Question 1:

Energy stored in an inductor is given by $\frac{1}{2}Li^2$*. Most of the energy is stored in*

(a) *The rate of change of flux in the air gap*
(b) *The mmf of the core material*
(c) *The permeance of the core*
(d) *The reluctance of the winding*

Solution: Correct option is (a)

Most of the energy is stored in the air gap.

Question 2:

Transformers with bifilar winding will have

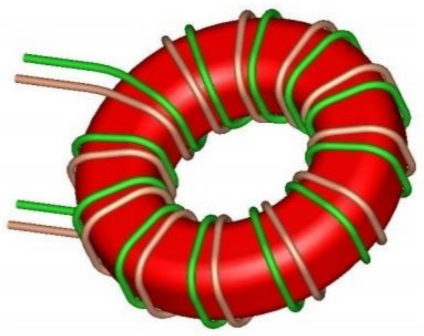
- *(a) Low interwinding capacitance, low leakage inductance and high insulation breakdown between windings*
- *(b) High interwinding capacitance, high leakage inductance and low insulation breakdown between windings*
- *(c) Low interwinding capacitance, low leakage inductance and low insulation breakdown between windings*
- *(d) High interwinding capacitance, low leakage inductance and low insulation breakdown between windings*

Solution: Correct option is (d)

Bifilar windings refer to a winding technique where two (or more) conductors are wound together in parallel around a core. These conductors are usually insulated from each other and can be connected in different configurations depending on the application. The term "bifilar" comes from *"bi-"* meaning two, and *"filar"* meaning thread or wire.

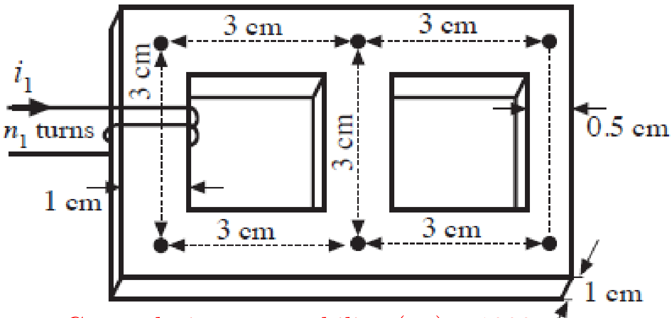
As the two conductors are placed close to each other they have very low leakage inductance and high interwinding capacitance. As the separation between the two parallel conductors is very low the insulation breakdown between the windings is low.

The figure below shows a simple bifilar winding wound around a toroidal core.



Question 3:

The core illustrated in Fig. (a) is 1 cm thick. All legs are 1 cm wide, except for the right-hand side vertical leg, which is 0.5 cm wide. You may neglect nonuniformities in the flux distribution caused by turning corners.

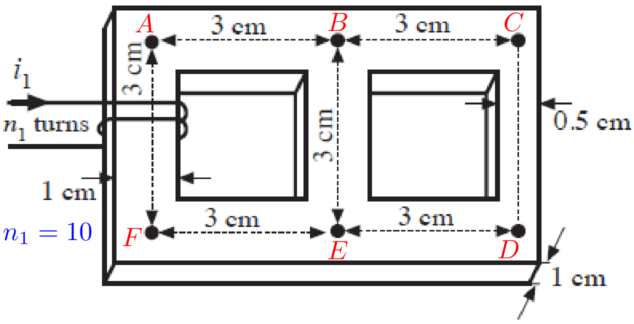


Core relative permeability (μ_r)= 1000

$n_1 = 10$

Determine the inductance of the winding in μH.

Solution: Answer range (109-111)



The mean magnetic path length $EFAB = l_1 = 9\ cm$

The area of cross-section of the core = $A_c = 1\ cm^2$

Thus, the reluctance of the path $EFAB$ is given by

$$\mathcal{R}_1 = \frac{l_1}{\mu_r \mu_o A_c} = \frac{9 \times 10^{-2}}{10^3 \times 4\pi \times 10^{-7} \times 10^{-4}} = 7.162 \times 10^5\ H^{-1}$$

The mean magnetic path length $BE = l_2 = 3\ cm$

Thus, the reluctance of the path BE is given by

$$\mathcal{R}_2 = \frac{l_2}{\mu_r \mu_o A_c} = \frac{3 \times 10^{-2}}{10^3 \times 4\pi \times 10^{-7} \times 10^{-4}} = 2.39 \times 10^5\ H^{-1}$$

