**Full Marks: 35**  
**Deadline: October 18, 2025**

### Question 1 (2 marks)

For a function , what is the difference between a **local minimizer** and a **global minimizer**?

### Question 2 (4 marks)

Verify whether the following functions are convex or not:

1. , defined by .
2. , defined by

$ f(x, y) = x^2 + 4xy + y^2 + x - y. $

### Question 3 (4 marks)

Verify whether the following matrices are **positive semidefinite** or not. Justify your answer.

$ A =

$

$ A =

$

### Question 4 (2 marks)

Let be a differentiable convex function. If , then prove that is a **global minimizer** of .

### Question 5 (3 marks)

Suppose you want to solve the problem  
$ \_{x, y} ; (x - 2)^2 + (x - 2y)^2. $  
You want to use the **steepest descent algorithm** with constant step length . Currently, you are at the point . Find the next iteration point.

### Question 6 (4 marks)

Find **local/global minimizers** of the following functions. Justify why they are local/global minimizers. (Also mention if it doesn’t have a local/global minimizer.)

* 1. defined by  
     $ f(x, y) = 8x + 12y + x^2 - 2y^2. $
  2. defined by  
     $ f(x, y) = 100(y - x2)2 + (1 - x)^2. $

### Question 7 (8 marks)

* Suppose four data points have been given in the plane. We want to find a quadratic curve such that  
  $ \_{i=1}^4 ( y\_i - a x\_i^2 - b x\_i - c )^2 $ is minimized. Write this as a **least squares problem** (i.e., express it as a quadratic optimization problem by identifying the design matrix, unknown vector, and the resulting quadratic form).
* For the particular choice of the following points, find the best-fitting quadratic curve in the least squares sense using **Python code**:

### Question 8 (2 marks)

Will **pure Newton’s method** be successful if we want to minimize  
$ f(x\_1, x\_2) = x\_1^3 + x\_1 x\_2 - x\_1^2 x\_2^2 $  
starting from the point ? Justify your answer.

### Question 9 (2 marks)

Let be defined by  
$ f(x\_1, x\_2) = 2x\_1^2 - 3x\_2^2. $  
Is it a function with a **Lipschitz continuous gradient**?

### Question 10 (4 marks)

Consider the function defined by  
$ f(x\_1, x\_2) = x\_1^2 + 3x\_1 x\_2 + x\_2^2. $  
Find **two distinct (linearly independent) descent directions** of this function at the point . Also, along **any one** of these directions, find a step length that satisfies **Armijo’s condition** with parameter .