Introduction

- Conceived by Lotfi Zadeh, UC Berkley in 1965.
- The classes of objects encountered in real world do not have precisely defined criteria of membership.
- For example, the class of animals such as dogs, horses, birds are clearly included in the ANIMAL CLASS. But objects such as starfish, bacteria etc. have an ambiguous status with respect to the ANIMAL CLASS.
- The same kind of ambiguity arises in the following cases.
  1. The “class of real numbers which are much greater than 1”.
  2. The “class of tall man”.
  3. The “class of young man”.

- Most human processes can be understood largely by imprecise human reasoning.
- The imprecision is a form of information that can be quite useful to humans.
- Not many problems require precision.
  a. Parking a car
  b. Washing clothes
  c. Judging beauty contestants
  d. Controlling traffic at intersection
  e. Understanding of a complex system

- Higher precision entails higher cost and also low tractability in a problem, for example TSP.
- As the number of cities grow, the problem quickly approaches combinatorial explosion.
- No computer exist today that can solve this problem through brute-force approach.

- Another analogous problem to TSP is fabrication of circuit boards, where precise laser drill millions of holes. Deciding in which order to drill the holes so as to minimize the drilling time.
- If feedback controllers could be programmed to accept noisy, imprecise input, they would be much more effective and perhaps easier to implement.

- Fuzzy systems are very useful in two general contexts:
  1. In situations involving highly complex systems whose behaviors are not well understood.
  2. In situations where an approximate but fast solution is warranted.

What is Fuzzy Logic

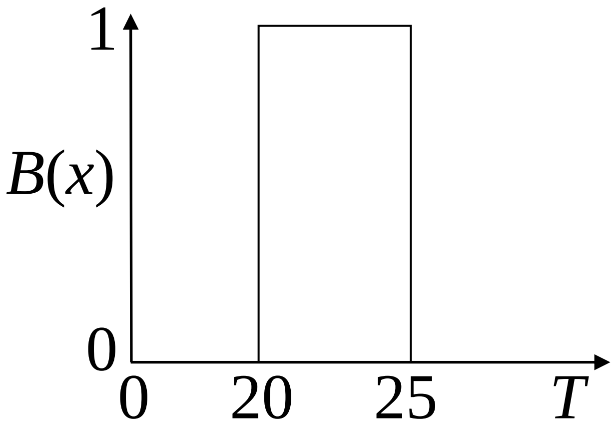
- It is a generalization of classical logic which manifests in crisp quantities.
- It represents concepts with unclear boundaries.
- Crisp logic deals with crisp sets having sharp boundaries. The inherent logic is Boolean in nature (i.e. either TRUE or FALSE).
- Fuzzy logic deals with fuzzy sets having indistinct boundaries. The inherent logic is multivalued in nature.

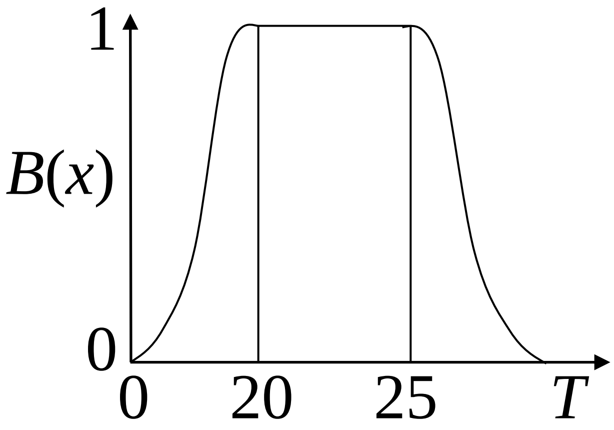
What is Fuzzy Logic

- In essence, fuzzy logic (FL) is focused on modes of reasoning which are approximate rather than exact.
- In fuzzy logic, everything, including truth, is allowed to be a matter of degree.
- Fuzzy logic has been an object of controversy, though to a lesser degree.
- The controversies are rooted in misperceptions, especially a misperception of the relation between fuzzy logic and probability theory.

What is Fuzzy Logic

- Consider the temperature on a sunny day.
- It can either be represented in terms of temperature values e.g. 20^0-25^0C.
- It can also be represented as a “warm” day with temperature, $T \in [20^0\ 25^0]$.





