Work, energy and power

Thu Jul 18 2024

Let's Solve.

Lets Solve

A B

Q. A particle is shifted from point (0, 0, 1 m) to point (1 m, 1 m, 2 m), under the simultaneous action of several forces. Two of them are

$$\vec{F}_1 = (2\hat{i} + 3\hat{j} - \hat{k})\,N \text{ and } \vec{F}_2 = (\hat{i} - 2\hat{j} + 2\hat{k})\,N$$

Find the work done by the resultant of these two forces.

$\vec{F}_1 + \vec{F}_2 = 3\hat{i} + 1\hat{j} + 1\hat{k} = \vec{F}$

$\vec{S} = \vec{r}_B - \vec{r}_A$

$= \hat{i} + \hat{j} + 2\hat{k} - \hat{k}$

$= 1\hat{i} + \hat{j} + 1\hat{k}$

$\vec{F} \cdot \vec{S} = 3 + 1 + 1 = 5$ Joules

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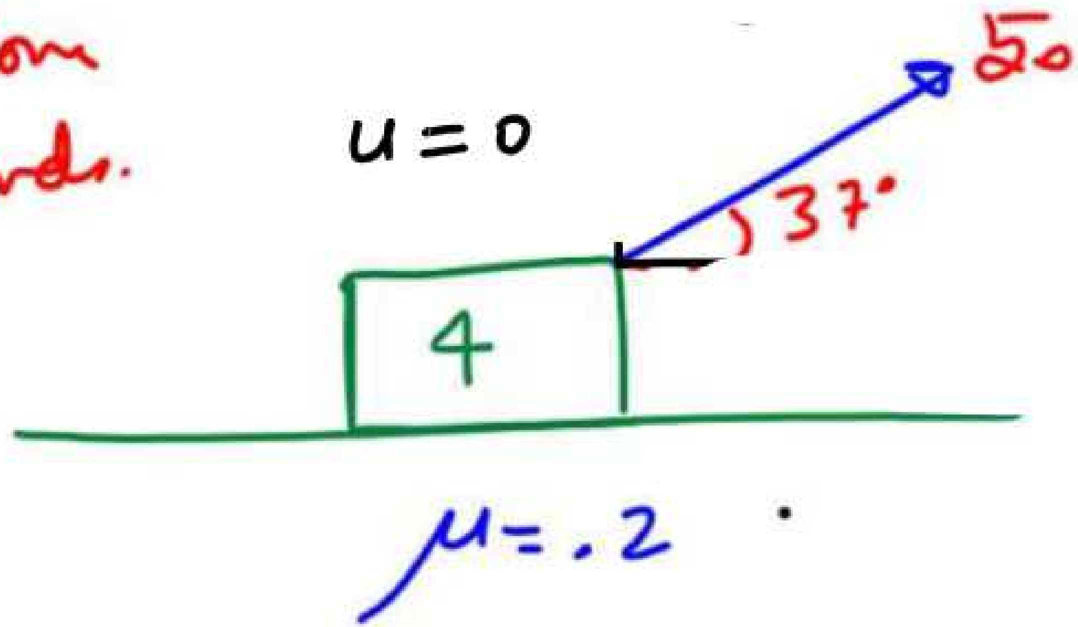
Q. Find all the forces and their work done in first two seconds.

Block starts from rest.

Also find the change in KE.



$$K_1 = \frac{1}{2}mu^2 = 0$$

$\mu = .2$

$N + 30 = 40$

$N = 10$

$fr_L = .2 \times 10 = 2$

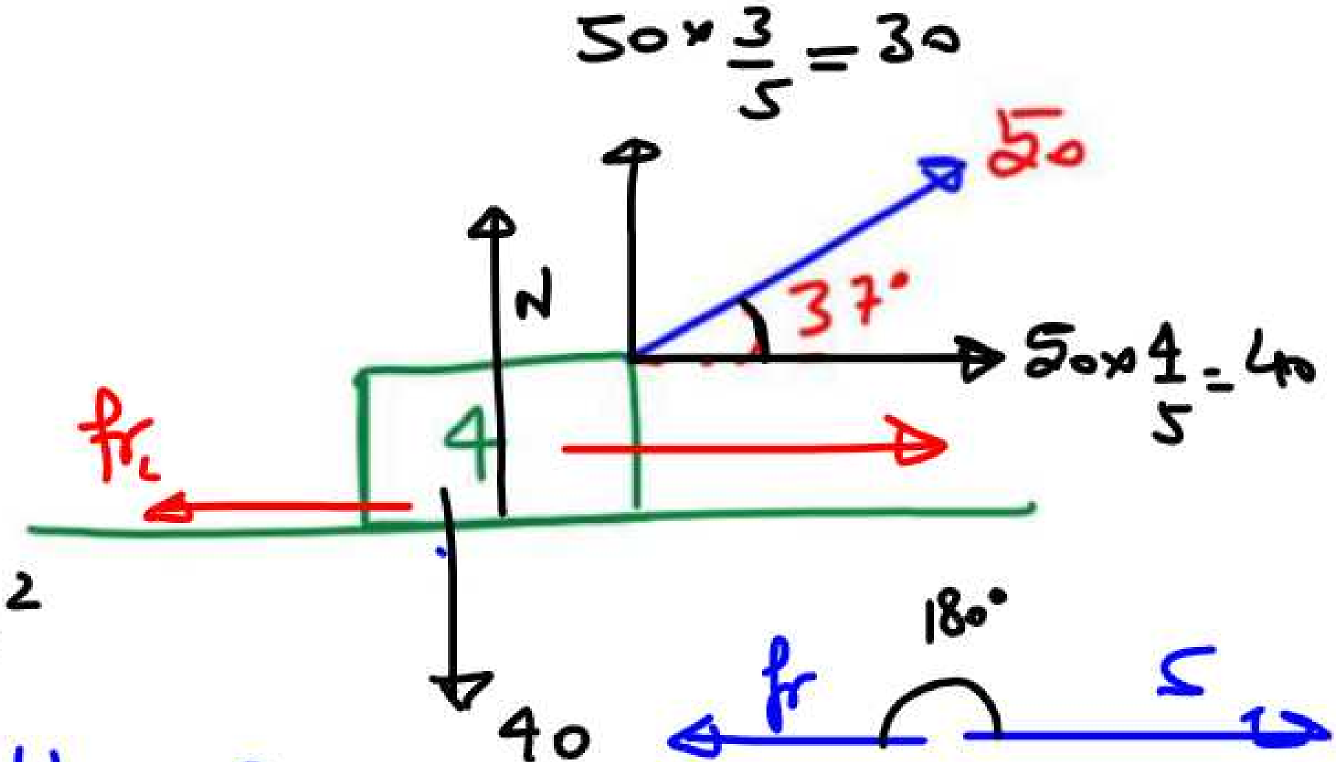
$F_{net} = 40 - 2 = 38$

$a_x = \dfrac{38}{4} = 9.5\,m/s^2$

$S_x = u_x t + \frac{1}{2} a_x t^2$

$S_x = \frac{1}{\cancel{2}} \times \frac{19}{\cancel{2}} \times \cancel{2} \times \cancel{2}$

