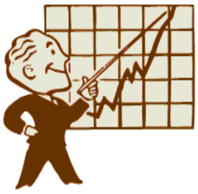


DFT - FFT Practice Problems

Q(1) Let $x[n]=\left\{\underset{\uparrow}{1} \quad 0 \quad 2 \quad 0 \quad 3 \quad 0 \quad 4 \quad 0\right\}$ Find DFT of x[n].

Solution : To find X[k]

By DFT, $X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk}$ where (i) $N = 8$ (ii) $W_N^1 = e^{-j\frac{2\pi}{N}}$

$X[k] = X[0] + X[1]\, W_N^{k} + X[2]\, W_N^{2k} + X[3]\, W_N^{3k} + X[4]\, W_N^{4k} + X[5] W_N^{5k} + X[6]\, W_N^{6k} + X[7]\, W_N^{7k}$

$X[k] = 1 + 2\, W_N^{2k} + 3\, W_N^{4k} + 4\, W_N^{6k}$

(i) $X[0] = 1 + 2 + 3 + 4$
$= 10$

(ii) $X[1] = 1 + 2\, W_N^{2} + 3\, W_N^{4} + 4\, W_N^{6}$
$= 1 + 2(-j) + 3(-1) + 4(j)$
$\therefore X[1] = -2 + 2j$

(iii) $X[2] = 1 + 2\, W_N^{4} + 3\, W_N^{8} + 4\, W_N^{12}$
$= 1 + 2(-1) + 3(1) + 4(-1)$
$\therefore X[2] = -2$

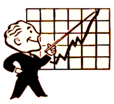
(iv) $X[3] = 1 + 2\, W_N^{6} + 3\, W_N^{12} + 4\, W_N^{18}$
$= 1 + 2(j) + 3(-1) + 4(-j)$
$\therefore X[3] = -2-2j$

(v) $X[4] = 1 + 2\, W_N^{8} + 3\, W_N^{16} + 4\, W_N^{24}$
$= 1 + 2(1) + 3(1) + 4(1)$
$\therefore X[4] = 10$

(vi) $X[5] = 1 + 2\, W_N^{10} + 3\, W_N^{20} + 4\, W_N^{30}$
$= 1 + 2(-j) + 3(-1) + 4(j)$
$\therefore X[5] = -2 + 2j$

(vii) $X[6] = 1 + 2\, W_N^{12} + 3\, W_N^{24} + 4\, W_N^{36}$
$= 1 + 2(-1) + 3(1) + 4(-1)$
$\therefore X[6] = -2$

(viii) $X[7] = 1 + 2\, W_N^{14} + 3\, W_N^{28} + 4\, W_N^{42}$
$= 1 + 2(j) + 3(-1) + 4(-j)$
$\therefore X[7] = -2 - 2j$



Q(2) Let $x[n] = \left\{ \underset{\uparrow}{1} \quad 2 \quad 3 \quad 4 \quad 0 \quad 0 \quad 0 \quad 0 \right\}$ Find 8 point DFT of x[n].

Solution : To find X[k]

By DFT, $X[k] = \sum_{n=0}^{N-1} x[n]\ W_N^{nk}$ where (i) $N = 8$ (ii) $W_N^1 = e^{-j\frac{2\pi}{N}}$

$X[k] = X[0] + X[1]\ W_N^{k} + X[2]\ W_N^{2k} + X[3]\ W_N^{3k} + X[4]\ W_N^{4k} + X[5] W_N^{5k} + X[6]\ W_N^{6k} + X[7]\ W_N^{7k}$

$X[k] = 1 + 2\ W_N^{k} + 3\ W_N^{2k} + 4\ W_N^{3k}$

....

(i) $X[0] = 1 + 2 + 3 + 4 = 10$

(ii) $X[1] = 1 + 2\ W_N^1 + 3\ W_N^2 + 4\ W_N^3$

$= 1 + 2(0.707 - j\ 0.707) + 3(-j) + 4(-0.707 - j\ 0.707)$

$\therefore X[1] = -0.414 - j\ 7.242$

(iii) $X[2] = 1 + 2\ W_N^2 + 3\ W_N^4 + 4\ W_N^6$

$= 1 + 2(-j) + 3(-1) + 4(j)$

$\therefore X[2] = -2 + j2$

(iv) $X[3] = 1 + 2\ W_N^3 + 3\ W_N^6 + 4\ W_N^9$

$= 1 + 2(-0.707 - j0.707) + 3(j) + 4(0.707 - j0.707)$

$\therefore X[3] = 2.414 - j\ 1.242$

(v) $X[4] = 1 + 2\ W_N^4 + 3\ W_N^8 + 4\ W_N^{12}$

$= 1 + 2(-1) + 3(1) + 4(-1)$

$\therefore X[4] = -2$

(vi) $X[5] = 1 + 2\ W_N^5 + 3\ W_N^{10} + 4\ W_N^{15}$

$= 1 + 2(-0.707 + j0.707) + 3(-j) + 4(0.707 + j0.707)$

$\therefore X[5] = 2.414 + j\ 1.24$

(vii) $X[6] = 1 + 2\ W_N^6 + 3\ W_N^{12} + 4\ W_N^{18}$

$= 1 + 2(j) + 3(-1) + 4(-j)$

$\therefore X[6] = -2 - j2$

(viii) $X[7] = 1 + 2\ W_N^7 + 3\ W_N^{14} + 4\ W_N^{21}$

$= 1 + 2(0.707 + j0.707) + 3(j) + 4(-0.707 + j0.707)$

$\therefore X[7] = -0.414 + j\ 7.242$

Q(3) **Let** $x[n] = \cos(n\frac{\pi}{2})\, u[n]$ **Find X[k]**

Solution : Given $x[n] = \cos\left(n\frac{\pi}{2}\right) u[n]$

Let $x[n] = \cos(nw)\, u[n]$ where $w = \frac{\pi}{2}$

i.e. $2\pi f = \frac{\pi}{2}$

$f = \frac{1}{4}$ $\Leftarrow$ *Rational Number*

That means x[n] is periodic with period N = 4.

Now, $x[(n)_{N=4}] = \{ \underset{\uparrow}{1}, \ 0 \ -1 \ 0 \}$

To find 4 point DFT of x[n]:

By DFT, $X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk}$ *where* $N = 4$ *and* $W_N^1 = e^{-j\frac{2\pi}{N}}$

$$X[k] = \begin{bmatrix} w_N^0 & w_N^0 & w_N^0 & w_N^0 \\ w_N^0 & w_N^1 & w_N^2 & w_N^3 \\ w_N^0 & w_N^2 & w_N^4 & w_N^6 \\ w_N^0 & w_N^3 & w_N^6 & w_N^9 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$$X[k] = \begin{bmatrix} 0 & k = 0 \\ 2 & \\ 0 & \\ 2 & \end{bmatrix}$$

Q(4) **Given** $x[n] = \sin\left(n\frac{\pi}{4}\right) u[n]$ Find X[k].

Solution :

Given $x[n] = \sin\left(n\frac{\pi}{4}\right) u[n]$

Let $x[n] = \sin(nw)\, u[n]$ were $w = \frac{\pi}{4}$ i.e. $2\pi f = \frac{\pi}{4}$

$f = \frac{1}{8}$ $\Leftarrow$ *Rational Number*

That means x[n] is periodic with period N = 8.

Now, $x[n] = \left(\underset{\uparrow}{0}, \ \frac{1}{\sqrt{2}}, \ 1, \ \frac{1}{\sqrt{2}}, \ 0, \ \frac{-1}{\sqrt{2}}, \ -1, \ \frac{-1}{\sqrt{2}} \right)$

To find 8point DFT of x[n]:

By DFT, $X[k] = \sum_{n=0}^{N-1} x[n] \ W_N^{nk}$ where N = 8,

$X[k] = X[0] + X[1] \, W_N^{k} + X[2] \, W_N^{2k} + X[3] \, W_N^{3k} + X[4] \, W_N^{4k} + X[5] W_N^{5k} + X[6] \, W_N^{6k} + X[7] \, W_N^{7k}$

$X[k] = 0 + \frac{1}{\sqrt{2}} \ W_N^{k} + W_N^{2k} + \frac{1}{\sqrt{2}} \ W_N^{3k} - \frac{1}{\sqrt{2}} \ W_N^{5k} - W_N^{6k} - \frac{1}{\sqrt{2}} \ 4 \ W_N^{7k}$

(i) $X[0] = 0 + \frac{1}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}} + 0 + \frac{-1}{\sqrt{2}} + -1 + \frac{-1}{\sqrt{2}} = 0$

(ii) $X[1] = \frac{1}{\sqrt{2}} \ W_N^{1} + W_N^{2} + \frac{1}{\sqrt{2}} \ W_N^{3} - \frac{1}{\sqrt{2}} \ W_N^{5} - W_N^{6} - \frac{1}{\sqrt{2}} \ 4 \ W_N^{7}$

$\therefore X[1] = -4j$

(iii) $X[2] = \frac{1}{\sqrt{2}} \ W_N^{2} + W_N^{4} + \frac{1}{\sqrt{2}} \ W_N^{6} - \frac{1}{\sqrt{2}} \ W_N^{10} - W_N^{12} - \frac{1}{\sqrt{2}} \ 4 \ W_N^{14}$

$\therefore X[2] = 0$

(iv) $X[3] = \frac{1}{\sqrt{2}} \ W_N^{3} + W_N^{6} + \frac{1}{\sqrt{2}} \ W_N^{9} - \frac{1}{\sqrt{2}} \ W_N^{15} - W_N^{18} - \frac{1}{\sqrt{2}} \ 4 \ W_N^{21}$

$\therefore X[3] = 0$

(v) $X[4] = \frac{1}{\sqrt{2}} \ W_N^{4} + W_N^{8} + \frac{1}{\sqrt{2}} \ W_N^{12} - \frac{1}{\sqrt{2}} \ W_N^{20} - W_N^{24} - \frac{1}{\sqrt{2}} \ 4 \ W_N^{28}$

$\therefore X[4] = 0$

(vi) $X[5] = \frac{1}{\sqrt{2}} \ W_N^{5} + W_N^{10} + \frac{1}{\sqrt{2}} \ W_N^{15} - \frac{1}{\sqrt{2}} \ W_N^{25} - W_N^{30} - \frac{1}{\sqrt{2}} \ 4 \ W_N^{35}$

$\therefore X[5] = 0$

(vii) $X[6] = \frac{1}{\sqrt{2}} \ W_N^{6} + W_N^{12} + \frac{1}{\sqrt{2}} \ W_N^{18} - \frac{1}{\sqrt{2}} \ W_N^{30} - W_N^{30} - \frac{1}{\sqrt{2}} \ 4 \ W_N^{42}$

$\therefore X[6] = 0$

(viii) $X[7] = \frac{1}{\sqrt{2}} \ W_N^{7} + W_N^{14} + \frac{1}{\sqrt{2}} \ W_N^{21} - \frac{1}{\sqrt{2}} \ W_N^{35} - W_N^{42} - \frac{1}{\sqrt{2}} \ 4 \ W_N^{49}$

$\therefore X[7] = 4j$

(**Properties of DFT**)

Q(5) A four point sequence x[n] has DFT X[k] = { 6, 2, 1, 2 }. Sequence p[n] is related to x[n] and has DFT P[k] = { 6, –2,1,–2} Find the relation between x[n] and p [n].

