Subjective Matter : For Self Study

1) Integration :

A function $\phi(x)$ is called a primitive or an antiderivative or indefinite integral of a function $f(x)$. If $\phi'(x) = f(x)$ or $\int f(x)dx = \phi(x) + c$, where c is constant of integration. It is the inverse operation of differentiation.

2) Integration formulae :

1) $\int 0.dx = c$

2) $\int dx = x + c$

3) $\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$

4) $\int e^x dx = e^x + c$

5) $\int a^x dx = \frac{a^x}{\log a} + c$

6) $\int \frac{1}{x} dx = \log|x| + c$

7) $\int \sin x\, dx = -\cos x + c$

8) $\int \cos x\, dx = \sin x + c$

9) $\int \sec^2 x . dx = \tan x + c$

10) $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

11) $\int \sec x . \tan x\, dx = \sec x + c$

12) $\int \operatorname{cosec} x . \cot x\, dx = -\operatorname{cosec} x + c$

13) $\int \cot x\, dx = \log|\sin x| + c$

14) $\int \tan x\, dx = -\log|\cos x| + c = \log|\sec x| + c$

15) $\int \sec dx = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + c$

$= \log[\sec x + \tan x] + c$

16) $\int \operatorname{cosec} x\, dx = \log \tan \frac{x}{2} + c$

$= \log[\operatorname{cosec} x - \cot x] + c$

17) $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$

18) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left(\frac{x-a}{x+a}\right) + c$

19) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left(\frac{a+x}{a-x}\right) + c$

20) $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + c$

21) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$

$= -\cos^{-1} x + c$

22) $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$

$= -\cot^{-1} x + c$

23) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$

$= -\operatorname{cosec}^{-1} x + c$

24) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left[x + \sqrt{x^2 + a^2}\right] + c$

25) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left[x + \sqrt{x^2 - a^2}\right] + c$

26) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

27) $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log\left[x + \sqrt{x^2 + a^2}\right] + c$

28) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left[x+\sqrt{x^2-a^2}\right]+c$

29) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$

30) $\int [f(x)]^n . f'(x)dx = \frac{[f(x)]^{n+1}}{n+1}+c, \quad n \neq -1$

31) $\int \frac{f'(x)}{f(x)}dx = \log[f(x)]+c$

32) $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)}+c$

33) **Integration by parts :**

If u and v be two function of x, then $\int u.v\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$

34) $\int e^x[f(x)+f'(x)]dx = e^x.f(x)+c$

3) **Theorems :** If $f(x)$ and $g(x)$ are integrable function of x and k is constant, then

(i) $\int k.f(x)dx = k\int f(x)dx$

(ii) $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

4) **Trigonometric substitutions :**

Expression	Substitution
(i) $\sqrt{x^2+a^2}$	$x = a\tan\theta$
(ii) $\sqrt{x^2-a^2}$	$x = a\sec\theta$
iii) $\sqrt{a^2-x^2}$	$x = a\sin\theta$
(iv) $\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$	$x = a\cos 2\theta$
(v) $\sqrt{\frac{x-a}{b-x}}$ or $\sqrt{(x-a)(x-b)}$	$x = a\cos^2\theta + b\sin^2\theta$

5) **Types of integrals :**

(1) Integration by substitutions :

(a) $\int [f(x)]^n . f'(x)dx$

(b) $\int \frac{f'(x)}{f(x)}dx$

(c) $\int \frac{f'(x)}{\sqrt{f(x)}}dx$

Method : Put $t = f(x)$, $dt = f'(x)\,dx$

(2) Integrals of the form :

(a) $\int \frac{dx}{ax^2+bx+c}$

(b) $\int \frac{dx}{\sqrt{ax^2+bx+c}}$

(c) $\int \sqrt{ax^2+bx+c}\,dx$

Method :

(1) Make the co-efficient of x^2 one if it is not $ax^2+bx+c = a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right]$

