

Backpropagation

$$\frac{\partial E_{total}}{\partial w_5}, \frac{\partial E_{total}}{\partial w_6}, \frac{\partial E_{total}}{\partial w_7}, \frac{\partial E_{total}}{\partial w_8}$$

$$E_{total} = E_{o1} + E_{o2} = \text{cost function}$$

cell here

$$E_{o1} = \frac{1}{2} (\text{target}_{o1} - \text{out}_{o1})^2$$

$$\text{out}_{o1} = \frac{1}{1 + e^{-\text{net}_{o1}}}$$

$$\text{net}_{o1} = w_5 * \text{out}_{h1} + w_6 * \text{out}_{h2} + b_2 * 1$$

$$\therefore \frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial \text{out}_{o1}} * \frac{\partial \text{out}_{o1}}{\partial \text{net}_{o1}} * \frac{\partial \text{net}_{o1}}{\partial w_5}$$

again,

$$\frac{\partial E_{total}}{\partial \text{out}_{o1}} * \frac{\partial \text{out}_{o1}}{\partial \text{net}_{o1}} = \delta_{o1}$$

Now, we will find out the partial derivatives of

$$\therefore E_{total} = \frac{1}{2} (\text{target}_{o1} - \text{out}_{o1})^2 + \frac{1}{2} (\text{target}_{o2} - \text{out}_{o2})^2$$

$$\begin{aligned} \left[\frac{\partial E_{total}}{\partial \text{out}_{o1}} \right] &= 2 * \frac{1}{2} (\text{target}_{o1} - \text{out}_{o1}) * (-1) + 0 \\ &= -(\text{target}_{o1} - \text{out}_{o1}) \\ &= -(.01 - .75136) \\ &= \boxed{.7413} \end{aligned}$$

again,

$$\begin{aligned} \text{out}_{o1} &= \frac{1}{1 + e^{-\text{net}_{o1}}} = \text{out}_{o1} (1 - \text{out}_{o1}) = \frac{\partial \text{out}_{o1}}{\partial \text{net}_{o1}} \\ &= .75136 * (1 - .75136) \\ &= .1868 \end{aligned}$$

Now, $\text{net}_{o1} = w_5 * \text{out}_{h1} + w_6 * \text{out}_{h2} + b_2 * 1$

$$\frac{\partial \text{net}_{o1}}{\partial w_5} = \text{out}_{h1} = .59326$$

$$\therefore \frac{\partial E_{total}}{\partial w_5} = .7413 * .1868 * .59326 = .0821$$

$$\therefore w_5^+ = w_5 - \eta * \frac{\partial E_{total}}{\partial w_5} = 0.4 - 0.5 * 0.082167 = .35891$$

w_6 :

$$\frac{\partial E_{total}}{\partial w_6} = \frac{\partial E_{total}}{\partial out_1} * \frac{\partial out_1}{\partial net_1} * \frac{\partial net_1}{\partial w_6}$$

where,

$$\frac{\partial E_{total}}{\partial out_1} * \frac{\partial out_1}{\partial net_1} = \delta_1$$

$$\textcircled{1} E_{total} = \frac{1}{2} E_{o1} + E_{o2} \text{ where } E_{o1} = \frac{1}{2} (\text{target}_{o1} - out_{o1})^2$$

$$\textcircled{2} out_{o1} = \frac{1}{1 + e^{-net_{o1}}}$$

$$\textcircled{3} net_{o1} = w_5 * out_{h1} + \boxed{w_6 * out_{h2}} + b * 1$$

$$\therefore \frac{\partial E_{total}}{\partial out_{o1}} = 0.74136507 * 0.1868 * \boxed{\frac{\partial net_{o1}}{\partial w_6}}$$

$$\therefore \frac{\partial net_{o1}}{\partial w_6} = w_5 * out_{h1} + w_6 * out_{h2} + b * 1$$

