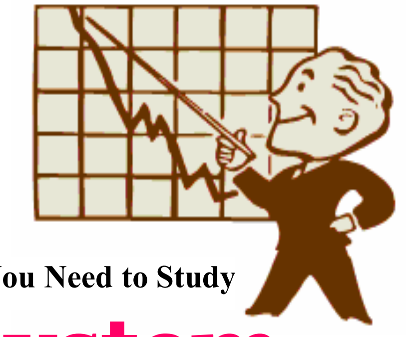
Chapter - 3



You Need to Study

Analysis of DT System

Dr. Kiran TALELE

- Academic : PhD
- Professional :
  - Dean-Students, Alumni & External Relations @ Bharatiya Vidya Bhavans' Sardar Patel Institute of Technology (SP-IT), Mumbai
  - Head-Academic Relation @ Sardar Patel Technology Business Incubator (SP-TBI), Mumbai
  - Treasurer-IEEE Bombay Section

https://www.linkedin.com/in/k-t-v-talele/

www.facebook.com/Kiran-Talele-1711929555720263

[1] THE FREQUENCY RESPONSE OF THE SYSTEM.

Definition. The *frequency response* of an LTI filter is defined as the spectrum of the output signal divided by the spectrum of the input signal.

The frequency Response of the system is determined by giving the sinusoidal input to the system. For sinusoidal input, output signal is also sinusoidal of the same frequency as the input signal. Thus in passing through the system the input signal is subjected only to an **amplitude scaling and a phase shift.**

The ratio of the output signal amplitude to the input signal amplitude is the amplitude response at the input frequency. The output phase minus the input phase is the phase response at the input frequency.

Consider a sampled sinusoidal sequence, $x[n] = A\ e^{j(nw+\theta)}$ as the input to the DT system.

The response of the system y[n] with impulse h[n] for input sequence x[n] is given by,

$$y[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m]$$

$$y[n] = \sum h[m]\, A.e^{j(wn+\theta)}$$

$$y[n] = A.\ e^{j(wn+\theta)} \sum_{-\infty}^{\infty} h[m]e^{-jmw}$$

$$y[n] = x[n]. \sum_{-\infty}^{\infty} h[m]e^{-jmw}$$

Let $y[n] = x[n]\ .\ G(w)$ Where $G(w) = \sum_{-\infty}^{\infty} h[m]e^{-jmw}$

The output sequence y[n] is the input sequence x[n] multiplied by a complex weighting factor G(w) that completely depends on input frequency w.

The multiplier G (w) is called the frequency response of the system.Since y[n] is also a sampled sinusoidal, we then have,

$$y[n] = B\ e^{j(wn+\phi)} = x[n]\ G(w)$$

$$B\ e^{j(wn+\phi)} = A\ e^{j(wn+\theta)}.G(w)$$

$$G(w) = \frac{B e^{j(wn+\phi)}}{A\ e^{j(wn+\theta)}} = \frac{B}{A}\ e^{j(\phi-\theta)}.$$

$$|\ G(W)| = \frac{B}{A} = \frac{Magnitude\ of\ Numerator}{Magnitude\ of\ Deno\min ator}$$

$$\angle G(W) = \phi - \theta = .\ \text{Angle of Numerator}\ \ \text{Angle of Denominator}$$

The frequency response of the system can be obtained Analytically OR Graphically.

(I) Frequency Response using Analytical Method :

The frequency response can be obtained from its Z – Transform. By substituting $Z = e^{jw}$.

$$G(W) = H(z)\Big|_{z = e^{jwt}} = H(e^{jwT})$$

i) **Magnitude Response** $= \dfrac{Magnitude\ of\ Numerator}{Magnitude\ of\ Denominator}$

$$\text{Where Magnitude} = \sqrt{(\text{Real})^2 + (\text{Imaginary})^2}$$

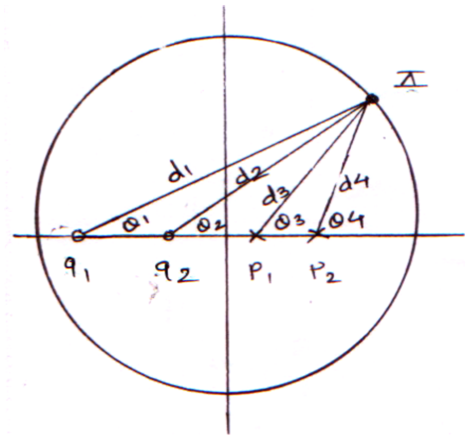
ii) **Phase Response = Angle of Numerator – Angle of denominator**

$$\text{Where angle} = \begin{cases} \tan^{-1}\left(\dfrac{Imaginary}{Real}\right) & When\ \ Real > 0 \\ \tan^{-1}\left(\dfrac{Imaginary}{Real}\right) + \pi & When\ \ Real < 0 \end{cases}$$

II) Frequency Response using Graphical Method :

In Graphical method, the frequency response at a given frequency w is determined by the ratio of the product of the zero vectors with the product of pole vectors.

