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RBE 500 - HW4

Q-1 $R = R_{x,\theta} \cdot R_{y,\phi}$ at $\theta = \pi/2$, $\phi = \pi/2$

$R = R_{x,\theta} \cdot R_{y,\phi}$ $\quad \dfrac{\partial R}{\partial \phi} = R_{x,\theta} \cdot \dfrac{\partial R_{y,\phi}}{\partial \phi}$

$$y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \quad S(y) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$R_{y,\phi} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$\partial R_{y,\phi} = \begin{bmatrix} -\sin\phi & 0 & \cos\phi \\ 0 & 1 & 0 \\ -\cos\phi & 0 & -\sin\phi \end{bmatrix}$$

$$\frac{\partial R}{\partial \phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} -\sin\phi & 0 & \cos\phi \\ 0 & 1 & 0 \\ -\cos\phi & 0 & -\sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} -\sin\phi & 0 & \cos\phi \\ \sin\theta\cos\phi & 0 & \sin\theta\sin\phi \\ -\cos\theta\cos\phi & 0 & -\cos\theta\sin\phi \end{bmatrix}$$

for $\theta = \pi/2$, $\phi = \pi/2$

$$\frac{\partial R}{\partial \phi} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$,,

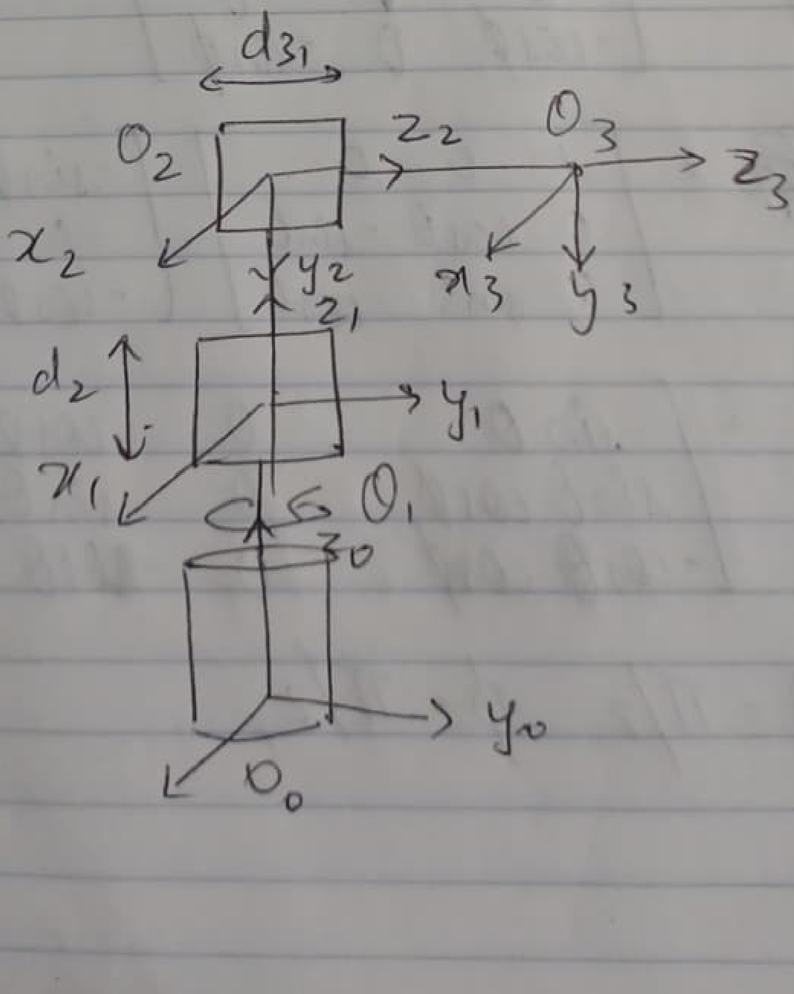
Q-4.10 $H_1^0 = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad V_1(t) = (3, 1, 0)$

$V_0(t) = ?$

$V_0 = R_1^0 V_1$ where R_1^0 is obtained from $H_1^0 = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix}$

$$V_0 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} = V_0$$,,

Q-4.15



DH parameters for]

	a	d	α	θ
link 1	0	l_1	0	θ_1
link 2	0	d_2	-90	0
link 3	0	d_3	0	0

$$H_i^{\circ} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus, $A_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & S\theta_1 . 0 & 0 \\ S\theta_1 & C\theta_1 & C\theta_1 . 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$H_2 = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ S\theta_1 & 0 & C\theta_1 & 0 \\ 0 & -1 & 0 & l_2 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & -S\theta_1 d_3 \\ S\theta_1 & 0 & C\theta_1 & C\theta_1 d_3 \\ 0 & -1 & 0 & l_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We know, for revolute joint

$$J_i = \begin{bmatrix} Z_{i-1}^0 \times (O_n^0 - O_{i-1}^0) \\ Z_{i-1}^0 \end{bmatrix}$$

for prismatic joint

$$J_i = \begin{bmatrix} Z_{i-1}^0 \\ 0 \end{bmatrix}$$

Here, Z_{i-1}^0 is obtained from 3^{rd} column of H_{i-1}^0

$O_{i-1}^0 = H_{i-1}^0$ is obtained from 4^{th} column i.e. translation column.

$Z_0^0 = [0 \ 0 \ 1]^T, \ O_0^0 = [0 \ 0 \ 0]^T$

$Z_1^0 = [0 \ 0 \ 1]^T, \ O_1^0 = [0 \ 0 \ l_1]^T$

$Z_2^0 = [-S\theta, C\theta, 0]^T, \ O_2^0 = [0 \ \ 0 \ \ l_1 + d_2]^T$

$Z_3^0 = [-S\theta, C\theta, 0]^T; \ O_3^0 = [-S\theta_1 d_3 \ \ C\theta_1 d_3 \ \ l_1 + d_2]$

$$J_1 = \begin{bmatrix} Z_0^0 (O_3^0 - O_0^0) \\ Z_0^0 \end{bmatrix}; \ J_2 = \begin{bmatrix} Z_1^0 \\ 0 \end{bmatrix}; \ J_3 = \begin{bmatrix} Z_2^0 \\ 0 \end{bmatrix}$$

Since J_1 is revolute and J_2 and J_3 are prismatic

$$J = \left[\begin{array}{c|c|c} Z_0^0 (O_3^0 - O_0^0) & Z_1^0 & Z_2^0 \\ Z_0^0 & 0 & 0 \end{array}\right]$$

$$J = \begin{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix}-s\theta_1 d_3\\ c\theta_1 d_3\\ l_1+d_2\end{bmatrix} & \begin{matrix}0\\0\\1\end{matrix} & \begin{matrix}-s\theta_1\\c\theta_1\\0\end{matrix} \\ \begin{matrix}0\\0\\1\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix}\end{bmatrix}$$

$$J = \begin{bmatrix} -c\theta_1 d_3 & 0 & -s\theta_1 \\ -s\theta_1 d_3 & 0 & c\theta_1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$\det J_{11} = 0$ for singularity

$$J_{11} = \begin{bmatrix} -c\theta d_3 & 0 & -s\theta_1 \\ -s\theta_1 d_3 & 0 & c\theta_1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\det J_{11} = 0$

$\Rightarrow c_1^2 d_3 + s_1^2 d_3 = 0$

$d_3 = 0$

So, the singularity will occur at $d_3 = 0$,