# The Birth of the Quadratic Formula

\*A short equation essay written entirely in

## 1 A historical teaser

Every high-school student meets the quadratic equation  
[ ax^{2}+bx+c=0,a ]  
but few realise that its closed-form solution, the *quadratic formula*,  
[ x=, ]  
was already known to the Babylonians in *algorithmic* form (c. 1800 BCE) and was later refined by Al-Khwarizmi (c. 820 CE) and Cardano (1545). In what follows we *derive* the formula in modern notation, inspect its discriminant, and illustrate why the complex plane is unavoidable when the discriminant is negative.

## 2 Completing the square

Begin with the generic monic form obtained by dividing by :  
[ x^{2}+x+=0. ]  
Translate the variable to kill the linear term: let  
[ x=y-. ]  
Substitution yields  
[ (y-)^{2}+(y-)+=0. ]  
Expand and collect terms:  
[ y^{2}-y++y-+=0 y^{2}=. ]  
Hence  
[ y= ]  
and back-substituting gives the celebrated formula  
[ x=. ]

## 3 The discriminant

Define  
[ =b^{2}-4ac. ]  
The sign of dictates the *nature* of the roots:  
[

]  
Thus the quadratic formula is a *complete classifier* of the root locus in the complex plane.

## 4 Complex roots and geometry

When write  
[ =i. ]  
The two roots become  
[ x\_{1,2}=i. ]  
In the Gaussian plane these points are mirror images across the real axis, and their *modulus* is  
[ |x\_{1,2}|= = =. ]  
Hence the distance of either root from the origin is , a fact often rediscovered in control-theory root-locus plots.

## 5 A symmetric remark

Viète’s formulas relate the *sum* and *product* of the roots to the coefficients without ever solving for the roots explicitly:  
[ x\_{1}+x\_{2}=-, x\_{1}x\_{2}=. ]  
These identities are the case of the general *elementary symmetric polynomials* that underpin Galois theory.

## 6 Epilogue

From Babylonian tablets to the complex plane, the quadratic formula  
[ ]  
has transcended its humble degree-2 origins to become the first explicit example of a *solution by radicals*, a theme that will echo through the cubic, the quartic, and finally shatter at the quintic. In the words of Hermann Weyl, “The quadratic formula is the seed from which the whole theory of algebraic equations grew.”