Solution :

Hint : P[k] = (–1)^k X[k]

For N=4, $W_N^2 = -1$

P[k] = W_N^{2k} **X[k]**

By Time shift property of IDFT,

P[n] = x[n – 2]

Q(6) Find the eight point DFT of the sequence $x[n] = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & 4 \le n \le 7 \end{cases}$

Compute the DFT of the following sequence using X[k] only.

(a) $p[n] = \begin{Bmatrix} 1 & n = 0 \\ 0 & 1 \le n \le 4 \\ 1 & 5 \le n \le 7 \end{Bmatrix}$ (b) $q[n] = \begin{Bmatrix} 0 & 0 \le n \le 1 \\ 1 & 2 \le n \le 5 \\ 0 & 6 \le n \le 7 \end{Bmatrix}$

Solution :

(a) To find X[k]

By DFT, $X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$ $\quad N = 8$

$X[k] = x[0]\, W_N^0 + x[1]\, W_N^k + x[2]\, W_N^{2k} + x[3]\, W_N^{3k} + 0 + 0 + 0 + 0$

$\mathbf{X[k] = 1 + W_N^k + W_N^{2k} + W_N^{3k}}$

At k =0, $\mathbf{X[0]} = 1 + 1 + 1 + 1 = \mathbf{4}$

At k =1 , $X[1] = 1 + W_8^1 + W_8^2 + W_8^3$

$X[1] = 1 + (0.707 – j\, 0.707) + (–j) + (–0.707 – j0.707)$

$\mathbf{X[1] = 1 – j2.414}$

At k=2, $\mathbf{X[2]} = 1 + W_8^2 + W_8^4 + W_8^6$

$X[2] = 1 + (–j) + (–1) + (j) = \mathbf{0}$

At k=3, $X[3] = 1 + W_8^3 + W_8^6 + W_8^9$

$X[3] = 1 + (–0.707 – j\, 0.707) + (j) + (0.707 – j0.707)$

$\mathbf{X[3] = 1 – j1.414}$

At k=4, $X[4] = 1 - 1 + 1 - 1 =$ **0**

By symmetry about $k = \frac{N}{2} = \frac{8}{2} = 4$

$X[k] = X^*[-k] = X^*[N-k]$

$X[k] = X^*[8-k]$

========================

K=5, $X[5] = X^*[3] =$ $\mathbf{1 + j1.414}$

K=6, $X[6] = X^*[2] = 0$

K=7, $X[7] = X^*[1] =$ $\mathbf{1 + j\,2.414}$

(b) To find P[k] using X[k]

$p[n] = \{1, 0, 0, 0, 0, 1, 1, 1\} = x[-n]$

$p[n] = x[-n]$

By DFT Time reversal property, $P[k] = X[-k]$

(c) **To find Q[k] using X[k]**

$q[n] = \{0, 0, 1, 1, 1, 1, 0, 0\}$

$q[n] = x[n - 2]$

By DFT Time shift property.

$Q[k] = W_N^{2k} X[k]$

Q(7) Let x[n] be four point sequence with X[k] = { 1, 2, 3, 4 }.
Find the DFT of the following sequences using X[k].
(a) $p[n] = x[n-1]$ (b) $q[n] = 2\,x[n-2]$

Solution :

(a) $p[n] = x[n-1]$ By Time shift property of DFT, $\mathbf{P[k]} = W_N^k\ \mathbf{X[k]}$ $P[k] = \{ 1, -2j, -3, 4j \}$.	(b) $q[n] = 2\,x[n-2]$ By Time shift property of DFT, $\mathbf{Q[k] = 2}\, W_N^{2k}\ \mathbf{X[k]}$ **For N=4,** $W_N^2 = -1$ $\mathbf{Q[k] = 2}\,(-1)^k\ \mathbf{X[k]}$ $Q[k] = \{ 2, -4, 6, -8 \}$

(c) To find R[k] :

Given $r[n] = x[n]+2x[n-1]+3x[n-2]$

By Time shift property of DFT,

$R[k] = X[k] + 2\,W_N^{k}\ X[k] + 3\ W_N^{2k}\ X[k]$

K=0, $R[0] = X[0] + 2\,W_N^{0}\ X[0] + 3\ W_N^{0}\ X[0]$

$\quad$ **R[0] = 1 + 2 (1) (1) + 3 (1) (1) == 6**

K=1, $R[1] = X[1] + 2\,W_N^{1}\ X[1] + 3\ W_N^{2}\ X[1]$

$\quad$ **R[1] = 2 + 2 (–j) (2) + 3 (–1) (2) == –4 –4j**

K=2, $R[2] = X[2] + 2\,W_N^{2}\ X[2] + 3\ W_N^{4}\ X[2]$

$\quad$ **R[2] = 3 + 2 (–1) (3) + 3 (1) (3) == 6**

K=3, $R[3] = X[3] + 2\,W_N^{3}\ X[3] + 3\ W_N^{6}\ X[3]$

$\quad$ **R[3] = 4 + 2 (j) (4) + 3 (–1) (4) == –8 + 8j**

Q(8) Let x[n] = {1, 2, 3, 4} and x[n] ←→ X[k].
Find inverse DFT of the following without using DFT/iDFT .
(a) $P[k]= e^{j\pi k}\ X[k]$ (b) $Q[k]=(-1)^k\ X[k]$

Solution :

(a) To find p[n] :

$P[k] = e^{j\pi k}\ X[k]$

Let $P[k] = W_N^{-mk} X[k]$ where m = 2

$P[k] = W_N^{-2k} X[k]$

By Time Shift Property of iDFT,

P[n] = x[n+2]

p[n] = { 3, 4, 1, 2 } ANS

(b) To find q[n] :

$Q[k]=(-1)^k\ X[k]$

For N=4, $W_N^{2} = -1$

Q[k] = W_N^{2k} **X[k]**

By Time Shift Property of iDFT,

q[n] = x[n–2]

q[n] = { 3, 4, 1, 2 } ANS

Q(9) Let x[n] be four point sequence with X[k] = { 1, 2, 3, 4 }.
Find the DFT of the following sequences using X[k].

(a) $p[n] = (-1)^n\ x[n]$ (b) $q[n] = x[n]\cos(\frac{n\pi}{2})$ (c) $r[n] = x[n] + \cos(\frac{n\pi}{2})$

Solution :

(a) To find P[k]

$p[n] = (-1)^n\ x[n]$

For N=4, $W_N^{2} = -1$

$p[n] = W_N^{2n}\ x[n]$

By frequency shift property of DFT,

P[k] = X[k+2] = { 3, 4, 1, 2 } ANS

(b) To find Q[k]

Given $q[n] = x[n] \cos(\frac{n\pi}{2})$

$$q[n] = \left(\frac{e^{j\frac{n\pi}{2}} + e^{-j\frac{n\pi}{2}}}{2} \right) x[n]$$

$$q[n] = \frac{1}{2}\left(e^{j\frac{n\pi}{2}}\ x[n] + e^{-j\frac{n\pi}{2}}\ x[n] \right)$$

Let $e^{j\frac{n\pi}{2}} = W_N^{-mn}$, $\quad e^{-j\frac{n\pi}{2}} = W_N^{mn}$

$$\therefore q[n] = \frac{1}{2}\left[W_N^{-mn}\ x[n] \,|\, W_N^{mn}\ x[n] \right]$$

By Frequency Shift Property of DFT,

$$\therefore Q[k] = \frac{1}{2}(X[k-m] + X[k+m])$$

To find m : $e^{jn\frac{\pi}{2}} = W_N^{-mn} = \left(W_N^{1}\right)^{-mn} = \left(e^{-j\frac{\pi}{2}} \right)^{mn} = e^{jn\frac{\pi}{2}m}$

By comparing, m = 1.

By substituting m=1 in P[k] we get,

$$Q[k] = \frac{1}{2}(X[k-1] + X[k+1])$$

$$Q[k] = \frac{1}{2}\left(\begin{bmatrix} 4 \\ 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} \right) \qquad \because Q[k] = \begin{bmatrix} 3 & k=0 \\ 2 & \\ 3 & \\ 2 & \end{bmatrix}$$

(c) To find R[k]

Given $r[n] = x[n] + \cos(\frac{n\pi}{2})$

$$r[n] = x[n] + \left(\frac{e^{j\frac{n\pi}{2}} + e^{-j\frac{n\pi}{2}}}{2} \right) u[n]_{4pt}$$

$$r[n] = x[n] + \frac{1}{2}\left(e^{j\frac{n\pi}{2}}\ u[n]_{4pt} + e^{-j\frac{n\pi}{2}}\ u[n]_{4pt} \right)$$

$$r[n] = x[n] + \frac{1}{2}\left(W_N^{-mn}\ u[n]_{4pt} + W_N^{mn}\ u[n]_{4pt} \right)$$

By Frequency Shift Property of DFT,

$$\therefore R[k] = X[k] + \frac{1}{2}(U[k-m] + U[k+m])$$

Where $U[k] = DFT\ \{ u[n](4pt)\} = 4\,\delta[k]$

To find m : $\quad e^{jn\frac{\pi}{2}} = W_N^{-mn} = \left(W_N^{1}\right)^{-mn} = \left(e^{-j\frac{\pi}{2}}\right)^{mn} = e^{jn\frac{\pi}{2}m}$

By comparing, m = 1.

By substituting m=1 in R[k] we get,

$R[k] = X[k] + \frac{1}{2}\left(U[k-1] + U[k+1] \right)$ $\qquad$ Put $U[k] = 4\,\delta[k]$

$R[k] = X[k] + \frac{1}{2}\left(4\,\delta[k-1] + 4\,\delta[k+1] \right)$

$$R[k] = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} + \frac{4}{2}\left(\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right)$$

$$R[k] = \begin{bmatrix} 1 & k=0 \\ 4 & \\ 3 & \\ 6 & \end{bmatrix} \quad \text{ANS}$$

Q(10) Let x[n] = { 1, 2, 3, 4 } and x[n] ←→ X[k]. Find inverse DFT of the following without using DFT/iDFT equations.
(a) X[k-2] (b) X[k+2]

Solution :

(a) Let P[k] = X [k – 2]

By frequency shift property of IDFT,

$$p[n] = W_N^{-2n}\; x[n]$$

$$p[n] = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$p[n] = \begin{bmatrix} 1 & n=0 \\ -2 & \\ 3 & \\ -4 & \end{bmatrix}$$

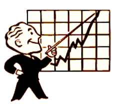
(b) Let Q[k] = X [k + 2]

By frequency shift property of IDFT,

$$q[n] = W_N^{2n}\; x[n]$$

$$q[n] = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$q[n] = \begin{bmatrix} 1 & n=0 \\ -2 & \\ 3 & \\ -4 & \end{bmatrix}$$



Q(11) For the sequences

$$x_1(n) = \cos\left(\frac{2\pi}{N}\right)n \qquad x_2(n) = \sin\left(\frac{2\pi}{N}\right)n \qquad 0 \le n \le N-1$$ Find $X_1[k]$ and $X_2[k]$.

