1 Mathematical Logic

Let's study.

1 Statement and its truth value.
2. Logical connective, compound statements.
3. Truth tables regation of statements and compound statements.
4. Statement pattern, logical equivalence.
5. Tautology, contradiction and contingency.
6. Quantifiers and quantified statements, Duality.
7. Application of logic to switching circuits, switching table.

Let's recall.

1.1.1 Introduction :

Mathematics is a logical subject and tries to be exact. For exactness, it requires proofs which depend upon proper reasoning. Reasoning requires logic. The word Logic is derived from the Greek word "LOGOS" which means reason. Therefore logic deals with the method of reasoning. In ancient Greece the great philosopher and thinker Aristotle started study of Logic systematically. In mathematics Logic has been developed by English Philosopher and mathematician George Boole (2 November 1815 - 8 December 1864)

Language is the medium of communication of our thoughts. For communication we use sentences. In logic, we use the statements which are special sentences.

1.1.2 Statement :

A statement is a declarative (assertive) sentence which is either true or false, but not both simultaneously. Statements are denoted by $p, q, r, ...$

1.1.3 Truth value of a statement :

Each statement is either true or false. If a statement is true then its truth value is 'T' and if the statement is false then its truth value is F.

Illustrations :

1) Following sentences are statements.

i) Sun rises in the East.

ii) $5 \times 2 = 11$

iii) Every triangle has three sides.

iv) Mumbai is the capital of Maharashtra.

v) Every equilateral triangle is an equiangular triangle.

vi) A natural number is an integer.

2) Following sentences are not statements.

i) Please, give your Pen.

ii) What is your name ?

iii) What a beautiful place it is !

iv) How are you ?

v) Do you like to play tennis ?

vi) Open the window.

vii) Let us go for tea

viii) Sit dows.

Note : Interrogative, exclamatory, command, order, request, suggestion are not statements.

3) Consider the following.

i) $\dfrac{3x}{2} - 9 = 0$

ii) He is tall.

iii) Mathematics is an interesting subject.

iv) It is black in colour.

Let us analyse these statements.

i) Fox $x = 6$ it is true but for other than 6 it is not true.

ii) Here, we cannot determine the truth value.

For iii) & iv) the truth value varies from person to person. In all the above sentences, the truth value depends upon the situation. Such sentences are called as open sentences. Open sentence is not a statement.

Solved examples

Q.1. Which of the following sentences are statements in logic ? Write down the truth values of the statements.

i) $6 \times 4 = 25$

ii) $x + 6 = 9$

iii) What are you doing ?

iv) The quadratic equation $x^2 - 5x + 6 = 0$ has 2 real roots.

v) Please, sit down

vi) The Moon revolves around the earth.

vii) Every real number is a complex number.

viii) He is honest.

ix) The square of a prime number is a prime number.

Solution :

i) It is a statement which is false, hence its truth value is F.

ii) It is an open sentence hence it is not a statement.

iii) It is an interrogative hence it is not a statement.

iv) It is a statement which is true hence its truth value is T.

v) It is a request hence it is not a statement.

vi) It is a statement which is true, hence its truth value is T.

vii) It is a statement which is true, hence its truth value is T.

viii) It is open sentence, hence it is not a statement.

ix) It is a statement which is false, hence its truth value is F.

1.1.4 Logical connectives, simple and compound statements :

The words or phrases which are used to connect two statements are called logical connectives. We will study the connectives 'and', 'or', 'if ..... then', 'if and only if ', 'not".

Simple and Compound Statements : A statement which cannot be split further into two or more statements is called a simple statement. If a statement is the combination of two or more simple statements, then it is called a compound statement.

"3 is a prime and 4 is an even number", is a compound statement.

"3 and 5 are twin primes", is a simple statement.

We describe some connectives.

1) Conjunction : If two statements are combined using the connective 'and' then it is called as a conjunction. In other words if p, q are two statements then 'p and q' is called as conjunction. It is denoted by '$p \wedge q$' and it is read as 'p conjunction q' or 'p and q'. The conjunction $p \wedge q$ is said to be true if and only if both p and q are true.

Truth table for conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Table 1.1

2) Disjunction : If two statements are combined by using the logical connective 'or' then it is called as a disjunction. In other words if p, q are two staements then 'p or q' is called as disjunction. It is denoted by '$p \vee$ q and it is read as 'p or q' or 'p disjunction q'.

Truth table for disconjunction

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

Table 1.2

The disjunction $p \vee$ q is false if and only if both p and q are false.

3) Conditional (Implication) : If two statements are combined by using the connective. 'if .... then', then it is called as conditional or implication. In other words if p, q are two statements then 'if p then q' is called as conditional. It is denoted by $p \to q$ or $p \Rightarrow q$ and it is read as 'p implies q' or 'if p then q'.

Truth table for conditional.

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Table 1.3

The conditional statement $p \to q$ is False only if p is true and q is false. Otherse it is true. Here p is called hypothesis or antecedent and q is called conclusion or consequence.

Note : The following are also conditional statement $p \to q$

i) p is sufficient for q

ii) q is necessary for p

iii) p implies q

iv) q follows from p

v) p only if q.

4) Biconditional (Double implication) :

If two statements are combined using the logical connective 'if and only if' then it is called as biconditional. In other words if p, q are two statements then 'p if and only if q' is called as biconditional. It is denoted by '$p \leftrightarrow q$' or $p \Leftrightarrow q$. It is read as 'p biconditional q' or 'p if and only if q'.

Truth table for biconditional.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Table 1.4

Biconditional statement $p \leftrightarrow q$ is true if p and q have same truth values. Otherwise it is False.

5) Negation of a statement : For any given statement p, there is another statement which is defined to be true when p is false, and false when p is true, is called the negation of p and is denoted by $\sim p$.

Truth table for negation.

p	~ p
T	F
F	T

Table 1.5

Note : Negation of negation of a statement is the statement itself. That is, $\sim(\sim p) = p$.

Solved examples

Ex.1: Express the following compound statements symbolically without examining the truth values.

i) 2 is an even number and 25 is a perfect square.

ii) A school is open or there is a holiday.

iii) Delhi is in India but Dhaka is not in Srilanks.

iv) $3 + 8 \geq 12$ if and only if $5 \times 4 \leq 25$.

Solution :

i) Let p : 2 is an even numder

q : 25 is a perfect square.

The symbolic form is $p \wedge q$.

ii) Let p : The school is open

q : There is a holiday

The symbolic form is $p \vee q$

iii) Let p : Delhi is in India

q : Dhaka is in Srilanka

The symbolic form is $p \wedge \sim q$.

iv) Let $p : 3 + 8 \geq 12$; $q : 5 \times 4 \leq 25$

The symbolic form is $p \leftrightarrow q$

Ex.2. Write the truth values of the following statements.

i) 3 is a prime number and 4 is a rational number.

ii) All flowers are red or all cows are black.

iii) If Mumbai is in Maharashtra then Delhi is the capital of India.

iv) Milk is white if and only if the Sun rises in the West.

Solution :

i) Let p : 3 is a prime number

q : 4 is a rational number.

Truth values of p and q are T and T respectively.

The given statement in symbolic form is $p \wedge q$.

The truth value of given statement is T.

ii) Let p : All flowers are red ; q : All cows are black.

Truth values of p and q are F and F respectively.

The given statement in the symbolic form is $p \vee q$

$\therefore p \vee q \equiv F \vee F$ is F

$\therefore$ Truth value of given statement is F.

iii) Let p : Mumbai is in Maharashtra

q : Delhi is capital of India

Truth values of p and q are T and T respectively.

The given statement in symbolic form is $p \rightarrow q$

$\therefore\ p \to q \equiv T \to T$ is T

$\therefore$ Truth value of given statement is T

iv) Let p : Milk is white; q : Sun rises in the West.

Truth values of p and q are T and F respectively.

The given statement in symbolic form is $p \leftrightarrow q$

$\therefore\ p \leftrightarrow q \equiv T \leftrightarrow F$ is F

$\therefore$ Truth value of given statement is F

Ex.3 : If statements p, q are true and r, s are false, determine the truth values of the following.

i) $\sim p \wedge (q \vee \sim r)$ ii) $(p \wedge \sim r) \wedge (\sim q \vee s)$

iii) $\sim (p \to q) \leftrightarrow (r \wedge s)$ iv) $(\sim p \to q) \wedge (r \leftrightarrow s)$

Solution :

i) $\sim p \wedge (q \vee \sim r) \equiv \sim T \wedge (T \vee \sim F) \equiv F \wedge (T \vee T) \equiv F \wedge T \equiv F$

Hence truth value is F.

ii) $(p \wedge \sim r) \wedge (\sim q \vee s) \equiv (T \wedge \sim F) \wedge (\sim T \vee F) \equiv (T \wedge T) \wedge (F \vee F) \equiv T \wedge F \equiv F.$

Hence truth value is F.

iii) $[\sim(p \to q)] \leftrightarrow (r \wedge s) \equiv [\sim (T \to T)] \leftrightarrow (F \wedge F) \equiv (\sim T) \leftrightarrow (F) \equiv F \leftrightarrow F \equiv T.$

Hence truth value is T

iv) $(\sim p \to q) \wedge (r \leftrightarrow s) \equiv (\sim T \to T) \wedge (F \leftrightarrow F) \equiv (F \to T) \wedge T \equiv T \wedge T \equiv T.$

Hence truth value is T .

Ex.4. Write the negations of the following.

i) Price increases

ii) $0! \neq 1$

iii) $5 + 4 = 9$

Solution :

i) Price does not increase

ii) $0! = 1$

iii) $5 + 4 \neq 9$

Exercise 1.1

Q.1. State which of the following are statements. Justify. In case of statement, state its truth value.

i) $5 + 4 = 13.$

ii) $x - 3 = 14.$

iii) Close the door.

iv) Zero is a complex number.

v) Please get me breafast.

vi) Congruent triangles are similar.

vii) $x^2 = x.$

viii) A quadratic equation cannot have more than two roots.

ix) Do you like Mathematics ?

x) The sun sets in the west

xi) All real numbers are whole numbers

xii) Can you speak in Marathi ?

xiii) $x^2 - 6x - 7 = 0$, when $x = 7$

xiv) The sum of cuberoots of unity is zero.

xv) It rains heavily.

Q.2. Write the following compound statements symbollically.

i) Nagpur is in Maharashtra and Chennai is in Tamilnadu

ii) Triangle is equilateral or isosceles.

iii) The angle is right angle if and only if it is of measure 90º.

iv) Angle is neither acute nor obtuse.

v) If Δ ABC is right angled at B, then m $\angle$A + m $\angle$C = 90º

iv) Hima Das wins gold medal if and only if she runs fast.

vii) x is not irrational number but is a square of an integer.