Definitions

- Fuzziness measures the degree to which an event occurs, not whether it occurs.
- It exists when the law of non-contradiction [$A \cap \tilde{A} = \Phi$] (or the law of excluded middle [$A \cup \tilde{A} = U$]) is violated.
- Fuzziness primarily describes partial truth or imprecision.

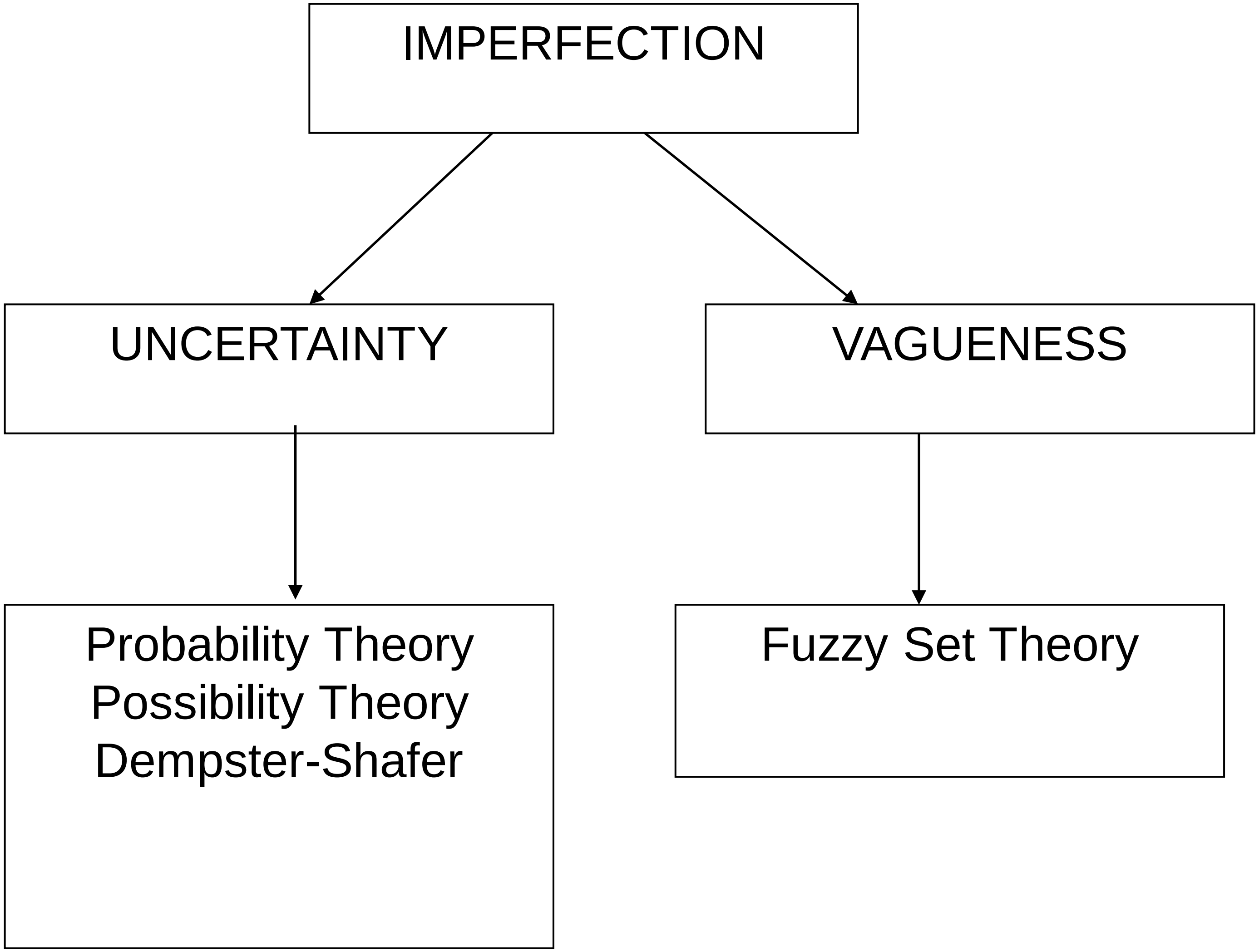
Definitions

- It means that everything is a matter of degree.

