10

Transportation Problem

"Commodities are transported while people travel"

10:1. INTRODUCTION

The **Transportation Problem** is one of the subclasses of L.P.Ps. in which the objective is to *transport* various quantities of a single homogeneous commodity, that are initially stored at various *sources* (*origins*), to different *destinations* in such a way that the total transportation cost is a minimum. To achieve this objective we must know the amount and location of available supplies and the quantities demanded. In addition, we must know the costs that result from transporting one unit of commodity from various origins to various destinations.

10:2. LP FORMULATION OF THE TRANSPORTATION PROBLEM

Consider the following transportation problem as an L.P.P. :

A company manufacturing air-coolers has two plants located at Hyderabad and Mumbai with a capacity of 300 units and 100 units per week respectively. The company supplies the air-coolers to its four showrooms situated at Bangalore, Chennai, Delhi and Ernakulam which have a maximum demand of 85, 150, 150 and 55 units respectively. Due to the differences in raw material cost and transportation cost, the *profit* per unit in rupees differs which is shown in the table below :

City	*Bangalore*	*Chennai*	*Delhi*	*Ernakulam*
Hyderabad	110	90	75	55
Mumbai	65	50	80	45

Plan the production programme so as to *maximize* the *profit*. The company may have its production capacity at both plants partly or wholly unused.

Formulate the above problem as an L.P.P.

Mathematical Formulation

Let x_{ij} represent the quantity of air-coolers to be transported from Hyderabad and Mumbai to Bangalore, Chennai, Delhi and Ernakulam respectively. The objective function is :

$$\text{Maximize} \quad z = 110x_{11} + 90x_{12} + 75x_{13} + 55x_{14} + 65x_{21} + 50x_{22} + 80x_{23} + 45x_{24}$$

The constraints are :

(*a*) $x_{11} + x_{12} + x_{13} + x_{14} = 300, \; x_{21} + x_{22} + x_{23} + x_{24} = 100$;

(*b*) $x_{11} + x_{21} = 85, \; x_{12} + x_{22} = 150, \; x_{13} + x_{23} = 150, \; x_{14} + x_{24} = 55$;

(*c*) $x_{ij} \geq 0$; $i = 1, 2$ and $j = 1, 2, 3, 4$.

In the above L.P.P. there are $m \times n = 2 \times 4 = 8$ decision variables, x_{ij} and $m + n = 6$ constraints, where m denotes the number of rows and n the number of columns.

In general, let there be m sources of supply, $S_i\,(i = 1, 2, \ldots, m)$ having $a_i\,(i = 1, 2, \ldots, m)$ units of supply to be transported among n destinations $D_j\,(j = 1, 2, \ldots, n)$ with $b_j\,(j = 1, 2, \ldots, n)$ units of demand. Also let c_{ij} = cost of transporting one unit of commodity from source i to destination j, and x_{ij} = quantity transported from source i to destination j.

Then, the problem is to determine the transportation schedule so as to minimize the total transportation cost satisfying supply and demand constraints.

Mathematically, the problem may be stated as a linear programming problem as follows :

$$\text{Minimize} \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij} \quad \text{subject to the constraints :}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \ldots, m \qquad \textit{(Supply constraint)}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \ldots, n \qquad \textit{(Demand constraint)}$$

and

$$x_{ij} \geq 0 \quad \text{for all } i \text{ and } j$$

Remark. Since all $x_{ij} \geq 0$, it follows that $a_i \geq 0$ and $b_j \geq 0$. Moreover, any $a_k = 0$ implies that $x_{kj} = 0$ and thus can be eliminated from the problem. Therefore by requiring $a_i > 0$ and $b_j > 0$, we are not restricting anything.

10:3. EXISTENCE OF SOLUTION IN T.P.

The general transportation problem always have a solution. In this section, we shall discuss the theorems on the existence of feasible solution, optimum solution and basic feasible solution.

Theorem 10-1 **(Existence of Feasible Solution).** *A necessary and sufficient condition for the existence of a feasible solution to the general transportation problem is that*

$$\textit{Total Supply} = \textit{Total Demand}, \textit{ i.e.,} \quad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \lambda \text{ (say).}$$

Proof. *The condition is necessary.* Let there exist a feasible solution to the T.P. Then, we have

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} a_i \quad \text{and} \quad \sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} b_j$$

yielding

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \lambda \text{ (say)}$$

Sufficiency. We assert that there exists a feasible solution given by $x_{ij} = a_i b_j / \lambda$ for all i and j. Clearly, $x_{ij} \geq 0$ since $a_i > 0$, $b_j > 0$ for all i and j.

Also,

$$\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} (a_i b_j / \lambda) = \frac{a_i}{\lambda} \sum_{j=1}^{n} b_j = a_i, \qquad i = 1, 2, \ldots, m$$

and

$$\sum_{i=1}^{m} x_{ij} = \sum_{i=1}^{m} (a_i b_j / \lambda) = \frac{b_j}{\lambda} \sum_{i=1}^{m} a_i = b_j, \qquad j = 1, 2, \ldots, n$$

Thus, x_{ij} satisfies all the constraints of the T.P. and hence is a feasible solution.

Corollary. **(Existence of an Optimum Solution).** *There always exists an optimum solution to a T.P.*

Proof. Let $\sum_i a_i = \sum_j b_j$, so that a feasible solution x_{ij} exists. It follows from the constraints of the problem that each x_{ij} is bounded, *viz.*,

$$0 \leq x_{ij} \leq \min.\ (a_i, b_j)$$

Thus the feasible region of the problem is closed, bounded and non-empty and hence there exists an optimum solution.

Theorem 10-2 **(Basic Feasible Solution)**. *The number of basic (decision) variables of the general transportation problem at any stage of feasible solution must be $m+n-1$.*

Proof. Consider the $m+n$ constraints of the transportation problem :

$$\sum_{i=1}^{m} x_{ij} = b_j \quad \text{and} \quad \sum_{j=1}^{n} x_{ij} = a_i \qquad (j = 1, 2, \ldots, n;\ i = 1, 2, \ldots, m)$$

Taking summation on both sides, these yield

$$\sum_{j=1}^{n-1} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n-1} b_j \quad \text{and} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} a_i$$

Subtracting the two, we have

$$\sum_{j=1}^{n-1} \sum_{i=1}^{m} x_{ij} - \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n-1} b_j - \sum_{i=1}^{m} a_i$$

or

$$\sum_{i=1}^{m} \left(\sum_{j=1}^{n-1} x_{ij} - \sum_{j=1}^{n} x_{ij} \right) = \sum_{j=1}^{n-1} b_j - \sum_{j=1}^{n} b_j \qquad \left(\because \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \right)$$

or

$$\sum_{i=1}^{m} x_{in} = b_n$$

which happens to be the last constraint of the problem.

This indicates that if the first $m+n-1$ constraints are satisfied, then $\sum_i a_i = \sum_j b_j$, ensures that the $(m+n)$th constraint will be automatically satisfied.

Thus, out of $m+n$ equations, we have only $(m+n-1)$ linearly independent equations. Therefore, a basic feasible solution will consist of at most $(m+n-1)$ positive variables, the rest being zero. Further a feasible solution involving exactly $(m+n-1)$ positive variables is known as **non-degenerate** basic feasible solution, otherwise it is said to be degenerate basic feasible.

Remarks 1. When the total demand is equal to total supply, the transportation problem is said to be *balanced* and otherwise *unbalanced.*

2. The allocated cells in the transportation table will be called *occupied* cells and empty cells will be called *non-occupied* cells.

10:4. DUALITY IN TRANSPORTATION PROBLEM

Since a transportation problem is itself a linear programming problem, there always exist another linear programming problem based upon the same data and having the same solution. Let the original transportation problem be the *primal problem* and its associated one be its *dual problem.*

For the general transportation problem, the primal-dual pair is :

Primal Problem. Consider the general transportation problem as

$$\text{Minimize} \quad z = \sum_i \sum_j x_{ij} c_{ij} \quad \text{subject to the constraints :}$$

$$\sum_i x_{ij} = a_i, \quad \sum_j x_{ij} = b_j; \quad x_{ij} \geq 0 \qquad \text{for all } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n$$

Dual Problem. Since the primal problem is in its standard form, dual problem will be as follows :

$$\text{Maximize} \quad z^* = \sum_i u_i a_i + \sum_j v_j b_j \quad \text{subject to the constraints :}$$

$$u_i + v_j \leq c_{ij} \qquad \text{for all } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n$$

$$u_i \text{ and } v_j \text{ unrestricted in sign for all } i, j.$$

where $u_i\,(i = 1, 2, \ldots, m)$ and $v_j\,(j = 1, 2, \ldots, n)$ are dual variables.

The dual solution of the transportation problem implicitly measures the comparative locational advantage of the sites of factories (source of capacities) and the delivered price of the product.

10:5. THE TRANSPORTATION TABLE

Since the T.P. is just a special case of general L.P.P., the application of simplex method would, no doubt, give an optimum solution to the problem. However, fortunately, *simplex like method* for solving such problems has been developed. Whenever it is possible to place the given L.P.P. in the transportation framework, it is far simpler to solve it by *Transportation Method* than by the simplex method. To facilitate presentation and solution, the transportation problem is generally portrayed in a tabular form as shown below :

		Destination				
		1	2	...	n	Supply
Origin	1	x_{11} c_{11}	x_{12} c_{12}	...	x_{1n} c_{1n}	a_1
	2	x_{21} c_{21}	x_{22} c_{22}	...	x_{2n} c_{2n}	a_2
	...	...	...	...	...	...
	m	x_{m1} c_{m1}	x_{m2} c_{m2}	...	x_{mn} c_{mn}	a_m
	Demand	b_1	b_2	...	b_n	

Table 10.1

The mn large squares are called the **cells.** The per unit cost c_{ij} of transporting from the ith origin O_i to the jth destination D_j is displayed in the lower right position of the (i, j)th cell. Any feasible solution to the T.P. is displayed in the table by variable x_{ij} at the upper left position of the (i, j)th cell. The various origin capacities and destination requirements are listed in the right most (outer) column and the bottom (outer) row respectively. These are called rim requirements.

Remark. In order to avoid difficulty to discriminate between degenerate and non-degenerate basic solutions, we shall adopt the convention that while displaying a basic feasible solution in a transportation table, the value of any particular x_{ij} is actually entered if, and only if, x_{ij} is a basic variable.

10:6. LOOPS IN TRANSPORTATION TABLES

***Definition* (Loop).** *In a transportation table, an ordered set of four or more cells is said to form a LOOP if (i) any two adjacent cells in the ordered set lie either in the same row or in the same column, and (ii) any three or more adjacent cells in the ordered set do not lie in the same row or the same column. The first cell of the set is considered to follow the last in the set, i.e., each cell (except the last) must appear only once in the ordered set.*

Consider the following two cases represented in Table 10.2(a) and 10.2(b). In Table 10.2(a) if we join the positive allocations by horizontal and vertical lines, then a closed loop is obtained. Here, the ordered set of cells forming a loop is

$$L = \{ (3, 2), (2, 2), (2, 3), (1, 3), (1, 5), (3, 5) \}$$

The loop in Table 10.2(*b*) is not allowed, because it does not satisfy the conditions of a loop.

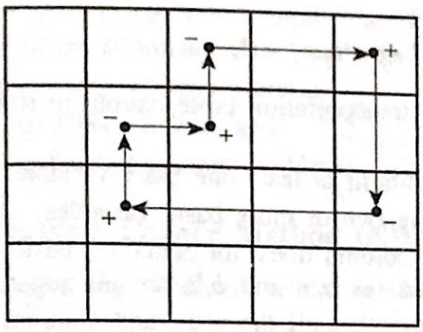

Table 10.2(*a*)

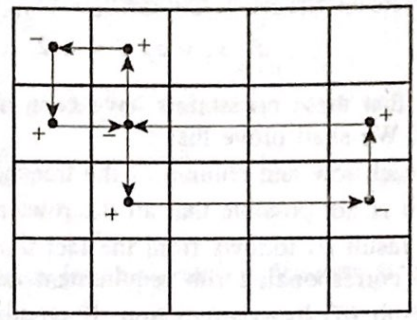

Table 10.2(*b*)

Remarks. *1.* Every loop has an even number of cells.

2. The allocations are said to be in independent positions if it is not possible to increase or decrease any independent individual allocation without changing the positions of these allocations, or violating the rim conditions. That is, a closed loop cannot be formed through these allocations.

3. Each row and column in the transportation table should have only one plus and minus sign. All cells that have a plus or minus sign, except the starting unoccupied (non-basic) cell are occupied (basic) cells.

4. Closed loops may or may not be square in shape.

Definition 2 **(Set containing a loop)**. *A set X of cells of a transportation table is said to contain a Loop if the cells of X or of a subset of X can be ordered (sequenced) so as to form a loop.*

Theorem 10-3 **(Linear Dependence and Loops)**. *Let X be a set of column vectors of the coefficient matrix of a T.P. Then, a necessary and sufficient condition for vectors in X to be linearly dependent is that the set of their corresponding cells in the transportation table contains a loop.*

Proof. Exercise for the reader.

Corollary. *A feasible solution to a T.P. is basic if, and only if, the corresponding cells in the transportation table do not contain a loop.*

Proof. Exercise for the reader.

Remark. The above corollary provides us a method to check graphically whether the current feasible solution to the T.P. is basic or not.

10:7. TRIANGULAR BASIS IN A T.P.

The reader may recall that a system of n linear equations $\mathbf{Ax} = \mathbf{b}$ is called a *triangular system* if the matrix $\mathbf{A}$ is triangular, *i.e.*, it is of the form

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & a_{22} & a_{23} & \dots & a_{2n} \\ 0 & 0 & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & a_{nn} \end{pmatrix}, \quad a_{ii} \neq 0 \quad \text{for each } i.$$

The system has a property that there is at least one equation having only one variable. This variable can be eliminated from the remaining equations, resulting in a reduced system of $(n-1)$ equations, which again happens to be triangular. Thus, a triangular system can be easily 'solved by back substitution'.

Definition **(Triangular basis)**. *A basis for the system* $\mathbf{Ax} = \mathbf{b}$ *is said to be a Triangular Basis if a triangular system is obtained when all the non-basic variables are set equal to zero in the system.*

Theorem 10-4. *All the basis for a transportation problem are triangular.*

Proof. Let the set of constraints of a T.P. be :

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1, 2, \ldots, m; \quad \text{and} \quad \sum_{i=1}^{m} x_{ij} = b_j,\ j = 1, \ldots, n.$$

Suppose that these constraints have been displayed in the transportation table having m rows and n columns. We shall prove that :

(*i*) each row and column of the transportation table contains at least one basic variable, and

(*ii*) it is not possible that all the rows and columns have two or more basic variables.

The result (*i*) follows from the fact that if any row or column does not contain a basic variable, then the corresponding rim requirement cannot be satisfied (as a_i's and b_j's are all non-zero). We prove result (*ii*) by contradiction. If possible, let us suppose that all the rows and columns have at least two basic variables.

Then since there are $m+n$ equations, the total number of basic variables will be at least $2m+2n$. Therefore, if N denotes the total number of basic variables, we must have $N \geq 2m$ and $N \geq 2n$.

Three cases now arise :

$$\text{If } m > n, \text{ then } N \geq 2m \Rightarrow N \geq m+n$$
$$\text{If } n > m, \text{ then } N \geq 2n \Rightarrow N > n+m$$
$$\text{If } m = n, \text{ then } N \geq 2m \Rightarrow N = n+m.$$

Thus, we observe that in every case $N \geq m+n$. But, as we already know, $N = m+n-1$, which is a contradiction. Thus, our assumption about the existence of at least two basic variables in each constraint equation is wrong. Combining (*i*) and (*ii*) we, therefore, conclude that there is at least one constraint equation that contains exactly one basic variable.

We can eliminate this basic variable from all the other constraints. This results in a reduced transportation table, with one row or column deleted and rim requirements readjusted. Repeating the above argument for this reduced transportation table we conclude that there is at least one row (or column) that contains exactly one basic variable in the reduced system. Eliminating this basic variable from the system and repeating the above argument for the new reduced system we see that the result again holds for this reduced system.

