FIR Filter Design Solved Examples

Q(1) **FIR** filter described by the difference equation : y(n) = x (n) + x (n – 4)

(a) Compute and sketch magnitude and phase response.

(b) Find its response to the input $x(n) = \cos\left(\frac{\pi}{2}n\right) + \cos\left(\frac{\pi}{4}n\right), -\infty < n < \infty.$

Solution :

(a) To find **M**agnitude and Phase Response

Given y (n) = x(n) + x (n – 4)

(i) By ZT,

$$Y(z) = X(z) + z^{-4} X(z)$$

$$Y(z) = X(z)\ (1 + z^{-4})$$

$$H(z) = 1 + z^{-4}$$

Put $z = e^{jw}$

$$H(e^{jw}) = 1 + e^{-j4w}$$

$$= e^{-j2w}\left[\ e^{j2w} + e^{-j2w}\right]$$

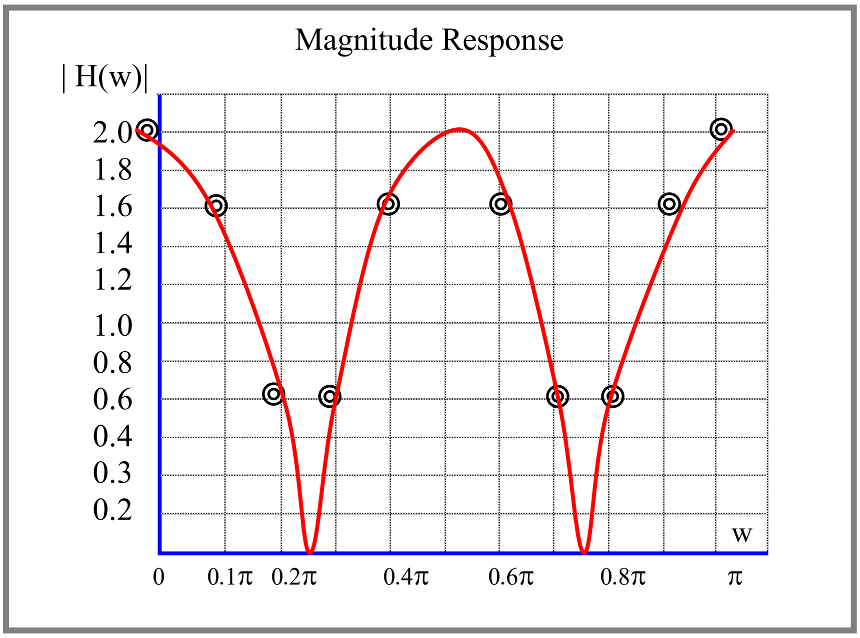
$$H(e^{jw}) = e^{-j2w}\left[\ 2\ \cos(2w)\ \right]$$

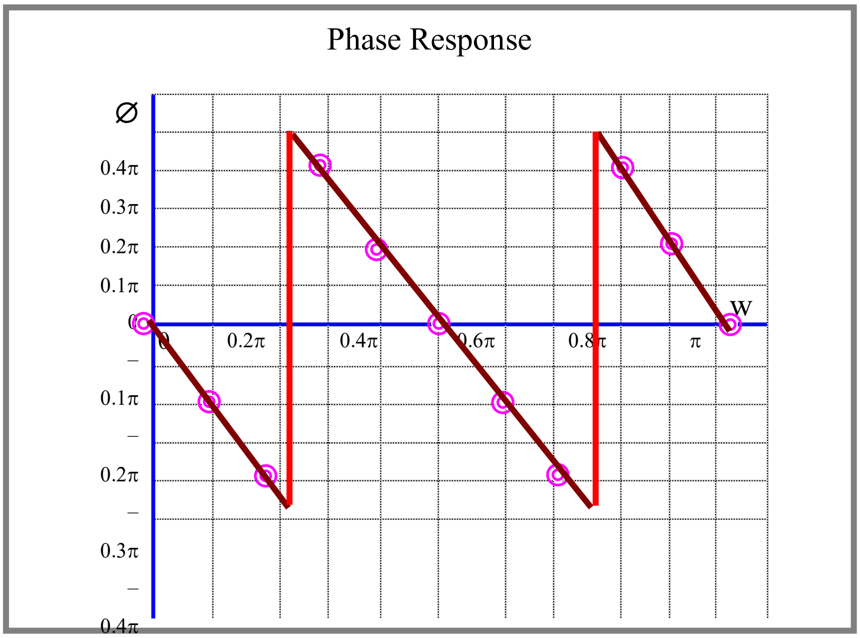
(i) Magnitude Response $M(w) = |\ Hr(w)\ | = |\ 2\cos(2w)\ |$

(ii) Phase Response : $\phi(w) = e^{-j2w}$

(iii) Phase : $\phi = -2w$

	W	Hr(w)	**Phase : ϕ**
1	0	2	0
2	0.1 π	1.62	– 0.2 π
3	0.2 π	0.62	– 0.4 π
4	0.3 π	– 0.62	– 0.6 π + π = 0.4 π
5	0.4 π	– 1.62	– 0.8 π + π = 0.2 π
6	0.5 π	– 2	– π + π = 0
7	0.6 π	–1.62	– 1.2 π + π = – 0.2 π
8	0.7 π	– 0.62	– 1.4 π + π = – 0.4 π
9	0.8 π	0.62	– 1.6 π + 2 π = 0.4 π
10	0.9 π	1.62	–1.8 π + 2 π = 0.2 π
11	π	2	– 2 π + 2 π = 0





Solution : (b) To find response :

The frequency components present in the input signal x(n) are,

$$w_1 = \frac{\pi}{2} \qquad \text{and} \qquad w_2 = \frac{\pi}{4}$$

At $w_1 = \frac{\pi}{2}$ $\quad M(w) = \left|2 \cos\left(2\frac{\pi}{2}\right)\right| = |2 \cos(\pi)| = 2$

$$\phi = -2\left(\frac{\pi}{2}\right) + \pi = 0$$

At $w_2 = \frac{\pi}{4}$ $\quad M(w) = \left|2 \cos\left(2\frac{\pi}{4}\right)\right| = \left|2 \cos(\frac{\pi}{2})\right| = 0$

$$\phi = -2\left(\frac{\pi}{4}\right) = -\frac{\pi}{2}$$

The Steady State Response due to x(n) is then given by,

$y\,(n) = 2\cos\left(n\frac{\pi}{2}\right)$ **ANS**

Q(2) Determine the frequency response of FIR filter defined by $y[n] = 0.25\ x[n] + x[n-1] + 0.25\,x[n-2]$. Calculate Phase Delay and Group Delay.

Solution :

$$y[n] = 0.25\ x[n] + x[n-1] + 0.25\,x[n-2]$$

$$Y(z) = 0.25\ X(z) + z^{-1}X(z) + 0.25\ z^{-2}\ X(z)$$

$$Y(z) = X(z)\ \left(0.25 + z^{-1} + 0.25\ z^{-2}\right)$$

$$H(z) = 0.25 + z^{-1} + 0.25\ z^{-2}$$

$$H(e^{jw}) = 0.25 + e^{-jw} + 0.25\ e^{-j2w}$$

$$H(e^{jw}) = e^{-jw}\left[\ 0.25\ \ e^{jw} + \ 1\ + 0.25\ e^{-jw}\ \right]$$

$$H(e^{jw}) = e^{-jw}\left[\ 1 + 0.5\ \cos(w)\ \right]$$

$$Group\ Delay\quad \tau_g = \frac{-d\phi(w)}{dw} == 1 \qquad\qquad Phase\ Delay\quad \tau_g = \frac{-\phi(w)}{w} == 1$$

Q(3) One of the zeros of antisymmetry FIR filter is at 0.5 ∠60. Show locations of other zeros. What is minimum order of this filter?

Solution :

ZEROs of a linear phase filter occur at reciprocal location

$z_0 = 0.5\ \angle 60^0$ or $0.5\ e^{\frac{\pi}{3}}$

$z_0^* = 0.5\ \angle -60^0$ or $0.5\ e^{-\frac{\pi}{3}}$

$\frac{1}{z_0} = 2\angle -60$ or $2e^{-\frac{\pi}{3}}$

$\frac{1}{z_0^*} = 2\ \angle 60^0$ or $2e^{\frac{\pi}{3}}$

No of ZEROS = 4 , so Order of filter is 4

Q(4) Show three possible POLE ZERO pattern of 4th order Linear Phase FIR filter with (a) symmetric h[n] and (b) Anti-symmetric h[n]

Solution : Linear Phase FIR filter

Order = 4

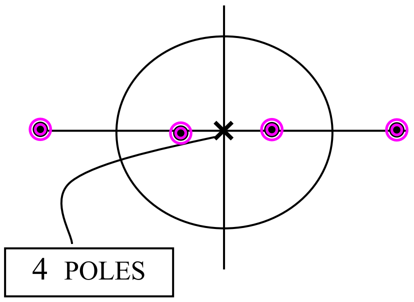
N – 1 = 4

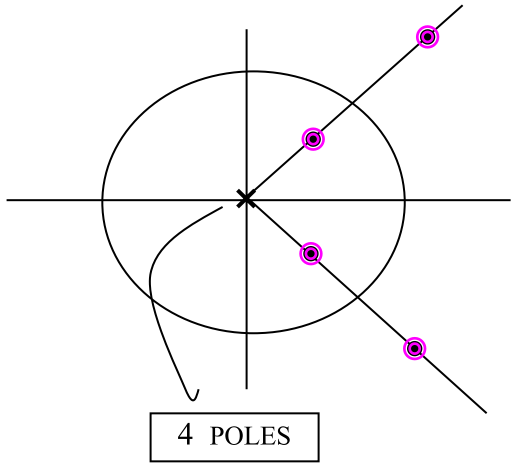
N = 5

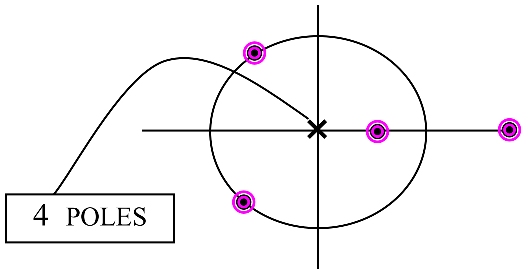
Let No of POLES = 4

Let No of ZEROS = 4

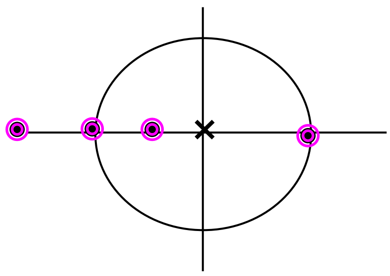
(a) Symmetric h[n]