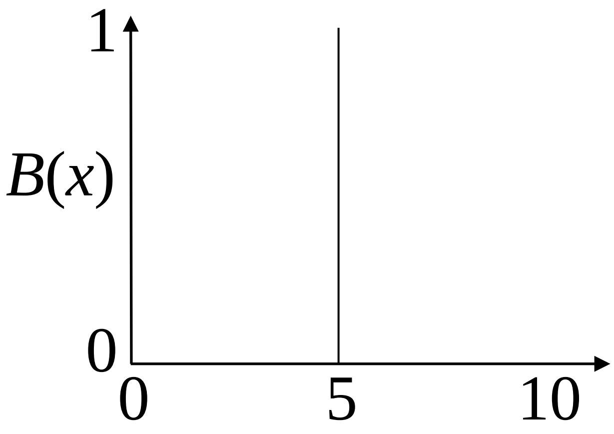
- It is a type of deterministic uncertainty. It deals with the soft meaning of concepts.

- It is related to computing with words, i.e. linguistic variables.

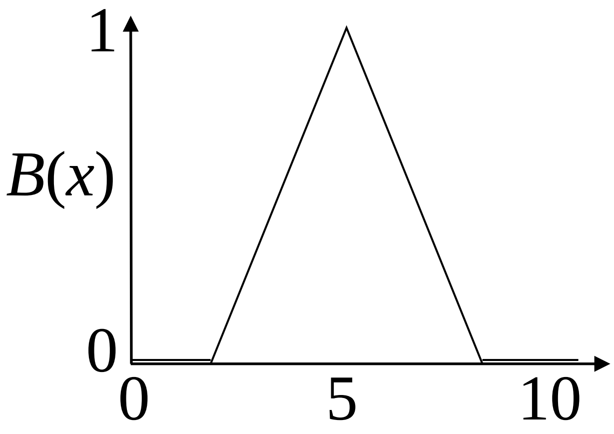
Probability & Fuzzy Logic



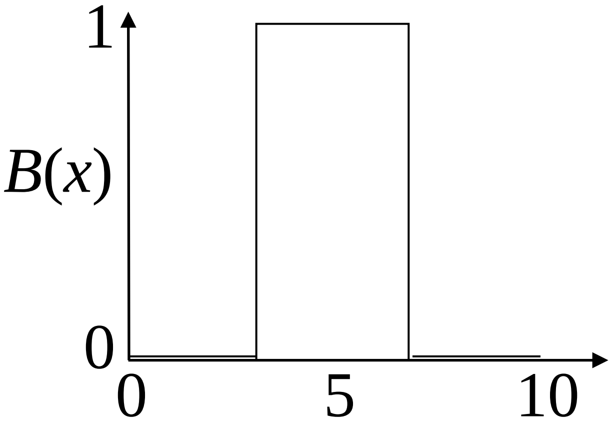
Probability & Fuzzy Logic



Certainty



Vagueness



Representation

- FL incorporates a simple, rule-based IF X AND Y THEN Z approach to solve a control problem rather than attempting to model a system mathematically.

- Terms like
  - "IF (process is too cool) AND (process is getting colder) THEN (add heat to the process)"
  - or "IF (process is too hot) AND (process is heating rapidly) THEN (cool the process quickly)"

  are used.

Fuzzy Membership

- In a crisp set, elements are either strictly contained in the set or are not strictly contained in the set.
- For a crisp set *C*, an element *X* is related as

  $C : X \in \{0,1\}$ implies either accept or reject, the Dichotomy concept.
- A fuzzy set comprises elements with a certain degree of containment in the set, referred to as the membership of that element.
- For a fuzzy set *F*, an element *X* is related as

  $F : X \in [0, 1]$ implies relaxed membership.

