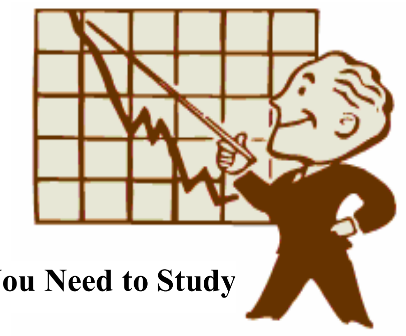

You Need to Study

• Digital IIR Filters •

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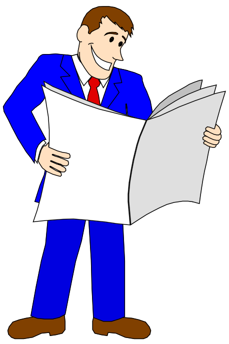
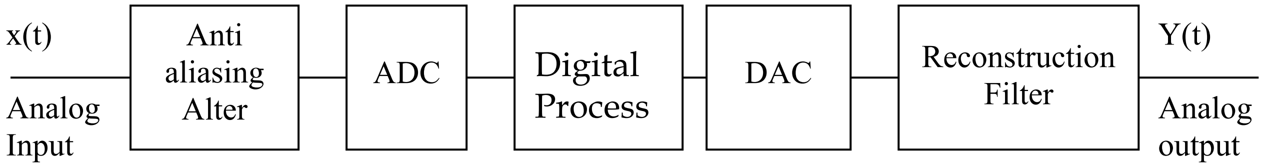


DIGITAL FILTERS

Q(1) What is Digital filter ?

Digital filter is a Discrete Time System which produces a discrete time output sequence y[n] for the discrete time input sequence x[n]. Digital filter is nothing but mathematical algorithm implemented in hardware or software.

Real time digital filter consist of processing of real time signal using digital device called digital processor.



As shown in figure, analog input signal is band limited using anti-aliasing filter which is then sampled and DT signal thus obtained is converted into digital signal using ADC. Digital processor, perform the operation depending upon the algorithm programmed in digital processor.

The output of the digital processor is converted into analog signal using DAC. Reconstruction filter is used to obtain the corresponding analog signal from the output DT signal.

Q(2) Explain the advantage and disadvantages of Discrete Time filter over the Analog filter.

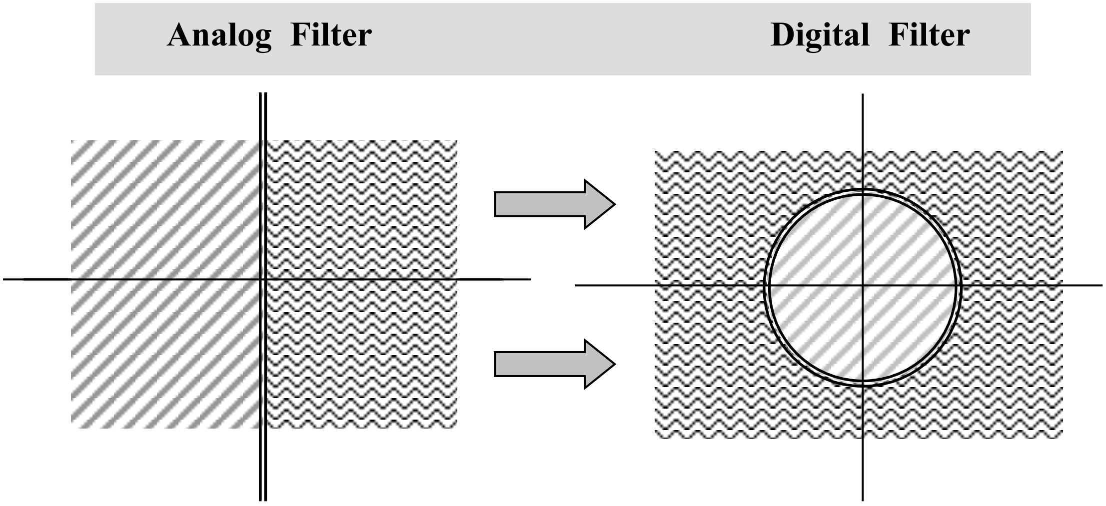
	Parameter	Analog filter	Digital Filter
1	Input/output signal	Analog	Digital(discrete time sequences)
2	Composition	Lumped elements such as R.L. and C or analog /C_s	Software + digital hardware
3	Filter representation	In terms of system components	By difference equation
4	Flexibility	Not flexible	Highly flexible
5	Portability	Not easily portable	Portable
6	Design objective and result	Specifications to values of R.L and C components	Specifications to difference equation
7	Environmental effects	Negligible effect of environmental parameters	Negligible effect of environmental parameters
8	Interference notes and other effects	Maximum effect	Minimum/negligible effect
9	Storage/maintenance failure	Difficult storage and maintenance and higher failure rate	Easier storage and maintenance and reduced failure rate

Q(3) What is the requirement of design of Digital IIR filter ?

⇨ Digital I I R filters are designed from Analog filters. The designed filter must be causal and stable. An analog filter H(s) is stable if all the poles lie in the left half of s-plane.

To obtain stable digital filter from stable analog filter, the filter design technique should have the following properties.

1. The $j\Omega$ axis in the s-plane should map onto the unit circle in the z-plane. This gives a direct relationship between the two frequency variables in the two domains.
2. The left-half plane of the s-plane should map into the inside of the unit circle in the z-plane to convert a stable analog filter into a stable digital filter.



Q(4) What are the Advantages of Digital filters-?

The following list gives some of the main advantages of digital over analog filters.

1. **A digital filter is *programmable***, i.e. its operation is determined by a program stored in the processor's memory. This means the digital filter can easily be changed without affecting the circuitry (hardware). An analog filter can only be changed by redesigning the filter circuit. (i.e. **Flexibility in parameter setting**)
2. **Digital filters are easily *designed, tested* and *implemented*** on a general-purpose computer or workstation.
3. The characteristics of analog filter circuits (particularly those containing active components) are subject to drift and are dependent on temperature. Digital filters do not suffer from these problems, and so are **extremely *stable*** with respect both to time and temperature.
4. Unlike their analog counterparts, **digital filters can handle *low frequency signals* accurately**. As the speed of DSP technology continues to increase, digital filters are being applied to high frequency signals in the RF (radio frequency) domain, which in the past was the exclusive preserve of analog technology.
5. **Digital filters are very much more *versatile* in their ability to process signals in a variety of ways**; this includes the ability of some types of digital filter to adapt to changes in the characteristics of the signal.

Q(5) What are Advantages of FIR Filters-?

