

Arrays 1D

1) Prefix Sum

2) Kadane's Algo. \rightarrow max subarray Sum.

$$\text{arr}[5] = \begin{array}{|c|c|c|c|c|} \hline 1 & 2 & 10 & -4 & 6 \\ \hline \end{array}$$

$$\text{Prefix_Sum}[5] = \begin{array}{|c|c|c|c|c|} \hline 1 & 3 & 13 & 9 & 15 \\ \hline \end{array}$$

$$\hookrightarrow \text{Pr}[i] \Rightarrow \text{Sum of } \{a[0], \dots, a[i]\}$$

$$\# \quad l=1, r=4 \quad \text{Sum} \Rightarrow \text{Pr}[4] - \text{Pr}[0]$$

arr[3][4]

	0	1	2	3
0				
1				
2				

Q1 Given a matrix of size $n \times m$. Given q queries
 Find the sum of the submatrix. You
 are given Top left & Bottom right
 coordinates.

Submatrix \Rightarrow Part of a matrix

\Rightarrow Defined by two 2 points.

	0	1	2	3
0	2	-1	3	2
1	3	2	6	2
2	10	9	8	2
3	4	-1	2	3
4	3	2	6	9

TL: (2,1)

BR: (4,2)

Sum = 26

Brute force: Traverse through the submatrix
 and find sum for each query.

TC for each query: $O(M \times N)$

$N \leq 10^3$

$M \leq 10^3$

$Q \leq 10^3$

TC: $O(QMN)$

SC: $O(1)$

$PP[n][m] = \text{Prefix Sum matrix}$

$PP[i][j] \Rightarrow$ Submatrix sum where
 $TL \Rightarrow (0,0)$ & $BR = (i,j)$

	0	1	2	3
0	2	-1	3	2
1	3	2	6	2
2	10	9	8	2
3	4	-1	2	3
4	3	2	6	9

$PP[2][1] \Rightarrow 25$

	0	1	2	3
0	2	-1	3	2
1	3	2	6	2
2	10	9	8	2
3	4	-1	2	3
4	3	2	6	9

$PP[3][2]$

$PP[0][1]$

	0	1	2	3
0	2	-1	3	2
1	3	2	6	2
2	10	9	8	2
3	4	-1	2	3
4	3	2	6	9

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						
6						
7						

TL $\Rightarrow (2,2)$

BR $\Rightarrow (5,4)$

$$\text{Sum} \Rightarrow P_j[S][4] -$$

$$P_j[1][4] -$$

$$P_j[S][1] +$$

$$P_j[1][1]$$

	b	b ₂
a ₁		
a ₂		

$$Q = \begin{matrix} \text{TL} & \text{BR} \\ (a_1, b_1) & (a_2, b_2) \end{matrix}$$

$$\text{Sum} = P_j[a_2][b_2] -$$

$$P_j[a_{1-1}][b_2] -$$

$$P_j[a_2][b_{1-1}] +$$

$$P_j[a_{1-1}][b_{1-1}]$$

$a_1 = 0$

TL $\Rightarrow (0, 2)$

BR $\Rightarrow (3, 4)$

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						
6						
7						

if $(a_1 == 0)$
don't need

need to deduct
the top
sub-matrix

or

add top

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						
6						
7						

$b_1 = 0$

TL $\Rightarrow (4, 0)$

BR $\Rightarrow (5, 2)$

ans \Rightarrow pf $[a_2] [b_2]$

pf $[a_{i-1}] [b_2] \Rightarrow$ check if $(a_1 \neq 0)$

pf $[a_2] [b_{i-1}] \Rightarrow$ check if $(b_1 \neq 0)$

$+$
pf $[a_{i-1}] [b_{i-1}] \Rightarrow$ check if
 $(a_1 \neq 0 \& b_1 \neq 0)$

Tc: $O(1)$

Sc: $O(M \times N)$

a_0	b_0	c_0
a_1	b_1	c_1
a_2	b_2	c_2

↓ Column wise Prefix-sum

a_0	$a_0 + b_0$	$a_0 + b_0 + c_0$
$a_0 + a_1$	$a_0 + b_0 + a_1 + b_1$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1$
$a_0 + a_1 + a_2$	$a_0 + b_0 + a_1 + b_1 + a_2 + b_2$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1 + a_2 + b_2 + c_2$

a_0	b_0	c_0
$a_0 + a_1$	$b_0 + b_1$	$c_0 + c_1$
$a_0 + a_1 + a_2$	$b_0 + b_1 + b_2$	$c_0 + c_1 + c_2$