Fuzzy Sets

- A fuzzy set is a set of ordered pairs.

$$\underline{A} = \{(x, \mu_{\underline{A}}(x)) \mid x \in X, \mu_{\underline{A}}(x) \in [0,1]\}$$

Here X is the universe of discourse. It is given by $X = \{x_1, x_2, x_{3,} ...., x_n\}$

According to Zadeh's notation:

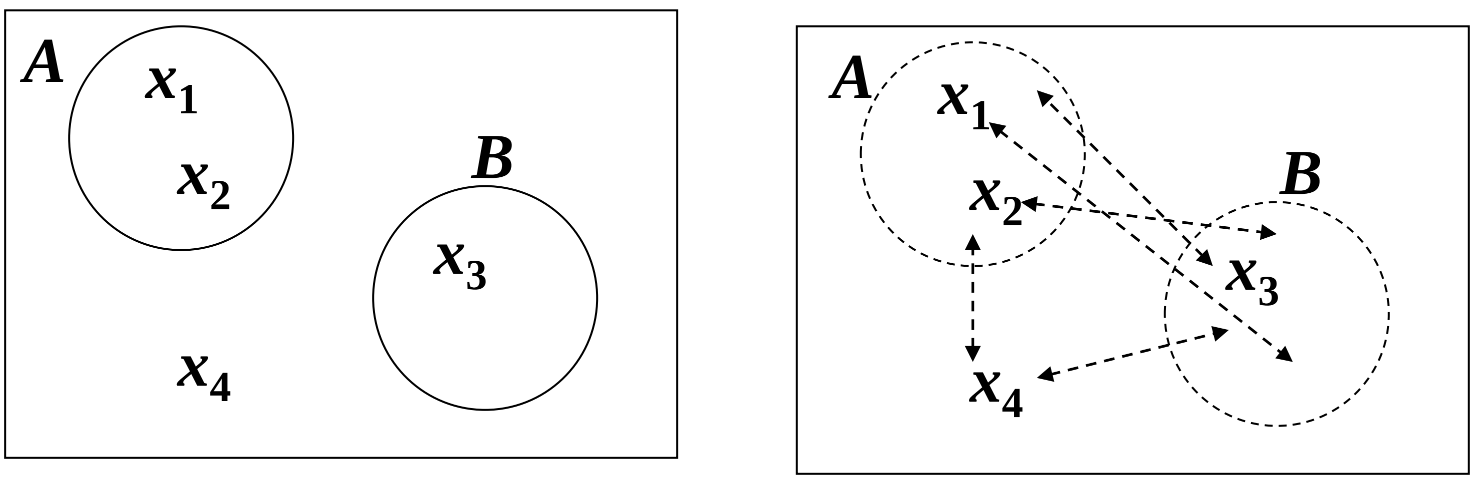
$$\underline{A} = \{(\frac{\mu_{\underline{A}}(x_i)}{x_i}) \mid x_i \in X\}$$

$$or \quad \underline{A} = \frac{\mu_{\underline{A}}(x_1)}{x_1} + \frac{\mu_{\underline{A}}(x_2)}{x_2} + \frac{\mu_{\underline{A}}(x_3)}{x_3} + ... + \frac{\mu_{\underline{A}}(x_n)}{x_n} = \sum_{i=1}^{n} \frac{\mu_{\underline{A}}(x_i)}{x_i}$$

Fuzzy Sets

- If X is continuous, $$\underline{A}(x) = \int_{x} \frac{\mu_{\underline{A}}(x)}{x}$$

$\sum$ $\int$ $+$ represent the union of membership degrees.



