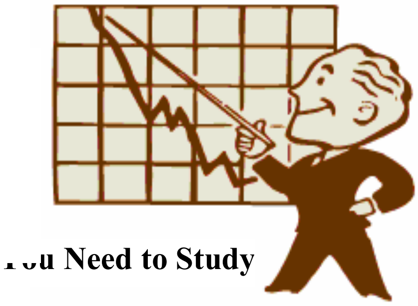
Digital FIR Filters



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Dr. Kiran TALELE

- Academic : PhD
- Professional :
  - Dean-Students, Alumni & External Relations @ Bharatiya Vidya Bhavans' Sardar Patel Institute of Technology (SP-IT), Mumbai
  - Head-Academic Relation @ Sardar Patel Technology Business Incubator (SP-TBI), Mumbai
  - Treasurer-IEEE Bombay Section

Mobile : 9987030881

https://www.linkedin.com/in/k-t-v-talele/

www.facebook.com/Kiran-Talele-1711929555720263

DIGITAL F I R FILTERS

Q(1) Explain the concept of Linear Phase and its importance.

ANS :

Consider a LPF with frequency response $H(e^{jw})$ given by

$$H(e^{jw}) = \begin{cases} e^{-jw\alpha} & |w| \le w_c \\ 0 & wc < w \le \pi \end{cases}$$

$$\frac{x[n]}{X(w)} \longrightarrow \boxed{H(z)} \longrightarrow \frac{y[n]}{Y(w)}$$

Let X(w) = DTFT { X[n] } ,

The FT of y[n] is then given by

Y(w) =X(w) . H(w)

Y(w) = X(w) . $e^{-jw\alpha}$

By iDTFT, $\boxed{y[n] = x[n - \alpha] \leftarrow \text{ o/p of filter}}$

Conclusion :

(1) The phase response of the filter can be linear or non linear.

(2) If the phase response of the filter is linear then the output of the filter is same as original input delayed by α.

The linear Phase does not alter the shape of the original signal. In this example, the phase response is , $\phi(w) = -\alpha.w$ where α is any constant.

(3) If the phase response of the filter is non linear the output is distorted version of input x[n].

I. If the Phase Response is Linear the output of the Filter during pass-band is delayed input.

II. If the phase Response is non Linear the output of the filter during pass-band is distorted one

The linear Phase characteristic is important when the phase distortion is not tolerable.

FIR Filter can be designed with linear phase characteristic. In application like data transmission, speech processing etc phase distortion can not be tolerated and here linear phase characteristic of FIR filter is useful.

Q(2) What is Phase Delay and Group Delay ?

Ans :

The phase delay and group delay. The phase delay (T_P) and group delay (Tg) of the filter are given by,

$$Tp = \frac{-\phi}{w} \quad and \quad Tg = \frac{d\phi}{dw}$$

The group delay Tg, is defined as the delayed response of the filter as a function of w to the signal. Linear Phase Filters are those
Filters in which the phase delay and group delay are constant, ie independent of frequency.
Linear Phase Filters are also called as constant time delay Filters.

Q(3) What is a linear phase filter ? What conditions are to be satisfied by the impulse response of FIR filter in order to have a linear phase ? Define phase delay and group delay.

Solution :

(1) The Linear Phase Filter has phase response linearly varying with frequency.

i.e. $\phi(w)$ is proportional to frequency w

(2) If the phase response of the filter is linear then the output of the filter is same as original input delayed by constant say ⍰. The linear Phase does not alter the shape of the original signal.

(3) For Linear Phase Filter the phase delay and group delay are constant over the frequency band.

The necessary condition to have linear phase is phase delay and group delay of the filter must be constant. That is possible when h[n] is either Symmetric or Anti-symmetric.

(i) When is symmetric i.e. h[n] = h[N−1−n] i.e.

$$\phi(w) = -\left(\tfrac{N-1}{2}\right).w$$

Let $\phi(w) = -\alpha.w$ where $\alpha = \frac{N-1}{2}$

$$\text{Phase Delay } \tau_p = \frac{-\Phi(w)}{w} = \alpha \text{ (constant)}$$

$$\tau_g = \frac{-d\Phi(w)}{d\,w} = \alpha \text{ (constant)}$$

(ii) When is Anti-symmetric i.e. h[n] = –h[N–1–n] i.e.

$$\phi(w) = \frac{\pi}{2} - \left(\tfrac{N-1}{2}\right).w$$

Let $\phi(w) = \frac{\pi}{2} - \alpha.w$ where $\alpha = \frac{N-1}{2}$

$$\tau_g = \frac{-d\,\Phi(w)}{d\ w} = \alpha \text{ (constant)}$$

The group Delay of the filter is defined as the delayed response of the filter as a function of w to a signal. $\tau_g = \dfrac{-d\,\Phi(w)}{d\ w}$

The Phase Delay of the filter is given by $\tau_p = \dfrac{-\Phi(w)}{w}$

➢ Position of definite ZEROS of a Linear Phase FIR filter.