1. **They can easily be designed to be "linear phase"** (and usually are). Put simply, linear-phase filters delay the input signal, but don't distort its phase.

2. **They are simple to implement**. On most DSP microprocessors, the FIR calculation can be done by looping a single instruction.

3. **They are suited to multi-rate applications**. By multi-rate, we mean either "decimation" (reducing the sampling rate), "interpolation" (increasing the sampling rate), or both. Whether decimating or interpolating, the use of FIR filters allows some of the calculations to be omitted, thus providing an important computational efficiency. In contrast, if IIR filters are used, each output must be individually calculated, even if it that output is discarded (so the feedback will be incorporated into the filter).

4. **They have desirable numeric properties**. In practice, all DSP filters must be implemented using "finite-precision" arithmetic, that is, a limited number of bits. The use of finite-precision arithmetic in IIR filters can cause significant problems due to the use of feedback, but FIR filters have no feedback, so they can usually be implemented using fewer bits, and the designer has fewer practical problems to solve related to non-ideal arithmetic.

5. **They can be implemented using fractional arithmetic**. Unlike IIR filters, it is always possible to implement a FIR filter using coefficients with magnitude of less than 1.0. (The overall gain of the FIR filter can be adjusted at its output, if desired.) This is an important consideration when using fixed-point DSP's, because it makes the implementation much simpler.

Q(6) What are the disadvantages of FIR Filters (compared to IIR filters)?

Compared to IIR filters, FIR filters sometimes have the disadvantage that they require more memory and/or calculation to achieve a given filter response characteristic.

Q(7) What are the advantages of IIR filters (compared to FIR filters)?

IIR filters can achieve a given filtering characteristic using less memory and calculations than a similar FIR filter.

Q(8) What are the disadvantages of IIR filters (compared to FIR filters)?

1) They are more susceptible to problems of finite-length arithmetic, such as noise generated by calculations, and limit cycles. (This is a direct consequence of feedback: when the output isn't computed perfectly and is fed back, the imperfection can compound.)
2) They are harder (slower) to implement using fixed-point arithmetic.
3) They don't offer the computational advantages of FIR filters for multirate (decimation and interpolation) applications.

Q(9) Compare FIR filters and IIR filters

	FIR filter	IIR filter
1	Length of h[n] is finite	Length of h[n] is In-finite
2	Provides exact linear phase.	Not linear phase.
3	Provides good stability.	Stability is not guaranteed.
4	Order required is higher.	Order required is lower.
5	Computationally not efficient.	Computationally more efficient.
6	More memory required for the storage of coefficients.	Less memory required fro storage of coefficients.
7	Requires more processing time.	Requires less processing time.
8	Requires M multiplications per output sample	Requires 2M + 1 multiplications per output sample.

➢ **I I R FILTER DESIGN**

Time Domain Method

(1) Impulse Invariant Method

(2) Step Invariant Method

Transform Domain Method

(1) Bilinear transformation (BLT)

(2) Matched Z Transform Method

Q(10) Give Invariant Method Design Procedure :-

I. Determine the Normalized Analog filter Transfer function H(s) that satisfies the specification for the desired filter.

II. Determine the analog filter output by tasking inverse laplace transform of Y(s)

ie y (t) = ILT {Y(s)} where Y(s) = X(s) . H(s) .

III. Sample the output of the analog filter .

$$y[nT] = y(t)\Big|_{t=nT}$$

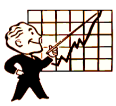
IV Obtain the Transfer function of the digital filter.

$$Y(z) = ZT\ \{y[nt]\}$$

Then $H(z) = \dfrac{Y(z)}{X(z).}$

For Impulse Invariant Method take $x(t) = \delta(t)$

For Step Invariant Method take $x(t) = u(t)$.



Q(11) Digital IIR Filter Design using Impulse Invariant Method :

I. Determine the normalized analog filter Transfer function H(s) that satisfies the specification for the desired filter.

II. Find X(s)

For Impulse Invariant Method ,

Take $x(t) = \delta(t)$

By LT, $X(s) = 1$

III. Find Y(s)

Now $Y(s) = X(s)\, H(s)$

$Y(s) = H(s)$

IV. Find y(t)

Now, $Y(s) = H(s)$

By Inverse LT,

$y(t) = h(t)$

V. Find x[n]

Now, $x(t) = \delta(t)$

Put $t = nT$

$x(nT) = \delta(nT)$

$x[n] = \delta[n]$

VI. Find X(z)

Now $x[n] = \delta[n]$

By ZT, $X(z) = 1$

VII. Find y[n]

Now, $y(t) = h(t)$

Put $t = nTs$

$y[nT] = h[nT]$

$y[n] = h[n]$

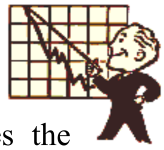
VIII. Find Y(z)

Now $y[n] = h[n]$

By ZT, $Y(z) = H(z)$

IX. Find H(z)

$H(z) = Y(z) / X(z)$



Q(12) Digital IIR Filter Design using Step Invariant Method :

I. Determine the normalized analog filter Transfer function H(s) that satisfies the specification for the desired filter.

II. Find X(s)

For Impulse Invariant Method ,

Take x(t) = u(t)

By LT, X(s) = 1/s

III. Find Y(s)

Now Y(s) = X(s) H(s)

Y(s) = H(s)/s

IV. Find y(t)

y(t) = ILT { Y(s) }

V. Find x[n]

Now, x(t) = u(t)

Put t = nT

x(nT) = u(nT)

x[n] = u[n]

VI. Find X(z)

Now x[n] = u[n]

By ZT, X(z) = z/(z-1)

VII. Find y[n]

To find y[n], put t=nT in y(t).

VIII. Find Y(z)

By ZT, Y(z) = ZT { y[n] }

IX. Find H(z)

H(z) = Y(z) / X(z)

(1) $$LT\ \left\{ e^{-at}\cos(bt)\ u(t) \right\} == \frac{S+a}{(S+a)^2+b^2}$$

(2) $$LT\ \left\{ e^{-at}\,Sin\,(bt)\ u(t) \right\} == \frac{b}{(S+a)^2+b^2}$$

(3) $$ZT\ \left\{ r^n\cos(nw)\ u(n) \right\} = \frac{z^2 - r\,z\,Cos(w)}{z^2 - 2\,r\,z\,Cos(w) + r^2}$$

(4) $$ZT\ \left\{ r^n\sin(nw)\ u(n) \right\} = \frac{r\,z\sin(w)}{z^2 - 2\,r\,z\,Cos(w) + r^2}$$

where **b** is Analog Resonant Frequency and r is Digital Resonant Frequency.

