Digital FIR Filter Design using Windowing Method

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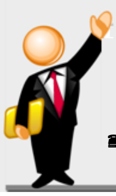

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ALGORITHM To Design Linear Phase F I R Filter Using Window function.

Given ➔ **W[n] N Wc Filter**

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Step-1 $H_d(w)$ — Desired Freq. Response

⬇

Step-2 $h_d(n)$ — Desired Impulse Response

⬇

Step-3 $h(n)$ — Finite Impulse Response

Linear Phase FIR Filter Using Window function

Consider a Linear Phase Low Pass FIR Filter with cutoff frequency = wc

(I) Find $H_d(w)$

Frequency Response ➔ $H_d(w) = |H_d(w)| \quad \Phi(w)$

Magnitude Response

Phase Response

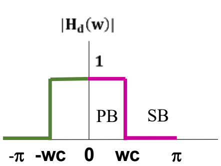
(I) Find $H_d(w)$

Frequency Response ➔ $H_d(w) = |H_d(w)| \quad \Phi(w)$

Magnitude Response

Phase Response

⊙ **Magnitude Spectrum (LPF):**



$$|H_d(w)| = \begin{cases} 1 & -wc \le w \le wc \\ 0 & \text{otherwise} \end{cases}$$

- **Phase Response :**

For a Linear phase FIR Filter with symmetric h(n),

$$\phi = -\left(\frac{N-1}{2}\right)w$$

$$\phi(w) = e^{j\phi}$$

Let $\phi(w) = e^{-j\alpha w}$

By substituting in $H_d(w)$ we get,

$$H_d(w) = \begin{bmatrix} e^{-j\alpha w} & -w_c \le w \le w_c. \\ 0 & Otherwise \end{bmatrix}$$

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(II) Find $h_d[n]$

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} Hd(e^{jw})\, e^{jnw}\, dw$$

Where $H_d(w) = \begin{bmatrix} e^{-j\alpha w} & -w_c \le w \le w_c \\ 0 & Otherwise \end{bmatrix}$

$$h_d[n] = \frac{1}{2\pi}\left[\int_{-wc}^{wc} e^{-j\alpha w} e^{jnw}\, dw\right]$$

$$h_d[n] = \frac{1}{2\pi}\left[\int_{-wc}^{wc} e^{j(n-\alpha)w}\, dw\right]$$

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(II) Find $h_d[n]$....

$$h_d[n] = \frac{1}{2\pi}\left[\int_{-wc}^{wc} e^{j(n-\alpha)w}\, dw\right]$$

$$h_d[n] = \frac{1}{2\pi}\left[\left\{\frac{e^{j(n-\alpha)w}}{(n-\alpha)j}\right\}_{-w_c}^{w_c}\right]$$

$$h_d[n] = \frac{1}{2\pi\, j\,(n-\alpha)}\left[\, e^{j(n-\alpha)w_c} - e^{-j(n-\alpha)w_c}\right]$$

$$h_d[n] = \frac{1}{\pi\,(n-\alpha)}\left[\frac{e^{j(n-\alpha)w_c} - e^{-j(n-\alpha)w_c}}{2j}\right]$$

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(II) Find $h_d[n]$....

$$h_d[n] = \frac{1}{\pi\,(n-\alpha)}\left[\frac{e^{j(n-\alpha)w_c} - e^{-j(n-\alpha)w_c}}{2j}\right]$$

$$h_d[n] = \frac{1}{\pi\,(n-\alpha)}\left[\,Sin((n-\alpha)w_c)\right]$$

$$h_d[n] = \frac{1}{\pi\,(n \cancel{-\alpha})}\left[\frac{Sin((n-\alpha)w_c)}{(n-\alpha)w_c}\right](n \cancel{-} \alpha)w_c$$

$$\boxed{h_d[n] = \frac{w_c}{\pi}\left[\frac{Sin((n-\alpha)w_c)}{(n-\alpha)w_c}\right]}$$

(III) Find h[n]

Linear Phase FIR filter is obtained by truncating infinite samples of $h_d[n]$ by using window function.

Linear Phase FIR filter with impulse response h[n] is given by, $h[n] = hd[n]\ w[n]$

Where,

$$h[n] = \left[\frac{wc}{\pi}\ \frac{\sin((n-\alpha)wc)}{(n-\alpha)wc\ )}\right]$$

W[n] is finite length causal window function

Find h[n] . . .

