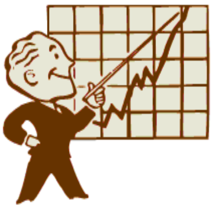


DT System Practice Problems

(Types of System)

Q(1) Is the system y[n] =ln(x[n]) Linear and Time Invariant ?

Solution :

(1) Linear System

A system that satisfies the superposition principle is called Linear System.
If a system is Linear then,
$T\{ a . x_1[n] + b\, x_2[n] \} = a_1\ T\{x_1[n]\} + a_2\, T\{x_2[n]\}$
If a system does not satisfy the superposition principle then it is Non Linear System.
$\therefore$ System is Not linear.

Example :

$y(n) = \ln(x[n])$

$x_1(n)$ —☐— $y_1(n) = \ln[x_1(n)]$

$x_2(n)$ —☐— $y_2(n) = \ln[x_2(n)]$

$x_1(n) + x_2(n)$ —☐— $y(n)$

where $y[n] = \ln\{x_1[n] + x_2[n]\}$
$y(n) \neq y_1(n) + y_2(n)$

Therefore system is NOT Linear System.

(2) Time Invariant System

A system is called Time Invariant if a time shift in the input signal causes a time shift in the output signal.

Ie If $x[n]$ —☐— $y[n]$

Then $x[n-m]$ —☐— $y[n-m]$.

Otherwise the system is Time Variant System.

Example :

$x[n]$ —☐— $y[n] = \ln(x[n])$

Delay by k,
$x[n-k]$ —☐— $y'[n] = \ln(x[n-k])$

Let y [n] = ln(x[n])

Delay by k,
y [n – k] = ln(x[n-k])

Since $y'[n] = y[n-k]$

Therefore System is Time Invariant.

(POLE-ZERO Plot)

Q(2) Draw POLE - ZERO plot and Sketch Magnitude Response and Phase Response of Finite Impulse Response Filter which is given by,
$h[n] = (0.5)^n$; $0 \le n \le 7$

Solution : $h[n] = (0.5)^n$; $0 \le n \le 7$

$$H(z) = \sum_{n=0}^{7} (0.5)^n z^{-n}$$

$$H(z) = \sum_{n=0}^{7} (0.5\, z^{-1})^n$$

$$H(z) = \frac{1-(0.5z^{-1})^8}{1-0.5z^{-1}}$$

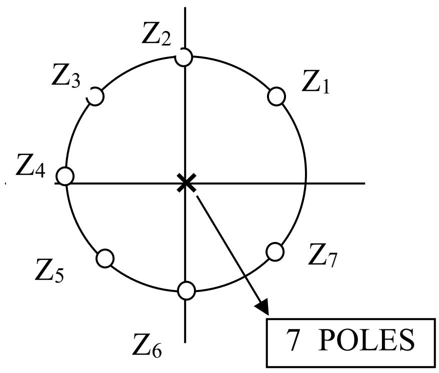
$$H(z) = \frac{z^8-(0.5)^8}{z^7(z-0.5)}$$

ZEROS : $Z^8 - (0.5)^8 = 0$

$Z^8 = (0.5)^8$

$Z^8 = (0.5)^8 e^{j2\pi k}$

$Z^8 = (0.5)^8 e^{j\pi k}$, k = 0, 1, … 7



k = 0 , $Z_o = 0.5$

k = 1 , $Z_1 = 0.5 \angle \pi/4$

k = 2 , $Z_2 = 0.5 \angle \pi/2$

k = 3 , $Z_3 = 0.5 \angle 3\pi/4$

k = 4 , $Z_4 = 0.5 \angle \pi$

k = 5 , $Z_5 = 0.5 \angle 0\pi/4$

k = 6 , $Z_6 = 0.5 \angle 3\pi/2$

k = 7 , $Z_7 = 0.5 \angle 7\pi/4$

The ZERO at $Z_o = 0.5$ cancels the pole at 0.5.

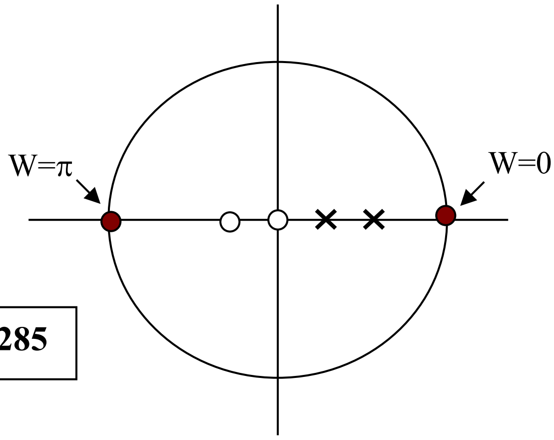
(Frequency Response)

Q(3) Given $H(z) = \dfrac{z(z+0.2)}{(z-0.5)(z-0.3)}$.

(a) Find Magnitude Response at w = 0 and w=π using graphical method.

(a) Verify your results using analytical method.

Solution(a): Graphical method



Given $H(z) = \dfrac{z(z+0.2)}{(z-0.5)(z-0.3)}$

POLES : $P_1 = 0.5$ and $P_2 = 0.3$
ZEROS : $Z_1 = 0$ and $Z_2 = -0.2$

(i) At w = 0

$$H(w) = \frac{d_1\ d_2}{d_3\ d_4} = \frac{(1)\ (1.2)}{(0.5)(0.7)} = \frac{1.2}{0.35} = \boxed{\mathbf{3.4285}}$$

(ii) At w = π

$$H(w) = \frac{d_1\ d_2}{d_3\ d_4} = \frac{(1)\ (0.8)}{(1.5)(1.3)} = \frac{0.8}{1.95} = \boxed{\mathbf{0.4102}}$$

Solution(a): Analytical Method

Given $H(z) = \dfrac{z(z+0.2)}{(z-0.5)(z-0.3)}$

(i) At w = 0 z=e^{jw} = 1 $\qquad H(w) = \dfrac{(1)(1+0.2)}{(1-0.5)(1-0.3)} == 3.4285$

(ii) At w = π z=e^{jw} = –1 $\qquad H(w) = \dfrac{(-1)(-1+0.2)}{(-1-0.5)(-1-0.3)} == 0.4102$

Q(4) Given $H(e^{jw}) = e^{-j2w}\,[\,2 + 1.8\ \cos(2w) + 1.2\cos(w)\,]$. Find h[n].

Solution :

$$H(e^{jw}) = e^{-j2w}\,[\,2 + 1.8\cos(2w) + 1.2\cos(w)\,]$$

$$\therefore H(e^{jw}) = e^{-j2w}\,[\,2 + 1.8\left(\frac{e^{j2w} + e^{-j2w}}{2}\right) + 1.2\left(\frac{e^{jw} + e^{-jw}}{2}\right)\,]$$

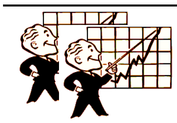
$$\therefore H(e^{jw}) = e^{-j2w}\,[\,2 + 0.9\,e^{j2w} + 0.9\,e^{-j2w} + 0.6\,e^{jw} + 0.6\,e^{-jw}\,]$$

$$\therefore H(e^{jw}) = 2\,e^{-j2w} + 0. + 0.9\,e^{-j4w} + 0.6\,e^{-jw} + 0.6\,e^{-j3w}$$

$$\therefore H(z) = 2\ z^{-2} + 0.9 + 0.9\ z^{-4} + 0.6\ z^{-1} + 0.6\,z^{-3}$$

$$\therefore H(z) = 0.9 + 0.6\,z^{-1} + 2\,z^{-2} + 0.6\ z^{-3} + 0.9\ z^{-4}$$

By iZT, **h[n] = { $\underset{\uparrow}{0.9}$, 0.6, 2, 0.6, 0.9 } ANS**



Q(5) Given $H(z) = \dfrac{0.7z^2 - 0.252}{z^2 - 0.1\ \ z - 0.72}$.

Find **Magnitude Response** and **Phase Response** of IIR filter.

Solution :

Given $$H(z) = \frac{0.7z^2 - 0.252}{z^2 - 0.1\ \ z - 0.72}$$

I I R Filter

Put $z = e^{jw}$ $$H(e^{jw}) = \frac{0.7e^{j2w} - 0.252}{e^{j2w} - 0.1\ \ e^{jw} - 0.72}$$

$$H(e^{jw}) = \frac{0.7[\cos(2w) + j\sin(2w)] - 0.252}{[\cos(2w) + j\sin(2w)] - 0.1\ [\cos(w) + j\sin(w)] - 0.72}$$

$$H(e^{jw}) = \frac{[0.7(\cos(2w) - 0.252] + j\,[0.7\sin(2w)]}{[\cos(2w) - 0.1\cos(w) - 0.72] + j[\sin(2w) - 0.1\sin(w)]}$$

(i) Magnitude Response $= \dfrac{Magnitude\ of\ Numerator}{Magnitude\ of\ Denominator}$

Where Magnitude $= \sqrt{(\mathrm{Real})^2 + (\mathrm{Imaginary})^2}$

$$\underset{\substack{Magnitude\\ Response}}{|H(e^{jw})|} = \frac{\sqrt{[0.7\cos(2w) - 0.252]^2 + [0.7\sin(2w)]^2}}{\sqrt{[\cos(2w) - 0.1\cos(w) - 0.72]^2 + [\sin(2w) - 0.1\sin(w)]^2}}$$

(ii) Phase Response = Angle of Numerator – Angle of denominator

$$\text{Where angle} = \begin{cases} \tan^{-1}\left(\dfrac{Imaginary}{Real}\right) & When\ \ Real > 0 \\ \tan^{-1}\left(\dfrac{Imaginary}{Real}\right) + \pi & When\ \ Real < 0 \end{cases}$$

$$\underset{\substack{\text{Phaze}\\ \text{response}}}{\angle H(e^{jw})} = \tan^{-1}\left[\frac{0.7\sin(2w)}{0.7\cos(2w) - 0.252}\right] - \tan^{-1}\left[\frac{\sin(2w) - 0.1\sin(w)}{\cos(2w) - 0.1\cos(w) - 0.72}\right]$$

Q(6) Obtain the plot of Magnitude Response and Phase Response of a filter with the following impulse response. h(n) = { –1, 2, –2, 1 } **[Marks=10]**

Solution :

$h(n) = \{ -1, 2, -2, 1 \}$

By ZT,

$H(z) = -1 + 2z^{-1} - 2z^{-2} + z^{-3}$

Put $z = e^{jw}$

$$H(e^{jw}) = -1 + 2e^{-jw} - 2e^{-j2w} + e^{-j3w}$$

$$H(e^{jw}) = e^{-j\frac{3}{2}w}\left[-e^{j\frac{3}{2}w} + 2\,e^{j\frac{1}{2}w} - 2\,e^{-j\frac{1}{2}w} + e^{-j\frac{3}{2}w}\right]$$

$$H(e^{jw}) = e^{-j\frac{3}{2}w}\left[-2j\,\sin\left(\tfrac{3}{2}w\right) + 4j\sin\left(\frac{1}{2}w\right)\right]$$

$$H(w) = e^{-j\frac{3}{2}w}\,e^{j\frac{\pi}{2}}\left[-2\,\sin\left(\frac{3}{2}w\right) + 4\sin\left(\frac{1}{2}w\right)\right]$$

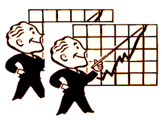
$$H(w) = e^{j\left(\frac{\pi}{2} - \frac{3}{2}w\right)}\left[-2\,\sin\left(\frac{3}{2}w\right) + 4\sin\left(\frac{1}{2}w\right)\right]$$

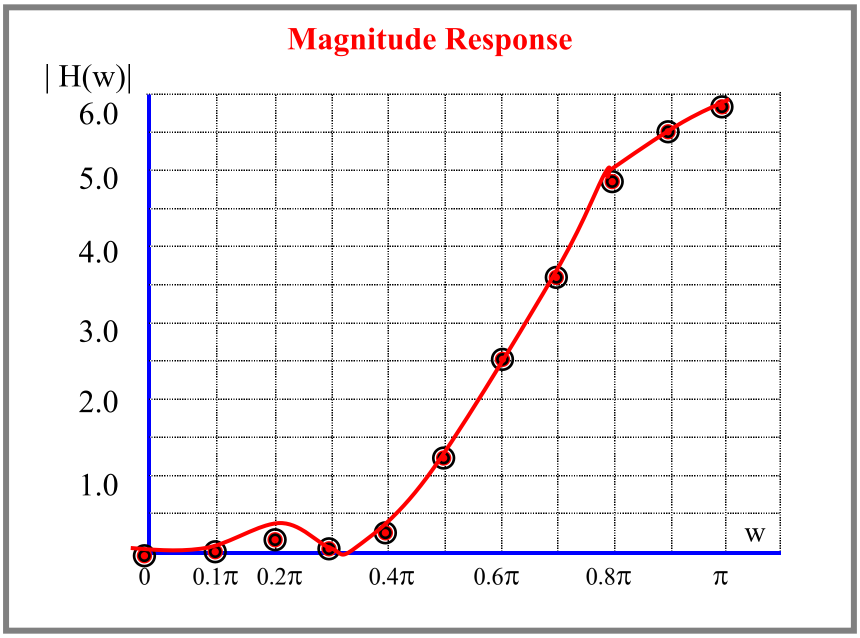
Now,

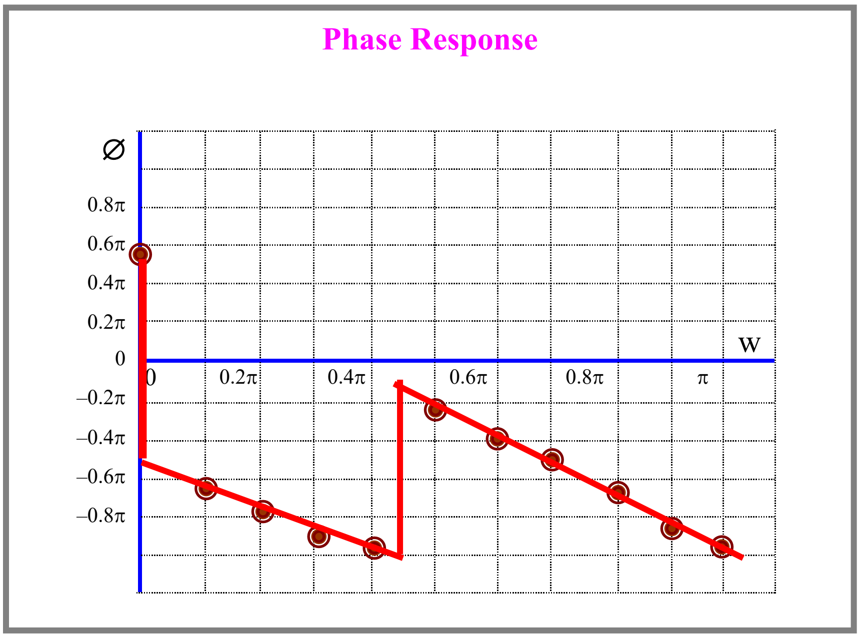
$$\text{Magnitude Response} \;:\left|\; -2\,\sin\left(\frac{3}{2}\mathrm{w}\right) + 4\sin\left(\frac{1}{2}\mathrm{w}\right)\right|$$

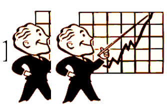
Phase : $\boldsymbol{\phi = 0.5\,\pi - 1.5\,\omega}$

W	$H_r(w)$	ϕ
0	0	$0.5\,\pi$
$0.1\,\pi$	– 0.28	$0.35\,\pi + \pi = 1.35\,\pi - 2\,\pi = -0.65\,\pi$
$0.2\,\pi$	– 0.38	$0.25\,\pi + \pi = 1.2\,\pi - 2\,\pi = -0.8\,\pi$
$0.3\,\pi$	– 0.16	$0.05\,\pi + \pi = 1.05\,\pi - 2\,\pi = -0.95\,\pi$
$0.4\,\pi$	0.45	$-0.1\,\pi$
$0.5\,\pi$	1.41	$-0.25\,\pi$
$0.6\,\pi$	2.62	$-0.4\,\pi$
$0.7\,\pi$	3.87	$-0.55\,\pi$
$0.8\,\pi$	4.98	$-0.7\,\pi$
$0.9\,\pi$	5.73	$-0.85\,\pi$
π	6.0	$-\pi$




Magnitude Response
| H(w)|
6.0
5.0
4.0
3.0
2.0
1.0
w
0
0.1π
0.2π
0.4π
0.6π
0.8π
π


Phase Response
∅
0.8π
0.6π
0.4π
0.2π
0
–0.2π
–0.4π
–0.6π
–0.8π
w
0
0.2π
0.4π
0.6π
0.8π
π



(Transient Response and Steady State Response)

Q(7) Given $H(z) = \frac{10\,z}{z-0.5}$ Find the Response of the System to the input

$x[n] = 10 - 5\sin(0.2\pi n) + 20\cos(0.4\pi n + 0.5\pi)$.

Solution :

To Find Steady State Response (SSR)

(i) Find the input signal frequencies

$W = \{0,\ 0.2\pi,\ 0.4\pi\}$

(ii) Find the Frequency Response

Now, $H(z) = \frac{z}{z-0.5}$ Put $z = e^{jw} = \cos w + j\sin w$

$$H(z) == \frac{10\,[\cos(w) + j\sin(w)]}{[\cos(w) - 0.5] + j\sin(w)}$$

(iii) Find magnitude and Phase value for every input signal frequency

1) At $w = 0$ $\quad H(w) = 20 \angle 0$

2) At $w = 0.2\pi$ $\quad H(w) = 15 \angle -0.45$

3) At $w = 0.4\pi$ $\quad H(w) = 10.30 \angle -0.51$

(iv) Find Steady state Response (SSR)

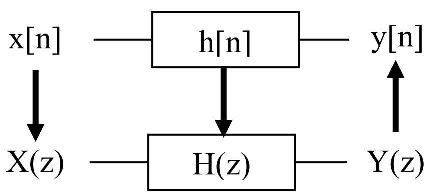
$y[n] = 10(20) - 5(15)\sin(0.2\pi n - 0.45) + 20(10.30)\cos(0.4\pi + 0.5\pi - 0.51)$.

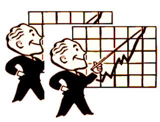
$y[n] = 200 - 75\sin(0.2\pi n - 0.45) + 206\cos(0.4\pi + 0.5\pi - 0.51)$.

Q(8) Given $H(z) = \frac{z}{z-0.5}$. Find the response of the system to the input

$x[n] = 10\cos\left(\frac{n\pi}{4}\right) u[n]$.

