Knowledge Representation and Reasoning

Introduction

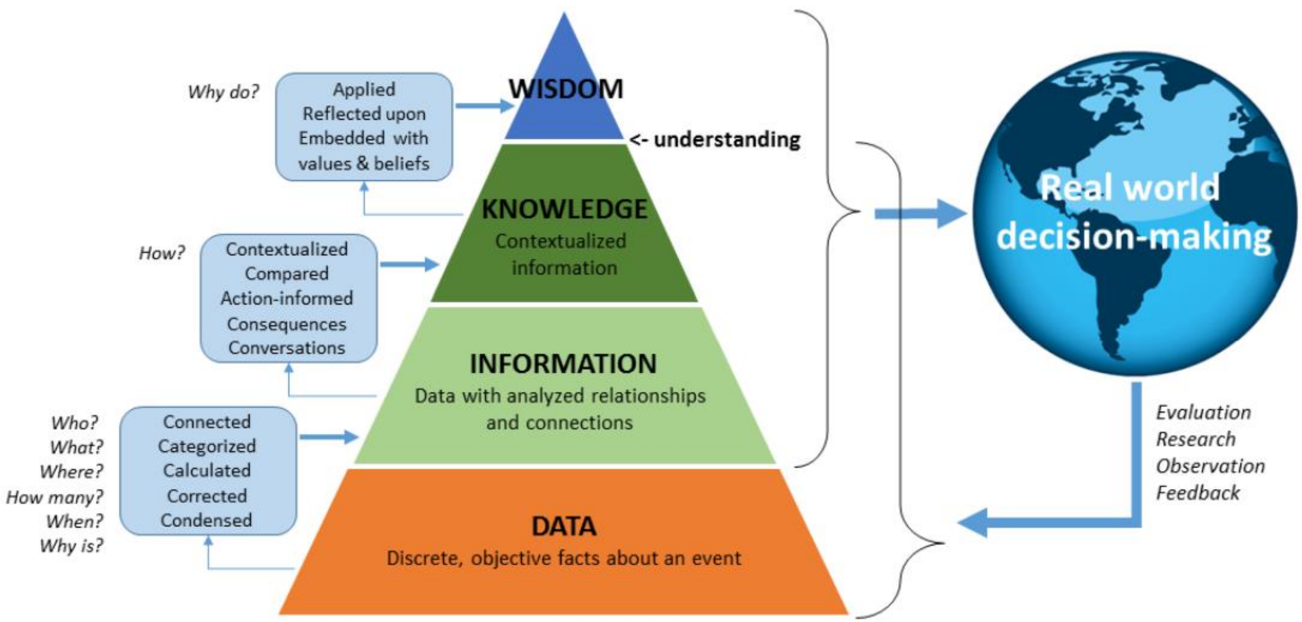
- Knowledge Representation (KR) is the field of artificial intelligence (AI) dedicated to representing information about the world in a form that a computer system can utilize to solve complex tasks such as diagnosing a medical condition
- Examples of knowledge representation models include logic, semantic nets, frames, scripts, and ontologies.
- Automated reasoning engines include Inference engines, theorem provers, classifiers etc

What is knowledge?

- Data: Data is raw facts about the world or events
- Information: Result of processing raw data
- Knowledge: Information gained through experience, reasoning, or acquaintance
- Examples: Driving your car, placement process at VIT-AP, etc.

Wisdom

- The ability to distinguish or judge what is true, right, or lasting
- Knowledge can exist without wisdom, but not the other way around
- Knowledge is knowing how to kill opponents in battlefield; wisdom is knowing what to do with an unarmed opponent


WISDOM
Why do?
Applied
Reflected upon
Embedded with
values & beliefs
<- understanding
KNOWLEDGE
Contextualized
information
How?
Contextualized
Compared
Action-informed
Consequences
Conversations
INFORMATION
Data with analyzed relationships
and connections
Who?
What?
Where?
How many?
When?
Why is?
Connected
Categorized
Calculated
Corrected
Condensed
DATA
Discrete, objective facts about an event
Real world
decision-making
Evaluation
Research
Observation
Feedback

Representation and Mapping

- Tommy is a dog
- Every dog has a tail

$\Downarrow$

Tommy has a tail

Representation and Mapping

- Tommy is a dog

dog(Tommy)

- Every dog has a tail

$\forall x: dog(x) \rightarrow hastail(x)$

$$\Downarrow$$

hastail(Tommy)

Tommy has a tail

Propositional Logic

Propositional logic

- **Logical constants**: true, false
- **Propositional symbols**: P, Q, S, ... (**atomic sentences**)
- Wrapping **parentheses**: (…)
- Sentences are combined by **connectives**:

$\wedge$ ...and	[conjunction]
$\vee$ ...or	[disjunction]
$\Rightarrow$...implies	[implication / conditional]
$\Leftrightarrow$..is equivalent	[biconditional]
$\neg$ ...not	[negation]

