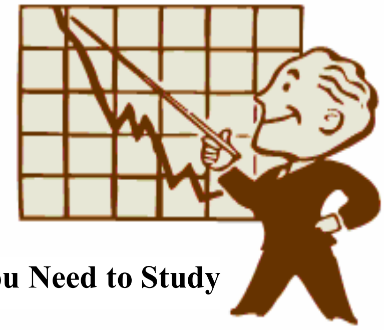
CHAPTER-2



You Need to Study

D F T + F F T

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[2.1] DFT- IDFT EQUATION

(A) Discrete Fourier Transform of x[n] is defined as,

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk}$$

Where i) N is Length of x[n]

ii) $W_N^{-1} = e^{j\frac{2\pi}{N}}$

(B) Inverse Discrete Fourier Transform of X[k] is defined as,

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, W_N^{-nk}$$

Where i) N is Length of X[k]

ii) $W_N^{-1} = e^{j\frac{2\pi}{N}}$

Q(1) Compute the DFT of a sequence $(-1)^n$ for N=4.

Solution :

Let $x[n] = (-1)^n$ for N=4 $\quad x[n] = : \{ 1, -1, 1, -1 \}$

By DFT, $\quad X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk} \quad where\ W_N^1 = e^{-j\frac{2\pi}{N}}$

$$X[K] = \begin{bmatrix} w_N^0 & w_N^0 & w_N^0 & w_N^0 \\ w_N^0 & w_N^1 & w_N^2 & w_N^3 \\ w_N^0 & w_N^2 & w_N^4 & w_N^6 \\ w_N^0 & w_N^3 & w_N^6 & w_N^9 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$X[k] = \begin{bmatrix} (1)+(-1)+(1)+(-1) \\ (1)+(j)+(-1)+(-j) \\ (1)+(1)+(1)+(1) \\ (1)+(-j)+(-1)+(j) \end{bmatrix}$$

$$X[k] = \begin{bmatrix} 0 & k=0 \\ 0 & \\ 4 & \\ 0 & \end{bmatrix} \qquad \text{ANS}$$

[2.2] Properties of DFT

[1] Scaling and Linearity property

Proof of Linearity Property:

By DFT, DFT $\{x[n]\} = \sum_{n=0}^{N-1} x[n]\, w_N^{nk}$

$$\therefore DFT\ \{a.x_1[n]+b\,x_2[n]\} = \sum_{n=0}^{N-1} \{a.x_1[n]+bx_2[n]\}\ w_N^{nk}$$

$$= \sum_{n=0}^{N-1} a.x_1[n]\ w_N^{nk} + \sum_{n=0}^{N-1} b\,x_2[n]\ w_N^{nk}$$

$$= a \sum_{n=0}^{N-1} x_1[n]\, w_N^{nk} + b \sum_{n=0}^{N-1} x_2[n]\, w_N^{nk}$$

DFT $\{a.x_1[n]+b\,x_2[n]\} = a.\ X_1[k] + b\,X_2[k]$ **Proved**.

[2] Periodicity Property :

Proof of Periodicity Property:

By definition, $X[k] = \sum_{n=0}^{N-1} x[n]\, w_N^{nk}$ ....(i)

Then

$$X\,[k+N] = \sum_{n=0}^{N-1} x[n]\, w_N^{n(k+N)} = \sum_{n=0}^{N-1} x[n]\ W_N^{nk}\ \ W_N^{n.N} \qquad \text{But } W_N^{N} = 1$$

$$\therefore X[k+N] = \sum_{n=0}^{N-1} x[n]\ W_N^{nk} \quad ....(ii)$$

From eq^n (i) and (ii)

X [k] = X [k + N] Hence X[k] is periodic with period = N

[3] Time Shift Property

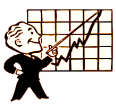
Proof of Time Shift Property:

By definition, $X[k] = \sum_{n=0}^{N-1} x[n]\, w_n^{nk}$

$$DFT\ \{x[n-m]\} = \sum_{n=0}^{N-1} x[n-m].\ \ w_N^{nk}$$

$$= \sum_{p=0}^{N-1} x[p]\, w_n^{(m+p)k} = \sum_{p=0}^{N-1} x[p]\, w_n^{mk}\, w_n^{pk}$$

$$= w_n^{mk}\ \ X[k]$$ **Proved**.



[4] Frequency Shift Property

Proof of Frequency Shift Property:

By definition, $\text{DFT}\ \{x[n]\} = \sum_{n=0}^{N-1} x[n]\ w_n^{nk}$

$$DFT\ \{W_N^{-mn}\ x[n]\} = \sum_{n=0}^{N-1} W_N^{-mn}\ x[n]\ W_N^{nk}$$

$$= \sum_{n=0}^{N-1} x[n]\ w_N^{n(k-m)}$$

$$= \mathbf{X[k\text{-}m]}$$ **Proved**.

[5] Time Reversal Property

Proof of Time Reversal Property :

By periodicity property, x [-n] = x [N- n]

By definition of DFT,

$$\text{DFT}\ \{X[-n]\} = \sum_{n=0}^{N-1} x[-n]\ W_N^{nk} = \sum_{n=0}^{N-1} x[N-n]\ W_N^{nk}$$

put N- n = m so $n = N - m$

$$DFT\ \{x[-n]\} = \sum x[m] W_N^{(N-m)k}$$

$$= \sum x[m] W_N^{-mk}\ W_N^{Nk}$$

$$= \sum x[m]\ W_N^{-mk}$$

$$= X[-k]$$

$$\text{DFT}\ \{x[-n]\} = X[-k] = [N-k]$$ **Proved**.

[6] Symmetry Property

Proof of Symmetry Property: Let x [n] be N point real valued sequence.

$$X[k] = \sum_{n=0}^{N-1} x[n]\, w_N^{nk}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\; e^{-j2\pi nk/N}$$

$$X[k] = \sum_{n=0}^{N-1} x[n] \left\{ \cos\left(\frac{2\pi nk}{N}\right) - j \sin\left(\frac{2\pi nk}{N}\right) \right\}$$

$$X[k] = \sum_{n=0}^{N-1} \left\{ x[n] \cos\left(\frac{2\pi nk}{N}\right) \right\} - j \sum_{n=0}^{N-1} \left\{ x[n] \sin\left(\frac{2\pi nk}{N}\right) \right\} \quad \ldots\ldots\ldots\ldots\ldots(i)$$

$$X^*[k] = \sum_{n=0}^{N-1} \left\{ x[n] \cos\left(\frac{2\pi nk}{N}\right) \right\} + j \sum_{n=0}^{N-1} \left\{ x[n] \sin\left(\frac{2\pi nk}{N}\right) \right\}$$

$$X^*[-k] = \sum_{n=0}^{N-1} \left\{ x[n] \cos\left(\frac{2\pi nk}{N}\right) \right\} - j \sum_{n=0}^{N-1} \left\{ x[n] \sin\left(\frac{2\pi nk}{N}\right) \right\} \quad \ldots\ldots\ldots\ldots(ii)$$

From equation (i) and (ii), $X[k] = X^*[-k]$ **Proved**.

[7] DFT Property of Even Signal and Odd Signal

If $x[n] = x[-n]$
Then
$X[k] = X[-k]$

i.e. If x[n] is EVEN
Then X[k] is EVEN

If $x[n] = -x[-n]$
Then
$X[k] = -X[-k]$

i.e. If x[n] is ODD
Then X[k] is ODD

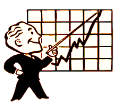
[8] Complex Conjugate Property

Proof of Complex sequence Property:

$$DFT\{x^*[n]\} = \sum_{n=0}^{N-1} x^*[n]\, W_N^{nk} = \left[\sum_{n=0}^{N-1} x[n]\, W_N^{-nk} \right]^*$$

$$= [X[-k]]^*$$

$$DFT\{x^*[n]\} = X^*[-k]$$ **Proved**



[9] Convolution Property

Proof of Convolution Property:

$$DFT\ \{x[n] \otimes h[n]\} = \sum_{n=0}^{N-1} \left(x[n] \otimes h[n]\right).W_N^{nk}$$

$$= \sum_{n=o}^{N-1} \sum_{m=0}^{N-1} x[m]h[n-m]\ w_N^{nk}$$

$$= \sum_{m=0}^{N-1} x[m] \left(\sum_{n=0}^{N-1} h[n-m]\ w_N^{nk} \right)$$

$$= \sum_{m=0}^{N-1} x[m].\ w_n^{mk}\ H[k]$$

$$\text{DFT}\ \{x[n] \otimes h[n]\} = X[k].H[k].$$ **Proved**

[11] Parseval's Energy Theorem

If $x[n] \longleftrightarrow X[k]$

Then $E = \sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$

Proof of Parseval's Theorem :

$$E = \sum_{n=0}^{N-1} |x[n]|^2 = \sum_{n=0}^{N-1} x[n].x^*[n]$$

$$Put \quad x^*[n] = \frac{1}{N} \sum_{k=0}^{N-1} X^*[k] W_N^{nk}$$

$$= \sum_{n=0}^{N-1} x[n] \left(\frac{1}{N} \sum_{k=0}^{N-1} X^*[k] w_n^{nk} \right)$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X^*[k]\ X[k]$$

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$ **Proved.**

NOTE: Parsevals' Energy Relation implies that the energy of the signal can be obtained in frequency domain and it depends on every frequency component present in the signal. Thus every frequency component of the signal contributes to the energy of the signal.

2.3 DISCRETE TIME FOURIER TRANSFORM

ENERGY DENSITY SPECTRUM OF DT APERIODIC SIGNALS

The energy of DT signal x[n] is E = $\sum_{n=-\infty}^{\infty} |x[n]|^2$

According to parseval's theorem, E = $\sum_{-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |x(w)|^2 \, dw$

Let Sx (w) = $|x(w)|^2$ = x (w) x* (w)

Sx (w) is the function of frequency and it is called energy density spectrum of x [n]. $E = \sum_{-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} Sx(w) \, dw.$

2.4 DISCRETE TIME FOURIER SERIES

The Fourier series of a periodic sequence $x_p[n]$ with period N is given by,

$$x_p[n] = \sum_{n=0}^{N-1} C_K \, e^{jwnk} \quad where \ w = \frac{2\pi}{N} \quad -\infty < n < \infty$$

The Fourier series coefficients are given by,

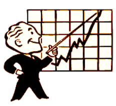
$$C_k = \frac{1}{N}\sum_{N=0}^{N-1} x_p[n] \, e^{-jwnk} \quad where \ w = \frac{2\pi}{N}$$

POWER DENSITY SPECTRUM OF PERIODIC DT SIGNALS

The average power of periodic DT signal is given by P = $\frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2$

According to Parseval's theorem, $P = \frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2 = \sum_{k=0}^{N-1} |C_k|^2$

The coefficients $|C_k|^2$ for k =0, 1, 2.....N-1 is the distribution of power as a function of frequency. It is called the power density spectrum of the DT periodic signal.