Solution :





(i) Find X(z)

For $\quad x[n] = 10\cos\left(n\frac{\pi}{4}\right)u(n)$

By ZT, $\quad X(z) = 10\left[\dfrac{z^2 - z\cos(\frac{\pi}{4})}{z^2 - 2z\cos\left(\frac{\pi}{4}\right)+1}\right] = 10\left[\dfrac{z^2 - 0.707\,z}{z^2 - 1.414\,z + 1}\right]$

(ii) Find Y(z)

Let $\quad Y(z) = H(z)\ X(z)$

$$Y(z) = \frac{z}{z-0.5}\ \frac{10\,z\,(z-0.707)}{z^2 - 1.44z + 1}$$

$$Y(z) = \frac{10\,z^2\,(z-0.707)}{(z-0.5)\,(z-p_1)\,(z-p_2)}$$ Where $P_1 = 0.707 + j0.707 = e^{j\frac{\pi}{4}}$

$P_2 = 0.707 - j0.707 = e^{-j\frac{\pi}{4}}$

(ii) Find y[n]

By PFE,

$$\frac{Y(z)}{z} = \frac{A}{z-0.5} + \frac{B}{z-P_1} + \frac{C}{z-P_3}$$

where $A = -1.035 \qquad B = 6.78\,e^{-j28.68}$ and $\qquad C = 6.78\,e^{j28.68}$

$$Y(z) = A\left[\frac{z}{z-0.5}\right] + B\left[\frac{z}{z-P_1}\right] + C\left[\frac{z}{z-P_3}\right]$$

By iZT, $\ y[n] = A\,(0.5)^n\,u[n] + B\,(P_1)^n\,u[n] + C\,(P_3)^n\,u[n].$

$$y[n] = -1.035\,(0.5)^n\,u[n] + 6.78\,e^{-j28.67}\left(e^{j\frac{\pi}{4}}\right)^n u[n] + 6.78\,e^{j28.68}\left(e^{j\frac{\pi}{4}}\right)^n u[n]$$

ANS : $\quad \mathbf{y[n] = -1.035\,(0.5)^n\ u[n] + 13.56}\ \cos\left[n\frac{\pi}{4} - 28.68^0\right]u[n]$

Q(9) Given $H(z) = 0.3 + 0.4\,z^{-1} + 0.5\,z^{-2} + 0.4\,z^{-3} + 0.3\,z^{-4}$

Find the response of the FIR filter to the following input signals.

(a) $x[n] = (\frac{1}{2})^n \cos(n\frac{\pi}{3})\ u[n].$

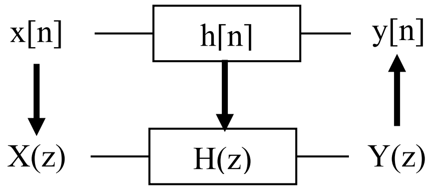
(b) $x[n] = (\frac{1}{2})^n \cos(n\frac{\pi}{3})$

Solution : (a) To find Response of the filter

(i) Input : $x[n] = (\frac{1}{2})^n \cos(n\frac{\pi}{3})\, u[n]$

Input is sinusoidal, Infinite Length and applied to the system at n = 0

Impulse Response is : $h[n] = \{ \underset{\uparrow}{0.3}, 0.4, 0.5, 0.4, 0.3 \}$ Finite Length



(i) Find Y(z) :

$$Y(z) = H(z)\, X(z)$$

$$Y(z) = (h_o + h_1 z^{-1} + h_2 z^{-2} + h_1 z^{-3} + h_o z^{-4})\, X(z)$$

$$Y(z) = 0.3\, X(z) + 0.4\, z^{-1} X(z) + 0.5\, z^{-2} X(z) + 0.4\, z^{-3} X(z) + 0.3\, z^{-4} X(z)$$

(iii) Find y[n]

By IZT, $y[n] = 0.3\, x[n] + 0.4\, x[n-1] + 0.5\, x[n-2] + 0.4\, x[n-3] + 0.3\, x[n-4]$

ANS :

$$\begin{aligned} y[n] = {} & 0.3\, (\tfrac{1}{2})^n \cos(n\tfrac{\pi}{3})\, u[n] \; + \\ & 0.4\, (\tfrac{1}{2})^{n-1} \cos(n-1)\tfrac{\pi}{3}\, u[n-1] \; + \\ & 0.5\, (\tfrac{1}{2})^{n-2} \cos(n-2)\tfrac{\pi}{3}\, u[n-2] \; + \\ & 0.4\, (\tfrac{1}{2})^{n-3} \cos(n-3)\tfrac{\pi}{3}\, u[n-3] \; + \\ & 0.3\, (\tfrac{1}{2})^{n-4} \cos(n-4)\tfrac{\pi}{3}\, u[n-4] \end{aligned}$$

(b) **Input :** $x[n] = (\frac{1}{2})^n \cos(n\frac{\pi}{3})$

$\Longrightarrow$ Input is sinusoidal, Infinite Length and applied to the system at $n = -\infty$

The output of the system is given by $y[n] = y_{tr}[n] + y_{ss}[n]$

But $y_{tr}[n] = 0 \quad \therefore \; y[n] = y_{ss}[n]$

To find $y_{ss}[n]$:

(i) Find Input signal frequency components : $w = \{ \frac{\pi}{3} \}$

(ii) Find Frequency Response

$$H(z) = 0.3 + 0.4\, z^{-1} + 0.5\, z^{-2} + 0.4\, z^{-3} + 0.3\, z^{-4}$$

Put $z = e^{jw}$

$$H(e^{jw}) = 0.3 + 0.4e^{-jw} + 0.5\,e^{-j2w} + 0.4\,e^{-j3w} + 0.3\,e^{-j4w}$$

$$H(e^{jw}) = e^{-j2w}\left(0.3\,e^{j2w} + 0.3\,e^{-j2w} + 0.4e^{jw} + 0.4\,e^{-jw} + 0.5\right)$$

$$H(e^{jw}) = e^{-j2w}\left(0.6\left[\frac{e^{j2w} + e^{-j2w}}{2}\right] + 0.8\left[\frac{e^{jw} + e^{-jw}}{2}\right] + 0.5\right)$$

$$H(e^{jw}) = e^{-j2w}[\,0.6\,\cos(2w) + 0.8\,\cos(w) + 0.5\,)\,]$$

(iii) Find magnitude and phase value for every input signal frequency :

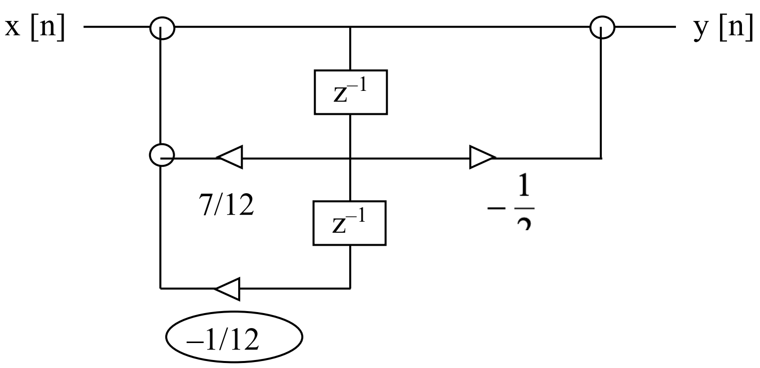
$w = \frac{\pi}{3}$ Magnitude : $|H(w)| = 0.601$ Phase : $\phi = \frac{-2\pi}{3}$

(iv) The steady state response of the system is then given by

ANS : y[n] = 0.601 $(\,\frac{1}{2}\,)^n \cos\left(n\frac{\pi}{3} + \frac{-2\pi}{3}\right)$

(Realization Diagram)

Q(10) Find Transfer function of the system **given below.**



Solution : Let Transfer function of the system H(z) be

$$H(z) = \frac{b_0 + b_1\,z^{-1} + b_2\,z^{-2}}{1 + a_1\,z^{-1} + a_2\,z^{-2}}$$

Where $b_0 = 1$ $b_1 = -\frac{1}{2}$

$-a_1 = 7/12$ $-a_2 = -1/12$

By substituting we get, $$H(z) = \frac{1 - \frac{1}{2}\,z^{-1}}{1 - \frac{7}{12}\,z^{-1} + \frac{1}{12}\,z^{-2}}$$

Q(11) Consider a casual, LTI system –

$$H(z) = \frac{1 - \frac{1}{2} z^{-1}}{\left(1 - z^{-1} + \frac{3}{16} z^{-1}\right)\left(1 + \frac{1}{4} z^{-1}\right)}$$

Realize the system in each of the following forms.

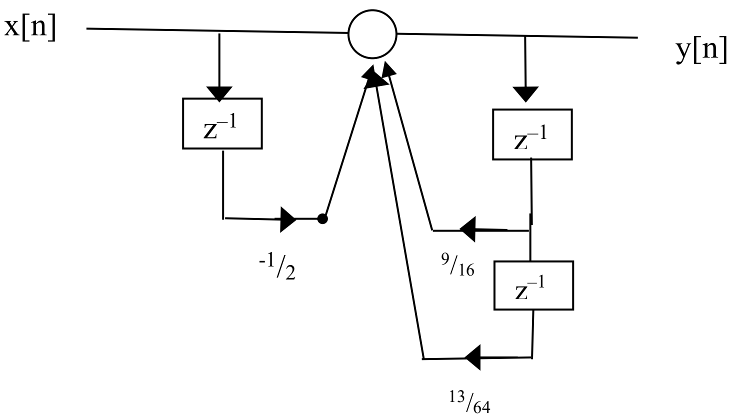
(a) Direct form I (b) Direct form II

(c) Cascade form (d) Parallel Form

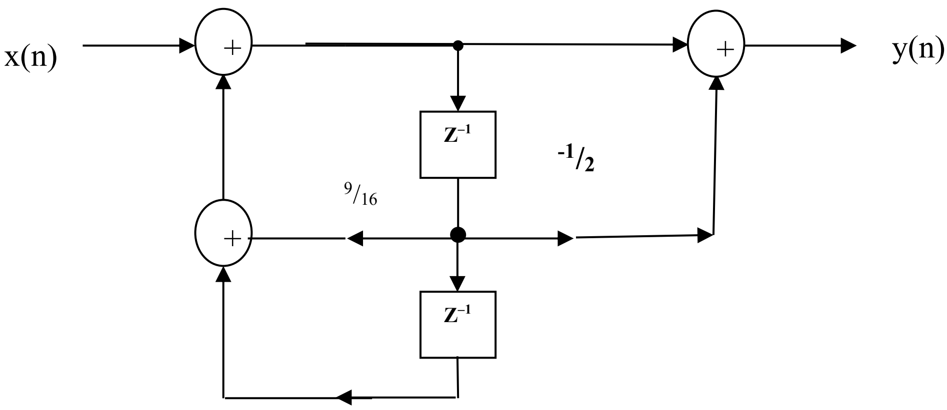
Solution :

(a) Direct form I

$$H(z) = \frac{1 - \frac{1}{2} z^{-1}}{\left(1 - \frac{13}{16} z^{-1}\right)\left(1 + \frac{1}{4} z^{-1}\right)} = \frac{1 - \frac{1}{2} z^{-1}}{1 - \frac{9}{16} z^{-1} - \frac{13}{64} z^{-2}}$$



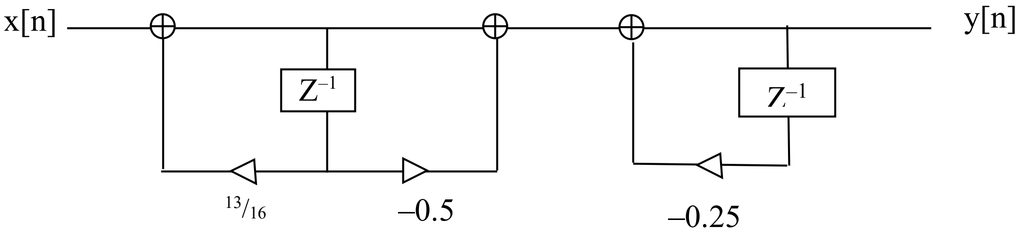
(b) Direct form II



(c) Cascade form

$$H(z)=\frac{1-\frac{1}{2}z^{-1}}{\left(1-\frac{13}{16}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

$$H(z)=\left[\frac{1-\frac{1}{2}z^{-1}}{1-\frac{13}{16}z^{-1}}\right]\left[\frac{1}{1+\frac{1}{4}z^{-1}}\right]$$



(d) Parallel form

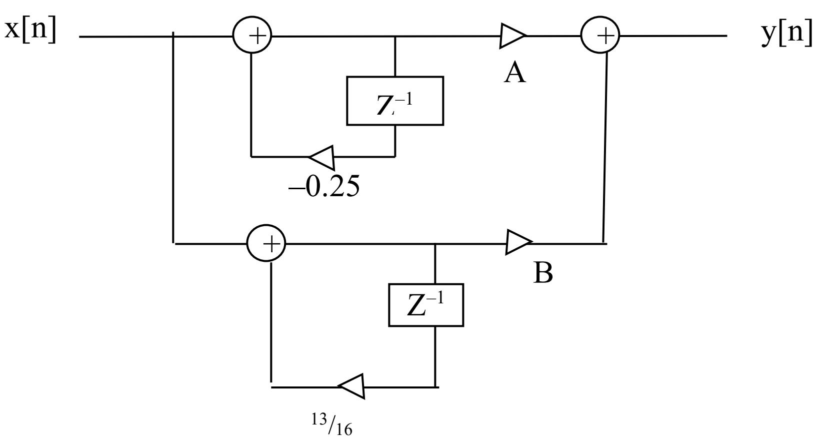
$$H(z)=\frac{1-\frac{1}{2}z^{-1}}{\left(1-\frac{13}{16}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)} \qquad H(z)=\frac{z\left(z-\frac{1}{2}\right)}{\left(z-\frac{13}{16}\right)\left(z+\frac{1}{4}\right)}$$

$$\frac{H(z)}{z}=\frac{\left(z-\frac{1}{2}\right)}{\left(z-\frac{13}{16}\right)\left(z+\frac{1}{4}\right)}$$

$$\frac{H(z)}{z}=\frac{A}{z+\frac{1}{4}}+\frac{B}{z-\frac{13}{16}}$$

$$A=\frac{H(z)}{z}\left(z+\frac{1}{4}\right)\ \text{at}\ z=\frac{-1}{4} \quad \text{and} \quad B=\frac{H(z)}{z}\left(z-\frac{13}{16}\right)\ at\ z=\frac{13}{16}$$

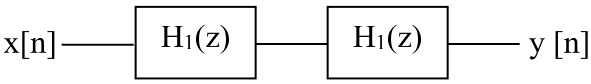
$$H(z)=\frac{A\,z}{z+\frac{1}{4}}+\frac{B\,z}{z-\frac{13}{16}}$$



Q(12) **Given** $H(z) = \dfrac{z^2}{z^2 - \frac{1}{4}}$ Show cascade and parallel form realization.

Solution :

(i) Cascade form Realization :

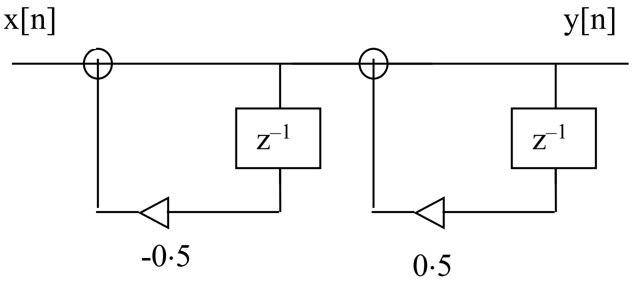


$$H(z) = \frac{z^2}{z^2 - 1/4} = \left[\frac{z}{z + 1/2}\right]\left[\frac{z}{z - 1/2}\right]$$

Let $\quad H(z) = H_1(z) \;\; H_2(z)$

Where $\quad H_1(z) = \dfrac{z}{z + 1/2} = \dfrac{1}{1 + \frac{1}{2} z^{-1}}$

$$H_2(z) = \frac{z}{z - 1/2} = \frac{1}{1 - \frac{1}{2} z^{-1}}$$

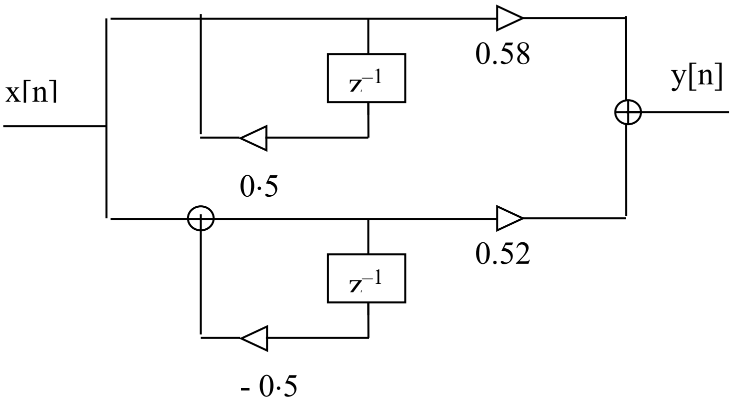


(ii) Parallel Form Realization.

$$H(z) = \frac{z^2}{z^2 - 1/4} = \frac{1}{2}\left[\frac{z}{z - 1/2}\right] + \frac{1}{2}\left[\frac{z}{z + 1/2}\right]$$

$$= \frac{0 \cdot 5}{1 - 0 \cdot 5\, z^{-1}} + \frac{0 \cdot 5}{1 + 0 \cdot 5 z^{-1}}$$

Let $\quad H(z) = H_1(z) + H_2(z)$



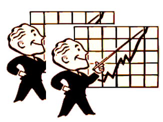
Q(13) Show cascade and parallel realization of the following casual LTI systems.

$$H(z) = \frac{10\, z\left(z - \frac{1}{2}\right)\; (z - \frac{2}{3}) \;\; (z + 2)}{(z - \frac{3}{4}) \;\; (z - \frac{1}{8}) \left(z - \frac{1}{2} - j\frac{1}{2}\right)\left(z - \frac{1}{2} + j\frac{1}{2}\right)}$$