Considering Rectangular window function

Rectangular window function is given by,

$$w[n] = \begin{bmatrix} 1 & 0 \le n \le N-1 \\ 0 & otherwise \end{bmatrix}$$

By substituting in h[n] we get,

$$h[n] = \left[\frac{wc}{\pi}\ \frac{\sin((n-\alpha)wc)}{(n-\alpha)wc\ )}\right]$$

$s[n] = \frac{w_c}{\pi}\left[\frac{Sin(n\,w_c)}{n\,w_c}\right]$

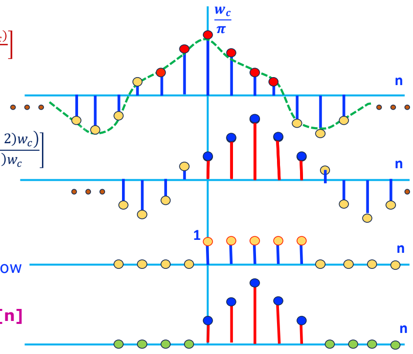
Let α = 2

$h_d[n] = \frac{w_c}{\pi}\left[\frac{Sin((n-2)w_c)}{(n-2)w_c}\right]$

W[n]

Rectangular Window

$h[n] = h_d[n]\ W[n]$



Examples of Window function :

(1) Rectangular Window

$$w[n] = \begin{cases} 1 & 0 \le n \le N-1 \\ 0 & otherwise \end{cases}$$

(2) Bartlet Window

$$w[n] = \begin{cases} 2n/(N-1) & 0 \le n \le (N-1)/2 \\ 2 - 2n/(N-1) & (N-1)/2 \le N-1 \\ 0 & otherwise \end{cases}$$

(3) Hanning Window

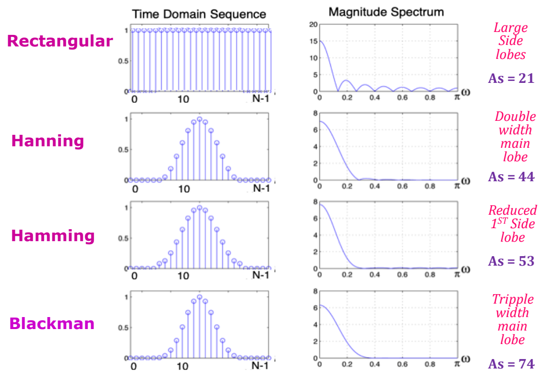
$$w[n] = \begin{cases} \left\{1 - \cos\left(\frac{2\pi n}{N-1}\right)\right\}/2 & 0 \le n \le N-1 \\ 0 & otherwise \end{cases}$$

(4) Hamming Window

$$w[n] = \begin{cases} 0.54 - 0.46\ \cos\left(\frac{2\pi n}{N-1}\right) & 0 \le n \le N-1 \\ 0 & otherwise \end{cases}$$

(5) Blackman Window

$$w[n] = \begin{cases} 0.42 - 0.5\ \cos\left(\frac{2\pi n}{N-1}\right) + 0.08\ \cos\left(\frac{4\pi n}{N-1}\right) & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$$



Characteristics of Window function :

	Window Function	Constant	As
1	Rectangular	C = 0.92	21
2	Bartlett	C = 2.1	25
3	Hanning	C = 3.21	44
4	Hamming	C = 3.47	53
5	Blackman	C = 5.71	74

Ref. "Digital Signal Processing : A Modern Introduction" by Ashok Ambardar, Cengage Learning India Edition 2007. Page 476

NOTE :

In FIR Filter, Order M = N-1 where N is Length of h[n]

In IIR Filter, Order N = No of POLEs

Q(1) Design 6^{th} order Linear Phase Low Pass FIR filter with cut off frequency $w_c = 0.75\pi$ radian using Hamming window

Solution :

Linear Phase LPF Design

Order N-1 = 6

N = 7

Cutoff frequency Wc = 0.75 π

✔ ✔ ✔ ✔

W[n] N Wc Filter (LPF)

ALGO.

(I) $H_d(w)$

↓

(II) $h_d(n)$

↓

(III) $h(n)$

Q(2) Given,

$$H(w) = \begin{cases} 1 & 0 \le F \le 800\,\text{Hz} \\ 0 & \text{otherwise} \end{cases}$$

Assume Sampling frequency Fs = 5000 Hz.
Design FIR filter for length N = 5 using Hamming window

Solution :

Digital Frequency $w = \left[\frac{800}{5000}\right] 2\pi$ **= 0.32 π**

$$H(w) = \begin{cases} 1 & 0 \le w \le 0.32\pi \\ 0 & \text{otherwise} \end{cases}$$

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Q(3) A filter is to be designed with the following desired frequency response, below-

$$H(w) = \begin{cases} 1 & 0 \le F \le 800\,\text{Hz} \\ 0 & \text{otherwise} \end{cases}$$

