i) Compute the average return, the standard deviation, and the correlation for Exxon and Darden.

For the period 2005-2009, here are the computed statistics for Exxon Mobil (XOM) and Darden Restaurants (DRI):

```
In [13]: # Calculating the average return and standard deviation for Exxon Mobil (XOM) and Darden Restaurants (DRI)
         average_xom = data['XOM'].mean()
         std_dev_xom = data['XOM'].std()
         average_dri = data['DRI'].mean()
         std_dev_dri = data['DRI'].std()

         # Calculating the correlation between XOM and DRI
         correlation_xom_dri = data['XOM'].corr(data['DRI'])

         print(average_xom), (std_dev_xom), (average_dri), (std_dev_dri), (correlation_xom_dri)

         0.9543722582833337

Out[13]: (None,
          10.049779345834818,
          1.422620666283333,
          10.630195656869258,
          0.5193835641540854)
```

For the period 2005-2009, here are the computed statistics for Exxon Mobil (XOM) and Darden Restaurants (DRI):

- Exxon Mobil (XOM)
- Average Monthly Return: 0.95%
- Standard Deviation of Returns: 10.05%

- Darden Restaurants (DRI)
- Average Monthly Return: 1.42%
- Standard Deviation of Returns: 10.63%

The correlation between the monthly returns of Exxon Mobil and Darden Restaurants is approximately 0.52, indicating a moderate positive relationship between their returns.

---

ii) If you can only invest in the risk-free asset and either Exxon or Darden, which one do you pick?

When deciding between investing in Exxon Mobil (XOM) or Darden Restaurants (DRI) alongside a risk-free asset, it's important to consider both the returns and the risk (volatility) associated with each investment. Here's a breakdown based on the data you provided:

- Exxon Mobil (XOM)
- Average Monthly Return: 0.95%
- Standard Deviation of Returns: 10.05%

- Darden Restaurants (DRI)
- Average Monthly Return: 1.42%

- Standard Deviation of Returns: 10.63%

Darden Restaurants shows a higher average monthly return than Exxon Mobil. However, it also has a slightly higher standard deviation, indicating it is more volatile or riskier.

The choice between these two would largely depend on your risk tolerance:

- Higher Risk Tolerance: If you are comfortable with taking on slightly more risk for a potentially higher return, Darden could be the preferable choice given its higher average return.
- Lower Risk Tolerance: If you prefer to minimize volatility while still aiming for reasonable returns, Exxon Mobil might be preferable due to its slightly lower standard deviation, though the return is also lower.

Thus, if maximizing expected return is your goal and you are comfortable with a slightly higher risk, Darden Restaurants might be the better option. If you prioritize stability slightly more, then Exxon Mobil could be the better fit.

---

iii) If you can only invest in the risk-free asset and either Exxon or Darden and your target standard deviation is 15% per annum, what will be your best possible expected monthly return? What weight does the risk-free asset take in your portfolio?

To determine the best possible expected monthly return and the weight of the risk-free asset in a portfolio with a target standard deviation of 15% per annum, we need to consider how mixing the risk-free asset with either Exxon Mobil (XOM) or Darden Restaurants (DRI) affects the portfolio's risk and return. The key steps include:

1. Convert the annual target standard deviation to a monthly value since our return data is monthly.
2. Calculate the expected return of a portfolio for each stock, consisting of a mix of the stock and the risk-free rate that achieves the target monthly standard deviation.
3. Calculate the corresponding weight of the risk-free asset in each portfolio.

Step 1: Convert Annual Standard Deviation to Monthly
The annual standard deviation is 15%, but we need to convert this to a monthly standard deviation. Since standard deviation scales with the square root of time, the monthly standard deviation is 15%/SQRT(12)

Step 2: Calculate Expected Portfolio Return for Each Stock
The portfolio's expected return can be calculated using:
$E(Rp) = w \times E(Rs)+(1-w) \times RF$
Where E(Rp) is the expected portfolio return, *w* is the weight of the stock in the portfolio, E(Rs) is the expected return of the stock, and RF is the risk-free rate (averaged and converted to monthly if given annually).

Step 3: Calculate Weight of the Risk-Free Asset
Given the target monthly standard deviation, the weight of the stock in the portfolio can be calculated using:
$\sigma p = w \times \sigma s$
where σp is the target portfolio standard deviation and σs is the standard deviation of the stock.

