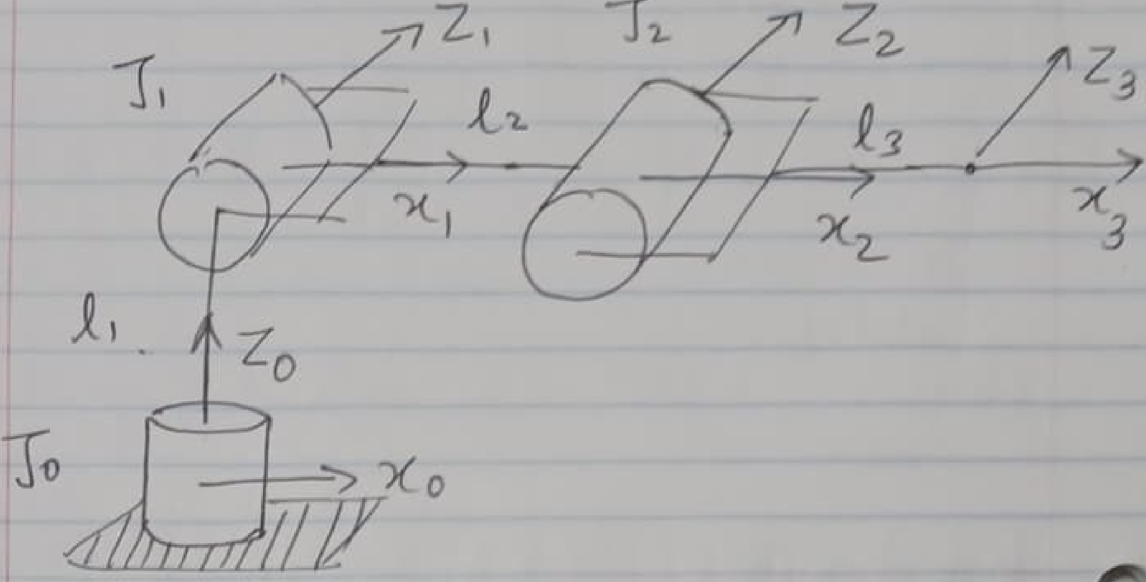
Prathamesh Mehta

HW Assign. 2 RBE 500

Q.1



Step 1, 2: Assign z and x axes

Step 3:- D-H table

	θ	α	a	d
link 1	θ_1^*	-90	0	l_1
link 2	θ_2^*	0	l_2	0
link 3	θ_3^*	0	l_3	0

Step 4:- Calculate the A_i

$$A_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \cos(-90) & \sin\theta_1 \sin(-90) & 0 \\ \sin\theta_1 & \cos\theta_1 \cos(-90) & -\cos\theta_1 \sin(-90) & 0 \\ 0 & \sin(-90) & \cos(-90) & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & +\cos\theta_1 & 0 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2\cos(0) & \sin\theta_2\sin(0) & l_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2\cos(0) & -\cos\theta_2\sin(0) & l_2\sin\theta_2 \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

