Kiran Talele's

Introduction to FIR Filter : Linear Please Comcept

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Sardar Patel Institute of Technology, Andheri, Mumbai

Dr. Kiran TALELE

Dean – Students, Alumni & External Relations

Bharatiya Vidya Bhavans'
Sardar Patel Institute of Technology Andheri(w) Mumbai

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Dr. Kiran TALELE
Teacher . Researcher . Guide . Mentor . Coach

@ **Bharatiya Vidya Bhavans' Sardar Patel Institute of Technology Andheri(w) Mumbai**

- **Associate Professor,** Electronics Engineering Department (1997)
- **Dean,** Students, Alumni & External Relations (2022)

@ **Sardar Patel Technology Business Incubator(SP-TBI), Funded by Department of Science & Technology(DST), Govt. of India**

- **Co-ordinator** (2015)

@ **IEEE Bombay Section**

- **Treasurer** (2020)
- **Executive Committee Member** (2015)

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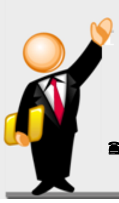
Concept of Linear Phase



Kiran.Tulshiram.Vasumati.Talele

9987030881 kiran.talele@spit.ac.in

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Q1. Show that FIR filter is always STABLE

ANS : Consider a Causal Digital FIR filter with impulse response

$$h[n]=\{\underset{\uparrow}{1}\quad 2\quad 3\quad 4\}$$

$$H(z)= 1+ 2z^{-1}+3z^{-2}+4z^{-3}$$

$$H(z) = \frac{1+ 2z^{-1}+3z^{-2}+4z^{-3}}{1}$$

$$H(z) = \frac{z^3\left(1+ 2z^{-1}+3z^{-2}+4z^{-3}\right)}{z^3\quad(1)}$$

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$$H(z) = \frac{z^3+2z^2+3z^1+4}{z^3}$$

POLES : $P_1 = P_2 = P_3 = 0$

All POLES are at ORIGIN

In FIR Filter POLES are always only at origin

For causal & stable system, all the POLES must lie INSIDE the unit circle

Therefore, FIR Filter is always STABLE

Q2. Plot Magnitude and Phase Spectrum of a Linear Phase FIR filter with h[n]={ 3,2,1,2,3 }

ANS :

$$h[n]=\{\underset{\uparrow}{3}\quad 2\quad 1\quad 2\quad 3\}$$ Length N = 5

$$H(z)=3+ 2z^{-1}+ z^{-2}+2z^{-3}+3z^{-4}$$

Put $z=e^{jw}$

$$H(e^{jw}) = 3+2e^{-jw}+e^{-j2w}+2e^{-j3w}+3e^{-j4w}$$

$$H(e^{jw}) = e^{-j\left(\frac{N-1}{2}\right)\omega}\Big[\quad\_\_\_\_\_\_\_\_\_\_\quad\Big]$$

Put $N=5$

$$H(e^{jw}) = 3 + 2e^{-jw} + e^{-j2w} + 2e^{-j3w} + 3e^{-j4w}$$

$$H(e^{jw}) = e^{-j2\omega}\left[3e^{j2\omega} + 2e^{j\omega} + 1 + 2e^{-j\omega} + 3e^{-j2\omega}\right]$$

$$H(e^{jw}) = e^{-j2\omega}\left[\,3(e^{j2\omega} + e^{-j2\omega}) + 2(e^{j\omega} + e^{-j\omega}) + 1\,\right]$$

$$H(e^{jw}) = e^{-j2\omega}\left[6\cos(2\omega) + 4\cos(\omega) + 1\,\right]$$

Frequency Response H(w)

Phase Response Φ(w)

Real Part of Frequency Response Hr(w)

Summary :

(1) Magnitude Response

$$|H(w)| = |\,6\cos(2\omega) + 4\cos(\omega) + 1\,|$$

(2) Phase Response $\phi(w) = e^{-j2\omega} = e^{j\phi}$

(3) Phase $\phi = -2w$

(4) Generalized Phase

$$\phi = \begin{cases} -2w & if \;\; \mathrm{Re}\{H(w)\} \ge 0 \\ -2w + \pi & if \;\; \mathrm{Re}\{H(w)\} < 0 \end{cases}$$