In [14]:
```
import numpy as np

# Convert annual target standard deviation to monthly
annual_target_std_dev = 15
monthly_target_std_dev = annual_target_std_dev / np.sqrt(12) / 100  # convert percentage to decimal

# Average monthly risk-free rate
average_rf = data['RF'].mean() / 100  # convert percentage to decimal

# Calculate the weight of the stock in the portfolio to meet the target std deviation
weight_xom = monthly_target_std_dev / std_dev_xom
weight_dri = monthly_target_std_dev / std_dev_dri

# Calculate the expected return of the portfolio
expected_return_xom = weight_xom * average_xom + (1 - weight_xom) * average_rf
expected_return_dri = weight_dri * average_dri + (1 - weight_dri) * average_rf

# Calculate the weight of the risk-free asset in each portfolio
weight_rf_xom = 1 - weight_xom
weight_rf_dri = 1 - weight_dri

print(monthly_target_std_dev), (expected_return_xom), (weight_rf_xom), (expected_return_dri), (weight_rf_dri)
```

```
0.04330127018922194
```

Out[14]:
```
(None,
 0.005079563486636896,
 0.9956913213017788,
 0.006762642621738448,
 0.9959265782505856)
```

The target monthly standard deviation for your portfolio, when converted from an annual target of 15%, is approximately 4.33%.

Here are the best possible expected monthly returns and the weights of the risk-free asset for each investment option:

- Exxon Mobil (XOM)
- Expected Monthly Return: 0.51%
- Weight of the Risk-Free Asset: 99.57%

- Darden Restaurants (DRI)
- Expected Monthly Return: 0.68%
- Weight of the Risk-Free Asset: 99.59%

Given these calculations, investing in Darden Restaurants, alongside the risk-free asset, would yield a higher expected monthly return of 0.68%, with a portfolio risk at the target standard deviation of 4.33% per month. The risk-free asset would constitute approximately 99.59% of the portfolio, slightly higher than the 99.57% for Exxon Mobil. This higher proportion of the risk-free asset in both cases ensures that the overall portfolio volatility remains within the desired target.

---

iv) Compute the average return, standard deviation, and the Sharpe ratio of the portfolio that invests 80% in Exxon and 20% in Darden

First, we need to find the following,

1. Average Return of the Portfolio ($E(Rp)$)
2. Standard Deviation of the Portfolio (σp)
3. Sharpe Ratio of the portfolio

In [15]:
```
# Define the weights for Exxon and Darden in the portfolio
weight_xom = 0.8
weight_dri = 0.2

# Calculate the average return of the portfolio
portfolio_avg_return = (weight_xom * average_xom) + (weight_dri * average_dri)

# Calculate the standard deviation of the portfolio
portfolio_std_dev = np.sqrt(
    (weight_xom ** 2 * std_dev_xom ** 2) +
    (weight_dri ** 2 * std_dev_dri ** 2) +
    (2 * weight_xom * weight_dri * std_dev_xom * std_dev_dri * correlation_xom_dri)
)

# Calculate the Sharpe Ratio of the portfolio
sharpe_ratio_portfolio = (portfolio_avg_return - average_rf) / portfolio_std_dev

print(portfolio_avg_return), (portfolio_std_dev), (sharpe_ratio_portfolio)
```

```
1.0480219398833337
```

Out[15]: (None, 9.322790835760626, 0.11231081890203541)

For the portfolio that invests 80% in Exxon Mobil (XOM) and 20% in Darden Restaurants (DRI), here are the calculated metrics:

- Average Monthly Return: 1.05%
- Standard Deviation of Returns: 9.32%
- Sharpe Ratio: 0.11

This Sharpe Ratio indicates the amount of excess return you are receiving for the extra volatility of holding the riskier asset mix compared to the risk-free asset. A Sharpe Ratio of 0.11 suggests a modest level of additional return per unit of risk taken.

---

v) Does the existence of Exxon and Darden violate the CAPM? (Hint: You will need to use the data on the market excess returns).

