.

Q(1) An analog domain filter has a transfer function $H(s) = \dfrac{b}{(s+a)^2 + b^2}$ filter is to be converted into digital filter so that its impulse response characteristics are retained. The sampling frequency is 100 Hz. Find the transfer function H(z).

Solution : By Impulse Invariant Method,

(i) Find h(t) by ILT

Given $H(s) = \dfrac{b}{(s+a)^2 + b^2}$

By ILT, $h(t) = e^{-at}\ \mathrm{Sin}\ (bt)\ \ u(t)$

(ii) Find h[n] by Sampling

Put t=nT

$$h(nT) = e^{-anT}\ \mathrm{Sin}\ (bnT)\ \ u(nT)$$

$$h(n) = \left(e^{-aT}\right)^n\ \mathrm{Sin}\ (nbT)\ \ u(n)$$

(iii) Find H(z) by ZT

$$H(z) = \frac{r\,z\sin(w)}{z^2 - 2\,r\,z\cos(w) + r^2} \quad \textbf{where} \quad r = \left(e^{-aT}\right) \quad and \quad w = bT$$

$$H(z) = \frac{\left(e^{-aT}\right)z\sin(bT)}{z^2 - 2\left(e^{-aT}\right)z\cos(bT) + e^{-2aT}} \quad \textbf{ANS}$$

Q(2) Determine H(z) by using Impulse Invariance technique for the analog system function

$$H(s) = \frac{1}{(S+0.5)\ (S^2 + 0.5S + 2)} \quad \textbf{(IMP)}$$

Solution : **By Impulse Invariance,** $H(s) \to h(t) \to h(n) \to H(z)$

(i) $H(s) = \dfrac{1}{(S+0.5)\ (S^2 + 0.5\ S + 2)}$

By Partial Fraction Expansion Method,

$$H(s) = \frac{A}{S+0.5} + \frac{BS+C}{S^2+0.5\,S+2} \quad \text{Where} \quad \mathbf{A} = \mathbf{0.5}$$

$$H(s) = \frac{0.5}{S+0.5} + \frac{BS+C}{S^2+0.5\;S+2}$$

$$\frac{1}{(S+0.5)\;(S^2+0.5\;S+2)} = \frac{0.5\;(S^2+0.5\,S+2)\,+\,(S+0.5)\;(BS+C)}{(S+0.5)\;(S^2+0.5\;S+2)}$$

$$\frac{1}{(S+0.5)\;(S^2+0.5\;S+2)} == \frac{0.5\;S^2+0.25\;\;S+1+BS^2+SC+0.5\;\;BS+0.5\;C}{(S+0.5)\;\;(S^2+0.5\;\;\;S+2)}$$

Equating Numerators

$1 = S^2\,(0.5+B) + S\,(0.25 + C + 0.5B) + (1 + 0.5C)$

$\therefore\; 0.5 + B = 0 \quad 1 + 0.5\,C = 1$

$\boxed{\mathbf{B = -\,0.5}}$ $\boxed{\mathbf{C = 0}}$

By substituting, $H(s) = \dfrac{0.5}{S+0.5} + \dfrac{-0.5\;S}{S^2+0.5\;S+2}$

$$H(s) \;= \frac{0.5}{S+0.5} + \frac{-0.5\,[(S+0.25)-0.25]}{(S+0.0.25)^2+(1.3919)^2}$$

$$H(s) = \frac{0.5}{S+0.5} - 0.5\left[\frac{S+0.25}{(S+0.25)^2+(1.3919)^2}\right] + 0.0898\left[\frac{1.3919}{(S+0.25)^2+(1.3919)^2}\right]$$

By I.L.T.,

$$h(t) = 0.5\;e^{-0.5t}\;u(t) - 0.5\,e^{-0.25t}\cos(1.3919t)\;u(t) + 0.0898\,e^{-0.25t}\sin(1.3919t)\;u(t)$$