Q.3. Write the truth values of the following.

i) 4 is odd or 1 is prime.

ii) 64 is a perfect square and 46 is a prime number.

iii) 5 is a prime number and 7 divides 94.

iv) It is not true that 5-3i is a real number.

v) If $3 \times 5 = 8$ then $3 + 5 = 15$.

vi) Milk is white if and only if sky is blue.

vii) 24 is a composite number or 17 is a prime number.

Q.4. If the statements p, q are true statements and r, s are false statements then determine the truth values of the following.

i) $p \vee (q \wedge r)$

ii) $(p \rightarrow q) \vee (r \rightarrow s)$

iii) $(q \wedge r) \vee (\sim p \wedge s)$

iv) $(p \rightarrow q) \wedge \sim r$

v) $(\sim r \leftrightarrow p) \rightarrow \sim q$

vi) $[\sim p \wedge (\sim q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)]$

vii) $[(\sim p \wedge q) \wedge \sim r] \vee [(q \rightarrow p) \rightarrow (\sim s \vee r)]$

viii) $\sim[(\sim p \wedge r) \vee (s \rightarrow \sim q)] \leftrightarrow (p \wedge r)$

Q.5. Write the negations of the following.

i) Tirupati is in Andhra Pradesh

ii) 3 is not a root of the equation $x^2 + 3x - 18 = 0$

iii) $\sqrt{2}$ is a rational number.

iv) Polygon ABCDE is a pentagon.

v) $7 + 3 > 5$

1.2 STATEMENT PATTERN, LOGICAL EQUIVALENCE, TAUTOLOGY, CONTRADICTION, CONTINGENCY.

1.2.1 Statement Pattern :

Letters used to denote statements are called statement letters. Proper combination of statement letters and connectives is called a statement pattern. Statement pattern is also called as a proposition. $p \rightarrow q$, $p \wedge q$, $\sim p \vee q$ are statement patterns. p and q are their prime components.

A table which shows the possible truth values of a statement pattern obtained by considering all possible combinations of truth values of its prime components is called the truth table of the statement pattern.

1.2.2. Logical Equivalence :

Two statement patterns are said to be equivalent if their truth tables are identical. If statement patterns A and B are equivalent, we write it as $A \equiv B$.

1.2.3 Tautology, Contradiction and Contingency :

Tautology : A statement pattern whose truth value is true for all possible combinations of truth values of its prime components is called a tautology. We denote tautology by t.

Statement pattern $p \vee \sim p$ is a tautology.

Contradiction : A statement pattern whose truth value is false for all possible combinations of truth values of its prime components is called a contradiction. We denote contradiction by c.

Statement pattern $p \wedge \sim p$ is a contradiction.

Contingency : A statement pattern which is neither a tautology nor a contradiction is called a contingency. $p \wedge q$ is a contingency.

Important table for all connectives :

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

☆ **In a statement pattern, different symbols are considered in the following priority**

$\sim, \vee, \wedge, \rightarrow, \leftrightarrow$

Solved Examples

Ex.1.: Construct the truth table for each of the following statement patterns.

i) $p \rightarrow (q \rightarrow p)$

ii) $(\sim p \vee q) \leftrightarrow \sim (p \wedge q)$

iii) $\sim (\sim p \wedge \sim q) \vee q$

iv) $[(p \wedge q) \vee \mathrm{r}] \wedge [\sim \mathrm{r} \vee (p \wedge q)]$

v) $[(\sim p \vee q) \wedge (q \rightarrow \mathrm{r})] \rightarrow (p \rightarrow \mathrm{r})$

Solution :

i) $p \to (q \to p)$

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

Table 1.7

ii) $(\sim p \vee q) \leftrightarrow \sim(p \wedge q)$

p	q	$\sim p$	$\sim p \vee q$	$p \wedge q$	$\sim (p \wedge q)$	$(\sim p \vee q) \leftrightarrow \wedge (p \wedge q)$
T	T	F	T	T	F	F
T	F	F	F	F	T	F
F	T	T	T	F	T	T
F	F	T	T	F	T	T

Table 1.8

iii) $\sim (\sim p \wedge \sim q) \vee q$

p	q	$\sim p$	$\sim q$	$\sim(p \wedge \sim q)$	$\sim(\sim p \wedge \sim q)$	$\sim(\sim p \wedge \sim q)$	$\sim(\sim p \wedge \sim q) \vee q$
T	T	F	F	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	T	F	F	F

Table 1.9

iv) $[(p \wedge q) \vee \text{r}] \wedge [\sim \text{r} \vee (p \wedge q)]$

p	q	r	~r	$p \wedge q$	$(p \wedge q) \vee \text{r}$	$\sim \text{r} \vee (p \wedge q)$	$[p \wedge q) \vee \text{r}] \wedge (\sim \text{r} \vee (p \wedge q)$
T	T	T	F	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	T	F	F
T	F	F	T	F	F	T	F
F	T	T	F	F	T	F	F
F	T	F	T	F	F	T	F
F	F	T	F	F	T	F	F
F	F	F	T	F	F	T	F

Table 1.10

v) $[(\sim p \vee q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

p	q	r	$\sim p$	$\sim p \vee q$	$q \rightarrow r$	$p \rightarrow r$	$(\sim p \vee q) \wedge (q \rightarrow r)$	$[(\sim p \vee q) \wedge (q \rightarrow r)] \rightarrow (q \rightarrow r)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	F	F	T	T	F	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

Table 1.11

Ex.2: Using truth tables, prove the following logical equivalences

i) $(p \wedge q) \equiv \sim (p \rightarrow \sim q)$

ii) $(p \leftrightarrow q) \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

iii) $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$

iv) $p \rightarrow (q \vee r) \equiv (p \rightarrow q) \vee (p \rightarrow r)$

i) **Solution :** (1) $(p \wedge q) \equiv \sim(p \rightarrow \sim q)$

I	II	III	IV	V	VI
p	q	$\sim q$	$p \wedge q$	$p \rightarrow \sim q$	$\sim(p \rightarrow \sim q)$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	F	T	F

Table 1.12

Columns (IV) and (VI) are identical $\therefore$ $(p \wedge q) \equiv \sim(p \rightarrow \sim q)$

ii) $(p \leftrightarrow q) \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

I	II	III	IV	V	VI	VII	VIII
p	q	~p	~q	$p \leftrightarrow q$	$(p \wedge q)$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
T	T	F	F	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	T	F	T	T

Table 1.13

Columns V and VIII are identical

$\therefore$ $(p \leftrightarrow q) \equiv (p \wedge q) \vee (\sim p \vee \sim q)$

(iii) $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$

I	II	III	IV	V	VI	VII
p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow r$	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	T	T	T
F	T	F	F	T	F	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

Table 1.14

Column (V) and (VII) are identical

$\therefore$ $(p \wedge \text{q}) \rightarrow \text{r} \equiv p \rightarrow (\text{q} \rightarrow \text{r})$

(iv) $p \rightarrow (q \vee \text{r}) \equiv (p \rightarrow q) \vee (p \rightarrow r)$

I	II	III	IV	V	VI	VII	VIII
p	q	r	$q \vee r$	$p \rightarrow (q \vee r)$	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \vee (p \rightarrow \text{r})$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

Table 1.15

Columns V and VIII are identical

$\therefore$ $p \rightarrow (\text{q} \vee \text{r}) \equiv (p \rightarrow \text{q}) \vee (p \rightarrow \text{r})$

Ex.3. Using truth tables, examine whether each of the following statements is a tautology or a contradiction or contingency.

i) $(p \wedge q) \wedge (\sim p \vee \sim q)$

ii) $[p \wedge (p \rightarrow \sim q)] \rightarrow p$

iii) $(p \rightarrow q) \wedge [(q \rightarrow \text{r}) \rightarrow (p \rightarrow \text{r})]$

iv) $[(p \vee q) \vee \text{r}] \leftrightarrow [p \vee \leftrightarrow q \vee \text{r})]$

Solution:

i) $(p \wedge q) \wedge (\sim p \ \vee \sim q)$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \vee \sim q$	$(p \wedge q) \wedge (\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

Table 1.16

All the truth values in the last column are F. Hence it is contradiction.

ii) $[p \wedge (p \to \sim q)] \to q$

p	q	$\sim q$	$p \to \sim q$	$p \wedge (p \to \sim q$	$p \wedge (p \to \sim q) \to q$
T	T	F	F	F	T
T	F	T	T	T	F
F	T	F	T	F	T
F	F	T	T	F	T

Table 1.17

Truth values in the last column are not identical. Hence it is contingency.

iii) $(p \to q) \wedge [(q \to \mathrm{r}) \to (p \to \mathrm{r})]$

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(q \to r) \to (p \to \mathrm{r})$	$(p \to q) \wedge (q \to r) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	F	T	T	T	F
T	F	F	F	T	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Table 1.18

Truth values in the last column are not same, hence it is contingency.

iv) $[(p \vee q) \vee \mathrm{r}] \leftrightarrow (p \vee (q \vee r)]$

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$q \vee r$	$p \vee (q \vee r)$	$[(p \vee q) \vee r] \leftrightarrow [p \vee (q \vee r)]$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	F	F	F	T

Table 1.19

All the truth values in the last column are T, hence it is tautology.

Exercie.1.2

Q.1. Construct the truth table for each of the following statement patterns.

i) $[(p \rightarrow q) \wedge q] \rightarrow p$

ii) $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$

iii) $(p \wedge q) \leftrightarrow (q \vee r)$

iv) $p \rightarrow [\sim (q \wedge r)]$

v) $\sim p \wedge [(p \vee \sim q) \wedge q]$

vi) $(\sim \mathrm{p} \rightarrow \sim \mathrm{q}) \wedge (\sim q \rightarrow \sim p)$

vii) $(q \rightarrow p) \vee (\sim p \leftrightarrow q)$

viii) $[p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \wedge q) \rightarrow r]$

ix) $p \rightarrow [\sim (q \wedge r)]$

x) $(p \vee \sim q) \rightarrow (r \wedge \mathrm{p})$

Q.2. Using truth tables prove the following logical equivalences.

i) $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

ii) $\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

iii) $p \leftrightarrow q \equiv \sim [(p \vee q) \wedge \sim (p \wedge q)]$

iv) $p \rightarrow (q \rightarrow p) \equiv \sim p \rightarrow (p \rightarrow q)$

v) $(\mathrm{p} \vee q) \rightarrow \mathrm{r} \equiv (p \rightarrow \mathrm{r}) \wedge (q \rightarrow \mathrm{r})$

vi) $p \rightarrow (q \wedge \mathrm{r}) \equiv (p \rightarrow q) \wedge (p \rightarrow \mathrm{r})$

vii) $p \rightarrow (q \wedge \mathrm{r}) \equiv \ (p \wedge q) \ \ (\mathrm{p} \rightarrow \mathrm{r})$

viii) $p \wedge (q \vee \mathrm{r}) \equiv (p \wedge q) \vee (p \wedge \mathrm{r})$

ix) $[\sim (p \vee q) \vee (p \vee q)] \wedge r \equiv r$

x) $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

Q.3. Examine whether each of the following statement patterns is a tautology or a contradiction or a contingency.

i) $(p \wedge q) \rightarrow (q \vee p)$

ii) $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$

iii) $[\sim (\sim p \wedge \sim q)] \vee q$

iv) $[(p \rightarrow q) \wedge q)] \rightarrow p$

v) $[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$

vi) $(p \leftrightarrow q) \wedge (p \rightarrow \sim q)$

vii) $\sim (\sim q \wedge p) \wedge q$

viii) $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$

ix) $(\sim p \rightarrow q) \wedge (p \wedge r)$

x) $[p \rightarrow (\sim q \vee r)] \leftrightarrow \sim [p \rightarrow (q \rightarrow r)]$

1.3 QUANTIFIERS, QUANTIFIED STATEMENTS, DUALS, NEGATION OF COMPOUND STATEMENTS, CONVERSE, INVERSE AND CONTRAPOSITIVE OF IMPLICATION.