(2) Add and substract $\left(\frac{b}{2a}\right)^2$

$$\therefore \quad ax^2+bx+c=a\left[x^2+\frac{b}{a}x+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}+\frac{c}{a}\right]$$

$$=a\left[\left(x+\frac{b}{2a}\right)^2\pm(k)^2\right] \qquad \text{where } k^2=\frac{c}{a}-\frac{b^2}{4a^2}$$

(3) Then use the suitable formula :

(i) $\int\frac{dx}{x^2+a^2}$ (ii) $\int\frac{dx}{x^2-a^2}$ (iii) $\int\frac{dx}{a^2-x^2}$

(iv) $\int\frac{dx}{\sqrt{x^2+a^2}}$ (v) $\int\frac{dx}{\sqrt{x^2-a^2}}$ (vi) $\int\frac{dx}{\sqrt{a^2-x^2}}$

(vii) $\int\sqrt{x^2+a^2}\,dx$ (viii) $\int\sqrt{x^2-a^2}\,dx$ (ix) $\int\sqrt{a^2-x^2}\,dx$

(3) Integrals of the form :

(a) $\int\frac{p(x)+q}{ax^2+bx+c}dx$ (b) $\int\frac{p(x)+q}{\sqrt{ax^2+bx+c}}dx$ (c) $\int[p(x)+q]\sqrt{ax^2+bx+c}\,dx$

Method : Here p, q, a, b, c are constants. We express numerator [p (x) + q] as follows :

$$\text{Numerator}=A\,\frac{d\,(\text{denominator})}{dx}+B$$

(4) Integrals of the form :

$$\int \frac{dx}{a+b\sin^2 x}, \int \frac{dx}{a+b\cos^2 x} \text{ or } \int \frac{dx}{a+b\sin^2 x+c\cos^2 x}$$

Method :

(i) Divide $\cos^2 x$ to both numerator and denominator.

(ii) Write $\sec^2 x = 1 + \tan^2 x$ in denominator [if $\sec^2 x$ is denominator]

(iii) Put $t = \tan x$ by this substitution integral reduce in the form $\int \frac{dx}{ax^2+bx+c}$

(iv) Evaluate it.

(5) Integrals of the form :

$$\int \frac{dx}{a+b\sin x}, \int \frac{dx}{a+b\cos x} \text{ or } \int \frac{dx}{a+b\sin x + c\cos x}$$

Method : Put $t = \tan\frac{x}{2}$, $dx = \frac{2dt}{1+t}$

$$\sin x = \frac{2t}{1+t^2} \text{ and } \cos x = \frac{1-t^2}{1+t^2}$$

(6) Integrals of the form :

$$\int \frac{dx}{a+b\sin 2x}, \int \frac{dx}{a+b\cos 2x} \text{ or } \int \frac{dx}{a+b\sin 2x + c\cos 2x}$$

Method : Put $t = \tan x$, $dx = \frac{dt}{1+t^2}$

$$\sin 2x = \frac{2t}{1+t^2} \text{ and } \cos 2x = \frac{1-t^2}{1+t^2}$$

(7) Integrals of the form : $\int \frac{dx}{a\sin x + b\cos x}$

Method : Divide and multiply $\sqrt{a^2+b^2}$ to denominator.

(8) Integrals of the form :

$$\int \frac{a\sin x + b\cos x}{c\sin x + d\cos x}\,dx \quad \text{and} \quad \int \frac{ae^x + b}{ce^x + d}\,dx$$

Method : Numerator = A [deri. of denominator] + B [denominator]

$+ Bx + k$

(9) Integration by parts :

If u and v are differentiable function of x, then

$$\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$$

Note : We can choose the first function according to the letters in the word LIATE. Where, L - stands for the logarithmic, I - inverse trigonometric, A - algebraic, T - trigonometric and E- exponential functions.