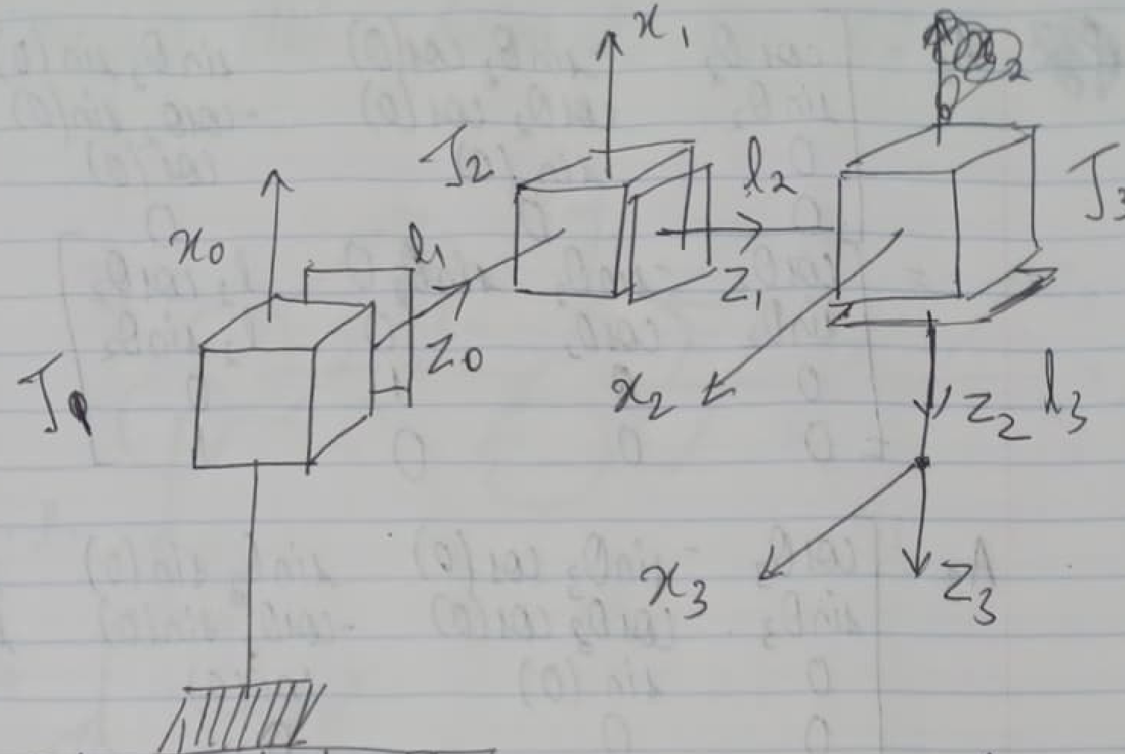
$$= \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & \cancel{\sin\theta_2}\ 0 & l_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & l_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3\cos(0) & \sin\theta_3\sin(0) & l_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_2\cos(0) & -\cos\theta_3\sin(0) & l_3\sin\theta_3 \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & l_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_2 & 0 & l_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_n^0 = A_1 * A_2 * A_3$$

Q-2



	~~α~~	~~θ~~	~~α~~	~~a~~	~~d~~
~~link 1~~					
~~link 2~~					
~~link 3~~					

Step 1: Assign x and z axes

Step 2:- D-H Table

	θ	α	a	d
link 1	0	-90	0	d_1^*
link 2	90	-90	0	d_2^*
link 3	0	0	0	d_3^*

Ai ~~Calculation~~

~~$A_1 = [\cos(0) - 90$~~

Step 3:- Calculate the A_i

~~$A_1 = [\cos(0) \quad 0 \; -\sin(0)\cos(d_1)$~~

$$A_1 = \begin{bmatrix} \cos(0) & -\sin(0)\cos(-90) & \sin(0)\sin(-90) & 0 \\ \sin(0) & \cos(0)\cos(-90) & -\cos(0)\sin(-90) & 0 \\ 0 & \sin(-90) & \cos(-90) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos(90) & -\sin(+90)\cos(-90) & \sin(90)\sin(-90) & 0 \\ \sin(90) & \cos(90)\cos(-90) & -\cos(90)\sin(-90) & 0 \\ 0 & \sin(-90) & \cos(-90) & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