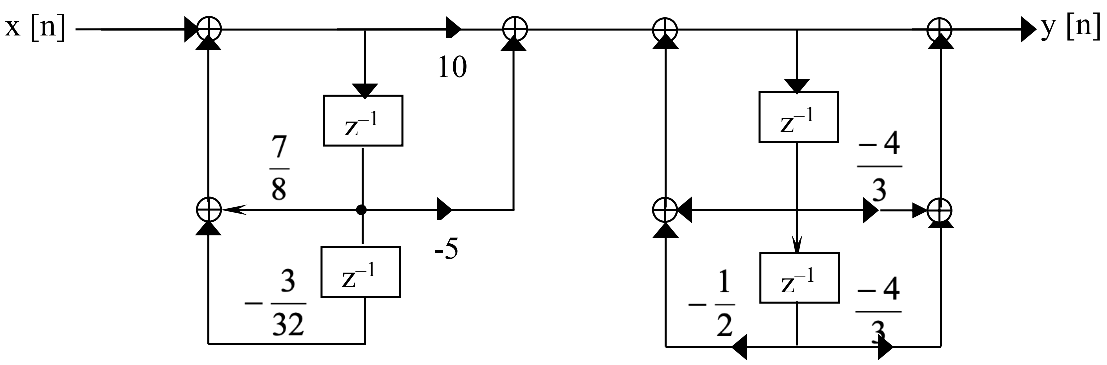
Solution: **Cascade Realization**

$$H(z) = \left[\frac{10z\left(z - \frac{1}{2}\right)}{z^2 - \frac{7}{8}z + \frac{3}{32}}\right]\left[\frac{\left(z - \frac{2}{3}\right)(z + 2)}{z^2 - z + 0 \cdot 5}\right]$$

$$H(z) = \left[\frac{10z^2 - 5z}{z^2 - \frac{7}{8}z + \frac{3}{32}}\right]\left[\frac{z^2 - \frac{4}{3}z - \frac{4}{3}}{z^2 - z + 0 \cdot 5}\right]$$



Let H (z) = $H_1(z)\,H_2(z)$



a) Parallel Realization,

$$\frac{H(z)}{z}=\frac{10\left(z-\frac{1}{2}\right)\left(2-\frac{2}{3}\right)(z+2)}{\left(z-\frac{3}{4}\right)\left(z-\frac{1}{8}\right)\left(z-\frac{1}{2}-j\frac{1}{2}\right)\left(z-\frac{1}{2}+j\frac{1}{2}\right)}$$

$$H(z)=A\left[\frac{z}{z-\frac{3}{4}}\right]+B\left[\frac{z}{z-\frac{1}{8}}\right]+C\left[\frac{z}{z-p_1}\right]+D\left[\frac{z}{z-p_2}\right]$$

Where

$p_1=\frac{1}{2}+j\frac{1}{2}$ $\qquad$ $p_2=\frac{1}{2}-j\frac{1}{2}$

A = 2·93 $\qquad$ B = 17·68

C = 12·25 – j 14·57 $\qquad$ D = 12·25 + j 14·57

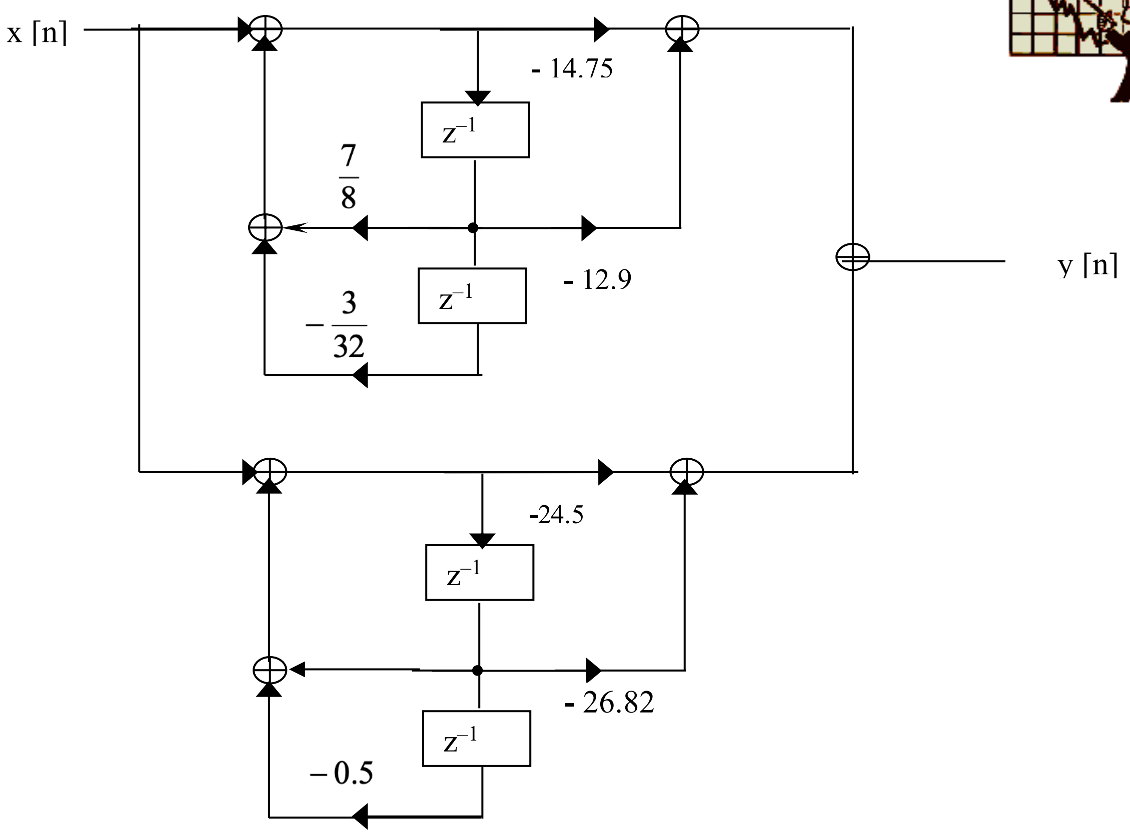
By substituting and simplifying we get,

$$H(z)=\left[\frac{-14\cdot 75z^2-12\cdot 90z}{z^2-\frac{7}{8}z+\frac{3}{32}}\right]+\left[\frac{24\cdot 50\,z^2+26\cdot 82\,z}{z^2-z+0\cdot 5}\right]$$

$$H(z)=\left[\frac{-14\cdot 57-12\cdot 90z^{-1}}{1-\frac{7}{8}z^{-1}+\frac{3}{32}z^{-2}}\right]+\left[\frac{-24\cdot 50+26\cdot 82z^{-1}}{1-z^{-1}+0\cdot 5z^{-2}}\right]$$

Let H(z) = $H_1(z) + H_2(z)$

Realization Diagram:



Q(14) **Given** $H(z) = -5 + \frac{5}{3}\left[\frac{z}{z-0\cdot 5}\right] + \frac{26}{6}\left[\frac{z}{z+0\cdot 1}\right]$

Show Direct form and Parallel form Realization.

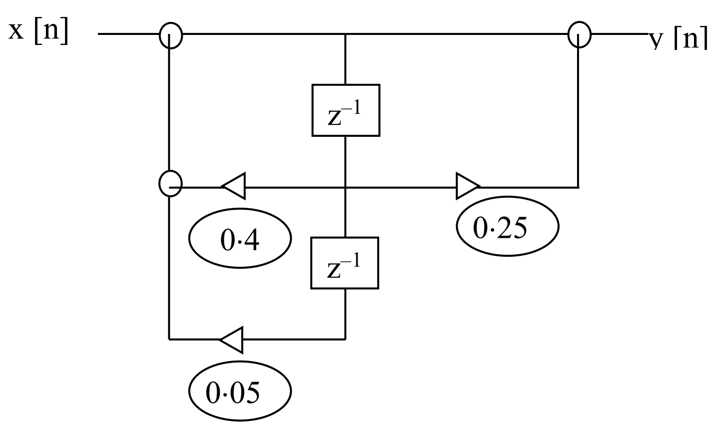
Solution : (I)Direct Form – II Realization

Given, $H(z) = -5 + \frac{5}{3}\left[\frac{z}{z-0\cdot 5}\right] + \frac{26}{6}\left[\frac{z}{z+0\cdot 1}\right]$

By solving,

$$H(z) = \frac{z^2 + 0\cdot 25}{(z-0\cdot 5)\ (z+0\cdot 1)}$$

$$H(z) = \frac{1 + 0\cdot 25z^{-1}}{1 - 0\cdot 4z^{-1} - 0\cdot 05z^{-2}}$$

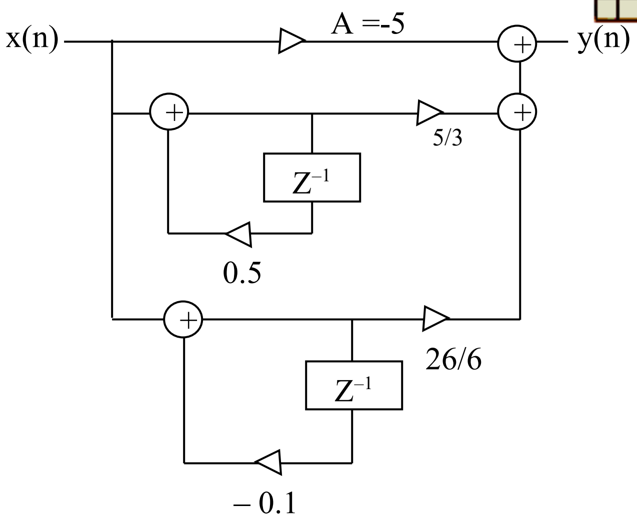


Solution :(II)Parallel form Realization.

$$H(z) = -5 + \frac{5/3\ z}{z-0\cdot 5} + \frac{26/6\ z}{z+0\cdot 1}$$

$$H(z) = -5 + \frac{5}{3}\left[\frac{z}{z-0\cdot 5}\right] + \frac{26}{6}\left[\frac{z}{z+0\cdot 1}\right]$$

Let $H(z) = -5 + H_1(z) + H_2(z)$



(Lattice Realizatiion)

Q(15) Given a three stage lattice filter with coefficients $k_1 = \frac{1}{4}$ $k_2 = \frac{1}{4}$ $k_3 = \frac{1}{3}$, Determine the FIR Filter coefficients for the direct form structure.

Solution.: $\mathbf{A_m(z) = A_{m-1}(z) + k_m z^{-1} B_{m-1}(z)}$

(i) For m = 1

$$A_1(z) = A_0(z) + k_1 z^{-1} B_0(z)$$

$$= 1 + k_1 z^{-1}$$

$$A_1(z) = 1 + \frac{1}{4} z^{-1}$$

$$\therefore B_1(z) = \frac{1}{4} + z^{-1}$$

(ii) For m = 2

$$A_2(z) = A_1(z) + k_2 z^{-1} B_1(z)$$

$$A_2(z) = 1 + \frac{1}{4} z^{-1} + \frac{1}{4} z^{-1}\left(\frac{1}{4} + z^{-1}\right)$$

$$A_2(z) = 1 + \frac{1}{4} z^{-1} + \frac{1}{16} z^{-1} + \frac{1}{4} + z^{-1}$$

$$A_2(z) = 1 + \frac{5}{16} z^{-1} + \frac{1}{4} + z^{-2}$$

$$\therefore B_2(z) = \frac{1}{4} + \frac{5}{16} z^{-1} + z^{-1}$$

(iii) For m = 3

$$A_3(z) = A_2(z) + k_3 z^{-1} B_2(z)$$

$$A_3(z) = 1 + \frac{5}{16} z^{-1} + \frac{1}{4} z^{-2} + \frac{1}{4} z^{-1}\left(\frac{1}{4} + \frac{5}{16} z^{-1} + z^{-2}\right)$$

$$A_3(z) = 1 + \frac{19}{48} z^{-1} + \frac{17}{48} z^{-2} + \frac{1}{3} + z^{-3}$$

For three stage lattice,

$$H(z) = A_3(z) = 1 + \frac{19}{48} z^{-1} + \frac{17}{48} z^{-2} + \frac{1}{3} + z^{-3}$$

By iZT $\quad h[n] = \left\{ \underset{\uparrow}{1}, \frac{19}{48}, \frac{17}{48}, \frac{1}{3} \right\}$

Q(16) Determine the FIR lattice coefficient of the filter with system Function

$$H(z) = 1 + \frac{13}{24} z^{-1} + \frac{5}{8} z^{-2} + \frac{1}{3} z^{-3}$$

Solution.: To find Latiice Coefficients $\{ k_1, k_2, k_3 \}$

(i) To find k_3

$$H(z) = A_3(z) = 1 + \frac{13}{24} z^{-1} + \frac{5}{8} z^{-2} + \frac{1}{3} z^{-3} \qquad \therefore\ k_3 = \frac{1}{3}$$

$B_3(z) =$

(ii) To find k_2 :

$$A_2(z) = \frac{A_3(z) - k_3 B_3(z)}{1 - k_3^2} = 1 + \frac{3}{8} z^{-1} + \frac{1}{2} z^{-2} \qquad \therefore\ k_2 = \frac{1}{2}$$

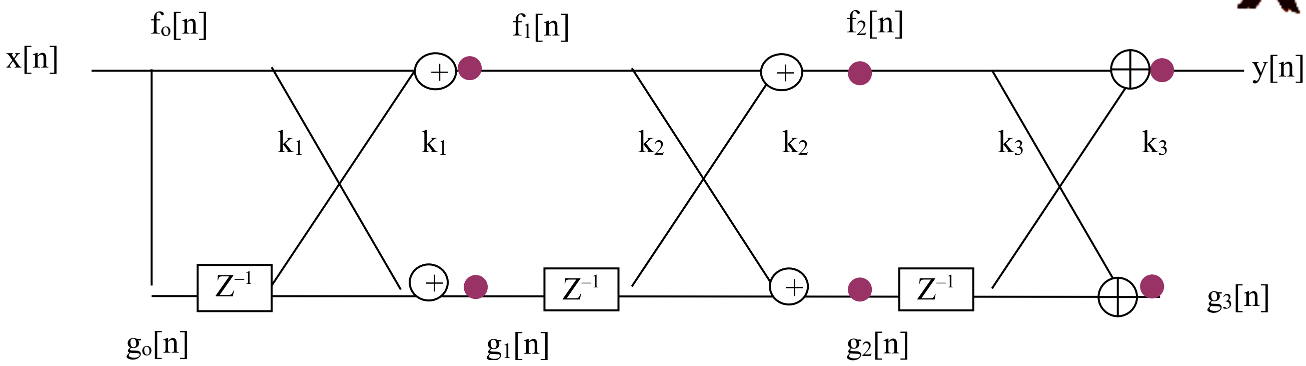
$$B_2(z) = \frac{1}{2} + \frac{3}{8} z^{-1} + z^{-2}$$

(iii) To find k1

$$A_1(z) = \frac{A_2(z) - k_2 B_2(z)}{1 - k_2^2}$$

$$A_1(z) = 1 + \frac{1}{4} z^{-1} \qquad \therefore\ k_1 = \frac{1}{4}$$

Lattice Realization Diagram of FIR filter :



Q(17) Given $H(z) = \dfrac{1}{1 + 0.9z^{-1} - 0.8z^{-2} + 0.5z^{-3}}$

Determine the equivalent lattice structure of All POLE IIR filter.

Solution.: All POLE System

For $N = 3$, $A_3(z) = 1 + 0.9z^{-1} - 0.8z^{-2} + 0.5z^{-3}$

(i.) To find k_3

$A_3(z) = 1 + 0.9z^{-1} - 0.8z^{-2} + 0.5z^{-3}$ $\quad \therefore\ k_3 = 0.5$

$B_3(z) = 0.5 - 0.8z^{-1} + 0.9z^{-2} + z^{-3}$

(ii) To find k_2

$$A_2(z) = \frac{A_3(z) - k_3 B_3(z)}{1 - k_3^2} = 1 + 1.73z^{-1} - 1.67z^{-2} \qquad \therefore\ k_2 = -1.67$$

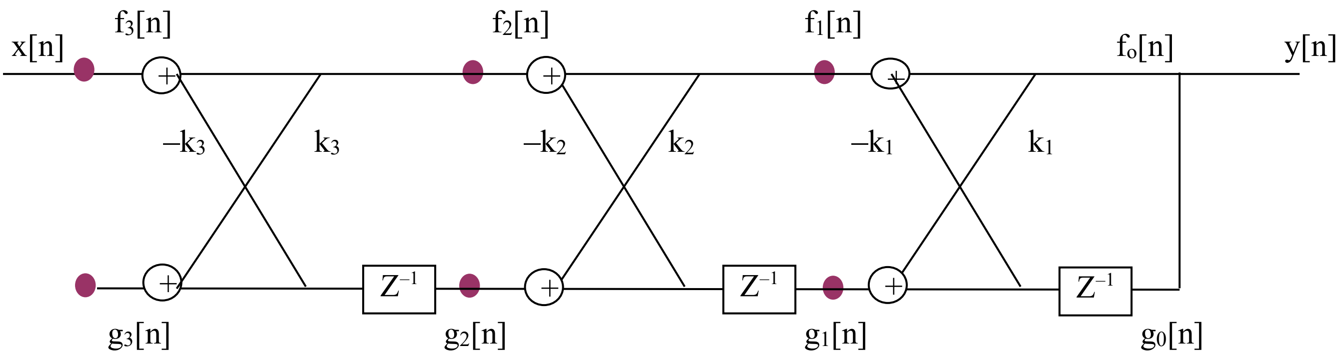
$B_2(z) = -1.67 + 1.73z^{-1} + z^{-2}$

(ii.) To find k_1,

$$A_1(z) = \frac{A_2(z) - k_2 B_2(z)}{1 - k_2^2} = 1 - 1.62z^{-2} \qquad \therefore\ k_1 = -1.62$$

$B_1(z) = -1.62 + z^{-1}$

Lattice Realization Diagram of All POLE IIR filter :



Q(18) Given $H(z) = \dfrac{1 + 2z^{-1} + 3z^{-2} + 2z^{-3}}{1 + 0.9z^{-1} - 0.8z^{-2} + 0.5z^{-3}}$

(a) Determine the equivalent lattice-ladder structure

(b) Check if the system is stable.