Case-1 : When h[n] is Symmetric with N EVEN [Type-2 Linear Phase Filter]

$$H(z) = z^{-(N-1)} \quad H\left(\frac{1}{Z}\right)$$

$$H(z) = z^{-ODD} \quad H\left(\frac{1}{Z}\right)$$

(1) At Z = –1 w = π	**(2) At Z = 1 w = 0**
$H(-1) = (-1)^{-ODD} \quad H(-1)$ $H(-1) = (-1) \quad H(-1)$ $H(-1) = -H(-1)$ $H(-1) + H(-1) = 0$ $2\,H(-1) = 0$ So, H(-1) = 0 $H(z)\big\vert_{z=-1} = 0$ That means there exists definite ZERO at z = –1	$H(1) = (1)^{-ODD} \quad H(1)$ $H(1) = (1) \quad H(1)$ $H(1) = H(1)$ No definite ZERO exists at z = 1

Case-2 : When h[n] is Symmetric with N ODD [Type-1 Linear Phase Filter]

$$H(z) = z^{-(N-1)} \quad H\left(\frac{1}{Z}\right) \qquad\qquad H(z) = z^{-EVEN} \quad H\left(\frac{1}{Z}\right)$$

(1) At Z = –1 w = π	**(2) At Z = 1 w = 0**
$H(-1) = (-1)^{-EVEN} \quad H(-1)$ $H(-1) = (1) \quad H(-1)$ $H(1) = H(1)$ No definite ZERO exists at z = –1	$H(-1) = (-1)^{-EVEN} \quad H(-1)$ $H(1) = (1) \quad H(1)$ $H(1) = H(1)$ No definite ZERO exists at z = 1

Case-3 : When h[n] is Anti-Symmetric with N EVEN [Type-4 Filter]

$$H(z) = -\,z^{-(N-1)} \quad H\left(\frac{1}{Z}\right) \qquad\qquad H(z) = -\,z^{-ODD} \quad H\left(\frac{1}{Z}\right)$$

(1) At Z = –1 w = π	**(2) At Z = 1 w = 0**
$H(-1) = -\,(-1)^{-ODD} \quad H(-1)$ $H(-1) = -\,(-1) \quad H(-1)$ $H(-1) = H(-1)$ No definite ZERO exists at z = –1	$H(1) = -\,(1)^{-ODD} \quad H(1)$ $H(1) = -\,(1) \quad H(1)$ $H(1) = -\,H(1)$ So, H(1) = 0 $\quad H(z)\big\|_{z=1} = 0$ That means there exists definite ZERO at z = 1

Case-4 : When h[n] is Anti-Symmetric with N ODD [Type-3 Linear Phase Filter]

$$H(z) = -z^{-(N-1)} \quad H\left(\frac{1}{Z}\right)$$

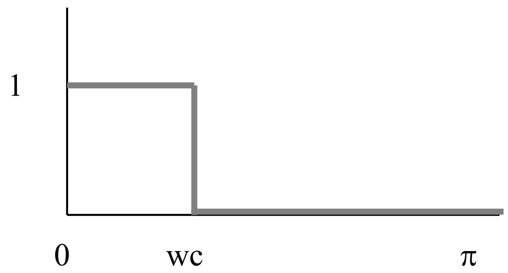
$$H(z) = -z^{-EVEN} \quad H\left(\frac{1}{Z}\right)$$

(1) At Z = –1 w = π	**(2) At Z = 1 w = 0**
$H(-1) = -\,(-1)^{-EVEN} \quad H(-1)$ $H(-1) = -\,(1) \quad H(-1)$ $H(-1) = -\,H(-1)$ So, H(-1) = 0 $H(z)\big\|_{z=-1} = 0$ That means there exists definite ZERO at z = –1	$H(1) = -\,(1)^{-EVEN} \quad H(1)$ $H(1) = -\,(1) \quad H(1)$ $H(1) = -\,H(1)$ So, H(1) = 0 $\quad H(z)\big\|_{z=1} = 0$ That means there exists definite ZERO at z = 1

Q(3) Anti-Symmetric h[n] can not be used for LPF design. Justify.

Solution :

Linear Phase LPF Design



$$H(e^{jw}) = \begin{cases} 1 & 0 \le w \le w_c \\ 0 & Otherwise \end{cases}$$

For LPF,

$$H(e^{jw}) \neq 0 \ \ for \ \ 0 \le w \le w_c$$

$$H(e^{jw}) \neq 0 \ \ at \ \ w = 0$$

i.e. $H(e^{jw})\big|_{w=0} \neq 0 \ \ for \ \ LPF$

Put z = e^{jw}
At w = 0 z = 1

$$H(z)\big|_{z=1} \neq 0 \ \ for \ \ LPF$$

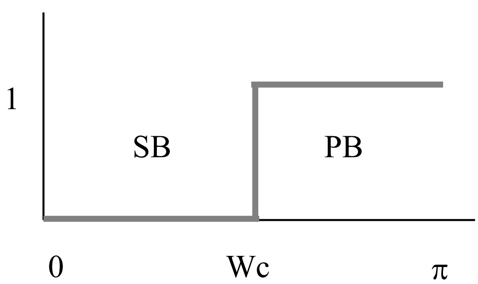
For Anti-symmetric h[n], there exists definite ZERO at z = 1. i.e. $H(z)\big|_{z=1} = 0$

That means, Anti-symmetric h[n] can-not be used for LPF design.

Q(4) Symmetric h[n] with N even and Anti-symmetric h[n] with N odd, can not be used for HPF design. Justify.

Solution :

Linear Phase HPF



$$H(e^{jw}) = \begin{cases} 1 & w_c \le w \le \pi \\ 0 & Otherwise \end{cases}$$

For HPF,

$$H(e^{jw}) \neq 0 \ \ for \ \ w_c \le w \le \pi$$

$$H(e^{jw}) \neq 0 \ \ at \ \ w = \pi$$

i.e. $H(e^{jw})\big|_{w=\pi} \neq 0 \ \ for \ \ HPF$

Put z = e^{jw}
At w = π z = –1

For symmetric h[n] with N even and Anti-symmetric h[n] with N odd, there exists definite ZERO at z = –1. i.e. $H(z)\big|_{z=-1} = 0$

- **That means, for symmetric h[n] with N even and Anti-symmetric h[n] with N odd, can not be used for HPF design.**

Q(5) Explain FIR Filter design Using Window Function
(Note on windowing method)

ANS :

FIR Filter is designed by truncating the infinite samples of IIR Filter. Let hd[n] denote the impulse response sequence of IIR Filter. To have the linear phase response, the impulse response must be symmetric or antisymmetric about some point in time. To simplify the analysis we consider hd[n] to be symmetric about n = 0.