Q(13) Explain the Mapping of POLES from s-plane to z–plane when Impulse Invariant Method is used for filter design.

Case-I When $\sigma = 0$, $r = 1$

Analog poles which lies on imaginary axis gets mapped onto the unit circle in the z-plane.

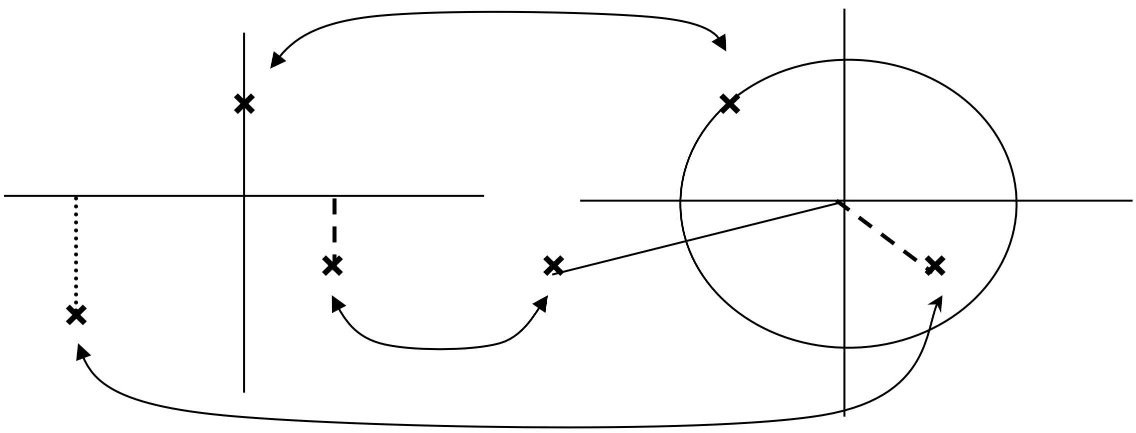
Case-II When $\sigma < 0$, $r < 1$,

Analog poles that lies on LEFT half of s-plane gets mapped INSIDE the unit circle in the z–plane.

Case–III When $\sigma > 0, r > 1$.

Analog poles that lies on RIGHT half of s-plane gets mapped OUTSIDE the unit circle in the z–plane.

ANALOG FILTER **DIGITAL FILTER**



Q(14) Explain the effect of Aliasing in Impulse Invariant Method.

The relation between analog filter pole and digital filter pole when impulse invariant technique is used for filter design is given by, $Z = e^{ST}$.

However the relation $Z = e^{ST}$ does not describe one to one mapping between s-plane and z-plane.

Case –1, Consider analog pole at $S_1 = \sigma + j\Omega$

Then $Z_1 = e^{S_1 T} = e^{(\sigma+j-2)\,T} = e^{\sigma T}\, e^{j\Omega T}$ ________ I

Case – 2.

Consider analog pole at $S_2 = \sigma + j\left(\Omega + \frac{2\pi}{T}\right)$

Then $Z_2 = e^{S_2 T} = e^{\left[\sigma + j\left(\Omega + \frac{2\pi}{T}\right)\right]T} = e^{\sigma T}\, e^{j\left(\Omega + \frac{2\pi}{T}\right)T}$

$= e^{\sigma T}\, e^{j\Omega T}\, e^{j2\pi}$ But $e^{j2\pi} = 1$

$\therefore \quad = e^{\sigma T}\, e^{j\Omega T}$ ______________ II

From eq^n I and II, Analog poles $S_1 \neq S_2$ But the corresponding digital poles $Z_1 = Z_2$.

That means, all frequencies $\left(\Omega + \frac{2\pi k}{T}\right)$ are mapped to the same point in the Z- plane.

The transformation maps all points in the s plane given by $S = \sigma + j\left(\Omega + \frac{2\pi k}{T}\right)$, $k = 0, \pm 1, \pm 2, \pm 3 \ldots$ on to a single point in the z – plane at $z = e^{ST}$. ie $Z = e^{\sigma T} \; e^{j\Omega T}$

The mapping implies that the interval $\frac{-\pi}{T} \leq \Omega \leq \frac{\pi}{T}$ maps into the corresponding values of $-\pi \leq w \leq \pi$ in the digital domain. Further the frequency interval $\frac{\pi}{T} \leq \Omega \leq \frac{3\pi}{T}$ of s-plane also maps into the interval $-\pi \leq w \leq \pi$ in the z-plane.

In general, any frequency in the interval $\frac{(2k-1)}{T} \leq \Omega \leq \frac{(2k+1)\pi}{T}$ will also map into the interval $-\pi \leq w \leq \pi$ in the z–plane. Thus the mapping from the analog frequency Ω to the digital frequency w is not one to one mapping which reflects the effect of aliasing due to sampling. A one to one mapping is thus possible only if freq. Ω lies in the principle range of $-\frac{\pi}{T} \leq \Omega \leq \frac{\pi}{T}$

Q(15) Impulse Invariant method is not suitable for HPF / BPF design. Justify.

In Impulse Invariant method, the relation between analog filter pole and digital filter pole and digital filter pole when impulse invariant technique is used for filter design is given by, $Z = e^{ST}$. However the relation $Z = e^{ST}$ does not describe one to one mapping between s-plane and z-plane. That means, all frequencies $\left(\Omega + \frac{2\pi k}{T}\right)$ are mapped to the same point in the z-plane. The transformation maps all points in the s plane given by $S = \sigma + j\left(\Omega + \frac{2\pi k}{T}\right)$, $k = 0, \pm 1, \pm 2, \pm 3 \ldots$ on to a single point in the z – plane at $z = e^{ST}$.

ie $Z = e^{\sigma T} \; e^{j\Omega T}$

The mapping implies that the interval $\frac{-\pi}{T} \leq \Omega \leq \frac{\pi}{T}$ maps into the corresponding values of $-\pi \leq w \leq \pi$ in the digital domain. Further the frequency interval $\frac{\pi}{T} \leq \Omega \leq \frac{3\pi}{T}$ of s-plane also maps into the interval $-\pi \leq w \leq \pi$ in the z-plane.

Thus the mapping from the analog frequency Ω to the freq. variable w in the digital domain is many to one. which reflects the effect of aliasing due to sampling. A one to one mapping is thus possible only if freq. Ω lies in the principle range of $-\frac{\pi}{T} \leq \Omega \leq \frac{\pi}{T}$.

That means if cut off frequency of analog filter Ωc is greater than $\frac{\pi}{T}$. then one to one mapping from analog filter frequency to digital filter frequency is not possible.