Q(2) Develop the relationship between Discrete Time Fourier Series of a periodic sequence $x_p[n]$ and DFT of a sequence x[n].

Solution : Relationship between DTFS and DFT

The fourier series representation of a periodic sequence $x_p[n]$ with fundamental period N is given by,

$$x_p[n] = \sum_{k=0}^{N-1} C_k \, e^{\frac{j2\pi nk}{N}} \;, \quad -\infty \le n \le \infty$$

Where the fourier series coefficients are given by,

$$C_k = \frac{1}{N} \sum x_p[n] e^{\frac{-j2\pi nk}{N}} \;, \qquad 0 \le k \le n-1$$

By DFT, $X[k] = \sum_{n=0}^{N-1} x[n] \, W_N^{nk} = \sum_{n=0}^{N-1} x[n] \, e^{\frac{-j2\pi nk}{N}}$

By comparing the above equations, If x[n] = $x_p[n]$ then **X [k] = N.C_k.**

Q(3) Develop the relationship between DTFT and DFT .

Solution : Relationship between DTFT and DFT.

Let x[n] be an a periodic finite Energy sequence.

The DTFT is given by, $X(w) = \sum_{n=0}^{N-1} x[n] \, e^{-jnw}$

If X(w) is sampled at N equally sampled frequencies,

Then

$$X[k] = X(w)\Big|_{w=\frac{2\pi k}{N}} \qquad 0 \le k \le N-1$$

$$X[k] = \sum_{N=0}^{N-1} x[n] \, e^{\frac{-j2\pi nk}{N}}$$

The original DT signal x[n] can be recovered without aliasing provided length of x[n] is less than or equal to N. The spectral components $\{X[k]\}$ corresponds to the spectrum of a periodic sequence of period N.

Q(4) Develop the relationship between ZT and DFT of Discrete Time Signal x[n]

Solution : Relationship between ZT and DFT.

The ZT of a sequence x [n] is given by,

$$X[z]=\sum_{-\infty}^{\infty} x[n]z^{-n}$$

If X(z) is sampled at N equally spaced points on the unit circle,

$$Z_k = e^{j2\pi k/N} \quad \text{for } 0 \le k \le N-1$$

We get , $X(z)\Big|_{z=e^{j2\pi k/N}} = \sum_{n=0}^{N-1} x[n]\ e^{\frac{-j2\pi nk}{N}} = X[k].$

That means,

$$X[k]=X(z)\Big|_{Z=e^{\frac{j2\pi k}{N}}}$$

$$X(z)=\sum_{n=0}^{N-1} X[n]z^{-n} \quad \text{when x [n] is N point sequence}$$

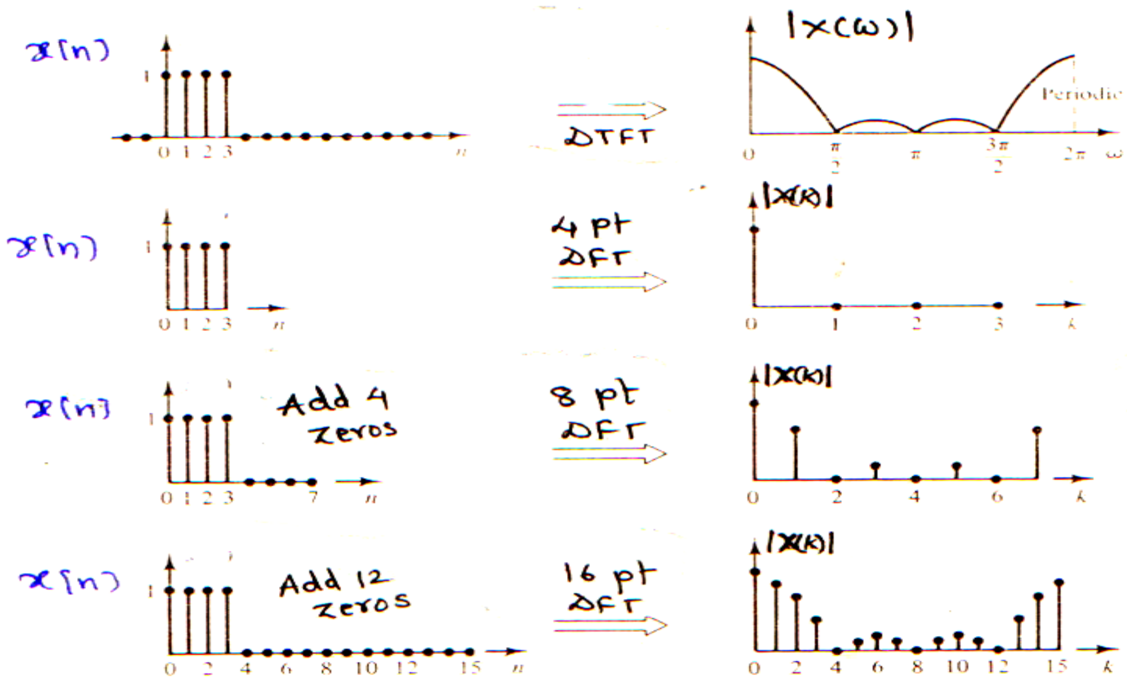
DFT computes the values of the Z transform for evenly spaced points around the unit circle for a given sequence.

Q(5) What is the effect of zero padding ?

ANS :

Let x[n] be N_1 pt sequence. If DFT of x[n] is taken then the sampled values of the fourier transform are spaced $\frac{2\pi}{N_1}$ apart. If x [n] is padded with N_2 zeros to give sequence of N values and if N pt DFT is taken then, the sampled values of fourier transform are spaced $\frac{2\pi}{N_1+N_2}$ apart. As more zeros are added, the DFT points are closely spaced samples of the furrier transform of the original sequence thus giving a better displayed version of the fourier Transform. i.e. just a better display of available information. The resolution of the spectrum increases.

The following figure gives spectrum of 4, 8 and 16 pt DFT of the zero padded original sequence.



2.3 Fast Fourier Transform

Why DFT is Slow ?

By DFT, $X[k] = \sum_{n=0}^{N-1} x[n] w_N^{nk}$

* Total Complex Multiplications = N^2

* Total Complex Additions = $N(N-1) = N^2 - N$

Let $P = a + jb$ and $Q = c + jd$

(1) $P \times Q = (a + jb)(c + jd)$
$= (ab - bd) + j(bc + ad)$ ← 4 Real Multiplications and 2 Real Additions

For 1 Complex Multiplications we require 4 Real Multiplications. and 2 Real Additions

(2) $P + Q = (a + jb) + (c + jd)$
$= (a + c) + j(b + d)$ ← 2 Real Additions

For 1 Complex Addition we require 2 Real Additions.

To find real Multiplications and Additions in DFT :

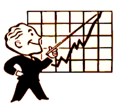
For N^2 Complex Multiplications we require $4N^2$ Real Multiplications and $2N^2$ Real Additions.
For (N^2-N) Complex Additions we require $2(N^2-N)$ Real Additions.

SO, Total Real Multilplications = $4N^2$
Total Real Additions = $2N^2 + 2(N^2-N) = 4N^2 - 2N$

➢ Computational Efficiency of Radix –2 Algorithms :-

By Radix –2 DIT-FFT / DIF-FFT Algorithms

- Total Complex Multiplications = $\frac{N}{2}\log_2 N$
- Total Complex Additions = $N\log_2 N$.
- Total Real Multilplications = $2N \log_2 N$
- Total Real Additions = $3N \log_2 N$



➢ Radix-2, DIT-FFT FLOWGRAPH FOR N = 8

[

By DFT, $X[k] = \sum_{n=0}^{N-1} x[n]\, w_N^{nk}$

By Decomposing N point DFT into two $\frac{N}{2}$ pt DFT'S,

$$X[k] = \sum_{n\ \text{even}} x[n] W_N^{nk} + \sum_{n\ \text{odd}} x[n] W_N^{nk}$$

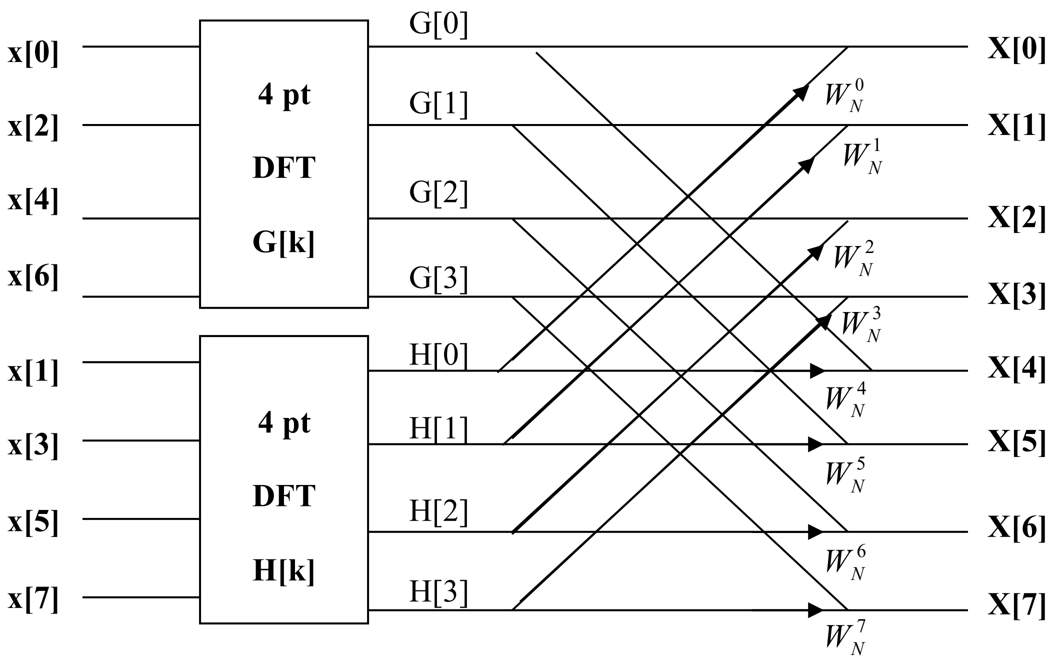
$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r]\, W_N^{2rk} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1]\, W_N^{(2r+1)k}$$

$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r]\, w_{\frac{N}{2}}^{rk} + W_N^k \sum_{r=0}^{\frac{N}{2}-1} x[2r+1]\, W_{\frac{N}{2}}^{rk}$$