$= 19m$



$W_N = 0$

$W_{mg} = 0$

$W_{fr} = fr \times S \times \cos 180°$

$= 2 \times 19 \times (-1)$

$= -38$

$W_F = 40 \times 19 = 760$

or $50 \times 19 \times \frac{4}{5} = 760$

$W_N + W_{mg} + W_{fr} + W_F$

$= 0 + 0 - 38 + 760$

$= 722$

$$v = u + at$$

$$= 0 + \frac{19}{\cancel{2}} \times \cancel{2}$$

$$V = 19 \, m/s$$

$$K_f = \frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}} \times 19 \times 19$$

$$= 722$$

$$K_f - K_i = 722 - 0$$

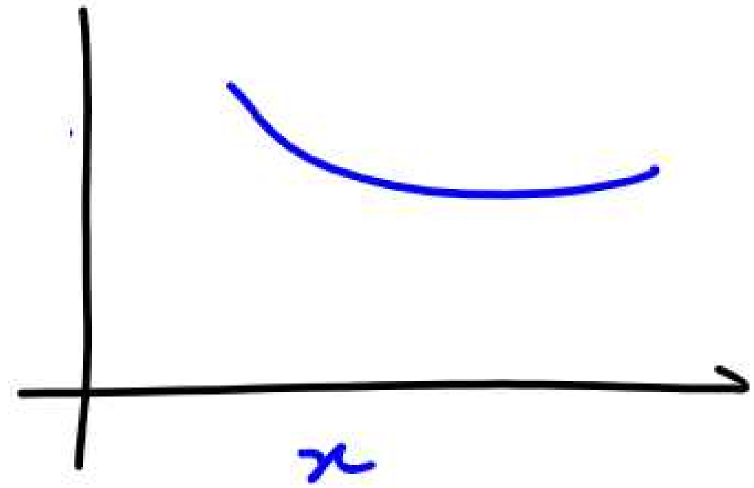
$$= 722$$

Area by F-x curve

* In a 1-D motion Area under F-x gives the work done.

Let's Solve.

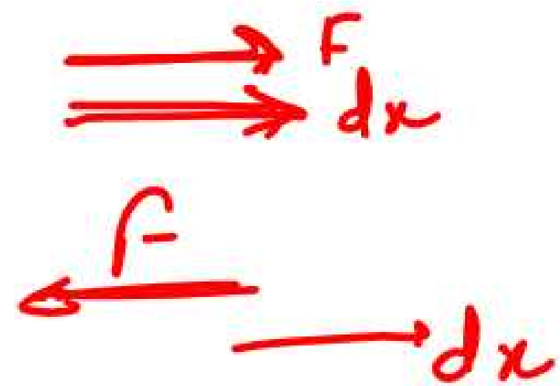
$$W = \int F\,dx$$



$$W = \int \vec{F} \cdot \vec{ds}$$

$$ds = dx$$

$$= \int F\,ds\cos\theta$$

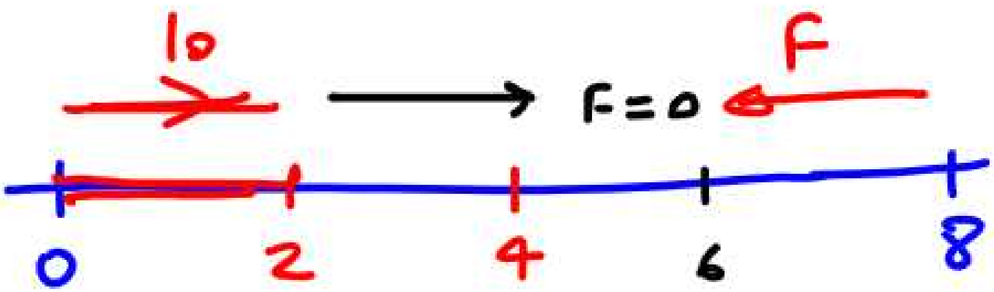


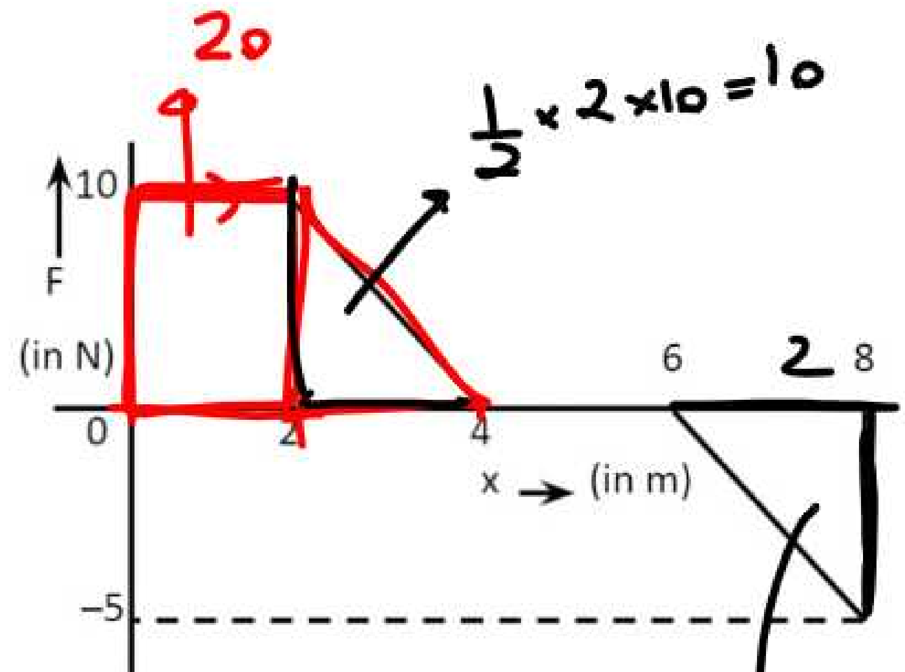
$$= \int F\,dx$$

$$= \int -F\,dx$$

Lets Solve

Q. A 5 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in the figure. Find the work done by this force as the block moves from the origin to x = 8m





