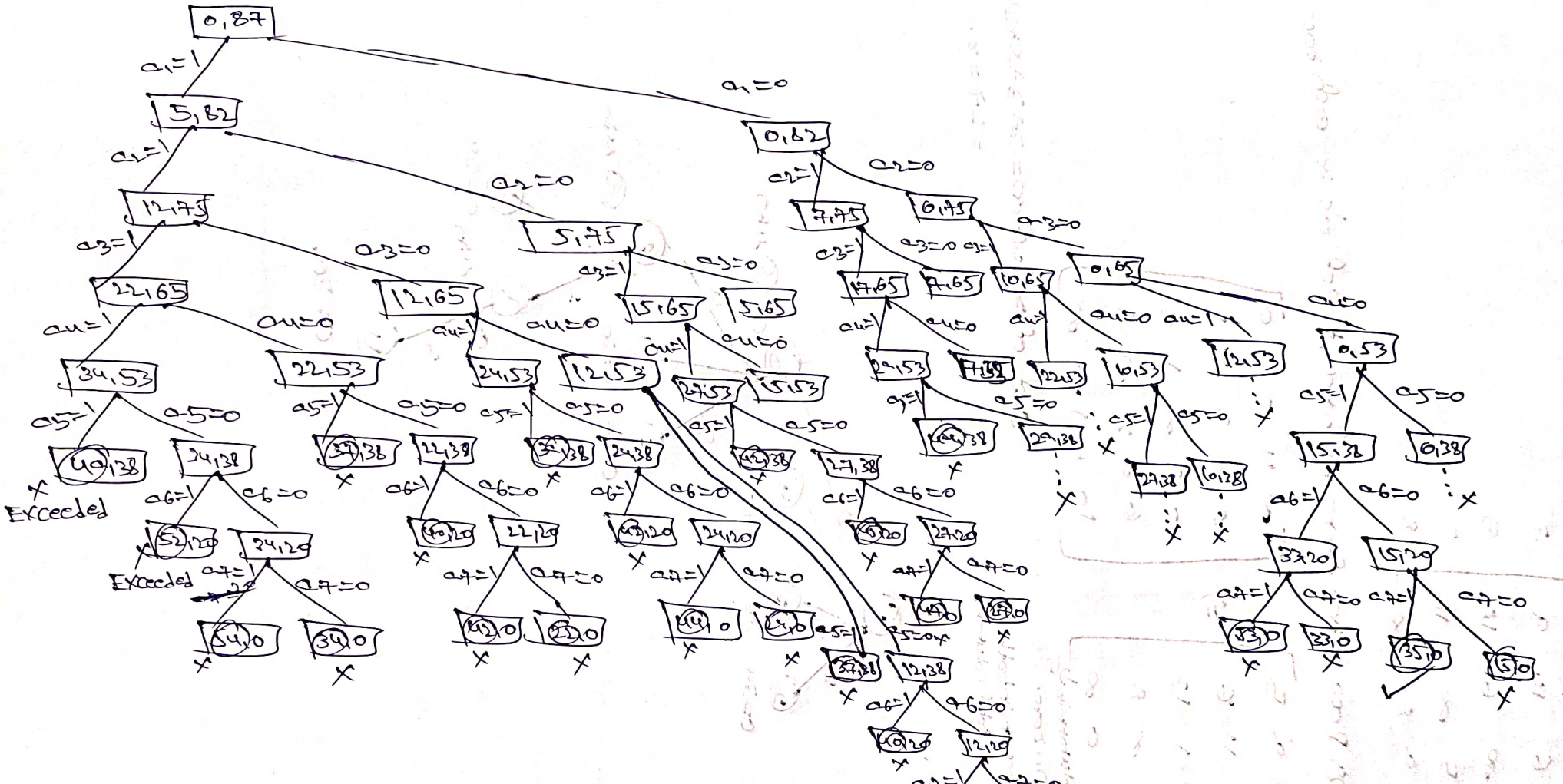
Assignment-2

1) Given 5, 7, 10, 12, 15, 18, 20

Total sum = 5+7+10+12+15+18+20 = 87

M = 35.



∴ The subset is:

$a_1 a_2 a_3 a_4 a_5 a_6 a_7$

{0 0 0 0 1 0 1}.

⇒ Sum = 15+20.

= 35.

2) Given.

$$\begin{array}{c|ccccc|c} & & & \text{Row min} & & & \\ 1 & \infty & 7 & 3 & 12 & 8 & 3 \\ 2 & 3 & \infty & 6 & 14 & 9 & 3 \\ 3 & 5 & 8 & \infty & 6 & 18 & 5 \\ 4 & 2 & 3 & 5 & \infty & 11 & 3 \\ 5 & 12 & 14 & 2 & 8 & \infty & 8 \end{array}$$

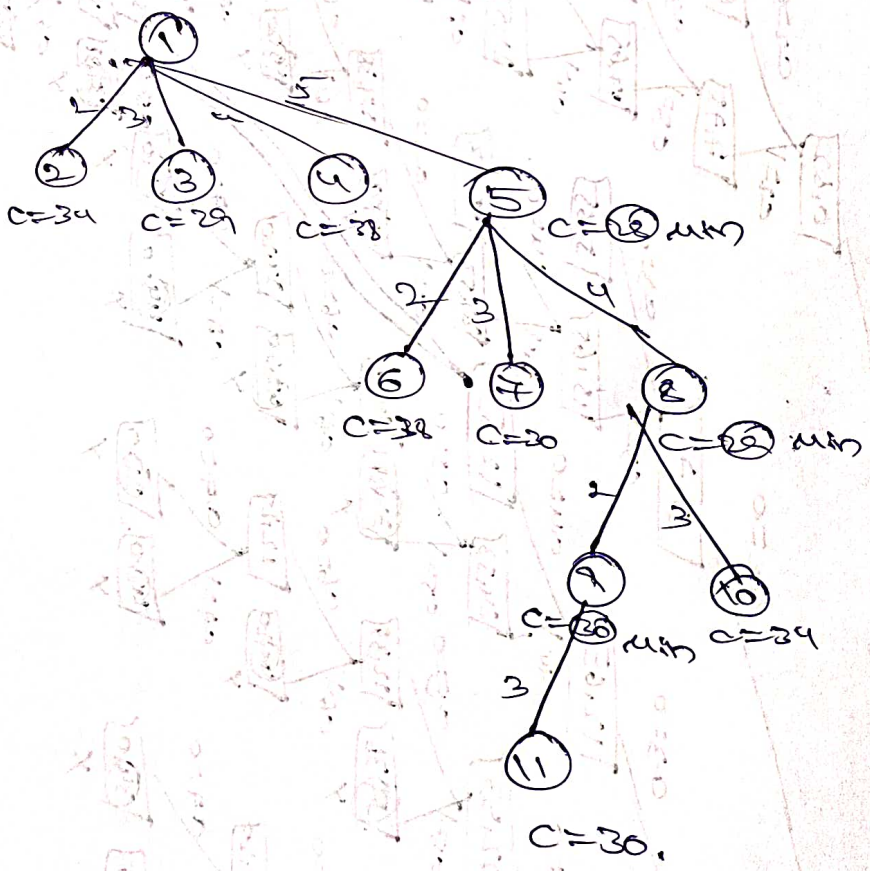
Subtracting row min from each row, to get 0's in each row.

