

## 1. Implementation of Rectified Linear Unit (RELU)

**Defintion of RELU layer**

$$y = \max(0, x) \quad (1)$$

**Forward Propagation of RELU layer**

$$RELU(x) = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

**Backward Propagation of RELU layer**

$$\frac{d}{dx}(RELU(x)) = \begin{cases} 1, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

## 2. Implementation of Softmax Layer

In a multi-class classification problem, softmax layer is used to calculate the probabilities in each class.

**Forward Propagation of softmax layer**

The softmax function for any value  $a_i$  is expressed as:

$$p_i = \frac{\exp(a_i)}{\sum_k \exp(a_k)} \quad (2)$$

The computation of softmax value can be huge, because exponential of even a moderately large value is very large number. In order to avoid this, the largest value of  $x$  is subtracted from each  $x$  value.

**Back Propagation of softmax layer**

We need to calculate the derivative and pass it through previous layers during back propagation.

$$\frac{\partial p_i}{\partial a_j} = \begin{cases} p_i(1 - p_j), & \text{if } i = j \\ -p_j p_i, & \text{if } i \neq j \end{cases}$$