To determine whether the existence of Exxon Mobil (XOM) and Darden Restaurants (DRI) violates the Capital Asset Pricing Model (CAPM), we need to examine whether their returns align with the predictions of CAPM based on their betas and the market returns. The CAPM states that the expected return of an asset should be proportional to its systematic risk (beta), as represented by the formula:

$E(Ri) = Rf + \beta i \times (E(Rm) - Rf)$

To assess whether Exxon and Darden align with or violate CAPM:

1. Calculate the beta for Exxon and Darden using the covariance of their returns with the market returns relative to the variance of the market returns.
2. Use these betas to calculate the expected returns according to CAPM.
3. Compare these CAPM-expected returns with the actual average returns of Exxon and Darden.

```
In [16]: # Calculate the excess returns for XOM, DRI, and the Market
         data['Excess_XOM'] = data['XOM'] - data['RF']
         data['Excess_DRI'] = data['DRI'] - data['RF']
         data['Excess_MKT'] = data['MKT'] - data['RF']

         # Calculate the covariance of XOM and DRI with the Market
         cov_xom_mkt = data['Excess_XOM'].cov(data['Excess_MKT'])
         cov_dri_mkt = data['Excess_DRI'].cov(data['Excess_MKT'])

         # Calculate the variance of the Market
         var_mkt = data['Excess_MKT'].var()

         # Calculate beta for XOM and DRI
         beta_xom = cov_xom_mkt / var_mkt
         beta_dri = cov_dri_mkt / var_mkt

         # Calculate the expected returns according to CAPM
         capm_expected_return_xom = average_rf + beta_xom * (data['Excess_MKT'].mean())
         capm_expected_return_dri = average_rf + beta_dri * (data['Excess_MKT'].mean())

         print(beta_xom), (capm_expected_return_xom), (beta_dri), (capm_expected_return_dri)

         1.0603088461430916
Out[16]: (None, 0.6949438064673202, 1.1888071174057981, 0.7790459250087617)
```

Here are the beta values and CAPM-expected returns for Exxon Mobil (XOM) and Darden Restaurants (DRI):

- Exxon Mobil (XOM)
- Beta: 1.06
- CAPM-Expected Monthly Return: 0.69%

- Darden Restaurants (DRI)
- Beta: 1.19
- CAPM-Expected Monthly Return: 0.78%

Comparing these CAPM-expected returns to the actual average monthly returns:

- Exxon Mobil (XOM) had an actual average monthly return of 0.95%.
- Darden Restaurants (DRI) had an actual average monthly return of 1.42%.

The actual returns of both Exxon and Darden are higher than their CAPM-expected returns. This discrepancy suggests that both stocks provided higher returns than predicted by their systematic risk alone, according to CAPM. This could be interpreted as a violation of CAPM if these

differences are statistically significant and consistent over time, as CAPM assumes that no asset should systematically earn a higher risk-adjusted return than its beta with the market would predict.

However, whether this represents a true violation depends on further analysis including statistical testing for significance and consideration of other factors such as changing risk premiums, unmodeled risk factors, or changes in risk-free rates over the period analyzed.

---

vi) Redo (iv) assuming that the correlation between Exxon and Darden is 0.6. What does it tell you about the benefits of diversification?

To redo the calculation of the standard deviation of a portfolio that invests 80% in Exxon Mobil (XOM) and 20% in Darden Restaurants (DRI) with an assumed correlation of 0.6, we will use the adjusted correlation coefficient for the portfolio's standard deviation.

The correlation coefficient between Exxon and Darden, now assumed to be 0.6. We will compare this new standard deviation with the previous value to evaluate the benefits of diversification.

In [17]:
```
# Updated correlation
correlation_xom_dri_updated = 0.6

# Calculate the standard deviation of the portfolio with updated correlation
portfolio_std_dev_updated = np.sqrt(
    (weight_xom ** 2 * std_dev_xom ** 2) +
    (weight_dri ** 2 * std_dev_dri ** 2) +
    (2 * weight_xom * weight_dri * std_dev_xom * std_dev_dri * correlation_xom_dri_updated)
)

print(portfolio_std_dev_updated)
```

```
9.469444498437767
```

With the updated correlation of 0.6 between Exxon Mobil (XOM) and Darden Restaurants (DRI), the standard deviation of the portfolio (investing 80% in Exxon and 20% in Darden) is approximately 9.47%. This compares to a standard deviation of 9.32% calculated with the original correlation of 0.52.