Therefore the filter such as HPF or BPF with cut off frequency of analog filter Ωc greater than $\frac{\pi}{T}$. can not be designed using impulse invariant method.

> **BILINEAR TRANSFORMATION (BLT) METHOD**
>
> Bilinear Transformation is a mapping of points from s-plane to corresponding points in the z-plane. The BLT transforms, the entire $j\Omega$ axis in the s-plane into one revolution of the unit circle in the z-plane ie. only once and therefore avoids the aliasing of frequency components

Q(16) Derive BLT equation

Consider the first order low pass analog filter with,

$$H(s) = \frac{1}{s+1} \quad \_\_\_\_\_\_ (i)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s+1}$$

$$SY(s) + Y(s) = X(s)$$

By Inverse LT,

$$y'(t) + y(t) = x(t)$$

$$\therefore \; y'(t) = -y(t) + x(t)$$

put t = nT,

$$y'[nT] = -y[nT] + x[nT] \quad \_\_\_\_\_\_ (ii)$$

By integral calculus,

$$y(t) = \int_{to}^{t} y'(t)\, dt + y(t_0.)$$

Let $\quad t = nT$

$\quad\quad\; to = (n-1)T$

By substituting,

$$y(nT) = \int_{(n-1)T}^{nt} y'(t)\,dt + y[(n-1)T]$$

Applying trapezoidal integral approximation rule,

$$y[nT] = \frac{T}{2}\left\{y'[nT] + y'[(n-1)T\right\} + y[(n-1)T] \quad \_\_\_\_\_\_ \text{(iii)`}$$

From eq^n (ii),

$$y'[nT] = -y[nT] + x[nT]$$

$$y'[(n-1)T] = -y[(n-1)T] + x[(n-1)T]$$

By substituting y'[nT] and y'[(n-1)T] in eq^n (iii) we get,

$$y[nT] = \frac{T}{2}\{(-y[nT] + x[nT]) + (-y[(n-1)T + x[(n-1)T])\} + y[(n-1)T]$$

i.e. $y[n] = \frac{T}{2}\{(-y[n] + x[n]) + (-y[(n-1) + x[(n-1)])\} + y[(n-1)]$

By ZT $\quad Y[z] = \frac{T}{2}\{(-Y[z] + X[z]) + (-z^{-1}Y[z] + z^{-1}X[z])\} + z^{-1}Y[z]$

$$Y(z) = Y(z)\left\{-\frac{T}{2} - z^{-1}\frac{T}{2} + z^{-1}\right\} = X(z)\left\{\frac{T}{2} + z^{-1}\frac{T}{2}\right\}$$

$$Y(z)\left\{1 + \frac{T}{2} + z^{-1}\frac{T}{2} - z^{-1}\right\} = X(z)\left\{\frac{T}{2} + z^{-1}\frac{T}{2}\right\}$$

$$Y(z)\left\{(1 - z^{-1}) + \frac{T}{2}(1 + z^{-1})\right\} = X(z)\left\{\frac{T}{2}(1 + z^{-1})\right\}$$

$$\frac{Y(z)}{X(z)} = \frac{\frac{T}{2}(1 + z^{-1})}{(1 - z^{-1}) + \frac{T}{2}(1 + z^{-1})} = \frac{1}{\dfrac{(1 - z^{-1})}{\frac{T}{2}(1 + z^{-1})} + 1}$$

$$H(z) = \frac{1}{\frac{2}{T}\frac{(z-1)}{(z+1)} + 1} \text{---------------(iv)}$$

By comparing H(s) and H(z) eq^n (i) & eq^n (iv) we get,

$$\boxed{H(z) = H(s)\Big|_{s = \frac{2}{T}\frac{(Z-1)}{(z+1)}}} \quad \textbf{(IMP)}$$

Q(17) Explain Mapping of points from s-plane to z–plane when BLT Method is used for filter design.

➡ BLT Transformation is characterized by, $s = \frac{2}{T}\frac{(z-1)}{(z+1)}$ And so $\therefore\ Z = \frac{2+ST}{2-ST}$

Put $z = r\,e^{jw}$ and $s = \sigma + j\Omega$

$$r\,e^{jw} = \frac{2 + (\sigma + j\Omega)\,T}{2-(\sigma + j\Omega)T} = \frac{(2+\sigma T)+j\,\Omega T}{(2-\sigma T)-j\,\Omega T}$$

$$r\,e^{jw} = \frac{\left(\frac{2}{T}+\sigma\right)+j\Omega}{\left(\frac{2}{T}-\sigma\right)-j\Omega}$$

$$|\,z\,| = r = \frac{\sqrt{\left(\frac{2}{T}+\sigma\right)^2 + \Omega^2}}{\sqrt{\left(\frac{2}{T}-\sigma\right)^2 + \Omega^2}} \quad \text{and } \angle Z = w = \tan^{-1}\left(\frac{\Omega}{\frac{2}{T}+\sigma}\right) - \tan^{-1}\left(\frac{-\Omega}{\frac{2}{T}-\sigma}\right)$$

Case-I : When $\sigma = 0,\ r = 1$

Analog poles which lies on imaginary axis gets mapped onto the unit circle in the z- plane.

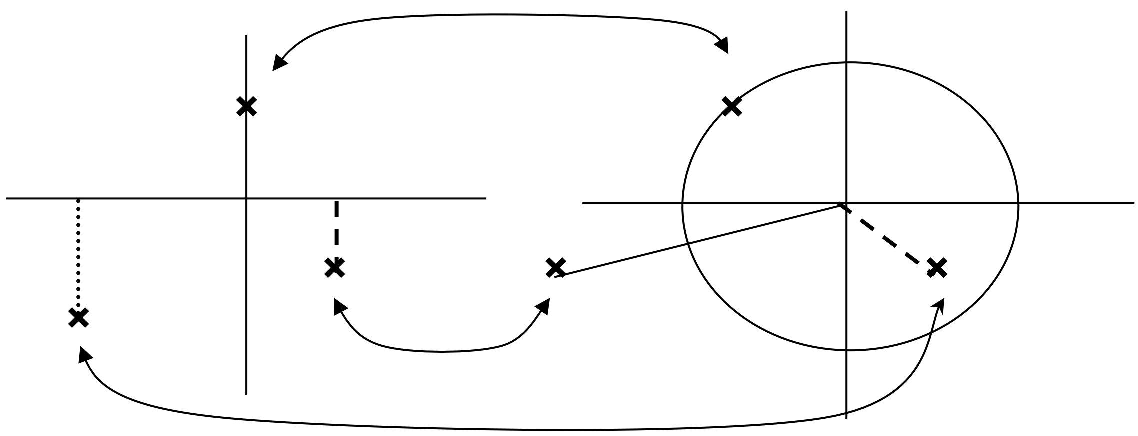
Case-II : When $\sigma < 0,\ r < 1,$

Analog poles that lies on LEFT half of s-plane gets mapped INSIDE the unit circle in the z–plane.