(ii) Put t = nT Let T = 1 sec $\therefore$ t = n

$$h(n) = 0.5\;e^{-0.5n}\;\;u(n) - 0.5\;e^{-0.25n}\cos\,(1.3919\;n)\;u(n) + 0.0898\;e^{-0.25}\sin(1.3919\;n)\;u(n)$$

(iii) By ZT,

$$H(z) = \frac{0.5\,z}{z-e^{-0.5}} - 0.5\left[\frac{z^2-e^{-0.25}z\cos(1.3919)}{z^2-2(e^{-0.25})\,z\cos(1.3919)+e^{-0.5}}\right]$$

$$+\,0.0898\left[\frac{e^{-0.25}z\sin(1.3919)}{z^2-2(e^{-0.25})\,z\cos(1.3919)+e^{-0.5}}\right]$$

Put $\quad e^{-0.5} = 0.6065 \quad \cos(1.3919) = 0.1779$
$e^{-0.25} = 0.7788 \quad \sin\,(1.3919) = 0.9840$

ANS : $H(z) = \dfrac{0.5\,z}{Z-0.6065} - 0.5\dfrac{z^2+0.1385\,z}{z^2-0.2770\,z+0.6065} + 0.0898\left[\dfrac{0.7614\;z}{Z^2-0.27707+0.6065}\right]$

Q(3) Convert the analog filter with system function $H(s) = \dfrac{s + 0.1}{(s+0.1)^2 + 16}$ into a digital IIR filter by means of a bilinear transformation. The digital filter is to resonate at frequency $w_r = \pi/2$.

ANS : Now, $H(s) = \dfrac{s + 0.1}{(s+0.1)^2 + 16}$

Let $H(s) = \dfrac{s+a}{(s+a)^2 + b^2}$ where b is analog resonant frequency.

By comparing we get, analog resonant frequency $b = 4$. **Let $\Omega r = b = 4$**

Digital resonant frequency $w_r = \pi/2$

Now, $\Omega r = \dfrac{2}{T} \tan\left(\dfrac{w_r}{2}\right)$

$4 = \dfrac{2}{T} \tan\left(\dfrac{\pi/2}{2}\right)$ By solving we get $T = 0.5$ sec

By BLT, Digital Filter is given by, $H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{(z-1)}{(z+1)}}$ Put $T = 0.5$ sec.

$$H(z) = \frac{0.128 + 0.006\, z^{-1} - 0.122\, z^{-2}}{1 + 0.0006\, z^{-1} - 0.975\, z^{-2}}$$ **ANS**

Q(4) A Digital IIR Low-Pass Filter required to meet the following specifications:

Passband ripple	: $\leq$ 0.5 dB	Stopband attenuation	: $\leq$ 40 dB
Passband edge	: = 1.2 kHz	Stopband edge	: = 2.0 kHz
Sample rate	: = 8.0 kHz		

Determine the required filter order for (a) A Digital Butterworth Filter
(b) A Digital Chebyshev Filter

Solution :

$A_p = 0.5$ dB $A_s = 40$ dB $w_p = 0.3\,\pi$ $w_p = 0.5\,\pi$ **L P F**

(a) A Digital Butterworth Filter

$$N = \frac{\log\left[\dfrac{10^{As/10} - 1}{10^{Ap/10} - 1}\right]^{1/2}}{\log\left[\dfrac{\Omega s}{\Omega p}\right]}$$

N = 8.38 $\cong$ **9**

(b) A digital Chebyshev filter

$$N = \frac{\cosh^{-1}\left[\dfrac{10^{As/10} - 1}{10^{Ap/10} - 1}\right]^{1/2}}{\cosh^{-1}\left[\dfrac{\Omega s}{\Omega p}\right]}$$

N = 4.09 $\cong$ **5**

Q(5) Given $H(s)=\dfrac{1}{S^2+S+1}$ describes the transfer function of a LPF with a passband of 1 rad/sec. Using Frequency transformations find the transfer function of the following filters.

