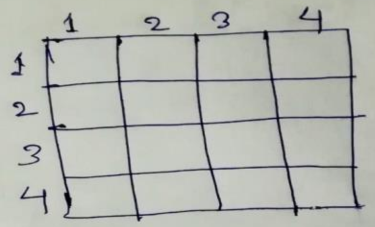
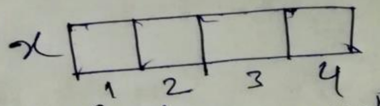
N-Queens Problem

N-Queens Problem

Q_1 Q_2 Q_3 Q_4





(column numbers)

An Instance of N-Queen problem is 4-Queens problem

- Here, 4-Queens will be given and it will move according to a given restrictions as followed in the game of chess.
- So, Queen can move horizontally, vertically, and diagonally.
- We have to place these 4-Queens in such a way that no two queens are under attack
- Queens are under attack if they are in
  1. same row
  2. same column
  3. same diagnol

- The question is: Is it possible to arrange them (i.e queens) such they are not under attack? (Yes it is possible)

- How many solutions are there?

- We want to know all those arrangements that satisfies these conditions.

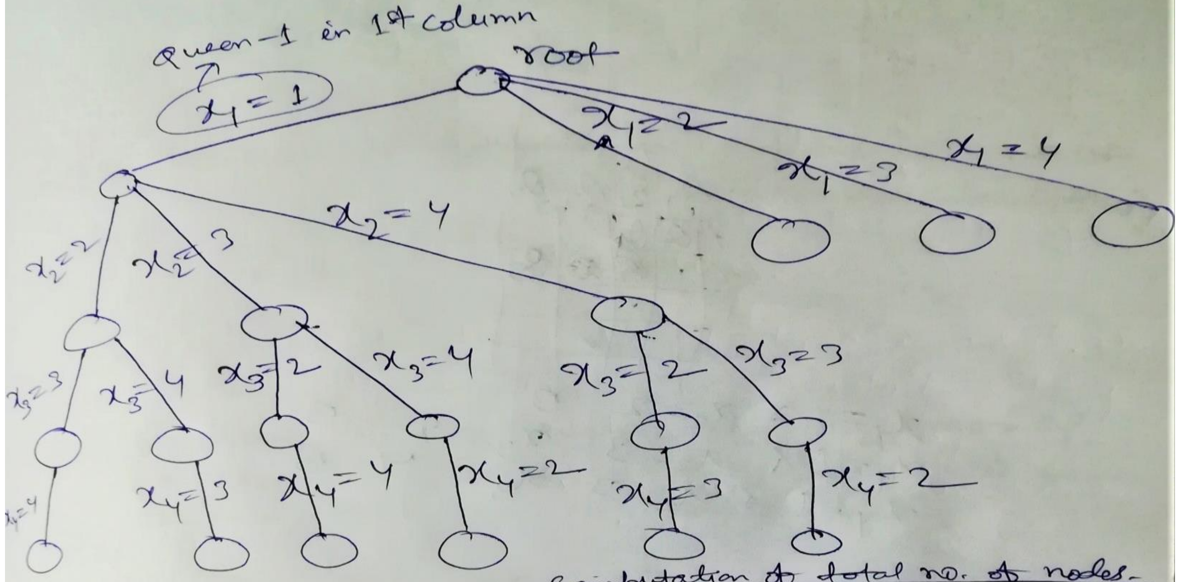
- For these problems backtracking is used. We want all those arrangements which satisfy the conditions (Not in same row, Not in same column, and Not in same diagonal)

- We not just interested in one solution (or optimal solution). Optimal solution can be found using dynamic programming.

- In how many ways one can place 4-queens in 16 cells. (Ans: $^{16}C_4$)

- To reduce the complexity, we suppose 1st queen can be placed in 1st row, 2nd queen can be placed in 2nd row, 3rd queen in 3rd row and 4th queen in 4th row.
- So, now we have to decide in which column queens can be placed.