1.3.1 Quantifiers and quantified statements.

Look at the following statements :

p : "There exists an even prime number in the set of natural numbers".

q : "All natural numbers are positive".

Each of them asserts a *condition* for some or all objects in a *collection*. Words "there exists" and "for all" are called quantifiers. "There exists is called existential quantifier and is denoted by symbol $\exists$. "For all" is called universal quantifier and is denoted by $\forall$. Statements involving quantifiers are called quantified statements. Every quantified statement corresponds to a *collection* and a *condition*. In statement p the collection is 'the set of natural numbers' and the condition is 'being even prime'. What is the condition in the statement q ?

A statement quantified by universal quantifier $\forall$ is true if all objects in the collection satisfy the condition. And it is false if at least one object in the collection does not satisfy the conditon.

A statement quantified by existential quantifier $\exists$ is true if at least one object in the collection satisfies the condition. And it is false if no object in the collection satisfies the condition.

Ex.1. If A = {1, 2, 3, 4, 5, 6, 7}, determine the truth value of the following.

i) $\exists\, x \in$ A such that $x - 4 = 3$

ii) $\forall\, x \in$ A , $x + 1 \geq 3$

iii) $\forall\, x \in$ A, $8 - x \leq 7$

iv) $\exists\, x \in$ A, such that $x + 8 = 16$

Solution :

i) For $x = 7$, $x - 4 = 7 - 4 = 3$

$\therefore$ $x = 7$ satisfies the equation $x - 7 = 3$

$\therefore$ The given statement is true and its truth value is T.

ii) For $x = 1$, $x + 1 = 1 + 1 = 2$ which is not greater than or equal to 3

$\therefore$ For $x = 1$ $x^2 + 1 \geq 3$ is not true.

$\therefore$ The truth value of given statement is F.

iii) For each $x \in$ A $8 - x \leq 7$

$\therefore$ The given statement is true.

$\therefore$ Its truth value is T.

iv) There is no x in A which satisfies $x + 8 = 16$.

$\therefore$ The given statement is false. $\therefore$ Its truth value is F.

1.3.2 Dual : We use letters t and c to denote tautology and contradiction respectively.

If two statements contain logical connectives like $\vee$, $\wedge$ and letters t and c then they are said to be duals of each other if one of them is obtained from the other by interchanging $\vee$ with $\wedge$ and t with c.

The dual of i) $p \vee$ q is $p \wedge$ q ii) t $\vee\, p$ is c $\wedge\, p$ iii) t $\wedge\, p$ is c $\vee\, p$

Ex.1. Write the duals of each of the following :

i) $(p \wedge q) \vee r$

ii) $t \vee (p \vee q)$

iii) $p \wedge [\sim q \vee (p \wedge q) \vee \sim r]$

iv) $(p \vee q) \wedge t$

v) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

vi) $p \wedge q \wedge r$

vii) $(p \wedge t) \vee (c \wedge \sim q)$

Solution :

i) $(p \vee q) \wedge r$

ii) $c \wedge (p \wedge q)$

iii) $p \vee [(\sim q \wedge (p \vee q) \wedge \sim r]$

iv) $(p \wedge q) \vee c$

v) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

vi) $p \vee q \vee r$

vii) $(p \vee c) \wedge (t \vee \sim q)$

1.3.3 Negations of compound statements :

Negation of conjunction : When is the statment "6 is even and perfect number" is false? It is so, if 6 is not even or 6 is not perfect number. The negation of $p \wedge q$ *is* $\sim p \vee \sim q$. The negation of "6 is even and perfect number" is "6 not even or not perfect number".

Activity : Using truth table verify that $\sim (p \wedge q) \equiv \sim p \vee \sim q$

Negation of disjunction : When is the statement "x is prime or y is even" is false? It is so, if x is not prime and y is not even. The negation of $p \vee q$ is $\sim p \wedge \sim q$. The negation of "x is prime or y is even" is "x is not prime and y is not even".

Activity : Using truth table verify that $\sim (p \wedge q) \equiv \sim p \vee \sim q$

Note : '$\sim (p \wedge q) \equiv \sim p \vee \sim q$' **and** '$\sim (p \vee q) \equiv \sim p \wedge \sim q$' **are called De'Morgan's Laws**

Negation of implication : Implication $p \rightarrow$ q asserts that "if p is true statement then q is true statement". When is an implication a true statement and when is it false? Consider the statement "If bakery is open then I will buy a cake for you." Clearly statement is false only when the bakery was open and I did not buy a cake for you. The conditional statement "If p then q" is false only in the case "p is true and q is false". In all other cases it is true. The negation of the statement "If p then q" is the statement "p and not q". i.e. p does not imply q

Activity : Using truth table verify that $\sim (p \rightarrow q) \equiv p \wedge \sim q$

Negation of biconditional : The biconditional p $\leftrightarrow$ q is the conjuction of statement p $\rightarrow$ q and q $\rightarrow$ p.

$\therefore \; p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\therefore$ The conditional statement $p \leftrightarrow q$ is false if $p \rightarrow q$ is false or $q \rightarrow p$ false.

The negation of the statement "p if and only if q " is the statement "p and not q, or q and not p".

$\therefore \; \sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim \mathrm{p})$

Activity : Using truth table verify that $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

1.3.4 Converse, inverse and contrapositive

From implication $p \rightarrow q$ we can obtain three implications, called converse, inverse and contrapositive.

$q \rightarrow p$ is called the converse of $p \rightarrow q$

$\sim p \rightarrow \sim q$ is called the inverse of $p \rightarrow q$.

$\sim \mathrm{q} \rightarrow \sim p$ is called the contrapositive of $p \rightarrow$ q.

Activity :

Prepare the truth table for $p \to q$, $q \to p$, $\sim p \to \sim q$ and $\sim q \to \sim p$. What is your conclusion from the truth table ?

i) .... $\equiv$ ....

ii) .... $\equiv$ ....

Ex.1) Write the negations of the following.

i) $3 + 3 < 5$ or $5 + 5 = 9$

ii) $7 > 3$ and $4 > 11$

iii) The number is neither odd nor perfect square.

iv) The number is an even number if and only if it is divisible by 2.

Solution :

i) Let $p : 3 + 3 < 5 : q : 5 + 5 = 9$

Given statement is $p \vee q$ and its negation is $\sim(p \vee q)$ and $\sim(p \vee q) \equiv \sim p \wedge \sim q$

$\therefore$ The negation of given statement is $3 + 3 \geq 5$ and $5 + 5 \neq 9$

ii) Let $p : 7 > 3$; $q: 4 > 11$

The given statement is $p \wedge q$

Its negation is $\sim (p \wedge q)$ and $\sim (p \wedge q) \equiv \sim p \vee \sim q$

$\therefore$ The negation of given statement is $7 \leq 3$ or $4 \leq 11$

iii) Let p : The number is odd

q : The number is perfect square

Given statement can be written as 'the number is not odd and not perfect square'

Given statement is $\sim p \wedge \sim q$

Its negation is $\sim (\sim p \wedge \sim q) \equiv p \vee q$

The negation of given statement is 'The number is odd or perfect square'.

iv) Let p : The number is an even number.

q : The number is divisible by 2

Given statement is $p \leftrightarrow q$

Its negation is $\sim (p \leftrightarrow q)$

But $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

$\therefore$ The negation of given statement is 'A number is even but not divisible by 2 or a number is divisible by 2 but not even'.

Ex.2. Write the negations of the following statements.

i) All natural numbers are rational.

ii) Some students of class X are sixteen year old.

iii) $\exists\, n \in$ N such that $n + 8 > 11$

iv) $\forall\, x \in$ N, $2x + 1$ is odd

Solution :

i) Some natural numbers are not rationals.

ii) No student of class X is sixteen year old.

iii) $\forall\ n \in$ N, $n + 8 \leq 11$

iv) $\exists\ x \in$ N such hat $2x + 1$ is not odd

Ex.3. Write the converse, inverse and contrapositive of the following statements.

i) If a function is differentiable then it is continuous.

ii) If it rains then the match will be cancelled.

Solution :

(1) Let p : A function is differentiable

q : A function is continuous.

$\therefore$ Given statement is $p \to q$

i) Its converse is $q \to p$

If a function is continuous then it is differentiable.

ii) Its inverse is $\sim p \to \sim q$.

If a function not differentiable then it is not continuous.

iii) Its contrapositive is $\sim q \to \sim p$

If a function is not continuous then it is not differentiable.

(2) Let p : It rains, q : The match gets cancelled.

$\therefore$ Given statement is $p \to q$

i) Its converse is $q \to p$

If the match gets cancelled then it rains.

ii) Inverse is $\sim p \to \sim q$

If it does not rain then the match will not be cancelled.

iii) Its contrapositive is $\sim q \to \sim p$.

If the match is not cancelled then it does not rain.