(10) Integral of the form :

$$\int e^x[f(x)+f'(x)]dx = e^x.f(x)+C$$

(11) Integration by partial fractions :

i) If $f(x)$ and $g(x)$ are two polynomials then $\frac{f(x)}{g(x)}$ is a rational function of x and $g(x) \neq 0$.

ii) If degree of $f(x)$ is less than the degree of $g(x)$ then the rational function is called proper rational function otherwise it is called improper rational function.

iii) If function is improper then divide $f(x)$ by $g(x)$ and this rational function can be written in the following form.

$$\frac{f(x)}{g(x)} = \text{Quotient} + \frac{\text{Remainder}}{g(x)}$$

29. $\int \tan^{-1}\sqrt{x}\,dx =$ [AMU 1990]

(1) $x.\tan^{-1}x - \frac{1}{2}\log(1+x^2)+c$

(2) $x\tan^{-1}\sqrt{x} - \frac{1}{2}\log(1+x^2)+c$

(3) $x\tan^{-1}\sqrt{x} - \sqrt{x} + \log(1+x)+c$

(4) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c$

30. $\int (x-1)e^{-x}\,dx =$ [S.C.R.A. 1990]

(1) $xe^{-x}+c$ (2) $xe^{x}+c$
(3) $-xe^{-x}+c$ (4) $-xe^{x}+c$

31. $\int (1-\cos x)\operatorname{cosec}^2 x\,dx =$ [I.I.T. 1990]

(1) $\tan\frac{x}{2}+c$ (2) $\frac{1}{2}\tan\frac{x}{2}+c$

(3) $\cot\frac{x}{2}+c$ (4) $2\tan\frac{x}{2}+c$

32. $\int \sqrt{1+\sin x}\,.f(x)dx = \frac{2}{3}(1+\sin x)^{3/2}+c$ then $f(x)$ is : [AMU 1990]

(1) $\cos x$ (2) $\tan x$
(3) $\sin x$ (4) 1

33. $\int \frac{\log(x+1)-\log x}{x(x+1)}dx =$

(1) $-\frac{1}{2}\left[\log(1+x)^2-(\log x)^2\right]+c$

(2) $-\frac{1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2+c$

(3) $\log\left(\frac{x}{x+1}\right)+c$

(4) $\log\left[\log\left(\frac{x}{x+1}\right)\right]+c$

34. $\int \frac{\sqrt[3]{x-x^3}}{x^4}dx =$

(1) $\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{4/3}+c$

(2) $-\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{4/3}+c$

(3) $\frac{8}{3}\left(\frac{1}{x^2}-1\right)^{4/3}+c$

(4) $-\frac{8}{3}\left(\frac{1}{x^3}-1\right)^{4/3}+c$

35. $\int [f(x).g''(x)-f''(x).g(x)]dx =$

(1) $f(x)g'(x)+f(x)g(x)+c$

(2) $f(x)g'(x)-f(x)g(x)+c$

(3) $f(x)g'(x)+f'(x)g(x)+c$

(4) $f(x)g'(x)-f'(x)g(x)+c$

36. $\int \operatorname{cosec}^2 x\,dx =$ [MP 1999]

✓ (1) $\cot x + c$ (2) $\tan^2 x + c$
(3) $-\cot x + c$ (4) $-\cot^2 x + c$

37. $\int \frac{x^2+1}{x(x^2-1)}dx =$

(1) $\log\left(\frac{x^2-1}{x}\right)+c$ (2) $\log\left(\frac{x}{x^2+1}\right)+c$

(3) $-\log\left(\frac{x^2-1}{x}\right)+c$ (4) $-\log\left(\frac{x}{x^2+1}\right)+c$

38. $\int x\,.\cos x^2 dx =$

(1) $-\frac{1}{2}\sin^2 x+c$ (2) $-\frac{1}{2}x^2+c$

(3) $\frac{1}{2}\sin^2 x+c$ (4) $\frac{1}{2}\sin x^2+c$

39. $\int \cos^2 x\,dx =$ [Delhi College of E.E. 1999]