$$\text{Magnitude Response} = \frac{Product\ of\ distance\ from\ zeros}{Product\ of\ distance\ from\ poles}$$



Example :

$$\text{Consider } H(z) = \frac{(z-q_1)(z+q_2)}{(z-p_1)(z-p_2)}$$

$$Magnitude\ Response = \frac{d_1 . d_2}{d_3 . d_4}$$

Phase Response = Summation of angles from ZEROS – Summation of angles from POLES.

$$\text{Phase Response} = (\theta_1 + \theta_2) - (\theta_3 + \theta_4)$$

Magnitude Spectrum is :	Phase Spectrum is :
(i) **Continuous** function of w for non periodic signal.	(i) **Continuous** function of w for non periodic signal.
(ii) **Symmetric** about w = 0	(ii) **Anti-Symmetric** about w = 0
(iii) **Periodic** with Period = 2π	(iii) **Periodic** with Period = 2π

IMPORTANT NOTE :

FIR Filter	IIR Filter
➢ **Poles are only at origin** ➢ **No. of Zeros = No. of Poles** **(1)** When all zeros are on Left side, filter is LPF e.g. $H(z) = \dfrac{(z+1)(z+½)(z+2)}{z^3}$ **(2)** When all Zeros are on Right side, filter is HPF. e.g. $H(z) = \dfrac{(z-1)(z-½)(z-2)}{z^3}$ **(3)** When zeros are both sides, filter is BPF. e.g. $H(z) = \dfrac{(z-1)(z+1)}{z^2}$	➢ **Poles can be anywhere in z-plane.** ➢ **No. of Zeros ≤ No. of Poles** **(1)** When All ZEROS are on Left side of POLES , then filter is LPF e.g. $H[z] = \dfrac{z}{z - 0.5}$ **(2)** When All ZEROS are on Right side of POLES , then filter is HPF e.g. $H[z] = \dfrac{z}{z + 0.5}$ (3) When ZEROS of the filter are outer sides of POLES,then filter is BPF e.g. (i) $H(z) = \dfrac{(z+1)(z-1)}{(z-0.5)(z+0.5)}$

Response of Complex Exponential and Sinusoidal Signal

Consider a complex exponential signal $x[n] = A\, e^{jnw} \quad -\infty < n < \infty$

Then $y[n] = x[n] * h[n]$

$y[n] = \sum x[m]\, h[n-m]$ where h[n] is impulse response of the system.

$$= \sum_{-\infty}^{\infty} x[m]\ A\ e^{jw(n-m)}$$

$$= \sum_{-\infty}^{\infty} x[m]\ A\ e^{-jmw} e^{jnw}$$

$$= A \sum_{-\infty}^{\infty} \left[x[m]\ e^{-jmw}\right] e^{jnw}$$

$y[n] = A\, H(w)\, e^{jnw}$ where H(w) is fourier Transform of h[n].

Output of the y[n] is also in the form of complex exponential with the same frequency as the input but altered by multiplicative factor H(w). As a result of this characteristics behaviour the exponential signal $x[n] = A\ e^{jnw}$ is called an eigen-function of the system.

Eigen-function of a system is an input signal that produces an output that differs from the input by a constant multiplicative factor. The multiplicative factor is called an eigen value of the system.***

❖ Steady State and Transient Responses to Sinusoidal Input Signals

Case-1 : If sinusoidal signal is applied to the system at $n = -\infty$, the response of the LTI system is the **Steady-State-Response** of the system. There is **No Transient Response** in this case.

The output of the system is given by $y[n] = y_{tr}[n] + y_{ss}[n]$

But $y_{tr}[n] = 0 \quad \therefore \quad y[n] = y_{ss}[n]$

Case-2 : If the sinusoidal signal is applied to the system at some finite time instant, say at $n = 0$, the response of the system consists of two terms, the **Transient Response** and the **Steady State Response.**

The transient response decays toward zero as n extends to ∞.

The output of the system is given by $y[n] = y_{tr}[n] + y_{ss}[n]$

[2] Linear Invariant System as Frequency Selective Filters

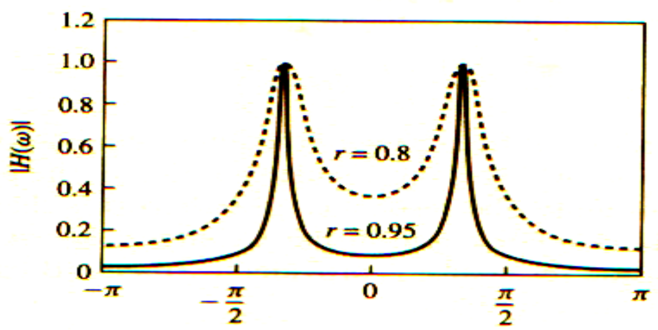
2.1. DIGITAL RESONATOR

A digital resonator is a special two pole Band-pass filter with the pair of complex conjugate poles located near the unit circle as shown below. Digital resonator is a narrowband band-pass filter. The name resonator refers to the fact that the filter has a large magnitude response (ie it resonates) in the vicinity of the pole location. The angular position determines the resonant frequency of the filter. Digital resonators are useful for bandpass filtering, speech generation etc.