Repeating the above argument again successively, we see that the system is completely solved.

10:8. SOLUTION OF A TRANSPORTATION PROBLEM

The solution to a transportation problem involves the following major steps :

Step 1. Formulate the given problem as a linear programming problem.

Step 2. Set up the given L.P.P. in the tabular form known as a transportation table.

Step 3. Examine whether total supply equals total demand. If not, introduce a dummy row/column having all its cost elements as zero and Supply/Demand as the (+ve) difference of Supply and Demand.

Step 4. Find an initial basic feasible solution that must satisfy all the supply and demand conditions.

Step 5. Examine the solution obtained in *step 4* for optimality, *i.e.*, examine whether an improved transportation schedule with lower cost is possible.

Step 6. If the solution is not optimum, modify the shipping schedule by including that unoccupied cell whose inclusion may result in an improved solution.

Step 7. Repeat *step 4* until no further improvement is possible.

We shall now discuss various methods available for finding an initial basic feasible solution and then attaining an optimum solution.

10:9. FINDING AN INITIAL BASIC FEASIBLE SOLUTION

There are several methods available to obtain an initial basic feasible solution. However, we shall discuss here the following three methods :

1. North-West Corner Method,
2. Least-Cost Method, and
3. Vogel's Approximation Method (or Penalty Method).

1. North-West Corner Method (*NWC Rule*)

It is a simple and efficient method to obtain an initial basic feasible solution. Various steps of the method are :

Step 1. Select the north-west (upper left hand) corner cell of the transportation table and allocate as much as possible so that either the capacity of the first row is exhausted or the destination requirement of the first column is satisfied, *i.e.*, $x_{11} = \text{min.}\ (a_1, b_1)$.

Step 2. If $b_1 > a_1$, we move down vertically to the second row and make the second allocation of magnitude $x_{21} = \text{min.}\ (a_2, b_1 - x_{11})$ in the cell (2, 1).

If $b_1 < a_1$, we move right horizontally to the second column and make the second allocation of magnitude $x_{12} = \text{min.}\ (a_1 - x_{11}, b_2)$ in the cell (1, 2).

If $b_1 = a_1$, there is a tie for the second allocation. One can make the second allocation of magnitude.

$$x_{12} = \text{min.}\ (a_1 - a_1, b_1) = 0 \text{ in the cell } (1, 2),$$

or

$$x_{21} = \text{min.}\ (a_2, b_1 - b_1) = 0 \text{ in the cell } (2, 1).$$

Step 3. Repeat *steps 1* and 2 moving down towards the lower/right corner of the transportation table until all the rim requirements are satisfied.

SAMPLE PROBLEM

1001. *Obtain an initial basic feasible solution to the following transportation problem using the north-west corner rule :*

	D	*E*	*F*	*G*	*Available*
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

[Pune M.B.A. 1999; Madras B.Com. 2005]

Solution. Since $\sum_i a_i = \sum_j b_j = 950$, there exists a feasible solution to the transportation problem. We obtain initial feasible solution as follows :

The transportation table of the given problem has 12 cells. Following north-west corner method, the first allocation is made in the cell (1, 1), the magnitude being $x_{11} = \text{min.}\ (250, 200) = 200$. The second allocation is made in the cell (1, 2) and the magnitude of the allocation is given by

$$x_{12} = \text{min.}\ (250 - 200, 225) = 50.$$

The third allocation is made in the cell (2, 2), the magnitude being $x_{22} = \text{min.}\ (300, 225 - 50) = 175$. The magnitude of fourth allocation in the cell (2, 3) is given by $x_{23} = \text{min.}\ (300 - 175, 275) = 125$. The fifth allocation is made in the cell (3, 3), the magnitude being $x_{33} = \text{min.}\ (400, 275 - 125) = 150$ and the sixth (last) allocation is made in the cell (3, 4) with magnitude $x_{34} = \text{min.}\ (400 - 150, 250) = 250$. Hence an initial basic feasible solution to the given T.P. has been obtained and is displayed in Table 10.3.

200 / 11	50 / 13	17	14	250
16	175 / 18	125 / 14	10	300
21	24	150 / 13	250 / 10	400
200	225	275	250	

Table 10.3

The transportation cost according to the above route is given by

$$z = 200 \times 11 + 50 \times 13 + 175 \times 18 + 125 \times 14 + 150 \times 13 + 250 \times 10 = 12{,}200.$$

Remarks 1. The cells which get allocation will be called **basic cells**.

2. The initial basic feasible solution obtained by means of north-west corner rule may be far from optimum, because the costs were completely ignored.

2. Least-Cost Method or Matrix Minima Method

This method takes into account the minimum unit cost and can be summarized as follows :

Step 1. Determine the smallest cost in the cost matrix of the transportation table. Let it be c_{ij}. Allocate x_{ij} = min. (a_i, b_j) in the cell (i, j).

Step 2. If $x_{ij} = a_i$ cross off the ith row of the transportation table and decrease b_j by a_i. Go to *step 3.*

If $x_{ij} = b_j$ cross off the jth column of the transportation table and decrease a_i by b_j. Go to *Step 3.*

If $x_{ij} = a_i = b_j$ cross off either the ith row or jth column but not both.

Step 3. Repeat *steps 1* and *2* for the resulting reduced transportation table until all the rim requirements are satisfied. Whenever the minimum cost is not unique, make an arbitrary choice among the minima.

SAMPLE PROBLEM

1002. *Obtain an initial basic feasible solution to the following T.P. using the matrix minima method :*

	D_1	D_2	D_3	D_4	*Capacity*
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	

where O_i and D_j denote ith origin and jth destination respectively.

Solution. The transportation table of the given T.P. has 12 cells. Following the least-cost method, the first allocation is made in the cells (3, 1), the magnitude being $x_{31} = 4$. This satisfies the requirement at destination D_1 and thus we cross off the first column from the table. The second allocation is made in the cell (2, 4) of magnitude x_{34} = min. (6, 8) = 6. Cross off the fourth column of the table. This yields Table 10.4. There is, again, a tie for the third allocation. We choose arbitrarily the cell (1, 2) and allocate x_{12} = min. (6, 6) = 6, there. Cross off either the second column or the first row. We choose to cross off the first row of the table. The next allocation of magnitude $x_{32} = 0$ is made in cell (3, 2). Cross off the second column getting Table 10.5.

1	2	3	4	6
4	3	2	6 / 0	8
4 / 0	2	2	4 / 1	10
4	6	8	6	

Table 10.4

	6 / 2	3		6
	3	2	6	2
4	0 / 2	2		6
	6	8		

Table 10.5

We choose arbitrarily again, to make the next allocation in cell (2, 3) of magnitude x_{23} = min. (2, 8) = 2. Cross off the second row. This gives Table 10.6. The last allocation of magnitude x_{33} = min. (6, 6) = 6 is made in the cell (3, 3).

	6 / 1	2	3	4	
	4	3	2 / 2	6 / 0	2
4	∈ / 0	6 / 2	2	1	6
	8				

Table 10.6

	6 / 1	2	3	4	
	4	3	2 / 2	6 / 0	
4	∈ / 0	6 / 2	2	1	

Table 10.7

Now, all the rim requirements have been satisfied and hence an initial feasible solution has been determined. This solution is displayed in transportation Table 10.7.

Since the cells do not form a loop, the solution is basic one. Moreover the solution is degenerate also. The transportation cost according to the above route is given by

$$z = 6\times 2 + 2\times 2 + 6\times 0 + 4\times 0 + 2\times \in + 6\times 2 = 28 + 2\in = 28 \text{ as } \in \to 0.$$

3. Vogel's Approximation Method (*VAM*)

The *Vogel's Approximation Method* takes into account not only the least cost c_{ij} but also the costs that *just exceed* c_{ij}. The steps of the method are given below :

Step 1. For each row of the transportation table identify the *smallest* and the *next-to-smallest* costs. Determine the *differencé* between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly, compute the differences for each column.

Step 2. Identify the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tie-breaking choice. Let the greatest difference correspond to ith row and let c_{ij} be the smallest cost in the ith row. Allocate the maximum feasible amount x_{ij} = min. (a_i, b_j) in the (i, j)th cell and cross off either the ith row or the jth column in the usual manner.

Step 3. Recompute the column and row differences for the reduced transportation table and go to *step 2*. Repeat the procedure until all the rim requirements are satisfied.

Remarks 1. A row or column "difference" indicates the minimum unit penalty incurred by failing to make an allocation to the smallest cost cell in that row or column.

2. It will be seen later that *VAM* determines an initial basic feasible solution which is very close to the optimum solution, that is, the number of iterations required to reach the optimum solution is smaller in this case.

SAMPLE PROBLEM

1003. *Use Vogel's Approximation Method to obtain an initial basic feasible solution of the transportation problem :*

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

Solution. Following *VAM*, the differences between the smallest and next-to-smallest costs in each row and each column are computed and displayed inside the parenthesis against the respective rows and columns. The largest of these differences is (5) and is associated with the first column of the transportation table. Since the minimum cost in the first column is $c_{11} = 11$, we allocate $x_{11} = \min.(250, 200) = 200$ in the cell (1, 1). This exhausts the requirement of the first column and, therefore, we cross off the first column. The row and column differences are now computed for the resulting reduced transportation Table 10.8, the largest of these is (5) which is associated with the second column. Since c_{12} (= 13) is the minimum cost we allocate $x_{12} = \min.(50, 225) = 50$.

200 / 11	13	17	14	250 (2)
16	18	14	10	300 (4)
21	24	13	10	400 (3)
200 (5)	225 (5)	275 (1)	250 (0)	

Table 10.8

50 / 13	17	14	50 (1)
18	14	10	300 (4)
24	13	10	400 (3)
225 (5)	275 (1)	250 (0)	

Table 10.9

This exhausts the availability of first row and, therefore, we cross off the first row. Continuing in this manner, the subsequent reduced transportation tables and the differences for the surviving rows and columns are shown below :

175 / 18	14	10	300 (4)
24	13	10	400 (3)
175 (6)	275 (1)	250 (0)	

125 / 14	10	125 (4)
13	10	400 (3)
275 (1)	250 (0)	

275 / 13	125 / 10	400
275	125	

Table. 10.10

Eventually, the basic feasible solution shown in Table 10.11 is obtained :

200 / 11	50 / 13	17	14
16	175 / 18	14	125 / 10
21	24	275 / 13	125 / 10

Table. 10.11

The transportation cost according to this route is given by

$$z = 200 \times 11 + 50 \times 13 + 175 \times 18 + 125 \times 10 + 275 \times 13 + 125 \times 10 = 12{,}075.$$

Remark. It may be observed that transportation cost by North-West Corner method was 12,200 (sample problem 1001) whereas in VAM (sample problem 1003) is 12,075.

1004. *Find the initial basic feasible solution to the following transportation problem using VAM, given the cost matrix :*

	D_1	D_2	D_3	D_4	*Supply*
S_1	20	25	28	31	200
S_2	32	28	32	41	180
S_3	18	35	24	32	110
Demand:	150	40	180	170	

[Osmania M.B.A. 2001]

Solution. Here, total demand is 540 and total supply is 490. Since, total demand ≠ total supply, we introduce a dummy row with its supply as (540 – 490), *i.e.*, 50 and take all the cost elements of this row as zero. Thus, the transportation table for the initial basic feasible solution of the given problem is :

	D_1	D_2	D_3	D_4	
S_1	20	25	28	31	200
S_2	32	28	32	41	180
S_3	18	35	24	32	110
Dummy	0	0	0	0	50
	150	40	180	170	

Table 10.12

Following VAM, the differences between the smallest and next to smallest costs in each row and each column are computed and displayed inside the parenthesis against the respective rows and columns. The largest of these differences is (31) and is associated with the fourth column of the transportation table. As the least cost in the 4th column is $c_{44} = 0$, we allocate x_{44} = min. (50, 170) = 50 in the cell (4, 4). This exhausts the supply of fourth row and, therefore we cross off this row.

The row and column differences of the reduced transportation table are now computed. The largest of these is (6) and is associated with the third row. Since least cost in third row is $c_{31} = 18$, we allocate x_{31} = min. (110, 150) = 110. This exhausts the supply of third row, and, therefore we cross off the third row.

The row and column differences of the resulting transportation table are now computed. The largest of these is (12) associated with the first column. As the least cost in the first column is $c_{11} = 20$, we allocate x_{11} = min. (200, 40) = 40. This exhausts the demand of first column, and, therefore we cross off the first column.

Again, the row and coloumn differences for the reduced transportation table are computed. The largest among these is (10) associated with fourth column. Thus x_{14} = min. (160, 120) = 120 is allocated to the cell (1, 4) being the least cost cell of the fourth column. As this allocation exhausts the demand of the 4th column, we cross off the fourth column for further consideration.

Continuing in this manner, the remaining allocations are : $x_{22} = 40$, $x_{13} = 40$ and $x_{23} = 140$.

The transportation table showing all the assignments is displayed in Table 10.13.

	D_1	D_2	D_3	D_4	Row penalty				
S_1	40 / 20		40 / 25	120 / 28 / 31	(5)	(5)	(5)	(6)	(3)
S_2		40 / 32	140 / 28	32 / 41	(4)	(4)	(4)	(4)	(4)
S_3	110 / 18		35	24 / 32	(6)	(6)	—	—	—
Dummy	0	0	0	50 / 0	(0)	—	—	—	—
Column penalty	(18)	(25)	(24)	(31)					
	(2)	(3)	(4)	(1)					
	(12)	(3)	(4)	(10)					
	—	(3)	(4)	(10)					
	—	(3)	(4)	—					

Table 10.13

The number of allocated cells in the above table are 7, which is equal to the required number $m+n-1$ (*i.e.*, $4+4-1=7$), therefore, this solution is non-degenerate basic feasible. The transportation cost associated with the above solution is :

$$\text{Total cost} = 40\times 20 + 40\times 28 + 120\times 31 + 40\times 28 + 140\times 32 + 110\times 18 + 50\times 0 = 13{,}220.$$

PROBLEMS

1005. Determine an initial basic feasible solution to the following transportation problem using North-West Corner method :

	D_1	D_2	D_3	D_4	*Availability*
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
Demand	16	18	31	25	

[Madras B.Com. (Nov.) 2006]

1006. Consider the following transportation problem :

Source	*Destination*				*Availability*
	1	*2*	*3*	*4*	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Requirement	60	40	30	110	240

Determine an initial basic feasible solution using the (*a*) row minima method, and (*b*) Vogel's approximation method.

[Delhi B.Sc. (Stat.) 1995]

1007. Obtain an initial basic feasible solution to the following T.P. using the Vogel's approximation method :

Warehouses	*Stores*				*Availability*
	I	*II*	*III*	*IV*	
A	5	1	3	3	34
B	3	3	5	4	15
C	6	4	4	3	12
D	4	−1	4	2	19
Requirement	21	25	17	17	80

[Mahatma Gandhi M.Com. 2010]

1008. Find an initial basic feasible solution to the following T.P. using (*a*) North-West Corner Rule and (*b*) Vogel's Approximation Method.

Factories	*Warehouses*					*Availability*
	W_1	W_2	W_3	W_4	W_5	
F_1	20	28	32	55	70	50
F_2	48	36	40	44	25	100
F_3	35	55	22	45	48	150
Requirement	100	70	50	40	40	

[Osmania M.B.A. 1999]

1009. Consider the following transportation table showing production and transportation costs, along with the supply and demand positions of factories/distribution centres :

	M_1	M_2	M_3	M_4	*Supply*
F_1	4	6	8	13	500
F_2	13	11	10	8	700
F_3	14	4	10	13	300
F_4	9	11	13	3	500
Demand	250	350	1,050	200	

(*a*) Obtain an initial basic feasible solution by using *VAM*.