$\frac{1}{2} \times 2 \times 10 = 10$

$\frac{1}{2} \times 2 \times -5$

$= -5$

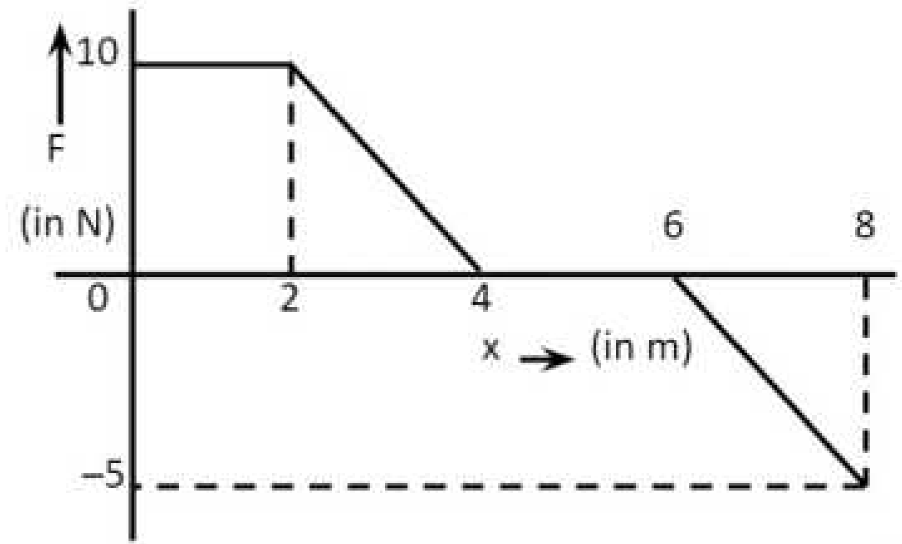
$W = 20 + 10 + (-5)$

$= 25$ Joules

Lets Solve

Q. A 5 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in the figure. Find the work done by this force as the block moves from the origin to x = 8m

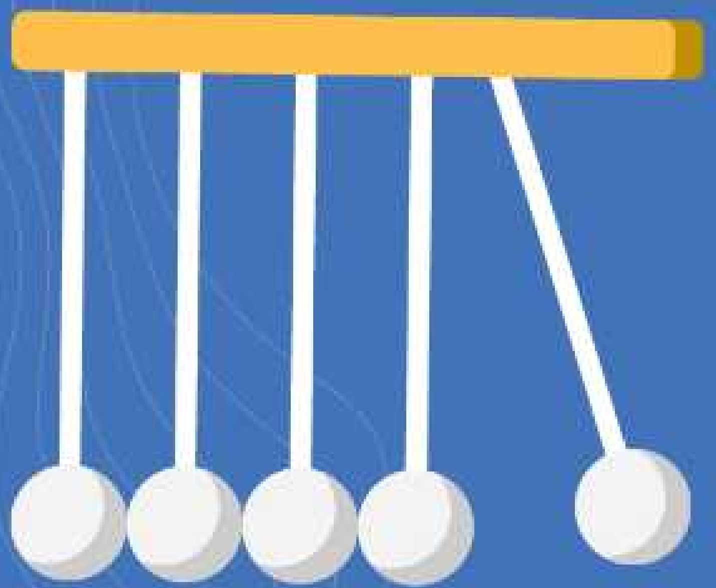
Solution:



The work from x = 0 and x = 8 is the area under the curve and area above x – axis is +ve and below x – axis is −ve.

$$W = \frac{1}{2}[2+4] \times 10 - \frac{1}{2} \times (2) \times 5$$
$$= 30 - 5$$
$$= 25J$$

Quiz.



Quiz

Chat Off

Ex. Figure shows the force F (in newton) acting on a body as a function of x. Calculate the work done in moving the body from x = 0 to x = 4.

A 20 J

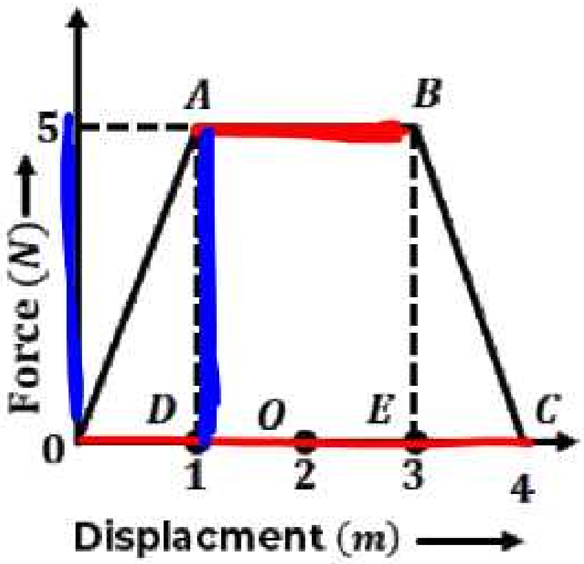
B 30 J

C 15 J

D 10 J

$Ar = \frac{1}{2}[4+2] \times 5$

$= 15$



QUIZ

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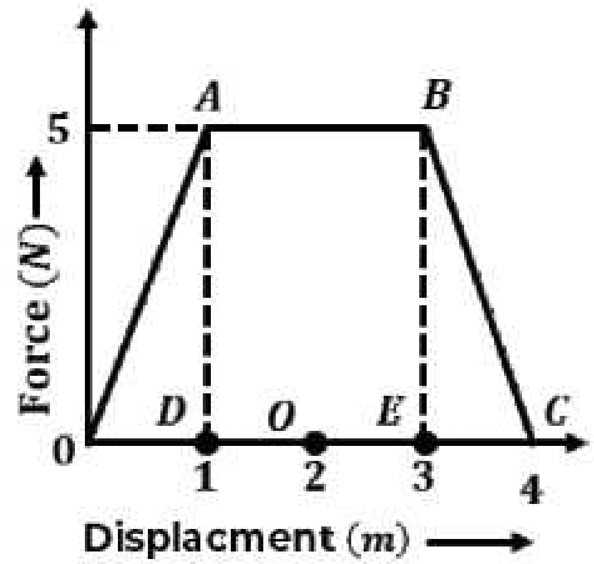