This slight increase in the standard deviation when using a higher correlation coefficient illustrates a key concept in portfolio theory: diversification benefits are reduced as the correlation between assets increases. Lower correlations between assets in a portfolio typically lead to better diversification benefits, demonstrated by a lower overall portfolio risk (standard deviation).

The example here shows that increasing the correlation reduces the diversification effect, resulting in a higher portfolio risk. This is because the asset returns are more synchronized, and thus less effective at smoothing out volatility through diversification.

---

vii) Estimate the market model for Exxon and the market model for Darden. Do either Exxon or Darden violate the CAPM by a statistically significant amount.

To estimate the market model for Exxon Mobil (XOM) and Darden Restaurants (DRI) and assess whether violates the Capital Asset Pricing Model (CAPM) by a statistically significant amount, we'll conduct a regression analysis for each company's excess returns against the market's excess returns.

If α is statistically significantly different from zero (typically using a 95% confidence interval), it suggests that the stock has systematically outperformed or underperformed what CAPM would predict, thereby violating the model.

In [19]:
```
import statsmodels.api as sm

# Prepare the regression data
X = sm.add_constant(data['Excess_MKT'])  # market excess returns as independent variable
Y_xom = data['Excess_XOM']               # Exxon's excess returns as dependent variable
Y_dri = data['Excess_DRI']               # Darden's excess returns as dependent variable

# Run the regression for Exxon
model_xom = sm.OLS(Y_xom, X).fit()

# Run the regression for Darden
model_dri = sm.OLS(Y_dri, X).fit()

# Get summary of the models
model_xom_summary = model_xom.summary()
model_dri_summary = model_dri.summary()

print(model_xom_summary), (model_dri_summary)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:              Excess_XOM   R-squared:                       0.350
Model:                             OLS   Adj. R-squared:                  0.339
Method:                  Least Squares   F-statistic:                     31.25
Date:                 Mon, 29 Apr 2024   Prob (F-statistic):           6.39e-07
Time:                         19:44:41   Log-Likelihood:                -210.26
No. Observations:                   60   AIC:                             424.5
Df Residuals:                       58   BIC:                             428.7
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.1632      1.064      0.153      0.879      -1.966       2.293
Excess_MKT     1.0603      0.190      5.590      0.000       0.681       1.440
==============================================================================
Omnibus:                       18.479   Durbin-Watson:                   1.530
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               26.331
Skew:                           1.132   Prob(JB):                     1.92e-06
Kurtosis:                       5.325   Cond. No.                         5.65
==============================================================================
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:             Excess_DRI   R-squared:                       0.394
Model:                            OLS   Adj. R-squared:                  0.383
Method:                 Least Squares   F-statistic:                     37.68
Date:                Mon, 29 Apr 2024   Prob (F-statistic):           8.05e-08
Time:                        19:44:41   Log-Likelihood:                -211.51
No. Observations:                  60   AIC:                             427.0
Df Residuals:                      58   BIC:                             431.2
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.5474      1.086      0.504      0.616      -1.627       2.722
Excess_MKT     1.1888      0.194      6.138      0.000       0.801       1.576
==============================================================================
Omnibus:                        6.263   Durbin-Watson:                   2.689
Prob(Omnibus):                  0.044   Jarque-Bera (JB):               10.109
Skew:                          -0.037   Prob(JB):                      0.00638
Kurtosis:                       5.009   Cond. No.                         5.65
==============================================================================
```

scroll output; double click to hide

Exxon Mobil (XOM)

- Alpha (Intercept): 0.1632
- Beta: 1.0603
- P-value for Alpha: 0.879
- Confidence Interval for Alpha: [-1.966, 2.293]

Darden Restaurants (DRI)

- Alpha (Intercept): 0.5474
- Beta: 1.1888
- P-value for Alpha: 0.616
- Confidence Interval for Alpha: [-1.627, 2.722]

Interpretation:

- Exxon Mobil (XOM) has an alpha of 0.1632, but the p-value of 0.879 indicates that this alpha is not statistically significant. This means we cannot reject the hypothesis that Exxon's returns are fully explained by its market risk (beta) as predicted by CAPM.
- Darden Restaurants (DRI) has a higher alpha of 0.5474, suggesting some level of performance above what CAPM would predict based on its beta alone. However, the p-value of 0.616 indicates that this alpha is also not statistically significant.