Sol^n.:(a) Lattice-Ladder Structure

Step (I) All POLE System

For N = 3, $A_3(z) = 1 + 0.9z^{-1} - 0.8z^{-2} + 0.5z^{-3}$

(i)To find k_3

$A_3(z) = 1 + 0.9z^{-1} - 0.8z^{-2} + 0.5z^{-3}$ $\quad \therefore\ k_3 = 0.5$

$B_3(z) = 0.5 - 0.8z^{-1} + 0.9z^{-2} + z^{-3}$

(ii) To find k_2

$$A_2(z) = \frac{A_3(z) - k_3 B_3(z)}{1 - k_3^2} = 1 + 1.73z^{-1} - 1.67z^{-2} \qquad \therefore\ k_2 = -1.67$$

$B_2(z) = -1.67 + 1.73z^{-1} + z^{-2}$

(iii.) To find k_1,

$$A_1(z) = \frac{A_2(z) - k_2 B_2(z)}{1 - k_2^2} = 1 - 1.62z^{-2} \qquad \therefore\ k_1 = -1.62$$

$B_1(z) = -1.62 + z^{-1}$

Step (II) All ZERO System.M = 3

(i) To find V_3

$C3(z) = 1+2z^{-1} + 3z^{-2} + 2z^{-3}$ So, $V_3 = 2$

(ii) To find V_2

$C_2(z) = C_3(z) - V_3\, B_3(z) = 3.6z^{-1} - 1.2z^{-2}$

$\therefore\ V_2 = -1.2$

(iv.) To find V_1

$C_1(z) = C_2(z) - V_2\, B_2(z) = -2.004 + 5.67z^{-1}$

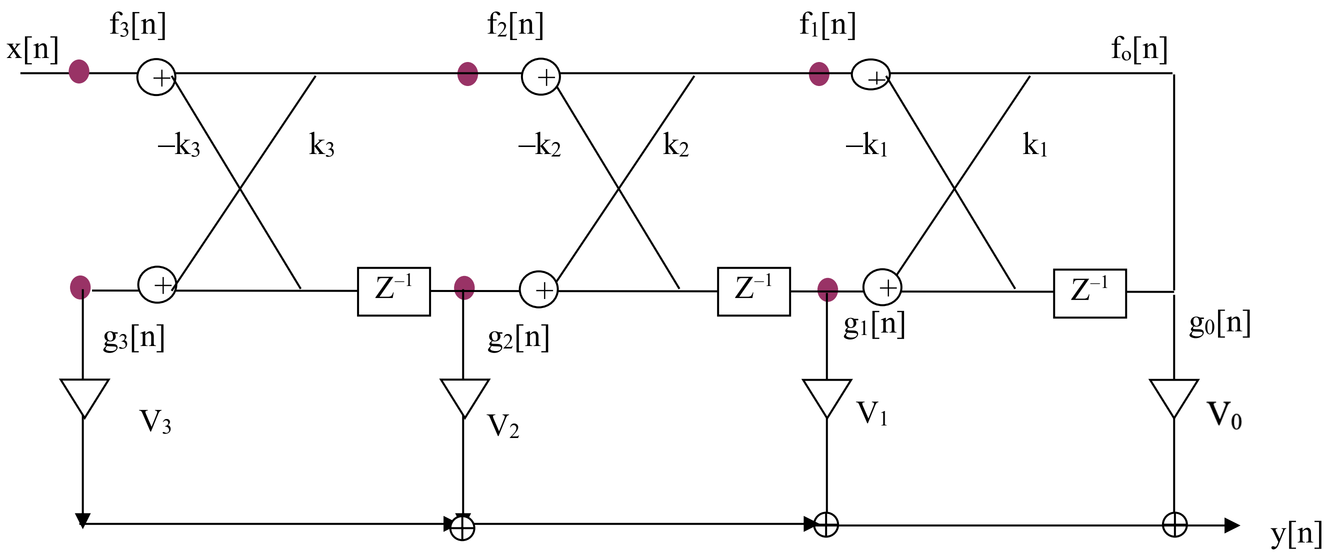
$\therefore\ V_1 = 5.67$

(v.) To find V_0

$C_0(z) = C_1(z) - V_1\, B_1(z) = -11.19$

$\therefore\ V_0 = -11.19$

Lattice Ladder Realization Diagram of IIR filter :



Q(19) A Discrete Time Invariant and Linear System is described by the difference equation y[n] = x[n] + 2 x[n-1] + x[n-2].
Obtain (a) Impulse Response
(b) Frequency Response
(c) Sketch Magnitude and Phase Response
(d) System Response to the Input $(-1)^n$ n u[n].

Solution : (a) To find Impulse Response

$$y[n] = x[n] + 2x[n-1] + x[n-2]$$

By ZT,

$$Y(z) = X(z) + 2Z^{-1}X(z) + Z^{-2}X(z)$$

$$\frac{Y(z)}{X(z)} = 1 + 2Z^{-1} + Z^{-2}$$

$$H(z) = 1 + 2Z^{-1} + Z^{-2}$$

By Inverse ZT,

$$h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2] == \left\{ \underset{\uparrow}{1}, \quad 2, \quad 1 \right\}$$

(b) To find Frequency Response

Now, $H(z) = 1 + 2Z^{-1} + Z^{-2}$

Put $z = e^{jw}$

$$H(e^{jw}) = 1 + 2e^{-jw} + e^{-2jw}$$

(c) To Sketch Magnitude and Phase Response

$$H(e^{jw}) = 1 + 2e^{-jw} + e^{-2jw}$$

$$H(e^{jw}) = e^{-jw}\left(e^{jw} + 2 + e^{-jw} \right)$$

$$H(e^{jw}) = e^{-jw}\left(2 + 2\cos(w) \right)$$

(i) Magnitude Response :

$$|H(w)| = |\ 2 + 2\cos(w)\ |$$

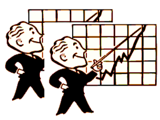
(ii) Phase Response : $\phi(w) = e^{-jw}$

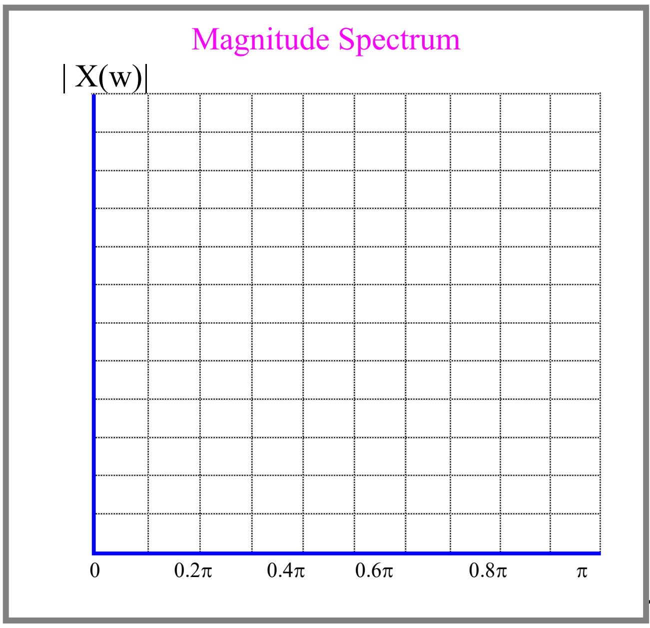
(iii) Phase : Φ = - jw

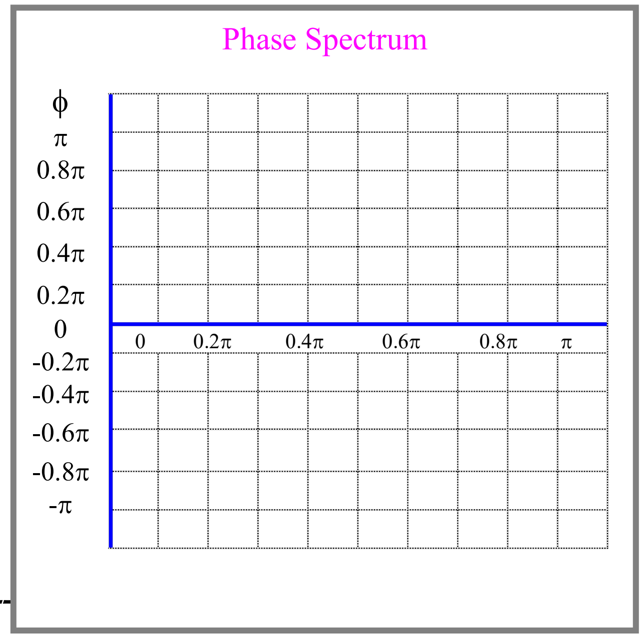
(iv) Generalized Phase :

$$\phi = \left\{ \begin{array}{ll} -jw & H_r(w) \geq 0 \\ -jw + \pi & H_r(w) < 0 \end{array} \right\}$$

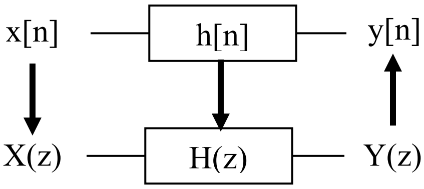
Sr No.	Freq. W	X(w)	Phase
1	0	4.00	0
2	0.1 π	3.90	-0.1 π
3	0.2 π	3.61	-0.2 π
4	0.3 π	3.17	-0.3 π
5	0.4 π	2.61	-0.4 π
6	0.5 π	2.00	-0.5 π
7	0.6 π	1.38	-0.6 π
8	0.7 π	0.82	-0.7 π
9	0.8 π	0.38	-0.8 π
10	0.9 π	0.09	-0.9 π
11	π	0	π







(d) To find response to the input x[n]=n $(-1)^n$ n u[n]



(iii) Find H(z)

$$y[n] = x[n] + 2x[n-1] + x[n-2]$$

By ZT,

$$Y(z) = X(z) \ + \ 2Z^{-1} \ X(z) \ + \ z^{-2} \ X(z)$$

$$Y(z) = X(z)(1 + \ 2Z^{-1} \ + \ z^{-2})$$

$$\frac{Y(z)}{X(z)} = (1 + \ 2Z^{-1} \ + \ z^{-2})$$

$$H(z) = 1 + \ 2Z^{-1} \ + \ z^{-2}$$

(i) Find X(z)

$$x[n] = (-1)^n \ n \ u[n]$$

By ZT,

$$X(z) = \frac{az}{(z-a)^2} \quad whare \ a = -1$$

$$X(z) = \frac{-z}{(z+1)^2}$$

(ii) Find Y(z)

$$Y(z) = X(z) \ H(z)$$

$$Y(z) = (1 + \ 2Z^{-1} \ + \ z^{-2}) X(z)$$

(iii) Find y[n]

$$Y(z) = (1 + 2Z^{-1} + z^{-2})\,X(z)$$
$$Y(z) = (1 + 2Z^{-1} + z^{-2})\,X(z)$$
$$Y(z) = X(z) + 2Z^{-1}\,X(z) + z^{-2}\,X(z)$$

By IZT, $y[n] = x[n] + 2x[n-1] + x[n-2]$

$$y[n] = n\,(-1)^n\,x[n] + 2\,(n-1)\;(-1)^{n-1}\,x[n-1] + (n-2)\,(-1)^{n-2}\,x[n-2]$$

Q(20) A LTI system is described by y[n] = 0.9 y[n–1] + x[n]. Find the expressions for the magnitude and phase response of the system

Solution :

$$y[n] = 0.9\,y[n-1] + x[n]$$

By ZT. $Y(z) = 0.9z^{-1}\,y(z) + X(z)$

$$\therefore\; Y(z)\,\{\,1 - 0.9\,z^{-1}\,\} = X(z)$$

$$\therefore\; \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.9z^{-1}}$$

$$\therefore\; H(z) = \frac{z}{z - 0.9}$$

Put $z = e^{jw}$ $\quad H(e^{jw}) = \dfrac{ejw}{e^{jw} - 0.9}$

$$H(e^{jw}) = \frac{\cos(w) + j\sin(w)}{[\cos(w) - 0.9] + j\sin(w)}$$

(i) Magnitude Response $= \dfrac{Magnitude\ of\ Numerator}{Magnitude\ of\ Denominator}$

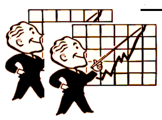
$$\underset{\substack{Magnitude\\ Response}}{|H(e^{jw})|} = \frac{\sqrt{[\cos(w)]^2 + [\sin(w)]^2}}{\sqrt{[\cos(w) - 0.9\,]^2 + [\sin(w)]^2}} = \frac{1}{\sqrt{1.81 - 1.8\,\cos(w)}}$$

(ii) Phase Response = Angle of Numerator – Angle of denominator

$$\underset{\substack{Phaze\\ response}}{\underline{|H(e^{jw})}} = \tan^{-1}\left[\frac{\sin(w)}{\cos(w)}\right] - \tan^{-1}\left[\frac{\sin(w)}{\cos(w) - 0.9}\right]$$

$$\underset{\substack{Phaze\\ response}}{\underline{|H(e^{jw})}} = \tan^{-1}[\,\tan(w)\,] - \tan^{-1}\left[\frac{\sin(w)}{\cos(w) - 0.9}\right]$$

$$\underset{\substack{Phaze\\ response}}{\underline{|H(e^{jw})}} = w - \tan^{-1}\left[\frac{\sin(w)}{\cos(w) - 0.9}\right]$$



Q(21) Determine the magnitude response phase response of the IIR system given by $y(n)+\frac{1}{2}\,y(n-1)=x(n)-x(n-1)$

Solution :

$$y(n)+\frac{1}{2}\,y(n-1)=x(n)-x(n-1)$$

By ZT, $Y(z)+\frac{1}{2}\,z^{-1}Y(z)=X(z)-z^{-1}X(z)$

$$Y(z)+\left(1+\frac{1}{2}\,z^{-1}\right)=X(z)\,(1-z^{-1})$$

$$\frac{Y(z)}{X(z)}=\frac{1-z^{-1}}{1+\frac{1}{2}z^{-1}}$$

$$H(z)=\frac{z-1}{z+\frac{1}{2}}$$

Put $z=e^{jw}=\cos(w)+j\sin(w)$

$$H(w)=\frac{[\cos(w)-1]+j\sin(w)}{\left[\cos(w)+\frac{1}{2}\right]+j\sin(w)}$$

(i) Magnitude Response $=\dfrac{\textit{Magnitude of Numerator}}{\textit{Magnitude of Denominator}}$

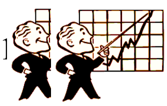
$$\underset{\substack{Magnitude\\ Response}}{|H(e^{jw})|}=\frac{\sqrt{[\cos(w)-1]^2+[\sin(w)]^2}}{\sqrt{[\cos(w)+0.5]^2+[\sin(w)]^2}}$$

$$\underset{\substack{Magnitude\\ Response}}{|H(e^{jw})|}=\frac{\sqrt{[\cos^2(w)-2\cos(w)+1+\sin^2(w)}}{\sqrt{\cos^2(w)+\cos(w)+0.25+\sin^2(w)]}}$$

$$\underset{\substack{Magnitude\\ Response}}{|H(e^{jw})|}=\frac{\sqrt{2-2\cos(w)}}{\sqrt{1.25+\cos(w)}}$$

(ii) Phase Response = Angle of Numerator – Angle of Denominator

$$\underset{\substack{Phaze\\ response}}{\angle H(e^{jw})}=\tan^{-1}\left[\frac{\sin(w)}{\cos(w)-1}\right]-\tan^{-1}\left[\frac{\sin(w)}{\cos(w)+0.5}\right]$$



Q(22) Given $H(z) = \dfrac{0.7(z^2 - 0.36)}{z^2 - 0.1\,z - 0.72}$

(a) Find the corresponding difference equation
(b) Find the transfer function.
(c) Realize the filter using parallel form
(d) Find the Impulse Response of the filter.
(e) Find the expression for magnitude and phase response of the filter.

Solution :

(a) To find difference equation

$$H(z) = \frac{0.7(z^2 - 0.36)}{z^2 - 0.1\,z - 0.72} = \frac{0.7\,z^2 - 0.252}{z^2 - 0.1\,z - 0.72}$$

$$\frac{Y(z)}{x(z)} = \frac{0.7 - 0.252\,z^{-2}}{1 - 0.1\,z^{-1} - 0.72\,z^{-2}}$$

Cross Multiply

$Y(z) - 0.1\,z^{-1}\,Y(z) - 0.72\,z^{-2}\,Y(z) = 0.7\,X(z) - 0.252\,z^{-2}\,X(z)$

By iZT,

$y(n) = 0.1\,y(n-1) + 0.72\,y(n-2) + 0.7\,x(n) - 0.252\,x(n-2)$ **(Ans)**

(b) To find Transfer Function,

$y(n) = 0.1\,y(n-1) + 0.72\,y(n-2) + 0.7\,x(n) - 0.252\,x(n-2)$

By ZT,

$Y(z) - 0.1\,z^{-1}\,Y(z) - 0.72\,z^{-2}\,Y(z) = 0.7\,X(z) - 0.252\,z^{-2}\,X(z)$

$$H(z) = \frac{0.7\,(z+0.6)\,(z-0.6)}{(z-0.9)\,(z+0.8)} \quad \textbf{(Ans)}$$

(c) Parallel Realization :

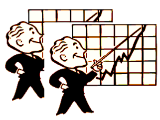
$$\frac{H(z)}{z} = \frac{0.7\,(z+0.6)\,(z-0.6)}{z\,(z-0.9)\,(z+0.8)}$$

$$\frac{H(z)}{z} = \frac{A}{z} + \frac{B}{z-0.9} + \frac{C}{z+0.8}$$

$$H(z) = A + \frac{B\,z}{z-0.9} + \frac{C\,z}{z+0.8}$$

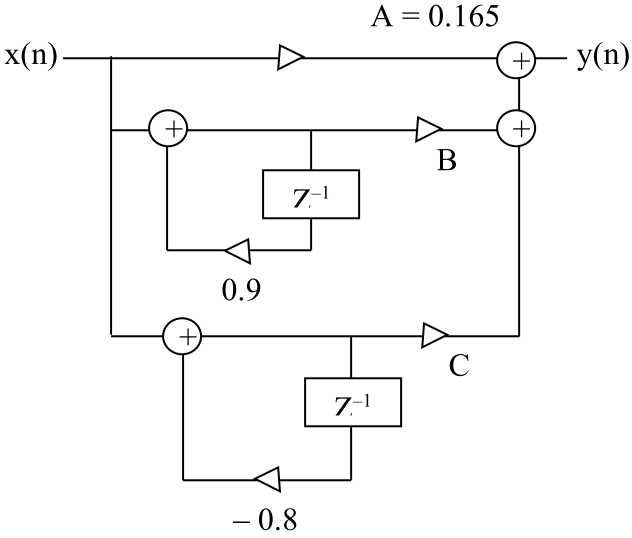
Where $A = 0.165$
$B = 0.2058$
$C = 0.1441$

Let $H(z) = A + H_1(z) + H_2(z)$
where,
$A = 0.165$



$$H1(z) = \frac{BZ}{z-0.9} = \frac{0.2058\ z}{1-0.9z^{-1}}$$

$$H_2(z) = \frac{CZ}{z+0.8} = \frac{0.1441\ z}{1+0.8z^{-1}}$$



(d) To find Impulse Response h(n)

$$\mathrm{H(z)} = 0.165 + 0.20\left[\frac{\mathrm{Z}}{\mathrm{z}-0.9}\right] + 0.14\left[\frac{\mathrm{Z}}{\mathrm{z}-0.8}\right]$$

By iZT, h(n) = 0.165δ[n] + 0.205 $(0.9)^n$ u(n) + 0.144 (- 0.8)^n u(n)

(e) To find Magnitude Response and Phase Response

Given $$H(z) = \frac{0.7z^2 - 0.252}{z^2 - 0.1\ \ z - 0.72}$$

Put $z = e^{jw}$ $$\mathrm{H}(e^{jw}) = \frac{0.7e^{j2w} - 0.252}{e^{j2w} - 0.1\ \ e^{jw} - 0.72}$$

$$H(e^{jw}) = \frac{0.7[\cos(2w) + j\sin(2w)] - 0.252}{[\cos(2w) + j\sin(2w)] - 0.1\ [\cos(w) + j\sin(w)] - 0.72}$$

$$H(e^{jw}) = \frac{[0.7(\cos(2w) - 0.252] + j\,[0.7\sin(2w)]}{[\cos(2w) - 0.1\cos(w) - 0.72] + j[\sin(2w) - 0.1\sin(w)]}$$

