Problem Set # 2

1. (12 total points) Recall the “flat tire” example from the first class. Two students (student 1 and student 2) missed an exam because they spent too much time partying the night before. They decide to ask the professor if they can take a make-up exam. The professor asks them why they missed the original exam, and the students say that they were headed to the exam when they got a flat tire. The professor agrees to allow them to take a make-up exam. The students receive the make-up exam and it contains one question: “Which tire?”. Suppose that each student chooses among four possibilities (driver’s front, driver’s rear, passenger’s front, and passenger’s rear). Assume the students cannot communicate prior to taking the make-up exam. Suppose that the students both receive a zero (0) if their answers do not match, but they both receive 100 if their answers do match.

a. (3 points) Draw this game in its normal form.

Answer:-

Student 1

Student 2	DF	DR	PF	PR
DF	100,100	0,0	0,0	0.0
DR	0,0	100,100	0,0	0,0
PF	0,0	0,0	100,100	0,0
PR	0,0	0,0	0,0	100,100

b. (3 points) Does either student have a dominant strategy? Explain.

Answer:-

Neither student has a dominant strategy. No one strategy is better than another strategy for either student no matter how the other student may play. Each strategy is the best for some strategy of the opponent.

c. (3 points) Does either student have a dominated strategy? Explain.

Answer:-

Neither student has a dominated strategy. No one strategy is worse than another strategy for either student no matter how the other student may play. Each strategy is the best for some strategy of the opponent.

d. (3 points) How many Nash Equilibria are there in this game? (If there are any at all.) Explain.

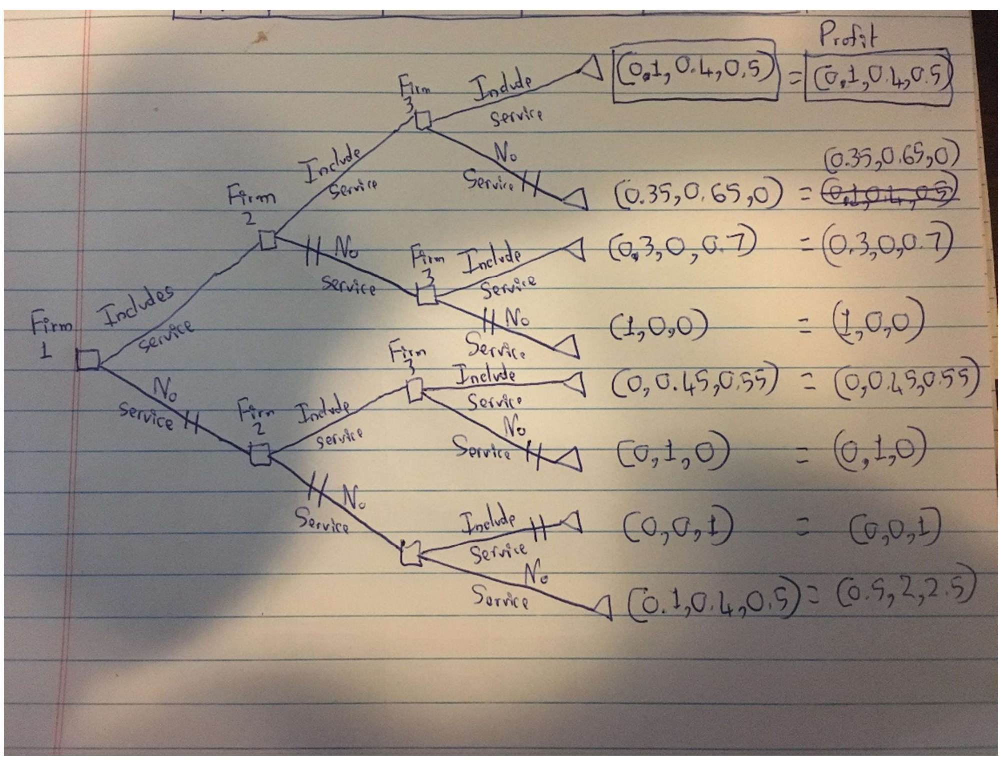
Answer:-

There are no Nash Equilibria in the game. This is because there is no optimal solution to this game. It is a game of pure chance and neither of the players can choose a good strategy which will give them a good outcome.