The N point impulse response of FIR can be obtained as,

$$h[n] = \begin{cases} hd[n] & 0 \le n \le N-1 \\ O & otherwise \end{cases}$$

To truncate the samples of hd[n], window function is used.

The impulse response of FIR Filter is then given by, **h[n] = hd[n] . w[n]**

where w[n] is a window function and is symmetric about n=0 to maintain the symmetry that is present in hd[n].

*

Q(6) What is a window in FIR filter design and why it is necessary ?

ANS :

For designing FIR filter, the impulse response is found from the desired frequency response $H_d(w)$. The ideal impulse response has infinite response has infinite terms. In order to truncate infinite values of ideal impulse response $h_d[n]$, to a finite values, window function is required. This is equivalent to truncating the no of terms in fourier series of $H_d(w)$.

From Gibb's phenomeno, we know that, a finite number of terms in fourier series gives a ringing effect at the abrupt discontinuities. Therefore, there is a need to smoothen the edges of a jump discontinuity.

In the theory of series, we know that this non-uniform convergence can be moderated through the use of less abrupt transition of the fourier series. By tapering the window smoothly to zero at each end, the height of the side lobe can be diminished. This is achieved at the expense of a wider main lobe and wider transition at the discontinuity.

For this purpose an appropriatde window function is used and multiplied with $h_d[n]$.

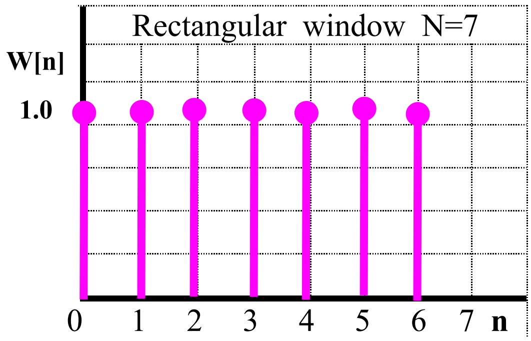
Thereofore, $h[n]=h_d[n]\ \ w[n]$ where w[n] is the window function used to smoothen out the edges of the discontinuity.

Q(7) Draw different types of windows used in designing FIR filters. Compare frequency domain characteristics of window functions.

Solution : The different types of window functions used in designing FIR filters are :

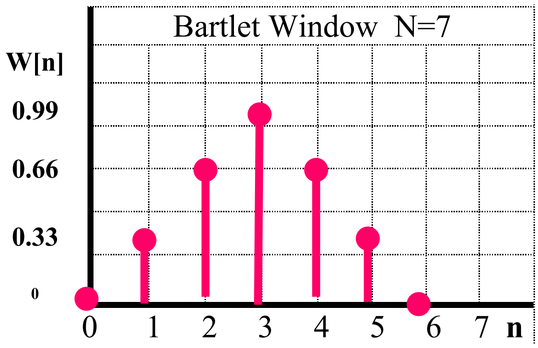
(1) Retangular window function

$$w[n] = \begin{cases} 1 & 0 \le n \le N-1 \\ 0 & otherwise \end{cases}$$



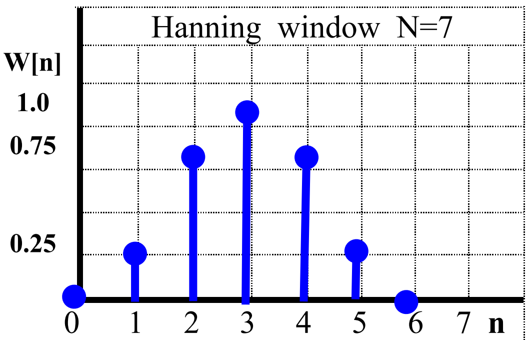
(2) Bartlet window function

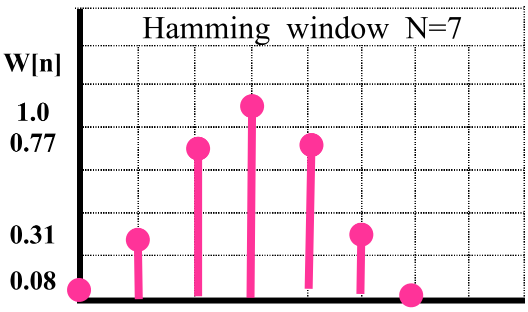
$$w[n] = \begin{cases} \dfrac{2n}{N-1} & 0 \le n \le \dfrac{N-1}{2} \\ 2 - \dfrac{2n}{N-1} & \dfrac{N-1}{2} \le n \le N-1 \\ 0 & otherwise \end{cases}$$

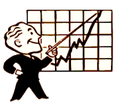


(3) Hanning window function

$$w[n] = \begin{cases} \left\{1 - \cos\left(\dfrac{2\pi n}{N-1}\right)\right\}/2 & 0 \le n \le N-1 \\ 0 & otherwise \end{cases}$$





