Mathematical Modeling and Representation of Systems Questions

MCQ Question 1

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Consider a linear time-invariant system whose input r(t) and output y(t) are related by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 4y\left(t\right) = 6r\left(t\right)$$

The poles of this system are at

1. +2j, -2j
2. +2, -2
3. +4, -4
4. +4j, -4j

Answer (Detailed Solution Below)

Option 1 : +2j, -2j

Mathematical Modeling and Representation of Systems MCQ Question 1 Detailed Solution

Concept:

A transfer function is defined as the ratio of Laplace transform of the output to the Laplace transform of the input by assuming initial conditions are zero.

TF = L[output]/L[input]

$$TF = \frac{C(s)}{R(s)}$$

For unit impulse input i.e. r(t) = δ(t)

⇒ R(s) = δ(s) = 1

Now transfer function = C(s)

Therefore, the transfer function is also known as the impulse response of the system.

Transfer function = L[IR]

IR = L^{-1} [TF]

Calculation:

Given the differential equation is,

$$\frac{d^2y(t)}{dt^2} + 4y\left(t\right) = 6r\left(t\right)$$

By applying the Laplace transform,

s^2 Y(s) + 4 Y(s) = 6 R(s)

$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{6}{s^2+4}$$

Poles are the roots of the denominator in the transfer function.

⇒ $s^2 + 4 = 0$

⇒ s = ±2j

MCQ Question 2 View this Question Online >

Let a causal LTI system be characterized by the following differential equation, with initial rest condition

$$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 10y\left(t\right) = 4x\left(t\right) + 5\frac{dx(t)}{dt}$$

where x(t) and y(t) are the input and output respectively. The impulse response of the system is (u(t) is the unit step function)

1. $2e^{-2t}u(t) - 7e^{-5t}u(t)$

2. $-2e^{-2t}u(t) + 7\,e^{-5t}u(t)$

3. $7e^{-2t}u(t) - 2\,e^{-5t}u(t)$

4. $-7e^{-2t}u(t) + 2\,e^{-5t}u(t)$

Answer (Detailed Solution Below)

Option 2 : $-2e^{-2t}u(t) + 7\,e^{-5t}u(t)$

Mathematical Modeling and Representation of Systems MCQ Question 2 Detailed Solution

$$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 10y\left(t\right) = 4x\left(t\right) + 5\frac{dx}{dt}$$

Apply Laplace transform on both sides,

$s^2Y(s) + 7sY(s) + 10Y(s) = 4\,X(s) + 5s\,X(s)$

$\Rightarrow (s^2 + 7s + 10)\,Y(s) = (4 + 5s)\,X(s)$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{(4+5s)}{s^2+7s+10}$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{4+5s}{(s+5)(s+2)} = \frac{7}{(s+5)} - \frac{2}{(s+2)}$$

Apply inverse laplace transform

$\Rightarrow y(t) = 7e^{-5t}u(t) - 2e^{-2t}\,u(t)$

MCQ Question 3

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Which of the following modelling methods uses Boolean operations?

1. Boundary representation

2. Constructive solid geometry
3. Surface modelling
4. Wireframe modelling

Answer (Detailed Solution Below)

Option 2 : Constructive solid geometry

Mathematical Modeling and Representation of Systems MCQ Question 3 Detailed Solution

Explanation:

- **Boolean operation is an important way in geometry modeling.**
- It is the main way to build a complex model from simple models, and it is widely used in computer-aided geometry design and computer graphics.
- Traditional Boolean operation is mainly used in solid modeling to build a complex solid from primary solid e.g. cube, column, cone, sphere, etc.
- With the development of computer applications, there are many ways to represent digital models, such as parametric surface, meshes, point model, etc.
- Models become more and more complex, and features on models such as on statutary artworks are more detailed.