2. (15 total points) Three firms are trying to sell photocopiers to the Paul Merage School of Business. Offers are made in sequence and publicly announced. First Firm 1 offers, then Firm 2, then Firm 3. Suppose that price is irrelevant. Instead, the key issue is whether the contract will include service (i.e. maintenance). Each firm gets a profit of 5 if service is not included in the contract but only 1 if service is included (i.e. providing service is costly). If all offers are equal with respect to service (i.e. if all offer service or if all offer no service), then Firm 1 gets the contract with probability .1, Firm 2 gets the contract with probability .4, and Firm 3 gets the contract with probability .5. If only some of the offers include service, the offers that do not include service are rejected with certainty, and the rejected firm's original probability of winning is split evenly among the remaining firms offering service. All firms know this information. For example: If Firm 1 does not offer service but Firms 2 and 3 offer service, then Firm 1's probability of winning the contract falls to 0, Firm 2's probability of winning increases to .45 = .4 + ½ (.1), and Firm 3's probability of winning increases to .55 = .5 + ½ (.1).

a. Draw this game in extensive form (where payoffs are the expected profit to each firm), and solve for the Rollback Equilibrium. Show your work.

Answer:-



3. (14 total points) Monitoring is one example where mixed strategies come into play. Consider the following payoffs for a manager and his employee. Find all Nash Equilibria including the mixed strategy Nash Equilibrium. What are the manager's and the employee's respective expected payoffs if each plays his/her mixed strategy?

		Manager	
		Monitor	Don't Monitor
Employee	Work	50, 90	50, 100
	Slack	0, -10	100, -100

Answer:-

In this example, there is no dominant strategy, no dominated strategy and no best response equilibrium.

Therefore mixed strategies should be used.

Assuming the manager will monitor with a probability P.

The employee's response :-

If P=1, the employee must choose work

If P=0, the employee must choose slack

Employee's payoff :-

E(work) = 50(P) + 50(1-P)

E(slack) = 0(P) + 100(1-P)

Employee should work when

50(P) + 50(1-P) > 0(P) + 100(1-P)

50 > 100 – 100P

100P > 50

P> 0.5

Assuming the employee will slack with a probability P.

The manager's response :-

If P=1, the manager must choose monitor

If P=0, the manager must choose don't monitor

Manager's payoff :-

E(monitor) = $-10(P) + 90(1-P)$

E(don't monitor) = $-100(P) + 100(1-P)$

Manager should monitor when

$-10(P) + 90(1-P) > -100(P) + 100(1-P)$

$-100P + 90 > -200P + 100$

$100P > 10$

$P > 0.1$

4. (20 total points) Consider two firms, firm 1 and firm 2, producing identical products so that they are forced to charge identical prices. The sole strategic choice of the firms is the amount they choose to produce, q1 and q2. Once the firms select their quantities, the resulting industry price is whatever price is required to "clear the market" which is the price at which consumers are willing to buy the total production, q1+q2. Suppose that the firms have no marginal costs of production and market demand is given by: P = 600 - q1 - q2.

a. (5 points)If the firms must make their output decisions simultaneously, how much will each firm produce?

b. (4 points) What will be the resulting industry price and profit for each firm?

c. (5 points) If the firms could achieve cooperation, what would their resulting profit be?

d. (6 points) What forces would work to achieve and maintain such cooperation? What forces would work against it?

Answer:-

a)

Here, p1=p2=p

Market demand(P) = 600-q1-q2

Revenue of firm 1 = price x quantity

= (600-q1-q2) x q1

dR1/dQ1 = 600 – 2q1 –q2=0

2q1+q2=600

Q1=(600-q2)/2

Revenue of firm 2 = price x quantity

= (600-q1-q2) x q2

dR2/dQ1 = 600 – 2q2 –q1=0

2q2+q1=600

Q2=(600-q1)/2

Optimal solution:-

Q1=(600-(600-q1)/2)/2

Q1= 200

Q2=200

b. ,c.

Market demand(P) = 600-q1-q2 = 200

Profit for each firm = 200*200=40000

d.

Forces which would work to achieve and maintain such cooperation:

The forces are q1 and q2 as any change in these will affect the firm’s profit.

Forces which would work against it:

If either firm decides to not cooperate and undercut the other, equilibrium will get changed.