The mean magnetic path length $BC = l_3 = 3\ cm$

Thus, the reluctance of the path BE is given by

$$\mathcal{R}_3 = \mathcal{R}_2 = \frac{l_3}{\mu_r \mu_o A_{c2}} = \frac{3 \times 10^{-2}}{10^3 \times 4\pi \times 10^{-7} \times 10^{-4}} = 2.39 \times 10^5\ H^{-1}$$

Similarly, the reluctance of the path ED is

$$\mathcal{R}_4 = \mathcal{R}_3 = \mathcal{R}_2 = \frac{l_3}{\mu_r \mu_o A_{c2}} = \frac{3 \times 10^{-2}}{10^3 \times 4\pi \times 10^{-7} \times 10^{-4}} = 2.39 \times 10^5\ H^{-1}$$

The width of the leg CD is 0.5 cm. Thus, the cross-sectional area of the core of this leg is half of A_c. Thus, we have the cross-sectional area of the leg (A_{c2}) and the mean length of the magnetic path as

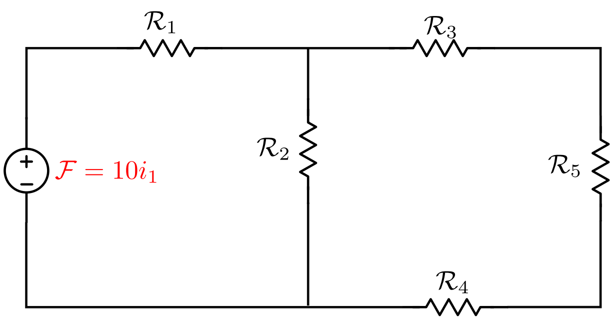
$$A_{c2} = 0.5\ cm^2\ ;\ l_5 = 0.3\ cm$$

Thus, the reluctance can be calculated as

$$\mathcal{R}_5 = \frac{l_5}{\mu_r \mu_o A_c} = \frac{3 \times 10^{-2}}{10^3 \times 4\pi \times 10^{-7} \times 0.5 \times 10^{-4}} = 4.77 \times 10^5\ H^{-1}$$

The magnetomotive force (MMF)=$\mathscr{F}$= $i_1 n_1 = 10 i_1$

The equivalent magnetic circuit can now be drawn as shown below



The reluctance as seen from the MMF source is

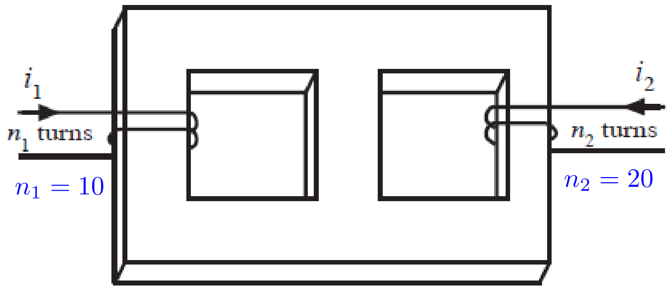
$$\mathcal{R}_{net} = \mathcal{R}_1 + \mathcal{R}_2 || (\mathcal{R}_3 + \mathcal{R}_4 + \mathcal{R}_5) = 9.07 \times 10^5\ H^{-1}$$

The inductance of the winding can then be calculated as

$$L = \frac{{n_1}^2}{\mathcal{R}_{net}} = \frac{100}{9.07 \times 10^5} = 110\ \mu H$$

Question 4:

A second winding is added to the same core described in Question 3. The new magnetic structure is illustrated in the figure below



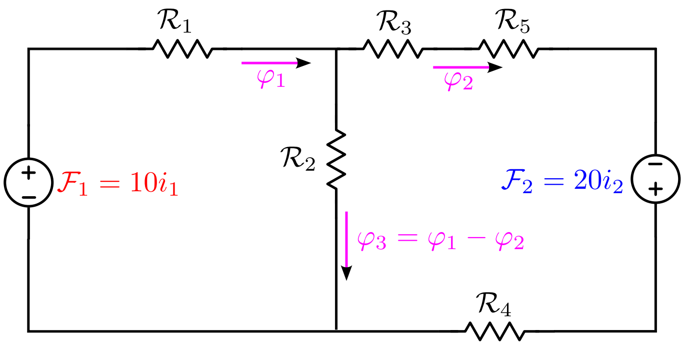
The electrical equations for this circuit may be written in the form

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix}$$

Find the value of L_{11} *in* μ*H*.