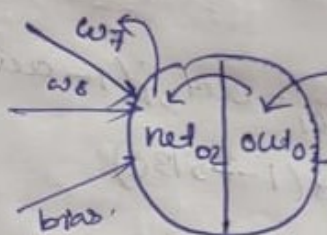
$$= out_{h2} = 0.59688$$

$$\therefore w_6^+ = w_6 - \eta \frac{\partial E_{total}}{\partial w_6}$$

$$= 0.45 - 0.5 * [0.74136507 * 0.1868 * 0.59688]$$

$$= 0.40866618$$

w_7 :



$$E_{o2} = \frac{1}{2} (\text{target}_{o2} - out_{o2})^2$$

$$E_{total} = E_{o1} + E_{o2}$$

$$\therefore \frac{\partial E_{total}}{\partial w_7} = \frac{\partial E_{total}}{\partial out_{o2}} * \frac{\partial out_{o2}}{\partial net_{o2}} * \frac{\partial net_{o2}}{\partial w_7}$$

where,

$$\frac{\partial E_{total}}{\partial out_{o2}} * \frac{\partial out_{o2}}{\partial net_{o2}} = \delta_{o2}$$

$$\textcircled{1} \frac{\partial E_{total}}{\partial out_{o2}} = 0 + 2 * \frac{1}{2} (\text{target}_{o2} - out_{o2}) * -1$$

$$= - (\text{target}_{o2} - out_{o2})$$

$$= - (0.99 - 0.772928) = -0.2170$$

$$\text{out}_2 = \frac{1}{1 + e^{-\text{net}_{o2}}}$$

$$\begin{aligned} \therefore \frac{\partial \text{out}_2}{\partial \text{net}_{o2}} &= \text{out}_{o2} (1 - \text{out}_{o2}) \\ &= 0.772928 (1 - 0.772928) \\ &= 0.1755 \end{aligned}$$

$$\begin{aligned} 3) \frac{\partial \text{net}_{o2}}{\partial w_7} &= w_7 + \text{out}_{h1} + w_8 + \text{out}_{h2} + \frac{1}{b} + 60 \\ &= \text{out}_{h1} = 0.59326 \end{aligned}$$

$$\begin{aligned} \frac{\partial E_{\text{total}}}{\partial w_7} &= -0.2170 * 0.1755 * 0.59326 \\ &= -0.022593 \end{aligned}$$

$$\begin{aligned} \therefore w_7^+ &= w_7 - \eta \frac{\partial E_{\text{total}}}{\partial w_7} \\ &= 0.50 - 0.5 * (-0.022593) \\ &= 0.5112965 \end{aligned}$$

$$\frac{\partial E_{\text{total}}}{\partial w_8} = \frac{\partial E_{\text{total}}}{\partial \text{out}_{o2}} * \frac{\partial \text{out}_{o2}}{\partial \text{net}_{o2}} * \frac{\partial \text{net}_{o2}}{\partial w_8}$$

① $\frac{\partial E_{\text{total}}}{\partial \text{out}_{o2}}$ and $\frac{\partial \text{out}_{o2}}{\partial \text{net}_{o2}}$ will be same as before

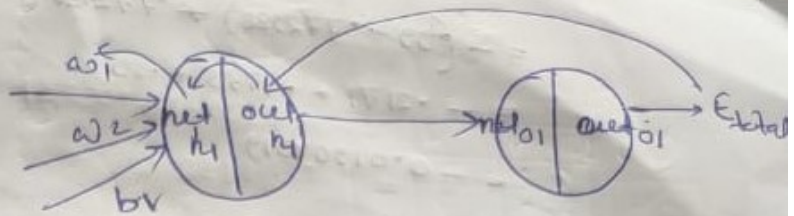
$$\begin{aligned} \text{But } \frac{\partial \text{net}_{o2}}{\partial w_8} &= w_7 + \text{out}_{h1} + w_8 + \text{out}_{h2} + b + 60 \\ &= \text{out}_{h2} \\ &= 0.59688 \end{aligned}$$

$$\begin{aligned} \therefore \frac{\partial E_{\text{total}}}{\partial w_8} &= -0.2170 * 0.1755 * 0.59688 \\ &= -0.022731279 \end{aligned}$$

$$\begin{aligned} \therefore w_8^+ &= w_8 - \eta \frac{\partial E_{\text{total}}}{\partial w_8} = 0.55 - 0.5 * (-0.0227312) \\ &= 0.55 + 0.0113656 \\ &= 0.5613656 \end{aligned}$$