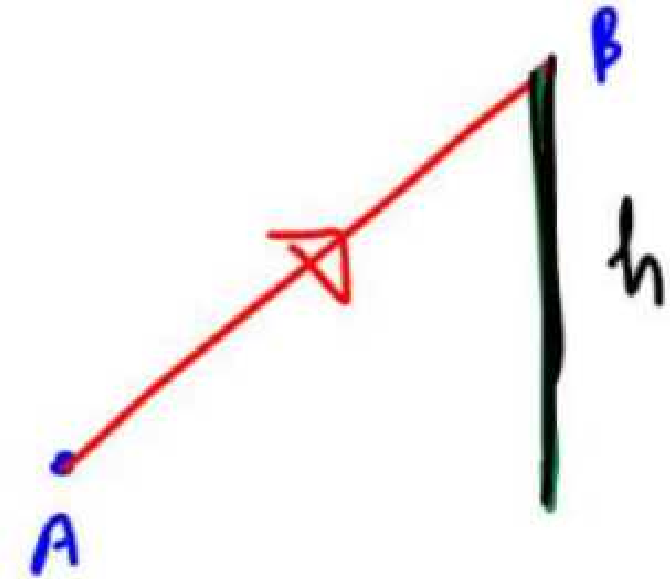
A. 20 J

B. 30 J

C. 15 J

D. 10 J

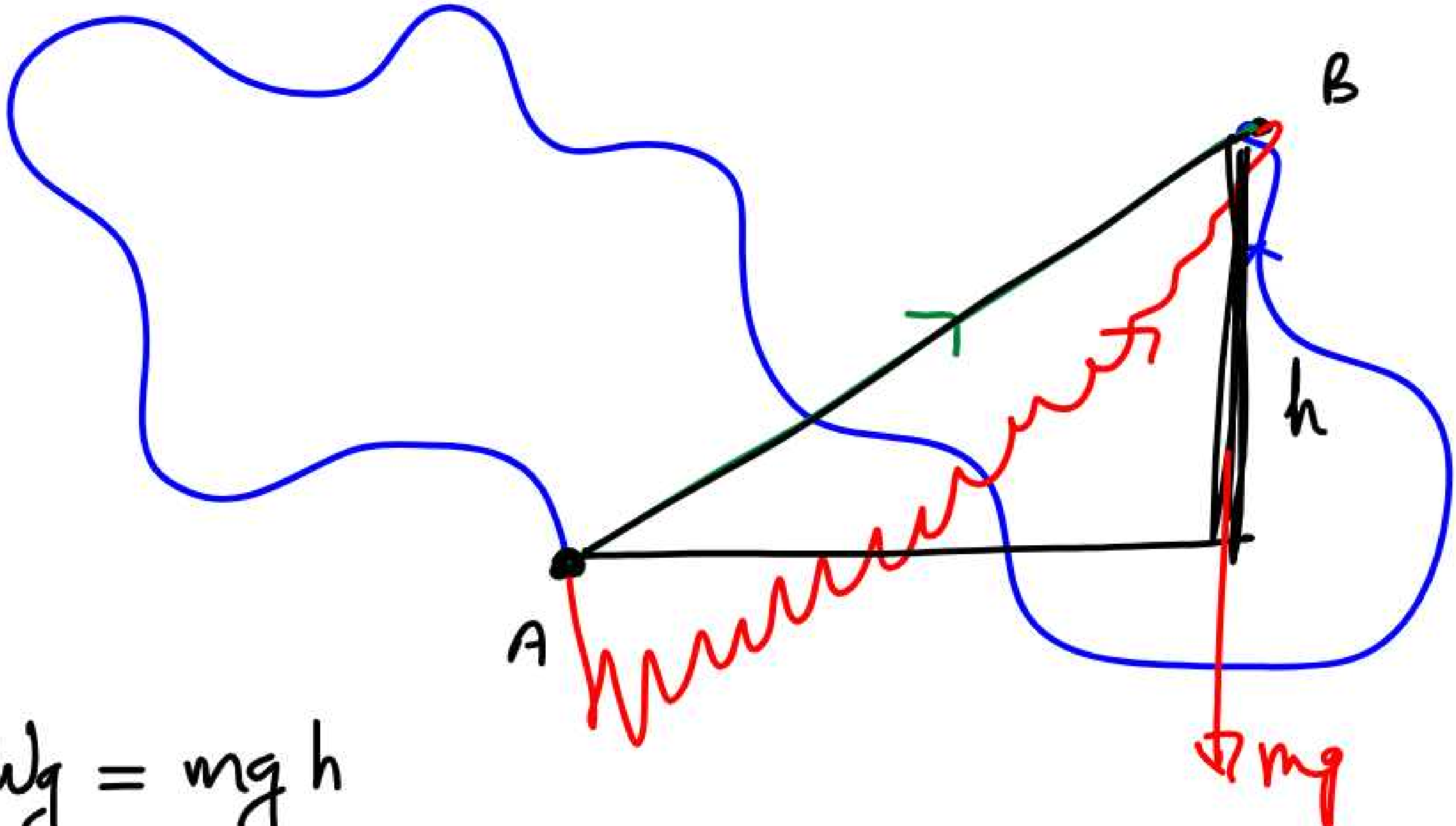
Work done by gravity



$$W_g = mgh$$

h = vertical displacement

- If particle moves up then W_g is negative
- " " " down " " " positive



$$W_g = mgh$$

Work done by spring force

$$W = \frac{k}{2}\left(x_i^2 - x_f^2\right)$$

x_i = Initial extension or compression

x_f = final " " "

k = spring constant

W = Work done by spring force

Let's Solve.

Lets Solve

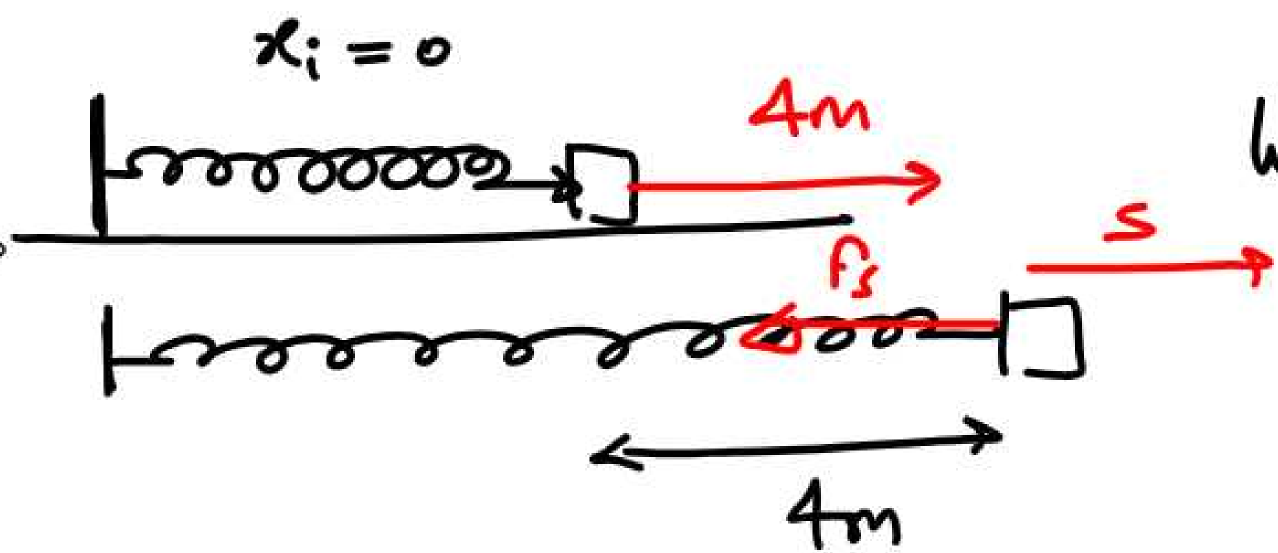
Q. If the force to stretch a spring is given by F = (100 N/m)x, how much work ~~does it take to stretch the spring 4 meters from rest?~~ is done by the spring