Case–III : When $\sigma > 0, r > 1.$

Analog poles that lies on RIGHT half of s-plane gets mapped OUTSIDE the unit circle in the z – plane.

ANALOG FILTER **DIGITAL FILTER**



Q(18) Explain Digital IIR filter Design using Matched Z-Transform Method.

ANS :

In Impulse Invariant Method, the relation between Analog filter POLE and Digital filter POLE is given by $z=e^{ST}$.

For $H(s) = \dfrac{1}{s+p}$ we get $H(z) = \dfrac{z}{z - e^{-pT}}$

The Matched Z Transform uses this mapping to convert each numerator and denominator terms of a factored H(s) to yield the digital filter H(z) in factored form as,

$$H(s) = K \frac{\prod_{i=1}^{M} (s - q_i)}{\prod_{k=1}^{N} (s - p_k)}$$

ZEROS : $q_1, q_2, q_3, \ldots$

POLES : $p_1, p_2, p_3, \ldots$

GAIN : K

$$H(z) = A \frac{\prod_{i=1}^{M} \left(s - e^{q_i T}\right)}{\prod_{k=1}^{N} \left(s - e^{p_k T}\right)}$$

Constant A is chosen to match the gains of H(s) and H(z) at some convenient frequency (typically DC)

In Matched Z-Transform, the POLES in the left half of the s-plane are matched inside the unit circle in the Z-plane. Thus matched z-Transform, preserves stability of analog filter.

As with the impulse invariant mapping the matched z-Transform also suffers from aliasing errors. Therefore it is NOT suitable for High pass and Band pass filter design.

Q(19) Differentiate between Analog filter and Digital filter.

ANS :

ANALOG FILTER	DIGITAL FILTER
(1) Analog filter processes analog signal input and generates analog output.	(1) Digital filter processes DT signal input and generates DT signal output.
(2) Analog filter is realized using Active and Passive components.	(2) Digital filter is realized using components such as adder, delay block, multiplier etc.
(3) Analog filter is described by a Differential Equation	(3) Digital filter is described by Difference Equation.
(4) The frequency response of digital filter is modified by changing the component values.	(4) The frequency response of digital filter is modified by changing filter coefficients.

Q(20) IIR filters have recursive realization always. Justify

ANS :

In case of IIR Filter, POLESs can be anywhere in the z plane. Due to pole position, the transfer function H(z) of IIR filter has the form,

$$H(z) = \frac{b_0 + b_1 + \cdots\cdots + b_m z}{1 + a_1 + \cdots\cdots + a_N z^{-N}}$$

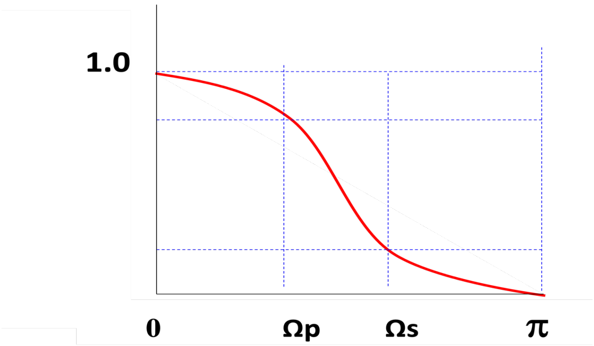
. This gives difference equation in the form,

$y[n] = b_0\, x[n] + b_1\, x\,[n-1] + \cdots\cdots b_m\, x[n-m] + a_1\, y\,[n-1] + \cdots\cdots + a_2\, y[n-N]$
output of the filter in-terms of past output leads to recursive realization

➢ Analog Butterworth LPF

- The Magnitude Response of Analog Butterworth Low Pass Filter is given by,

$$|H(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}}$$



- The Magnitude Response is maximmally flat in both passband and stop band.
- There are NO ripples in pass band and stop band.
- The magnitude response of Butterworth filter decreases monotonically as the frequency increases.
- As N gets larger, Magnitude Response approaches an Ideal Low Pass Frequency response.
- At $\Omega=0$, $|\mathbf{H(j\Omega)}| = 1$ for all N
- At $\Omega=\Omega_c$, $|\mathbf{H(j\Omega)}| = 0.707$ for all N

Q(21) Derive the equation to find POLES of Analog Butterworth LPF.

ANS :

The magnitude response of Analog Butterworth LPF is given by,

$$|H(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}}$$

Where N is order of filter and Ωc is analog filter cutoff frequency.

To find POLES :

Now $s = j\,\Omega$

So $\Omega = \frac{s}{j}$

$$|\mathrm{H}(s)| = \frac{1}{\sqrt{1+\left(\frac{s}{j\Omega_c}\right)^{2N}}}$$

$$\sqrt{1+\left(\frac{s}{j\Omega_c}\right)^{2N}} = 0$$

$$1+\left(\frac{s}{j\Omega_c}\right)^{2N} = 0$$

$$\left(\frac{s}{j\Omega_c}\right)^{2N} = -1$$

$$(s)^{2N} = (j\Omega c)^{2N} \ (-1)$$

$$(s)^{2N} = (e^{j\frac{\pi}{2}}\Omega c)^{2N} \ \left(e^{j\pi}\right)$$

$$s = (e^{j\frac{\pi}{2}}\Omega c) \ \left(e^{j\pi}\right) (1)$$

$$s^{2N} = (\Omega c)^{2N} \ e^{jN\pi} \ \left(e^{j\pi}\right)\left(e^{j2\pi k}\right)$$

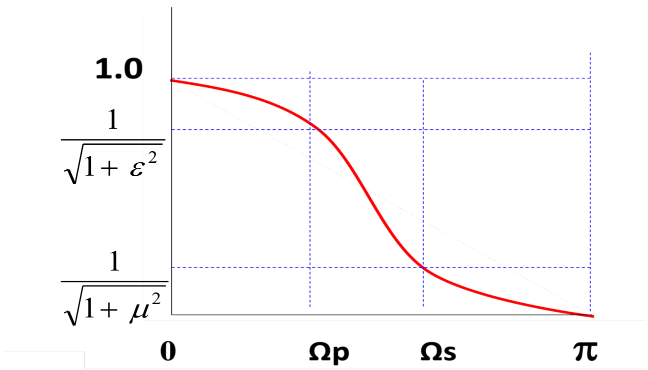
$$s^{2N} = (\Omega c)^{2N} \ e^{j\pi(N+1+2k)}$$

$$S_k = \Omega c \ e^{\frac{j\pi(N+1+2k)}{2N}} \quad for \ \ 0 \le k \le N-1$$

Q(22) Derive the formula to find the ORDER of Analog Butterworth LPF.