Exercise 1.3

Q.1. If A = {3, 5, 7, 9, 11, 12}, determine the truth value of each of the following.

i) $\exists\ x \in$ A such that $x - 8 = 1$

ii) $\forall\ x \in$ A, $x^2 + x$ is an even number

iii) $\exists\ x \in$ A such that $x^2 < 0$

iv) $\forall\ x \in$ A, x is an even number

v) $\exists\ x \in$ A such that $3x + 8 > 40$

vi) $\forall\ x \in$ A, $2x + 9 > 14$

Q.2. Write the duals of each of the following.

i) $p \vee (q \wedge r)$ ii) $p \wedge (q \wedge r)$ iii) $(p \vee q) \wedge (r \vee s)$ iv) $p \wedge \sim q$

v) $(\sim p \vee q) \wedge (\sim r \wedge s)$ vi) $\sim p \wedge (\sim q \wedge (p \vee q) \wedge \sim r)$

vii) $[\sim (p \vee q)] \wedge [p \vee \sim (q \wedge \sim s)]$ viii) $c \vee \{p \wedge (q \vee r)\}$ ix) $\sim p \vee (q \wedge r) \wedge t$ x) $(p \vee q) \vee c$

Q.3. Write the negations of the following.

i) $x + 8 > 11$ or $y - 3 = 6$

ii) $11 < 15$ and $25 > 20$

iii) Qudrilateral is a square if and only if it is a rhombus.

iv) It is cold and raining.

v) If it is raining then we will go and play football.

vi) $\sqrt{2}$ is a rational number.

vii) All natural numbers are whole numers.

viii) $\forall$ n $\in$ N, $n^2 + n + 2$ is divisible by 4.

ix) $\exists\, x \in$ N such that $x - 17 < 20$

Q.4. Write converse, inverse and contrapositive of the following statements.

i) If $x < y$ then $x^2 < y^2$ $(x, y \in R)$

ii) A family becomes literate if the woman in it is literate.

iii) If surface area decreases then pressure increases.

iv) If voltage increases then current decreases.

1.4 SOME IMPORTANT RESULTS :

1.4.1.

i) $p \to q \equiv \sim p \vee q$

ii) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

iii) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

iv) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

I	II	III	IV	V	VI	VII	VIII
p	q	$\sim p$	$p \to q$	$q \to p$	$p \leftrightarrow q$	$\sim p \vee q$	$p \to q \wedge q \to p$
T	T	F	T	T	T	T	T
T	F	F	F	T	F	F	F
F	T	T	T	F	F	T	F
F	F	T	T	T	T	T	T

Table 1.20

Columns (IV, VII) and (VI, VIII) are identical.

$\therefore$ $p \to q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$ are proved.

Activity :

Prove the results (iii) and (iv) by using truth table.

1.4.2. Algebra of statements.

Idempotent Law	$p \wedge p \equiv p, \quad p \vee p \equiv p$
Commutative Law	$p \vee q \equiv q \vee p \quad p \wedge q \equiv q \wedge p$
Associative Law	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r \equiv p \wedge q \wedge r$
	$p \vee (q \vee r) \equiv (p \vee q) \vee r \equiv p \vee q \vee r$
Distributive Law	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
De Morgan's Law	$\sim(p \wedge q) \equiv \sim p \vee \sim q, \sim(p \vee q) \equiv \sim p \wedge \sim q$
Identity Law	$p \wedge T \equiv p, p \wedge F \equiv F, p \vee F \equiv p, p \vee T \equiv T$
Complement Law	$p \wedge \sim p \equiv F, p \vee \sim p \equiv T$
Absorption Law	$p \vee (p \wedge q) \equiv p, p \wedge (p \vee q) \equiv p$
Conditional Law	$p \rightarrow q \equiv \sim p \vee q$
Biconditional Law	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p) \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

Table 1.21

Solved Examples

Ex.1. Write the negations of the following stating the rules used.

i) $(p \vee q) \wedge (q \vee \sim r)$

ii) $(p \rightarrow q) \vee r$

iii) $p \wedge (q \vee r)$

iv) $(\sim p \wedge q) \vee (p \wedge \sim q)$

v) $(p \wedge q) \rightarrow (\sim p \vee r)$

Solution :

i) $\sim [(p \vee q) \wedge (q \vee \sim r)] \equiv \sim (p \vee q) \vee \sim (q \vee \sim r)$ [DeMorgan's law]

$\equiv (\sim p \wedge \sim q) \vee (\sim q \wedge \text{r})$ [DeMorgan's law]

$\equiv (\sim q \wedge \sim p) \vee (\sim q \wedge r)$ [Commutative law]

$\equiv \sim q \wedge (\sim p \vee r)$ [Distributive law]

ii) $\sim [(p \rightarrow q) \vee r] \equiv \sim (p \rightarrow q) \wedge \sim r$ [DeMorgan's law]

$\equiv (p \wedge \sim q) \wedge \sim r$ $[\sim (p \rightarrow q) \equiv p \wedge \sim q]$

iii) $\sim [p \wedge (q \vee r)] \equiv \sim p \vee \sim (q \vee r)$ [DeMorgan's law]

$\equiv \sim p \vee (\sim q \wedge \sim r)$ [DeMorgan's law]

iv) $\sim [(\sim p \wedge q) \vee (p \wedge \sim q)] \equiv \sim (\sim p \wedge q) \wedge \sim (p \wedge \sim q)$ [DeMorgan's law]

$\equiv (p \vee \sim q) \wedge (\sim p \vee q)$ [DeMorgan's law]

v) $\sim [(p \wedge q) \rightarrow (\sim p \vee r)] \equiv (p \wedge q) \wedge \sim (\sim p \vee r)$ $[\sim (p \rightarrow q) \equiv p \wedge \sim q]$

$\equiv (p \wedge q) \wedge [p \wedge \sim r]$ [DeMorgan's law]

$\equiv q \wedge p \wedge p \wedge \sim r$ [Associative law

$\equiv q \wedge p \wedge \sim r$ [Idempotent law]

Ex.2. Rewrite the following statements without using if ...... then.

i) If prices increase then the wages rise.

ii) If it is cold, then we wear woolen clothes.

Solution :

i) Let p : Prices increase

q : The wages rise.

The given statement is $p \to q$

but $p \to q \equiv \sim p \vee q$

The given statement can be written as

'Prices do not increase or the wages rise'.

ii) Let p : It is cold, q : We wear woollen clothes.

The given statement is $\mathrm{p} \to q$

but $\mathrm{p} \to q \equiv \sim \mathrm{p} \vee q$

The given statement can be written as

It is not cold or we wear woollen clothes.

Ex.3. Without using truth table prove that :

i) $p \leftrightarrow q \equiv \sim (p \wedge \sim q) \wedge \sim (q \wedge \sim p)$

ii) $\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

iii) $\sim p \wedge q \equiv (p \vee \mathrm{q}) \wedge \sim p$

Solution :

i) We know that

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$ [Conditional law]

$\equiv \sim (p \wedge \sim q) \wedge \sim (q \wedge \sim \mathrm{p})$ [Demorgan's law]

ii) $\sim (p \vee q) \vee (\sim p \wedge q) \equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ [Demorgan's law]

$\equiv \sim p \wedge (\sim q \vee q)$ [Distributive law]

$\equiv \sim p \wedge \mathrm{T}$ [Complement law]

$\equiv \sim p$ [Identity law]

iii) $(p \wedge q) \wedge \sim p \equiv \sim p \wedge (p \vee q)$ [Commutative law]

$\equiv (\sim p \wedge p) \vee (\sim p \wedge q)$ [Distributive law]

$\equiv \mathrm{F} \vee (\sim p \wedge q)$ [Complement law]

$\equiv \sim p \wedge q$ [Identity law]

Exercise 1.4

Q.1. Using rules of negation write the negations of the following with justification.

i) $\sim q \rightarrow p$

ii) $p \wedge \sim q$

iii) $p \vee \sim q$

iv) $(p \vee \sim q) \wedge r$

v) $p \rightarrow (p \vee \sim q)$

vi) $\sim (p \wedge q) \vee (p \vee \sim q)$

vii) $(p \vee \sim q) \rightarrow (p \wedge \sim q)$

viii) $(\sim p \vee \sim q) \vee (p \wedge \sim q)$

Q.2. Rewrite the following statements without using if .. then.

i) If a man is a judge then he is honest.

ii) It 2 is a rational number then $\sqrt{2}$ is irrational number.

iii) It $f(2) = 0$ then $f(x)$ is divisible by $(x - 2)$.

Q.3. Without using truth table prove that :

i) $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

ii) $(p \vee q) \wedge (p \vee \sim q) \equiv p$

iii) $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$

iv) $\sim [(p \vee \sim q) \rightarrow (p \wedge \sim q)] \equiv (p \vee \sim q) \wedge (\sim p \vee q)$

Application of Logic to switching circuits :

We shall study how the theory of Logic can be applied in switching network. We have seen that a logical statement can be either true or false i.e. it can have truth value either T or F.

A similar situation exists in various electrical devices. For example, an electric switch can be on or off. In 1930 Claude Shannan noticed an analogy between operation of switching circuits and operation of logical connectives.

In an electric circuit, switches are connected by wires. If the switch is 'on', it allows the electric current to pass through, it. If the switch is 'off', it does not allow the electric current to pass through it. We now define the term 'switch' as follows.

Switch : A switch is a two state device used to control the flow of current in a circuit.

We shall denote the switches by letters S, S_1, S_2, S_3 .... etc.

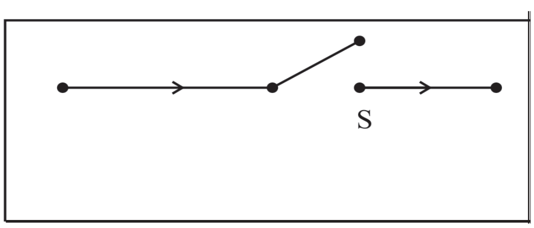


Fig. 1.1

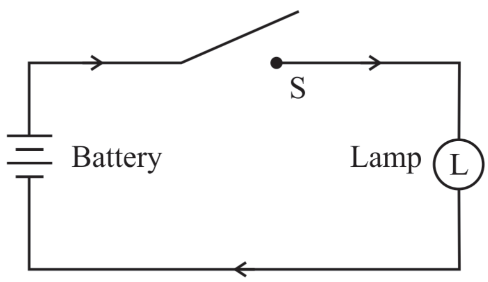


Fig. 1.2

In figure 1.2, we consider a circuit containing an electric lamp L, controlled by a switch S.

When the switch S is closed (i.e. on), then current flows in the circuit and hence the lamp glows. When the switch S is open (i.e. off), then current does not flow in the circuit and subsequently the lamp does not glow.

The theory of symbolic logic can be used to represent a circuit by a statement pattern. Conversely for given statement pattern a circuit can be constructed. Corresponding to each switch in the ciruict we take a statement letter in statement pattern. Switches having the same state will be denoted by the same letter and called equivalent switches. Switches having opposite states are denoted by S and S'. They are called complementary switches. In circuit we don't show whether switch is open or closed. In figure 1.3 switch S_1 corresponds to statement letter p in the corresponding statement pattern.

We write it as p : switch S_1 and $\sim p$: switch S'_1

The correspondence between switch S_2 and statement letter q is shown as q : switch S_2 and $\sim q$: switch S'_2.