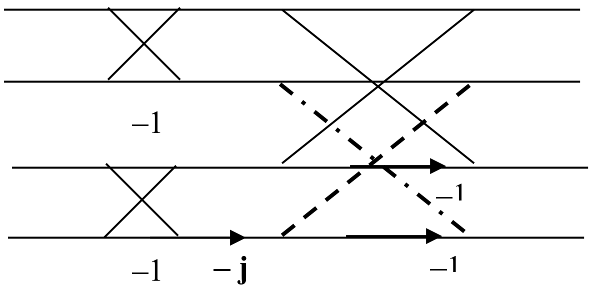
Let $X[k] = G[k] + W_N^k\, H[k]$

Where $G[k] = DFT\{x[2r]\}$ and $H[k] = DFT\{x[2r+1]\}$

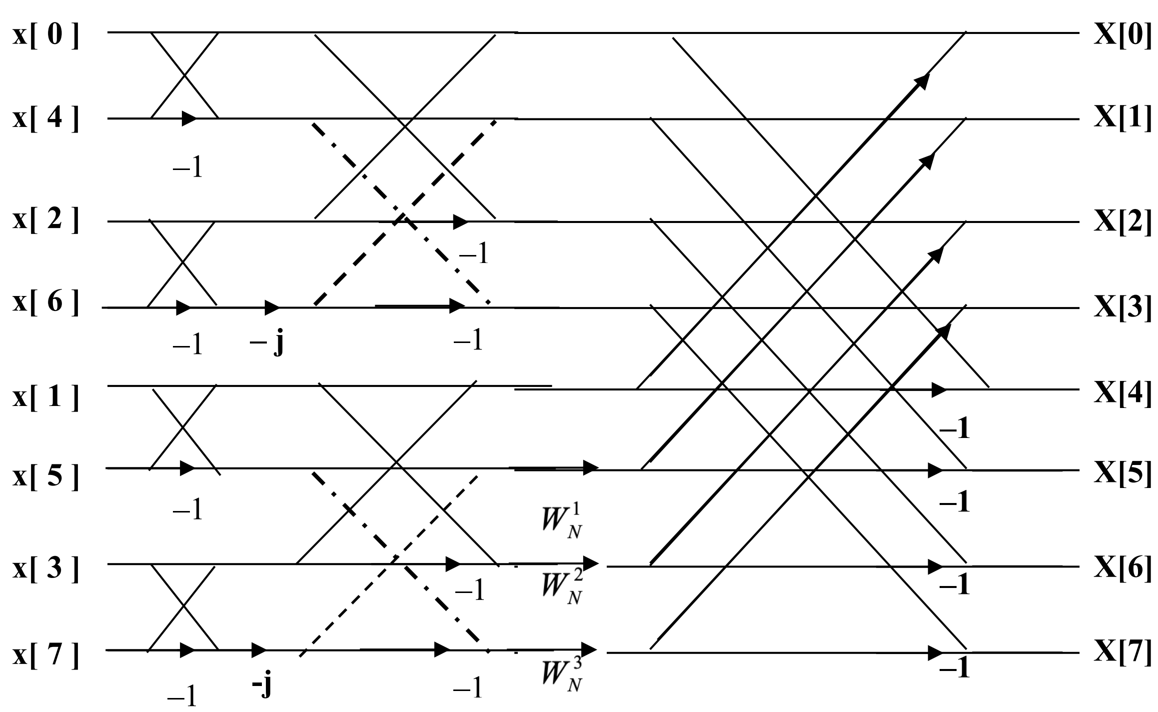
$$G[k] = DFT\begin{bmatrix} x[0] \\ x[2] \\ x[4] \\ x[6] \end{bmatrix} \qquad H[k] = DFT\begin{bmatrix} x[1] \\ x[3] \\ x[5] \\ x[7] \end{bmatrix}$$



DIT FFT flowgraph For N = 4 is given by,



By substituting we get,



➢ Radix-3 DIT-FFT Algorithm : FFT flowgraph for N=3

By DFT $X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk}$ where N =3

$$X[k] = x[0] + x[1]\, W_N^{k} + x[2]\, W_N^{2k}$$

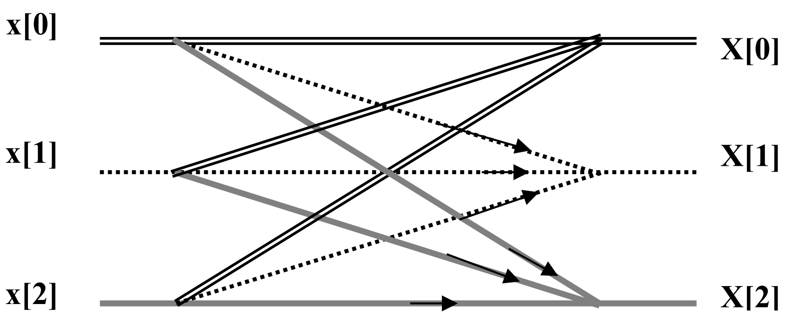
- -

k= 0, $X[0] = x[0] + x[1] + x[2]$

k = 1, $X[1] = x[0] + x[1]\, W_N^{1} + x[2]\, W_N^{2}$

k = 2, $X[2] = x[0] + x[1]\, W_N^{2} + x[2]\, W_N^{4}$

FFT flowgraph for N = 3 is given by,



Q(6) Develop DIT FFT Algorithm for decomposing DFT for N= 6 and draw the flowgraph for the following two cases. (a) Using Two 3 pt DFT's (b) Using Three 2 pt DFT's

Solution :

(a) DIT FFT flowgraph for N = 6 using Two 3 pt DFT's :

By DFT

$$X[k] = \sum_{n=0}^{N-1} X[n]\, W_N^{nk}$$

By Decomposing 6 pt DFT into Two 3pt DFT's

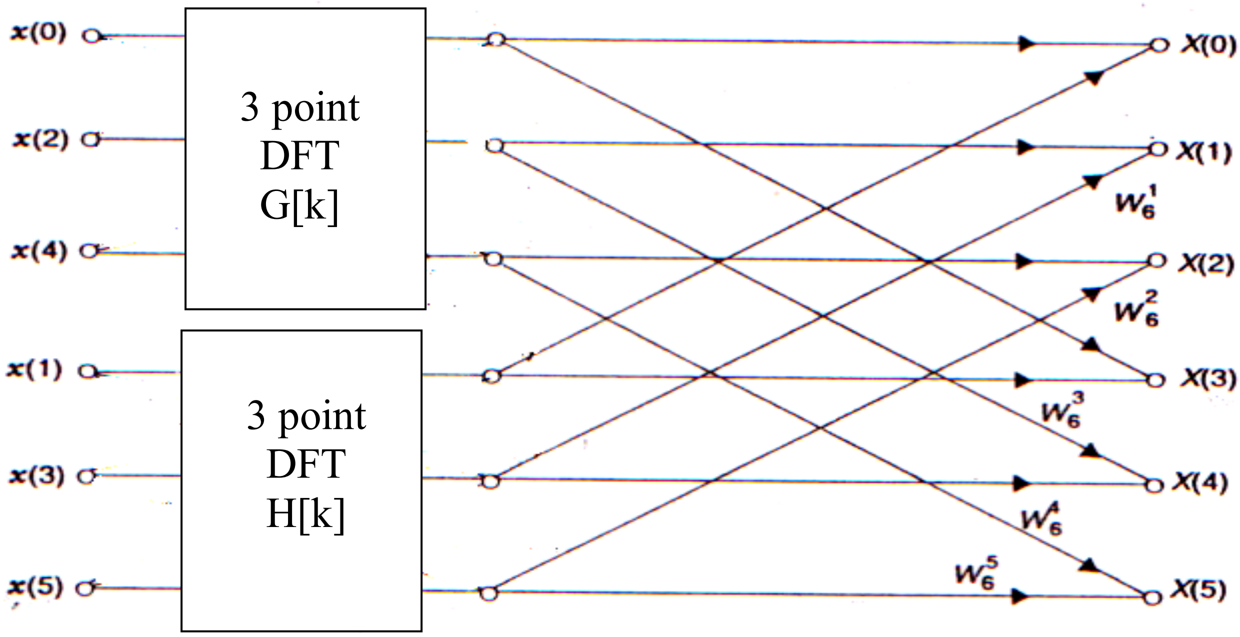
$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r]\ W_N^{2rk} + \sum_{r=0}^{\frac{N}{2}-1} x\left[2r+1\right] W_N^{(2r+1)k}$$

$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r]\ W_{\frac{N}{2}}^{rk} + W_N^{k} \sum_{r=0}^{\frac{N}{2}-1} x[2r+1]\ W_{\frac{N}{2}}^{rk}$$

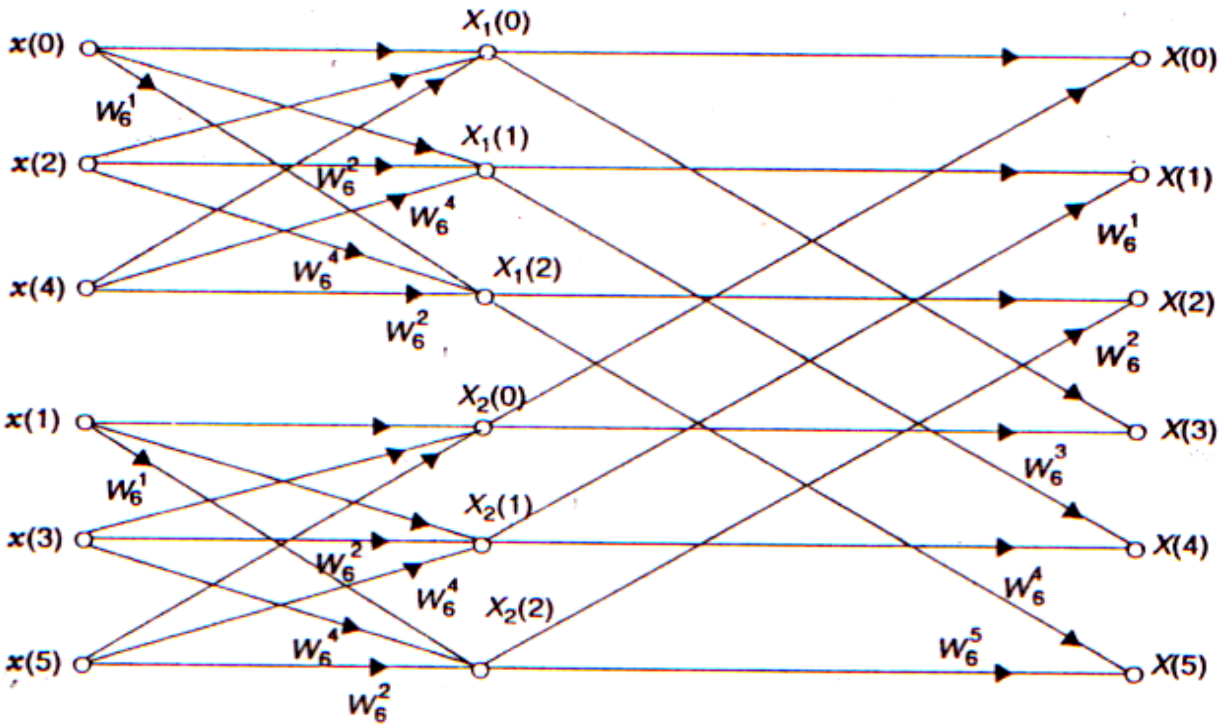
$$X[k] = G[k] + W_N^{k}\ H[k]$$

Where $G[k] = DFT\left\{X[2r]\right\}$ $\quad H[k] = DFT\left\{X[2r+1]\right\}$

$$G[k] = DFT\begin{bmatrix} X[0] \\ X[2] \\ X[4] \end{bmatrix} \qquad H[k] = DFT\begin{bmatrix} X[1] \\ X[3] \\ X[5] \end{bmatrix}$$