$\rightarrow k$

$k = 100\ N/m$

$x_i = 0$

$x_f = 4$



$x_f = 4m$

$$W_{sp} = \frac{k}{2}\left[x_i^2 - x_f^2\right]$$

$$= \frac{100}{2}\left[0^2 - 4^2\right]$$

$$= -\frac{100}{\cancel{2}} \times \cancel{16}\,8$$

$$= -800\,J$$

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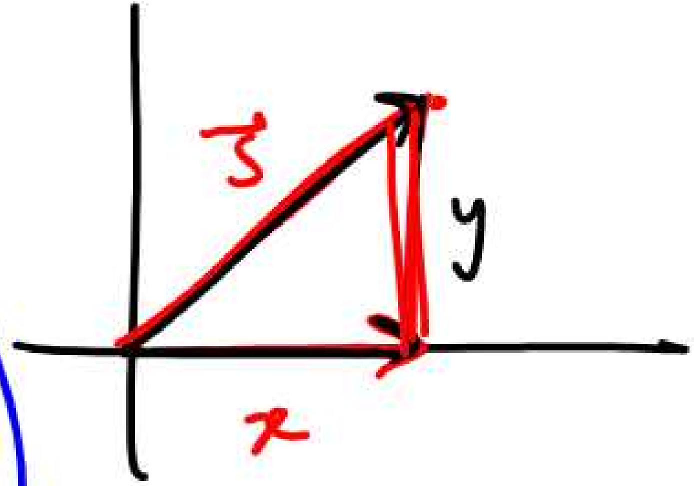
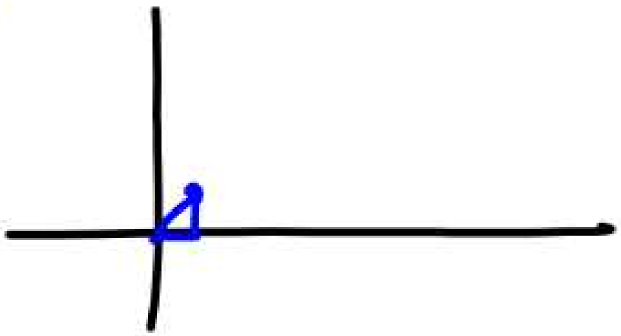
$\vec{s} = x\hat{i} + y\hat{j}$

$\vec{ds} = dx\hat{i} + dy\hat{j}$

$+ dz\hat{k}$





Work done by a variable force

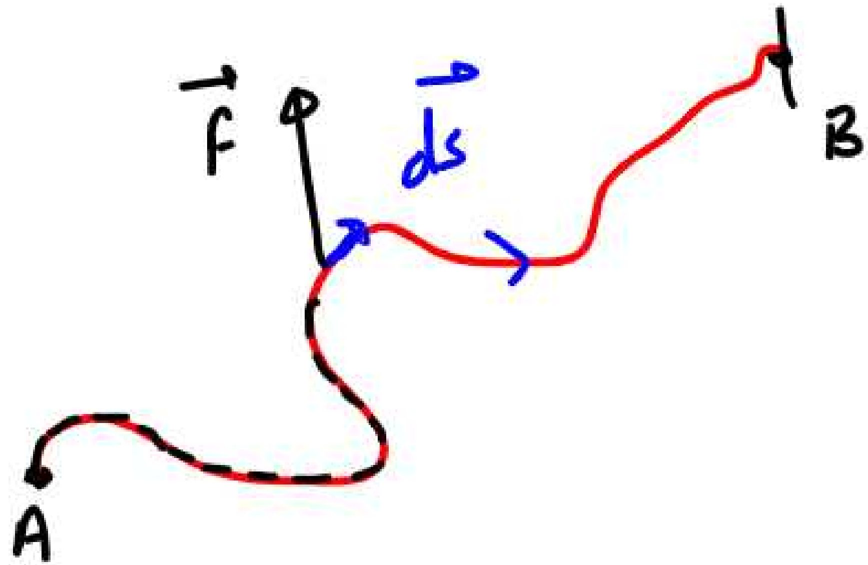
$$W = \vec{F} \cdot \vec{S}$$

$$dW = \vec{F} \cdot \vec{ds}$$

$$W = \int \vec{F} \cdot \vec{ds}$$

$$\int_0^W dW = \int \vec{F} \cdot \vec{ds}$$

$$W = \int \vec{F} \cdot \vec{ds}$$

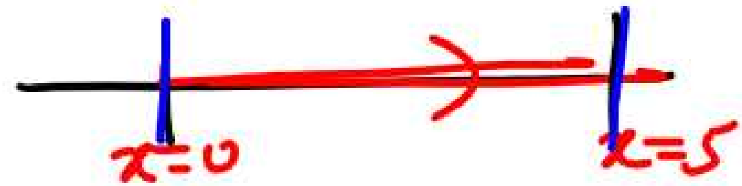


Let's Solve.

Lets Solve

Q. A force $F = (4.0\,x\hat{i} + 3.0\,y\hat{j})$ N acts on particle which moves in the x-direction from the origin to x = 5.0 m. Find the work done on the object by the force.