We don't know the actual states of switches in the circuit. We consider all possible combinations of states of all switches in the circuit and prepare a table, called "Input Output table", which is similar to truth table of the corresponding statement pattern.

☆ In an Input-output table we represent '1' when the state of the switch is 'on' and '0' when the state of the switch is 'off'.

1.5.1. Two switches in series.

Two switches S_1 and S_2 connected in series and electric lamp 'L' as shown in fig 1.3.

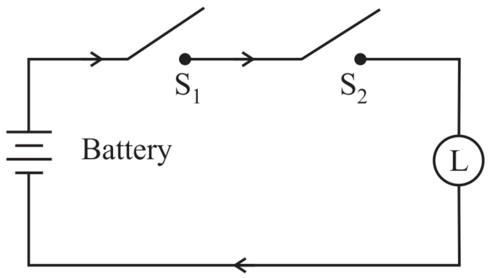


Fig. 1.3

Let p : The switch S_1

q : The switch S_2

L : The lamp L

Input output table (switching table) for $p \wedge q$.

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

Table 1.22

1.5.2 Two switches in parallel :

Two switches S_1 and S_2 are connected in parallel and electric lamp L is as shown in fig. 1.4

Let p : The switch S_1

q : The switch S_2

L : The lamp L

Input - output table. for p ∨ q.

p	q	p ∨ q
1	1	1
1	0	1
0	1	1
0	0	0

Table 1.23

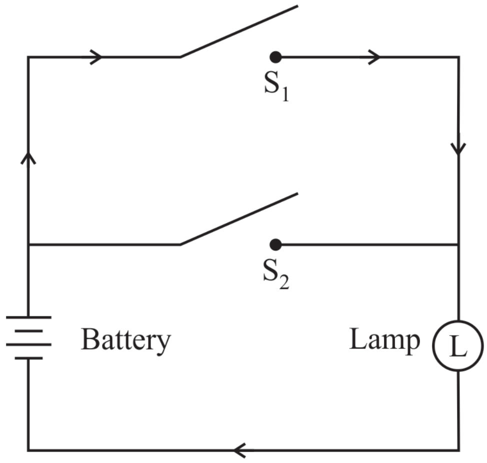


Fig. 1.4

Q.1. Express the following circuits in the symbolic form of logic and write the input-output table.

i)

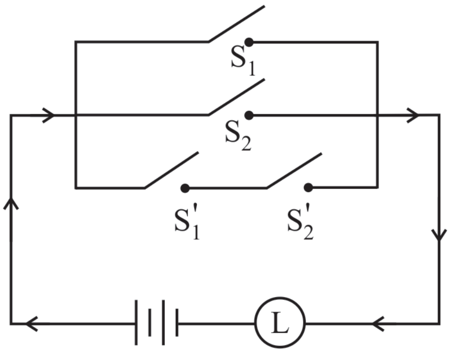


Fig. 1.5

ii)

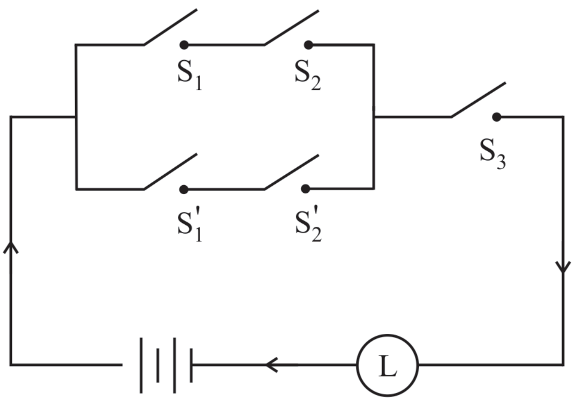


Fig. 1.6

iii)

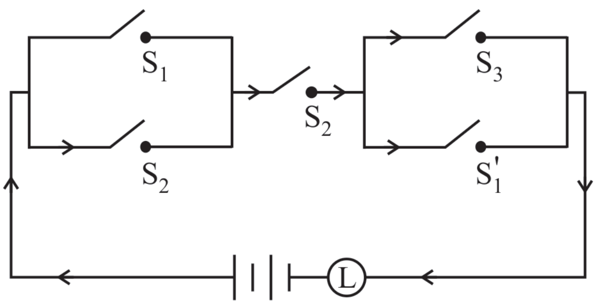


Fig. 1.7

Solution :

i) Let p : The switch S_1 q : The switch S_2 L : The lamp L

Given circuit is expressed as $(p \lor q) \lor (\sim p \land \sim q)$

Solution :

p	q	$\sim p$	$\sim q$	$p \lor q$	$\sim p \land \sim q$	$(p \lor q) \lor (\sim p \sim q)$
1	1	0	0	1	0	1
1	0	0	1	1	0	1
0	1	1	0	1	0	1
0	0	1	1	0	1	1

Table 1.24

ii) Let p : The switch S_1 is closed

q : The switch S_2 r : The switch S_3 L : The lamp L

The symbolic form is $[(p \land q) \lor (\sim p \land \sim q)] \land r$

p	q	r	~p	~q	$p \land q$	$(p \land q) \lor (\sim p \land \sim q)$	$(p \land q) \lor (\sim p \land \sim q)$	$[(p \land q) \lor (\sim p \land \sim q)] \land r$
1	1	1	0	0	1	0	1	1
1	1	0	0	0	1	0	1	0
1	0	1	0	1	0	0	0	0
1	0	0	0	1	0	0	0	0
0	1	1	1	0	0	0	0	0
0	1	0	1	0	0	0	0	0
0	0	1	1	1	0	1	1	1
0	0	0	1	1	0	1	1	0

Table 1.25

iii) Let p : The switch S_1 q : The switch S_2

r : The switch S_3 L : The lamp L

The symbolic form of given circuit is $(p \lor q) \land q \land (r \lor \sim p)$

p	q	r	~p	$p \lor q$	$r \lor \sim p$	$(p \lor q) \land \sim q$	$(p \lor q) \land q \land (r \lor \sim p)$
1	1	1	0	1	1	1	1
1	1	0	0	1	0	1	0
1	0	1	0	1	1	0	0
1	0	0	0	1	0	0	0
0	1	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	0	1	1	0	1	0	0
0	0	0	1	0	1	0	0

Table 1.26

Ex.2. Construct switching circuits of the following.

i) $[(p \vee (\sim p \wedge q)] \vee [(\sim q \wedge r) \vee \sim p]$

ii) $(p \wedge q \wedge r) \vee [p \vee (q \wedge \sim r)]$

iii) $[(p \wedge r) \vee (\sim q \wedge \sim r)] \vee (\sim p \wedge \sim r)$

Solution :

Let p : The Switch S_1

q : The switch S_2

r : The switch S_3

The circuits are as follows.

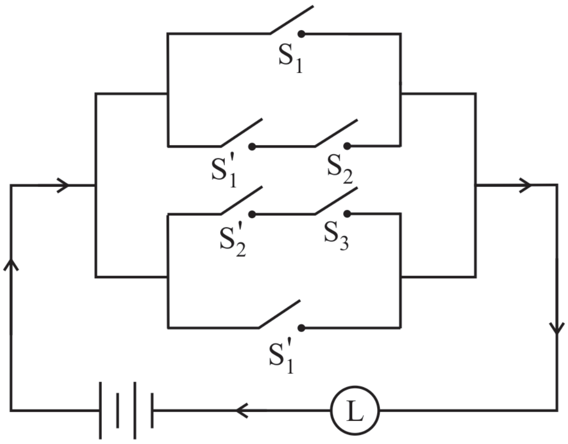


Fig. 1.8

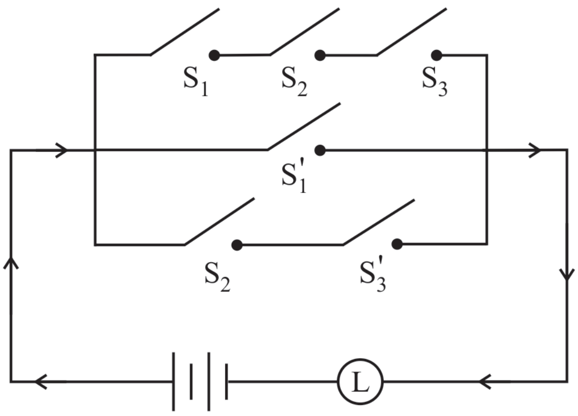


Fig. 1.9

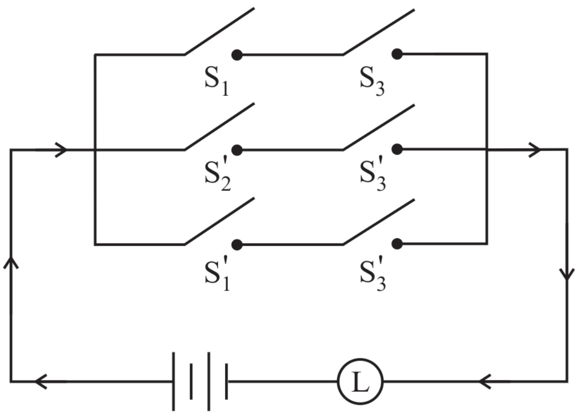


Fig. 1.10

Ex.3. Give an alternative arrangement for the following circuit, so that the new circuit has minimum switches.

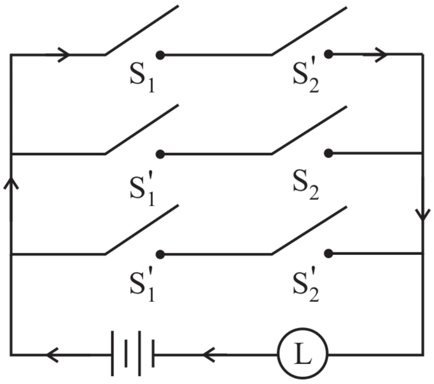


Fig. 1.11

Let p : The switch S_1

q: The switch S_2

The symbolic form is $(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

Consider $(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

$\equiv (p \wedge \sim q) \vee [\sim \mathrm{p} \wedge (q \vee \sim q)]$ [Distributive Law]

$\equiv (p \wedge \sim q) \vee [\sim \mathrm{p} \wedge \mathrm{T}]$ [Complement Law]

$\equiv (p \wedge \sim q) \vee \sim \mathrm{p}$ [Identity Law]

$\equiv \sim p \vee (\mathrm{p} \wedge \sim q)$ [Commutative Law]

$\equiv (\sim p \vee p) \wedge (\sim p \wedge \sim q)$ [Distributive Law]

$\equiv \mathrm{t} \wedge (\sim p \vee \sim q)$ [Law of Complement]

$\equiv \sim p \vee \sim q$ [Identity Law]

The alternative arrangement for the given circuit is as follows :

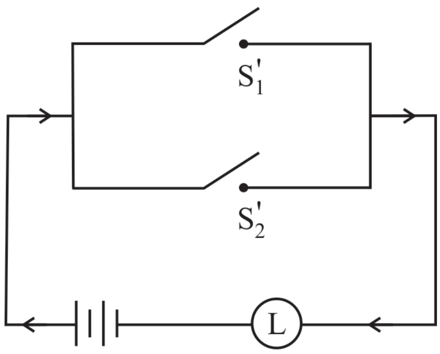


Fig. 1.12

Ex.4. Express the following switching circuit in the symbolic form of Logic. Construct the switching table and interpret it.

