Numerical Optimization – Assignment 1

Deadline: 18 October 2025

### Question 1 (2 marks)

**Local vs. global minimizer**  
A point is  
- a **local minimizer** of if there exists a neighborhood such that  
 for all ;  
- a **global minimizer** if  
 for **every** .

### Question 2 (4 marks)

**Convexity check**  
(i) on .  
 for all , hence **convex**.

1. .  
   .  
   Determinant , so **not convex**.

### Question 3 (4 marks)

**Positive semidefinite**  
.  
Leading principal minors: , , .  
All minors non-negative, so **positive semidefinite**.

.  
Leading minors: , , .  
Hence **positive semidefinite**.

### Question 4 (2 marks)

**Global minimizer**  
Let differentiable convex, .  
Convexity: for all .  
Thus is **global minimizer**.

### Question 5 (3 marks)

**Steepest descent**  
Minimize .  
Gradient: .  
At : .  
Constant step :  
.

### Question 6 (4 marks)

**Minimizers**  
(i) .  
.  
Hessian: (indefinite), so **saddle**, **no local/global minimizer**.

1. .  
   Unique global minimizer: .

### Question 7 (8 marks)

**Least-squares quadratic**  
Given points , .  
Minimize .  
Design matrix:  
,  
, .  
Solve .

For data sets:  
(i) .  
(ii) .  
Python code:

import numpy as np def fit\_quad(x,y): X = np.vander(x,3)[:,::-1] beta = np.linalg.lstsq(X,y,rcond=None)[0] return beta

### Question 8 (2 marks)

**Newton success**  
.  
Starting : ,  
Hessian singular, **pure Newton diverges**; **not successful**.

### Question 9 (2 marks)

**Lipschitz gradient**  
.  
.  
, so **Lipschitz gradient**.

### Question 10 (4 marks)

**Descent directions**  
.  
At : .  
Two independent descent directions:  
, .  
Along : ,  
.  
Armijo with :  
.  
Choose .