- **Literal**: atomic sentence or negated atomic sentence

Examples of PL sentences

- P means "It is hot."
- Q means "It is humid."
- R means "It is raining."
- $(P \land Q) \rightarrow R$

  "If it is hot and humid, then it is raining"
- $Q \rightarrow P$

  "If it is humid, then it is hot"
- A better way:

  Hot = "It is hot"

  Humid = "It is humid"

  Raining = "It is raining"

Propositional logic (PL)

- A simple language useful for showing key ideas and definitions
- User defines a set of propositional symbols, like P and Q.
- User defines the **semantics** of each propositional symbol:
  - P means “It is hot”
  - Q means “It is humid”
  - R means “It is raining”
- A sentence (well formed formula) is defined as follows:
  - A symbol is a sentence
  - If S is a sentence, then $\neg$S is a sentence
  - If S is a sentence, then (S) is a sentence
  - If S and T are sentences, then $(S \vee T)$, $(S \wedge T)$, $(S \rightarrow T)$, and $(S \leftrightarrow T)$ are sentences
  - A sentence results from a finite number of applications of the above rules

Some terms

- The meaning or **semantics** of a sentence determines its **interpretation**.
- Given the truth values of all symbols in a sentence, it can be “evaluated” to determine its **truth value** (True or False).
- A **model** for a KB is a “possible world” (assignment of truth values to propositional symbols) in which each sentence in the KB is True.

More terms

- A **valid sentence** or **tautology** is a sentence that is True under all interpretations, no matter what the world is actuallylike or how the semantics are defined. Example: “It’s raining or it’s not raining.”
- An **inconsistent sentence** or **contradictio**n is a sentence that is False under all interpretations. The world is never like what it describes, as in “It’s raining and it’s not raining.”
- **P entails Q**, written P |= Q, means that whenever P is True, so is Q. In other words, all models of P are also models of Q.

Truth tables

And

p	q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

Or

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

If . . . then

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Not

p	$\sim p$
T	F
F	T

Truth tables II

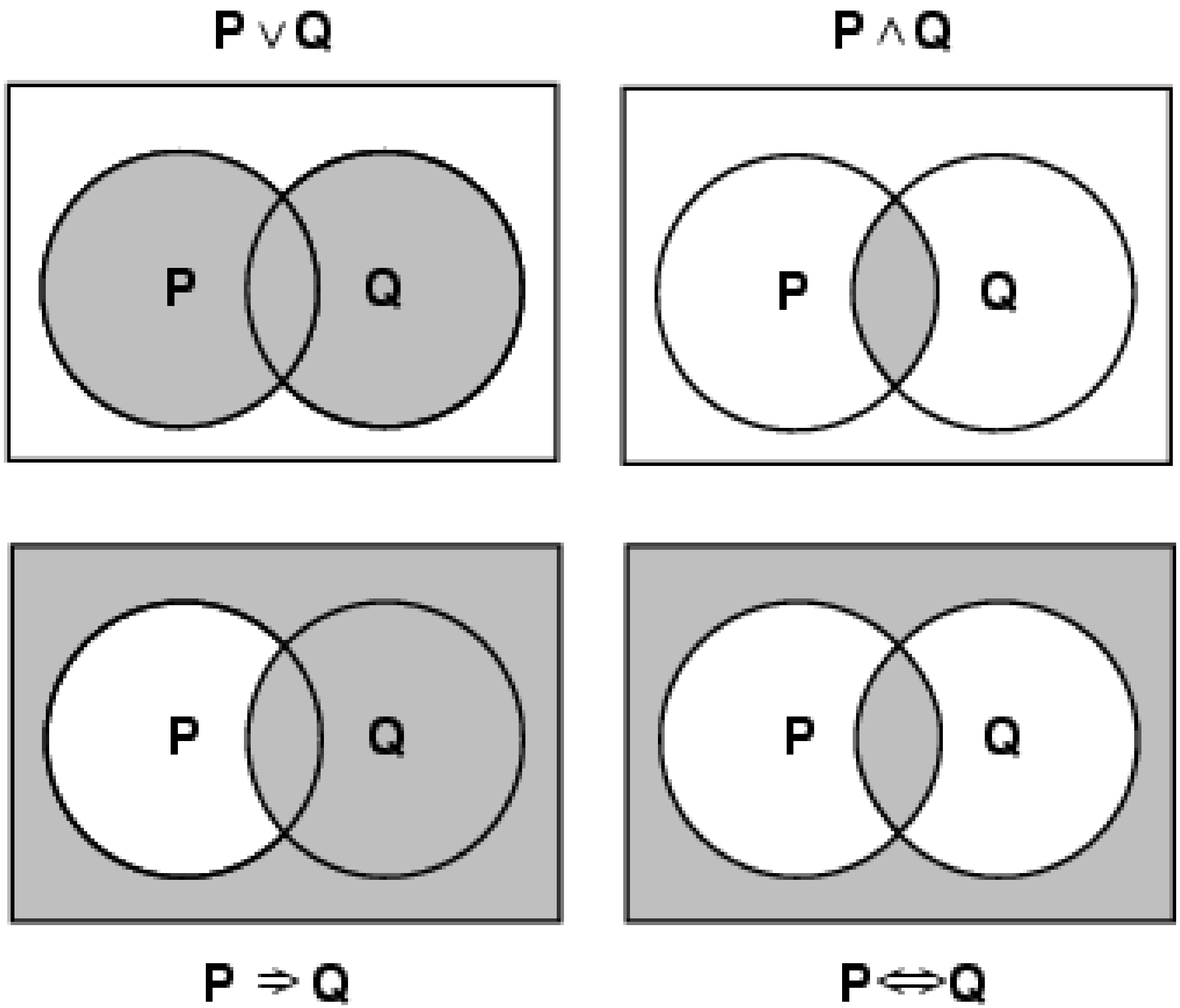
The five logical connectives:

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
False	*False*	*True*	*False*	*False*	*True*	*True*
False	*True*	*True*	*False*	*True*	*True*	*False*
True	*False*	*False*	*False*	*True*	*False*	*False*
True	*True*	*False*	*True*	*True*	*True*	*True*

A complex sentence:

P	H	$P \vee H$	$(P \vee H) \wedge \neg H$	$((P \vee H) \wedge \neg H) \Rightarrow P$
False	*False*	*False*	*False*	*True*
False	*True*	*True*	*False*	*True*
True	*False*	*True*	*True*	*True*
True	*True*	*True*	*False*	*True*

Models of complex sentences



Tautology: A compound proposition which is always true.

e.g., $P \vee \neg P$

Contradiction: A compound proposition which is always false.

e.g., $P \wedge \neg P$

Contingency: compound proposition which is sometimes true and sometimes false

e.g., $(P \vee Q)$; $(P \wedge Q)$ etc.

Satisfiability: A compound proposition is satisfiable if there is atleast one true results in its truth table.

e.g., All the tautologies are satisfiable

Unsatisfiability: Not even a single true results in its truth table.

e.g., contradiction is always unsatisfiable.

Prove that the following complex propositions are tautologies using truth table.

(i) $(P \rightarrow Q) \Rightarrow P \rightarrow (P \wedge Q)$

(ii) $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R) \Rightarrow R$

Propositional logic is a weak language

- No notion of objects
- Hard to identify "individuals" (e.g., Mary, 3)
- No notion of relations among objects
- Can't directly talk about properties of individuals or relations between individuals (e.g., "Bill is tall")
- Generalizations, patterns, regularities can't easily be represented (e.g., "all triangles have 3 sides")
- First-Order Logic (abbreviated FOL or FOPC) is expressive enough to concisely represent this kind of information

  FOL adds relations, variables, and quantifiers, e.g.,

  - *"Every elephant is gray":* $\forall$ x (elephant(x) $\rightarrow$ gray(x))
  - *"There is a white alligator":* $\exists$ x (alligator(X) ^ white(X))

First-Order Logic

First-order logic

- First-order logic (FOL) models the world in terms of
  - **Objects,** which are things with individual identities
  - **Properties** of objects that distinguish them from other objects
  - **Relations** that hold among sets of objects
  - **Functions,** which are a subset of relations where there is only one "value" for any given "input"
- Examples:
  - Objects: Students, lectures, companies, cars ...
  - Relations: Brother-of, bigger-than, outside, part-of, has-color, occurs-after, owns, visits, precedes, ...
  - Properties: blue, oval, even, large, ...
  - Functions: father-of, best-friend, Square-root, one-more-than ...

User provides

- **Constant symbols,** which represent individuals in the world
  - Mary
  - 3
  - Green
- **Function symbols,** which map individuals to individuals
  - father-of(Mary) = John
  - color-of(Sky) = Blue
- **Predicate symbols,** which map individuals to truth values
  - greater(5,3)
  - green(Grass)
  - color(Grass, Green)

FOL Provides

- **Variable symbols**
  - E.g., x, y, foo
- **Connectives**
  - Same as in PL: not ($\neg$), and ($\wedge$), or ($\vee$), implies ($\rightarrow$), if and only if (biconditional $\leftrightarrow$)
- **Quantifiers**
  - Universal $\forall$**x** or **(Ax)**
  - Existential $\exists$**x** or **(Ex)**

Sentences are built from terms and atoms

- A **term** (denoting a real-world individual) is a constant symbol, a variable symbol, or an n-place function of n terms.
  - x and $f(x_1, ..., x_n)$ are terms, where each x_i is a term.
  - A term with no variables is a **ground term**
- An **atomic sentence** (which has value true or false) is an n-place predicate of n terms
- A **complex sentence** is formed from atomic sentences connected by the logical connectives:
  - $\neg P$, $P \vee Q$, $P \wedge Q$, $P \rightarrow Q$, $P \leftrightarrow Q$ where P and Q are sentences
- A **quantified sentence** adds quantifiers $\forall$ and $\exists$
- A **well-formed formula (wff)** is a sentence containing no "free" variables. That is, all variables are "bound" by universal or existential quantifiers.
  - $(\forall x)P(x,y)$ has x bound as a universally quantified variable, but y is free.