Determine the filter coefficients using Hamming window

Solution : FIR Filter Design

✔	✗	✗	✗
W[n]	**N**	**Wc**	**Filter Type**

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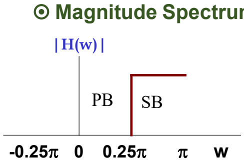
⊙ Frequency Response :

$$H(w) = \begin{cases} 1 & 0 \le F \le 800\,\text{Hz} \\ 0 & \text{otherwise} \end{cases}$$

⊙ Magnitude Spectrum :



⊙ Phase Response :

$$\phi(w) = e^{j\phi} = e^{-j3w}$$

Phase : $\left[\frac{800}{5000}\right] 2\pi$

$$\phi = -\left(\frac{N-1}{2}\right) w = -3w$$

By Solving we get N = 7

Linear Phase HPF
Wc = 0.25π N = 7

✔	✔	✔	✔
W[n]	**N**	**Wc**	**Filter (HPF)**

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Q(4) Given Ap = 1 dB As = 40 dB
Wp = 0.2 π Ws = 0.8 π

Design a digital filter using appropriate window function.

Solution: Given Ap = 1 dB As = 40 dB
Wp = 0.2 π Ws = 0.8 π

window function ➔ Linear Phase FIR Filter

✔ ✔ ✔ ✔

Ap As w_p w_s

ALGORITHM To Design Linear Phase F I R Filter Using Window function.

Given ➔ **Ap As w_p w_s**

W[n] N Wc Filter

Step-1 $H_d(w)$ Desired Frequency Response

Step-2 $h_d(n)$ Desired Impulse Response

Step-3 $h(n)$ Impulse Response

(HW) Design a causal digital High Pass filter using windowing technique to meet the following specifications :

Stopband edge : = 2 KHz
Stopband Attenuation : $\geq$ 40 dB
Passband edge : = 9.5 KHz
Sampling frequency : = 25 KHz

Q(4) Design sixth order Linear Phase Band Pass FIR filter with pass-band frequencies $w_1 = 0.25\pi$ and $w_2 = 0.65\pi$ using Blackman window function.

Solution :

Order M = N-1 == 6

Therefore, N = 7 and so, $\alpha = 3$

- Band Pass Filter, with $w_1 = \mathbf{0.25\pi}$ and $w_2 = \mathbf{0.65\pi}$
- Blackman window function.

Linear Phase Realization

Consider a Linear Phase FIR filter with Symmetric **h[n]**

Let h[n] = { b_o b_1 b_2 b_3 b_2 b_1 b_o }

By ZT,

$$H[z] = b_o + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_2 z^{-4} + b_1 z^{-5} + b_0 z^{-6}$$

$$\frac{Y[z]}{X[z]} = \frac{b_o + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_2 z^{-4} + b_1 z^{-5} + b_0 z^{-6}}{1}$$

Cross Multiply,

$$Y(z) = X[z] \left(b_o + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_2 z^{-4} + b_1 z^{-5} + b_0 z^{-6} \right)$$

$$Y(z) = b_o X[z] + b_1 z^{-1} X[z] + b_2 z^{-2} X[z] + b_3 z^{-3} X[z] + b_2 z^{-4} X[z] + b_1 z^{-5} X[z] + b_0 z^{-6} X[z]$$

By IZT,

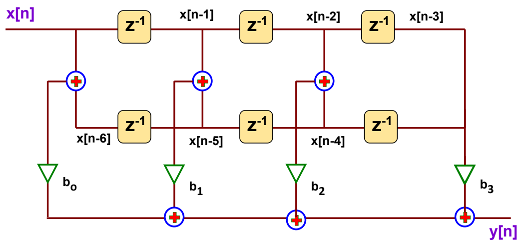
$$y[n] = b_o x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3] + b_2 x[n-4] + b_1 x[n-5] + b_0 x[n-6]$$

This is a Non recursive Filter

$$y[n] = b_o \left(x[n] + x[n-6] \right) + b_1 \left(x[n-1] + x[n-5] \right) + b_2 \left(x[n-2] + x[n-4] \right) + b_3 \, x[n-3]$$

$$y[n] = b_o \left(x[n] + x[n-6] \right) + b_1 \left(x[n-1] + x[n-5] \right) + b_2 \left(x[n-2] + x[n-4] \right) + b_3 \, x[n-3]$$

Linear Phase Realization Diagram :



Total Number of Delay Blocks : 5
Total Number of Multipliers : 4

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- He is a Treasurer of IEEE Bombay Section and Mentor for Startup Incubation & Intellectual Asset Creation.
- He received incentives for excellent performance in academics and research from Management of S.P.I.T. in 2008-09. He is a recipient of P.R. Bapat IEEE Bombay Section Outstanding Volunteer Award 2019.

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