Generation of State-Space tree



Computation of total no. of nodes-

$$\underbrace{1}_{\text{level-0}} + \underbrace{4}_{\text{level-1}} + \underbrace{4 \times 3}_{\substack{12 \\ \text{level-2}}} + \underbrace{4 \times 3 \times 2}_{\substack{24 \\ \text{level-3}}} + \underbrace{4 \times 3 \times 2 \times 1}_{\substack{24 \\ \text{level-4}}}$$

$$= 1 + \sum_{i=0}^{3} \left[\prod_{j=0}^{i} (4-j) \right] = \underline{65}$$

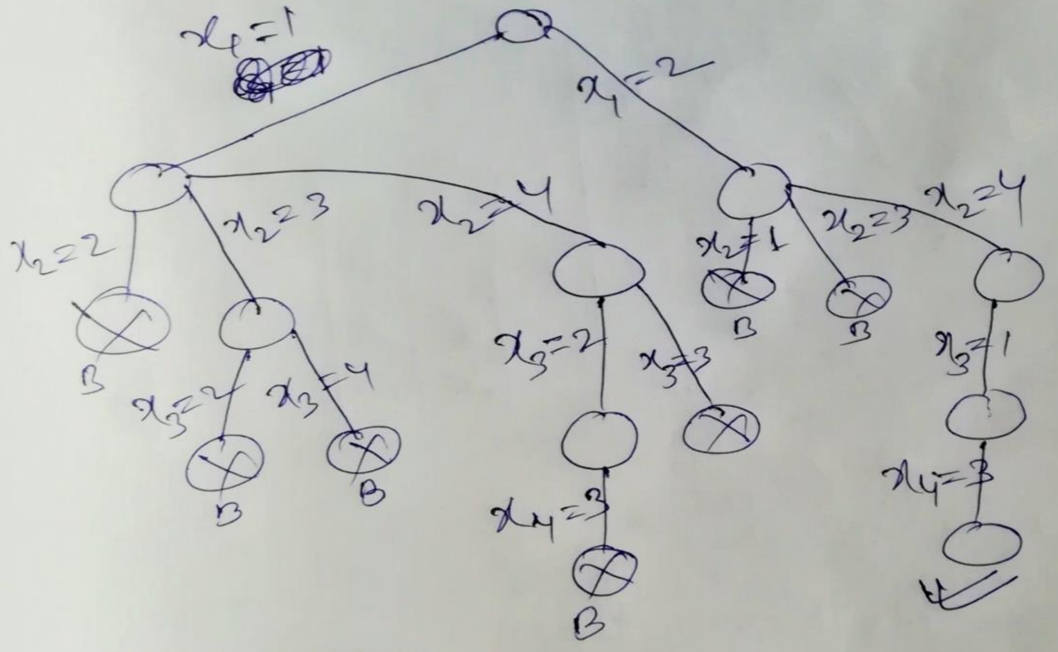
	1	2	3	4
1	~~Q_1~~	Q_1		
2		~~Q_2~~	~~Q_2~~	Q_2
3		Q_3	~~Q_3~~	~~Q_3~~
4			Q_4	~~Q_4~~

4^{th} queen can not be moved further.
So move third queen in 4^{th} column & fourth queen(Q_4) in third column.
Now, the Q_3 can't be move further. So, move Q_2 and accordingly Q_3 & Q_4 and so on (i.e move Q_1 and accordingly move Q_2, Q_3 & Q_4)

Generalized formula for any no. of queens.
(for the number of nodes calculations)

$$= 1 + \sum_{i=0}^{N-1} \left[\prod_{j=0}^{i} (N-j) \right]$$

Redraw the tree by applying the bounding function (i.e Not in same row, Not in same column, and Not in same diagonal).