Quantifiers

- **Universal quantification**
  - $(\forall x)P(x)$ means that P holds for **all** values of x in the domain associated with that variable
  - E.g., $(\forall x)$ dolphin(x) $\rightarrow$ mammal(x)
- **Existential quantification**
  - $(\exists\, x)P(x)$ means that P holds for **some** value of x in the domain associated with that variable
  - E.g., $(\exists\, x)$ mammal(x) $\wedge$ lays-eggs(x)
  - Permits one to make a statement about some object without naming it

Quantifiers

- Universal quantifiers are often used with "implies" to form "rules":

  $(\forall x)\ student(x) \rightarrow smart(x)$ means "All students are smart"

- Universal quantification is *rarely* used to make blanket statements about every individual in the world:

  $(\forall x) student(x) \wedge smart(x)$ means "Everyone in the world is a student and is smart"

- Existential quantifiers are usually used with "and" to specify a list of properties about an individual:

  $(\exists x)\ student(x) \wedge smart(x)$ means "There is a student who is smart"

- A common mistake is to represent this English sentence as the FOL sentence:

  $(\exists x)\ student(x) \rightarrow smart(x)$

  – But what happens when there is a person who is *not* a student?

Quantifier Scope

- Switching the order of universal quantifiers *does not* change the meaning:
  - $(\forall x)(\forall y)P(x,y) \leftrightarrow (\forall y)(\forall x)\ P(x,y)$
- Similarly, you can switch the order of existential quantifiers:
  - $(\exists x)(\exists y)P(x,y) \leftrightarrow (\exists y)(\exists x)\ P(x,y)$
- Switching the order of universals and existentials *does* change meaning:
  - Everyone likes someone: $(\forall x)(\exists y)\ likes(x,y)$
  - Someone is liked by everyone: $(\exists y)(\forall x)\ likes(x,y)$
  - There is a person who loves everyone in the world: $\exists x\ \forall y\ Loves(x,y)$
  - Everyone in the world is loved by at least one person: $\forall y\ \exists x\ Loves(x,y)$

Universal quantification

- $\forall$<*variables*> <*sentence*>

Everyone at VIT-AP is smart:
$\forall x\ At(x, VIT\text{-}AP) \Rightarrow Smart(x)$

- $\forall x\ P$ is true in a model *m* iff *P* is true with *x* being each possible object in the model

- Roughly speaking, equivalent to the conjunction of instantiations of *P*
-

$$
\begin{aligned}
& At(KingJohn, VIT\text{-}AP) \Rightarrow Smart(KingJohn) \\
\wedge\ & At(Richard, VIT\text{-}AP) \Rightarrow Smart(Richard) \\
\wedge\ & At(VIT\text{-}AP, VIT\text{-}AP) \Rightarrow Smart(VIT\text{-}AP) \\
\wedge\ & \ldots
\end{aligned}
$$

A common mistake to avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
- Common mistake: using $\wedge$ as the main connective with $\forall$:

  $\forall x\ At(x, VIT\text{-}AP) \wedge Smart(x)$

  means "Everyone is at VIT-AP and everyone is smart"

Existential quantification

- $\exists$<*variables*> <*sentence*>

- Someone at VIT-AP is smart:
- $\exists x$ At(x, VIT-AP) $\wedge$ Smart(x)

- $\exists x$ *P* is true in a model *m* iff *P* is true with *x* being some possible object in the model

- Roughly speaking, equivalent to the disjunction of instantiations of *P*
-

$$
\begin{aligned}
&\quad \text{At(KingJohn, VIT-AP)} \wedge \text{Smart(KingJohn)} \\
&\vee \text{At(Richard, VIT-AP)} \wedge \text{Smart(Richard)} \\
&\vee \text{At(VIT-AP, VIT-AP)} \wedge \text{Smart(VIT-AP)} \\
&\vee \ldots
\end{aligned}
$$

Another common mistake to avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:
-

$$\exists x \text{ At}(x, \text{VIT-AP}) \Rightarrow \text{Smart}(x)$$

is true if there is anyone who is not at VIT-AP!