(a) A LPF with passband freq = 10 rad/sec

Solution : $$H_{LPF}(s)=\hat{H}(s)\Big|_{S=\frac{S}{10}}$$
$$=\frac{100}{S^2+10S+100}$$

(b) A HPF with a cutoff freq of 10 rad/sec

Solution : $$\hat{H}_{HPF}(S)=\overline{\hat{H}}_{LPF}(S)\Big|_{S=\frac{1}{S}}=\frac{S^2}{S^2+S+1}$$
$$\hat{H}_{HPF}(S)=\overline{\hat{H}}_{HPF}(S)\Big|_{S=\frac{S}{10}}$$
$$H_{HPF}(S)=\frac{\left[{}^{S}/_{10}\right]^2}{\left[{}^{S}/_{10}\right]^2+\left[{}^{S}/_{10}\right]+1}=\frac{S^2}{S^2+10S+100}$$

(c) A BPF with a pass band of 10 rad/sec and a centre freq of 100 rad/sec

Solution : $$H_{BPF}(s)=H_{LPF}(s)\Big|_{S=\frac{S^2+\Omega_o^2}{SB}}$$ where $B=10$ and $\Omega_o=100$
$$H_{BPF}(s)=\frac{1}{S^2+S+1}\Big|_{S=\frac{S^2+10000}{10\,S}=\frac{S^2+10^4}{10\,S}}$$
$$=\frac{1}{\left[\dfrac{S^2+10^4}{10S}\right]^2+\left[\dfrac{S^2+10^4}{10S}\right]+1}$$
$$H_{BPF}(s)=\frac{100\,S^2}{S^4+10\,S^3+20100S^2+10^5\,S+10^8}$$

(d) A Band Stop filter with a stopband of 2 rads/sec and a centre freq of 10 rads/sec

Solution :
$$H_{BSF}(s)=\hat{H}_{LPF}(s)\Big|_{S=\frac{SB}{S^2+\Omega_o^2}}$$ where $B=2$ and $\Omega_o=10$
$$H_{BSF}(s)=\frac{1}{S^2+S+1}\Big|_{S=\frac{10\,S}{S^2+10000}=\frac{10\,S}{S^2+10^2}}$$

Q(6) A Digital Butterworth is required to meet the following specifications

Pass band ripple	$\le$	1 dB
Stop band attenuation	$\ge$	40 dB
Pass band edge	=	4 KHz
Stop band edge	=	6 KHz
Sampling rate	=	24 KHz

Find the filter order and cutoff freq. if
(a) Impulse Invariant Method is used
(b) BLT technique is used.

Solution :

Solution (a) Impulse Invariant Method

(1) Calculate Ωp, Ωs

In IIM, $w = \dfrac{\Omega}{Fs}$ $\quad \therefore \Omega = Fs\ w$

$$\therefore \Omega = 24000\ w$$

(i) $\Omega p = 24000\ wp == 25132.74$ rad/sec

(ii) $\Omega s = 24000\ \text{ws} == 37699.11$ rad/sec

(2) Calculate Filter order N

$$N = \frac{\log \left[\dfrac{10^{As/10} - 1}{10^{Ap/10} - 1} \right]^{1/2}}{\log \left[\dfrac{\Omega s}{\Omega p} \right]} = 13.02 \qquad \text{Let } \mathbf{N == 14}$$

(3) Calculate Analog filter Ωc

$$\Omega c = \frac{\Omega_p}{\left(10^{Ap/10} - 1\right)^{\frac{1}{2N}}} = \mathbf{26375.32} \text{ rad/sec}$$