$$\vec{F} = 4x\,\hat{i} + 3y\,\hat{j}$$



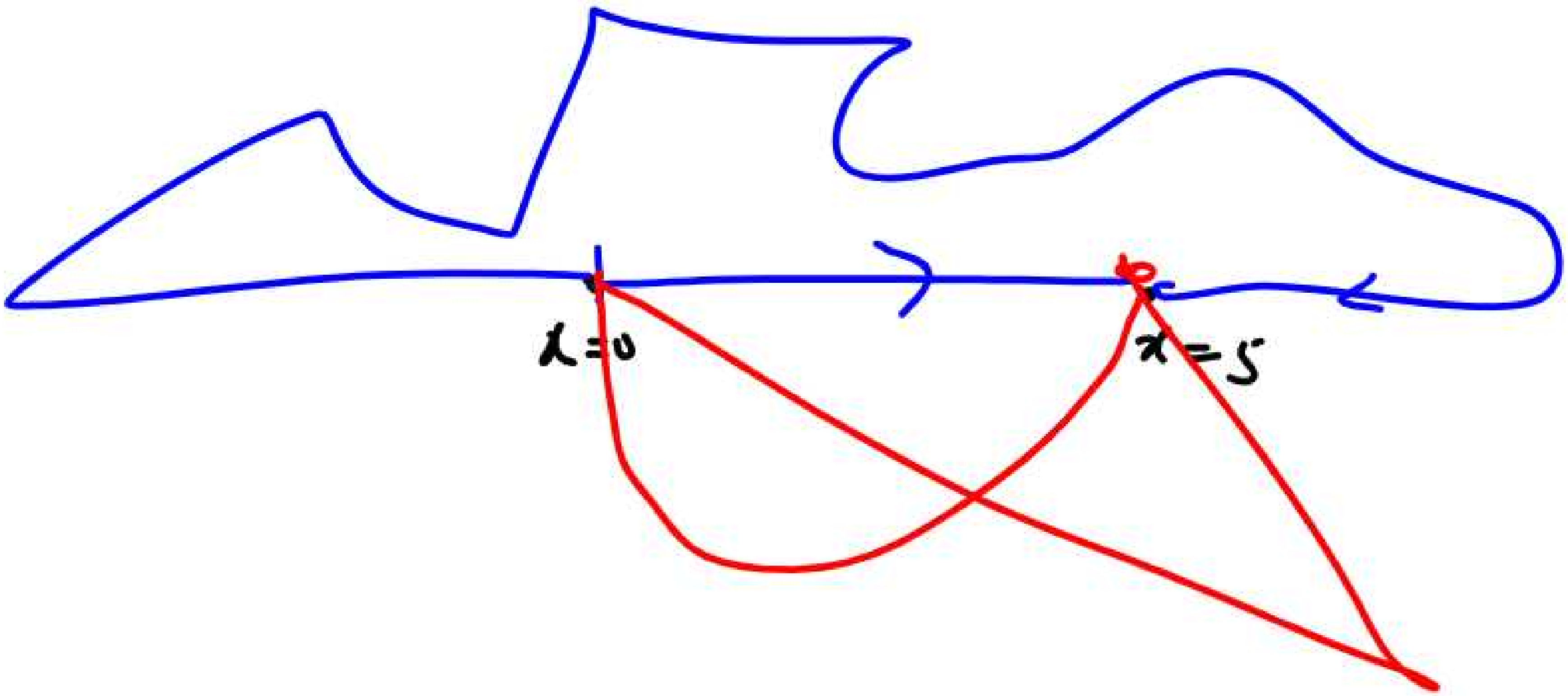
$$W = \int \vec{F} \cdot \vec{ds}$$

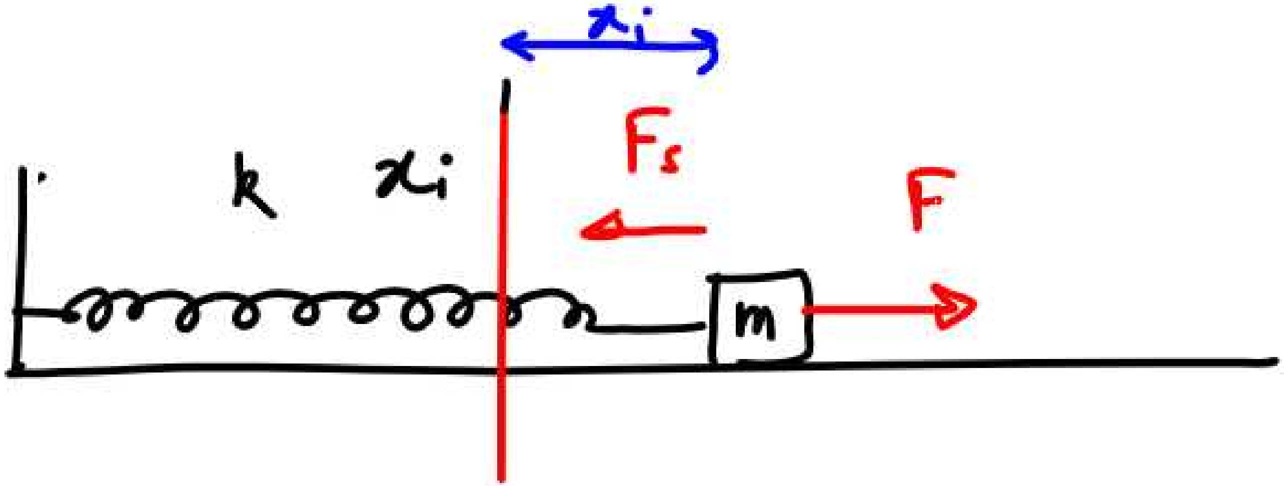
$$\vec{ds} = dx\,\hat{i}$$

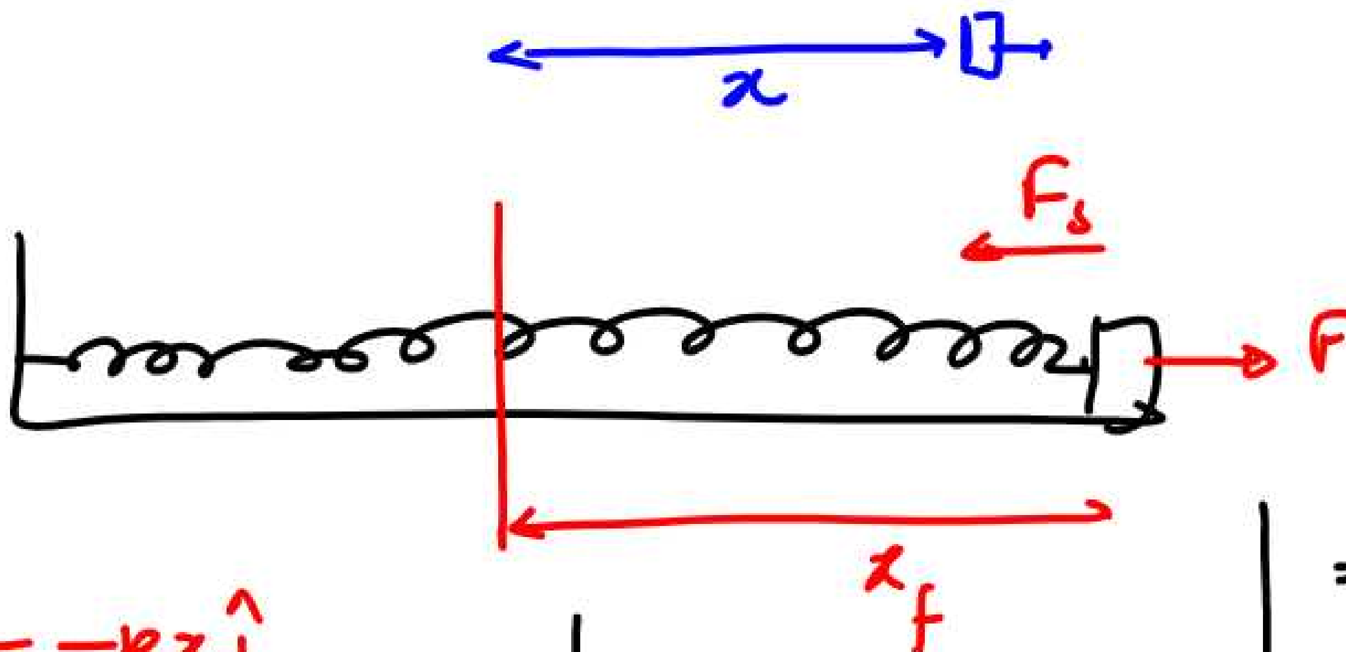
$$= \int (4x\,\hat{i} + 3y\,\hat{j}) \cdot (dx\,\hat{i})$$