Connections between All and Exists

We can relate sentences involving $\forall$ and $\exists$ using De Morgan's laws:

$$(\forall x)\ \neg P(x) \leftrightarrow \neg(\exists x)\ P(x)$$

$$\neg(\forall x)\ P \leftrightarrow (\exists x)\ \neg P(x)$$

$$(\forall x)\ P(x) \leftrightarrow \neg\ (\exists x)\ \neg P(x)$$

$$(\exists x)\ P(x) \leftrightarrow \neg(\forall x)\ \neg P(x)$$

Quantified inference rules

- Universal instantiation
  - $\forall x\ P(x)\ \therefore\ P(A)$
- Universal generalization
  - $P(A) \wedge P(B) \ldots \therefore\ \forall x\ P(x)$
- Existential instantiation
  - $\exists x\ P(x)\ \therefore P(F)$ $\leftarrow$ **skolem constant F**
- Existential generalization
  - $P(A)\ \therefore\ \exists x\ P(x)$

Translating English to FOL

Every gardener likes the sun.

$\forall x\ gardener(x) \rightarrow likes(x,Sun)$

You can fool some of the people all of the time.

$\exists x\ \forall t\ \ person(x) \wedge time(t) \rightarrow can\text{-}fool(x,t)$

You can fool all of the people some of the time.

$\forall x\ \exists t\ (person(x) \rightarrow time(t) \wedge can\text{-}fool(x,t))$

$\forall x\ (person(x) \rightarrow \exists t\ (time(t) \wedge can\text{-}fool(x,t)))$

Equivalent

All purple mushrooms are poisonous.

$\forall x\ (mushroom(x) \wedge purple(x)) \rightarrow poisonous(x)$

No purple mushroom is poisonous.

$\neg\exists x\ purple(x) \wedge mushroom(x) \wedge poisonous(x)$

$\forall x\ \ (mushroom(x) \wedge purple(x)) \rightarrow \neg poisonous(x)$

Equivalent

There are exactly two purple mushrooms.

$\exists x\ \exists y\ mushroom(x) \wedge purple(x) \wedge mushroom(y) \wedge purple(y) \wedge \neg(x=y) \wedge \forall z\ (mushroom(z) \wedge purple(z)) \rightarrow ((x=z) \vee (y=z))$

Clinton is not tall.

$\neg tall(Clinton)$

X is above Y iff X is on directly on top of Y or there is a pile of one or more other objects directly on top of one another starting with X and ending with Y.

$\forall x\ \forall y\ above(x,y) \leftrightarrow (on(x,y) \vee\ \exists z\ (on(x,z) \wedge above(z,y)))$

More Examples

- Everyone who loves all animals is loved by someone
- Anyone who kills an animal is loved by no one
- Jack loves all animals.

$$\forall x ((\forall y \; animal(y) \rightarrow loves(x,y)) \rightarrow (\exists z \; loves(z,x)))$$

$$\forall x ((\exists y \; animal(y) \wedge Kills(x,y)) \rightarrow (\forall z \; \neg loves(z,x)))$$

$$\forall x (animal(x) \rightarrow loves(Jack, x))$$

More Examples

- “John has at least two umbrellas”

there exists x: (there exists y: (Has(John, x) AND IsUmbrella(x) AND Has(John, y) AND IsUmbrella(y) AND NOT(x=y))

- “John has at most two umbrellas”

for all x, y, z: ((Has(John, x) AND IsUmbrella(x) AND Has(John, y) AND IsUmbrella(y) AND Has(John, z) AND IsUmbrella(z)) => (x=y OR x=z OR y=z))

More Examples

- "Duke's basketball team defeats any other basketball team"

for all x: ((IsBasketballTeam(x) AND NOT(x=BasketballTeamOf(Duke))) => Defeats(BasketballTeamOf(Duke), x))

- "Every team defeats some other team"

for all x: (IsTeam(x) => (there exists y: (IsTeam(y) AND NOT(x=y) AND Defeats(x,y))))

CLAUSAL FORM

(1) LITERAL: A literal is either an atomic sentence or negation of an atomic sentence.

E.g., $P(a)$, $\neg P(b)$

(2) A clausal sentence is either a literal or a disjunction of literals

e.g., $P(a)$, $\neg P(b)$, $P(a) \vee \neg P(b)$

(3) A clause is a set of literals

$\{P(a)\}$, $\{\neg P(b)\}$, $\{P(a), \neg P(b)\}$

Resolution

- Unification
- For example,

$$\frac{\begin{array}{l}\neg Rich(x) \vee Unhappy(x)\\ Rich(Ken)\end{array}}{Unhappy(Ken)}$$

  with $\theta = \{x/Ken\}$

- Apply resolution steps to $CNF(KB \wedge \neg\alpha)$; complete for FOL

Conversion to Conjunctive Normal Form (CNF) or Clausal Form

Everyone who loves all animals is loved by someone:

$$\forall x\ [\forall y\ Animal(y) \implies Loves(x,y)] \implies [\exists y\ Loves(y,x)]$$

1. Eliminate biconditionals and implications

$$\forall x\ [\neg\forall y\ \neg Animal(y) \lor Loves(x,y)] \lor [\exists y\ Loves(y,x)]$$