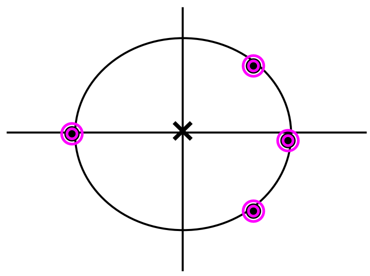






(b) Anti-symmetric h[n] with N odd has definite zeros at z = 1 and z = -1





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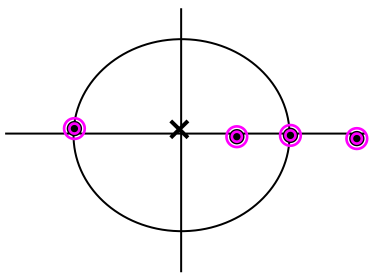
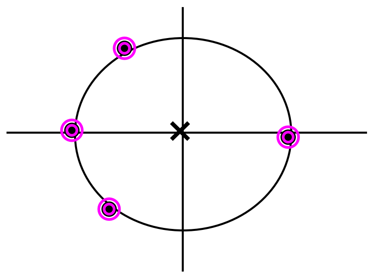




Q(5) One of the ZEROS of a causal linear phase FIR filter is at 0.5 ∠60. Show the locations of the other zeros and hence find transfer function and impulse response of the filter.

Solution :

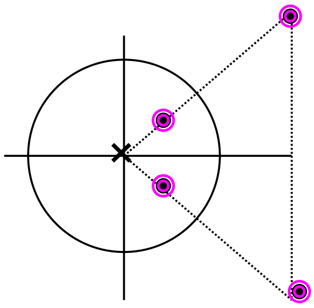
ZEROs of a linear phase filter occur at reciprocal location

$$z_0 = 0.5 \ \angle 60^0 \quad \text{or} \quad 0.5\ e^{\frac{\pi}{3}}$$

$$z_0^* = 0.5 \ \angle -60^0 \quad \text{or} \quad 0.5\ e^{-\frac{\pi}{3}}$$

$$\frac{1}{z_0} = 2 \angle -60 \quad \text{or} \quad 2e^{-\frac{\pi}{3}}$$

$$\frac{1}{z_0^*} = 2 \ \angle 60^0 \quad \text{or} \quad 2e^{\frac{\pi}{3}}$$



To find H[z] :

$$H(z) = \frac{\left(z - \frac{1}{2} e^{\frac{\pi}{3}}\right)\left(z - \frac{1}{2} e^{-\frac{\pi}{3}}\right)(z - 2\ e^{\frac{\pi}{3}})\ (z - 2e^{-\frac{\pi}{3}})}{z^4}$$

$$H(z) = \frac{\left[z^2 - \frac{1}{2}\ (e^{\frac{\pi}{3}} + e^{-\frac{\pi}{3}})\ 2 + \frac{1}{4}\right]\left[z^2 - 2\ (e^{\frac{\pi}{3}} + e^{-\frac{\pi}{3}})\ z + 4\right]}{z^4}$$

$$H(z) = \frac{\left[z^2 - \cos\frac{\pi}{3} z + \frac{1}{4}\right]\left[z^2 - 4\ \cos\frac{\pi}{3} z + 4\right]}{z^4}$$

$$H(z) = \frac{\left[z^2 - \frac{1}{2} z + \frac{1}{4}\right][z^2 - 2z + 4]}{z^4}$$

$$H(z) = \frac{z^4 - 2.5z^3 + 5.25\ z^2 - 2.5\ z + 1}{z^4}$$

$$H(z) = 1 - 2.5\ z^{-1} + 5.25\ z^{-2} - 2.5\ z^{-3} + z^{-4}$$

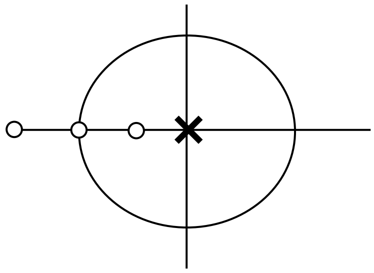
By I ZT, $h(n) = \left\{ \underset{\uparrow}{1}\ \ -2.5,\ \ 5.25,\ \ -2.5, 1 \right\}$

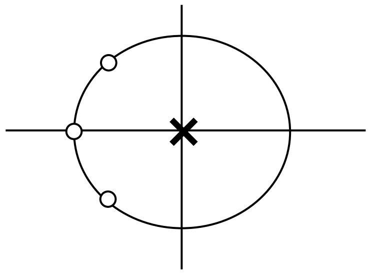
Q(6) Draw POLE-ZERO location of third order linear phase LPF & HPF.

Solution : Linear Phase FIR filter

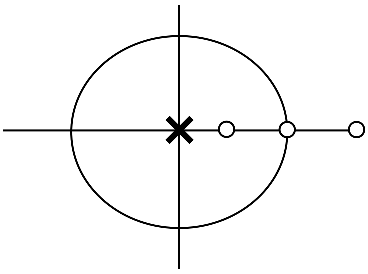
Order = 3 | Let no of POLES = 3
$N - 1 = 3$ | No of ZEROS = 3
$N = 4$ (EVEN)

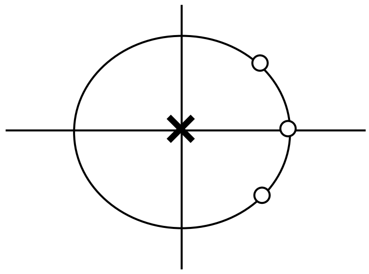
I. Linear Phase LPF with Symmetric h[n] and N EVEN
(Definite ZERO at $z = -1$)





II. Linear Phase HPF with Anti-Symmetric h[n] and N EVEN.
(Definite ZERO at $z = 1$)



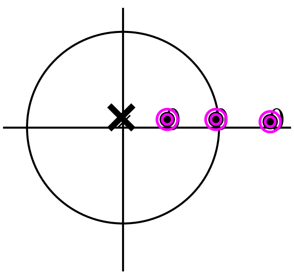


Q(7) **An antisymmetric filter has one ZERO at $z = \frac{1}{2}$. What is the minimum order of this filter? Justify your answer.**

Solution :

ZEROs of a linear phase filter occur at reciprocal location $z_0 = \frac{1}{2}$. $\therefore = \frac{1}{z_0} = 2$.

For Antisymmetric h[n] there exists definite ZERO at $z = 1$.



Total No of ZERO = 3
$\therefore$ Order = 3
i.e. N-1 = 3
so N = 4 (Even)

Minimum order is 3.

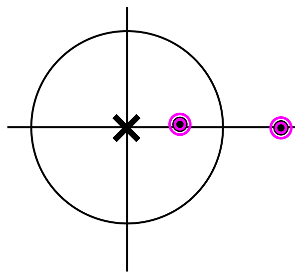
Q(8) One of the zeros of a causal linear phase FIR filter lies at $z = \frac{1}{2}$ Find the location of the other zero and hence find the transfer function and impulse response of the filter.

Solution:

Zero of a linear phase filter occur at reciprocal location $z_0 = \frac{1}{2}$. $\therefore = \frac{1}{z_0} = 2$.

$$H(z) = \frac{\left(z - \frac{1}{2}\right)(z-2)}{z^2} = \frac{z^2 - \frac{5}{2}z + 1}{2}$$

$$H(z) = 1 - \frac{5}{2}z^{-1} + z^{-2}$$



ANS : $h(n) = \left\{ 1, \frac{-5}{2}, 1 \right\}$

Q(9) One of the zeros of a third order causal linear phase High pass FIR filter lies at $z = \frac{1}{2}$ Find the location of the other zeros and hence find the transfer function and impulse response of the filter.

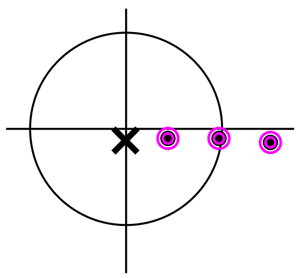
Solution :

For HPF, consider antisymmetric h[n] with N even. For Anti symmetric h[n] with N even there exists definite zero at z = 1.

