NCERT: Class XII

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13.2.4 A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not.

Solution:

Let X_i be the event when coin is tossed and Y_j be the event when the die is rolled. Then,

Random variable	Value	Description
X_i	{0,1}	0:Head, 1:Tail
Y_j	{1,2,3,4,5,6}	Number on the die

The sample space is given by,

$$S = \{X_i, Y_j\} = \{(0,1),(0,2),(0,3),(0,4),(0,5),(0,6),\\ (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\}$$

Given A is the event 'head appears on the coin' and B is the event '3 on the die', then

$$\begin{aligned}\Pr(\mathrm{A}) &= \Pr(0, Y_j) && (13.2.4.1)\\ &= \Pr(0,1)+\Pr(0,2)+\Pr(0,3)+\Pr(0,4)+\Pr(0,5)+\Pr(0,6) && (13.2.4.2)\\ &= 1/12+1/12+1/12+1/12+1/12+1/12 && (13.2.4.3)\\ &= 6/12 = 1/2 && (13.2.4.4)\end{aligned}$$

and,

$$\begin{aligned}\Pr(\mathrm{B}) &= \Pr(X_i, 3) && (13.2.4.5)\\ &= \Pr(0,3)+\Pr(1,3) && (13.2.4.6)\\ &= 1/12+1/12 && (13.2.4.7)\\ &= 2/12 = 1/6 && (13.2.4.8)\end{aligned}$$

$$\begin{aligned}\Pr(\mathrm{AB}) &= \Pr(0,3) && (13.2.4.9)\\ &= 1/12 && (13.2.4.10)\\ & && (13.2.4.11)\end{aligned}$$

Since, $\Pr(\mathrm{AB}) = \Pr(\mathrm{A}).\Pr(\mathrm{B})$, A and B are independent events.