By substituting FFT flowgraph for N=3 we get,



(b) DIT FFT flowgraph for N = 6 using Three 2 pt DFT's

ANS :

By DFT $X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk}$

By Decomposing 6 pt DFT into Three 2 pt DFT's

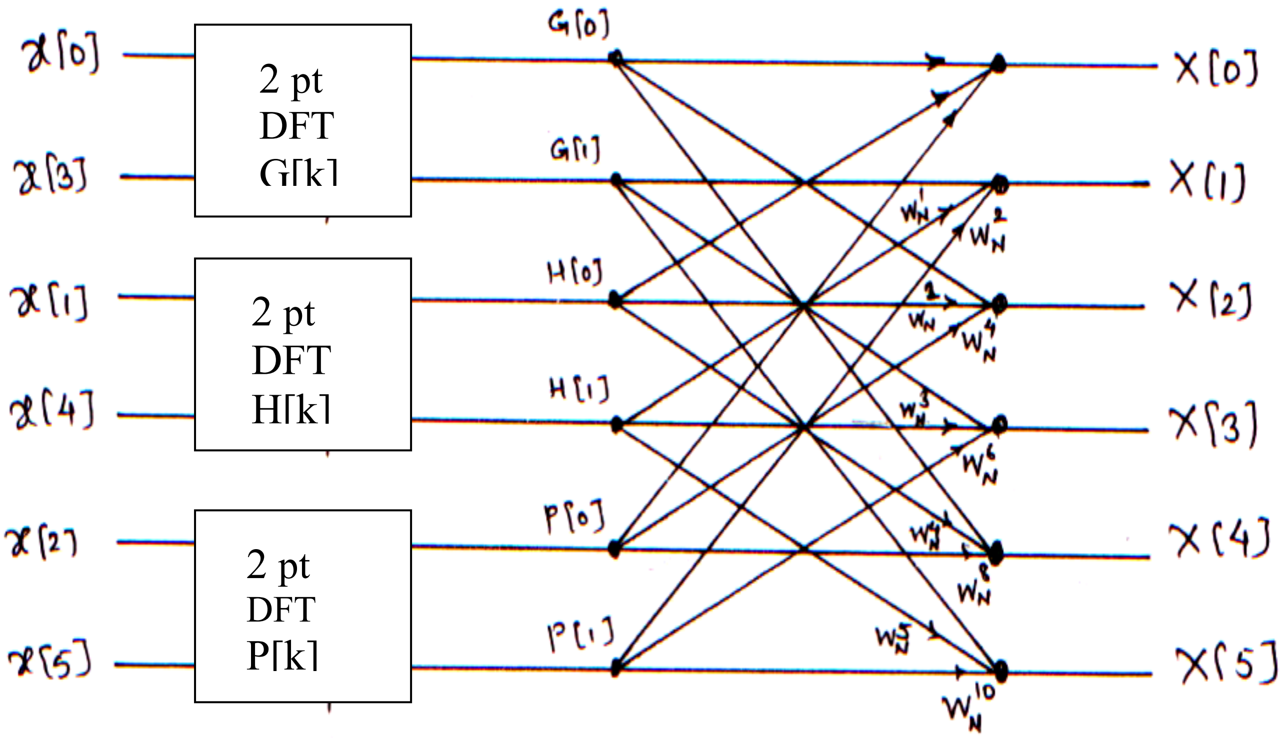
$$X[k] = \sum_{r=0}^{\frac{N}{3}-1} x[3r]\ W_N^{3rk} + \sum_{r=0}^{\frac{N}{3}-1} x[3r+1]\ W_N^{(3r+1)k} + \sum_{r=0}^{\frac{N}{3}-1} X[3r+2]\, W_N^{(3r+2)k}$$

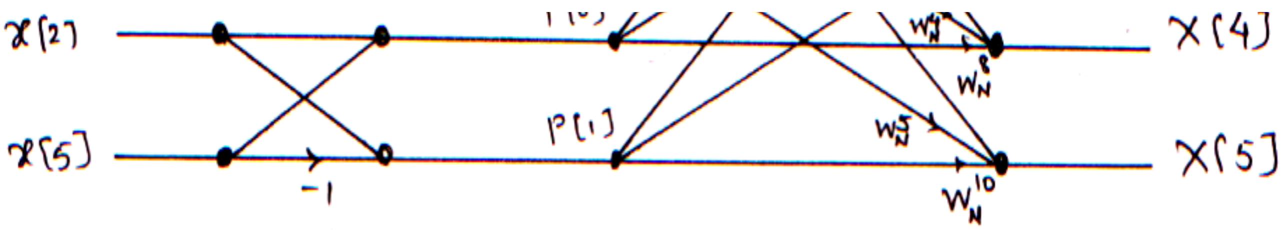
$$X[k] = \sum_{r=0}^{\frac{N}{3}-1} x[3r]\ W_{\frac{N}{3}}^{rk} + W_N^{k} \sum_{r=0}^{\frac{N}{3}-1} x[3r+1]\, W_{\frac{N}{3}}^{rk} + W_N^{2k} \sum_{r=0}^{\frac{N}{3}-1} x[3r+2]\, W_{\frac{N}{3}}^{rk}$$

$$X[k] = G[k] + W_N^{k}\, H[k] + W_N^{2k}\, P[k].$$

Where $G[k] = DFT\,\{X[3r]\}$ $\quad H[k] = \mathrm{DFT}\,\{X[3r+1]\}$ $\quad P[k] = \mathrm{DFT}\,\{X[3r+2]\}$

$$G[k] = DFT\begin{bmatrix} X[0] \\ X[3] \end{bmatrix} \qquad H[k] = DFT\begin{bmatrix} X[1] \\ X[4] \end{bmatrix} \qquad P[k] = DFT\begin{bmatrix} X[2] \\ X \end{bmatrix}$$





➢ DIF-FFT flowgraph for N=8

STEP – I. To find X [k] for k even.

By DFT, $$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk}$$

$$X[2r] = \sum_{n=0}^{N-1} x[n]\, W_n^{2rn}$$

$$X[2r] = \sum_{n=0}^{\frac{N}{2}-1} x[n]\, W_N^{2rn} + \sum_{n=\frac{N}{2}}^{N-1} x[n]\, W_N^{2rn}$$

$$X[2r] = \sum_{n=0}^{\frac{N}{2}-1} x[n]\, W_N^{2rn} + \sum_{n=0}^{\frac{N}{2}-1} X\left[n+\frac{N}{2}\right] W_N^{2r\left(n+\frac{N}{2}\right)}$$

$$X[2r] = \sum_{n=0}^{N/2-1} X[n]\, W_{N/2}^{rn} + \sum_{n=0}^{N/2-1} X\left[n+\frac{N}{2}\right] W_{N/2}^{rn}$$

$$X[2r] = \sum_{n=0}^{N/2-1} \left\{x[n] + x\left[n+\frac{N}{2}\right]\right\} W_{N/2}^{rn}$$

$$X[2r] = \sum_{n=0}^{N/2-1} g[n]\, W_{N/2}^{rn}$$

$$\boxed{X[2r] = DFT\,\{g[n]\}}$$

STEP- II To find X[k] For K odd.

$$X[2r+1] = \sum_{n=0}^{N-1} x[n]\, W_N^{n[2r+1]}$$

$$X[2r+1] = \sum_{n=0}^{N/2-1} x[n]\, W_N^{n(2r+1)} + \sum_{n=N/2}^{N-1} x[n] W_N^{n(2r+1)}$$

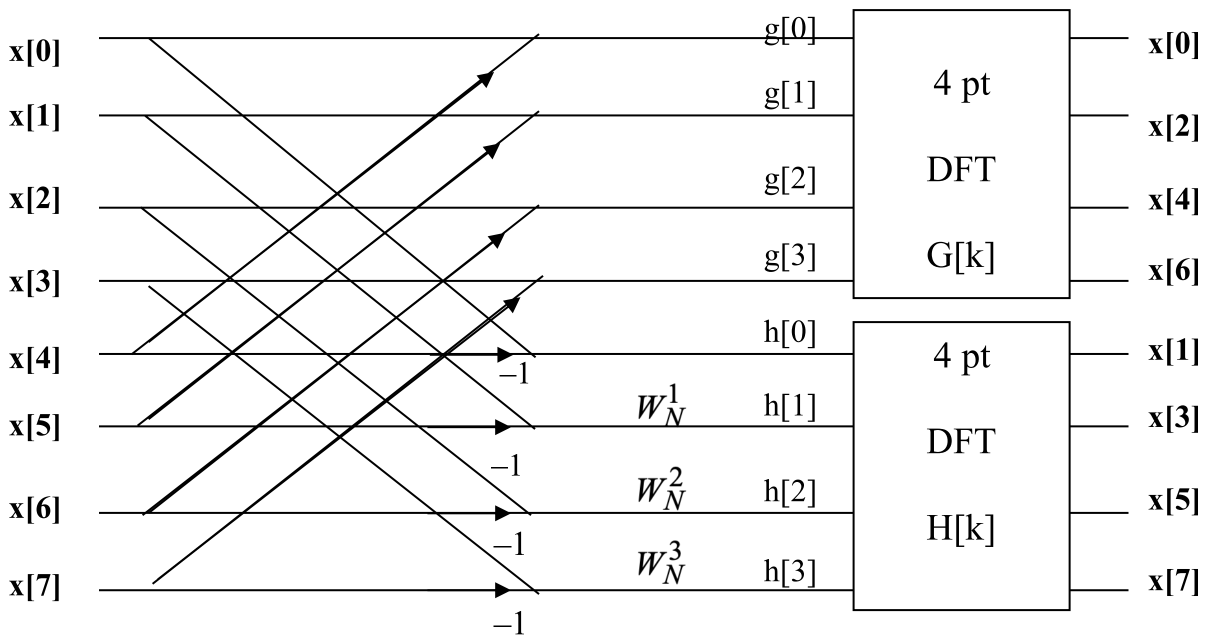
$$X[2r+1] = \sum_{n=0}^{N/2-1} x[n]\, W_N^{n(2r+1)} + \sum_{n=0}^{N/2-1} x\left[n+\frac{N}{2}\right] W_N^{(n+N/2)(2r+1)}$$

$$X[2r+1] = \sum_{n=0}^{N/2-1} x[n]\ W_N^n\ W_{N/2}^{rn} - \sum_{n=0}^{N/2-1} x\left[n+\frac{N}{2}\right] W_N^n\ \ W_{N/2}^{rn}$$