(*b*) Find out an optimal solution for the above given problem. *[Kerala M.Com. 1994]*

10:10. TEST FOR OPTIMALITY

Following the determination of an initial basic feasible solution to a transportation problem, the next question is how to arrive at the optimum solution. The basic technique is the same as in the simplex method, namely :

(*i*) Determining the net evaluations for non-basic variables (unoccupied cells);

(*ii*) choosing that net evaluation which may improve the current basic feasible solution;

(*iii*) determining the current occupied cell which leaves the basis (*i.e.*, becomes unoccupied) and repeating (*i*) through (*iii*) until an optimum solution is attained.

Determining the Net Evaluations (*the uv method*)

Unlike the simplex method, the net evaluations or opportunity costs for a T.P. can be determined more efficiently by using an indirect method which makes use of the properties of the primal and dual problems. Consider the following m-origin, n-destination T.P.

Determine x_{ij} so as to

$$\text{Minimize} \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij} \quad \text{subject to the constraints :}$$

$$a_i - \sum_{j=1}^{n} x_{ij} = 0 \qquad i = 1, 2, \ldots, m$$

$$b_j - \sum_{i=1}^{m} x_{ij} = 0 \qquad j = 1, 2, \ldots, n$$

$$x_{ij} \geq 0 \qquad \text{for all } i \text{ and } j.$$

Using duality in transportation problems, constraints of the dual problem are :

$$u_i + v_j \leq c_{ij};$$

where u_i ($i = 1, 2, \ldots, m$) and v_j ($j = 1, 2, \ldots, n$) are dual variables and are unrestricted in sign.

Recall from chapter on Simplex Method, that for any standard primal LPP with basis **B** and associated cost vector $\mathbf{c}_B$, the associated solution to its dual is given by $\mathbf{u} = \mathbf{c}_B \mathbf{B}^{-1}$. Thus, if $\mathbf{a}_j$ is the

jth column of the primal constraint matrix then an expression for evaluating the net evaluation is given by $z_j - c_j = \mathbf{c}_B \mathbf{B}^{-1} \mathbf{a}_j - c_j$ for all j. In our present case, the associated dual solution can be represented as

$$(\mathbf{u}, \mathbf{v}) = (u_1, \ldots, u_m, v_1, \ldots, v_n)$$

and hence the net evaluations are given by

$$\begin{aligned} z_{ij} - c_{ij} &= (\mathbf{u}, \mathbf{v})\, \mathbf{a}_{ij} - c_{ij} \\ &= (u_1, \ldots, u_m, v_1, \ldots, v_n)\, [\mathbf{e}_i + \mathbf{e}_{m+j}] - c_{ij} \\ &= u_i + v_j - c_{ij}\,; \qquad i = 1, \ldots, m\,;\ j = 1, 2, \ldots, n \end{aligned}$$

where $\mathbf{a}_{ij}$ is the column vector of the constraint matrix associated with the variable x_{ij}.

Now, since the net evaluations must vanish for the basic variables, it follows that $z_{ij} - c_{ij} = u_i + v_j - c_{ij}$ for all non-basic cells (i, j), where u_i and v_j obey the relation $u_r + v_s = c_{rs}$ for all basic cells (r, s). Except for the degeneracy case, these are $m+n-1$ dual equations in $m+n$ dual unknowns, for the $m+n-1$ basic cells. We can assign arbitrary value to one of these unknowns u_r and v_s and solve *uniquely* for the remaining $m+n-1$ variables. After an arbitrary assignment, say $u_1 = 0$, the rest of the values are obtained by simple addition and subtraction. Once all the u_i and v_j have been determined, the net evaluations for all the non-basic cells are determined by using the relation $z_{ij} - c_{ij} = u_i + v_j - c_{ij}$.

Selecting the Entering Variable

Since our objective is to minimize the cost of transportation, the current basic feasible solution will not be optimum so long as any of the net evaluations is positive. Thus, if all $z_{ij} - c_{ij}$ are non-positive, the current solution is an optimum one, otherwise using simplex criterion, we choose that variable x_{rs}, *i.e.*, cell (r, s) to enter the basis for which the net evaluation $z_{rs} - c_{rs} = \max_{i,j} \{z_{ij} - c_{ij} > 0\}$.

Selecting the Leaving Variable

The next step is to determine the leaving basic variable (basic cell) and then to determine the new improved basic solution. For it, we proceed as follows :

We identify a closed loop which starts and ends at the *entering* non-basic cell (r, s) connecting only the basic cells. Such closed path exists and is unique for any non-degenerate basic solution. By interchangeably subtracting and adding the amount to be allocated to the entering variable (say θ) from and to the successive *end points* of the closed loop, the supply and demand constraints can always be satisfied. However, the value of θ must be limited by the minimum quantity which will render non-negative values for all the basic variables in the new solution. The value of θ can be easily seen from the allocations made in the cells of the loop. Assign – and + signs alternately to the cells of the loop, starting with a '+' sign for the entering cells. Then θ = minimum of the allocations made in the cells having a '–' sign. Add this value of θ to all cells having '+' sign and subtract the same from the cells having a '–' sign. This consequently determines the *leaving* variable.

Remark. The procedure of $\pm\theta$ adjustments around the entering cell and then choosing the leaving basic cell on the basis of the maximum possible value of θ chosen, can be very conveniently accomplished without bringing θ into the picture at all.

10.11. ECONOMIC INTERPRETATION OF u_i's AND v_j's

The dual variables u_j's and v_j's represent the shadow prices (value of the commodity) for the supply centres and demand centres. The value of each variable u_i measures the comparative implicit contribution (value or location advantages) of a unit of capacity at supply centre i and therefore may be termed *location rent*. Similarly, the value of each variable v_j measures the comparative implicit

contribution of an additional unit of commodity transported to demand centre j and therefore may be termed *market price*.

10:12. DEGENERACY IN TRANSPORTATION PROBLEM

A basic feasible solution for the general transportation problem must consist of $(m+n-1)$ occupied cells. The basic solution will be called degenerate when the number of occupied cells is less than the required number, $m+n-1$. Degeneracy can occur in initial solution or it may arise in some subsequent iterations. We now discuss a procedure to deal with the problem of degeneracy.

Case 1. *Degeneracy at the Initial Solution.* To resolve degeneracy at the initial solution, a very small quantity $\in$ (>0) is allocated in an unoccupied cell so as to get $m+n-1$ number of unoccupied cells. In a minimization transportation problem it is better to allocate $\in$ to unoccupied cells that have lowest transportation costs. In some cases, $\in$ must be added in one of those unoccupied cells which make possible the determination of u_i and v_j uniquely.

The quantity $\in$ is considered to be so small that if it is transferred to an occupied cell, it does not change the quantity of allocation. That is, $x_{ij}+\in = x_{ij}-\in = x_{ij}$ but $\in-\in = 0$. Also $\in$ does not affect the total transportation cost of the allocation. Hence the quantity $\in$ is used to evaluate unoccupied cells and once the purpose is over, $\in$ must be removed from the scene.

Case 2. *Degeneracy at Subsequent Iterations.* To resolve degeneracy which occur during optimality test, the quantity $\in$ may be allocated to one or more cells which have become unoccupied recently to have $m+n-1$ number of occupied cells in the new solution. It may be removed once the purpose is over.

10:13. TRANSPORTATION ALGORITHM (*MODI Method*)

Various steps involved in solving any transportation problem may be summarized in the following iterative procedure :

Step 1. Find the initial basic feasible solution by using any of the three methods discussed above.

Step 2. Check the number of occupied cells. If there are less than $m+n-1$, there exists degeneracy and we introduce a very small positive assignment of $\in (\approx 0)$ in suitable independent positions, so that the number of occupied cells is exactly equal to $m+n-1$.

Step 3. For each occupied cell in the current solution, solve the system of equations $u_i + v_j = c_{ij}$ starting initially with some $u_i = 0$ or $v_j = 0$ and entering successively the values of u_i and v_j in the transportation table margins.

Step 4. Compute the net evaluations $z_{ij} - c_{ij} = u_i + v_j - c_{ij}$ for all unoccupied basic cells and enter them in the lower left corners of the corresponding cells.

Step 5. Examine the sign of each $z_{ij} - c_{ij}$. If all $z_{ij} - c_{ij} \le 0$, then the current basic feasible solution is an optimum one. If at least one $z_{ij} - c_{ij} > 0$, select the unoccupied cell, having the largest positive net evaluation to enter the basis.

Step 6. Let the unoccupied cell (r, s) enter the basis. Allocate an unknown quantity, say θ, to the cell (r, s). Identify a loop that starts and ends at the cell (r, s) and connects some of the basic cells. Add and subtract interchangeably, θ to and from the transition cells of the loop in such a way that the rim requirements remain satisfied.

Step 7. Assign a maximum value to θ in such a way that the value of one basic variable becomes zero and the other basic variables remain non-negative. The basic cell whose allocation has been reduced to zero, leaves the basis.

Step 8. Return to *step 3* and repeat the process until an optimum basic feasible solution has been obtained.

SAMPLE PROBLEMS

1010. *Find the starting solution in the following transportation problem by Vogel's Approximation Method. Also obtain the optimum solution :*

	D_1	D_2	D_3	D_4	*Supply*
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
Demand	3	3	2	2	

Solution. *Step 1. Initial Basic Feasible Solution.*

Following *Vogel's Approximation Method*, the differences between the two successive lowest costs for each row and each column are computed. These are written besides the corresponding rows or columns under the heading 'row difference' or 'column difference' :

					Row difference
	$\boxed{3}$ 3	7	6	$\boxed{2}$ 4	(1) (1) (1) ①
	2	4	$\boxed{2}$ 3	$\boxed{\epsilon_1}$ 2	(1) (2) (2) —
	4	$\boxed{3}$ 3	8	$\boxed{\epsilon_2}$ 5	(1) (1) (1) ①
Column difference	(1) (1) (1) ①	(1) (1) ④ —	③ — — —	(2) ② (1) (1)	

Table 10.14

The largest row difference or column difference is 3 which corresponds to third column. Allocating as much as possible to the lowest cost cell in the third column gives $x_{23} = 2$. This satisfies the supply at S_2 and the requirement at D_3. So arbitrarily we cross off the third column and consider $\epsilon_1 (\to 0)$ as the small quantity to be supplied from S_2. The row and column differences are now recomputed for the reduced cost matrix, and we choose the largest of these, *i.e.*, 2 which corresponds to fourth column. We therefore allocate $x_{24} = \epsilon_1$ and second row is now eliminated. Again, we compute the row and column differences for the reduced 2×3 transportation table and choose the largest of these, *i.e.*, 4 corresponding to the second column. We thus allocate $x_{32} = 3$ and eliminate second column while consider $\epsilon_2 (\to 0)$ being the negligible positive quantity to be supplied from S_3. Again, computing the column and row differences in the reduced 2×2 cost matrix, we assign $x_{11} = 3$ and thus eliminate first column. This leaves now only fourth column, which can be filled by inspection and thus we assign $x_{14} = 2$ and $x_{34} = \epsilon_2$.

The transportation cost according to above allocation is given by

$$\begin{aligned} \text{Total cost} &= 3 \times 3 + 2 \times 4 + 2 \times 3 + \epsilon_1 \times 2 + 3 \times 3 + \epsilon_2 \times 5 \\ &= 32 + 2\epsilon_1 + 5\epsilon_2 \\ &= 32 \text{ as } \epsilon_1 \to 0 \text{ and } \epsilon_2 \to 0. \end{aligned}$$

Step 2. From the initial basic feasible solution obtained in *step 1*. We observe that unknown quantities ϵ_1, $\epsilon_2 (> 0)$ have been allocated in the unoccupied cells (2, 4) and (3, 4) respectively to overcome the danger of degeneracy.

Step 3. We now compute the numbers u_i $(i = 1, 2, 3)$ and v_j $(j = 1, 2, 3, 4)$ using successively the equations $u_i + v_j = c_{ij}$ for all the occupied cells. For this we arbitrarily assign $v_4 = 0$. Thus we have

$$u_1 + v_4 = c_{14} \Rightarrow u_1 + 0 = 4 \Rightarrow u_1 = 4$$
$$u_2 + v_4 = c_{24} \Rightarrow u_2 + 0 = 2 \Rightarrow u_2 = 2$$
$$u_3 + v_4 = c_{34} \Rightarrow u_3 + 0 = 5 \Rightarrow u_3 = 5$$

Given u_1, u_2 and u_3, values of v_1, v_2 and v_3 can be calculated as shown below :

$$u_1 + v_1 = c_{11} \Rightarrow 4 + v_1 = 3 \Rightarrow v_1 = -1$$
$$u_2 + v_3 = c_{23} \Rightarrow 2 + v_3 = 3 \Rightarrow v_3 = 1$$
$$u_3 + v_2 = c_{32} \Rightarrow 5 + v_2 = 3 \Rightarrow v_2 = -2$$

Step 4. The net evaluations for each of the unoccupied cells are now determined :

$$z_{12} - c_{12} = u_1 + v_2 - c_{12} = 4 + (-2) - 7 = -5$$
$$z_{13} - c_{13} = u_1 + v_3 - c_{13} = 4 + 1 - 6 = -1$$
$$z_{21} - c_{21} = u_2 + v_1 - c_{21} = 2 + (-1) - 2 = -1$$
$$z_{22} - c_{22} = u_2 + v_2 - c_{22} = 2 + (-2) - 4 = -4$$
$$z_{31} - c_{31} = u_3 + v_1 - c_{31} = 5 + (-1) - 4 = 0$$
$$z_{33} - c_{33} = u_3 + v_3 - c_{33} = 5 + 1 - 8 = -2$$

Step 5. Since all $z_{ij} - c_{ij} \leq 0$, the current basic feasible solution is an optimum one.

It is possible to present a more compact form for computing the unknown u_i's and v_j's and then evaluate the net evaluations for each of the unoccupied cells in a more convenient way by working in the transportation table margins :

	D_1	D_2	D_3	D_4	u_i
S_1	3	3 -5 7	-1 6	2 4	4
S_2	-1 2	2 -4 4	2 3	ϵ_1 2	2
S_3	0 4	3 3	-2 8	ϵ_2 5	5
v_j	-1	-2	1	0	

Table 10.15

The optimum solution is :

$$x_{11} = 3,\ x_{14} = 2,\ x_{23} = 2,\ x_{24} = \epsilon_1,\ x_{32} = 3 \text{ and } x_{34} = \epsilon_2.$$

The transportation cost associated with the optimum schedule is :

$$3 \times 3 + 2 \times 4 + 2 \times 3 + 2 \times \epsilon_1 + 3 \times 3 + 5 \times \epsilon_2,$$

i.e., $$32 + 2\epsilon_1 + 5\epsilon_2 \text{ or } 32 \text{ since } \epsilon_1 \to 0 \text{ and } \epsilon_2 \to 0.$$

1011. *Given $x_{13} = 50$ units, $x_{14} = 20$ units, $x_{21} = 55$ units, $x_{31} = 30$ units, $x_{32} = 35$ units and $x_{34} = 25$ units. Is it an optimal solution to the transportation problem :*

					Available units
	6	1	9	3	70
	11	5	2	8	55
	10	12	4	7	90
Required units :	85	35	50	45	

If not, modify it to obtain a better feasible solution.