MCQ Question 4

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For a tachometer, ie θ(t) is the an rotor displacement in radians, e(t) is the output voltage and k_t is the tachometer constant in V/rad/sec, then the transfer function $\frac{E(S)}{\theta(S)}$ will be:

1. $K_t s^2$
2. $\frac{k_t}{s}$
3. $K_t s$
4. K_t

Answer (Detailed Solution Below)

Option 3 : $K_t s$

Mathematical Modeling and Representation of Systems MCQ Question 4 Detailed Solution

Output = e(t)

Input = (t)

$$e\,(t) \propto \frac{d\theta(t)}{dt}$$

Taking LT,

$E(s) = K_t s\, \theta(s)$

$$\Rightarrow \text{TF} = \frac{E(S)}{\theta(S)} = k_t s$$

MCQ Question 5

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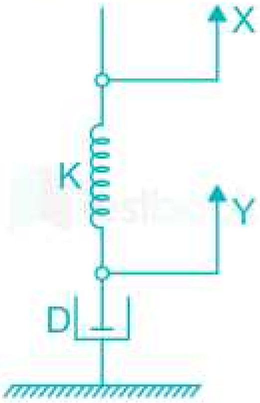
The mechanical system shown in the figure below has its pole(s) at:



1. -K/D
2. -D/K
3. -DK
4. 0, -K/D

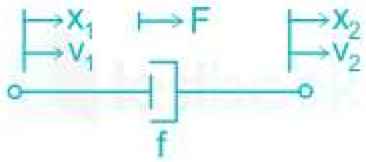
Answer (Detailed Solution Below)

Option 1 : -K/D

Mathematical Modeling and Representation of Systems MCQ Question 5 Detailed Solution

Concept:

Damping force:



$$F = f\frac{d(x_1 - x_2)}{dt} = f\,(v_1 - v_2)$$

F: Damper force

f: Damper constant

x_1, x_2: Displacement at side 1 and side 2 of the damper

v_1, v_2: Velocity at side 1 and side 2

Spring force

$$F = k\,(x_1 - x_2) = k\int(v_1 - v_2)dt$$

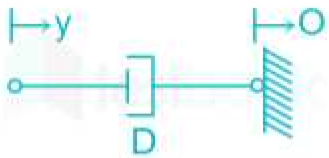
k: Spring constant

Calculation:

Method 1:

Given damper constant is D and the Spring constant is k

Assuming that velocities at side 1 and 2



K ∫(y - x) dt + D (y − 0) = 0

Dy +k ∫ y dt − k ∫ x dt = 0

Applying the Laplace Transform

$$DY\,(s) + \frac{k}{s}Y\,(s) - \frac{k}{s}X\,(s) = 0$$

$$Y\,(s)\left[\frac{Ds+k}{s}\right] = \frac{k}{s}X\,(s)$$

$$\frac{Y(s)}{X(s)} = \frac{k}{Ds+k}$$

Pole is present at **s = - k / D**

Method 2:

Given damper constant is D and the Spring constant is k

Assuming displacement at side 1 and 2

$k\left(y-x\right)+D\frac{d(y-0)}{dt}=0$

Applying the Laplace Transform

k Y(s) − k X(s) + D sY(s) = 0

Y(s) (k + Ds) − k X(s) = 0

$\frac{Y(s)}{X(s)}=\frac{k}{Ds+k}$

Pole is present at **s = - k / D**

MCQ Question 6

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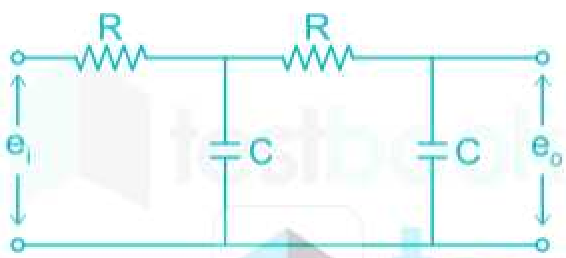