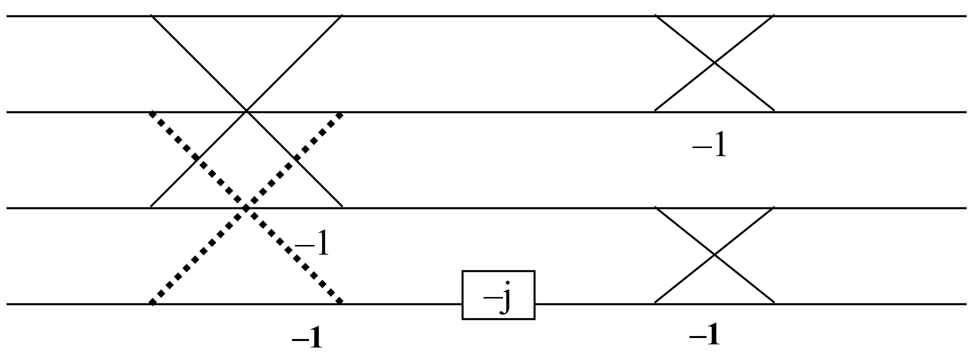
$$X[2r+1] = \sum_{n=0}^{N/2-1} \left\{\left(x[n]-x\left[n+\frac{N}{2}\right]\right) W_N^n\right\}\ \ W_{N/2}^{rn}$$

$$X[2r+1] = \sum_{n=0}^{N/2-1} h[n] W_{N/2}^{rn}$$

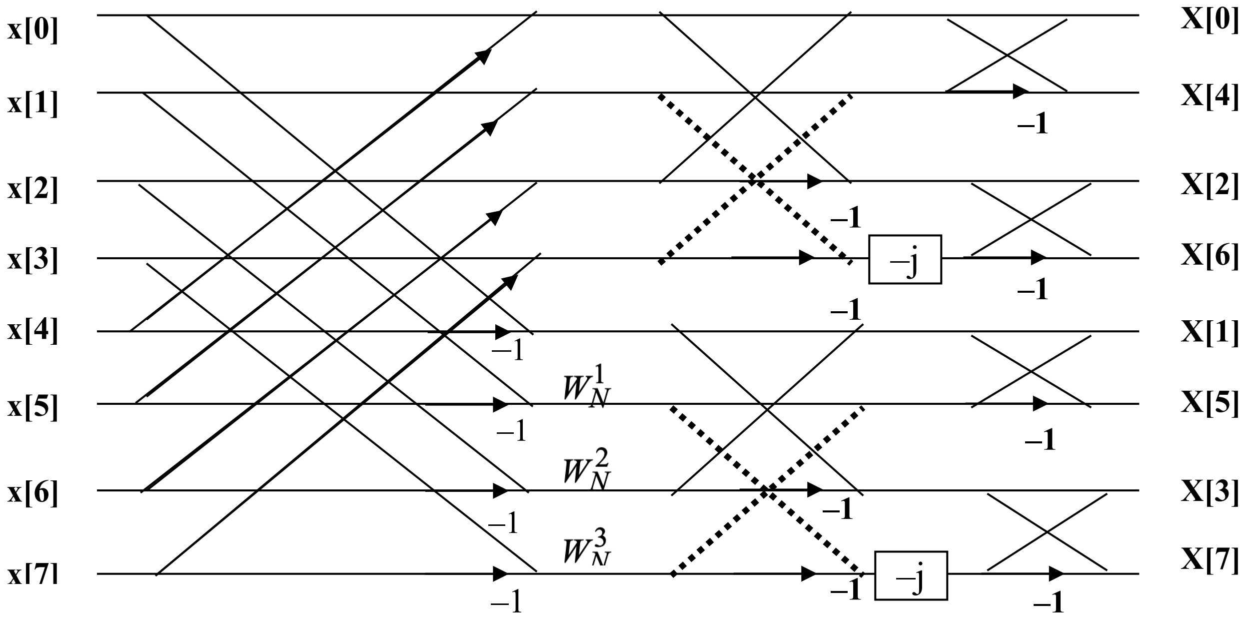
$$\boxed{X[2r+1] = \text{DFT}\ \{h[n]\}}$$



DIF-FFT flowgraph for N=4 is given by,



By substituting 4 point DIF-FFT flowgraph we get



Q(7) Let x[n]= {x[0], x[1], x[2], x[3]} and h[n] = {h[0], h[1], h[2]}. Give step by step procedure to obtain Circular Convolution using FFT–IFFT.

Solution :

(i) Let 'L' be the length of x[n] and 'M' be the length of h[n]. Then L = 4 and M = 3

To find Circular Convolution,

Select N = max(L, M)

N = 4

Let **N = 4** for radix-2 FFT algorithms,

(ii) Zero pad x[n] and h[n] to make their length equal to 4.

Let x[n] ={x[0], x[1], x[2], x[3]} and h[n] = {h[0], h[1], h[2], 0}.

(iii) Find FFT of both x[n] and h[n] using 4 point DIT-FFT algorithm.

(iv) Let Y[k] = X[k] H[k]

(v) Take IFFT of Y[k] to give y[n]

$$y[n] = \frac{1}{N}\left(\text{FFT}\{ Y^*[k] \}\right)^*$$

➢ APLICATIONS OF D F T / F F T

(I) Linear Filtering : To find output of a digital FIR filter for any given input say x[n].

(II) Spectral Analysis : To find magnitude spectrum and phase spectrum of signal.

Q(8) Given x[n] = {x[0], x[1], x[2], x[3]} and h[n] = {h[0], h[1], h[2]}.
Both are non-periodic finite length sequences. Give step by step procedure to obtain linear convolution using FFT–IFFT.

Solution :

(i) Let 'L' be the length of x[n] and 'M' be the length of h[n]. Then L = 4 and M = 3

To find Linear convolution using circular convolution using FFT-iFFT,

Select $N \geq L + M - 1$

$N \geq 4 + 3 - 1$

$N \quad 6$

Let **N = 8** for radix-2 FFT algorithms,

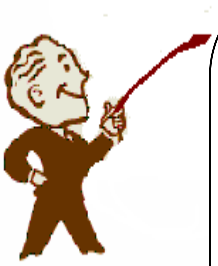
(ii) Zero pad x[n] and h[n] to make their length equal to 8.

Let x'[n] ={x[0], x[1], x[2], x[3], 0, 0, 0, 0 } and h'[n] = {h[0], h[1], h[2] 0, 0, 0, 0, 0}.

(iii) Find FFT of both x'[n] and h'[n] using 8 point DIT-FFT algorithm.

(iv) Let Y[k] = X'[k] H'[k]

(v) Take IFFT of Y[k] to give y[n] $\quad y[n] = \frac{1}{N}\left(\text{FFT}\{ Y^{*}[k] \}\right)^{*}$



- Limitations of LC by FFT Algorithms

  1). It is NOT suitable for Real Time Applications where entire input signal is not available.
  Examples include
  (i) ECG Monitoring system
  (ii) Digital Telephone System
  (iii) Weather Monitoring System

  2) It is NOT suitable for Long Data Sequence.
  Examples include Digital Song in the form of wave file (Fs = 44.1 KHz) + ECG/Weather Monitoring Systems. In most of the real Time applications data is Long sequence.

Filtering of Long Data Sequence

Overlap Add Method [May- 2005 Q7(c) Marks=12]

Let x[n] be the sequence that is to be convolved with finite impulse response h[n] of length M.

In overlap add method x[n] is decomposed into non overlapping sequence of length L. Thus x[n] may be written a sum of shifted finite length sequences of length M

such that $x[n] = \sum_{i=0}^{\infty} X i[n-iL]$ where $x_i[n] = \begin{cases} x[n+iL] & 0 \le n \le L-1 \\ 0 & otherwise \end{cases}$

Therefore, the linear convolution of x[n] with h[n] is

$$y[n] = x[n] * h[n] = \sum_{i=0}^{\infty} Xi[n-iL] * h[n] = \sum_{i=0} yi[n-iL]$$

Where yi[n] is given by, $y_i[n] = xi[n] * h[n]$.

Example : Consider a digital filter with M pt h[n]. Consider long data sequence x[n].

By decomposing x[n] into L point sequences we get, $x_1[n]$, $x_2[n]$, $x_3[n]$, etc …..

Overlap Add fast Convolution Algorithm :-

i) Decompose x[n] into L point sequences

ii) Select N

ii) Append h[n] with (N-M) zeros and find H [k] using N pt FFT flowgraph.

iii) Append each input signal data block by (N-L) zeros and find DFT of each block using N pt FFT algorithm.

iv) Let Yi [k] = Xi [k] . H [k] for i = 0,1…..∞

v) Obtain yi[n] by N pt iFFT Algorithm.

vi) Find y[n] ∴ $y[n] = \sum_{i=0}^{N-1} yi[n-iL] = y_0[n] + y_1[n-L] + y_2[n-2L] + ..$

✪ Overlap Save Method

Let x[n] be the sequence that is to be convolved with finite impulse response h[n] of length M.

In overlap save method, input data sequence x[n] is decomposed into number of L point sequences. To find Linear Convolution Select N such that $N \geq L + M + 1$. Each data block begins with the last (N-L) values in the previous data block, except the first data block which begins with (N–L) zeros.

> ### Overlap Save fast Convolution Algorithm :
>
> i) Decompose x[n] into L point sequences
>
> ii) Select $N \geq L + M + 1$
>
> iii) Begin each decomposed input sequence with (N–L) values of previous sequence
>
> iv) Append h[n] with (N–M) zeros and find H[k] using N point FFT.
>
> v) Perform N – point FFT on the selected data block $X_i[n]$.
>
> vi) Then $Yi[k] = Xi[k] \cdot H[k]$.
>
> vii) Perform N point iFFT of Yi[k].
>
> viii) Discard the first (N–L) values of $y_i[n]$ and save the remaining values of $y_i[n]$.
>
> ix) y[n] is obtained by concatenating all the saved values of $y_i[n]$

Q. What is in place calculation ?

ANS:

An "in place" FFT is simply an FFT that is calculated entirely inside its original sample memory.