Crisp Set **Fuzzy Set**

Set Theoretic Operations vis-a-vis the fuzzy sets

- Cardinality $|\underline{A}| = \sum_{x \in X} \mu_{\underline{A}}(x)$

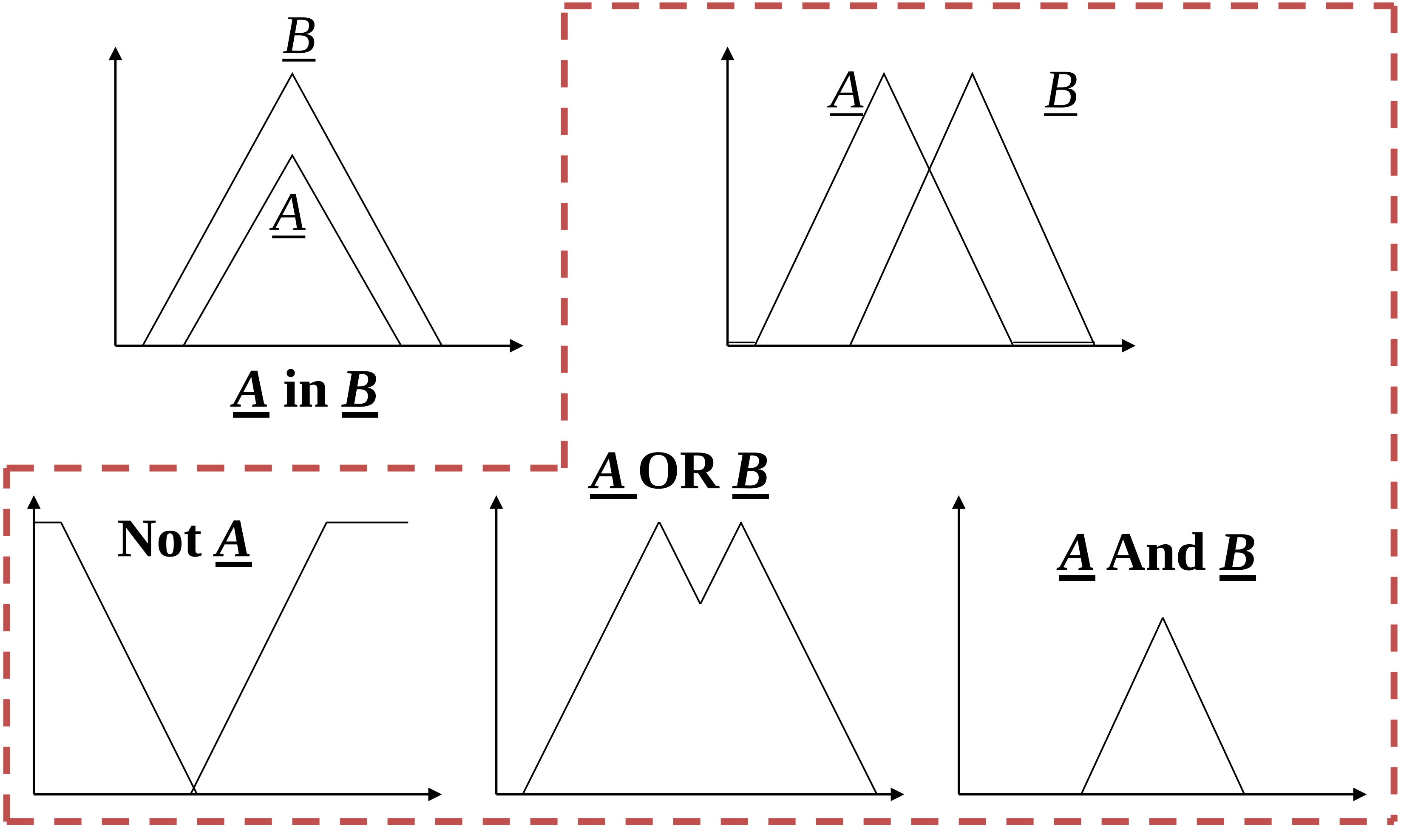
- Subset $\underline{A} \subseteq \underline{B} \Rightarrow \mu_{\underline{A}} \leq \mu_{\underline{B}}$

- Complement $\overline{\underline{A}} = X - \underline{A} \Rightarrow \mu_{\overline{\underline{A}}}(x) = 1 - \mu_{\underline{A}}(x)$

- Union $\underline{C} = \underline{A} \cup \underline{B} \Rightarrow \mu_{\underline{C}}(x) = \max(\mu_{\underline{A}}(x), \mu_{\underline{B}}(x)) = \mu_{\underline{A}}(x) \vee \mu_{\underline{B}}(x)$
- Intersection

$$\underline{C} = \underline{A} \cap \underline{B} \Rightarrow \mu_{\underline{C}}(x) = \min(\mu_{\underline{A}}(x), \mu_{\underline{B}}(x)) = \mu_{\underline{A}}(x) \wedge \mu_{\underline{B}}(x)$$