Solution : (a) To find $X_1[k]$

Given $\mathbf{x_1[n]} = \cos\left(\frac{2\pi n}{N}\right)$

$$x_1[n] = \left(\frac{e^{j\frac{2\pi n}{N}} + e^{-j\frac{2\pi n}{N}}}{2}\right) \; u[n]_{Npt}$$

$$x_1[n] = \frac{1}{2}\left(e^{j\frac{2\pi n}{N}} \; u[n]_{Npt} \; + e^{-j\frac{2\pi n}{N}} \; u[n]_{Npt} \right)$$

$$x_1[n] = \frac{1}{2}\left(W_N^{-mn} \; u[n]_{Npt} \; + W_N^{mn} \; u[n]_{Npt} \right)$$

By frequency shift property of DFT,

$$\therefore X_1[k] = \frac{1}{2}\left(U[k-m] + U[k+m]\right)$$

Where $U[k] = DFT\ \{ u[n](Npt)\} = N\ \delta[k]$

By substituting in $X_1[k]$ we get,

$$X_1[k] = \frac{1}{2}\left(N\ \delta[k-1] + N\ \delta[k+1] \right)$$

$$X_1[k] = \frac{N}{2}[\delta(k-1) + \delta(k+1)]$$

Solution : (a) To find $X_2[k]$

Given $\mathbf{X_2[n]} = \sin\left(\frac{2\pi n}{N}\right)$

$$x_2[n] = \left(\frac{e^{j\frac{2\pi n}{N}} - e^{-j\frac{2\pi n}{N}}}{2j}\right) \; u[n]_{Npt}$$

$$x_2[n] = \frac{1}{2j}\left(e^{j\frac{2\pi n}{N}} \; u[n]_{Npt} \; - e^{-j\frac{2\pi n}{N}} \; u[n]_{Npt} \right)$$

$$x_2[n] = \frac{1}{2j}\left(W_N^{-mn} \; u[n]_{Npt} \; - W_N^{mn} \; u[n]_{Npt} \right)$$

By Frequency Shift Property of DFT,

$$\therefore X_2[k] = \frac{1}{2j}\left(U[k-m] - U[k+m]\right)$$

Where $U[k] = DFT\ \{ u[n](Npt)\} = N\ \delta[k]$

By substituting in $X_2[k]$ we get,

$$X_2[k] = \frac{1}{2j}(\, N\,\delta[k-1] - N\,\delta[k+1]\,)$$

ANS : $$X_2[k] = \frac{N}{2j}[\delta(k-1) - \delta(k+1)]$$

Q(12) A 4 point sequence x[n] has DFT X[k] = { 6, 2, 1, 2 }. Sequence p[n] is related to x[n] has DFT P[k] = { 1, 2, 6, 2}. Find the relation between x[n] and p[n].

Solution :

To find relation between x[n] and p[n].

Given P[k] = { 1, 2, 6, 2}.

P[k] = X[k–2]

By DFT frequency shift property of IDFT,

$$p[n] = W_N^{-2n}\; x[n]$$

$\therefore$ $p[n] = (-1)^n\, x[n]$ ANS

Q(13) Given a[n] = { 1, 2, 3, 4 }.

(a) Find A[k].

(b) Let b[n] = { 1, 4, 3, 2 } Find B[k] using A[k].

(c) Let c[n] = { 2, 6, 6, 6 } Find C[k] using A[k].

(d) Let d[n] = { 2, 1, 4, 3 } Find D[k] using A[k].

Solution :

(b) To find B[k] using A[k]

b[n] = a[-n]

By Time Reversal Property of DFT,

B[k] = A[–k]

$$B[k] = \begin{bmatrix} 10 & k=0 \\ -2-2j & \\ -2 & \\ -2+2j & \end{bmatrix} \quad \text{ANS}$$

(c) To find C[k] using A[k]

C[n] = a[n] + b[n]

By Linearity Property of DFT,

C[k] = A[k] + B[k]

$$C[k] = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} + \begin{bmatrix} 10 \\ -2-2j \\ -2 \\ -2+2j \end{bmatrix} == \begin{bmatrix} 20 & k=0 \\ -4 & \\ -4 & \\ -4 & \end{bmatrix} \quad \text{ANS}$$

(d) To find D[k] using A[k].

d[n] = b[n-1]

By Time Shift Property of DFT,

D[k] = $W_N^k\, B[k]$

$$D[k] = \begin{bmatrix} 1 \\ -j \\ -1 \\ j \end{bmatrix} \begin{bmatrix} 10 \\ -2 - 2j \\ -2 \\ -2 + 2j \end{bmatrix} \begin{array}{l} k=0 \\ \\ \\ \\ \end{array} \qquad D[k] = \begin{bmatrix} 10 \\ -2 + 2j \\ 2 \\ -2 - 2j \end{bmatrix} \begin{array}{l} k=0 \\ \\ \\ \\ \end{array}$$ ANS

Q(14) Given x(n) = { 1, 1 , 1, 1, 0 , 0, 0, 0 }. Let X[k] be DFT of x[n].

(a) Let a [n] = { 1, 0, 0, 0, –1, 0, 0, 0} Find A[k] using X[k]

(b) Let b[n] = { 2, 0, 0, 0, 0, 2, 2, 2 } Find B[k] using X[k]

Solution :

(a) To find a[n] : a[n] = x[n] –x[n–1] By Linearity Propertyof DFT A[k] = X[k] - $W_N^k\, X[k]$	(b) To find b[n] : b[n] = 2 x[–n] By Time ReversalProperty, B[k] = 2 X[-k]

Q(15) Let p[n] and q[n] are sequences of length four such that p[n] is time reversed version of q[n] and p[n] = {1, 2, 3, 4} .

If x[n] = p[n] – j q[n]. Find X[k] in terms of P[k].

Solution :

x[n] = p[n] – j q[n].

Put q[n] = p[-n]

x[n] = p[n] – j p[–n].

By Linearity and Time reversal property of DFT, X[k] = P[k] – j P[-k]

Q(16) Let x[n] be 4 point sequence with X[k] = {1, 2, 3, 4}.

Find the FFT of the following sequences using X[k] and not otherwise

(a) x[-n] (b) x[–n+1] (c) x[–n–1]

Solution :

(a) To find DFT {x[-n]}

Let y[n] = x[-n]

By Time Reversal Property of DFT, Y[k] = X[-k] $\therefore$ $Y[k] = \begin{bmatrix} 1 \\ 4 \\ 3 \\ 2 \end{bmatrix} \begin{array}{l} k=0 \\ \\ \\ \\ \end{array}$

(b) To find DFT {x[-n+1]}

Let y(n) = x[-n]

Put n = n – 1

y(n–1) = x[–n + 1]

By Time Shift Property of DFT, $W_N^K \; Y[k] = \text{DFT} \; \{x(-n+1)\}$

$$\text{DFT} \; \{x(-n+1)\} = W_N^k \; Y[k]$$

$$DFT \; \{x(-n+1)\} = W_N^k \; X[-k]$$

$$\text{DFT} \; \{x(-n+1)\} = \begin{bmatrix} 1 \\ -j \\ -1 \\ j \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 3 \\ 2 \end{bmatrix} == \begin{bmatrix} 1 & k=0 \\ -4j & \\ -3 & \\ 2j & \end{bmatrix}$$

(c) To find DFT {x[-n–1]}

Let y(n) = x[– n]

Put n = n + 1

y[n + 1]= x[–n–1]

$$W_N^{-K} \; Y[k] = \text{DFT} \; \{x[-n-1]\}$$

$$\text{DFT} \; \{x[\text{-n -1}]\} = W_N^{-K} Y[k]$$

$$\text{DFT} \; \{x[\text{-n -1}]\} = W_N^{-k} \; X[\text{-k}]$$

$$\text{DFT} \; \{x[\text{-n -1}]\} = \begin{bmatrix} 1 \\ j \\ -1 \\ -j \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 3 \\ 2 \end{bmatrix} == \begin{bmatrix} 1 & k=0 \\ 4j & \\ -3 & \\ -2j & \end{bmatrix}$$

Q(17) Let x[n] = { 1, 2, 3, 4 } and x[n] ←→ X[k]. Find inverse DFT of the following without using DFT/iDFT equations.
(a) X[-k] (b) X[–k+2] (c) X[–k–2]}

Solution:

(a) To find IDFT { X[–k] }

Let P[k] = X[-k]

By Time Reversal Property of IDFT,

p[n] = x[-n]

p[n]= = { 1, 4, 3, 2 }

(b) To find IDFT { X[–k+2] }

Let Q[k] = X[-k]
put k = k – 2

Q[k – 2] = X[- k + 2]

By Frequency Shift property of IDFT,

$$W_N^{-2n}\ q(n) = \text{iDFT } \{ X[-k+2] \}$$

$$\text{iDFT } \{ X[-k+2] \} = W_N^{-2n}\ q[n]$$

$$\text{iDFT } \{ X[-k+2] \} = W_N^{-2n}\ x[-n]$$

$$\text{iDFT } \{ X[-k+2] \} == \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 3 \\ 2 \end{bmatrix} == \begin{bmatrix} 1 & n=0 \\ -4 & \\ 3 & \\ -2 & \end{bmatrix}$$

(c) To find IDFT { X[–k–2] }

Let R[k] = X[-k]
Put k = k + 2

R[k + 2] = X[– k– 2]

By Frequency Shift property of IDFT,

$$W_N^{2n}\ r[n] = iDFT\ \{X[-k-2]\}$$

$$\text{iDFT } \{X[-k-2] = W_N^{2n}\ r[n]$$

$$= W_N^{2n}\ \ x[-n]$$

$$\text{iDFT } \{X[-k-2] = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 3 \\ 2 \end{bmatrix} == \begin{bmatrix} 1 & n=0 \\ -4 & \\ 3 & \\ -2 & \end{bmatrix}$$

Q(18) Let X[k] = { 1, –2, 1–j, 2j, 0, ….} is the 8 point DFT of a real valued sequence. What is the 8 point DFT Y[k] such that $y[n] = (-1)^n\ x[n]$?

