Experiment 4 : Fast Fourier transform

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Experiment No.	4

AIM:	The aim of this experiment is to implement computationally Fast Algorithms.
OBJECTIVE:	1. Develop a program to perform FFT of N point Signal. 2. Calculate FFT of a given DT signal and verify the results using mathematical formula. 3. Computational efficiency of FFT.
INPUT SPECIFICATION:	**1.** Length of first Signal N **2.** DT Signal values
PROBLEM DEFINITION:	(1) Take any four-point sequence x[n]. Find FFT of x[n] and IFFT of {X[k]}. (2) Calculate Real and Complex Additions & Multiplications involved to find X[k]
THEORY:	**Discrete Fourier Transform (DFT)** 1. **Definition:** ○ The DFT is a mathematical algorithm used to convert a discrete sequence of values into its frequency domain representation. The DFT computes the Fourier coefficients for a sequence of NNN discrete points. 2. **Formula:** ○ The DFT of a sequence x[n] is given by: $X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi}{N}kn}$ ○ Here, X[k] is the frequency domain representation of the sequence, and the summation involves all Npoints of the input sequence.

3. **Complexity:**
    - The direct computation of the DFT requires O(N^2) operations because each of the N output values requires a summation over N input values.
4. **Usage:**
    - The DFT is used to analyze the frequency components of a discrete signal. However, due to its computational complexity, it is often impractical for large datasets.

Fast Fourier Transform (FFT)

1. **Definition:**
    - The FFT is an efficient algorithm for computing the DFT. It significantly reduces the computational complexity of the DFT, making it practical for larger datasets.
2. **Algorithm:**
    - The FFT utilizes the divide-and-conquer strategy to break down the DFT computation into smaller DFTs. It reorganizes the computation to reduce redundant operations and improve efficiency.
3. **Complexity:**
    - The FFT reduces the complexity of computing the DFT from O(N^2) to O(NlogN), making it much faster for large N. This efficiency gain is achieved by exploiting symmetries and periodicities in the DFT computation.
4. **Variants:**
    - There are various FFT algorithms, such as the Cooley-Tukey algorithm, which is the most commonly used, and others like the Radix-2, Radix-4, and mixed-radix FFT algorithms. Each variant is optimized for different types of inputs and applications.
5. **Usage:**
    - The FFT is widely used in practical applications where large datasets need to be transformed quickly, such as in real-time signal processing, audio analysis, image processing, and communication systems

Theoretical solution

Question : A= [6,12,7,14]

Solution :
N=4
X[k] = [39, -1 + 2j, -13, -1 - 2j]
Magnitude : [39 , 2.24 , 13 , 2.24]

Question : A= [6,12,7,14,8,16,9,18]
Solution :
N=8

```
X[k] = [90.000  + j    0.000
  -2.000  + j7.657
  -2.000  + j4.000
  -2.000  + j3.657
 -30.000  + j0.000
  -2.000  + j-3.657
  -2.000  + j-4.000
  -2.000  + j-7.657]
```

Magnitude : [90,7.91,4.47,4.1681,30,4.168,4.47,7.91]

Developing algorithm using programming language

PROBLEM STATEMENT:	Write a program in any programming language to perform the linear convolution of two signals with length L and M respectively.

