Constraint Satisfaction Problem

Constraint satisfaction problems (CSPs)

- Standard search problem: state is a "black box“ – any data structure that supports successor function and goal test
- CSP:
  - state is defined by variables X_i with values from domain D_i
  - goal test is a set of constraints specifying allowable combinations of values for subsets of variables

- Allows useful general-purpose algorithms with more power than standard search algorithms

Constraint satisfaction problems (CSPs)

- What is a CSP?
  - Finite set of variables $X_1, X_2, \ldots, X_n$
  - Nonempty domain of possible values for each variable $D_1, D_2, \ldots, D_n$
  - Finite set of constraints $C_1, C_2, \ldots, C_m$
    - Each constraint C_i limits the values that variables can take,
    - e.g., $X_1 \neq X_2$
  - Each constraint C_i is a pair <scope, relation>
    - Scope = Tuple of variables that participate in the constraint.
    - Relation = List of allowed combinations of variable values.
      May be an explicit list of allowed combinations.
      May be an abstract relation allowing membership testing and listing.

CSP benefits

- Standard representation pattern
- Generic goal and successor functions
- Generic heuristics (no domain specific expertise).

Example: Map-Coloring

- Variables *WA, NT, Q, NSW, V, SA, T*
- Domains D_i = {red,green,blue}
- Constraints: adjacent regions must have different colors
- e.g., WA ≠ NT, or (WA,NT) in {(red,green),(red,blue),(green,red), (green,blue),(blue,red),(blue,green)}

CRYPTARITHMETIC PROBLEM

→ Type of constraint satisfaction problem (CSP)

→ Constraints : (i) No two letters have same value.

(ii) Sum of digits must be as shown in problem.

(iii) There should be only one carry forward.

→ Digits can be assigned to a word/alphabet in the range (0–9).

→ e.g.:

```
  T O
+ G O
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O U T
```

letter	Digit.
T	
O	
G	
U	

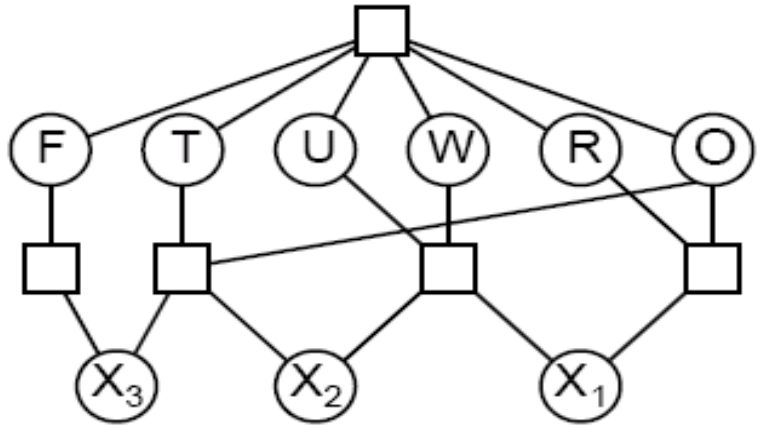
Hint:

→ Starts from the left most digit = 1

→ Leftmost value ≠ 0 (zero)

CSP Example: Cryptharithmetic puzzle

$$\begin{array}{r} \,T\;W\;O \\ +\;T\;W\;O \\ \hline F\;O\;U\;R \end{array}$$



Variables: $F\ T\ U\ W\ R\ O\ X_1\ X_2\ X_3$
Domains: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
Constraints
alldiff(F, T, U, W, R, O)
$O + O = R + 10 \cdot X_1$, etc.

Solve the following problem

1.

SEND

+MORE

MONEY

2. BASE

+BALL

GAMES