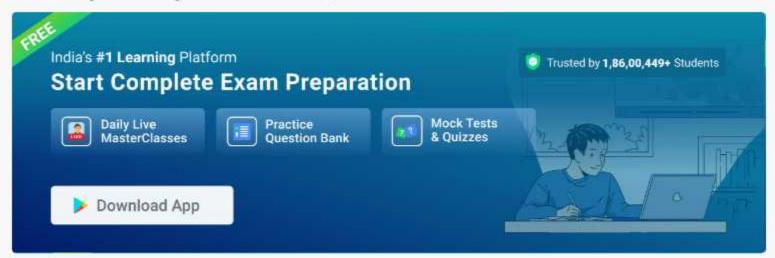
# State Space Representation Questions



#### MCQ Question 1

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A discrete system is represented by the difference equation 
$$\begin{bmatrix} X_1 \left(k+1\right) \\ X_2 \left(k+1\right) \end{bmatrix} = \begin{bmatrix} a & a-1 \\ a+1 & a \end{bmatrix} \begin{bmatrix} X_1 \left(k\right) \\ X_2 \left(k\right) \end{bmatrix}$$

It has initial conditions  $X_1(0) = 1$ ;  $X_2(0) = 0$ . The pole locations of the system for a = 1, are

1 ± j0

2. -1 ± j0

3.  $\pm 1 + j0$ 

4. 0 ± j1

# Answer (Detailed Solution Below)

Option 1:1 ± j0

# State Space Representation MCQ Question 1 Detailed Solution

## Concept:

The poles of the system are the roots of the characteristic equation.

The characteristic equation is: |z| - A| = 0

#### Calculation:

From the given representation, the matrix A

$$A = \begin{bmatrix} a & a-1 \\ a+1 & a \end{bmatrix}$$

$$zI-A=egin{bmatrix} z & 0 \ 0 & z \end{bmatrix}-egin{bmatrix} a & a-1 \ a+1 & a \end{bmatrix}$$

$$= \begin{bmatrix} z-a & -a+1 \\ -a-1 & z-a \end{bmatrix}$$

For a = 1,

Characteristic equation: |zl - A| = 0

$$\Rightarrow$$
  $(z-1)^2 = 0$ 

$$\Rightarrow$$
 z = 1 + i0

The roots of characteristic equation gives the system poles.



## MCQ Question 2

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A second-order linear time-invariant system is described by the following state equations  $\frac{d}{dt}x_1\left(t\right)+2x_1\left(t\right)=3u\left(t\right)$   $\frac{d}{dt}x_2\left(t\right)+x_2\left(t\right)=u\left(t\right)$ 

where x1(t) and x2(t) are the two-state variables and u(t) denotes the input. If the output  $c(t) = x_1(t)$ , then the system is:

- 1. controllable but not observable
- 2. observable but not controllable
- 3. both controllable and observable
- neither controllable nor observable

## Answer (Detailed Solution Below)

Option 1: controllable but not observable

## State Space Representation MCQ Question 2 Detailed Solution

## Concept:

## Controllability:

A system is said to be controllable if it is possible to transfer the system state from any initial state  $x(t_0)$  to any desired state x(t) in a specified finite time interval by a control vector u(t)

## Kalman's test for controllability:

$$\dot{x} = Ax + Bu$$

$$Q_c = [B AB A^2B ... A^{n-1} B]$$

Qc = controllability matrix

If |Qc| = 0, system is not controllable

If |Q<sub>c</sub>|≠ 0, system is controllable

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## Observability:

A system is said to be observable if every state  $x(t_0)$  can be completely identified by measurement of output y(t) over a finite time interval.

## Kalman's test for observability:

$$Q_0 = [C^T A^T C^T (A^T)^2 C^T .... (A^T)^{n-1} C^T]$$

Q<sub>0</sub> = observability testing matrix

If  $|Q_0| = 0$ , system is not observable

If  $|Q_0| \neq 0$ , system is observable.