$$\begin{bmatrix} \infty & 4 & 0 & 9 & 5 \\ 0 & \infty & 3 & 11 & 6 \\ 0 & 3 & \infty & 1 & 13 \\ 6 & 0 & 2 & \infty & 8 \\ 6 & 6 & 1 & 0 & \infty \end{bmatrix} \Rightarrow \begin{bmatrix} \infty & 4 & 0 & 9 & 0 \\ 0 & \infty & 3 & 11 & 1 \\ 0 & 3 & \infty & 1 & 8 \\ 6 & 0 & 2 & \infty & 3 \\ 6 & 6 & 1 & 0 & \infty \end{bmatrix}$$

Column min 5

5th column has no zero.
So, subtract column min to all total column.

Cost of reduction $= 3+3+5+3+8+5$
$= 27 = r$.



(1,2)

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \cancel{7}^{2} & \cancel{11}^{10} & \cancel{1}^{0} \\ 0 & \infty & \infty & 1 & 8 \\ \cancel{4}^{4} & \infty & \cancel{7}^{0} & \infty & \cancel{8}^{1} \\ 10 & \infty & 1 & 0 & \infty \end{bmatrix} \begin{matrix} \\ \to 1 \\ \\ \to 2 \\ \\ \end{matrix} \quad \delta = 1+2 = 3.$$

$C(1,2) + \gamma + \delta = 4+27+3$
$= 34.$

(1,3)

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & \cancel{1}^{0} \\ \infty & \cancel{8}^{2} & \infty & \cancel{1}^{0} & \cancel{4}^{7}\cancel{6} \\ 6 & 0 & \infty & \infty & \cancel{4}^{2} \\ 10 & 6 & \infty & 0 & \infty \end{bmatrix} \begin{matrix} \\ \\ \to 1 \\ \\ \\ \end{matrix} \quad \delta = 1+1 = 2$$

$C(1,3) + \gamma + \delta = 0+27+2$
$= 29.$

(1,4)

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 3 & \infty & \cancel{1}^{0} \\ 0 & 3 & \infty & \infty & \cancel{4}^{7} \\ \infty & 0 & 2 & \infty & \cancel{7}^{2} \\ \cancel{4}^{9} & \cancel{4}^{5} & \cancel{1}^{0} & \infty & \infty \end{bmatrix} \begin{matrix} \\ \\ \\ \\ \to 1 \end{matrix} \quad \delta = 1+1 = 2$$

$C(1,4) + \gamma + \delta = 0+27+2$
$= 38$

(1,5)

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \cancel{7}^{2} & 11 & \infty \\ 0 & 3 & \infty & 1 & \infty \\ 6 & 0 & \cancel{1}^{1} & \infty & \infty \\ \infty & 6 & \cancel{1}^{0} & 0 & \infty \end{bmatrix} \to \delta = 1$$

$C(1,5) + \gamma + \delta = 0+27+1$
$= 28.$

(5,2)

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \cancel{7}^{0} & \cancel{11}^{8} & \infty \\ 0 & \infty & \infty & \cancel{1}^{0} & \infty \\ \cancel{4}^{5} & \infty & \cancel{1}^{0} & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} \\ \to 2 \\ \\ \to 1 \\ \\ \end{matrix} \quad \delta = 2+1+1 = 4.$$

$C(5,2) + C(5) + \delta$
$= 6+28+4 = 38.$

(5,3)

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & \infty \\ \infty & \cancel{4}^{2} & \infty & \cancel{1}^{0} & \infty \\ 6 & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} \\ \\ \to 1 \\ \\ \\ \end{matrix} \quad \delta = 1$$

$C(5,3) + C(5) + \delta$
$= 1+28+1 = 30.$

(5,4)

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \cancel{7}^{1} & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & 0 & \cancel{1}^{0} & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \quad \delta = 1$$

$C(5,4) + C(5) + \delta = 0+28+1$
$= 29.$

(4,2)

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \cancel{1}^{0} & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} \\ \to 1 \\ \\ \\ \\ \end{matrix} \quad \delta = 1$$

$C(4,2) + C(5) + \delta$
$= 0+29+1$
$= 30.$

(4,3)

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \cancel{4}^{0} & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} \\ \\ \to 3 \\ \\ \\ \end{matrix} \quad \delta = 3.$$

$C(4,3) + C(5) + \delta$
$\cancel{= 29}$
$= 2+29+3$
$= 34.$

(2,3)

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \hat{g} = 0$$

$c(2,3) + c(a) + \hat{g}$

$= 0 + 30 + 0$

$= 30.$

Path $\Rightarrow$ $1 \to 5 \to 4 \to 2 \to 3 \to 1.$

$8 + 8 + 3 + 6 + 5 = 30$

$\therefore$ optimal cost $= 30$

3) Given, $N=5$, $m=12$

$(P_1, P_2, P_3, P_4, P_5) = (10, 15, 6, 8, 4)$

$(W_1, W_2, W_3, W_4, W_5) = (4, 6, 2, 4, 2)$.

LIFO Branch & Bound, D-Search

As, knapsack is maximize problem, branch & bound is minimize problem. We use -ve sign for profits.

So, $(P_1, P_2, P_3, P_4, P_5) = (-10, -15, -6, -8, -4)$.