(4) Calculate Digital filter Wc :

$$Wc = \frac{\Omega c}{Fs} = \frac{26375.32}{24000.}$$

W_c = **1.09 radian**

Solution (b) BLT Method

(1) Calculate $\Omega p, \Omega s$

$$\Omega p = \frac{2}{T}\tan\left(\frac{w_p}{2}\right)$$

$$\Omega p = \frac{2}{1/24000}\tan\left(\frac{\pi/3}{2}\right)$$

$$\Omega p = 27712.8$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{w_s}{2}\right)$$

$$\Omega_s = \frac{2}{1/24000}\tan\left(\frac{\pi/2}{2}\right)$$

$$\Omega s = 48000$$

(2) Calculate Filter order N

$$N = \frac{\log\left[\dfrac{10^{As/10}-1}{10^{Ap/10}-1}\right]^{1/2}}{\log\left[\dfrac{\Omega s}{\Omega p}\right]} = 9.61 \qquad \text{Let } \mathbf{N == 10}$$

(3) Calculate Analog filter Ωc

$$\Omega c = \frac{\Omega_p}{\left(10^{Ap/10}-1\right)^{\frac{1}{2N}}} = \mathbf{29649.7} \ \text{rad/sec}$$

(4) Calculate Digital filter Wc

$$\Omega_C = \frac{2}{T}\tan\left(\frac{\omega_C}{2}\right)$$

$$\frac{\Omega_C T}{2} = \tan\left(\frac{\omega_C}{2}\right)$$

$$Wc = 2\tan^{-1}\left(\frac{\Omega_C T}{2}\right)$$

Wc = **1.106** radian

Q(7) Design a first order high pass DT Butterworth filter whose cutoff frequency is 1 kHz at the sampling rate of 10^4 sample/sec.

HINT :

1. Transfer function for 1st order H.P Filter : $H(s) = \dfrac{s}{s+1}$
2. Digital Cut-off frequency $\omega_C = 2\pi f_C = 0.2\pi$ rad
3. Prewarp frequency
$$\Omega_C = \frac{2}{T_s}\tan\left(\frac{\omega_C}{2}\right) = \frac{2}{T_s}\tan\left(\frac{0.2\pi}{2}\right) = 6498.39 \ \text{rad/sec}$$
4. By Denormalization $H_{HPF}(s) = H_{LPF}(s)\Big|_{s=\frac{S}{\Omega_C}=\frac{S}{6498.39}}$
5. By BLT transformation, $H(z) = H(s)\Big|_{s=\frac{2(z-1)}{T(z+1)}}$

Q(8) Design and realize a Low Pass Filter using the Bilinear Transformation Method to satisfy the following characteristics.

(i) Monotonic Stop Band and Pass Band.
(ii) −3 dB cutoff frequency of 0.5π
(iii) Stop Band Attenuation of 15 dB at 0.65π

Solution :

STEP-1 Design Analog Butterworth LPF Filter.

(1) Calculate Ωp Ωs

$$\Omega p = \frac{2}{T}\tan\left(\frac{wp}{2}\right) = 2\ \tan\left(\frac{0\cdot 5\pi}{2}\right) = 2\ \text{rad /sec}$$

$$\Omega s = \frac{2}{T}\tan\left(\frac{ws}{2}\right) = 2\ \tan\left(\frac{0\cdot 75\pi}{2}\right) = 4\cdot 828\ \text{rad /sec}$$

(2) Calculate filter order N

$$\underset{LPF}{N} = \frac{\log\left[\frac{10^{As/10}-1}{10^{AP/10}-1}\right]^{\frac{1}{2}}}{\log\left[\frac{\Omega s}{\Omega p}\right]} == 2$$

(3) Calculate Normalized LPF

LPF **N = 2** $\Omega c = 1$ rad/sec

------ Calculation of H(s) from POLES ----------

$$\hat{H}(s) = \frac{1}{s^2 + \sqrt{2}\ s + 1}$$

(4) Calculate Denormalized LPF

$$H(s) = \hat{H}(s)\Big|_{S=\frac{S}{\Omega_C}} \quad \text{where} \quad \Omega_c = \frac{\Omega_p}{\left(10^{Ap/10}-1\right)^{\frac{1}{2N}}} = 2\ \text{rad/sec}$$

