Single-Step Comparison of Gradient Descent and Newton’s Method

Gradient Descent and Newton’s Method are two fundamental approaches to finding the minimum of a function.  
Here we perform **one complete iteration** of both algorithms on the quadratic

starting from the initial point

### 1. Gradient Descent

#### 1.1 Gradient

#### 1.2 Steepest-descent direction at

Evaluate the gradient at :

The **direction of steepest descent** is the negative gradient:

### 2. Newton’s Method

#### 2.1 Hessian matrix

#### 2.2 Newton direction

For a quadratic with constant positive-definite Hessian, the inverse is

Hence the Newton direction is

Newton’s step points **directly to the global minimizer** for this quadratic, whereas gradient descent takes a direction that merely reduces the function value.