Kiran Talele's

Introduction to FIR Filter : POLES & ZERO

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NOTE-1: For Linear Phase FIR filter, h[n] is either Symmetric OR Anti-symmetric.

Symmetric h[n]

h[n] = { 1, 2, 3, 2, 1 } N = 5 ODD

h[n] = { 1, 2, 2, 1 } N = 4 EVEN

Antisymmetric h[n]

h[n] = { 1, 2, 0, -2, -1 } N = 5 ODD

h[n] = { 1, 2, -2, -1 } N = 4 EVEN

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NOTE-2: **When h[n] is either Symmetric OR Antisymmetric, ZEROS of the filter are always in Reciprocal order.**

i.e. If Z_1 is ZERO of the filter,

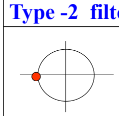
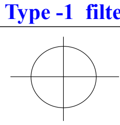
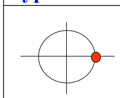
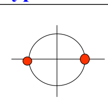
Then $^1/_{Z1}$ is also a ZERO of the filter

NOTE-3 : If ZEROS of the filter are in reciprocal order, Then filter is Linear Phase FIR filter

$$H(z) = \frac{\left(z-\frac{1}{2}\right)(z-2)}{Z^2} \qquad H(z) = \frac{(z+1)(z-1)}{Z^2}$$

$$H(z) = \frac{(z+1)\left(Z-\frac{1}{2}\right)(z-2)}{Z^3}$$

Position of Definite ZEROS

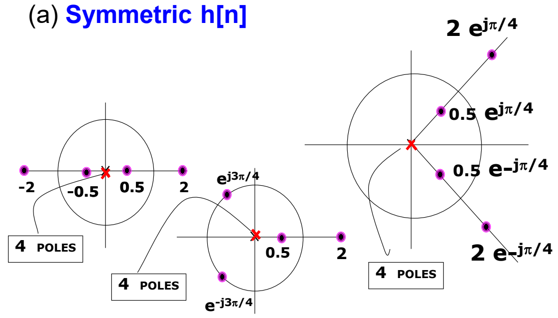
I. When h[n] is Symmetric	
Type -2 filter : N EVEN	Type -1 filter : N ODD
	
II. When h[n] is Anti-symmetric	
Type -4 filter : N EVEN	Type -3 filter : N ODD
	

(**Q1**) Show three possible POLE ZERO pattern of 4th order Linear Phase FIR filter with (a) symmetric h[n] (b) Anti-symmetric h[n]

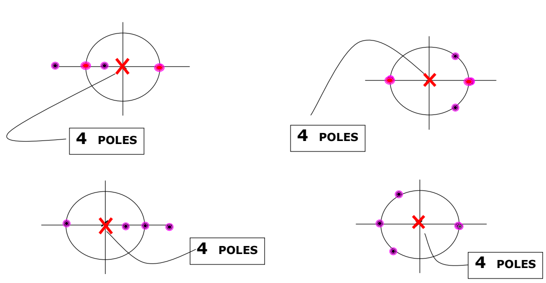
Solution :

- Linear Phase FIR filter   Order = 4
  Let No of POLES = 4
- Let No of ZEROS = 4
- N-1 = 4
- N = 5

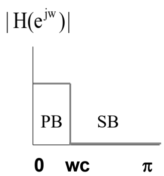
(a) **Symmetric h[n]**



(b) Anti-symmetric h[n] with N odd has definite zeros at $z = 1$ and $z = -1$



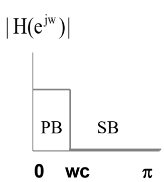
(**Q2**) Anti-symmetric h[n] can-not be used for LPF design.Justify.



$$H(e^{jw}) = \begin{cases} 1 & 0 \le w \le w_c \\ 0 & Otherwise \end{cases}$$

$$H(e^{jw}) \neq 0 \text{ for } 0 \le w \le w_c$$

$$H(e^{jw}) \neq 0 \text{ at } w = 0$$



$$H(e^{jw}) \neq 0 \quad at \quad w = 0$$

$$H(e^{jw})\Big|_{w=0} \neq 0 \quad for \quad LPF$$

Put $z = e^{jw}$

At $w = 0$ $z = 1$

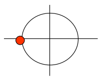
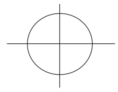
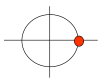
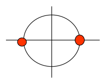
$$H(z)\Big|_{z=1} \neq 0 \quad for \quad LPF$$

- For Anti-symmetric h[n], there exists definite ZERO at z = 1.

$$H(z)\Big|_{z=1} = 0$$

Therefore, Anti-symmetric h[n] can-not be used for LPF design.