Design of digital resonator : To have a peak at $\omega = \omega_o$ select the complex conjugate poles at $p_1 = r\,e^{j\omega o}$ and $p_2 = r\,e^{-j\omega o}$ and zeros at $Z_1 = 1$ and $Z_2 = -1$

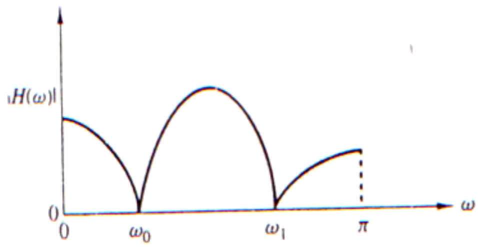
The system function and the magnitude response is given by,

$$H(z) = \frac{G(z-1)(z+1)}{(z - re^{j\omega_o})(z - re^{-j\omega_0})} = \frac{G(z-1)(z+1)}{z^2 - 2\,r\,z\,\cos(\omega_o) + r^2}$$



2.2 NOTCH FILTER

A notch filter is a filter that contains one or more deep notches or ideally a perfect nulls in the frequency response characteristics. The following figure illustrates the frequency characteristics of a notch filter with nulls at frequencies ω_o and ω_1.



Notch filters are useful in many applications where the specific frequency components must be eliminated. For example, instrumentation and recording systems require that the power line frequency of 60 Hz and its harmonics be eliminated.

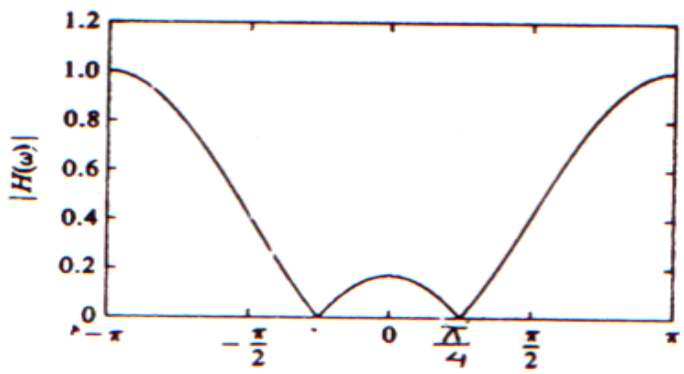
To create a null in the frequency response of a filter at a frequency ω_o, we simply introduce a pair of complex-conjugate zeros on the unit circle at an angle ω_o.

That is $Z_1 = e^{j\omega o}$ $Z_2 = e^{-j\omega o}$

Thus the system function of FIR notch filter is simply,

$$H(z) = \frac{G\,(z-e^{j\omega_o})(z-e^{-j\omega_o})}{z^2}$$

Magnitude response of FIR notch filter having a null at $\omega = \frac{n\ \pi}{4}$



The problem with FIR notch filter is that the other frequency components around the desired NULL are severely attenuated.

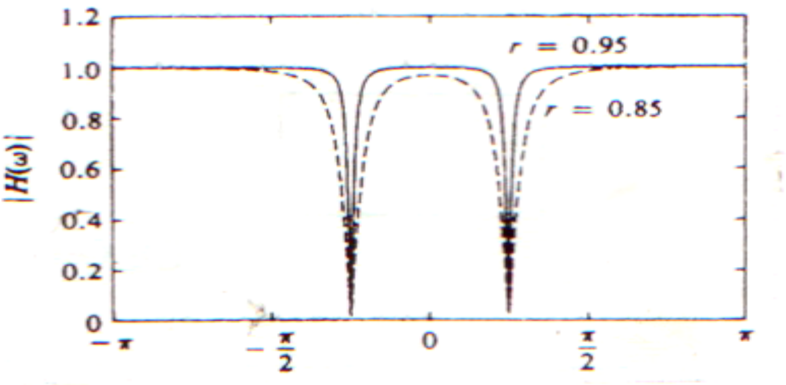
Suppose a pair of complex conjugate poles at at $p_1 = r\,e^{j\omega o}$ and $p_2 = r\,e^{-j\omega o}$

The effect of pole is to introduce a resonance in the vicinity of the null that reduces the bandwidth of the null.

The system function of the resulting filter is

$$H(z) = \frac{G\,(z-e^{j\omega_o})(z-e^{-j\omega_o})}{(z-r\,e^{j\omega_o})(z-re^{-j\omega_o})} = \frac{z^2 - 2\,z\,\cos(\omega_o)\ +\ 1}{z^2 - 2\,r\,z\,\cos(\omega_o)\ +\ r^2}$$

The magnitude response of the filter with $\omega_o = \frac{\pi}{4}$ and r = 0.85.