Solution: Answer range (109-111)

On adding the second winding the magnetic circuit gets modified as shown in the figure below



From Faraday's law and the first line of the matrix equation we have

$$v_1 = n_1 \frac{d\varphi_1}{dt} = L_{11} \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

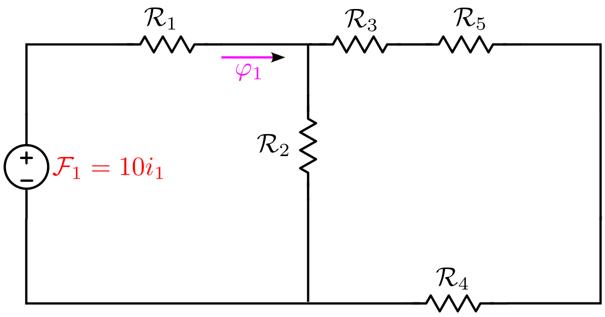
Thus, we have integrating both sides we get

$$\varphi_1 = \frac{L_{11}}{n_1} i_1 + \frac{L_{12}}{n_1} i_2$$

Thus, we have

$$L_{11} = \left. \frac{\varphi_1 n_1}{i_1} \right|_{i_2=0}$$

Now, if we make $i_2 = 0$, the magnetic circuit becomes



The reluctance as seen from the MMF source is

$$\mathcal{R}_{net} = \mathcal{R}_1 + \mathcal{R}_2||(\mathcal{R}_3 + \mathcal{R}_4 + \mathcal{R}_5) = 9.07 \times 10^5\ H^{-1}$$

Now, we can write

$$10i_1 = \mathcal{R}_{net} \times \varphi_1$$

Thus, we have

$$L_{11} = \left.\frac{\varphi_1 n_1}{i_1}\right|_{i_2=0} = \frac{10n_1}{\mathcal{R}_{net}} = \frac{100}{9.07 \times 10^5} = 110\ \mu H$$

Question 5:

In continuation with Question 4, Find the value of L_{12} in μH.

Solution: Answer range (43.5 – 44.5)

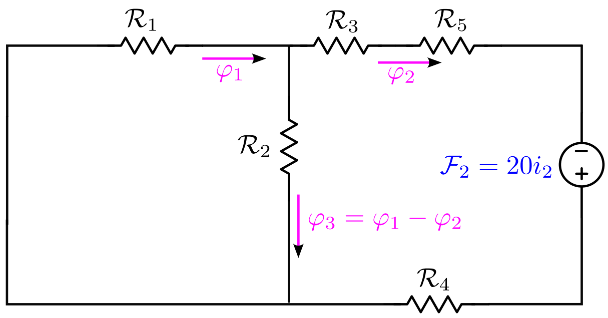
As discussed in the solution of *Question 4*

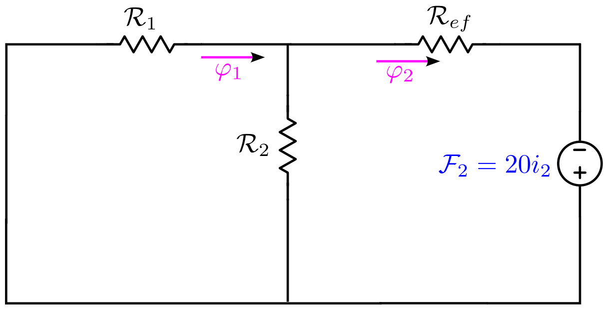
$$\varphi_1 = \frac{L_{11}}{n_1} i_1 + \frac{L_{12}}{n_1} i_2$$