ANS :

The magnitude response of Analog Butterworth LPF is given by,



$$|\mathrm{H}(j| = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}}$$

(i) To find Ap

$$\text{At } \Omega = \Omega p : \quad |H(j\Omega p)| = \frac{1}{\sqrt{1 + \left(\frac{\Omega p}{\Omega_c}\right)^{2N}}}$$

$$Ap = 0 - 20 \log \left(\frac{1}{\sqrt{1 + \left(\frac{\Omega p}{\Omega_c}\right)^{2N}}} \right)$$

$$Ap = 20 \log \left(\sqrt{1 + \left(\frac{\Omega p}{\Omega_c}\right)^{2N}} \right)$$

$$Ap = 10 \log \left(1 + \left(\frac{\Omega p}{\Omega_c}\right)^{2N} \right)$$

$$Ap/10 = \log \left(1 + \left(\frac{\Omega p}{\Omega_c}\right)^{2N} \right)$$

$$10^{\left(Ap/10\right)} = 1 + \left(\frac{\Omega p}{\Omega_c}\right)^{2N}$$

$$10^{Ap/10} - 1 = \left(\frac{\Omega p}{\Omega_c}\right)^{2N}$$

(ii) To find As

$$\text{At } \Omega = \Omega s : \quad |H(j\Omega s)| = \frac{1}{\sqrt{1 + \left(\frac{\Omega s}{\Omega_c}\right)^{2N}}}$$

$$As = 0 - 20 \log \left(\frac{1}{\sqrt{1 + \left(\frac{\Omega s}{\Omega_c}\right)^{2N}}} \right)$$

$$As = 20 \log \left(\sqrt{1 + \left(\frac{\Omega s}{\Omega_c}\right)^{2N}} \right)$$

$$As = 10 \log \left(1 + \left(\frac{\Omega s}{\Omega_c}\right)^{2N} \right)$$

$$As/10 = \log \left(1 + \left(\frac{\Omega s}{\Omega_c}\right)^{2N} \right)$$

$$10^{\left(As/10\right)} = 1 + \left(\frac{\Omega s}{\Omega_c}\right)^{2N}$$

$$10^{As/10} - 1 = \left(\frac{\Omega s}{\Omega_c}\right)^{2N}$$

(ii) To find N

Dividing equation II by I we get,

$$\frac{10^{A_s/10}-1}{10^{A_p/10}-1} = \left(\frac{\Omega s}{\Omega p}\right)^{2N}$$

$$\log\left[\frac{10^{A_s/10}-1}{10^{A_p/10}-1}\right] = 2N \log\left[\frac{\Omega s}{\Omega p}\right]$$

$$N = \frac{\log\left[\frac{10^{A_s/10}-1}{10^{A_p/10}-1}\right]}{2\log\left[\frac{\Omega s}{\Omega p}\right]}$$

Q(23) Derive the equation to find CUTOFF frequency of Analog Butterworth LPF.

ANS :

$$\text{At } \Omega = \Omega p \; : \quad |H(j\Omega p)| = \frac{1}{\sqrt{1+\left(\frac{\Omega p}{\Omega_c}\right)^{2N}}}$$

$$Ap = 0 - 20 \log\left(\frac{1}{\sqrt{1+\left(\frac{\Omega p}{\Omega_c}\right)^{2N}}}\right)$$

$$Ap = 20 \log\left(\sqrt{1+\left(\frac{\Omega p}{\Omega_c}\right)^{2N}}\right)$$

$$Ap = 10 \log\left(1+\left(\frac{\Omega p}{\Omega_c}\right)^{2N}\right)$$

$$Ap/10 = \log\left(1+\left(\frac{\Omega p}{\Omega_c}\right)^{2N}\right)$$

$$10^{\left(Ap/10\right)} = 1+\left(\frac{\Omega p}{\Omega_c}\right)^{2N}$$

$$10^{Ap/10}-1 = \left(\frac{\Omega p}{\Omega_c}\right)^{2N}$$

$$\left(\frac{\Omega p}{\Omega_c}\right)^{2N} = 10^{Ap/10}-1$$

$$\frac{\Omega p}{\Omega c} = \left(10^{Ap/10}-1\right)^{\frac{1}{2N}}$$

$$\frac{\Omega c}{\Omega p} = \frac{1}{\left(10^{Ap/10} - 1\right)^{\frac{1}{2N}}}$$

$$\Omega c = \frac{\Omega p}{\left(10^{Ap/10} - 1\right)^{\frac{1}{2N}}}$$

$$\Omega c = \frac{1}{\left(10^{Ap/10} - 1\right)^{\frac{1}{2N}}}$$

*** Another method to find Normalized H(s) for LPF:**

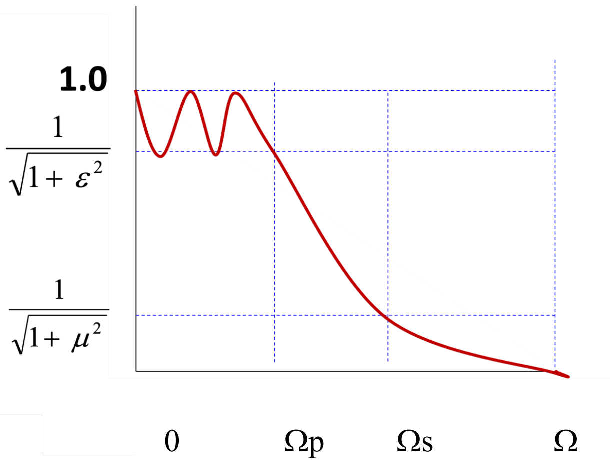
$$\overline{H}(s) = \frac{1}{a_0 + a_1 S^1 + \ldots\ldots + ..a_N\ S^N}$$ **where** $a_0 = a_N == 1$

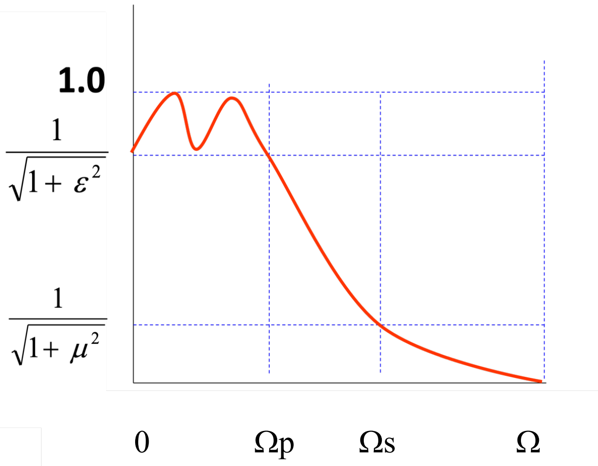
And $$a_k = \left[\frac{\cos\left\{\frac{(k-1)\pi}{2N}\right\}}{\sin\left(\frac{k\pi}{2N}\right)}\right] a_{k-1}$$ where N is the filter order.