$$H(s) = \frac{1}{s^2+\sqrt{2}\ s+1}\Bigg|_{S=\frac{S}{2}}$$

$$\therefore\ \hat{H}(s) == \frac{4}{s^2+2\sqrt{2}s+4}$$

STEP – 2 Design Digital Butterworth L P F

By BLT,

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{(z-1)}{(z+1)}} \qquad \textbf{Put } T = 1 \text{ sec}$$

$$H(z) = \frac{4}{\left[\frac{2(z-1)}{(z+1)}\right]^2 + 2\sqrt{2}\left[\frac{2(z-1)}{z+1}\right] + 4}$$

$$H(z) = \frac{4}{\left[\frac{2(z-1)}{(z+1)}\right]^2 + 2\sqrt{2}\left[\frac{2(z-1)}{z+1}\right] + 4}$$

$$H(z) = \frac{4(z+1)^2}{4(z-1)^2 + 4\sqrt{2}\,(z-1)(z+1) + 4(z+1)^2}$$

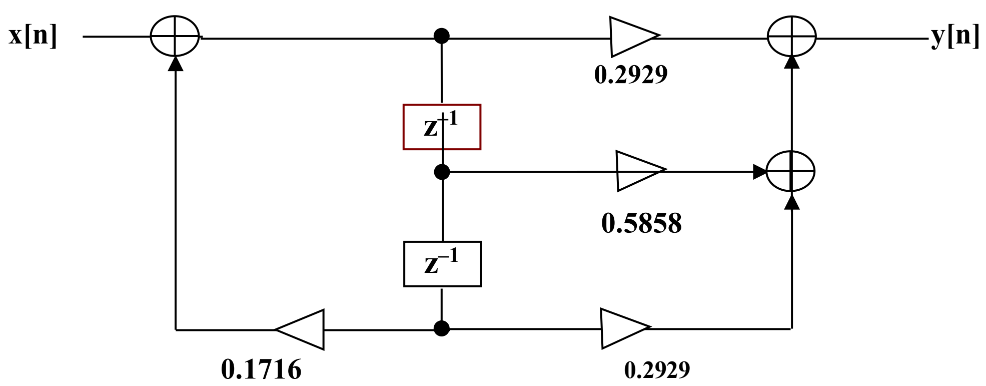
$$H(z) = \frac{4(z^2+2z+1)}{4(z^2-2z+1) + 4\sqrt{2}\,(z^2-1) + 4(z^2+2z+1)}$$

$$H(z) = \frac{(z^2+2z+1)}{(z^2-2z+1) + 1.414(z^2-1) + (z^2+2z+1)}$$

$$H(z) = \frac{z^2+2z+1}{3.414\,z^2 - 0.586}$$

$$H(z) = \frac{0\cdot 2929 + 0\cdot 5858\,z^{-1} + 0\cdot 2929\,z^{-2}}{1 - 0\cdot 1716\,z^{-2}}$$

Butterworth Filter Realization Diagram :



Q(9) Design a Digital Butterworth filter that satisfies the following constraint using bilinear transformation. Assume T = 1s.

$$0.9 \le |H(e^{jw})| \le 1 \; ; \quad 0 \le w \; \frac{\pi}{2}$$

$$|H(e^{jw})| \le 0.2 \quad ; \quad \frac{3\pi}{4} \le w \le \pi.$$

Solution :

(i) Ap = 0.9151 (ii) As = 13.97 LPF

(iii) wp = **π/2** (iv) ws = **3 π/4** (v) Fs = 1 Hz

STEP – 1 Design Analog Butterworth L P F

(1) Calculate Ω_p, Ω_s

i) $\Omega_p = \frac{2}{T} \tan\left(\frac{w_p}{2}\right)$ Put T = 1 $\Omega_p = 2$ rad/sec	ii) $\Omega_s = \frac{2}{T} \tan\left(\frac{w_s}{2}\right)$ Put T = 1 $\Omega_s = 4.828$ rad/sec