Position of Definite ZEROS,…..

I. When h[n] is Symmetric	
Type -2 filter : N EVEN	Type -1 filter : N ODD
 LPF ✓ HPF ✗	 LPF ✓ HPF ✓ BPF ✓
II. When h[n] is Anti-symmetric	
Type -4 filter : N EVEN	Type -3 filter : N ODD
 LPF ✗ HPF ✓	 LPF ✗ HPF ✗ BPF ✓

(Q3) One of the zeros of a third order causal linear phase Low Pass FIR filter lies at z= - 0.5

- Find the location of the other zeros
- Find transfer function of filter.

Solution :

Linear Phase LPF

Order = 3

Given ZERO at z_1 = - 0.5

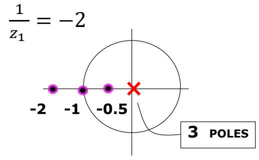
Given ZERO at z_1 = -0.5

For Symmetric h[n] with N even there exists a definite zero at z = -1

ZEROS of a linear phase filter occurs at reciprocal location.

For ZERO at z_1 = -0.5,

There exists ZERO at $\frac{1}{z_1} = -2$



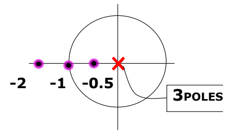
$$H(z) = \frac{\left(z+\frac{1}{2}\right)(z+2)\ (z+1)}{z^3}$$

$$H(z) = \frac{\left(z^2+\frac{5}{2}z+1\right)\ (z+1)}{z^3}$$

$$H(z) = \frac{z^3+\frac{7}{2}z^2+\frac{7}{2}z+1}{z^3}$$

$$H(z) = 1+\frac{7}{2}z^{-1}+\frac{7}{2}z^{-2}+z^{-3}$$

$$h[n] = \{\ 1, \frac{7}{2}, \frac{7}{2}\ , 1\ \}$$



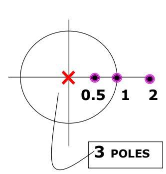
- (**Q4**) **One of the zeros of a third order causal linear phase High pass FIR filter lies at z=0.5**
- **Find the location of the other zeros**
- **Find the transfer function of filter.**

Solution :

N –1 = 3 So N = 4 (Even)

- For Anti symmetric h[n] with N even there exists definite zero at z = 1.
- ZEROS of a linear phase filter occur at reciprocal location

$$For\ z_0 = \frac{1}{2},\ \ \frac{1}{z_0} == 2$$

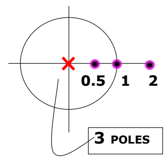


$$H(z)=\frac{\left(z-\frac{1}{2}\right)(z-2)\ (z-1)}{z^3}$$

$$H(z)=\frac{\left(z^2-\frac{5}{2}z+1\right)\ (z-1)}{z^3}$$

$$H(z)=\frac{z^3-\frac{7}{2}z^2+\frac{7}{2}z-1}{z^3}$$

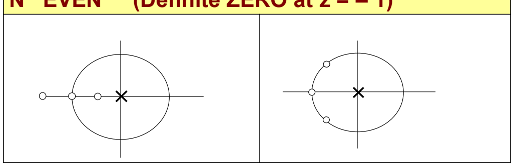
$$H(z)=1-\frac{7}{2}z^{-1}+\frac{7}{2}z^{-2}-z^{-3}$$



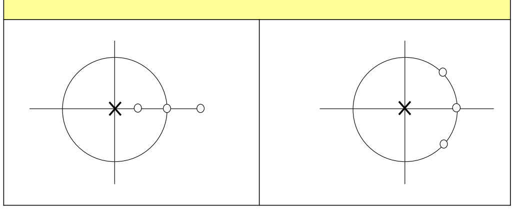
(**Q5**) Draw POLE-ZERO location of third order Linear Phase LPF & HPF.