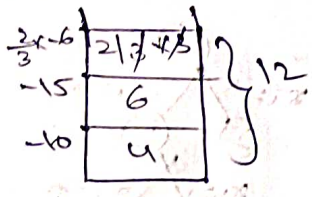
$\hat{c}$ - Lower bound

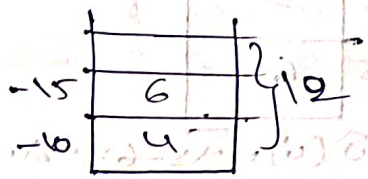
$\hat{U}$ - Upper bound

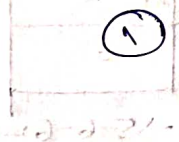
If $\hat{c}(n) > \hat{U}$ is false, Node can be extended.

If $\hat{U}(n) < \hat{U}$ is false, $\hat{U}$ is not updated.

Node 1,





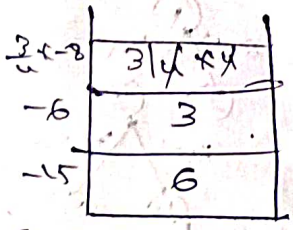


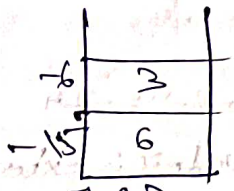
$\hat{c}(1) = -10-15-4 = -29$ $\hat{U}(1) = -10-15 = -25 = \hat{U}$.

$\hat{c}(1) > \hat{U} \rightarrow$ False. Node 1 is extended.

$\hat{U}(1) < \hat{U} \rightarrow$ False, $\hat{U}$ is not updated.

Node 3, $x_1 = 0$

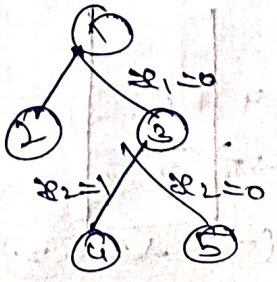




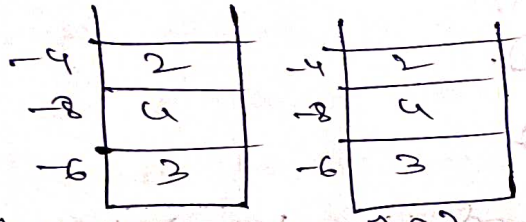
$\hat{c}(3) = -15-6-6 = -27$ $\hat{U}(3) = -15-6 = -21$

$\hat{c}(3) > \hat{U} \rightarrow$ False. Node 3 is extended.

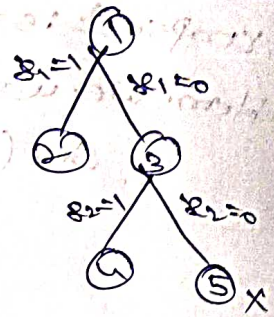
$\hat{U}(3) < \hat{U} \rightarrow$ False. $\hat{U}$ is not updated.



Node 5, $x_1=0, x_2=0$



$\hat{c}(5) = -6-8-4 = -18$ $\hat{v}(5) = -6-8-4 = -18.$

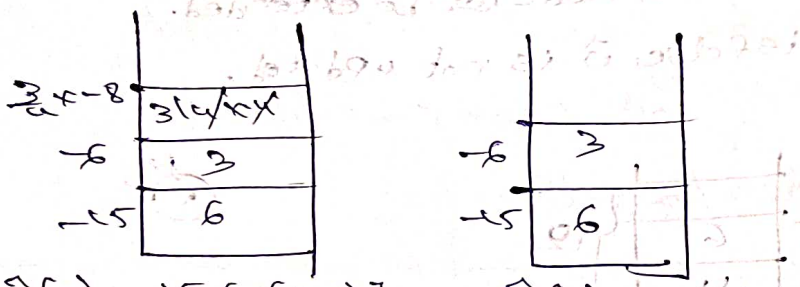


$\hat{c}(5) > \hat{v} \rightarrow$ true. Node 5 is not extended.

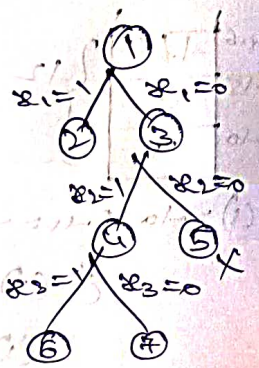
Kill Node 5.

$\hat{v}(5) < \hat{v} \rightarrow$ False. $\hat{v}$ is not updated.

Node 4, $x_1=0, x_2=1$



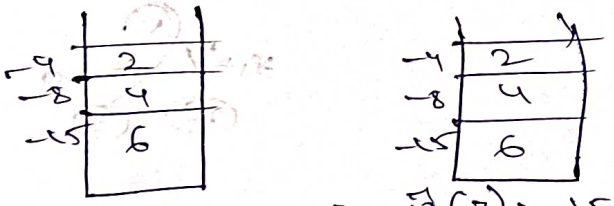
$\hat{c}(4) = -15-6-6 = -27$ $\hat{v}(4) = -15-6 = -21.$



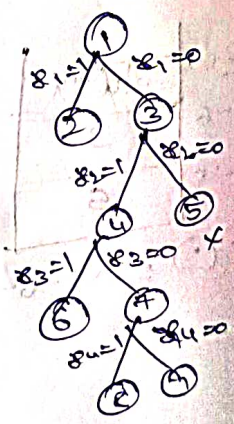
$\hat{c}(4) > \hat{v} \rightarrow$ False. Node 4 is extended.

$\hat{v}(4) < \hat{v} \rightarrow$ False. $\hat{v}$ not updated.

Node 7, $x_1=0, x_2=1, x_3=0.$



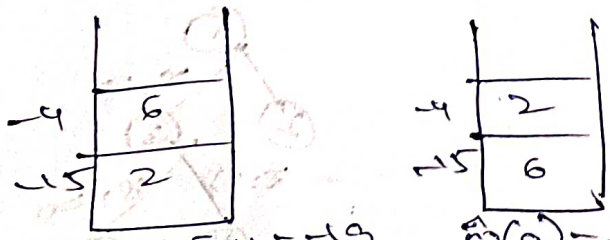
$\hat{c}(7) = -15-8-4 = -27$ $\hat{v}(7) = -15-8-4 = -27$



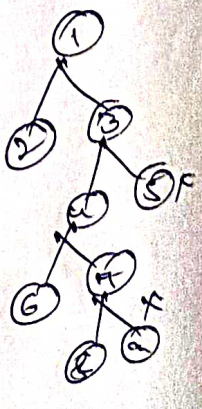
$\hat{c}(7) > \hat{v} \rightarrow$ False. Node 7 is extended.

