I = (P x R x T)/100

- Add Interest to Principal for Next Period: After calculating the Simple Interest for the first period, add this interest to the principal amount. This sum becomes the new principal for the next period.

- Repeat for Each Period: Repeat the process of calculating Simple Interest on the new principal for each subsequent period.

- Total Compound Interest: After the final period, subtract the original principal from the final amount to get the total Compound Interest earned over the entire period.

Question: Aman has deposited Rs. 1,000 into a savings account that earns compound interest at an annual rate of 5%. If the interest is compounded annually, how much money will be in the account after 2 years?

Solution: Principal(P) = 1,000 R = 5% p.a T = 2 years

SI for 1st years = (P x R xT)/100 = (1000x5x1)/100 = 50 Amount for 1^{st} year = 1,000+50 = 1,050

Now, Principal(P') = 1050 R = 5% p.a T = 2 years SI for 2nd year = (P' x R xT)/100 = (1050x5x1)/100 = 52.5 Amount after 2^{nd} year = 1,050+52.5 = 1102.5

Hence, C.I for 2 years = Amount for 2 years - Principal(P) C.I for 2 years = 1102.5 - 1000 = 102.5

Using Formulae:

Where,

CI = compound Interest

P = principal

r = rate of interest

n = number of times interest is compounded per year

t = time (in years)

Question: Aman has deposited Rs. 1,000 into a savings account that earns compound interest at an annual rate of 5%. If the interest is compounded annually, how much money will be in the account after 2 years?

Solution: Principal(P) = 1,000 R = 5% p.a n=1 t = 2 years (compounded annually)

$C.I = P\{1+(r/100)\}^{nt} - P$ $C.I = 1000\{1+(5/100)\}^{(1*2)} - 1000$ $C.I = 1000\{1+(1/20)\}^2 - 1000$ $C.I = 1000\{(21/20)^2\} - 1000$ $C.I = 1102.5 - 1000$ $C.I = 102.5$

Half yearly Compound Interest

If the principal invested be P and the rate of interest is R % per annum which is compounded half-yearly for 't' years

As it is compounded half-yearly, the principal will be changed at the end of 6 months, and interest earned till then will be added to the principal and then this becomes the new principal.

Hence, Rate of Interest(R) becomes half. Time(t) becomes double.

Note

$A = P(1 + R/200)^{2t}$

$CI = A - P$

Question Find the compound interest on Rs. 1000 at the rate of 8 % per annum for 1 year. When interest is compounded half-yearly?

Solution: Principal(P) = 1000 Rate(R) = 8% p.a Time(t) = 1 (2 half years)

$A = P\{1 + R/(2*100)\}^{2t}$

$= 1000\ (1 + 8/200)^2$

$= 1000(26/25)^2$

$= 1081.6$

Compound Interest = A - P = 1081.6 – 1000 = 81.6

Quarterly Compound Interest

If the principal invested be P and the rate of interest is R % per annum which is compounded quarterly for 't' years

As it is compounded quarterly, the principal will be changed at the end of 4 months, and interest earned till then will be added to the principal and then this becomes the new principal.

Hence, Rate of Interest(R) becomes one-fourth of itself. Time(t) becomes four times of itself.

Note

$A = P(1 + R/400)^{4t}$
$CI = A - P$

Question Find the compound interest on Rs. 10000 at the rate of 8 % per annum for half years. When interest is compounded quarter year?

Solution: Principal(P) = 10000 Rate(R) = 8% per annum->2% per quarter year Time(t) = (½) y → (2 quarter years)

$A = P \times \{1 + R/(4*100)\}^{2n}$
$= 10,000 * (1 + 8 / (4x100))^2$
$= 10,000 * (51/50)^2$
= 10,404 Compound Interest = A - P = 10,404 – 10,000 = 404

Difference between Compound Interest and Simple Interest

Compound Interest (CI)	Simple Interest (SI)
CI is the interest that is calculated both on the principal and the previously earned interest.	SI is the interest that is calculated only on the principal.
For the same principle, Rate, and Time period CI > SI	For the same principle,Rate, and Time period SI < CI
$A = P(1 + R/100)^{nT}$	SI = (P×R×T) / 100

Question: Find the difference between Compound Interest and Simple Interest when principal = Rs 8000, rate = 10% per annum and time = 2 years.

Solution: principal = Rs 8000, rate = 10% per annum time = 2 years.