2. Move $\neg$ inwards: $\neg\forall x, p \equiv \exists x\ \neg p$, $\neg\exists x, p \equiv \forall x\ \neg p$:

$$\forall x\ [\exists y\ \neg(\neg Animal(y) \lor Loves(x,y))] \lor [\exists y\ Loves(y,x)]$$
$$\forall x\ [\exists y\ \neg\neg Animal(y) \land \neg Loves(x,y)] \lor [\exists y\ Loves(y,x)]$$
$$\forall x\ [\exists y\ Animal(y) \land \neg Loves(x,y)] \lor [\exists y\ Loves(y,x)]$$

3. Standardize variables: each quantifier should use a different one

$$\forall\, x\ \ [\exists\, y\ \ Animal(y) \land \neg Loves(x,y)] \lor [\exists\, z\ \ Loves(z,x)]$$

4. Skolemize: a more general form of existential instantiation.
   Each existential variable is replaced by a Skolem function
   of the enclosing universally quantified variables:

$$\forall\, x\ \ [Animal(F(x)) \land \neg Loves(x, F(x))] \lor Loves(G(x), x)$$

5. Drop universal quantifiers:

$$[Animal(F(x)) \land \neg Loves(x, F(x))] \lor Loves(G(x), x)$$

6. Distribute $\land$ over $\lor$:

$$[Animal(F(x)) \lor Loves(G(x), x)] \land [\neg Loves(x, F(x)) \lor Loves(G(x), x)]$$

Example Knowledge

- *The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.*
- Prove that Col. West is a criminal

Steps for Resolution

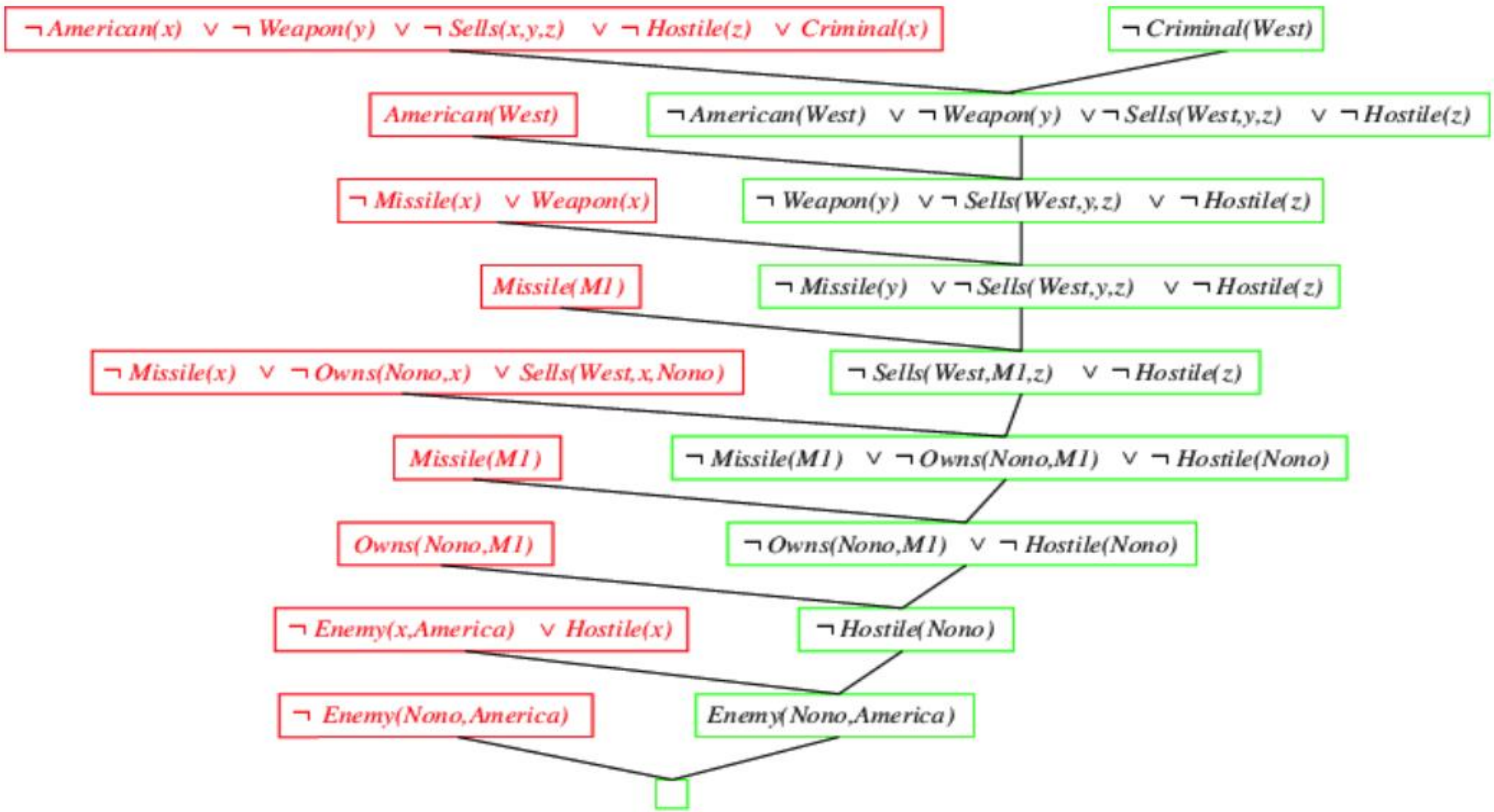
1. Conversion of facts into first-order logic.
2. Convert FOL statements into CNF
3. Negate the statement which needs to prove (proof by contradiction)
4. Draw resolution graph (unification).