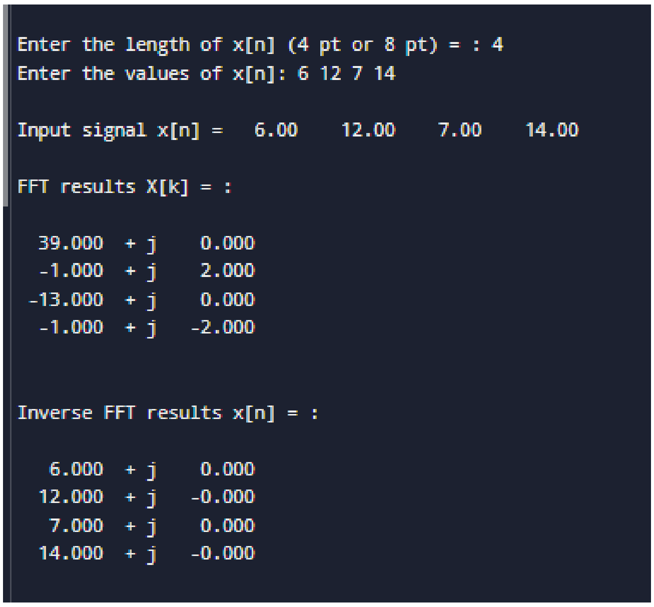
PROGRAM:

```
#include <stdio.h>
#include <math.h>

#define SIZE 8

void FFT_4_Point(int N, float input[SIZE][2], float output[SIZE][2]);
void FFT_8_Point(int N, float input[SIZE][2], float output[SIZE][2]);
void Inverse_FFT(int N, float input[SIZE][2], float output[SIZE][2]);

int main() {
    int i, n;
    float input[SIZE][2], output[SIZE][2];

    // Initialize input and output arrays
    for(i = 0; i < SIZE; i++) {
        output[i][0] = 0;
        output[i][1] = 0;
    input[i][0] = 0;
    input[i][1] = 0;
}

// Get the input signal length
printf("\nEnter the length of x[n] (4 pt or 8 pt) = : ");
scanf("%d", &n);

if (n != 4 && n != 8) {
    printf("Length must be 4 or 8 for this implementation.\n");
    return 1;
}

// Get the input signal values
printf("Enter the values of x[n]: ");
for(i = 0; i < n; i++) {
    scanf("%f", &input[i][0]);
}

// Display the input signal
printf("\nInput signal x[n] = ");
for(i = 0; i < n; i++) {
    printf("  %4.2f  ", input[i][0]);
}

// Perform FFT
if (n == 4) {
    FFT_4_Point(n, input, output);
} else {
    FFT_8_Point(n, input, output);
}

// Display FFT results
printf("\n\nFFT results X[k] = :\n");
for(i = 0; i < n; i++) {
    printf("\n %7.3f  + j  %7.3f", output[i][0], output[i][1]);
}
printf("\n\n");

// Perform Inverse FFT
Inverse_FFT(n, output, input);

// Display Inverse FFT results
```

```
    printf("\nInverse FFT results x[n] = :\n");
    for(i = 0; i < n; i++) {
        printf("\n %7.3f  + j  %7.3f", input[i][0], input[i][1]);
    }
    printf("\n\n");

    return 0;
}

void FFT_4_Point(int N, float input[SIZE][2], float output[SIZE][2]) {
    float temp[SIZE][2];
    int k;
    float angle = 2 * M_PI / N;

    // Stage 1
    temp[0][0] = input[0][0] + input[2][0];
    temp[0][1] = input[0][1] + input[2][1];

    temp[1][0] = input[0][0] - input[2][0];
    temp[1][1] = input[0][1] - input[2][1];

    temp[2][0] = input[1][0] + input[3][0];
    temp[2][1] = input[1][1] + input[3][1];

    temp[3][0] = input[1][0] - input[3][0];
    temp[3][1] = input[1][1] - input[3][1];

    // Stage 2
    for (k = 0; k < N; k++) {
        output[k][0] = temp[k % 2][0] + (temp[k % 2 + 2][0] * cos(angle * k)
+ temp[k % 2 + 2][1] * sin(angle * k));
        output[k][1] = temp[k % 2][1] + (temp[k % 2 + 2][1] * cos(angle * k) -
temp[k % 2 + 2][0] * sin(angle * k));
    }
}

void FFT_8_Point(int N, float input[SIZE][2], float output[SIZE][2]) {
    float G[4][2], H[4][2], temp[SIZE][2];
    int k;
    float angle = 2 * M_PI / N;

    // Split input into two 4-point sequences
    for (k = 0; k < 4; k++) {
        G[k][0] = input[2 * k][0];
        G[k][1] = input[2 * k][1];
        H[k][0] = input[2 * k + 1][0];
        H[k][1] = input[2 * k + 1][1];
    }

    // Perform 4-point FFT on both sequences
    FFT_4_Point(4, G, G);
    FFT_4_Point(4, H, H);

    // Combine the results
    for (k = 0; k < 4; k++) {
        output[k][0] = G[k][0] + (H[k][0] * cos(angle * k) + H[k][1] * sin(angle * k));
        output[k][1] = G[k][1] + (H[k][1] * cos(angle * k) - H[k][0] * sin(angle * k));

        output[k + 4][0] = G[k][0] + (H[k][0] * cos(angle * (k + 4)) + H[k][1] * sin(angle * (k + 4)));
        output[k + 4][1] = G[k][1] + (H[k][1] * cos(angle * (k + 4)) - H[k][0] * sin(angle * (k + 4)));
    }
}

void Inverse_FFT(int N, float input[SIZE][2], float output[SIZE][2]) {
    int i;
    float temp[SIZE][2];

    // Take the conjugate of the input
    for (i = 0; i < N; i++) {
        temp[i][0] = input[i][0];
        temp[i][1] = -input[i][1];
    }

    // Perform FFT on the conjugate
    if (N == 4) {
        FFT_4_Point(N, temp, output);
    } else {
        FFT_8_Point(N, temp, output);
    }

    // Take the conjugate again and divide by N
    for (i = 0; i < N; i++) {
        output[i][0] = output[i][0] / N;
        output[i][1] = -output[i][1] / N;
    }
}
```