(4) Calculate filter order N

$$N = \frac{\log \left[\frac{10^{As/10} - 1}{10^{Ap/10} - 1}\right]^{1/2}}{\log \left[\frac{\Omega s}{\Omega p}\right]} = 1.966 \qquad \text{Let } \mathbf{N = 2}$$ NOT Correct *****

(5) Calculate Normalized LPF

LPF **N = 2** Ωc = 1 rad/sec

POLES : $S_K = \Omega c \, e^{j\pi\left(\frac{N+1+2k}{2N}\right)}$

$$S_K = e^{j\pi\left(\frac{3+2k}{4}\right)}$$

----

k=0, $S_0 = e^{j\frac{3\pi}{4}}$

k=1, $S_1 = e^{-j\frac{3\pi}{4}}$

Now, $H(s) = \frac{1}{(s-s_0)(s-s_1)}$ $\qquad \hat{H}(s) = \frac{1}{s^2 + \sqrt{2}\, s + 1}$

---

(6) Calculate De-normalized LPF

$$H(s) = \hat{H}(s)\Big|_{S=\frac{S}{\Omega_c}} \quad \text{where} \quad \Omega_c = \frac{\Omega_p}{\left(10^{Ap/10} - 1\right)^{\frac{1}{2N}}} = 2.8738 \ \text{rad/sec}$$

$$H(s) = \frac{1}{s^2 + \sqrt{2}\ s + 1}\Bigg|_{S=\frac{S}{2.8738}}$$

$$H(s) = \frac{1}{\left[\frac{s}{2.8738}\right]^2 + \sqrt{2}\left[\frac{s}{2.8738}\right] + 1}$$

$$H(s) = \frac{8.2592}{s^2 + 4.0641\ s + 8.2592}$$

STEP – 2 Design Digital Butterworth L P F

By BLT, Digital Filter is given by,

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{(z-1)}{(z+1)}} \qquad \textbf{Put}\ T = 1\ \text{sec}$$

$$H(z) = \frac{8.2592}{\left[\frac{2(z-1)}{(z+1)}\right]^2 + 4.0641\left[\frac{2(z-1)}{(z+1)}\right] + 8.2592}$$

$$H(z) = \frac{8.2592(z-1)^2}{2(z-1)^2 + 8.182(z-1) + 8.2592\ (z-1)^2}$$

$$H(z) = \frac{8.2592(z^2-2z+1)}{2(z^2-2z+1) + 8.182(z-1) + 8.2592(z^2-2z+1)}$$

$$H(z) = \frac{8.2592(z^2-2z+1)}{10.2592z^2 - 12.3364\ z + 2.0772}$$

$$H(z) = \frac{8.2592(z^2-2z+1)}{10.2592(\ z^2 - 1.2024\ z + 0.2024\ )}$$

$$H(z) = \frac{0.805(z^2-2z+1)}{z^2 - 1.2024\ z + 0.2024}$$

Q(10) An IIR Digital Low Pass Filter is required to meet the following specifications :

Passband ripple ≤ 0.5 dB Stopband attenuation ≥ 40

Passband edge : 1.2 KHz Stopband edge : 2.0 KHz

Find the filter order of Digital Chebychev filter. Take Fs = 8 KHz

Solution :

(i) Ap = 0.5 dB (ii) As = 40 dB (iii) Fs = 8 KHz

(iv) wp = 0.3π (v) ws = 0.5π (vi) LPF

(1) Calculate Ωp, Ωs

$$\Omega p = \frac{2}{T}\tan\left(\frac{wp}{2}\right) = \frac{2}{1/8000}\tan\left(\frac{0.3\pi}{2}\right) = \boxed{8152.4 \ \text{rad/sec}}$$

$$\Omega s = \frac{2}{T}\tan\left(\frac{ws}{2}\right) = \frac{2}{1/8000}\tan\left(\frac{0.5\pi}{2}\right) = \boxed{16000 \ \text{rad/sec}}$$