$\hat{v}(7) < \hat{v} \rightarrow$ True. $\hat{v}$ is updated.

Node 9, $x_1=0, x_2=1, x_3=0, x_4=0.$



$\hat{c}(9) = -15-4 = -19$ $\hat{v}(9) = -15-4 = -19.$

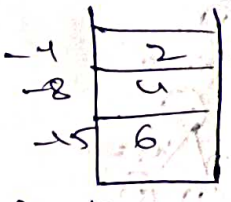


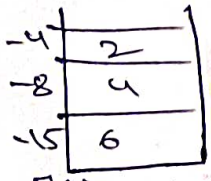
$\hat{c}(9) > \hat{v} \rightarrow$ True. Node 9 is not extended.

Kill Node 9.

$\hat{v}(9) < \hat{v} \rightarrow$ False. $\hat{v}$ is not updated.

Node 8, $x_1=0, x_2=1, x_3=0, x_4=1$

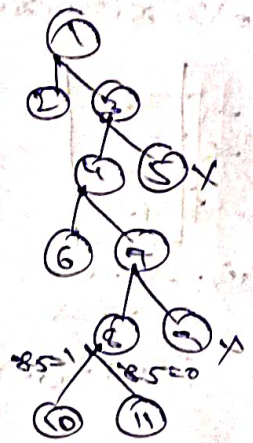




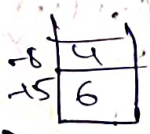
$\hat{c}(8) = -15-8-4 = -27$ $\hat{u}(8) = -15-8-4 = -27$

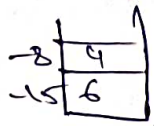
$\hat{c}(8) > \hat{u} \rightarrow$ False. Node 8 is extended.

$\hat{u}(8) < \hat{u} \rightarrow$ False. $\hat{u}$ is not updated.



Node 11, $x_1=0, x_2=1, x_3=0, x_4=1, x_5=0$.



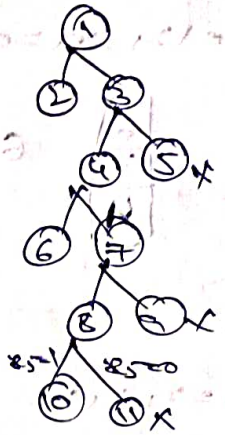


$\hat{c}(11) = -15-8 = -23$ $\hat{u}(11) = -15-8 = -23$.

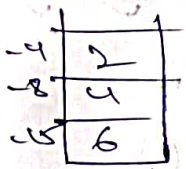
$\hat{c}(11) > \hat{u} \rightarrow$ True. Node 11 is not extended.

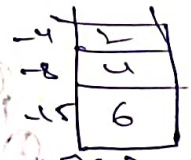
Kill Node 11.

$\hat{u}(11) < \hat{u} \rightarrow$ False. $\hat{u}$ is not updated.



Node 10, $x_1=0, x_2=1, x_3=0, x_4=1, x_5=1$.



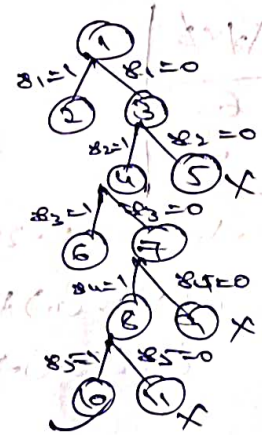


$\hat{c}(10) = -15-8-4 = -27$ $\hat{u}(10) = -15-8-4 = -27$

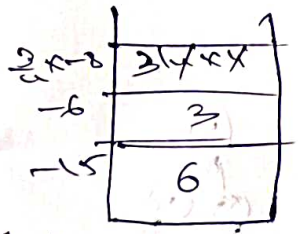
$\hat{c}(10) > \hat{u} \rightarrow$ False. Node 10 is extended.

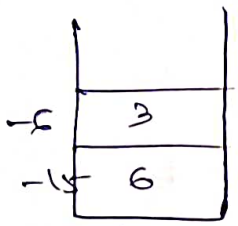
$\hat{u}(10) < \hat{u} \rightarrow$ False. $\hat{u}$ is not updated.

Node 10 is extended. But no more objects are there. So, it is a active node.



Node 6, $x_1=0, x_2=1, x_3=1$

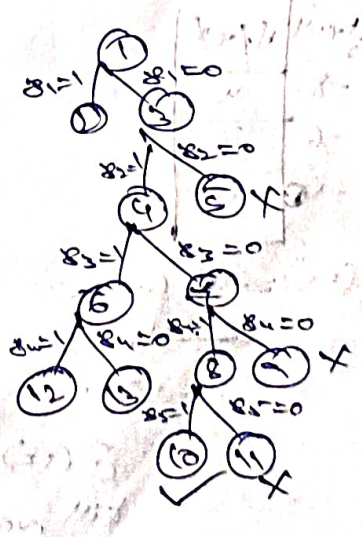




$\hat{c}(6) = -15-6-6 = -27$ $\hat{u}(6) = -15-6 = -21$.

$\hat{c}(6) > \hat{u} \rightarrow$ False. Node 6 is extended.

$\hat{u}(6) < \hat{u} \rightarrow$ False. $\hat{u}$ not updated.



Node 13, $x_1=0, x_2=1, x_3=1, x_4=0$

-4	2
-6	3
-15	6

$\hat{c}(13) = -15-6-4$
$= -25$

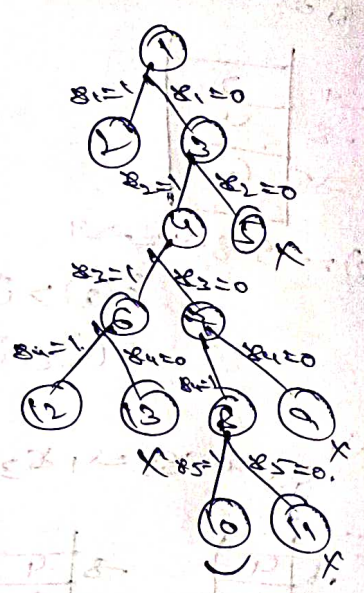
-4	2
-6	3
-15	6

$\hat{o}(13) = -15-6-4$
$= -25$

$\hat{c}(13) > \hat{U} \rightarrow$ True. Node 13 is not extended.