(1) $\frac{1}{2}\left(x-\frac{1}{2}\sin 2x\right)+c$

(2) $\frac{1}{2}\left(1+\frac{1}{2}\sin 2x\right)+c$

(3) $\frac{1}{2}\left(x+\frac{1}{2}\sin 2x\right)+c$

(4) $(x+\sin 2x)+c$

40. $\int (1+4x+6x^2+4x^3+x^4)dx =$ [CET 1990]

(1) $4+12x+12x^2+4x^3+c$

(2) $\frac{(1+x)^5}{5}+c$

(3) $x+2x^2+2x^3+6x^4+4x^5+c$

(4) None of these

41. $\int(1+5x+10x^2+10x^3+5x^4+x^5)dx$ **[CET 89]**

(1) $5+20x+30x^2+20x^3+5x^4+c$

(2) $x+\frac{5x^2}{2}+\frac{10x^3}{3}+\frac{10x^4}{4}+\frac{5x^5}{5}+\frac{5x^6}{6}+c$

(3) $\frac{(1+x)^6}{3!}+c$

(4) None of these

42. $\int\sqrt{x}.e^{\sqrt{x}}\,dx=$ **[EAM CET 1991]**

(1) $2\sqrt{x}-e^{\sqrt{x}}-4\sqrt{x}\,e^{\sqrt{x}}+c$

(2) $(2x-4\sqrt{x}+4)e^{\sqrt{x}}+c$

(3) $(1-4\sqrt{x})e^{\sqrt{x}}+c$

(4) None of these

43. $\int x^2e^{2x}\,dx=$ **[PET Raj. 1992]**

(1) $e^{2x}(2x^2-2x+1)+c$

(2) $\frac{1}{4}e^{2x}(2x^2-2x+1)+c$

(3) $\frac{1}{4}e^{2x}(2x^2+2x-1)+c$

(4) None of these

44. $\int\frac{e^{2x}+1}{e^{2x}-1}dx \quad \because x\in R_0$ **[PET Raj. 1990]**

(1) $\log|e^x+e^{-x}|+c$ (2) $2\log|e^x-e^{-x}|+c$

(3) $\log|e^x-e^{-x}|+c$ (4) None of these

45. $\int\frac{e^x(x-1)}{(x+1)^3}dx=$

(1) $\frac{e^x}{x+1}+c$ (2) $-\frac{e^x}{x+1}+c$

(3) $\frac{e^x}{(x+1)^2}+c$ (4) $-\frac{e^x}{(x+1)^2}+c$

46. $\int\frac{e^{1/x}}{x^2}dx=$ **[CET 1990]**

(1) $\int\frac{e^{1/x}}{x^2}dx=$ (2) $\frac{1}{4}e^{1/x}+c$

(3) $-e^{1/x}+c$ (4) None of these

47. $\int\frac{e^{mx}}{e^{mx}+1}dx$ **[CET 1990]**

(1) $e^{mx}+1+c$ (2) $\log(e^{mx}+1)+c$

(3) $m(e^{mx}+1)+c$ (4) $\frac{1}{m}\log(e^{mx}+1)+c$

48. $\int\frac{dx}{e^x+e^{-x}}$ **[CET 1993]**

(1) $\sin^{-1}(e^x)+c$ (2) $\tan^{-1}(e^x)+c$

(3) $\cos hx+c$ (4) $\log|e^x+e^{-x}|+c$

49. If $\int e^x\left[\frac{x-1}{x^2}\right]dx=\frac{ke^x}{x}$ for $x\neq0$ then k is :

(1) 0 (2) –1

(3) 1 (4) not defined.