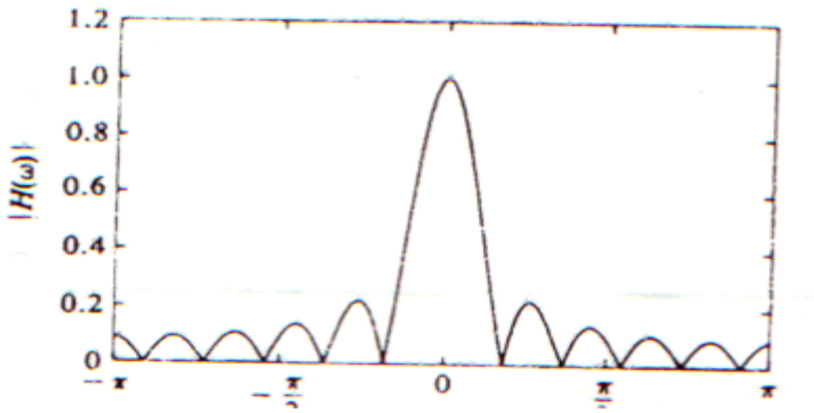


2.3 COMB FILTER

Comb filter is a notch filter in which the nulls occur periodically across the frequency band. Comb filter has applications in a wide range of practical systems such as in the rejection of power line harmonics

Comb filter has zeros on the unit circle at $e^{\frac{j2\pi k}{M+1}}$ for k = 1, 2, 3, …M

Magnitude response of comb filter with M = 10 is given by,



2.4 ALL PASS FILTER

An all pass filter is defined as a system that has a constant magnitude response for all frequencies, that is $|H(\omega)| = 1$ for $0 \le \omega \le \pi$.

The simplest example of an all pass filter is a pure delay system with system function $H(z) = z^{-k}$. All pass filter passes all signals without modification. It simply adds delay of k samples.

All pass filter is also described by the system function $H(z) = \dfrac{a_N + a_{N-1} z^{-1} + a_{N-2} z^{-2} + \dots + a_1 z^{-N+1} + z^{N}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{N}}$ where all coefficients are real.

If we define polynomial A(z) as $A(z) = \sum_{k=0}^{N} a_k z^{-k}$ with $a_o = 1$

Then H(z) can be expressed as $A(z) = z^{-N} \dfrac{A(z^{-1})}{A(z)}$

Since $|H(w)|^2 = H(z)\ H(z^{-1})|_{z=e^{jw}} = 1$, the system is an all pass filter.

Application :

All pass filter finds application as Phase Equalizer. When placed in cascade with a system that has an undesired phase response, a phase equalizer is designed to compensate for the poor phase characteristics of the system to produce an overall Linear phase Response.

2.5 DIGITAL SINUSOIDAL OSCILLATOR

A Digital sinusoidal oscillator can be viewed as a limiting form of a two pole resonator for which the complex conjugate poles lie on the unit circle.

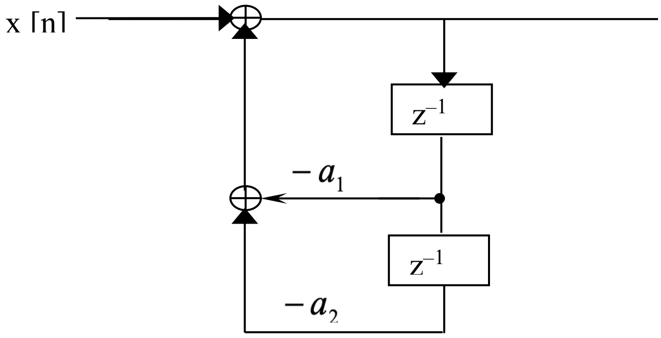
Consider the complex poles at $P_1 = r\, e^{j\omega o}$ and $P_2 = r\, e^{-j\omega o}$

$$\text{System function } H(z) = \frac{b_o\ z}{z^2 - 2\ r\ z\ \cos(\omega_o)\ +\ r^2}$$

If b_o = is set to A $\sin(w_o)$ Then $h[n] = A \sin(n\, w_o)\, u[n]$

Thus the impulse response of the second order system with complex conjugate poles on the unit circle is is a sinusoid and the system is called a Digital Sinusoidal generator.
A digital sinusoidal oscillator is a basic component of a digital synthesizer.

Block Diagram Representation : y[n]



2.6 INVERTIBILITY OF LINEAR TIME INVARIANT SYSTEM

A system is said to be invertible if there is a one to one correspondence between its input and output signals. The inverse system with input y[n] and output x[n] is denoted by ‘T^{-1}.

Then, $w[n] = h_I[n] * h[n] * x[n] = x[n]$

This implies that, $h[n] * h_I[n] = \delta[n]$

By ZT, $H(z)\ H_I(z) = 1$

$$H_I(z) = \frac{1}{H(z)}$$

If H(z) has a rational system function, $H(z) = \frac{B(z)}{A(z)}$

Then Inverse system $H_I(z) = \frac{A(z)}{B(z)}$

Thus the zeros of H(z) become the poles of the inverse system and vice versa.