$$= \int_0^5 4x\,dx = \frac{\cancel{4}^{\,2}\, x^2}{\cancel{2}}\Bigg|_0^5$$

$$= 2\,[25 - 0) = 50 \text{ Joule}$$


x=0
x=5





$F_s = -kx\hat{i}$

$$W = \int \vec{F_s} \cdot \vec{ds}$$

$$= \int -kx\hat{i} \cdot dx\hat{i}$$

$$= \int_{x_i}^{x_f} -kx\,dx$$

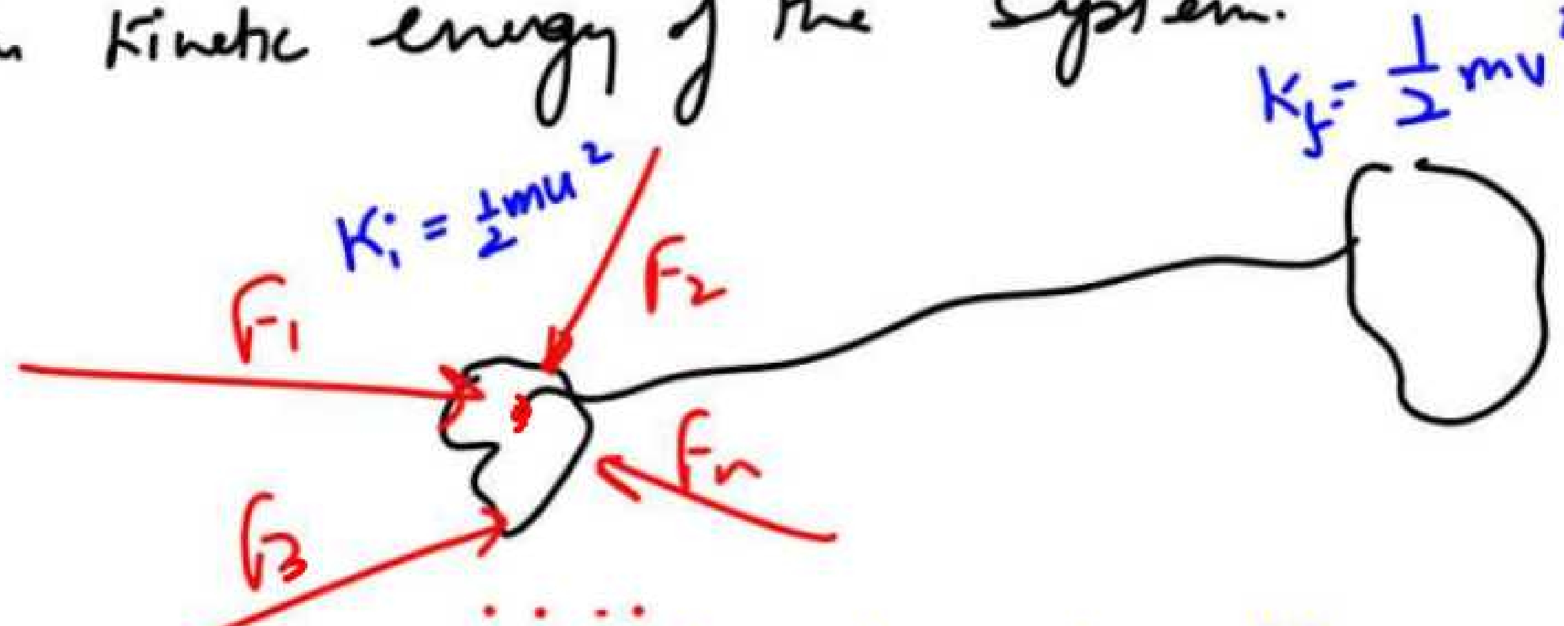
$$= -\frac{kx^2}{2}\Bigg|_{x_i}^{x_f}$$

$$= -\frac{k}{2}\left[x_f^2 - x_i^2\right]$$

$$= \frac{k}{2}\left[x_i^2 - x_f^2\right]$$

Work Energy theorem

- Work done by all the forces acting on a system is equal change in the kinetic energy of the system.



$$W_1 + W_2 + W_3 \ldots W_n = K_f - K_i$$

Let's Solve.

Quiz

Ex. Calculate the amount of work done in raising a glass of water weighing 0.5 kg through a height of 20 cm. ($g = 10\ m/s^2$)