Solution. Considering the given solution as an initial basic feasible solution, we now compute the numbers u_i $(i = 1, 2, 3)$ and v_j $(j = 1, 2, 3, 4)$ using successively the equations $u_i + v_j = c_{ij}$ for all the occupied cells. For this, we arbitrarily assign $u_3 = 0$. Thus, we have

$$u_3 + v_1 = c_{31} \Rightarrow 0 + v_1 = 10 \Rightarrow v_1 = 10$$
$$u_3 + v_2 = c_{32} \Rightarrow 0 + v_2 = 12 \Rightarrow v_2 = 12$$
$$u_3 + v_4 = c_{34} \Rightarrow 0 + v_4 = 7 \Rightarrow v_4 = 7$$

Given v_1, v_2, v_4; values of u_2 and u_1 can be calculated as shown below :

$$u_2 + v_1 = c_{21} \Rightarrow u_2 + 10 = 11 \Rightarrow u_2 = 1$$
$$u_1 + v_4 = c_{14} \Rightarrow u_1 + 7 = 3 \Rightarrow u_1 = -4$$

Now, using the value of u_1, we get

$$u_1 + v_3 = c_{13} \Rightarrow -4 + v_3 = 9 \Rightarrow v_3 = 13.$$

The net evaluations for each of the unoccupied cells are now determined as follows

$$z_{11} - c_{11} = u_1 + v_1 - c_{11} = -4 + 10 - 6 = 0$$
$$z_{12} - c_{12} = u_1 + v_2 - c_{12} = -4 + 12 - 1 = 7$$
$$z_{22} - c_{22} = u_2 + v_2 - c_{22} = 1 + 12 - 5 = 8$$
$$z_{23} - c_{23} = u_2 + v_3 - c_{23} = 1 + 13 - 2 = 12$$
$$z_{24} - c_{24} = u_2 + v_4 - c_{24} = 1 + 7 - 8 = 0$$
$$z_{33} - c_{33} = u_3 + v_3 - c_{33} = 0 + 13 - 4 = 9$$

According to optimality criterion, since all $z_{ij} - c_{ij} \geq 0$, current basic feasible solution is not optimum. Maximum of $z_{ij} - c_{ij}$, *i.e.*, $z_{23} - c_{23}$ indicates that the total transportation cost can be reduced in multiple of 12 by shifting an allocation to this cell. Thus we assign θ units to the unoccupied cell (2, 3). Since the requirements and availability are to be satisfied, we make $+\theta$ and $-\theta$ adjustments in the occupied cells in such a way that a closed path is formed that starts and ends at the assigned cell (2, 3).

Now, in order to maintain feasibility, locate the occupied cell with $(-\theta)$ assignment that has the smallest allocation. This determines the maximum number of units that can be shifted along the closed path. Here, the occupied cells (1, 3), (2, 1) and (3, 4) have $(-\theta)$ assignments. Among these the cell (3, 4) is selected because it has the smallest allocation, *i.e.*, 25 units. Thus we allocate θ (= 25) units. By doing this, cell (3, 4) becomes unoccupied and cell (2, 3) as occupied in the next iteration.

Initial Iteration. Cell (2, 3) enters the basis and cell (3, 4) leaves the basis. See Table 10.16.

First Iteration. Cell (1, 2) enters the basis and cell (1, 3) leaves the basis. See Table 10.17.

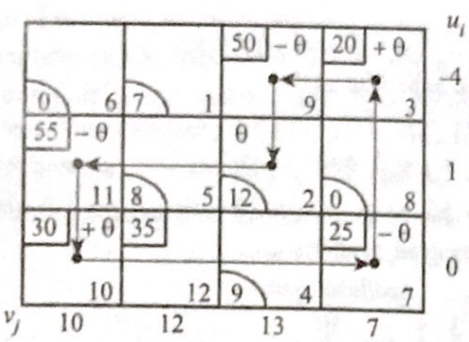

Table 10.16

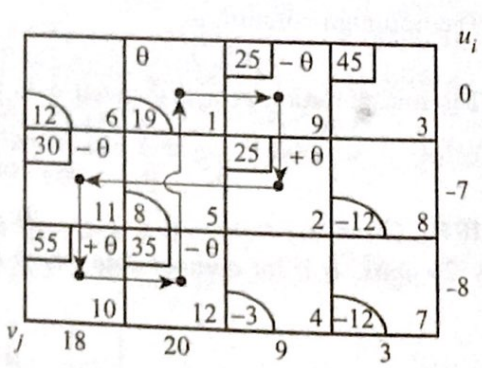

Table 10.17

Second Iteration. Cell (2, 2) enters and cell (2, 1) leaves the basis. See Table 10.18.

Third Iteration. Cell (3, 4) enters and cell (3, 2) leaves the basis. See Table 10.19.

					u_i
	25		45		0
−7 / 6		1 / −19	9	3	
5 / −θ	θ	50			12
11	8 / 5		2 / 7	8	
80 / +θ	10 / −θ				0
10	12	−14 / 4	−4 / 7		
v_j −1	1	−10	3		

Table 10.18

					u_i
	25 / +θ		45 / −θ		0
−7 / 6		1 / −11	9	3	
	5	50			4
−8 / 11		5	2 / −1	8	
85	5 / −θ		θ		11
10		12 / 5	4 / 7	7	
v_j −1	1	−2	3		

Table 10.19

Final Iteration. Optimum Solution.

					u_i
	30		40		0
0 / 6		1 / −11	9	3	
	5	50			4
−1 / 11		5	2 / −1	8	
85			5		4
10	−7 / 12	−2 / 4		7	
v_j 6	1	−2	3		

Table 10.20

Thus the optimum allocation is :

$$x_{12} = 30,\ x_{14} = 40,\ x_{22} = 5,\ x_{23} = 50,\ x_{31} = 85 \text{ and } x_{34} = 5.$$

The transportation cost according to above allocation is :

$$30 \times 1 + 40 \times 3 + 5 \times 5 + 50 \times 2 + 85 \times 10 + 5 \times 7, \textit{ i.e.}, 1160.$$

1012. *ABC Limited has three production shops supplying a product to five warehouses. The cost of production varies from shop to shop and cost of transportation from one shop to a warehouse also varies. Each shop has a specific production capacity and each warehouse has certain amount of requirement. The costs of transportation are as given below :*

Shop	*Warehouse*					*Supply*
	I	*II*	*III*	*IV*	*V*	
A	6	4	4	7	5	100
B	5	6	7	4	8	125
C	3	4	6	3	4	175
Demand	60	80	85	105	70	400

The cost of manufacture of the product at different production shops is :

Shop	*Variable cost*	*Fixed cost*
A	14	7,000
B	16	4,000
C	15	5,000

Find the optimum quantity to be supplied from each shop to different warehouses at minimum total cost. **[Delhi M.B.A. 2008; Nagpur M.B.A. (Nov.) 1998]**

Solution. In the transportation cost matrix, the data pertaining to fixed costs is of no use. We shall include only the transportation costs plus the variable costs as shown below :

Shop	Warehouse					Supply
	I	*II*	*III*	*IV*	*V*	
A	6 + 14 = 20	4 + 14 = 18	4 + 14 = 18	7 + 14 = 21	5 + 14 = 19	100
B	5 + 16 = 21	6 + 16 = 22	7 + 16 = 23	4 + 16 = 20	8 + 16 = 24	125
C	3 + 15 = 18	4 + 15 = 19	6 + 15 = 21	3 + 15 = 18	4 + 15 = 19	175
Demand	60	80	85	105	70	400

Using *Vogel's Approximation Method*, the following initial basic feasible solution to the transportation problem is obtained :

						Supply
	20	15 / 18	85 / 18	21	19	100
	21	20 / 22	23	105 / 20	24	125
	60 / 18	45 / 19	21	18	70 / 19	175
Demand	60	80	85	105	70	

Table 10.21

Now using *MODI* method, the optimum solution as given in Table 10.22, is obtained where $c_{ij} = u_i + v_j$ for basic cells and $z_{ij} - c_{ij} = u_i + v_j - c_{ij}$ for non-basic cells.

						u_i
	-3 / 20	15 / 18	85 / 18	-5 / 21	-1 / 19	0
	0 / 21	20 / 22	-1 / 23	105 / 20	-2 / 24	4
	60 / 18	45 / 19	-2 / 21	-1 / 18	70 / 19	1
v_j	17	18	18	16	18	

Table 10.22

Since all the net evaluations, $z_{ij} - c_{ij} \leq 0$, an optimum solution is attained, *viz.*, $x_{12} = 15$, $x_{13} = 85$, $x_{22} = 20$, $x_{24} = 105$, $x_{31} = 60$, $x_{32} = 45$ and $x_{35} = 70$. The transportation cost associated with this solution is : $= 15 \times 18 + 85 \times 18 + 20 \times 22 + 105 \times 20 + 60 \times 18 + 45 \times 19 + 70 \times 19 = 7{,}605$.

Total cost = Transportation cost + Fixed cost = 7,605 + 16,000 = 23,605.

Hence, an optimum solution is to transport

shop *A* → 15 units to II and 85 units to III,
shop *B* → 20 units to II and 105 units to IV,
shop *C* → 60 units to I, 45 units to II and 70 units to V.

Total minimum cost will be Rs. 23,605.

10:14. STEPPING STONE SOLUTION METHOD

This method is an alternative procedure for the solution of a transportation problem. Various steps involved in this method are summarized below :

Step 1. Find the initial basic feasible solution of the transportation problem.

Step 2. Check the number of occupied cells. If there are less than $m+n-1$, there exists degeneracy and we introduce a very small quantity $\in (\to 0)$ in suitable independent positions, so that the number of occupied cells is exactly equal to $m+n-1$.

Step 3. Compute improvement index for each of the unoccupied cells. This is computed by calculating the opportunity cost of an unoccupied cell. This means that if we shift one unit from a cell containing positive shipment to the unoccupied cell, that will be the net cost. If all the unoccupied cells have positive improvement index, then the given solution is an optimum solution.

Step 4. If there are several unoccupied cells with negative improvement indices, then we select the cell having the largest negative improvement index and shift the maximum possible units to that cell without violating the supply and demand constraints. After it, again go to *step 3.*

SAMPLE PROBLEM

1013. *A company has factories at F_1, F_2 and F_3 which supply warehouses at W_1, W_2 and W_3. Weekly factory capacities are 200, 160 and 90 units respectively. Weekly warehouse requirements are 180, 120 and 150 units respectively. Unit shipping costs (in rupees) are as follows :*

Factory	*Warehouse*			*Supply*
	W_1	W_2	W_3	
F_1	16	20	12	200
F_2	14	8	18	160
F_3	26	24	16	90
Demand	180	120	150	450

Determine the optimum distribution for this company to minimize shipping costs.

[Osmania M.Com. 2010]

Solution. Following *Vogel's Approximation Method*, an initial basic feasible solution is obtained in the table given below :

	W_1	W_2	W_3	*Supply*
F_1	140 / 16	20	60 / 12	200
F_2	40 / 14	120 / 8	18	160
F_3	26	24	90 / 16	90
Demand	180	120	150	

Table 10.23

The Stepping Stone method starts with an evaluation of each of the unoccupied cells, *viz.*, (F_1, W_2), (F_2, W_3), (F_3, W_1), (F_3, W_2) to examine whether it would improve the initial solution by introducing any of these cells into the current solution basis. To demonstrate the application of this method let us evaluate the unoccupied cell (F_1, W_2) of Table 10.23 and assign θ units to be shipped from factory F_1 to warehouse W_2 [Note that an addition of a unit to cell (F_1, W_2) adds Rs. 20 to the cost]. Since the requirements of the warehouse must be satisfied, we adjust the allocation of cell (F_1, W_1) by subtracting θ units from the initial assignment of 140 units which decreases the cost by Rs. 16 per unit. In order to make this adjustment and to satisfy the supply restriction at factories F_1 and F_2, we subtract θ units from cell (F_2, W_2) and then add θ units to cell (F_2, W_1). The closed path followed in making these adjustments is given below :

By following the closed path and analysing the unit shipment, we can compute the incremental cost evaluation of unoccupied cell (F_1, W_2) as follows :

Cell (i, j)	*Change in x_{ij}*	*Change in total cost*
(F_1, W_2)	+1	$+20 \times 1$
(F_1, W_1)	−1	-16×1
(F_2, W_1)	+1	$+14 \times 1$
(F_2, W_2)	−1	-8×1
		+10 Net change

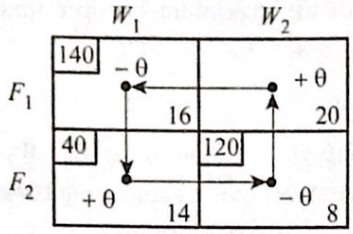

Table 10·24

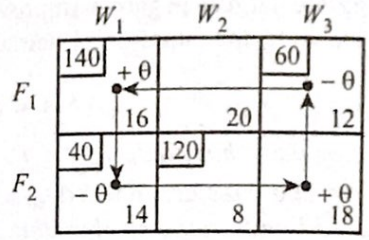

Table 10·25

This indicates that shift of 1 unit into unoccupied cell (F_1, W_2) increases the transportation cost by Rs. 10. Obviously, cell (F_1, W_2) is not included in the assignment. Let us compute the opportunity cost for one more unoccupied cell (F_2, W_3). As before, we assign θ units to cell (F_2, W_3) and make necessary adjustments in cells (F_1, W_3), (F_1, W_1) and (F_2, W_1). The closed path associated with this assignment is shown in Table 10.25.

The net change in cost is :

$$(18 \times 1 - 12 \times 1 + 16 \times 1 - 14 \times 1), \text{ i.e., Rs. 8 per unit.}$$

Since this route is more expensive than our initial solution, we shall not plan any change based on this evaluation. We now evaluate the remaining unoccupied cells (F_3, W_1) and (F_3, W_2). The path suggested for these is shown below :

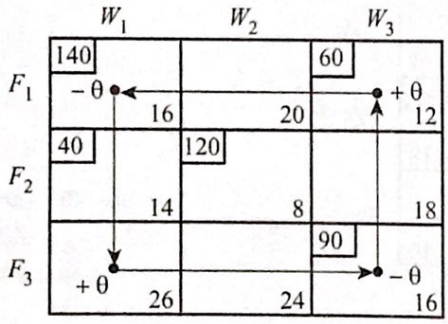

Table 10.26

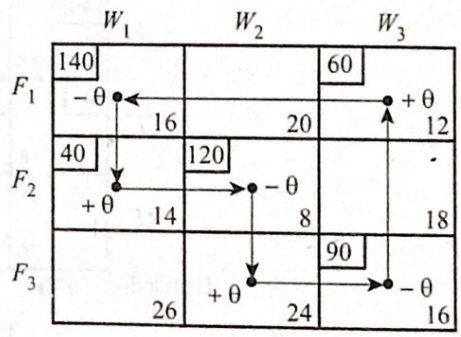

Table 10.27

The closed path and an incremental cost change for each of unoccupied cells are given below :

Unoccupied cell	*Closed path*	*Net change in cost (in Rs.)*
(F_1, W_2)	$(F_1, W_2) \to (F_2, W_2) \to (F_2, W_1) \to (F_1, W_1)$	$+20 - 8 + 14 - 16 = 10$
(F_2, W_3)	$(F_2, W_3) \to (F_1, W_3) \to (F_1, W_1) \to (F_2, W_1)$	$+18 - 12 + 16 - 14 = 8$
(F_3, W_1)	$(F_3, W_1) \to (F_3, W_3) \to (F_1, W_3) \to (F_1, W_1)$	$+26 - 16 + 12 - 16 = 6$
(F_3, W_2)	$(F_3, W_2) \to (F_3, W_3) \to (F_1, W_3) \to (F_1, W_1) \to (F_2, W_1) \to (F_2, W_2)$	$+24 - 16 + 12 - 16 + 14 - 8 = 10$

Since, none of the unoccupied cells in the above table is associated with a negative value, the total transportation costs will either remain the same or increase if a shift of the unit is made to any one of the unoccupied cells. Hence, the preceding table provides an *optimum plan* as follows :

$$F_1 : x_{11} = 140,\ x_{12} = 0,\ x_{13} = 60;\ F_2 : x_{21} = 40,\ x_{22} = 120,\ x_{23} = 0$$

$$F_3 : x_{31} = 0,\ x_{32} = 0,\ x_{33} = 90$$

with total transportation cost = Rs. 5,920.