Furthermore, if H(z) is FIR system then $H_I(z)$ is an all pole system or if H(z) is an all pole system then $H_I(z)$ is FIR system.

Q(1) Determine the inverse of the system with impulse response $h[n] = \left(\frac{1}{2}\right)^n u[n]$.

Solution : . $h[n] = \left(\frac{1}{2}\right)^n u[n]$

By iZT, $H(z) = \dfrac{z}{z - \frac{1}{2}}$

$$H_I(z) = \frac{1}{H(z)} = 1 - \frac{1}{2}z^{-1}$$

By iZT, $h_I[n] = \delta[n] - \frac{1}{2}\delta[n-1]$.

Q(2) Determine the inverse of the system with impulse response $h[n] = \delta[n] - \frac{1}{2}\delta[n-1]$.

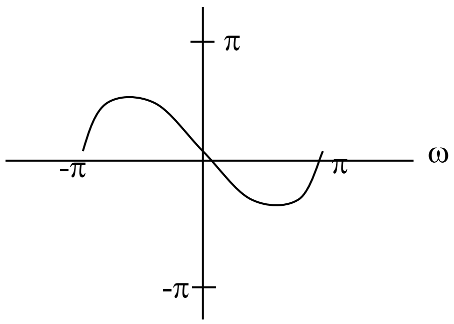
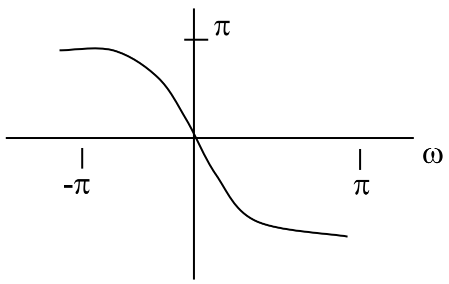
Solution. : By ZT,

$$H(z) = 1 - \frac{1}{2}z^{-1}, |z| > 0$$

$$H_I(z) = \frac{1}{H(z)} = \frac{1}{1 - \frac{1}{2}z^{-1}} = \frac{z}{z - \frac{1}{2}} \qquad |z| > \frac{1}{2}$$

➢ MINIMUM PHASE, MAXIMUM PHASE AND MIXED PHASE SYSTEM

Consider two FIR Systems characterized by system functions,

(I)	$H_1(z) = 1+\frac{1}{2}z^{-1} = \; = \left(\frac{z+\frac{1}{2}}{z}\right)$	(II)	$H_2(z) = \frac{1}{2}+z^{-1} \quad = \frac{1}{2}\left(\frac{z+2}{z}\right)$
	By iZT $h_1[n] = \{1, \frac{1}{2}\}$		By iZT, $h_2[n] = \{1/2, 1\}$
i)	Zero: $z_1 = -½$	i)	Zero: $z_1 = -2$
ii)	Magnitude Response :	ii)	Magnitude Response :
	$H_1(z) = \left(\frac{z+\frac{1}{2}}{z}\right) = \left(\frac{e^{jw}+\frac{1}{2}}{e^{jw}}\right)$		$H_2(z) = \frac{1}{2}\left(\frac{z+2}{z}\right) = \frac{1}{2}\left(\frac{e^{jw}+2}{e^{jw}}\right)$
	$H_1(z) = \left(\frac{\cos(w)+j\sin(w)+\frac{1}{2}}{\cos(w)+j\sin(w)}\right)$		$H_2(z) = \frac{1}{2}\left(\frac{cow(w)+j\sin(w)+2}{cow(w)+j\sin(w)}\right)$
	$\lvert H_1(w)\rvert = \sqrt{\frac{5}{4}+\cos(w)}$		$\lvert H_2(w)\rvert = \sqrt{\frac{5}{4}+\cos(w)}$
iii)	Phase Response:	iii)	Phase Response :
	$\theta_1(w) = \tan^{-1}\left(\frac{\sin(w)}{\frac{1}{2}+\cos(w)}\right) - w$		$\theta_2(w) = \tan^{-1}\left[\frac{\sin(w)}{1+\frac{1}{2}\cos(w)}\right] - w$
			

For $H_1(w)$ with zero inside the unit circle, the net phase change $\theta_1(\pi) - \theta_1(0) = 0$ i.e. minimum phase. For $H_2(w)$ with zero outside the unit circle, the net phase change, $\theta_2(\pi) - \theta_2(0) = \pi$ i.e. max. phase. Therefore, $H_1(z)$ is minimum phase system and $H_2(z)$ is maximum phase system.

FIR system with M zeros, $H(w) = b_0\,(1-z_1 e^{-jw})\,(1-z_2 e^{-jw})$ -------------- $(1 - Zm\, e^{-jw})$
When all zeros are inside the unit circle, each term in the product corresponds to a real valued zero, will undergo a net phase change of zero between $\omega = 0$ and $\omega = \pi$. Also each pair of complex conjugate factors in $H(w)$ will undergo a net phase change of zero.

Therefore, $\angle$ **H(π) – $\angle$ H(0) = 0** Hence, the system is called a **Minimum Phase** System.