Set Theoretic Operations



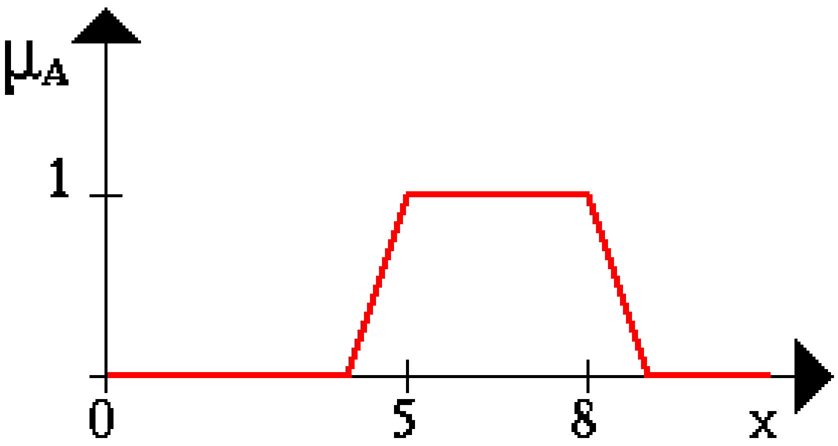
Properties of Fuzzy sets visa-vis classical sets

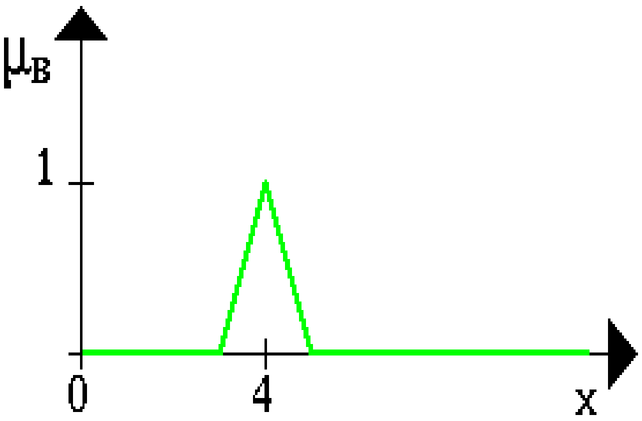
- Commutativity $\underline{A} \cup \underline{B} = \underline{B} \cup \underline{A}; \quad \underline{A} \cap \underline{B} = \underline{B} \cap \underline{A}$
- Idempotency $\underline{A} \cup \underline{A} = \underline{A}; \quad \underline{A} \cap \underline{A} = \underline{A}$
- Associativity $\underline{A} \cup (\underline{B} \cup \underline{C}) = (\underline{A} \cup \underline{B}) \cup \underline{C}; \quad \underline{A} \cap (\underline{B} \cap \underline{C}) = (\underline{A} \cap \underline{B}) \cap \underline{C}$
- Distributivity

$$\underline{A} \cup (\underline{B} \cap \underline{C}) = (\underline{A} \cup \underline{B}) \cap (\underline{A} \cup \underline{C}); \quad \underline{A} \cap (\underline{B} \cup \underline{C}) = (\underline{A} \cap \underline{B}) \cup (\underline{A} \cap \underline{C})$$