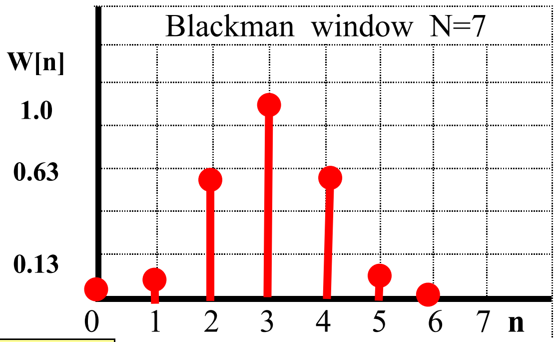


(4) Hamming window function

$$w[n] = \begin{cases} 0.54 - 0.46 \cos\left(\dfrac{2\pi n}{N-1}\right) & 0 \le n \le N-1 \\ 0 & otherwise \end{cases}$$

(5) Blackman window function

$$w[n] = \begin{cases} 0.42 - 0.5 \cos\left(\dfrac{2\pi n}{N-1}\right) + 0.08 \cos\left(\dfrac{4\pi n}{N-1}\right) & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$$



	Window Function	**Transition width [C/N]**	**Attenuation in Stop Band [As]**
1	Rectangular	**C = 0.92**	**21**
2	Bartlett	**C = 2.1**	**25**
3	Hanning	**C = 3.21**	**44**
4	Hamming	**C = 3.47**	**53**
5	Blackman	**C = 5.71**	**74**

Frequency Domain characteristics of window functions are :

Ref. "Digital Signal Processing : A Modern Introduction" by Ashok Ambardar, Cengage Learning India Edition 2007. Page 476

Q(8) Explain Gibb's phenomenon. State its significance in FIR filter design. State how it can be reduced. [Marks-10]

ANS :
The causal FIR filter is obtained by simply truncating the impulse response coefficients of the ideal filters using window function. i.e. h[n] = hd[n] w[n] where hd[n] is the impulse response of Ideal filter which is of infinite length and w[n] is causal finite length window function.

For. Ex. When window Rectangular window is used then,

$$W_R[n] = \begin{cases} 1 & 0 \le n \le N-1 \\ 0 & otherwise \end{cases}$$

When hd[n] is multiplied by rectangular window function, hd[n] samples are abruptly truncated and h[n] = 0 for n> N-1. Due to abrupt truncation of sample values of hd[n] we get an oscillatory behavior in their respective magnitude responses which is referred to as the Gibbs phenomenon.

The FIR filter obtained by truncation can be expressed as:

$$h_t[n] = h_d[n] \cdot \omega[n]$$

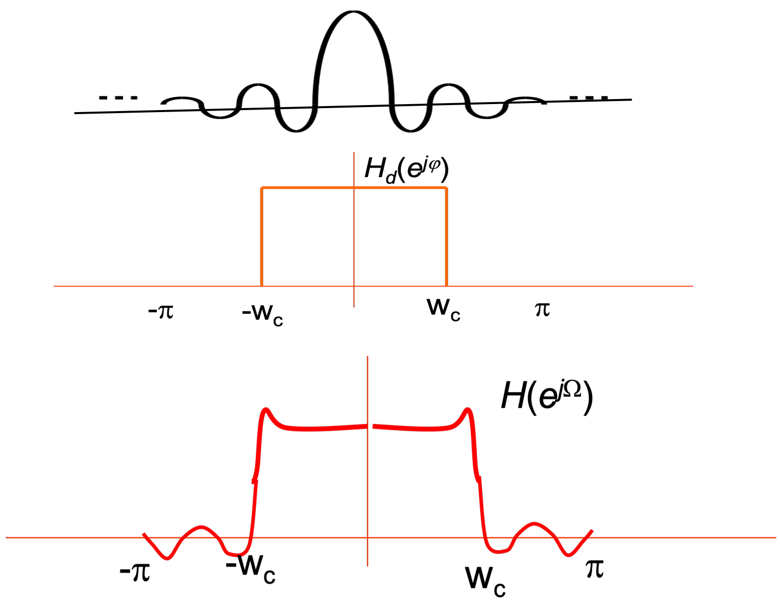
By DTFT,

$$H\ (e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\varphi})\ W_R(e^{j(\omega-\varphi)})d\varphi$$

So there are two basic reason of the oscillatory behavior:

(1) the impulse response of a ideal filter is infinitely long and not absolutely summable.

(2) the rectangular window has an abrupt transition to zero.



Gibbs phenomenon can be reduced by ,

(1) using a window that tapers smoothly to zero at each end.

(2) providing a smooth transition from the passband to the stopband.

This **reduces** the **height** of the **sidelobes** but **increases** the **main lobe width**.

The commonly used fixed windows are Bartlett Window, Hannning window, Hamming window, Blackman window. These fixed windows have fixed passband and stopband ripples levels.

Q(9) What are Advantages and Disadvantages of Window Method of Filter Design.

ANS :

- Advantages:

  1. An Important advantage of Window method is its simplicity. It is simple to apply and simple to understand.
  2. It involves minimum amount of computational effort, even for more complicated Kaiser Window.

- Dis-Advantages :

  1. The major disadvantage is Lack of flexibility. Both the peak pass-band and stop band ripples are approximately equal, so that the designer may end up with either too small pass-band ripple or too large stop-band attenuation.
  2. Because of the effect of convolution of the spectrum of the window function and the desired response, the pass-band and stop-band edge frequencies cannot be precisely specified.
  3. For a given window (except Kaiser Window) the maximum ripple amplitude in the filter response is fixed regardless of how large we take N. Thus stop-band attenuation for a given window is fixed.
  4. In some applications, the expressions for the desired filter response, $H_D(w)$ will be too complicated for $h_d[n]$ to be obtained analytically.

Q(10) What are the desirable characteristics of window Function ?