# Application:

$$_{\mathsf{Given}}\,\dot{\mathtt{x}}_{1}=-2\mathtt{x}_{1}+3\mathrm{U}$$

$$\dot{x}_{2}=-x_{2}+U_{\text{ and }}c\left( t\right) =x_{1}$$

$$_{..}\,A=\begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Controllability matrix Q<sub>c</sub> = [B AB]

$$\begin{bmatrix} 3 & -6 \\ 1 & -1 \end{bmatrix}$$

$$|Q_c| = (-3) - (-6) = 6 - 3 = 3 \neq 0$$

:. System is controllable

Observability matrix  $Q_0 = [C^T A^TC^T]$ , i.e.

$$Q_0 = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$$

 $|Q_0|=0$  = not observable

:. The system is controllable but not observable



## MCQ Question 3

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The value of A matrix in  ${}^{\displaystyle X=AX}$  for the system described by  ${}^{\displaystyle y'}+2y'+3y=0$ 

1. 
$$\begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}$$

3. 
$$\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$4. \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}$$

# Answer (Detailed Solution Below)

Option 4 : 
$$\begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}$$

## State Space Representation MCQ Question 3 Detailed Solution

## Concept:

## State-space representation:

State equation:  $\dot{X}\left(t\right)=A_{n imes n}X\left(t\right)+B_{n imes n}U\left(t\right)$ 

Output equation:  $Y\left(t\right) = C_{q \times n} X\left(t\right) + D_{q \times p} U\left(t\right)$ 

If the transfer function is in the form of

$$TF = \frac{b[C_n s^n + C_{n-1} s^{n-1} + \dots + C_1 s + C_0]}{s^{n+1} + a_n s^n + \dots + a_1 s + a_0}$$

Then, the above transfer function can be represented as:

#### Controllable Canonical Form:

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_n \end{bmatrix} \begin{bmatrix} b \\ b \end{bmatrix}$$

$$C = \begin{bmatrix} C_0 & C_1 & C_2 & \dots & C_n \end{bmatrix}$$

#### Observable Canonical Form:

$$C = [0 \quad 0 \quad 0 \quad \dots \quad b]$$

## Application:

The given system is y'+2y'+3y=0

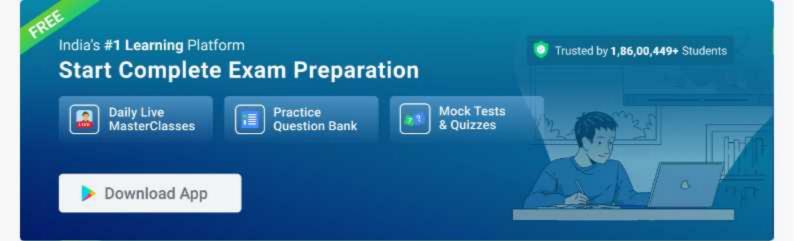
By applying the Laplace transform, we get

$$s^2 Y(s) + 2s Y(s) + 3 Y(s) = 1$$
  

$$\Rightarrow Y(s) = \frac{1}{s^2+2s+3}$$

So, the state space representation for the above transfer function is:

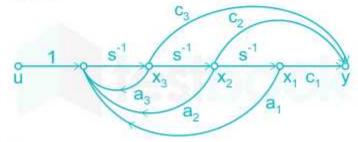
$$A = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$



#### MCQ Question 4

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Consider the state space system expressed by the signal flow diagram shown in the figure.



# The corresponding system is

- 1. always controllable
- 2. always observable
- 3. always stable
- 4. always unstable

# Answer (Detailed Solution Below)

Option 1: always controllable

# State Space Representation MCQ Question 4 Detailed Solution

# Concept:



Where v = differentiation of v<sub>1</sub>

Thoron annormation of m

#### Analysis:

The state equation and output equation as:

State equation:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = a_1 x_1 + a_2 x_2 + a_3 x_3 + u$$

Output equation:

$$y = c_1 x_1 + c_2 x_2 + c_3 x_3$$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_1 & a_2 & a_3 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} u$$

$$y = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Here,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_1 & a_2 & a_3 \end{bmatrix} \text{ is state matrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ is input matrix}$$

And  $C = [c_1 \ c_2 \ c_3]$  is the output matrix.