Zero of a linear phase filter occur at reciprocal location $z_0 = \frac{1}{2}$. $\therefore = \frac{1}{z_0} = 2$.

$$H(z) = \frac{\left(z - \frac{1}{2}\right)(z-2)\ (z-1)}{z^3}$$

$$H(z) = \frac{\left(z^2 - \frac{5}{2}z + 1\right)(z-1)}{z^3}$$



$$H(z) = \frac{z^3 - \frac{7}{2}z^2 + \frac{7}{2}z - 1}{z^3}$$

$$H(z) = 1 - \frac{7}{2}z^{-1} + \frac{7}{2}z^{-2} - z^{-3}$$

ANS : $h(n) = \left\{ 1, \frac{-7}{2}, \frac{7}{2}, -1 \right\}$

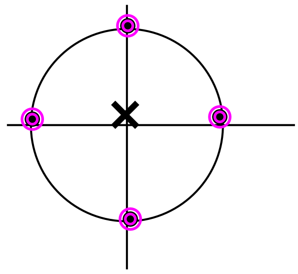
Q(10) Obtain the POLE-ZERO plot of a Causal Symmetrical Linear Phase FIR filter with ODD number of Coefficients, assuming smallest length, if it is known to have ZEROs at $z = j$, $z = \mathbf{1}$ and $z = \mathbf{-1}$.

Solution :

For a Causal Symmetrical Linear Phase FIR filter with ODD number of Coefficients, there are NO definite ZEROs.

ZEROS of a Linear Phase filter always occur at reciprocal order.

For a ZERO at $z = j$, there will be another ZERO at $z = -j$.



No of ZEROs = 4
Order N-1= 4

Q(11) Determine the coefficients of High Pass linear phase FIR filter of length N = 4 which has frequency response such that, $\left|H\left(\pi/4\right)\right| = 1/2$ and $\left|H\left(\frac{3\pi}{4}\right)\right| = 1.$

Solution :

For high pass filter with N = 4 (even) h[n] must be Antisymmetric.

Let $\quad h[n] = \{ h_0, h_1, -h_1, -h_o \}$

By ZT, $H(z) = h_0 + h_1 z^{-1} - h_1 z^{-2} - h_o z^{-3}$
By ZT, $H(z) = h_0 + h_1 z^{-1} - h_1 z^{-2} - h_o z^{-3}$

Put $z = e^{jw}$,

$$H(e^{jw}) = h_0 + h_1 e^{-jw} - h_1 e^{-j2w} - h_o e^{-j3w}$$

$$H(e^{jw}) = e^{-j3/2\,w}\left[h_0 e^{+j\frac{3}{2}w} + h_1 e^{j1/2\,w} - h_1 e^{-j\frac{1}{2}w} - h_0 e^{-j\frac{3}{2}w}\right]$$

$$H(w) = e^{-j\frac{3}{2}w}\left[2jh_0 \sin\left(\frac{3}{2}w\right) - 2j\, h_1 \sin\left(\frac{1}{2}w\right)\right]$$

$$H(w) = e^{-j\frac{3}{2}w}\; e^{j\frac{\pi}{2}}\left[2\, h_0 \sin\left(\frac{3}{2}w\right) - 2h_1 \sin\left(\frac{1}{2}w\right)\right]$$

(i) At $w = \pi/4$

$$\left|H\left(\frac{\pi}{4}\right)\right| = 2\, h_0 \sin\left(\frac{3\pi}{8}\right) - 2\, h_1 \sin\left(\frac{\pi}{8}\right)$$

$$1/2 = 1.85\, h_0 + 0.75\, h_1 \quad \_\_\_\_\_\_\_\_\text{(i)}$$

(ii) At $w = \dfrac{3\pi}{4}$

$$\left| H\left(\frac{\pi}{4}\right) \right| = 2\,h_0 \sin\left(\frac{9\pi}{8}\right) - 2h_1 \sin\left(\frac{3\pi}{8}\right)$$

$$1 = -0.765\,h_o + 1.85\,h_1 \quad \text{-----------(ii)}$$

Solving equation (i) & (ii) we get, $h_o = 0.04$ and $h_1 = 0.56$

ANS : $h[n] = \{\, 0.04 ,\ 0.56,.\ -0.56,\ -0.04 \,\}$ for $n \geq 0$.

Q(12) A High Pass Linear Phase FIR filter has a magnitude response :-
$|\ H(e^{jw})\ | = 4\ \sin(aw) - 3\ \sin(bw)$
Find values of ‘a’ and ‘b’ assuming minimum value of N. Obtain corresponding Impulse Response.

Solution :

Given $|\ H(e^{jw})\ | = 4\ \sin(aw) - 3\ \sin(bw)$ --------------(I)

For High Pass filter, Consider Antisymmetric h[n] with N=4 (EVEN),
$h\,[n] = \{\, h_0, h_1, -h_1, -h_o \,\}$
Let $h\,[n] = \{\, h_0, h_1, -h_1, -h_o \,\}$

By ZT, $H\,(z) = h_0 + h_1\ z^{-1} - h_1\ z^{-2} - h_o\, z^{-3}$
By ZT, $H\,(z) = h_0 + h_1\ z^{-1} - h_1\ z^{-2} - h_o\, z^{-3}$

Put $z = e^{jw}$,

$$H\,(e^{jw}) = h_0 + h_1\ e^{-jw} - h_1\ e^{-j2w} - h_o\, e^{-j3w}$$

$$H\,(e^{jw}) = e^{-j3/2 w}\left[h_0\, e^{+j\frac{3}{2}w} + h_1 e^{j1/2 w} - h_1 e^{-j\frac{1}{2}w} - h_0 e^{-j\frac{3}{2}w} \right]$$

$$H(w) = e^{-j\frac{3}{2}w}\left[2\,j h_0 \sin\left(\frac{3}{2}w\right) - 2j\ h_1 \sin\left(\frac{1}{2}w)\right) \right]$$

$$H\,(w) = e^{-j\frac{3}{2}w}\ e^{j\frac{\pi}{2}}\ \left[2\ h_0\ \sin\ (1.5\ w) - 2\,h_1\ \sin\ (0.5\ w) \right]$$

The Magnitude Response is given by,

$$|\ H(e^{jw})\ | = |\ 2\ h_0\ \sin\ (1.5\ w) - 2\,h_1\ \sin\ (0.5\ w)\ | \quad \text{----------------(II)}$$

By comparing equation I and II we get,

a = 1.5 and b = 0.5 **$h_0 = 2$ and $h_1 = 1.5$** **ANS**

So, h[n] = { 2, 1.5, - 1.5, -2 }

Q(13) One of the zeros of an anti-symmetric FIR filter lies at $Z = 0.2e^{j\pi/3}$.
What is the minimum order of this filter? Show all zero locations ?.

Solution :

Zeros of a linear phase filter occur at reciprocal location

$$z_0 = 0.2\ e^{j\frac{\pi}{3}} \qquad z_0^* = 0.2\ e^{-j\frac{\pi}{3}}$$

$$\frac{1}{z_0} = 5\ e^{-j\frac{\pi}{3}} \qquad \frac{1}{z_0^*} = 5\ e^{j\frac{\pi}{3}}$$

For anti-symmetric FIR filter with N Even there exists fixed ZERO at z = –1

For anti-symmetric FIR filter with N Odd there exists fixed ZERO at z=–1 and z=1

For minimum order of filter consider fixed ZERO at z=–1

So Total No of ZEROS = 4+1 ==5 , Therefore Order of filter is 5

Q(14) Determine the coefficients of a linear phase FIR filter

$$y[n] = bo\ x[n] + b_1\ x[n-1] + b_2\ x[n-2] \quad \text{such that :}$$

(a) It rejects completely a frequency component at $w_o = 2\pi/3$.
(b) Its frequency response is normalized so that H(w) = 1 at w=0.
(c) Compute and sketch the magnitude response and phase response

Solution :

$$y[n] = bo\ x[n] + b_1\ x[n-1] + b_2\ x[n-2]$$
$$Y(z) = b_oX(z) + b_1\ z^{-1}X(z) + b_1\ z^{-2}\ X(z)$$
$$Y(z) = X(z)\ (\ b_o + b_1\ z^{-1} + b_1\ z^{-2}\ )$$
$$\frac{Y(z)}{X(z)} = b_o + b_1\ z^{-1} + b_2\ z^{-2}$$
$$H(z) = b_o + b_1\ z^{-1} + b_2\ z^{-2}$$

Put $z = e^{jw}$

$$H(z) = b_o + b_1\ e^{-jw} + b_2\ e^{-j2w}$$

For Linear Phase FIR filter with symmetric h[n],
h[n]=h[N-1-n]
i.e. $b_0 = b_2$

$$H(w) = b_o + b_1\ e^{-jw} + b_0\ e^{-j2w}$$
$$H(w) = e^{-jw}\left(b_o\ e^{jw} + b_1 + b_0\ e^{-jw}\right)$$
$$H(w) = e^{-jw}\ \left\{\ 2b_o\ \cos(w) + b_1\ \right\}$$

(i) It rejects completely a frequency component at $w_o = 2\pi/3$.

That means, $H(w)\big|_{w=\frac{2\pi}{3}} = \left\{ 2b_o \cos\left(\frac{2\pi}{3}\right) + b_1 \right\} = 0$

$-b_o + b_1 = 0$

$\therefore b_o = b_1$ ---((1))

(ii) Its frequency response is normalized so that H(w) = 1 at w=0.