Solution :

By symmetry property of real sequence, $X[k] = X^*[-k]$

X[k] = { 1, –2, 1–j, 2j, 0, -2j, 1+j, -2}

To find Y[k] : $y[n] = (-1)^n\ x[n]$

y[n] = $(W_N^4)^n$ x[n]

y[n] = W_N^{4n} x[n]

By frequency shift property of DFT , $\therefore$ Y[k] = X[k + 4]

$\therefore$ Y[k] = { $\underset{\uparrow}{0}$, **-2j, 1+j, -2, 1, –2, 1–j, 2j** }

Q(19) A four point DT signal x(n) is given by x(n) = [1, 2, 0, 2]. A student found the DFT of this sequence as X[k] = [5, (-1 + j2), -3, (-1 –j2)] Guess whether this answer is correct or not, without performing DFT. Justify your guess.

Solution:

Here, x[n] = x[–n]
Therefore x[n] is an even signal.
By even signal property of DFT,
If x[n] = x[–n] Then X[k] = X[–k]
That means X[k] must be even.
However, Given X[k] is not Even. i.e. X[k] ≠ X[–k]

Q(20) Given x[n] = { (1 + j), (2 + j2), (3 + j3), (4 + j2) }
(a) Find X[k].
(b) Find DFT of x*[n]using X[k] and not otherwise.

Solution (a) To find X[k]

****

ANS $$X[k] = \begin{bmatrix} 10 & + & 8j & k=0 \\ -2 & & & \\ -2 & & & \\ -2 & - & 4j & \end{bmatrix}$$

(b) To find DFT of x*[n]

Let p[n] = x*[n]

By DFT, P[k] = X*[–k]

$$P[k] = \begin{bmatrix} 10 & - & 8j & k=0 \\ -2 & + & 4j & \\ -2 & & & \\ -2 & & & \end{bmatrix}$$

Q(21) If x[n] = { 1, 2, 3, 4 } and h[n] = { 5, 6, 0, 0 }

(a) Find circular convolution using DFT-IDFT.
(b) Find circular convolution using Time Domain Method.

Solution : (a) To Find Circular Convolution using DFT

Let $y[n] = x[n] \otimes h[n]$
By convolution property of DFT,
$Y[k] = X[k]\ H[k]$
Then $y[n] = \text{IDFT}\ \{\ Y[k]\ \}$

Step 1 : Find X[k]

$$\text{By DFT,}\quad X[k] = \sum_{n=0}^{N-1} x[n]\ W_N^{nk}$$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad X[k] = \begin{bmatrix} 10 & k=0 \\ -2 + 2j & \\ -2 & \\ -2 - 2j & \end{bmatrix}$$

Step 2 : Find H[k]

$$\text{By DFT,}\quad H[k] = \sum_{n=0}^{N-1} h[n]\ W_N^{nk}$$

$$H[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 0 \\ 0 \end{bmatrix} \qquad H[k] == \begin{bmatrix} 11 & k=0 \\ 5-j6 & \\ -1 & \\ 5+j6 & \end{bmatrix}$$

Step 3 : Find Y[k]

Y[k] = X[k] H[k]

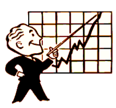
$$Y[k] = \begin{bmatrix} 10 \\ -2 + 2j \\ -2 \\ -2 - 2j \end{bmatrix} \begin{bmatrix} 11 \\ 5-j6 \\ -1 \\ 5+j6 \end{bmatrix} \quad Y[k] == \begin{bmatrix} 110 & k=0 \\ 2+22j & \\ 2 & \\ 2-22j & \end{bmatrix}$$

Step 4 : Find y[n]

$$\text{By iDFT,}\quad y[n] = \frac{1}{N} \sum_{k=0}^{3} W_N^{-nk}\ Y[k]$$

$$y[n] = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 110 \\ 2+22j \\ 2 \\ 2-22j \end{bmatrix}$$

$$y[n] = \frac{1}{4} \begin{bmatrix} 116 \\ 64 \\ 108 \\ 39 \end{bmatrix} \qquad y[n] = \begin{bmatrix} 29 \\ 16 \\ 27 \\ 38 \end{bmatrix}$$



Q(22) Let x[n] be 4 point sequence with X[k] = {1, 2, 3, 4}.
Find the DFT of $p[n]=x[n]\otimes x[n]$ using X[k] and not otherwise.

Solution : To find P[k],

Given $p[n]=x[n]\otimes x[n]$

By DFT, P[k] = X[k] X[k]

$P[k] = \{\ \underset{\uparrow}{1},\ 4,\ 9,\ 16\ \}$ ANS

Q(23) Obtain linear convolution of two sequences using circular convolution.

$x(n) = u(n) - u(n-3)$

$h(n) = u(n-1) + u(n-2) - u(n-4) - u(n-5)$

Solution :

$x(n) = u(n) - u(n-3)$

$x[n] = \{\ \underset{\uparrow}{1},\ 1,\ 1\ \}$ Length L = 3

$h(n) = u(n-1) + u(n-2) - u(n-4) - u(n-5)$

$h[n] = \{\ \underset{\uparrow}{0},\ 1,\ 2,\ 2,\ 1\ \}$ Length M = 5

(i) To find LC by CC select $N \geq L+M-1$

$N \geq 7$

Select N = 7.

(ii) Zero padding

$x[n] = \{\ \underset{\uparrow}{1},\ 1,\ 1,\ 0,\ 0,\ 0,\ 0\ \}$ $h[n] = \{\ \underset{\uparrow}{0},\ 1,\ 2,\ 2,\ 1,\ 0,\ 0\ \}$

(iii) Find y[n]

$$y[n] = \sum_{m=0}^{N-1} x[m]\, h[n-m]$$

$$y[n] = \begin{bmatrix} 0 & 0 & 0 & 1 & 2 & 2 & 1 \\ 1 & 0 & 0 & 0 & 1 & 2 & 2 \\ 2 & 1 & 0 & 0 & 0 & 1 & 2 \\ 2 & 2 & 1 & 0 & 0 & 0 & 1 \\ 1 & 2 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} == \begin{bmatrix} 0 & n=0 \\ 1 & \\ 3 & \\ 5 & \\ 5 & \\ 3 & \\ 1 & \end{bmatrix} \text{ANS}$$

Q(24) Obtain 4 point Circular Convolution using DFT

$x(n) = \delta(n) + 2\delta(n-1) - \delta(n-2)$ $h(n) = 2\delta(n) + 3\delta(n-1).$

Solution :

$x(n) = \delta(n) + 2\delta(n-1) - \delta(n-2)$ $\quad x[n] = \{ \underset{\uparrow}{1}, 2, 1 \}$ Length : L = 3

$h(n) = 2\delta(n) + 3\delta(n-1).$ $\quad h[n] = \{ \underset{\uparrow}{2}, 3 \}$ Length : M=2

(i) Select N N= 4 (Given)

(ii) Zero padding

$x[n] = \{ \underset{\uparrow}{1}, 2, 1, 0 \}$ $\qquad h[n] = \{ \underset{\uparrow}{2}, 3, 0, 0 \}$

(iii) Find CC by DFT

Let $y[n] = x[n] \otimes h[n]$

By convolution property of DFT,

$Y[k] = X[k]\, H[k]$

Then $y[n] = \text{IDFT}\{ Y[k] \}$

Step 1 : **Find X[k]]**

By DFT, $X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk}$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} == \begin{bmatrix} 4 \quad k=0 \\ -2j \\ 0 \\ 2j \end{bmatrix}$$

Step 2 : Find H[k]

By DFT, $H[k] = \sum_{n=0}^{N-1} h[n]\, W_N^{nk}$

$$H[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} == \begin{bmatrix} 5 \quad k=0 \\ 2-3j \\ -1 \\ 2+3j \end{bmatrix}$$

Step 3 : **Find Y[k]**

Y[k] = X[k] H[k]

$$Y[k] = \begin{bmatrix} 4 \quad k=0 \\ -2j \\ 0 \\ 2j \end{bmatrix} \begin{bmatrix} 5 \quad k=0 \\ 2-3j \\ -1 \\ 2+3j \end{bmatrix} === \begin{bmatrix} 20 \quad k=0 \\ -6-4j \\ 0 \\ -6+4j \end{bmatrix}$$

Step 4 : **Find y[n]**

By iDFT, $y[n] = \frac{1}{N} \sum_{k=0}^{3} W_N^{-nk}\, Y[k]$

$$y[n] = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 20 \quad k=0 \\ -6-4j \\ 0 \\ -6+4j \end{bmatrix}$$

$$y[n] = \frac{1}{4}\begin{bmatrix} 8 \quad k=0 \\ 28 \\ 32 \\ 12 \end{bmatrix} \qquad y[n] = \begin{bmatrix} 2 \\ 7 \\ 8 \\ 3 \end{bmatrix} \quad ANS$$

Q(25) If $x(n) \xleftrightarrow{DFT} X[k]$ Determine N pt DFT of the following sequences in terms of X[k].

(a) $p[n] = x[n]\cos\left(\frac{2\pi nk}{N}\right)$ **(b)** $q[n] = x[n]\sin\left(\frac{2\pi nk}{N}\right)$

Solution (a)

$$p[n] = x[n]\cos\left(\frac{2\pi nk}{N}\right)$$

$$p[n] = x[n]\left[\frac{1}{2}\left(e^{\frac{j2\pi ko}{N}} + e^{\frac{-j2\pi ko}{N}}\right)\right]$$

$$p[n] = \frac{1}{2}\left[e^{\frac{j2\pi nko}{N}}x[n] + e^{\frac{-j2\pi ko}{N}}x(n)\right]$$

$$P[n] = \frac{1}{2}\left(W_N^{-nko}x[n] + W_N^{nko}x[n]\right)$$

By DFT Frequency shift property,

ANS : $P[k] = \frac{1}{2}(X[k-k_o] + X[k+k_o])$

Solution (b)

$$q(n) = x(n)\left[\sin\left(\frac{2\pi nko}{N}\right)\right]$$

$$q[n] = x[n]\left[\frac{1}{2j}\left(e^{\frac{j2\pi nko}{N}} - e^{\frac{-j2\pi nko}{N}}\right)\right]$$

$$q[n] = \frac{1}{2j}\left[e^{\frac{j2\pi nko}{N}}x[n] - e^{\frac{-j2\pi nko}{N}}x(n)\right]$$

$$q[n] = \frac{1}{2j}\left(W_N^{-nko}x[n] - W_N^{nko}x[n]\right)$$

By DFT Frequency shift property,

ANS : $Q(k) = \frac{1}{2j}(X[k-ko] - X[k+ko])$

Q(26) For the sequences

$$x_1(n) = \cos\left(\frac{2\pi}{N}\right)n \qquad x_2(n) = \sin\left(\frac{2\pi}{N}\right)n \qquad 0 \le n \le N-1$$

Determine the N- point circular convolution of x_1 (n) and x_2 (n).