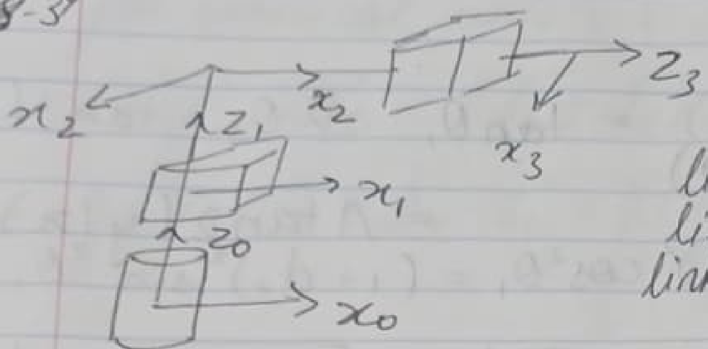
$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \cos(0) & -\sin(0)\cos(0) & \sin(0)\sin(0) & 0 \\ \sin(0) & \cos(0)\cos(0) & -\cos(0)\sin(0) & 0 \\ 0 & \sin(0) & \cos(0) & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_n^0 = A_1 * A_2 * A_3 \quad ''$$

Q-8-3)



	a	α	d	θ
link 1	0	0	1	θ_1^*
link 2	0	$-90°$	d_2^*	$-90°$
link 3	0	0	$1+d_3^*$	0

$$T_1^0 = A_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1+d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = T_0^1 \, T_2^1 \, T_3^2 \cancel{= [\sin\theta_1 \quad 0 \quad \cos\theta_1}$$

Assuming the end-effector position to be (x, y, z)

$x = (1 + d_3^*) \cos\theta_1$
$y = (1 + d_3^*) \sin\theta_1$
$z = 1 + d_2^*$

Then, $d_2^* = z - 1$

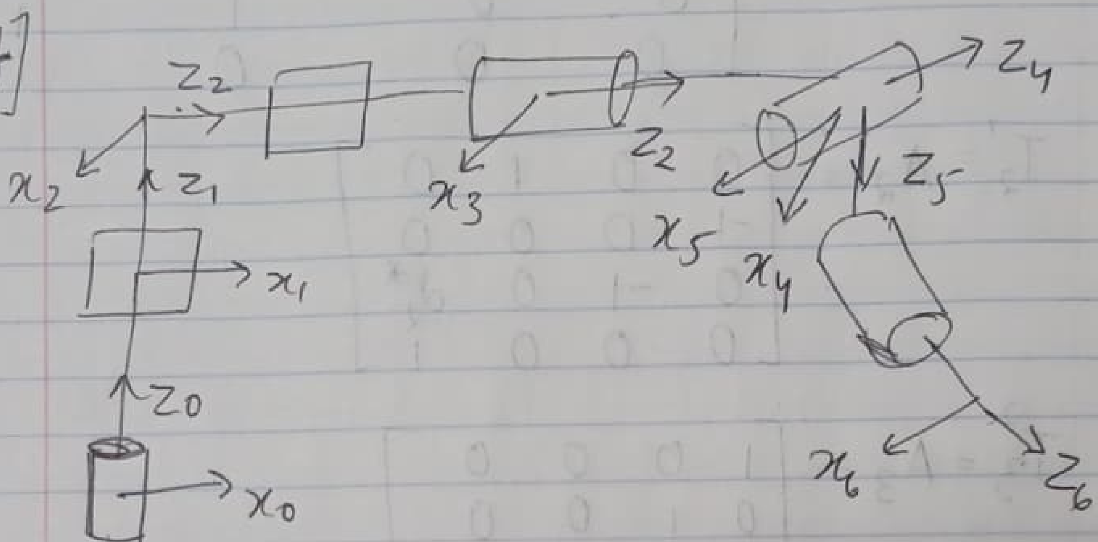
$$\frac{y}{z} = \frac{\sin\theta_1 (1+d_3^*)}{\cos\theta_1 (1+d_3^*)} = \tan\theta_1 \Rightarrow \theta_1 = \tan^{-1}(y/x)$$

$$= A\tan2(y/x)$$

$$\Rightarrow x^2 + y^2 = (1+d_3)^2 \cos^2\theta_1 = (1+d_3)^2 \sin^2\theta_1$$

$$x^2 + y^2 = (1+d_3)^2 \Rightarrow d_3 = \sqrt{x^2+y^2} - 1$$

Q-4]