Calculating Compound Interest for 2 years:

Interest for first year = (8000 × 10 × 1)/100 = 800

Amount at the end of first year = 8000 + 800 = 8800

Interest for second year = (8800 × 10 × 1) / 100 = 880

Amount at the end of second year = 8800 + 880 = 9680

Compound Interest = 9680 – 8000 = 1680

Calculating Simple Interest for 2 years:

S.I for 2 years = (8000 × 10 × 2)/100 = 1600

Difference b/w C.I and S.I

= C.I – S.I = 1680 – 1600 = 80

Concepts:

- Time
- Efficiency
- Work
- Rules and Tricks

Time

Duration taken to complete a particular task.

Efficiency

It is defined as work done per unit time. Efficiency inversely proportional to the time taken.

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Efficiency ∝ 1/Time
```

Work

It can be defined as product of time and efficiency.

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Work = Time*Efficiency
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Note

If a piece of work is done in x number of days, then the work done in one day = 1/x

Rules and Tricks

Rule 1:

```
A
```

can do a task in

x days

, and

B

can do it in

y days

. If they work together, In how many days can they complete the task?
Number of days taken by

A

= x Number of days taken by

B

= y
Amount of work done by A in 1 day = 1/x Amount of work done by B in 1 day = 1/y Both A and B can complete part of work in 1 day = 1/x + 1/y Time taken by A and B together to complete the task= xy/(x+y)

Another method:

A

can do a task in

x days

, and

B

can do it in

y days

. If they work together,In how many days can they complete the task?
Number of days taken by

A

= x Number of days taken by

B

= y
Total work can be assumed as LCM(x,y) Efficiency of

A

= LCM/(Days taken by A) = LCM/x Efficiency of

B

= LCM/(Days taken by B) =LCM/y
Time taken by A and B to complete the task = LCM / (Efficiency of A + Efficiency of B)

Question: Aman can do a task in 8 days and Ajeet can do a task in 12 days .If they work together,In how many days can they complete the task?

Solution:

Number of days taken by Aman = 8 Number of days taken by Ajeet = 12

Total work done by Aman and Ajeet= LCM (8,12) = 24 units Efficiency of Aman = LCM/(Days taken by A) = 24/8 = 3 units/day Efficiency of Ajeet = LCM/(Days taken by B) = 24/12 = 2 units/day

Time taken by Aman and Ajeet to complete the task = LCM / (Efficiency of Aman + Efficiency of Ajeet) = 24/(3+2) =(24/5) days **Rule 2:**

A

can do a task in

x days

,

B

can do in

y days

and

C

can do in

z days

. If they work together, In how many days can they complete the task?
Number of days taken by

A

= x Number of days taken by

B

= y Number of days taken by

C

= z
Total work can be assumed as LCM(x,y,z)

Efficiency of

A

= LCM/(Days taken by A) Efficiency of

B

= LCM/(Days taken by B) Efficiency of

C

= LCM/(Days taken by C)
Time taken by A ,B and C to complete the task = LCM / (Efficiency of A + Efficiency of B+ Efficiency of C)

Question: A can do a task in 2 days and B can do a task in 3 days , C can do a task in 6 days .If they work together, In how many days can they complete the task?

Solution:

Number of days taken by A = 2 Number of days taken by B = 3 Number of days taken by C = 6

Total work done by A ,B and C =LCM(2,3,6) = 6 units Efficiency of A = LCM/(Days taken by A) = 6/2 = 3 units/day Efficiency of B = LCM/(Days taken by B) = 6/3 = 2 units/day Efficiency of C =

LCM/(Days taken by C) = 6/6 = 1 unit/day Time taken by A ,B and C to complete the task= LCM / (Efficiency of A + Efficiency of B+ Efficiency of C) = 6/(3+2+1) = 6/6 = 1 day.

Concepts:

- Inlet Pipe
- Outlet Pipe
- Rules and Tricks

Inlet Pipe

A pipe which fill up the tank is known as inlet.

Outlet Pipe

A pipe which empties the tank is known as outlet.

Note
If pipe(A) is filled a tank in x hours, then the part of tank filled in one hour= 1/x

Rules and Tricks:

Rule 1:

If

`pipe(A)`

can fill a tank in

`x hours`

and

`pipe(B)`

can fill it in

`y hours`

. If they are open together to fill the tank, how many hours will it take to fill the tank?

Number of hours taken by

`A`

= x Number of hours taken by

`B`

= y

Parts of tank filled by

A

in 1 hour = 1/x Parts of tank filled by

B

in 1 hour = 1/y Both

A

and

B

together fill the tank in 1 hour = 1/x + 1/y Time taken by