Kill Node 13.

$\hat{o}(13) < \hat{U} \rightarrow$ False. $\hat{U}$ is not updated.



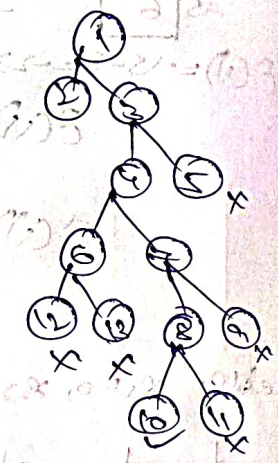
Node 12, $x_1=0, x_2=1, x_3=1, x_4=1$.

6
3
4

$6+3+4 = 13 > 12(m)$

It is infeasible solution.

So, kill node 12.



Node 2, $x_1=1$

$2/3 \times -6$	$2/3 \times 3$
-15	6
-10	4

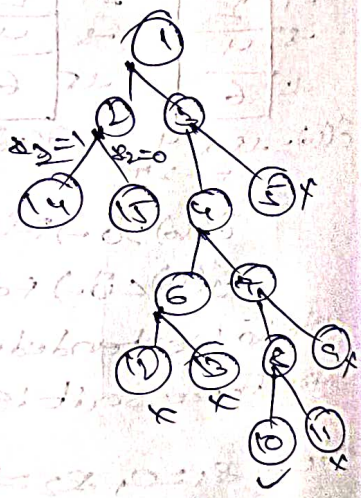
$\hat{c}(2) = -10-15-4 = -29$

-15	6
-10	4

$\hat{o}(2) = -10-15 = -25$

$\hat{c}(2) > \hat{U} \rightarrow$ False. Node 2 is extended.

$\hat{o}(2) < \hat{U} \rightarrow$ False. $\hat{U}$ not updated.



Node 15, $x_1=1, x_2=0$.

$\frac{1}{2} \times -4$	$1/2 \times 4$
-8	4
-6	3
-10	4

$\hat{c}(15) = -10-6-8-2$
$= -26$.

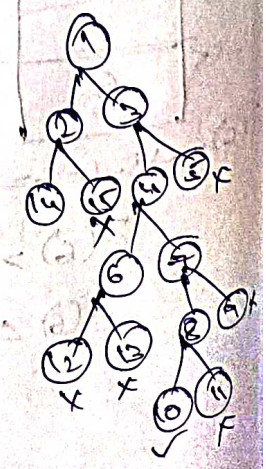
-8	4
-6	3
-10	4

$\hat{o}(15) = -10-6-8$
$= -24$.

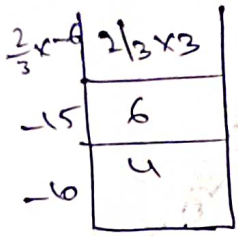
$\hat{c}(15) > \hat{U} \rightarrow$ True. Node 15 is not extended.

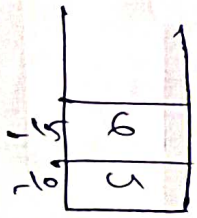
Kill node 15.

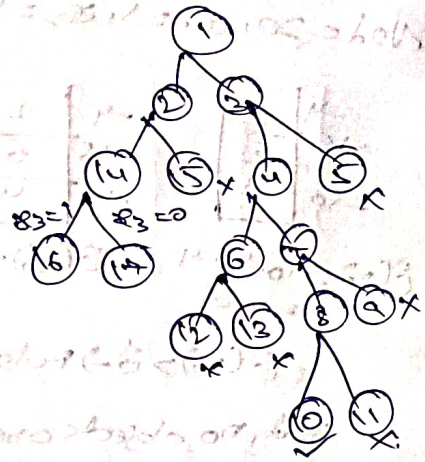
$\hat{o}(15) < \hat{U} \rightarrow$ False. $\hat{U}$ not updated.



Node 14, $x_1=1$, $x_2=1$.





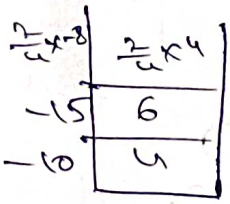


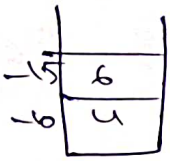
$\hat{c}(14) = -10-15-4 = -29$ $\quad$ $\hat{v}(14) = -10-15 = -25$

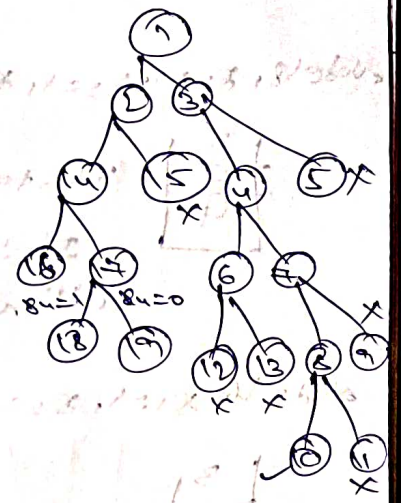
$\hat{c}(14) > \hat{v} \rightarrow$ False. Node 14 is extended.

$\hat{v}(14) < \hat{v} \rightarrow$ False. $\hat{v}$ not updated.

Node 17, $x_1=1$, $x_2=1$, $x_3=0$





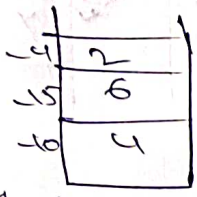


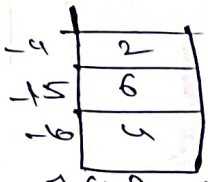
$\hat{c}(17) = -10-15-4 = -29$ $\quad$ $\hat{v}(17) = -10-15 = -25$.