(2) Find Filter order N.

$$N = \frac{\cosh^{-1}\left[\dfrac{10^{As/10}-1}{10^{Ap/10}-1}\right]^{1/2}}{\cosh^{-1}\left[\dfrac{\Omega s}{\Omega p}\right]}$$

$$N = \frac{\cosh^{-1}\left[\dfrac{10^{4}-1}{10^{0.05}-1}\right]^{1/2}}{\cosh^{-1}\left[\dfrac{16000}{8152.4}\right]}$$

N = 4.903

Let N = $\boxed{5}$ ANS.

Q(11) Design a Digital Chebyshev filter that satisfies the following constraint using bilinear transformation. Assume T = 1s.

$$0.8 \leq |H(e^{jw})| \leq 1 \ ; \quad 0 \leq w \leq 0.2\pi$$

$$|H(e^{jw})| \leq 0.2 \ ; \quad 0.6\pi \leq w \leq \pi$$

Solution :

(i) Ap = 1.93 (ii) As = 13.97 (iii) Fs = 1 Hz

(iv) wp = 0.2π (v) ws = 0.6π (vi) LPF

Step–1 Design Analog Chebyshev L P F

(1) Calculate Ω_p, Ω_s

i) $\Omega_p = \frac{2}{T} \tan\left(\frac{w_p}{2}\right)$ Put T = 1 $\Omega_p = 0.6498$ rad/sec	ii) $\Omega_s = \frac{2}{T} \tan\left(\frac{w_s}{2}\right)$ Put T = 1 $\Omega_s = 2.752$ rad/sec

(2) Calculate filter order N

$$N = \frac{\operatorname{Cosh}^{-1}\left[\frac{10^{As/10}-1}{10^{Ap/10}-1}\right]^{1/2}}{\operatorname{Cosh}^{-1}\left[\frac{\Omega s}{\Omega p}\right]} = 1.208$$

Let **N = 2** (EVEN)

(3) Calculate Normalized Analog LPF

(i) Find Ɛ

$$\varepsilon = \sqrt{10^{Ap/10} - 1} == 0.75$$

(ii) Find u

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} == 3$$

(iii) Find a

$$a = \left(\frac{\mu^{\frac{1}{N}} - \mu^{\frac{-1}{N}}}{2}\right) == 0.3752$$

(iv) Find b

$$b = \left(\frac{\mu^{\frac{1}{N}} + \mu^{\frac{-1}{N}}}{2}\right) == 0.75$$

(v) Find ϕ_k

$$\Phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad for \;\; k = 1, 2, \ldots N$$

$$\Phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{4} \quad for \;\; k = 1, 2$$

$\Phi_1 = \frac{3\pi}{4}$ Cos(0.75π) = - 0.707 Sin(0.75π) = 0.707

(vi) Find S_k

$$S_k = a \cos \Phi_k + jb \sin \Phi_k$$

$S_1 = -0.2653 + j\,0.53$
$S_2 = -0.2653 - j\,0.53$

(vii) Find $Q_p(s)$

$Q_p(s) = (S-S_1)\ (S-S_2)$

$Q_p(s) = (S + 0.2653 - j\,0.53)\ (S + 0.2653 + j\,0.53)$

$Q_p(s) = (S + 0.2653)^2 + (0.53)^2$

$Q_p(s) = S^2 + 0.5306\ S + 0.3516$

(viii) Find K

For N Even, $K = \dfrac{Qp(0)}{\sqrt{1+\varepsilon^2}}$ == 0.2814

(ix) Find $\hat{H}(s)$

$$\hat{H}(s) = \frac{K}{Qp(s)}$$

$$\hat{H}(s) = \frac{0.2814}{S^2 + 0.5306\ S\ 0.3516}$$

(4) Calculate De-normalized Analog LPF

$$H(s) = \hat{H}(s)\Bigg|_{S=\frac{S}{\Omega_c}=\frac{S}{\Omega p}=\frac{S}{0.649}}$$

Step–2 Design Digital Chebyshev L P F

By BLT,

$$H(z) = H(s)\Bigg|_{s=\frac{2}{T}\frac{(z-1)}{(z+1)}}$$ **Put** T = 1 sec

Q(12) Design a Digital Chebyshev filter that satisfies the following constraint using bilinear transformation. Assume T = 1s.