$H(w)\big|_{w=0} = \{ 2b_o \cos(0) + b_1 \} = 1$

$2b_o + b_1 = 1$ ------((2))

Put $b_o = b_1$

$2b_q + b_0 = 1$

$b_o = \frac{1}{3}$

Finally, $b_o = b_1 = b_2 = \frac{1}{3}$

Therefore, $H(z) = \frac{1}{3} + \frac{1}{3}z^{-1} + \frac{1}{3}z^{-2}$

(iii) Magnitude Response and Phase Response

Now, $H(w) = e^{-jw}\{ 2b_o \cos(w) + b_1 \}$

$H(w) = e^{-jw}\{ \frac{2}{3}\cos(w) + \frac{1}{3} \}$

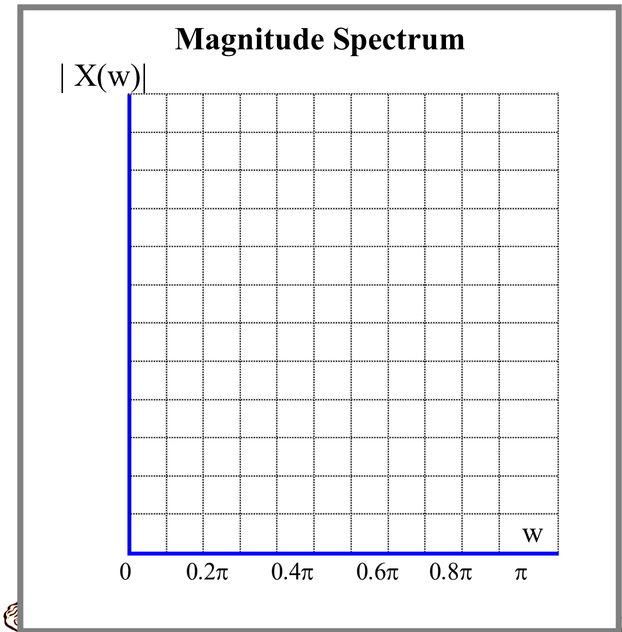
(i) Magnitude Response

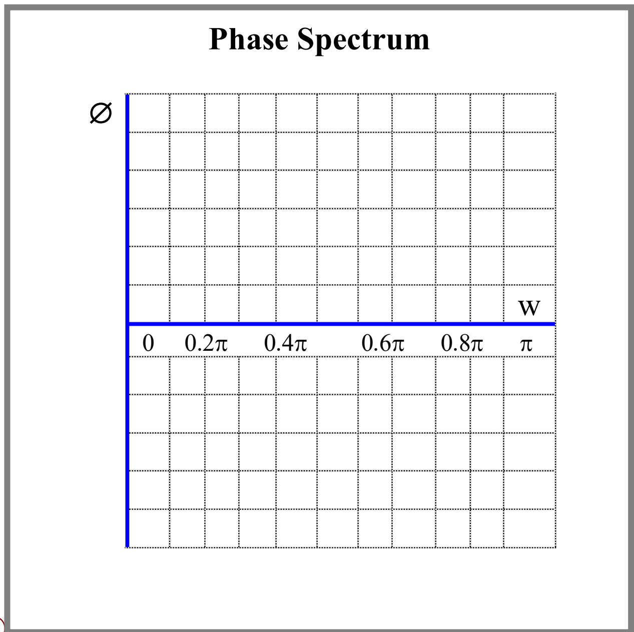
$|H(w)| = |Hr(w)| = \{ \frac{2}{3}\cos(w) + \frac{1}{3} \}$

(ii) Phase Response : $\phi(w) = e^{-jw}$

(iii) Phase : $\phi = -w$

W	$H_r(w)$	ϕ
0	0.99	0
0.1 π	**0.96**	–0.1 π
0.2 π	**0.86**	–0.2 π
0.3 π	**0.72**	–0.3 π
0.4 π	**0.53**	–0.4 π
0.5 π	**0.33**	–0.5 π
0.6 π	**0.12**	–0.6 π
0.7 π	**– 0.05**	–0.7 π+π = 0.3 π
0.8 π	**– 0.20**	–0.8 π+π = 0.2 π
0.9 π	**– 0.29**	–0.9 π+π = 0.1 π
π	**– 0.32**	–π+π = 0





Q(15) Given Linear Phase FIR filter Impulse Response h[n] = {0.318, 0.45, 0.5, 0.45, 0.318 }

Find Response of the filter

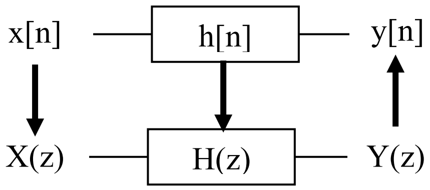
(a) Input : $x[n] = (\frac{1}{2})^n \cos(n\frac{\pi}{3})\, u[n]$ (b) **Input :** $x[n] = (\frac{1}{2})^n \cos(n\frac{\pi}{3})$

Solution : (a) To find Response of the filter

Given Input : $x[n] = (\frac{1}{2})^n \cos(n\frac{\pi}{3})\, u[n]$

Input is sinusoidal, Infinite Length and applied to the system at n = 0

Impulse Response is **:** h[n] = { 0.318, 0.45, 0.5, 0.45, 0.318 } Finite Length



Find H(z) :

Let $h[n] = \{ h_o, h_1, h_2, h_1, h_o \}$

By ZT, $H(z) = h_o + h_1 z^{-1} + h_2 z^{-2} + h_1 z^{-3} + h_o z^{-4}$

Find Y(z) :

$Y(z) = H(z)\, X(z)$

$Y(z) = (h_o + h_1 z^{-1} + h_2 z^{-2} + h_1 z^{-3} + h_o z^{-4})\, X(z)$

$Y(z) = h_o X(z) + h_1 z^{-1} X(z) + h_2 z^{-2} X(z) + h_1 z^{-3} (z) + h_o z^{-4} X(z)$

By IZT, $y[n] = h_o x[n] + h_1 x[n-1] + h_2 x[n-2] + h_1 x[n-3] + h_o x[n-4]$

ANS:
$$\begin{aligned} y[n] = \; & 0.32\ (\tfrac{1}{2})^n \cos(n\tfrac{\pi}{3})\ u[n] \; + \\ & 0.45\ (\tfrac{1}{2})^{n-1} \cos(n-1)\tfrac{\pi}{3}\ u[n-1] \; + \\ & 0.50\ (\tfrac{1}{2})^{n-2} \cos(n-2)\tfrac{\pi}{3}\ u[n-2] \; + \\ & 0.45\ (\tfrac{1}{2})^{n-3} \cos(n-3)\tfrac{\pi}{3}\ u[n-3] \; + \\ & 0.32\ (\tfrac{1}{2})^{n-4} \cos(n-4)\tfrac{\pi}{3}\ u[n-4] \end{aligned}$$

(b) To find Response of the filter

Given Input : $x[n] = (\tfrac{1}{2})^n \cos(n\frac{\pi}{3})$

Input is sinusoidal, Infinite Length and applied to the system at $n = -\infty$

The output of the system is given by $y[n] = y_{tr}[n] + y_{ss}[n]$

But $y_{tr}[n] = 0 \quad \therefore \quad y[n] = y_{ss}[n]$

To find $y_{ss}[n]$:

Find Input signal frequency components : $w = \{ \frac{\pi}{3} \}$

Find freq response : $H(e^{jw}) = e^{-j2w}[\ 0.636\ \cos(2w)\ + 0.9\ \cos(w) + 0.5\ )\ ]$

Find magnitude and phase value for every input signal frequency :

Magnitude : M_1 = Phase : ϕ_1 =

The steady state response of the system is then given by

ANS : y[n] =

Q(16) The linear phase constraint on FIR filters places constraints on the unit sample response and the location of the zeros of the system function. In the table below, indicate with a check with filter types could successfully be used to approximate the given filter type. Justify your answer.

Solution :

*	Type -I	Type -II	Type -III	Type -IV
Low-pass filter	√	√		
High-pass filter	√			√
Band-pass filter	√		√	
Band-stop filter	√			
Differentiator				√

(i) Low-pass filter

For LPF, $H(e^{jw})\Big|_{w=0} \neq 0 \qquad i.e.\ H(z)\big|_{z=1} \neq 0$

For Type-III and Type-IV FIR filter with Antisymmetric h[n] there exists definite ZERO at z=1. $i.e.\ H(z)\big|_{z=1} = 0$

Therefore, Type- III and Type-IV are not suitable for LPF design.

Type –I and Type-II FIR filters are suitable for LPF design.

(ii) High-pass filter

$$For\ HPF,\quad H\left(e^{jw}\right)\Big|_{w=\pi} \neq 0 \qquad i.e.\ \ H(z)\big|_{z=-1} \neq 0$$

For Type-II and Type-III FIR filter has definite ZERO at z = –1.

$$i.e.\ \ H(z)\big|_{z=-1} = 0$$

Therefore, Type- II and Type-III are not suitable for HPF design.

Type –I and Type-IV FIR filters are suitable for HPF design.

(iii) Band-pass filter

$$For\ BPF,\quad H\left(e^{jw}\right)\Big|_{w=0} = 0 \qquad and \quad H\left(e^{jw}\right)\Big|_{w=\pi} = 0$$

For Type-III FIR filter there has definite ZERO at z=–1 and z=1.

and Type-1 has NO definite ZERO anywhere. Therefore Type–I and Type-III FIR filters are suitable for BPF design

(iv) Band-stop filter

$$For\ BSF,\quad H\left(e^{jw}\right)\Big|_{w=0} \neq 0 \qquad and \quad H\left(e^{jw}\right)\Big|_{w=\pi} \neq 0$$

For Type-I FIR filter there has NO definite ZERO,

Type=II filter has definite ZERO at z=–1,

Type-III has definite ZERO at z=1and at z=–1,

Type-IV has definite ZERO at z=1.

Therefore Type–I FIR filter is suitable for BSF design

(v) Differentiator

The magnitude response of Differentiator is HPF. Type-IV has ZERO at z=1. Therefore they are more suitable to approximate ideal differentiator design.

(Filter Design)

Q(17) Design FIR filter to satisfy the following specifications :-

Pass-band edge frequency	:	1.5 KHz
Transition width	:	0.5 KHz
Stop-band attenuation	:	75 dB
Sampling frequency	:	8 KHz

Use Hamming window function.