The transfer function of the network shown above is

1. $\frac{1}{s^2T^2+2sT+1}$

2. $\frac{1}{s^2T^2+3sT+1}$

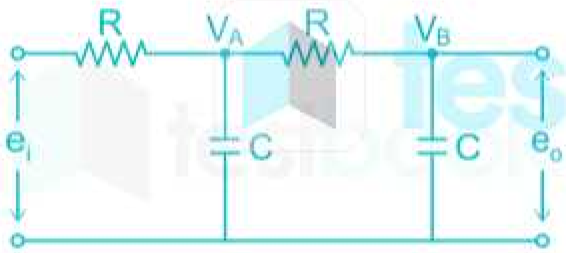
3. $\frac{1}{s^2T^2+sT+1}$

4. $\frac{1}{s^2T^2+1}$

Answer (Detailed Solution Below)

Option 2 : $\frac{1}{s^2T^2+3sT+1}$

Mathematical Modeling and Representation of Systems MCQ Question 6 Detailed Solution



By applying KCL at node A,

$$\frac{V_A-e_i}{R}+\frac{V_A-V_B}{R}+\frac{V_R}{X_C}=0$$

$$\Rightarrow V_A\left[\frac{2}{R}+\frac{1}{X_C}\right]-\frac{e_i}{R}-\frac{V_B}{R}=0 \quad \text{...1)}$$

By applying KCL at node B,

$$\frac{V_B-V_A}{R}+\frac{V_B}{X_c}=0$$

$$V_B\left[\frac{1}{R}+\frac{1}{X_c}\right]=\frac{V_A}{R}\Rightarrow V_A=V_B\left[1+\frac{R}{X_c}\right] \quad \text{...2)}$$

From equation 1) and 2)-

$$\Rightarrow V_B\left[1+\frac{R}{X_C}\right]\left[\frac{2}{R}+\frac{1}{X_c}\right]-\frac{V_B}{R}=\frac{e_i}{R}$$

$$\Rightarrow V_B\left[\frac{2}{R}+\frac{1}{X_c}+\frac{2}{X_c}+\frac{R}{X_c^2}-\frac{1}{R}\right]=\frac{e_i}{R} \quad \text{...3)}$$

As we can see from the circuit diagram, $V_B = e_0$

$$\Rightarrow e_0\left[\frac{1}{R}+\frac{3}{X_c}+\frac{R}{X_c^2}\right]=\frac{e_i}{R}$$

$$\Rightarrow \frac{e_0}{e_i}=\frac{1}{R\left[\frac{1}{R}+\frac{3}{X_c}+\frac{R}{X_c^2}\right]}$$

$$=\frac{1}{(CsR)^2+3RCs+1}$$

Time constant, T = RC

Time constant, T = RC

$$h(T) = \frac{1}{T^2 s^2 + 3Ts + 1}$$

MCQ Question 7

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The transfer function of tachometer is of the form

1.
$\frac{K}{s(s+1)}$

2.
$\frac{K}{(s+1)}$

3.
$\frac{K}{s}$

4.
K.s

Answer (Detailed Solution Below)

Option 4 :

K.s

Mathematical Modeling and Representation of Systems MCQ Question 7 Detailed Solution

Output = e(t)

Input = (t)

$$e(t) \propto \frac{d\theta(t)}{dt}$$

Taking LT,

$E(s) = K_t s\ \theta(s)$

$$\Rightarrow \mathrm{TF} = \frac{E(S)}{\theta(S)} = k_t s$$

MCQ Question 8

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Which one of the following statements related to modeling of system dynamics is NOT true?

1. The transfer function is not changed by a linear transformation of state
2. A given state description can be transformed to a controllable canonical form if the controllability matrix is nonsingular
3. A change of state by a nonsingular linear transformation does not change the
4. Zeros cannot be computed from its state description matrices

Answer (Detailed Solution Below)

Option 3 : A change of state by a nonsingular linear transformation does not change the

Mathematical Modeling and Representation of Systems MCQ Question 8 Detailed Solution

Modeling of Dynamic System:

- A dynamic system is a kind of **system** whose **behavior is a function of time**.
- **Static system analysis does not give an accurate analysis** while **Dynamic system analysis does give an accurate analysis**.