→

a_0	$a_0 + b_0$	$a_0 + b_0 + c_0$
$a_0 + a_1$	$a_0 + b_0 + a_1 + b_1$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1$
$a_0 + a_1 + a_2$	$a_0 + b_0 + a_1 + b_1 + a_2 + b_2$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1 + a_2 + b_2 + c_2$

↓ Row wise prefix Sum

TC: Column wise Prefix Sum + Row-wise Prefix sum

$$O(m \times n) + O(n \times m) = O(mn)$$

TC of Original Ques $\Rightarrow O(mn + 9)$

Q2 Given a matrix $[N \times M]$, find max submatrix sum, where submatrix starts from row=0 & ends at row=n-1

	0	1	2	3	4	5
0	3	2	3	4	-6	4
1	5	5	-5	2	2	-7
2	-4	-3	1	-2	1	4

↓

-2	4	-1	5	-3	1
----	---	----	---	----	---

⇒ Apply Kadane's

⇒ 8

Tc: $O(m \times n)$

Sc: $O(m)$

Q2 Given a matrix $[N \times M]$, find max submatrix sum, where submatrix ~~starts~~ starts from row=0 & end anywhere.

	0	1	2	3	4	5
0	2	-4	1	3	-1	2
1	1	3	2	-7	3	3
2	0	-1	1	3	4	-7
3	1	-2	-6	4	-4	6

Start end

row 0 0

2	-4	1	3	-1	2
---	----	---	---	----	---

= 5

row 0 1

3	-1	3	-4	2	5
---	----	---	----	---	---

⇒ 8

row 0 2

3	-2	4	-1	6	-2
---	----	---	----	---	----

⇒ 10

row 0 3

4	-3	-2	3	2	4
---	----	----	---	---	---

⇒ 9

Pseudo

$s \Rightarrow 0$

$Sum[m] \Rightarrow 0$; $max_sum = Integer.Min;$

for (int end = 0; end < n; end++) {

$O(m)$ for (int j = 0; j < m; j++) {

$Sum[j] = Sum[j] + arr[end][j];$

}

$O(m)$ \Leftarrow int curr_sum \Leftarrow Kadane(Sum, m);

$max_sum \Rightarrow \max(max_sum, curr_sum);$

}

return max_sum;

$Tc: (nm)$

$Sc: O(m)$

Q2 Given a matrix $[N \times M]$, find max submatrix sum.

	0	1	2	3	4	5
0	2	-4	1	3	-1	2
1	1	3	2	-7	3	3
2	0	-1	1	3	4	-7
3	1	-2	-6	4	-4	6

Start

end

1

1

1	3	2	-7	3	3
---	---	---	----	---	---

$\Rightarrow 6$

1

2

1	2	3	-4	7	-4
---	---	---	----	---	----

$\Rightarrow 9$

1

3

Pseudo

max_sum = Integer.MIN;

for (int s = 0; s < n; s++) {

Sum [m] \Rightarrow 0;

for (int end = s; end < n; end++) {

$O(m)$ for (int j = 0; j < m; j++) {

} Sum [j] = Sum [j] + arr[end] [j];

$O(m)$ { int curr_sum \Rightarrow kadane (Sum, m);
max_sum \Rightarrow max (max_sum, curr_sum);

}

return max_sum;

TC: $O(n \times n \times m) = O(n^2 m)$

SC: $O(m)$

Double

#

0	1	2	3	4	5	6	7
2	3	-6	4	5	1	1	

$$l = 3$$

$$r = 4$$

$$\text{Sum} = 4$$

$$s = 3$$