Solution :

Linear Phase LPF with Symmetric h[n] and N EVEN (Definite ZERO at z = – 1)



Linear Phase HPF with Anti-Symmetric h[n] and N EVEN. (Definite ZERO at z = 1)



(Q6) An antisymmetric filter has one ZERO at z=0.5 . What is the minimum order of this filter?

Solution :

- Zeros of a linear phase filter occur at reciprocal location

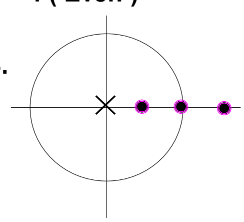
$$z_0 = \frac{1}{2} \qquad \therefore \frac{1}{z_0} = 2$$

- **Total No of zeros = 3**

  order = 3

  i.e. N-1 = 3 so N = 4 (Even)

  Minimum order is 3.



(Q7) Determine the coefficients of High Pass linear phase FIR filter of length N = 4 which has frequency response such that, and

$$\left| H\left(\pi/4\right) \right| = 1/2$$

$$\left| H\left(\frac{3\pi}{4}\right) \right| = 1$$

Solution :

HPF Linear Phase FIR filter

Length N = 4

Given frequency response :

$$\left| H\left(\pi/4\right) \right| = 1/2 \qquad \left| H\left(\frac{3\pi}{4}\right) \right| = 1$$

For **High Pass Filter** with N = 4 (even)
h[n] must be Anti-symmetric.

Let h [n] = { h0, h1, –h1, –ho }

By ZT, $H(z) = h_0 + h_1 z^{-1} - h_1 z^{-2} - h_o z^{-3}$

Put $z = e^{jw}$,

$H(e^{jw}) = h_0 + h_1 e^{-jw} - h_1 e^{-j2w} - h_o e^{-j3w}$

$$H(e^{jw}) = e^{-j\frac{3}{2}w}\left[h_0 e^{+j\frac{3}{2}w} + h_1 e^{j1/2 w} - h_1 e^{-j\frac{1}{2}w} - h_0 e^{-j\frac{3}{2}w}\right]$$

$$H(w) = e^{-j\frac{3}{2}w}\left[2\,j\,h_0 \sin\left(\frac{3}{2}w\right) - 2\,j\;h_1 \sin\left(\frac{1}{2}w\right)\right]$$

$$H(w) = e^{-j\frac{3}{2}w}\left[2\;h_0 \sin\left(\frac{3}{2}w\right) - 2\,h_1 \sin\left(\frac{1}{2}w\right)\right]$$

$$|\,H(w)| = \left[2\;h_0 \sin\left(\frac{3}{2}w\right) - 2\,h_1 \sin\left(\frac{1}{2}w\right)\right]$$

- (i) At $w = \pi/4$

$$\left| H\left(\frac{\pi}{4}\right) \right| = 2\ h_0\ \sin\left(\frac{3\pi}{8}\right) - 2\,h_1\ \sin\left(\frac{\pi}{8}\right)$$

½ = 1.85 h0 + 0.75 h1 __(I)

(ii) At $w = \frac{3\pi}{4}$

$$\left| H\left(\frac{\pi}{4}\right) \right| = 2\,h_0\ \sin\left(\frac{9\pi}{8}\right) - 2h_1\ \sin\left(\frac{3\pi}{8}\right)$$

1 = – 0.765 ho + 1.85 h1 ------(II)

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½ = 1.85 h0 + 0.75 h1 __(I)

1 = – 0.765 ho + 1.85 h1 ------(II)

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- He is a Treasurer of IEEE Bombay Section and Mentor for Startup Incubation & Intellectual Asset Creation.
- He received incentives for excellent performance in academics and research from Management of S.P.I.T. in 2008-09. He is a recipient of P.R. Bapat IEEE Bombay Section Outstanding Volunteer Award 2019.

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