A

and

B

to fill a tank = xy/(x+y)

Another method:

If

pipe(A)

can fill a tank in

x hours

and

pipe(B)

can fill it in

y hours

. If they are opened together,how many hours will it take to fill the tank?

Number of hours taken by

A

= x Number of hours taken by

B

= y

Total capacity can be assumed as LCM(x,y)

Efficiency of

A

= LCM/(hours taken by A) Efficiency of

B

= LCM/(hours taken by B)

Time taken by pipe

A

and

B

to fill the tank = LCM / (Efficiency of A + Efficiency of B)

Question : If pipe A and pipe B can fill a cistern in 40 and 60 hours respectively. Both the pipes are opened together, how long will it take to fill the cistern?

Solution

Number of hours taken by A = 40 Number of hours taken by B = 60

Total capacity of cistern= LCM(40,60) = 120 L (L =liters) Efficiency of A = LCM/(hours taken by A) = 120/40 = 3 L/hr (hr=hour) Efficiency of B = LCM/(hours taken by B) = 120/60 = 2 L/hr

Time taken by pipe A and B to fill the cistern = LCM / (Efficiency of A + Efficiency of B) = 120/(3+2) = 120/5 = 24 hrs (or) 24 hours.

Rule 2:

If

pipe(A)

can fill a tank in

x hours

,

pipe(B)

can fill in

y hours

and

pipe(C)

can empty it in

z hours

. If they are open together to fill the tank, how many hours will it take to fill the tank?

Number of hours taken by

A

= x Number of hours taken by

B

= y Number of hours taken by

C

= -z (- indicate negative work)

Total capacity of the tank = LCM (x,y,z)

Efficiency of

A

= LCM/(hours taken by A) Efficiency of

B

= LCM/(hours taken by B) Efficiency of

C

= LCM/(hours taken by C) Time taken by pipe

A

,

B

and

C

to fill the tank = LCM / (Efficiency of A + Efficiency of B+ Efficiency of C)

Question: Pipes A and B can fill a tank in 12 and 18 hours respectively. Pipe C can empty it in 30 hours. If all three pipes are opened together, then the tank will be filled in how many hours?

Solution:

Number of hours taken by A = 12 Number of hours taken by B = 18 Number of hours taken by C = -30

Total capacity of tank = LCM(12,18,30) = 180 L Efficiency of A = LCM/(hours taken by A) = 180/12 = 15 L/hr Efficiency of B = LCM/(hours taken by B) = 180/18 = 10 L/hr Efficiency of C = LCM/(hours taken by C) = 180/(-30) = -6 L/hr Time taken by A,B and C to fill the tank = LCM / (Efficiency of A + Efficiency of + Efficiency of C) = 180/(15+10-6) = 180/19 hrs (or) 180/19 hours.

Concepts:

- Mixture
- Alligation

Mixture:

When we are mixing or combining two or more different substances then the resultant solution is known as mixture.

Example: Suppose the

Beaker(A)

has 10 liters of milk and the

Beaker(B)

has 5 liters of water. Now, put the milk(10 liter) and water(5 liter) in beaker(c) Then we can say that the resultant substance in beaker C is a mixture of milk and water with a quantity of 15 liters.

Alligation:

Alligation is a rule which is used to solve problems related to mixture and ingredient. It helps us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of desired price.

Alligation Rule :

When two elements are mixed to make a mixture and one of the elements is cheaper and other one is dearer then,

Question: In what ratio must rice(A) at Rs 10/kg be mixed with rice(B) sold at Rs. 12/kg, so that the mixture be worth Rs. 11 per kg?

Solution: Using the rule of alligation, we have:

C.P. of 1 kg of rice(A) = Rs. 10 → cheaper value(c) C.P. of 1 kg of rice(B) = Rs. 12 → dearer value(d) CP of 1 kg of rice(C) = Rs. 11 → mean value(m)

Quantity of Cheaper rice: Quantity of Dearer rice= (d – m)/ (m – c) Quantity of Cheaper rice: Quantity of Dearer rice= (12 – 11)/(11 – 10) Quantity of Cheaper rice: Quantity of Dearer rice= 1:1

Ratio of rice at Rs 10/kg and rice at Rs. 12/kg = 1:1

Question: In what ratio must a grocer mix two varieties of pulses costing Rs.15 /Kg and Rs.20/Kg respectively so as to get a mixture worth Rs.18/Kg.

Solution Using the rule of alligation, we have:

C.P. of 1 kg of pulse(A) = Rs. 15 → cheaper value(c) C.P. of 1 kg of pulse(B) = Rs. 20 → dearer value(d) CP of 1 kg of pulse(C) = Rs. 18 → mean value(m)

Quantity of Cheaper pulse: Quantity of Dearer pulse= (d – m)/ (m – c) Quantity of Cheaper pulse: Quantity of Dearer pulse= (20 – 18)/(18 – 15) Quantity of Cheaper pulse: Quantity of Dearer pulse= 2:3