Sr. NO	Freq. W	$H_r(w)$	Phase ϕ	Phase ϕ
1	0	11.00	0	0
2	0.1 π	9.66	– 0.2 π	– 0.2 π
3	0.2 π	6.09	– 0.4 π	– 0.4 π
4	0.3 π	1.50	– 0.6 π	– 0.6 π
5	0.4 π	– 2.62	– 0.8 π + π = 0.2 π	0.2 π
6	0.5 π	– 5.00	– 1.0 π + π = 0	0
7	0.6 π	– 5.09	– 1.2 π + π = –0.2 π	–0.2 π
8	0.7 π	– 3.20	– 1.4 π + π = –0.4 π	–0.4 π
9	0.8 π	– 0.38	– 1.6 π + π = –0.6 π	–0.6 π
10	0.9 π	2.05	– 1.8 π + (2π) = 0.2 π	0.2 π
11	π	3.00	– 2.0 π + (2π) = 0	0

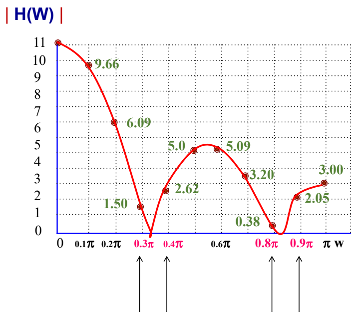
Note :

Range of freq. w is (-π, π]

Range of phase ϕ is (-π, π]

$$\phi = \begin{cases} -2w & if \;\; \mathrm{Re}\{H(w)\} \ge 0 \\ -2w + \pi & if \;\; \mathrm{Re}\{H(w)\} < 0 \end{cases}$$

Magnitude Spectrum



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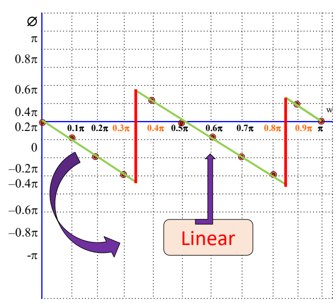
10	0.9 π	2.05
11	π	3.00

Phase Spectrum



Sr. NO	Freq. W	Phase ϕ
1	0	0
2	0.1 π	– 0.2 π
3	0.2 π	– 0.4 π
4	0.3 π	– 0.6 π
5	0.4 π	0.2 π
6	0.5 π	0
7	0.6 π	−0.2 π
8	0.7 π	−0.4 π
9	0.8 π	−0.6 π
10	0.9 π	0.2 π
11	π	0

Q3. What is Linear Phase ?

ANS : The Linear Phase means phase varies linearly with frequency

i.e. ϕ is proportional to frequency w

Q3. What is Linear Phase Filter?

ANS : The Linear Phase Filter has phase response linearly varying with frequency

i.e. ϕ is proportional to frequency w

Q(4) **What is the advantage of Linear Phase ?**

ANS : If the phase response of the filter is **Linear**
Then
The output of the filter is same as
original input delayed by some constant, say α.

There is NO distortion at the output.

Q(5) What is Phase Delay and Group Delay ?
ANS :

- **The phase delay (tp) and group delay (tg) of the filter are given by,**

$$tp = \frac{-\phi}{w} \quad and \quad tg = \frac{-d\phi}{dw}$$

Ex. Given $y[n] = 0.25\ x[n] + x[n-1] + 0.25\,x[n-2]$
Calculate Phase Delay and Group Delay.