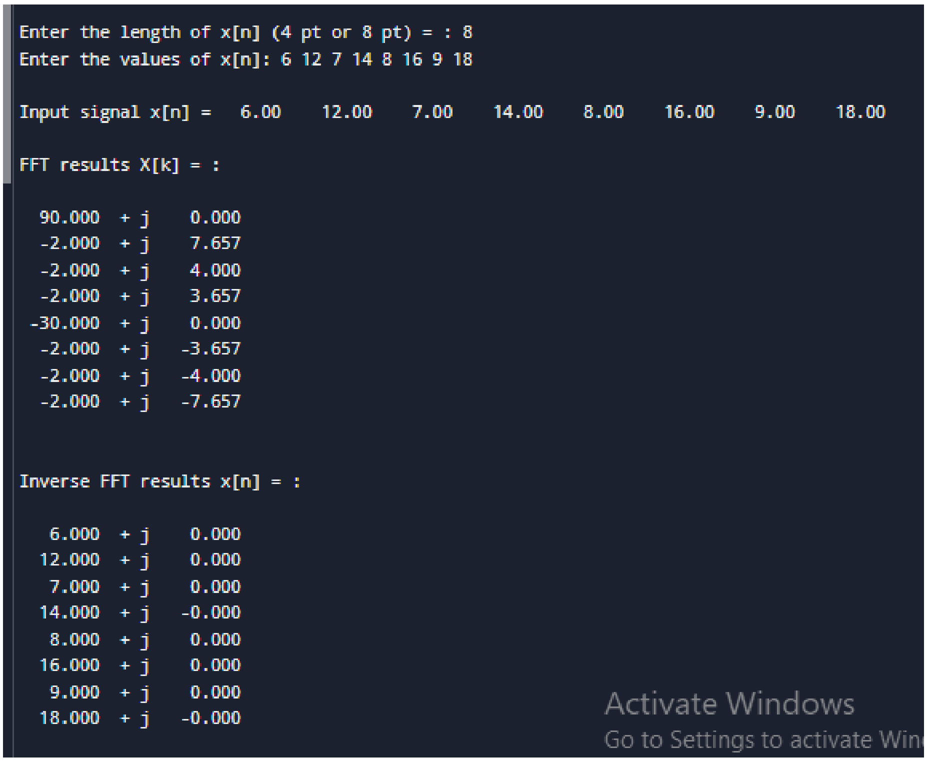
RESULT:

CASE 1 : FOUR POINT FFT AND IFFT



```
Enter the length of x[n] (4 pt or 8 pt) = : 4
Enter the values of x[n]: 6 12 7 14

Input signal x[n] =   6.00    12.00    7.00    14.00

FFT results X[k] = :

  39.000  + j    0.000
  -1.000  + j    2.000
 -13.000  + j    0.000
  -1.000  + j   -2.000


Inverse FFT results x[n] = :

   6.000  + j    0.000
  12.000  + j   -0.000
   7.000  + j    0.000
  14.000  + j   -0.000
```

CASE 2 : EIGHT POINT FFT AND IFFT



```
Enter the length of x[n] (4 pt or 8 pt) = : 8
Enter the values of x[n]: 6 12 7 14 8 16 9 18

Input signal x[n] =   6.00    12.00    7.00    14.00    8.00    16.00    9.00    18.00

FFT results X[k] = :

  90.000  + j    0.000
  -2.000  + j    7.657
  -2.000  + j    4.000
  -2.000  + j    3.657
 -30.000  + j    0.000
  -2.000  + j   -3.657
  -2.000  + j   -4.000
  -2.000  + j   -7.657


Inverse FFT results x[n] = :

   6.000  + j    0.000
  12.000  + j    0.000
   7.000  + j    0.000
  14.000  + j   -0.000
   8.000  + j    0.000
  16.000  + j    0.000
   9.000  + j    0.000
  18.000  + j   -0.000
```

CONCLUSION:

1. Computational Efficiency in DFT

For N=4N :

- **Total Real Multiplications:** 4×4^2 = 64
- **Total Real Additions:** 4×4^2−2×4=64−8=56

For N=8N :

- **Total Real Multiplications:** 4×82=256
- **Total Real Additions:** 4×8^2−2×8=256−16=240

2. Computational Efficiency in FFT

For N=4:

- **Total Real Multiplications:** 2×4×log2(4) =2×4×2 =16
- **Total Real Additions:** 3×4×log2(4) =3×4×2 =24

For N=8 :

- **Total Real Multiplications:** 2×8×log2(8) =2×8×3 =48
- **Total Real Additions:** 3×8×log2(8) =3×8×3 =72

3. FFT Performance

The FFT produces fast results due to:

- **Less Computations:** The FFT significantly reduces the number of multiplications and additions compared to the DFT. For N=4N and N=8, the reduction in operations is substantial.

The FFT dramatically reduces computational complexity compared to the DFT. For small N like 4 and 8, the difference is clear and substantial. As N increases, the efficiency of the FFT becomes even more pronounced, making it the preferred choice for larger datasets. The FFT achieves this efficiency through a reduction in the total number of computations and the potential for parallel processing, which enhances performance in practical applications.