50. $\int\frac{(x+1)e^x}{\cos^2(xe^x)}dx=$ **[PET Raj. 1988]**

(1) $\tan(xe^x)+c$ (2) $\tan^{-1}(xe^x)$

(3) $\sqrt{\tan(xe^x)}+c$ (4) None of these

51. $\int e^x\frac{(1+\sin x)}{(1+\cos x)}dx$ **[PET Raj. 1991]**

(1) $\log\tan x+c$ (2) $\sin(\log x)+c$

(3) $e^x.\tan\frac{x}{2}+c$ (4) $e^x.\cot x+c$

52. $\int\frac{ax+b}{cx+d}dx=$

(1) $\frac{ax}{c}+\frac{bc-ad}{c^2}\log(cx+d)+c$

(2) $\frac{ax}{c}n\frac{bcnad}{c^2}\log(cx+d)+c$

(3) $\frac{ax}{c}-\frac{bc-ad}{c^2}\tan^{-1}\left(\frac{cx+d}{2}\right)+c$

(4) None of these

53. $\int\frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx=$

(1) $\sin^{-1}\left(\frac{x}{a}\right)+c$

(2) $\frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2}+c$

(3) $\sin^{-1}\left(\frac{x}{a}\right)^{3/2}+c$

(4) None of these

44. (2) 45. (3) 46. (2) 47. (4)

54. $\int \frac{x^2+x+1}{x+1}dx =$ [CET 1991]

(1) $\frac{x^2}{2}+\log(x+1)+c$

(2) $\frac{x^3}{3}+\log(x+1)+c$

(3) $\frac{x^4}{4}+\frac{x^3}{3}+\log x+c$

(4) None of these.

55. $\int \frac{x^3-7x+6}{x^2+3x}dx$

(1) $\frac{x^2}{2}-3x+2\log x+c$

(2) $\frac{x^2}{2}+3x+2\log x+c$

(3) $\frac{x^2}{2}-3x-2\log x+c$

(4) None of these

56. $\int \frac{dx}{(x+1)\sqrt{x+2}}=$

(1) $\log\left[\frac{\sqrt{x+2}+1}{\sqrt{x+2}-1}\right]+c$

(2) $\log\left[\frac{\sqrt{x-2}+1}{\sqrt{x+2}-1}\right]+c$

(3) $\log\left[\frac{x+2+1}{\sqrt{x+2}+1}\right]+c$

(4) $\log\left[\frac{\sqrt{x+2}-1}{\sqrt{x+2}+1}\right]+c$

57. If $\int \frac{dx}{x\sqrt{1-x^3}}=a\log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right|+c$ then a =

(1) $\frac{1}{3}$ (2) $-\frac{1}{3}$

(3) $\frac{2}{3}$ (4) $-\frac{2}{3}$

58. The value of λ for which

$\int \frac{4x^3+\lambda 4^x}{4^x+x^4}dx=\log|4^x+x^4|$ is

(1) 1 (2) $\log_4 e$

(3) $\log_e 4$ (4) 4

59. $\int \log x . dx =$ [MNR, CET 1992, BIT (Ranchi)]

(1) $\log x + c$

(2) $\log[\log(\log x)] + c$

(3) $x \log x + c$

(4) $x(\log x - 1) + c$

60. $\int \frac{1}{x\log x}dx =$ [CEEE(Kurukshetra)]

(1) $\log x + c$

(2) $x(\log x - 1) + c$

(3) $\log(\log x) + c$

(4) $x(\log x + 1) + c$

61. $\int \frac{\log(1+x^2)}{x^2}dx =$ [Osmania University]

(1) $-\frac{\log(1+x)}{x}+2\tan^{-1}x+c$

(2) $\frac{\log(1+x^2)}{x}+2\tan^{-1}x+c$

(3) $-\frac{\log(1+x^2)}{x}+2\tan^{-1}x+c$

(4) $-\frac{\log(1+x^2)}{x}-2\tan^{-1}x+c$

62. $\int \frac{dx}{x\log_a x}=\log_e a.\log_e[\log_e x]$ is true for :

