ENPM 667 : PROJECT 1

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Paper summary

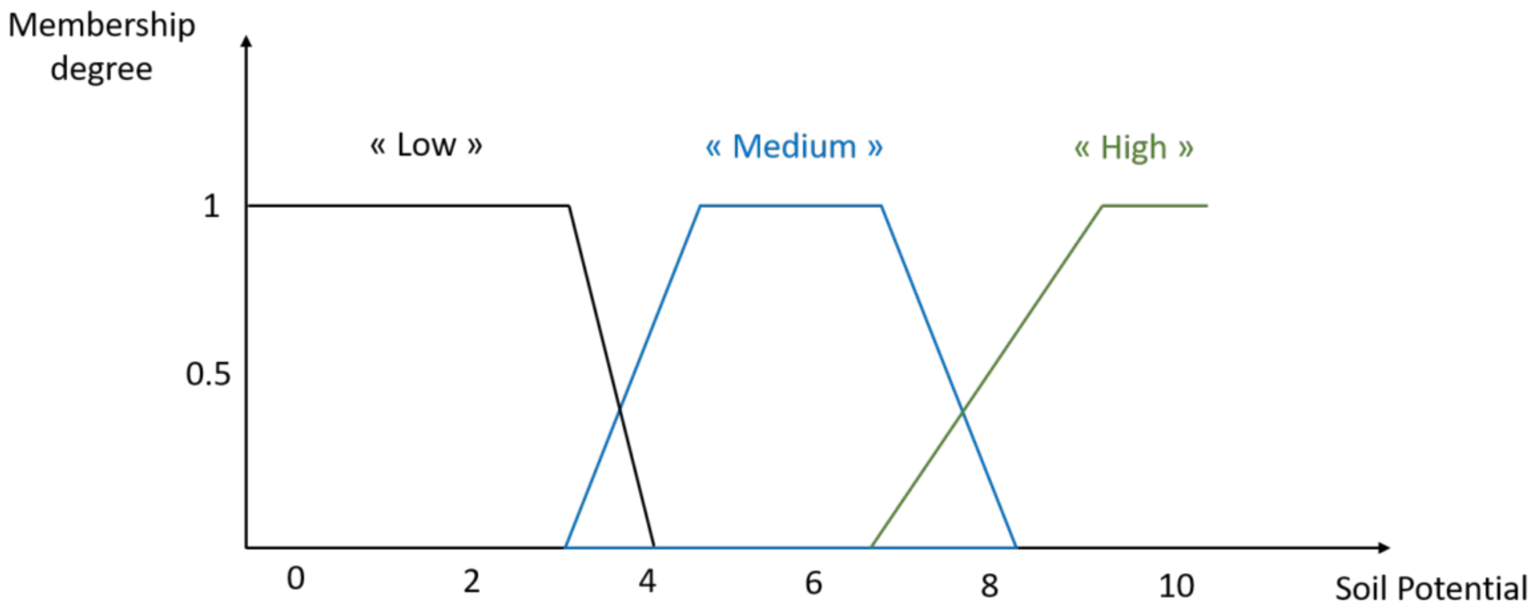
Paper - ‘Fuzzy Gain Scheduling of PID controller’

- We perform a comparison of the 2 different tuning methods of PID controllers.
- Zeigler-Nichols tuning method, generates fixed gain parameters for the PID controller.
- Fuzzy Logic method, which generates the PID parameters dynamically according to a set of rules based on expert knowledge of how the response should ideally be.

Introduction to Fuzzy Logic

Understanding Fuzzy Logic

- Definition: Fuzzy Logic is a form of multi-valued logic derived from fuzzy set theory to deal with reasoning that is approximate rather than precise.
- Binary vs. Fuzzy: Unlike binary logic that only permits true or false values (0 or 1), fuzzy logic encompasses a range of values between 0 and 1, representing the degree of truth.
- Origins: Conceptualized by Lotfi A. Zadeh in 1965, fuzzy logic extends classical set theory to handle partial membership.
- Real-World Application: Used in systems that require an inexact solution such as control systems, pattern recognition, and decision making

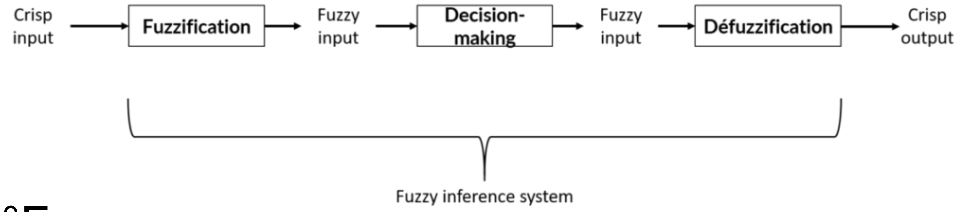


Reference: https://www.aspexit.com/fuzzy-logic-or-the-extension-of-classical-logic/

An example of 3 classes of soil potential, and therefore 3 membership functions to the soil potential variable

Fuzzy inference systems

A fuzzy inference system is a system composed of three large bricks: fuzzification, decision making and defuzzification. It's a framework for processing data through fuzzy logic to arrive at a definite conclusion.



Crisp Input Defined:

- Exact values with no ambiguity.
- Example: Temperature is exactly 70°F.

Introduction to Fuzzification:

- Transforms crisp data into fuzzy sets.
- Assigns a membership degree to each input

- Membership Functions:
    - Define how inputs relate to fuzzy set terms: "low," "medium," "high."
    - Can be triangular, trapezoidal, bell-shaped, etc.
    - Determine input's degree of membership.
- Fuzzy Values:
    - The outcome of mapping crisp inputs to fuzzy sets.
    - Represent inputs with degrees of truth.
- Fuzzy Inference System Decision-Making:
    - Applies logic rules to fuzzy inputs.
    - Outputs fuzzy results for further processing.
- Inference Engine Operations:
    - Assesses applicable rules using membership values.
    - Aggregates rule outputs into a single fuzzy set.
- Defuzzification Overview:
    - Converts fuzzy output to a crisp, actionable result.
    - Centroid method to be discussed: balances all outcomes for a precise decision.

Centroid method(or Centre of Gravity) of Defuzzification

Overview

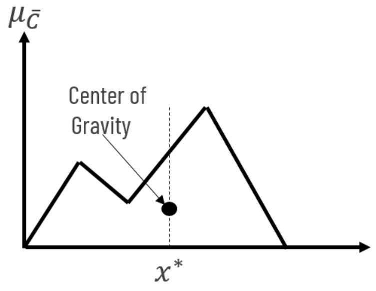
- Most prevalent and intuitively appealing method for defuzzification
- Balances a fuzzy set to find a crisp, precise output value

Principle

- Identifies the point x∗that divides the membership function's area into two equal masses

Process

- Calculates the center of gravity for the fuzzy set
- Utilizes the aggregate area under the membership function



$$x^* = \frac{\sum_{i=1}^{n} A_i . x_i}{\sum_{i=1}^{n} A_i}$$

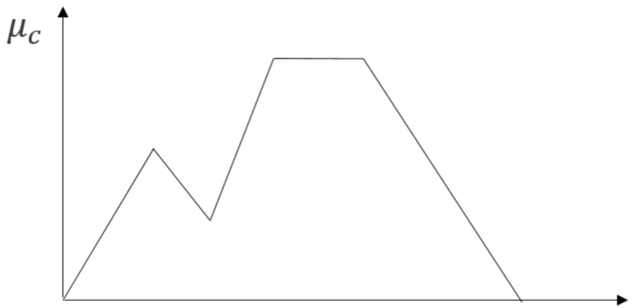
Centroid method(cont.)

Sub-Area Calculation

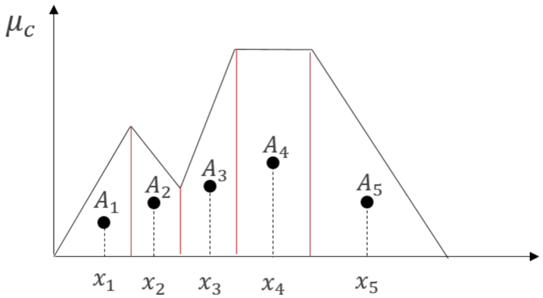
- Divides the overall area into shapes (e.g., triangles, trapezoids)
- Determines the area and centroid for each shape

Defuzzified Output

- Summation of all sub-area centroids weighted by their respective areas
- Yields a single, discrete value representing the combined control action

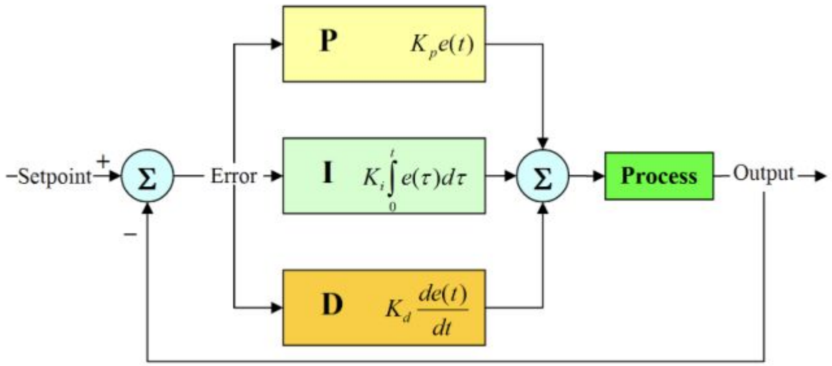


Aggregated fuzzy output (before division)



Aggregated fuzzy output (after division)

PID controller



The PID controller is a combination of the proportional controller, the integral controller, and the derivative controller.