A) 4 J

B) 3 J

C) 2 J

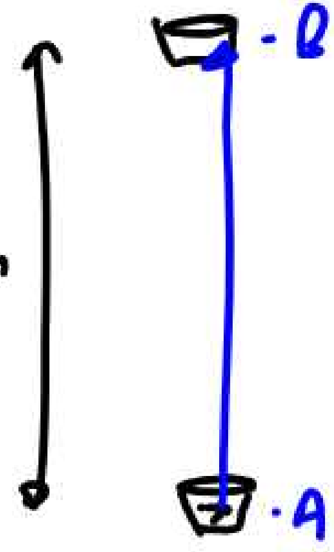
D) 1 J

$$\boxed{W_g = mgh}$$

$$= -\frac{1}{2} \times 10 \times .2$$

$$= -1$$

$H = .2m$



$K_f = 0$

$\Rightarrow K_i = 0$

$m = \frac{1}{2} kg$

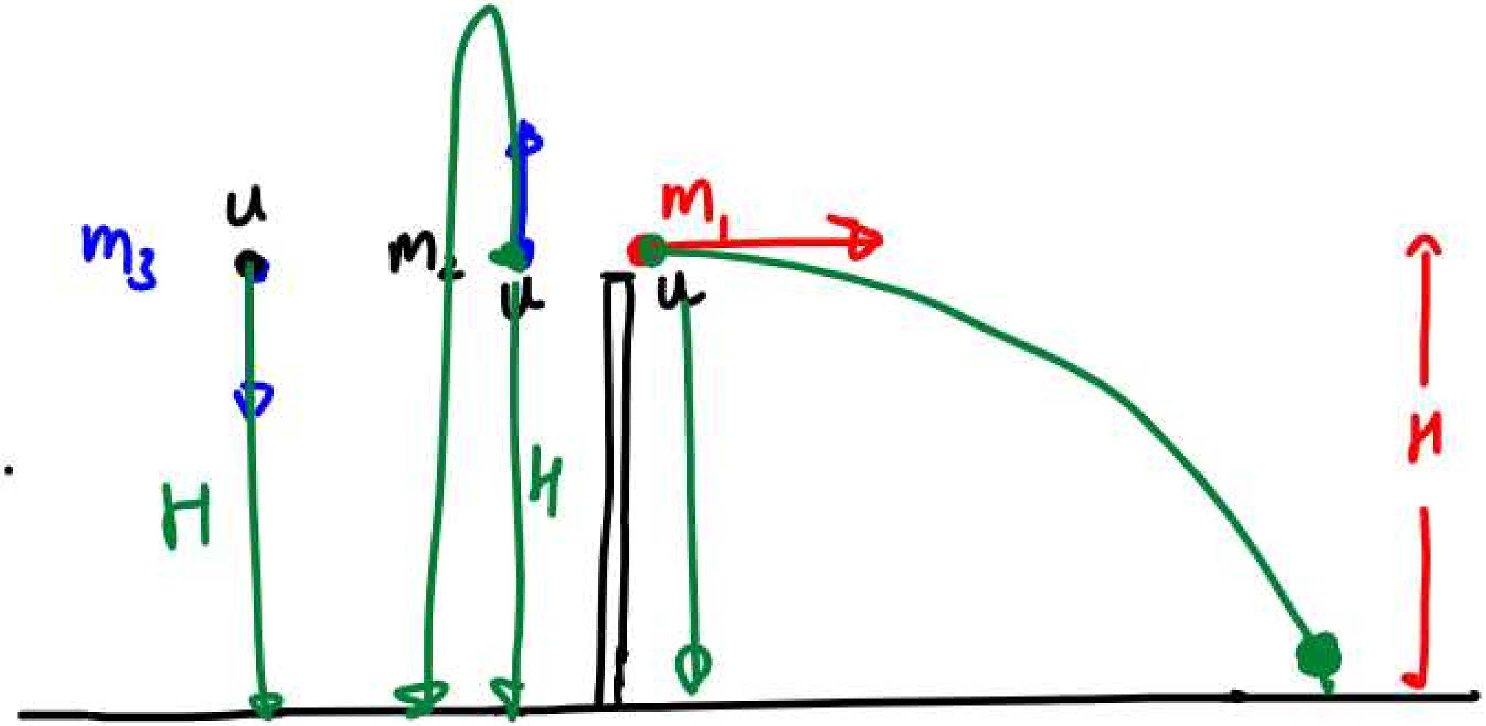
$$W_{Ext} + W_g = \Delta K$$

$$W_{ext} + W_g = 0$$

$$\Rightarrow W_{ext} + (-1) = 0$$

$$\boxed{W_{ext} = 1}$$

just before hitting the ground compare the KE.



a) $K_1 = K_2 = K_3$

b) $K_1 > K_2 > K_3$

c) $K_3 > K_2 > K_1$

d) Data is insufficient

$$W_g = K_f - K_i$$

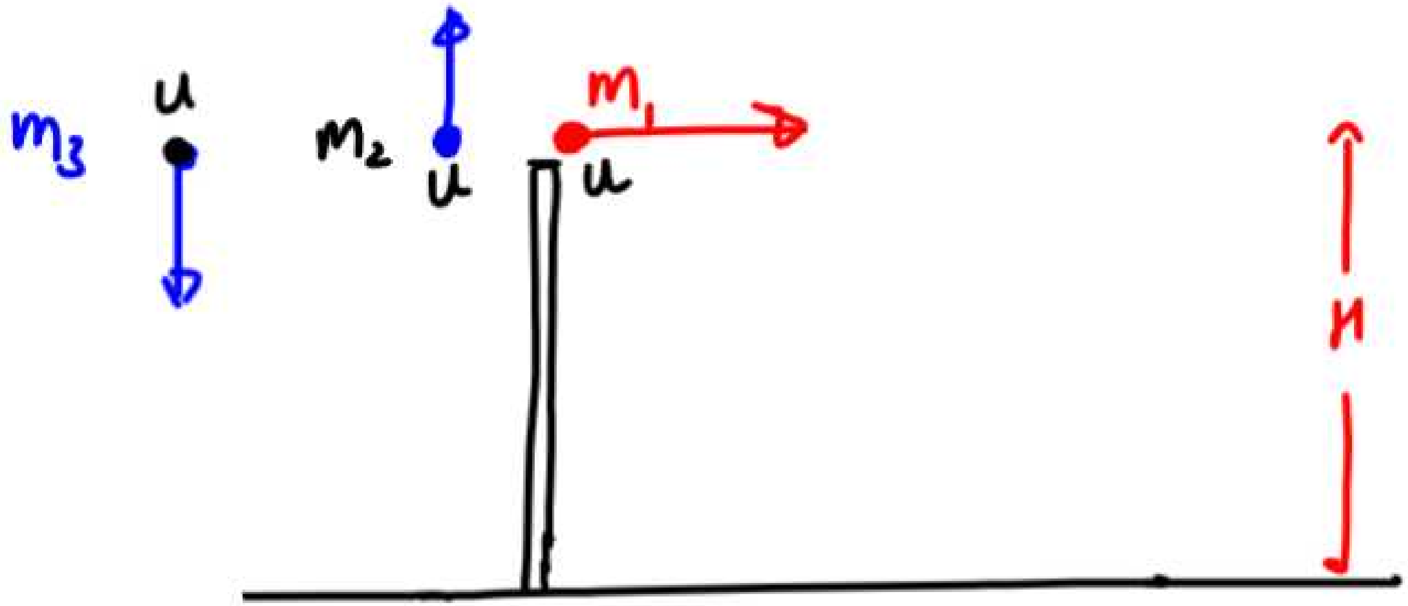
$$mgH = K_f - \frac{1}{2}mu^2$$

$$\boxed{K_f = \frac{1}{2}\cancel{m}u^2 + \cancel{m}gH}$$

$$\frac{1}{2}\cancel{m}v^2$$

$$\boxed{v^2 = u^2 + 2gH}$$

just before hitting the ground
Compare the speeds



a) $v_1 = v_2 = v_3$

b) $v_1 > v_2 > v_3$

c) $v_3 > v_2 > v_1$

d) Data is insufficient