For first stage one array is required to store input sample values, another array is required to store output values of stage-1. For stage-2 second array becomes input array and first array becomes output array and so on. Thus throughout only two arrays are required. Since the values in the next stage are stored in place of input array of previous stage, it does not require additional buffer memory.

⌘ Frequently Asked Questions on DFT+FFT

(1) Define Discrete Fourier Transform of x[n].

Ans : $$X[k] = \sum_{n=0}^{N-1} x[n] \; W_n^{\;nk}$$

(2) What is the interpretations of DFT coefficients ?

Ans : DFT gives N values of Fourier Transforms of DT signal x[n] at $w = \frac{2\pi k}{N}$ *for* $k = 0, 1, 2, \ldots\ldots N-1$.

They are equally spaced with frequency spacing of $\frac{2\pi}{N}$

(3) How many complex multiplications and additions are required to find DFT ?

Ans : By DFT

(i) Complex Multiplications $= N^2$

(ii) Complex Additions $= N(N-1)$

(4) How many real multiplications and additions are required to find DFT.

Ans :

Let P = a + j b and Q = c + j d

(1) P X Q = (a + j b) (c + j d)
= (ab – bd)+ j (bc + ad) ← 4 Real Multiplications and 2 Real Additions

For 1 Complex Multiplications we require 4 Real Multiplications. and 2 Real Additions

(2) P + Q = (a + j b) + (c + j d)
= (a + c)+ j (b + d) ← 2 Real Additions

For 1 Complex Addition we require 2 Real Additions

Now, In DFT, Total Complex Multiplications = N^2 and Total Complex Additions $= N(N-1)$

So, For N^2 Complex Multi. we require **4** N^2 **Real Multi. and 2** N^2 **Real Additions**

For (N^2-N) Complex Additions we require $2(N^2-N)$ Real Additions.

In summary, Total Real Mulitplications = **4** N^2

Total Real Additions = **2** $N^2 + 2(N^2-N) == 4N^2 - 2N$

(5) How many real multiplications and additions are required to find DFT of 32 point signal.?

Ans : By DFT **[Dec-2011 Q5(b) Marks=2]**

(i) Real Multiplications $= 4N^2 = 4(32)^2 = 4096$

(i) Real Additions $= 4N^2 - 2N = 40321$

(6) How many complex multiplications and additions are required to find FFT ?

Ans : By DFT

(i) Complex Multiplications $= \frac{N}{2} \log_2 N$

(ii) Complex Additions $= N \; \log_2 N$

(7) How many real multiplications and additions are required to find DFT of 32 point signal using FFT algorithm?

Ans : By FFT

(i) Real Multiplications $= 2\ N \log_2 N = 320$

(ii) Real Additions $= 3\ N\ \log_2 N = 480$

(8) What is Scaling and Linearity property of DFT ?

Ans : Scaling Property : If signal is multiplied by constant Then DFT is also multiplied by the same constant. i.e. DFT $\{ a\ x_1[n] \} = a\ X_1[k]$

Linearity Property : If signals are added, Then DFT's are also added.

i.e. DFT $\{ a\ x_1[n] + b\ x_2[n] \} = a\ X_1[k] + b\ X_2[k$

(9) What is the DFT of $\delta[n]$?

Ans : DFT $\{ \delta[n] \} = 1$

(10) What is the DFT of N pt signal u[n] ?

Ans : DFT $\{u[n] \} = N\ \delta[k]$

(11) What is the DFT of 4 pt x[n] where $x[n] = \delta[n] + u[n]$?

Ans : $X[k] = 1 + 4\ \delta[k]$

$= \{ 5, 1, 1, 1 \}$

(12) What is periodicity property of DFT ?

Ans : DFT equation produces periodic results with period = N

i.e. $X[k] = X[k+N] = X[k \text{ MOD } N] = X[((k))]$

Inverse DFT equation produces periodic results with period = N

i.e. $x[n] = x[n+N] = x[n \text{ MOD } N] = x[((n))]$

(13) Why DFT results are periodic ?

Ans : DFT results are periodic because twiddle factor is periodic with period = N

(14) DFT gives discrete spectrum or continuous spectrum ? Justify ?

Ans : DFT gives discrete spectrum.

If the signal is periodic then spectrum is discrete and if the signal is non-periodic then spectrum is continuous. DFT assumes that input signal is periodic and therefore DFT gives discrete spectrum.

(15) What do you know about spectrum is Discrete or continuous?

Ans : Continuous spectrum is defined for every value of frequency. Discrete spectrum is defined only at discrete values of frequencies ie. Not defined for every value of frequency.

(16) Find DFT of x[n] where $x[n] = u[n] + 2\ u[n-2] - 3\ u[n-4]$

Ans : Here $x[n] = \{ 1, 1, 3, 3 \}$ By DFT $X[k] = \{8, -2+2j, 2, -2-2j \}$

(17) Find DFT of 10 pt x[n] where x[n] = δ[n] + δ[n-5] ?

Ans : $X[k] = 1 + W_N^{5k} \quad = 1 + (-1)^k$

(18) What is Time shift and frequency shift property of DFT ?

Ans : $DFT\{x[n-m]\} = W_N^{mk} \; X[k]$

$$DFT\left\{W_N^{-mn} x[n]\right\} = X[k-m]$$

(19) What is symmetry property of DFT ?

Ans : If x[n] ←→ X[k] Then $X[k] = X^*[-k]$.

i.e. If x[n] is real valued signal, then real part of X[k] is symmetric about k = N/2 and Imaginary part of X[k] is Anti-symmetric about k = N/2.

(20) What is DFT property of EVEN signal ?

Ans : If x[n] is Even , Then X[k] is also Even

i.e.. If x[n] = x[–n] Then X[k] = X[–k]

(21) What is the DFT of real and even signal.?

Ans : If x[n] is Real and Even, Then X[k] is also Real and Even

Eg. x[n] = { 1, 2, 3, 2 }

X[k] = { 8, -2, 0, -2 }

(22) What is the DFT of Imaginary and Even signal ?

Ans : If x[n] is Imaginary and Even

Then X[k] is also Imaginary and Even

Eg. x[n] = { j, 2j, 3j, 2j }

X[k] = { 8j, -2j, 0, -2j }

(23) What is DFT property of ODD signal ?

Ans : If x[n] = – x[–n] Then X[k] = – X[–k]

i.e. If x[n] is Odd , Then X[k] is also Odd.

(24) What is the DFT of real and Odd signal ?

Ans : If x[n] is Real and Odd, Then X[k] is also Imaginary and Odd

Eg. x[n] ={ 0, 2, 0, –2 }

X[k] = { 0, – 4j, 0, 4j }

(25) What is the DFT of Imaginary and Odd signal ?

Ans : If x[n] is Imaginary and Odd

Then X[k] is also Real and Odd

Eg. x[n]={ 0, 2j, 0, – 2j }

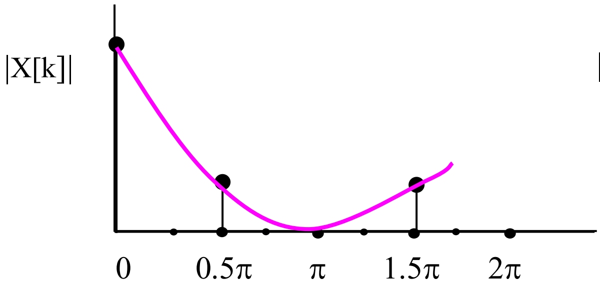
X[k] = { 0, 4, 0, – 4 }

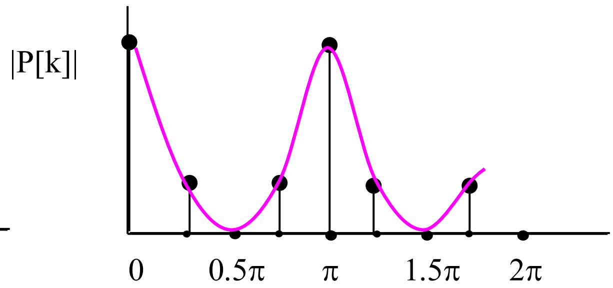
(26) If DT signal is **expanded** in time domain what will be the effect in frequency domain?

Ans : Expansion in time domain corresponds to Compression in frequency domain.

Eg. $x[n] = \{1,2,3,2\}$ $X[k] = \{8, -2, 0, -2\}$

Let $p[n] = \{1, 0, 2, 0, 3, 0, 2, 0\}$ Then $P[k] = \{8, -2, 0, -2, 8, -2, 0, -2\}$



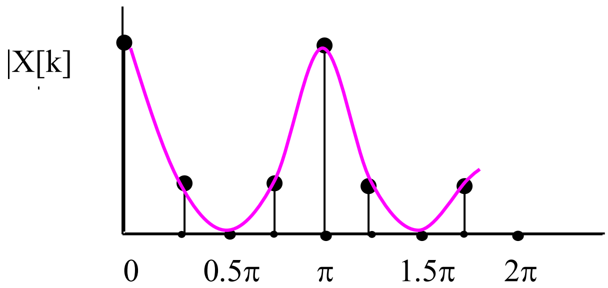


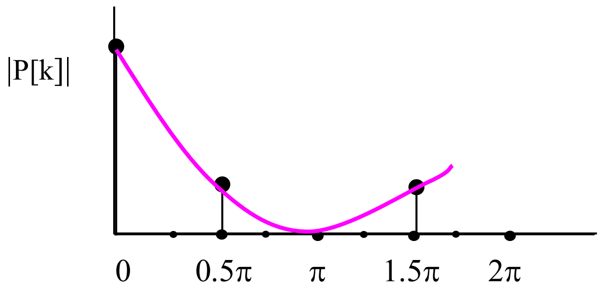
(27) If DT signal is **compressed** in time domain what will be the effect in frequency domain?

Ans : Compression in time domain corresponds to Expansion in frequency domain.

Eg. $x[n] = \{1, 0, 2, 0, 3, 0, 2, 0\}$ $X[k] = \{8, -2, 0, -2, 8, -2, 0, -2\}$

Let $p[n] = \{1,2,3,2\}$ Then $P[k] = \{8, -2, 0, -2\}$

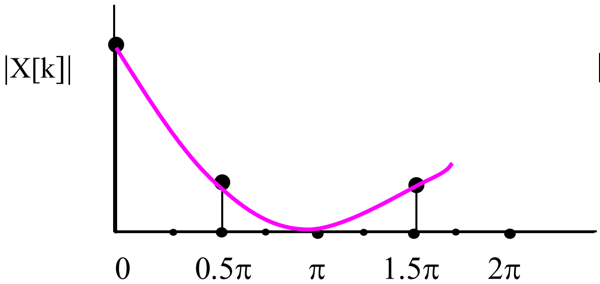


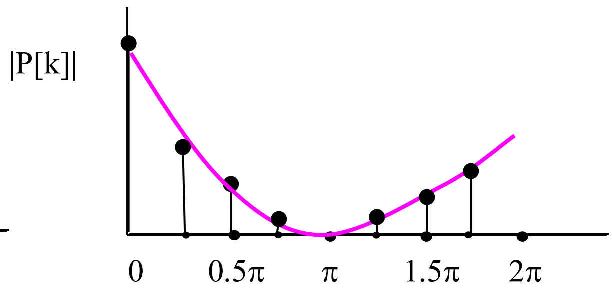


(28) If DT signal is **appended** by zeros in time domain what will be the effect in frequency domain?