Solution :

$F_{PB} = 1.5$ KHz $F_s = 8$ KHz $W_p = 0.375\ \pi$

$A_s = 75$ db B = 500 Hz Assuming Low Pass Filter,

Transition width = $F_{SB} - F_{PB} = 0.5$ K

$F_{SB} = F_{PB} + 0.5$ K

$F_{SB} = 1.5$ KHz$+0.5$ K

$F_{SB} = 2$ KHz $W_s = 0.5\ \pi$

(1) Calculate N

For LPF, $N \geq \dfrac{C}{fs - fp}$

i. For Hamming window C=3.467

ii. $fs - fp = \dfrac{0.5\ KHz}{8\ KHz} == 0.0625$

By substituting we get, $N \geq \dfrac{3.47}{0.0625}$

$N \geq 50.24$ Let N = 51 $\therefore \alpha = 25$

(2) Calculate w_c

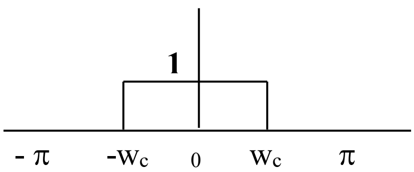
$$w_c = \frac{w_p + w_s}{2} = \frac{0.375\pi + 0.5\pi}{2} == 0.437\pi$$

To find filter coefficients h[n]: -------------------

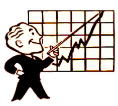
Step 1. Find Hd(w)

Let Hd(w) = | Hd(w) | $e^{j\phi}$

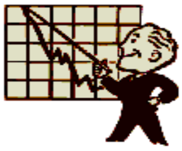
Where (i) **Magnitude response :**



$$H_d(\omega) = \begin{cases} 1 & for\ -w_c \leq |\omega| \leq w_c \\ 0 & otherwise \end{cases}$$



(ii) **Phase Response** : $e^{j\phi}$



For Linear Phase LPF with symmetric h[n]

$$\phi = -\left(\frac{N-1}{2}\right) w = -\alpha w$$

$$\therefore e^{j\phi} = e^{-j\alpha w}$$

By substituting, $H_d(w) = \begin{cases} e^{-j\alpha w} & -w_c \le |\omega| \le w_c \\ 0 & otherwise \end{cases}$

where $\alpha = 25$ and $w_c = 0.437\pi$

Step 2. Find hd[n]

------Derivation of $h_d[n]$-----

$$\boxed{h_d[n] = \frac{w_c}{\pi} \frac{\sin(n-\alpha)w_c}{(n-\alpha)w_c}}$$ where $\alpha = 25$ and $w_c = 0.437\pi$

Step 3. Find h[n]

Linear phase FIR filter with impulse response h[n] is given by,

$h[n] = h_d[n]\ w[n]$

$$h[n] = \left[2.282\left(\frac{\sin(n-25)\,0.437\pi}{(n-25)\,0.437\pi}\right)\right]\left[0.54 - 0.46\cos\left(\frac{2\pi n}{50}\right)\right] \quad \text{for } 0 \le n \le 50$$

Q(18) (a) Desired frequency response of linear phase FIR filter is given below-

$$H_d(e^{jw}) = \begin{cases} 0 & for\ |w| \le \frac{\pi}{2} \\ e^{-j2w} & \frac{\pi}{2} \le |w| \le \pi \end{cases}$$

Using Hamming window. Design the filter

(b) Plot Magnitude and Phase response of the filter.

Solution :

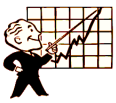
Phase $\phi = -2w$

For linear phase FIR Filter with symm h (n),

$$\phi = -\left(\frac{N-1}{2}\right) = -2w \quad \therefore\ N = 5. \qquad \alpha = \frac{N-1}{2} = 2$$

Step–1 : Find $H_d(w)$

$$H_d(w) = \begin{cases} 0 \quad ; & -w_c \le w \le w_c \\ e^{-j\alpha w} \quad ; & w_c < |w| \le \pi . \end{cases}$$



Step–2 Find $h_d[n]$

By Inverse DTFT, $$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} Hd(e^{jw})\, e^{jnw}\, dw$$

------Derivation of $h_d[n]$ for HPF -----

$$\therefore\ h_d[n] = \frac{\sin(n-\alpha)\pi}{(n-\alpha)\pi} - \frac{w_c}{\pi}\,\frac{\sin(n-\alpha)w_c}{(n-\alpha)w_c}$$

where $\alpha = 2$ and $w_c = \frac{\pi}{2}$

Step–3 Find h[n]

Linear phase FIR filter with impulse response h(n) is given by

$h[n] = h_d[n]\, w(n)$

By substituting,

$$h[n] = \left[\frac{\sin(n-2)\pi}{(n-2)\pi} - \frac{1}{2}\,\frac{\sin\left((n-2)\frac{\pi}{2}\right)}{(n-2)\frac{\pi}{2}}\right]\left[0.54 - 0.46\ \cos\left(\frac{2\pi n}{4}\right)\right]$$

$$h[n] = \begin{bmatrix} 0 \\ -0.3183 \\ 0.5 \\ -0.3183 \\ 0 \end{bmatrix}\begin{bmatrix} 0.08 \\ 0.54 \\ 1.0 \\ 0.54 \\ 0.08 \end{bmatrix} = \begin{bmatrix} 0 & n=0 \\ -0.1718 & \\ 0.5 & \\ -0.1718 & \\ 0 & \end{bmatrix}$$

Q 3 (b) Plot Magnitude Response and Phase Response of designed filter. ;

Solution :

Let h [n] = { h_0, h_1, h_2, h_1, h_0 }

By ZT, $H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_1 z^{-3} + h_0 z^{-4}$

Put $z = e^{jw}$

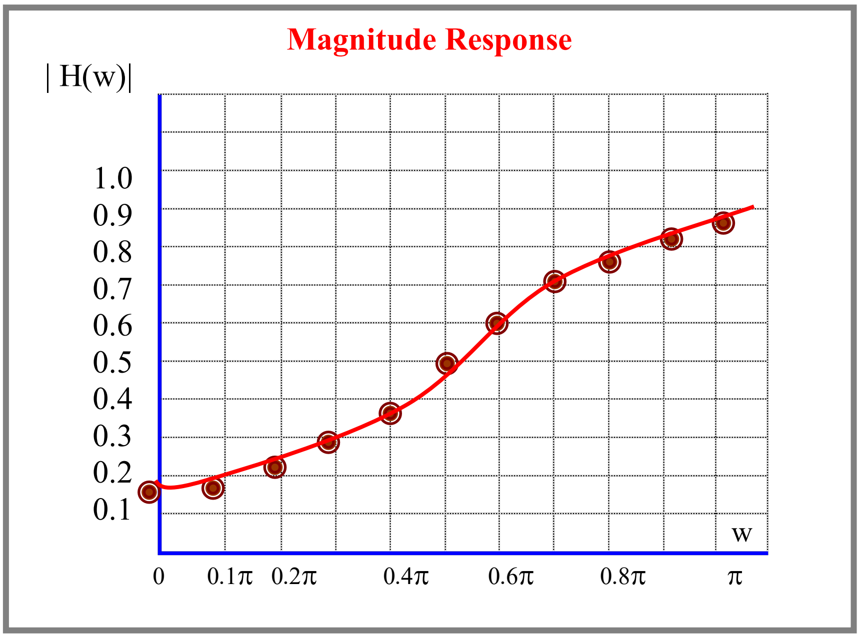
$$H(e^{jw}) = h_o + h_1 e^{-jw} + h_2 e^{-j2w} + h_1 e^{-jw} + h_0 e^{-j4w}$$

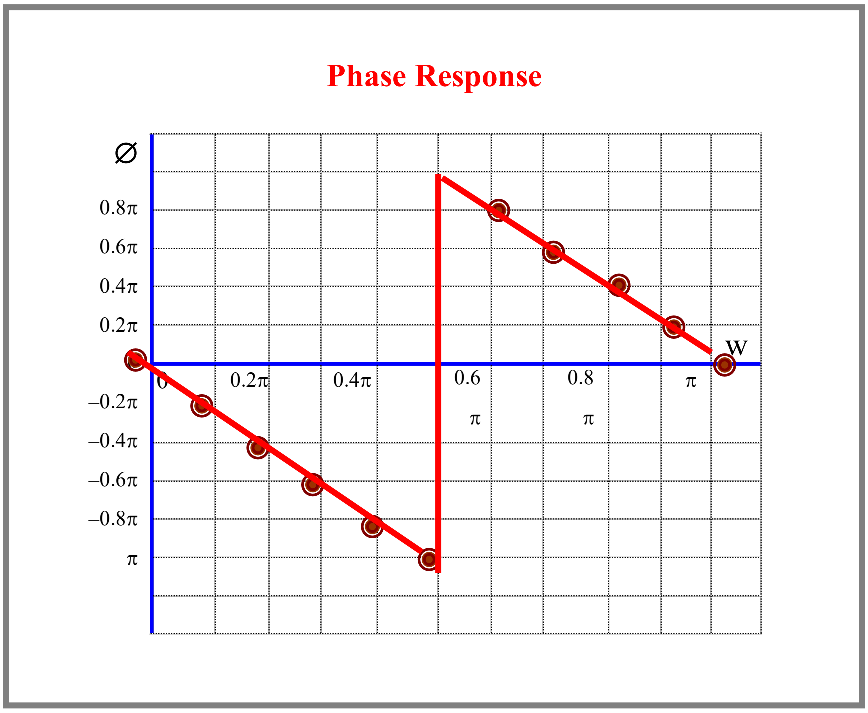
$$= e^{-j2w}\left[h_0 e^{j2w} + h_1 e^{jw} + h_2 + h_1 e^{-jw} + h_0 e^{-j2w}\right]$$