Solution : To find Phase Delay and Group Delay

$$y[n] = 0.25\ x[n] + x[n-1] + 0.25\,x[n-2]$$

$$Y(z) = 0.25\ X(z) + z^{-1}X(z) + 0.25\ z^{-2}\ X(z)$$

$$Y(z) = X(z)\ \left(0.25\ +\ z^{-1} + 0.25\ z^{-2}\right)$$

$$H(z) = 0.25\ +\ z^{-1} + 0.25\ z^{-2}$$

$$H(e^{jw}) = 0.25\ +\ e^{-jw} + 0.25\ e^{-j2w}$$

$$H(e^{jw}) = e^{-jw}\left[\, 0.25\ e^{jw} + \ 1\ + 0.25\ e^{-jw}\,\right]$$

$$H(e^{jw}) = e^{-jw}\left[\, 1 + 0.5\ \cos(w)\,\right]$$

$$H(e^{jw}) = e^{j\phi}\left[\mathrm{Hr(w)}\right]$$

$$\phi(\mathrm{w}) = e^{-j\omega} = e^{j\phi'} \text{ Where } \phi = -w$$

- ***Phase Delay*** $t_p = \frac{-\phi}{w}$ **== 1**
- ***Group Delay*** $t_g = \frac{-d\phi}{dw}$ **== 1**

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NOTE : The necessary condition to have Linear Phase is Group Delay of the filter must be constant. ie independent of frequency

- That is possible only when h[n] is either **Symmetric or Anti-symmetric**

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For Symmetric h[n] :

- Phase : $\phi = -\left(\frac{N-1}{2}\right)w$
- **Phase Delay :** $\frac{-\phi}{w} = \left(\frac{N-1}{2}\right)$
- **Group Delay :** $\frac{-d\phi}{dw} = \left(\frac{N-1}{2}\right)$ **CONSTANT**

Example of Symmetric h[n]:

h[n] = { 1, 2, 3, 2, 1 } N = 5 ODD

h[n] = { 1, 2, 2, 1 } N = 4 EVEN

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For Anti-Symmetric h[n] :

- Phase : $\phi = \frac{\pi}{2} - \left(\frac{N-1}{2}\right)w$
- **Phase Delay :** $\frac{-\phi}{w} = \frac{-0.5\pi}{w} + \left(\frac{N-1}{2}\right)$
- **Group Delay :** $\frac{-d\phi}{dw} = \left(\frac{N-1}{2}\right)$ ⟸ CONSTANT

Example of Antisymmetric h[n]:

h[n] = { 1, 2, 0, -2, -1 } N=5 ODD

h[n] = { 1, 2, -2, -1 } N=4 EVEN

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For a Linear PHASE FIR filter h[n] must be

either Symmetric OR Antisymmetric

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Note :

If the Phase Response is Linear the output of the Filter during pass-band is delayed input.

Example :

Consider a LPF with frequency response $H(e^{-jw\alpha})$ given by

$$H(e^{jw}) = \begin{cases} e^{-jw\alpha} & |w| \le w_c \\ 0 & wc < |w| \le \pi \end{cases}$$

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$$\frac{x[n]}{X(w)} \longrightarrow \boxed{H(e^{jw})} \longrightarrow \frac{y[n]}{Y(w)}$$

Let Y(w) = X(w) . H(w)

Y(w) = X(w) . $e^{-jw\alpha}$

By iDTFT,

y[n] = x[n – α] ← o/p of filter

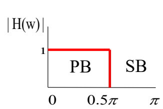
Q6. Consider a Linear Phase LPF with frequency response,

$$H(w) = \begin{cases} e^{-j2w} & 0 \le w \le 0.5\pi \\ 0 & \text{otherwise} \end{cases}$$

Show that if the phase response of the filter is Linear, then output of filter is delayed version of input signal

A Linear Phase LPF with frequency response,

$$H(w) = \begin{cases} e^{-j2w} & 0 \le w \le 0.5\pi \\ 0 & \text{otherwise} \end{cases}$$

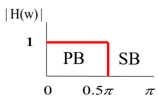


$|H(w)| = 1 \quad \text{for} \quad 0 \le w \le 0.5\pi$

$\phi = -2w$

$\phi \;\; \alpha \;\; w \;\; (\textit{Hence}, \text{Linear Phase})$

Consider $x[n] = 3\,\mathrm{Sin}(0.1\pi n) - 4\,\mathrm{Cos}(0.2\pi n)$
where $w_1 = 0.1\pi$ and $w_2 = 0.2\pi$