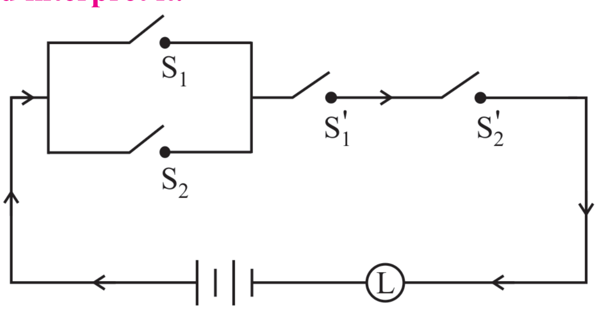


Fig. 1.13

Solution :

Let p : The switch S_1

q : The switch S_2

The symbolic form of the given switching circuit is $(p \vee q) \wedge (\sim p) \wedge (\sim q)$

The switching table.

p	q	~ p	~ q	p ∨ q	(p ∨ q) ∧ (~ p)	(p ∨ q) ∧ (~ p) ∧ (~ q)
1	1	0	0	1	0	0
1	0	0	1	1	0	0
0	1	1	0	1	1	0
0	0	1	1	0	0	0

Table 1.27

Last column contains all 0, lamp will not glow irrespective of the status of the switches.

Ex.5. Simplify the given circuit by writing its logical expression. Also,write your conclusion.

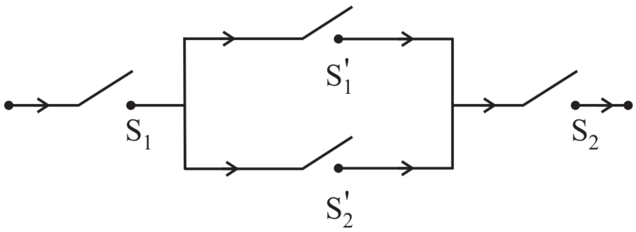


Fig. 1.14

Let p : The switch S_1

q: The The swtch S_2

The logical expression for the given circuit is $p \wedge (\sim p \vee \sim q) \wedge q$

Consider

$p \wedge (\sim p \vee \sim q) \wedge q$

$\equiv$ $[p \wedge (\sim p \vee \sim q] \wedge q$ [Associative Law]

$\equiv$ $[(p \wedge \sim p) \vee (p \vee \sim q)] \wedge q$ [Distributive Law]

$\equiv$ $[F \vee (p \wedge \sim q)] \wedge q$ [Complement Law]

$\equiv$ $(p \wedge \sim q) \wedge q$ [Identity Law]

$\equiv$ $p \wedge (\sim q \wedge q)$ [Associative Law]

$\equiv$ $p \wedge F$ [Complement Law]

$\equiv$ F [Identity Law]

Conclusion : The lamp will not glow irrespective of the status of the switches.

Ex. 6 : In the following switching circuit,

i) Write symbolic form ii) Construct switching table iii) Simplify the circuit

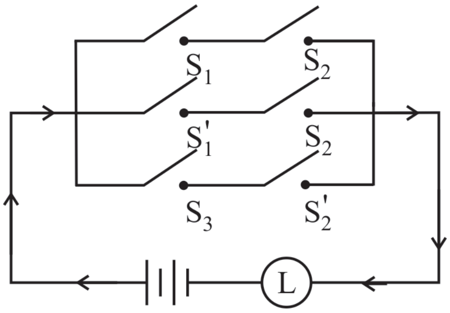


Fig. 1.15

Solution : Let p : The switch S_1. q : The switch S_2. r : The switch S_3

1) The symbolic form of given circuit is $(p \wedge q) \vee (\sim p \wedge q) \vee (r \wedge \sim q)$.

ii) Switching Table :

P	q	r	~ p	~ q	~ p ∧ q	~ p ∧ q	r ∧ ~ q	(p ∧ q) ∨ (~p ∧ q) ∨ (r ∧ ~ q)
1	1	1	0	0	1	0	0	1
1	1	0	0	0	1	0	0	1
1	0	1	0	1	0	0	1	1
1	0	0	0	1	0	0	0	0
0	1	1	1	0	0	1	0	1
0	1	0	1	0	0	1	0	1
0	0	1	1	1	0	0	1	1
0	0	0	1	1	0	0	0	0

Table 1.28

iii) Consider $= (p \wedge q) \vee (\sim p \wedge q) \vee (r \wedge \sim q)$

$= [(p \vee \sim p) \wedge q] \vee [(r \wedge \sim q)]$ [Distributive Law]

$= (T \wedge q) \vee (r \vee \sim q)$ [Complement Law]

$= q \vee (r \wedge \sim q)$ [Identity Law]

$= (q \vee r) \wedge (q \vee \sim q)$ [Distributive Law]

$= (q \vee r) \wedge T$ [Complement Law]

$= (q \vee r)$ [Identify Law]

Simplified circuit is :

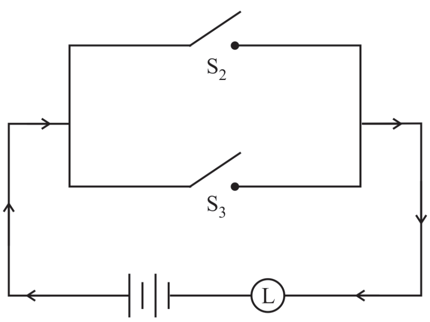


Fig. 1.16

Q.1. Express the following circuits in the symbolic form of logic and writ the input-output table.

(i)

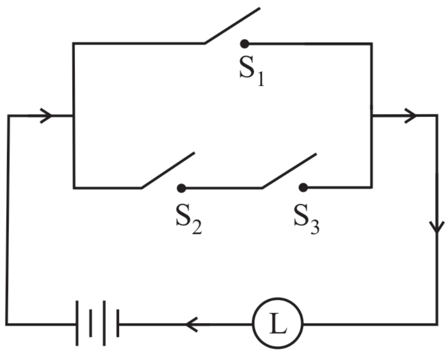


Fig. 1.17

(ii)

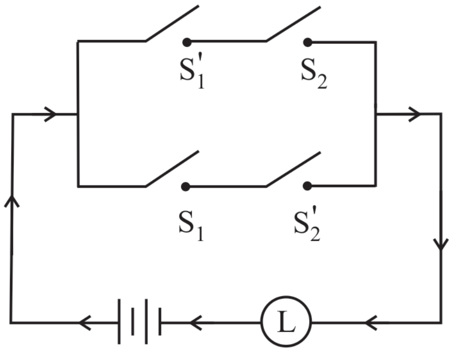


Fig. 1.18

(iii)

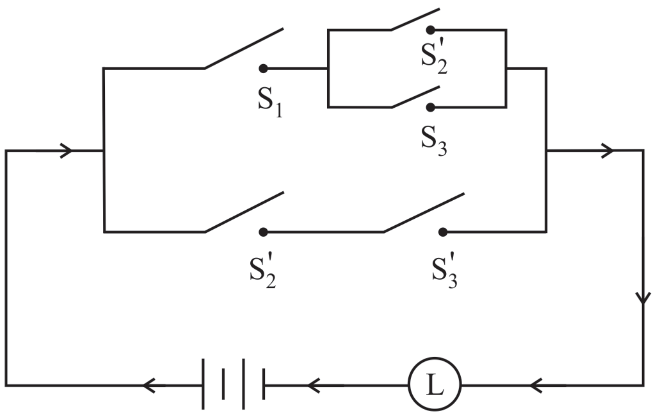


Fig. 1.19

(iv)

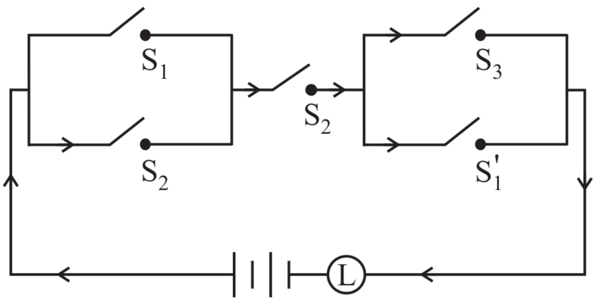


Fig. 1.20

(v)

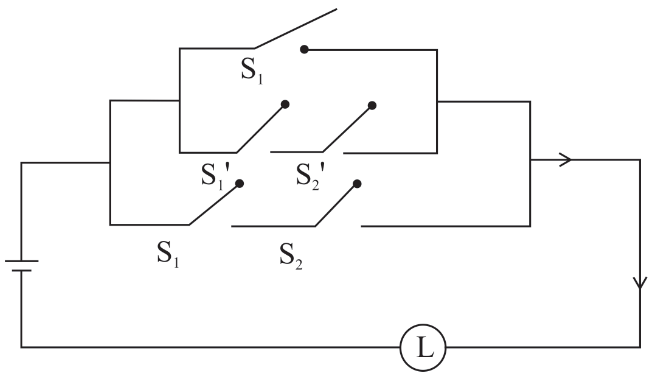


Fig. 1.21

(vi)

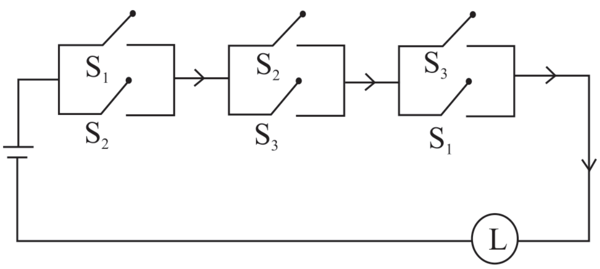


Fig. 1.22

Q.2. Construct the switching circuit of the following :

i) $(\sim p \wedge q) \vee (p \wedge \sim r)$

ii) $(p \wedge q) \vee [\sim p \wedge (\sim q \vee p \vee r)]$

iii) $(p \wedge r) \vee (\sim q \wedge \sim r)] \wedge (\sim p \wedge \sim r)$

iv) $(p \wedge \sim q \wedge r) \vee [\mathrm{p} \wedge (\sim q \vee \sim r)]$

v) $\mathrm{p} \vee (\sim p) \vee (\sim q) \vee (p \wedge q)$

vi) $(p \wedge q) \vee (\sim p) \vee (p \wedge \sim q)$

Q.3. Give an alternative equivalent simple circuits for the following circuits :