Ratio of pulse at Rs 15/kg and rice at Rs. 20/kg = 2:3

Question: If a shopkeeper mixes two varieties of sweets costing Rs.400 /Kg and Rs.x /Kg respectively in the ratio of 7:5 so as to get a mixture worth Rs. 650/Kg?

Solution Using the rule of alligation, we have:

C.P. of 1 kg of sweet(A) = Rs. 400 → cheaper value(c) C.P. of 1 kg of sweet(B) = Rs. x → dearer value(d) CP of 1 kg of sweet(C) = Rs. 650 → mean value(m)

Quantity of Cheaper pulse: Quantity of Dearer pulse= (d – m)/ (m – c) Quantity of Cheaper pulse: Quantity of Dearer pulse= (x – 650)/(650 – 400) 7:5 = (x – 650)/(650 – 400) 7/5 = (x – 650)/ 250 250*7 = 5x – 3250 1750 = 5x – 3250 5x = 1750 + 3250 x = 1000 Hence, the value of x is Rs. 1000.

Concepts

Distance

Distance

is the total length of the path traveled by an object. Distance is measured in units like

meters

,

kilometers

,

miles

, etc.

Time

Time

refers to the duration over which an object has been moving to cover a certain distance. It's a fundamental quantity measured in

seconds

,

minutes

,

hours

, etc.

Speed

Speed

is a measure of how fast something is moving. It's calculated as the distance covered per unit of time. The standard unit is

meters per second (m/s)

and

kilometers per hour km/hr

.

Relationship Between Speed, Time and Distance

Note
Speed(S) = Distance(D)/Time(T)

Unit Conversion

Kilometers per hour(km/hr) to Meters per second(m/s)

Note
x km/hr = x*(5/18) m/s

Question: Convert 54 Km/hr to m/s?

Solution: 54 km/hr = 54*(5/18)m/s = 15m/s

Meters per second(m/s) to Kilometers per hour(km/hr)

Note
x m/s = x*(18/5) km/hr

Question: Convert m/s to km/hr?

Solution: 25 m/s = 25*(18/5) = 90km/hr

Average Speed

The average speed is determined by the formula

Note
Average Speed = (Total distance traveled)/(Total time taken)

Question: A truck covers a distance of 900 km in 30 hours and next 500 km in 40 hours. What is the average speed of the truck?

Solution: Total distance = 900 + 500 =1400 Km Total time= 30 + 40 = 70 hr

Speed = Total distance/Total time Speed = 1400/70 km/hr = 20 km/hr

Case 1:

When a body travels at a speeds of x and y for equal distances, The average speed is (2xy)/(x+y)

Question: Raman goes Delhi to Pune at a speed of 40km/h and comes back at a speed of 60 km/h. Find his average speed of the journey.

Solution: let x = 40 km/hr, y = 60 km/hr Average Speed = 2xy/(x+y) = (2x40x60)/(40+60) = 48 km/hr

Case 2:

When the body travels at speeds of x and y for equal time durations, the average speed is (x+y)/2.

Question: When a car travels at a speed of 48 km/hr for an hour and then at a speed of 36 km/hr for the next 1 hour, what is the average speed of the car for the entire journey?

Solution: let x = 48 km/hr , y = 36 km/hr Average Speed = (x+y)/2 = (48+36)/2 = 84/2 = 42 km/hr

Concepts:

- Relative Speed

Relative Speed:

Relative speed

is the speed of one object or body in relation to another object. It tells us how fast one thing is moving compared to another thing.
If two objects are travelling opposite to each other then the relative speed is sum of individual speeds

Note
Speed of object(A) = x km/hr Speed of object(B) = y km/hr Relative Speed = (x+y) km/hr

Question: Two cars, initially 140 kilometers apart, are driving opposite to each other. The first car is traveling at 45 km/h, and the second car is moving at 25 km/h.After how much time will they meet?

Solution: First car(A) travelling at speed = 45 km/hr Second car(B) travelling at speed = 25 km/hr

Relative Speed of A and B = (45+25) km/hr = 70 km/hr Time = Total distance / Relative speed = 140/70 = 2hr

If two objects are travelling in the same direction then the relative speed is difference between individual speeds.

Note

Speed of body(A) = x km/hr Speed of body(B) = y km/hr

When Speed(A) > Speed(B) Relative Speed = (x – y) km/hr

When Speed(B) > Speed(A) Relative Speed = (y – x) km/hr

Question: Two bikes, initially 50 kilometers apart, are driving in the same direction. The bike(A) is traveling at 40 km/h, and the bike(B) is moving at 30 km/h.After how much time will they meet?

Solution: First car(A) travelling at speed = 40km/hr Second car(B) travelling at speed = 30km/hr

Relative Speed of A and B = (40 – 30) km/hr = 10 km/hr Time = Total distance / Relative speed = 50/10 = 5hr