(i) Magnitude Response $= \dfrac{Magnitude\ of\ Numerator}{Magnitude\ of\ Denominator}$

Where Magnitude $= \sqrt{(\mathrm{Real})^2 + (\mathrm{Imaginary})^2}$

$$\underbrace{|H(e^{jw})|}_{Magnitude\ Response} = \frac{\sqrt{[0.7\cos(2w) - 0.252]^2 + [0.7\sin(2w)]^2}}{\sqrt{[\cos(2w) - 0.1\cos(w) - 0.72]^2 + [\sin(2w) - 0.1\sin(w)]^2}}$$

(ii) Phase Response = Angle of Numerator – Angle of denominator

$$\text{Where angle} = \begin{cases} \tan^{-1}\left(\dfrac{Imaginary}{Real}\right) & When \quad Real > 0 \\ \tan^{-1}\left(\dfrac{Imaginary}{Real}\right) + \pi & When \quad Real < 0 \end{cases}$$

Q(23) Given y [n] = 0.5 y [n – 1] + x [n]

(a) Determine response of the system to the input x[n] = $(1/3)^n$ u[n]

(b) Determine transient and steady state response of the system to input x[n] = $(1/3)^n$ u[n]

(c) Determine zero state response of the system to the input x[n]=$(1/3)^n$ u[n]

Solution : (a) To find Response of the system

(i) Given y[n] = 0.5 y[n – 1] + x [n]

By ZT, $Y(z) = 0.5\, z^{-1}\, Y(z) + x(z)$

$$Y(z)\,(1 - 0.5\, z^{-1}) = x(z)$$

$$H(z) = \frac{1}{1-0.5z^{-1}} = \frac{z}{z-0.5}$$

(ii) Given $x[n] = (1/3)^n\, u[n]$

$$\text{By ZT , } X(z) = \frac{z}{z-(1/3)^n}$$

(iii) Let Y(z) = H(z) . X(z)

$$Y(z) = \frac{z}{z-{}^1/_2}\;\frac{z}{z-{}^1/_3}$$

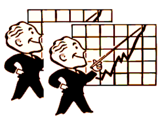
$$Y(z) = \frac{z^2}{\left(z-{}^1/_2\right)-\left(z-{}^1/_3\right)}$$

(iv) $$\frac{Y(z)}{z} = \frac{z^2}{\left(z-{}^1/_2\right)\left(z-{}^1/_3\right)}$$

By PFE,

$$\frac{Y(z)}{z} = \frac{A}{z-{}^1/_2} + \frac{B}{z-{}^1/_3}$$

$$\text{where } A = \frac{Y(z)}{z}\left(z-{}^1/_2\right)\Bigg|_{z={}^1/_2} = 3 \quad \text{and} \quad B = \frac{Y(z)}{z}\left(z-{}^1/_3\right)\Bigg|_{z={}^1/_3} = -2$$



By substituting,

$$Y(z) = A\left[\frac{z}{z-{}^{1}/_{2}}\right] + B\left[\frac{z}{z-{}^{1}/_{3}}\right]$$

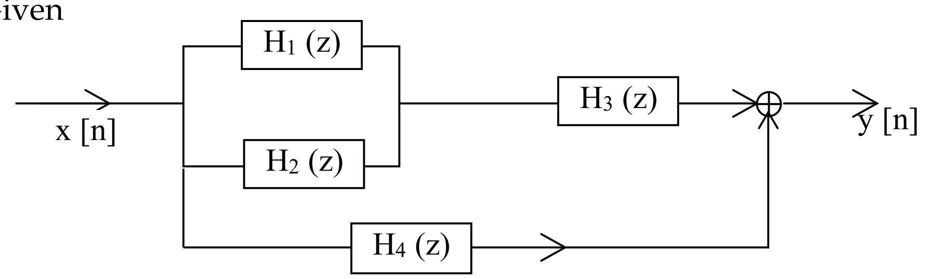
$$Y(z) = 3\left[\frac{z}{z-{}^{1}/_{2}}\right] - 2\left[\frac{z}{z-{}^{1}/_{3}}\right]$$

By iZT : $y[n] = 3({}^{1}/_{2})^{n}\ u[n] - 2({}^{1}/_{3})^{n}\ u[n]$ **(ANS)**

Solution : (b) Same as part (a) **ANS** $\therefore y[n] = 3({}^{1}/_{2})^{n}\ u[n] - 2({}^{1}/_{3})^{n}\ u[n]$

Solution : (c) Same as part (a) ANS : $y[n] = 3({}^{1}/_{2})^{n}\ u[n] - 2({}^{1}/_{3})^{n}\ u[n]$

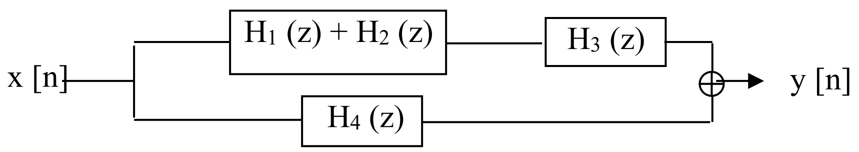
Q(24) Given

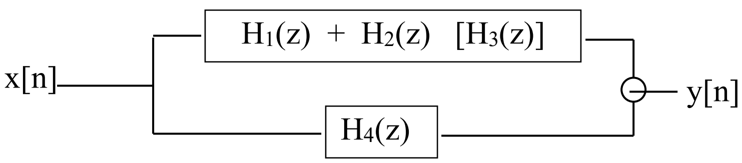


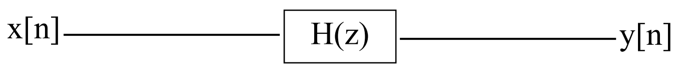
$H_1(z)$ has pole at 0·6 and zero at 0
H_2 (z) has a zero at – 2 and a pole at 0 and $H_3(z) = z^{-4}$, $H_4(z) = 2$.

(a) Find the overall impulse response.
(b) Plot impulse response found in the question 4(a).
(c) Realize the system of Question (a).
(d) How do we find stability of a non causal system?

Solution : (a) To find impulse response







$H(z) = H_4(z) + H_1(z)\, H_3(z) + H_2(z)\, H_3(z)$

Where

$$H_1(z) = \frac{G_1 Z}{Z - 0.6} \qquad H_2(z) = \frac{G_2(Z+2)}{z} \qquad H_3(z) = z^{-4} \qquad H_4(z) = 2$$

Let $G_1 = G_2 = 1$

$$H_1(z) = \frac{Z}{Z - 0.6} \qquad\qquad H_2(z) = \frac{Z+2}{z} = 1 + 2z^{-1}$$

By iZT,

$$h_1[n] = (0.6)^n \; u[n] \qquad h_2[n] = \{\underset{\uparrow}{1}, 2\} \qquad h_3[n] = \{0, 0, 0, 0, 1\}$$

$H(z) = 2 + H_1(z)\, z^{-4} + (\, 1 + 2\, z^{-1}\,)\, z^{-4}$

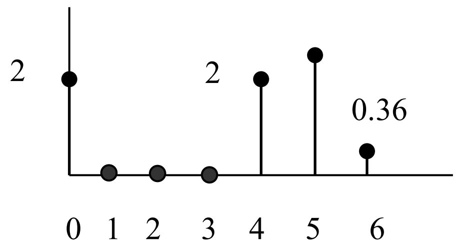
$H(z) = 2 + H_1(z)\, z^{-4} + z^{-4} + 2z^{-5}$

By iZT,

$h[n] = 2\delta[n] + h_1\,[\,n - 4] + \delta[\,n - 4] + 2\,\delta[n - 5]$

$h[n] = 2\,\delta[n] + (0.6)^{n-4}\, u[n - 4] + \delta[\,n - 4] + 2\,\delta[n - 5]$ ANS:

Solution : (b) Plot of Impulse Response **$h[n] = \{\, \underset{\uparrow}{2},\ 0,\ 0,\ 0,\ 2,\ 2.6,\ 0.36\ \text{-----}\,\}$**



Solution : (c) To find Transfer Function

$$H(z) \;=\; H_4(z) + h_1(z)\, H_3(z) + H_2(z)\, H_3(z)$$

$$H(z) \;=\; 2 + \frac{z}{z - 0.6} z^{-4} + \frac{(z+2)}{z} z^{-4}$$

$$H(z) \;=\; 2 + \frac{z}{z^4 (z - 0.6)} + \frac{z+2}{z^5}$$

$$H(z) \;=\; \frac{2\,(z^5 (z - 0.6) + z^2 + (z+2)\,(z - 0.6)}{z^5 (z = 0.6)}$$

$$H(z) \;=\; \frac{2z^6 - 1.2z^5 + z^2 + z^2 + 1.4z - 1.2}{z^6 - 0.6z^5}$$

$$H(z) \;=\; \frac{2 - 1.2z^{-1} + 2z^{-4} + 1.4z^{-5} - 1.2z^{-6}}{1 - 0.6z^{-1}}$$

Solution : (d) Condition for Stability : If ROC includes unit circle, then system is

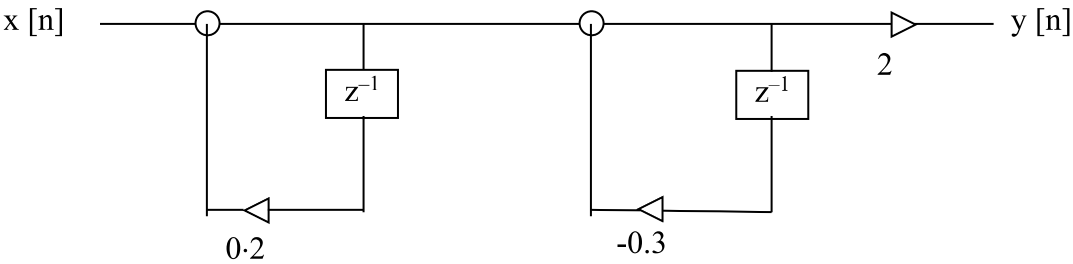
STABLE.

(i) When h[n] is Causal Signal, system is Causal System.
For stable and Causal System, all the poles must lie INSIDE the unit circle.

(ii) When h[n] is Anti-causal signal, system is Anti-causal System.
For stable and Anti-causal system, all the poles must lie OUTSIDE the unit circle.

(iii) When h[n] is Both-sided signal, system is Both-sided System If ROC includes unit circle then System is STABLE.

Q(25) The figure given below shows a cascade form realization of digital filter.



(a) Find the difference equation of the total system in terms of input x[n] and output y[n].
(b) Find the impulse response of the filter.
(c) Show parallel form realization.
(d) Find magnitude squared response of the system transfer function at frequencies $\omega = 0$ and $\omega = \pi$. by graphical method.
(e) Find the response of a system for a step input u[n].

Solution : (a) To find Difference Equation of the total system

Let $H(z) = H_1(z)\ H_2(z)$ Where $H_1(z) = \dfrac{1}{1-0\cdot 2z^{-1}}$ And $H_2(z) = \dfrac{2}{1+0\cdot 3z^{-1}}$

$$\therefore H(z) = H_1(z)H_2(z) = \frac{2}{(1-0\cdot 2z^{-1})(1+0\cdot 3z^{-1})}$$

$$\therefore H(z) = \frac{Y(z)}{Z(z)} = \frac{2}{1+0\cdot 1z^{-1} - 0\cdot 06z^{-2}}$$

$$Y(z)\ (1+0\cdot 1z^{-1} - 0\cdot 06\, z^{-2}] = 2X(z)$$

By IZT, $y[n] = 0\cdot 06\, y[n-2] - 0\cdot 1y[n-1] + 2x[n]$ **(ANS)**

Solution : (b) To find Impulse Response

$$H(z) = \frac{2}{(1-0\cdot 2z^{-1})\ (1+0\cdot 3z^{-1})} == \frac{2z^2}{(z-0\cdot 2)\ (z+0\cdot 3)}$$

$$\therefore \quad \frac{H(z)}{z} = \frac{2z}{(z-0\cdot 2)(z+0\cdot 3)}$$

$$\therefore \frac{H(z)}{z} = \frac{A}{z-0\cdot 2} + \frac{B}{z+0\cdot 3}$$

$$H(z) = \frac{4}{5}\left[\frac{z}{z-0\cdot 2}\right] + \frac{6}{5}\left[\frac{z}{z+0\cdot 3}\right]$$

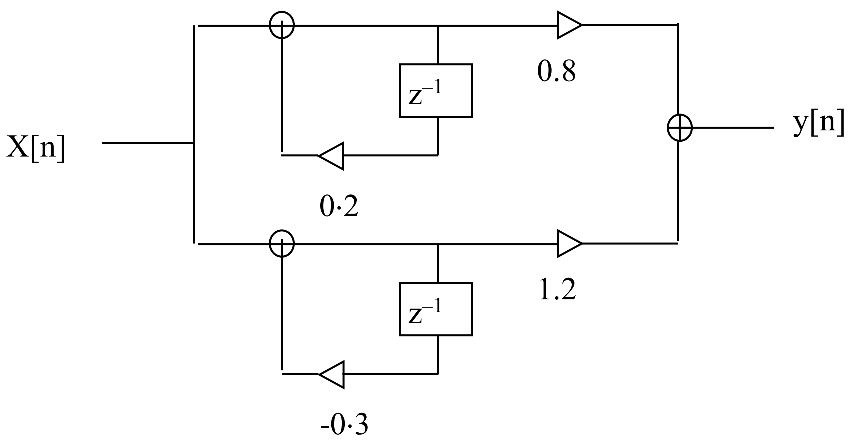
By IZT, $\therefore h[n] = \frac{4}{5}(0\cdot 2)^n u[n] + \frac{6}{5}(-0\cdot 3)^n u[n]$ **(ANS)**

Solution : (c) To find Parallel Realization

For parallel realization, we will express $H(z) = H_1(z) + H_2(z)$

Using result of part (b) $H(z) = \dfrac{\frac{4}{5}}{1-0\cdot 2z^{-1}} + \dfrac{\frac{6}{5}}{1+0\cdot 3z^{-1}}$

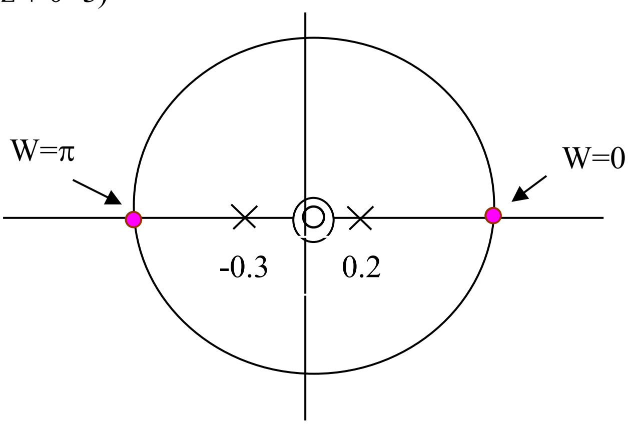
$$H(z) = \frac{0.8}{1-0\cdot 2z^{-1}} + \frac{1.2}{1+0\cdot 3z^{-1}}$$



Solution : (d) To find Squared Magnitude Response

We have ,

$H(z) = \dfrac{2z^2}{(z-0\cdot 2)(z+0\cdot 3)}$ The pole – zero plot of H(z) is as follows



(i) Magnitude | H (z) | at ω = 0 is

$$\left|H(e^{jw})\right| = G\frac{\text{Product of distances from zeros to } \omega = 0 \text{ on unit circle}}{\text{Product of distances from poles to } \omega = 0 \text{ on unit circle}}$$

$$= \frac{2|1||1|}{|1\cdot 3||0\cdot 8|} = \frac{2}{1\cdot 04} = 1\cdot 92$$

$\therefore$ Magnitude squared response is $\left|H(e^{jw})\right|^2 = (1\cdot 92)^2 = \boxed{3.6864}$

(ii) Magnitude $|H(z)|$ at $\omega = \pi$ is

$$\left|H(e^{jw})\right| = G\frac{\text{Product of distances from zeros to } \omega = \pi \text{ on unit circle}}{\text{Product of distances from poles to } \omega = \pi \text{ on unit circle}}$$

$$= \frac{2|1||1|}{|0\cdot 7|\ |1\cdot 2|} = \frac{2}{0\cdot 84} = 2\cdot 38$$

$\therefore$ Magnitude squared response is $= (2\cdot 38)^2 = \boxed{5.6644}$

Solution : (d) To find Step Response

For $x[n] = u[n]$ $\quad X(z) = \dfrac{z}{z-1}$

we have $\quad H(z) = \dfrac{2z^2}{(z-02)(z+0\cdot 3)}$

$$Y(z) = H(z)\ X(z)$$

$$\therefore\ Y(z) = \frac{2z^2}{(z-0\cdot 2)(z+0\cdot 3)}\ \frac{z}{z-1}$$

$$\therefore\ \frac{Y(z)}{z} = \frac{A}{z-0\cdot 2} + \frac{B}{z+0\cdot 3} + \frac{C}{z-1}$$

$$A = \left.\frac{2z^2}{(z+0\cdot 3)\ (z-1)}\right/_{z=0\cdot 2} = \frac{2\times 0\cdot 04}{(0\cdot 5)\ (-0\cdot 8)} = \frac{1}{5} = -0\cdot 2$$

$$B = \left.\frac{2z^2}{(z-0\cdot 2)\ (z-1)}\right/_{z=-0\cdot 3} = 0277$$

$$C = \left.\frac{2z^2}{(z-0\cdot 2)(z+0\cdot 3)}\right/_{z=1} = 1\cdot 923$$

$$Y(z) = -0\cdot 2\left[\frac{z}{z-0\cdot 2}\right] + 0\cdot 277\left[\frac{z}{z+0\cdot 3}\right] + 1\cdot 923\left[\frac{z}{z-1}\right]$$

By iZT,

$y[n] = -0\cdot 2(0\cdot 2)^n u[n] + 0\cdot 277(-0\cdot 3)^n u[n] + 1\cdot 923\ u[n]$ **(Ans)**

Q(26) A certain DT LTI filter has following data:
POLES at 0·2 and 0·6. ZEROS at – 0·4 and origin.
Gain of the filter is 5.

Show (a) Direct form–II realization and Cascade realization.

(b) If register length is 4 bits including sign bit, calculate the effect on poles and zeros of the filter due to finite word length of register.