Thus, we have

$$L_{12} = \left.\frac{\varphi_1 n_1}{i_2}\right|_{i_2=0}$$

Now, if we make $i_1 = 0$, the magnetic circuit becomes

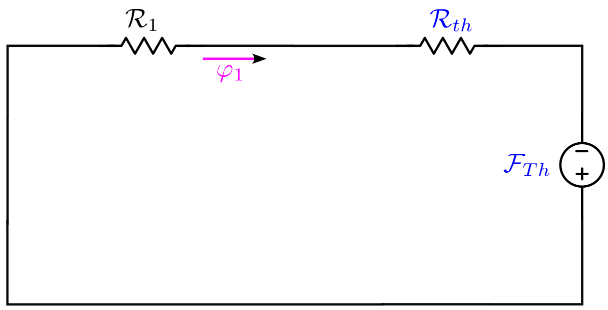




Where,

$$\mathcal{R}_{ef} = \mathcal{R}_3 + \mathcal{R}_4 + \mathcal{R}_5 = 9.55 \times 10^5\ H^{-1}$$

By applying Thevenin's theorem we can redraw the circuit as



Where,

$$\mathcal{R}_{th} = \frac{\mathcal{R}_{ef} \times \mathcal{R}_2}{\mathcal{R}_{ef} + \mathcal{R}_2} = 1.91 \times 10^5\ H^{-1}$$

$$\mathcal{F}_{th} = \frac{\mathcal{R}_2}{\mathcal{R}_{ef} + \mathcal{R}_2} \times \mathcal{F}_2 = 4i_2$$

Now, φ_1 can be expressed as

$$\varphi_1 = \frac{\mathcal{F}_{th}}{\mathcal{R}_{th} + \mathcal{R}_1} = \frac{4i_2}{9.07 \times 10^5}$$

Thus, we now have

$$L_{12} = \left.\frac{\varphi_1 n_1}{i_2}\right|_{i_2=0} = \frac{4i_2}{9.07 \times 10^5} \times \frac{n_1}{i_2} = 44.1\ \mu H$$

Question 6:

In continuation with Question 4, Find the value of L_{21} in μH.

Solution: Answer range (43.5 – 44.5)

From the second line of the matrix equation and Faraday's law we have

$$v_2 = n_2 \frac{d\varphi_2}{dt} = L_{21} \frac{di_1}{dt} + L_{22} \frac{di_2}{dt}$$

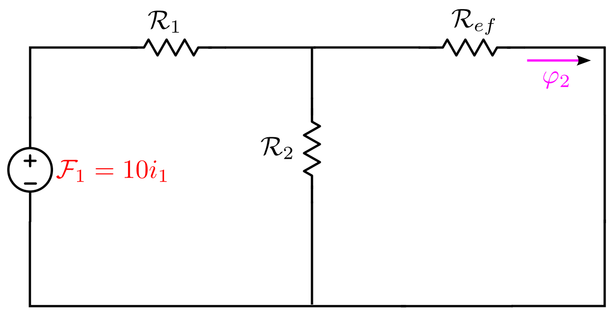
Thus, we have integrating both sides we get

$$\varphi_2 = \frac{L_{21}}{n_2} i_1 + \frac{L_{22}}{n_2} i_2$$

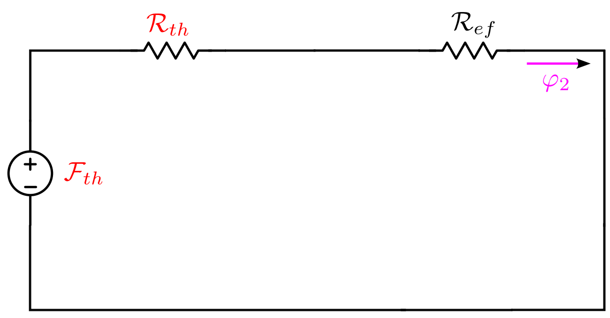
Thus, we have

$$L_{21} = \frac{\varphi_2 n_2}{i_1}\bigg|_{i_2=0}$$

Now, if we make $i_2 = 0$, the magnetic circuit becomes



By applying Thevenin's theorem, the circuit can be redrawn as



Where,

$$\mathcal{R}_{th} = \frac{\mathcal{R}_1 \times \mathcal{R}_2}{\mathcal{R}_1 + \mathcal{R}_2} = 1.79 \times 10^5\ H^{-1}$$

$$\mathcal{F}_{th} = \frac{\mathcal{R}_2}{\mathcal{R}_1 + \mathcal{R}_2} \times \mathcal{F}_1 = 2.5i_1$$