$w_2, w_3 \text{ \& } w_9$

Backpropagation



$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h1}} * \frac{\partial out_{h1}}{\partial net_{h1}} * \frac{\partial net_{h1}}{\partial w_1}$$

$$\text{But, } \frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial out_{h1}}$$

$$E_{o1} = \frac{1}{2} (target_{o1} - out_{o1})^2$$

$$out_{o1} = \frac{1}{1 + e^{-net_{o1}}}$$

$$net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$$

$$\therefore \frac{\partial E_{o1}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial out_{h1}}$$

$$\begin{aligned} \text{Now, } \frac{\partial E_{o1}}{\partial out_{o1}} &= \left[2 * \frac{1}{2} (target_{o1} - out_{o1}) * -1 \right] \\ &= -(target_{o1} - out_{o1}) \\ &= -(0.01 - 0.75136) \\ &= 0.74136 \end{aligned}$$

$$\frac{\partial out_{o1}}{\partial net_{o1}} = out_{o1} (1 - out_{o1}) = 0.1868$$

$$\frac{\partial net_{o1}}{\partial out_{h1}} = w_5 = 0.40$$

$$\therefore \frac{\partial E_{o1}}{\partial out_{h1}} = 0.7413 * 0.1868 * 0.40 = 0.05538$$

$$\text{Similarly, } \frac{\partial E_{o2}}{\partial out_{h1}} = \frac{\partial E_{o2}}{\partial out_{o2}} * \frac{\partial out_{o2}}{\partial net_{o2}} * \frac{\partial net_{o2}}{\partial out_{h1}}$$

$$E_{o2} = \frac{1}{2} (target_{o2} - out_{o2})^2$$

$$out_{o2} = \frac{1}{1 + e^{-net_{o2}}}$$

$$net_{o2} = w_7 * out_{h1} + w_8 * out_{h2} + 1 * b_2$$

$$\begin{aligned} \therefore \frac{\partial E_{o2}}{\partial out_{h1}} &= \frac{\partial E_{o2}}{\partial out_{o2}} * \frac{\partial out_{o2}}{\partial net_{o2}} * \frac{\partial net_{o2}}{\partial out_{h1}} \\ &= -(target_{o2} - out_{o2}) * out_{o2} (1 - out_{o2}) * w_7 \end{aligned}$$

$$\begin{aligned}
 &= - (0.99 - 0.772928) * 772928 (1 - 0.772928) * 0.50 \\
 &= - 0.2171 * 0.1755 * 0.50 \\
 &= - 0.01904919
 \end{aligned}$$

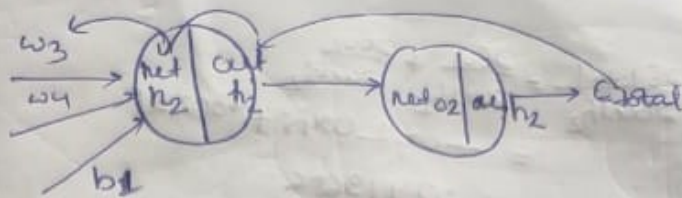
$$\begin{aligned}
 \therefore \frac{2E_{total}}{2\omega_{th}} &= \frac{2E_{o1}}{2\omega_{th}} + \frac{2E_{o2}}{2\omega_{th}} = 0.055399 + (-0.0090419) \\
 &= 0.03635 \rightarrow \omega_{th}(1 - \omega_{th}) \\
 &= 0.2413007
 \end{aligned}$$

$$\begin{aligned}
 \therefore \frac{2E_{total}}{2\omega_1} &= \frac{2E_{total}}{2\omega_{th}} * \frac{2\omega_{th}}{2\omega_{th}} * \frac{2\omega_{th}}{2\omega_1} \\
 &= 0.03635 * \left(\frac{0.1868}{0.1868} \right) * \left[\frac{\omega_1 * i_1 + \omega_3 * i_2 + b_1 * i_1}{2\omega_1} \right] \\
 &= 0.03635 * 0.2413007 * 0.05 \\
 &= 0.0004385
 \end{aligned}$$

$$\begin{aligned}
 \therefore \omega_1^+ &= \omega_1 - \eta * \frac{2E_{total}}{2\omega_1} \\
 &= 0.15 - 0.5 * 0.0004385 \\
 &= 0.149780716
 \end{aligned}$$

$$\begin{aligned}
 \text{Hij } \omega_2 \\
 \frac{2E_{total}}{2\omega_2} &= \frac{2E_{total}}{2\omega_{th}} * \frac{2\omega_{th}}{2\omega_{th}} * \frac{2\omega_{th}}{2\omega_2} \\
 &= 0.03635 * 0.2413007 * i_2 \\
 &= 0.03635 * 0.2413007 * 0.10 \\
 &= 0.000877
 \end{aligned}$$

$$\begin{aligned}
 \therefore \omega_2^+ &= \omega_2 - 0.5 * 0.000877 \\
 &= 0.20 - 0.000438 \\
 &= 0.1995614
 \end{aligned}$$