Ans : (i) The Fourier Transform of the window function $W(e^{jw})$ should have a small width of main lobe containing as much of the total energy.

(ii) The Fourier Transition of the window function $W(e^{jw})$ should have side lobes that decrease in energy rapidly as w tends to ⍰⍰.

➢ ALGORITHM To Design Linear Phase F I R Filter Using Window Function

I. Accept the specifications : Ap, As, wp, ws

II. Select the window function such that window function stop-band Attenuation exceeds the specified As.

III. Select the number of points in the window to satisfy the transition width for the type of window function.

Transition width Of desired filter ≥ ition width dow function

$$f_2 - f_1 \geq \frac{C}{N}$$

$$\text{i.e. } N \geq \frac{C}{fs - fp} \quad \text{for LPF}$$

$$N \geq \frac{C}{fp - fs} \quad \text{for HPF}$$

IV. Select the cut off frequency w_c : Take $w_c = \dfrac{w_p + w_s}{2}$

V. The impulse response h[n] of desired Linear Phase FIR is then given by,

$$\mathbf{h[n] = h_d[n] . w[n]}$$

1	**LPF**	$hd[n] = \dfrac{wc}{\pi} \dfrac{\sin(n-\alpha)wc}{(n-\alpha)wc}$
2	**HPF**	$hd[n] = \dfrac{\sin(n-\alpha)\pi}{(n-\alpha)\pi} - \dfrac{wc}{\pi} \dfrac{\sin(n-\alpha)wc}{(n-\alpha)wc}$
3	**BPF**	$hd[n] = \dfrac{w_2}{\pi} \dfrac{\sin(n-\alpha)w_2}{(n-\alpha)w_2} - \dfrac{w_1}{\pi} \dfrac{\sin(n-\alpha)w_1}{(n-\alpha)w_1}$
4	**BRF / BSF**	$hd[n] = \dfrac{\sin(n-\alpha)\pi}{(n-\alpha)\pi} + \dfrac{w_1}{\pi} \dfrac{\sin(n-\alpha)w_1}{(n-\alpha)w_1} - \dfrac{w_2}{\pi} \dfrac{\sin(n-\alpha)w_2}{(n-\alpha)w_2}$

Q(11) Show that FIR filter can also be realized using IIR filters.
(i.e. **FREQUENCY SAMPLING REALIZATION of FIR filter**)

ANS :

By ZT, $$H(z) = \sum_{n=0}^{N-1} h[n]\ z^{-n}$$

$$H(z) = \sum_{n=0}^{N-1} \left\{ \frac{1}{N} \sum_{k=0}^{N-1} H[k]\ W_N^{-nk} \right\} Z^{-n}$$

$$H(z) = \sum_{k=0}^{N-1} H[k] \left\{ \frac{1}{N} \sum_{n=0}^{N-1} W_N^{-nk}\ Z^{-n} \right\}$$

$$H(z) = \sum_{k=0}^{N-1} H[k] \left\{ \frac{1}{N} \sum_{n=0}^{N-1} \left(W_N^{-k}\ Z^{-1} \right)^n \right\}$$

$$H(z) = \sum_{k=0}^{N-1} H[k].\ \frac{1}{N}.\ \sum_{n=0}^{N-1} \left(e^{\frac{j2\pi k}{N}}\ Z^{-1} \right)^n$$

$$H(z) = \frac{1}{N} \sum_{k=0}^{N-1} H[k]\ .\ \frac{1 - \left(e^{\frac{j2\pi k}{N}}\ Z^{-1} \right)^N}{1 - e^{\frac{j2\pi k}{N}}\ Z^{-1}}$$

$$H(z) = \frac{1}{N} \sum_{k=0}^{N-1} H[k]\ .\ \frac{1 - e^{j2\pi k}\ Z^{-N}}{1 - e^{\frac{j2\pi k}{N}}\ Z^{-1}}$$

But $e^{j2\pi k} = 1$

$$H(z) = \frac{1}{N} \left(1 - z^{-N} \right) \sum_{k=0}^{N-1} \frac{H[k]}{1 - e^{\frac{j2\pi k}{N}}\ z^{-1}}$$

Let $$H(z) = \frac{1}{N}\ H_1(z) \cdot H_2(z)$$

Where $H_1(z) = (1 - z^{-N})$ ← FIR Filter

And $H_2(z) = \sum_{k=0}^{N-1} \frac{H[k]}{1 - e^{\frac{j2\pi k}{N}}\ z^{-1}}$ ← Parallel Bank of IIR Filters.

Frequency sampling realization is realization of FIR (i.e. Non recursive) filter using IIR (i.e. Recursive) filters.

Q(4) A stable, causal FIR filter has its poles lying anywhere inside the unit circle in the z plane : State True Or False. Justify your answer.

ANS : FALSE

In case of causal FIR Filter, poles are always only at origin.i.e. $|\text{pole}| = 0$.

For causal and stable filter all the poles must lie inside the unit circle.

Therefore, FIR filters are always stable filter with poles only at origin.

Q(5) State true or false and justify your Answer. If a linear phase filter having Anti symmetric even number of coefficients, then the filter acts like a band pass filter only.

ANs : FALSE

Anti-symm h[n] with N even has definite zero at z = 1. i.e. w = 0 $H(z)\Big|_{\substack{z=1\\ w=0}} = 0$

That means low frequency components will get attenuated and zero frequency components will not get passed. However, the filter can pass high frequency components, therefore it can be used for HPF design also.

Q(6) $H_1(z)$ and $H_2(z)$ both have zeros at (+ 0.5) and (– 0.2). However $H_1(z)$ has both the poles at origin; whereas $H_2(z)$ has only one pole and it is situated at origin. Both the systems are causal FIR systems. State True or False and Justify your answer.