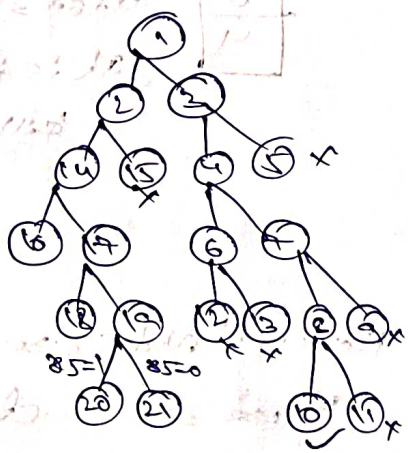
$\hat{c}(17) > \hat{v} \Rightarrow$ False. Node 17 is extended.

$\hat{v}(17) < \hat{v} \Rightarrow$ False. $\hat{v}$ not updated.

Node 19, $x_1=1$, $x_2=1$, $x_3=0$, $x_4=0$.





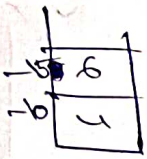


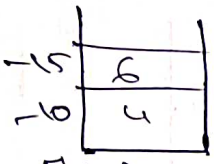
$\hat{c}(19) = -10-15-4 = -29$ $\quad$ $\hat{v}(19) = -10-15-4 = -29$

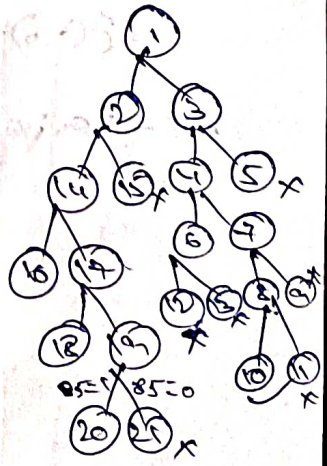
$\hat{c}(19) > \hat{v} \rightarrow$ False. Node 19 is extended.

$\hat{v}(19) < \hat{v} \rightarrow$ True. $\hat{v}$ is updated.

Node 21, $x_1=1$, $x_2=1$, $x_3=0$, $x_4=0$, $x_5=0$.







$\hat{c}(21) = -10-15 = -25$ $\quad$ $\hat{v}(21) = -10-15 = -25$

$\hat{c}(21) > \hat{v} \rightarrow$ True $\Rightarrow$ Node 21 is not extended.

Kill Node 21.

$\hat{v}(21) < \hat{v} \rightarrow$ False. $\hat{v}$ is not updated.

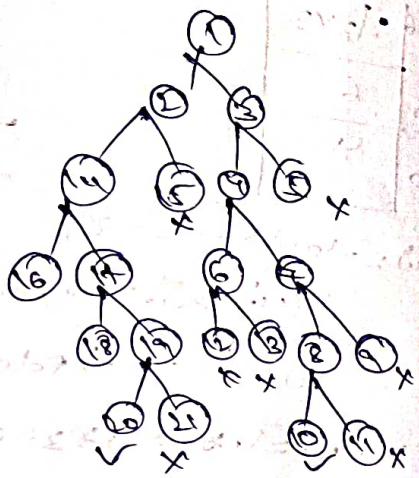
Node 20, $x_1=1, x_2=1, x_3=0, x_4=0, x_5=1$

-4	2
-15	6
-10	4

-4	2
-15	6
-10	4

$\hat{c}(20) = -10-15-4$
$= -29$

$\hat{u}(20) = -10-15-4$
$= -29$



$\hat{c}(20) > \hat{u} \rightarrow$ False $\Rightarrow$ node 20 is extended.

But, no objects are there.

So, it is active node.

$\hat{u}(20) > \hat{u} \rightarrow$ False $\Rightarrow$ $\hat{u}$ is not updated.

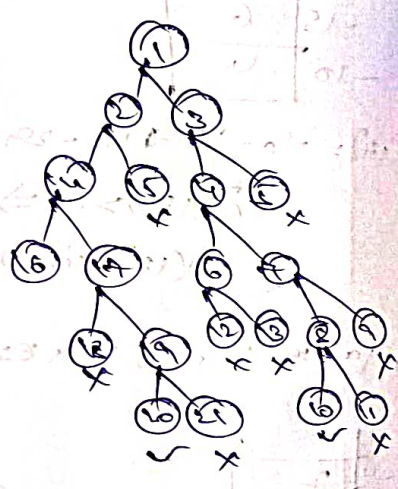
Node 18, $x_1=1, x_2=1, x_3=0, x_4=1$

4
6
4

$4+6+4=14 > 12(m)$.

It is infeasible solution

Kill node 18 ~~is killed~~



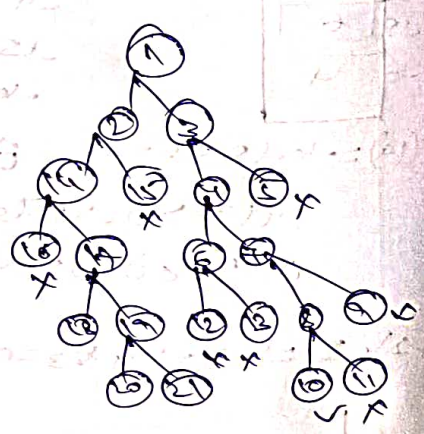
Node 16, $x_1=1, x_2=1, x_3=1$

3
6
4

$4+6+3=13 > 12(m)$

It is infeasible solution

Kill node 16.



Here, all nodes are explored.

There are 2 active nodes.

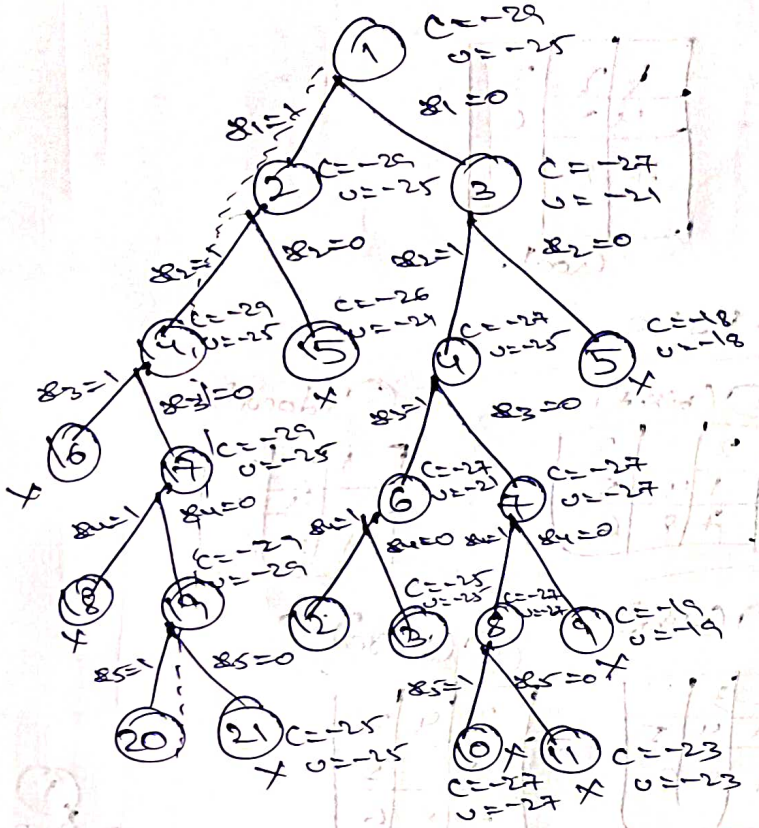
Node 20	Node 10
$\hat{c}(20) = -29$	$\hat{c}(10) = -27$.