Solution :

Let $y[n] = x_1[n] \otimes x_2[n]$

By convolution property of DFT,

$Y[k] = X_1[k]\, X_2[k]$

$$Y[k] = \left(\frac{N}{2j}[\delta(k-1) - \delta(k+1)]\right)\left(\frac{N}{2}[\delta(k-1) + \delta(k+1)]\right)$$

$$Y[k] = \left(\frac{N^2}{4j}[\delta(k-1) - \delta(k+1)]\right)$$

$$Y[k] = \frac{N}{2}\left(\frac{N}{2j}[\delta(k-1) - \delta(k+1)]\right)$$

Then y[n] = IDFT { Y[k] }

ANS : $$y[n] = \frac{N}{2}\sin\left(\frac{2\pi n}{N}\right)$$

Q(27) Compute the energy of N pt sequence, $x[n] = \cos\left(\frac{2\pi n k}{N}\right),\ 0 \le n \le N-1$

Solution :

$$\text{Let } x(n) = \frac{1}{2}\left[e^{\frac{j2\pi nk}{N}} + e^{\frac{-j2\pi nk}{N}}\right] \text{ and } \qquad x^*[n] = \frac{1}{2}\left[e^{\frac{-j2\pi nk}{N}} + e^{\frac{j2\pi nk}{N}}\right]$$

$$x[n]\,x^*[n] = \frac{1}{4}\left[2 + e^{\frac{j4\pi nk}{N}} + e^{\frac{-j4\pi nk}{N}}\right]$$

$$E = \sum_{n=-\infty}^{\infty} x[n]\,x^*[n] \quad = \sum_{n=0}^{N-1} \frac{1}{4}[2 + e^{\frac{j4\pi nk}{N}} + e^{\frac{-j4\pi nk}{N}}] \quad = \frac{N}{2} \text{ ANS}$$

Q(28) Given x(n) = { 1, 1 , 1, 1, 0, 0, 0, 0}. Let X[k] be 8 point DFT of X[k].
Find DFT of the following sequences in terms of X[k].

A).a [n] = { 0, 0, 0, 0, 1, 1, 1, 1 }
B).b [n] = { 1, 0, 0, 0, 0, 1, 1, 1}
C).c [n] = { 1, 0, 0, 0, –1, 0, 0, 0}
D).d[n] = { 1, 1, 1, 1, –1, –1, –1, –1}
E). e [n] = { 1, 1, 1, 1, 1, 1, 1, 1}
F). f [n] = { 0, 0, 1, 1, 1, 1, 0, 0}
G). g [n] = { 1, –1, 1, –1, 0, 0, 0, 0}
H). p [n] = { 1, 0.5, 0.5, 0.5, 0, 0.5, 0.5, 0.5}

Solution :

(A) Let a[n] = x[n – 4]
By DFT Time shift property,
$A[K] = W_N^{4k} X[k]$
$A[k] = (-1)^k X[k]$

(B) Let b[n] = x[– n]
By DFT and Time reversal property,
B[k] = X [– k]

(C) Let C[n] = b[n] – a [n]
By DFT Linearity property,
C[k] = B[k] – A[k]

(D) Let d[n] = x[n] – a [n]
By DFT, Linearity property
D[k] = X[k] – A[k]

(E) Let e[n] = x [n] + a[n]
E[k] = X [k] + A[k]

(F) Let F(n) = x(n – 2)
By DFT Time shift property,
$F[k] = W_N^{2k} X[k]$

(G) Let $g(n) = (-1)^n x(n)$
$g[n] = W_N^{4n} x(n)$
By DFT frequency shift property
G [k] = X[k – 4]

(H) Let $p(n) = \frac{1}{2}[x(n) + x(-n)]$
By DFT,
$P[k] = \frac{1}{2}[X(k) + X(-k)]$
$P[k] = X_e[k] = \text{Real}\{X[k]\}$

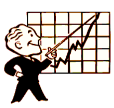
Q(29) Find DFT of the following signals

(a) $x_1[n] = \{ \underset{\uparrow}{1} \quad 2 \quad 3 \quad 4 \}$
(b) $x_2[n] = \{ 1 \quad \underset{\uparrow}{2} \quad 3 \quad 4 \}$
(c) $x_3[n] = \{ 1 \quad 2 \quad \underset{\uparrow}{3} \quad 4 \}$

Solution :

(a) Given $x_1[n] = \{ \underset{\uparrow}{1} \quad 2 \quad 3 \quad 4 \}$

By DFT, $X_1[k] = \sum_{n=0}^{N-1} x_1[n]\, W_N^{nk}$ *where* $N = 4$ *and* $W_N^1 = e^{-j\frac{2\pi}{N}}$



$$X_1[k] = \begin{bmatrix} w_N^0 & w_N^0 & w_N^0 & w_N^0 \\ w_N^0 & w_N^1 & w_N^2 & w_N^3 \\ w_N^0 & w_N^2 & w_N^4 & w_N^6 \\ w_N^0 & w_N^3 & w_N^6 & w_N^9 \end{bmatrix} \begin{bmatrix} x_1[0] \\ x_1[1] \\ x_1[2] \\ x_1[3] \end{bmatrix}$$

$$X_1[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad X_1[k] = \begin{bmatrix} 10 & k=0 \\ -2 + 2j & \\ -2 & \\ -2 - 2j & \end{bmatrix}$$ **ANS**

(b) Given $x_2[n] = \{ 1 \quad \underset{\uparrow}{2} \quad 3 \quad 4 \}$

By DTFT, $X_2(w) = \sum_{n=-\infty}^{\infty} x_2[n]\ e^{-jnw}$

$$X_2(w) = x_2[-1]\, e^{jw} + x_2[0] + x_2[1]\, e^{-jw} + x_2[2]\, e^{-j2w}$$

$$X_2(w) = e^{jw} + 2 + 3\, e^{-jw} + 4\, e^{-j2w}$$

$$X_2(w) = [2 + 4\cos(w) + 4\cos(2w)] - j[\ 2\sin(w) + 4\sin(2w)]$$

But $X[k] = X(w)\big|_{w=\frac{2\pi k}{N}}$ where N =4

$$X_2[k] = X_2(w)\big|_{w=\frac{\pi k}{2}}$$

$$X_2[k] = \left[\ 2 + 4\cos\left(\frac{\pi k}{2}\right) + 4\cos(\pi k)\right] - j\left[\ 2\sin\left(\frac{\pi k}{2}\right) + 4\sin(\pi k)\right]$$

$$X_2[k] = \begin{bmatrix} 10 & k=0 \\ -2 - 2j & \\ 2 & \\ -2 + 2j & \end{bmatrix}$$

(c) By DTFT, $X_3(w) = \sum_{n=-\infty}^{\infty} x_3[n]\ e^{-jnw}$

$$X_3(w) = x_3[-2]\, e^{j2w} + x_3[-1] e^{jw} + x_3[0] + x_3[1]\, e^{-jw}$$

$$X_3(w) = e^{jw} + 2 + 3\, e^{-jw} + 4\, e^{-j2w}$$

$$X_3(w) = [2 + 4\cos(w) + 4\cos(2w)] - j[\ 2\sin(w) + 4\sin(2w)]$$

But $X[k] = X(w)\big|_{w=\frac{2\pi k}{N}}$ where N =4

$$X_3[k] = X_3(w)\big|_{w=\frac{\pi k}{2}}$$

$$X_3[k] = \left[2 + 4\cos\left(\frac{\pi k}{2}\right) + 4\cos(\pi k) \right] - j\left[2\sin\left(\frac{\pi k}{2}\right) + 4\sin(\pi k) \right]$$

$$X_3[k] = \begin{bmatrix} 10 & k=0 \\ -2 - 2j & \\ 2 & \\ -2 + 2j & \end{bmatrix}$$

Q(30) Given $x[n] = \{ \underset{\uparrow}{1} \quad 2 \quad 3 \quad 4 \}$

(a) Find X[k] by using DFT equation.
(b) Find X[k] by using DTFT equation.

Solution :(a) To find X[k] by using DFT equation

Given $x[n] = \{ \underset{\uparrow}{1} \quad 2 \quad 3 \quad 4 \}$

By DFT, $X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk}$ *where* $(i)\ N = 4 \quad (ii)\ W_N^1 = e^{-j\frac{2\pi}{N}}$

$$X[k] = \begin{bmatrix} w_N^0 & w_N^0 & w_N^0 & w_N^0 \\ w_N^0 & w_N^1 & w_N^2 & w_N^3 \\ w_N^0 & w_N^2 & w_N^4 & w_N^6 \\ w_N^0 & w_N^3 & w_N^6 & w_N^9 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad X[k] = \begin{bmatrix} 10 & k=0 \\ -2 + 2j & \\ -2 & \\ -2 - 2j & \end{bmatrix} \quad \textbf{ANS}$$

(b) **To find X[k] from DTFT X(W)**

By DTFT, $X(w) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jnw}$

$$X(w) = x[0] + x[1]\, e^{-jw} + x[2]\, e^{-j2w} + x[3]\, e^{-j3w}$$

$$X(w) = 1 + 2\, e^{-jw} + 3\, e^{-j2w} + 4\, e^{-j3w}$$

$$X(w) = [1 + 2\cos(w) + 3\cos(2w) + 4\cos(3w)] - j[2\sin(w) + 3\sin(2w) + 4\sin(3w)]$$

$$X[k] = X(w)\Big|_{w=\frac{2\pi k}{N}} \quad \text{where N = 4}$$

$$X[k] = X(w)\big|_{w=\frac{\pi k}{2}}$$

$$X[k] = \left[1 + 2\cos\left(\frac{\pi k}{2}\right) + 3\cos(\pi k) + 4\cos\left(\frac{3\pi k}{2}\right)\right] - j\left[2\sin\left(\frac{\pi k}{2}\right) + 3\sin(\pi k) + 4\sin\left(\frac{3\pi k}{2}\right)\right]$$

At k=0, $X[0] = 10$

At k=1, $X[1] = \left[1 + 2\cos\left(\frac{\pi}{2}\right) + 3\cos(\pi) + 4\cos\left(\frac{3\pi}{2}\right)\right] - j\left[2\sin\left(\frac{\pi}{2}\right) + 3\sin(\pi) + 4\sin\left(\frac{3\pi}{2}\right)\right]$

$X[1] = -2 + 2j$

At k=2, $X[2] = \left[1 + 2\cos(\pi) + 3\cos(2\pi) + 4\cos(3\pi)\right] - j\left[2\sin(\pi) + 3\sin(2\pi) + 4\sin(3\pi)\right]$

$X[2] = -2$

At k=3, $X[3] = \left[1 + 2\cos\left(\frac{3\pi}{2}\right) + 3\cos(3\pi) + 4\cos\left(\frac{9\pi}{2}\right)\right] - j\left[2\sin\left(\frac{3\pi}{2}\right) + 3\sin(3\pi) + 4\sin\left(\frac{9\pi}{2}\right)\right]$

$X[3] = -2 - 2j$

ANS :

$$X[k] = \begin{bmatrix} 10 & & & k = 0 \\ -2 & + & 2j & \\ -2 & & & \\ -2 & - & 2j & \end{bmatrix}$$

Q(31) A 2nd order FIR filter has impulse response $h[n] = \{ \underset{\uparrow}{2}, 2, 1 \}$. Determine the output sequence response to the following input sequence $x[n] = \{ \underset{\uparrow}{3}\ 0, -2, 0, 2, 1, 0, -2, -1, 0\}$ using (a) The overlap add method (b) The overlap save method.