Solution :

Given, $P_1 = 0{\cdot}2$ $\quad z_1 = -0{\cdot}4$ $\quad G = 5.$ $\quad P_2 = 0{\cdot}6$ $\quad z_2 = 0$

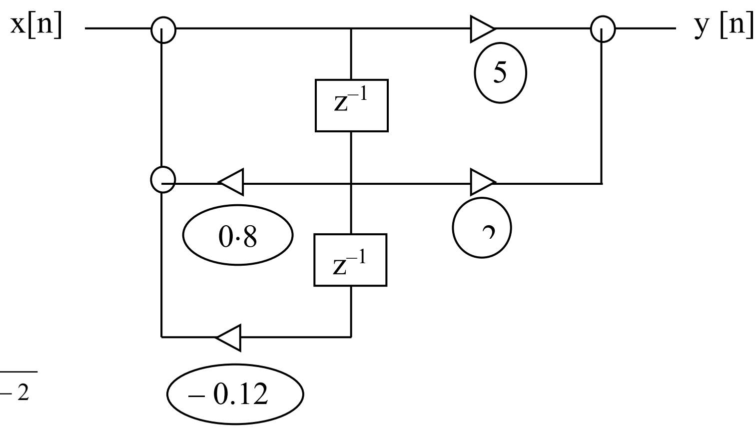
(a) Direct form – II Realization

$$H(z) = \frac{G(z-z_1)(z-z_2)}{(z-p_1)(z-p_2)}$$

$$H(z) = \frac{5(z+0\cdot4)(z)}{(z-0\cdot2)(z-0\cdot6)}$$

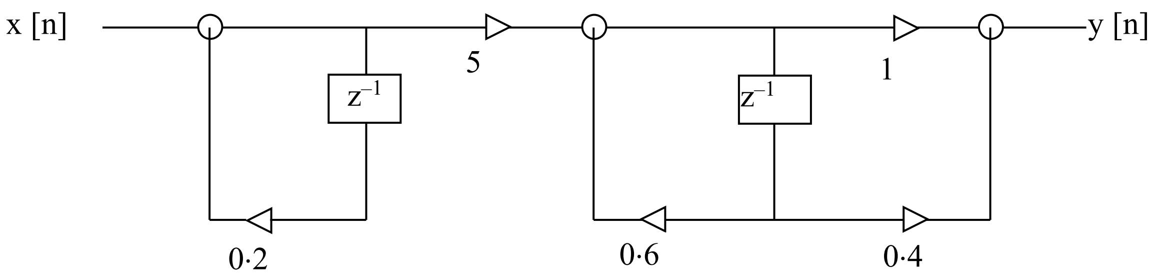
$$H(z) = \frac{5z^2+2z}{z^2-0\cdot8z+0\cdot12}$$

$$H(z) = \frac{5+2z^{-1}}{1-0\cdot8z^{-1}+0\cdot12z^{-2}}$$



(b) Cascade Realization.

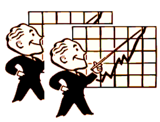
$$H(z) = \frac{5z}{z-0\cdot2}\,\frac{z+0\cdot4}{z-0\cdot6} = \left(\frac{5}{1-0\cdot2z^{-1}}\right)\left(\frac{1+0\cdot4z^{-1}}{1-0\cdot6z^{-1}}\right)$$



(b) Quantization

$$H(z) = 5\frac{1+0.4z^{-1}}{1-0\cdot8z^{-1}+0.12\,z^{-2}}$$

$$H(z) = \frac{bo+b_1z^{-1}}{1+a_1z^{-1}+a_2\,z^{-2}}$$



$a_1 = -0{\cdot}8$ $-a_1 = 0{\cdot}8$	$a_2 = 0{\cdot}12$ $-a_2 = -0{\cdot}12$	$b_1 = 0.4$
$0{\cdot}8 \times 2 = 1.6$	$0 \cdot 12 \times 2 = 0{\cdot}24$	$0.4 \text{ x } 2 = 0.8$
$0{\cdot}6 \times 2 = 1.2$	$0 \cdot 24 \times 2 = 0{\cdot}48$	$0.8 \text{ x } 2 = 1.6$
$0{\cdot}2 \times 2 = 0 \cdot 4$	$0 \cdot 48 \times 2 = 0 \cdot 96$	$0.6 \text{ x } 2 = 1.2$
Coeff in Binary **0 1 1 0**	Coeff in Binary **1 0 0 0**	Coeff in Binary **0 0 1 1**
Modified Coeff. $-a_1' = 110 = 0 \cdot 75$	Modified Coeff. $-a_2' = 000 = 0$	Modified Coeff. $-b_1' = 011 = 0 \cdot 375$

Modifed T.F. $H_1'(z) = 5\dfrac{1+0.375z^{-1}}{1-0 \cdot 75z^{-1}} = \dfrac{5z+1.875}{z-0 \cdot 75}$

Poles : $P_1 = 0{\cdot}75$ P_2 = Does not exist.
$Z_1 = 0.375$ Z_2 = Does not exist'

Q(27) Consider an LTI system, initially at rest, described by the difference equation :

$$y(n) = \frac{1}{4}y(n-2) + x(n)$$

(a) Determine the impulse response, h(n) of the system.

(b) What is the response of the system to the input signal ?

$$x(n) = \left[\left(\frac{1}{2}\right)^n + \left(-\frac{1}{2}\right)^n\right]u(n)$$

(c) Determine the direct form II, parallel form and cascade form realization of the systems.

(d) Determine and draw, magnitude response and phase response of system

Solution :(a) To find h[n] :

Given $y(n) = \frac{1}{4}\ y(n-2) + x(n)$

By ZT, $Y(z) = \frac{1}{4} Z^{-2}\ Y(z) + x(z)$

$$Y(z)\left(1-\frac{1}{4}z^{-2}\right) = x(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{1-\frac{1}{4}z^{-2}}$$

$$H(z) = \frac{z^2}{z^2 - 1/4}$$

$$\frac{H(z)}{z}=\frac{z}{(z-1/2)\,(z+1/2)}$$

By PFE, $\dfrac{H(z)}{z}=\dfrac{A}{z-1/2}+\dfrac{B}{z+1/2}$

$$H(z)=\frac{1}{2}\left[\frac{z}{z-1/2}\right]+\frac{1}{2}\left[\frac{z}{z+1/2}\right]$$

By iZT,

ANS : $h[n]=\dfrac{1}{2}\left(\dfrac{1}{2}\right)^{n}u[n]+\dfrac{1}{2}\left(-\dfrac{1}{2}\right)^{n}u[n].$

Solution : (b) To find y[n] :

Given $x[n]=\left[\left(\dfrac{1}{2}\right)^{n}+\left(\dfrac{-1}{2}\right)^{n}\right]u[n]$

$$x[n]=\left(\frac{1}{2}\right)^{n}u[n]+\left(-\frac{1}{2}\right)^{n}u[n]$$

By ZT, $X(z)=\dfrac{z}{z-1/2}+\dfrac{z}{z+1/2}=\dfrac{z(z+1/2)+z(z-1/2)}{(z-1/2)\ \ (z+1/2)}=\dfrac{2z^{2}}{(z^{2}-1/4)}$

$Y(z)=H(z)\,X(z)$

$$Y(z)\ =\frac{z^{2}}{z^{2}-1/4}\ \ \frac{2z^{2}}{z^{2}-1/4}$$

$$Y(z)=\frac{2z^{4}}{(z^{2}-1/4)^{2}}=\frac{2z^{4}}{[(z+1/2)\,(z-1/2)]^{2}}$$

$$\frac{Y(z)}{z}=\frac{2z^{3}}{(z+1/2)^{2}\,(z-1/2)^{2}}$$

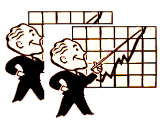
By PFE,

$$\frac{Y(z)}{z}=\frac{A}{z+1/2}+\frac{B}{(z+1/2)}+\frac{C}{(z-1/2)}+\frac{D}{(z-1/2)^{2}}$$

Where

$$B=\frac{Y(z)}{z}(z+\frac{1}{2})^{2}\Bigg|_{z=\frac{-1}{2}}=\boxed{-\frac{1}{4}}$$

$$D=\frac{Y(z)}{z}(z-1/2)^{2}\Bigg|_{z-\frac{1}{2}}=\boxed{\frac{1}{4}}$$



To Find A and C :

$$\frac{Y(z)}{z}=\frac{A}{z+1/2}+\frac{B}{(z+1/2)^2}+\frac{C}{z-1/2}+\frac{D}{(z-1/2)^2}$$

$$\frac{2z^3}{(z+1/2)^2\,(z-1/2)^2}=\frac{A}{z+1/2}+\frac{-1/4}{(z+1/2)^2}+\frac{C}{z-1/2}+\frac{1/4}{(z-1/2)^2}$$

Put $z=0$

$$0=\frac{A}{1/2}+\frac{-1/4}{(1/2)^2}+\frac{C}{-1/2}+\frac{1/4}{(-1/2)^2}$$

$A=C$

Put $z=1$

$$\frac{2}{\left(\frac{9}{4}\right)\left(\frac{1}{4}\right)}=\frac{A}{3/2}+\frac{-1/4}{9/4}+\frac{C}{1/2}+\frac{1/4}{1/4}$$

By solving we get, $C = 1$ and $A=1$

$$Y(z)=A\left[\frac{z}{z+1/2}\right]+B\left[\frac{z}{(z+1/2)^2}\right]+C\left[\frac{z}{z-1/2}\right]+D\left[\frac{z}{(z-1/2)^2}\right]$$

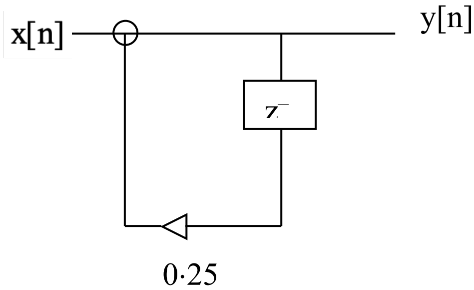
$$Y(z)=\left[\frac{z}{z+1/2}\right]-\frac{1}{4}\left[\frac{z}{(z+1/2)^2}\right]+\left[\frac{z}{z-1/2}\right]+\frac{1}{4}\left[\frac{z}{(z-1/2)^2}\right]$$

By iZT, $y[n]=\left(-\frac{1}{2}\right)^n u[n]-\frac{1}{4}n\left(-\frac{1}{2}\right)^{n-1}u[n]+\left(\frac{1}{2}\right)^n u[n]+\frac{1}{4}n\left(\frac{1}{2}\right)^{n-1}u[n]$

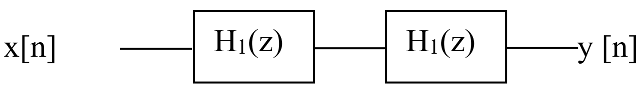
Solution : (c) Realization Diagram :

(i)Direct form realization

$$H(z)=\frac{z^2}{z^2-\frac{1}{4}}=\frac{1}{1-\frac{1}{4}z^{-2}}=\frac{b_0}{1+a_2\,z^{-2}}$$



(ii) Cascade form Realization :

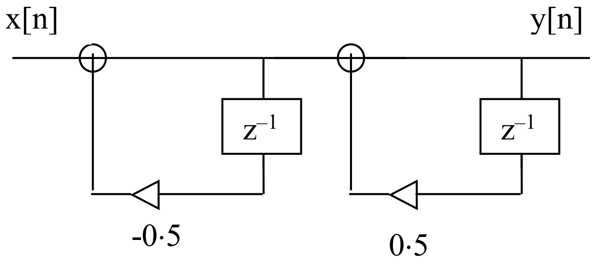


$$H(z) = \frac{z^2}{z^2 - 1/4} = \left[\frac{z}{z+1/2}\right]\left[\frac{z}{z-1/2}\right]$$

Let $\quad H(z) = H_1(z)\ H_2(z)$

Where $\quad H_1(z) = \frac{z}{z+1/2} = \frac{1}{1+\frac{1}{2}z^{-1}}$

$$H_2(z) = \frac{z}{z-1/2} = \frac{1}{1-\frac{1}{2}z^{-1}}$$



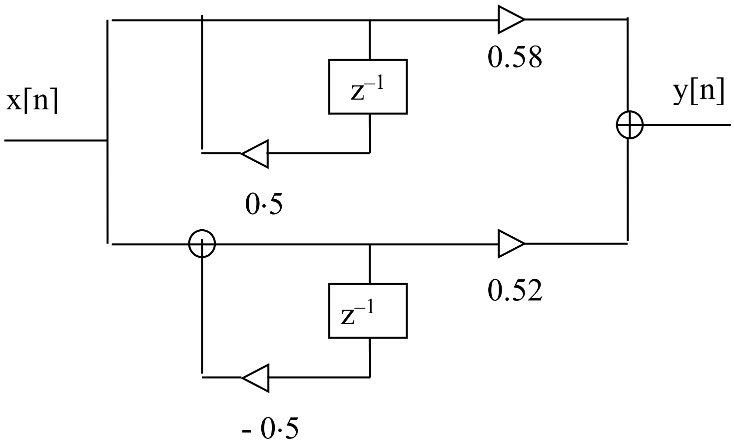
(iii) Parallel Form Realization.

$$H(z) = \frac{z^2}{z^2 - 1/4}$$

$$H(z) = \frac{1}{2}\left[\frac{z}{z-1/2}\right] + \frac{1}{2}\left[\frac{z}{z+1/2}\right]$$

$$H(z) = \frac{0\cdot 5}{1-0\cdot 5\,z^{-1}} + \frac{0\cdot 5}{1+0\cdot 5z^{-1}}$$

Let $\quad H(z) = H_1(z) + H_2(z)$



Q(28) Given $\quad H(z) = 5 + \frac{3}{z+\frac{1}{3}} + \frac{z+2}{z-\frac{1}{2}}$

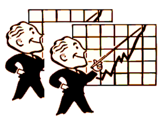
(a) Obtain Difference Equation.
(b) Find the impulse response of the system.
(c) Plot POLE-ZERO plot and comment on Stability.

Solution (a) To find Difference Equation:

Given, $\quad H(z) = 5 + \frac{3}{z+\frac{1}{3}} + \frac{z+2}{z-\frac{1}{2}}$

$$H(z) = \frac{5\left(z+\frac{1}{3}\right)\left(z-\frac{1}{2}\right) + 3\left(z-\frac{1}{2}\right) + (z+2)\left(z+\frac{1}{3}\right)}{\left(z+\frac{1}{3}\right)\left(z-\frac{1}{2}\right)}$$

$$H(z) = \frac{5\left(z^2 - \frac{1}{6}z - \frac{1}{6}\right) + 3z - \frac{3}{2} + \left(z^2 + \frac{7}{3}z + \frac{2}{3}\right)}{\left(z^2 - \frac{1}{6}z - \frac{1}{6}\right)}$$



$$H(z) = \frac{6z^2 + \frac{27}{6}z - \frac{5}{3}}{\left(z^2 - \frac{1}{6}z - \frac{1}{6}\right)}$$

To find Difference Equation

$$\frac{Y(z)}{X(z)} = \frac{6 + \frac{27}{6}z^{-1} - \frac{5}{3}z^{-2}}{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}$$

$$Y(z) - \frac{1}{6}z^{-1}Y(z) - \frac{1}{6}z^{-2}Y(z) = 6X(z) + \frac{27}{6}z^{-1}X(z) - \frac{5}{3}z^{-2}X(z)$$

By ZT, $y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = 6x[n] + \frac{27}{6}x[n-1] - \frac{5}{3}x[n-2]$

$$y[n] = 6x[n] + \frac{27}{6}x[n-1] - \frac{5}{3}x[n-2] + \frac{1}{6}y[n-1] + \frac{1}{6}y[n-2]$$

Solution (b) To find Impulse Response

Now, $H(z) = 5 + \frac{3}{z + \frac{1}{3}} + \frac{z+2}{z - \frac{1}{2}}$

$$H(z) = 5 + 3z^{-1}\left(\frac{z}{z + \frac{1}{3}}\right) + \frac{z}{z - \frac{1}{2}} + \frac{2}{z - \frac{1}{2}}$$

$$H(z) = 5 + 3z^{-1}\left(\frac{z}{z + \frac{1}{3}}\right) + \frac{z}{z - \frac{1}{2}} + 2z^{-1}\left(\frac{z}{z - \frac{1}{2}}\right)$$

By IZT,

$$h[n] = 5\delta[n] + 3\left(\frac{-1}{3}\right)^{n-1} u[n-1] + \left(\frac{1}{3}\right)^{n} u[n] + 2\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

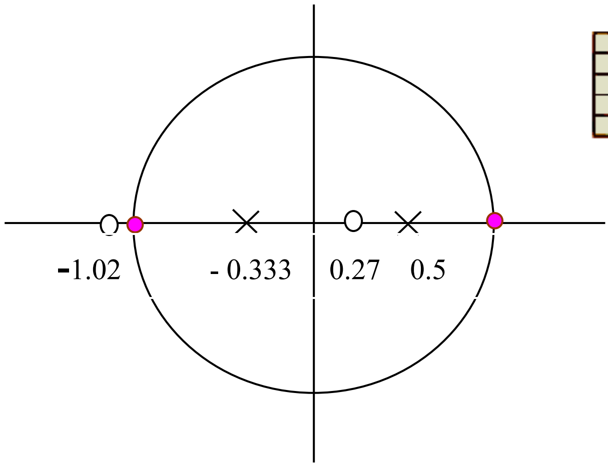
(i) POLE-ZERO Plot.

$$H(z) = \frac{6z^2 + \frac{27}{6}z - \frac{5}{3}}{\left(z^2 - \frac{1}{6}z - \frac{1}{6}\right)}$$

$$H(z) = \frac{6\left(z^2 + \frac{3}{4}z - \frac{5}{18}\right)}{\left(z^2 - \frac{1}{6}z - \frac{1}{6}\right)}$$

$$H(z) = \frac{6\left(z^2 + \frac{3}{4}z - \frac{5}{18}\right)}{\left(z^2 - \frac{1}{6}z - \frac{1}{6}\right)}$$

$$H(z) = \frac{6(z-0.2718)(z+1.0218)}{\left(z+\frac{1}{3}\right)\left(z-\frac{1}{2}\right)}$$

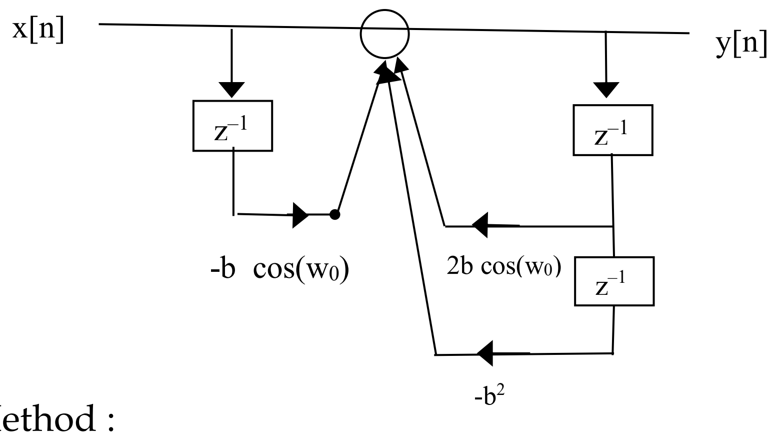


Q(29) Given $H(z) = \dfrac{1 - b\cos(w_o)z^{-1}}{1 - 2b\cos(w_o)z^{-1} + b^2 z^{-2}}$

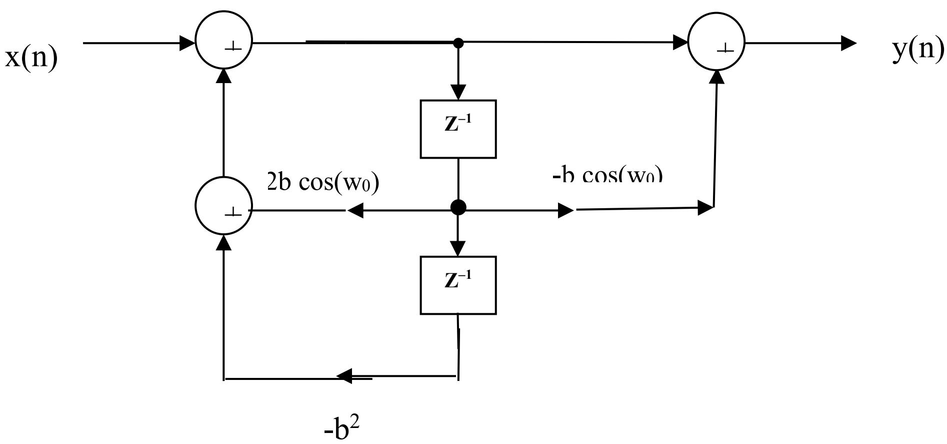
Show DF-I and DF-II Realization.