Ans : Eg. $x[n] = \{1,2,3,2\}$ $X[k] = \{8, -2, 0, -2\}$ N=4 pt

Let $p[n] = \{1, 2, 3, 2, 0, 0, 0, 0\}$ N=8 pt





As the length of signal increases, the frequency spacing decreases.

The number of points per unit length i.e. resolution of the spectrum increases.

Therefore the approximation error in the representation of the spectrum decreases.

(29) What is convolution property of DFT ?

Ans : Convolution in time domain corresponds to multiplication in frequency domain.

If $x[n] \longleftrightarrow X[k]$ and

$h[n] \longleftrightarrow H[k]$

Then

$$\text{DFT}\{x[n] \otimes h[n]\} = X[k]\,H[k]$$

(30) What is correlation property of DFT ?

Ans : If $x[n] \longleftrightarrow X[k]$ and

$h[n] \longleftrightarrow H[k]$

Then

$$\text{DFT}\{\, x[n] \circ h[n] \,\} = X[k]\; H^*[k]$$

(31) How to find energy of signal from its DFT ?

Ans : According to parseval's energy theorem, Energy of the signal is given by,

$$E = \frac{1}{N} \sum_{k=0}^{N-1} |\, X[k]\,|^2$$

(32) Are FFT's limited to sizes that are powers of 2?

Ans : No. The most common and familiar FFT's are "radix 2". However, other radices are sometimes used, which are usually small numbers less than 10. For example, radix-4 is especially attractive because the "twiddle factors" are all 1, -1, j, or -j, which can be applied without any multiplications at all.

Also, "mixed radix" FFT's also can be done on "composite" sizes. In this case, you break a non-prime size down into its prime factors, and do an FFT whose stages use those factors. For example, an FFT of siz 1000 might be done in six stages using radices of 2 and 5, since 1000 = 2 * 2 * 2 * 5 * 5 * 5. It might also be done in three stages using radix 10, since 1000 = 10 * 10 * 10.

(33) What is an FFT "radix"?

Ans : The "radix" is the size of an FFT decomposition. In the example above, the radix was 2. For single-radix FFT's, the transform size must be a power of the radix. In the example above, the size was 32, which is 2 to the 5th power.

(34) What is an "in place" FFT?

Ans : An "in place" FFT is simply an FFT that is calculated entirely inside its original sample memory. In other words, calculating an "in place" FFT does not require additional buffer memory (as some FFT's do.)

(35) What is "bit reversal"?

Ans : "Bit reversal" is just what it sounds like: reversing the bits in a binary word from left to write. Therefore the MSB's become LSB's and the LSB's become MSB's. But what does that have to do with FFT's? Well, the data ordering required by radix-2 FFT's turns out to be in "bit reversed" order, so bit-reversed indexes are used to find order of input and output.

It is possible (but *slow*) to calculate these bit-reversed indices in software; however, bit reversals are trivial when implemented in hardware. Therefore, almost all DSP processors include a hardware bit-reversal indexing capability (which is one of the things that distinguishes them from other microprocessors.)

(36) How efficient is the FFT?

Ans : The DFT takes N^2 operations for N points. Since at any stage the computation required to combine smaller DFTs into larger DFTs is proportional to N, and there are $\log_2(N)$ stages (for radix 2), the total computation is proportional to $N \log_2(N)$. Therefore, the ratio between a DFT computation and an FFT computation for the same N is proportional to $N / \log_2(N)$. In cases where N is small this ratio is not very significant, but when N becomes large, this ratio gets very large. (Every time you double N, the numerator doubles, but the denominator only increases by 1.)

(37) FFT is faster than DFT. Justify.

Ans : FFT produces fast results because calculations are reduced by decomposition technique.

In FFT, N pt DFT is decomposed into two N/2 pt DFT's, N/2 pt DFT is decomposed into N/4 pt DFT's and so on… **Decomposition reduces calculations.** FFT algorithms are implemented using parallel processing techniques. Because calculations are done in parallel, FFT produces fast results.

Complex Multiplications :

	DFT	**FFT**
N	N^2	$\frac{N}{2}\log_2 N$
16	256	32
32	1,024	80
64	4,096	192
256	65,536	1,024
512	2,62,144	2,304
1024	10,48,576	5,120

(38) What do you mean by Decimation ?

Ans : Decimation means Sampling.

(39) Why Radix-2 algorithms are fast compared to radix-3 algorithms. ?

Ans : In FFT, N pt DFT is decomposed into two N/2 pt DFT's, N/2 pt DFT is decomposed into N/4 pt DFT's and so on… **Decomposition reduces calculations** This process continues till further decomposition is not possible. In radix-2 last level of decomposition is when the length of signal becomes 2 pt.

For minimum calculations there must be maximum levels of decompositions. In Radix-2 algorithms, we get maximum levels of decompositions and therefore radix-2 algorithms requires less calculations. Radix-2 algorithms are fast algorithms.

(40) Which algorithm is more powerful : DIT-FFT or DIF-FFT ?

Ans : Computationally, both the algorithms are exactly same.

(41) What is the order of input and output sequence in 8 pt DIT-FFT ?

Ans : x[n] = { x[0], x[4], x[2], x[6], x[1], x[5], x[3], x[7] }

X[k] = { X[0], X[1], X[2], X[3], X[4], X[5], X[6], X[7] }

(42) What is the order of input and output sequence in 8 pt DIF-FFT?

Ans : x[n] = { x[0], x[1], x[2], x[3], x[4], x[5], x[6], x[7] }

X[k] = { X[0], X[4], X[2], X[6], X[1], X[5], X[3], X[7] }

(43) What are the applications of FFT. ?

Ans : (i) Linear Filtering i.e. to find output of digital filter for any given input sequence

(ii) Spectral Analysis i.e. to find magnitude spectrum and phase spectrum

(iii) Circular Correlation ie to find degree of similarity between two signals.

(44) How to find CC using DFT ?

Ans : To find CC of x[n] and h[n] using DFT,

(i) Select N

Let N = max(L,M) where L is the length of x[n] and M is length of h[n],

(ii) Append x[n] by (N-L) zeros and Append h[n] by (N-M) zeros

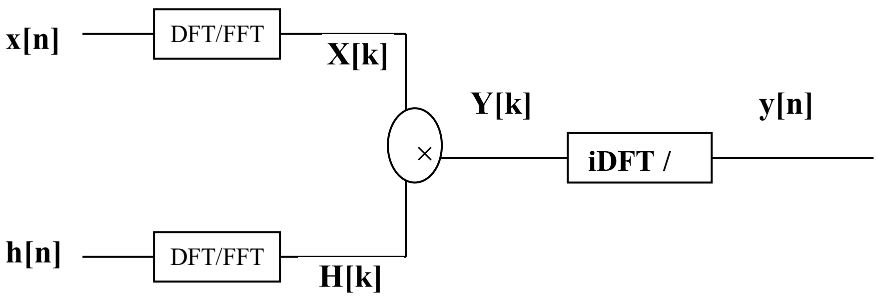
(iii) Find X[k] where $X[k] = \sum_{n=0}^{N-1} x[n]\ W_N^{nk}$

(iv) Find H[k] where $H[k] = \sum_{n=0}^{N-1} h[n]\ W_N^{nk}$

(v) Let Y[k] = X[k] H[k].

(vi) Find y[n] where $y[n] = \frac{i}{N} \sum_{K=0}^{N-1} Y[k]\ W_N^{-nk}$

→→ Always explain wrt diagram



(45) How to find CC using FFT ?

Ans : To find CC of x[n] and h[n] using FFT,

(i) Select N

Let N = max(L,M) where L is the length of x[n] and M is length of h[n],

(ii) Append x[n] by (N-L) zeros and Append h[n] by (N-M) zeros

(iii) Find X[k] by using N point DIT-FFT / DIF-FFT flowgraph

(iv) Find H[k] by using N point DIT-FFT / DIF-FFT flowgraph

(v) Let Y[k] = X[k] H[k].

(vi) Find y[n] by Inverse FFT.

By Inverse FFT, y[n] = $\frac{1}{N} \left(FFT \left\{ Y^*[k] \right\} \right)^*$

→→ Always explain wrt diagram.

(46) How to find LC using CC ?

Ans : To find LC of x[n] and h[n] using CC,

(i) /Select N:

Let N ≥ L + M – 1 where L is the length of x[n] and M is length of h[n],

(ii) Append x[n] by (N-L) zeros and Append h[n] by (N-M) zeros

(iii) Perform N point Circular convolution of x[n] and h[n]

(47) How to find LC using DFT /FFT ?

Ans : : To find LC of x[n] and h[n] using DFT/FFT,

(i) Select N

Let $N \geq L + M - 1$ where L is the length of x[n] and M is length of h[n],

(ii) Append x[n] by (N-L) zeros and Append h[n] by (N-M) zeros

(iii) Perform N point Circular convolution of x[n] and h[n] using DFT/FFT.

Find N point X[k] and H[k]

Let Y[k] = X[k] H[k].

Find y[n] by Inverse DFT/FFT.

→→ Always explain wrt diagram.

(48) How to find output of the filter using DFT ?

Ans : Output of the filter is Linear convolution of impulse response with the input of the signal.

To find output means to find LC by DFT.

(49) How to find output of the IIR filter using DFT / FFT?

Ans : Output of the filter is Linear convolution of impulse response with the input of the signal.

To find output means to find LC by DFT/FFT. Length of h[n] in IIR filter is infinite. So, DFT/FFT implementation of infinite length signals is not possible.

(50) What is the length of linearly convolved signals ?

Ans : Length of linearly convolved signal is always equal to $N = L + M - 1$ where L is length of first signal and M is length of second signal.

(51) How to find output of the FIR filter using FFT ?

Ans : In FIR filter length of h[n] is finite. Output of the filter is always Linear convolution of impulse response with the input of the signal. To find output i.e. to find LC by FFT

(i) Select N

Let $N \geq L + M - 1$ where L is the length of x[n] and M is length of h[n],

(ii) Append x[n] by (N-L) zeros and Append h[n] by (N-M) zeros

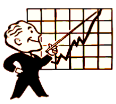
(iii) Perform N point Circular convolution of x[n] and h[n] using FFT.