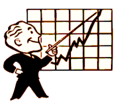
$$H(e^{jw}) = e^{-j2w}\left[2h_0\ \cos(2w) + 2h_1\ \cos(w) + h_2\right]$$

$$H(w) = e^{-j2w}\left[-0.3437\ \cos(w) + 0.5\ \right]$$

W	$H_r(w)$	ϕ
0	0.15	0
0.1π	0.17	-0.2π
0.2π	0.22	-0.4π
0.3π	0.29	-0.6π
0.4π	0.39	-0.8π
0.5π	0.50	$-\pi$
0.6π	0.61	$-1.2\pi + 2\pi = 0.8\pi$
0.7π	0.70	$-1.4\pi + 2\pi = 0.6\pi$
0.8π	0.78	$-1.6\pi + 2\pi = 0.4\pi$
0.9π	0.82	$-1.8\pi + 2\pi = 0.2\pi$
π	0.84	$-2\pi + 2\pi = 0$







Q 6(a) The desired response of a low pass filter is

$$H_d(e^{jw}) = \begin{cases} e^{-j3w} & for\ \frac{-3\pi}{4} \le w \le \frac{3\pi}{4} \\ 0 & \frac{3\pi}{4} \le |w| \le \pi \end{cases}$$

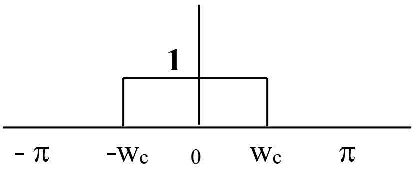
Determine H (e^{jw}) for M=7 using Hamming Window.

Solution :

Step 1. Find Hd(w)

Let Hd(w) = | Hd(w) | e^{jw}

Where (i) **Magnitude response :**



$$\mathbf{H_d(\omega)} = \begin{cases} 1 & for\ -w_c \le |\omega| \le w_c \\ 0 & otherwise \end{cases}$$

(ii) Phase Response : e^{jw}

For Linear Phase LPF with symmetric h[n]

$$\phi = -\left(\frac{N-1}{2}\right) w = -\alpha w$$

$$\therefore e^{j\phi} = e^{-j\alpha w}$$

By substituting, $H_d(w) = \begin{cases} e^{-j\alpha w} & -w_c \le |\omega| \le w_c \\ 0 & otherwise \end{cases}$

where $\alpha = 3$ and $w_c = \frac{3\pi}{4}$

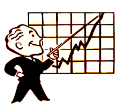
Step 2. Find hd[n]

By Inverse DTFT, $h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(w) e^{jnw}\, dw$

------Derivation of $h_d[n]$ for LPF -----

$$\boxed{h_d[n] = \frac{w_c}{\pi} \frac{\sin(n-\alpha)w_c}{(n-\alpha)w_c}}$$

where $\alpha = 3$ and $w_c = \frac{3\pi}{4}$



Step 3. Find h[n]

Linear phase FIR filter with impulse response h[n] is given by,

$$h[n] = h_d[n]\ \ w[n]$$

$$h[n] = \left[\frac{3}{4} \frac{\sin(n-3)\frac{3\pi}{4}}{(n-3)\frac{3\pi}{4}} \right] \left[0.54 - 0.46\ \cos\left(\frac{2\pi n}{6}\right) \right]$$

$$h[n] = \begin{bmatrix} 0.075 \\ -0.159 \\ 0.225 \\ 0.750 \\ 0.225 \\ -0.159 \\ 0.075 \end{bmatrix} \begin{bmatrix} 0.08 \\ 0.31 \\ 0.77 \\ 1.0 \\ 0.77 \\ 0.31 \\ 0.08 \end{bmatrix} = \begin{bmatrix} 0.0060 \\ -0.0493 \\ 0.1733 \\ 0.7500 \\ 0.1733 \\ -0.0493 \\ 0.0060 \end{bmatrix}$$

Q(19) Consider the following specifications for a Low Pass Filter.

$$0.99 \le |H(e^{jw})| \le 1.01 \quad \text{for} \quad 0 \le w \le 0.3\pi$$

$$|H(e^{jw})| \le 0.01 \quad \text{for} \quad 0.35\pi \le w \le \pi$$

Design a Linear Phase FIR filter to meet these specifications using the window design method.

Solution :

(i) Ap = 20 log(1.01) – 20 log(0.99) == 0.1737 dB

(ii) As = 20 log(1.01) – 20 log(0.01) == 40.086 dB

(iii) w_p = 0.3π (iv) w_s = **0.35π** (v) F_s = 1 Hz (vi) LPF

(1) Select window function

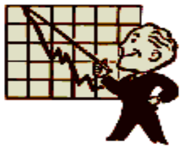
For Hamming window function As= 53 dB > 40.086 dB(required value)
∴ Select Hamming window function.

(2) Calculate N

$$N \ge \frac{C}{f_2 - f_1}$$

(i) For hamming window function **C=3.47**

(ii) $f_2 - f_1 = f_s - f_p$



$w_p = 0.3\pi = 2\pi f_p \quad \therefore \quad f_p = 0.15$

$w_s = 0.35\pi = 2\pi f_p \quad \therefore \quad f_s = 0.175$

$$N \geq \frac{3.47}{0.175 - 0.15}$$

$$N \geq 125.6$$

Let N = 127 (odd)

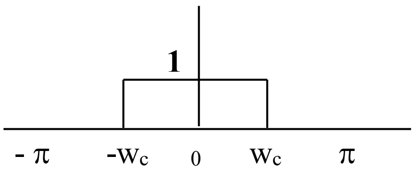
(3) Calculate w_c

$$w_c = \frac{w_p + w_s}{2} == 0.325\pi \ rad$$

Step 1.: Find Hd(w)

Let $Hd(w) = |Hd(w)| \ e^{j\phi}$

Where (i) **Magnitude response :**



$$\mathbf{H_d(\omega)} = \begin{cases} 1 & for \ -w_c \leq |\omega| \leq w_c \\ 0 & otherwise \end{cases}$$

(ii) Phase Response : $e^{j\phi}$

For Linear Phase LPF with symmetric h[n]

$$\phi = -\left(\frac{N-1}{2}\right) w = -\alpha w$$

$$: \ e^{j\phi} = e^{-j\alpha w}$$

By substituting, $$H_d(w) = \begin{cases} e^{-j\alpha w} & -w_c \leq |\omega| \leq w_c \\ 0 & otherwise \end{cases}$$

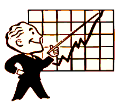
where $\alpha = 63$ and $w_c = 0.325\pi$

Step 2. Find hd[n]

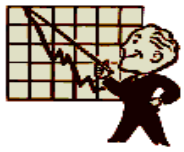
By Inverse DTFT, $$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(w)e^{jnw} \, dw$$

------Derivation of $h_d[n]$ for LPF -----

$$\boxed{h_d[n] = \frac{w_c}{\pi}\frac{\sin(n-\alpha)w_c}{(n-\alpha)w_c}}$$ where $\alpha = 63$ and $w_c = 0.325\pi$



Step 3. Find h[n]



Linear phase FIR filter with impulse response h[n] is given by,

$h[n] = h_d[n]\ \ w[n]$

$$h[n] = \left[0.325 \left(\frac{\sin(n-63)0.325\pi}{(n-83)0.325\pi} \right) \right] \quad \left[0.54 - 0.46\ \ \cos\left(\frac{2\pi n}{126} \right) \right]$$

for $0 \le n \le 126$

Q(20) Desired frequency response of a low pass filter is

$$H_d(e^{jw}) = \begin{cases} e^{-j3w} & for\ \frac{-3\pi}{4} \le w \le \frac{3\pi}{4} \\ 0 & Otherwise \end{cases}$$

Determine $H(e^{jw})$ for M = 7 using Blackman window.

Solution :

Step 1. Find Hd(w)

$$H_d(e^{jw}) = \begin{cases} e^{-j3w} & for\ \frac{-3\pi}{4} \le w \le \frac{3\pi}{4} \\ 0 & Otherwise \end{cases}$$

Step 2. Find $h_d[n]$

By Inverse DTFT, $h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(w) e^{jnw}\, dw$

------Derivation of $h_d[n]$ for LPF -----

$$\boxed{h_d[n] = \frac{w_c}{\pi} \frac{\sin(n-\alpha)w_c}{(n-\alpha)w_c}}$$

where $\alpha = 3$ and $w_c = \frac{3\pi}{4}$

Step 3. Find h[n]

Linear phase FIR filter with impulse response h[n] is given by,

$h\,[n] = h_d[n]\ \ w[n]$

$$h[n] = \left[\frac{3}{4} \frac{\sin(n-3)\frac{3\pi}{4}}{(n-3)\frac{3\pi}{4}} \right] \left[0.42 - 0.5\ \cos\left(\frac{2\pi n}{6}\right) + 0.08\ \cos\left(\frac{4\pi n}{6}\right) \right]$$

$$\mathrm{h[n]} = \begin{bmatrix} 0.075 \\ -0.159 \\ 0.225 \\ 0.750 \\ 0.225 \\ -0.159 \\ 0.075 \end{bmatrix} \begin{bmatrix} 0 \\ 0.13 \\ 0.63 \\ 1.00 \\ 0.63 \\ 0.13 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.02067 \\ 0.14175 \\ 0.75000 \\ 0.14175 \\ -0.02607 \\ 0 \end{bmatrix}$$

To find $H(e^{jw})$

(i) Find H(z)