On the other hand, when all zeros are outside the unit circle, a real valued zero will contribute a net change of 'π' radians as the freq. varies from $\omega = 0$ to $w = \pi$ and each pair of complex conjugate zero will contribute a net change of 2π radians.

Therefore, $\angle$ **H(π) – $\angle$ H(0) = M. π** Hence the system is called **Maximum Phase** System. Which is the Largest possible phase change for FIR system with M zeros. Hence the system is called Maximum Phase System.

FIR system with M zeros, if some of the zeros are inside the unit circle and remaining zeros outside the unit circle, then the system is called mixed phase system or non-minimum phase system. Minimum Phase characteristic implies a min. delay function while a max. phase characteristic implies that the delay characteristic is also maximum.

- When zeros of the filter are INSIDE the unit circle filter is called Minimum Phase Filter. Concept : For Minimum Phase filter $\phi(\pi) - \phi(0) = 0$
- When all zeros of the filter are OUTSIDE the unit circle filter is called maximum phase filter Concept : For Maximum Phase filter $\phi(\pi) - \phi(0) = \pm m\,\pi$
- When filter is neither minimum phase nor maximum phase filter,then it is mixed phase filter

Q(3) Determine the zeros and indicate whether the system is Min. Phase, Max. Phase or Mixed Phase.

(a) $H_1(z) = 6 + z^{-1} - z^{-2}$ (c) $H_3(z) = 1\ 5/2\ z^{-1} - 3/2\ z^{-2}$

(b) $H_2(z) = 1 - z^{-1} - 6z^{-2}$ (d) $H_4(z) = 1 + 5/2\ z^{-1} - 2/3\ z^{-2}$

Solution :

(a) $H_1(z) \Rightarrow Z_1, Z_2 : -½\ , 1/3$: Min. Phase.

(b) $H_2(z) \Rightarrow Z_{1,\,2} = -2,\ 3$: Max. Phase.

(c) $H_3(z) \Rightarrow Z_{1,\,2} = -1/2,\ 3$: Mixed Phase

(d) $H_4(z) \Rightarrow Z_{1,\,2} = -2,\ 1/3$: Mixed Phase.

Q(4) Determine whether the following systems are of Minimum Phase, Miximum Phase or Mixed Phase type.

Solution:

(a) $H_1(z) = z^2 + 2z - 8 \quad = (z + 4)\ (z - 2)$

Hence, the zeros are at $z = -4$ and $z = 2$.

As both the zeros are outside the unit circle, this system is Maximum Phase System.

(b) $H_2(z) = 3z^2 + \frac{1}{2}z - \frac{1}{2} \quad = (3z - 1)\left(z + \frac{1}{2}\right)$

Hence the zeros are at $z = \frac{1}{3}$ and $z = -\frac{1}{2}$.

Both the zeros are inside the unit circle, hence it is Minimum Phase System.

◈ Frequently Asked Questions

(1) Why ZT is used for frequency domain analysis of DT systems instead of DTFT ?

Ans : DTFT of every input signal is not possible. DTFT of u[n] is not possible because u[n] is not an energy signal. However ZT of u[n] is possible. Therefore ZT is used for analysis.

(2) What is the ZT of δ[n] and u[n]

Ans : ZT {δ[n]}=1 and ZT{u[n]} = z/(z-1)

(3) What is the ZT of x[n] = $(2)^n$ u[n]

Ans : X(z) = z/(z-2) ROC : | z | > 2

$$ZT\left(a^n u[n]\right) = \begin{cases} \dfrac{z}{z-a} & |z| > |a| \\ 0 & Otherwise \end{cases}$$

$$ZT\left(a^n u[-n-1]\right) = \begin{cases} \dfrac{-z}{z-a} & |z| < |a| \\ 0 & Otherwise \end{cases}$$

(4) Let x[n] = $(4)^n$ u[n]

What is X(z) at z = 6 and z = 2 ?

Ans : X(z) = z/(z-4) ROC : | z | > 4

(i) At z = 6 X(z) = 6/2 = 3

(ii) At z = 2 X(z) = ∞

(5) What is the concept of ROC ?

Ans : ROC gives the set of values of Z for which X(z) is finite. Every value of Z in the ROC gives X(z) finite.

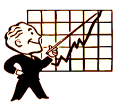
(6) What is the ROC condition for causal signal. ? Why ? Justify with example.

Ans : ROC is |z| > | Largest value of POLE |

Ex x[n] = $(2)^n$ u[n] + $(3)^n$ u[n]

NOTE : **If x[n] is right handed sequence, the ROC extends outward from the outermost finite pole in** $X(z)$ *to* $z = \infty$

	Sequence	ROC
1	x[n] =	Entire Z-plane
2	x[n] =	|Z| > 0
3	x[n] = a^n u[n]	|Z| > |a|
4	x[n] = a^n u[n] + b^n u[n]	|Z|> max{ |a |,|b| }
5	x[n] = $(-3)^n$ u[n] + $(2)^n$ u[n]	|Z| > 3



(7) What is the ROC condition for Anti-causal signal? Why ? Justify with example.