Solution.

(a) Overlap Add Method

h[n] = { 2, 2, 1 }

x[n] = { 3, 0, –2, 0, 2, 1, 0, –2, –1, 0 }

By decomposing x[n] into L= 5 point sequences we get,

Let $x_1[n]$ = { 3, 0, –2, 0, 2 } and $x_2[n]$ = { 1, 0, –2, –1, 0 }

(i) Find $y_1[n]$ $\therefore$ $y_1[n]$ = {6, 6, –1, –4, 2, 4, 2} ANS

(ii) Find $y_2[n]$ $\therefore$ $y_2[n]$ = {2, 2, –3, –6, –4, –1}

(iii) Find $y[n] = y_1[n] + y_2[n-5]$

$\therefore$ y[n] = { 6, 6, –1, –4, 2, (4+2), (2+2), –3, –6, –4, –1}

(b) Overlap Save Method

Solution :

(i) Find $y_1[n]$ $\therefore$ $y_1[n] = \{4, 2, 6, 6, -1, -4, 2\}$

(ii) Find $y_2[n]$ $\therefore$ $y_2[n] = \{-1, 4, 6, 4, -3, -6, -4\}$

(iii) Find $y_3[n]$ $\therefore$ $y_3[n] = \{-2, -2, -1, 0, 0, 0, 0\}$

(iv) To find y[n]

Discarding 1st (M–1 = 2) values of $y_1[n]$, $y_2[n]$ and $y_3[n]$ we get ,

$$\mathbf{y[n] = \{ \underset{\uparrow}{6},\ 4,\ -1,\ -4,\ 2,\ 6,\ 4,\ -3,\ -6,\ -4,\ -1 \}}$$ **ANS**

Q(32) Given that $x[n] = \{ (1+2j), (1+j), (2+j), (2+2j) \}$

(a) Find X [k] using DIT–FFT / DIF–FFT algorithm.

ANS $$X[k] = \begin{bmatrix} 6+6j & k=0 \\ -2+2j & \\ 0 & \\ 0 & \end{bmatrix}$$

(b) Using the results in and not otherwise find the DFT of p[n] and q[n] where

$p[n] = \{1, 1, 2, 2\}$ and $q[n] = \{2, 1, 1, 2\}$.

Solution : (i) To find P[k] using X[k]

$x[n] = P[n] + j\,q[n]$ … (1)

$x[n] = P[n] - j\,q[n]$ … (2)

Adding equation (1) and (2)

$$2P[n] = x[n] + x^*[n]$$

$$P[n] = \frac{1}{2}\{x[n] + x^*[n]\}$$

$$\text{BY DFT, } P[k] = \frac{1}{2}\left\{X[k] + X^*[-k]\right\}$$

$$\therefore P[k] = \frac{1}{2}\left\{ \begin{bmatrix} 6+j6 \\ -2+j2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 6-j6 \\ 0 \\ 0 \\ -2-j2 \end{bmatrix} \right\} \qquad \therefore P[k] = \begin{bmatrix} 6 & k=0 \\ -1+j & \\ 0 & \\ -1-j & \end{bmatrix}$$

(ii) To find Q[k] using X[k]

Subtracting equation (2) from (1)

$$2j\,q[n] = x[n] - x^*[n]$$

$$\therefore q[n] = \frac{1}{2j}\{x[n] - x^*[n]\}$$

$$\text{By DFT, } Q[k] = \frac{1}{2j}\left\{X[k] - X^*[-k]\right\}$$

$$\therefore Q[k] = \frac{1}{2j}\left\{ \begin{bmatrix} 6+j6 \\ -2+j2 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 6-j6 \\ 0 \\ 0 \\ -2-j2 \end{bmatrix} \right\}$$

$$\therefore Q[k] = \begin{bmatrix} 6 & k=0 \\ 1+j & \\ 0 & \\ 1-j & \end{bmatrix}$$

Q(33) Let p[n] = {1, 2, 3, 4} and q[n]= {5, 6, 7, 8 }. Find DFT of each of the sequence using 4 pt FFT only once.

Solution :

(i) Let $x[n] = p[n] + j\,q[n]$

$\therefore x[n] = \{ 1 + j5, 2+ j6, 3 + j7, 4 + j8 \}$

(ii) Find X[k] by four point FFT, flowgraph

$$\therefore X[k] = \begin{bmatrix} 10+j26 & k=0 \\ -4 & \\ -2-j2 & \\ -j4 & \end{bmatrix}$$

(iii) To find P[k]

Hint : Derive $p[n] = \frac{1}{2}\{ x[n] + x^*[n] \}$

Then By DFT, $P[k] = \frac{1}{2}\{ X[k] + X^*[-k] \}$

ANS : $$\therefore P[k] = \begin{bmatrix} 10 & k=0 \\ -2+2j & \\ -2 & \\ -2-2j & \end{bmatrix}$$

(iv) To find Q[k]

Hint : Derive $\therefore q[n] = \frac{1}{2j}\{ x[n] - x^*[n] \}$

Then By DFT, $Q[k] = \frac{1}{2j}\{ X[k] - X^*[-k] \}$

ANS : $$\therefore Q[k] = \begin{bmatrix} 26 & k=0 \\ -2+2j & \\ -2 & \\ -2-2j & \end{bmatrix}$$

Q(34) Let x[n] = {1, 0, 2, 0, 3, 0, 2, 0}. Find X[k] using DIT-FFT using X[k] and not otherwise find the DFT of p[n] = {1, 2, 3, 2}

Solution :

(a) To find X[k] using 8 pt DITFFT flowgraph

ANS : X[k] = { 8, -2, 0, -2, 8, -2 , 0, -2 }

(b) To find P[k] using X[k]

x[2r] = {1, 2, 3, 2} = p[n]

By DFT, X[2r] = P[k]

$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r] W_{\frac{N}{2}}^{rk} + W_N^{k} \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] W_{\frac{N}{2}}^{rk}$$

$X[k] = G[k] + W_N^k\, H[k]$

Where $G[k] = DFT\ \{x[2r]\} = DFT\ \{p[n]\} = P[k]$

$H[k] = DFT\ \{x[2r+1]\} = X[2r+1] = 0$

$\therefore\ X[k] = P[k] + 0 = P[k]$

$k = 0 \quad X[0] = P[0] = 8$

$k = 1 \quad X[1] = P[1] = -2$

$k = 2 \quad X[2] = P[2] = 0$

$k = 3 \quad X[3] = P[3] = -2$

Q(35) Let x[n] = {a, b, c, d} and the corresponding DFT X[k] = {A, B, C, D}. Find the DFT of the following sequences using X[k] only .

(a) p[n] = {a, 0, b, 0, c, 0, d, 0} (b) q[n] = {a, 0, 0, b, 0, 0, c, 0, 0, d, 0 ,0}

(c) r[n] = {a, b, c, d, 0, 0, 0, 0} Find R[k] only for k even.

Solution :

(a) To find P[k]

Let p[2r] = {a, b, c, d} = x[n]

P[2r + 1] = {0, 0, 0, 0} = 0

By DIT-FFT

$$P[k] = \sum_{r=0}^{\frac{N}{2}-1} p[2r] W_N^{2rk} + \sum_{r=0}^{\frac{N}{2}-1} p[2r+1] W_N^{(2r+1)k}$$

$$P[k] = \sum_{r=0}^{\frac{N}{2}-1} p[2r] W_{\frac{N}{2}}^{rk} + W_N^{k} \sum_{r=0}^{\frac{N}{2}-1} p[2r+1] W_{\frac{N}{2}}^{rk}$$

Let $P[k] = G[k] + W_N^k\ H[k]$

where,

$G[k] = DFT\{p[2r]\} = DFT\{x[n]\} = X[k]$

$H[k] = DFT\{p[2r+1]\} = 0$

$\therefore P[k] = G[k] = X[k]$

$P[k] = \{ \underset{\uparrow}{A}, B, C, D, A, B, C, D \}$ ANS

(b) To find Q[k]

Given $q[n] = \{a, 0, 0, b, 0, 0, c, 0, 0, d, 0, 0\}$

Let $q[3r] = \{a, b, c, d\} = x[n]$

$q[3r+1] = \{0, 0, 0, 0\} = 0$

$q[3r+2] = \{0, 0, 0, 0\} = 0$

By DIT-FFT

$$Q[k] = \sum_{r=0}^{3} q[3r]W_N^{3rk} + \sum_{r=0}^{3} q[3r+1]W_N^{(3r+1)k} + \sum_{r=0}^{3} q[3r+2]W_N^{(3r+2)k}$$

$$Q[k] = \sum_{r=0}^{3} q[3r]W_{\frac{N}{3}}^{rk} + W_N^{k}\sum_{r=0}^{3} q[3r+1]W_{\frac{N}{3}}^{rk} + W_N^{2k}\sum_{r=0}^{3} q[3r+2]W_{\frac{N}{3}}^{rk}$$

Let $Q[k] = G[k] + W_N^k H[k] + W_N^{2k} p[k]$

Where $G[k] = DFT\{q[3r]\} = DFT\{x[n]\} = X[k]$

$H[k] = DFT\{q[3r]\} = DFT\{0\} = 0$

$P[k] = DFT\{q[3r+2]\} = DFT\{0\} = 0$

$\therefore Q[k] = X[k]$

$Q[k] = \{ \underset{\uparrow}{A}, B, C, D, A, B, C, D, A, B, C, D \}$ ANS

(c) To find R[k]

Given $r[n] = \{a, b, c, d, 0, 0, 0, 0\}$

By DIF-FFT, To find R[k] for K even,

Let $R[2r] = DFT\{g[n]\} = G[k]$

where $g[n] = r[n] + r[n + \frac{N}{2}]$ Put $N = 8$

$g[n] = r[n] + r[n+4]$

$g[0] = g[0] + g[4] = a + 0 = a$

$g[1] = g[1] + g[5] = b + 0 = b$

$g[2] = g[2] + g[6] = c + 0 = c$

$g[3] = g[3] + g[7] = d + 0 = d$

$\therefore$ g[n] = { a, b, c, d } = x[n]

By DFT G[k] = X[k]

$\therefore$ R[2r] = G[k] = X[k]

$$\therefore R[k] \text{ for k even values } = \begin{pmatrix} A & k=0 \\ B & k=2 \\ C & k=4 \\ D & k=6 \end{pmatrix}$$

Q(36) A sequence x[n] = { x[0], x[1], x[2], x[3] } Let DFT { x[n] } = X[k] = { 1, 2 , 3, 2 }

(a) Identify the signal type.