* Find N point X[k] and H[k] by using FFT flowgraph.
* Let Y[k] = X[k] H[k].
* Find y[n] by Inverse FFT. $y[n] = \frac{1}{N}\left(FFT\ \{Y^*[k]\}\right)^*$

→→ Always explain wrt diagram.

(52) What is periodic convolution ?

Ans : Periodic convolution is convolution of two periodic signals of the same period. When two periodic signals are periodic with common period, periodic convolution is similar to circular convolution.



(53) What is the difference between circular convolution and periodic convolution ?

Ans : In periodic convolution input signals are originally periodic with common value of period.

In circular convolution, if input signals are not periodic then they are assumed to be periodic with period = N where N = max(L,M) where L is the length of first signal and M is length of second signal.

(54) What do you mean by aliasing in circular convolution ?

Ans : In circular convolution if value of $N < L+M-1$ then last M-1 values of y[n] wraps around gets added with first M-1 values of y[n]. This is called aliasing.

(55) Why FFT is used to find output of FIR filter ? Justify.

Ans : FFT produces fast results because in practical applications FFT algorithms are implemented using parallel processing techniques. Because in FFT calculations are done in parallel, FFT produces fast results.

(56) What are the limitations of filtering by FFT algorithms? Justify.

Ans : (i) NOT suitable for real time applications :

FFT algorithms are implemented using parallel processing techniques. When FFT is used input is applied in parallel i.e simultaneously. For real time applications entire input signal is not available. So FFT algorithms can not be used.

(ii) NOT suitable for Long Data Sequence.

As the length of the input sequence increases, the no of stages in FFT will also increase proportionally and so the delay increases, processing time at each stage increases.

(57) How to find output of FIR filter for long input sequence.

Ans : In FIR filter length of h[n] is finite. Output of the filter is always Linear Convolution of impulse response with the input of the signal. To find output of digital FIR filter FFT technique is used. But for Long data sequence, direct FFT technique is not suitable.

For long data sequence, Overlap Add Method using FFT or Overlap Save Method using FFT is used.

(58) How to find output of FIR filter for real time input signal.?

Ans : In real time application entire input is not available and input signal has to be processed online. Length of input signal depends on application. It can be long sequence also.

In FIR filter length of h[n] is finite. Output of the filter is always Linear Convolution of impulse response with the input of the signal.

To find output of digital FIR filter, Overlap Add Method using FFT or Overlap Save Method using FFT is used.

(59) How to find output of IIR filter for real time input signal.?

Ans : In real time application entire input is not available and input signal has to be processed online. Length of input signal depends on application. It can be long sequence also.

In IIR filter length of h[n] is infinite. Output of the filter is always Linear Convolution of impulse response with the input of the signal. To find output of digital IIR filter, Overlap Add Method using FFT or Overlap Save Method using FFT can not be used.

Output of digital IIR filter is calculated using difference equation recursively.

(60) How to find output of IIR filter for long input sequence.?

Ans : In IIR filter length of h[n] is infinite. Output of the filter is always Linear Convolution of impulse response with the input of the signal. To find output of digital IIR filter, Overlap Add Method using FFT or Overlap Save Method using FFT can not be used.

Output of digital IIR filter is calculated using difference equation recursively.

(61) What is DTFT ?

Ans : DTFT is Fourier Transform of DT signal that converts the sampled DT signal from time domain to frequency domain. Frequency domain representation parameters are magnitude and phase. DTFT gives frequency response that includes magnitude response and phase response.

(62) If DTFT is Fourier Transform of DT signal then What is DFT ?

Ans : DFT is frequency sampling of DTFT. When DTFT is sampled in frequency domain we get DFT.

(63) Describe the relation between DFT and DTFT.

Ans : DFT is frequency sampling of DTFT. When DTFT is sampled in frequency domain with frequency spacing of $w = \frac{2\pi}{N}$ we get DFT coefficients.

i.e. $X[k] = X(w)\Big|_{w=\frac{2\pi k}{N}}$

(64) Why DFT ? What is the need of Sampling DTFT ?

Ans : In digital domain for processing, input has to be discrete. For frequency domain analysis, DT signal is converted to frequency domain. Frequency domain representation of DT signal is continuous, NOT discrete. For processing in digital domain we need to take sampled values. The frequency samples thus obtained are called DFT coefficient. That is what DFT is.

(65) How to find DFT of infinite length sequence ?

Ans : To find DFT of infinite length sequence x[n]:

(i) Find DTFT of x[n] i.e. $X(w) = \sum_{n=-\infty}^{\infty} x[n]\ e^{-jnw}$

(ii) Find DFT by frequency sampling DTFT. i.e. $X[k] = X(w)\Big|_{w=\frac{2\pi k}{N}}$

DFT coefficients can be obtained by evaluating DFT equation.

(66) Find DTFT and Energy Density Spectrum of x[n] = u[n].

Ans : Energy of u[n] is infinite. Therefore u[n] is not energy signal.

Fourier Transform is defined only for energy signal.

(67) DTFT gives continuous spectra or discrete spectra?.

Ans : When signal is periodic spectrum is Discrete. If the signal is not-periodic then spectrum is always continuous.

DTFT is fourier transform of Non-periodic signals. Therefore DTFT gives continuous spectra.

(68) What is the necessary condition to find DTFT of any signal. ?

Ans : To find DTFT of any signal the necessary condition is, signal must be an energy signal. It must be absolutely summable.

(69) What is Power Density Spectrum of Periodic DT Signals ?

Ans :

The average power of periodic DT signal is given by $P = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$

According to Parseval's theorem, $P = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2 = \sum_{k=0}^{N-1} |C_k|^2$

The coefficients $|C_k|^2$ for k =0, 1, 2…..N-1 is the distribution of power as a function of frequency is called the power density spectrum of the DT periodic signal

(70) What is Energy Density Spectrum of DT Aperiodic Signals

Ans :

The energy of DT signal x[n] is $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$

According to parseval's theorem, $E = \sum_{-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(w)|^2 dw$

Let $S_x(w) = |X(w)|^2 = X(w)\, X(w)^*$

Sx (w) is the function of frequency and it is called energy density spectrum of x [n].

$$E = \sum_{-\infty}^{\infty} |x[n]|^2 = = \frac{1}{2\pi} \int_{-\pi}^{\pi} Sx(w)\, dw.$$

(71) How to use FFT algorithm to find IDFT ?

Ans : By IFFT equation we get, $x[n] = \frac{1}{N} \left(FFT\{ X^*[k] \}\right)^*$

Algo : (i) Find $X^*[k]$

(ii) Find FFT ($X^*[k]$) using DIT-FFT/DIF-FFT flowgraph,
Here same flowgraph is required to find FFT $\{X^*[k]\}$result..

(iii) Find x[n]

(72) What is the difference between DFT and DTFS ?

Ans: DFT is frequency sampling of DTFT. When DTFT is sampled in frequency domain with frequency spacing of $w = \frac{2\pi}{N}$ we get DFT coefficients.

Discrete Time Fourier Series is a representation of a periodic signal using frequency domain coefficients.

(73) What is the relation between DFT and DTFS ?

Ans :

The relation between DFT and DTFS igiven by,

$$\mathbf{X[k] = N.C_k}$$

Where the fourier series coefficients $\mathbf{C_k}$ are given by,

$$C_k = \frac{1}{N}\sum x_p[n]e^{\frac{-j2\pi nk}{N}}, \qquad 0 \le k \le n-1$$

(74) What is the relation between DFT and DTFT ?

Ans :

DFT X[k] is frequency sampling of DTFT X(w) such that,

$$X[k] = X(w)\Big|_{w=\frac{2\pi k}{N}} \qquad 0 \le k \le N-1$$

(75) What is the relation between DTFT and ZT ?

Ans :

$$X[z] = X(w)\Big|_{Z=e^{jw}}$$

(76) What is the relation between DFT and ZT ?

Ans :

$$X[k] = X(z)\Big|_{Z=e^{\frac{j2\pi k}{N}}}$$

(77) What is the effect of zero padding ?

Ans :

Adding or padding zeros to the end of a sequence gives a better display of available information. The resolution of the spectrum increases. DFT points more closely spaced on frequency scale. The frequency spacing therefore decreases.

(78) How to find DFT of Two N point Real Sequence using a single N point FFT ?

Ans :

(I) Let p[n] and q[n] be two real N – point sequences.

Let $x[n] = p[n] + j\,q[n]$ _______(i)

$x^*[n] = p[n] - j\,q[n]$ _______ (ii)

By eqⁿ (i) + eqⁿ (ii) we get,

$x[n] + x^*[n] = 2\,p[n]$

$p[n] = \frac{1}{2}\left(x[n] + x^*[n]\right)$

By DFT,

$P[k] = \frac{1}{2}\left(X[k] + X^*[-k]\right)$

By eqⁿ (i) - eqⁿ (ii) we get,

$x[n] - x^*[n] = 2j\;q[n]$

$q[n] = \frac{1}{2j}\left(x[n] - x^*[n]\right)$

By DFT

$Q[k] = \frac{1}{2j}\left(X[k] - X^*[-k]\right)$

(II) Find X[k] using N point FFT Flowgraph.

(III) Find P[k] and Q[k] using X[k] by evaluating the above derived equations.

ie. $P[k] = \frac{1}{2}\left(X[k] + X^*[-k]\right)$ and $Q[k] = \frac{1}{2j}\left(X[k] - X^*[-k]\right)$

(79) How to find DFT of 2N point DFT of real valued sequence using a single N point FFT Algorithm?

Ans :

(1) Let a [n] be a real valued sequence of length 2N.

By DIT FFT equation, $A[k] = G[k] + W_N^k\ H[k]$ _______ I

where $G[k] = DFT\,\{g[n]\} = DFT\,\{a[2r]\}\ \ for\ \ r = 0,1.....(N-1)$

and $H[k] = DFT\,\{h[n]\} = DFT\,\{a[2r+1]\}$

(2) Let $x[n] = g[n] + j\,h[n]$ ______ i

Then $x^*[n] = g[n] - jh\,[n]$ ______ ii

By eqⁿ (i) + eqⁿ (ii),

$x[n] + x^*[n] = 2g[n]$

$g[n] = \frac{1}{2}\left(x[n] + x^*[n]\right)$

By DFT

$G[k] = \frac{1}{2}\left(X[k] + X^*[-k]\right)$_____ II

By eqⁿ (i) -eqⁿ (ii)

$x[n] - x^*[n] = 2jh[n]$

$h[n] = \frac{1}{2j}\left(x[n] - x^*[n]\right)$

By DFT

$H[k] = \frac{1}{2j}\left(X[k] - X^*[-k]\right)$____III