PROBLEMS

1014. Solve the following transportation problems :

From	*To*			*Available*
	A	*B*	*C*	
I	50	30	220	1
II	90	45	170	3
III	250	200	50	4
Requirement	4	2	2	

[Madras B.Com. 2004]

1015. The following data is given :

		Destinations			
		1	*2*	*3*	*Capacities*
Sources	*1*	2	2	3	10
	2	4	1	2	15
	3	1	3	X	40
Demands		20	15	30	

The cost of shipment from third source to the third destination is not known. How many units should be transported from sources to destinations so that the total cost of transporting all the units to their destinations is a minimum ? ***[IAS 1994]***

1016. Solve the following transportation problem :

From	*To*			*Available*
	A	*B*	*C*	
I	6	8	4	14
II	4	9	8	12
III	1	2	6	5
Demand	6	10	15	

[Meerut M.Sc. (Math.) 1999]

1017. Three fertilizer factories *X, Y* and *Z* located at different places of the country produce 6, 4 and 5 lakh tonnes of urea respectively. Under the directive of the Central Government, they are to be distributed to 3 States *A, B* and *C* as 5, 3 and 7 lakh tonnes respectively. The transportation cost per tonne in rupees is given below :

	A	*B*	*C*
X ...	11	17	16
Y ...	15	12	14
Z ...	20	12	15

Find out suitable transportation pattern at minimum cost. ***[Delhi B.Sc. (Stat.) 2003]***

1018. A company has three warehouses W_1, W_2 and W_3. It is required to deliver a product from these warehouses to three customers *A, B* and *C*. The warehouses have the following units in stock.

Warehouse	W_1	W_2	W_3
No. of units	65	42	43

and customer requirements are :

Customer	*A*	*B*	*C*
No. of units	70	30	50

The table below shows the costs of transporting one unit from warehouse to the customer :

		Warehouse		
		W_1	W_2	W_3
Customer	A	5	7	8
	B	4	4	6
	C	6	7	7

Find the optimum transportation route. *[C.A. (Final) May 1999]*

1019. Consider a transportation problem with $m = 3$ and $n = 4$, where :

$c_{11} = 2$, $c_{12} = 3$, $c_{13} = 11$, $c_{14} = 7$

$c_{21} = 1$, $c_{22} = 0$, $c_{23} = 6$, $c_{24} = 1$

$c_{31} = 5$, $c_{32} = 8$, $c_{33} = 15$, $c_{34} = 9$

Suppose $S_1 = 6$, $S_2 = 1$ and $S_3 = 10$, whereas $D_1 = 7$, $D_2 = 5$, $D_3 = 3$ and $D_4 = 2$. Apply the transportation simplex method to find out an optimal solution.

[Pondicherry B.Sc. (Comp.) 2001; Madras B.Com. (Nov.) 2005]

1020. Solve the following transportation problem :

Source	Destination				Available
	1	2	3	4	
1	21	16	25	13	11
2	17	18	14	23	13
3	32	27	18	41	19
Requirement	6	10	12	15	43

[Madras M.B.A. (Nov.) 2006, Osmania M.B.A. (Nov.) 2002]

1021. Solve the following transportation problem :

Origin	Destination				Availability
	D_1	D_2	D_3	D_4	
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
Requirement	20	40	30	10	

[Purvanchal MCA 1996; Madras B.Com. (Nov.) 2007]

1022. Find the optimal solution to the following transportation problem :

	D_1	D_2	D_3	D_4	Available
O_1	23	27	16	18	30
O_2	12	17	20	51	40
O_3	22	28	12	32	53
Required	22	35	25	41	123

The cell entries are unit transportation costs.

1023. A company has three factories F_i ($i = 1, 2, 3$) from which it transports the product to four warehouses W_j ($j = 1, 2, 3, 4$). The unit cost of production at the three factories are Rs. 4, 3, 5 respectively. Given the following information on unit costs of transportation, capacities at the three factories and of the requirement at the four warehouses, find the optimum allocation.

Factory	Unit cost of production Rs./unit	Transportation cost Rs./unit				Capacity
		W_1	W_2	W_3	W_4	
F_1	4	5	7	3	8	300
F_2	3	4	6	9	5	500
F_3	5	2	6	4	5	200
Requirements		200	300	400	100	1,000

1024. A company has three cement factories *A*, *B* and *C*, and four area distributors *W*, *X*, *Y* and *Z*. With identical costs of production at the three factories, the only variable cost involved is the transportation cost. The monthly production capacity (in tonne) of the three factories, monthly demand of the four distributors, and the transportation cost per tonne (in rupees) from the different factories to different distribution centres, are given below :

Factory	*Distributor*				*Monthly availability*
	W	*X*	*Y*	*Z*	
A	20	25	50	10	45,000
B	45	50	15	40	50,000
C	22	10	45	35	55,000
Monthly demand	50,000	40,000	30,000	30,000	

Suggest an optimal transport schedule and find the minimum transportation cost. ***[Delhi M.Com. 1999]***

1025. A company has three factories at Amethi, Baghpat and Gwalior; and four distribution centres at Allahabad, Mumbai, Kolkata and Delhi. With identical cost of production at the three factories the only variable cost involved is transportation cost. The production at the three factories is 5,000 tonnes; 6,000 tonnes and 2,500 tonnes respectively. The demand at four distribution centres is 6,000 tonnes; 4,000 tonnes; 2,000 tonnes and 1,500 tonnes respectively. The transportation costs per tonne from different factories to different centres are given below :

Factory	*Distribution centre*			
	Allahabad	*Mumbai*	*Kolkata*	*Delhi*
Amethi	3	2	7	6
Baghpat	7	5	2	3
Gwalior	2	5	4	5

Suggest the optimum transportation schedule and find the minimum cost of transportation.
[Delhi M.B.A. (PT) 2008; Jiwaji M.Com. 2009; Panjab Tech. Univ. M.B.A. 2011]

1026. A wholesale company has three warehouses from which supplies are drawn for four retail customers. The company deals in a single product, the supplies of which at each warehouse are :

Warehouse no.	*Supply (units)*	*Customer no.*	*Demand (units)*
1	20	1	15
2	28	2	19
3	17	3	13
		4	18

Conveniently, total supply at the warehouses is equal to total demand from the customers. The following table gives the transportation costs per unit shipment from each warehouse to each customer.

Warehouse	*Customer*			
	1	*2*	*3*	*4*
1	3	6	8	5
2	6	1	2	5
3	7	8	3	9

Determine what supplies to despatch from each of the warehouses to each customer so as to minimize overall transportation cost. ***[BHU M.Com. 2010]***

1027. A company is spending Rs. 1,000 on transportation of its units from three plants to four distribution centres. The supply and demand of units with unit cost of transportation are given as :

Plant	*Distribution centre*				*Capacity (tons)*
	D_1	D_2	D_3	D_4	
P_1	19	30	50	12	7
P_2	70	30	40	60	10
P_3	40	10	60	20	18
Demand	5	8	7	15	35

What can be the maximum saving by optimum scheduling? ***[Madras M.B.A. (April) 2006; Annamalai M.B.A. 2006]***

1028. The following table shows all the necessary information on the available supply to each warehouse, the requirement of each market and the unit transportation cost from each warehouse to each market :

Warehouse	*Market*				*Supply*
	I	*II*	*III*	*IV*	
A	5	2	4	3	22
B	4	8	1	6	15
C	4	6	7	5	8
Requirement	7	12	17	9	

The shipping clerk has worked out the following schedule from experience : 12 units from *A* to *II*, 1 unit from *A* to *III*, 9 units from *A* to *IV*, 15 units from *B* to *III*, 7 units from *C* to *I* and 1 unit from *C* to *III*.

(*a*) Check and see if the clerk has the optimal schedule.

(*b*) Find the optimal schedule and minimum total shipping cost.

(*c*) If the clerk is approached by a carrier of route *C* to *II* who offers to reduce his rate in the hope of getting some business, by how much must the rate be reduced before the clerk should consider giving him an order ?

[Madras M.B.A. (Nov.) 2006; Delhi M.B.A. (Nov.) 2008; Panjab Tech. Univ. M.B.A. (Dec.) 2009]

1029. The following table gives one of the possible solutions to a transportation problem involving three sources and four destinations :

	D_1	D_2	D_3	D_4
S_1	10	$\boxed{200}$ 6	$\boxed{100}$ 18	23
S_2	$\boxed{150}$ 4	9	$\boxed{50}$ 13	10
S_3	7	13	$\boxed{50}$ 15	$\boxed{350}$ 5

Answer the following questions :

(*i*) Is the above solution degenerate?

(*ii*) Is the above solution optimal? Is it a unique solution? If yes, why?

(*iii*) What is the opportunity cost of transporting one unit from source S_1 to destination D_4?

(*iv*) How would the cost be affected if it is decided to transport one unit from S_2 to D_2? *[Delhi MIB 2011]*

1030. Solve the following transportation problem to find the minimum transportation cost :

Source	*Destination*					*Available*
	D_1	D_2	D_3	D_4	D_5	
S_1	4	7	3	8	2	4
S_2	1	4	7	3	8	7
S_3	7	2	4	7	7	9
S_4	4	7	2	4	7	2
Required	8	3	7	2	2	

1031. A manufacturer has distribution centres at *X*, *Y* and *Z*. These centres have availability of 40, 20 and 40 units respectively of his product. His retail outlets at *A*, *B*, *C*, *D* and *E* require 25, 10, 20, 30 and 15 units respectively. The transportation cost (in rupees) per unit between each centre outlet is given below :

Distribution centre	*Retail outlet*				
	A	*B*	*C*	*D*	*E*
X	55	30	40	50	50
Y	35	30	100	45	60
Z	40	60	95	35	30

Determine the optimal distribution to minimize the cost of transportation.

[Delhi M.B.A. (Nov.) 2003; Pune M.B.A. 1997]

1032. Solve the following transportation problem :

	To						
	9	12	9	6	9	10	5
	7	3	7	7	5	5	6
From	6	5	9	11	3	11	2
	6	8	11	2	2	10	9
	4	4	6	2	4	2	22

[Meerut M.Sc. (Math.) 2000; Annamalai M.B.A. 2003]

1033. The table below records transportation costs per unit of a product from origins O_1, O_2, O_3 and O_4 to destinations D_1, D_2, D_3, D_4 and D_5. The capacities of the four origins are respectively 55, 45, 30 and 50, while the requirements of the five destinations are respectively 40, 20, 50, 30 and 40. Employing Vogel's Approximation method, or otherwise, make the initial allocations to the origins to satisfy the requirements of the destinations and test the optimality of these allocations. Use MODI method for obtaining an optimal solution that would minimize the total cost of transportation.

Origin	*Destination*					*Capacity*
	D_1	D_2	D_3	D_4	D_5	
O_1	12	4	9	5	9	55
O_2	8	1	6	6	7	45
O_3	1	12	4	7	7	30
O_4	10	15	6	9	1	50
Requirement	40	20	50	30	40	

Find the total cost implied by the solution. Is the solution unique? If not, find an alternative solution as well.

[Delhi M.Com. 2008]

1034. The following table gives the cost of transporting material from supply points A, B, C and D to demand points E, F, G, H and J :

$$\begin{array}{cc} & \text{To} \\ & \begin{array}{ccccc} E & F & G & H & J \end{array} \\ \text{From} \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 8 & 10 & 12 & 17 & 15 \\ 15 & 13 & 18 & 11 & 9 \\ 14 & 20 & 6 & 10 & 13 \\ 13 & 19 & 7 & 6 & 12 \end{bmatrix} \end{array}$$

The present allocation is as follows :

A to E — 90; A to F — 10; B to F — 150; C to F — 10;
C to G — 50; C to J — 120; D to H — 210; D to J — 70.

(*a*) Check if this allocation is optimum. If not, find an optimum schedule.

(*b*) If in the above problem the transportation cost from A to G is reduced to 10, what will be the new optimum schedule ?

[Annamalai M.B.A. 2002; Delhi BFIA 2007]

10:15. SOME EXCEPTIONAL CASES

In the preceding sections, we discussed transportation problems for minimizing the cost or time or distance when total demand equals total supply, *i.e.*, the balanced problems.

In real life there may be problems, where total demand may not equal total supply—the objective may be of maximization, there may be restriction on the transportation routes, and so on. This section deals with the resolution of such difficulties.

Unbalanced Transportation Problem

A necessary and sufficient condition for the existence of feasible solution to the general transportation problem is that the total demand must equal the total supply, *i.e.*,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

However, sometimes there may be more demand than the supply and *vice versa* in which case the problem is said to be unbalanced. It may occur in the following situations :

(*i*) $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$ and (*ii*) $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$

In *case 1*, we introduce a *dummy destination* in the transportation table. The cost of transporting to this destination are all set equal to zero. The requirement at this dummy destination is then assumed to be equal to

$$\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$$

In *case 2*, we introduce a *dummy source* in the transportation table. The cost of transporting from this source to any destination are all set equal to zero. The availability at this dummy source is assumed to be equal to

$$\sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i.$$

Alternative Optimal Solutions

The existence of alternative optimal solutions can be determined by an inspection of the opportunity costs, $z_{ij} - c_{ij}$ for the unoccupied cells. If an unoccupied cell in an optimal solution has opportunity cost of zero, then an alternative optimal solution can be formed with another set of allocations without increasing the total transportation cost.

Prohibited Transportation Routes

The situation may arise such as road hazards (snow, flood, etc.), traffic regulations, etc., when it is not possible to transport goods from certain sources to certain destinations. Such type of problems can be handled by assigning a very large cost say M to that route (or cell).

Maximization Transportation Problem

In general, transportation method is used for minimization problems. However it may also be used to solve problems in which objective is to maximize when we consider the unit profit (or payoff) p_{ij} instead of unit cost c_{ij} associated with each route (i, j). The solution procedure for solving such problems is summarized below.

Step 1. Convert the given problem into that of minimisation by replacing each element of the transportation table by its difference from the maximum element of the table.

Step 2. Find an initial feasible solution using any of the three methods.

Step 3. Use *MODI* method for finding an optimum solution.

SAMPLE PROBLEMS

1035. *A company manufacturing air-coolers has two plants located at Mumbai and Kolkata with a weekly capacity of 200 units and 100 units, respectively. The company supplies air-coolers to its 4 showrooms situated at Ranchi, Delhi, Lucknow and Kanpur which have a demand of 75, 100, 100 and 30 units, respectively. The cost of transportation per unit (in Rs.) is shown in the following table :*

	Ranchi	*Delhi*	*Lucknow*	*Kanpur*
Mumbai	90	90	100	100
Kolkata	50	70	130	85

Plan the production programme so as to minimize the total cost of transportation.

[Panjab M.B.A. 2007; Pune M.Com. 2007]

Solution. Since the total demand (of 305 units) at Ranchi, Delhi, Lucknow and Kanpur is greater than the total capacity (of 300 units) at Mumbai and Kolkata, by (305 – 300) = 5 units, the problem is unbalanced. A dummy plant with its weekly capacity of 5 units with zero transportation cost from this plant to all demanding cities is added.

Using *Vogel's Approximation Method (VAM)*. The initial basic feasible solution is obtained and displayed in Table 10.28.

	Ranchi	*Delhi*	*Lucnow*	*Kanpur*	u_i
Mumbai	-20 (90)	75 (90)	95 (100)	30 (100)	0
Kolkata	75 (50)	25 (70)	-50 (130)	-5 (85)	-20
Dummy	-70 (0)	-10 (0)	5 (0)	0 (0)	-100
v_j	70	90	100	100	

Table 10.28

Since there are 6 (*i.e.*, 4 + 3 – 1) allocations, the solution is non-degenerate. Using MODI's algorithm, we observe that all $z_{ij} - c_{ij} \leq 0$ (*i.e.*, all the opportunity costs of the unoccupied cells are non-positive). Therefore, the optimum solution is reached and the associated cost associated with this solution is Rs. 24,750.

Alternate Optimal Solution

An inspection of the opportunity costs ($z_{ij} - c_{ij}$) of the unoccupied cells in the optimum Table 10.28 reveals that $z_{34} - c_{34} = 0$. This indicates the existence of an alternative optimum solution. Therefore, if we allocate an unknown quantity θ to the unoccupied cell (3, 4), the iterative transportation table becomes :

	75	95 (+θ)	30 (–θ)	
90	90	100	100	
75	25			
50	70	130	85	
		5		
0	0	–θ 0	+θ 0	

Table 10.29

				u_i
	75	100	25	
-20 (90)	90	100	100	0
75	25			
50	70	-50 (130)	-5 (85)	-20
			5	
-30 (0)	-10 (0)	0 (0)	0 (0)	-100
v_j 70	90	100	100	

Table 10.30

The maximum quantity that can be allocated to the cell (3, 4) is 5. After substituting θ = 5, the alternative solution is obtained in Table 10.30.

