Cyclicity Of Numbers

The cyclicity of any number is mainly focused on its unit digit. Every unit digit has its own repetitive pattern when raised to any power.

The concept of cyclicity of numbers can be learned by figuring out the unit digits of all the single-digit numbers from 0 to 9 when raised to certain powers.

These numbers can be broadly classified into three categories listed as follows:

1. Digits 0, 1, 5, and 6: Here, when each of these digits is raised to any power, the unit digit of the final answer is the number itself.

Examples:

1. $5^2 = 25$: *Unit digit is 5, the number itself.*

2. $1^6 = 1$: *Unit digit is 1, the number itself.*

3. $0^4 = 0$: *Unit digit is 0, the number itself.*

4. $6^3 = 216$: *Unit digit is 6, the number itself.*

Question 1: Find the unit digit of 416^{345}.

Answer: Simply find 6^{345} which will give 6 as a unit digit, hence the unit digit of 416^{345} is 6.

Question 2: Find the unit digit of 235^{34566}.

Answer: Find 5^{34566} which will give 5 as a unit digit, hence the unit digit of 235^{34566} is 5.

2. Digits 4 and 9: Both of these two digits, 4 and 9, have a cyclicity of two different digits as their unit digit.

Examples:

1. $4^2 = 16$: Unit digit is 6.

2. $4^3 = 64$: Unit digit is 4.

3. $4^4 = 256$: Unit digit is 6.

4. $4^5 = 1024$: Unit digit is 4.

5. $9^2 = 81$: Unit digit is 1.

6. $9^3 = 729$: Unit digit is 9.

It can be observed that the unit digits 6 and4 are repeating in an odd-even order. So, 4 has a cyclicity of 2. Similar is the case with 9.

It can be generalized as follows:

- $4^{odd} = 4$: If 4 is raised to the power of an odd number, then the unit digit will be 4.
- $4^{even} = 6$: If 4 is raised to the power of an even number, then the unit digit will be 6.
- $9^{odd} = 9$: If 9 is raised to the power of an odd number, then the unit digit will be 9.
- $9^{even} = 1$: If 9 is raised to the power of an even number, then the unit digit will be 1.

Below are some questions based on the above concept:

Question 1: Find the unit digit of 414^{23}.
Answer: 23 is an odd number, so $4^{odd}=4$, hence the unit digit is 4.
Question 2: Find the unit digit of 29^{82}.
Answer: 82 is an even number, so $9^{even}=1$, hence the unit digit is 1.
3. Digits 2, 3, 7, and 8: These numbers have a cyclicity of four different numbers.
Examples:
1. $2^1 = 2$: Unit digit is 2.
2. $2^2 = 4$: Unit digit is 4.
3. $2^3 = 8$: Unit digit is 8.
4. $2^4 = 16$: Unit digit is 6.
5. $2^5 = 32$: Unit digit is 2.
6. $2^6 = 64$: Unit digit is 4.
It can be observed that the unit digits 2, 4, 8, 6 repeats themselves after a period of four numbers. Similarly,

- The cyclicity of 3 has 4 different numbers: 3, 9, 7, 1.
- The cyclicity of 7 has 4 different numbers: 7, 9, 3, 1.
- The cyclicity of 8 has 4 different numbers: 8, 4, 2, 6.

Below are some questions based on the above concept:
Question 1: Find the unit digit of 257^{345}.
Answer: 345 % 4 = 1, so 7^1, hence the unit digit is 7.
Question 2: Find the unit digit of 423^{43}.
Answer: 43 % 4 = 3, so 3^3, hence 7 is the unit digit.
Question 3: Find the unit digit of 28^{146}.
Answer: 146 % 4 = 2, so 8^2, hence the unit digit is 4.

Cyclicity Table:

Number	Cyclicity	Power Cycle
1	1	1
2	4	2, 4, 8, 6
3	4	3, 9, 7, 1
4	2	4, 6
5	1	5
6	1	6
7	4	7, 9, 3, 1
8	4	8, 4, 2, 6
9	2	9, 1
10	1	0