ANS : FALSE

$$H_1(z) = \frac{(z-0.5)(z+0.2)}{z^2} == \frac{z^2+1.5\,z-1}{z^2} = 1+1.5\,z^{-1} + z^{-2}$$

By iZT, $h_1[n] = \{1,\ 1.5,\ 1\}$ for $n \geq 0$ Causal Finite Length

$$H_2(z) = \frac{(z-0.5)(z+0.2)}{z} = \frac{z^2+1.5\,z-1}{z} = z+1.5 + z^{-1}$$

By iZT,

$h_2[n] = \{1,\ 1.5,\ 1\}$ for $-1 \geq n \geq -1$

$h_2[n]$ is Both sided Finite Length

Q(12) Give the procedure to find FIR filter coefficients using Optimum Method.

ANS :

Step-1: Accept Pass-band (Fp) and Stop-band frequencies (Fs), Pass-band ripple (δ_p)and Stop-band Attenuation(δs) and Sampling Frequency F.

Step-2 : Normalize each edge frequency by dividing it by sampling frequency and determine normalized transition width Δf.

Step-3 : Calculate Filter Length N

$$N = \frac{Dm\,(\delta_p, \delta_s)}{\Delta f} - f\,(\delta_p, \delta_s)\Delta f\ + 1$$

Where $f(\delta p, \delta s) = 11.0217 + 0.5\,12\,44\ [\log_{10} \delta p, -\log_{10} \delta s\ ]$

$a_1 = 5.309 \times 10^{-3}$ $a_2 = 7.114 \times 10^{-2}$

$a_1 = -4.761 \times 10^{-1}$ $a_4 = -2.66 \times 10^{-3}$

$a_1 = -5.941 \times 10^{-1}$ $a_6 = -4.278 \times 10^{-1}$

Step-4 : Obtain the weights for each band from the ration of pass band to stop-band ripple.

Step-5 : Input the parameters to the optimum design program to obtain the coefficients.

For a given set of specifications i.e. fp, fs, N, ratio between pass-band and Stop-band ripple, the optimum method involves the following key steps :

I. Use the Remez exchange algorithm to find the optimum set of extremal frequencies.

II. Determine the frequency Response using extremal frequencies.

III. Obtain the impulse response Coefficients.

Step-6 : Check the pass-band ripple and stop-band attenuation produced by the program.

Step-3 : If the specifications are not satisfied, increase the value of N and repeat the procedure 5 and 6. Obtain the frequency response to ensure that it satisfies the specifications.

► Always Remember This……..

[A] For Linear Phase filter h[n] must be either Symmetric or Antisymmetric.

Examples of Linear phase filters	Examples of non Linear phase filters
h[n] = { 3, 2, 1, 2, 3 }	h[n] = { 1, 2, 3, 1, 2, 3 }
h[n] = { 1, 2, 2, 1 }	h[n] = { 3, 2, 1, 2, 3 }
h[n] = { 1, -2, 0, 2, -1 }	h[n] = { 3, 2, 1, -2, -3 }
h[n] = δ[n] + δ[n-3]	h[n] = { 3, -2, 2, 3 }

[B] When h[n] is either Symmetric OR Antisymmetric, **ZEROS** of the filter are always in **Reciprocal order**.

i.e. If Z_1 is ZERO of the filter, Then $\frac{1}{z_1}$ is also a ZERO of the filter.

[C] If ZEROS of the filter are in reciprocal order, then filter is Linear Phase FIR filter.

[D] For linear Phase FIR filter.

a) ZEROS are always in reciprocal order (ie linear Phase)
b) POLES are always only at origin (ie FIR)

ex h[n] = { 1, –2.5, 1} $H(z) = \dfrac{\left(z-\frac{1}{2}\right)(z-2)}{Z^2}$

h [n] = { 1, 0, -1} $H(z) = \dfrac{(z+1)\,(z-1)}{Z^2}$

h [n] = {1, -1.5, -1.5, 1} $H(z) = \dfrac{(z+1)\left(Z-\frac{1}{2}\right)(z-2)}{Z^3}$

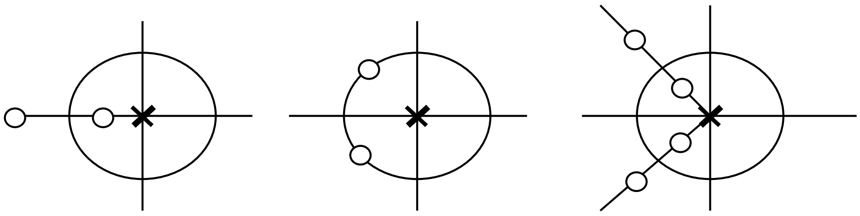
[E] When zeros of the filter are INSIDE the unit circle filter is called Minimum Phase Filter.
Concept : For Minimum Phase filter $\phi(\pi) - \phi(0) = 0$

[F] When all zeros of the filter are OUTSIDE the unit circle filter is called maximum phase filter.
Concept : For Maximum Phase filter $\phi(\pi) - \phi(0) = \pm\, m\, \pi$

[G] When System is Neither Minimum Phase Nor Maximum Phase, Then System is Mixed Phase System.