➢ Analog Chebyshev Low-Pass Filter

- There are two types of Chebyshev filters
- Chebyshev-1 filters are all POLE filters that exhibit equiripple behavior in the pass band and a monotonic characteristics in the stop band.
- Chebyshev-2 filters contain both POLES and ZEROs and exhibits a monotonic behavior in the passband and an equiripple behavior in the stopband.
- The transtition width of chebyshev filter is narrow as compared to transition width of Butterworth filter.

Magnitude Spectrum :





Order of Chebyshev-1 Analog filter is given by,

$$\text{LPF} \quad N = \frac{\cosh^{-1}\sqrt{\left[\dfrac{10^{As/10}-1}{10^{Ap/10}-1}\right]}}{\cosh^{-1}\left[\dfrac{\Omega s}{\Omega p}\right]} \qquad \text{HPF} \quad N = \frac{\cosh^{-1}\sqrt{\left[\dfrac{10^{As/10}-1}{10^{Ap/10}-1}\right]}}{\cosh^{-1}\left[\dfrac{\Omega p}{\Omega s}\right]}$$

Q(24) Distinguish between Butterworth filter and Chebyshev Filter.

Butterworth Filter	Chebyshev Filter
(1) It is called maximally flat because the frequency response of filter has NO ripples in the pass-band or stop-band	(1) The frequency response has ripples in the pass-band and/or stop-band.
(2) The transition band is generally wide .and it is not applicable for sharp cut-off characteristics.	(2) The transition band is generally narrow and it is more applicable for sharp cutoff characteristics.
(3) In general, a larger order is needed to meet design specifications.	(3) In general, a smaller order is needed to meet design specifications.
(4) In the POLE ZERO plot, the POLES lie on a Circle.	(4) In the POLE ZERO plot, the POLES lie on an ellipse.
(5)Requires more hardware components for realization.	(5) Requires Less hardware components for realization.

Q(25) Distinguish between Impulse Invariant Method and BLT Method.

ANS :

Impulse Invariant Method (IIM)	Bilinear Transformation Method(BLT)
(1) In IIM Method, the relation between analog filter POLE and Digital filter POLE is given by $Z=e^{ST}$	**(1)** In BLT Method, the relation between analog filter POLE and Digital filter POLE is given by $S = \frac{2(z-1)}{T(z+1)}$
(2) The relation between Analog freq. and Digital freq. is given by, $W = \frac{\Omega}{Fs}$	(2) The relation between Analog freq. and Digital freq. is given by, $W = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)$
(3) Frequency relation doesn't give one to one mapping between analog filter freq. Ω and digital filter freq. W. which reflects the effect of aliasing due to sampling.	(3) Frequency relation gives one to one mapping between analog filter freq. Ω and digital filter freq. W
(4). Due to aliasing, HPF or BPF with cut off frequency of analog filter Ωc greater than $\frac{\pi}{T}$. can not be designed using impulse invariant method	(4). BLT method is suitable for all types of filter design.

➢ Frequently Asked Questions

(1) State any two properties of Butterworth filter.

Ans : i. The magnitude response of Butterworth filter decreases monotonically as the frequency Ω increases from 0 to ∞.

ii. The magnitude response of Butterworth filter closely approximates the ideal response as the order N increases.

iii. The POLEs of Butterworth filter lies on Circle.

(2) What are the characteristics of Chebyshev filter?

Ans : i. The magnitude response of Chebyshev filter exhibits ripple in passband or in stopband according to type.

ii. The POLEs of Butterworth filter lies on Ellipse.

(3) Distinguish between the frequency response of Chebyshev Type-1 and Type-2 filter.

Ans : Type-1 Chebyshev filters are all POLE filters that exhibits equiripple behavior in the passband and a monotonic behavior in the stopband.

Type-2 Chebyshev filters contains both POLEs and ZEROs and exhibits a monotonic behavior in the passband and equiripple behavior in the stopband.

(4) How to convert Analog LPF to LPF with specified Cutoff frequency?

Ans :

$$H_{LPF}(s) = \hat{H}(s)\Big|_{S=\frac{S}{\Omega c}}$$

(5) How to convert Analog LPF to HPF with specified Cutoff frequency?

Ans :

$$H_{HPF}(s) = H_{LPF}(s)\Big|_{S=\frac{\Omega_c}{s}}$$

(6) How to convert Analog LPF to BPF with specified Cutoff frequency?

Ans :

$$H_{BPF}(s) = H_{LPF}(s)\Big|_{S=\frac{S^2+\Omega_o^2}{SB}}$$

(7) How to convert Analog LPF to Band Stop Filter with specified Cutoff frequency?

Ans :

$$H_{BSF}(s) = \hat{H}_{LPF}(s)\Big|_{S=\frac{SB}{S^2+\Omega_o^2}}$$

(8) What are the input specifications required to design Analog filter?

Ans : Attenuation in Pass Band and Stop-Band.

Pass Band and Stop-Band Edge Frequencies

Sampling Frequency

(9) How to calculate the cutoff frequency of analog butterworth filter ?

Ans :

$$\Omega_C = \frac{\Omega_p}{\left(10^{Ap/10} - 1\right)^{\frac{1}{2N}}}$$

(10) What is the cutoff frequency of analog Chebyshev filter ?

Ans : For Analog Chebyshev Filter,

Cutoff frequency = Pass Band Frequency.

(11) What is Digital filter ?

Ans : Digital filter is a discrete time System which produces a discrete time output sequence y[n] for the discrete time input sequence x [n]. Digital filter is nothing but mathematical algorithm implemented in hardware or software.

(12) What is Real time Digital filter?

Ans : Real time digital filter consist of processing of real time signal using digital device called digital processor.

(13) What are the Advantages of Digital filters-?