Ans : ROC is $|z| <$ | Lowest value of POLE |

Ex $\quad x[n] = (2)^n\, u[-n-1] + (3)^n\, u[-n-1]$

NOTE : **If x[n] is Left handed sequence, the ROC extends inward from the innermost finite pole in** $X(z)$ to $z = 0$

	Sequence	ROC
1	$x[n] = \{1, 2, 3, \underset{\uparrow}{0}\}$	$\|Z\| < \infty$
2	$x[n] = a^n\, u[-n-1]$	$\|Z\| < \|a\|$
3	$x[n] = a^n\, u[-n-1] + b^n\, u[-n-1]$	$\|Z\| < \min\{\|a\|, \|b\|\}$
4	$x[n] = (-3)^n\, u[-n-1] + (2)^n\, u[-n-1]$	$\|Z\| < 2$

(8) What is the ROC condition for Both-sided signal. ? Why ? Justify with example.

Ans : ROC condition for both sided signal is bounded between two POLES.

Ex $\quad x[n] = (2)^n\, u[n] + (3)^n\, u[-n]$

NOTE : **If x[n] is two sided sequence, the ROC consist of a ring in the Z plane, bounded by interior and exterior pole.]**

	Sequence	ROC
1	$x[n] = a^n\, u[n] + b^n\, u[-n-1]$	$\|b\| > \|z\| > \|a\|$
2	$x[n] = (2)^n\, u[n] + (3)^n\, u[-n-1]$	$3 > \|z\| > 2$
3	$x[n] = (3)^n\, u[n] + (2)^n\, u[-n-1]$	Not possible
4	$x[n] = (2)^n u[n] + (3)^n u[n] + (-4)^n u[-n-1] + (5)^n u[-n-1]$	$4 > \|z\| > 3$

(9) What is DT system ?

Ans : A DT system is a device or algorithm that operates on a DT signal according to some well defined rule, to produce another DT signal. In general a DT system can be thought as a set of operations performed on the input signal x[n] to produce the output signal y[n].

(10) What are the classification of DT systems ?

Ans : Systems are classified as,

(1) Static (Memorylees) / Dynamic (Memory System) :-

(2) Linear / Non Linear System.

(3) Causal / Non Causal System

(4) Time Invariant / Time Variant System.

(5) Stable / Unstable system

(11) Explain classification of DT system

(1)Static (Memorylees) / Dynamic (Memory System) :-

A DT system is called static or memoryless if it output at any instant depends on the input sample at the same time and not on past or future samples of the input. If the system is not static then it is dynamic.

(2)Linear / Non Linear System.

A system that satisfies the superposition principle is called Linear System.

If a system is Linear then,

$T\{a \, . \, x_1[n] + b \, x_2[n]\} = a_1 \; T\{x_1[n]\} + a_2 \, T\{x_2[n]\}$

If a system does not satisfy the superposition principle then it is Non Linear System.

(3)Causal / Non Causal System

A system is said to be causal if the output of the system at any time depends only on present and past values of input and does not depend on future values of input.

If the system is not causal then it is Non casual. For non causal system output depends on future values of input.

(4)Time Invariant / Time Variant System.

A system is called Time Invariant if a time shift in the input signal causes a time shift in the output signal. Otherwise the system is Time Variant System.

(5)Stable / Unstable system.

A system is said to be bounded input, bounded output stable if and only if every bounded input produces a bounded output.

(12) What is Impulse response ? Step response ?

Ans : Impulse Response is output of the system when input is $\delta[n]$.

Step Response is output of the system when input is u[n].

(13) What is zero input response ?

Ans : If the initial state of the system is NOT zero and the input x[n] = 0 to all n, then the output of the system with zero input is called the zero input response or natural response or free response of the system.

(14) What is zero state response ?

Ans : If the initial state of the system is zero and the input $x[n] \neq 0$ then the output of the system with non zero input is called the zero state response or forced response of the system.

(15) What is zero state step response ?

Ans : If the initial state of the system is zero and the input x[n]=u[n] then the output of the system is called zero step response of the system.

(16) What is Transient response ?

Ans : Transient response of the system is the response of the system that decays to zero.

(17) What is Steady State Response ?

Ans : Everlasting response of the system that depends on magnitude response and phase response of the system is steady state response of the system.

(18) What is Infinite Impulse Response ?

Ans : When length of h[n] is infinite it is called infinite impulse response. E.g. $h[n] = (\frac{1}{2})^n u[n]$

(19) What is Finite Impulse Response ?

Ans : When length of h[n] is finite it is called finite impulse response, E.g. $h[n] = \{ \underset{\uparrow}{1}\ 2,\ 3,\ \ 4 \}$

(20) What is frequency response ?

Ans : Frequency response means magnitude response and phase response.

(21) What is Magnitude Response ?