PID controller

Each component of a PID controller complements the others, addressing their limitations to minimize errors and optimize system performance effectively.

Proportional (P)

- Action: Generates output directly proportional to the error, which is the discrepancy between the setpoint and the process variable.
- Purpose: Offers quick error correction with response intensity matching the error size.
- Effect: High gains may cause instability; low gains might result in weak responses.

Integral (I) Block:

- Action: Integrates the error over time, outputting in proportion to total error.
- Purpose: Eliminates steady-state error left by the P component.
- Effect: Achieves setpoint accuracy, but excessive use can cause instability.

Derivative (D) Block:

- Action: Outputs according to the error's rate of change.
- Purpose: Foresees error trends and applies damping to minimize overshoot.
- Effect: Stabilizes the system but can be disrupted by noise in error measurement.

Fuzzy Logic Controller

The fuzzy logic controller is a tool that is available in MATLAB, that allows us to implement fuzzy logic.

To implement fuzzy tuning for the PID controller. We assign fuzzy sets to the error and first difference of the error.

The error and error difference fuzzy sets are as follows:

NB, NM, NS, ZO, PS, PM, PB.

Here N - Negative, P - Positive and S - Small, M - Medium, and B - Big, ZO - approximately zero.

Therefore, NB is Negative big, PS - positive small. Etc.

Fuzzy Logic Controller

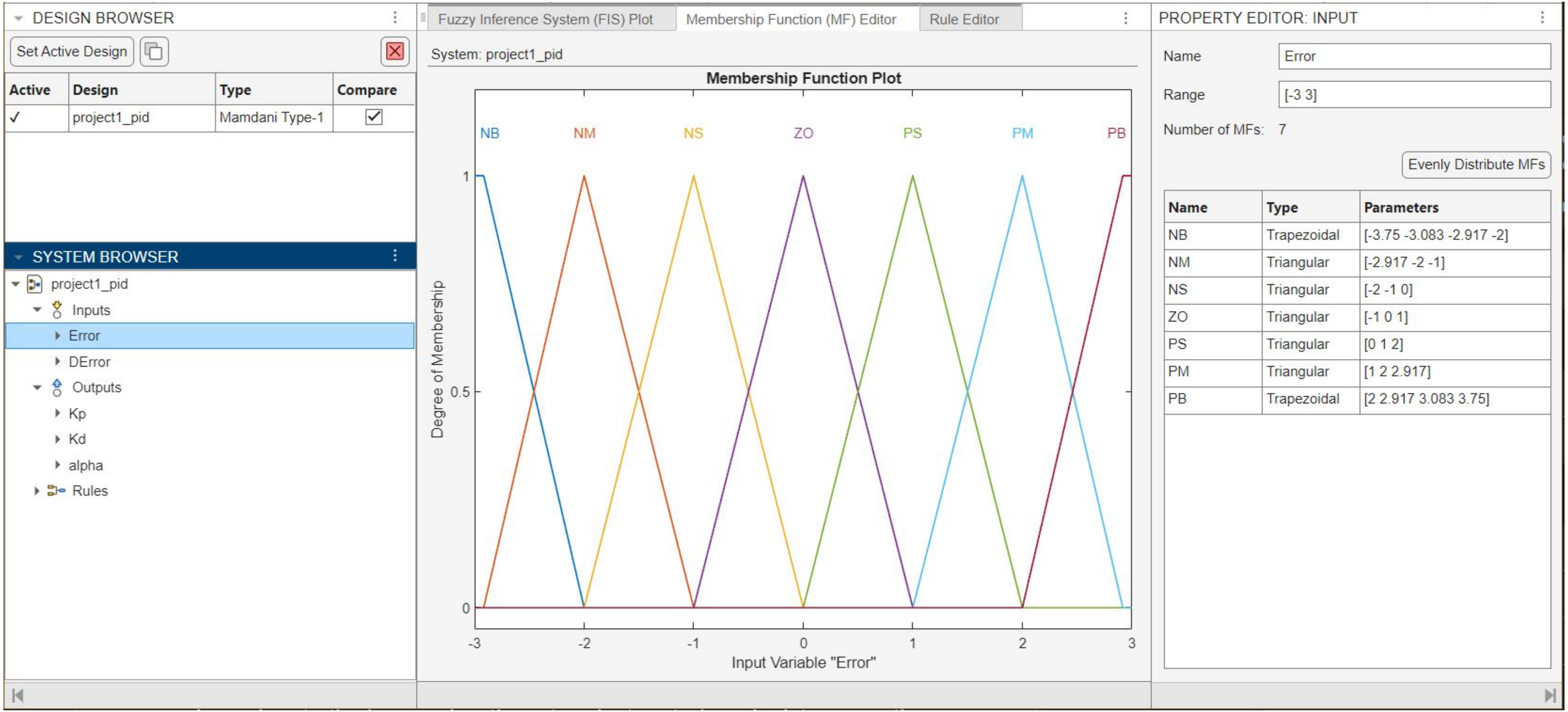
Here the fuzzy sets, describe the kind of error that is present, that is, if its is negative or positive. If the magnitude is small, medium, or big. It helps classify the error $e(k)$ and the change in error $\Delta e(k)$ into these categories.

The error $e(k)$'s membership functions of the fuzzy set are described as follows:

The membership function μ of $e(k)$ has the range [0,1].

We used triangular functions to describe the the membership function of each of the fuzzy sets as shown in the next slide.

The values of $e(k)$ range from [-3,3].



This the membership function graph of error e(k)

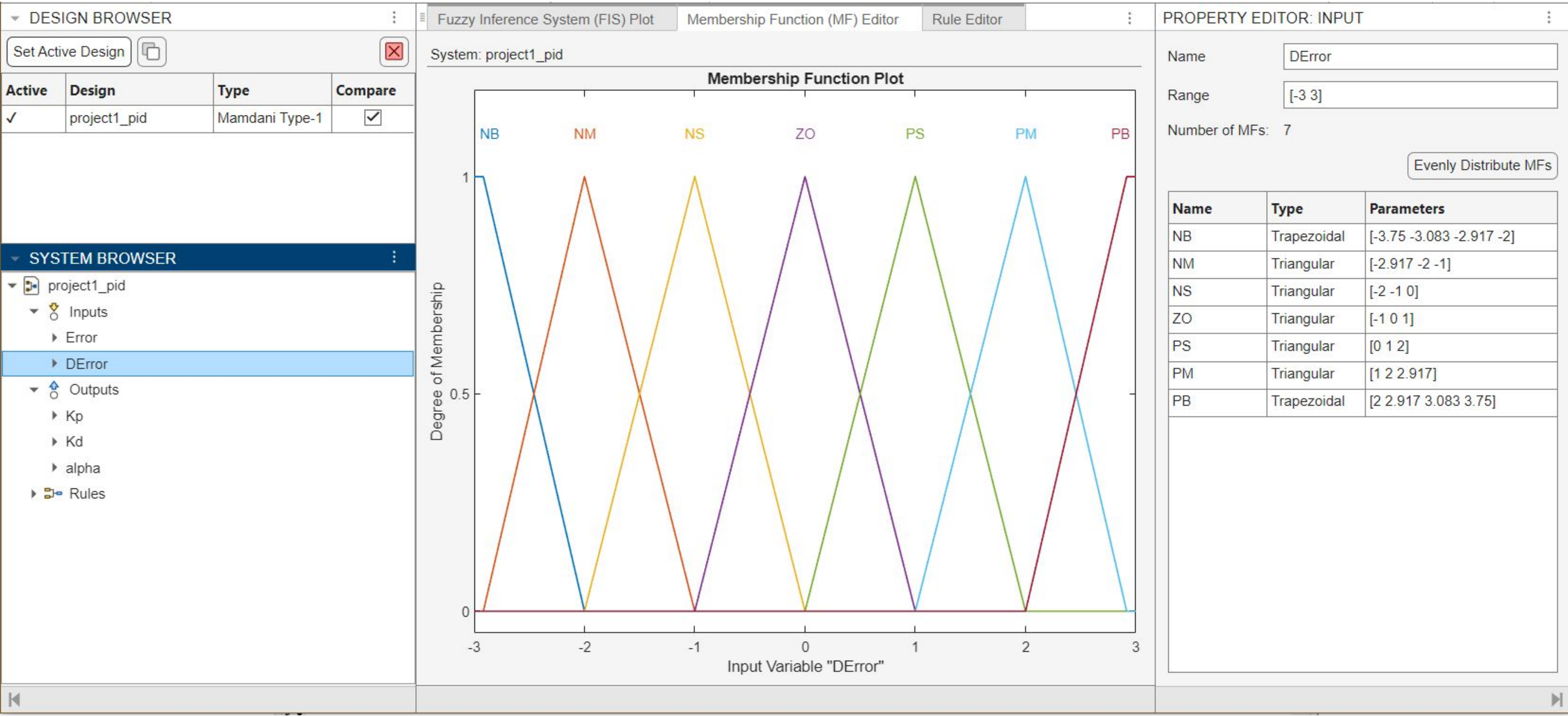
Fuzzy Logic Controller

The error difference Δe(k)'s membership functions of the fuzzy set are described as follows:

The membership function μ has the range [0,1].

We used triangular functions to describe the the membership function of each of the fuzzy sets as shown in the next slide.

The values of the Δe(k) range from [-3,3]



This the membership function graph of error Δe(k)

Fuzzy Logic Controller

The outputs of the fuzzy controller are also fuzzy sets:

They yield the normalized value of the Proportional Gain Kp’, normalized Derivative Gain Ki’, and the α.

Since the values of Kp’ and Kd’ are normalized their values will be in the range [0,1].

The fuzzy sets or categories the values of Kp’ or Kd’ can fall into are

Small (S) or Big (B).

Fuzzy Logic Controller

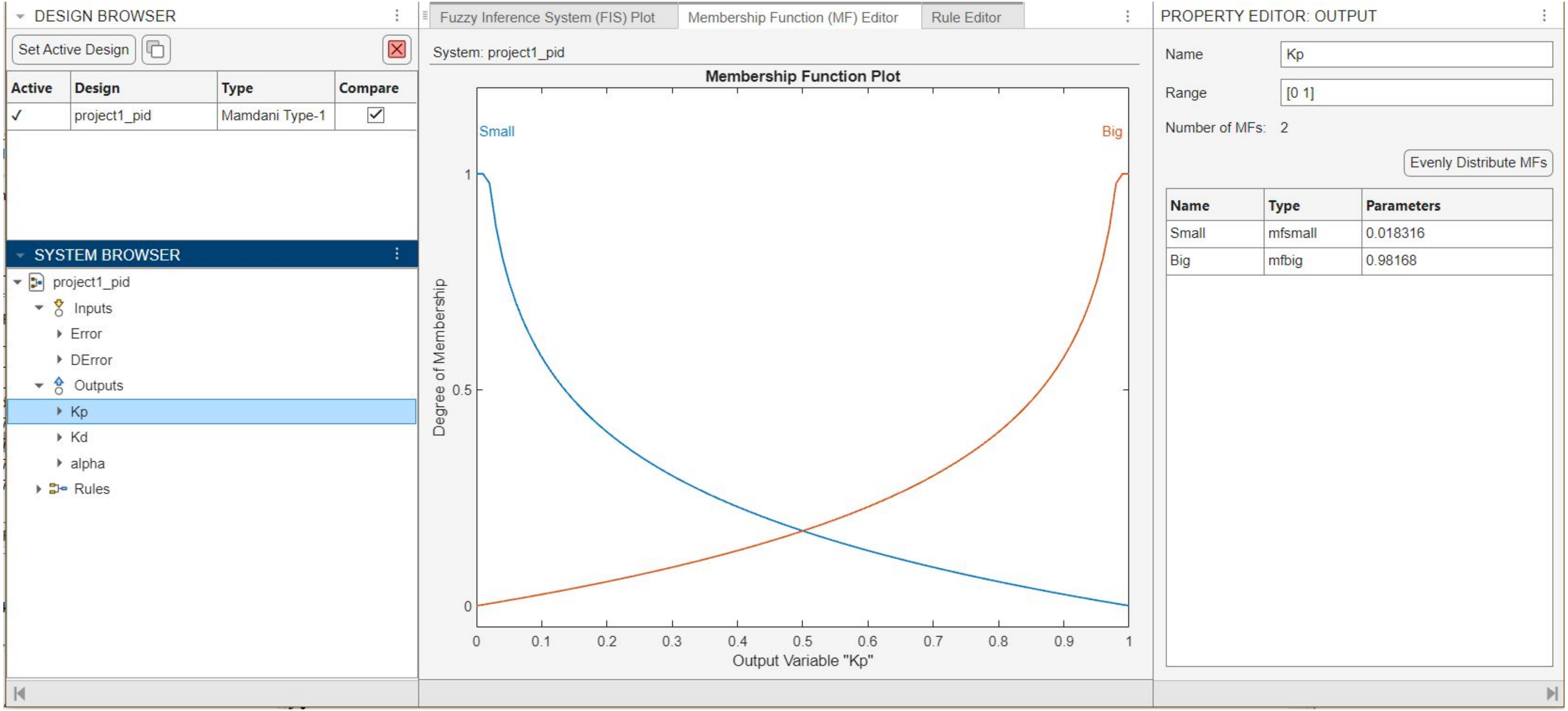
The value of the membership function for Kp' and Kd' have the custom functions:

- $\mu small(x) = (-1/4)\ln(x)$ or $xsmall(\mu) = e\verb|^|(-4\mu)$
- $\mu big(x) = (-1/4)\ln(1 - x)$ or $xbig(\mu) = 1 - e\,(-4\mu)$

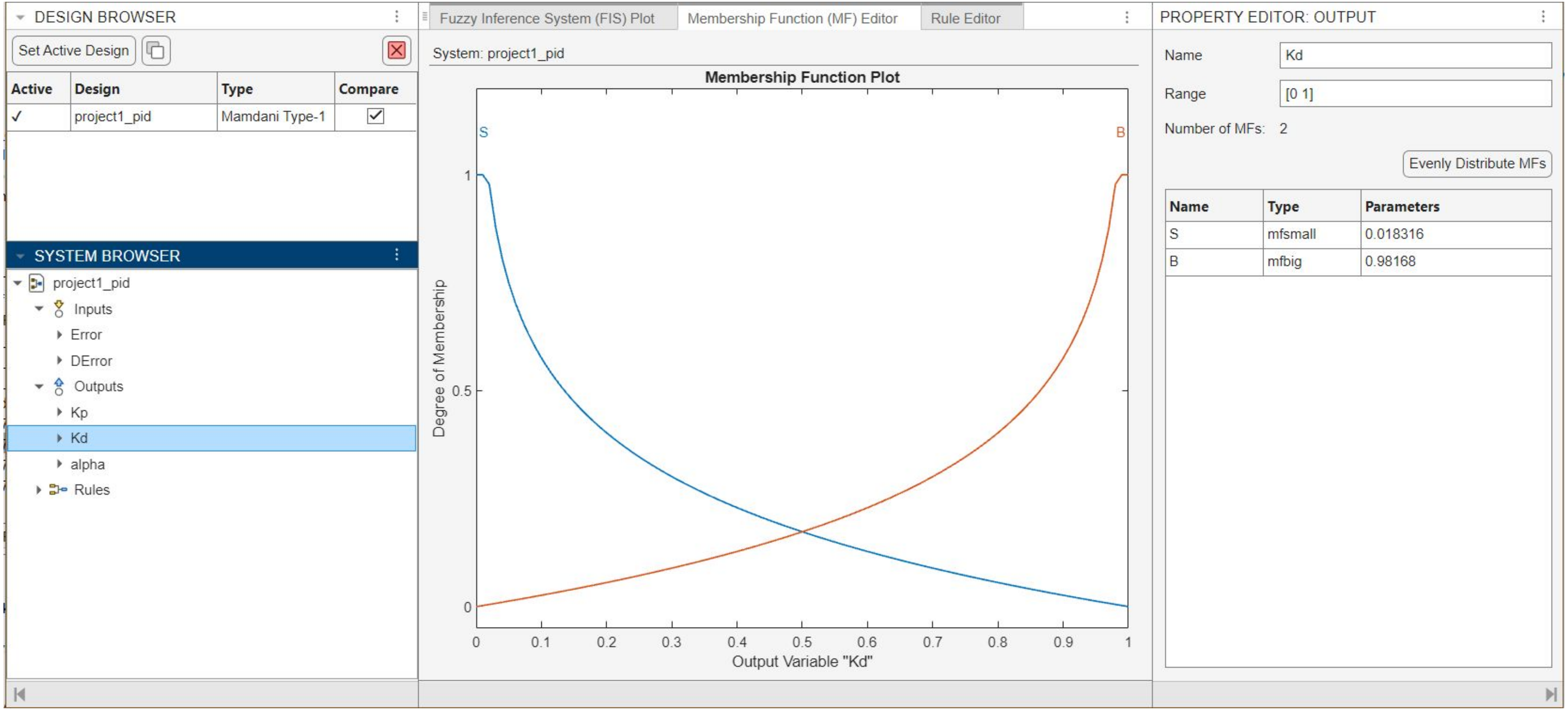
The membership graph of normalized Kp' and Kd' are shown in the next slide:

- The value of μ ranges from [0,1].
- The values of Kp' or Kd' range from [0,1].