Now,

After state modeling the system, let's check for controllability & observability.

#### Kalman's Test:

(i) For controllability:

$$Q_c = [B AB A^2B ...]$$

If  $|Q_c| \neq 0 \Rightarrow$  Controllable

= 0 ⇒ Uncontrollable

$$_{\left( \text{ii}\right) }Q_{0}=\left| \begin{array}{c} C\\CA\\[1mm] CA2 \end{array} \right|$$

 $|Q_0| \neq 0 \Rightarrow Observable$ 

= 0 ⇒ Unobservable

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
 
$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 \\ 1 \\ \lfloor a_3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 & 1 \\ a_1 & a_2 & a_3 \\ a_1a_3 & a_1 + a_2a_3 & a_2 + a_3^2 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0 & 0 & 1 & 0 \\ a_1 & a_2 & a_3 & 0 \\ a_1a_3 & a_1 + a_2a_3 & a_2 + a_3^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 1 \\ a_3 \\ \left\lfloor a_2 + a_3^2 \right\rfloor \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & a_3 \\ 1 & a_3 & a_2 + a_3^2 \end{bmatrix}$$

$$|Q_c| = 0 = 0 + 1 \, (0 - 1)$$

$$|Q_c| = -1$$

Since  $|Q_c| \neq 0$ , hence our system is always controllable.

Since it is MCQ type Question we need not check other options. Option A is correct.

But let's check for the observability.

$$Q_0 = egin{bmatrix} C & C \ CA \end{bmatrix}$$

$$CA = \begin{bmatrix} C_1 & C_2 & C_3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_1 & a_2 & a_3 \end{bmatrix}$$

 $CA = [a_1c_3 c_1 + a_2c_3 c_2 + c_3a_3]$ 

$$CA^2 = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & a_2 & a_3 \\ a_1 a_3 & a_1 + a_2 a_3 & a_1 + a_2^2 \end{bmatrix}$$

$$CA^2 = \begin{bmatrix} a_1 a_3 c_3 & a_2 c_2 + a_1 c_3 + a_2 a_3 c_3 & c_1 + c_2 a_3 + a_2 c_3 + a_3^2 c_3 \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} C_1 & c_2 & c_3 \\ a_1c_3 & c_1 + a_2c_3 & c_2 + c_3a_3 \\ a_1a_3c_3 & a_2c_2 + a_1c_3 + a_2a_3c_3 & c_1 + c_2a_3 + a_2c_3 + a_3^2c_3 \end{bmatrix}$$

Since we do not know about the nature of  $a_1$ ,  $a_2$ ,  $a_3$ , and  $c_1$ ,  $c_2$ ,  $c_3$  whether they are positive or negative numbers we cannot comment on  $|Q_0|$  & hence we cannot comment on observability. Similar is the case for stability since for stable system all the roots of characteristic equation must lie in the left half of s-plane characteristic equation: |SI - A| = 0

$$[SI-A] = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_1 & a_2 & a_3 \end{bmatrix}$$

$$=\begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ -a_1 & -a_2 & s-a_3 \end{bmatrix}$$

$$|SI - A| = S(s^2 - a_3s - a_2) + (-a_1)$$

$$|SI - A| = s^3 - a_3 s^2 - a_2 s - q_1 = 0$$

Now, again we cannot comment on stability because we do not know the nature of  $a_1$ ,  $a_2$  &  $a_3$  Hence, only option A is correct.