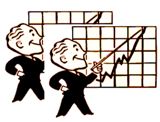
ANS:

(a) Direct Form – I Realization



(b) Direct form-II Method :





Q(30) A causal first order Digital Filter is described by the system function

$$H(z) = b_0 \frac{1 + b z^{-1}}{1 + a z^{-1}}$$

(a) Sketch the direct form-I and direct form-II realization of this filter and find the corresponding difference equations.

(b) For a = 0.5 and b = –0.6, sketch the pole zero pattern. Is the system stable? Why ?

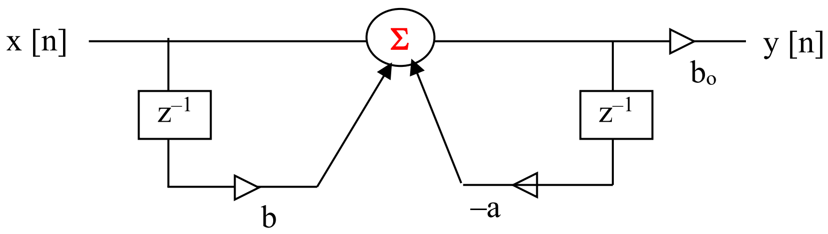
(c) For a = –0.5 and b = 0.5, determine b_0 so that the maximum value of |H(w)| is equal to 1.

(d) Sketch the magnitude response and phase response of the filter obtained in part (c) .

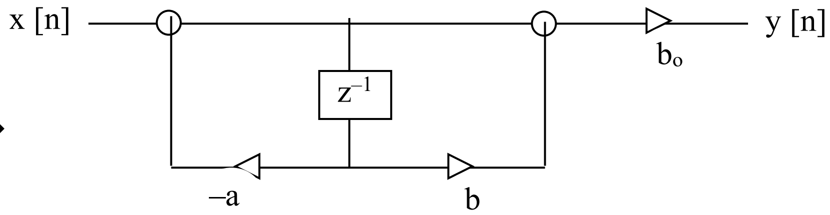
Solution : (a) Direct form-I and Direct form-II realization diagram

$$H(z) = b_0 \frac{1 + b z^{-1}}{1 + a z^{-1}}$$

Direct Form-I →



Direct Form-II →



Solution : (b) To Sketch the POLE ZERO pattern

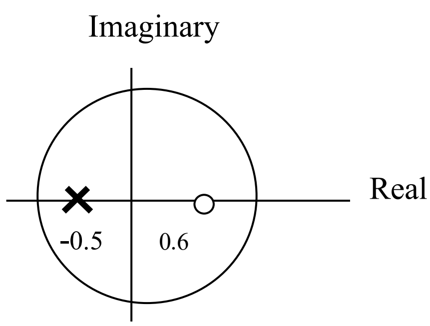
Given, $H(z) = b_0 \frac{1 + b z^{-1}}{1 + a z^{-1}}$

For a = 0.5 and b = –0.6,

$$H(z) = b_0 \frac{1 - 0.6 z^{-1}}{1 + 0.5 z^{-1}}$$

POLE = – 0.5

ZERO = 0.6



Solution : (c) To find b_0

$$H(z) = b_0 \frac{1 + b z^{-1}}{1 + a z^{-1}}$$

For a = –0.5 and b = 0.5, $H(z) = b_0 \dfrac{1 + 0.5 z^{-1}}{1 - 0.5 z^{-1}}$

POLE = 0.5 and ZERO = –0.5 Filter is Low Pass Filter
For LPF, Max Value occurs at w = 0.
At w = 0, z = 1.

$$H(w)\big|_{w=0} = b_0 \frac{1 + 0.5}{1 - 0.5} == 1\ (Given)$$ $\therefore$ **b_0 = 1/3 ANS**

Solution : (d) Sketch the Magnitude Response and Phase Response.

Now, $H(z) = b_0 \dfrac{1 + b z^{-1}}{1 + a z^{-1}}$

For a = –0.5 and b = 0.5, **b_0 = 1/3**

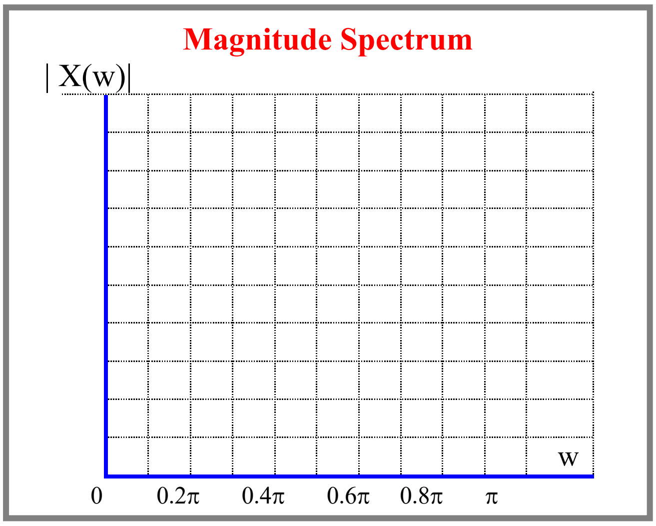
$$H(z) = \frac{1}{3}\left(\frac{1 + 0.5 z^{-1}}{1 - 0.5 z^{-1}}\right)$$

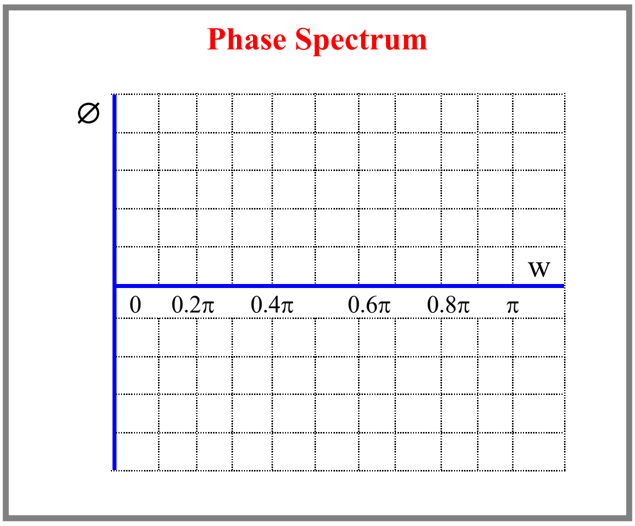
Put z = e^{jw}

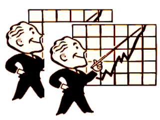
$$H(z) = \frac{1}{3}\left(\frac{1 + 0.5\cos(w) - j\ 0.5\sin(w)}{1 - 0.5\cos(w) + j\ 0.5\sin(w)}\right)$$

$$H(z) = \frac{1}{3}\left[\frac{(1 + 0.5\cos(w)) - j\ 0.5\sin(w)}{(1 - 0.5\cos(w)) + j\ 0.5\sin(w)}\right]$$

w	|H(w)|	ϕ
0		
0.2π		
0.4π		
0.6π		
0.8π		
π		







Q(31) Let h[n] be the unit impulse response of a Low Pass Filter with a cutoff frequency w_c, what type of filter has a unit sample response $g[n] = (-1)^n h[n]$. ?

Solution :

Consider a Low Pass Filter with $h[n]=(0.5)^n u[n]$

Then $G(z) = \dfrac{z}{z-0.5}$ **POLE = 0.5**

$g[n] = (-1)^n h[n]$

$g[n] = (-1)^n (0.5)^n u[n] = (-0.5)^n u[n]$

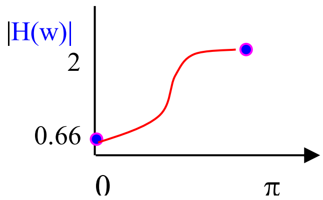
By ZT, $G(z) = \dfrac{z}{z+0.5}$ **POLE = – 0.5**

(i) At w=0, z=1

$|H(w)| = 0.66$

(ii) At w=π, z=–1

$|H(w)| = 2$



That means g[n] is impulse response of High Pass Filter. **ANS**

Q(32) For following linear shift invariant systems, draw pole zero diagram and identify the filter type based on pass band.

(a) $H(z) = \dfrac{Z^{-1} - a^*}{1 - a z^{-1}}$ *where* $|a| < 1$ **(b)** $H(z) = A \dfrac{1 - z^{-4}}{1 + a^4 z^{-4}}$

Solution : (a)

$$H(z) = \frac{Z^{-1} - a^*}{1 - a z^{-1}} \quad where\ |a| < 1$$

$$H(z) = \frac{1 - a^* z}{z - a} \quad Let\ a = 0.5$$

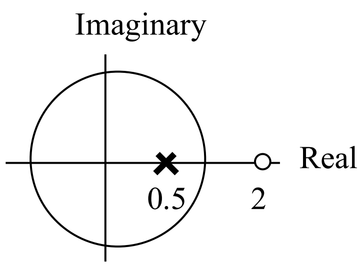
$$H(z) = \frac{1 - 0.5^* z}{z - 0.5} = \frac{1 - 0.5z}{z - 0.5}$$

[1] POLE-ZERO diagram

$H(z) == \dfrac{1-0.5z}{z-0.5}$

ZEROS: $Z_1 = 2$

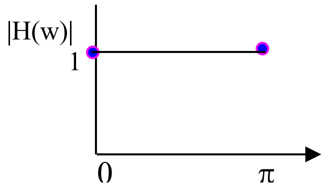
POLES : $P_1 = 0.5$



[2] Magnitude Spectrum

$$H(z) == \frac{1-0.5z}{z-0.5}$$

1. **At w = 0, z = 1**
   $|H(w)| = 1$
2. **At w = π, z = -1**
   $|H(w)| = 1$



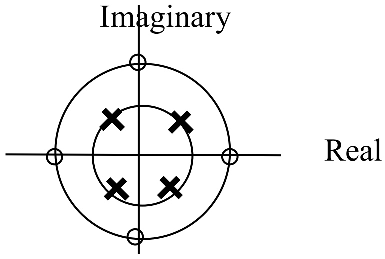
Filter is ALL Pass Filter

Solution : (b) $H(z) = A \dfrac{1-z^{-4}}{1+a^4 z^{-4}}$

$$H(z) = A \frac{z^4-1}{z^4+a^4} \quad \textbf{Let a =0.5}$$

[1] POLE-ZERO diagram : $H(z) = A \dfrac{z^4-1}{z^4+(0.5)^4}$

ZEROS	POLES
$z^4-1=0$	$z^4+(0.5)^4=0$
$z^4=1$	$z^4=-(0.5)^4$
$z^4 = e^{j2\pi k}$	$z^4=(0.5)^4 e^{j\pi} e^{j2\pi k}$
$z_k = e^{j\frac{\pi k}{2}}$	$z_k=(0.5)e^{j\pi\left(\frac{2k+1}{4}\right)}$
k=0 , $z_0=1$	**k=0 ,** $z_0=0.5e^{j\frac{\pi}{4}}$
k=1, $z_1=e^{j\frac{\pi}{2}}$	**k=1,** $z_1=0.5e^{j\frac{3\pi}{4}}$
k=2, $z_2=e^{j\pi}$	**k=2,** $z_2=0.5e^{j\frac{5\pi}{4}}$
k=3, $z_3=e^{j\frac{3\pi}{2}}$	**k=3,** $z_3=0.5e^{j\frac{7\pi}{4}}$

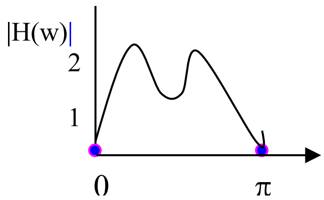


[2] Magnitude Spectrum

$$H(z) = A \frac{z^4-1}{z^4+(0.5)^4} \quad \textbf{Let A =1 Put z = e}^{\textbf{jw}}$$

$$H(w) = \frac{e^{j4w}-1}{e^{j4w}+0.0625} == \frac{[\cos(4w)-1]+j\,[\sin 4w)]}{[\cos(4w)+0.0625]+j\,[\sin(4w)]}$$

W	|H(w)|	ϕ
0	0	0
0.2π	2.002	0.35
0.4π	1.151	-1.000
0.6π	1.151	1.000
0.8π	2.002	-0.353
π	0	0



Filter is Band Pass Filter.

Q(33) Consider filter with transfer function $H(z) = \dfrac{z^{-1} - a}{1 - a\,Z^{-1}}$

Identify the type of filter and justify it.

Solution : **Given** $H(z) = \dfrac{z^{-1} - a}{1 - a\,Z^{-1}}$

$$H(z) = \frac{1 - az}{z - a} = \frac{-az + 1}{z - a}$$

$$H(z) = \frac{-a\left(z - \dfrac{1}{a}\right)}{z - a}$$

POLE : Z = a **ZERO : Z = 1/a**

Here, POLE = 1/ ZERO Therefore filter is **ALL PASS IIR FILTER.**

Q(34) Determine the zeros of the following FIR systems and indicate whether the system is minimum phase, maximum phase, or mixed phase.

$H_1(Z) = 6 + Z^{-1} - Z^{2}$

$H_2(Z) = 1 - Z^{-1} - 6Z^{-2}$

$H_3(Z) = 1 - (5/2)Z^{-1} - (3/2)Z^{-2}$

$H_4(Z) = 1 + (5/3)Z^{-1} - (2/3)Z^{-2}$

Comment on the stability of the minimum and maximum phase system

Solution :

(i) $H_1(Z) = 6 + Z^{-1} - Z^{2}$

$$H_1(z) = \frac{6z^2 + z - 1}{z^2} = \frac{6\left(z + \dfrac{1}{2}\right)\left(z - \dfrac{1}{3}\right)}{z^2}$$

ZEROS $\mathbf{Z_1} = -\tfrac{1}{2}$ $\mathbf{Z_2} = \tfrac{1}{3}$

As both the zeros are INSIDE the unit circle, this system is **Minimum Phase** System.

(ii) $H_2(Z) = 1 - Z^{-1} - 6Z^{-2}$

$$H_2(z) = \frac{z^2 - z - 6}{z^2} = \frac{(z+2)(z-3)}{z^2}$$

Hence the zeros are at $z = \dfrac{1}{3}$ and $z = -\dfrac{1}{2}$.

ZEROS $\mathbf{Z_1} = -2$ $\mathbf{Z_2} = \mathbf{3}$

As both the zeros are OUTSIDE the unit circle, this system is **Maximum Phase** System.

(iii) $H_3(Z) = 1 - (5/2)Z^{-1} - (3/2)Z^{-2}$

$$H_3(z) = \frac{z^2 - 2.5\,z - 1.5}{z^2} = \frac{(z+0.5)(z-3)}{z^2}$$

ZEROS **Z_1** $= -0.5$ **Z_2 = 3**

For this system one zero is inside the unit circle and one is outside the unit circle, hence the system is Mixed Phase System.

(iv) $H_4(Z) = 1 + (5/3)Z^{-1} - (2/3)Z^{-2}$

$$H_4(z) = \frac{z^2 + \frac{5}{3}z - \frac{2}{3}}{z^2} = \frac{\left(z - \frac{1}{3}\right)(z+2)}{z^2}$$

ZEROS **$Z_1 = \frac{1}{3}$** **$Z_2 = -2$**

For this system one zero is inside the unit circle and one is outside the unit circle, hence the system is Mixed Phase System.

Q(35) State whether following statement is true or false, justify your answer in 2 to 3 sentences. "A stable filter is always causal"

ANS : FALSE

A stable filter can be causal, or Non-causal (i.e. Anti-causal or Both-sided)

For stable filter, ROC must include unit circle.

For causal & stable filter all poles must lie inside the unit circle eg.

$$H(z) = \frac{z}{z-0.5} \quad |z| > 0.5$$

For Anti-causal & stable filter all poles must lie outside the unit circle eg

$$H(z) = \frac{z}{z-2} \quad |z| > 2$$

Ex. Both sided & stable filter $H(z) = \frac{z}{z-0.5} + \frac{z}{z-2} \quad 2 > |z| > 0.5$

Q(36) Realize a Digital Sinusoidal Generator with the help of block diagram.

