**Full Marks: 35**  
**Deadline: October 18, 2025**

### Question 1 (2 marks)

For a function $ f : ^n $, what is the difference between a **local minimizer** and a **global minimizer**?

### Question 2 (4 marks)

Verify whether the following functions are convex or not:

* 1. $ f : $, defined by $ f(x) = e^x $.
  2. $ f : ^2 f(x, y) = x^2 + 4xy + y^2 + x - y. $

### Question 3 (4 marks)

Verify whether the following matrices are **positive semidefinite** or not. Justify your answer.

$ A =

$

$ A =

$

### Question 4 (2 marks)

Let $ f : ^n $ be a differentiable convex function. If $ f(x) = 0 $, then prove that $ x $ is a **global minimizer** of $ f $.

### Question 5 (3 marks)

Suppose you want to solve the problem  
$ \_{x, y} ; (x - 2)^2 + (x - 2y)^2. $  
You want to use the **steepest descent algorithm** with constant step length $ = 0.3 $. Currently, you are at the point $ (0, 3) $. Find the next iteration point.

### Question 6 (4 marks)

Find **local/global minimizers** of the following functions. Justify why they are local/global minimizers. (Also mention if it doesn’t have a local/global minimizer.)

* 1. $ f : ^2 $ defined by  
     $ f(x, y) = 8x + 12y + x^2 - 2y^2. $
  2. $ f : ^2 $ defined by  
     $ f(x, y) = 100(y - x2)2 + (1 - x)^2. $

### Question 7 (8 marks)

* Suppose four data points $ {(x\_i, y\_i) ^2 i = 1, 2, 3, 4} $ have been given in the plane. We want to find a quadratic curve $ y = ax^2 + bx + c $ such that  
  $ \_{i=1}^4 ( y\_i - a x\_i^2 - b x\_i - c )^2 $ is minimized. Write this as a **least squares problem** (i.e., express it as a quadratic optimization problem by identifying the design matrix, unknown vector, and the resulting quadratic form).
* For the particular choice of the following points, find the best-fitting quadratic curve in the least squares sense using **Python code**:
  + 1. $ {(1, 10), (-1, 4), (3, 32), (-2, 7)} $
    2. $ {(1, 10), (-1, 5), (2, 10), (3, 15)} $

### Question 8 (2 marks)

Will **pure Newton’s method** be successful if we want to minimize  
$ f(x\_1, x\_2) = x\_1^3 + x\_1 x\_2 - x\_1^2 x\_2^2 $  
starting from the point $ (1, 1) $? Justify your answer.

### Question 9 (2 marks)

Let $ f : ^2 $ be defined by  
$ f(x\_1, x\_2) = 2x\_1^2 - 3x\_2^2. $  
Is it a function with a **Lipschitz continuous gradient**?

### Question 10 (4 marks)

Consider the function $ f : ^2 $ defined by  
$ f(x\_1, x\_2) = x\_1^2 + 3x\_1 x\_2 + x\_2^2. $  
Find **two distinct (linearly independent) descent directions** of this function at the point $ (1, 1) $. Also, along **any one** of these directions, find a step length that satisfies **Armijo’s condition** with parameter $ c\_1 = 0.75 $.