3 dB ripple in pass band at 0.2 π
25 dB attenuation in stop band at 0.45π

Solution :

(i) Ap = 3 dB (ii) As = 25 dB (iii) Fs = 1 Hz

(iv) wp = 0.2 **π** (v) ws = 0.45 **π** (vi) LPF

Step–1 Design Analog Chebyshev L P F

(1) Calculate Ω_p, Ω_s

i) $\Omega_p = \frac{2}{T}\tan\left(\frac{w_p}{2}\right)$ Put T = 1 Ω_p = 0.6498 rad/sec	ii) $\Omega_s = \frac{2}{T}\tan\left(\frac{w_s}{2}\right)$ Put T = 1 Ω_s = 1.71 rad/sec

(2) Calculate filter order N

$$N = \frac{\mathrm{Cosh}^{-1}\left[\dfrac{10^{As/10}-1}{10^{Ap/10}-1}\right]^{1/2}}{\mathrm{Cosh}^{-1}\left[\dfrac{\Omega s}{\Omega p}\right]} =$$

Let **N = 3** (ODD)

(3) Calculate Normalized Analog LPF

(i) Find ε

$$\varepsilon = \sqrt{10^{Ap/10}-1} == 1.0$$

(ii) Find u

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} == 2.414$$

(iii) Find a

$$a = \left(\frac{\mu^{\frac{1}{N}} - \mu^{\frac{-1}{N}}}{2}\right) == 0.1935$$

(iv) Find b

$$b = \left(\frac{\mu^{\frac{1}{N}} + \mu^{\frac{-1}{N}}}{2} \right) == 0.678$$

(v) Find ϕ_k

$$\Phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad for \;\; k = 1,\, 2,\ldots N$$

$$\Phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{6} \quad for \;\; k = 1,\, 2,\, 3$$

$\Phi_1 = \frac{4\pi}{6}$ Cos(ϕ_1) = - 0.5 Sin(ϕ_1) = 0.866

$\Phi_1 = \pi$ Cos(ϕ_1) = - 1 Sin(ϕ_1) = 0

$\Phi_1 = \frac{-4\pi}{6}$

(vi) Find S_k

$$S_k = a \cos \Phi_k + jb \sin \Phi_k$$

S_1 = - 0.09675 + j 0.587
S_2 = - 0.1935
S_3 = - 0.09675 - j 0.587

(vii) Find $Q_p(s)$

$Q_p(s)$ = (S-S_1) (S-S_2) (S-S_3)

$Q_p(s)$ = (S + 0.1935) (S + 0.09675 - j 0.587) (S + 0.09675 - j 0.587)

$Q_p(s)$ = (S+ 0.1935) $(S + 0.09675)^2 + (0.587)^2$

$Q_p(s)$ = (S+ 0.1935) (S^2 + 0.1935 S + 0.354)

(viii) Find K

For N ODD, $K = Qp(0)$
K = (0.1935) (0.354)
K == **0.0685**

(ix) Find H(s)

$$H(s) = \frac{K}{Qp(s)}$$

$$H(s) = \frac{0.0685}{(S+0.1935)\ (S^2 + 0.1935\,S + 0.354)}$$

(5) Calculate De-normalized Analog LPF

$$H(s) = \hat{H}(s)\Big|_{S = \frac{S}{\Omega_c} = \frac{S}{\Omega p} = \frac{S}{0.649}}$$

Step–2 Design Digital Chebyshev L P F

By BLT,

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\frac{(z-1)}{(z+1)}}$$ **Put** T = 1 sec

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