Example Knowledge Base

- . . . it is a crime for an American to sell weapons to hostile nations:
  $American(x) \land Weapon(y) \land Sells(x,y,z) \land Hostile(z) \implies Criminal(x)$
- Nono . . . has some missiles, i.e., $\exists\, x\ Owns(Nono, x) \land Missile(x)$:
  $Owns(Nono, M_1)$ and $Missile(M_1)$
- . . . all of its missiles were sold to it by Colonel West
  $\forall\, x\ \ Missile(x) \land Owns(Nono, x) \implies Sells(West, x, Nono)$
- Missiles are weapons:
  $Missile(x) \Rightarrow Weapon(x)$
- An enemy of America counts as "hostile":
  $Enemy(x, America) \implies Hostile(x)$
- West, who is American . . .
  $American(West)$
- The country Nono, an enemy of America . . .
  $Enemy(Nono, America)$

- Rules
  - $American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \implies Criminal(x)$
  - $Missile(M_1)$ and $Owns(Nono, M_1)$
  - $\forall x \;\; Missile(x) \wedge Owns(Nono, x) \implies Sells(West, x, Nono)$
  - $Missile(x) \Rightarrow Weapon(x)$
  - $Enemy(x, America) \implies Hostile(x)$
  - $American(West)$
  - $Enemy(Nono, America)$
- Converted to CNF
  - $\neg American(x) \vee \neg Weapon(y) \vee \neg Sells(x,y,z) \vee \neg Hostile(z) \vee Criminal(x)$
  - $Missile(M_1)$ and $Owns(Nono, M_1)$
  - $\neg Missile(x) \vee \neg Owns(Nono, x) \vee Sells(West, x, Nono)$
  - $\neg Missile(x) \vee Weapon(x)$
  - $\neg Enemy(x, America) \vee Hostile(x)$
  - $American(West)$
  - $Enemy(Nono, America)$
- Query: $\neg Criminal(West)$

Resolution Proof



Step 1: Remove Implications

$$P \rightarrow Q \equiv \neg P \vee Q$$

$$P \leftrightarrow Q \equiv (\neg P \vee Q) \wedge (P \vee \neg Q)$$

Step 2: (Negations In)

$$\neg \neg P \equiv P$$

$$\neg (P \wedge Q) \equiv \neg P \vee \neg Q$$

$$\neg (P \vee Q) \equiv \neg P \wedge \neg Q$$

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$

Step 3: Standardize Variables

$$\forall x P(x) \vee \forall x Q(x) \longrightarrow \forall (x) P(x) \vee \forall y Q(y)$$

Step 4: $\exists x\, P(x) \equiv P(a)$ $\left[\underline{\text{Existentials}}\ \text{Out}\right]$

$$\forall x\,(P(x) \wedge \exists z\, Q(x,y,z)) \equiv \forall x\,(P(x) \wedge Q(x,y,f(x,y))$$

Step 5: $\forall x\,(P(x) \wedge Q(x,y,f(x,y))) \equiv P(x) \wedge Q(x,y,f(x,y))$ $\boxed{\underline{\text{Alls Out}}}$

Step 6:

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

$$(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$$

$$P \vee (P_1 \vee \ldots \vee P_n) \equiv (P \vee P_1 \vee \ldots P_n)$$

$$(P_1 \vee \ldots \vee P_n) \vee P \equiv (P_1 \vee P_2 \vee \ldots \vee P_n \vee P)$$

$$P \wedge (P_1 \wedge \ldots \wedge P_n) \equiv (P \wedge P_1 \wedge \ldots \wedge P_n)$$

$$(P_1 \wedge \ldots \wedge P_n) \wedge P \equiv (P_1 \wedge P_2 \ldots \wedge P_n \wedge P)$$

Distribution.

Step 7: $P_1 \wedge \ldots \wedge P_n \Longrightarrow P_1, \ldots, P_n$

$$P_1 \vee \ldots \vee P_n \equiv \{P_1, \ldots, P_n\}$$

Operators Out.