(1) all $x \in R$ (2) $x > 1$

(3) $x > e$ (4) for no real x

63. $\int[\log(\log x)+(\log x)^{-2}]dx$ is

(1) $x\log(\log x)+c$

(2) $\frac{1}{x\log(\log x)}+\frac{x}{\log x}+c$

(3) $\frac{x}{(\log x)^2}-\frac{x}{\log x}+c$

(4) $x\log(\log x)-\frac{x}{\log x}+c$

64. $\int \tan^3 x\,dx =$

(1) $\frac{1}{2}\tan^2 x + \log\cos x + c$

(2) $3\tan^2 x \cdot \sec^2 x + c$

(3) $\frac{\tan^4 x}{4} + \sec^2 x + c$

(4) None of these

65. $\int \frac{\sin x}{\sin 3x}dx =$

(1) $\frac{1}{2\sqrt{3}}\log\left[\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right]+c$

(2) $\frac{1}{2\sqrt{3}}\log\left[\frac{\sqrt{3}-\tan x}{\sqrt{3}+\tan x}\right]+c$

(3) $\frac{1}{\sqrt{3}}\log\left[\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right]+c$

(4) $-\frac{1}{\sqrt{3}}\log\left[x+\sqrt{\sqrt{3}+\tan x}\right]+c$

66. $\int \sec^3 x\,dx =$ **[PET (Raj.) 90, 92]**

(1) $3\sec^3 x \cdot \tan x + c$

(2) $\frac{1}{2}\tan x.\sec x + \frac{1}{2}\log[\tan x + \sec x] + c$

(3) $\frac{1}{2}\sec^2 x + \log[\tan x + \sec x] + c$

(4) None of these

67. $\int \cos\sqrt{x}\,dx =$ **[IIT, PET (Raj.) 93]**

(1) $2\left[\sqrt{x}.\sin\sqrt{x} + \cos\sqrt{x}\right] + c$

(2) $\sqrt{x}\sin\sqrt{x} - \cos\sqrt{x} + c$

(3) $2\left[\sqrt{2}\sin\sqrt{2} - \cos\sqrt{x}\right] + c$

(4) None of these.

68. $\int\left[\sqrt{\tan x} + \sqrt{\cot x}\right]dx =$ **[IIT 1989]**

(1) $\sqrt{2}\sin^{-1}(\sin x + \cos x) + c$

(2) $2\sin(\sin x - \cos x) + c$

(3) $2\sin^{-1}(\sin x + \cos x) + c$

(4) $\sqrt{2}\sin^{-1}(\sin x - \cos x) + c$

69. $\int \sin 4x.\cos 2x\,dx =$

(1) $\frac{\cos^3 2x}{6} + c$

(2) $\frac{\sin^3 2x}{6} + c$

(3) $\frac{1}{12}[\cos 6x + 3\cos 2x] + c$

(4) None of these

70. $\int \tan^6 x.\sec^2 x\,dx =$ **[CEE (Kar.) 1990]**

(1) $\frac{\tan^7 x}{7} + c$

(2) $7\tan^7 x + c$

(3) $\tan^7 x + c$

(4) None of these

71. $\int \tan^5 x\,dx =$ **[CEE (Karn.) 1991]**

(1) $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \log\cos x + c$

(2) $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \log\sec x + c$

(3) $\frac{\tan^6 x}{6} + c$

(4) None of these

72. $\int\left[\sec^2\left(\frac{x}{2}\right) + \mathrm{cosec}^2\left(\frac{x}{2}\right)\right]dx$ **[PET (Raj.) 91]**

(1) $2\left[\tan\left(\frac{x}{2}\right) + \cot\left(\frac{x}{2}\right)\right] + c$

(2) $2\left[\tan\left(\frac{x}{2}\right) - \cot\left(\frac{x}{2}\right)\right] + c$

(3) $\tan\frac{x}{2} - \cot\left(\frac{x}{2}\right) + c$

(4) None of these

73. $\int \cos x\sqrt{1+\cos 2x}\,.dx =$

(1) $\frac{1}{\sqrt{2}}[x + \sin x\cos x] + c$

(2) $\sqrt{2}(\sin x - x\cos x) + c$

(3) $\sqrt{2}(x + \sin x.\cos x) + c$

(4) $\sqrt{2}(\sin x + \sin x.\cos x) + c$

98. $\int \frac{x}{(a^2-x^2)^{3/2}} dx =$

(1) $\frac{x}{\sqrt{a^2-x^2}}+c$ (2) $\frac{1}{\sqrt{a^2-x^2}}+c$

(3) $\frac{-1}{\sqrt{a^2-x^2}}+c$ (4) None of these.