## MCQ Question 5

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Let 
$$X' = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ 0 & 2 \end{bmatrix} X + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} U$$

U = [b, 0] X

Where b is an unknown constant. This system is

- 1. Observable for all values of b
- 2. Unobservable for all values of b
- 3. Observable for all non-zero values of b
- 4. Unobservable for all non-zero values of b

#### Answer (Detailed Solution Below)

Option 3: Observable for all non-zero values of b

## State Space Representation MCQ Question 5 Detailed Solution

Concept:

State space representation:

$$\dot{x}(t) = A(t) x(t) + B(t) u(t)$$

$$y(t) = C(t) x(t) + D(t) u(t)$$

y(t) is output

u(t) is input

x(t) is a state vector

A in a quotam matrix

## This representation is continuous time-variant.

## Controllability:

A system is said to be controllable if it is possible to transfer the system state from any initial state  $x(t_0)$  to any desired state x(t) in a specified finite time interval by a control vector u(t)

## Kalman's test for controllability:

$$\dot{x} = Ax + Bu$$

$$Q_c = \{B AB A^2B ... A^{n-1} B\}$$

Qc = controllability matrix

If  $|Q_c| = 0$ , system is not controllable

If |Qc|≠ 0, system is controllable

## Observability:

A system is said to be observable if every state  $x(t_0)$  can be completely identified by measurement of output y(t) over a finite time interval.

## Kalman's test for observability:

$$Q_0 = [C^T A^T C^T (A^T)^2 C^T .... (A^T)^{n-1} C^T]$$

Q<sub>0</sub> = observability testing matrix

If  $|Q_0| = 0$ , system is not observable

If  $|Q_0| \neq 0$ , system is observable.

#### Calculation:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; C = \begin{bmatrix} b & 0 \end{bmatrix}$$

Observability:

$$CA = \begin{bmatrix} b & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} b & 2b \end{bmatrix}$$

If  $|Q_0| \neq 0$ , system is observable.

Observable for all non-zero values of b.



#### MCQ Question 6

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# Consider a system described by the state model

$$\dot{X} = egin{bmatrix} 2 & 1 \ -1 & 2 \end{bmatrix} X + egin{bmatrix} 1 \ 1 \end{bmatrix} U$$

 $Y = [1 \ 1] X$ 

The system is



- 1. controllable but not observable
- 2. uncontrollable and observable
- both controllable and observable
- neither controllable nor observable

#### Answer (Detailed Solution Below)

Option 3: both controllable and observable

## State Space Representation MCQ Question 6 Detailed Solution

Concept:

State space representation:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$

$$y(t) = C(t)x(t) + D(t)u(t)$$

y(t) is output

u(t) is input

x(t) is a state vector

This representation is continuous time-variant.

# Controllability:

A system is said to be controllable if it is possible to transfer the system state from any initial state  $x(t_0)$  to any desired state x(t) in a specified finite time interval by a control vector u(t)

## Kalman's test for controllability:

$$\dot{x} = Ax + Bu$$

$$Q_c = \{B AB A^2B ... A^{n-1} B\}$$

Qc = controllability matrix

If  $|Q_c| = 0$ , system is not controllable

If |Q<sub>c</sub>|≠ 0, system is controllable

## Observability:

A system is said to be observable if every state  $x(t_0)$  can be completely identified by measurement of output y(t) over a finite time interval.

## Kalman's test for observability:

$$Q_0 = [C^T A^T C^T (A^T)^2 C^T .... (A^T)^{n-1} C^T]$$

Qn = observability testing matrix

If  $|Q_0| = 0$ , system is not observable

If  $|Q_0| \neq 0$ , system is observable.

#### Calculation:

$$A = egin{bmatrix} 2 & 1 \ -1 & 2 \end{bmatrix}, \ B = egin{bmatrix} 1 \ 1 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Controllability:

$$AB = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Controllability matrix,

$$M = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$$

$$|M| = -2$$

Therefore, the system is controllable.

Observability:

$$CA = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \end{bmatrix}$$

Observability matrix,

$$N = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

|N| = 2

Therefore, the system is observable



#### MCQ Question 7

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The dynamic model of a pendulum is given by  $\frac{d^2\theta}{dt^2} + 400\theta = 100T$ , where  $\theta$  is the displacement in rad / s and T is the applied torque in N-m. Its representation in time scale state variable form  $\dot{\bf X}$  =  $\alpha$  X +  $\beta$ u can have the constants.