(i)

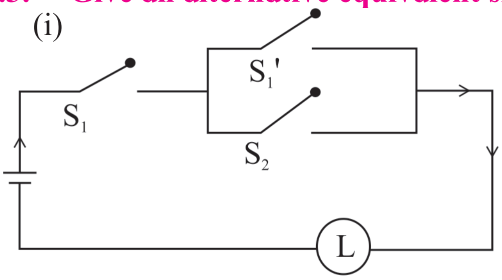


Fig. 1.23

(ii)

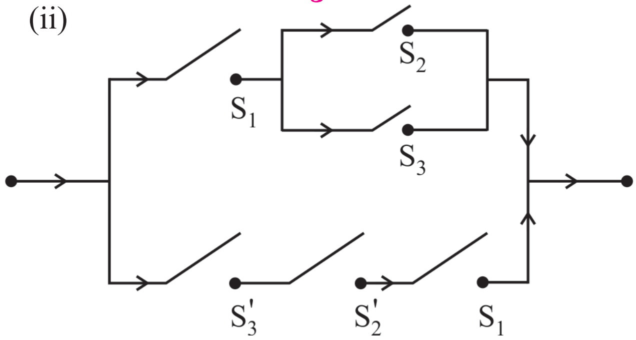


Fig. 1.24

Q.4. Write the symbolic form of the following switching circuits construct its switching table and interpret it.

i)

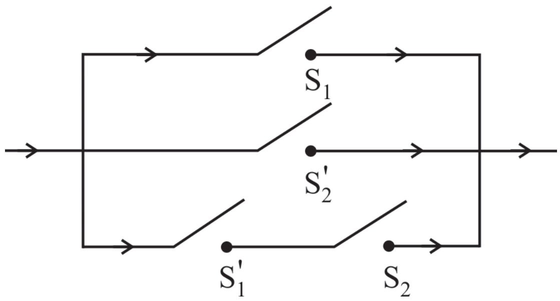


Fig. 1.25

ii)

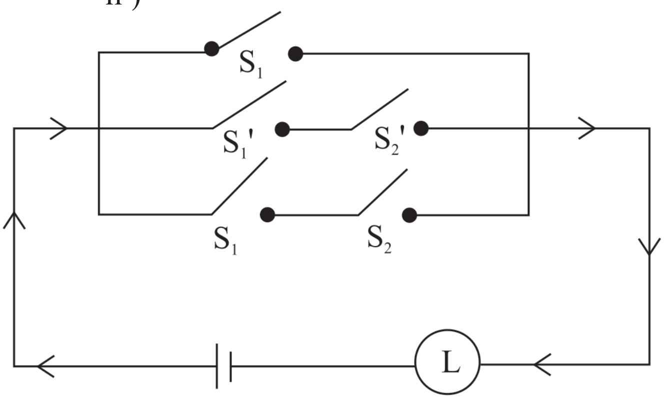


Fig. 1.26

(iii)

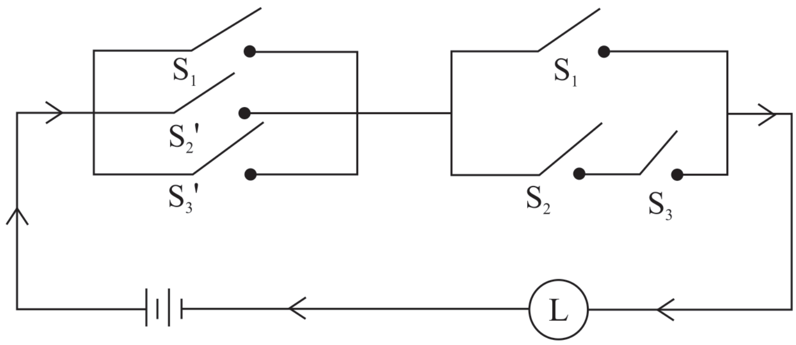


Fig. 1.27

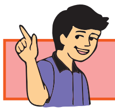
Q.5. Obtain the simple logical expression of the following. Draw the corresponding switching circuit.

i) $p \vee (q \wedge \sim q)$

ii) $(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)]$

iii) $[p\ (\vee (\sim q) \vee \sim r)] \wedge (p \vee (q \wedge r)$

iv) $(p \wedge q \wedge \sim p) \vee (\sim \mathrm{p} \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$



Let's remember!

1) A declarative sentence which is either true or false, but not both simultaneously is called a statement.

Sr. No.	Connective	Symbolic Form	Name of Compound Statement	Hint in truth table	Negation
i)	and	$p \wedge q$	Conjunction	$\mathrm{T} \wedge \mathrm{T} \equiv \mathrm{T}$	$\sim p \vee \sim q$
ii)	or	$p \vee q$	Disjunction	$\mathrm{F} \vee \mathrm{F} \equiv \mathrm{F}$	$\sim p \wedge \sim q$
iii)	if... then	$p \rightarrow q$	Conditional	$\mathrm{T} \rightarrow \mathrm{F} \equiv \mathrm{F}$	$p \wedge \sim q$
iv)	if and only if	$p \leftrightarrow q$	Biconditional	$\mathrm{T} \leftrightarrow \mathrm{T} \equiv \mathrm{T}$ $\mathrm{F} \leftrightarrow \mathrm{F} \equiv \mathrm{T}$	$(p \wedge \sim q) \vee (q \wedge \sim p)$

Table 1.29

3) In the truth table of the statement pattern if all truth values in the last column

a) are 'T' then it is tautology.

b) are 'F' then it is contradiction.

4) In the truth table of the statement pattern if some entries are 'T' and some are 'F' then it is called as contingency.

5) The symbol $\forall$ stands for 'for all' or 'for every'. It is universal quantifier. The symbol $\exists$ stands for 'for some' or 'for one' or 'there exists at least one'. It is called as existential quantifier.

6) Algebra of statements.

Idempotent Law	$p \wedge p \equiv p, \qquad p \vee p \equiv p$
Commutative Law	$p \vee q \equiv q \vee p \qquad p \wedge q \equiv q \wedge p$
Associative Law	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r \equiv p \wedge q \wedge r$
	$p \vee (q \vee r) \equiv (p \vee q) \vee r \equiv p \vee q \vee r$
Distributive Law	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
De Morgan's Law	$\sim(p \wedge q) \equiv \sim p \vee \sim q, \sim(p \vee q) \equiv \sim p \wedge \sim q$
Identity Law	$p \wedge T \equiv p, p \wedge F \equiv F, p \vee F \equiv p, p \vee T \equiv T$
Complement Law	$p \wedge \sim p \equiv F, p \vee \sim p \equiv T$
Absorption Law	$p \vee (p \wedge q) \equiv p, p \wedge (p \vee q) \equiv p$
Conditional Law	$p \rightarrow q \equiv \sim p \vee q$
Biconditional Law	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p) \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

7) If $p \rightarrow q$ is conditional then its converse is $q \rightarrow p$, inverse is $\sim p \rightarrow \sim q$ and contrapositive is $\sim q \rightarrow \sim p$.

8) Switching circuits :

i) Switches in series

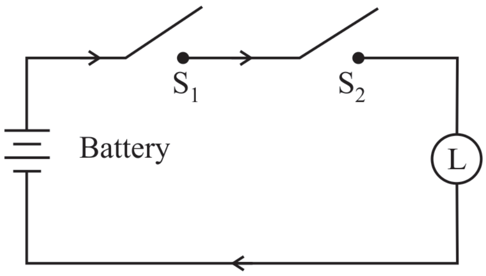


Fig. 1.28

ii) Switches in parallel.

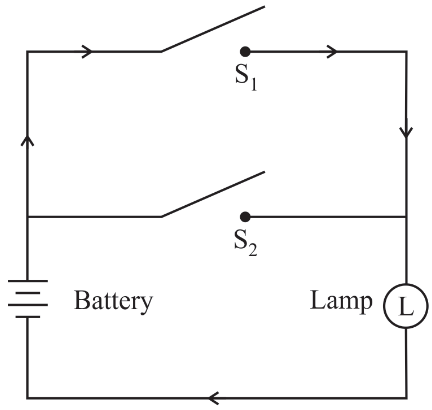


Fig. 1.29

Input-output table

p	q	p ∧ q	p ∨ q
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	0

Table 1.30

Miscellaneous Exercise 1

I] Select and write the correct answer from the given alternatives in each of the following questions :

i) If $p \wedge q$ is false and $p \vee q$ is true, the ________ is not true.

A) $p \vee q$ B) $p \leftrightarrow q$ C) $\sim p \vee \sim q$ D) $q \vee \sim p$

ii) $(p \wedge q) \rightarrow r$ is logically equivalent to ________.

A) $p \rightarrow (q \rightarrow r)$ B) $(p \wedge q) \rightarrow \sim r$ C) $(\sim p \vee \sim q) \rightarrow \sim r$ D) $(p \vee q) \rightarrow r$

iii) Inverse of statement pattern $(p \vee q) \rightarrow (p \wedge q)$ is ________.

A) $(p \wedge q) \rightarrow (p \vee q)$ B) $\sim(p \vee q) \rightarrow (p \wedge q)$

C) $(\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$ D) $(\sim p \vee \sim q) \rightarrow (\sim p \wedge \sim q)$

iv) If $p \wedge q$ is F, $p \rightarrow q$ is F then the truth values of p and q are ________.

A) T, T B) T, F C) F, T D) F, F

v) The negation of inverse of $\sim p \rightarrow q$ is ________.

A) $q \wedge p$ B) $\sim p \wedge \sim q$ C) $p \wedge q$ D) $\sim q \rightarrow \sim p$

vi) The negation of $p \wedge (q \rightarrow r)$ is ________.

A) $\sim p \wedge (\sim q \rightarrow \sim r)$ B) $p \vee (\sim q \vee r)$

C) $\sim p \wedge (\sim q \rightarrow \sim r)$ D) $\sim p \vee (\sim q \wedge \sim r)$

vii) If A = {1, 2, 3, 4, 5} then which of the following is not true?

A) $\exists\, x \in$ A such that $x + 3 = 8$ B) $\exists\, x \in$ A such that $x + 2 < 9$

C) $\forall\, x \in$ A, $x + 6 \geq 9$ D) $\exists\, x \in$ A such that $x + 6 < 10$

Q.2. Which of the following sentences are statements in logic? Justify. Write down the truth value of the statements :

i) $4! = 24$.

ii) π is an irrational number.

iii) India is a country and Himalayas is a river.

iv) Please get me a glass of water.

v) $\cos^2\theta - \sin^2\theta = \cos 2\theta$ for all $\theta \in R$.

vi) If x is a whole number the $x + 6 = 0$.