From the above table, we observe that although the total transportation cost remains same, *viz.*, Rs. 24,750, the alternative optimum allocation is :

$$x_{12} = 75,\ x_{13} = 100,\ x_{14} = 25,\ x_{21} = 75 \text{ and } x_{22} = 25.$$

1036. (*a*) *Consider the following transportation problem :*

Factory	Godowns 1	2	3	4	5	6	Stock available
A	7	5	7	7	5	3	60
B	9	11	6	11	—	5	20
C	11	10	6	2	2	8	90
D	9	10	9	6	9	12	50
Demand	60	20	40	20	40	40	

It is not possible to transport any quantity from factory B to Godown 5.

Determine :

(*i*) *Initial solution by Vogel's Approximation Method.*

(*ii*) *Optimum Basic Feasible Solution.*

(*iii*) *Is the optimum solution unique? If not, find an alternative optimum basic feasible solution.*

Solution. (*i*) Since it is not possible to transport any quantity from Factory *B* to Godown *5*, we assign a very high cost to the cell (2, 5), say *M*. Then using Vogel's Approximation Method to find the initial basic feasible solution, we obtain Table 10.31.

Since number of occupied cells is 8, *i.e.*, less than $(4+6-1)$ there is degeneracy in the initial solution. To overcome degeneracy, we allocated a small quantity $\in$ (>0) in the cell (1, 5) being the unoccupied cell having the lowest transportation cost.

(*ii*) To test the initial solution for optimality, we use *MODI* method.

							Row Differences
		20			∈	40	
	7	5	7	7	5	3	(2) (2) (4) (6)
10			10				
	9	11	6	11	M	5	(1) (1) (1) (3)
			30	20	40		
	11	10	6	2	2	8	(0) (4) (2) —
50							
	9	10	9	6	9	12	(3) (0) (0) (0)
Column Differences	(2)	(5)	(0)	(4)	(3)	(2)	

Table 10.31

The iterative transportation tables are as follows :

Initial Iteration. Introduce cell (1, 1) and drop cell (1, 5).

							u_i
+θ	20			∈ −θ	40		
5 7	5 2	7	-2 7	5	3		0
10		10 +θ					
−θ 9	-9 11	6	-9 11	2-M M	-5 5		-3
		30	20	40 +θ			
-2 11	-8 10	−θ 6	2	2	-8 8		-3
50							
9	-8 10	-3 9	-4 6	-7 9	-12 12		-3
v_j 12	5	9	5	5	3		

Table 10.32

							u_i
$\in$	20				40		0
7	5	-3 7	-7 7	-5 5		3	
10		10					2
9	-4 11	6	-9 11	2-M M	0	5	
		30	20	40			2
-2 11	-3 10	6	2	2	-3	8	
50							2
9	-3 10	-3 9	-4 6	-7 9	-7	12	
v_j 7	5	4	0	0	3		

Table 10.33

Final Iteration. Optimum Solution. See Table 10.33.

In the above transportation table, since all the net evaluations are non-positive, an optimum solution has been obtained. Hence, the optimum solution is

$$x_{11} = 0,\ x_{12} = 20,\ x_{16} = 40,\ x_{21} = 10,\ x_{23} = 10$$

$$x_{33} = 30,\ x_{34} = 20,\ x_{35} = 40 \text{ and } x_{41} = 50.$$

The minimum transportation cost is

$$1120 + 7\in \text{ or } 1120 \text{ as } \in \to 0.$$

(*iii*) Since $z_{26} - c_{26} = 0$, there exists an alternative optimum solution. Letting the unoccupied cell (2, 6) enter the basis it is easily seen that the currently occupied cell (1, 1) becomes unoccupied in the next iteration. In the revised transportation table, the following alternative optimum solution is obtained :

$$x_{12} = 20,\ x_{16} = 40,\ x_{21} = 10,\ x_{23} = 10,\ x_{26} = 0,\ x_{33} = 30,\ x_{34} = 20,$$

$$x_{35} = 40, \text{ and } x_{41} = 50 \text{ with minimum transportation cost as } 1120.$$

1036 (*b*). *Find an optimum solution to the following Transportation Problem :*

Factory	*Warehouse*				*Capacity*
	D	*E*	*F*	*G*	
A	42	48	38	37	160
B	40	49	52	51	150
C	39	38	40	43	190
Demand	80	90	110	160	

[Delhi B.Sc. (Stat.) 2006; Bharthidasan B.Sc. (Maths.) 1995]

Solution. As the total supply exceeds the total demand, we introduce a dummy warehouse with its requirement as 500 – 440, *i.e.*, 60 units. The resulting balanced transportation problem is:

	D	*E*	*F*	*G*	*H*	
A	42	48	38	37	0	160
B	40	49	52	51	0	150
C	39	38	40	43	0	190
	80	90	110	160	60	

Using Vogel's Approximation Method for initial basic feasible solution and *MODI* method for Optimum solution, we get the Table 10.34.

						u_i
	(–ve)	(–ve)	ϵ	160	(–ve)	–13
	42	48	38	37	0	
	80	10	(–ve)	(–ve)	60	0
	40	49	52	51	0	
	(–ve)	80	110	(–ve)	(–ve)	–11
	39	38	40	43	0	
v_j	40	49	51	50	0	

Table 10.34

The optimum solution is

$$x_{14} = 160,\ x_{21} = 80,\ x_{22} = 10,\ x_{32} = 80,\ \text{and}\ x_{33} = 110.$$

The minimum transportation cost according to the above schedule is 17,050.

1037. *ABC Company wishes to develop a monthly production schedule for the next three months. Depending upon the sales commitments, the company can either keep the production constant, allowing fluctuations in inventory, or inventories can be maintained at a constant level, with fluctuating production. Fluctuating production necessitates in working overtime, the cost of which is estimated to be double the normal production cost of Rs. 12 per unit. Fluctuating inventories result in inventory carrying cost of Rs. 2 per unit. If the company fails to fulfil its sales commitment, it incurs a shortage cost of Rs. 4 per unit per month. The production capacities for the next three months are shown below :*

Month	*Production capacity*		*Sales*
	Regular	*Overtime*	
1	50	30	60
2	50	0	120
3	60	50	40

Determine the optimal production schedule. ***[Delhi M.B.A. 2001; Himachal B.Tech. (Mech.) 2006]***

Solution. The cost elements in each cell are determined as follows :

(*i*) If items are produced in a month for sales during the same month, then there will be no inventory carrying cost. Thus, total cost will be either the normal production cost or overtime production cost. Thus, the cost elements for cells (R_1, 1), (R_2, 2) and (R_3, 3) are Rs. 12 each and for cells (O_1, 1), (O_2, 3) are Rs. 24 each.

(*ii*) If items are produced in a month for sales during subsequent month in addition to the production costs (normal or overtime), inventory carrying cost at the rate of Rs. 2 per month will be incurred.

Cell	*Production cost (Rs.)*	*Inventory carrying cost (Rs.)*	*Total cost (Rs.)*
(R_1, 2)	12	2	14
(R_1, 3)	12	4	16
(O_1, 2)	24	2	26
(O_1, 3)	24	4	28
(R_2, 3)	12	2	14

(*iii*) If the company fails to fulfil its sales commitment, it incurs a shortage cost of Rs. 4 per unit per months, in addition to the production costs (normal or overtime), carrying (storage) cost at the rate of Rs. 2 per month will be incurred.

Cell	*Production cost (Rs.)*	*Shortage cost (Rs.)*	*Total cost (Rs.)*
$(R_2, 1)$	12	4	16
$(R_3, 2)$	12	4	16
$(R_3, 1)$	12	8	20
$(O_3, 2)$	24	4	28
$(O_3, 1)$	24	8	32

Using the above information, the given problem can be presented in tabular form as follows :

Month	*Total cost (Rs.)*					*Demand*
	R_1	O_1	R_2	R_3	O_3	
M_1	12	24	16	20	32	60
M_2	14	26	12	16	28	120
M_3	16	28	14	12	24	40
Supply	50	30	50	60	50	

Since, total supply exceeds total demand, we add a dummy month with its capacity 20 units having zero transportation costs from dummy month to R_1, O_1, R_2, R_3 and O_3.

Using now *Vogel's Approximation Method*, the initial basic feasible solution is obtained as displayed in table below :

						u_i
	50 / 12	10 / 24	-6 / 16	-6 / 20	-6 / 32	-2
	0 / 14	20 / 26	50 / 12	20 / 16	30 / 28	0
	-6 / 16	-6 / 28	-6 / 14	40 / 12	0 / 24	-4
	-14 / 0	-2 / 0	-16 / 0	-12 / 0	20 / 0	-28
v_j	14	26	12	16	28	

Table 10.35

Since there are $4+5-1$ $(=8)$ allocations, the solution is non-degenerate. Using *MODI's algorithm*, we observe that all $z_{ij} - c_{ij} \leq 0$ (*i.e.*, all opportunity costs in the unoccupied cells are non-positive). Therefore, the optimum solution is reached and the associated total minimum cost is Rs. 3,400.

Optimum production schedule is :

Production in		*No. of units*	*For use of*
M_1	R_1	50	50 units in M_1
	O_1	30	10 units in M_1 and 20 in M_2.
M_2	R_2	50	50 units in M_2
M_3	R_3	60	20 units in M_2 and 40 in M_3
	O_3	30	30 units in M_2.

1038. *ABC Enterprises is having three plants manufacturing dry-cells, located at different locations. Production cost differs from plant to plant. There are five sales offices of the company located in different regions of the country. The sales prices can differ from region to region. The shipping cost from each plant to each sales office and other data are given below :*

Production Data

Production cost per unit (Rs.)	*Max. capacity in no. of units*	*Plant no.*
20	150	1
22	200	2
18	125	3

Shipping Costs (in Rs.)

	Sales Offices				
	A	*B*	*C*	*D*	*E*
Plant 1	1	1	5	9	4
Plant 2	9	7	8	3	6
Plant 3	4	5	3	2	7

Demand and Sales Prices

Demand	80	100	75	45	125
Sales Price (Rs.)	30	32	31	34	29

Find the production and distribution schedule most profitable to the company.

[Delhi B.Sc. (Stat.) 2005, M.Com. 2006, M.I.B. 2008; Pune M.Com. 2009]

Solution. In order to solve the transportation problem, we use the given information to derive the profit matrix. This is done as follows :

Profit = Sales Price – Production Cost – Shipping Cost.

Thus, if we transport one unit of dry-cell from each of the three plants to each of the 5 sales offices, the following profit matrix is obtained :

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 9 & 11 & 6 & 5 & 5 \\ -1 & 3 & 1 & 9 & 1 \\ 8 & 9 & 10 & 14 & 4 \end{bmatrix} \end{array}$$

Here, if we transport one unit from plant 2 to sales office D, the profit obtained is :

Profit = Rs. 34 – Rs. 22 – Rs. 3.

Above matrix is converted into a loss matrix by subtracting each element of the profit matrix from 14 (largest element). Thus, we have

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 5 & 3 & 8 & 9 & 9 \\ 15 & 11 & 13 & 5 & 13 \\ 6 & 5 & 4 & 0 & 10 \end{bmatrix} \end{array}$$

Since the total demand (of 425 units) at five sales offices is less than the total supply (of 475 units) at the three plants, the problem is unbalanced. A dummy sales office with its demand of 50 units with zero transportation cost from each of the three plants to this sales office is added.

Using *Vogel's Approximation Method* to this balanced problem, the initial basic feasible solution is obtained and displayed in Table 10.36 :

50 / 5	100 / 3	8	9	9	0
25 / 15	11	13	5	125 / 13	50 / 0
5 / 6	5	75 / 4	45 / 0	10	0

Table 10.36

Since there are 8 allocations, the solution is non-degenerate. Using *MODI*'s algorithm, we have following iterative transportation Table 10.37 :

							u_i
50	100						
5	3	-5	8	-10	9	-6	9	-10	0	5
25 $-\theta$				$+\theta$	125	50	15
15	2	11	0	13	4	5	13	0
5			75	45			6
$+\theta$	6	-1	5	4	$-\theta$	0	-6	10	-9	0
v_j	0	-2	-2	-6	-2	-15	

Table 10.37

Initial Iteration. Introduce cell (2, 4) and drop cell (2, 1).

Final Iteration. Optimum Solution.

							u_i
50	100						
5	3	-5	8	-10	9	-2	9	-6	0	-6
				25	125	50	0
-4	15	-2	11	-4	13	5	13	0
30			75	20			-5
6	-1	5	4	0	-2	10	-5	0
v_j	11	9	9	5	13	0	

Table 10.38

Since all $z_{ij} - c_{ij} \leq 0$, Table 10.38 will give the optimum solution. The optimum allocations are given below :

$$x_{11} = 50,\ x_{12} = 100,\ x_{24} = 25,\ x_{25} = 125,\ x_{31} = 30,\ x_{33} = 75 \text{ and } x_{34} = 20.$$

$$\text{Total Profit} = 50 \times 9 + 100 \times 11 + 25 \times 9 + 125 \times 1 + 30 \times 8 + 75 \times 10 + 20 \times 14$$
$$= 450 + 1100 + 225 + 125 + 240 + 750 + 280$$
$$= \text{Rs. } 3{,}170.$$

PROBLEMS

1039. National Oil Company (NOC) has three refineries and four depots. Transportation cost per ton, capacities, and requirements are given below :

	D_1	D_2	D_3	D_4	*Capacity (tons)*
R_1	5	7	13	10	700
R_2	8	6	14	13	400
R_3	12	10	9	11	800
Requirement	200	600	700	400	

Determine optimum allocation of output. *[Gujarat M.B.A. 1996]*

1040. Four gasoline dealers *A*, *B*, *C* and *D* require 50, 40, 60 and 40 KL of gasoline respectively. It is possible to supply these from locations 1, 2 and 3 which have 80, 100 and 50 KL respectively. The cost (in Rs.) for shipping every KL is shown in the following table :

		A	B	C	D
Location	1	7	6	6	6
	2	5	7	6	7
	3	8	5	8	6

Determine the most economical supply pattern. *[Jammu M.B.A. 1996]*

1041. A company has three warehouses in cities *A*, *B* and *C*. These warehouses have the following quantities of the product in stock :

Distributorships :	*A*	*B*	*C*
Capacity :	100 units	80 units	80 units

The four customers have the demands as follows :

(*1*) 60 units, (*2*) 120 units, (*3*) 50 units, (*4*) 40 units.

If cost is 20 paise per km. to transport this product from a distributor to customer. The distance between warehouses and customers can be obtained from a map or suitable route tables as follows :

$$\text{Warehouse} \quad \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \text{Customer} \\ \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 270 & 230 & 310 & 690 \\ 100 & 450 & 400 & 320 \\ 300 & 540 & 350 & 570 \end{bmatrix} \end{array}$$

Which warehouse should deliver how much product to which customer so that the total transportation cost becomes a minimum?