Thus, we can calculate φ_2 as

$$\varphi_2 = \frac{\mathcal{F}_{th}}{\mathcal{R}_{th} + \mathcal{R}_{ef}} = \frac{2.5 i_1}{11.34 \times 10^5}$$

Thus, we can calculate L_{21} as

$$L_{21} = \left. \frac{\varphi_2 n_2}{i_1} \right|_{i_2=0} = \frac{2.5 i_1}{11.34 \times 10^5} \times \frac{n_2}{i_1} = 44.09\ \mu H$$

Question 7:

In continuation with Question 4, Find the value of L_{22} in μH.

Solution: Answer range (351-354)

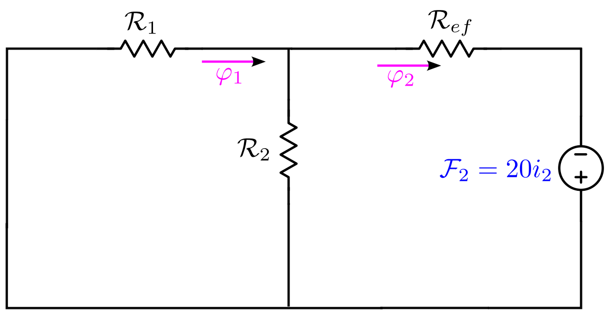
We have already seen in the solution to previous question that

$$\varphi_2 = \frac{L_{21}}{n_2} i_1 + \frac{L_{22}}{n_2} i_2$$

Thus, we have

$$L_{22} = \left.\frac{\varphi_2 n_2}{i_2}\right|_{i_1=0}$$

As we have already seen in the solution to *Question 5,* if we make $i_1 = 0$, the magnetic circuit becomes



The net reluctance as seen from the MMF source $\mathcal{F}_2$ can be calculated as

$$\mathcal{R}_{net} = \mathcal{R}_{ef} + \mathcal{R}_1 || \mathcal{R}_2 = 11.34 \times 10^5 \ H^{-1}$$

Thus, we can calculate φ_2 as

$$\varphi_2 = \frac{\mathcal{F}_2}{\mathcal{R}_{net}} = \frac{20 i_2}{11.34 \times 10^5}$$

Thus, we can calculate L_{22} as

$$L_{22} = \left.\frac{\varphi_2 n_2}{i_2}\right|_{i_1=0} = \frac{20 i_2}{11.34 \times 10^5} \times \frac{n_2}{i_2} = 352.73 \ \mu H$$

Question 8:

The manufacturer's data sheet includes the plot for the core loss of the ferrite material as shown in the figure below. This plot passes through the points

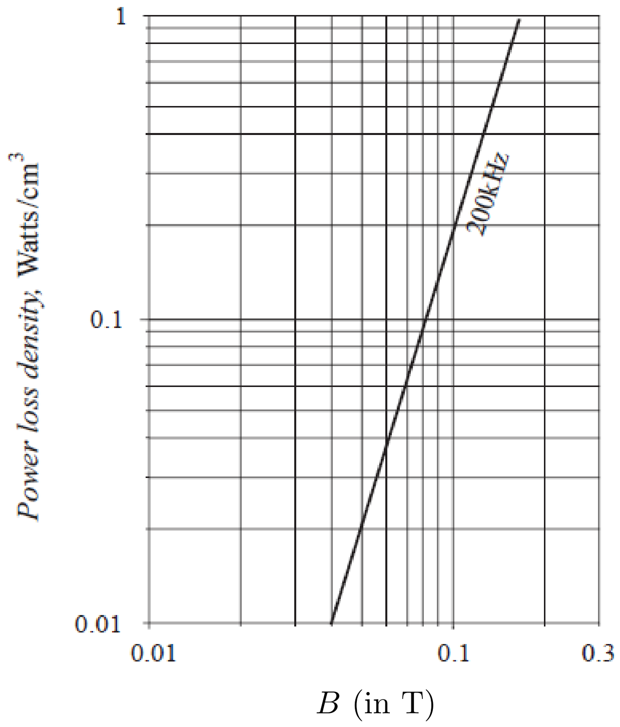
- *0.2* W/cm^3 *at* $\Delta B = 0.1\,T$
- *0.01*W/cm^3 *at* $\Delta B = 0.04T$

The Steinmetz empirical equation for the core loss at 200 kHz of the form

$$(power\ loss\ density) = K_{fe} B_{max}^{\beta}$$

where B_{max} *is the peak sinusoidal flux density in Tesla and the power loss density is expressed in* W/cm^3.