Ans : The following list gives some of the main advantages of digital over analog filters.

1. A digital filter is *programmable*,
2. Digital filters are easily *designed, tested* and *implemented* on a general-purpose computer or workstation.
3. Digital filters are Stable filters

(14) What is Infinite Impulse Response (I I R) filter ?

Ans : If the impulse response of the system is of infinite duration, the system is said to be I I R filter system.

<u>Ex.</u> $h[n] = \left(\frac{1}{2}\right)^n u[n]$. Lenth $= \infty$

(15) What is Finite Impulse Response (FIR) filter ?

Ans : If the impulse response of the system s of finite duration then the system is said to be FIR system.

Ex : $h_1[n] = \{1,2,3,4\}$ Length = 4 (finite)

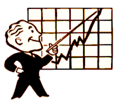
$$h_2[n] = \left\{\overset{\uparrow}{1},2,3,2,1\right\}$$ Length = 5 (fnite)

$\uparrow$

(16) What are Advantages of FIR Filters-?

Ans:

1. They can easily be designed to be "linear phase"
2. They are suited to multi-rate applications.
3. They have desirable numeric properties.
4. They can be implemented using fractional arithmetic.
5. They are simple to implement.



(17) What are the *disadvantages* of FIR Filters (compared to IIR filters)?

Ans : Compared to IIR filters, FIR filters sometimes have the disadvantage that they require more memory and/or calculation to achieve a given filter response characteristic.

(18) What are the advantages of IIR filters (compared to FIR filters)?

Ans : IIR filters can achieve a given filtering characteristic using less memory and calculations than a similar FIR filter.

(19) What are the disadvantages of IIR filters (compared to FIR filters)?

Ans :

1) They are more susceptible to problems of finite-length arithmetic, such as noise generated by calculations, and limit cycles.
2) They are harder to implement using fixed-point arithmetic.
3) They don't offer the computational advantages of FIR filters for multirate (decimation and interpolation) applications.

(20) What is the relation between Analog filter POLE and Digital filter POLE when impulse invariant technique is used for filter design.

Ans :

$Z = e^{ST}$

(21) What is the relationship between Analog filter frequency and digital filter frequency when impulse invariant technique is used for filter design.

Ans : $W = \Omega T$

(22) Why Impulse Invariant method is not suitable for HPF / BPF design?

Ans : The mapping from the analog frequency Ω to the freq. variable w in the digital domain is many to one. which reflects the effect of aliasing due to sampling. A one to one mapping is thus possible only if freq. Ω lies in the principle range of $-\frac{\pi}{T} \leq \Omega \leq \frac{\pi}{T}$.

That means if cut off frequency of analog filter Ωc is greater than $\frac{\pi}{T}$. then one to one mapping from analog filter frequency to digital filter frequency is not possible. **Therefore the filter such as HPF or BPF with cut off frequency of analog filter Ωc greater than $\frac{\pi}{T}$. can not be designed using impulse invariant method.**

(23) What is the relation between Analog filter pole and digital filter pole when BLT method is used for filter design.

Ans : $S = \dfrac{2(z-1)}{T(z+1)}$

(24) What is the relationship between Analog filter frequency and digital filter frequency when BLT method is used for filter design.

Ans : $\Omega = \dfrac{2}{T}\tan\left(\dfrac{w}{2}\right)$

(25) Explain the Mapping of points from s-plane to z–plane

(26) Explain frequency warping in BLT.

Ans : In BLT, Analog filter frequency in the rage { 0 to ∞) is compressed to Digital filter frequency in the rage [0 to π]. This frequency compression is called frequency warping.

(27) Frequency warping is needed to perform in BLT technique but not in impulse invariance technique OR In BLT there is no aliasing

Ans :

Bilinear Transformation is a mapping of points from s-plane to corresponding points in the z-plane. The BLT transforms, the entire $j\Omega$ axis in the s-plane into one revolution of the unit circle in the z-plane ie. only once and therefore avoids the aliasing of frequency components.

(28) Explain how to find output of digital IIR filter in real time application.

Ans : In real time applications, output of IIR filter can be obtained by evaluating difference equation.

(29) What do you mean by invariant ?

Ans : Invariant means, Not variant, ie. Doesn't change.

(30) Can we use Overlap Add Method and Overlap Save Method to find output of IIR filter for long data sequence.

Ans : No.

(31) What are the quantization errors due to finite word length registers in Digital filters ? **[May-2011 Q1 (b) Marks=5]**

Ans **:**

When the Digital systems are implemented either in hardware or software, the filter coefficients are stored in binary registers. These registers can accomodate only a finite number of bits and hence, the filter coefficients have to be truncated or round-off in order to fit into these registers. Truncation or rounding off the coefficients is called quantization. This process of quantization introduces error in the system performance. The position of the POLE and ZERO gets shifted to new location.

Rounding and Truncation Errors : The magnitude of quantization Error depends on the number of bits truncated.

(32) Why IIR filter cannot have a linear phase :

Ans : The physically realizable and stable IIR filter cannot have a linear phase. For a filter to have a linear phase, the condition is h (n) = h (N-1-n) and the filter would have a mirror image pole outside the unit circle for every pole inside the unit circle. This results in an unstable filter. As a result, a causal and stable IIR filter cannot have a linear phase.

(33) What are the Advantages and Dis-advantages of BLT Method of filter design?

Ans :

Advantages :

1. The BLT provides one-to-one mapping.

2. Stable continuous systems can be mapped into realizable, stable digital systems.

3. There are NO aliasing.

Dis-Advantages :

1. The mapping is highly non-linear producing frequency compression at high frequencies.

2. Impulse Response as well as the phase response of analog filter is NOT preserved in a digital filter.

(34) Compare between Analog Butterworth Filter and Analog Chebyshev Filter.

Ans :

(i.) The magnitude response of Butterworth filter decreases monotonically as the frequency increases. There are NO ripples in pass band and stop band.
The magnitude response of Chebyshev filter exhibits ripple pass band and stopband depending on its type.
Chebyshev-1 filter shows ripple in Passband and NO ripple in Stop band.
Chebyshev-2 filter shows ripple in Stopband and NO ripple in Pass band.

(ii.) The transition band in butterworth filter is more compared to Chebyshev filter.

(iii.) The number of POLES in Butterworth filter are more i.e. order of filter in butterworth filter is more as compared to chebyshev filter for the same specifications.

(iv.) The POLES of Butterworth filter lie on a circle, where as the POLES of the Chebyshev lie on the ellipse.

(v.) Butterworth filter requires More hardware components as compared to chebyshev filter due to larger value of order.