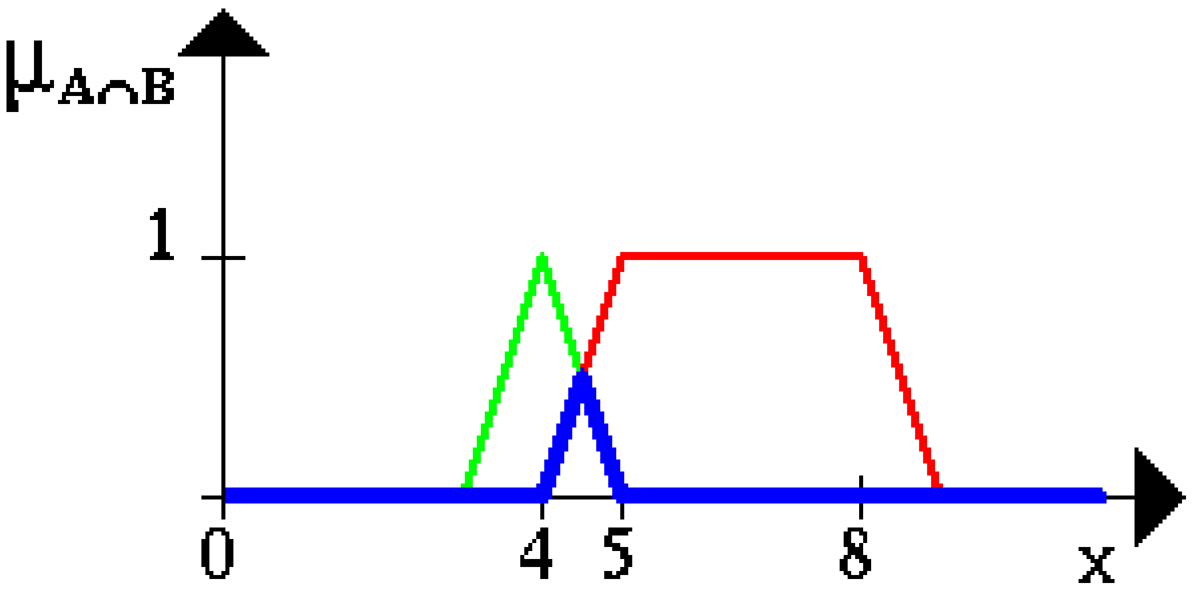
- De Morgan's Law $\overline{\underline{A} \cup \underline{B}} = \overline{\underline{A}} \cap \overline{\underline{B}}; \quad \overline{\underline{A} \cap \underline{B}} = \overline{\underline{A}} \cup \overline{\underline{B}}$
- Law of absorption $\underline{A} \cup (\underline{A} \cap \underline{B}) = \underline{A}; \quad \underline{A} \cap (\underline{A} \cup \underline{B}) = \underline{A}$

Let *A* be a fuzzy interval *between* 5 *and* 8 and *B* be a fuzzy number *about* 4

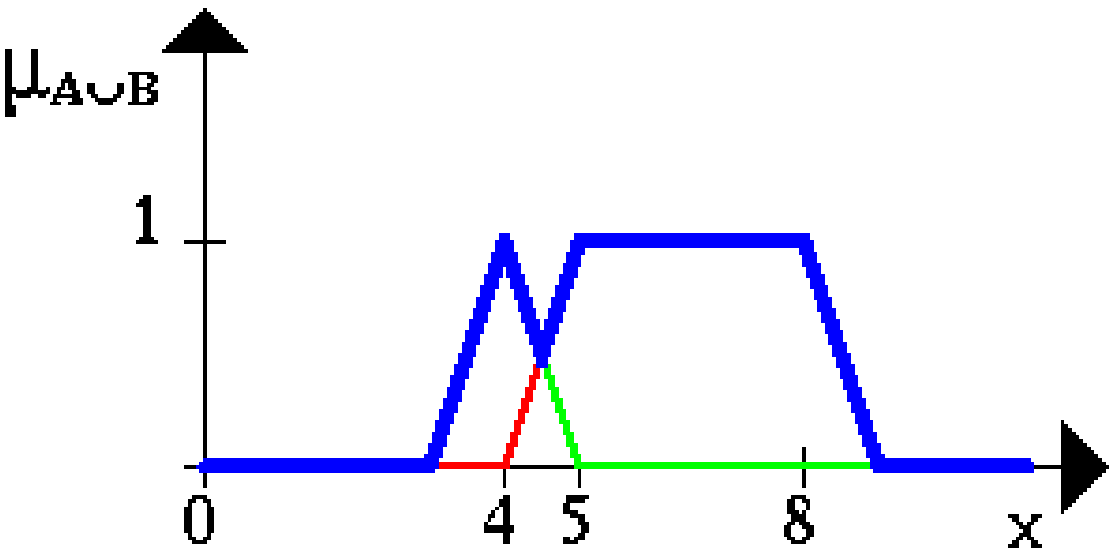




fuzzy set *between* 5 *and* 8 **AND** *about* 4



The Fuzzy set *between* 5 *and* 8 **OR** *about* 4 is shown in the next figure



NEGATION of the fuzzy set *A*