(b) Let p[n] = { x[0] , x[1], x[2], x[3], 0, 0, 0, 0 }. Find P[k] only for even values of k. Justify your answer.

Solution :

(a) Since X[k] is real and even , therefore x[n] is also real and even. i.e. x[n] =x[–n]

(b) To find P[k] only for even values of k.

Given p[n] = { x [0] , x [2], x [3], 0, 0, 0, 0 }.

By DIF-FFT :
$$P[2r] = X[2r] = \sum_{n=0}^{\frac{N}{2}-1} x[n]\ W_{N/2}^{2rn} + \sum_{n=0}^{\frac{N}{2}-1} x\left[n+\frac{N}{2}\right] W_N^{2r(n+\frac{N}{2})}$$

$$P[2r] = \sum_{n=0}^{\frac{N}{2}-1} x[n]\ W_{N/2}^{rn} + \sum_{n=0}^{\frac{N}{2}-1} x\left[n+\frac{N}{2}\right] W_N^{rn}$$

$$P[2r] = \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] + x\left[n+\frac{N}{2}\right]\right) W_{\frac{N}{2}}^{rn}$$

$$P[2r] = \sum_{n=0}^{\frac{N}{2}-1} g[n]\quad W_N^{rn}$$

DFT { g [n] } = G [k]

$$g[n] = \left(x[n] + x\left[n+\frac{N}{2}\right]\right) \quad N = 8$$

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g [0] = x [0] + x [4] = x [0] + 0 = x [0]

g [1] = x [1] + x [5] = x [1] + 0 = x [1]

g [2] = x [2] + x [6] = x [2] + 0 = x [2]

g [3] = x [3] + x [7] = x [3] + 0 = x [3]

$\therefore$ g [n] = x [n]

$\therefore$ By DFT , G [k] = X[k]

$\therefore$ P [2 r] = X[k]

$\therefore$ r=0, $\quad P[0] = X[0] = 1$

r=1, $\quad P[2] = X[1] = 2$

r=2, $\quad P[4] = X[2] = 3$

r=3, $\quad P[6] = X[3] = 2$

Q(37) Let $y[n] = x_1[n] + x_2[n]$ where $x_2[n]$ is such that $Y[k] = \begin{bmatrix} 2X_1[k] & \text{for } k \text{ even} \\ 0 & \text{for } k \text{ odd} \end{bmatrix}$

Find $x_2[n]$ without performing iFFT. Prove the DFT property you used.

Solution :

$$y[n] = x_1[n] + x_2[n]$$

where

$$Y[k] = \begin{bmatrix} 2X_1[k] & \text{for } k \text{ even} \\ 0 & \text{for } k \text{ odd} \end{bmatrix}$$

$\therefore$ we can write Y[k] as, $\quad Y[k] = X_1[k] + (-1)^k X_1[k] \ldots (1)$

Consider given expression for y[n],

$y[n] = x_1[n] + x_2[n]$

By DFT,

$Y[k] = X_1[k] + X_2[k] \ldots (2)$

Comparing equations (1) & (2), $X_2[k] = (-1)^k X_1[k]$

But we know that $W_N^{\frac{N}{2}} = -1$

$$\therefore X_2[k] = \left(W_N^{\frac{N}{2}}\right)^k X_1[k]$$

$$X_2[k] = W_N^{\frac{Nk}{2}} X_1[k]$$

Consider Time shift property of DFT,

$$\text{DFT}\{x[n-m]\} = W_N^{mk} X[k] \quad \text{where} \quad m = \frac{N}{2}$$

$\therefore$ By applying Time shift property of iDFT,

$$x_2[n] = x[n-m]$$

Put $m = \dfrac{N}{2}$, So $x_2[n] = x\left[n - \dfrac{N}{2}\right]$

Q(38) Let y[n] be a 8 point sequence obtained from x[n] such that $y[n] = \begin{bmatrix} x[\frac{n}{2}] & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd} \end{bmatrix}$

where $x[n] = \{ 1, 2, 3, 4 \}$. Find y[k] in turns of X[k] without performing DFT/FFT

Solution : To find y[n],

$$\text{Now} \quad y[n] = \begin{bmatrix} x[\frac{n}{2}] & \text{for n even} \\ 0 & \text{for n odd} \end{bmatrix}$$

$y[0] = x[0] = 1$	$y[4] = x[2] = 3$
$y[1] = 0$	$y[5] = 0$
$y[2] = x[1] = 2$	$y[6] = x[3] = 2$
$y[3] = 0$	$y[7] = 0$

$$\therefore \; y[n] = \{ \underset{\uparrow}{1}, 0, 2, 0, 3, 0, 2, 0 \}$$

By Decimating In Time, we get,

Let $y[2r] = \{1, 2, 3, 2 \} = x[n]$

$y[2r + 1] = \{ 0,0,0,0 \} = 0$

$$Y[k] = \sum_{r=0}^{\frac{N}{2}-1} y[2r]W_N^{2rk} + \sum_{r=0}^{\frac{N}{2}-1} y[2r+1]W_N^{(2r+)k}$$

$$Y[k] = \sum_{r=0}^{\frac{N}{2}-1} y[2r]W_{N/2}^{rk} + W_N^{k} \sum_{r=0}^{\frac{N}{2}-1} y[2r+1]W_{\frac{N}{2}}^{rk}$$

Let $Y[k] = G[k] + W_N^k H[k]$

where

(i) $G[k] = \text{DFT } \{y[2r] = \text{DFT } \{x[n]\} = X[k]$

(ii) $H[k] = \text{DFT } \{y[2r + 1] = 0$

By substituting,

$$\therefore \; P[k] = G[k] + W_N^k H[k]$$

$\therefore$ $P[k] = X[k]$ for $k = 0, 1, \ldots 7$

$P[k] = \{ \underset{\uparrow}{8}, -2, 0, -2, 8, -2, 0, -2 \}$ ANS

Q(39) (a) If $x[n] = \{ 1 + j5, 2 + j6, 3 + j7, 4 + j8 \}$ Find X[k] using DIF-FFT.

(b) Using the results obtained in (a) and not otherwise, find DFT of the following sequences :- $x_1[n] = \{ 1, 2,, 3, 4 \}$ and $X_2[n] = \{ 5, 6, 7, 8 \}$

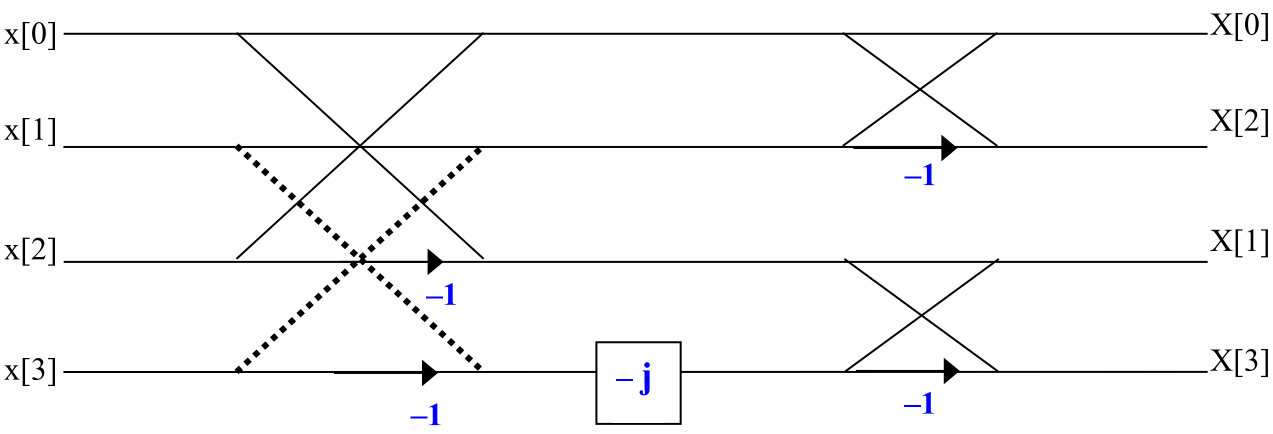
Solution :

(a) To find X[k]

Let $x[n] = x_1[n] + j\, x_2[n]$

$\therefore x[n] = \{ 1 + j5,\ 2 + j6,\ 3 + j7,\ 4 + j8 \}$

Find X[k] by four point DIF-FFT, flowgraph



$$\therefore X[k] = \begin{bmatrix} 10 + j26 & k = 0 \\ -4 & \\ -2 - j2 & \\ -j4 & \end{bmatrix}$$

(b) To find X_1[k]

$x[n] = x_1[n] + j\, x_2[n] \ \ldots (1)$

$x[n] = x_1[n] - j\, x_2[n] \ \ldots (2)$

Adding equation (1) and (2)

$2\, x_1[n] = x[n] + x^*[n]$

$X_1[n] = \frac{1}{2} \{ x[n] + x^*[n] \}$

BY DFT, $\quad X_1[k] = \frac{1}{2} \{ X[k] + X^*[-k] \}$

$$\therefore X_1[k] = \frac{1}{2} \left\{ \begin{bmatrix} 10 + j\,26 \\ -4 \\ -2 - 2j \\ -4j \end{bmatrix} + \begin{bmatrix} 10 - j\,26 \\ 4j \\ -2 + 2j \\ -4 \end{bmatrix} \right\} \qquad \therefore X_1[k] = \begin{bmatrix} 10 & k = 0 \\ -2 + 2j & \\ -2 & \\ -2 - 2j & \end{bmatrix}$$

(c) To find X_2[k]

Subtracting equation (2) from (1)

$2\,j\,x_2\,[n] = x\,[n] - x^*\,[n]$

$\therefore x_2\,[n] = \dfrac{1}{2\,j}\{\,x[n] - x^*[n]\,\}$

By D F T

$$X_2\,[k] = \frac{1}{2j}\left\{\,X[k] - X^*[-k]\,\right\}$$

$$\therefore X_2[k] = \frac{1}{2j}\left\{\begin{bmatrix}10 + j\,26\\ -4\\ -2-2j\\ -4j\end{bmatrix} - \begin{bmatrix}10 - j\,26\\ 4j\\ -2+2j\\ -4\end{bmatrix}\right\} : \quad \therefore X_2[k] = \begin{array}{c}\begin{bmatrix}26\\ -2+2j\\ -2\\ -2-2j\end{bmatrix}\end{array}\begin{array}{l}k=0\\ \\ \\ \\ \end{array}$$

Q(40) The real sequence of length 8 is given as x[n] = {1, 2, 2, 0, 1, 1, 1 } Find the 8 point DFT X[k], by using 4 point DFTs only.