Example 1:

$\exists y(\, g(y) \wedge \forall z(\, r(z) \rightarrow f(y,z)))$

$\exists y(\, g(y) \wedge \forall z(\neg r(z) \vee f(y,z)))$ — Step 1 (Implication, I)

I $\exists y(\, g(y) \wedge \forall z(\neg (r(z) \vee f(y,z)))$ — Step 2 (Negations In, N)

N $\exists y(\, g(y) \wedge \forall z(\neg r(z) \vee f(y,z)))$ — Step 3 (Standardize, S)

S $\exists y(\, g(y) \wedge \forall z(\neg r(z) \vee f(greg, z)))$ — Step 4 (Existentials Out, E)

E $g(greg) \wedge \forall z(\neg r(z) \vee f(greg, z)))$ — Step 5 (Alls out, A)

A $g(greg) \wedge (\neg (r(z) \vee f(greg, z)))$ — Step 6 (Distribution, D)

D $g(greg) \wedge (\neg (r(z) \vee f(greg, z)))$

O $\{g(greg)\}$

$\{\neg r(z), f(greg, z)))$ — Step 7 (Operators Out, O)

Example

$$\forall y\,(\neg g(y) \vee \exists z\,(r(z) \wedge \neg f(y,z)))$$

E $\forall y\,(\neg g(y) \vee (r(h(y)) \wedge \neg f(y, h(y))))$

A $\neg g(y) \vee (r(h(y)) \wedge \neg f(y, h(y)))$

D $(\neg g(y) \vee r(h(y))) \wedge (\neg g(y) \vee \neg f(y, h(y)))$

O $\{\neg g(y), r(h(y))\}$

$\{\neg g(y), \neg f(y, h(y))\}$

Example of prove by Resolution

1. Anyone whom Mary loves is a football star.
   $\forall x\ (LOVES(Mary,x) \rightarrow STAR(x))$

2. Any student who does not pass does not play.
   $\forall x\ (STUDENT(x) \land \neg\ PASS(x) \rightarrow \neg\ PLAY(x))$

3. John is a student.
   $STUDENT(John)$

4. Any student who does not study does not pass.
   $\forall x\ (STUDENT(x) \land \neg\ STUDY(x) \rightarrow \neg\ PASS(x))$

5. Anyone who does not play is not a football star.
   $\forall x\ (\neg\ PLAY(x) \rightarrow \neg\ STAR(x))$

6. (Conclusion) If John does not study, then Mary does not love John.
   $\neg\ STUDY(John) \rightarrow \neg\ LOVES(Mary,John)$

(1) $\forall x\,(LOVE(Mary, x) \rightarrow STAR(x))$

$\boxed{\neg LOVE(Mary, x) \vee STAR(x)}$

(2) $\forall x\,(STUDENT(x) \wedge \neg PASS(x) \rightarrow \neg PLAY(x))$

$\boxed{\neg STUDENT(x) \vee PASS(x) \vee \neg PLAY(x)}$

(3) $\boxed{STUDENT(John)}$

(4) $\forall x\,(STUDENT(x) \wedge \neg STUDY(x) \rightarrow \neg PASS(x))$

$\boxed{\neg STUDENT(x) \vee STUDY(x) \vee \neg PASS(x)}$

(5) $\forall x\,(\neg PLAY(x) \rightarrow \neg STAR(x))$

$\boxed{PLAY(x) \vee \neg STAR(x)}$

(6)

$\neg STUDY(John) \rightarrow \neg LOVE(Mary, John)$

$STUDY(John) \vee \neg LOVE(Mary, John)$

Negation of (6). $\boxed{\underbrace{\boxed{\neg STUDY(John)}}_{6(a)} \wedge \underbrace{\boxed{LOVE(Mary, John)}}_{6(b)}}$

1 & 5

$\neg\, love(Mary, x) \lor play(x)$ [¬7]

2

$\neg\, student(x) \lor pass(x) \lor \neg\, love(Mary, x)$

3

$pass(John) \lor \neg\, love(Mary, John)$

6(b)

$pass(John)$

4

$study(John) \lor \neg\, student(John)$

6(a)

$\neg\, student(John)$

3

$\square$ or $\{\phi\}$.

Home Work

1. Every coyote chases some roadrunner.
   $\forall x\ (COYOTE(x) \rightarrow \exists y\ (RR(y) \wedge CHASE(x,y)))$

2. Every roadrunner who says ``beep-beep" is smart.
   $\forall x\ (RR(x) \wedge BEEP(x) \rightarrow SMART(x))$

3. No coyote catches any smart roadrunner.
   $\neg\ \exists x\ \exists y\ (COYOTE(x) \wedge RR(y) \wedge SMART(y) \wedge CATCH(x,y))$

4. Any coyote who chases some roadrunner but does not catch it is frustrated.
   $\forall x\ (COYOTE(x) \wedge \exists y\ (RR(y) \wedge CHASE(x,y) \wedge \neg\ CATCH(x,y)) \rightarrow FRUSTRATED(x))$

5. (Conclusion) If all roadrunners say ``beep-beep", then all coyotes are frustrated.
   $(\forall x\ (RR(x) \rightarrow BEEP(x)) \rightarrow (\forall y\ (COYOTE(y) \rightarrow FRUSTRATED(y)))$