Solution: Consider a Digital Sinusoidal Generator with output $y[n] = r^n \cos(nw)\, u[n]$ and input x[n] = δ[n]

By ZT , $Y(z) = \dfrac{z^2 - r z \cos(w)}{z^2 - 2 r z \cos(w) + r^2}$

$Y(z) = \dfrac{1 - r\cos(w) z^{-1}}{1 - 2r\cos(w) z^{-1} + r z^{-2}}$ ---------((1))

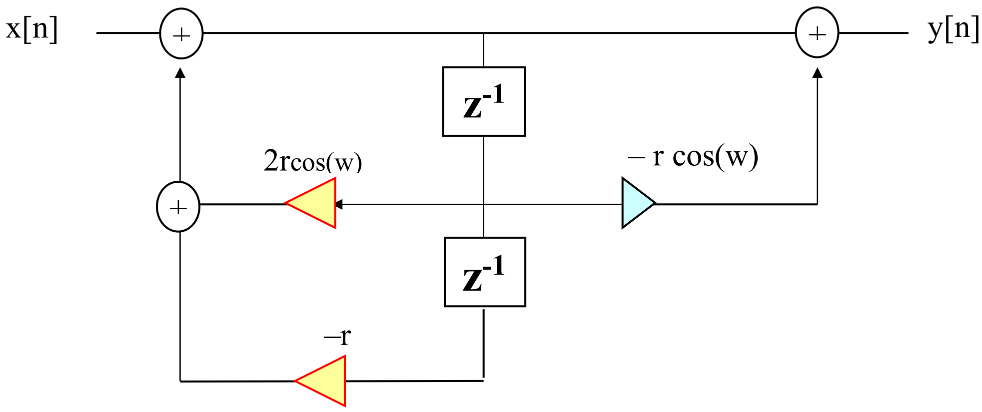
x[n] = δ[n]

By ZT ,X(z)=1 -----------((2))

From eq (1) and (2)

$H(z) = \dfrac{Y(z)}{X(z)}$

$H(z) = \dfrac{1 - r\cos(w) z^{-1}}{1 - 2r\cos(w) z^{-1} + r z^{-2}}$ ➔ This is Transfer function of a Digital Sinusoidal Generator



Q(37) A two POLE filter has the system function $H(z) = \dfrac{bo}{(1 - p z^{-1})^2}$. Determine the values of b_0 and P such that the frequency response H(w) satisfies the condition H(0)=1 and $\left| H\left(\frac{\pi}{4}\right) \right|^2 == \frac{1}{2}$.

Solution :

(i) At $w = 0$, $z = 1$ $\quad \therefore \quad H(0) = \dfrac{bo}{(1-p)^2} = 1$

Hence $\quad bo = (1 - p)^2$

(ii) At $w = \pi/4$ $\qquad z = \dfrac{1}{\sqrt{2}} + j\dfrac{1}{\sqrt{2}}$ $\quad \therefore H\left(\dfrac{\pi}{4}\right) = \dfrac{bo}{(1-pz^{-1})^2}$

$$H\left(\tfrac{\pi}{4}\right) = \frac{bo}{\left(1 - \frac{1}{\sqrt{2}}p - j\frac{1}{\sqrt{2}}p\right)^2}$$ Put $bo = (1 - p)^2$

$$\left|H\left(\frac{\pi}{4}\right)\right| = \frac{(1-p)^2}{\left[\left(1 - \frac{1}{\sqrt{2}}p\right)^2 + \left(\frac{1}{\sqrt{2}}p\right)^2\right]}$$

$$\left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{(1-p)^4}{\left[\left(1 - \frac{1}{\sqrt{2}}p\right)^2 + \left(\frac{1}{\sqrt{2}}p\right)^2\right]^2}$$

$$\left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{(1-p)^4}{\left[1 + \frac{1}{2}p^2 - \frac{2}{\sqrt{2}}p + \frac{1}{2}p^2\right]^2}$$

$$\left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{(1-p)^4}{\left[1 + p^2 - \frac{2}{\sqrt{2}}p\right]^2} == \frac{1}{2}$$

Taking square root on both sides we get,

$$\frac{(1-p)^2}{\left(1 + p^2 - \sqrt{2}p\right)} = \frac{1}{\sqrt{2}}$$

$$\sqrt{2}\,(1-p)^2 = (1 + p^2 - \sqrt{2}\,p)$$

$$\sqrt{2}\,(1 + p^2 - 2p) = 1 + p^2 - \sqrt{2}\,p$$

$$1 + p^2 - 2p = 0.707 + 0.707\,p^2 - p$$

$$p^2(1 - 0.707) - p + (1 - 0.707) = 0$$

$$p^2\,(0.293) - p + 0.293 = 0$$

$$p^2 - 3.413\,p + 1 = 0$$

By solving we get, **$p = 3.089$** OR **$p = 0.323$** **ANS**

➢ Drill Problems

Q(38) The Magnitude Response of a system is given below. Find the Transfer Function H(z) of the System.

$$\left|H(e^{jw})\right| = \frac{1}{\sqrt{1+a^2-2a\cos\omega}}$$

Solution :

$$\left|H(e^{jw})\right|^2 = \frac{1}{1+a^2-2a\cos\omega}$$

$$= \frac{1}{1+a\,e^{jw}a\ e^{-jw}-2a\left(\dfrac{e^{jw}+e^{-jw}}{2}\right)}$$

$$= \frac{1}{1-ae^{-jw}+a\ e^{jw}a\ e^{-jw}-a\ e^{jw}}$$

$$= \frac{1}{(1-a\ e^{-jw})\ (1-a\ e^{jw})}$$

$$= H(e^{jw})\ H(e^{-jw})$$

$$H(e^{jw}) = \frac{1}{1-a\ e^{-jw}}$$

$$H(z) = \frac{1}{1-a\ z^{-1}} = \frac{z}{z-a}$$ **ANS**

Q(39) The z – transform of the sequence x(n) = u(n) – u(n –7) is sampled at five points on the unit circle as follows:

$$X[k] = X(Z)\,|_{z=e^{\frac{j2\pi k}{5}},\ k=0,1,2,3,4}$$

Determine the inverse DFT x(n) of X[k].

Solution : Given x(n) = u(n) – u(n – 7)

By ZT, $X(z) = \dfrac{z}{z-1} - z^{-7}\dfrac{a}{z-1} = \dfrac{z}{z-1} - \dfrac{z^{-6}}{z-1}$

$$X(z) = \frac{z-z^{-6}}{z-1} = \frac{z^7-1}{z^6(z-1)}$$

$$X(z) = \frac{z^6+z^5+z^4+z^3+z^2+z+1}{z^6}$$

$$X(z) = 1+z^{-1}+z^{-2}+z^{-3}+z^{-4}+z^{-5}+z^{-6}$$

Put $z = e^{jw}$

$H(e^{jw}) = 1 + e^{-jw} + e^{-j2w} + e^{-j3w} + e^{-j4w} + e^{-j5w} + e^{-j6w} = H(w)$

Put $w = \dfrac{2\pi k}{N} = \dfrac{2\pi k}{5}$

$$X[k] = H(w)\Big|_{w} = \frac{2\pi k}{N}$$

$$X[k] = 1 + e^{-j}\left(\frac{2\pi k}{N}\right) + e^{-j2}\left(\frac{2\pi K}{N}\right) + e^{-j4}\left(\frac{2\pi k}{N}\right) + e^{-j5}\left(\frac{2\pi k}{N}\right) + e^{-j6}\left(\frac{2\pi k}{N}\right)$$

Put $w_N^1 = e^{-j\frac{2\pi}{N}}$

$$X[K] = 1 + W_N^K + W_N^{2K} + W_N^{3K} + W_N^{4K} + W_N^{5K} + W_N^{6K}$$

$$X[0] = 1 + W_N^0 + W_N^0 + W_N^0 + W_N^0 + W_N^0 + W_N^0 = 7$$

$$X[1] = 1 + W_N^1 + W_N^2 + W_N^3 + W_N^4 + W_N^5 + W_N^6 = 0$$

$$X[2] = 1 + W_N^2 + W_N^4 + W_N^6 + W_N^8 + W_N^{10} + W_N^{12} = 0$$

$$X\,[3] = 0$$

$$X\,[4] = 0$$

$$x'[n] = \frac{1}{N}\sum_{K=0}^{N-1} X[k] W_N^{-nK} = \frac{1}{5}\sum_{K=0}^{4} X[K\} W_N^{-nK}$$

$$x'[n] = \frac{1}{5}[x(0) + x[1]\, W_N^{-n} + x[2] W_N^{-2n} + X[3] W_N^{-3n} + X[4] W_N^{-4n}]$$

$$x'[n] = \frac{1}{5}[7 + 0 + 0 + 0 + 0] = 1.4$$

$$x'[n] = \{1\cdot 4,\ 1\cdot 4,\ 1\cdot 4,\ 1\cdot 4,\ 1\cdot 4\}$$

Q(40) Suppose that H(z) and G(z) are rational and have minimum phase. Which of the following filters have minimum phase.

(i) H(z) G(z) (ii) H(z) + G(z)

Solution :

H(z) and G(z) are rational and have minimum phase.

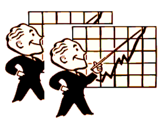
For minimum phase system, all ZEROS are always inside the unit circle.

Let $H(z) = \dfrac{z - \frac{1}{2}}{z}$ and $G(z) = \dfrac{z - \frac{1}{3}}{z}$

(i) Let P(z) = H(z) G(z)

$$P(z) = \left(\frac{z - \frac{1}{2}}{z}\right)\left(\frac{z - \frac{1}{3}}{z}\right)$$

Since all ZEROS are inside the unit circle, System is Minimum phase system.



(ii) Let Q(z) = H(z) + G(z)

$$Q(z) = \left(\frac{z-\frac{1}{2}}{z}\right) + \left(\frac{z-\frac{1}{3}}{z}\right)$$

$$Q(z) = \frac{z\left(z-\frac{1}{2}\right) + z\left(z-\frac{1}{3}\right)}{z^2}$$

$$Q(z) = \frac{z^2 - \frac{1}{2}z + z^2 - \frac{1}{3}z}{z^2}$$

$$Q(z) = \frac{2z^2 - \frac{5}{6}z}{z^2} == \frac{2\left(z-\frac{5}{12}\right)}{z}$$

Since ZERO is inside the unit circle, System is Minimum phase system.

Q(41) In a DSP system sampled at 1 kHz a notch band-pass filter at 125 Hz is to be added. Show the necessary extra poles and zeros to achieve this.

Solution : Sampling frequency F_s = 1000 Hz. The band pass frequency F_{BP} = 125 Hz.

$$f_{BP} = \frac{F_{PB}}{F_s} = \frac{25}{1000} = \frac{1}{8} \qquad \omega_{BP} = 2\pi f_{BP} = 2\pi \times \frac{1}{8} = \frac{\pi}{4} \text{ rad}$$

Here we want to pass a band of frequencies near $\omega = \frac{\pi}{4}$ Hence we should place pole at $\omega = \frac{\pi}{4}$. This pole will allow the system to pass frequencies near $\omega = \frac{\pi}{4}$, As shown in figure and pole is place at $\omega = \frac{\pi}{4}$ [i.e. z = 0.707 + j 0.707]. The filter will be realizable if all the poles and zeros occurs in complex conjugate pairs. Hence we should place conjugate poles. Z = (0.707 – j 0.707) at $\omega = \frac{\pi}{4}$

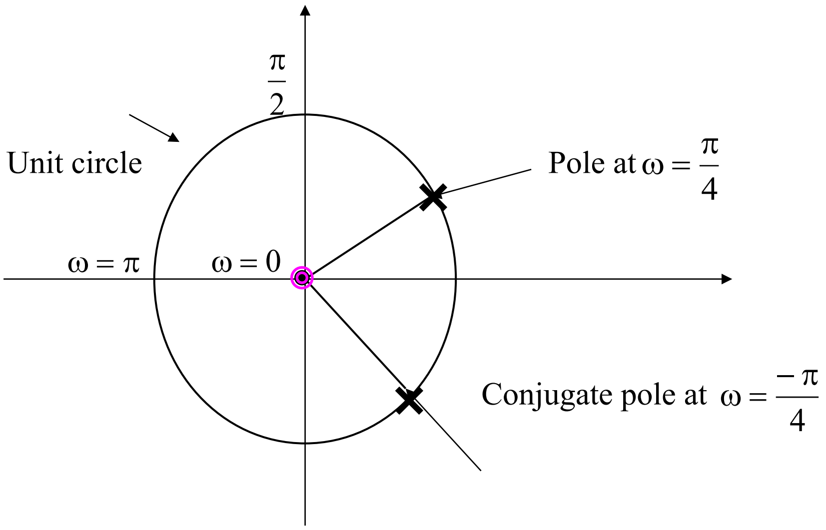
To make system physically realizable , the number of poles and zeros should be equal since we have placed two poles, we should also place two zeros. Normally such zeros are placed at origin z = 0. These zeros do not affect the frequency response. Since they are placed at z= 0.

The poles and zeros of the band pass filter as follows.

Poles at $z = 0.707 \pm j\,0.707$ and Zeros at z = 0 (two zeros)

$\therefore$ The system function $H(z) = \frac{(z-z_1)\,(z-z_2)}{(z-p_1)(z-P_1)}$

$$\therefore H(z) = \frac{(z-0)\,(z-0)}{(z-[0.707+0.707j])\,[z-(0.707-0.707j)]} == \frac{z^2}{z^2 - 1.414\,z + 1}$$



Q(42) Design a two POLE Bandpass filter that has the centre of its passband at $w = \pi/2$, zero in its frequency response characteristics at $w=0$ and $w = \pi$ and its magnitude response is $^1/\sqrt{2}$ at $w = 4\pi/9$.

Solution: The filter must have poles at, $P_1, P_2 = r\,e^{\pm j\pi/2}$ and zero at $z_1 = 1$, $z_2 = -1$

The system function is $$H(z) = \frac{G(z-1)\,(z+1)}{(z-re^{j\pi/2})\,\,(z-re^{-j\pi/2})} \quad = \frac{G(z^2-1)}{z^2+r^2}$$

(i) To find G

Let $|H(e^{jw})| = 1$ at $w = \pi/2$

When $w = \pi/2$ $z = j$

$$|H(e^{jw})| \quad = \frac{G(j^2-1)}{j^2+r^2} \quad = \; 1 \qquad \therefore \qquad G = \frac{1-r^2}{2}$$

(ii) To find r

$$|H(e^{jw})| = \frac{1}{\sqrt{2}} \qquad \text{at } w = \frac{4\pi}{9}$$

$$H(z) = \frac{G(z^2-1)}{z^2+r^2} = 1$$

Put $z = e^{jw}$

$$H(e^{jw}) = \frac{G(e^{2jw}-1)}{e^{j2w}+r^2} = 1$$

$$H(e^{jw}) = \frac{(1-r^2)}{2}\,\frac{[\;\cos\,(2w + j\sin\,(2w) - 1\;]}{[\;\cos\,(2w) + j\sin\,(2w) + r^2\;]}$$

Put w = 4π/9

$$H(e^{jw}) = \frac{(1-r^2)}{2}\ \frac{[\cos(8\pi/9) + j\sin(8\pi/9) - 1]}{[\cos(8\pi/9) + j\sin(8\pi/9) + r^2]}$$

$$H(e^{jw}) = \frac{(1-r^2)}{2}\ \frac{-1.93969 + j\,0.34202}{-0.93969 + j\,0.34202 + r^2}$$

$$|H(e^{jw})| = \frac{(1-r^2)}{2}\ \frac{1.969}{0.9999} = \frac{1}{\sqrt{2}} \qquad \underline{\therefore\ \ r = 0.47525}$$

$$r^2 = 0.7 \quad G = \frac{1-r^2}{2} = 0.15$$

By substituting, $H(z) = \dfrac{G(z^2-1)}{z^2+r^2}$ $\qquad H(z) = \dfrac{0.15(z^2-1)}{z^2+0.7}$ $\qquad$ ANS

Q(43) Consider a moving averager $y(n) = \dfrac{1}{M+1}\displaystyle\sum_{k=0}^{M} x(n-k)$ Show the pole zero diagram of the system for $M = 7$. Also find an expression of the magnitude response of the filter.

Solution : - To find $H(z)$:

$$y(n) = \frac{1}{M+1}\sum_{k=0}^{M} x(n-k)$$

By ZT,

$$Y(z) = \frac{1}{M+1}\sum_{k=0}^{M} z^{-k}\,x(z)$$

$$= \frac{1}{M+1}\left[\frac{1-(z^{-1})^{M+1}}{1-(z^{-1})}\right]x(z)$$

$$H(z) = \frac{1}{M+1}\left[\frac{1-(z^{-1})^{M+1}}{1-(z^{-1})}\right]$$

Put $M = 7$

$$H(z) = \frac{1}{8}\left[\frac{1-z^{-8}}{1-z^{-1}}\right] = \frac{z^8-1}{8(z-1)z^7}$$

$$H(z) = \frac{1}{8}\ \frac{(z^4+1)\,(z^2+1)\,(z+1)}{z^7}$$

POLES :

$$P_1 = P_2 = P_3 = P_4 = P_5 = P_6 = P_7 = 0$$

ZEROS :

(i) $z^4 + 1 = 0$

$z^4 = -1$

$z^4 = e^{j\pi}\ (1)$

$= e^{j\pi}\ e^{j\,2\pi k}$

$z^4 = e^{j\pi\,(2k+1)}$

$z_k = e^{j\pi\left(\frac{2k+1}{4}\right)}$

$K = 0,\ Z_0 = e^{j\pi/4}$

$K = 1,\ Z_1 = e^{j3\pi/4}$

$K = 2,\ Z_2 = e^{j5\pi/4} = e^{-j3\pi/4}$

$K = 3,\ Z_3 = e^{j7\pi/4} = e^{-j\pi/4}$

Total No of ZEROs = 7

ZEROS : $1,\ \pm j,\ \frac{1}{\sqrt{2}}(1 \pm j),\ \frac{1}{\sqrt{2}}(-1 \pm j)$

(ii) $z^2 + 1 = 0$

$z^2 = -1$

$\therefore$ zeros at $z = j$ and $z = -j$

(iii) $z + 1 = 0$

zero at $z = -1$

To find magnitude response :

$$H(z) = \frac{1}{8}\left[1 + z^{-1} + z^{-2} + z +^{-3} + z^{-4} + z^{-5} + z^{-6} + z^{-7}\right]$$

Put $z = e^{jw}$

$$H(e^{jw}) = \frac{1}{8}\left[1 + e^{-jw} + e^{-2jw} + e^{-j3w} + e^{-j4w} + e^{-j5w} + e^{-j7w}\right]$$

$$= \frac{1}{8} e^{-j\frac{7}{2}w}\left[e^{j\frac{7}{2}w} + e^{j\frac{5}{2}w} + e^{j\frac{3}{2}w} + e^{j\frac{1}{2}w} + e^{-j\frac{1}{2}w} + e^{-j\frac{3}{2}w} + e^{-j\frac{5}{2}w} + e^{-j\frac{7}{2}w}\right]$$

$$H(w) = \frac{1}{8}\, e^{-j\frac{7}{2}w}\left[2\cos\left(\frac{7}{2}w\right) + 2\cos\left(\frac{5}{2}w\right) + 2\cos\left(\frac{3}{2}w\right) + 2\cos\left(\frac{1}{2}w\right)\right]$$

$\rightarrow$ Magnitude Response $M(w) = \frac{1}{4}\left[\cos\left(\frac{7}{2}w\right) + \mathrm{cows}\left(\frac{5}{2}w\right) + \cos\left(\frac{3}{2}w\right) + \cos\left(\frac{1}{2}w\right)\right]$

$\rightarrow$ Phase Response $\phi(w) = e^{-j7/2\,w}$

$\therefore$ Phase $\phi = -\left(\frac{7}{2}\right)w$ if $M(w) \geq 0$

$$\Phi = -(3.5)w + \pi \quad if \quad M(w) < 0$$