As -29 is minimum, node 20 is considered

& node 10 is killed.

— Contd.

Final state Space Tree:



$\therefore x = (1,1,0,0,1)$.

$\therefore$ Maximum Profit $= (10\times1)+(15\times1)+(6\times0)+(3\times0)+(4\times1)$

$= 10+15+0+0+4$

$= 29$.

Maximum weight $= (4\times1)+(6\times1)+(3\times0)+(4\times0)+(2\times1)$

$= 4+6+0+0+2$

$= 12 = M$.

4) Given,

Objective function Maximize $z = 2x_1 - x_2 + 2x_3 + 0s_1 + 0s_2 + 0s_3$ — (1)

Subj. to the constraints

$2x_1 + x_2 + 0x_3 + 1s_1 + 0s_2 + 0s_3 = 10$ — (2)

$1x_1 + 2x_2 - 2x_3 + 0s_1 + 1s_2 + 0s_3 = 20$ — (3)

$0x_1 + 1x_2 + 2x_3 + 0s_1 + 0s_2 + 1s_3 = 5$ — (4)

where, $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.

For finding basic variables,

Put $x_1, x_2, x_3 = 0$ in eq. (2), (3) & (4) then we get, $s_1 = 10, s_2 = 20, s_3 = 5$.

Table-1:

It-1		C_j	2	-1	2	0	0	0	
Basic	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Min ratio (x_B/x_1)
s_1	0	10	2	1	0	1	0	0	$10/2 = 5$
s_2	0	20	1	2	-2	0	1	0	$20/1 = 20$
s_3	0	5	0	1	2	0	0	1	—
z_j	—	—	0	0	0	0	0	0	
$z_j - c_j$	—	—	-2	1	-2	0	0	0	

Incoming vector = -2 (Max. Negative $z_j - c_j$)

Outcoming vector = 5 (Min. +ve & non zero)

Entering → x_1
Departing → s_1
Key Ele → 2

Pivot (key) is 2. (Intersection of I.V & O.V).

$R_1(new) = R_1(old) \div 2$

$R_2(new) = R_2(old) - R_1(new)$

$R_3(new) = R_3(old)$

It-2		C_j	2	-1	2	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	min ratio (X_B/x_3)
x_1	2	5	1	0.5	0	0.5	0	0	—
s_2	0	15	0	1.5	-2	-0.5	1	0	—
s_3	0	5	0	1	2	0	0	1	$\frac{5}{2} = 2.5 \rightarrow$
z_j	—	—	2	1	0	1	0	0	
$z_j - c_j$	—	—	0	2	-2 ↑	1	0	0	

I.V = 2.5, O.V = -2, key = 2, Entering → x_3
Departing → s_3.

$R_3(new) = R_3(old) \div 2$
$R_1(new) = R_1(old)$
$R_2(new) = R_2(old) + 2R_3(new)$

It-3		C_j	2	-1	2	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	min ratio
x_1	2	5	1	0.5	0	0.5	0	0	
s_2	0	20	0	2.5	0	-0.5	1	1	
x_3	2	2.5	0	0.5	1	0	0	0.5	
z_j	—	—	2	2	2	1	0	1	
$z_j - c_j$	—	—	0	3	0	1	0	1	

Since all, $z_j - c_j \geq 0$.

Hence, optimal solution is arrived with value of variables as:

$x_1 = 5$, $x_2 = 0$, $x_3 = 2.5$.

$\therefore$ Max $z = 15$.

5) Given

	2	3
1	4	5
7	8	6

Initial

1	2	3
4	5	6
7	8	

Goal.

BFS search i.e. FIFO.

1 → 2 (right) → 3 (down)

	2	3
1	4	5
7	8	6

2		3
1	4	5
7	8	6

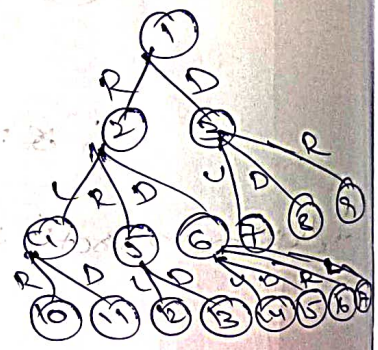
1	2	3
	4	5
7	8	6

2 → 4 (left) → 5 (right) → 6 (down)

	2	3
1	4	5
7	8	6

2	3	
1	4	5
7	8	6

2	4	3
1		5
7	8	6



3 → 7 (up) → 8 (Down) → 9 (right)

	2	3
1	4	5
7	8	6

1	2	3
7	4	5
	8	6

1	2	3
4		5
7	8	6

4 → 10 (right) → 11 (down)

2		3
1	4	5
7	8	6

1	2	3
	4	5
7	8	6

5 → 12 (left) → 13 (down)

2		3
1	4	5
7	8	6

2	3	5
1	4	
7	8	6

6 → 14 (up) → 15 (Down) → 16 (right) → 17 (left)

2		3
1	4	5
7	8	6

2	4	3
1	8	5
7		6

2	4	3
1	5	
7	8	6

2	4	3
	1	5
7	8	6

7 → 18 (Right) → 19 (Down)

2		3
1	4	5
7	8	6

1	2	3
	4	5
7	8	6

2 → 20 (right) → 21 (up)