$$\frac{\partial E_{total}}{\partial w_3} = \frac{\partial E_{total}}{\partial out_{h2}} * \frac{\partial out_{h2}}{\partial net_{h2}} * \frac{\partial net_{h2}}{\partial w_3}$$

$$\text{But, } \frac{\partial E_{total}}{\partial out_{h2}} = \frac{\partial E_{o1}}{\partial out_{h2}} + \frac{\partial E_{o2}}{\partial out_{h2}}$$

$$E_{o2} = \frac{1}{2} (\text{target}_{o2} - \text{out}_{o2})^2$$

$$\text{out}_{o2} = \frac{1}{1 + e^{-net_{o2}}}$$

$$net_{o2} = w_7 * out_{h1} + w_8 * out_{h2} + b_2 * 1$$

$$\therefore \frac{\partial E_{o2}}{\partial out_{h2}} = \frac{\partial E_{o2}}{\partial out_{o2}} * \frac{\partial out_{o2}}{\partial net_{o2}} * \frac{\partial net_{o2}}{\partial out_{h2}}$$

$$\begin{aligned} \therefore \frac{\partial E_{o2}}{\partial out_{o2}} &= \left[2 * \frac{1}{2} (\text{target}_{o2} - \text{out}_{o2}) \right] * -1 \\ &= -(\text{target}_{o2} - \text{out}_{o2}) \\ &= -(0.99 - 0.772928) = -0.2170 \end{aligned}$$

$$\begin{aligned} \frac{\partial out_{o2}}{\partial net_{o2}} &= out_{o2} (1 - out_{o2}) \\ &= 0.772928 (1 - 0.772928) \\ &= 0.1755 \end{aligned}$$

$$\begin{aligned} \frac{\partial net_{o2}}{\partial out_{h2}} &= w_8 \\ &= 0.55 \end{aligned}$$

$$\begin{aligned} \therefore \frac{\partial E_{o2}}{\partial out_{h2}} &= -0.2170 * 0.1755 * 0.55 \\ &= -0.020945 \end{aligned}$$

$$\text{Hly, } E_{o1} = \frac{1}{2} (\text{target}_{o1} - \text{out}_{o1})^2$$

$$\text{out}_{o1} = \frac{1}{1 + e^{-net_{o1}}}$$

$$net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$$

$$\begin{aligned} \therefore \frac{\partial E_{o1}}{\partial out_{h2}} &= \frac{\partial E_{o1}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial out_{h2}} \\ &= -(\text{target}_{o1} - \text{out}_{o1}) * out_{o1} (1 - out_{o1}) * w_6 \\ &= 0.7413 * 0.7868 * 0.45 = 0.06231 \end{aligned}$$

$$1. \frac{\partial E_{total}}{\partial \omega_{th2}} = \frac{\partial E_{o1}}{\partial \omega_{th2}} + \frac{\partial E_{o2}}{\partial \omega_{th2}}$$

$$= -0.0020945 + 0.06231$$

$$= 0.0413686$$

$$\therefore \frac{\partial E_{total}}{\partial \omega_3} = \frac{\partial E_{total}}{\partial \omega_{th2}} * \frac{\partial \omega_{th2}}{\partial \omega_3} * \frac{\partial \omega_3}{\partial \omega_3}$$

$$= 0.0413686 * 1.755 * 0$$

$$= 0.0413686 * \omega_{th2}(1 - \omega_{th2}) * i$$

$$= 0.0413686 * 0.59688(1 - 0.59688) * 0.0$$

$$= 0.000995387 - 0.000497693$$

$$\omega_3^+ = \omega_3 - \eta \frac{\partial E_{total}}{\partial \omega_3}$$

$$= 0.25 - 0.5 * 0.000995387$$

$$= 0.24975$$

$$\underline{\omega_4 = 0.3}$$

$$\frac{\partial E_{total}}{\partial \omega_4} = \frac{\partial E_{total}}{\partial \omega_{th2}} * \frac{\partial \omega_{th2}}{\partial \omega_4} * \frac{\partial \omega_4}{\partial \omega_4}$$

$$= 0.0413686 * 0.240614 * 0.2$$

$$= 0.0413686 * 0.240614 * 0.10$$

$$= 0.000995386$$

$$\therefore \omega_4^+ = \omega_4 - 0.5 * 0.000995386$$

$$= 0.30 - 0.000497693$$

$$= 0.299502307$$