Small (S)


DESIGN BROWSER
Set Active Design
Active | Design | Type | Compare
✓ | project1_pid | Mamdani Type-1
SYSTEM BROWSER
project1_pid
Inputs
Error
DError
Outputs
Kp
Kd
alpha
Rules
Fuzzy Inference System (FIS) Plot
Membership Function (MF) Editor
Rule Editor
System: project1_pid
Membership Function Plot
Small
Big
Degree of Membership
Output Variable "Kp"
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
PROPERTY EDITOR: OUTPUT
Name
Kp
Range
[0 1]
Number of MFs: 2
Evenly Distribute MFs
Name | Type | Parameters
Small | mfsmall | 0.018316
Big | mfbig | 0.98168

Big (B)



DESIGN BROWSER
Set Active Design
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SYSTEM BROWSER
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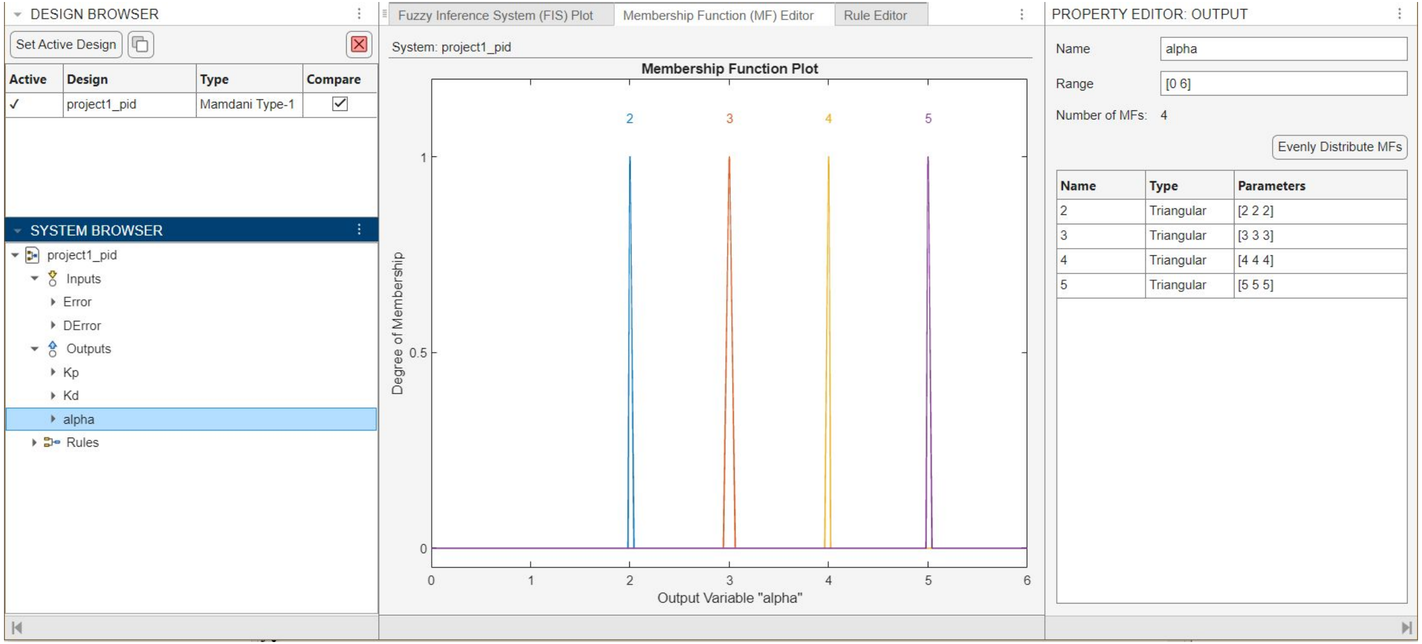
Fuzzy Inference System (FIS) Plot
Membership Function (MF) Editor
Rule Editor
System: project1_pid
Membership Function Plot
S
B
Degree of Membership
Output Variable "Kd"
PROPERTY EDITOR: OUTPUT
Name
Kd
Range
[0 1]
Number of MFs: 2
Evenly Distribute MFs
Name
Type
Parameters
S
mfsmall
0.018316
B
mfbig
0.98168

Fuzzy Logic Controller

The fuzzy output α has singleton membership function, and its values range from

$\alpha = \{2,3,4,5\}$



Fuzzy Logic Controller

The fuzzy logic controller outputs (Kp’,Kd’, and α) depend on the fuzzy rule base. These rules are based on the expert knowledge of how the system’s response should behave. Based on this knowledge the outputs are generated.

The rules depend on the inputs and the outputs of the fuzzy logic controller.

There are a total of 49 rules applied to this Fuzzy Logic Controller.

The inputs are the error $e(k)$ and $\Delta e(k)$.

The outputs are Kp’, Kd’, and α.

The rules are written in this form:

If $e(k)$ is Ai, and $\Delta e(k)$ is Bi, then Kp’ is Ci, Kd’ is Di, and $\alpha = \alpha i$.

Fuzzy Logic Controller

Here the Ai is the fuzzy set = {NB,NM,NS,ZO,PS,PM,PB}

Here Bi is the fuzzy set = {NB,NM,NS,ZO,PS,PM,PB}

Ci is the fuzzy set = {Small, Big}

Di is the fuzzy set = {Small, Big}

α is the fuzzy set = {2,3,4,5}

The fuzzy set describes the type of value they are.

Fuzzy Logic Controller

Based on the expert knowledge base, these were the rules that we set for the fuzzy logic controller for Kp’:

TABLE I
FUZZY TUNING RULES FOR K_p'

		$\Delta e(k)$						
		NB	NM	NS	ZO	PS	PM	PB
	NB	B	B	B	B	B	B	B
	NM	S	B	B	B	B	B	S
	NS	S	S	B	B	B	S	S
$e(k)$	ZO	S	S	S	B	S	S	S
	PS	S	S	B	B	B	S	S
	PM	S	B	B	B	B	B	S
	PB	B	B	B	B	B	B	B

Fuzzy Logic Controller

Based on the expert knowledge base, these were the rules that we set for the fuzzy logic controller for Kd’:

TABLE II
FUZZY TUNING RULES FOR K'_d

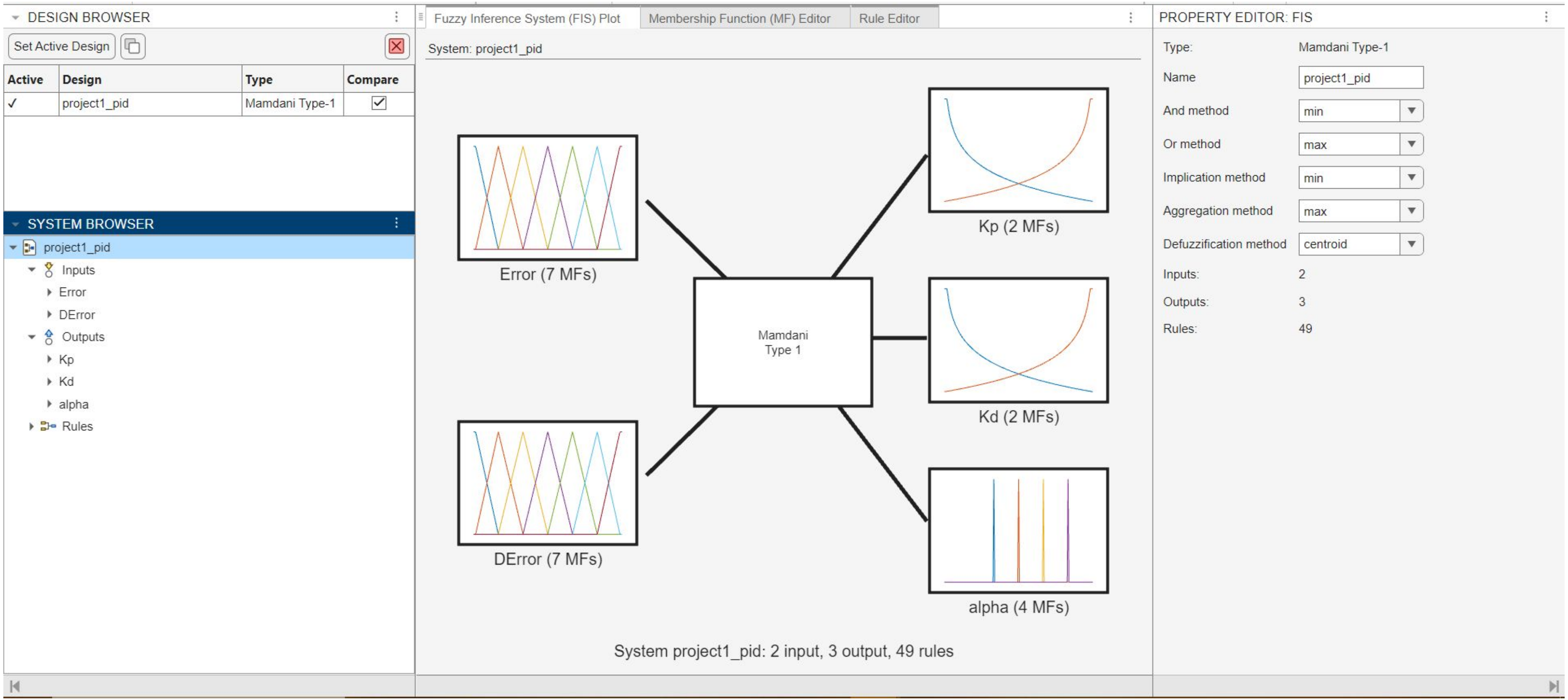
		$\Delta e(k)$						
		NB	NM	NS	ZO	PS	PM	PB
	NB	S	S	S	S	S	S	S
	NM	B	B	S	S	S	B	B
	NS	B	B	B	S	B	B	B
$e(k)$	ZO	B	B	B	B	B	B	B
	PS	B	B	B	S	B	B	B
	PM	B	B	S	S	S	B	B
	PB	S	S	S	S	S	S	S

Fuzzy Logic Controller

Based on the expert knowledge base, these were the rules that we set for the fuzzy logic controller for α:

TABLE III
FUZZY TUNING RULES FOR α

		$\Delta e(k)$						
		NB	NM	NS	ZO	PS	PM	PB
	NB	2	2	2	2	2	2	2
	NM	3	3	2	2	2	3	3
	NS	4	3	3	2	3	3	4
$e(k)$	ZO	5	4	3	3	3	4	5
	PS	4	3	3	2	3	3	4
	PM	3	3	2	2	2	3	3
	PB	2	2	2	2	2	2	2



Final form of the fuzzy logic controller- 2 inputs e(k) and Δe(k) and the 3 outputs Kp’, Kd’, and α.

Fuzzy Logic Controller

This fuzzy logic controller will dynamically produce the values for Kp’, Kd’, and α.

We use a function to convert the normalized values to the actual values.

Using the formula:

$Kp = Kp' (Kpmax - Kpmin) + Kpmin$

$Kd = Kd' (Kdmax - Kdmin) + Kdmin$

$Ki = Kp^2 / (\alpha . Kd)$

Where the values of Kpmax = 0.6Ku , Kpmin = 0.32Ku

Kdmin = 0.08KuTu and Kdmax = 0.15KuTu

Zeigler - Nichols Tuning Method

The value of Ku and Tu are determined by Zeigler-Nichols tuning method.

It is done as follows:

1. Initially, set the parameters Kp = 0, Kd = 0, and Ki = 0 for the PID controller and check the step response of the system.

2. Gradually increase the value of KP until sustained oscillation is observed in the step response of the system.

3. The gain at which these sustained oscillations occur is the Ultimate gain (Ku).

4. The time period of oscillation (Tu) can also be obtained from the graph.

Second Order System

The system is defined using:

Applying Zeigler-Nichol's method,

We have found that the

ultimate gain Ku = 4.68

Time period of the sustained oscillation Tu = 3.28 s

Proportional Gain Kp = 2.808,

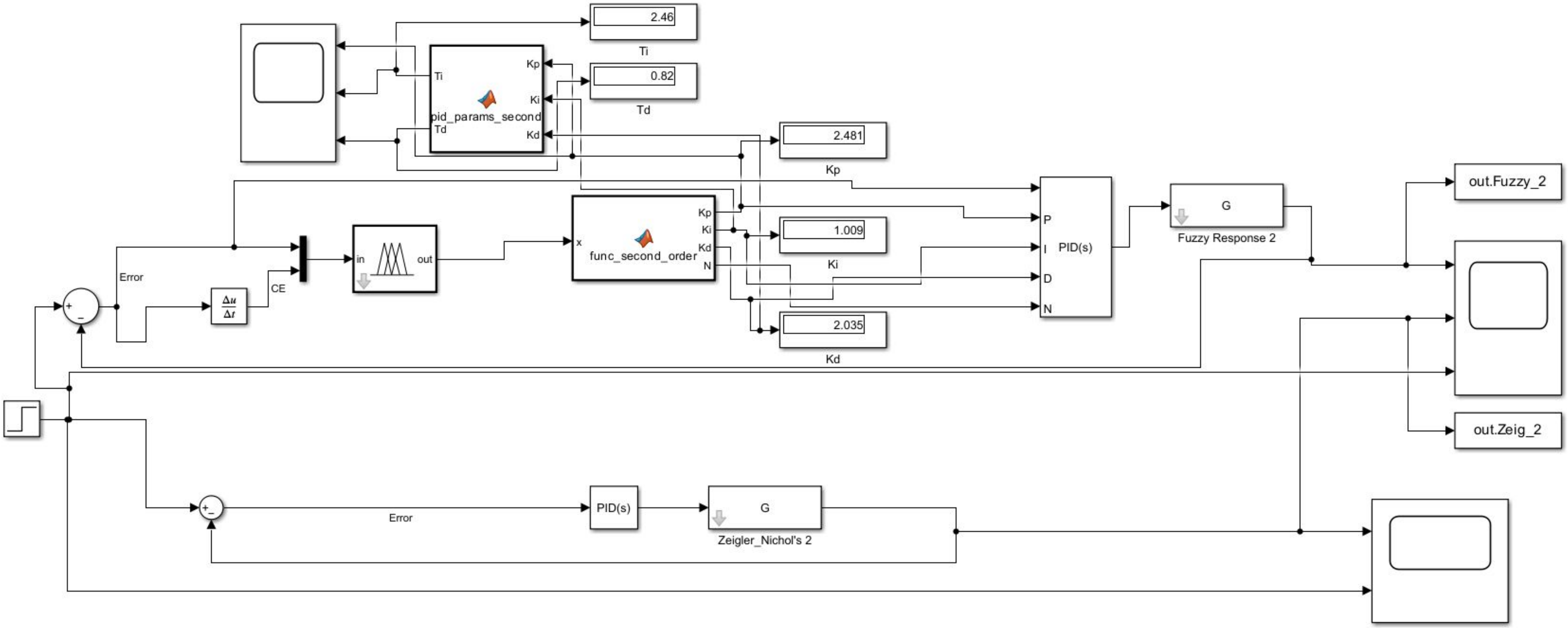
Derivative gain Kd = 1.15128,

Integral gain Ki = 1.7122,

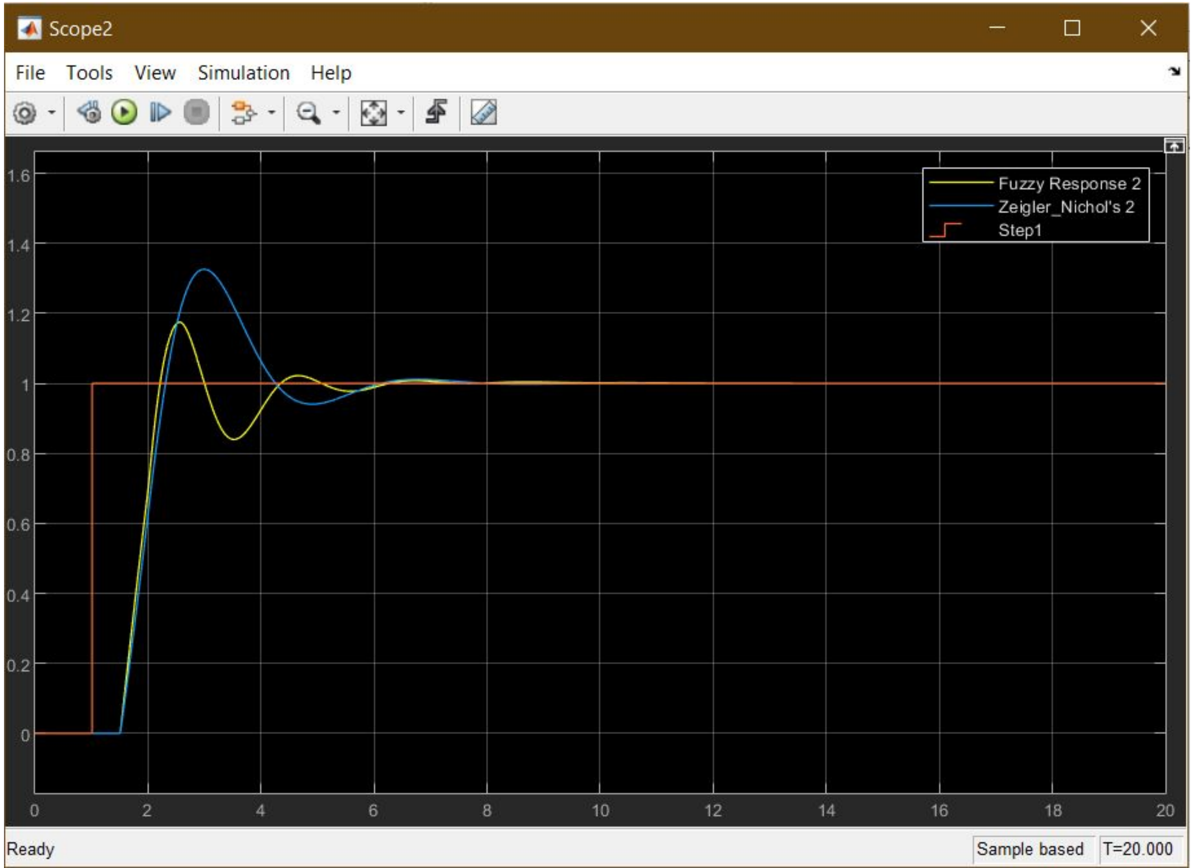
Integral Time constant = 3.28 s,

Derivative Time constant = 0.41 s.