1	2	3
7	4	5
8		6

1	2	3
	4	5
7	8	6

4 → 22 (up) → 23 (Down) → 24 (right) → 25 (left)

1		3
4	2	5
7	8	6

1	2	3
4	8	5
7		6

1	2	3
4	5	
7	8	6

1	2	3
	4	5
7	8	6

6 → 26 (right) → 27 (left) → 28 (down)

2	3	
1	4	5
7	8	6

	2	3
1	4	5
7	8	6

2	4	3
1		5
7	8	6

11 → 29 (up) → 30 (Down) → 31 (right)

	2	3
1	4	5
7	8	6

1	2	3
7	4	5
	8	6

1	2	3
4	1	5
7	8	6

12 → 32 (right) → 33 (left) → 34 (Down)

2	3	
1	4	5
7	8	6

	2	3
1	4	5
7	8	6

2	4	3
1		5
7	8	6

13 → 35 (up) → 36 (Down) → 37 (left)

2	3	
1	4	5
7	8	6

2	3	5
1	4	6
7	8	

2	3	5
1		4
7	8	6

(4) → (38) (right) → (39) (left) → (40) (down)

2	3	
1	4	5
7	8	6

	2	3
1	4	5
7	8	6

2	4	3
1		5
7	8	6

(5) → (41) (right) → (42) (left) → (43) (up)

2	4	3
1	8	5
7	6	

2	4	3
1	8	5
	7	6

2	4	3
1		5
7	8	6

(6) → (44) (up) → (45) (Down) → (46) (left)

2	4	
1	5	3
7	8	6

2	4	3
1	5	6
7	8	

2	4	3
1		5
7	8	6

(7) → (47) (up) → (48) (down) → (49) (right)

	4	3
2	1	5
7	8	6

2	4	3
7	1	5
	8	6

2	4	3
1		5
7	8	6

(8) → (50) (right) → (51) (left) → (52) (Down)

2	3	
1	4	5
7	8	6

	2	3
1	4	5
7	8	6

2	4	3
1		5
7	8	6

(9) → (53) (up) → (54) (Down) → (55) (right)

	2	3
1	4	5
7	8	6

1	2	3
7	4	5
	8	6

1	2	3
4		5
7	8	6

(55) → (56) (right) → (57) (left) → (58) (up)

1	2	3
7	4	5
8	6	

1	2	3
7	4	5
	8	6

1	2	3
7		5
8	4	6

(21) → (59) (up) → (60) (Down) → (61) (Right)

	2	3
1	4	5
7	8	6

1	2	3
7	4	5
	8	6

1	2	3
4		5
7	8	6

(22) → (62) (right) → (63) (left) → (64) (Down)

1	2	
4	2	5
7	8	6

1	1	3
4	2	5
7	8	6

1	2	3
4		5
7	8	6

(23) → (65) (right) → (66) (left) → (67) (up)

1	2	3
4	8	5
7	6	

1	2	3
4	8	5
	7	6

1	2	3
4		5
7	8	6

(24) → (68) (up) → (69) (left) → (70) (Down)

1	2	
4	5	3
7	8	6

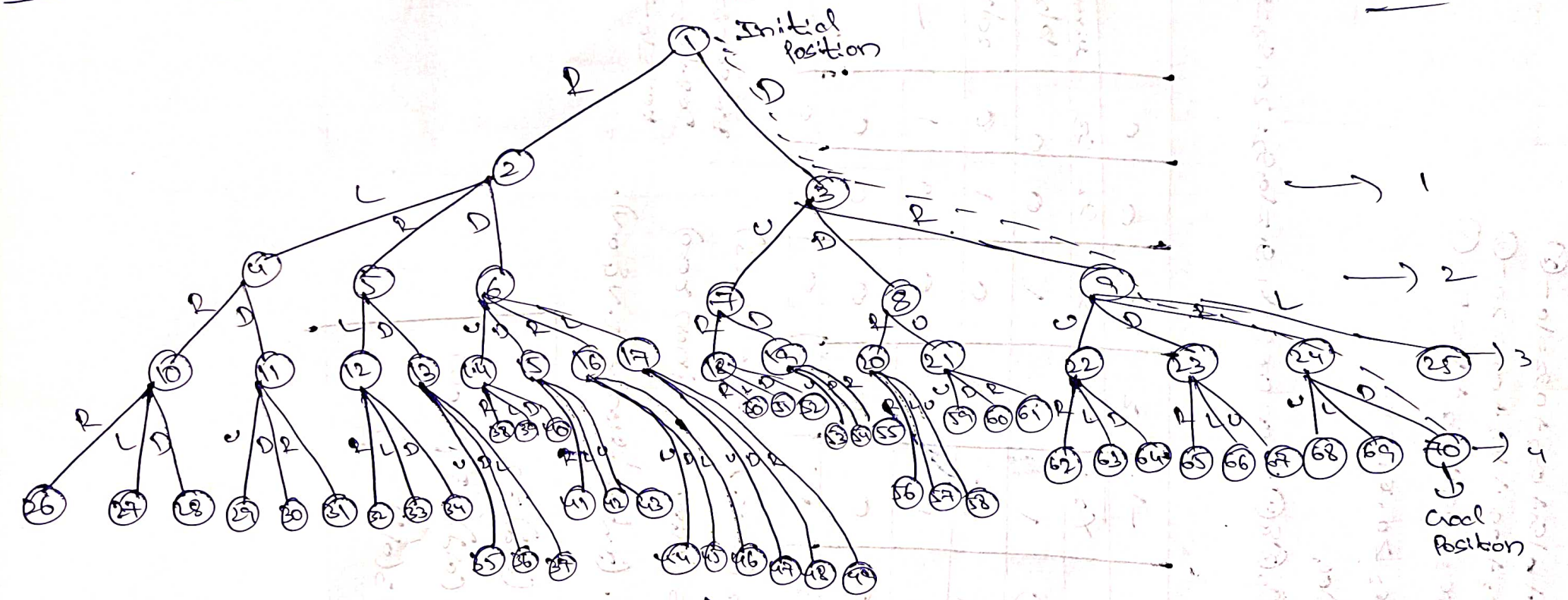
1	2	3
4		5
7	8	6

1	2	3
4	5	6
7	8	

Goal

Contd.

Final State Space Tree,



So, Moves are: 1) Down
2) Right
3) Right
4) Down.

∴ No. of states to reach goal position = 4.