1042. A company has three plants at locations *A*, *B* and *C* which supply to warehouses located at *D*, *E*, *F*, *G* and *H*. Monthly plant capacities are 800, 500 and 900 units respectively. Monthly warehouse requirements are 400, 400, 500, 400 and 800 units respectively. Unit transportation costs (in rupees) are given below :

$$\text{From} \quad \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \text{To} \\ \begin{array}{ccccc} D & E & F & G & H \end{array} \\ \begin{bmatrix} 5 & 8 & 6 & 6 & 3 \\ 4 & 7 & 7 & 6 & 5 \\ 8 & 4 & 6 & 6 & 4 \end{bmatrix} \end{array}$$

Determine an optimum distribution for the company in order to minimize the total transportation cost.
[Delhi B.Sc. (Stat.) 2000, 1998; Bharathidasan B.Sc. (Math.) 1995]

1043. The *ABC* Tool Company has a sales force of 25 men who work out from three regional offices. The company produces four basic product lines of hand tools. Mr. Jain, the sales manager feels that 6 salesmen are needed to distribute product line 1, 10 salesmen to distribute product line 2, 4 salesmen for product line 3 and 5 salesmen for product line 4. The cost (in Rs.) per day of assigning salesmen from each of the offices for selling each of the product lines are as follows :

	Product lines			
	1	2	*3*	*4*
Regional office *A*	20	21	16	18
Regional office *B*	17	28	14	16
Regional office *C*	29	23	19	20

At the present time, 10 salesmen are allocated to office *A*, 9 salesmen to office *B*, and 7 salesmen to office *C*. How many salesmen should be assigned from each office to sell each product line in order to minimize costs? Identify alternative optimum solutions, if any. *[Delhi M.B.A. 2001; Delhi M.Com. 2006]*

1044. Priyanshu Enterprise has three factories at locations *A*, *B* and *C* which supply three warehouses located at *D*, *E* and *F*. Monthly factory capacities are 10, 80 and 15 units respectively. Monthly warehouse requirements are 75, 20 and 50 units respectively. Unit shipping costs (in Rs.) are given below :

Factory	Warehouse		
	D	*E*	*F*
A	5	1	7
B	6	4	6
C	3	2	5

The penalty costs for not satisfying demand at the warehouse *D, E* and *F* are Rs. 5, Rs. 3 and Rs. 2 per unit respectively. Determine the optimal distribution for Priyanshu, using any of the known algorithms.

[Delhi PG Dip. in Glob. Bus. Oper. 2010]

1045. A company has three factories at *A, B* and *C* which supply warehouses at *D, E, F* and *G* respectively. Monthly production capacities of these factories are 250, 300 and 400 units respectively. If overtime production is utilised, factories *A* and *B* can produce 50 and 75 additional units with an incremental cost of Rs. 4 and 5 respectively. The current warehouse requirements are 200, 225, 275 and 300 units, respectively. Unit transportation costs (in Rupees) from factories to warehouses are as follows :

From	*To*			
	D	*E*	*F*	*G*
A	11	13	17	14
B	16	18	14	10
C	21	24	13	10

Determine the optimum distribution for this company, to minimize cost.

[Pune M.B.A. 1999; Shivaji M.B.A. 1996]

1046. A donor agency is interested in supplying a special drug to three cities *A, B* and *C*. It is able to identify only two manufacturing units *X* and *Y*, who can supply the drug to these three cities. Drug demands of these cities *A, B* and *C* are 150, 250 and 100 units respectively. Manufacturing capacities of the plants *X, Y* are 200 and 150 units. Transportation costs are given below :

From Plant *X* to City *A* — Rs. 4,000 per unit
From Plant *X* to City *B* — Rs. 6,000 per unit
From Plant *X* to City *C* — Rs. 8,000 per unit
From Plant *Y* to City *A* — Rs. 9,000 per unit
From Plant *Y* to City *B* — Rs. 5,000 per unit
From Plant *Y* to City *C* — Rs. 7,000 per unit

Shortfall in demand can be met through plants located in the respective cities themselves without any transportation cost but at additional price of Rs. 3,000 per unit for city *A*, for Rs. 7,000 per unit for City *B* and Rs. 10,000 per unit for City *C*.

Give the drug supply schedule which the donor agency should follow to minimize the cost.

[Delhi PG Dip. Glob. Bus. Oper. 2011]

1047. The Purchase Manager, Mr. Shah, of the State Road Transport Corporation must decide on the amount of fuel to buy from three possible vendors. The corporation refuels its buses regularly at the four depots within the area of its operations.

The three oil companies have said that they can furnish up to the following amounts of fuel during the coming month : 275,000 litres by oil company *1*; 50,000 litres by oil company *2*; and 660,000 litres by oil company *3*. The required amount of the fuel is 110,000 litres by depot *2*; 20,000 litres at depot *2*; 330,000 litres at depot *3*; and 440,000 litres at depot *4*.

When the transportation costs are added to the bid price per litre supplied, the combined cost per litre for fuel from each vendor servicing a specific depot is shown below :

	Company 1	*Company 2*	*Company 3*
Depot *1*	5.00	4.75	4.25
Depot *2*	5.00	5.50	6.75
Depot *3*	4.50	6.00	5.00
Depot *4*	5.50	6.00	4.50

Determine the optimum transportation schedule. *[Delhi M.B.A. (Nov.) 1997]*

1048. *XYZ* Tobacco Company purchases tobacco and stores in warehouses located in the following four cities :

Warehouse location	*Capacity (tonnes)*
City *A*	90
City *B*	50
City *C*	80
City *D*	60

The warehouses supply tobacco to cigarette companies in three cities that have the following demand :

Cigarette company	*Demand (tonnes)*
Bharat	120
Janta	100
Red Lamp	110

The adjoining railroad shipping costs per tonne (in hundred rupees) have been determined :

From	To		
	Bharat	Janta	Red Lamp
A	7	10	5
B	12	9	4
C	7	3	11
D	9	5	7

Because of railroad construction, shipments are temporarily prohibited from warehouse at city *A* to Bharat Cigarette Company.

(*i*) Find the optimum distribution for *XYZ* Tobacco Company.

(*ii*) Are there multiple optimum solutions ? If there are alternative optimum solutions, identify them.

[Panjab M.B.A. 1997; Delhi M.B.A. (Nov.) 2009]

1049. A manufacturer of jeans is interested in developing an advertising campaign that will reach four different age groups. Advertising campaigns can be conducted through T.V., radio and magazines. The following table gives the estimated cost in paise per exposure for each age group according to the medium employed. In addition, maximum exposure levels possible in each of the media, namely T.V., radio, magazines are 40, 30 and 20 millions respectively. Also the minimum desired exposure within each age group, namely *13–18, 19–25, 26–35,* 36 and older are 30, 25, 15 and 10 millions. The objective is to minimize the cost of attaining the minimum exposure level in each age group.

Media	Age groups			
	13–18	*19–25*	*25–35*	*36 and older*
T.V.	12	7	10	10
Radio	10	9	12	10
Magazine	14	12	9	12

Formulate the above as a transportation problem and find the optimal solution. *[Delhi M.B.A. (March) 2011]*

1050. Solve the following transportation problem to maximize profit and give criteria for optimality :

Origin	Destination Profit (Rs. per unit)				Supply
	1	*2*	*3*	*4*	
A	40	25	22	33	100
B	44	35	30	30	30
C	38	38	28	30	70
Demand	40	20	60	30	

[Gurunank Dev B.Com. 2007]

1051. Maximize the following T.P. using highest profit method :

	D_1	D_2	D_3	D_4	D_5	D_6	Supply
F_1	35	41	28	16	20	12	320
F_2	14	21	28	30	15	24	180
F_3	45	18	17	29	26	19	200
Demand	125	125	100	100	175	75	

[Osmania M.B.A. 2000]

1052. A company produces a small component for all industrial products and distributes it to five wholesalers at a fixed price of Rs. 2.50 per unit. Sales forecasts indicate that monthly deliveries will be 3,000, 3,000, 10,000, 5,000 and 4,000 units to wholesalers *1, 2, 3, 4* and *5* respectively. The monthly production capabilities are 5,000, 10,000, 12,500 at plants *1, 2* and *3* respectively. The direct costs of production of each unit are Rs. 1.00, Rs. 0.90 and Rs. 0.80 at plants *1, 2* and *3* respectively. The transportation costs of shipping a unit from a plant to a wholesaler are given below :

Plant	Wholesaler				
	1	*2*	*3*	*4*	*5*
1	0.05	0.07	0.10	0.15	0.15
2	0.08	0.06	0.09	0.12	0.14
3	0.10	0.09	0.08	0.10	0.15

Find how many components each plant supplies to each wholesaler in order to maximize profit.

[Delhi B.Sc. (Stat.) 2000; Meerut M.Sc. (Math.) 2010]

1053. A particular product is manufactured in factories *A, B, C* and *D* and is sold at centres *1, 2* and *3*. The cost (in rupees) of product per unit and capacity (in kg.) per unit time of each plant is given below :

Factory	*Cost (Rs.) per unit*	*Capacity (kg.) per unit*
A	12	100
B	15	20
C	11	60
D	13	80

The sale price in rupees per unit and the demand in kg. per unit time are as follows :

Sales centre	*Sale price (Rs.) per unit*	*Demand (kg.) per unit*
1	15	120
2	14	140
3	16	60

Find an optimum sales distribution. *[Bangalore M.B.A. 2007]*

[*Hint.* Profit = Sales Price – Cost Price.]

1054. A departmental store wishes to purchase the following types of sarees :

Types of sarees	:	*A*	*B*	*C*	*D*	*E*
Quantity	:	150	100	75	250	200

Tenders are submitted by four different manufacturers who undertake to supply not more than the quantities mentioned below (all types of sarees combined) :

Manufacturer	:	*W*	*X*	*Y*	*Z*
Total quantity	:	300	250	150	200

The store estimates that its profit per saree will vary with the manufacturer as shown below :

Manufacturer	*Saree*				
	A	*B*	*C*	*D*	*E*
W	275	350	425	225	150
X	300	325	450	175	100
Y	250	350	475	200	125
Z	325	275	400	250	175

Use transportation technique to determine how the orders should be placed? What is the maximum profit?

[Pune M.B.A. 1999; Punjabi M.B.A. 1996]

1055. *XYZ* and Co. has provided the following data seeking your advice on optimum investment strategy :

Investment made at beginning of year	*Net return data (in Paise) of selected investments*				*Amount available (Lacs)*
	P	*Q*	*R*	*S*	
1	95	80	70	60	70
2	75	65	60	50	40
3	70	45	50	40	90
4	60	40	40	30	30
Maximum Investment (Lacs)	40	50	60	60	—

The following additional information is also provided :

- *P, Q, R* and *S* represent the selected investments.
- The company has decided to have four years investment plan.
- The policy of the company is that amount invested in any year will remain so until the end of the fourth year.
- The values (Paise) in the table represent net return on investment of one Rupee till the end of the planning horizon (for example, a Rupee invested in Investment *P* at the beginning of year 1 will grow to Rs. 1.95 by the end of the fourth year, yielding a return of 95 paise).

Using the above, determine the optimum investment strategy. *[Delhi B.Com. (Hons.) 2010]*

1056. A company has four factories situated in four different locations in the country and four sales agencies located in four other locations in the country. The cost of production (Rs. per unit), the sales price, capacities and monthly requirement are given below :

Factory	Sales agency				Monthly capacity (units)	Cost of production
A	7	5	6	4	10	10
B	3	5	4	2	15	15
C	4	6	4	5	20	16
D	8	7	6	5	15	15
Monthly requirement (units)	8	12	18	22	—	—
Sales price	20	22	25	18	—	—

Find the monthly production and distribution schedule which will maximize profit.

[*Hint :* Profit = Sales Price – Production Cost – Shipping Cost.] *[C.A. Final (May) 1996]*

1057. Three refineries with maximum daily capacities of 6, 5 and 8 million gallons of gasolene supply three distribution areas with daily demands of 4, 8 and 4 million gallons, through a network of pipelines. The transportation cost (in Rs.) per 100 gallons is given below :

		Distribution area		
		1	2	3
Refinery	1	120	180	—
	2	300	100	80
	3	200	250	120

Note : The refinery *1* is not connected to distribution area *3*.

Any surplus production at refineries *1* and *2* must be diverted to other areas by trucks. The resulting transportation cost per 100 gallons is Rs. 150 from refinery *1* and Rs. 220 from refinery *2*. The refinery *3* can divert its surplus to a chemical process within its own plant at no cost. Formulate the problem as transportation model and find the optimal solution. *[Kerala B.E. (Prod.) 1994]*

1058. A company has factories at four different places (1, 2, 3 and 4) which produces 200, 175, 105 and 230 items respectively and supply items to warehouses *A*, *B*, *C*, *D* and *E*. Monthly warehouse requirements are 110, 90, 120, 230 and 160 respectively. Unit shipping costs are given in the following table (the costs are given in rupees):

		To				
		A	B	C	D	E
From	1	13	—	31	8	20
	2	14	9	17	26	10
	3	25	11	12	17	15
	4	10	21	13	—	17

Shipments from 1 to *B* and from 4 to *D* are not possible. Determine the optimum distribution plan to minimize shipping cost. *[Madras M.C.A. 2002, M.B.A. 2008]*

1059. A contractor has undertaken construction at 5 different locations in a city. The cost of transportation of unit material for location j (= *1, 2, 3, 4, 5*) are given below. At location i (= *1, 2, 3*) stocks of material of 150, 200 and 170 units respectively are available. 300 units are to be sent to location *4* and the rest to location *5*. Find the cheapest way of doing this :

From	To				
	1	2	3	4	5
1	—	8	9	15	12
2	6	—	7	21	11
3	12	9	—	17	18
4	13	8	14	—	10
5	7	6	12	10	—

1060. A company has four factories F_1, F_2, F_3 and F_4 manufacturing the same product. Production and raw material costs differ from factory to factory, and are given in the following table in the first two rows. The transportation costs from the factories to sales depots S_1, S_2, S_3 are also given.

The last two columns in the table give the sale price and the total requirement at each depot. The production capacity of each factory is given in the last row :

Item		Factories F_1	F_2	F_3	F_4	Sales price per unit	Requirement
Production cost per unit		15	18	14	13		
Raw material cost per unit		10	9	12	9		
Transportation cost per unit	S_1	3	9	5	4	34	80
	S_2	1	7	4	5	32	120
	S_3	5	8	3	6	31	150
Production capacity		10	150	50	100		

Determine the most profitable production and distribution schedule and the corresponding profit. The surplus should be taken to yield zero profit. *[C.A. Final (Nov.) 2002; Delhi M.B.A. (PT.) Nov. 2009]*

[*Hint :* Profit = Sales price – production cost – raw material cost – transportation cost. Using this construct profit table]

1061. A manufacturer must produce a certain product in sufficient quantity to meet contracted sales in next four months. The production facilities available for this product are limited, but by different amounts in respective months. The unit cost of production also varies in each month.

The product may be produced in one month and then held for sale in a later month, but at an estimated storage cost of Re. 1 per unit per month. No storage cost is incurred for goods sold in the same month in which they are produced. There is no initial inventory and none is desired at the end of 4 months. Given the following table, show how much to produce in each of four months in order to minimize total cost. :

Month	Contracted sales (in units)	Maximum production (in units)	Unit cost of production (in Rs.)
1	20	40	14
2	30	50	16
3	50	30	15
4	40	50	17

Formulate and solve the above problem as a transportation problem. *[Delhi M.B.A. 1996]*

[*Hint :* Transportation problem is as follows :

	To (Months)				Production
From (Months)	14	15	16	17	40
	—	16	17	18	50
	—	—	15	16	30
	—	—	—	17	50
Sales	20	30	50	40]	

1062. The demand and production costs vary from month to month in an industry. The following table contains budgeted information of a firm in this industry on the quantity demanded, the production cost per unit and the production capacity in each of the coming five months.

Month	January	February	March	April	May
Demand	200	250	150	80	120
Production cost	24	27	32	30	34
Capacity	250	225	250	200	225

It is known that the production in any month can meet demand in that month or can be held for the future. The holding cost is Rs. 5 per unit per month.

Using this information :

(*i*) Formulate a transportation model to assist in the production planning, and

(*ii*) Determine what would the optimal plan be. *[Delhi M.B.A. (Nov.) 2003]*

10:16. TIME-MINIMIZATION TRANSPORTATION PROBLEM

Time-minimization transportation problems are similar to the problems discussed so far, except that the unit transportation cost c_{ij} is replaced by the unit time t_{ij} required to transport the items from ith origin to jth destination.

An initial basic feasible solution (feasible plan) for this problem can be obtained by using *Vogel's Approximation Method* or any of the methods. Now, if T_j be the largest transportation time associated with the jth feasible plan, then we have to determine that plan which gives minimum T_j.

A systematic procedure for the solution of such problems is as follows :

Step 1. Determine an initial basic feasible solution. Enter the solution in the upper left corners of the basic cells.