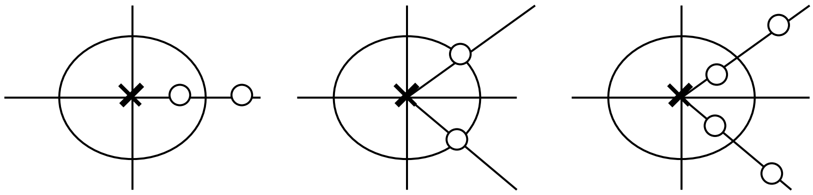
[H] When all zeros of the FIR filter are LEFT side of POLES, filter is LOW PASS FIR FILTER.

eg. FIR Filter



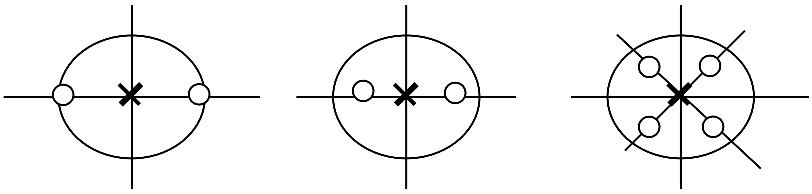
[I] When all zeros of the FIR filter are RIGHT side of ZEROS, filter is HIGH PASS FIR FILTER.

eg. FIR Filter



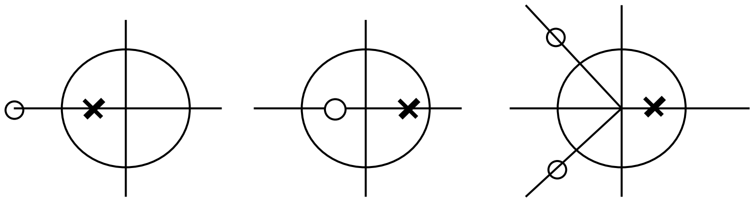
[J] When zeros of the FIR filter are Both sides of POLES, Then filter is BAND PASS FIR FILTER.

eg. FIR Filter



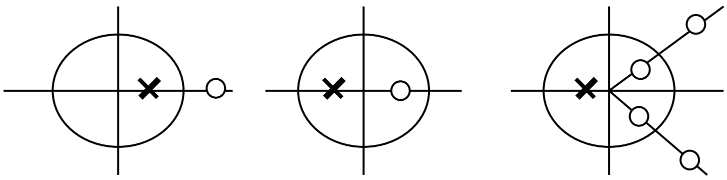
[K] When All ZEROS are on Left side of POLES , then filter is LPF

eg. IIR Filter



[L] When All ZEROS are on Right side of POLES , then filter is HPF

eg. IIR Filter



[M] When ZEROS of the filter are outer sides of POLES, then filter is BPF

e.g. (i) $H(z) = \dfrac{(z+1)\,(z-1)}{(z-0.5)(z+0.5)}$

[N] When POLES and ZEROS of the filter are in reciprocal order, filter is ALL PASS FILTER.

Eg. IIR filter , $H\,(z) = \dfrac{Z-2}{Z-0.5}$ POLE $P_1 = 0.5$ ZERO $Z_1 = 2$

Here, ZERO = $1/\text{POLE}$ $\therefore$ **Filter is All Pass IIR Filter.**

[O] When Numerator Coefficients and Denominator coefficients of H(z) are in **Reverse Order** Filter is ALL PASS FILTER.

Eg. $H(z) = \dfrac{3+2z^{-1}+z^{-2}}{1+2z^{-1}+3z^{-2}}$ (IIR)

Numerator Coefficients : [3, 2, 1]
Denominator Coefficients : [1, 2, 3]

$\therefore$ **Filter is All Pass IIR Filter.**

➢ Frequently Asked Questions

(1) What is Finite Impulse Response (FIR) filter ?

Ans : If the impulse response of the system s of finite duration then the system is said to be FIR system.

Ex : $h_1[n] = \{1,2,3,4\}$ Length = 4 (finite)

$$h_2[n] = \begin{Bmatrix} \uparrow \\ 1,2,3,2,1 \\ \end{Bmatrix} \quad \text{Length} = 5 \text{ (fnite)}$$

$$\uparrow$$

(2) What are Advantages of FIR Filters-?

Ans:

1) They can easily be designed to be "linear phase"
2) They are suited to multi-rate applications.
3) They have desirable numeric properties.
4) They can be implemented using fractional arithmetic.
5) They are simple to implement.

(3) What are the *disadvantages* of FIR Filters (compared to IIR filters)?

Ans : Compared to IIR filters, FIR filters sometimes have the disadvantage that they require more memory and/or calculation to achieve a given filter response characteristic.

(4) Compare FIR filters and IIR filters

	FIR filter	**IIR filter**
1	Provides exact linear phase.	Not linear phase.
2	Provides good stability.	Stability is not guaranteed.
3	Order required is higher.	Order required is lower.
4	Computationally not efficient.	Computationally more efficient.
5	More memory required for the storage of coefficients.	Less memory required fro storage of coefficients.
6	Requires more processing time.	Requires less processing time.
7	Requires N multiplications per output sample	Requires 2N + 1 multiplications per output sample.

(5) What is a *linear phase* filter?

Ans : "Linear Phase" refers to the condition where the phase response of the filter is a linear (straight-line) function of frequency.

(6) Explain the concept of Linear Phase and its importance.

Ans : **I**. If the Phase Response is Linear the output of the Filter during pass-band is delayed input.
II. If the phase Response is non Linear the output of the filter during pass-band is distorted one

The linear Phase characteristic is important when the phase distortion is not tolerable.
FIR Filter can be designed with linear phase characteristic. In application like data transmission, speech processing etc phase distortion can not be tolerated and here linear phase characteristic of FIR filter is useful.

(7) What is the advantage of Linear Phase ?

Ans : This results in the *delay* through the filter being the same at all frequencies. Therefore, the filter does not cause "phase distortion" or "delay distortion".