Now $h[n] = = \{ h_o,\ h_1,\ h_2, h_3, h_2, h_1,\ h_o \}$

By ZT, H(z) = $h_o + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} + h_2 z^{-4} + h_1 z^{-5} + h_0 z^{-6}$

(ii) Find H(w)

Put $z = e^{jw}$ $\quad H(e^{jw}) = h_0 + h_1\ e^{-jw} + h_2\ e^{-j2w} + h_3\ e^{-j3w} + h_2\ e^{-j4w} + h_1\ e^{-j5w} + h_0\, e^{-j6w}$

Substituting,

$$H(w) = e^{-j3w}\ [\ h_0\ e^{j3w} + h_1\ e^{j2w} + h_2\, e^{jw} + h_3 + h_2\ e^{-jw} + h_1\ e^{-2w} + h_0\, e^{-j3w}\ ]$$

$$H(w) = e^{-j3w}[\ h_0\,(e^{j3w} + e^{-j3w}) + h_1\ (e^{j2w} + e^{-j2w}) + h_2\,(e^{jw} + e^{-jw}) + h_3\,]$$

$$H(w) = e^{-j3w}[\ 2\ h_0\ \cos(3w) + 2\ h_1 \cos(2w) + 2h_2 \cos(w) + h_3]$$

$$H(w) = e^{-j3w}[\ -0.04134\cos(2w) + 0.2835\,\mathrm{coss}(w) + 0.75\,]\ \mathrm{ANS}$$

Q(21) A low pass filter has the response

$$H(e^{jw}) = \begin{cases} 2\, e^{-jw_0} & |w| \le \dfrac{\pi}{2} \\ 0 & \textit{Otherwise} \end{cases}$$

Find h(n) for transition width < $\pi/32$, calculate the window length and the value of α for (i) Rectangular window (ii) Hamming window.

Solution :

Transition width = $\dfrac{\pi/32}{2\pi} = \dfrac{1}{64}$

(i) Rectangular window :

Transition width of filter $\ge$ Transition width of rectangular window.

$\dfrac{1}{64} \ge \dfrac{C}{N}$ where C = 0·92 for rectangular window

$\therefore$ N $\ge$ C $*$ 64

N $\ge$ 0·94 $\times$ 64

N $\ge$ 58·88 Let **N = 59** $\boldsymbol{\alpha}$ **=** $\dfrac{N-1}{2}$ **= 29**

(ii) Hamming Window

Transition width of filter $\ge$ Transition width of Hamming window.

$\dfrac{1}{64} \ge \dfrac{C}{N}$

N $\ge$ 64 $\times$ C Where C = 3· 47 for hamming window

N $\ge$ 200·96 Let **N = 201** $\boldsymbol{\alpha}$ **=** $\dfrac{N-1}{2}$ **= 100**

◉ **To find h[n]**

Step-1 Find Hd(w)

$$H(e^{jw}) = \begin{cases} 2\, e^{-j\alpha\, w_c} & |w| \le w_c \\ 0 & wc \le |w| \le \pi \end{cases}$$

Step-1 Find hd(n)

------Derivation of $h_d[n]$ for LPF -----

$$\boxed{h_d[n] = \frac{2w_c}{\pi}\,\frac{\sin(n-\alpha)w_c}{(n-\alpha)w_c}}$$

where $W_c = \pi/2$ $\alpha = 29$ for rectangular window.

And $\alpha = 100$ for Hamming window.

Step-3 Find h[n] using Rectangular window

$h[n] = h_d[n]\ w[n]$ where $w[n] = 1$ for rectangular window.

$$h[n] = \left[\frac{\sin(n-29)\frac{\pi}{2}}{(n-29)\frac{\pi}{2}} \right] \quad \textbf{ANS}$$

Step-4 Find h[n] using Hamming window

$h[n] = h_d[n]\ w[n]$

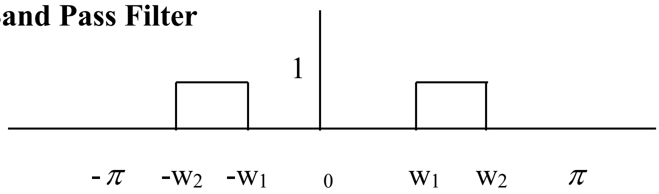
$$h[n] = \left[\frac{\sin(n-100)\frac{\pi}{2}}{(n-100)\frac{\pi}{2}} \right] \left[0.54 - 0.46\ \cos\left(\frac{2\pi\, n}{200} \right) \right]$$

for $0 \le n \le 99$

Q(22) Consider the magnitude response of ideal filters as shown below. Find $h_d[n]$ in each case.

Solution :

(a) Band Pass Filter



Solution : By Inverse DTFT, $h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(w)\ e^{jnw} dw$

$$= \frac{1}{2\pi} \left[\int_{-w_2}^{-w_1} H(w) e^{jnw}\ dw + \int_{w_1}^{w_2} H(w) e^{jnw}\ dw \right]$$

$$h_d[n] = \frac{1}{2\pi} \left[\int_{-w_2}^{-w_1} (1) e^{jnw}\ dw + \int_{w_1}^{w_2} (1) e^{jnw}\ dw \right]$$

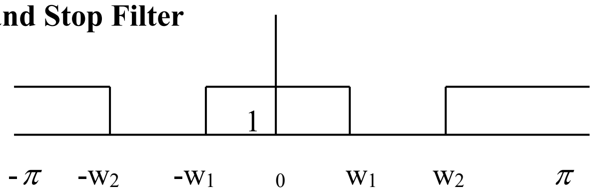
$$h_d[n] = \frac{1}{2\pi} \left[\left\{ \frac{e^{jnw}}{nj} \right\}_{-w_2}^{-w_1} + \left\{ \frac{e^{jnw}}{nj} \right\}_{w_1}^{w_2} \right]$$

$$h_d[n] = \frac{1}{2\pi} \left[\frac{e^{-jnw_1} - e^{-jnw_2}}{nj} + \frac{e^{jnw_2} - e^{jnw_1}}{nj} \right]$$

$$h_d[n] = \frac{1}{\pi n} [\ \sin(nw_2) - \sin(nw_1)]$$

$$\boxed{h[n] = \frac{w_2}{\pi} \frac{\sin(nw_2)}{nw_2} - \frac{w_1}{\pi} \frac{\sin(nw_1)}{nw_1}}$$

(b) Band Stop Filter



Solution : By Inverse DTFT,

$$h_d[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi} H(w)e^{jnw}dw$$

$$h_d[n]=\frac{1}{2\pi}\left[\int_{-\pi}^{-w_2} H(w)e^{jnw}\,dw+\int_{-w_1}^{w_1} H(w)e^{jnw}dw+\int_{w_2}^{\pi} H(w)\,e^{jnw}dw\right]$$

$$h_d[n]=\frac{1}{2\pi}\left[\int_{-\pi}^{-w_2} (1)e^{jnw}\,dw+\int_{-w_1}^{w_1} (1)e^{jnw}dw+\int_{w_2}^{\pi} (1)e^{jnw}dw\right]$$

$$h_d[n]=\frac{1}{2\pi}\left[\left\{\frac{e^{jnw}}{nj}\right\}_{-\pi}^{-w_2}+\left\{\frac{e^{jnw}}{nj}\right\}_{-w_1}^{w_1}+\left\{\frac{e^{jnw}}{nj}\right\}_{w_2}^{\pi}\right]$$

$$h_d[n]=\frac{1}{2\pi}\left[\frac{e^{-jnw_2}-e^{-jn\pi}}{nj}+\frac{e^{jnw_1}-e^{-jnw_1}}{nj}+\frac{e^{jn\pi}-e^{jnw_2}}{nj}\right]$$

$$h_d[n]=\frac{1}{\pi n}\left[\frac{e^{jn\pi}-e^{-jn\pi}}{2j}+\frac{e^{jnw_1}-e^{-jnw_1}}{2j}-\left(\frac{e^{jnw_2}-e^{-jnw_2}}{2j}\right)\right]$$

$$h_d[n]=\frac{1}{\pi n}\left[\sin(n\pi)+\sin(nw_1)-\sin(nw_2)\right]$$

$$\boxed{\therefore\; h_d[n]=\frac{\sin(n\pi)}{n\pi}+\frac{w_1}{\pi}\,\frac{\sin(nw_1)}{nw_1}-\frac{w_2}{\pi}\,\frac{\sin(nw_2)}{nw_2}}$$

Q(23) Using frequency sampling realization, realize the filter which has following transfer function.