W	| H(w) |	Phase $\phi = -2w$
0.1π	1	$\phi_1 = -0.2\pi$
0.2π	1	$\phi_2 = -0.4\pi$

To find output y[n] :

$y[n] = 3\,(\,|H(w_1)|\,)\,\mathrm{Sin}[\,0.1\pi\,n + (\phi_1)\,] - 4\,(\,|H(w_2)|\,)\,\mathrm{Cos}[\,0.2\pi\,n + (\,\phi_2)\,]$

$y[n] = 3\,(1)\,\mathrm{Sin}[\,0.1\pi n + (\,-0.2\pi\,)\,] - 4\,(1)\,\mathrm{Cos}[\,0.2\pi n + (\,-0.4\pi\,)\,]$

$y[n] = 3\;\mathrm{Sin}[\,0.1\pi(n-2)\,] - 4\,\mathrm{Cos}[\,0.2\pi\,(n-2)\,]$

$y[n] = x[n-2]$

Here, output is delayed version of input

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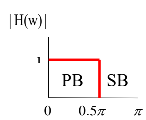
Q(7) Show that if the phase response of the filter is NOT Linear, then output of filter is **distorted version** of input signal

ANS : Consider a Non Linear Phase LPF with frequency response,

$$H(w) = \begin{cases} e^{-j3w^2} & 0 \le w \le 0.5\pi \\ 0 & \text{otherwise} \end{cases}$$



$$|H(w)| = 1 \quad \text{for} \quad 0 \le w \le 0.5\pi$$

$$\phi = -3w^2 \quad (\textbf{Non - Linear Phase})$$

Dr. Kiran TALELE 9987030881 talelesir@gmail.com 26

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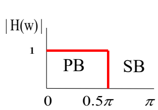
Consider $x[n] = 3\,\mathrm{Sin}(0.1\pi n) - 4\,\mathrm{Cos}(0.2\pi n)$
where $w_1 = 0.1\pi$ and $w_2 = 0.2\pi$



W	| H(w) |	Phase $\phi = -3w^2$
0.1π	1	$\phi_1 = -0.03\,\pi^2$
0.2π	1	$\phi_2 = -0.12\,\pi^2$

To find output y[n] :

$y[n] = 3\,(1)\,\mathrm{Sin}[\,0.1\pi n + (-0.03\,\pi^2)\,] - 4\,(1)\,\mathrm{Cos}[\,0.2\pi n + (-0.12\,\pi^2)\,]$

Here, output is Distorted version of input

Dr. Kiran TALELE 9987030881 talelesir@gmail.com 27

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NOTE : **For Linear Phase filter h[n] must be either Symmetric or Antisymmetric**

Examples of Linear Phase filters	Examples of Non Linear Phase filters
h[n] = { 3, 2, 1, 2, 3 }	h[n] = { 1, 2, 3, 1, 2, 3 }
h[n] = { 1, 2, 2, 1 }	h[n] = { 3, 2, 1, 1, 2 }
h[n] = { 1, -2, 0, 2, -1 }	h[n] = { 3, 2, 1, -2, -3 }
h[n] = { 1, 2, 0, 2, 1}	h[n] = { 3, -2, 2, 3 }

Dr. Kiran TALELE

- Academic : PhD
- Professional :
  - Dean-Students, Alumni & External Relations @ Bharatiya Vidya Bhavans' Sardar Patel Institute of Technology (SP-IT), Mumbai
  - Co-ordinator @ Sardar Patel Technology Business Incubator (SP-TBI), Mumbai
  - Treasurer-IEEE Bombay Section

091-9987030881 www.talelesir.com talelesir@gmail.com

https://www.linkedin.com/in/k-t-v-talele/

www.facebook.com/Kiran-Talele-1711929555720263

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Anudaan Jagruti