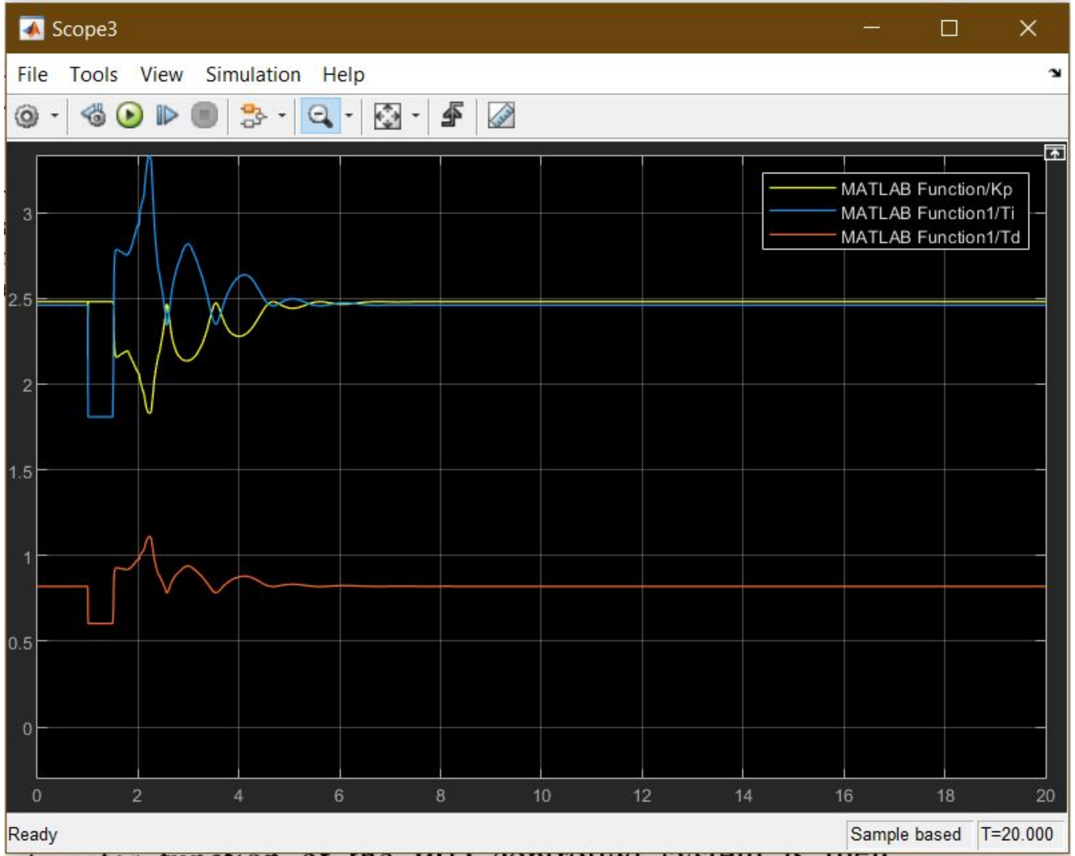
Second Order system



Second order system response



Second order system Parameter range of Kp, Ti, and Td



Third Order System

Applying the Zeigler-Nichols method,

We have found that the ultimate gain Ku = 3.65,

Time period of the sustained oscillation Tu = 2.06s

Proportional Gain KP = 2.19,

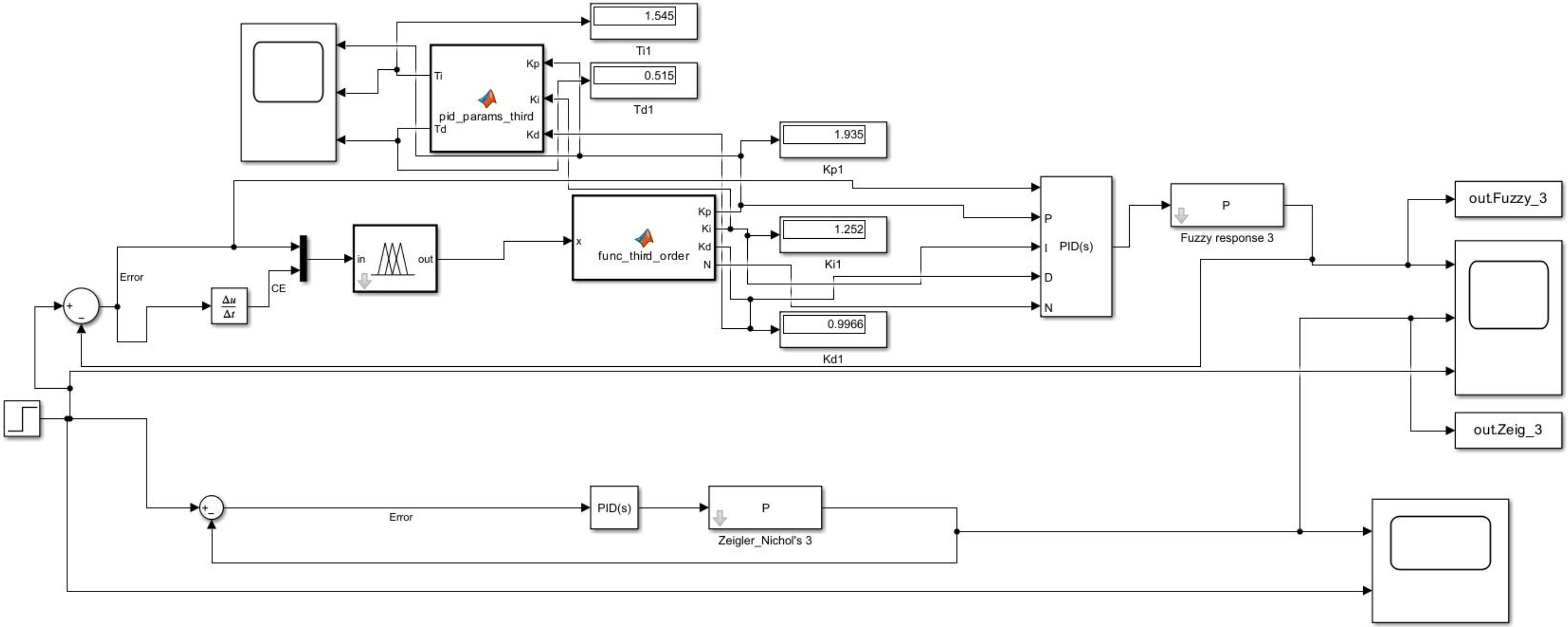
Derivative gain KD = 0.56502,

Integral gain KI = 2.1262,

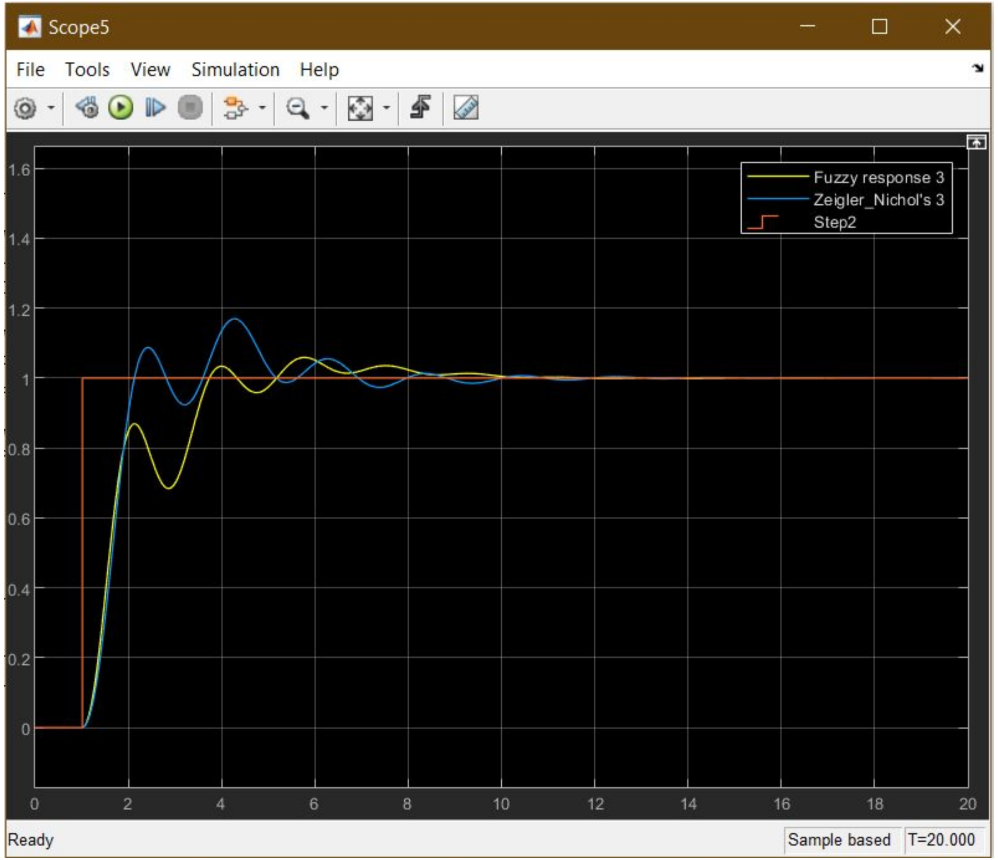
Integral Time constant TI = 1.03s,

Derivative Time constant TD = 0.258s.