In conclusion, neither Exxon Mobil nor Darden Restaurants violate the CAPM by a statistically significant amount based on this analysis. The alphas for both companies do not significantly differ from zero, indicating that their returns align with what CAPM would predict given their market sensitivities (betas). This analysis supports the notion that market risk (beta) predominantly explains the returns of these stocks, and any deviations are not significant enough to constitute a systematic violation of CAPM.

viii) Redo (iii) assuming that now you can invest in Exxon, Darden, and the risk-free asset simultaneously. What are the weights of Exxon, Darden, and the risk-free asset in the portfolio that delivers the best possible return for the target standard deviation of 15% per annum?

To determine the optimal portfolio weights for Exxon Mobil (XOM), Darden Restaurants (DRI), and a risk-free asset that together achieve a target standard deviation of 15% per annum, we'll need to solve a portfolio optimization problem. This involves maximizing the portfolio's expected return for a given level of risk (standard deviation).

1. Convert the Annual Standard Deviation to Monthly: Since the returns are given monthly, we need to convert the annual target standard deviation into a monthly figure.

2. Formulate the Optimization Problem:

- Objective: Maximize the portfolio's expected return.
- Constraint: The portfolio's standard deviation should be equal to the target monthly standard deviation.

In [20]:
```
from scipy.optimize import minimize

# Convert the annual target standard deviation to a monthly standard deviation
monthly_target_std_dev = annual_target_std_dev / np.sqrt(12) / 100  # convert percentage to decimal

# Define the function to calculate the portfolio standard deviation
def portfolio_std_dev(weights, std_dev_xom, std_dev_dri, correlation):
    w_xom, w_dri = weights
    w_rf = 1 - w_xom - w_dri
    # Compute the portfolio standard deviation
    return np.sqrt((w_xom * std_dev_xom) ** 2 + (w_dri * std_dev_dri) ** 2 +
                   2 * w_xom * w_dri * std_dev_xom * std_dev_dri * correlation)

# Define the objective function (negative of the portfolio's return to minimize)
def negative_portfolio_return(weights, avg_ret_xom, avg_ret_dri, rf_rate):
    w_xom, w_dri = weights
    w_rf = 1 - w_xom - w_dri
    return -(w_xom * avg_ret_xom + w_dri * avg_ret_dri + w_rf * rf_rate)

# Constraints (portfolio standard deviation equal to target, sum of weights equal to 1)
constraints = [
    {'type': 'eq', 'fun': lambda weights: portfolio_std_dev(weights, std_dev_xom, std_dev_dri, correlation_xom_dri)
    {'type': 'eq', 'fun': lambda weights: sum(weights) - 1}
]

# Bounds for weights (no short selling, weights between 0 and 1)
bounds = [(0, 1), (0, 1)]

# Initial guess (equally distributed weights)
initial_guess = [0.5, 0.5]

# Run the optimizer
result = minimize(negative_portfolio_return, initial_guess, args=(average_xom, average_dri, average_rf),
                  constraints=constraints, bounds=bounds, method='SLSQP')

print(result.x), (-result.fun)
```

```
[0.52905987 0.47094013]
```

Out[20]:
```
(None, 1.174889225677219)
```

The optimization results indicate the optimal weights for the portfolio that includes Exxon Mobil (XOM), Darden Restaurants (DRI), and the risk-free asset to achieve the target standard deviation of 15% per annum:

- Weight of Exxon Mobil (XOM): 52.91%
- Weight of Darden Restaurants (DRI): 47.09%
- Weight of the Risk-Free Asset: Essentially 0% (since the sum of the weights of XOM and DRI is approximately 100%)

The expected return of this portfolio is approximately 1.17% per month.

This result demonstrates the potential benefits of diversification between the two stocks, with no need to allocate to the risk-free asset to achieve the target risk level of 15% annualized standard deviation. The absence of the risk-free asset in the optimal portfolio suggests that the combination of these two stocks at these specific weights provides the necessary diversification and risk-return trade-off to meet the desired risk profile without needing to reduce exposure through a risk-free investment.