Q.3. Write the truth values of the following statements :

i) $\sqrt{5}$ is an irrational but $3\sqrt{5}$ is a complex number.

ii) $\forall\ n \in N$, $n^2 + n$ is even number while $n^2 - n$ is an odd number.

iii) $\exists\ n \in N$ such that $n + 5 > 10$.

iv) The square of any even number is odd or the cube of any odd number is odd.

v) In ΔABC if all sides are equal then its all angles are equal.

vi) $\forall\ n \in N$, $n + 6 > 8$.

Q.4. If A = {1, 2, 3, 4, 5, 6, 7, 8, 9}, determine the truth value of each of the following statement :

i) $\exists\ x \in A$ such that $x + 8 = 15$.

ii) $\forall\ x \in A$, $x + 5 < 12$.

iii) $\exists\ x \in A$, such that $x + 7 \geq 11$.

iv) $\forall\ x \in A$, $3x \leq 25$.

Q.5. Write the negations of the following :

i) $\forall\ n \in A$, $n + 7 > 6$.

ii) $\exists\ x \in A$, such that $x + 9 \leq 15$.

iii) Some triangles are equilateral triangle.

Q.6. Construct the truth table for each of the following :

i) $p \to (q \to p)$

ii) $(\sim p \vee \sim q) \leftrightarrow [\sim(p \wedge q)]$

iii) $\sim(\sim p \wedge \sim q) \vee q$

iv) $[(p \wedge q) \vee r] \wedge [\sim r \vee (p \wedge q)]$

v) $[(\sim p \vee q) \wedge (q \to r)] \to (p \to r)$

Q.7. Determine whether the following statement patterns are tautologies contradictions or contingencies :

i) $[(p \to q) \wedge \sim q)] \to \sim p$

ii) $[(p \vee q) \wedge \sim p] \wedge \sim q$

iii) $(p \to q) \wedge (p \wedge \sim q)$

iv) $[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$

v) $[(p \wedge (p \to q)] \to q$

vi) $(p \wedge q) \vee (\sim p \wedge q) \vee (p \vee \sim q) \vee (\sim p \wedge \sim q)$

vii) $[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$

viii) $(p \to q) \vee (q \to p)$

Q.8. Determine the truth values of p and q in the following cases :

i) $(p \vee q)$ is T and $(p \wedge q)$ is T

ii) $(p \vee q)$ is T and $(p \vee q) \rightarrow q$ is F

iii) $(p \wedge q)$ is F and $(p \wedge q) \rightarrow q$ is T

Q.9. Using truth tables prove the following logical equivalences :

i) $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

ii) $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$

Q.10. Using rules in logic, prove the following :

i) $p \leftrightarrow q \equiv \sim (p \wedge \sim q) \vee \sim (q \wedge \sim p)$

ii) $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

iii) $\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

Q.11. Using the rules in logic, write the negations of the following :

i) $(p \vee q) \wedge (q \vee \sim r)$

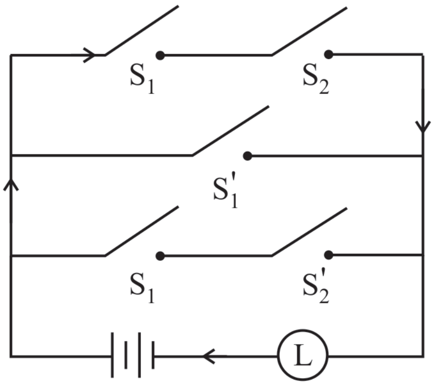
ii) $p \wedge (q \vee r)$

iii) $(p \rightarrow q) \wedge r$

iv) $(\sim p \wedge q) \vee (p \wedge \sim q)$

Q.12. Express the following circuits in the symbolic form. Prepare the switching table :

(i)



(ii)

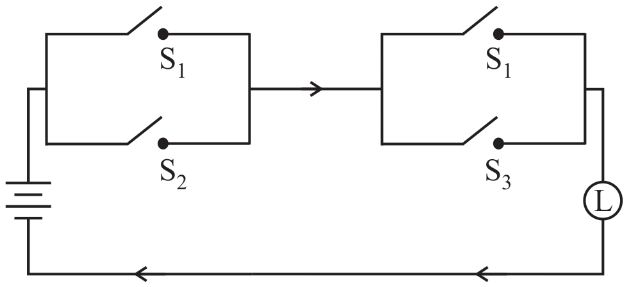
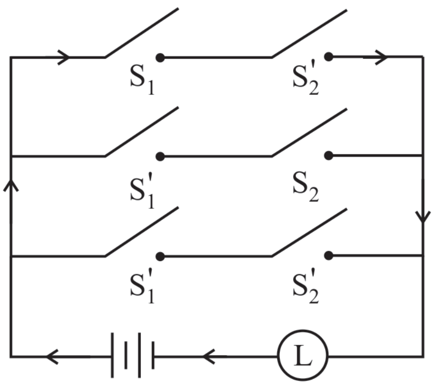


Fig. 1.30 **Fig. 1.31**

Q.13. Simplify the following so that the new circuit has minimum number of switches. Also, draw the simplified circuit.

(i)



(ii)

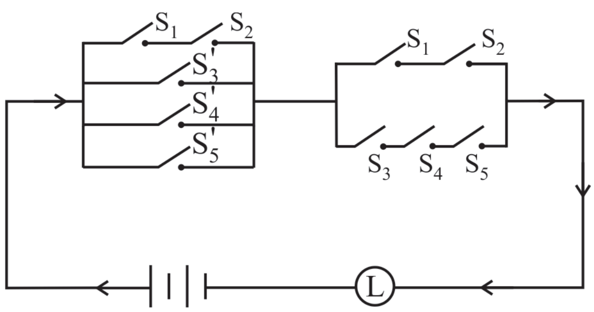


Fig. 1.32 **Fig. 1.33**

Q.14. Check whether the following switching circuits are logically equivalent - Justify.

(A) i)

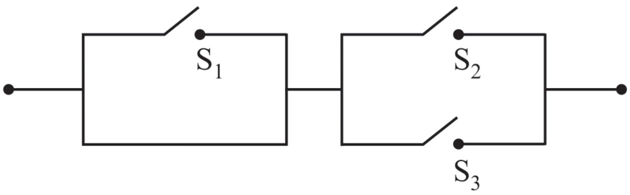


Fig. 1.34

ii)

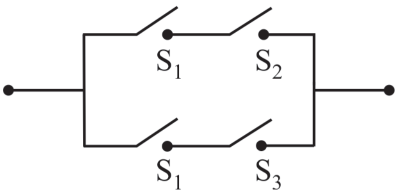


Fig. 1.35

(B) i)

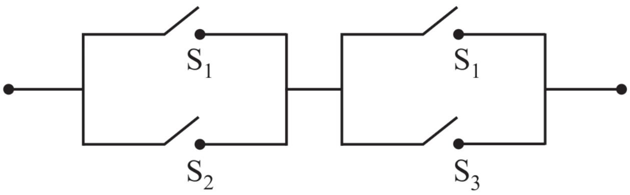


Fig. 1.36

ii)

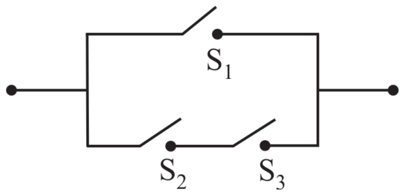


Fig. 1.37

Q.15. Give alternative arrangement of the switching following circuit, has minimum switches.

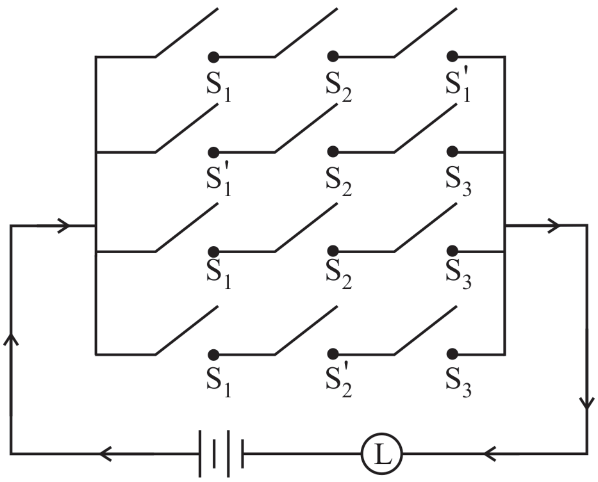


Fig. 1.38

Q.16 Simplify the following so that the new circuit circuit.

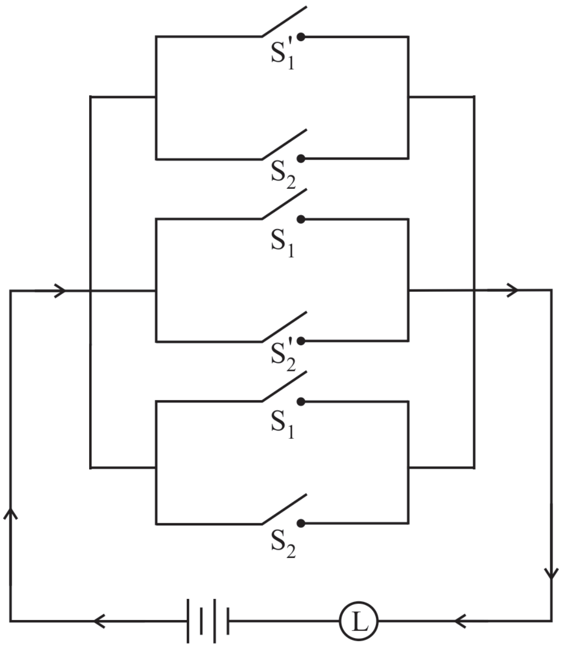


Fig. 1.39

Q.17. Represent the following switching circuit in symbolic form and construct its switching table. Write your conclusion from the switching table.

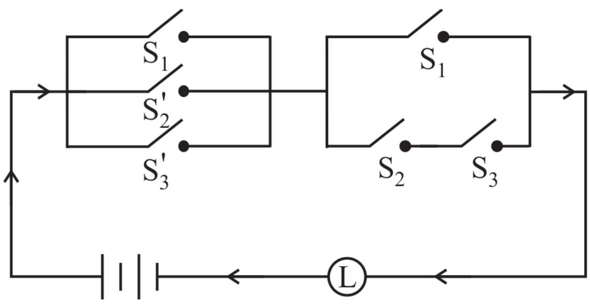


Fig. 1.40

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