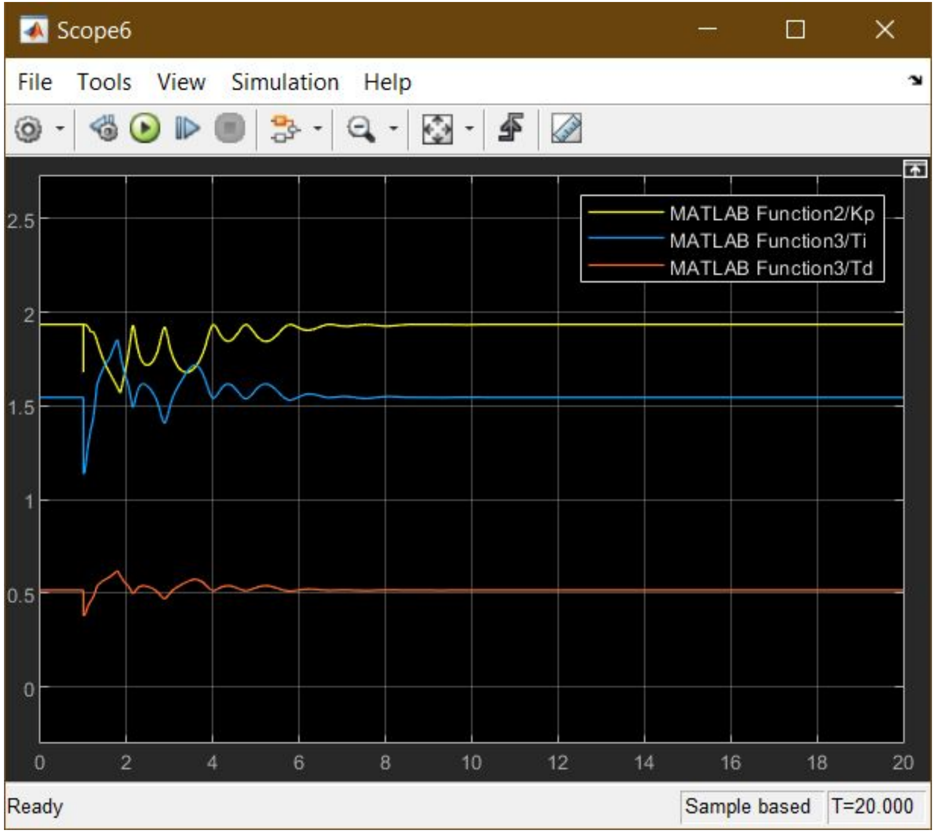
Third order system



Third Order System Response



Third Order System Parameter range of Kp, Ti, and Td



Fourth Order system

Applying the Zeigler-Nichols method,

We have found that the ultimate gain Ku = 5.12,

Time period of the sustained oscillation Tu = 2.704s

Proportional Gain KP = 3.072,

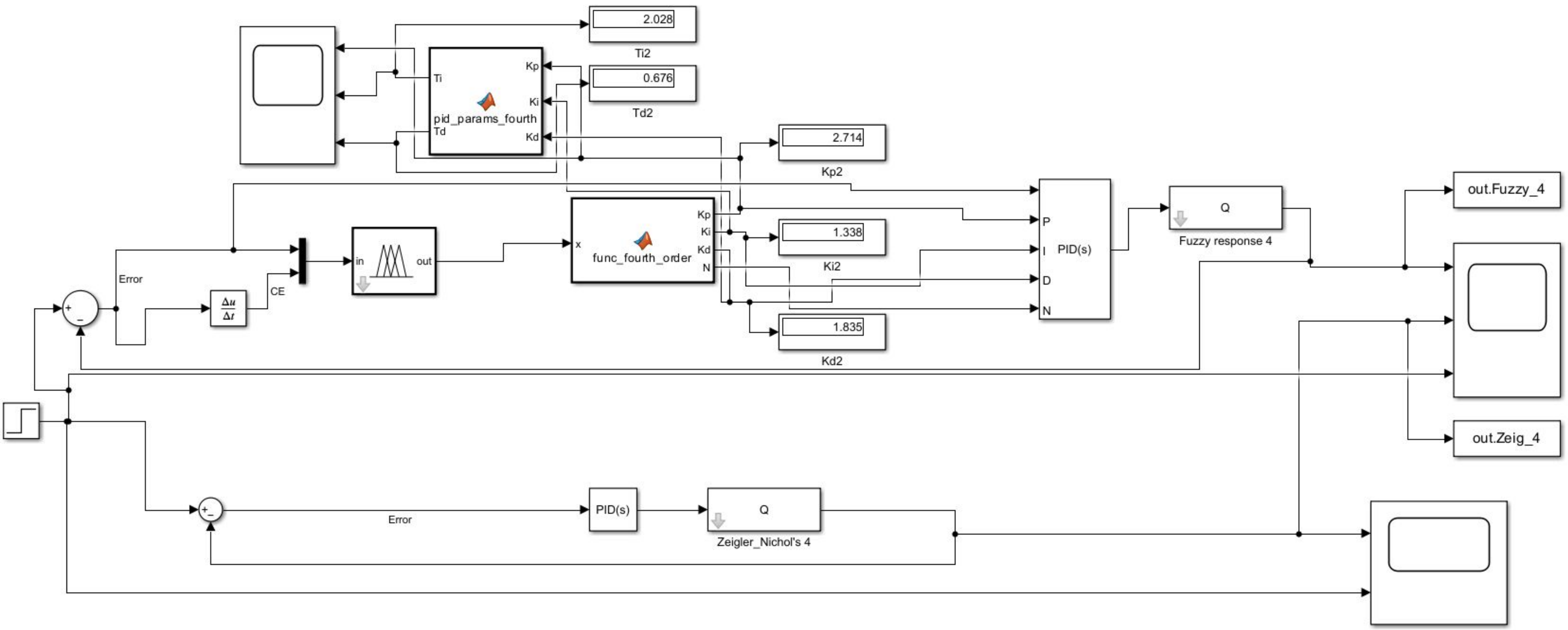
Derivative gain KD = 1.0383,

Integral gain KI = 2.272189,

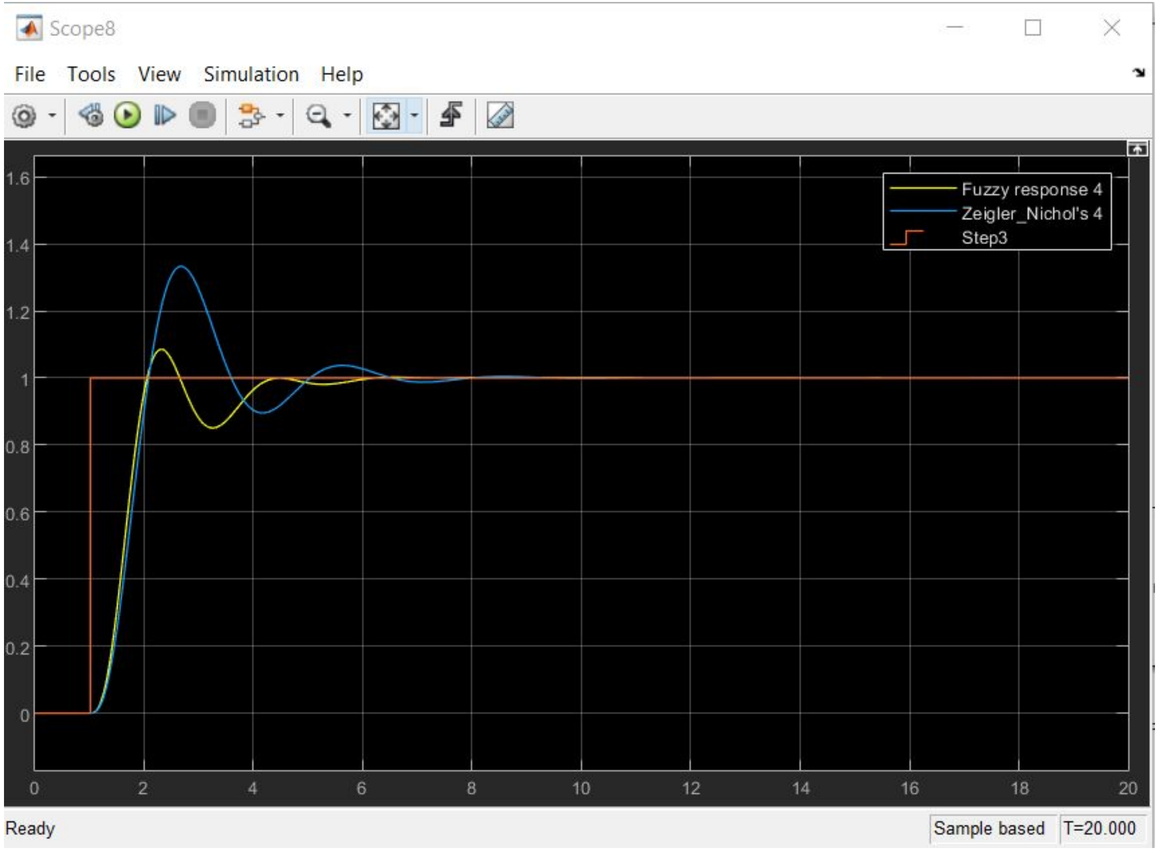
Integral Time constant TI = 1.352s,

Derivative Time constant TD = 0.338s.

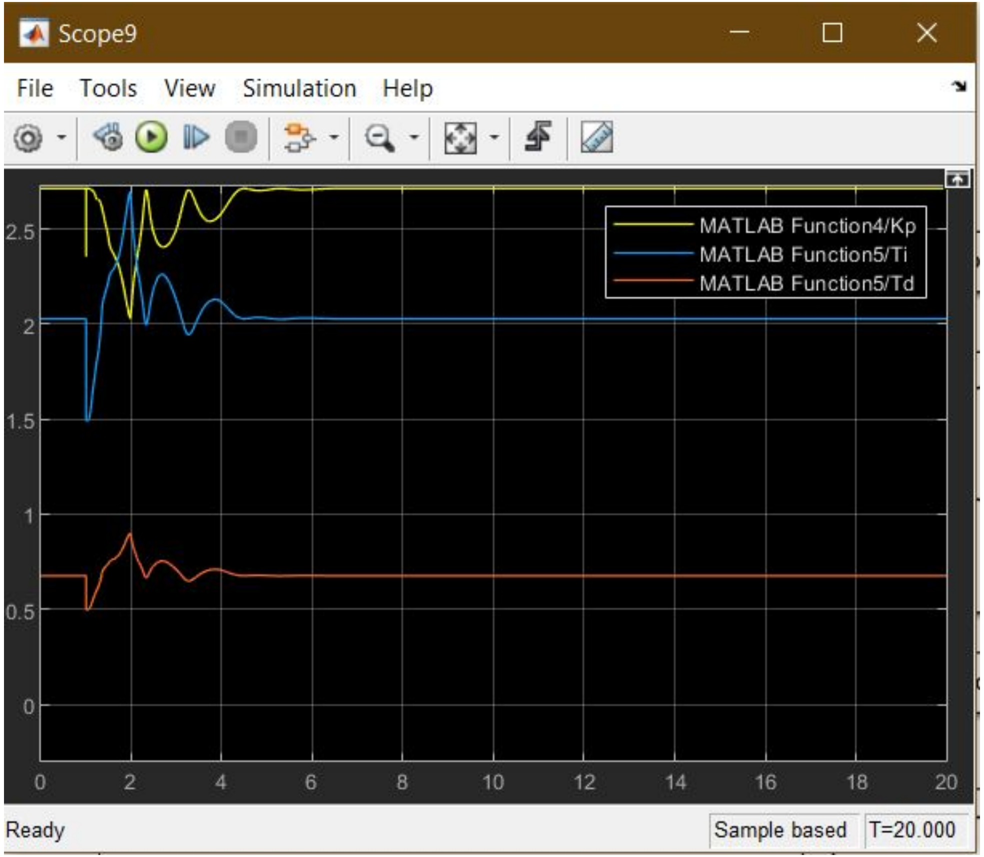
Fourth Order System



Fourth Order System Response



Fourth Order System Parameter range of Kp, Ti, and Td



The results of second order system: Fuzzy Tuning

RiseTime: 0.5541,

TransientTime: 5.7650,

SettlingTime: 5.7650

SettlingMin: 0.8397

SettlingMax: 1.1749

Overshoot: 17.4911

Undershoot: 0

Peak: 1.1749

PeakTime: 2.5607

The results of second order system: Zeigler-Nichols Tuning

RiseTime: 0.6226

TransientTime: 5.7314

SettlingTime: 5.7314

SettlingMin: 0.9101

SettlingMax: 1.3257

Overshoot: 32.5699

Undershoot: 0

Peak: 1.3257

PeakTime: 3.0042

The results of third order system: Fuzzy Tuning

RiseTime: 2.2246

TransientTime: 8.0915

SettlingTime: 8.0915

SettlingMin: 0.9074

SettlingMax: 1.0590

Overshoot: 5.8957

Undershoot: 0

Peak: 1.0590

PeakTime: 5.7669

The results of third order system: Zeigler-Nichols Tuning

RiseTime: 0.7175

TransientTime: 7.6319

SettlingTime: 7.6319

SettlingMin: 0.9157

SettlingMax: 1.1696

Overshoot: 16.9708

Undershoot: 0

Peak: 1.1696

PeakTime: 4.2840

The results of fourth order system: Fuzzy Tuning

RiseTime: 0.6242

TransientTime: 4.1124

SettlingTime: 4.1124

SettlingMin: 0.8508

SettlingMax: 1.0854

Overshoot: 8.5360

Undershoot: 7.4074e-31

Peak: 1.0854

PeakTime: 2.3215

The results of fourth order system: Zeigler-Nichols Tuning

RiseTime: 0.6554

TransientTime: 6.1335

SettlingTime: 6.1335

SettlingMin: 0.8955

SettlingMax: 1.3329

Overshoot: 33.2852

Undershoot: 1.6412e-32

Peak: 1.3329

PeakTime: 2.6809

Conclusion

It is quite evident that the fuzzy gain scheduling results in a satisfactory transient response as compared to the Zeigler-Nichols tuning method. The data from the step response of the second and fourth order system shows that the Fuzzy tuner is better compared to the Zeigler-Nichols method.

For the third order, the Rise time, transient time, settling time, and peak time are a tad higher for the fuzzy tuning method as compared to the Zeigler-Nichols tuning method.

Video of Simulation

https://drive.google.com/file/d/1TAnO4tuq2YZUzF-BhV0cf1uP8aNIAJP4/view?usp=sharing