2022-July Session-07-26-2022-shift-1-1-15

AI24BTECH11019-KOTHA PRATHEEK REDDY

1) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(3x) - f(x) = x$. If $f(8) = 7$, then $f(14)$ is equal to

a) 4
b) 10
c) 11
d) 16

2) Let O be the origin and A be the point $z_1 = 1+2i$. If B is the point z_2, $Re(z_2) < 0$,such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true?

a) $\arg z_2 = \pi - \tan^{-1} 3$
b) $\arg(z_1 - 2z_2) = -\tan^{-1} \frac{4}{3}$
c) $|z_2| = \sqrt{10}$
d) $|2z_1 - z_2| = 5$

3) If the system of linear equations.

$$8x + y + 4z = -2$$
$$x + y + z = 0$$
$$\lambda x - 3y = \mu$$

has infinitely many solutions, then the distance of the point $\left(\lambda, \mu, -\frac{1}{2}\right)$ from the plane $8x + y + 4z + 2 = 0$ is

a) $3\sqrt{5}$
b) 4
c) $\frac{26}{9}$
d) $\frac{10}{3}$

4) Let A be a 2×2 matrix with $\det(A) = -1$ and $\det((A + I)(Adj(A) + I)) = 4$. Then the sum of the diagonal elements of A can be:

a) -1
b) 2
c) 1
d) $-\sqrt{2}$

5) The odd natural number a, such that the area of the region bounded by $y = 1, y = 3, x = 0, x = y^a$ is $\frac{364}{3}$, equal to:

a) 3
b) 5
c) 7
d) 9

6) Consider two G.Ps. $2, 2^2, 2^3, \ldots$ and $4, 4^2, 4^3, \ldots$ of 60 and n terms respectively. If the geometric mean of $60 + n$ terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to:

a) 560
b) 1540
c) 1330
d) 2600

7) If the function $f(x) = \begin{cases} \frac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x - \cos x}, & x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \\ k, & x = 0 \end{cases}$
is continuous at $x = 0$, then k is equal to:

a) 1
b) -1
c) e
d) 0

8) If $f(x) = \begin{cases} x + a, & x \leq 0 \\ |x-4|, & x > 0 \end{cases}$ and
$g(x) = \begin{cases} x + 1, & x < 0 \\ (x-4)^2 + b, & x \geq 0 \end{cases}$
are continuous and $\mathbb{R}$, then $(f \circ g)(2) + (f \circ g)(-2)$ is equal to:

a) -10
b) 10
c) 8
d) -8

9) Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 7, & x \leq 1 \\ -2x + \log_2\left(b^2 - 4\right), & x > 1 \end{cases}$ Then the set of all values of b, for which $f(x)$ has maximum value at $x = 1$, is:

a) $(-6, -2)$
b) $(2, 6)$
c) $[-6, -2) \cup (2, 6]$
d) $[-\sqrt{6}, -2) \cup (2, \sqrt{6}]$

10) If $a = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{2n}{n^2+k^2}$ and $f(x) = \sqrt{\frac{1-\cos x}{1+\cos x}}, x \in (0, 1)$, then:

a) $2\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$
b) $f\left(\frac{a}{2}\right) f'\left(\frac{a}{2}\right) = \sqrt{2}$
c) $\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$
d) $f\left(\frac{a}{2}\right) = \sqrt{2} f'\left(\frac{a}{2}\right)$

11) $\frac{dy}{dx} + 2y \tan x = \sin x, 0 < x < \frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of $y(x)$ is

a) $\frac{1}{8}$
b) $\frac{3}{4}$
c) $\frac{1}{4}$
d) $\frac{3}{8}$

12) A point P moves so that the sum of squares of its distances from the points $(1, 2)$ and $(-2, 1)$ is 14. Let $f(x, y) = 0$ be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the points C, D. Then the area of the quadrilateral $ABCD$ is equal to

a) $\frac{9}{2}$
b) $\frac{3\sqrt{17}}{2}$
c) $\frac{3\sqrt{17}}{4}$
d) 9

13) Let the tangent drawn to the parabola $y^2 = 24x$ at the point (α, β) is perpendicular to the line $2x + 2y = 5$. Then the normal to the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ at the point $(\alpha + 4, \beta + 4)$ does NOT pass through the point:

a) $(25, 10)$
b) $(20, 12)$
c) $(30, 8)$
d) $(15, 13)$

14) The length of the perpendicular from the point $(1, -2, 5)$ on the line passing through $(1, 2, 4)$ and parallel to the line $x + y - z = 0 = x - 2y + 3z - 5$ is:

a) $\sqrt{\frac{21}{2}}$
b) $\sqrt{\frac{9}{2}}$
c) $\sqrt{\frac{73}{2}}$
d) 1

15) Let $\mathbf{a} = \alpha\hat{i} + \hat{j} - \hat{k}$ and $\mathbf{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$. If the projection of $\mathbf{a} \times \mathbf{b}$ on the vector $-\hat{i} + 2\hat{j} - 2\hat{k}$ is 30, then α is equal to

a) $\frac{15}{2}$
b) 8
c) $\frac{13}{2}$
d) 7