$$T_6^0 = \begin{bmatrix} R_6^0 & O_6^0 \\ 0 & 1 \end{bmatrix}$$
(Assume)

$$= \begin{bmatrix} r_{11} & r_{21} & r_{31} & x_p \\ r_{12} & r_{22} & r_{32} & y_p \\ r_{13} & r_{23} & r_{33} & z_p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

for spherical wrist,

$$O_4^0 = O_6^0 - (l_6 + l_5) Z_6^0$$

$$\therefore O_4^0 = O_3^0 + l_4 \cdot Z_3^0 = \begin{bmatrix} (1+d_3)\cos\theta_1 \\ (1+d_3)\sin\theta_1 \\ 1+d_2 \end{bmatrix} + \begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \\ 0 \end{bmatrix} l_4$$

$$= \begin{bmatrix} (1+d_3+l_4)\cos\theta_1 \\ (1+d_3+l_4)\sin\theta_1 \\ 1+d_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_p - (l_5+l_6)\cdot r_{31} \\ y_p - (l_5+l_6)\cdot r_{32} \\ z_p - (l_5+l_6)\cdot r_{33} \end{bmatrix}$$

$$d_2 = z_p - (l_5 + l_6) \cdot r_{33} - 1$$

Take $y_5 = y_p - (l_5 + l_6) \cdot r_{32}$

$x_5 = x_p - (l_5 + l_6) \cdot r_{31}$

$$d_3 = \sqrt{x_5^2 + y_5^2} - 1 - l_4$$

$$\theta_1 = Atan2(y_5, x_5)$$

R_6^0 is $R_3^0 = R_0^3$, $R_6 = R_3^{0\,T} \cdot R_6^0$

$$R_6^3 = \begin{bmatrix} \sin\theta_1 & -\cos\theta_1 & 0 \\ 0 & 0 & -1 \\ \cos\theta_1 & \sin\theta_1 & 0 \end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{bmatrix}$$

Use DH table to get R_6^3

a	α	d	θ
0	$-90°$	l_4	$90+\theta_4$
0	$90°$	0	θ_5
0	0	l_5+l_6	θ_6

$$R_4^3 = \begin{bmatrix} \sin\theta_4 & 0 & -\cos\theta_4 \\ \cos\theta_4 & 0 & -\sin\theta_4 \\ 0 & -1 & 0 \end{bmatrix}$$

$$R_5^4 = \begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 \\ \sin\theta_5 & 0 & -\sin\theta_5 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_6^5 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_6^3 = R_4^3 \cdot R_5^4 \cdot R_6^5$$

$$\cancel{= [\sin\theta_4 \cos\theta_5 \cos\theta_6 - \sin\theta_6 \cos 4}$$

$\therefore$ $\theta_4 = A\tan 2(\cos\theta_1 r_{32} - \sin\theta_1 r_{31}, -r_{33})$

$\theta_5 = A\tan 2(-r_{33} / \cos\theta_4, \cos\theta_1 r_{31} + \sin\theta_1 r_{32})$