$$H\left(\frac{2\pi k}{16}\right)=1 \quad ; \quad k=0,1$$

$$=0.5 \quad ; \quad k=2$$

$$=0 \quad ; \quad k=3 \text{ to } 15.$$

Solution : Frequency Sampling Realization

$$H(z) = \frac{1}{N} H_1(z)\ H_2(z)$$

Where (1) $N = 6$

(2) $H_1(z) = 1 - z^{-N} = 1 - z^{-16}$

(3) $H_2(z) = \sum_{k=0}^{N-1} \dfrac{H[K]}{1 - e^{-j\frac{2\pi k}{N}} z^{-1}}$

$$H_2(z) = \frac{H[0]}{1 - z^{-1}} + \frac{H[1]}{1 - e^{-j\frac{2\pi}{16}} z^{-1}} + \frac{H[15]}{1 - e^{j\frac{2\pi\pi 1}{16}} z^{-1}} + \frac{H[2]}{1 - e^{-j\frac{2\pi\pi}{16}} z^{-1}} + \frac{H[14]}{1 - e^{-j\frac{2\pi\pi 1}{16}} z^{-1}}$$

$$H_2(z) = \frac{1}{1 - z^{-1}} + \left[\frac{1}{1 - e^{-j\frac{\pi}{8}} z^{-1}} + \frac{1}{1 - e^{+j\frac{\pi}{8}} z^{-1}}\right] + \left[\frac{0.5}{1 - e^{-j\frac{\pi}{4}} z^{-1}} + \frac{0.5}{1 - e^{j\frac{\pi}{4}} z^{-1}}\right]$$

$$H_2(z) = \frac{1}{1 - z^{-1}} + \left[\frac{1 - e^{j\frac{\pi}{8}} z^{-1} + 1 - e^{-j\frac{\pi}{8}} z^{-1}}{\left(1 - e^{-j\frac{\pi}{8}} z^{-1}\right)\left(1 - e^{j\frac{\pi}{8}} z^{-1}\right)}\right] + \left[\frac{0.5 - 0.5\, e^{j\frac{\pi}{4}} z^{-1} + 0.5 - 0.5\, e^{-j\frac{\pi}{4}} z^{-1}}{\left(1 - e^{-j\frac{\pi}{4}} z^{-1}\right)\left(1 - e^{-j\frac{\pi}{4}} z^{-1}\right)}\right]$$

$$H_2(z) = \frac{1}{1 - z^{-1}} + \left[\frac{2 - z^{-1}\left(e^{j\frac{\pi}{8}} + e^{-j\frac{\pi}{8}}\right)}{1 - z^{-1}\left(e^{j\frac{\pi}{8}} + e^{-j\frac{\pi}{8}}\right) + z^{-2}}\right] + \left[\frac{1 - 0.5\, z^{-1}\left(e^{j\frac{\pi}{4}} + e^{-j\frac{\pi}{4}}\right)}{1 - z^{-1}\left(e^{j\frac{\pi}{4}} + e^{-j\frac{\pi}{4}}\right) + z^{-2}}\right]$$

$$H_2(z) = \frac{1}{1 - z^{-1}} + \left[\frac{2 - 2z^{-1}\cos(\pi/8)}{1 - 2\cos\left(\frac{\pi}{8}\right) z^{-1} + z^{-2}}\right] + \left[\frac{1 - z^{-1}\cos(\pi/4)}{1 - 2\cos\left(\frac{\pi}{4}\right) z^{-1} + z^{-2}}\right]$$

$$H_2(z) = \frac{1}{1 - z^{-1}} + \left[\frac{2 - 1.847\, z^{-1}}{1 - 1.847\, z^{-1} + z^{-2}}\right] + \left[\frac{1 - 0.707\, z^{-1}}{1 - 1.414\, z^{-1} + z^{-2}}\right]$$

$$H_2(z) = \frac{b_0}{1 + a_1 z^{-1}} + \left[\frac{b_1\ z^{-1}}{1 + a_1\ z^{-1} + a_2 z^{-2}}\right] + \left[\left[\frac{b_0 + b_1\ z^{-1}}{1 + a_1\ z^{-1} + a_2 z^{-2}}\right]\right]$$

Realization Diagram :

Q(24) Sketch the POLE-ZERO plot of $X(z) = z^2 + 2.5z - 2.5z^{-1} - z^{-2}$

If x(n) is a linear phase sequence? If yes plot phase response for around 10 (ten) different values of w.

Solution :

$X(z) = z^2 + 2.5z - 2.5z^{-1} - z^{-2}$

By IZT,

x[n] ={ 1, 2.5, 0, -2.5, -1 } for $-2 \geq n \geq 2$

$X(z) = z^2 + 2.5z - 2.5z^{-1} - z^{-2}$

Put z = e^{jw}

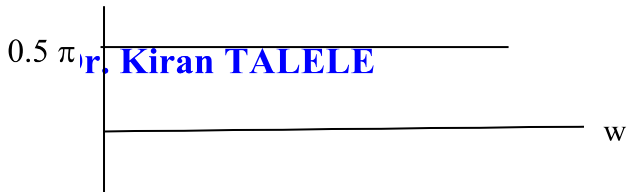
$X(e^{jw}) = e^{j2w} + 2.5e^{jw} - 2.5e^{-jw} - e^{-j2w}$

$X(w) = (e^{j2w} - e^{-j2w}) + 2.5(e^{jw} - e^{-jw})$

$X(w) = 2j\sin(2w) + 5j\sin(w)$

$X(w) = e^{j\frac{\pi}{2}}\left[2\sin(2w) + 5\sin(w)\right]$

$X(w) = e^{j\frac{\pi}{2}}\left[2\sin(2w) + 5\sin(w)\right]$

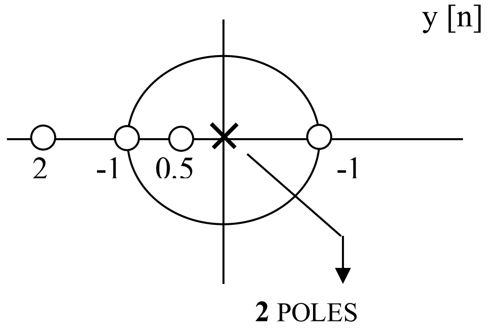


Phase Spectrum :

POLE-ZERO Plot :

$$X(z) = \frac{z^2 + 2.5z - 2.5z^{-1} - z^{-2}}{1}$$

$$X(z) = \frac{z^4 + 2.5z^3 - 2.5z - 1}{z^2}$$



Q(25) The unit sample response of An FIR filter is ,

$$h[n] = \begin{cases} \alpha^n & 0 \le n \le 6 \\ 0 & Otherwise \end{cases}$$

(a) Draw the direct form implementation of this system.

(b) Determine system function and use this to draw a flow graph that is cascade of FIR system with IIR system.

(c) For both of this implementations, determine the number of multiplications and additions required to compute each output value and the number of storage registers that are required.

Solution :

(a) Direct form implementation

$$h[n] = \begin{cases} \alpha^n & 0 \le n \le 6 \\ 0 & Otherwise \end{cases}$$

By ZT,

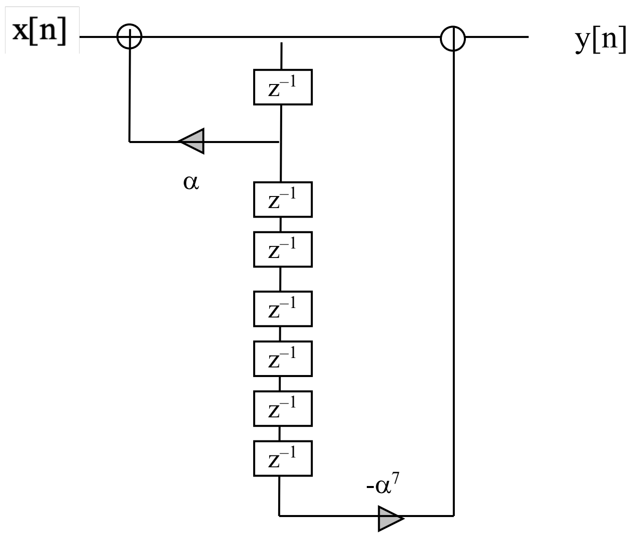
$$H(z) = \sum_{n=-\infty}^{\infty} h[n]\, z^{-n}$$

$$H(z) = \sum_{n=0}^{6} \alpha^n\, z^{-n}$$

$$H(z) = \sum_{n=0}^{6} \left(\alpha\, z^{-1}\right)^n$$

$$H(z) = \frac{1 - \left(\alpha z^{-1}\right)^7}{1 - \alpha z^{-1}}$$

$$H(z) = \frac{1 - \alpha^7 z^{-7}}{1 - \alpha z^{-1}}$$

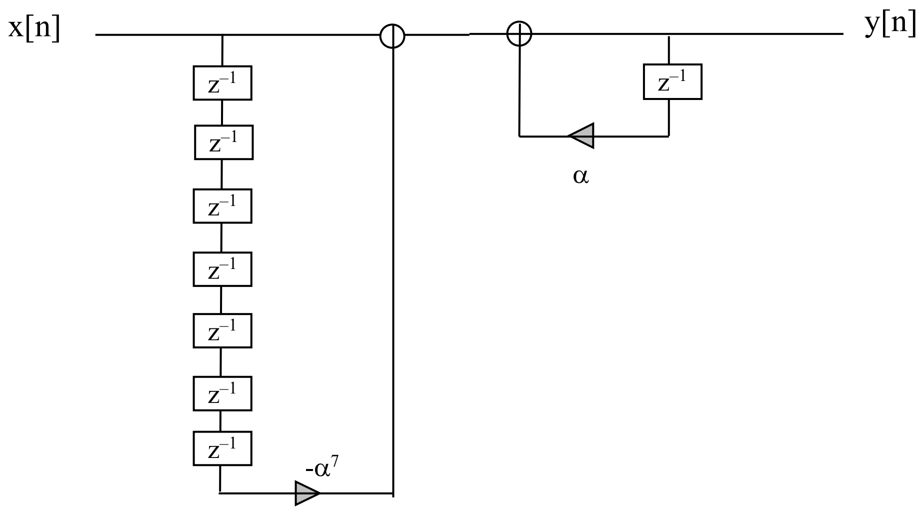


Solution : (b) Cascade form implementation

$$H(z)=\frac{1-\alpha^{7}z^{-7}}{1-\alpha z^{-1}}$$

$$H(z)=\left[1-\alpha^{7}z^{-7}\right]\left[\frac{1}{1-\alpha z^{-1}}\right]$$

Let $H(z)=H_1(z)\ H_2(z)$



Solution : (c)

Sr No	Parameter	Direct Form	Cascade form
1	Number of Multiplications	**2**	**2**
2	Number of Additions	**2**	**2**
3	Number of Delay Blocks	**7**	**7**