99. $\int \frac{x}{(x+2)\sqrt{x+1}} dx$

(1) $2\sqrt{x+1}-\tan^{-1}\sqrt{x+1}+c$

(2) $2\left[\sqrt{x+1}-2\tan^{-1}\sqrt{x+1}\right]+c$

(3) $2\sqrt{x+1}+4\tan^{-1}\sqrt{x+1}+c$

(4) None of these

100. $\int \frac{x^2}{(x^3-1)(x^3+4)} dx$

(1) $\frac{1}{5}\log\left|\frac{x^3-1}{x^3+4}\right|+c$ (2) $\frac{1}{3}\log\left|\frac{x^3-1}{x^3+4}\right|+c$

(3) $\frac{1}{15}\log\left|\frac{x^3-1}{x^3+4}\right|+c$ (4) None of these.

101. $\int \frac{x\,dx}{(3x^2+2)(x-2)} =$

(1) $\frac{1}{7}\log|x-2|+\frac{1}{14}\log|3x^2+2| - \frac{1}{7\sqrt{6}}\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt{2}}\right)+c$

(2) $\frac{1}{7}\log|x-2|-\frac{1}{14}\log|3x^2+2|+c$

(3) $\frac{1}{7}\log|x-2|-\frac{1}{14}\log|3x^2+2| + \frac{1}{7\sqrt{6}}\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt{2}}\right)+c$

(4) None of these

102. $\int \frac{\sec^2 x}{\tan^2 x-4\tan x+3} dx =$

(1) $\frac{1}{2}\log\left|\frac{\tan x-3}{\tan x-1}\right|+c$

(2) $\log\left|\frac{\tan x-1}{\tan x-3}\right|+c$

(3) $\frac{1}{2}\log\left|\frac{\tan x-1}{\tan x-3}\right|+c$

(4) None of these

103. $\int e^{\sqrt{x}} dx =$

(1) $e^{\sqrt{x}}(x-1)+c$ (2) $e^{\sqrt{x}}+c$

(3) $2e^{\sqrt{x}}(x-1)+c$ (4) None of these

104. $\int \frac{dx}{e^x+e^{-x}+2} =$

(1) $\frac{1}{e^x+1}+c$ (2) $\frac{2}{e^x+1}+c$

(3) $\frac{-1}{e^x+1}+c$ (4) $\frac{-2}{e^x+1}+c$

105. If $\int \sqrt{1+\sin x}\, f(x).dx = \frac{2}{3}(1+\sin x)^{3/2}+c$ then $f(x)$ is **[AMU 1990]**

(1) $\cos x$ (2) $\tan x$

(3) $\tan x$ (4) 1

106. $\int \frac{dx}{3-2x-x^2}$ **[UPS, EAT 2000]**

(1) $\frac{1}{4}\log\left[\frac{3+x}{1-x}\right]+c$

(2) $\frac{1}{2}\log\left[\frac{3+x}{1-x}\right]+c$

(3) $\frac{1}{3}\log\left[\frac{3+x}{1-x}\right]+c$

(4) $\log\left(\frac{1-x}{3+x}\right)+c$

107. $\int \frac{\sin x}{3+4\cos^2 x} dx =$ **[Karnat. CET 2000]**

(1) $\frac{-1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+c$

(2) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+c$

(3) $\log(3+4\cos^2 x)+c$

(4) $\frac{-1}{2\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{\sqrt{3}}\right)+c$