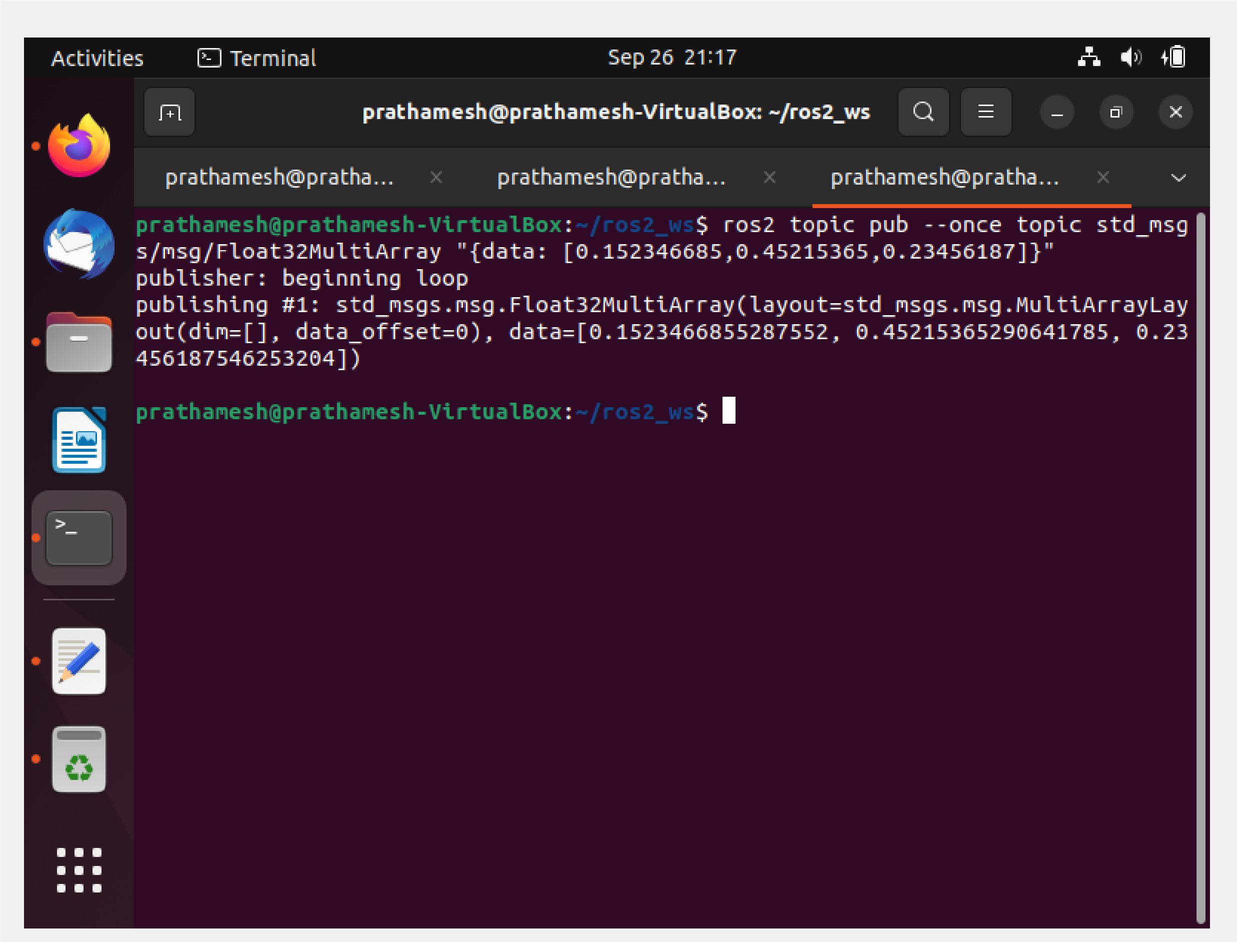
$\theta_6 = A\tan 2(\cos\theta_1 r_{21} + \sin\theta_1 r_{22}, -\cos\theta_1 r_{11} - \sin\theta_1 r_{12})$

In the implementation mentioned, we make use of the std_msgs library.