- **Example:** An aircraft is subjected to time-varying stress during the flight through turbulent air hard landing is an example of a Dynamic system.
- Dynamic system analysis is more complex than static system analysis since the conclusion based on static system analysis is not correct.

- In the dynamic system,
  the transfer function does not change by a linear transformation of the state.
- The dynamic **modeling starts with the physical component description** of the system's understanding of component behavior to create the mathematical model.
- A given state description in the **mathematical model can be transformed to a controllable canonical form if the controllability matrix is non-singular**. And **zero cannot be computed from this matrix**.

The dynamic system state variable is used for:

- **Analysis, Identification**, and **Synthesis** of Dynamic System.
- **Predict** the **future behavior** (Y) of the system when subjected to future input variables (U) and present (X).

MCQ Question 9:

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Consider a linear time-invariant system whose input r(t) and output y(t) are related by the following differential equation:

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The poles of this system are at

1. +2j, -2j

2. +2, -2

3. +4, -4

4. +4j, -4j

Answer (Detailed Solution Below)

Option 1 : +2j, -2j

Mathematical Modeling and Representation of Systems MCQ Question 9 Detailed Solution

Concept:

A transfer function is defined as the ratio of Laplace transform of the output to the Laplace transform of the input by assuming initial conditions are zero.

TF = L[output]/L[input]

$$TF = \frac{C(s)}{R(s)}$$

For unit impulse input i.e. r(t) = δ(t)

⇒ R(s) = δ(s) = 1

Now transfer function = C(s)

Therefore, the transfer function is also known as the impulse response of the system.

Transfer function = L[IR]

IR = L^{-1} [TF]

Calculation:

Given the differential equation is,

$$\frac{d^2y(t)}{dt^2} + 4y\left(t\right) = 6r\left(t\right)$$

By applying the Laplace transform,

s^2 Y(s) + 4 Y(s) = 6 R(s)

$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{6}{s^2+4}$$

Poles are the roots of the denominator in the transfer function.

⇒ s^2 + 4 = 0

⇒ s = ±2j

MCQ Question 10:

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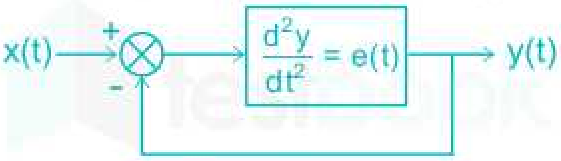
For the system given figure, e(t) is the error between input x(t) and output y(t)



If x(t) = u(t) and all the initial conditions are zero, then e(t) will be

1. −sin t
2. −cos t
3. sin t
4. cos t

Answer (Detailed Solution Below)

Option 4 : cos t

Mathematical Modeling and Representation of Systems MCQ Question 10 Detailed Solution

From the block diagram,

e(t) = -y(t) + x(t)

$$\Rightarrow \frac{d^2y}{dt^2} = -y\left(t\right) + x\left(t\right)$$

$$\Rightarrow \frac{d^2y}{dt^2} = -y\left(t\right) + u\left(t\right)$$

By applying Laplace transform,

$$\Rightarrow s^2 y(s) = -y(s) + \frac{1}{s}$$

$$\Rightarrow y(s) = \frac{1}{s(1+s^2)}$$

$$e(t) = \frac{d^2 y}{dt^2}$$

$\Rightarrow$ E(s) = s^2 y(s)

$$\Rightarrow E(s) = s^2 \times \frac{1}{s(1+s^2)} = \frac{s}{1+s^2}$$

By applying inverse Laplace transform,

$\Rightarrow$ e(t) = cos t