Step 2. Determine T_j for this basic feasible solution and cross out all the non-basic cells for which $t_{ij} \geq T_j$.

Step 3. Construct a loop for the basic cells corresponding to T_j in such a way that when the values at the corner cells are shifted around, the value at this cell tends towards zero and no variable becomes zero. If no such closed path can be formed, the solution under test is optimal, otherwise move on to the next step.

Step 4. Go to *step 2* and repeat the procedure until an optimum basic feasible solution is attained.

SAMPLE PROBLEM

1063. *If the matrix elements represent the unit transportation times, solve the following transportation problem :*

From	*To*				*Available*
	D_1	D_2	D_3	D_4	
O_1	10	0	20	11	15
O_2	1	7	9	20	25
O_3	12	14	16	18	5
Required	12	8	15	10	45

Solution. Using Vogel's Approximation Method an initial basic feasible solution is given in Table 10.39.

10	[8] 0	20	[7] 11
[12] 1	7	[13] 9	20
12	14	[2] 16	[3] 18

Table 10.39

The numbers written at the lower right corners of each cell represent the corresponding times. The time for this feasible plan are

$$t_{12} = 0,\ t_{14} = 11,\ t_{21} = 1,\ t_{23} = 9,\ t_{33} = 16 \text{ and } t_{34} = 18.$$

$\therefore\ T_1 = \max.\{0, 11, 1, 9, 16, 18\} = 18$. Clearly, all the shipments for this plan are completed within the time $T_1 = 18$. So, we cross out the cell (1, 3) and (2, 4) as $t_{13} > T_1$ and $t_{24} > T_1$. Now the closed path (loop) for the cell (3, 4) for which $t_{34} = T_1 = 18$ is shown in Table 10.40.

It is evident from Table 10.40 that only 3 units be shifted around. The next feasible plan with times is shown in Table 10.41.

Here
$$T_2 = \max.\{t_{12}, t_{14}, t_{21}, t_{23}, t_{32}, t_{33}\}$$
$$= \max.\{0, 11, 1, 9, 14, 16\} = 16.$$

10	[8] 0	20	[7] 11
[12] 1	7	[13] 9	20
12	[2] 14	16	[3] 18

Table 10.40

10	[5] 0	20	[10] 11
[12] 1	7	[13] 9	20
12	[3] 14	16	[2] 18

Table 10.41

As all the shipments for the feasible solution are completed within time $T_2 = 16$, we cross out the cell (3, 4), since $t_{34} > T_2$. The closed loop starts from the cell (3, 3) as shown in Table 10.41.

Clearly, only 2 units can be shifted around. The next feasible plan, therefore, is as shown in Table 10.42.

Here
$$T_3 = \text{max. } \{t_{12}, t_{14}, t_{21}, t_{23}, t_{31}, t_{32}\}$$
$$= \text{max. } \{0, 11, 1, 9, 12, 14\} = 14.$$

Since $t_{32} = T_3 = 14$, the closed loop starts from the cell (3, 2) as shown in Table 10.42. It is clear that only 3 units are shifted around. The next feasible solution, therefore, is as shown in Table 10.43.

10	[5] 0	20	[10] 11
[10] 1	7	[15] 9	
12	[3] 14	16	18

Table 10.42

10	[5] 0	20	[10] 11
[7] 1	7	[3] 9	[15] 20
[5] 12	14	16	18

Table 10.43

Here
$$T_4 = \text{max. } \{t_{12}, t_{14}, t_{21}, t_{22}, t_{23}, t_{31}\}$$
$$= \text{max. } \{0, 11, 1, 7, 9, 12\} = 12.$$

Now, we cannot form any other closed loop without increasing the present minimum shipping time. Hence, the feasible plan at this stage is optimal.

Thus, all the shipments can be made within 12 times units.

10:17. TRANSHIPMENT PROBLEMS

A transportation problem in which available commodity frequently moves from one source to another source or destination before reaching its actual destination is called a *transhipment problem*. Such a problem cannot be solved as such by the usual transportation algorithm, but slight modification is required before applying it to the transhipment problem. Following are the chief characteristics of the transhipment problems :

(*a*) The number of sources and destinations in the transportation problem are m and n respectively. In transhipment problem, however, we have $m + n$ sources and destinations.

(*b*) If S_i denotes the ith sources and D_j denotes the jth destinations then commodity can move along the route $S_i \to D_i \to D_j$, $S_i \to S_j \to D_i \to D_j$, $S_i \to D_i \to S_i \to D_j$, or in various other ways. Clearly, transportation cost from S_i to S_i is zero and the transportation costs from S_i to S_j or S_i to D_i do not have to be symmetrical, *i.e.*, in general $S_i \to S_j \neq S_j \to S_i$.

(*c*) In solving the transhipment problem, we first find the optimum solution to the transportation problem, and then proceed in the same fashion as in solving the transportation problem.

(*d*) The basic feasible solution contains $2m + 2n - 1$ basic variables, if we omit the variables appearing in the $m + n$ diagonal cells, we are left with $m + n - 1$ basic variables.

SAMPLE PROBLEM

1064. *A firm having two sources S_1 and S_2 wishes to ship its products to two destinations D_1 and D_2. The number of units available at S_1 and S_2 are 5 and 25 respectively and the product demanded at D_1 and D_2 are 20 and 10 units respectively. The firm, instead of shipping directly from sources to destinations, decides to investigate the possibility of transhipment. The unit transportation costs (in rupees) are given in the following table. Find the optimal shipping schedule :*

		Source		Destination		Available
		S_1	S_2	D_1	D_2	
Source	S_1	0	2	3	4	5
	S_2	2	0	2	4	25
Destination	D_1	3	2	0	1	—
	D_2	4	4	1	0	—
	Demand	—	—	20	10	

Solution. The transportation problem can be extended, which includes four supply points and four demand points. Since all the demand may concentrate at any one supply or demand point, the fictitious supply and demand quantities may be assumed for each of these supply (source) points and demand (destination) points. These quantities may be regarded as *buffer stocks*. Each of these buffer stocks must, at least, be equal to the total supply (or demand) in the original problem. For the given problem the buffer stock will be 30 units. The role of buffer stock in the transhipment problem is only to convert the problem into a standard form and once the optimum solution is obtained, necessary adjustments will be made to conform to the original problem.

Using Vogel's Approximation Method for an initial basic feasible solution and *MODI* method for an optimum solution to the extended transportation problem we obtain the transportation Table 10.44.

	S_1	S_2	D_1	D_2	u_i
S_1	30			5	0
	0	-1 / 2	0 / 3	4	
S_2		30	25		-1
	-3 / 2	0	2	-1 / 4	
D_1			25	5	-3
	-6 / 3	-4 / 2	0	1	
D_2				30	-4
	-8 / 4	-7 / 4	-2 / 1	0	
v_j	0	1	2	3	

Table 10.44

Since all $z_{ij} - c_{ij} \leq 0$ for unoccupied cells, the solution obtained above is optimum.

The allocations in the diagonal cells are ignored, since they do not have any physical meaning (*i.e.*, no transportation). The remaining allocations can be interpreted as follows :

(*a*) Transport 25 units from S_2 to destination D_1. It increases the capacity of S_2 to 55 units including 30 units as buffer stock.

(*b*) From S_1, transport 5 units to destination D_2, and

(*c*) Out of 25 units available at D_1, transport 5 units to D_2.

PROBLEMS

1065. Consider the following transhipment problem with two sources and two destinations, the costs for shipment in rupees are given below. Determine an optimum shipping schedule :

		Source		*Destination*		*Supply*
		S_1	S_2	D_1	D_2	
Source	S_1	0	1	6	2	4
	S_2	1	0	8	2	5
Destination	D_1	0	7	0	3	—
	D_2	1	0	1	0	—
	Demand	—	—	3	6	

1066. Consider a firm having two factories. The firm is to ship its products from the factories to three retail stores. The number of units available at factories X and Y are 200 and 300 respectively, while those demanded at retail stores A, B and C are 100, 150 and 250 respectively. Rather than shipping directly from factories to retail stores, it is added to investigate the possibility of transhipment. The transportation cost (in rupees) per unit is given below :

		Factory		*Retail store*		
		X	Y	A	B	C
Factory	X	0	8	7	8	9
	Y	6	0	5	4	3
Retail store	A	7	2	0	5	1
	B	1	5	1	0	4
	C	8	9	7	8	0

Find an optimal shipping schedule.

MULTIPLE CHOICE QUESTIONS

1. The transportation problem deals with the transportation of
(*a*) a single product from several sources to a destination.
(*b*) a multi-product from several sources to several destinations.
(*c*) a single product from several sources to several destinations.
(*d*) a single product from a source to several destinations.

2. The transportation problem is balanced, if
(*a*) total demand and total supply are equal and the number of sources equals the number of destinations.
(*b*) total demand equals total supply irrespective of the number of sources and destinations.
(*c*) number of sources matches with the number of destinations.
(*d*) none of the routes is prohibited.

3. The initial solution of a T.P. can be obtained by applying any known method. However, the only condition is that
(*a*) the solution must be optimum.
(*b*) the solution should be non- degenerate.
(*c*) the rim conditions are satisfied.
(*d*) all of the above.

4. The solution to a transportation problem with m-sources and n-destinations is feasible, if the number of allocations are
(*a*) $m + n - 1$
(*b*) $m + n + 1$
(*c*) $m + n$
(*d*) $m \times n$.

5. While solving a transportation problem, the occurrence of degeneracy means that
   (*a*) total supply equals total demand.
   (*b*) the solution so obtained is not feasible.
   (*c*) the few allocations become negative.
   (*d*) none of the above.
6. In Vogel's Approximation method
   (*a*) the cost difference indicate the penalties for not using the respective least cost routes.
   (*b*) initial solution to transportation problem is not applicable, if some routes are prohibited.
   (*c*) degeneracy never occurs.
   (*d*) none of the above.
7. The initial solution of a T.P. obtained by
   (*a*) North-West corner rule would invariably be optimum.
   (*b*) least cost method does not provide the least cost solution to a T.P.
   (*c*) VAM would invariably be very near to optimum solution.
   (*d*) MODI method is infeasible.
8. In transportation problem
   (*a*) a solutions is said to degenerate if the number of occupied cells is smaller than the number of rows plus the number of columns minus one (1).
   (*b*) a degenerate solution may or may not be optimum.
   (*c*) to remove degeneracy, an infinitesimally small quality is placed in each of the required number of independent cells.
   (*d*) all of the above hold.
9. The dummy source or destination in a T.P. is introduced to
   (*a*) prevent solution to become degenerate. (*b*) to satisfy rim conditions.
   (*c*) ensure that total cost does not exceed a limit. (*d*) solve the balanced transportation problem.
10. If we were to use opportunity cost value for non- basic cell to test optimality, it should be
   (*a*) most negative number (*b*) most positive number
   (*c*) equal to zero (*d*) any value.
11. In transportation problem
   (*a*) if all the cost elements, c_{ij}, are multiplied by a constant, the total cost of transportation in the optimum solution shall be multiplied by the same constant.
   (*b*) if some $u_i + v_j - c_{ij}$ is equal to zero in the optimum solution, then the problem has multiple optimum solutions.
   (*c*) if certain routes are prohibited then their cost elements are replaced by M (an extremely large value).
   (*d*) all of the above.
12. Which of the following is not correct ?
   (*a*) The $u_i + v_j - c_{ij}$ value of an unoccupied cell indicates the net change in cost of re-allocating one unit through the route involved.
   (*b*) The maximum quantity that can be re-allocated in a closed path is equal to the minimum quantity in the cells bearing negative sign.
   (*c*) Any of the $m + n - 1$ number of occupied cells would allow determining whether a given solution is optimum or not.
   (*d*) In time minimization problem, the cost c_{ij} is replaced by the unit time t_{ij}.
13. Which of the following is not correct ?
   (*a*) Multiple optimum solutions are indicated if there are multiple zeroes for u_i and v_j values.
   (*b*) For an optimum solution to a transportation problem, the u_i and v_j values represent the optimum values of the dual problem.
   (*c*) If each cost element in a transportation problem is increased or decreased by a constant amount, it will not effect the optimum solution to the problem.
   (*d*) A maximization transportation problem is first converted into a minimization one by subtracting each value of the given matrix from the largest value.

14. For a transhipment problem, choose the statement which is not correct :
   (*a*) The problem allows for the shipment of goods from one source to another, and from one destination to another.
   (*b*) There is no real distinction between sources and destinations.
   (*c*) An 'm' source, 'n' destination transportation problem, when written as a transhipment problem would have $m + n$ sources and n destinations.
   (*d*) A transhipment problem is not likely to involve a lower cost than a transportation problem, in a given situation.

ANSWERS

1. (*c*)	**2.** (*b*)	**3.** (*c*)	**4.** (*a*)	**5.** (*b*)
6. (*a*)	**7.** (*c*)	**8.** (*d*)	**9.** (*b*)	**10.** (*a*)
11. (*d*)	**12.** (*c*)	**13.** (*a*)	**14.** (*c*).	

REVIEW QUESTIONS

1. What is transportation problem ? *[Bharathidasan M.Com. 2007; Madurai M.Com. 2002]*
2. Explain transportation problem and show that it can be considered as an LPP. *[Delhi B.Sc. (Stat.) 2003; Nagarjuna B.Sc. (Stat.) 1994]*
3. Prove that a necessary and sufficient condition for the existence of a feasible solution to a $m \times n$ transportation problem is
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j,$$
where a_i and b_j denote the availability and requirement at ith origin and jth destination respectively.
4. What is a balanced transportation problem? What are its applications? *[Panjab Tech. Univ. M.B.A. 2007]*
5. Describe an unbalanced transportation table. *[Delhi B.Sc. (Stat.) 2004]*
6. What are the common methods to obtain an initial basic feasible solution for a transportation problem whose cost and requirement table is given? Give a stepwise procedure for one of them. *[Delhi B.Sc. (Stat.) 1995]*
7. Explain North-West Corner method to solve transportation problem for an initial solution. *[Kerala M.Com. 1994; Panjab M.B.A. 2006, B.Com. 2004]*
8. Explain briefly (*i*) North-West Corner rule, (*ii*) Minimum matrix method and (*iii*) Vogel's approximation method, for finding an initial basic feasible solution for a transportation problem. *[Meerut M.Sc. (Math.) 2000]*
9. Prove that every standard cost minimising T.P. has a feasible solution. *[Meerut M.Sc. (Math.) 1996]*
10. What are the conditions to be satisfied for proceeding with optimality test in a transportation problem. *[Annamalai M.B.A. 2009]*
11. Give a brief outline of the procedure for solving a transportation problem.
12. How the problem of degeneracy arises in a transportation problem? Explain how does one overcome it? *[Meerut M.Sc. (Math.) 2001; Panjab Tech. Univ. M.B.A. (Dec.) 2010]*
13. What is stepping stone in transportation problem ? *[Osmania M.B.A. 2001; Panjab B.Com. 2005]*
14. Having obtained the initial basic feasible solution, discuss the method of obtaining the optimum solution in transportation problem. What is the procedure in the case of degeneracy? *[Delhi B.Sc. (Stat.) 1996]*
15. What is meant by unbalanced transportation problem ? *[Annamalai M.B.A. (Nov.) 2002]*
16. Explain how the profit maximization transportation problem can be converted to an equivalent cost minimization transportation problem.
17. What is a transhipment problem ? Explain how a transhipment problem can be formulated and solved as a transportation problem ? *[Delhi M.B.A. (PT.) 2008; Annamalai M.B.A. 2010]*
18. Write a short note on Time-Minimization in transportation problem. *[Delhi M.B.A. (Nov.) 2007]*
19. Explain the following in the context of T.P. :
   (*i*) Stepping Stone Method, (*ii*) Degenerate Transportation Problem, and (*iii*) Modified Distribution Method. *[IGNOU (MCA) 2003; Pondicherry M.B.A. 2006]*
20. What is the indicator of multiple optimal solution to a transportation problem in case of maximization? How do you find out the alternate optimal solution in such a case? *[Delhi MIB 2011]*