108. $\int \frac{x^3-1}{x^3+x}dx =$ **[MP CET 2001]**

(1) $x - \log x + \log(x^2+1) - \tan^{-1}x + c$

(2) $x - \log x + \frac{1}{2}\log(x^2+1) - \tan^{-1}x + c$

(3) $x + \log x + \frac{1}{2}\log(x^2+1) + \tan^{-1}x + c$

(4) None of these

109. The value of the integral $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x}dx$ is **[PU CET 2002, IIT 1995]**

(1) $\sin x - 6\tan^{-1}(\sin x) + c$

(2) $\sin x - 2(\sin x)^{-1} + c$

(3) $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + c$

(4) $\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + c$

110. $\int \frac{\left[\sqrt{1+x^2}+x\right]^n}{\sqrt{1+x^2}}dx =$

[PB CET 2002, Rookee 1993]

(1) $\left(\sqrt{1+x^2}+x\right)^n + c$

(2) $\frac{1}{n}\left(\sqrt{1+x^2}+x\right)^n + c$

(3) $\frac{\left(\sqrt{1+x^2}+x\right)^{n+1}}{n+1} + c$

(4) None of these

111. The integral $\int \frac{dx}{(1+\sin x)^{1/2}} =$ **[IIIT Hy. 2002]**

(1) $\sqrt{2}\log\left|\cos\left(\frac{3\pi}{8}-\frac{x}{4}\right)\right| + c$

(2) $\sqrt{2}\log\left|\operatorname{cosec}\left(\frac{\pi}{4}+\frac{x}{2}\right)-\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right| + c$

(3) $\sqrt{2}\log\left|\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)\right| + c$

(4) $\sqrt{2}\log\left|\sec\left(\frac{\pi}{4}-\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right| + c$

112. $\int \frac{\cos x - \sin x}{\cos x + \sin x}(2+2\sin 2x)dx$ is equal to

[IIIT Allahabad 2002, IIT 1997]

(1) $\sin 2x + c$ (2) $\tan 2x + c$

(3) $\cos 2x + c$ (4) None of these

113. $\int \frac{\sin x + \cos x}{\sin(x-\alpha)}dx =$

[IIIT Hydrabad 2002, Rookee 1997]

(1) $(\cos\alpha - \sin\alpha)(x-\alpha) + (\cos\alpha + \sin\alpha)\log|\sin(x-\alpha)| + c$

(2) $(\cos\alpha + \sin\alpha)(x-\alpha) + (\cos\alpha - \sin\alpha)\log|\sin(x-\alpha)| + c$

(3) $(\cos\alpha + \sin\alpha)(x+\alpha) + (\cos\alpha - \sin\alpha)\log|\sin(x+\alpha)| + c$

(4) None of these

114. $\int \frac{dx}{(2x-7)\sqrt{x^2-7x+12}} =$ **[PB CET 02, IIT 97]**

(1) $2\sec^{-1}(2x-7) + c$

(2) $\sec^{-1}(2x-7) + c$

(3) $\frac{1}{2}\sec^{-1}(2x-7) + c$

(4) None of these

115. $\int \frac{e^{2x}-2e^x}{e^{2x}+1}dx =$ **[Him. U.C.E.T. 02]**

(1) $\log(e^{2x}+1) - \tan^{-1}(e^x) + c$

(2) $\frac{1}{2}\log(e^{2x}+1) - \tan^{-1}(e^x) + c$

(3) $\frac{1}{2}\log(e^{2x}+1) - 2\tan^{-1}(e^x) + c$

(4) None of these

116. $\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax + B\log|\sin(x-\alpha)| + c$ then the value of (A, B) is **[AIEEE 2004]**

(1) $(-\sin\alpha, \cos\alpha)$

(2) $(\sin\alpha, \cos\alpha)$

(3) $(\cos\alpha, \sin\alpha)$

(4) $(-\cos\alpha, \sin\alpha)$