Solution : To find X[k]

Given x[n] = { 1, 2, 2, 2, 0, 1, 1, 1 }

By Decimating In Time,

$x(2r) = \{1,2,0,1\}$ and $x(2r+1) = \{2,2,1,1\}$

By DFT, $X[k] = \sum_{n=0}^{N-1} x[n]\; W_N^{nk}$

$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r]\, W_N^{2rk} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1]\; W_N^{(2r+1)k}$$

$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r]\;\; W_{\frac{N}{2}}^{rk} + W_N^k \sum_{r=0}^{\frac{N}{2}-1} x[2r+1]\; W_{\frac{N}{2}}^{rk}$$

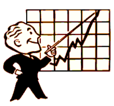
$$X[k] = DFT\{x(2r)\} + W_N^k\;\; DFT\{x(2r+1)\}$$

Let $X[k] = G[k] + W_N^k\; H[k]$

Where $G[k] = DFT\{x(2r)\}$

And $H[k] = DFT\{x(2r+1)\}$

(i) To find G [k] :



$$DFT\{x(2r)\} = \sum_{r=0}^{\frac{N}{2}-1} x(2r)\; W_{\frac{N}{2}}^{rk}$$

$$G[k] \;=\; \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \qquad G[k] = \begin{bmatrix} 4 \quad k=0 \\ 1-j \\ -2 \\ 1+j \end{bmatrix}$$

(ii) To Find H [k] :

$$DFT\{x(2r+1)\} = \sum_{r=0}^{\frac{N}{2}-1} x(2r+1)\, W_{\frac{N}{2}}^{rk}$$

$$H[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$H[k] = \begin{bmatrix} 6 \quad k=0 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

(iii) Find X [k]

$$X[k] = G[k] + W_N^k\, H[k]$$

$$X[k] = \begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \\ 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix} + \begin{bmatrix} 1 \\ 0.707 - j0.707 \\ -j \\ -0.707 - j0.707 \\ -1 \\ -0.707 + j0.707 \\ j \\ 0.707 + j0.707 \end{bmatrix} \begin{bmatrix} 6 \\ 1-j \\ 0 \\ 1+j \\ 6 \\ 1-j \\ 0 \\ 1+j \end{bmatrix} \qquad X[k] \;=\; \begin{bmatrix} 10 \quad k=0 \\ 1-j\ 2.414 \\ -2 \\ 1-j\ 0.414 \\ -2 \\ 1+j\ 0.414 \\ -2 \\ 1+j\ 2.414 \end{bmatrix}$$

Q(41) Consider a finite duration sequence $x(n) = \{0, 1, 2, 3, 4\}$.

Find the sequence y(n) with five point DFT Y(k) = Re | X(k) |.

Solution : **To find y[n]**

Given Y(k) = Re | X(k) |.

Y(k) = Xe[k].

By IDFT, $y[n] = x_e[n]$

$y[n] = ½ \{ x[n] + x[-n] \}$

$$y[n] = \frac{1}{2}\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} \right\} = \begin{bmatrix} 0 & n=0 \\ 2.5 & \\ 2.5 & \\ 2.5 & \\ 2.5 & \end{bmatrix}$$ **ANS**

Q(42) Assume that a complex multiply takes 1 micro sec and that the amount of time to compute a DFT is determined by the amount of time to perform all the multiplications.

(a) How much time does it take to compute a 1024 point DFT directly**?**

(b) How much time is required if FFT is used.

Solution :

(a) By DFT

Total Complex multiplications = $N^2 = (1024)^2$

Time required to compute N^2 Multiplications $= (1024)^2$ micro sec =1.048576 Sec

(b) By FFT

Total Complex multiplications = $N/2 \ \log_2 N = (512) \log_2(1024) = 5120$

Time required to compute 5120 Multiplications =(5120)micro sec =0.005120 Sec

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❖ Drill Problems on DFT + FFT

Q(43) A signal x(t) that is band-limited to 10 KHz is sampled with a sampling frequency of 20 KHz. The DFT of **N=**1000 samples of x[n] is then computed.

(a) To what analog frequency does the index k=150 corresponds. ?

(b) What is the spacing between the spectral samples ?

Solution :

Given Fm=10 KHz and Fs=20 KHz **N=**1000

(a) $w = \frac{2\pi k}{N} = \frac{2\pi k}{1000}$

$2\pi f = \frac{2\pi k}{1000}$

Digital frequency $f = \frac{k}{1000} == \frac{150}{1000}$

To find Analog frequency F,

$f = \frac{F}{Fs} == \frac{F}{20000}$

$F = 20000\ f$

$F = 20000\left(\frac{150}{1000}\right) == 3000\ Hz$ **ANS**

(b) Frequency spacing $= \frac{2\pi}{N} = \frac{2\pi}{1000} == 0.002\,\pi$ **ANS**

Q(44) Determine the number of bits required to compute an FFT of 2048 points with an SNR of 30 dB.

Solution :

Signal to Noise ratio is given by,

$SNR\ =\ 4\ N\ 2^{-2B}$

Where B is no of bits required, and N is length of signal.

Put N= 2048

SNR = 30 db

$30\ =\ 4\ (2048)\ 2^{-2B}$

B = 4.046

Let No of bits required = **4** **ANS**

Q(45) Explain the Symmetry and Periodicity properties of a phase factor.

Solution :

Phase factor $W_N^1 = e^{-j\frac{2\pi}{N}}$

(1) Phase Factor is Periodic i.e $W_N^{k+N} = W_N^k$

Proof : Now $W_N^1 = e^{-j\frac{2\pi}{N}}$

$$W_N^k = \left(e^{-j\frac{2\pi}{N}} \right)^k$$

$$W_N^{k+N} = \left(e^{-j\frac{2\pi}{N}} \right)^{k+N}$$

$$W_N^{k+N} = e^{-j\frac{2\pi k}{N}} \, e^{-j2\pi} \qquad \text{Put } = e^{j2\pi} = 1$$

$$W_N^{k+N} = e^{-j\frac{2\pi k}{N}}$$

$$W_N^{k+N} = W_N^k$$ **Hence Phase factor is periodic.**

(1) Phase Factor is 0dd Symmetric i.e $W_N^{k+\frac{N}{2}} = -W_N^k$

Proof : Now $W_N^k = \left(e^{-j\frac{2\pi}{N}} \right)^k$

$$W_N^{k+\frac{N}{2}} = \left(e^{-j\frac{2\pi}{N}} \right)^{k+\frac{N}{2}}$$

$$W_N^{k+\frac{N}{2}} = e^{-j\frac{2\pi k}{N}} \, e^{-j\pi} \qquad \text{Put } = e^{-j\pi} = -1$$

$$W_N^{k+\frac{N}{2}} = -\, e^{-j\frac{2\pi k}{N}}$$

$$W_N^{k+\frac{N}{2}} = -W_N^k$$ **Hence Phase factor is 0dd symmetric.**

Q(46) For the given sequence x[n]={ 2, 0, 0, 1 }. Perform the following operations:-

(a) Find out 4 point DFT of x[n].

(b) Plot x[n], its periodic extension $x_p[n]$ and $x_p[n-3]$

(c) Add phase angle in (a) with factor $\left[\frac{-2\pi r k}{N}\right]$ where N=4, r=3, k=0,1,2,3.

Then find x′[n].

(d) Comment on results you had in point (b) and (c).

Soln (a): To find X[k]

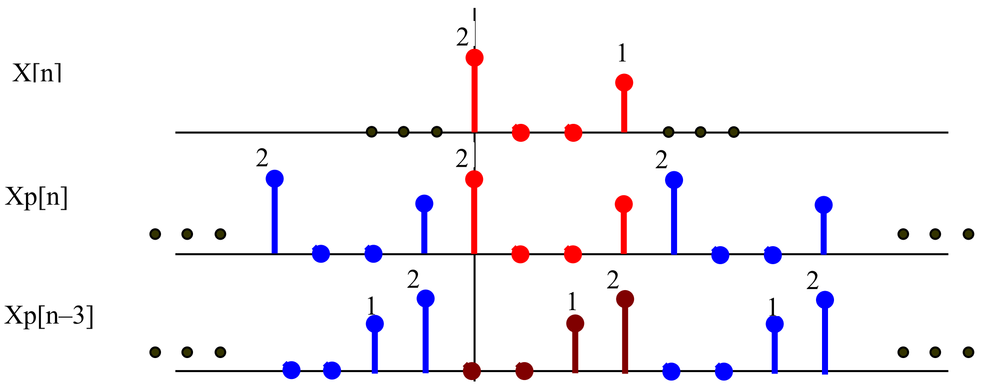
By DFT, $X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk}$

$$X[K] = \begin{bmatrix} w_N^0 & w_N^0 & w_N^0 & w_N^0 \\ w_N^0 & w_N^1 & w_N^2 & w_N^3 \\ w_N^0 & w_N^2 & w_N^4 & w_N^6 \\ w_N^0 & w_N^3 & w_N^6 & w_N^9 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$X[k] = \begin{bmatrix} 3 & k=0 \\ 2+j & \\ 1 & \\ 2-j & \end{bmatrix} \quad \textbf{ANS}$$

Soln (b): To plot x[n], $x_p[n]$ and $x_p[n-3]$



Soln (c): To add phase angle $\phi_k = \left[\frac{-2\pi rk}{N}\right]$ where N=4, r=3,

	$\phi_k = \left[\frac{-2\pi 3k}{4}\right] == -4.71\,k$
k=0	$\phi_0 = 0$
k=1	$\phi_1 = -4.71$
k=2	$\phi_2 = -9.42$
k=3	$\phi_3 = -14.13$

$$\text{Now, } X[k] = \begin{bmatrix} 3 & = & 3\angle 0 \\ 2+j & = & 2.236\angle 0.4636 \\ 1 & = & 1\angle 0 \\ 2-j & = & 2.236\angle -0.4636 \end{bmatrix}$$

By adding Phase Angle, we get,

$$X[k] = \begin{bmatrix} 3\angle 0 & k=0 \\ 2.236\angle -4.2464 & \\ 1\angle -9.42 & \\ 2.236\angle -14.5936 & \end{bmatrix} == \begin{bmatrix} 3 & k=0 \\ -1+2j & \\ -1 & \\ -1-2j & \end{bmatrix}$$

By iDFT, $\text{x[n]} = \frac{1}{N}\sum_{n=0}^{N-1} X[k]\,W_N^{-nk}$

$$x[n] = \frac{1}{4}\begin{bmatrix} w_N^0 & w_N^0 & w_N^0 & w_N^0 \\ w_N^0 & w_N^{-1} & w_N^{-2} & w_N^{-3} \\ w_N^0 & w_N^{-2} & w_N^{-4} & w_N^{-6} \\ w_N^0 & w_N^{-3} & w_N^{-6} & w_N^{-9} \end{bmatrix}\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix}$$

$$x[n] = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 3 \\ -1+2j \\ -1 \\ -1-2j \end{bmatrix}$$

$$x[n] = \begin{bmatrix} 0 & n=0 \\ 0 & \\ 1 & \\ 2 & \end{bmatrix}$$

Solution (d) Comment on results you had in point (b) and (c).

x[n] obtained in (c) is same as x_p[n-3] in (b)