1. 
$$\alpha = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}; \beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$_{2.}\ \alpha = \left[\begin{array}{cc} 0 & 1 \\ -4 & 0 \end{array}\right];\ \beta = \left[\begin{array}{c} 1 \\ 0 \end{array}\right]$$

3. 
$$\alpha = \begin{bmatrix} 0 & 0 \\ 4 & 1 \end{bmatrix}; \ \beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$_{4.} \ \alpha = \left[ \begin{array}{cc} 0 & 0 \\ -4 & 1 \end{array} \right]; \ \beta = \left[ \begin{array}{c} 1 \\ 0 \end{array} \right]$$

# Answer (Detailed Solution Below)

Option 1 : 
$$\alpha = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}$$
;  $\beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ 

## State Space Representation MCQ Question 7 Detailed Solution

Let 
$$x_1 = \theta$$

$$\Rightarrow \dot{x}_1 = d\theta / dt ---- (1)$$

And consider  $x_2 = d\theta / dt$  and u = T

$$\Rightarrow \dot{x}_2 = d^2\theta / dt^2$$

From the given dynamic model of a pendulum, we get

$$\frac{d^2\theta}{dt^2} = -400\theta + 100T$$

$$\Rightarrow \dot{x}_2 = -400 x_1 + 100 u ----- (2)$$

From equations (1) and (2), we can get the state model as

$$\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ -400 & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ 100 \end{bmatrix} \mathbf{U}$$

$$\Rightarrow \begin{bmatrix} \vec{X}_1 \\ \vec{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

On comparing the above state model with  $\dot{X} = \alpha X + \beta u$ , we get

$$\Rightarrow \alpha = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}; \ \beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



## MCQ Question 8:

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Given the homogeneous state space equation  $\dot{x}=\begin{bmatrix}0&1\\-1&-2\end{bmatrix}x$  and the initial state

value 
$$x(0) = \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

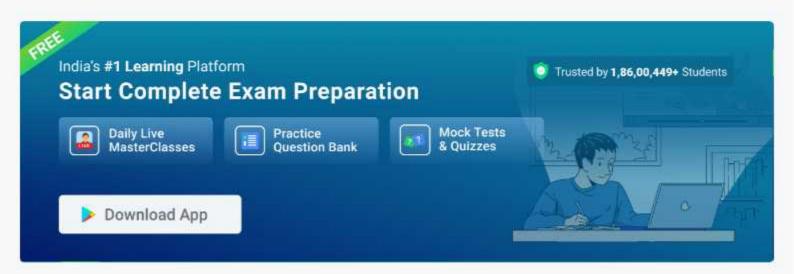
The steady state values of  $x_{ss1} = \lim x_1(t)$  and  $x_{ss2} = \lim x_2(t)$  are

- 1. 0,0
- 2. 10,0
- 3. 10, -10
- 4. 0, -10

## Answer (Detailed Solution Below)

Option 1:0,0





#### MCQ Question 9:

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Which of the following is not an advantage of state space approach in analyzing the performance of a control system?

- 1. State space approach can be used to represent non linear systems having backlash.
- 2. State space approach can handle system with nonzero initial conditions.
- 3. Multiple input/output system can be handled effectively by state space approach.
- The physical interpretation of the model in state space approach is easily obtained.

#### Answer (Detailed Solution Below)

Option 4: The physical interpretation of the model in state space approach is easily obtained.



#### MCQ Question 10:

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# A second order system is given by

$$x = egin{bmatrix} 1 & 1 \ -3 & -2 \end{bmatrix} egin{bmatrix} x_1 \ x_2 \end{bmatrix} + egin{bmatrix} 1 \ 0 \end{bmatrix} u \ y = egin{bmatrix} 1 & 0 \end{bmatrix} egin{bmatrix} x_1 \ x_2 \end{bmatrix}$$

- 1. The system is state controllable and output controllable
- 2. The system is state controllable but not output controllable
- 3. The system is output controllable but not state controllable
- 4. The system is neither state controllable nor output controllable