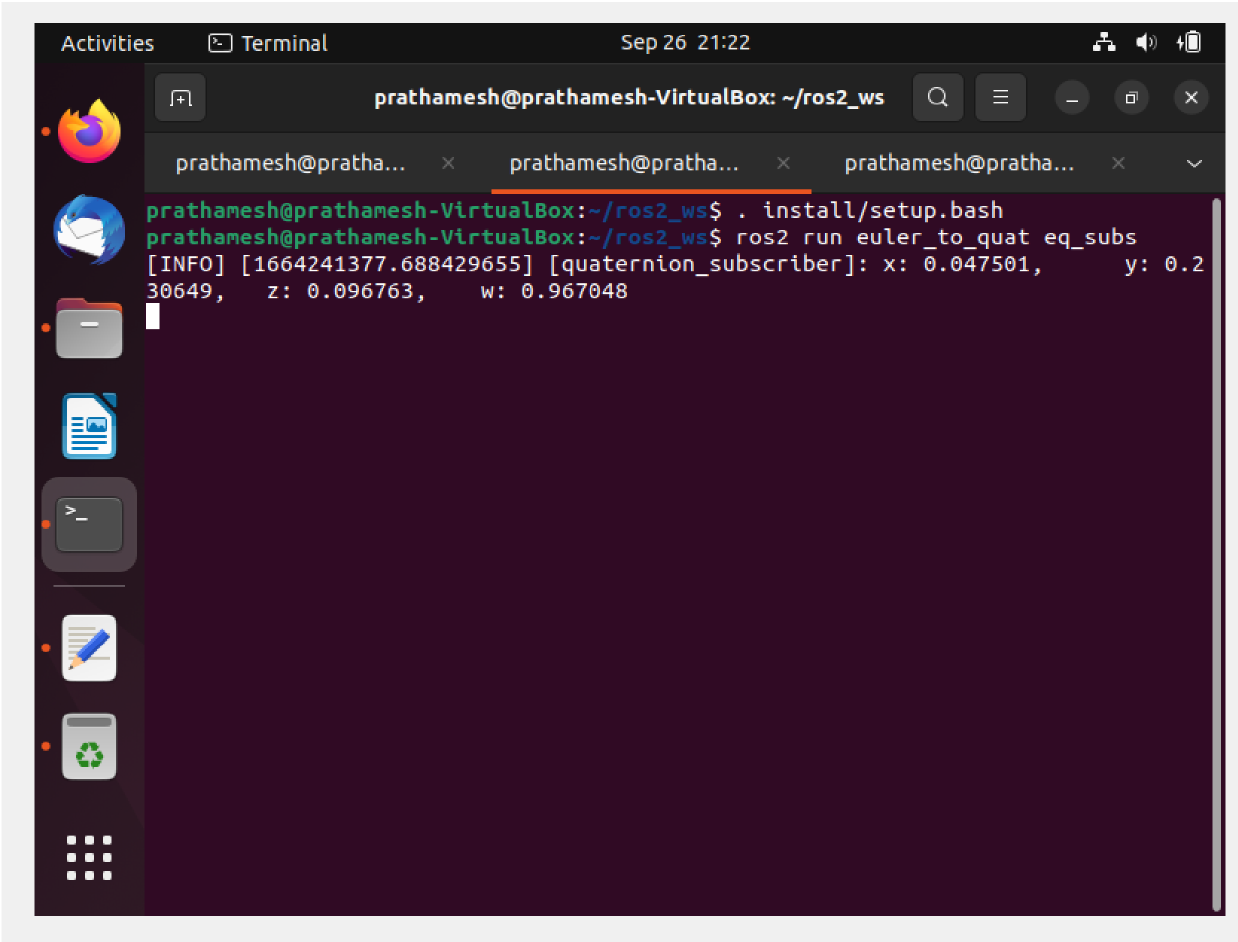
There is an input in form of euler ~~and~~ values i.e. (angle values corresponding to yaw, pitch and roll) in accordance to xyz euler angles then this input is converted to quaternion ~~[illegible]~~ with the help of the conversion formula and thus the output is a quaternion value set consisting of x, y, z, w values for the inputted euler angles.

The output is printed out using %f as its float in nature.

The publisher publishes this converted quaternion value and the ~~send~~ subscriber ~~sets~~ receives the same and displays it into the subscriber node. ~~This is~~



Publisher Node



Subscriber Node