x				
	2	4	1	3

	1	2	3	4
1		Q_1		
2				Q_2
3	Q_3			
4			Q_4	

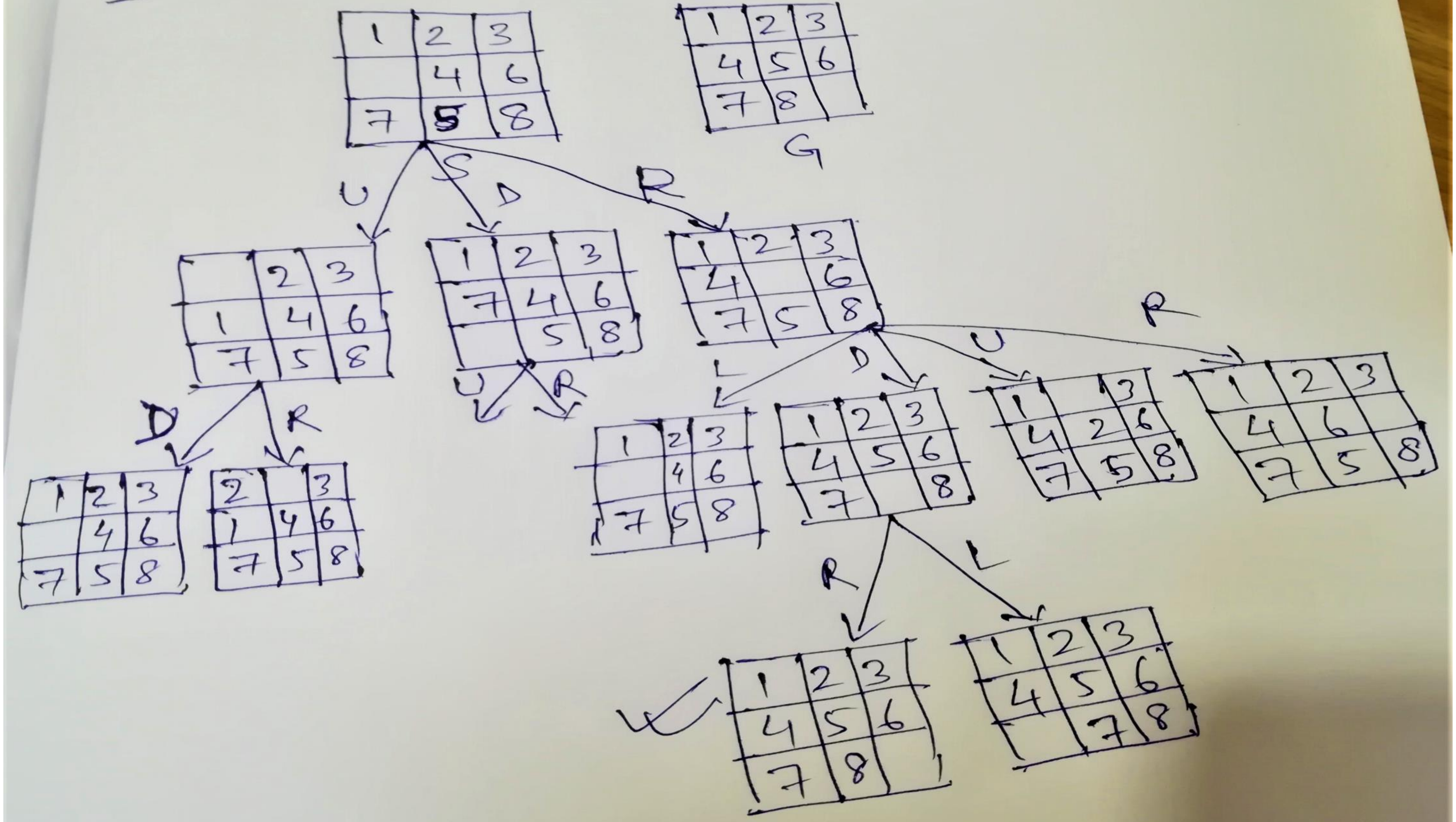
one of the solution

2, 4, 1, 3

	1	2	3	4
1			Q_1	
2	Q_2			
3				Q_3
4		Q_4		

3, 1, 4, 2

8-puzzle problem (Agent based Approach)



Brute force search | Blind Search (Uninformed)

Breadth first search.

$O(b^d)$

b : branching factor

d : depth.

How do we calculate average branching factor

X	$	X
$	O	$
X	$	X

$$\frac{4\times2 + 4\times3 + 4\times1}{9}$$

$$= \frac{24}{9} = \lceil 2.66 \rceil \simeq 3$$