(8) Show that if the Phase Response is Linear the output of the Filter during pass-band is delayed input.

⇨

Consider a LPF with frequency response $H(e^{-jw\alpha})$ given by

$$H(e^{jw}) = \begin{cases} e^{-jw\alpha} & |w| \le w_c \\ 0 & wc < |w| \le \pi \end{cases}$$

$$\frac{x[n]}{X(w)} \longrightarrow \boxed{H(e^{jw})} \longrightarrow \frac{y[n]}{Y(w)}$$

Let X(w) = DTFT { X[n] } ,

The FT of y[n] is then given by

Y(w) =X(w) . H(w)

$Y(w) = X(w) \cdot e^{-jw\alpha}$

By iDTFT, **$y[n] = x[n - \alpha]$** ← o/p of filter

(9) What is the role of window in the design of FIR filter ? Name the few types of windows.

Ans : FIR filter is designed by truncating infinite samples of hd[n] by using window function. Examples of window function include, Hamming window, Bartlet Window, Hanning window, Blackman window etc.

(10) Why rectangular window is not preferred for FIR filter design ?

Ans : Rectangular window function has As = 21 db which is very small compared to other window function. Larger value of As desired.

(11) Is the following filter a linear phase filter. If yes, what is the type of filter ? It's transfer function is given by $H(z) = 1 - z^{-4}$.

Ans : By IZT h[n] = { 1, 0, 0, 0, –1 } Since h[n] is anti-symmetric, filter is a linear phase FIR filter. Antisymmetric h[n] with N odd is suitable only for Band Pass Filter.

(i) At w = 0, z = 1 : H(w) = 0

(ii) At w = π, z = – 1: H(w) = 0

(iii) At w = π/2, z = j: H(w) = 2

(12) Why anti-symmetric h[n] is not suitable for LPF filter design ?

(13) Why symmetric h[n] with N even and anti-symmetric h[n] with N odd is not suitable for HPF design ?:

(14) Explain Linear phase FIR filter design using window.

Ans : In windowing method, FIR filter with impulse response h[n] is obtained by truncating infinite samples of desired impulse response hd[n] using window function.

(15) Explain frequency Sampling method of FIR filter design ?

Ans :

In frequency sampling method, a desired frequency response is sampled at $w = \dfrac{2\pi k}{N}$ *For* $k = 0, 1, 2, \ldots\ldots N-1$.

i.e. $H[k] = H_d(w)\Big|_{w = \frac{2\pi k}{N}}$ *For* $k = 0, 1, 2, \ldots\ldots N-1$

The samples thus obtained are identified as DFT coefficients.

The filter coefficients are then obtained by taking IDFT of this set of samples.

The filter coefficients are given by $h[n] = \dfrac{1}{N} \sum_{k=0}^{N-1} H[k]\, W_N^{-nk}$

(16) What is the advantage of Frequency Sampling Realization ?

Ans : The frequency sampling realization of filter is computationally more efficient than the direct form realization.

Justification : When the desired frequency response characterization of the FIR filter is narrowband, most of the coefficients H[k] are zero. The corresponding filter sections can be eliminated and only the filters with non zero coefficients need to be retained. The net result is a filter that requires fewer computations (multiplications and additions) than the corresponding direct form realization. Thus frequency Sampling realization is more efficient realization.

(17) Why IIR filters are called as recursive filters ?

Ans : In IIR filter output dépends on output values.
e.g. y[n] = x[n] + x[n-1] + y[n] + y[n-1].

Therefore **IIR filters** are also called as **Recursive** Filters

(18) Why FIR filters are called as Non-recursive filters?

Ans : In FIR filter output depends only on input values. It doesn't depend on output values.
e.g. y[n] = x[n] + x[n-1]

Therefore **FIR filters** are also called as **Non-Recursive** Filters.

(19) Explain how to find output of digital FIR filter in real time application.

Ans : In real time applications, output of FIR filter is obtained using overlap add method / overlap save method.

(20) Explain how to find output of digital IIR filter in real time application.

Ans : In real time applications, output of IIR filter can be obtained by evaluating difference equation.

(21) Can we use Overlap Add Method and Overlap Save Method to find output of IIR filter for long data sequence.

Ans : No.

(22) What is Phase Delay and Group Delay ?

Ans : The phase delay and group delay. The phase delay (T_p) and group delay (Tg) of the filter are given by,

$$Tp = \frac{-\varphi}{w} \quad and \quad Tg = \frac{-d\varphi}{dw}$$

The group delay Tg, is defined as the delayed response of the filter as a function of w to the signal. Linear Phase Filters are those Filters in which the phase delay and group delay are constant, ie independent of frequency. Linear Phase Filters are also called as constant time delay Filters.

(23) How to choose IIR filter and FIR filter ?

Ans : Use IIR filter when only requirements are sharp cutoff filters and high throughputs. IIR filters especially chebyshev filters gives fewer coefficients than FIR.

Use FIR filter if number of filter coefficients is not too large and if little or No phase distortion is desired.

(24) Why IIR filter cannot have a linear phase :

Ans : The physically realizable and stable IIR filter cannot have a linear phase. For a filter to have a linear phase, the condition is h (n) = h (N-1-n) and the filter would have a mirror image pole outside the unit circle for every pole inside the unit circle. This results in an unstable filter. As a result, a causal and stable IIR filter cannot have a linear phase.

(25) What are the desirable characteristics of window Function ?

Ans : (i) The Fourier Transform of the window function $W(e^{jw})$ should have a small width of main lobe containing as much of the total energy.

(ii) The Fourier Transition of the window function $W(e^{jw})$ should have side lobes that decrease in energy rapidly as w tends to □□.