Find the numerical value of K_{fe} *rounded off to two decimal places.*



Solution: Answer range (370-372)

The Steinmetz equation is of the form

$$P = K_{fe} B^{\beta}$$

The curve given to us in the question is plotted in logarithmic scale. So, taking log on both sides we end up getting

$$\log P = \log K_{fe} + \beta \log B$$

Given that the curve passes through two points as

- $P_1 = 0.2\ W/cm^3 \qquad B_1 = 0.1\ T$
- $P_2 = 0.01\ W/cm^3 \qquad B_2 = 0.04\ T$

Putting these values in the above equation we get

$$\log P_1 = \log K_{fe} + \beta \log B_1$$

$$\log P_2 = \log K_{fe} + \beta \log B_2$$

Thus, we can express β from the above equation as

$$\beta = \frac{\log P_1 - \log K_{fe}}{\log B_1} = \frac{\log P_2 - \log K_{fe}}{\log B_2}$$

Solving the above equation, we can obtain the expression for $\log K_{fe}$ as

$$\log K_{fe} = \left(\frac{\log P_1}{\log B_1} - \frac{\log P_2}{\log B_2}\right) \times \frac{\log B_1 \times \log B_2}{\log B_2 - \log B_1} = 2.57$$

Thus, we can obtain the value of

$$K_{fe} = 10^{2.57} = 371.91$$

Question 9:

For the core loss data of Question 8, enter the numerical value of β (rounded off to two decimal places).

Solution: Answer range (3.2-3.3)

As given in *Question 8,* the power loss curve passes through the point

- $P_1 = 0.2\, W/cm^3 \qquad B_1 = 0.1\, T$

In solution to *Question 8,* we have already obtained the value of K_{fe} as

$$K_{fe} = 371.91$$

Putting these values into Steinmetz equation we can write

$$P = K_{fe} B^{\beta}$$

Taking log on both sides we end up getting

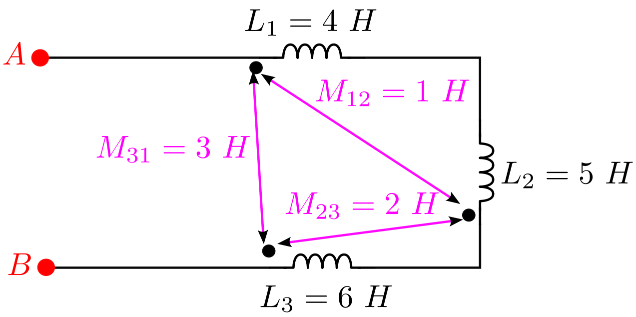
$$\log P_1 = \log K_{fe} + \beta \log B_1$$

Thus, the value of β can be obtained as

$$\beta = \frac{\log P_1 - \log K_{fe}}{\log B_1} = 3.27$$

Question 10:

The effective inductance of the circuit across the terminals $A - B$ in the figure shown below is



(a) 9 H
(b) 21 H
(c) 11 H
(d) 6 H

Solution: Correct option is (c)

Apply KVL we can write

$$v_{AB} = \left(L_1\frac{di_1}{dt} - M_{12}\frac{di_1}{dt} - M_{31}\frac{di_1}{dt}\right) + \left(L_2\frac{di_1}{dt} - M_{12}\frac{di_1}{dt} + M_{23}\frac{di_1}{dt}\right) + \left(L_3\frac{di_1}{dt} - M_{31}\frac{di_1}{dt} + M_{23}\frac{di_1}{dt}\right)$$

Thus, we can write the above equation as

$$v_{AB} = (L_1 + L_2 + L_3)\frac{di_1}{dt} + (2M_{23} - 2M_{12} - 2M_{31})\frac{di_1}{dt}$$

Thus, the equivalent inductance as seen from the terminals $A - B$ is

$$L_{eq} = L_1 + L_2 + L_3 + 2M_{23} - 2M_{12} - 2M_{31} = 11\ H$$