Ans : $$\text{Magnitude Response} = \frac{\textit{Magnitude of Numerator}}{\textit{Magnitude of Denominator}}$$

Where $$\textit{Magnitude} = \sqrt{(\text{Real})^2 + (\text{Imaginary})^2}$$

(22) What is Phase Response?

Ans : Phase Response = Angle of Numerator – Angle of Denominator

Where
$$\textit{Angle} = \begin{cases} \tan^{-1}\left(\dfrac{\text{Imaginary}}{\text{Real}}\right) & \text{When Real} > 0 \\ 180 + \tan^{-1}\left(\dfrac{\text{Imaginary}}{\text{Real}}\right) & \text{When Real} < 0 \end{cases}$$

(23) How to obtain Frequency Response Graphically ?

Ans : In Graphical method, the frequency response at a given frequency w is determined by the ratio of the product of the zero vectors with the product of pole vectors.

$$\text{Magnitude Response} = \frac{\text{Product of distance from zeros}}{\text{Product of distance from poles}}$$

Phase Response = Summation of angles from ZEROS – Summation of angles from POLES.

(24) Magnitude spectrum is continuous or discrete ?

Ans : If the signal is periodic then magnitude spectrum is discrete and If the signal is not-periodic then spectrum is continuous function of w.

(25) What is a digital resonator ?

Ans : A digital resonator is essentially a narrowband bandpass filter.

(26) What is eigen value of the system ?

Ans : Eigen-function of a system is an input signal that produces an output that differs from the input by a constant multiplicative factor. The multiplicative factor is called an eigen value of the system.

(27) How to find value of DT signal at infinity. ?

Ans : By final value theorem we can find x[∞]. $x(\infty) = \lim_{z \to 1} \left(\frac{z-1}{z} \right) X(z)$

(28) What is Transfer function of DT system ?

Ans : The Z – Transform H(z) of an impulse response h[n] is known as the system function or transfer function of the system

(29) What are different realization methods of digital filters ?

Ans :

	IIR FILTER	LINEAR PHASE FIR FILTER.
1	Direct Form Realization a) DF-I b) DF-II	Direct Form Realization -DF-I -DF-II
2	Lattice Realization	Lattice Realization
3		Linear Phase Realization
4		Frequency Sampling Realization

(30) What is canonic structure ?

Ans : If the number of delays in the realization block diagram is equal to the order of the transfer function, then the realization structure is called canonic otherwise it is called non-canonic.

(31) What is the advantage of direct form –II method of realization ?

Ans : DF-II method of realization requires LESS no of delay block.

(32) What is the advantage of Linear Phase Realization ?

Ans : Linear Phase method of realization requires LESS no of multipliers.

(33) What is the advantage of cascade connection of systems?

Ans : In cascade form, the shift from the actual POLE location due to quantization is LESS. So, quantization error is less.

(34) What is the difference equation of DT system ?

Ans : Output in terms of past/present , input/output of the system is called difference of the system.

Eg y[n] = y[n–1] + y[n–1] + x[n] + x[n–1]

(35) What is transform domain stability condition ?

Ans : If ROC includes unit circle, then system is stable.

(36) What is stability condition for causal and stable system?

Ans : For causal and stable system, all the POLES must lie INSIDE the unit circle.

(37) What do you mean by stable system ?

Ans : If the system is stable then output of the system depends on the input that is applied and characteristics of the system. Mathematically, output should be always finite.

(38) What will happen if the system is not stable. ?

Ans : If the system is stable then output of the system depends on the input that is applied and characteristics of the system. Mathematically, output should be always finite. We get finite desirable output only when system is stable.

If the system is not stable, output will not depend on the input, output will not depend on the characteristics of the system. In that case we get undesired, distorted, noisy output.

(39) What is Minimum Phase System ?

Ans : For any system If $\angle \mathbf{H(\pi)} - \angle \mathbf{H(0)} = \mathbf{0}$ Then system is called a **Minimum Phase** System.

When All zeros are inside the unit circle, the net phase change $\theta_1(\pi) - \theta_1(0) = 0$ i.e. minimum phase.

(40) What is Maximum Phase System ?

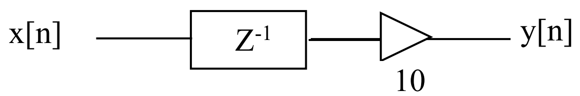
Ans : For any system If $\angle \mathbf{H(\pi)} - \angle \mathbf{H(0)} = \mathbf{M\pi}$ Then system is called a **Maximum Phase** System.

When All zeros are outside the unit circle, the net phase change $\theta_1(\pi)-\theta_1(0) = M\pi$ i.e. Maximum phase.

NOTE: If the system is Neither Minimum Phase NOR Maximum Phase Then System is Mixed Phase System.

Minimum Phase characteristic implies a min. delay function while a maximum phase characteristic implies that the delay characteristic is also maximum.

(41) Find the output of the following system



(42) Impulse response of Digital Low Pass filter is given by h[n] ={ 3, 2, 1, 2, 3 }. What will be the output of the filter for any given input x[n] ?

Ans :